AN ASSESSMENT OF THE TEACHING OF CALCULUS: A CASE OF THREE SECONDARY SCHOOL TEACHERS OF MATHEMATICS IN CHINSALI DISTRICT, ZAMBIA

BY

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DECLARATION

I **John Simovwe,** do hereby declare that this dissertation is my own work, and that all the works of other persons used have been duly acknowledged, and that it has never been previously submitted for a degree at the University of Zambia or any other University.

Signature: …… Date: ……

CERTIFICATE OF APPROVAL

This dissertation of **John Simovwe** is approved as partial fulfillment of the requirements for the award of the degree of Master of Education in Mathematics Education by the University of Zambia.

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ABSTRACT

This descriptive qualitative case study focused on the assessment of the teaching of Calculus at senior secondary school level in Chinsali district of Zambia. The study was motivated by the Zambian School certificate candidates' poor performance in questions based on the newly introduced topic of Calculus as well as the lack of research literature concerning the causes of the said poor performance. Specifically, teachers' subject matter knowledge in the area of secondary school Calculus was investigated. In addition, the teaching strategies which mathematics teachers employed to teach the topic were examined. Furthermore, mathematical challenges faced by the teachers when teaching Calculus were explored. Three purposively selected teachers of senior secondary mathematics participated in this study. A teacher was chosen from each of the three conveniently sampled secondary schools (sites) in Chinsali district. Data was collected through questionnaires, semi-structured interviews and video recorded lesson observations. Questionnaires were analysed question by question to determine the teachers' depth of understanding of the subject matter while the transcribed interview data and video recorded observations were explored for similar answer segments for interpretation. This study revealed that sampled teachers did not have in-depth understanding of subject matter knowledge of secondary school Calculus. They could not, for example, solve problems using the chain rule, product rule and quotient rule. Likewise, the teachers had difficulties to find the equation of a normal as well as the area bounded by a curve. Furthermore, teacher-centered methods, to the exclusion of other teaching methods, were used by the three teachers to teach Calculus. They never prepared lesson plans and solely relied on text books when teaching. Apart from that, this study reviewed that teachers had difficulties to explain concepts on Calculus to the learners. This scenario corroborated the finding relating to the teachers' shallow subject matter knowledge of the topic. It is therefore recommended that intensive regular in-service courses for teachers of mathematics are conducted in the area of Calculus as a way of fostering teachers' subject matter knowledge and teaching skills. It is also recommended that internal monitoring of lessons by school administrators be intensified to ensure that teachers prepare for lessons.

Keywords: Assessment, Calculus, Teaching strategies, Mathematical Challenges, Subject Matter Knowledge, Senior Secondary School

DEDICATION

This work is dedicated to my beloved wife Joyce Lungu and my Son Salifyanji Simovwe. Their presence gave me hope and encouragement both spiritually and emotionally during this study. To my father, Mr. Clestone Simovwe and my mother Mrs. Evy Namwila Simovwe, I will always remember the encouragements and hope and support that they gave me throughout my study.

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ACRONYMS AND ABBREVIATIONS

CDC-Curriculum Development Centre

CPD-Continuing Professional Development

DEBS-District Education Board Secretary

ECZ-Examinations Council of Zambia

HOD-Head of Department

MCK-Mathematical Content Knowledge

MKT Mathematical Knowledge for Teaching

MESVTEE-Ministry of Education, Science, Vocational Training and Early Education

MOE-Ministry of Education

MOGE-Ministry of General Education

PCK-Pedagogical Content Knowledge

TIMSS-Trends in International Mathematics and Science Study

CHAPTER ONE: INTRODUCTION

1.1 Overview

This chapter contains the background of the study, statement of the problem, the aim of the study, research objectives and research questions. It further contains a theoretical framework, the conceptual framework, and significance of the study, delimitations, limitations and operational definition of terms.

1.2 Background

Mathematics is one of the compulsory subjects in Zambian schools. It is taught in all grades at secondary school level as part of the curriculum (Ministry of Education (MOE), 1996). Since mathematics is about pattern and structure, and if integrated in different areas of science and technology, this pattern and structure can be used to explain and manage natural happenings and environmental problems which have two distinct features; namely the quality and the quantity of environmental resources.

In 2016, Zambia as a country recorded an improvement in the mean performance of candidates in mathematics at senior secondary school level (see Table 1.1).

Table 1.1: *Grade 12 candidates' performance in mathematics.*

Subject	Candidates who sat for examinations in 2016	Average Mean Performance in 2015 (%)	Average Mean Performance in 2016 (%)	Total Marks	Highest Mark Attained	Lowest Mark Attained	Number Of Pupils who were Absent
Mathematics Paper 1	126220	11.8	24.1	80	77	0	2536
Mathematics Paper 2	126027	21.86	24.6	100	100	0	2727
Mathematics Subject	125914	17.4	24.39	180	177	0	2839

Source: *2016 Examinations Council of Zambian Report*

From Table 1.1 it is apparent that the performance of learners in 2015 and 2016 was still unsatisfactory after the introduction of new topics in the mathematics syllabus. Despite mathematics being one of the most important subjects in the Zambian Education System as well as one of the oldest fields of study

in the history of mankind, the performance of learners in Zambia has been unsatisfactory for several years (ECZ, 2016). This view is supported by the information shown in table 1.1.

Chinsali district of Muchinga province is not exemptional to this problem of poor performance of learners in mathematics. Table 1.2 below shows learners performance in mathematics in Chinsali district among the selected secondary schools.

Table 1.2: *Grade 12 candidates' performance in mathematics in Chinsali District (2016 and 2017).*

School	**Performance in the year** 2016	**Performance in the year** 2017
Kenneth Kaunda secondary	47%	55%
Kalwala Day Secondary	35%	46%
Chinsali Day Secondary	30%	35.4%
Mishishi Day Secondary	31%	48%
Chinsali Girls Secondary	52%	44%
Mulakupikwa Day secondary	33%	41%

Table 1.2 shows that learners in Chinsali district have not been performing very well in mathematics. Nevertheless, there are still other concerns that require attention. For example, the Zambian Secondary School Mathematics Syllabus shows that topics such as Number and Numeration, Equations and In equations, Angles, and Formulae are not taught at senior secondary school, but at junior secondary school level (MOE, 2013). Furthermore, new topics have been introduced in the senior secondary school mathematics syllabus including Introduction to Calculus, Graphs of cubic functions, Composite functions, and Computer and Calculator. In spite of this progressive development, the 2016 Examination Council of Zambia (ECZ) examiners' report revealed that candidates in the ordinary level mathematics at national level including Chinsali district in particular performed poorly in some of the new topics specifically, Calculus.

This calls for investigations in order to establish the cause for this unsatisfactory performance of learners in mathematics. Moreover, how the subject is being taught in schools is largely affected by the teachers' competency of the subject knowledge (Toh, 2009). It is not clear in Chinsali district what may have led to learners' poor performance in mathematics. It is also not clear whether teachers of mathematics in Chinsali district have competences to teach these newly introduced topics such as Calculus. However, one of the important players in matters of learners' performance is a teacher. This

is because; the failure in mathematics by the learners could have been due to lack of competency in teachers' subject matter knowledge, and also in the use of teaching strategies when teaching Calculus. There could have been several other factors that could have made learners to perform poorly in mathematics, but since there is no study that has been conducted in Chinsali district to determine what may have led to poor performance in learners, this study came in to try to find out whether the teachers that handle this topic have the competences in subject matter knowledge and also in the use of appropriate teaching strategies in the teaching of Calculus at Secondary School level.

1.3 Statement of the problem

Candidates' poor performance in the Zambian school certificate mathematics national examinations has been of great concern to stakeholders (MOE, 1996). Besides, Examination Council of Zambia (ECZ) reports usually show that candidates perform poorly in mathematics in national examinations. For example, the ECZ (2016) report reveals that only 24.39% of candidates passed mathematics at senior secondary school level. The poor performance of learners in mathematics in national examinations has negative effects on pupils' progression chances in mathematics related higher education programs (MOE, 1996). Of interest is the adverse reality that most Zambian secondary school learners have challenges in newly introduced topics such as Calculus. Specific learners' difficulties in Calculus are evidenced in examiners' reports, for example, the ECZ (2017) report. While the Zambian context has no reported study that has investigated the cause of learners' poor performance in Calculus, there are international studies suggesting that teachers of mathematics do lack comprehensive understanding of Calculus (Tol, 2009). Most importantly, in Zambia, and Chinsali in particular, it is not yet known whether secondary school teachers of mathematics have comprehensive understanding of school Calculus concepts. Likewise, it is not known how effective teachers of mathematics are in the area of teaching Calculus. Therefore, this study focused on assessment of the teaching of Calculus by three selected teachers of mathematics in Chinsali district.

1.4 The purpose of the study

The purpose of this study was to assess the teaching of Calculus by three selected Senior Secondary School Teachers of Mathematics. The study assessed teachers' subject matter knowledge in the area of secondary school Calculus and explored the teaching strategies which they employed to teach Calculus. Furthermore, an investigation of the mathematical challenges faced by teachers of mathematics when teaching calculus was conducted.

1.5 Objectives of the study

The following were the objectives of the study:

i. To investigate teachers' subject matter knowledge in the area of secondary school Calculus.
ii. To explore the teaching strategies employed by teachers of mathematics when teaching Calculus at secondary school level.
iii. To establish the mathematical challenges faced by teachers of mathematics as they teach Calculus.

1.6 Research questions

The study answered the following questions:

i. Do teachers of mathematics have comprehensive understanding of subject matter knowledge in the area of secondary school Calculus?
ii. What teaching strategies do teachers of mathematics use to teach calculus in Secondary Schools?
iii. What mathematical challenges do teachers of mathematics face as they teach Secondary School Calculus?

1.7 Theoretical framework

This study was guided by content-based and social constructivism theory of learning and teaching. Content-based theory was propounded by Haberman and Stinnettt (1973). This theory was later used by other scholars such as: Shulman (1987) and Changwe (2017). Changwe (2017, p. 24) argued that if one is to judge the quality or effectiveness of any teacher education programme, one of the criteria that can be used is to examine the content that teachers are exposed to as well as the products of the programme. Besides, content based theory informed the study in order to investigate the subject matter knowledge in the area of secondary school Calculus.

Social constructivism theory as propounded by Vygotsky's (1978) was also used in this study in order to explore teaching strategies used by teachers and to investigate the mathematical challenges faced by teachers when teaching. This theory places emphasis on social and linguistic influences on learning, and in particular on the role of the teacher in the education process. It has been argued that much of the current discourse on learning and teaching in mathematics education draws on the developmental theories of Piaget and Vygotsky (Brodie, 1994). Brodie (1994) further argued that, there are many

similarities in their ideas, including the crucial notion of an active learner constructing knowledge in the interaction with the physical and social environment.

Furthermore, Vygotsky conceives of the zone of proximal development as central to instructional enhancement and classroom change in Mathematics. Vygotsky (1978) posits that the zone of proximal development is the distance between the actual development level as determined by independent problem solving under adult guidance or collaboration of more capable peers (Murray, & Arroyo,2002). Vygotsky (1978) goes on to opine that the ZPD is the current or actual level of development of the learner and the next level attainable through the use of mediating semiotic and environmental tools and capable adult or peer facilitation. In other words, it is what a child can do alone at a particular point in time. The second, "potential development" is defined as that which a child can achieve if given the benefit of support during the task. Vygotsky (1978, p.86) further states that, “ZPD is the ability to solve problems, "under adult guidance or in collaboration with more capable peers.” It is for this reason that, the teacher should use different strategies such as learner centered in the teaching of Calculus through the use of ZPD as proposed by Vygotsky.

Mediation according to Vygotsky refers to the part played by other significant people in the learners’ lives, people who enhance their learning by selecting and shaping the learning experiences presented to them. Vygotsky (1978) (as cited in O’Neil, 2011) claims that the secret of effective learning lies in the nature of the social interaction between two or more people with different levels of skills and knowledge. This involves helping the learner to move into and through the next layer of knowledge or understanding. Therefore, key to the concept of mediation is that the teacher should have competence in the subject matter knowledge of Calculus, in order to facilitate the learning process for the learners to be able to provide necessary feedback to the teacher in areas where they would need closer assistance. Teachers who are constructivists are aware of the role of prior knowledge in students' learning, recognising that students are not blank slates or empty vessels waiting to be filled with knowledge. Instead, they believe that students bring with them a lot of prior experiences, knowledge and beliefs that they use in constructing new understandings (Jones, 2002).

Depiction from this theory, we see that teaching of calculus at secondary school level is crucial to the learners. Therefore, it requires social interaction between a teacher and the learners, and the interaction amongst the learners themselves, thus; learners require the support of social constructivism in the teaching of calculus at secondary school level in order for them to develop their potential and the understanding of the concepts found in calculus to the fullest.

Conceptual framework

The purpose of a conceptual framework is to categorize and describe concepts relevant to a study and the relationships among them (Miles and Huberman, 1994). Figure 1.1 shows the conceptual framework which was used for the current study. The conceptual framework illustrates the link between the assessment of the teaching of Calculus and the conceptualized aspects which were investigated (teachers' subject matter knowledge, teaching strategies and mathematical challenges in the teaching of calculus in secondary schools) that may result to effective teaching of Calculus.

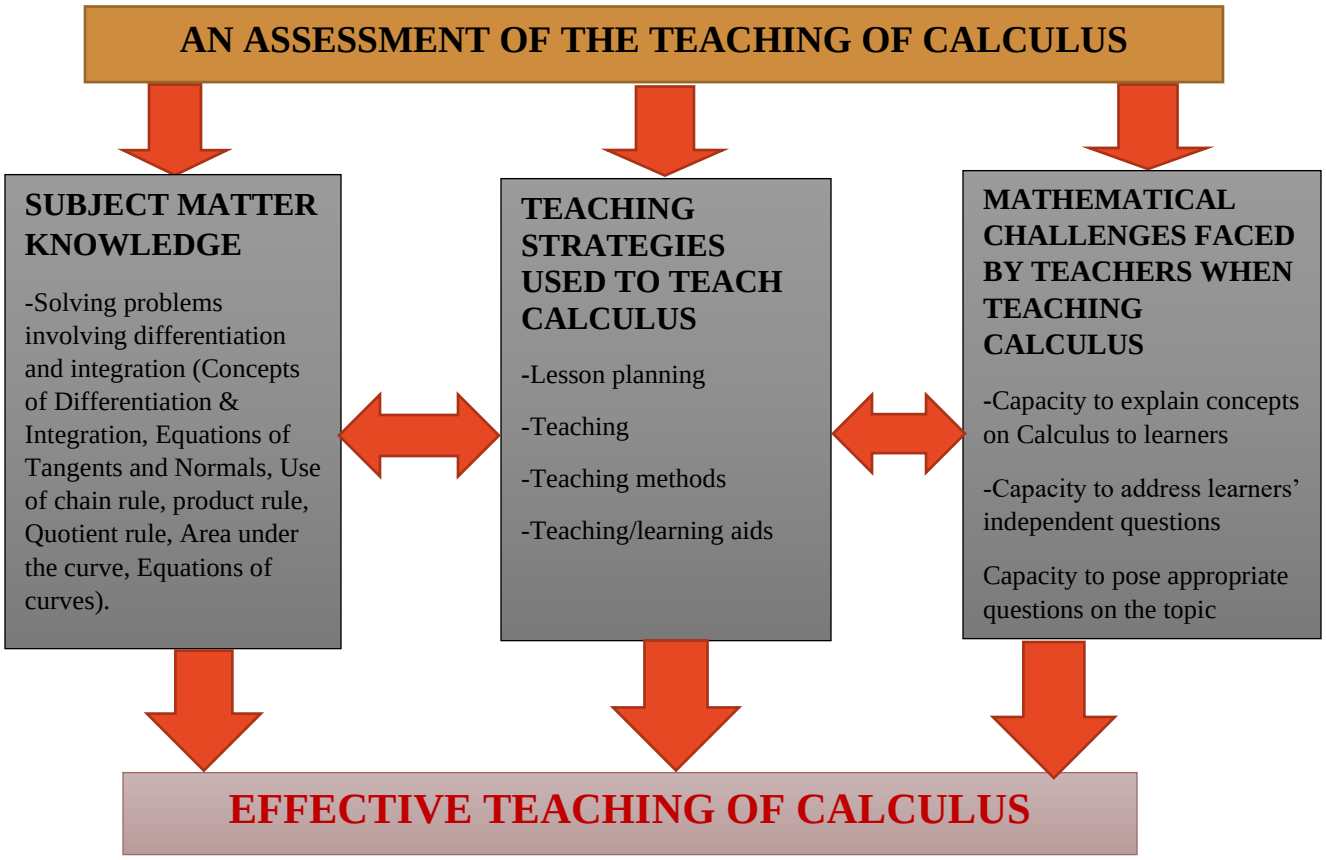


Figure 1.1: Conceptual Framework

Subject Matter Knowledge

Ma (1999) defines Subject matter knowledge as teachers' ability to use their knowledge of mathematics to explore the mathematics topics and present the content of topics in the methods that can build successful mathematics learning. With regard to the present study, subject matter knowledge meant teachers' capacity to solve questions correctly based on secondary school differentiation and integration with the intended applications. Teachers were particularly assessed concerning their understanding of, for example, the chain, product, and quotient rules.

Strategies used to teach Calculus

Ma explained that a teacher in transforming knowledge of content should use different representations, help students to make connection between different representations to solve mathematics problems, identify students' wrong thinking, and be able to give responses towards students' questions (Ma, 1999). Regarding this study, teaching strategies refer to lesson planning, teaching methods, teaching and learning aids, procedures and processes that a teacher uses during instructions in the classroom. In addition, the teacher must know what teaching methods are available, what strengths and weaknesses these methods have, what purpose each method serves and how to use the methods (Chifwa, 2015).

Mathematical challenges faced by teachers when teaching Calculus

Mathematical challenges are challenges faced by teachers of mathematics as they teach Calculus specifically on solving problems that involve differentiation and integration. Thus, this study investigates the capacity of teachers to explain concepts on Calculus, to address learners' independent questions and also to pose appropriate questions on the topic.

1.8 Significance of the study

The findings of this study might provide information to the stake holders in the Ministry of General Education and teachers with the information that might help in coming up with the appropriate strategies in order to improve the teaching of calculus in ordinary mathematics. Through observing teachers of mathematics as well as through interviewing them, the study has brought out findings that may contribute to the existing literature on the teaching of Calculus by teachers of mathematics globally and on mathematics teacher education. Education researchers may also use the results to build on many more studies that might be carried out on the same study area and may also provide a rich ground for further research based on the gaps left out by this study.

1.9 Delimitations of the study

This study was conducted in Chinsali district of Muchinga province. Three selected secondary teachers of mathematics were targeted. The reason for this scope of study was based on the fact that Chinsali District consists of many secondary schools currently being upgraded from primary schools to secondary schools. Taking into account of this fact, the researcher was confident that relevant data would be collected from the right respondents.

1.10 Limitations of the study

There was unwillingness by some of my participants especially during classroom observations. Due to limited time and resources available, the researcher could not carry out a study of such a magnitude. This restricted the results of this study to be generalised beyond the population of the study.

1.11 Operational definition of terms

The process of defining concepts is essential because it allows for specific concepts to be described and explained in a manner that pertains to the study. To ensure uniformity and understanding of this study, the following definition of terms used in the study were provided:

Calculus: A branch of mathematics concerned with the study of such concepts as the rate of change of one variable quantity with respect to another.

Content Knowledge: refers to the body of knowledge and information that teachers teach and that students are expected to learn in a given subject or content area such as mathematics.

Constructivism: Refers to the theory of learning which states that learners construct their own knowledge from the environment around them through experiencing things.

Curriculum: All the planned learning experiences offered to the learner or student under the guidance of the educational institution.

Mathematical Content Knowledge (MCK): Specialised body of knowledge that helps the teacher to assess learners' mathematical thinking in order to offer appropriate guidance.

Pedagogical Content Knowledge (PCK): A skill that enables the teacher to present the fundamental mathematical concepts and teaching methods to the learner in a comprehensive manner.

Subject matter knowledge: refers to teacher's ability of using his knowledge of mathematics to explore the mathematics topics and present the content of topics in the methods that can build successful mathematics learning.

Teaching strategies: refer to structures, system, methods, techniques, procedures and processes that a teacher uses during instructions in the classroom.

1.13 Research outline

The document composed of six chapters, i.e.

CHAPTER ONE: This chapter gives an introduction and background to the study. It highlights the problem statement, the research objectives and questions and significance of the study. The theoretical and conceptual framework is also explained.

CHAPTER TWO: This chapter gives an illustration of the related literature.

CHAPTER THREE: Chapter three looks at the research design and methodology undertaken by the author in conducting this study. It describes the methods and instruments used to collect data. The chapter further illustrates ethical aspects of the study and the limitations of this study are brought to the attention of the researcher.

CHAPTER FOUR: The findings obtained from the research instruments are presented in this chapter.

CHAPTER FIVE: In this chapter, the author discusses the findings obtained in this study. Some of the findings are compared and contrasted with findings from previous research. This allows the reader to see the similarities and differences between the findings of this study and those of previous research.

CHAPTER SIX: This chapter begins by reminding the researcher of the research questions and then gives a summary of the main findings. It makes recommendations and ends with suggestions on a possible area for future research.

1.13 Chapter Summary

In this chapter, the researcher has presented the background of the study which had set the context of the study and a justification of why this study was important to be carried out. The background led to the description of the statement of the problem. Additionally, the researcher explained the aim of the study, objectives, research questions, theoretical framework where the study was guided, conceptual framework, significance of the study, delimitations, limitations and operational definition of terms. The chapter that follows focused on reviewing of related literature that provided gaps and background to this study.

CHAPTER TWO: REVIEW OF RELATED LITERATURE

2.1 Overview

The researcher in the previous chapter gave an introduction and background to the study. In this review of literature, the researcher will give an understanding of the context of this study as well as providing an insight into the findings of previous researchers. The chapter begins with the definition of Calculus and basic information on the introduction of new topics in senior secondary school mathematics syllabus. This is followed by the teaching and learning outcomes on Calculus for senior secondary schools of Zambia. Next, teachers' subject matter knowledge on secondary school Calculus and this is followed by related literature towards teaching strategies that teachers use in the teaching of Calculus. The section further reviews related literature concerning mathematical challenges faced by teachers in the teaching of calculus at senior secondary school level. As part of this justification of this study, an integrated overview on the research findings on the assessment of the teaching of Calculus in secondary schools is presented thereafter.

2.2 The definition of Calculus and the basic information on the introduction of Calculus in the senior secondary school mathematics syllabus

Tall (1992) defined Calculus as the rate of change and the rules of differentiation with integration as the inverse process. Calculus traditionally focuses on mastery of symbolic methods for differentiation and applying these to solve a range of problems (Tall, 2006). Calculus is important at the Secondary School level because understanding calculus is an essential step in understanding how the world works. It is a foundation on which other skills can be built and is often a requirement or prerequisite to higher institutions such as colleges and universities. Nevertheless, it is a challenging area regardless of which School offering the instruction (Petropoulou, 2016).

Calculus has been introduced into the Senior Secondary School Curriculum in many countries. Moreover, studies (Amit and Vinner, 1990) have also shown that generally students performed poorly in conceptual tasks in calculus. Both learners and teachers put their emphasis on procedures and avoided the concepts related to Calculus (Amit and Vinner, 1990). How the subject is being taught in schools is largely affected by the teachers' competency of the subject knowledge (Thompson, 1992 and Toh, 2007). For this reason, this study focuses on the assessment of the teaching of Calculus by three secondary school mathematics teachers of Chinsali district.

2.3 Teaching and Learning outcomes on Calculus

Teaching of mathematics is influenced by many factors such as society, time, conceptions and ideas of students, curriculum, inner structure of the mathematical domain of knowledge, to mention but just a few. Mathematics teachers have to make decisions based on choices which are partly implicit and teachers may even be unaware of them. Mohan (2010) defined teaching as a process of assisting learners acquire new knowledge and skills. It consists of showing and guiding the learners in performance of tasks and then measuring their results. Das (2007) on the other hand defined teaching as the creation of a situation which facilitates learning. This was one of the areas the current study tried to address regarding the teaching of Calculus in secondary schools as enshrined in the revised senior secondary school mathematics curriculum.

Calculus course and its teaching strategies such as the traditional teaching methods are basically skills-based which result in rote, manipulative learning. In this style of teaching and learning, there is too much drill and recitation and many learners may retain little of both techniques and ideas in calculus in the long run. Thus, this results in instrumental understanding or unsupported procedural knowledge. Skemp (1976) describes "instrumental understanding" as knowing rules without knowing why they work. As pointed out by Skemp, skills-based courses are very efficient if and only if criterion is the ability to perform routine manipulations. It means if an application problem requires conceptual knowledge, then most students might find it difficult to solve the problem. In fact, it is found that such students suffer from their pursuit of a meaningless, ritualistic manipulation of symbols. Based on the current study, the question that still demand a quick response was on how teachers of mathematics were teaching this important topic calculus in the manner that learners understand fully all the concepts found in calculus.

The traditional calculus and its teaching strategies have also become a litany of procedures and template problems which too often results only in giving students some rather routine practice in algebraic manipulations. In addition, students with weaker backgrounds are usually driven away in frustration over the manipulations required, even if they are able to understand the basic ideas of calculus' (Tucker and Leitzel 1995, p. 57).

According to Tall (1986) the calculus traditionally focuses on mastery of symbolic methods for differentiation and integration and applying these to solve a range of problems. The differential and integral calculus is considered as one of the greatest inventions in mathematics. Previous related concepts like the concept of a variable and the concept of function are necessary for the understanding of calculus concepts Kidron (2014). The differential and integral calculus is based upon the

fundamental concept of limits. In regard to this study, the researcher was trying to find out from teachers of mathematics whether or not they would teach the concept of functions to the learners before they teach differentiation and integration.

According to MOE (2013) the general outcomes in the teaching of Calculus to the learners is to build an understanding and appreciation of Calculus concepts and computational skills in order to apply them in everyday life and also for further studies. Therefore, a summary of the learning outcomes for the topic Calculus as anticipated in the Zambian mathematics secondary school curriculum is provided in Table 2.1.

Table 2.1: The summary of learning outcomes, knowledge and skills on Calculus for Zambian senior secondary school mathematics syllabus.

TOPIC	**SUB TOPIC**	**SPECIFIC OUTCOME**	**KNOWLEDGE**	**SKILLS**
Introduction to Calculus	Differentiation	Explain the concept of differentiation	Explain the concept of differentiation	Interpretation of differentiation
		Differentiate functions from the first principles	Differentiate functions from the first principles (Limits)	
		Use the formula for differentiation	Product rule, chain rule and quotient rule ($y = ax^n$, $\frac{dy}{dx} = nax^{n-1}$)	
		Calculate equations of tangents and normal		
	Integration	Explain integration	Indefinite integrals	Integration
		Find Indefinite Integral	Arbitrary constant	
		Evaluate simple definite integrals	Definite integrals	Application of definite
		Find the area under the curve	Stationary points	Integrals
			Tangents and Normal	
			Explain integration as a reverse of differentiation	Estimation of area under the curve
			Rule of integration ($\int ax^n dx = \frac{ax^{n+1}}{n+1} + c$)	
			Area under the curve	

Based on the learning outcomes as shown from Table 2.1, a teacher should know and comprehend a particular topic which is easy or difficult for the students and a conception or misconception which

the students may have related to a particular topic. The next section will focus on different views of scholars on teachers' subject matter knowledge on Calculus.

2.4 Teachers' subject matter knowledge on calculus

In the Zambian context, the subject matter knowledge on Calculus in Chinsali district has never been reviewed by other researchers because of being new in the senior secondary school syllabus. Calculus is one of the most interesting topic in mathematics that should be enjoyed by teachers when teaching. This can only be done if teachers of mathematics have comprehensive understanding of this topic. Tol (2009) carried out a study in Singapore on in- service mathematics teachers' content knowledge of calculus and related concept. His research findings showed that, teachers did not have good understanding of calculus concepts that were needed for teaching. The study which was carried out by Tol is important to the current study as it pointed out the weaknesses in teacher's understanding of Calculus for effective teaching. However, the context of his study which was done in Singapore is different from that of Zambia.

Additionally, at the center of effective teaching lie two critical components, that is, subject matter knowledge and pedagogy and the relationship between the two. Beaton and et al (1996) argued that, effective teaching as a complex endeavor requiring knowledge about the subject matter of mathematics, the ways students learn and effective pedagogy in mathematics. Based on the scholar's findings it is clear that for one to teach mathematics effectively, one needs to know the subject matter to be taught. Subject matter knowledge is of critical importance for teachers.

Numerous studies, from Shulman (1987) to Khakbaz (2015), have indicated that teachers require different types of knowledge in the classroom. PCK was proposed by Shulman (1987) as an essential component of teacher knowledge, defined as a "special amalgam of content and pedagogy that is … understanding" (p.8). Later worked by, for example, Khakbaz (2015), confirms that it is necessary to go beyond the subject and examine how the teacher interprets the subject matter for example, on this topic Calculus and how this is linked to their role in facilitating learning in the classroom. Shulman (1986) explained that, Pedagogical Content Knowledge (PCK) is a kind of specific knowledge as the basic knowledge for teacher which includes the connection of knowledge and skill of representation, analogy, examples, demonstration of a material in order to be understood by the students.

Despite the above cited scholars having not looked at mathematics and Calculus in particular, the current study considered their work to be vital in the area of teachers' subject matter knowledge in the area of secondary school Calculus. These studies cited by scholars above however, did not focus on

teachers' subject matter knowledge in the area of secondary school Calculus in Chinsali district. This was also one of the areas this current study tried to investigate.

According to Shulman (1986) mathematical knowledge to students with understandable technique is the core of PCK. In addition, it is explained that teacher's PCK is an important element to be an effective mathematics teacher. Teacher's mastery on the lesson or mathematical content is important for teacher's success in teaching. A teacher cannot be expected to explain a mathematical concept, if he does not have a complete comprehension of that concept. On the other hand, mastering the lesson that will be taught is not enough to gain the objective intended on students (Shulman, 1986). Thus, the teacher who is expected to teach Calculus effectively is a teacher who masters the lesson that will be taught and able to explain Calculus concepts so that it can be understood by the learners.

Ponte and Chapman (2006) draw distinction between these terms and refers to 'knowledge' as a wide network of concepts, images, and intelligent abilities possessed by human beings; 'beliefs' as incontrovertible personal 'truth' held by everyone, drawing from experiences and fantasy; and 'conceptions' as the underlying organizing frames of concepts, having essentially a cognitive nature. Thus 'knowledge' is considered as a broader category, including conceptions, beliefs, insights, mental images and understandings that people possess either derived from formal or practical experiences.

Shulman (1986) distinguished between two kinds of understanding of the subject-matter that teachers need to have. These are 'knowing that' and 'knowing why'. The teacher needs to understand that something is so and also why it is so. 'Knowing that' is the most basic level of subject-matter knowledge. It includes declarative knowledge of rules, algorithms, procedures and concepts related to specific mathematical topics in the school curriculum. 'Knowing that' is important as a basis for adequate pedagogical content knowledge. 'Knowing why' is knowledge which pertains to the underlying meaning and understanding of why things and the way they are Even and Tirosh (1995). Perhaps mathematics teachers require specialised knowledge for them to be able to assess pupils understanding of Calculus concept in mathematics effectively.

In addition, the notion of PCK was advanced by Shulman (1986) in which he proposed the consideration of the relationship between teacher content knowledge and teacher pedagogical knowledge. He argued that PCK is an amalgam of content knowledge and pedagogical knowledge that enables the transformation of the subject matter knowledge into pedagogically useful forms. It is the content knowledge that deals with teaching. Ball and Bass (2000) indicated that the mathematics content and pedagogical knowledge which teachers learn in the colleges and universities was normally not the knowledge most useful for teaching secondary school mathematics. Based on the scholars'

findings, it is clear that no suggestion of what they thought could be the best mathematics subject matter and pedagogical knowledge for secondary school teachers was made. Conversely, their findings acted as a basis where the current study was to be grounded as the researcher tried to ascertain the subject matter knowledge of teachers of mathematics were exposed to in Chinsali district to teach secondary school Calculus.

Thompson (1984) and Ernest (1988) based on their wider experience of working in the field of mathematics education claimed that any attempt in improving the quality of mathematics teaching and learning must begin with an understanding of the conceptions held by teachers. Lerman (1990) supported this view and asserted that unless teachers' knowledge about mathematics, mathematics teaching and learning are examined, "little will be achieved in terms of development and change in the mathematics classroom (p. 54). One of the key reasons is that knowledge, beliefs and conceptions teachers' hold play significant role in shaping their thinking and behaviours which influence their teaching of Calculus to the learners in the classroom.

Mathematical content knowledge (MCK) is simply the specialised body of knowledge that helps the teacher to assess learners' mathematical thinking in order to offer appropriate guidance (Changwe, 2017). Mathematical content knowledge and pedagogical content knowledge (PCK) are two cardinal concepts that are interchangeably used by several scholars. Fennema and Franke (1992) distinguished the two concepts by stating that knowledge of mathematics and knowledge of mathematics presentations are related to MCK while the knowledge of students and knowledge of teaching are related to PCK. This means that having a deeper knowledge of calculus and knowledge of pupils is necessary for the teacher to teach mathematics effectively. Teachers really need to have the appropriate mathematical content knowledge both on what they are expected to teach and beyond learners' mathematical knowledge. The findings of Fennema and Franke shows that they only focused on teachers' subject matter knowledge of mathematics in general however, they did not look at teachers' subject matter knowledge in the area of secondary school Calculus. It is for this reason, this study was trying to the investigate teachers' subject matter knowledge in the area of secondary school Calculus in Chinsali district.

Shulman (1987) revealed that MKT cannot just be effective on its own but requires the accompaniment of both the content and pedagogical knowledge. Besides, the MKT helps student teachers to become mathematically proficient and eventually learn to teach in the manner that can equally help their learners to become mathematically proficient (Hiebert, Morris and Glass, 2003). It is clear from what was reported that teachers really need to have the appropriate mathematical content knowledge both

on what they are expected to teach and beyond learners' mathematical knowledge. Similarly, Silvernam and Thomson (2008) looked at MKT as the profound knowledge of mathematics and methods of representing it to the learners. The current study therefore, assessed teachers' subject matter knowledge of calculus in the teaching of mathematics in Chinsali district.

Furthermore, it is also believed that the views teachers hold about the subject, if unchallenged usually leads to the failure of curriculum reforms (Goldin, Rosken and Torner, 2009). Therefore, Hersh (1979) concluded that "the issue then, is not, what is the best way to teach, but what is mathematics really all about controversies about high school teaching cannot be resolved without confronting problems about the nature of mathematics" (p.33). Thus, implementing new curriculum along with facilitating teachers to challenge their views about the nature of Calculus, Calculus teaching and learning will contribute in bringing change in teachers' thinking and teaching practices.

Shulman (1986) noted that mathematics include, knowledge of concepts, theories, ideas, organisational frameworks, knowledge of evidence and proof as well as established practices and approaches toward developing such knowledge. Amplifying on Shulman's statement, it means that a teacher of mathematics in schools needs to have a deeper knowledge of differentiation and integration and their applications. Pfund and Duit (2000) and the National Research Council (2000), submitted that, the cost of not having a comprehensive base of content knowledge can be prohibitive, for example, students can receive incorrect information and develop misconceptions about the content area.

Teachers must know the subject they teach. Indeed, there may be nothing more foundational to teacher competency. The reason is simple: Teachers who do not themselves know a subject well are not likely to have the knowledge they need to help students learn this content (Shulman, 1987). This means that teachers are able to teach effectively when they have enough knowledge about calculus. In addition, teachers need to know calculus in various ways useful for, among other things, making mathematical sense of student work and choosing powerful ways of representing the subject so that it is understandable to the learners.

Shulman (1986) further pointed out that, it seems unlikely that just knowing more advanced mathematics will satisfy all of the content demands of teaching. In fact, elementary teachers' mathematics course attainment does not predict their students' achievement gains (National Mathematics Advisory Panel, 2008). What seem most important is about knowledge and the ability to use the mathematics required in the teaching. Ball, Thames and Phelps, (2008, p. 402) stated that, "the fundamentals of subject matter knowledge for teaching is by establishing a practice-based

conceptualization of it, by elaborating sub domains, and by measuring and validating knowledge of those domains." What distinguishes this sort of mathematical knowledge from other knowledge of mathematics is that it is subject matter knowledge needed by teachers for specific tasks of teaching calculus and clearly subject matter knowledge. These tasks of teaching depend on mathematical knowledge, and, significantly, they have aspects that do not depend on knowledge of students or of teaching. The question that still remain unattended to is, do teachers of mathematics have comprehensive understanding of subject matter knowledge in the area of secondary school Calculus? This was what the researcher through this study sought to investigate teachers' subject matter knowledge in the area of secondary school Calculus in Chinsali district.

Moreover, with regard to teachers' subject matter knowledge in the area of secondary school calculus, this study focuses on investigating whether or not teachers of mathematics have the ability to solve problems involving Differentiation and Integration such as concepts of differentiation and integration, Equations of Tangents and Normals, Use of Chain rule, Product rule, Quotient rule, Area under the curve and Equations of curves. The next section will focus on the teaching strategies in the teaching of Calculus at senior secondary school level.

2.5 Teaching strategies in the teaching of calculus at senior secondary school level

In Zambian context, mathematics senior secondary school syllabus encourages a learner-centered approach in the teaching of mathematics (Ministry of Education, 2013). Ministry of Education (2013, p. 2) the revised curriculum further stated that, the role of the teacher may be that of a facilitator of learning who provides appropriate scaffolding of that process by asking probing questions, providing appropriate resources and leading class discussions as well as designing students assessments. The strategy strives to transform the traditional teacher centered mathematics classroom situation into student centered environment completely where learners are allowed to construct new knowledge through the specific outcomes learned, thinking processes such as communication, reasoning and problem solving (Ministry of Education, 2013). Based on these assertions, the researcher came in to explore the teaching strategies employed by teachers in the teaching of Calculus and these strategies includes: Lesson planning, teaching methods and also teaching and learning aids.

Musonda (2009) carried out a study on reviewing Learner-centered approach in the teaching of mathematics at Nkrumah and Copperbelt secondary teachers colleges. This study established that, student performance in mathematics can be affected either positively or negatively by the lecturers' method of teaching. As such lecturers should use methods of teaching that involve students. The researcher restricted himself on learner-centered approach in the teaching of mathematics in colleges,

but he did not look at the assessment of teaching strategies in the teaching of Calculus in secondary schools.

Moreover, there are a variety of methods in the teaching of calculus. In order for a teacher to decide what teaching method to use and have a variety of activities during the lesson, the teacher must know what teaching methods are available, what strengths and weaknesses these methods have, what purpose each method serves and how to use the methods (Chifwa, 2015). It is not straight forward to say which one is the most effective way to teach students. In most cases, a combination of different teaching strategies on the topic calculus generally may lead to good teaching outcomes. This is because; in the classroom there are pupils who are fast learners and slow learners. Therefore, teachers should work with strategies that are suitable for themselves, to the pupils they are teaching and their subject matter knowledge. This statement is supported by Zhang (2003, p. 101) who argued that student-centered teaching and teacher-centered teaching strategies are also very helpful in leading to good teaching and learning outcomes.

According to Nyaumwe, Bappoo, Buzuzi and Kasiyandima (2004, p. 33), traditional approaches, which involve "teacher-centered instructional methods that do not make learners develop conceptual understanding of mathematics", have been criticized because they do not encourage problem-solving skills in learners. Instructional methods based mainly on teacher talk, do not involve much questioning, discussion or individual development of understanding. In contrast, a learner-centered teaching approach is one that supports learners in developing mathematical reasoning, while encouraging them to perceive the teacher as someone who is there to help them make sense of mathematics while creating contexts which help them develop meaning in mathematics (Brodie, 2006, p. 543 ; Yashau, Mji & Wessels, 2005, p. 20). However, learner-centered discourse is much harder to achieve in practice than it appears to be in policy. Chisholm and Leyendecker (2008, p. 197) note that learner-centered education is one of the most pervasive ideas; yet it is very hard for them to take root in the classroom. Based on the arguments by various scholars the current study tried to explore teaching strategies used by teachers when teaching in the senior secondary schools of Chinsali district.

Furthermore, an approach requires teachers to have a variety of skills, as well as a sound knowledge of mathematics content. The use of a variety of teaching approaches and styles is recommended, because it can "encourage adapt-ability and lifelong learning in the teaching–learning process" (Vaughn & Baker, 2001, p. 610). Shulman (1986, p. 9), in his seminal definition of pedagogic content knowledge, articulates that "there are no single most powerful forms of representation, the teacher must have at hand a veritable armamentarium of alternative forms of representation." Shulman's

definition focuses the need for teachers to have at their disposal a variety of ways to represent the subject matter, in order to make it meaningful to their learners. Based on the current study, the question that still demand a quick response was on what teaching strategies do teachers of mathematics use to teach Calculus in the senior secondary schools?

Ball and Bass (2008) identified teacher's knowledge deal with the students' difficulties and learning strategies which are suitable to overcome those problems in learning mathematics as a part of PCK. Ma (1999) gave a definition that PCK in mathematics learning is a teacher's ability using his or her knowledge of mathematics to explore the mathematics topics and present the content of the topics in the methods that can build successful mathematics learning. Furthermore, Ma (1999) further explained that, a teacher should be able to demonstrate the width, depth, and interconnection of content knowledge and utilize all of them to express mathematical solution. Based on the assertions by scholars above, it can be concluded that knowledge of content or knowledge of main material owned by a mathematics teacher should be transformed by utilizing various resources such as textbook with the concept presentation that is easy to be understood by the learners.

In addition, a teacher in transforming knowledge of content should use different representations, help students to make connection between different representations to solve mathematical problems, identify students' wrong thinking, and be able to give respond toward learners' questions.Teaching methods are chosen on the basis of fitness for a particular purpose (Petty, 2009). The teacher first clarifies the purposes of the lesson and then chooses activities which will achieve these purposes. A number of factors determine what strategies a teacher should use to accomplish a given learning outcome. These factors may include age and academic level of pupils, amount of time available, physical environment, availability of teaching and learning resources as well as the topic being presented. A variety of teaching methods increase student attention and interest. It also helps the teacher to manage the class well (Petty, 2009). Different teaching methods develop different skills in the learners as they learn calculus.

Koehler and Mishra (2007) submitted that a teacher with deep pedagogical knowledge understands how students construct knowledge and acquire skills and how they develop habits of mind and positive dispositions towards learning. Additionally, having a deeper knowledge of differentiation and integration is of no use if the teacher cannot transmit this knowledge to students in multiple ways. Possessing this knowledge covers the core business of teaching and learning. It is this knowledge that helps the teacher to become aware of common misconception and common mistakes in mathematics and bringing them to the attention of the students. Ball (2005) went a step further and noted that

teachers need ways to see into the subject matter through the eyes, hearts and minds of the learners. In other words, the teacher needs to accommodate the students when teaching and find the simplest way of transmitting the subject matter.

Tall (1992) stated that various hypotheses have been put forward suggesting ways in which students' understanding might be improved and one of them is active learning by the learners, either independently or with the help of class discussion, could arrive at some methods and facts of the calculus. Chifwa (2015) in her study revealed that teaching strategies may be classified in different ways: those in which the teacher has direct control are called teacher-centred which include lecture, teacher demonstrations and teacher questioning and those in which the teacher involve the learners when teaching are called learner-centered such as discussion. Teacher-centered is where by learning is not reinforced by physical activities and the pupils are only mentally involved in the lesson. Petty (2009) and Mohan (2012) described lecture method as a teaching method used for the explanation of content by a teacher for passive assimilation by students. The teacher is the fountain of knowledge and the learners are empty tins that need to be filled. In a lecture, there is little room or no room at all of asking questions by the learners.

Lecture method is one of the teacher centred methods of teaching. Davar (2012) and MESVTEE (2014) mentioned that lecture method can be used when teaching a large number of students, introducing a new topic in which students have little previous knowledge, communicating basic facts, terminologies or promoting initial understanding of the concepts, summarising certain scientific concepts and principles at the end of the lesson, giving historical accounts of scientific events or history of great scientists. The following are some of the advantages of a lecture method; efficient in resource requirement, one person can teach a large number of students at the same time, provides a role model by providing an insight into the thinking and problem solving abilities of the teacher, less time consuming and simplifies work of the teacher, it can be adapted to the level of the class, it is a rapid method of presenting material (Petty, 2009; Davar, 2012).

In this study, lecture method is one of the methods that can be used when introducing the topic calculus because learners have little prior knowledge. This method promotes initial understanding of concepts and principles that are found in calculus. However, Chifwa (2015) further explained that, lecture method has a number of limitations when applied to the teaching of calculus; Students' involvement in the lesson is very minimal, minimises feedback from the students, teachers cannot be sure how much the students have understood, identification of individual's specific learning difficulties is also a challenge. In addition, other disadvantages are, evaluation of the methodology, lesson and oneself is

difficult, students get easily distracted and inattentive, scientific skills cannot be developed, retention is very low and learners are not given the opportunity to use the ideas being taught. A lecture method is not suitable for slow learners and those pupils who have language problems, it can be boring, the students are not actively involved in the lesson, the concentration span for students is short (Petty, 2009; Davar, 2012). For this method to be very effective it should be used together with other methods such as group work, pairing, demonstration and discussion.

Demonstration is another teacher-centered method that can be used in teaching calculus. Muzumara (2008) defined a demonstration as a repetition of a series of planned activities which are designed to illustrate a certain phenomenon or event. Petty (2009) defined a demonstration as 'showing how' demonstrations help the teacher to make some information clear. It helps the teacher to introduce certain topics brilliantly and clearly to pupils so that they can, on their own, carry out the activities or illustrations (Muzumara, 2008). A good demonstration has the following characteristics: clearly defined aims and objectives, can be observed clearly by every learner and involves the class at every stage, has a logical order of presentations, stimulates inquiry and curiosity in pupils and can easily be performed by learners themselves (Muzumara, 2008; Mohan, 2010; Davar, 2012).

On the other hand concerning teacher-centered teaching method is that, the teacher plays a leading role and transfers information. He or she is regarded as the authoritative expert, the main source of knowledge, and the focal point of all activity. The student is the passive recipient of the information already acquired by the teacher. The teacher selects from the discipline the information to be taught, studied, and learned (Committee on Undergraduate Science Education, 1997). In information transmission, the focus is on facts and skills, but not on the relationships between them. The prior knowledge of students is not considered to be important and it is assumed that students do not need to be active in the teaching-learning process (Trigwell and Prosser, 1996).

Zhang (2003) in his study had shown that the teacher-centered teaching method has many advantages, such as the ability to deliver a large amount of information quickly. It is an easy and safe way for teachers to teach, most students are accustomed to it, and so on. The traditional teaching method is still considered a preferable teaching method by many teachers, and a lot of success cases illustrate its effectiveness. Besides, the teacher-centered teaching method has some disadvantages, such as not providing an active learning environment for students, diminishing the students' interest and leading in most cases to students adopting a surface learning approach by focusing on rote memory and reproduction, and so on. For this reason, the disadvantages of teacher-centered teaching method can be supported by using learner-centered teaching method in the teaching of Calculus.

Zhang (2003) stated that learner-centred teaching strategies actively involve pupils in the lesson. These strategies are the most recommended in teaching because they involve learners, both in hands-on and minds- on activities. There are many concepts, ideas and theories in calculus. Some of them are hard to learn and would be better taught with student-centred teaching strategies. Student-centred teaching strategies give student time to think and can be used to make students active and interactive in the teaching of calculus. Students should participate in the learning processes and become active learners. Learners learn best when they are engaged in the learning processes. These methods stress learning by doing and becoming actively involved in experiencing, formulating ideas and solving problems (Muzumara, 2008; Petty, 2009). Hence learners become critical thinkers and problem solvers. Learner centered strategies are supported by the theory of constructivism.

A discussion is one of the learner-centered teaching methods that can be used in the teaching of Calculus. During classroom discussions, pupils learn how to express themselves clearly, to justify opinions and to tolerate different views. During discussions, learners also get a chance to ask for clarifications, to examine their own thinking, to evaluate ideas and to put together personal viewpoints. Some learners assume responsibility by taking leadership roles in the group. This method enables learners to collaboratively construct their knowledge (Mohan, 2010; Davar, 2012; MESVTEE, 2014). In addition, Muzumara (2008) and Petty (2009) agreed that during class discussions, pupils learn a number of skills such as being open to new ideas, making eye contact with the speaker, being attentive, organising thoughts, speaking clearly, taking notes, allowing speakers to express their thoughts without interruption and respecting other people's ideas. The teacher's role during class discussion is to encourage learners to express their opinions and not to inform or force the teacher's opinion on learners.

Learner-centered teaching focuses on the student and, in particular, on the cognitive development of the student. The teacher's goal is to help learners grasp the development of knowledge as a process rather than a product. The focus of classroom activities and assignments is on the student-centered process of inquiry itself, not on the products of inquiry (Zhang, 2003). Students create their own conceptual or cognitive models. Content, teaching style, and methods are adapted to aid the cognitive and intellectual growth of students. Student-centered teaching combines an understanding of the way that humans process information with other factors that affect learning such as attitudes, values, beliefs, and motivation' (Committee on Undergraduate Science Education,1997).

Zhang (2003) further stated that learner-centered teaching may increase student involvement by drawing them into the learning process and help students make the transition from passive listeners to

active participants in their own learning. Students learn best if they are engaged in active learning, if they are forced to deal with observations and concepts before terms and facts, and if they have the sense that they are part of a community of learners in a classroom environment that is very supportive of their learning (Fraser, 1986; McDermott 1991). Studies (Fraser, 1986; McDermott 1991) have shown that learner-centered teaching leads to a strong tendency for students to adopt a deep learning approach which involve focusing on meaning and understanding and then results in good teaching and learning outcomes.

Teaching calculus in the child's zone of proximal development is viewed as one of the strategy as the way of improving performance in Mathematics. According to Denhere, Chinyoka and Mambeu (2013, p. 371) teaching in the Zone of Proximal Development where the child's learning is mediated and scaffold by the teacher or expert adult or knowledgeable peer makes learning more meaningful, easier, manageable, effective and efficient. Besides the major challenge that the theory presents to teachers is that it is difficult to identify every learner's zone of proximal development. Despite this limitation, the ZPD if appropriately applied could improve Mathematics teaching and learning in the schools.

Teachers should be convinced that the capable peers possess the Mathematics ability before they are allowed to assist others. If they are not really capable, chances are that they may teach the less capable wrongly. Teachers are the experts who should be able to organise challenging mathematical tasks that do not cause frustration or less motivation (Denhere, Chinyoka and Mambeu, 2013). This is because; students are motivated to acquire knowledge by actively participating in the learning process. Active learning will engage the students' mind in constructing meaningful knowledge of lasting benefit. Active participation by students helps them construct a better framework from which to generalise their knowledge.

The more expert partner should provide support at the moments where maturing functions are inadequate. With collaboration, direction or some kind of help, the learner is able to accomplish more mathematical tasks independently (Vygotsky, 1987). It is important to understand that a learner is able to perform a certain task alone, while in collaboration, is able to perform a greater number of tasks. The high rate of mathematical failure may be an indication that not sufficient collaboration is done to assist the learners' mathematical understanding. Teachers have the propensity of giving students voluminous homework, without ascertaining whether the learner will be able to get instructional assistance from adults at home.

Zhang (2003) further added that, self-study is an important method for students' learning. Students are responsible for their learning processes independently while teachers are only the guides to students'

learning. Therefore, in calculus topic, there is some content, such as differentials for approximation, Newton's method for finding solutions to equations, and so on, which are suitable to learn through self-study. Through learning this content by themselves, students can solidify the new material they have learnt, enhance their learning, problem solving skills and develop their abilities to work on the questions independently. Sometimes, self-study inspires interests in the subject. Students can develop abilities of lifelong learning through this method.

Boaler (2006, 2016, p. 143–150) advises that teaching should draw upon rich mathematical activities, which have high intellectual demand, instead of resorting to rote learning, so that it can inculcate a positive mindset towards mathematics. Studies further argue that the connection of mathematics to real-world contexts gives teachers the opportunity of making mathematics seem more accessible and enjoyable to learners (Miller, 2009:4).

Researchers indicate that traditional methods, especially in teaching introductory statistics courses, are often viewed as unproductive, and result in students getting nervous about coursework because they consider statistics as a difficult field (Smith & Martinez-Moyano, 2012, p. 107). Instead, researchers advocate that small-group or co-operative learning should replace traditional methods in order to encourage more critical engagement with statistics concepts (Garfield, 1993, p. 30; Roseth, Garfield & Ben-Zvi, 2008, p. 2–4). In recent years, there has been an increased emphasis on using real-life settings in the mathematics and statistics classroom so that learners can connect to the subject (Steen, 2001). In teaching statistics in particular, a data-driven approach can be very useful. Real data can be used to emphasise statistical principles and procedures, rather than using a traditional theoretical approach where the importance is on identifying the correct formula and performing a calculation (North, Gal & Zewotir, 2014, p. 1). The next section will focus on the mathematical challenges in the teaching of Calculus.

2.6 Mathematical challenges in the teaching of Calculus

The ECZ (2016) mathematics examiner's report indicated that most learners had challenges in new topics such as the introduction to Calculus. Tol (2005) articulated that, teachers' challenges in calculus concepts could have developed when they were students. Thus, existing literature on students' challenges in calculus and related concepts provides a suitable framework for the study of teachers' content knowledge in calculus. Thus, this study intended to investigate mathematical challenges faced by teachers in the teaching of Calculus at senior secondary school level. These mathematical challenges includes whether or not teachers of mathematics have the capacity to explain concepts on

Calculus to the learners, to address learners' independent questions and to pose appropriate questions on the topic.

Students' difficulties in calculus stem from their learning difficulties in dealing with topics on functions, graphs and other related algebra concepts (Judson & Nishimori, 2005). In the study carried out by Judson and Nishimori (2005), many students had an immature understanding of functions which could have led to misconceptions in problems involving application of differentiation and integration. Usiskin (2003) believed that learners may perform better in calculus if they have been given early exposure to concepts involving inequality, summation, and other algebraic concepts. Instructions in secondary school calculus classrooms often emphasize procedural knowledge grounded in algebra (Morris, 1999). Hence, it is not surprising that students neglect the conceptual part of calculus and only consider the computational part, thereby making calculus learning meaningless (Bezuidenhout, 2001; Davis and Vinner, 1986; Toh, 2007a, p. 74). The question that still remain unattended to is whether or not teachers have the capacity to explain Calculus concepts when teaching the learners.

According to Aspinwell and Miller (1997) students regard computation as the essential outcome of calculus and thus end their study of calculus with little conceptual understanding. The sequence of teaching material in the secondary school calculus content could be another source of learning difficulties for students. Take for example, the teaching of integral calculus in schools. The most common approach in schools is to define integration as the antiderivative. Following that, one derives at evaluating the definite integral with the Fundamental Theorem of Calculus (Anatoli, 2008). As a result of this sequencing of teaching materials, most students do not acquire appropriate comprehension of the definite integral concept (Anatoli, 2008; Orton, 1983; Sealey, 2006; Thomas and Hong, 1996).

The difficulties that students encounter in calculus concepts (Davis and Vinner, 1986) and teachers' misconceptions in limit concepts (Akkoc and Huillet, 2005) can be explained by the existence of a gap between the concept definition and the concept image of the limit concept (Tall and Vinner, 1981). The calculus concept of limit is not "commonsensical" to them as it conflicts the use of this term in daily life (Davis and Vinner, 1986; Tall and Vinner, 1981). Studies done by Mastorides and Zachariades (2004) shows that, in-service teachers specifically have difficulties with the concepts related to limits and continuity of functions.

Thompson (1992) refers to conceptions as conscious or subconscious beliefs, understanding, meaning, mental images, and preferences. In this study 'knowledge' is considered as a broader concept which includes teachers' conscious and unconscious beliefs, meanings, mental images, understandings, preferences, beliefs and conceptions constructed or developed through enculturation, education and schooling. However, many teachers are unable to provide conceptual explanations for the procedural tasks they perform. For example, a common finding of these studies is that pre-service teachers lack an understanding of measurement division and are prone to rely only on a sharing interpretation of division (Simon, 1993).

In addition, Makgakga and Makwakwa (2015), did a study on exploring learners' difficulties in solving Grade 12 differential Calculus. The results revealed that 52% of the learners performed better on finding the derivative of functions by using the rules of differentiation, compared with 23% of the learners who performed better on finding the derivative of functions by using first principles. Although the above cited scholars looked at exploring learners' difficulties in solving Grade 12 differential Calculus, they did not focus on teachers' difficulties when teaching Calculus. The current study therefore, was one of the areas this study tried to investigate.

Research has shown that many students have difficulties in learning some of the key concepts of calculus (Artigue, Batanero and Kent, 2007) and it does not help that traditional calculus courses tend to focus more on algebraic drill and practice on calculus problems without understanding the underlying concepts (Gordon, 2004). Many researchers have expressed their concern that over-emphasis on algebraic drill and practice method will produce students who are only able to regurgitate what has been taught and duplicate it in examinations (Gordon, 2004; Zachariades *etal*., 2007).

The study conducted by Gordon (2004) revealed that algebraic solutions of problems and lengthy derivations of formulas are commonly expected among students in calculus. Gordon also pointed out that some calculus teachers felt that they were teaching algebra rather than the concepts of calculus. This phenomenon is also noted by Axtell (2006) who felt that conventional teaching of calculus fails to produce students who are able to understand the fundamentals of calculus. Teachers seem to focus more on procedures than understanding of the concepts among the students in teaching calculus (Zachariades and *et al,* 2007). As a result, students approach differentiation or integration problems by simply applying the steps they have to memorise without a good grasp of the calculus concepts.

Chabulembwa (2014) looked at the challenges of teaching and learning mathematics at the Copperbelt University, in Zambia. The study reviewed that students did not have positive attitude towards mathematics and they had poor foundations in mathematics. The scholar restricted himself on the

challenges of teaching and learning mathematics, little is known on the challenges faced by teachers when teaching Calculus in the senior secondary schools of Chinsali district.

2.7 Summary of literature review

The major aim of this chapter was to review relevant literature pertaining to teachers' subject matter knowledge on calculus, the teaching strategies used when teaching Calculus, and mathematical challenges faced by teachers in the teaching of calculus in senior secondary schools. The researcher attempted to carry out this study bearing in mind and having reviewed what other scholars had written both in Zambia and internationally on the teaching of Calculus. As observed in the literature reviewed no study has been conducted in Zambia to assess the teaching of calculus in ordinary mathematics particularly in secondary schools of Chinsali district. There is a knowledge gap which this study attempted to contribute to. This has made the current study necessary. The next chapter outlines the research design and methodology that of this study.

CHAPTER THREE: RESEARCH DESIGN AND METHODOLOGY

3.1 Overview

This section contains how the study was conducted. It outlines the procedures and strategies which were used to collect and analyse data. It consists of the description of the research design, study area or site, study population, study sample and sampling techniques, data collection instruments, data collection procedures and data analysis, trustworthiness and ethical considerations.

3.2 Research design

Research design can be referred to as the plan of the study that will lead to the attainment of objectives. Research designs are the specific procedure involved in the research process which are; data collection, data analysis and report writing (Patton, 2002 and Creswell, 2012). This study employed a qualitative research approach which is descriptive in nature. This approach to research was the most appropriate for achieving the purpose of the study which was to assess the teaching of Calculus by three selected senior secondary school teachers of mathematics in Chinsali district of Zambia. Patton (2002) described descriptive qualitative research as a type of research which describes the phenomena as they exist, and that it is used to identify and obtain information on the characteristics of a particular problem. Burns & Grove (2001) described a qualitative approach as a systematic subjective approach used to describe life experiences and situations to give them meaning.

In terms of design, this study was a case study. Yins (1994) define a case study as "an empirical inquiry that investigates a contemporary phenomenon within its real-life context, especially when the boundaries between phenomenon and context are clearly evident" (p. 13). On the other hand, researchers, Merriam (1988) and Wolcott (1992) saw a case study in terms of end-product and defined a case study as an intensive description and analysis of a unit or set, rather than a strategy or method. For example, an assessment through questionnaire, observation and interviews were conducted concerning concepts of Calculus that are well defined in the Zambian secondary school curriculum. Consequently, a case study allowed for the purpose of the concepts in innovations to assess and explore teachers' subject matter knowledge in Calculus and the appropriateness of teaching strategies as well as the mathematical challenges in the teaching of Calculus. This type of design may be appropriate because the topic on calculus is new in the mathematics syllabus and has never been addressed with a certain sample or group of people in Chinsali district.

3.3 Research Sites

The study was conducted in Chinsali district of Muchinga province of Zambia at the three selected secondary schools. The reason for selecting Chinsali district was that the researcher was of the view that, since Chinsali district being the head-quarter of Muchinga province; it requires attention in order to know how competent teachers are in the area of secondary school Calculus. Another reason was that, teachers in Chinsali district were easily available to the researcher.

3.4 Study population

Polit and Hungler (1999) refer to the population as an aggregate or totality of all the objects, subjects or members that conform to a set of specifications. This means that a population refers to the larger group from which the sample is taken. In this study, the targeted population included senior secondary school teachers of mathematics in Chinsali district. These teachers of mathematics were targeted because they are the ones who have been teaching Calculus since 2016.

3.5 Study Sample

A sample is a subset of a population selected to participate in the study, it is a fraction of the whole selected to participate in the research project (Polit and Hungler, 1999). According to Creswell (2014) qualitative research is context bound and uses a small sample size. In this study, the sample comprised of three selected secondary school teachers of mathematics in selected schools in Chinsali district of Muchinga province, Zambia. The researcher selected the participants who had experience with the phenomenon and also who were willing to participate and to share their thoughts in this study. Three teachers of mathematics from three secondary schools were selected because Calculus is the topic that contains many sub topics as shown from Table 2.1 whereby it was required by the researcher to observe six lessons from each teacher of mathematics.

3.6 Sampling Procedure

Purposive sampling method was used in order to choose participants who provided the best information to answer the research questions (Kumar, 1996 & Creswell, 2003). Purposive sampling was used to select three teachers of mathematics in Chinsali district. Bearing in mind different types of purposive sampling, the researcher specifically used conveniently purposive sampling. Kombo and Tromp (2006) referred to this type of purposive sampling because it aims at picking a small sample with similar characteristic in order to describe some particular subgroup in depth. In order to select the three teachers who participated in this study, the researcher did a pilot study from different Secondary

Schools in Chinsali district. Firstly, the researcher talked to the HOD for mathematics who gave the information to the researcher for teachers who were teaching grade 12 at that particular time. Thereafter, the researcher talked to the teachers and asked them if they were willing to participate in the study and the researcher took note of the teachers who were willing to participate in the study until finally he came up with three teachers. These three teachers were purposively sampled because: (1) these are the teachers who were willing to participate in this research, (2) they were also the same teachers who were teaching Calculus to grade twelve at that time when the researcher was collecting data, and (3) these are the teachers with degree holders in mathematics who provided with necessary information which was required by the researcher. Furthermore, Chinsali district is the district that comprised of 6 senior secondary schools of which 2 are boarding secondary schools and 4 are day secondary schools. Teachers who were purposively sampled are teachers from 2 day secondary school representing 4 day secondary schools of Chinsali district and 1 boarding secondary school representing 2 boarding secondary schools of Chinsali district. Therefore, purposive sampling helped the researcher to select the participants to the convenience of the researcher.

3.7 Data Collection Methods

In this study, data was collected through questionnaire, semi-structured interviews and video recorded observations from teachers of mathematics in Chinsali district. Therefore, the researcher used methods that are appropriate for qualitative research. The reasons for the choice of these methods are explained below.

3.7.1 Questionnaire

In this study, the questionnaire items were administered to the sampled teachers of mathematics. The questionnaire was intended to investigate teachers' subject matter knowledge of secondary school Calculus. Six questions on Calculus were adapted from 2016 and 2017 Zambian Grade 12 paper 1 and paper 2 mathematics final examination past papers. Grade 12 mathematics textbooks were also used as a source of the adapted question items. This was done to facilitate the validity of the items. The draft questionnaire was piloted involving teachers of mathematics in Chinsali district who bore similar characteristics to the sample. The idea was to enhance the clarity and quality of questions. After the pilot study, expert judgment by mathematics teachers and lecturers was sought to as one way of ensuring content validity of the questionnaire. For further details of the questionnaire, refer to Appendix A.

3.7.2 Semi - structured Interviews

According to Patton, (2002) Cohen, Manion and Morrison (2007) and Leavy (2014) semi-structured interview guide is a written list of questions or topics that need to be covered by the interviewer in order to get a systematic, comprehensive and detailed understanding of a topic. Semi-structured interviews give room for further probing which can help one to get detailed information (Patton, 2002). Interviews were principally used in this study because of their flexibility which enabled the researcher to rephrase the questions and to probe further in order to clearly get the actual views of the respondents. Semi-structured interview guide was administered to the sampled teachers of mathematics. The aforementioned was intended to investigate teachers' subject matter knowledge of secondary school Calculus, to explore teaching strategies employed by teachers of mathematics in the area of secondary school Calculus and to investigate the mathematical challenges faced by teachers of mathematics as they teach Calculus. Besides, the draft of the semi-structured interviews was developed before the final one and the same draft was piloted involving teachers of mathematics in Chinsali district in order to assess whether or not the questions were answering the research objectives and also to determine the duration of the interviews. Subsequently, the questions which were not answering the research objectives were removed. This was done in order to ensure the content validity of the semi-structured interview guide. For details on the semi structured interviews refer to Appendix B.

3.7.3 Lesson observations

According to Leavy (2014) lesson observation is a research method that provides information about actual behaviour. Lesson observation schedule was use to observe the three sampled teachers of mathematics. It was anticipated to investigate teachers' subject matter knowledge in the area of secondary school Calculus, to explore the teaching strategies employed by teachers of mathematics as they teach Calculus and to investigate mathematical challenges faced by teachers as they teach secondary school Calculus. Lesson observation schedule was adapted from the observation schedule which is used by administrators to observe teachers in secondary schools of Zambia. Further, the draft of the Lesson observation schedule was developed before the final one and the same draft was piloted involving teachers of mathematics in Chinsali district. This was done in order to ensure the content validity of the Lesson observation schedule. Thereafter, the researcher entered the classroom simply to observe as a non-participant and note how the teacher was interacting with learners during the teaching and learning of Calculus. The researcher did not participate in the actual teaching and learning process. The classroom observation and the descriptions of the lesson's activities allowed the researcher an opportunity to understand what actually happens in mathematics lessons, the behavior

of learners and the teachers' pedagogical content knowledge for the teaching of Calculus to the learners. Six lessons from each teacher were observed by the researcher which gave the total of eighteen lessons. This method has been used by other researchers investigating classroom interactions (Sinyangwe and Chilangwa, 1995; Nkoya, 2006 and Kalumba, 2012). For further detail of the lesson observation schedule refer to Appendix C.

3.8 Data Collection Procedure

Data collection took place in the first and second term of 2018 academic year between January and June. In order to have data collected, the researcher requested for permission from the Ethics Committee of the University of Zambia and thereafter, the District Education Board Secretary for Chinsali district. Permission was also word orally from HOD and teachers. The purpose of this research was explained to the DEBS in Chinsali, Head teachers of the sampled schools and teachers. This was done in order for the researcher to be given permission to freely interact with the selected respondents without any interference. The researcher also had to ask for consent from the respondents to enable them make an informed decision on whether they could participate in the study or not. The researcher asked the Head of Mathematics Department at each selected school for schedules of periods when teachers would be teaching Calculus in grade 12. The schedules from each school were noted down.

The researcher had begun by administering a questionnaire to the sampled teachers of mathematics who were willing to participate in this study. These three teachers answered the questionnaire at different time and in terms confidentiality, these teachers did not know their fellow teachers who were targeted in this study and they were also from different schools. The questionnaire intended to investigate teachers' subject matter knowledge in the area of secondary school Calculus.

Secondly, semi-structured interviews were administered to the sampled teachers of mathematics from three selected secondary schools in Chinsali district who were teaching Calculus at that particular time. During interviews, the explanations and validations of reasoning as well as the understanding of different representations of Calculus concepts were stimulated from the three sampled teachers of mathematics. Semi-structured interviews allowed for flexibility in the wording of questions and allowed for justification of the data from the questionnaire and follow ups were also made of their explanations.

Thirdly, as a way of getting first-hand information on how Calculus is taught by the senior secondary school teachers of mathematics, the researcher asked for permission from the school administration in

the selected Senior Secondary Schools in Chinsali district to observe Calculus lessons from the sampled teachers of mathematics. This was done in order to compare their responses in the test and also in the semi- structured interviews to the actual classroom practices. Howie and Grayson (2002) observed that, teachers do not practice what they say, they are of the view that the best way to find out about the strategies teachers use, is to observe what they actually do in class. During each lesson observation the researcher was sited at the back of the class from the beginning to the end of each lesson. Six lessons from each teacher were observed by the researcher which gave the total of eighteen lessons.

3.9 Data Analysis

The method of analysis involved qualitative methods. In qualitative method similar information were grouped into similar themes for ease of analysis and in making conclusions. Qualitative data collected from the sampled teachers of mathematics was analysed thematically. Creswell (2004) asserted that thematic analysis categorises related topics, and major themes are identified to provide rich deep description of the phenomena under study. Questionnaires were analysed item by item to determine the teachers' depth of understanding of the subject matter while the transcribed interview data and video recorded observations were explored for similar answer segments for interpretation. Therefore, the section below explains how Qualitative data from a questionnaire, semi-structured interviews and lesson observation schedule was analysed and interpreted into themes.

3.9.1 Questionnaire

This study used a questionnaire which was intended to provide answers to research question on, do teachers of mathematics have comprehensive understanding of subject matter knowledge in the area of secondary school Calculus? To analyse the questionnaire, the researcher had begun with the scoring of question by question because a questionnaire was mainly comprised of calculations on Calculus by using an adapted marking rubric as it is shown from the table below.

Table 3.1 below shows the marking rubric for scoring a questionnaire (Adapted from the works of Malambo, 2016).

Marking rubric for scoring

No	Explanation of areas for giving marks	Marks given
1	Appropriate method used to solve questions that involve the use of chain rule, product rule, quotient rule and area under the curve clearly up to the correct answer.	Total marks were given
2	Appropriate method used to solve questions as stated above but arriving at the wrong answer when solving	One mark was deducted from total marks
3	Appropriate method used partially and without arriving at a correct answer	Two marks were deducted from the total marks
4	Demonstrating lack of understanding of the question and by using wrong method in the process of solving.	No marks were given

Table 3.2 shows the allocation of marks from each test question.

Number of questions	Marks allocated
1. (a)	4
I. (b)	3
II. (a)	4
2. (b)	3
III. (a)	4
3. (b)	3
IV. (a)	4
4 .(b)	4
V. (a)	2
5 .(b)	3
VI. (a)	3
6 .(b)	3
Total Marks	40

After scoring the questionnaire the responses from the sampled teachers of mathematics were consolidated for analysis according to the following categories which were linked with the conceptual framework such as the understanding of Calculus concepts and solving problems patterning to differentiation and integration. Full marks were awarded for vital calculations and few marks were awarded for suitable steps even when the final answer was incorrect.

3.9.2 Semi-structured interviews

The analysis of data through semi-structured interviews intended to provide answers to the research questions such as; do teachers of mathematics have comprehensive understanding of subject matter knowledge in the area of secondary school Calculus, what teaching strategies do teachers of mathematics use to teach Calculus in secondary schools and what mathematical challenges do teachers of mathematics face as they teach secondary school Calculus? After the interviews, the audio-recorded interviews were transcribed, giving a total of 3 transcripts from the sampled teachers of mathematics. The three sampled teachers of mathematics were coded as teacher A, teacher B and teacher C. Thereafter, the transcripts were read several times in order to acquire a sense of data. Following this process, a content analysis was used to explore the transcripts in relation to research questions as stated above. Merrian (2009) describes content analysis as a technique that involves the search for themes and patterns. In relation to this study, content analysis was employed in order to identify segments of data in the transcripts that would provide answers to specific interview questions. Thereafter, the same transcripts were also explored for commonalities and differences from the sampled teachers' responses and interpreted into themes according to the research questions.

3.9.3 Lesson observation schedule

Lesson observation schedule which was observed by the researcher to the sampled teachers of mathematics intended to provide answers to the research questions such as; do teachers of mathematics have comprehensive understanding of subject matter knowledge in the area of secondary school Calculus, what teaching strategies do teachers of mathematics use to teach Calculus in secondary schools and what mathematical challenges do teachers of mathematics face as they teach secondary school Calculus? The three sampled teachers who were observed were coded as teacher A, teacher B and teacher C. All lessons observed were video-recoded and transcribed by the researcher and read through all the responses. After reading through, the researcher gathered the collected data by grouping the responses that were sounding similar. Thereafter, the lessons were interpreted into themes according to the research questions. In a nutshell, the researcher used description and content analysis in analysing qualitative data to have the research questions answered.

3.10 TRUSTWORTHINESS

In qualitative research, the concepts such as credibility, dependability and transferability have been used to describe various aspects of trustworthiness (Patton, 1987; Polit and Hungler, 1999 & Long and Johnson, 2000). Trustworthiness in this study was achieved through giving a clear and distinctive

description of the: research context, selection and characteristics of respondents, data collection as well as the procedure for data analysis. Creswell (2008) pointed out that the background of the researcher has some influence and significance on the findings. It is for this reason this step allowed the researcher to seek clarity with the participants and verify whether the research instruments were answering the research questions and this was undertaken to enhance the credibility of the data collected. In this regard, this study were gathered through a questionnaire, semi-structured interviews and lesson observation schedule from the sampled teachers of mathematics. Some of the aspect that were assessed in the questionnaire were explored further during interviews. To enhance the trustworthiness of the interviews, the draft interview schedule was piloted in order to facilitate the credibility of the interviews. The three sampled teachers were asked similar questions during interviews. At the beginning of each interview session, the sampled teachers were encouraged to express their views without any fear. Thereafter, the three sampled teachers were observed by the researcher.

3.11 Ethical Considerations

Ethical considerations are of the utmost importance when one is conducting research involving human participants (Goddard & Melville, 2001). In this study the following ethical measures were adhered to in the process of data collection, analysis and dissemination. Written permission to conduct the research at sites was sought and obtained from the District Education Board Secretary Officer of Chinsali. Permission was also obtained in writing, from the Head teachers and teachers to conduct the study. The aims and objectives of the study were explained verbally to the teachers by the researcher prior to their participation. Assurance was given to the sampled teachers of mathematics in Chinsali district that the information gathered is for academic purposes and therefore will be treated with anonymity and confidentiality. To guarantee confidentiality, anonymity, non-identifiable and non-traceability of the participants, the researcher used codes instead of names. Additionally, Informed consent is a key issue in research ethics. No one should be a participant or a source of information in a research project unless they have agreed to be so on the basis of a complete understanding of what their participation will involve and the purpose and use of the research. In this regard to this study, informed consent was sought from all the three sampled teachers and they were allowed to withdraw from the research, or ask for the information they have provided to be withdrawn, at any time; at least up until the point when the data have been processed and written up.

CHAPTER FOUR: PRESENTATION OF FINDINGS

4.1. Overview

In the previous chapter, the researcher described the research methodology, which was employed in this study in order to come up with the findings which are presented in this chapter. The main purpose of this study was to assess the teaching of Calculus at senior secondary school level. The themes that are presented in this chapter emerged from the data collected from test questionnaire, semi- structured interviews and lesson observations. The presentation of data was intended to provide answers to the research questions. The research questions are:

- Do teachers of mathematics have comprehensive understanding of subject matter knowledge in the area of secondary school Calculus?
- What teaching strategies do teachers of mathematics use to teach calculus in secondary schools?
- What mathematical challenges do teachers of mathematics face as they teach secondary school Calculus?

4.2. Background characteristics of the Participants

The participants who took part in this study had to indicate their brief background information for the purpose of analysis regarding their working experience and professional qualification. The responses were collected from semi-structured interviews as shown below.

Interviewer: how long have you been teaching mathematics at Senior Secondary School level?

Teacher A: 3 years

Teacher B: 2 years

Teacher C: 10 years

Interviewer: what is your professional qualification?

Teacher A: degree

Teacher B: degree

Teacher C: degree

In regard to the working experience of teachers of mathematics, the findings have shown that only teacher C has been teaching mathematics for a long time whilst teacher A and teacher B have been teaching for 2 years and 3 years respectively. The findings have also shown that all the three teaches observed were degree holders in regard to their professional qualification.

4.2.1: Description of Participants

The following is a brief description of each participant. In keeping with the confidentiality commitment to the participants, names shall not be used, instead, the researcher will just name the participants as teacher A, teacher B and teacher C.

Teacher A: Teacher A was the first teacher who answered the questionnaire which was administered to him. After answering the questionnaire, he was interviewed then observed. However, the same teacher was teaching a pure class and he has been teaching for 3 years at the time the researcher was collecting data from him. Six questions from the questionnaire were administered to him, 23 questions were interviewed and 6 lessons were observed. He holds a Bachelors' degree in mathematics and taught additional mathematics to a grade 12 class.

Teacher B: Teacher B was the second teacher who answered the questionnaire, interviewed and being observed. The same teacher was teaching ordinary mathematics to the grade 12 and he has been teaching mathematics for 2 years only at the time the researcher was collecting data. Six questions from the questionnaire were also administered to him, 23 questions were interviewed and 6 lessons were observed. He holds a Bachelors' degree in mathematics and he taught mathematics to a grade 12 class.

Teacher C: Teacher C was the third teacher to answer the questionnaire which was administered to him. Thereafter, he was interviewed and also being observed, he was the same teacher who has been teaching mathematics for 10 years. Six questions from the questionnaire were administered to him, 23 questions were interviewed and 6 lessons were observed. He holds a Bachelor's degree in mathematics and he taught mathematics to a grade 12 class at the time the researcher was collecting data.

4.3 Research Findings from the Pilot Study

The pilot study was done in Chinsali district and two schools were chosen purposively. During pilot study, teachers of mathematics from sampled secondary schools participated willingly. This was mainly done to assess the reliability of the research instruments and to give the researcher experience especially on the part of a questionnaire, semi-structured interviews and lesson observations. Pilot study enabled the researcher to find out whether the research instruments were measuring what they were expected to measure, whether the questions could provoke a response as well as to check for the clarity of the wording and if different respondents could interpret the questions in a similar way. All the data collection methods as indicated in chapter three were used in the pilot study. Having done the pilot study, the collections were made in order for the instrument to be in line with the research

objectives. However, the changes which were made to the instruments as a result of the pilot study were only on the part of lesson observation. The researcher intended to observe 9 lessons from each participants but he only observed 6 lesson from the participants due to time constraints on the part of the participants.

4.4 Actual Research Findings for the Targeted Sample

The research questions guided the researcher to formulate and organise the research instruments in this study. Being a purely qualitative study, all the three research questions implored for qualitative data. Data was collected through a questionnaire, semi-structured interviews and lesson observations. The questionnaires, were analysed question by question to determine the teachers' depth of understanding of the subject matter knowledge while the transcribed interview data and video recorded lesson observations were explored for similar answer segments for interpretation. The collected data would be presented according to research objectives which are in line with the conceptual framework.

4.5 The subject matter knowledge of teachers of mathematics in the area of secondary school Calculus

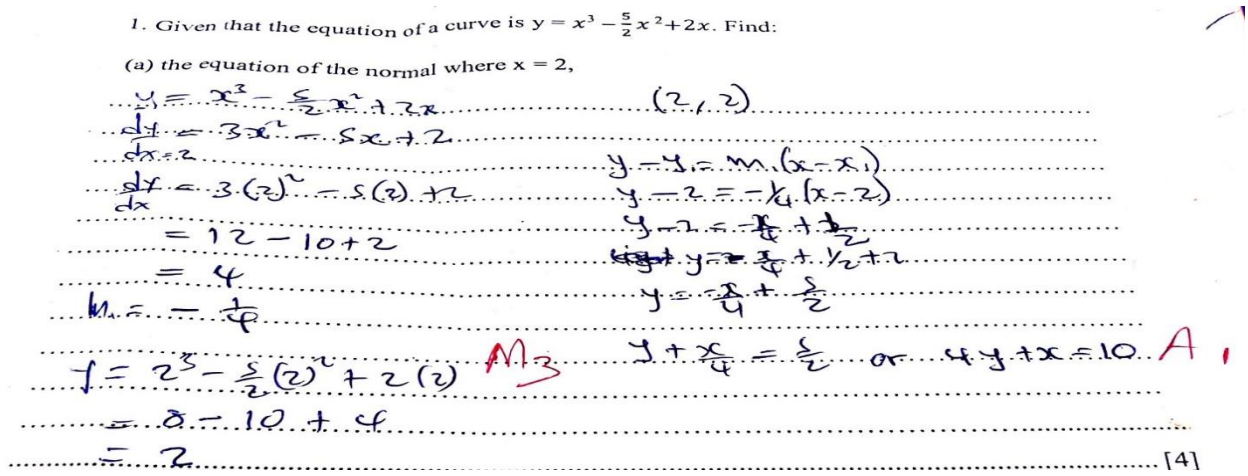
In an attempt to provide answers to research objective stated above, the data collected was presented according to the conceptual framework and the research instruments that were used to collect this data were questionnaires, semi-structured interviews and lesson observations as indicated below.

4.5.1 Solving Calculus problems involving differentiation and integration

The data presented and analysed had begun by using a questionnaire which was administered to all the sampled teachers of mathematics. A questionnaire consists of Calculus questions which were administered to the sampled teachers of mathematics and the main objective of this questionnaire was to assess teachers' subject matter knowledge in the area of secondary school Calculus. These three teachers were purposively sampled. A questionnaire consists of 6 Calculus questions from differentiation and integration that assessed teachers' in-depth understanding of the subject matter in the area of secondary school Calculus. These questions totalled 40 marks and each question assessed a particular aspect of Calculus, as found in the Zambian secondary school mathematics curriculum. However, a questionnaire was analysed question by question for mathematics teachers' performance. Individual questions will be explained first followed by teachers' performance.

Question 1(a)

Given that the equation of a curve $y = x^3 - \frac{5}{2}x^2 + 2x$. Find the equation of the normal where $x = 2$. This question required the mathematics teachers to differentiate the equation of the curve first and then, find the gradient of the tangent and the gradient of the normal by substituting x with 2, thereafter, this question was required to find the value of y from the original equation of the curve and finally, to find the equation of the normal at the point (2, 2) into $y - y_1 = m_1(x - x_1)$. The purpose of this question was to assess the content knowledge of teachers of mathematics in this area by finding the equation of the normal. The findings shows that only one teacher was knowledgeable in this area, this is because he answered this question correctly while other two teachers had solved this question partially, meaning that they have little knowledge in the finding of the equation of the normal. Figure 4.1 below shows the way one of the teachers answered this question correctly.


1. Given that the equation of a curve is $y = x^3 - \frac{5}{2}x^2 + 2x$. Find:

(a) the equation of the normal where x = 2,

$y = x^3 - \frac{5}{2}x^2 + 2x$ (2, 2)

$\frac{dy}{dx} = 3x^2 - 5x + 2$

$\frac{dy}{dx} = 3(2)^2 - 5(2) + 2$

$= 12 - 10 + 2$

$= 4$

$m = -\frac{1}{4}$

$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{1}{4}(x - 2)$

$y - 2 = -\frac{x}{4} + \frac{1}{2}$

$y = -\frac{x}{4} + \frac{1}{2} + 2$

$y = -\frac{x}{4} + \frac{5}{2}$

$y + \frac{x}{4} = \frac{5}{2}$ or $4y + x = 10$ A1

$y = 2^3 - \frac{5}{2}(2)^2 + 2(2)$ M3

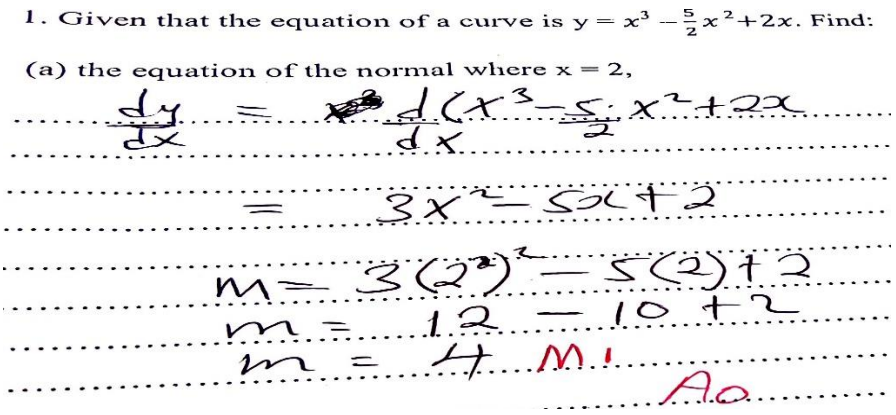
$= 8 - 10 + 4$

$= 2$

[4]

Figure 4.1: Answer from one of the teachers who answered correctly

An example of a weak answer from other two teachers is presented below in figure 4.2


1. Given that the equation of a curve is $y = x^3 - \frac{5}{2}x^2 + 2x$. Find:

(a) the equation of the normal where x = 2,

$\frac{dy}{dx} = \frac{d}{dx}(x^3 - \frac{5}{2}x^2 + 2x$

$= 3x^2 - 5x + 2$

$m = 3(2^2) - 5(2) + 2$

$m = 12 - 10 + 2$

$m = 4$ M1

A0

Figure 4.2: Answer from one of the two teachers.

Question 1 (b)

Given that the equation of the curve is $y = x^3 - \frac{5}{2}x^2 + 2x$. find the coordinates of the stationary points. This question required the teachers of mathematics to differentiate the given curve and equate it to zero in order to find the values of x and thereafter to find the values of y by substituting the values of x into the given curve. This question was assessed in order to find out whether the teachers of mathematics were competent in this area. The results suggested that only one out the three teachers performed well and other two teachers failed to find the coordinates of the stationary point from the given curve, which shows that other two teachers of mathematics did not understand the Calculus concepts in this area. Figure 4.3 below shows how one of the teachers performed well in finding the coordinates of the stationary point from the given curve.

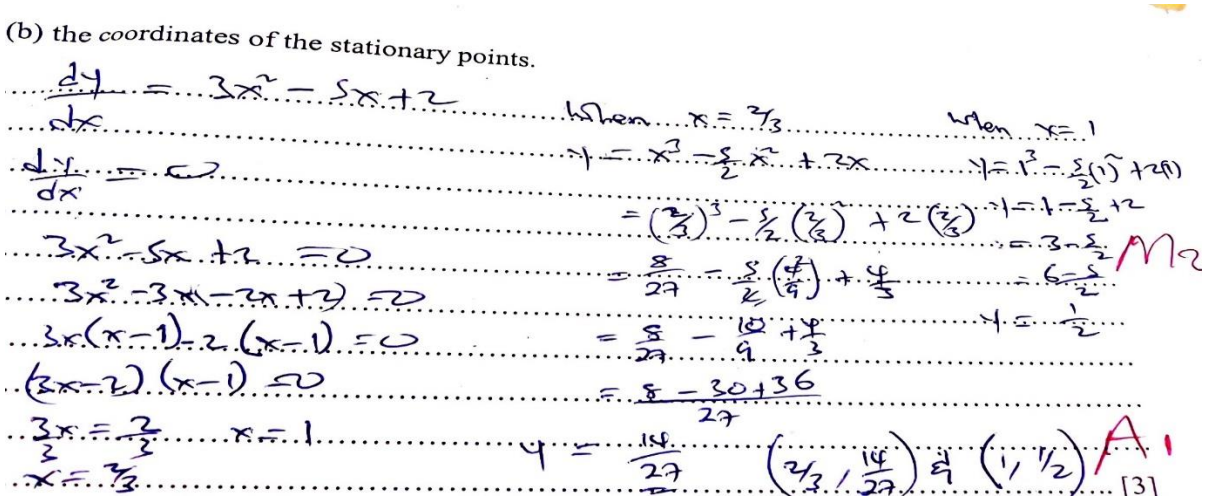


Figure 4.3: Answer from one of the teachers who answered correctly

Figure 4.4 below shows the way one of the teachers out of the three failed even to attempt to find the coordinates of the stationary point from the given curve.

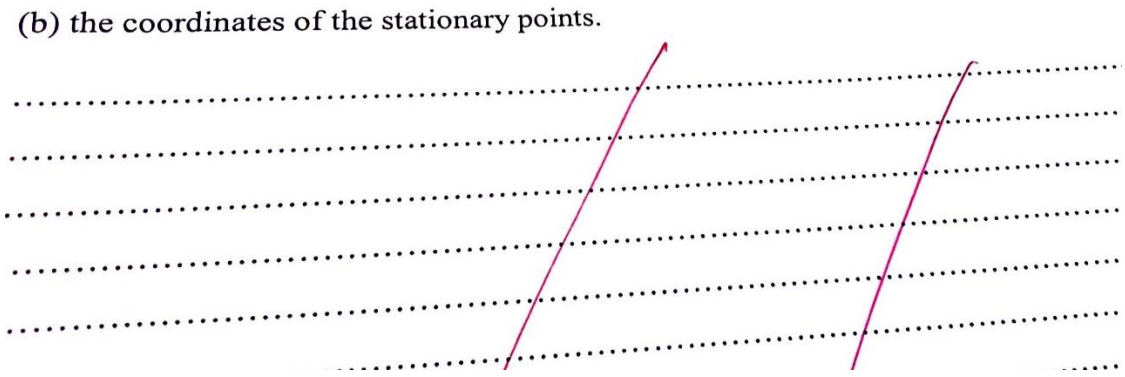


Figure 4.4: Answer from one of the two teachers

Question 2(a)

Find the coordinates of the points on the curve $y = 4x^3 - 6x^2 - 24x + 3$ where the gradient is zero. This question assessed the mathematics teachers' ability where they were expected to differentiate the equation of the curve and thereafter, to find the values of x by equating the expression to zero. The results show that only teacher A was successful at providing complete and accurate answers which shows that he was knowledgeable on this question. The results also shows that teacher B failed to solve this question which shows that he has difficulties in understanding the concepts on this question concerning finding of the coordinates when the gradient is zero, whilst teachers C answered this question partially which shows that he just had an idea. With regard to this question it was discovered that some teachers did not know that finding the coordinates of the stationary point from the given curve is the same as finding the coordinates from the given curve when the gradient is zero and the only difference is the language itself which is found in Calculus.

Figure 4.5 below shows an example which was done correctly from one of the sampled teachers.

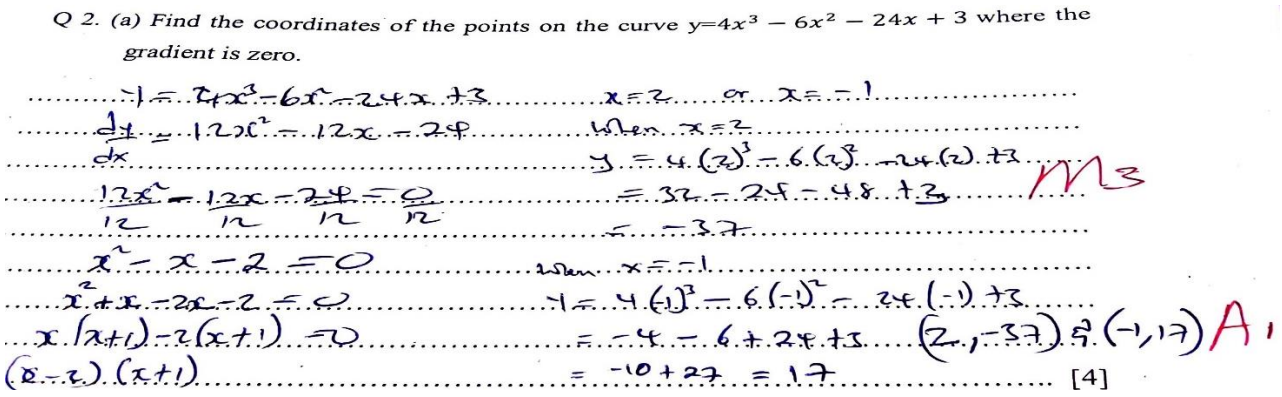


Figure 4.5: Answer from one of the teachers who answered correctly

Figure 4.6 below shows the way one of the teachers from the sampled teachers failed to find the coordinates of the stationary point from the given curve.

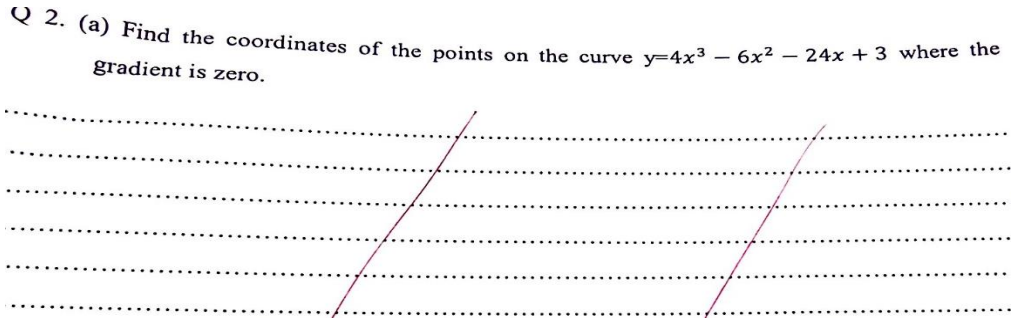


Figure 4.6: Answer from one of the teachers who did not even attempt the question.

Question 2 (b)

Evaluate $\int_{-1}^{2}(4x^3 - 3x^2 + 2x)dx$. This question assesses teachers of mathematics whether they have ability to integrate and to evaluate. The response from this item was that, teacher A and teacher B performed moderately well. They were able to integrate and to evaluate. Moreover, teacher C failed to integrate completely that made him to evaluate wrongly. These results show that teacher C was incompetent in this area.

An example of two teachers answered correctly is presented below in Figure 4.7.

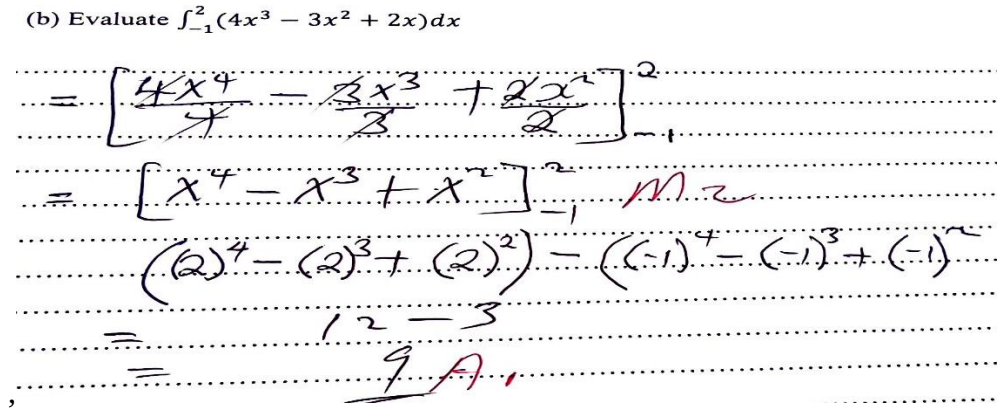


Figure 4.7: Answer from two teachers who answered correctly

An example of a weak answer from other teacher is presented below in figure 4.8

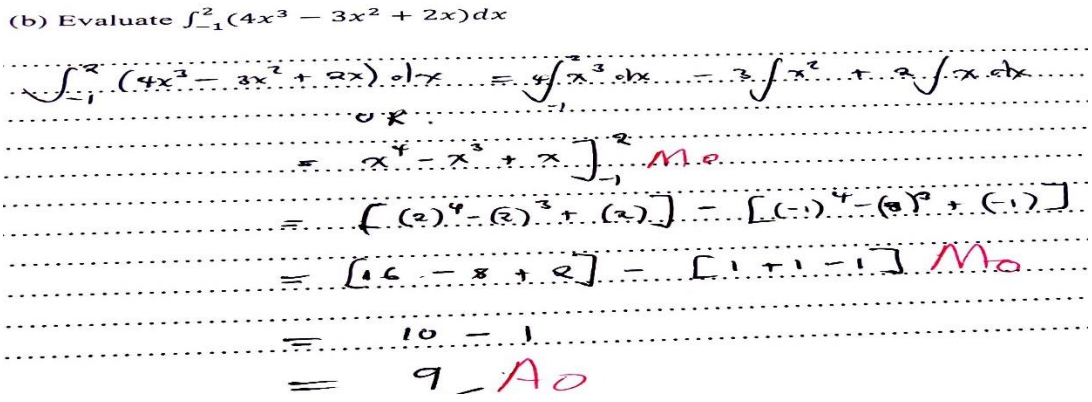


Figure 4.8: Answer from other teacher who answered wrongly

Question 3(a)

Differentiate $4x^2 + 2x$ by using the first principle. This question assessed the teacher's ability to differentiate by using the first principle. The results show that all the three teachers performed well in this area of differentiating by using first principle as presented in Figure 4.9 below.

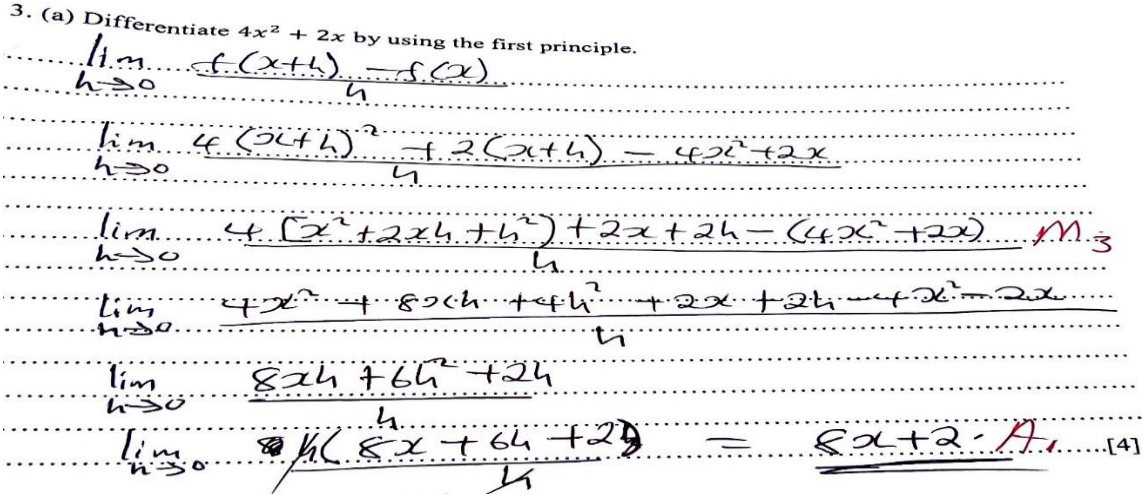


Figure 4.9: Answer from one of the teachers who answered correctly.

Question 3(b)

Find the derivative of $y = \sqrt{9x-3}$ with respect to x. this question also assessed teachers' ability to differentiate this item by using chain rule. To begin with, this item required to express $y = \sqrt{9x-3}$ in the form $y = (9x-3)^{1/2}$ and then differentiate by using chain rule. The results shows that out of the three teachers, two of them understood the concept in this area because they were able to differentiate this question by using chain rule correctly whilst the other teacher did not know how to differentiate this question by using chain rule because he failed completely and he didn't even attempt to solve it as it is shown from the figure 4.10 below.

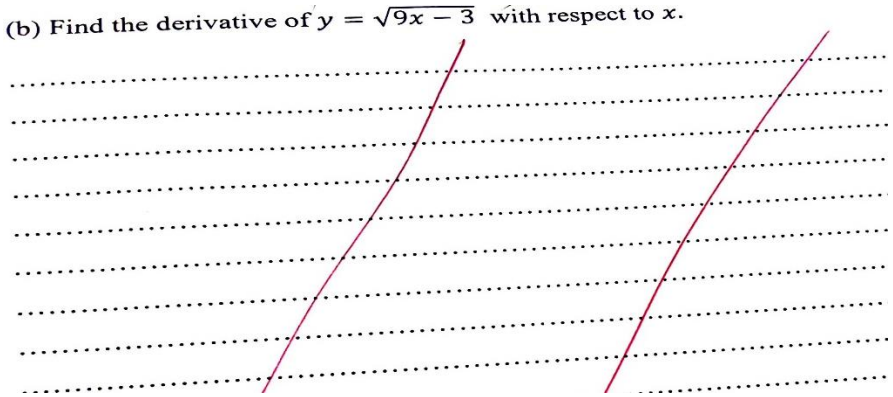


Figure 4.10. Answer from one the two teachers who failed to attempt this question

An example of a correct answer from one of the teachers is presented below in figure 4.11

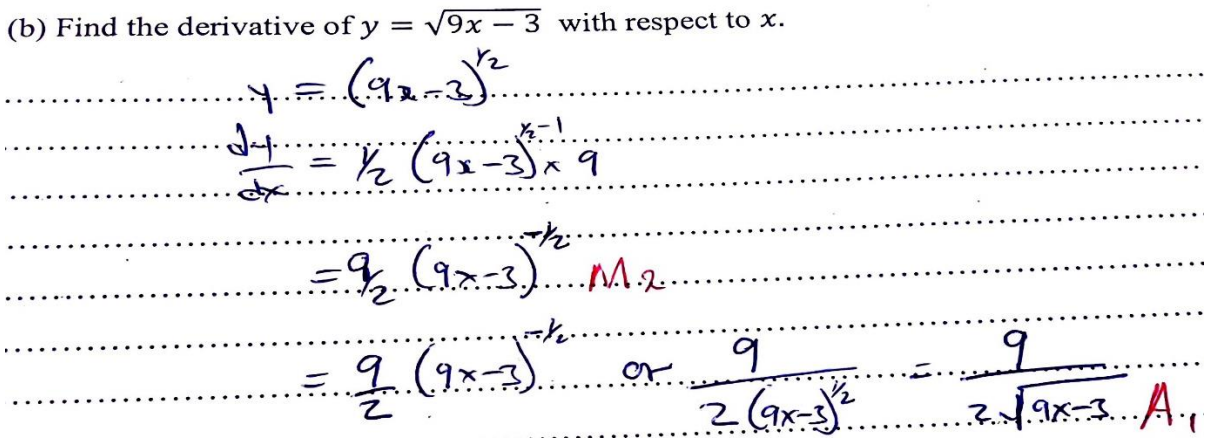


Figure 4.11: Answer from one of the teachers who answered correctly

Question 4(a)

Find the gradient of the curve $y = x^2(x-4)^3$ at the point (1, 0). This question required the teachers of mathematics to differentiate this question by using product rule and thereafter to find the gradient of the tangent. Thus, this question assessed the teachers' ability whether they are competent in differentiating by using product rule. The findings from this question show that only one teacher from the sampled teachers was competent in this area and other two teachers from the sample failed to solve this question completely. From these results, it can be argued that teachers performed poorly in this area. This is because; they were not able to differentiate this question by using product rule.

An example of how other two teachers failed to solve this question completely is presented below in figure 4.12

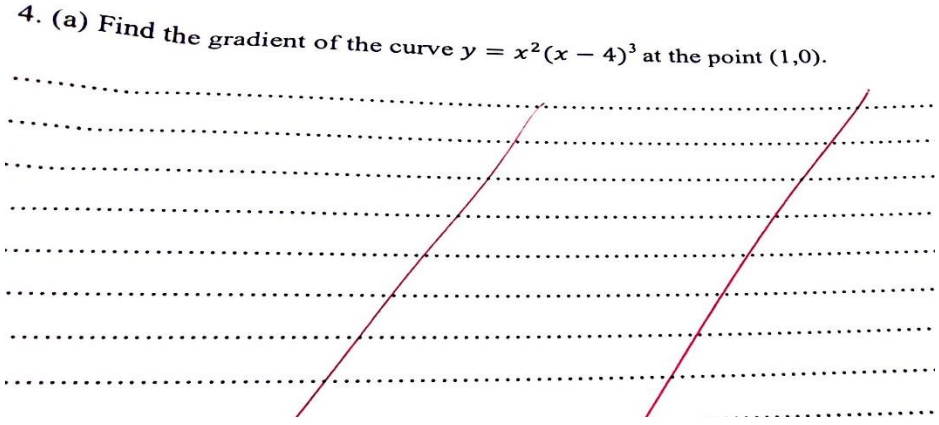


Figure 4.12 *Answer from one the two teachers who failed to attempt this question*

Question 4(b)

Find the gradient of the normal given that $y = \frac{5x-3}{x+2}$ at the point (-1, 2).

This question assessed the subject matter knowledge of teachers of mathematics in the area of differentiating by using quotient rule. Teachers of mathematics were required first to differentiate this question by using quotient rule and then find the gradient of the normal and equation of the normal. The results suggest that two teachers from the sample were incompetent in differentiating this question by using quotient rule. These results seems to be consistent with question 4(a) in which only one teacher from the sample was able to find the equation of the normal whilst other two teachers had difficulties in differentiating question 4(a) by using product rule that made them not to find the equation of the normal. An example of a correct answer is presented below in figure 4.13

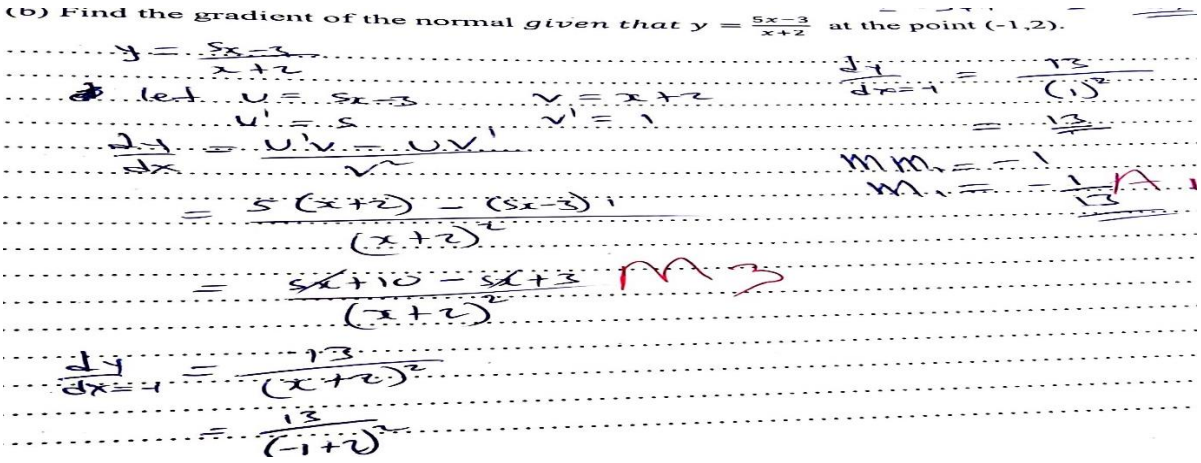


Figure 4.13: Answer from one of the teachers who answered correctly

Another figure below shows how other two teachers lacked subject matter knowledge in this area.

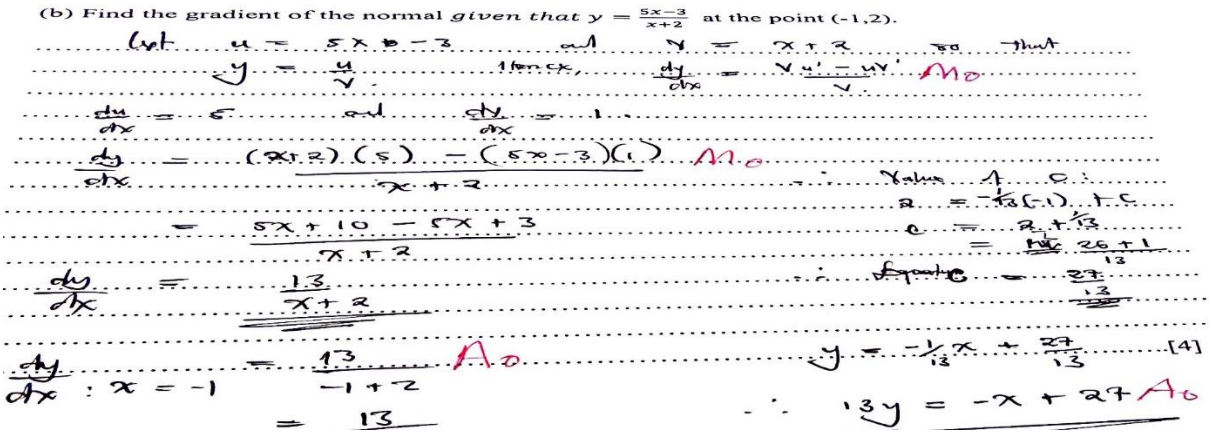


Figure 4.14 Answer from one of the two teachers who answered wrongly

Question5 (a)

Given that y= $4x^2 - \frac{2}{x^2}$, find$\frac{dy}{dx}$.

This question required to differentiate by using the formula of differentiation. The results show that this is another question where all the sampled teachers targeted performed very well as shown from figure 4.15 below:

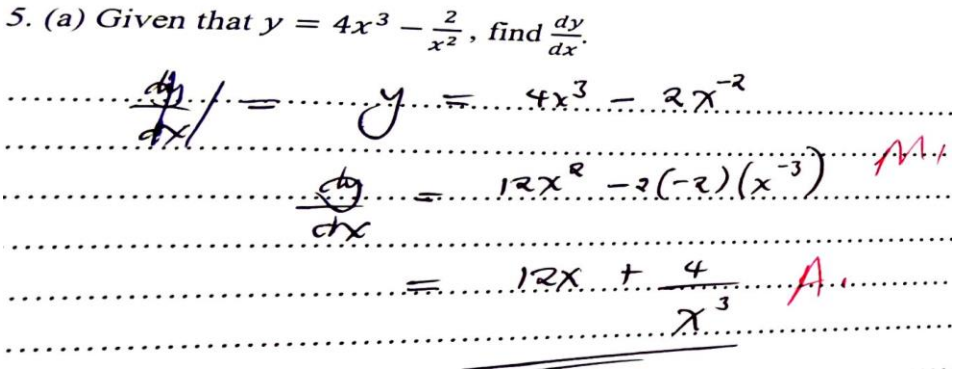


Figure 4.15 shows how all the three teachers were able to solve question 5 (a)

Question 5(b) integrate $\int \frac{6x^6-3x^3}{x} dx$

The purpose of this question \was to assess whether these teachers of mathematics could integrate. These results seem to suggest a lack of competence of the teachers of mathematics about in the area of integration. The results also show that only one teacher was able to integrate this question whilst other two teachers failed to integrate. An example on how one of the teachers was knowledgeable in this area is presented below in Figure 4.16.

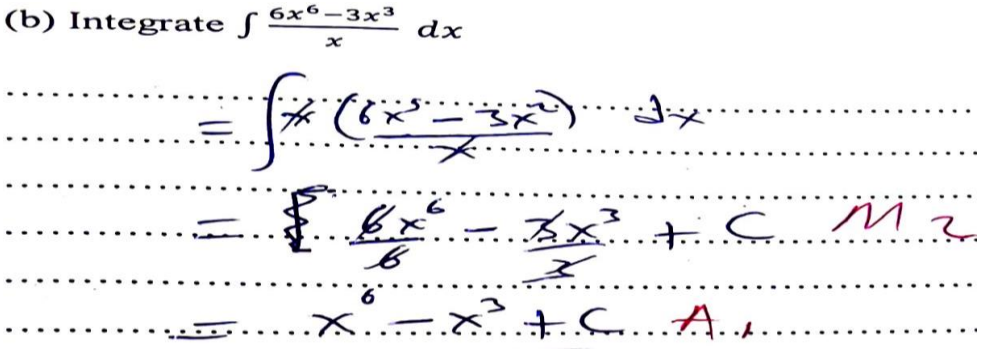


Figure 4.16: Answer from one of the teachers who answered correctly

Another example which shows how other two teachers lacked subject matter knowledge in this area is presented below in Figure 4.17.

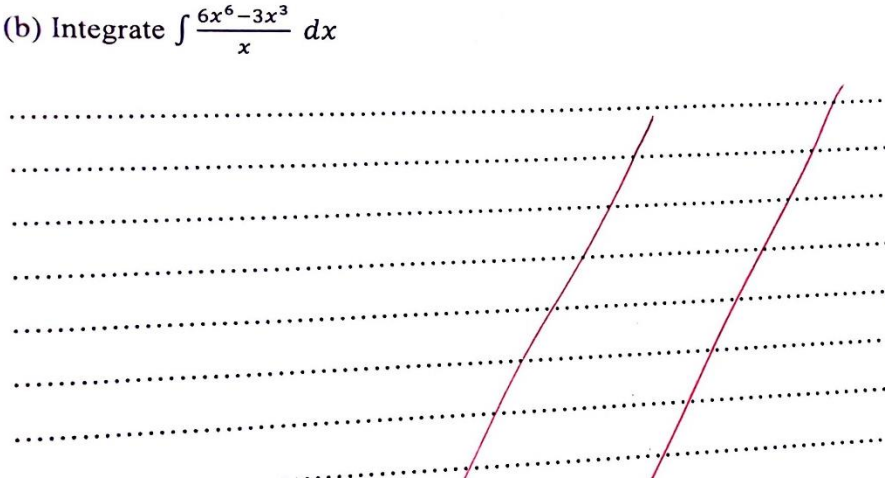


Figure 4.17: Answer from one of two teachers who failed to attempt this question.

Question 6(a)

A curve is such that $\frac{dy}{dx} = 12x^3 - 8x$. Given that it passes through (2, -1) find its equation. This question required the teachers of mathematics to integrate the expression and thereafter to find the value of C by substituting where there is x and y with the point (2,-1) in order to find its equation.

The findings of this study shows that all the three teachers lacked subject matter knowledge in finding the equation of the curve. They had difficulties when integrating $12x^3 - 8x$. These results also show that only one teacher out of three teachers had an idea but the rest had completely no idea in finding the equation of the curve as shown from the figure below.

An example on how two teachers from the sampled teachers of mathematics failed to solve this question is shown below in Figure 4.18

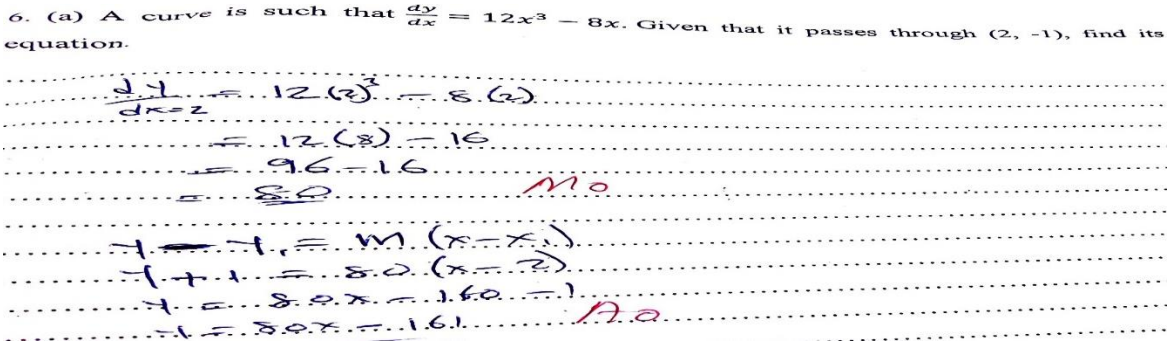


Figure 4.18: Answer from one of the two teachers who answered wrongly.

Figure 4.19 below shows how one of the sampled teachers had an idea in this area when finding the equation of the curve.

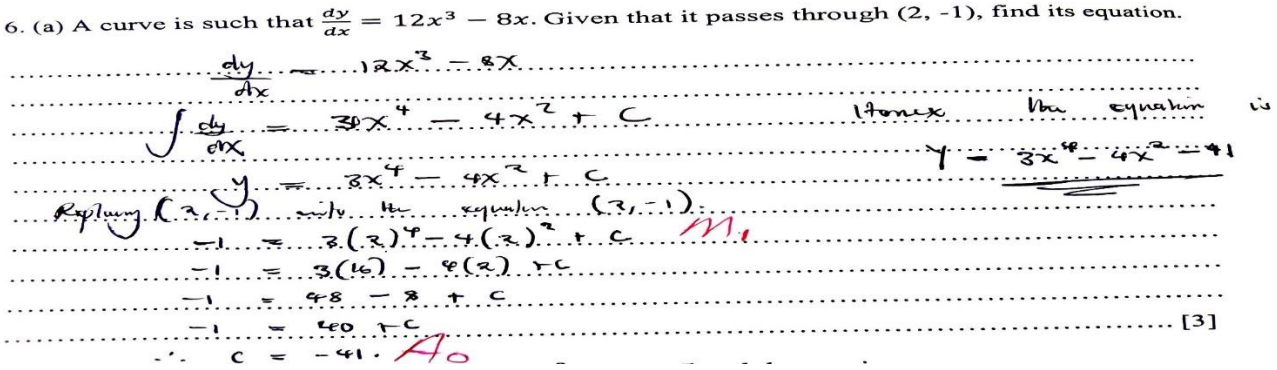


Figure 4.18: shows an answer which was answered partially.

Questions 6(b) find the area bounded by the curve y=3x^3- 2x -5 and the x-axis.

This question required to integrate first, thereafter it was required to find the limits that can be used to find the area bounded by the curve by evaluating the integral, in order to find the area bounded by the curve. These results suggest that two teachers lacked subject matter knowledge when finding the area bounded by the curve. Only one teacher managed or was able to find the area bounded by the curve as shown from the figure 4.20 below.

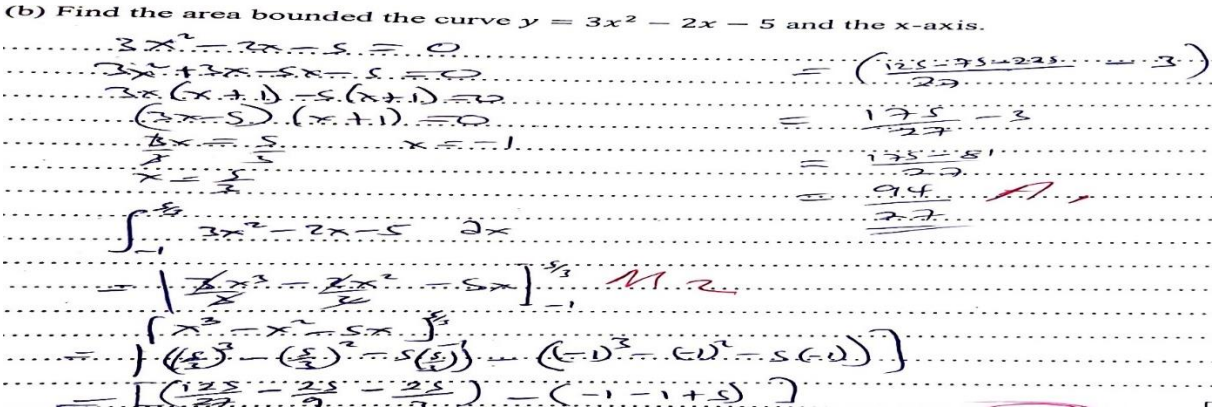


Figure 4.20 Answer from one of the teachers who answered correctly.

An example below which shows the incompetence in the area of finding the area under the curve by other two teacher is presented in Figure 4.21

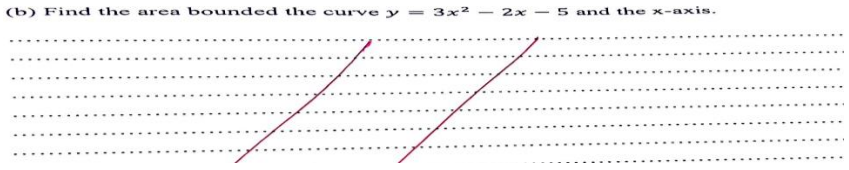


Figure 4.21: Answer from one of the two teachers who failed to attempt this question.

The summary of the responses from the questionnaire.

The questionnaire consisted of 12 questions that assessed the subject matter knowledge of teachers in the area of Calculus. These questions totaled 40 marks and each question assessed a particular aspect of Calculus as found in the Zambian senior secondary school mathematics curriculum. The scores which were obtained by sampled teachers of mathematics in order to investigate the subject matter knowledge of Calculus are summarized in table 4.1 below.

QUESTION	TEACHER A	TEACHER B	TEACHER C
1 (a)	4	1	2
1 (b)	3	0	2
2 (a)	4	0	2
2 (b)	3	3	0
3 (a)	4	4	4
3 (b)	3	0	3
4 (a)	4	0	1
4 (b)	4	0	0
5 (a)	2	1	2
5 (b)	3	0	0
6 (a)	0	0	1
6 (b)	3	0	0
Total Score	37 out of 40	09 out of 40	17 out of 40

Table 4.1: Sampled teachers' scores obtained from the questionnaire.

Table 4.1 shows that only one teacher who scored above 20 marks was knowledgeable in the area of Calculus, the other two teachers scored below 20 marks which shows that they lacked subject matter knowledge in the area of secondary school Calculus. Having presented the data from questionnaire, I now present the data from semi-structured interviews.

4.5.2 Teachers' understanding of the definition of the term Calculus

The research instrument which was used to present data concerning the definition of Calculus was semi-structured interviews. Semi-structured interviews was administered to sampled teachers of mathematics and it was intended to provide answers on how teachers of mathematics could define

Calculus according to their own understanding and their responses from the sampled teachers of mathematics are shown below.

Definition of Calculus

With regard to the definition of Calculus, this study reviewed that, all the three teachers failed to define this term Calculus to the fullest. Although there were some partial understanding from Teacher A and Teacher C, Teacher B failed completely to define the term Calculus for instance, this was evidenced in the following excerpt below.

Interviewer: *what do you understand by the term Calculus?*

Teacher A: *Calculus it is just a topic that deals with analysing the situations or which involves may be three parts or two parts that involves differentiation and integration.*

Teacher B: *The definition of Calculus yaaa I have forgotten.*

Teacher C *responded that: umm Calculus though I have never payed much attention to this, Calculus rely help us to deal with things, let's say practically when we look at the aspect for example the aspect of this country it is in the curve way so when it comes to the aspect, you will be able to determine the gradient of those curves in form of polynomials, that is how I understand about that.*

The responses from the three teachers interviewed showed that teachers lacked the understanding of this term Calculus.

4.5.3 The understanding of two main Calculus concepts such as differentiation and integration

This part presents the findings of the semi-structured interviews that were conducted with the three selected senior secondary school teachers of mathematics. Semi-structured interviews also intended to investigate the understanding of two main Calculus concepts such as differentiation and integration. The sampled teachers of mathematics who were interviewed were the same teachers who answered the questionnaire on Calculus.

The description of the two main concepts that are found in Calculus

To begin with, the researcher wanted to know whether or not the secondary school teachers of mathematics have knowledge on the main concepts found in Calculus and it was discovered that among the three teachers of mathematics, Teacher A is the only teacher who described the main concepts that are found in Calculus whilst teacher B and teacher C failed to describe. For instance, this was evidenced in the following excerpt below for Teacher A and one of the other two teachers.

Interviewer: *Describe the two main concepts that are found in Calculus*.

Teacher A: *ok I can describe them since the two main concepts that are found there it is the differentiation and integration, I can describe the differentiation part as the process of finding the change in y coordinates as well as the change in x coordinates or to differentiate a certain equation with respect to a certain variable which is found there. Then when we talk of integration we are just trying to take back whatever something that has been differentiated to its origin or maybe the equation has been differentiated and now we want to come up with the original equation*.

Teacher B: *the two main concepts of Calculus are differentiation and integration, differentiation is the reverse of integration.*

How these two main concepts that are found in Calculus can be applied to real life situation?

With regard to how these two main concepts that are found in Calculus can be applied to real life situation, Teacher A is the only teacher who tried to explain whilst other two teacher, their explanations was shallow. For instance, this is evidenced from excerpt below for Teacher A and one of other two teachers.

Interviewer: how do this two main concepts that are found in Calculus can be applied to real life situation?

Teacher A: *to real life situation, let me start with differentiation part, when we talk of differentiation part, it helps us to find or it is applicable when we talk of movement of the car, by applying the change or may be by applying the brakes that change in movement, it is called differentiated part, then when we talk of integration part it is now the situation whereby we are trying to accelerate after changing that speed maybe something like that.*

Teacher C: *new life situation application of calculus, like people say to everything that we do is mathematics. It is normally inculcate the aspect of judgmental on the way we understand about the problem before we understand about the problem before we are quick to react to the reaction which has happened we have to take time to say where the problem is coming from. It will help us to identify which one is the problem and which one is a solution suppose you have a critical problem how are you going to apply. In real life situation you have to take time and analyse why should we analyse, you have to analyse so that you are able to perceive and receive the information which you will be given, sometimes you might be dealing with a problem or solution because on the differentiation or integration, on the two there is problem and solution, now how do you get, and if you have a solution how do you get back to a problem and if you have a problem how do you get back to the solution?*

The level of appropriateness at which Calculus content are taught in Senior Secondary Schools to the learners

Teacher A said that, *it is appropriate because, they just learn the few concepts, yes they learn differentiation part as well as integral part but they don't go in detail as in now start differentiating the cosine function as well as sine function, they only target or concentrate much on just simple equations and by doing so it's just give them an insight of what they will be doing when they go to higher institutions, it's not so much involving*. Whilst Teacher C contradict to that of Teacher A and Teacher B: he indicated that, *appropriate, I like Levy Vygotsky concept where he was saying on the issue of scaffolding by saying on each and every information can be taught well at any time provided, that as it has been taught well, so when we talk of to say it is appropriate, it depends with the tutor, the facilitator or a teacher on how best he or she disseminating the information. It doesn't matter what information they are feeding to the pupils as long as they are disseminating or feeding the right Information, pupils will be able to understand well.*

An explanation on how the sampled teachers of mathematics found the equation of the normal from the questionnaire which they responded.

Teacher A: *To find the equation of the normal I first found gradient from the given function, thereafter I found the gradient of the normal then the equation.*

Teacher C: *equation of the normal suppose you have two lines and those two lines are meeting at an angle of 90 degree then will say to say that, the two lines are perpendicular to each other, so will look at the aspect where since we are looking at the perpendicular we have to equate the two gradient's the gradient of one line multiply by the gradient of another line and give us negative one as the answer may be from that perspective, will able to determine the equation of the normal.*

Stating the formula for differentiation by using chain rule, product rule and quotient rule which was used from the test they wrote.

Teacher A: responded that, *when we talk of chain rule it is just given by* $\frac{dy}{dx} =$ $lets\ say\ if\ we\ have\ a\ function\ y = (2x-1)^3$ *the formula that can be used is first we drop n which is n multiplied by the differentiation of what is inside the brackets, so let's if in the brackets we have a we say* $\frac{dy}{dx}$ *of a, then open brackets, it is just the same functions which is a to power n-1. A product rule if we have a function f(x) and we also have a function g(x) then when applying the formula that we use is* $\frac{dy}{dx} = f.g' + g.f'$. *meaning that in the first part we differentiate, we keep then we differentiate the second function plus then when we do it vice versa meaning we keep now the second*

function and try to differentiate the second function. For the quotient rule the formula which I used it is just a situation whereby I squared the numerator just like $\frac{g.f^{'}-f.g^{'}}{g^2}$**.**

Teacher C: *That one I have forgotten the formula which I used aaa is it plus or divide.*

An explanation on how to find the area under the curve

Teacher A: *The area was found by integrating the function which I had been given after integrating the function, I found the upper limit as well as the lower limit and then I replaced the upper limit the answer that I got I subtracted by the lower limit in the same function that had been integrated*.

Teacher B and Teacher C failed to explain on how they found the area under the curve which showed lack of understanding in the area of secondary school Calculus as shown in figure 4.20 where they even failed to attempt it.

Explanation on how the sampled teachers of mathematics solved question 6 (a), when finding the equation from the curve which was $\frac{dy}{dx} = 12x^3 - 8x$**. Given that it passes through (2, -1).**

Teacher A: *I tried to solve but I forgot the concept thus why I didn't do very well on this part.*

Teacher B: *I did not even attempt this question because I did not have an idea*.

Teacher C: *I did not know that I had to integrate thus why I did not do well in that area.*

In summarizing research question 1 which states that, 'do teachers of mathematics have comprehensive understanding of subject matter knowledge in the area of secondary school Calculus? This study reviewed that, from all the sampled teachers of mathematics, only one teacher was knowledgeable in the area of secondary school Calculus whilst other two teachers lacked the understanding of secondary school Calculus. This was evidenced when the questionnaire and semi-structured interviews were administered to them.

4.6 Teaching strategies employed by teachers of mathematics when teaching Calculus at secondary school level.

From the definition of teaching strategies, this study referred teaching strategies to structures, system, methods, techniques, procedures processes that a teacher uses during instructions in the classroom. It is for this reason, a researcher in this study focused on exploring the preparedness of lessons, teaching methods, lesson introduction, lesson progression, teaching and learning aids from the sampled teachers of mathematics. Therefore, to answer the research objective above, data on lesson planning, lesson introductions, lesson progressions, teaching methods, questioning skills, teaching and learning aids, lesson evaluations and summary was collected through interviews and lesson observation schedule. This is presented below.

4.6.1 Lesson planning

The findings from the three teachers observed, showed that none of them used lesson plans even if they were told to do so, instead, they were using text books to teach the learners in the classroom as it is shown from the figure below.

Figure 4.22 the use mathematics text book when teaching by one of the three teachers.

The figure above is an example which shows the way teacher A was using the text book to teach instead of lesson plan. This study reviewed that all the three teachers observed lacked the preparedness of lessons when teaching Calculus to the learners. **(Lesson observation, 2018)**

4.6.2 Teaching

Lesson introduction

In all the lessons observed the intended outcomes of the lesson were not told to the learners at the beginning of the lesson. All the three teachers observed introduced their lessons by reviewing the previous lessons using question and answer technique. In this regard, teachers were asked the questions orally and the pupils gave oral answers.

To begin with, teachers were interviewed concerning the teaching method they would use when introducing the topic and the responses were as follows:

Interviewer: *what method would you use when introducing differentiation and integration when teaching?*

Teacher A: *lecture method*

Teacher B: *questions and answers*

Teacher C: *class discussion*

Interviewer: *Explain why you have adopted such kind of methods?*

Teacher A: *This is because there is a part whereby they don't know anything so you just have to first explain certain points without involving them, if you involve them, you waste a lot of time because they don't know the concept, so not until they become familiar with the topic.*

Teacher B: *When you look at question and answers that is the only way to know whether they have prerequisite knowledge about the topic. If they have no knowledge that is when you can know again where to start and it gives a hint on how you begin a lesson, whether you just go straight to learner centered or first discuss with the learners.*

Teacher C: *This is because learners are engaged in a lesson and to know what learners know and what they don't know.*

The findings from the three teachers observed were that, teacher A was using lecture method when introducing the lessons and this was in line with his responses from the interview. The response from teacher B concerning the teaching method when introducing the lessons, is different from what he was observed in the actual teaching in the classroom. He was using lecture method when teaching although he responded that the method supposed was to be questions and answers. The response from teacher C also contradict with what he was observed in the classroom concerning the teaching method to use when introducing the lesson, he was also using lecture method compared with what he responded to that the method supposed was to be class discussion.

The example below shows the way one of the teachers were using teacher-centered and the way they were introducing the lessons when teaching secondary school Calculus to the learners.

Teacher A: *good morning pupils?*

Pupils: *good morning sir.*

Teacher A: *today we are looking at integration*

Teacher A: $when\ you\ are\ given\ y = ax^n \qquad then \quad \frac{dy}{dx} = nax^{n-1}$

$$\int ax^n dx \qquad = \frac{ax^{n-1}}{n+1}$$

Teacher A; *On the part of differentiation the power is being subtracted by 1. But under integration we add the power and the power is what you divide there, now because we are integrating here and we are dealing with the combination of one or more thing, we are putting them together we also then introduce the constant c, so for every integration c is always added representing the constant.*

Teacher A: *Note, under integration we have two types of integration; we have definite and indefinite integral.*

Teacher A: *What do you understand the meaning of indefinite?*

Pupils: *silent*

Teacher A: *Indefinite is something which has no end meaning that there is no specific value under indefinite integral meaning that when you integrate the given function there is no exactly value because they are no limit, it is limitless. That's what we are going to start looking at indefinite integrals.*

As shown from the conversation above, what was happening in the classroom when teacher A was being observed, it is clear that learners were not told the outcomes of his lesson when introducing the topic. These results also shows that teacher A was unproductive in the use of learner-centered teaching method when teaching Calculus to the learners. The use of teacher-centered and the introduction of the topic to the learners when teaching a lesson was one of the teaching strategies used to all the three sampled teachers of mathematics as shown from the excerpt above.

Therefore, the findings have clearly shown that all the three teachers observed were using teacher-centered when introducing the topic and the reasons have been stated above. The findings from all the three teachers observed have also clearly shown that the introduction of lessons were not good whereby learners were not told the intended outcome of the lesson. For this reason, it has been shown that teachers lacked the knowledge on how lessons should be introduced to the learners especially on this important topic Calculus.

Lesson progression

The interviewees gave different views concerning the teaching method to be used in the process of teaching differentiation and integration as stated below.

Interviewer: *what teaching method would you use in the process of teaching differentiation and integration?*

Teacher A: *learner-centered*

Teacher B: *discussion and learner-centered*

Teacher C: *class discussion and explanations*

Interviewer: *explain why you have adopted such methods?*

Teacher A: *on the part of learner centered this is where you involve the learners as well because calculus involves the concept of coordinate geometry where we used to find gradient, so that when they have that knowledge of how to find the gradient you can be able to engage them.*

Teacher B: *in order to know whether learners have some pre-requisite knowledge.*

Teacher C: *learners are engaged.*

Nevertheless, the responses from the interviewees concerning the teaching method they use when teaching Calculus, their responses contradict with the method which they were using in the classroom when they were being observed by the researcher. They neither used learner-centered nor class discussion throughout the lesson from the beginning up to the end of the lesson instead they were using teacher-centered. The study reviewed that all the sampled teachers of mathematics were using teacher-centered approach in the teaching of differentiation and integration. For instance, the use of teacher-centered approach in the process of teaching can further be illustrated in the excerpt below from one the teachers observed in all the lessons.

Topic: Differentiating by using first principle

Teacher A: *Good morning pupils?*

Pupils: *Good morning sir.*

Teacher A: *How are you?*

Pupils: *We are fine and how are you?*

Teacher A: *I am fine.*

Teacher A: Today *we are going to monitor the difference which is between two points; they went a particle of something which is changing from one point to another, so meaning that there is either an increase or decrease. So in other words we are just talking at a change which is taking place between two things. So now we want to determine the change which is taking place from point P to point Q. so in this case we have one series in the X-axis from point P to Q. The change to determine the gradient between two points will then look at the change which is taking place between in the Y-axis and in the X-axis. So this is the reason why this triangle has been drawn there. So this triangle is showing the change in Y and also the change in X. This is what it is showing.*

Teacher A: *So in this case P should have a point or the coordinates, we should have the coordinates for P and also the coordinates for Q. So using the information that we have, we say the coordinates for P in this case where P is, we are going to have P(X, X^2) as a coordinates for P because what we have in this case is, there was a change from this point to that point, so meaning that there was an increase. This particle was increasing from P going to Q. this is why this increase has been shown indicated by this letter h, meaning that in the Y-axis moving from here to there you have the change which is what h. so since there is an increase then that increase indicated that by the addition of this h. so to determining the new point for Q, this new point for Q is just going to be Q[x^2, $(x+h)^2$].*

So that taking the formula for the gradient, the gradient between two points we say $m = \frac{QR}{PR}$ *so that we have the change in y so here it was indicated the change in x. So QR which is in y-axis then PR which is in the x-axis.*

Teacher A: *So from here we have QR which is the change in Y over change in X, now we know the actual formula not so?*

Pupils: *Yes*

Teacher A: *We know the actual formula which is given by what* $\frac{y_2-y_1}{x_{2-x_1}}$. *This one is suppose to be what x+h from here we point Q the* x_2 *because x is also increasing in the X-axis there is also a change from here x to x+h which simply means that h is representing the different which is in there. So there is an increase in the X-axis are together?*

Pupils: yes.

Teacher A: *thus why there is an addition of h, in this case, let us put it in this way, you want to know the new points there, there is an increase of h from here to there, then a new point is just going to be what?*

Pupils: *(x+h).*

Teacher A: *it is the same as you have the X-axis you have this point which is 2 let say this X and the length between 2 and x is 5, so the new point will be what?*

Pupils 7.

Teacher A: *how are you getting that 7?*

Pupils: *by adding 2 and 5.*

Teacher A: *Because from where 2 is you add another 5 then you come to 7. So in another words it is 2 + this change which is the increase. So this is the same thing which is happening here. So equally in the Y-axis the same thing is happening, why, we begin to wonder here, what is here is y value is there ngatwatitumone kabili ama (if we can see then the) points they meet. So this is the function of the curve for y so y is also changing ok, it is also going to change from there to there ok, so in this case, this x to get now the second point there it is the same as you get this (x+h) which is the increase and substitute it where x is to get the second point there, so it is going to be* $(x+h)^2$*. So now we use that now to substitute in this formula.*

The teacher as he was solving on the board he wrote; $M = \frac{QR}{PR} = \frac{\Delta Y}{\Delta X} = \frac{y_2 - y_1}{x_{2-}\, x_1}$

$$= \frac{(x+h)^2 - x^2}{(x+h) - x} = 2x + h$$

Notation

If the equation of the curve is y = fx. The gradient function will be f'(x). Taking point A on y=f(x) the coordinates of A is [x, f(x)] and the coordinates of B is [x+h, f(x+h)].

Now we can look at this example using the first principle to differentiate this function $y = 3x^2$*. So we have this function as* $3x^2$*. Now what you should know is the point that you are supposed to have here. Now using* $\frac{dy}{dx} = \frac{lim\, f(x+h) - f(x)}{h \to 0 \qquad h}$

So this y is representing f(x+h). What it means is that where there is x we replace with (x+h). after solving the answer was 6x.

Teacher: Any question on how to differentiate using first principle.

Pupil: where did 6 came from?

Teacher: this is because hx + hx =2hx and now when we come to multiply with 3 it will be 6hx.

He also gave another example that if $y = 2 - x^2$*. Find* $\frac{dy}{dx}$ *by using first principle.*

Solution

So if you want you can even say f(x+h) = 2- x^2 then in this where there is x you replace where there is x with x+h. therefore, =2-$(x+h)^2$. So expanding this one so we have f(x+h) = 2-(x^2+2hx+h^2). He continued solving until he reached the final answer $\frac{dy}{dx} = -2x$. **(Lesson observation, 2018)**

4.6.3 Teaching methods

Findings from all the three teachers observed in all the lessons have shown that teachers were using teacher-centered approach when teaching the learners throughout the lesson instead of learner-centered approach except one lesson from one of the teachers who used learner-centered approach although he used it wrongly as shown from figure 4.3 about the lesson which was taught to the learners.

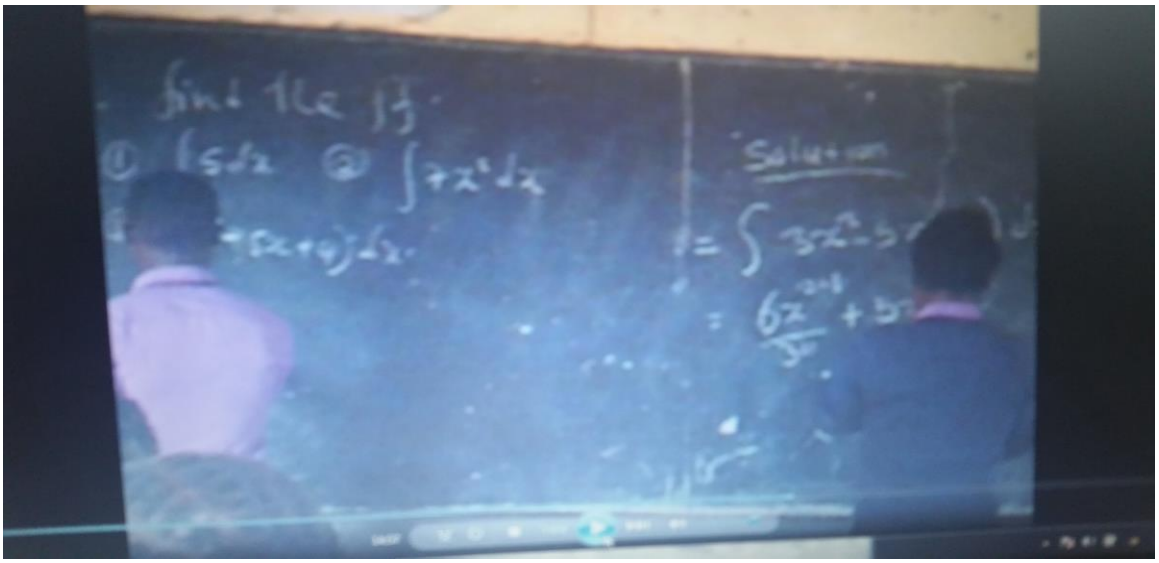


Figure 4.23 shows the use of learner-centered approach wrongly by Teacher B

The picture above shows the way teacher B was using learner centered teaching method wrongly whereby two pupils allowed to solve on the board at the same time with different questions. **(Lesson observation, 2018)**

One of the lesson below shows the way teacher B used learner-centered approach wrongly by allowing 2 pupils as shown from figure 4.3 to solve the question on the board at the same time.

Topic: Integration.

Teacher B: *Good morning pupils?*

Pupils: *Good morning sir.*

Teacher B: *How are you?*

Pupils: *We are fine and how are you?*

Teacher B: *I am fine.*

Teacher B: *When you are given* $\int kdx$ *it means you differentiate with respect to x or I mean you integrate with respect to x, now when you are integrating they are only two rules. So rule number 1. When you are integrating a constant the answer you get is that constant times x + what C. So when you are integrating* $\int kdx = kx + c$ *for example* $\int 6dx = 6x + c$ *ninshi wacita (then you have done) integrate.*

Then rule number 2 which is talking about a variable which has the power 1, power 2, power 3. When you are integrating it is the opposite of differentiation, so in this case we are going to have

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad \text{that is integration. for example } \int 5x^2 dx \text{ so we are going to have}$$

$\frac{5x^{2+1}}{2+1} + c = \frac{5x^3}{3} + c$ *integration emobalelosha (that is what they mean). Then there are cases in situation when you have this* $\int x^3 + 2x^2 + 4dx$ *how do we integrate that one, so even in that case we integrate each, each,* $= \frac{x^{3+1}}{3+1} + \frac{2x^{2+1}}{2+1} + 4x + c = \frac{x^4}{4} + \frac{2x^3}{3} + 4x + c$ *. When you are integrating the constant just multiply with x.*

Teacher B: *Anyone to go and solve question 1*

Pupil: *Let me solve number 2*

Teacher B: *uwakweshako (someone to try) number 1. 2 pupils were solving on the board at the same time silently without explaining.*

Teacher B: *if you want them to be easy for you kucitafye (you just have to) understand utumalutunga, tubili (the two rules). Rule number 1, whenever you are integrating the constant, multiply that constant with x ninshiwacita (then you have) integrate you are done. Then whenever you are integrating variables with powers for example, X^2 so in that case you add 1 with that power then you also divide panshi apa (down here).*

$$\int 3x^2 - 5x + 4\, dx$$

$$= \frac{3x^{2+1}}{2+1} - \frac{5x^{1+1}}{1+1} + 4x + c \; = \; \frac{3x^3}{3} - \frac{5x^2}{2} + 4x + c \quad = \; x^3 - \frac{5}{2}x^2 + 4x + c$$

You end there

Teacher B: *When we come now to the definite one where we find the actual value, this is why we introduce the upper limit and the lower limit like what we have done there.* ***(Lesson observation, 2018).***

The table below shows the summery of responses from the three teachers when they were asked by the researcher the kind of methods they would use when introducing differentiation and integration and in the process of teaching? (**Semi-structured interview, 2018)**

Table 4.2 Teaching strategies in the teaching of secondary school Calculus

THE PARTICIPANTS	THEIR RESPONSES
TEACHER A	Lecture method and learner centered
TEACHER B	Question and answer, discussions and learner centered
TEACHER C	Class discussion and explanations

Table 4.2 showed that all the three teachers observed were using teacher-centred when teaching instead of learner-centered approach from the introduction and also in the process of teaching.

Lesson Evaluation and Conclusion

All the lessons observed were not evaluated to find out if the learners had attained the lesson objectives. In all the lessons that were observed the teachers did not summarize the main points of the lesson. The lessons ended abruptly without conclusion.

4.6.4 Teaching and Learning Aids

All the teachers that were observed did not use any kind of teaching aids apart from the chalk board and the text book. It was surprising to observe that teachers did not use even the charts especially when teaching the learners on finding the equation of the tangent and equation of the normal so that learners could see the difference between the line of tangent and the normal.

In conclusion, the tame below shows the summary from the findings which were answering research question 2 concerning teaching strategies that teachers used when teaching Calculus in the selected senior secondary schools of Chinsali district.

Table 4.3 shows the summary of responses from the sampled teachers of mathematics.

Themes (Teaching strategies)	The Findings from the sampled teachers of mathematics
Lesson Planning	All the sampled teachers observed did not prepare any lesson plan instead they relied on text books.
Lesson introduction and progression	The introduction and the progression of the lessons observed were throughout teacher-centred approach.
Teaching methods	Teaching methods were teacher-centered such as lecture method instead of learner-centered such as discussion.
Lesson Evaluation and Conclusion	In all the lessons observed were not evaluated and teachers did not summarise the main points of the lesson.
Teaching and Learning aids	All the sampled teachers observed did not use any teaching aid apart from chalk board and the text book.

Table 4.3 shows that teaching strategies used by sampled teachers of mathematics was not in accordance with the curriculum which emphasise the use of constructivism in the teaching of mathematics and Calculus in particular at senior secondary school level.

4.7 Mathematical challenges in the teaching of Calculus

In order to investigate the mathematical challenges that teachers of mathematics encountered when teaching Calculus to secondary school learners, semi-structured interviews as well as lesson observations were used to analyse data collected from the sampled teachers of mathematics. The data presented was also in line with the conceptual framework as stated below.

4.7.1 Capacity to explain concepts on secondary school Calculus to the learners

Semi-structured interviews which were administered to the sampled teachers of mathematics intended to provide answers concerning the capacity of teachers to explain concepts found in Calculus as shown below.

Interviewer: *What are some of the mathematical challenges do you face when teaching Calculus?*

Teacher A: *They are certain concepts which are straight forward but it is difficult to explain to the learners such as differentiating by using chain rule, product rule and quotient rule. In addition, in some books there is shallow knowledge, where it becomes difficult to deliver*.

Teacher B: *There are certain areas which needs time, so lack of enough time. Some books are not accurate, so it become difficult to get the concepts and deliver to the learners for example, pure and applied mathematics is difficult to teach. Being a new topic it is difficult to deliver to the learners.*

Teacher C: *For learners to accept calculus for them, they think that it is a difficult topic, so it takes time for them to get engaged into learning. The topic being new in the syllabus they need a lot of exposure.*

Furthermore, the sampled teachers of mathematics who were interviewed gave various difficulties that they face when teaching Calculus to the learners for example, teacher C who has been teaching for 10 years responded that: *this topic Calculus because of being new in the mathematics syllabus, it is difficult to find suitable teaching and learning aids to use in order to help the learners to comprehend. It is also difficult to improvise.*

Teacher B also found it difficult to improvise appropriate teaching and learning aids for teaching Calculus on their own for this particular topic. He said that, *"improvising teaching and learning aids requires being creative and resourceful but due to lack of enough time it makes it difficult for us to find time to improvise teaching and learning aids."* **(Teacher B).**

Furthermore, teacher B cited that: *this topic Calculus it's wide and time allocated is not enough for it to be taught well*. He also complained that*: mostly Calculus is schemed to be taught in Grade 12 in the third term when most of learners have their minds in panic state. Hence, learners' concentration towards learning is reduced.*

4.7.2 Capacity to address learners' independent questions

In order to determine teachers' ability to address learners' independent questions, the sampled teachers of mathematics were observed. The findings were that teachers of mathematics lacked the capacity to address learners, independent questions especially on finding the equation of the normal as shown from this example below;

Topic: Finding the equation of the normal.

Teacher B: *Good morning pupils?*

Pupils: *Good morning sir.*

Teacher B: *How are you?*

Pupils: *We are fine and how are you?*

Teacher B: *I am fine.*

Teacher B: *When you are given the equation of a curve is* $y = x^2 - 3x - 4$. *Find the gradient of the normal.*

Teacher B: *Ok now, lets us come to this one, find the gradient of the normal.*

To find the gradient of the normal is to multiply with the reciprocal. When you find that number you multiply with negative.

Pupils: Nomba basa natusanga (now sir have we found) gradient?

Teacher B: gradient twacisangaiyi (this is the one we have found), kwisabulafye iyi (you just have to get this) number times negative.

Tukesapwisha pa (we will finish on)Monday. (Lesson observation, 2018)

Researcher: next Monday which he said we will continue, he never even taught the pupils on how to find the gradient of the normal and the equation of the normal, he just went on another topic which was integration.

4.7.3 Capacity to pose appropriate questions on the topic

This category of analysis assessed teachers' capacity to pose appropriate questions on the topic when teaching. In order to provide answers on this category sampled teachers were observed. During lessons, the response was that all the sampled teachers did not give room to the learners to ask question because their teaching was teacher- centred. In addition, most of the questions teachers asked were low order questions which were not effective in making learners to critically think about the topic at hand. For example, instead of learners being asked to state the symbol for integration, one of the sampled teachers asked the learners as shown below;

Teacher C: this integration $\int dx$ ici cisimbo ici mwalicimonapo ai (this symbol have you ever seen it)?

Pupils: tatwacimonapo (we have never seen it).

Teacher C: this symbol means integration. **(Lesson observation, 2018)**

Failure to pose appropriate questions to learners by teachers, made the learners not to fully participate in the lessons. This was as a result of lack of unpreparedness of lessons by teachers.

Table 4.4 shows the summary of responses for research question 3 from the sampled teachers of mathematics on mathematical challenges that they face when teaching calculus.

Mathematical challenges	**Teachers' responses on challenges that they face**
Capacity to explain concepts found in Calculus to the learners	They are certain concepts which are straight forward but it is difficult to explain to the learners such as differentiating by using chain rule, product rule and quotient rule. In addition, in some books there is shallow knowledge, where it become difficult to deliver.
Capacity to address learners' independent questions	The findings were that teachers of mathematics lacked the capacity to address learners, independent questions especially on finding the equation of the normal. This study also established that teachers of mathematics had difficulties when solving Calculus questions to the learners that involve differentiation and integration.
Capacity to pose appropriate questions on the topic	The findings were that questions that teachers asked were low order questions which were not effective in making learners to critically think about the topic at hand. The study also reviewed that due to lack enough time during lessons, teachers were unable to pose questions to the learners frequently in the classroom.

4.8 Summary

The research findings based on three research questions had indicated that all the sampled teachers of mathematics lacked subject matter knowledge in the area of secondary school Calculus and also in the use of teaching strategies in the teaching of Calculus. It was also discovered they had various mathematical challenges in the area of secondary school Calculus.

Table 4.5 below shows the summary of findings from all the three research questions.

Research Questions	Findings from the sampled teachers of mathematics
Do teachers of mathematics have comprehensive understanding of subject matter knowledge in the area of secondary school Calculus?	This study reviewed that, from all the sampled teachers of mathematics, only one teacher was knowledgeable in the area of secondary school Calculus whilst other two teachers lacked the understanding of secondary school Calculus.
What teaching strategies employed by teachers when teaching Calculus at secondary school level?	This study reviewed that the sampled teachers were not preparing lessons plans instead they relied on text books, they were using teacher-centered approach when teaching instead of learner-centered and they did not use or improvise any teaching aid when teaching Calculus to the learners.
What mathematical challenges faced by teachers when teaching Calculus to the learners?	The mathematical challenges were; failure to explain Calculus concepts to the learners, failure to address learners' independent questions and failure to pose appropriate questions to the learners.

The researcher in the next chapter will discuss the research findings that have been presented in this chapter.

CHAPTER FIVE: DISCUSSION

5.1 Overview

The researcher in the previous chapter presented the findings of the study which investigated an assessment of the teaching of Calculus by three selected senior secondary school teachers of mathematics in Chinsali district. In accordance with the use of related literature presented in chapter two, research objectives and the theoretical framework or conceptual framework, the researcher in this chapter will venture into discussing the research findings. As earlier indicated in chapter one, this study adopted the social constructivism theory of learning and teaching as theoretical approaches.

5.2 Subject matter knowledge of teachers of mathematics in the area of secondary school Calculus.

In order to discuss the research objective stated above, the data presented was discussed according to the conceptual framework as indicated below.

5.2.1 Solving Calculus problems involving differentiation and integration

This study recognized that, from all the sampled teachers of mathematics, only one teacher was knowledgeable in the area of secondary school Calculus whilst other two teachers lacked the understanding of secondary school Calculus. These findings contradict with Shulman (1987) who revealed that Mathematical Knowledge for Teaching (MKT) cannot just be effective on its own but require the accompaniment of both the content and pedagogical knowledge for a teacher of mathematics to teach effectively.

In order to determine teachers' subject matter knowledge in the area of secondary school Calculus, the questionnaire was used and it was analysed question by question. The responses were that, two teachers of mathematics from the sampled teachers failed to solve some concept found in calculus such as finding the equation of the tangents and normal, the use of chain rule, product rule, quotient rule, finding the area under the curve and finding the equations of the curves.

Finding the equation of the Tangents and Normals

This study established that only one teacher was able to find the equation of the tangent and normal while the other two teachers from the sampled teachers were failed find the equation of the tangent and normal from the given equation of the curve $y = x^3 - \frac{5}{2}x^2 + 2x$. This question was assessed in order to find out whether teachers of mathematics were knowledgeable in this area. Shulman (1986)

noted that mathematics include knowledge of concepts, theories, ideas, organisational frameworks, knowledge of evidence and proof as well as established practices and approaches toward developing such knowledge. In regard to the statement above, the finding from this study contradict with that of Shulman due to the fact that some teachers lacked knowledge on concepts found in Calculus.

The use of the Chain rule, Product rule and Quotient rule

The sampled teachers were given to differentiate the equation of the curve $y = \sqrt{9x - 3}$ by using chain rule. This question assessed the sampled teachers of mathematics weather they are knowledgeable in this area and it was reviewed that only Teacher B failed to differentiate completely while Teacher A and Teacher C were able to differentiate this question which shows that they understood the concept in this area.

In order to determine teachers' in-depth understanding of the subject matter in this area, the sampled teachers were give the question from the questionnaire to find the gradient of the curve $y = x^2(x - 4)^3$ at the point (1, 0). This question required the teachers to differentiate by using product rule and thereafter to find the gradient of the tangent. Thus, this question assessed the teachers' ability whether they are knowledgeable in differentiating this question by using product rule. The findings from this question shows that only one teacher from the sampled teachers of mathematics was knowledgeable in this area whilst other two teachers from the sampled teachers failed to solve this question completely. From these results, it can be argued that teachers performed poorly on grounds that they were not able to differentiate this question by using product rule.

Additionally, Teacher B and Teacher C failed to differentiate the equation of the curve $y = \frac{5x-3}{x+2}$ by using quotient rule while only one Teacher was able to differentiate as indicated in Figure 4.13 and Figure 4.14 shows that teachers lacked subject matter knowledge in the area of secondary school Calculus. The results also shows that only one teacher from the sampled teachers was knowledgeable in this area. These findings are in harmony with Pfund and Duit (2000) and the National Research Council (2000), as it is shown from the literature review.

Finding the area under the curve

Teachers of mathematics, who were sampled in this study, were given to find the area bounded by the curve $y = 3x^3 - 2x - 5$ and the x-axis. It was established that two teachers failed to find the area under the curve meaning that they were not familiar in this area of the topic while one teacher managed to find the area under the curve.

Finding the equation of the curve

This study established that all the sampled teachers of mathematics lacked in-depth understanding on the subject matter especially in this area. This is because they had difficulties when integrating $12x^3 - 8x$, as a result all the sampled teachers failed to find the equation of the curve. These findings contradict with Khakbaz (2015), who confirmed that it is necessary to go beyond the subject matter knowledge in order to examine how teachers interprets the subject matter for example on this topic Calculus and how this is linked to their role in facilitating learning in the classroom.

5.2.2 Definition of Calculus

Teachers' understanding of the definition of the term Calculus

Concerning the definition of Calculus, this study reviewed that, only one teacher from the sampled teachers of mathematics defined Calculus whilst other teachers were unsuccessful. Failure to define Calculus by some teachers shows that they lacked the understanding of subject matter knowledge in the area of secondary school Calculus. This finding is similar to Tol (2009) who discovered that teachers did not have good understanding of calculus concepts.

5.2.3 The understanding of two main Calculus concepts such as differentiation and integration

The description of the two main concepts that are found in Calculus

This study reviewed that among the three teachers of mathematics who were interviewed teacher B and teacher C failed to describe the two main concepts that are found in Calculus whilst teacher A is the only one who was able to describe the two main concepts that are found in Calculus such as differentiation and integration as it is presented in the findings. The findings contradict with Thompson (1984) and Ernest (1988) based on their wider experience of working in the field of mathematics education, they claimed that any attempt in improving the quality of mathematics teaching and learning must begin with an understanding of the conceptions held by teachers.

Moreover, effective teaching of Calculus requires the understanding of knowledge of teachers in order for them to have a better understanding when teaching to the learners. According to content based theory, In relation to the theoretical framework, the findings of this study has shown that sampled teachers of mathematics lacked the understanding of subject matter knowledge in the area of secondary school Calculus. These findings contradict with content based theory as stated in chapter one which resulted for failure from sampled teachers of mathematics to fully deliver the subject matter knowledge of Calculus to the learners.

5.3 Teaching strategies employed by the three teachers of mathematics when teaching Calculus at secondary school level

The findings from this study were discussed according to the conceptual framework as shown below.

5.3.1 Lesson planning

The study established that all the three teachers observed did not prepare lesson plans instead they relied on text books when teaching. This study collaborated with Chifwa (2015) who established that fifty percent (50%) of the teachers observed had prepared lesson plans and fifty percent had not done so.

Lesson planning is the hallmark of any teaching and learning as it prepares the teacher for lesson delivery. From the findings of this study, it can be seen that all the sampled teachers went to the classes relatively unprepared. Failure to plan or inadequate planning for a lesson entailed that a lot of learning time was wasted during the lessons and the delivery of content was compromised, to the disadvantage of the learners (Jones, 1998).

This failure to plan led to unsystematic and ineffective lesson presentation because the teachers did not set clear objectives for each lesson. This lack of planning also led to teachers introducing the lessons using question and answer only which did very little to arouse the interests of the learners. As a result, learners did not participate fully in the lessons. Moreover, the teacher's unpreparedness impacted negatively on the confidence of teachers during lesson presentation (Petty, 2009). The findings of this study especially in this area whereby teachers, because of being unpreparedness for lessons has made the education system to lack behind in the improvement of performance of pupils especially in senior secondary schools of Chinsali district.

5.3.2 Teaching

Lesson Introduction

The findings from the three teachers observed were that, teacher A was using lecture method when introducing the lessons and this was in line with his responses from the interview. The response from teacher B concerning the teaching method to use when introducing a lesson is different from what he did when he was observed in the actual teaching in the classroom. He was using lecture method when teaching although he responded that the method is supposed to be questions and answers. The response from teacher C also contradict with what he was observed in the classroom concerning the teaching method to use when introducing the lesson, he was also using lecture method compared with what he responded that the method supposed to be class discussion.

Lesson progression

This study established that all the sampled teachers of mathematics were still using lecture method when teaching in the lesson progression. Nevertheless, the responses from the interviewees concerning the teaching method they were using when teaching Calculus, their responses contradict with the method which they were using in the classroom when they were observed by the researcher. They neither used learner-centered nor class discussion throughout the lesson from the beginning up to the end of the lesson instead they were using teacher-centered. These findings are similar to Chifwa, (2015) as presented in the literature review.

In addition, Teacher C anticipated that, Calculus requires more attention to the learners because of being new in the syllabus. Therefore, the teaching of Calculus requires the use Zone of Proximal Development (ZPD) as proposed by Vygotsky who conceives of the Zone of Proximal Development as central to instructional enhancement and classroom change in Mathematics. The findings of this study shows that teachers were not competent enough in the use of learner-centered teaching due to lack of understanding of subject matter knowledge of Calculus. As a result these findings contradict with the theory of social constructivism as proposed by Vygotsky (1978).

5.3.3 Teaching methods

The study established that all the three teachers observed were still using teacher centered methods in the teaching of calculus. The responses from the interviewees concerning the teaching method they used when teaching Calculus, contradicted with the methods they were using in the classroom when they were being observed by the researcher. They neither used learner-centered nor class discussion throughout the lesson from the beginning up to the end of the lessons but rather made extensive use of teacher-centred methods. This is similar to Chifwa, (2015) who argued that the teaching of genetics requires that teachers of biology are eclectic in their methods. They did not do practical work. Lack of questions from the learners could point to the fact that the teaching failed to capture the interests of the learners, which spilled over to the other parts of the lessons.

Contrary to the findings of this study, Nkoya (2006) reported that, teachers used a variety of teaching methods, where on average, teachers used at least three learner centered methods such as group work (practical), demonstrations, discussions, project, problem solving, question and answer in their teaching. The implications of the lack of variety of teaching methods are that teachers did not cater for individual learning differences of the learners. It is common knowledge that lack of differentiated learning results in monotony in lesson delivery which in turn adversely impacts on learner

concentration and interest. In the current study, this was evident by the lack of questions from the learners in almost all the lessons observed.

Consequently, figure 4.23 shows the way teacher B was using learner centered teaching method wrongly whereby two pupils were allowed to solve on the board at the same time with different questions. These findings shows that teachers of mathematics were not competent in the use of learner-centered approach. The responses from the three teachers interviewed indicated that the teaching of Calculus could be improved by using learner centered methods of teaching as it is in agreement with the constructivist thought. Learners who are actively involved in a lesson construct their own knowledge. Therefore, teachers should facilitate this construction of knowledge by providing enabling environments for knowledge construction.

Lesson Evaluation and Conclusion

All the lessons observed were not evaluated to find out if the learners had attained the lesson objectives. It was discovered that teachers did not evaluate learners as a result there was no feedback from the learners. This could explain the poor performance in Calculus. It was difficult to tell whether the lesson objectives were achieved or not. Evaluating a lesson provides feedback on the effectiveness of instruction and gives students a measure of their progress (Muzumara, 2008).

Teaching and Learning aids

Additionally, all the three teachers who were observed during this study did not use any teaching aids, there was ineffective teaching because learners found it difficult to remember everything that the teacher talked about. The non-use of teaching and learning aids made it difficult for learners to understand concepts and ideas especially for an abstract topic like Calculus. Teachers could have used charts when teaching Calculus especially on finding the equation of the tangent and the equation of the normal in order for them to see the difference between the equation of the tangent and the normal. This finding is similar to Chifwa, (2015).

5.4 Mathematical challenges faced by the three teachers of mathematics as they taught Calculus.

The data presented was also discussed in line with the conceptual framework as stated below.

5.4.1 Capacity to explain concepts on secondary school Calculus to the learners

This study further established that all the sampled teachers of mathematics were failing to explain Calculus concepts to the learners, for example, one of the sampled teachers during interviews said that, there were certain concepts which were straight forward but it was difficult to explain to the

learners such as differentiating by using chain rule, product rule and quotient rule. In addition, in some books there was shallow knowledge, where it became difficult to teach the learners.

Besides, teacher C who has been teaching for 10 years responded that, as teachers have challenges in finding suitable teaching and learning aids to use in order to help the learners comprehend Calculus as a topic. He further added that it was also difficult to improvise teaching materials when teaching Calculus as indicated in chapter 4.7.1. For this reason, these challenges faced by teachers of mathematics need to be dealt with in order to improve the performance of learners in mathematics especially in the area of secondary school Calculus. Therefore, the incapability of teachers to explain calculus concepts when teaching may result to unsatisfactory pupil's performance on this important topic. These mathematical challenges that teachers face requires serious attention to teachers in order to improve the teaching of this important topic Calculus by helping them on how to handle this topic Calculus in various secondary schools.

5.4.2 Capacity to address learners' independent questions

In order to determine teachers' ability to address learners' independent questions, the sampled teachers of mathematics were observed. The findings were that teachers of mathematics did not address learners, independent questions especially when finding the equation of the normal. This study also established that teachers of mathematics had difficulties when solving Calculus questions to the learners that involve differentiation and integration. Some of the difficulties faced by teachers of mathematics for instance included the inability by teachers to find the equation of normal and also in finding the area bounded by the curve. Failure to find the equation of the normal and the area bounded by the curve by sampled teachers lead to the incapability to address learners' independent questions. This coincided with Axtell (2006) who argued that teachers seemed to focus more on procedures than understanding of the concepts among the students in teaching calculus. However, Swangberg (2016) stated that it was uncalled for, for a teacher to fail to deliver in class. A teacher is supposed to possess high knowledge content in the topics as this will instill confidence in the learners.

5.4.3 Capacity to pose appropriate questions on the topic

This category of analysis assessed teachers' capacity to pose appropriate questions on the topic when teaching. In order to provide answers on this category sampled teachers were observed. During lessons, the response was that all the sampled teachers did not give room to the learners to ask question because their teaching was teacher-centered. In addition, most of the questions teachers asked were low order questions which were not effective in making learners to critically think about the topic at hand. The study also reviewed that due to lack enough time during lessons, teachers were unable to

pose questions to the learners frequently in the classroom. Therefore, lack of enough time to complete the whole sub topics in a specific period of time and failure to deliver certain calculus concepts to the learners, sampled teachers were unable to pose appropriate questions to the learners as observed by Changwe (2017).

Furthermore, all the sampled teachers of mathematics did not comply with social constructivism theory of learning and teaching as proposed by Vygotsky (1978) who placed great emphasis on social and linguistic influences on learning, and in particular on the role of the teacher in the educative process. This is because they did not have the capacity to explain concepts that are found in Calculus, they were unable to address learners' independent questions and also to pose appropriate question to the learners. Having used this theory in the context of the current investigation, the researcher is of the view that this could work towards addressing the issue of mathematical challenges faced by teachers when teaching secondary school Calculus to the learners.

5.5. Summary

All in all, this study revealed that some teachers have no reading culture, always using old traditional teaching methods which are not applicable in the 21st century, which may result to learner's unsatisfactory performance in mathematics. Additionally, the findings of this study reviewed that most of the teachers in senior secondary schools have neglected the issue of CPD meeting which may help them to increase their SMK in some of the challenging topics such as Calculus. Besides, in Zambian context, mathematics senior secondary school syllabus encourages a learner-centered approach in the teaching of mathematics according to what has been enshrined in the revised mathematics senior secondary school syllabus (Ministry of education, 2013). The findings of this study have shown that most teachers do not know how to apply social constructivism in the teaching of Calculus. The next chapter, present the summary of findings, recommendations.

CHAPTER SIX: CONCLUSION AND RECOMMENDATIONS

6.1. Overview

The researcher in chapter four presented the research findings which were later discussed in chapter five to ensure that the three research questions raised in chapter one are addressed. In this chapter, the researcher presents the summary of the main research findings which answered the research questions and finally recommendations are given. Since the major key player in the matters of teaching mathematics is a teacher, and in order to enhance effective teaching and learning in Secondary Schools, the researcher has suggested a number of areas that may require further research to enhance quality in the teaching of Calculus.

6.2 Conclusion

In conclusion, being the final chapter of this study, it is important to note that the study looked at an assessment of the teaching of Calculus by three selected Senior Secondary School teachers of mathematics in Chinsali district, Zambia. The researcher wanted to establish if teachers of mathematics in the senior secondary schools were competent in the area of secondary school Calculus. The researcher also wanted to explore the teaching strategies in the teaching of Calculus and the mathematical challenges that teachers of mathematics face as they teach Calculus. The researcher in the sub-sections below presents the main findings as mirrored by the research questions.

6.2.1. The subject matter knowledge of the three teachers of mathematics in the area of Secondary School Calculus.

This study concluded that all the three teachers lacked the understanding of subject matter knowledge in calculus and this contributed to unsatisfactory teaching of calculus. This was proved during questionnaire, semi-structured interviews and lesson observations where teachers were discovered to be unable to teach the learners on differentiating by using chain rule, product rule and quotient rule. This study also reviewed that the two teachers of mathematics from the sampled teachers failed to define the term Calculus and they failed to explain the two main concepts that are found in Calculus such as differentiation and integration.

6.2.2. Teaching strategies employed by teachers of mathematics when teaching Calculus

This study revealed that all the three teachers observed did not prepare lesson plans and they were in the habit of using teacher-centered methods from the beginning up to the end of the lesson, instead of using learner centered teaching methods. The findings of this study also revealed that all the three

teachers observed were not using lesson plans to teach instead, they relied on mathematics text books. For this reason, it can be concluded that poor or lack of planning contributed to ineffective lesson presentation because it made the lesson presentations unsystematic. It was further established that teachers did not use any teaching and learning aids apart from the chalkboard and the text books. This also resulted in ineffective teaching of Calculus.

6.2.3. Mathematical challenges faced by teachers of mathematics as they taught Calculus

This study established that teachers had challenges in the teaching of calculus; among others were failure to teach Calculus effectively to the learners and lack of sufficient time to complete the sub topics in calculus at a specific period of time. It was also discovered that some teachers had difficulties in delivering some Calculus concepts especially when finding the area under the curve. This study also revealed that learners tended to forget some Calculus concepts and also developed negative attitude towards this topic considering that it was quite new to them and they also considered it to be quite difficult and very challenging. In addition, what emerged from this study is that the three teachers of Mathematics had different perceptions in the way they understood calculus concepts.

This study makes an important contribution to various stake holders of which the researcher has brought to right over what is really happening in the teaching of Calculus by senior secondary school teachers of mathematics. The originality of the study is appropriate in that no such study has been conducted in Chinsali district of Muchinga province to assess the teaching of Calculus by senior secondary school teachers of mathematics and this study informed by content based theory and social constructivism theory.

6.3 Recommendations

The following recommendations arose from the study are:

i. This research recommends that, regular short in-service courses for teachers of mathematics should be conducted as well as (CPD) meetings in order to enhance subject matter knowledge of teachers in the teaching of calculus.
ii. This study further recommends that internal monitoring of lessons by Mathematics Heads of Department in schools should be intensified in order to ascertain that teachers were preparing lesson plans and ensure that teachers involved learners in the teaching discourse of calculus. .

iii. Additionally, this research recommends that differential calculus be taught in grade 11 and integral calculus in grade 12 in order to give learners time to learn the concepts involved adequately.

iv. By and large, head teachers should involve the teachers of mathematics in the procurement of the teaching and learning resources (including textbooks) since they usually understand the teaching materials that may be urgently required and the materials that would benefit the learners.

6.4. Suggestions for further Research

The following suggestions are made for further research:

This research recommends that further research be conducted in order to assess the misconceptions that learners face as they learn Calculus and also to investigate learner's perception and attitudes towards Calculus.

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APPENDICES

APPENDIX A: Questionnaire for Teachers

I am a post graduate student carrying out an academic research study where your participation is very much important for the success of this study. The study is entitled 'an assessment of the teaching of calculus: a case of three secondary school mathematics teachers in Chinsali district Zambia**.**' Therefore, you are requested to answer the questions according to your own understanding in the spaces provided. Be assured that the information that you will provide shall be treated with high level of confidentiality and will only be used for the purpose of this study.

INSTRUCTIONS

(i) You are **not** expected to indicate your name or the name of any other person in this test.

(ii) Calculators are not allowed to be used in this test.

(iii) Time allowed is 1 hour and total marks are 40.

(iv)There are 8 questions in this paper, answer all questions.

1. Given that the equation of a curve is y = $x^3 - \frac{5}{2}x^2 + 2x$. Find:

(a) the equation of the normal where x = 2,

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(b) the coordinates of the stationary points.

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Q 2. (a) Find the coordinates of the points on the curve y=$4x^3 - 6x^2 - 24x + 3$ where the gradient is zero.

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(b) Evaluate $\int_{-1}^{2}(4x^3 - 3x^2 + 2x)dx$

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3. (a) Differentiate $4x^2 + 2x$ by using the first principle.

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(b) Find the derivative of $y = \sqrt{9x - 3}$ with respect to x.

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4. (a) Find the gradient of the curve $y = x^2(x - 4)^3$ at the point (1,0).

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(b) Find the gradient of the normal $given\ that\ y = \frac{5x-3}{x+2}$ at the point (-1,2).

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5. (a) Given that $y = 4x^3 - \frac{2}{x^2}$, find $\frac{dy}{dx}$.

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(b) Integrate $\int \frac{6x^6-3x^3}{x}\ dx$

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6. (a) A curve is such that $\frac{dy}{dx} = 12x^3 - 8x$. Given that it passes through (2, -1), find its equation.

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(b) Find the area bounded the curve $y = 3x^2 - 2x - 5$ and the x-axis.

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APPENDIX B: Semi-structured interviews for teachers

Sir/Madam good morning/afternoon and welcome to this interview? I am a Masters Student at the University of Zambia and I am currently conducting a research titled:"**An assessment of the teaching of calculus: a case of three secondary school mathematics teachers in Chinsali District Zambia."**

May I also take this opportunity to appreciate you for accepting to be interviewed and may I also let you know that the responses that I am going to record from this interview shall be solely used for academic purposes. Furthermore, information collected will be treated with the highest level of confidentiality. Therefore, feel free to answer the questions and ask where you are not clear.

Background characteristics of the Participants

1. How long have you been teaching Calculus at secondary school level in mathematics?
2. What Qualification do you possess in mathematics?

The content knowledge of teachers of mathematics in the area of secondary school Calculus.

3. According to your own thinking, is this topic Calculus important to teach the learners in Secondary schools? Give reasons for your answer.
4. What do you understand by the term Calculus?
5. Describe the two main concepts that are found in Calculus.
6. The concepts that you have mentioned in question 3, how can they be applied to real life situation?
7. Is the level at which Calculus content and teaching methods are taught in Senior Secondary Schools appropriate to the learners?
8. State the formula for differentiation by using chain rule, product rule and quotient rule which you used from the test you wrote.
9. From the test which you wrote from question 6 (b), explain how you found the area under the curve.
10. From the test which you wrote explain on how you found the equation of the normal?
11. Explain how you solved question 6 (a), when finding the equation from the curve which was $\frac{dy}{dx} = 12x^3 - 8x$. Given that it passes through (2, -1).

The teaching strategies employed by teachers of mathematics when teaching Calculus at secondary school level.

12. Suggest which topics would you like to teach the learners before you introduce Calculus?
13. What kind of methods would you use when introducing differentiation?
14. What kind of methods would you use when introducing integration?
15. Explain why you have adopted such kind of methods?
16. How often do you assess the pupils in a week, such as giving homework and assignments to them?
17. What strategies have you come up with in order to help the learners who are not doing well?
18. What teaching aids do you think can be used to teach differentiation effectively?
19. What teaching aids do you think can be used to teach Integration effectively?

The mathematical challenges faced by teachers of mathematics as they taught Calculus.

20. What are some of the challenges do you face when teaching calculus at Secondary school level?
21. Are there any sub topics in calculus that you face difficulties when teaching?
22. If yes, what could be some of the reasons why you face such difficulties when teaching such sub topics?
23. Suggest how these difficulties (named in Q.20) can be overcome?

APPENDIX C: Lesson observation schedule for three teachers in selected secondary schools.

OBSERVER: …… SCHOOL: …….

TOPIC: ……. DATE: ……..

SUB TOPIC ……. DURATION: ……

CLASS …… NO. OF LEARNERS: GIRLS …….. BOYS …… TOTAL ……

RATINGS: 1=Unacceptable, 2= Satisfactory, 3= Good 4= Very Good.

AREA OF ASSESSMENT	ITEMS BEING ASSESSED		RATING				COMMENTS
			1	2	3	4	
PLANNING	1	Schemes of work					
	2	Lesson Planning					
	3	Punctuality					
PRESENTATION	4	Introduction of the lesson					
	5	The use of teaching aids					
	6	The use of teaching methods in the introduction of the lesson.					
	7	Teacher preparedness for the lesson					
LESSON ORGANISATION AND PUPIL ASSESSMENT	8	Use of pupils practical activity					
	9	Marking of pupils work done					
	10	Remedial work					

	11	Appropriateness of the content to the learners					
COMMUNICATION	12	Teacher's questioning techniques skills					
	13	Pupil's participation in the learning process					
	14	Teacher-pupil interaction					
	15	Use of chalk board					
	16	Voice projection					
	17	Use of appropriate vocabulary					
	18	Does the teacher allow chorus answers in the process of teaching?					
	19	The use of teaching methods in the process of teaching					
OTHERS	20	Was the lesson enjoyed by the pupils					
	21	Knowledge of subject matter					
	22	Were the pupils able to answer teacher's questions?					
	23	Where the materials helpful and relevant to the learners?					
	24	Where there a discussion among the pupils?					

	25	Does the application of calculus in the classroom in line with mathematics syllabus?					
OVERAL RATING		OVERAL RATING					

General comments:…….

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Areas to improve: ……..

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Name of the observer: …….

Signature: ……

APPENDIX D: Papers Presented

1. The researcher presented a proposal at the departmental level, which was scheduled in June, 2017. The proposal was titled an assessment of the teaching of Calculus: A case of three selected secondary school teachers of mathematics in Chinsali district, Zambia.

2. The researcher presented oral and poster presentation on the 31st October, 2018 which was held at the University of Zambia. The paper was titled, "an assessment of the teaching of Calculus: A case of three selected secondary school teachers of mathematics in Chinsali district, Zambia".

APPENDIX E: Work Schedule

Date	Task
May-August 2017	Preparation for fieldwork • Submission of proposal
August - December 2017	Submission for ethical clearance
February – May 2018	Data collection • Questionnaires • Lesson observations • Semi-structured interviews
May – July 2018	Data Analysis
July – August 2018	Presentation of findings
September – December 2018	Submission for examination

APPENDIX F: Ethical Clearance

THE UNIVERSITY OF ZAMBIA

DIRECTORATE OF RESEARCH AND GRADUATE STUDIES

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Approval of Study

2nd November, 2018

REF. No. HSSREC: 2018-JULY-027

Mr. Simovwe John
The University of Zambia
P.O Box 32379
LUSAKA

Dear Mr. Simovwe,

RE: "AN ASSESSMENT OF THE TEACHING OF CALCULUS: A CASE OF THREE SECONDARY SCHOOL MATHEMATICS TEACHERS IN CHINSALI DISTRICT"

The University of Zambia Humanities and Social Sciences Research Ethics Committee IRB resolved to **approve** this study and your participation as Principal Investigator for a period of one year.

Review Type	**Expedited Review**	**Approval No. HSSREC: HSSREC: 2018-JULY-027**
Approval and Expiry Date	Approval Date: 2nd November, 2018	Expiry Date: 1st November, 2019
Protocol Version and Date	Version-Nil	2nd November, 2018
Information Sheet, Consent Forms and Dates	• English.	To be provided
Consent form ID and Date	Version	To be provided
Recruitment Materials	Nil	Nil

1

Excellence in Teaching, Research and Community Service

There are specific conditions that will apply to this approval. As Principal Investigator it is your responsibility to ensure that the contents of this letter are adhered to. If these are not adhered to, the approval may be suspended. Should the study be suspended, study sponsors and other regulatory authorities will be informed.

Conditions of Approval

- Provide information sheets and consent letters as these were not attached. The information sheets should have had the essential features included. Please use the WHO templates which you could download at www.who.int/rpc/research_ethics/informed_consent/en/). REC would appreciate if the PI could customise the WHO templates and include the domains of what the submitted protocol is positing on tools and the sampling units (people who have been or shall be participating in this study.

- No participant may be involved in any study procedure prior to the study approval or after the expiration date.

- All unanticipated or Serious Adverse Events (SAEs) must be reported to the IRB within 5 days.

- All protocol modifications must be IRB approved by an application for an amendment prior to implementation unless they are intended to reduce risk (but must still be reported for approval). Modifications will include any change of investigator/s or site address or methodology and methods. Many modifications entail minimal risk adjustments to a protocol and/or consent form and can be made on an Expedited basis (via the IRB Chair). Some examples are: format changes, correcting spelling errors, adding key personnel, minor changes to questionnaires, recruiting and changes, and so forth. Other, more substantive changes, especially those that may alter the risk-benefit ratio, may require Full Board review and approval. In all cases, except where noted above regarding subject safety, any changes to any protocol document or procedure must first be approved by the IRB before they can be implemented.

- All protocol deviations must be reported to the IRB within 5 working days.

- All recruitment materials must be approved by the IRB prior to being used.

- Principal investigators are responsible for initiating Continuing Review proceedings. Documents must be received by the IRB at least 30 days before the expiry date. This is for the purpose of facilitating the review process. Any documents received less than 30 days before expiry will be labelled "late submissions" and will incur a penalty.

- Every 6 (six) months a progress report form supplied by The University of Zambia Humanities and Social Sciences Research Ethics Committee IRB must be filled in and submitted to us. There is a penalty of K500.00 for failure to submit the report.

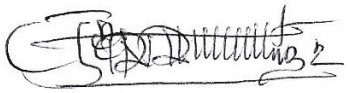
- The University of Zambia Humanities and Social Sciences Research Ethics Committee IRB does not "stamp" approval letters, consent forms or study documents unless requested for in writing. This is because the approval letter

clearly indicates the documents approved by the IRB as well as other elements and conditions of approval.

Should you have any questions regarding anything indicated in this letter, please do not hesitate to get in touch with us at the above indicated address.

On behalf of The University of Zambia Humanities and Social Sciences Research Ethics Committee IRB, we would like to wish you all the success as you carry out your study.

Yours sincerely,



Dr. Jason Mwanza
BA, MSoc, Sc., PhD
CHAIRPERSON
THE UNIVERSITY OF ZAMBIA HUMANITIES AND
SOCIAL SCIENCES RESEARCH ETHICS COMMITTEE IRB