TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE OF PROBABILITY: THE CASE OF TEACHERS OF MATHEMATICS IN CHONGWE DISTRICT LUSAKA PROVINCE, ZAMBIA

BY

Gift Muzamaya Sinzala

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DECLARATION

I, Gift Muzamaya Sinzala, declare that this dissertation represents my own work that it has not been previously submitted for a degree at the University of Zambia or any other University and that all published work or materials incorporated in this report have been acknowledged.

Signed ______________________________________

Date ____

THE UNIVERSITY OF ZAMBIA APPROVAL

This dissertation by Gift Muzamaya Sinzala has been accepted as fulfilling part of the requirements for the award of the degree of Masters of Education in Mathematics Education of the University of Zambia.

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Supervisor :……………………………………Date………………Signature:……………

Chairperson, Board of Examiners: ………………..….Date………...Signature: …….........

DEDICATION

I dedicate this work to my late mother, Regina Sinzala who I would have loved to see me through the whole programme. My wonderful Angela Nyirenda without whom I would not have reached this far. My sons Choolwe Mwape Sinzala, Tumbikani Mwape Sinzala, Lubomba Mwape Sinzala, Philip Mwape Sinzala and my daughter Rabecca Sinzala Mwape for their support. I also dedicate this piece of work to my friends Brian Mumba, Nsokolo Chali, Joseph Tembo, Rodric N'gandu and Milupi Musiwa for their patience, love and encouragement.

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LIST OF ABBREVIATIONS AND ACRONYMS

SMKDT:	Subject Matter Knowledge Diagnostic Test
MOE:	Ministry of Education
ECZ:	Examination Council of Zambia
MESVTEE:	Ministry of Education, Science, Vocational Training and Early Education
ZAMSTEP:	Zambia Mathematics and Science Teacher Education Project
CPD:	Continuing Professional Development
SBCPD:	School-Based Continuing Professional Development
SMASTE:	Strengthening Science, Mathematics and Technology Education.
AIEMS:	Action to Improve English, Mathematics and Science
MOGE:	Ministry of General Education
ZECF:	Zambia Education Curriculum Framework

ABSTRACT

This was a qualitative multiple case study that investigated mathematics teachers' pedagogical content knowledge of probability in selected secondary schools of Chongwe District, Zambia. A sample of seventeen (17) senior secondary school mathematics teachers was purposefully selected for participation in the study. A subject matter knowledge diagnostic test (SMKDT) based on Probability was administered to the sample to assess their subject matter knowledge of Probability. Documents such as teachers' schemes of work, lesson plans, and learners' exercise books were also analyzed followed by lesson observations and semi-structured interviews on all the 17 teachers. The intention of the observations and interviews was to investigate teachers' ability to use pedagogical strategies and enabling prompts as they taught probability. Data obtained from the SMKDT was analyzed using elements of the subject matter knowledge category of the study's conceptual framework. The elements included teachers' ability to make clear conceptual connections, interpret, and justify their reasoning in probability. In respect of data collected from lesson observations and interview schedules, the pedagogical strategies and enabling prompts categories of the conceptual framework were employed for analysis of teachers' ability to select and use appropriate multiple pedagogical strategies. Based on the analysis of the findings given in the SMKDT, the study showed that the sampled teachers were able to reflect the subject matter knowledge of probability in their processes of making conceptual connections, solving, interpretations, explanations and justifications of major mathematical concepts in relation to probability. However, majority of the sampled teachers (9) could not build on and extend beyond their understanding of the subject matter, instead they exhibited their own informal understanding of the subject matter knowledge of probability, which was a clear indication of a lack of understanding of the subject matter of probability. Likewise, in their lesson presentation and follow-up interviews, ten teachers showed less ability to use multiple pedagogical strategies and enabling prompts with regarding to the unpacking of the subject matter knowledge of probability in a comprehensive way to the learners. This was characteristic of incapacity to facilitate learners' clear conceptual understanding of probability. In regard to this, only seven teachers were able to demonstrate their ability to employ multiple pedagogical strategies to engage learners and use multiple forms of mathematical imagery to simplify and explain on the subject matter as they taught concepts such as independence and mutually exclusive events. Therefore, the implication of these findings indicate that despite teachers having taught for ten years and above, their experience could not guarantee them the development of the conceptualized components of PCK necessary for teaching probability of this study's conceptual framework. It is, therefore, recommended that teachers be provided with opportunities during training for in-depth study of probability concepts so that they could comprehensively teach learners probability with understanding. Additionally, issues of teachers' capacity to use multiple pedagogical strategies should be considered by serving secondary school mathematics teachers in their various continuous professional development activities.

KEYWORDS: Pedagogical Content Knowledge, Pedagogical Enabling Prompts, Probability, Pedagogical Teaching strategies, Subject Matter Knowledge, Socially Agreed Conventional Meanings of Terms, Socially Agreed Qualitative Terms, Zambia.

CHAPTER ONE: INTRODUCTION

1.1 Introduction

This research focused on the senior secondary school mathematics teachers' subject matter knowledge and their ability to use multiple pedagogical strategies in the teaching of probability concepts in selected secondary schools of Chongwe District, Lusaka Province, Zambia. The chapter highlights the background and the context of the study and justification of carrying out research on probability, statement of the problem, purpose and objectives, research questions, and the significance of the study. Furthermore, the delimitation and limitation of the study as well as the theoretical and conceptual frameworks are explained. Thereafter a summary of the chapter is provided.

1.2 Background of the Study

One of the major concerns of the education system for Vision 2030 relates to the matter of quality (National Development Plan, 2017). Among the factors that contribute to the quality of education is the mastery of the subject matter knowledge of teachers (Ministry of Education (MOE), 1996). Teacher quality is globally accepted as a significant aspect of effective teaching and learning in schools (Darling-Hammond, 2000). This simply means that teachers of mathematics should have a clear understanding of the subject matter knowledge for them to effectively teach learners. Consequently, mastery of the subject matter knowledge is critical for effective teaching of mathematics topics that are challenging to learners. Nevertheless, topics such as Probability, Calculus, and Trigonometry have been posing challenges to understand the majority of learners at both local and international levels (Examination Council of Zambia [ECZ], 2017; Nilsson, 2015 & Stohl, 2005). In regard to the above claim, international research asserts that the difficulties learners have been encountering in the learning of probability concepts have been confirmed by a number of recent studies (Mutodi & Ngirande, 2014). Probability is relatively defined as the study of random events (Mutodi & Ngirande, 2014). In the past, this topic was regarded as an enrichment topic that was only given to intellectually gifted learners during mathematics competitions (Department of Education, South Africa National Curriculum Statement for Grades 10-12 [DoE], 2012). Nevertheless, the topic has become a topical issue in the field of mathematics, and the need to include it in the nations' mathematics secondary school curriculum has increased in order to meet the needs of people for their daily survival (Batanero, Henry & Parzysz, 2005; New Zealand Council of Educational Research, 2011). In Zambia, Probability is an old topic and integral part of the Zambian mathematics senior secondary school curriculum (MOE, 1996). Currently, the topic is both taught at the junior and senior level in order for the learners to develop an early understanding of the Probability concepts and language as they proceed to the senior and higher education levels respectively (Ministry of Education, Science, Vocational Training and Early Education [MESVETEE], 2013). The need to make the Zambian mathematics curriculum relevant to the needs of an individual and society advocated for the inclusion of Probability both in the Senior and Junior Secondary School Mathematics syllabus (MESVETEE, 2013).

According to the MESVTEE (2013) Probability plays an important role in terms of inculcating mathematical reasoning in learners once the subject matter is clearly taught. The topic also enhances learners' analytical skills and probability solving skills (MOE, 1996; MESVTEE, 2013). It is also important to note that the topic plays an important role in the decision-making process in situations that are uncertain and also helps to improve learners' computational skills in solving probability problems and other challenges that they may encounter in this modern society (MESVTEE, 2013). Probability further develops the ability of citizens to be able to interpret whatever they see in ordinary life and put it into reality (Jones, 2004). Furthermore, understanding of Probability helps learners to appreciate their social environment and become competent in calculating and determining the various chances for specific events to occur without necessarily referring to the probability scale (Department of Education, South African National Curriculum Statement for Grades 10-12 [DoE], 2008). In addition, a clear understanding of Probability has also the potential to develop learners' ability to collect, summarize, display and analyze data. As such, learners can easily use knowledge of Statistics and Probability to justify, predict and critically interrogate the data findings and draw conclusions, and thereafter communicate the results to the relevant authorities.

At the global level, the topic has been useful in analyzing games of chance (football betting), genetics, weather prediction, and other everyday events (Gal, 2005; Jones, 2005). Probability has been also influential in providing conceptual bases in other disciplines such as biological sciences, economics, meteorological, political, and social activities (games and sports) settings (Gal, 2005; Jones, 2005). This is a clear indication that today's learners require an understanding of probability principles and laws to enhance their computational skills in other subject areas. Despite this importance of Probability, the majority of learners do not fully understand

Probability (ECZ, 2017; Jones & Tarr, 2007). It is as a result of the above reason that selection of Probability as the topic worthy of studying was necessitated. This study was also motivated by the need for learners in the 21st century to acquire skills of interpretation and analysis of uncertain events using Probability knowledge.

However, the current performance of Zambian learners is far from satisfactory, especially in the key areas of mathematics and science and this has been seen in the candidates' final Grade Twelve examination results. On average, less than two-thirds of candidates obtain a full pass in school certificate each year (MOE 1996). MOE (1996) further reports that ''cumulatively, one-third of boys and two-thirds of girls have registered complete fail in mathematics since 1987, while only half of the boys and one-fifth of the girls have managed to obtain a pass or better in mathematics'' (p. 53). It is this learners' poor performance background that has led to the exploration of ways of enhancing mathematics teachers' subject matter knowledge and their teaching skills. In response to the above, the Ministry of General Education has been involved in a number of teacher professional development projects with a view to improving the teachers' subject matter knowledge and their teaching skills in Mathematics and Sciences through Continuous Professional Development (CPD) programs. These projects include Zambia Mathematics and Science Teacher Education Project (ZAMSTEP), Action to Improve English, Mathematics, and Science (AIEMS), and Strengthening Mathematics, Science and Technology Education (SMASTE) (MOE, 2013). Currently, the Ministry of General Education is promoting Lesson study cycles, Continuous Professional Development (CPD), School-Based Continuous Professional Development (SBCPD) programs (MESVTEE, 2014). All these projects have been put in place in order to improve teachers' subject matter knowledge and their teaching skills so that they could be in a position to effectively teach learners challenging topics and improve performance in mathematics. In view of this, the Zambian national policy on education states that when well-informed teachers teach the subjects they are trained to teach, the results of learners tend to improve significantly (MOE, 1996).

Nevertheless, research studies have shown continued failure rates in probability and other topics by many learners (Bulut, 1994; ECZ, 2018; Isik, Kaplan & Zehir, 2011; Sezgin-Memrum, 2008). Internationally, learners have continued finding it difficult to use their understanding of Probability to solve probability problems correctly especially those concepts whose meanings are tied to their application (Kazima & Adler, 2006). For example, learners struggle to use their understanding of probability to determine the concept of independence using the coin (Kazima & Adler, 2006). Furthermore, international studies have also shown that most of the learners had challenges to apply their understanding of laws of probability in solving of probability problems due to their inadequacy in verbal and cognitive ability to express themselves freely in the language of Probability (Green, 1983; Hansen, McCann & Meyers, 1985; Kaplan, 2008; Mutodi & Ngirande, 2014). Learners further showed the inability to use word problem concepts to express probability problem statements into an algebraic expression. In this case, learners found it challenging to convert word probability problem statements into algebraic sentences (expressions) using the word problem concept, for instance writing in the form 1-n (Hansen, McCann & Meyers, 1985). This means that learners have challenges to use their understanding to make the conceptual connection of algebraic expression with Probability concepts which is one of the pre-requisite topics of Probability. This shows that learners do not have a clear understanding of how to convert word probability problem statements into an algebraic expression. Internationally, this has been frustrating learners' desire to learn Probability concepts which in turn has resulted in poor performance in the topic.

In Zambian, learners are performing poorly in Probability at Grade 12 final examinations (ECZ, 2015). For example, Grade Twelve candidates confuse the addition rules for mutually exclusive events P (A or B) =P(A)+P(B) with multiplication rules for independent events P (A and B)=P(A) P(B) (ECZ, 2016). The candidates (learners) have challenges to differentiate the basic addition rules of mutually exclusive events with the multiplication rules of independent events (ECZ, 2016). Furthermore, candidates have also shown challenges in the use of the concepts of unions (U), intersection (∩), and complements (A'UB') of sets to solve probability problems (ECZ, 2017). This implies that candidates have challenges of using their understanding to make clear conceptual connections of unions, intersections, and complements with the subject matter of Probability, which in turn affects their procedural knowledge of solving probability problems. However, learners' poor performance in the mathematics final examinations has become a major source of concern to the majority of stakeholders such as non-governmental organizations like Forum for African Women Educationalists of Zambia (FAWEZA), Ministry of Education, Teacher unions and Parents (MOE, 1996; MOGE, 2016). In line with the above challenges, stakeholders such as the Ministry of General Education (MOE, 1996) argued that the candidates' failure could be caused by limited numbers of teachers of mathematics and teaching materials in these secondary schools. In respect to this, there is no empirical evidence to support such a view, especially that many degree-holding teachers of mathematics have been deployed in secondary schools. However, the teaching and learning of Probability does not appear to be easy and has been posing challenges to international teachers of mathematics and learners because of its disparity between intuition and conceptual development and some concepts are too tied to the application (Batanero & Diaz, 2009; Jones & Tarr, 2007). Thus, research suggests that candidates' poor performance in Probability could be attributed to a number of factors that may include mathematics teachers' deficiencies in the subject matter knowledge of Probability (Jones, 2005; Stohl, 2005). International studies have also revealed that mathematics teachers experience various difficulties in varying their teaching and instruction strategies using teaching and learning materials such as coin, spinners and dice to simulate the concept of probability and other topics (Boyacioglu, Erduran & Alkan, 1996; Bulut, Ekicive & Iseri, 1999; Sizgen – Memnum, 2008). It is in light of this, that the study investigated the Zambian mathematics teachers' subject matter knowledge, their ability to use multiple pedagogical strategies and enabling prompts in the teaching of Probability, with a view to establishing whether they are part of the factors that contribute to teachers' failure to teach Probability and other related concepts effectively.

1.3 Statement of the Problem

Although mathematics is one of the core subjects in the Zambian secondary school curriculum, the subject is characterized with learner poor results at Grade 12 final examinations level, particularly in Probability (ECZ, 2018; Zambia Education Curriculum Framework [ZECF], 2013). The failure by candidates is of concern to stakeholders such as the Ministry of Education, teacher unions, and parents (MOE, 1996; MOGE, 2016). In spite of the concerns by stakeholders, there is no reported study that has suggested reasons for candidates' failure particularly in topics such as probability. Even though reasons for Zambian learners' poor performance in Probability are not known, some international studies have claimed that teachers of mathematics have demonstrated significant challenges with regard to the teaching of the topic (Jones 2005; Stohl, 2005). This scenario could be one of the causes of learners' poor performance. However, the poor results that Zambian candidates have continued recording in secondary school probability are indicative of underlying problems that require investigation of which no research has been reported. As a result, this created a gap that motivated the researcher to investigate Mathematics Teachers' Pedagogical Content Knowledge of Probability.

1.4 Purpose of the Study

The purpose of the study was to investigate secondary school mathematics teachers' Pedagogical Content Knowledge of Probability. The sampled teachers were investigated for their understanding of the subject matter knowledge of probability in relation to their ability to solve, interpret, use correct notation symbol, explain and justify their mathematical views. The study also investigated teachers' enabling conditions for the successful teaching of probability. The sample was further investigated for their ability to use the identified pedagogical strategies effectively in the teaching of probability concepts.

1.5 Research Objectives

The study was guided by the following objectives;

1. To investigate mathematics teachers' subject matter knowledge of Probability.
2. To assess pedagogical strategies which mathematics teachers use to teach Probability
3. To assess pedagogical enabling prompts that mathematics teachers use to support the teaching of probability

1.6 Research Questions

This study sought to answer the following research questions;

1. What subject matter knowledge of Probability do mathematics teachers have?
2. What pedagogical strategies do teachers of mathematics use to teach Probability concepts?
3. What pedagogical enabling prompt that mathematics teachers use to support the teaching of Probability?

1.7 Significance of the Study

This study's findings have an implication to contribute to the existing Pedagogical Content Knowledge literature on teachers' understanding of probability concepts. The findings of the study have also an important implication in providing probability instruction which different researchers can take as reference to further their related studies in probability, whereas teacher training institutions can also use these research findings to develop their curriculum in terms of content knowledge to be taught to the student teachers. Furthermore, stakeholders such as the

Ministry of General Education and mathematics educators are likely to be informed concerning mathematics teachers' PCK of Probability.

1.7.1 Delimitation of the study

The study was conducted with teachers of mathematics teaching in Chongwe District of Lusaka Province, Zambia.

1.7.2 Limitations of the study

The giving of the test to the teachers in order to collect the required subject matter knowledge of probability was received with mixed fillings as most of them expressed unwillingness to sit for the test. In regard to this, the researcher took time to discuss with individual sampled teachers on the importance of the research which was meant for the academic progression of learners and themselves. However, the researcher and the teachers managed to reach a consensus and they accepted to write the test which allowed the researcher to proceed with investigating teachers' understanding of the subject matter knowledge of probability. In addition, data collection was conducted in the third term of a school calendar which posed a challenge to access all the initially sampled teachers, as they were attending meetings in their zone areas on different days on the guidelines on how to handle examinations. The researcher had to reschedule the days when he was supposed to meet them in their schools for the test, observation, and interview schedules to other convenient days. Therefore, this allowed all the sampled teachers to participate in the research. Besides this, because of the nature of PCK, it was difficult to identify and relate the teachers' understanding of the subject matter knowledge of probability in relation to the conceptualized components of PCK of this study's conceptual framework as they were only able to employ a small part of its accumulation of PCK in the manner they were responding to the SMKDT question, the quality of the subject matter they presented. However, the researcher took time to read the journals that have looked at PCK for teaching probability and, thereafter the researcher was able to compare notes with what was presented in the SMKDT, recorded videos and audios during lesson presentation and follow up interviews. Furthermore, because of a large sample and the type of research approach which was involved in this study, it was time consuming and involving to analyze each case of the seventeen teachers' responses given in the preliminary observation interviews, SMKDT, lesson presentation and in the follow-up interviews. However, the researcher had to extend his completion period in order to have enough time to analyze the teachers' responses one by one.

1.8 Theoretical Framework of the Study

The theoretical framework underpinned this study was situated in the model of Pedagogical Content Knowledge as outlined by Shulman (1986, 1987). In directing the study on investigating mathematics teachers' subject matter knowledge, pedagogical strategies and enabling prompts used in teaching of probability, the researcher used the model of pedagogical content knowledge (PCK), a type of knowledge which could help to provide an understanding of the nature of the subject matter knowledge and pedagogical strategies teachers have in specific areas of mathematics topics (Marks, 1990). In regard to this, Shulman's (1986) seminal work proposes that teachers must possess the Pedagogical Content Knowledge (PCK) which allows them to understand the subject matter and how to present it to the learners in a comprehensive way using various ways. This implies that teachers with PCK have the ability to transform the subject matter knowledge into simplified forms of representations, illustrations, examples, explanations and demonstrations. He further state that Pedagogical Content Knowledge (PCK) also provides an understanding of what makes certain topics to be difficult or less challenging to the learners. It also helps understanding of the conceptions and preconceptions that learners of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons (p.9). Shulman (1987) further states that teachers must not only understand the subject matter, but they must also be abreast with deep knowledge of how to represent and present the subject matter concepts using real-life situations that could enhance learners' understanding and retention of the concepts. In this regard, Shulman (1987) state that ‘‘teachers require an understanding of how particular topics, problems, or issues are organized, represented, and adapted to diverse interests and abilities of learners, and presented for instruction’’ (p. 8). This means that teachers are required to combine the knowledge of the subject matter and pedagogy to comprehensively transform the subject matter to the learners (Shulman, 1986, 1987). Furthermore, teachers are required to have knowledge of specific pedagogical strategies to engage learners, and for teaching particular concepts of probability, and means of supporting their understanding of the subject matter while engaged in a lesson. Similarly, Ball, Thames and Phelps (2008) also insist that the organizing principles and structures and rules for establishing what is legitimate to do must be understood by teachers. Teachers are not only expected to know that a concept is as it is but they should also know why it is as it is (Shulman, 1986). Therefore, it is out of the above assertions that brought the notion of pedagogical content knowledge to the fore of this study.

According to Shulman (1986) defined pedagogical content knowledge (PCK) as comprising the most useful forms of representing and formulating the subject matter that make the teaching of that subject matter comprehensible to the learners and others such as use of different useful forms of representations for example mathematical ideas, most powerful analogies, illustrations, examples, explanations and demonstrations (Ball, Thames & Phelps, 2008). Niess (2005) also cited Ball, Thames and Phelps (2008) and defined PCK as the intersection of knowledge of the subject with knowledge of teaching and learning. While, Lowery (2002) defined PCK as ‘‘that domain of teachers’ knowledge that combines subject matter knowledge and knowledge of pedagogy’’ (p.69). However, with reference to the above definitions, PCK comes out as a product of changing the subject matter to what is learnable to the learners and thus creating an understanding of what is a difficult or easy of the subject matter of the topic. As such, PCK is viewed as the most important component of a teacher’s knowledge and is central to successful teaching of challenging topics and has attracted a lot of research in terms of teacher knowledge (Abd-El-Khalick, 2006; Konig et al, 2016). In light of the above, a large number of researchers have conceptualized and identified various components that constitute PCK and have viewed PCK as an integration of those components on which the subject matter knowledge was investigated (Brijlall, 2008; Ball, Thames & Phelps, 2008; Ozden, 2008). In this case, it is important to note that, Shulman (1986, 1987) was the first one to conceptualise that PCK consist of the knowledge of the subject matter, students’ understanding, curriculum, and pedagogy or instructional strategies.

Furthermore, in a quest to understand what subject matter knowledge teachers need to have for comprehensive planning and teaching of a topic, Ball, Thames and Phelps (2008) also developed six domains of mathematical knowledge for teaching (MKT) frameworks which were built on Shulman’s ideas of content knowledge concerning pedagogical content knowledge. These frameworks include; Common Content Knowledge (CCK), Specialized Content Knowledge (SCK), Horizon Content Knowledge (HCK), Knowledge of Content and Teaching (KCT), Knowledge of Content and Students (KCS), Knowledge of Content and Curriculum(KCC). It is important to note that the Common Content Knowledge (CCK), Specialized Content Knowledge (SCK), and Horizon Content Knowledge (HCK) domains were theorized by Shulman as subject matter knowledge, and the last three which includes Knowledge of Content and Teaching (KCT), Knowledge of Content and Students (KCS), Knowledge of Content and Curriculum (KCC) were conceptualized by Shulman as the Pedagogical Content Knowledge (Malambo, 2020).

Consequently, in order to investigate mathematics teachers' PCK of probability in this study, the researcher conceptualised PCK into three components namely the subject matter knowledge, pedagogical strategies and enabling prompts (See Figure 1.1, Section 1.8.1). However, these conceptualized components of PCK of this study were used to help to provide an understanding of mathematics teachers in terms of how they could bridge the subject matter knowledge with the practices of teaching in the teaching of probability concepts to the learners in a comprehensible way. Hence, this study adopted the theoretical model of PCK and made use of it as outlined by Shulman (1986, 1987).

It is therefore important to note that the inclusion of pedagogical enabling prompts in the conceptualised component of PCK suggest that, mathematics teachers should not only have the understanding of the subject matter knowledge and the ability to engage learners in a lesson, but should have the ability to design means of supporting and assisting learners to understand the major mathematical concepts through the use of designed enabling prompts. This is because the use of enabling prompts ensures that learners are not deprived of the opportunities to use their own approach to the problem and to think creatively about how they could solve it (Clarke, Roche, Cheessman & Sullivan, 2014). In regard to this, Sullivan, Walker, Boercek and Rennie (2015) state that the use of pedagogical enabling prompts plays a major role in assisting learners to easily grasp the interconnections and relationships among different aspects of the subject matter knowledge of abstract topics. This is because it simplifies what looked to be difficulty to the learners.

Park and Oliver (2008) also argued in line with Shulman's (1987) notion of PCK that, ''it is the teachers' understanding and enactment of how to help learners to understand specific subject matter using multiple instructional strategies, representations, and assessments while working within the contextual, cultural, and social limitation in the learning environment'' (p.264). This shows that enabling prompts are a crucial element in the teaching of abstracts topics like probability, for they provide an entry point into how to solve the given task by making sure that learners understands the level of challenge of the task. In addition, enabling prompts encourages the use of various forms of representations and concrete visual cues which makes learners to gain clear understanding of what the task is asking them to do rather than just engaging the learners in a lesson. Therefore, to teach comprehensively the secondary school mathematics, teachers of mathematics should combine the subject matter and the pedagogical knowledge as well as enabling prompts so that they can demonstrate an understanding of how particular topics are organized, represented, and adapted to diverse interests and abilities of learners (Shulman, 1987). This means that teachers of mathematics are supposed to acquire competence in both general pieces of knowledge of teaching and subject matter knowledge to teach effectively. However, the argument of this study is how do we investigate the understanding and ability in planning that we expect the teachers to have in terms of knowledge of teaching, of defining concepts, of selecting relevant examples and exercises, of choosing the sequence in treating a specific topic and of distinguishing between wrong and correct strategies in the teaching and solving of, for example, probability problems (Brijlall & Isaac, 2011). In this study, the teachers' pedagogical content knowledge (PCK) has been conceptualized as a reflection of teachers' subject matter knowledge, teachers' knowledge of specific pedagogical strategies for teaching particular concepts and ability in designing enabling prompts that would supports learners' understanding of probability once engaged in the lesson in the teaching of probability by (Shulman, 1986, 1987).

In order to be specific in terms of subject matter knowledge, pedagogical strategies and enabling prompts, the researcher developed generic descriptors for each dependent variable which facilitated this study in describing the mathematics teachers' nature of the subject matter knowledge of probability they held, their ability to use multiple, and engaging pedagogical strategies, and enabling prompts that helped to support learners' understanding of probability. In this regard, the teachers were expected to exhibit the ability to recognize, identify, explain, interpret and justify the mathematical ideas. The teachers were further expected to define probability terms using socially agreed qualitative terms and use of notations, and multiple strategies to comprehensively teach on specific concepts of Probability. It is important to note that, the expectations of the generic descriptors were also in line with the Zambian national policy on education (MOE, 1996) that ''the essential competencies required in every teacher are mastery of the material that is to be taught, and skill to communicating that material to pupils'' (p.108). This means that the developed conceptual framework recognized the necessity of secondary school mathematics teachers the need of having a sufficient understanding of the organization of concepts, principles, and procedures of mathematics topics and their link with the pre-requisite topics that they teach in secondary schools.

1.8.1 Conceptual Framework

Orodho (2009) states that the development of a conceptual framework sets a platform for the presentation of the research questions that drive the study. However, this study's conceptual framework was developed from the works of Hill, Ball and Schilling (2008); Krauss, Neubrand, Blum and Baumert, (2008); Malambo (2015) and Shulman (1986, 1987). The following is Figure 1.1 showing the developed conceptual framework.

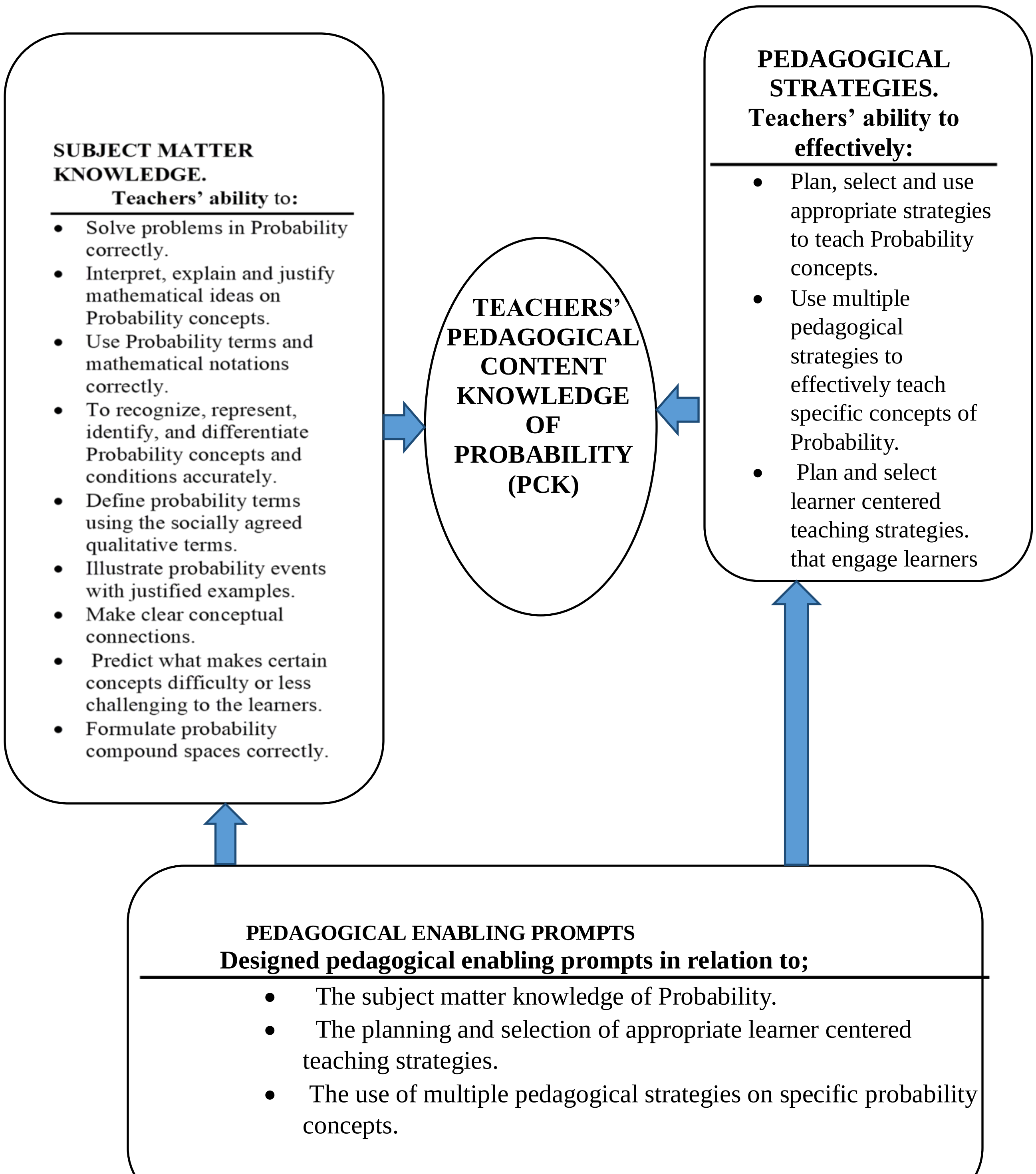


Figure 1.1

Figure 1: Conceptual Framework

1.8.2 Operational Definition of Terms

Pedagogical Content Knowledge (PCK) refer to the domains of teachers' knowledge such as Knowledge of Content and Teaching (KCT), Knowledge of Content and Students (KCS), Knowledge of Content and Curriculum (KCC) that combines subject-matter knowledge and knowledge of pedagogy (Ball, Thames & Phelps, 2008; Shulman, 1986, 1987).

Multiple pedagogical instructions refer to the teachers' knowledge of using various teaching strategies and ideas that bring effective teaching of Probability concepts to the learners (Shulman, 1987).

Teaching Strategies. These are methods that teachers employ to unpack the subject matter in ways that keep learners engaged and practicing different skills (MOE, 1996)

Pedagogical strategies: In this study refers to the general ways of presenting the subject matter to the learners using various ways of engaging the learners in a lesson (Ball, Thames & Phelps, 2008; Shulman, 1986, 1987)

Subject matter knowledge implies having a clear understanding of the organizational structure of the subject matter of the topic as well as the ability to recognize, and apply correctly the concepts, facts and procedures when teaching (Wilson, Shulman & Richert, 1987).

Senior Secondary School Teachers: These are teachers who provide teaching services to learners from grades 8-12 (MOE, 1996).

Pedagogical enabling prompts: These are strategies designed to support learners to engage in active learning experiences about the initial goal of the given task, which involve reducing the number of steps, simplifying the complexity of the question, and use of various concrete forms of representation in problem-solving and means of scaffolding the learners' level of performance who cannot proceed with the task (Sullivan, Walker, Borcek & Rennie, 2015).

Socially agreed qualitative words. These refers to words that are used to convey socially agreed conventional meanings of the occurrence of probability events with reference to the probability scale and include words such as most likely, almost certain, random, less likely, certain, impossible, likelihood, never, probably, likely, risk, odds, chance, fair, and so on (Baer, 2008; Mutodi & Ngirande, 2014).

Simulation refers to a methodological aspect or aid used to demonstrate either physically or use of technology to describe the occurrence of the probability concept such as determining the actual independence of a single after tossing the coin twice (Khazanov & Prado, 2010; Stohl, 2005). However, simulation can either be done using a computer or tossing of a coin, rolling a dice.

1.9 Summary of Chapter One

Chapter one presented the background, the context of the study, and a justification for researching Probability. Thereafter, the background is followed by the statement of the problem which is presented with the nature of the subject matter knowledge of probability and pedagogical challenges in the use of multiple instructional strategies held by teachers of mathematics as they taught probability concepts. Furthermore, the chapter presents the aim of the study, objectives, research questions, significance, and delimitation of the study, limitation of the study, theoretical and conceptual framework on which the study is grounded. The chapter also presents the operational definition of terms. Therefore, the chapter that follows discusses the literature reviewed from different scholars that looked at teachers' pedagogical content knowledge of probability.

CHAPTER TWO: REVIEW OF RELATED LITERATURE

2.1 Introduction

The previous chapter presented the introductory information to the study, including the background information of the study, statement of the problem, and the theoretical framework. This chapter focuses on the literature which generally looks at the nature of the subject matter knowledge of probability held by secondary school teachers of mathematics as they teach Probability concepts. To have a further clear picture of mathematics teachers' subject matter knowledge of Probability and ability to use multiple instructional pedagogical strategies, this study also consulted other literature that focused on learners' understanding of probability concepts to triangulate the findings from other reviewed literature. The study also reviewed literature that focused on teachers' PCK in the teaching of mathematics. The literature reviewed included journals, Examination Council of Zambia Examiners' reports (2014-2019), and other sources with a bearing on the study. Thereafter a summary of the chapter is provided. Therefore, Section 2.2 presents the topics which were reviewed in support of this study on teachers' PCK of probability.

2.2 Overview of Literature Reviewed about Teachers' PCK of Probability

Much of the literature which was reviewed in this study on teachers' PCK of probability was based on mathematics topics such as Algebra, Geometry, Tangent functions, and Probability and study projects in mathematics education courses. However, studies that were carried out on probability as the focus of this study show that teachers exhibited subject matter knowledge, while others exhibited challenges in their subject matter knowledge for teaching probability. On the latter, this is because at the time the research studies were conducted, the probability was relatively a new topic in many countries' curricular, and as such majority of teachers had no clear understanding of probability (Kazima & Adler, 2006). For example, in South Africa probability was introduced as a topic of study in mathematics for the first time in the senior phases (Grade 7-9) in 1992 and this resulted in teachers having challenges presented in the reviewed literature (Laridon, 1995). This means that these studies reviewed were conducted when the probability was relative a new topic to teachers, however, their findings are cardinal for the current study to be well-grounded, despite being carried out in Algebra, Geometry and Calculus. The following Section 2.3 presents the importance of mathematics teachers having pedagogical content knowledge.

2.3. Teachers' Pedagogical Content Knowledge in the Teaching of Mathematics

In his presidential speech, Shulman (1987) specified seven categories of professional knowledge required for teaching which included content knowledge, general pedagogical knowledge, curricular knowledge, knowledge of learners, knowledge of educational contexts, knowledge of educational aims, goals, and purpose, and pedagogical content knowledge (PCK). In regard to this, Shulman (1986, 1987) concluded by stating that ‘‘besides content knowledge and curricular knowledge, teachers need a third knowledge PCK. This because PCK goes beyond knowledge of the subject matter per se to the dimension of subject matter knowledge for teaching’’ (P. 9). Because of this, PCK as a construct has become the main focus of many research studies in terms of teacher knowledge for teaching in different fields (Abd-El-Khalick, 2006; Ball, Thames & Phelps, 2008). Shulman (1986) defined that ‘‘PCK is the ability to make the most useful forms of representation of those (mathematical) ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations. In other words, it is the most useful ways of representing and formulating the subject matter that makes it comprehensible to the learners, which also includes an understanding of what makes the learning of specific topics easy or difficult in terms of the conceptions and preconceptions that learners of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons’’ (p. 9). Consequently, Shulman (1986) conceptualised PCK to be a blend of subject matter and pedagogy. This means that teachers should not only have subject matter knowledge but should be able to explain the subject to the learners with clarity if they are to be effective in their teaching (Malambo, 2015). In a similar context, based on Shulman's (1986) notion of PCK, Ball and Bass (2000) regard the ability to unpack the mathematics from the constructs, concepts, analogies, metaphors and images as being an important aspect of PCK. Other scholars have also conceptualised and argued that PCK is the understanding and enactment on how to help learners to understand specific subject matter using multiple instructional strategies, representations, and assessments while working within the contextual, cultural, and social limitations in the learning environment (Park & Oliver, 2008). Furthermore, Magnusson, Krajcik and Borko (1999) have conceptualised PCK as the teachers' understanding of how to help learners understand specific subject matter which includes knowledge of how particular subject matter topics and problems are organised, represented and could be adapted to the diverse interests and abilities of learners, and then presented for instruction. In this study, as earlier indicated in Section 1.8, PCK has been conceptualised under three categories, which includes teachers' subject matter knowledge, teachers' pedagogical strategy choices, and enabling prompts designed to support learners' understanding in the teaching of probability. The nature of the conceptualised components of PCK held by teachers of the study's conceptual framework are explained in the reviewed literature in the following Sections 2.4, 2.5 and 2.7.

2.4 Teachers' Subject Matter Knowledge for teaching Probability

In Zambia, studies which have been conducted concerning mathematics teachers' understanding of mathematics topics and concepts have been conducted in topics such as Calculus, Earth Geometry, Tangent function, Algebra and other topics (Changwe, 2017; Malambo, 2020 & Nalube, 2014). Despite these studies having not been conducted on mathematics teachers' pedagogical content knowledge of probability, their findings are cardinal for the current study to be well-grounded. Regarding this, Malambo (2020) investigated mathematics student teachers of a particular university on their nature of understanding of the tangent function. The findings of the study suggested that the university student teachers lacked relational understanding of the tangent function despite having studied advanced mathematics. Likewise, Changwe (2017) conducted a study on the effectiveness of mathematics teacher education curriculum at one of the university in Zambia in preparation for secondary school student teachers of mathematics using a mixed-method approach and collected data using both open and closed-ended questions on the student and in-service teachers' confidence to teach secondary school mathematics based on their tertiary education program. The study found that student and in-service teachers of mathematics exhibited knowledge gaps in the presentation of mathematics topics' subject matter. He further indicated that the majority of the sampled teachers avoided teaching topics such as probability and Calculus. Nalube (2014) also carried out a qualitative case study on mathematics student teachers of a particular university in Zambia and analysed the extent to which Learner Mathematical Thinking (LTM) is considered in the area of Algebra at teacher education level.

The findings of her study show that teacher-educators' privileged selections of learner mathematical thinking (LMT) is weakly classified and framed, hence implicit messages being relayed to the student teachers.

In terms of PCK, researchers such as Papaieronymon (2009) analyzed the recommendations for the required subject matter knowledge for teaching probability concepts provided by the four American Organisations which included the American Mathematics Society, the American Statistical Association, the Mathematical Association of America and the National Council of Teachers of Mathematics. In his analysis, he identified the suggested subject matter knowledge for teaching probability from the sampled teachers which were recommended by the four American professional organisations. The findings from the analysis of the recommendations also showed that there was sixty-six percent of the recommendations related to the subject matter content knowledge whereas twenty-four percent referred to pedagogical content knowledge and only ten percent referred to curricular knowledge.

Similarly, Hill, Ball and Schilling (2008) designed and analysed measures which assessed mathematics teachers' knowledge of content and students (KCS) based on Shulman's (1986) conception of KCS as a branch of PCK that contains teachers' knowledge of what makes certain concepts easy or difficult for students to learn and which preconceptions and initial ideas do students typically bring with them concerning content being presented. In this regard, Hill and his colleagues conceptualised and measured knowledge of content and students through writing, piloting, and analysing results from multiple-choice items and found that students teachers were able to identify learners' mathematical thinking such as learners' understanding of content, development sequences, and common computational strategies and common errors that learners were likely to make, which is one of the subject matter knowledge for teaching abstract topic, and also a conceptualized components of PCK for teaching mathematics. In addition, Even and Tirosh (2002) also carried out a research project on teachers' understanding of learners' thinking of the mathematics. In their study of in-service elementary school teachers were presented with a model of learners' thinking of world problems such as basic addition, subtraction, multiplication, and division. The analysis of the findings found that teachers were able to recognise differences among world problems, identify strategies that learners were to use to solve different mathematics problems. The teachers were also able to recognise learners' errors and explain the causes of the errors as well as suggesting possible strategies that could provide solutions.

Furthermore, the teachers were able to organize the strategies according to the learners' levels of thinking as they taught. Therefore, the findings of this study indicate that the teachers were able to exhibit the subject matter knowledge for an effective way of teaching abstract mathematics topics, in particular topics like probability. This means that the teachers exhibited the required conceptualised components of PCK for teaching mathematics, which is also the main focus of this study.

Faaiz (2008) investigated 250 teachers' probabilistic reasoning in their learning and teaching of probability in an advanced certificate in education (ACE) courses. The findings revealed that teachers had variations in understanding of learning probability. However, more than half of teachers were able to give reasons for learners' misconceptions in terms of formal or mathematical probability, namely, the position or order of outcomes, heads/tails, and tails/heads. Teachers were also able to distinguish between the outcome of the heads/tails and tails/heads in the case of tossing two coins. Nevertheless, the ability by teachers to distinguish between heads/tails and tails/heads signified understanding of the subject matter knowledge for teaching probability. In a similar context, Swenson (1997) investigated teachers' PCK for teaching probability and found that teachers held traditional views about mathematics and learning and teaching of mathematics, lacked understanding of key mathematical ideas in probability instruction and knowledge, and skills needed to promote learners' higher level of learning as well as an integrated understanding of the nature of the reform in their probability instructions. Furthermore, Batanero, Henry and Parzysz (2005) in their study of the nature of chance and probability, challenges teachers found in teaching of secondary school Probability, found that majority of mathematics teachers exhibited knowledge gaps in their unpacking of the subject matter to the learners. The majority of the teachers showed lack of ability to use probability terms and notations correctly as a result of having the subject matter knowledge of probability which is fragmented, compartmentalized, and poorly organized towards teaching abstract topics. These findings suggest that teachers did not have in-depth understanding of probability concepts. Notably, it is very difficult for teachers to teach what they do not know and understand (Avong, 2013). This entails that learners are likely to have a higher possibility of acquiring incorrect mathematical concepts which could continue contributing to unsatisfactory-performance in mathematics as a subject, particularly in Probability. Therefore, this could be a reason why

Shulman (1987) argue that teachers who hold inaccurate information or had lack of deep conceptual knowledge in their subject matter were likely to pass on the same incorrect information to their learners and generation to come. This means that there is a likelihood of the continuous poor performance of learners in mathematics especially in abstract topics like probability. Mathematics teachers are expected to be knowledgeable in the subject matter knowledge of probability concepts and pedagogical teaching skills of teaching abstract topics like Probability. This, however, could be achieved when teachers of mathematics are in a position to communicate the required knowledge of concepts in a clear, informative, and precise manner to their learners (Idowu, 2016; Soer, 2009). Therefore, as it has been seen in the studies reviewed about the PCK of probability, the teachers exhibited knowledge gaps in their unpacking of the subject matter knowledge of probability concepts (Batanero, Henry & Parzysz, 2005; Swenson, 1997) as compared to the recent findings of Papaieronymon (2009) who found teachers having the conceptualized pedagogical content knowledge for teaching probability. This means that there has been an improvement in the subject matter knowledge for teaching probability. However, Sections 2.4.1 and 2.4.2 presents the reviewed literature and explains on the subdivided descriptors of the teachers' subject matter knowledge of the study's conceptual framework.

2.4.1 Teachers' ability to define, interpret, explain and justify their mathematical views to the learners

Kazima and Adler (2006) conducted a study involving mathematical knowledge for teaching and problem solving of probability in practice in South Africa. The analysis of the eight lessons the teachers presented revealed that each of the six aspects of mathematical problem solving required by the mathematics teachers which included defining, explaining, representing, working with learners' ideas to restructure tasks, and questioning were exhibited even though they were uneven in their presentation. In terms of pedagogy in which teachers were required to engage learners in their lesson presentations, there were many instances that teachers exhibited and worked with learners' ideas in the restructuring of the given tasks as they taught probability which other researchers have conceptualized as part of PCK (Even 1990). Shulman (1986) states that teachers are required to define for students the accepted truths in a domain, explain why a particular proposition is deemed warranted, why it is worth knowing, and how it relates to the other propositions, both within the discipline and outside, and both in theory and practice. In addition, Ball, Bass and Hill (2004) also state that it is an important aspect in mathematical problem solving for mathematics teachers to make judgments about mathematical tasks and modifying them accordingly. This means that teachers have the subject matter knowledge of teaching the subject

Parallel to the above findings of study, Nilsson (2015) carried out a survey study on correlation between Swedish teachers' content knowledge of probability and their level of education, years of teaching, and self-assessments of probability concepts. The findings of his study found that the majority of teachers of mathematics had low confidence and ability to explain the subject matter knowledge of Probability to the learners in a comprehensible way. Mathematics teachers showed knowledge gaps in terms of explaining and justifying whether the tossing of a 'fair' coin as well as proving a coin to be 'fair' could have been either theoretical or experimental probability. In this case, it was found that teachers of mathematics could not provide a comprehensive explanation and justification on how the tossing of a fair coin as well as proving a coin to be 'fair' could be proved using theoretical and experimental probability respectively. This shows that the majority of teachers of mathematics exhibited knowledge gaps in terms of their ability to explain and justify the mathematical concept on whether the tossing of a coin and proving of a coin to be fair was theoretical or an experimental probability. Similarly, to the above study, Krauss, Neubrand, Blum and Baumert (2008) also carried out a study on teachers' pedagogical content knowledge and their number of years of experience in the teaching of abstract topics. The findings of the study show that there was no positive correlation between teachers' knowledge base for teaching and their years of teaching experience. Teachers exhibited weakness and lacked a deep conceptual understanding of mathematics.

Ball (1990) investigated mathematics teachers' understanding of Probability that they bring to the classrooms. The study indicates that 78% of secondary school mathematics teachers were found to have less understanding in terms of identifying, justifying, and making conceptual connecting of major mathematics concepts and principles to the laws of probability, irrespective of their higher level of education attained. In this case, teachers were required to be in a position to interpret, explain and justify their mathematical views on Probability concepts as well as to have an understanding of making conceptual connections of probability concepts rather than reciting memorized concepts, procedures, and rules.

Ives (2007, 2009) carried out a study using a qualitative approach on the definitions and meaning of probability terms such as independent and dependent events, mutually exclusive events, randomness, and Probability. The study found that the majority of the definitions and meanings of terms were shallow and incomplete defined which made them be incorrectly defined. Teachers of mathematics need to have correct knowledge of definitions and meanings of terms used in Probability. This is because the meanings of some probability terms are usually tied to the mathematical application of their concepts, for example, terms such as the independent events for any of the two events of A and B that are independent, are defined as the multiplicative rule of P (A and B) which means P (A) × P (B). This further means that event 'A' does not affect the possible happening of event 'B', then 'A' and 'B' are independent events that require multiplying their separate probabilities to obtain the probability of both happenings. While mutually exclusive events are defined as the addition of P (A or B) which means P (A) + P (B). This also means that sets A and B are mutually exclusive events and cannot happen at the same time and requires adding of their separate probabilities to get the probability of either of them happening. Therefore, it can be seen that their meanings are completely attached to their mathematical application of probability concepts (Batanero & Diaz, 2009). This means that teachers of mathematics need to have a better understanding of the definitions of probability terms that are used to define and describe the Probability terms (Baer, 2008). It is also important to note that, learners' understanding of the meaning of probability-related terms has a strong bearing on their retention of the concepts.

Dean and Illowsky (2012) also stated that understanding the meaning of probability terms is a very important step in developing the ability to be able to understand probability concepts and solve probability problems correctly. Although teachers do not need high levels of mathematical knowledge for them to teach secondary school mathematics, they do require a profound understanding of the basic mathematics that they teach at the secondary school level, which includes a deep grasp of the interconnections and relationships among different aspects of the subject matter knowledge of Probability. Ma (1999) states that understanding the different meanings of probability terms is cardinal for the teachers to acquire the ability to relate different meanings of probability concepts to practical situations as they teach learners. In addition to this, different meanings of Probability should be clearly understood by teachers to correct learners' intuitive ideas of chance and Probability as well as the subjective view of probability as a degree of belief. This means that mathematics teachers need to be well vested with the ability to use the socially agreed conventional meanings of probability terms in defining Probability events.

However, the above-reviewed literature shows that teachers understood the concepts of probability differently.

2.4.2 Teachers' Knowledge of Conceptual Connections

Brijlall (2014) carried out a qualitative research study at one of an educational faculty University in South Africa and investigated the in-service mathematics teachers' pedagogical content knowledge (PCK) for teaching probability in the middle school using open-ended questionnaires to complete. In his study, the teachers were required to list three lessons and solve probability problems which were given in the questionnaires. The analysis of the responses showed that there were overwhelming positive responses such that teachers were able to indicate that the lessons can be sequenced starting with the definitions of probability terms with a task, secondly, the probability scale with a task, and thirdly problem solving involving probability and a task. In terms of problem-solving involving probability concepts, teachers demonstrated their conceptual understanding of probability and this was evident in the manner they selected the procedures that they used to solve the questions. It is also important to note that the teachers further performed satisfactorily in the data handling module in which sixty-three out of eighty-sixing teachers were able to score marks above seventy percent. This means that the teachers had three domains of PCK in their responses which include common content knowledge (CCK), specialised content knowledge (SCK), and knowledge of content and teaching (KCT) (Ball, Thames & Phelps, 2008). Therefore, the findings of this study reveal that mathematics teachers exhibited the subject matter knowledge of probability in their responses necessary for teaching probability concepts.

Furthermore, Ma (1999) carried out a study on teachers' understanding of fundamental mathematics in China and the United States. The findings of her study indicate that Chinese teachers were having a great understanding of conceptual knowledge of elementary mathematics, while their counterparts, the teachers from the United States of America were more proficient in procedural knowledge. About this, Even (1990) brings the notions of the strength of the concept knowledge about the connectedness of a concept to other mathematical concepts at the same time level and beyond, as an important part of PCK in the teaching of probability. In this regard, teachers must be knowledgeable of both the procedural and the conceptual connections of the topics being taught, because their conceptual organization would provide their knowledge of solving Probability concepts. Shulman (1987) and Even (1990) also argued that teachers of mathematics need to know and understand more of their subject matter because teaching is the transformation of knowledge into a form that learners can easily comprehend. Even (1990) further suggests that teachers' knowledge about the connectedness of concepts to other key mathematical concepts can assist learners to understand the concept of Probability comprehensively.

Danisman and Tanisli (2017) investigated mathematics teachers' subject content knowledge of Probability in Turkey and found that all the three sampled teachers of mathematics were able to state the prior knowledge which teachers were required to use to teach probability. Teachers were able to indicate that learners required prior knowledge of ratios, proportions, percentages, fractions, and rational number concepts. However, only one teacher was able to remind learners about the basic prior concepts of probability at the beginning of the lesson, while the remaining two teachers showed limited understanding of the subject matter knowledge of using pre-requisite knowledge of fractions, ratios, percentages, sets, and rational number concepts to represent and solve probability problems. This implies that the majority of mathematics teachers could not make conceptual connections with the pre-requisite topic concepts of Probability and this acts as a stumbling block to the learners' understanding of probability concepts. Wessels and Nieuwondt (2011) in their study revealed that the majority of South African teachers of mathematics were observed with limited knowledge background in the subject matter of knowledge of the pre-requisite topics to probability.

Khazanov and Prado (2010) carried out a survey project study on student teachers who were studying statistics on how to correct students' misconceptions and conceptual understanding about probability concepts in an introductory college statistics course. In their study, student teachers were taught directly on the misconceptions teachers and secondary school learners have in probability such as representativeness, equiprobability bias, and outcome orientation and they responded positively to the course. The above results obtained suggest that it is possible to improve student teachers' conceptual understanding of probability and correct their misconceptions by targeting the misconceptions directly. This was further observed in teachers' efforts in which they successfully addressed secondary school learners' erroneous concepts about probability in their teaching after completing their college course in statistics.

However, the findings of Khazanov and Prado (2010) support Shulman (1986,1987) since they identified knowledge that teachers need to have to teach probability effectively and also an important means of developing PCK in teachers involved in the teaching of probability (Even, 1990). In another study, Khazanov and Gourgey (2005) surveyed statistics student teachers and found that majority of them agreed that secondary school learners' erroneous concepts and beliefs about probability need to be addressed by teaching directly on the teachers who are involved in the teaching of secondary school probability to the learners. However, their agreement to teach directly on learners' erroneous concepts and beliefs about probability showed an important component of PCK in the teaching of probability. This is because teachers were able to realize that, the teaching of probability for conceptual understanding implies a major shift from simply providing formulas, rules, and calculations to address learners' erroneous intuitions and preconceptions.

A related study was conducted by Swenson (1997) using a qualitative research study on mathematics teachers' PCK of Probability. The findings of her study were different from the findings of the above researchers and show that mathematics teachers lacked explicit knowledge and ability to make conceptual connections of probability concepts with other key mathematical concepts and ideas that were needed to be emphasised during the teaching and learning of Probability. Mathematics teachers showed a lack of explicit knowledge in their explanation and justification of probability concepts with the help of examples as they attempted to explain the concepts of independence and dependence to the learners using the experiment of the dice and coin pattern about the daily life situation. Consistently with the above explanations, Kaplan (2008) posits that learners' failure to solve probability problems is a result of mathematics teachers' insufficient knowledge background of rational number concepts and proportional reasoning which assists in interpreting probability concepts to the learners. In agreement with Kaplan (2008), international research has also indicated that most of teachers of mathematics have little experience with probability and share with their students a variety of probability misconceptions and challenges with pre-requisite knowledge concepts of probability which could be the cause of large number of students across the world to have challenges to use their understanding to apply the concepts that form Probability (Batanero, Henry & Parzysz, 2005; Jones & Tarr 2007; Kaplan, 2008). However, lack of adequate knowledge reviewed from the above literature, shows that it has the potential to affect learners' understanding of Probability. However, it is important to note that teachers' lack of understanding of probability in the above-reviewed literature is that by time research were conducted, the probability was relatively a new topic in many countries' curricular, and as such majority of the teacher had no clear understanding of probability (Kazima & Adler 2006) in contrast with this current study. Furthermore, it is also important to note that the findings of Swenson (1997), Wessels and Nieuwondt (2011) were inconclusive on what kind of mathematical concepts teachers had challenges with to make conceptual connections of concepts used in probability and other topics except for concepts of the pre-requisites topics of probability which are other means of representing probability such as in terms of fractions, and percentages. Consequently, Sections 2.5 and 2.6 presents the reviewed literature on the second conceptualised component of PCK and explains on the subdivided descriptors of the study's conceptual framework.

2.5 Reviewed literature on pedagogical strategies and materials used in the teaching Probability concepts

A research study done by Batanero, Canizares and Godino (2005) found that simulation of coins and the use of multiple teaching and learning materials have been helping teachers to explain on the probability concepts and provide simultaneous help in terms of improving learners' and teachers' knowledge of probability concepts. The use of simulation as a strategy allows teachers to be conscious of the incorrect intuitions that surround their learners and themselves, at the same time improves their learners' understanding of probability concepts. Borovenik (2011) argues that simulation of teachings materials such as dice or coin is a vital strategy in the teaching of probability concepts for it helps to improve learners' probability intuitions and materialise probability problems. The learners are physically allowed to observe and become involved in the demonstration of trials, for example determining the actual independence of each toss of a coin, which is a focus of this study. It is therefore important to note that, learners can build their understanding of the subject matter knowledge of probability through learner engagement in a lesson. This could be independent solving of probability problems or involvement in an experiment or concept simulation. Therefore, this kind of teaching strategy has the potential to promote effective teaching and learning among teachers and learners and this could also help to clear the misconception on probability concepts that are in the minds of learners.

Mutodi and Ngirande (2014) also state that in a high school, learners should have the ability to solve the probabilities of compound events and understand conditional and independent events correctly. However, this could be achieved when teachers of mathematics can engage learners in games and experiments that support the development and understanding of the everyday language of probability. For example, the use of an experimental approach to teaching

Probability through experiments and simulations are useful means which could make learners easily understand the law of large and small numbers such as 1 and 0.1 respectively. This also makes learners easily grasp the connections between the notions of relative frequency and Probability, since they are fully involved in the experiment. Therefore, it is important to note that understanding of Probability concepts occurs as a continuous constructive process that allows learners to acquire and relate different meanings of probability concepts to a real-life situation which in turn allow them (learners) to attain the final stage of mathematical formalism and axiomatic (Batanero & Diaz, 2012).

Furthermore, teachers' ability to use various teaching strategies could be one of the key strategies that could assure understanding of probability concepts among learners. Given this, Steinbring (1990) suggests that it is important for mathematics teachers to have an understanding of the nature of the subject matter knowledge involved in probability concepts and the ability to use various ways of teaching the topic. This means that mathematics teachers must have the ability to teach probability concepts based on much more interpretative activities that would engage learners in the lesson rather than based on a hierarchical and cumulative amount of concepts that are learned in a linear sequence and are more complex. In a similar context, Ball, Thames and Phelps (2008) state that interpreting students' thinking, whether in a whole-class discussion or on written homework or quiz, is an essential part of engaging learners in the learning of the subject matter. Furthermore, explaining on major mathematical ideas in whatever the approach or style of teaching, writing assessment questions, drawing a clear diagram and choosing of a counterexample are central in terms of engaging learners in a lesson. This, however, entails that mathematics teachers need to make a major shift in emphasis from simply providing formulas, rules, and procedures for calculating probability problems correctly to a level where learners are taught probability for conceptual understanding through learner involvement such as the use of multiple teaching strategies that could help to address learners' erroneous intuitions and preconceptions (Garfied, 1995; Konold, 1995 & Sharma, 2006).

Mutodi and Ngirande (2014) further in their study recommended that to teach probability concepts effectively, teachers need to develop a habit of engaging the learners in the topic discussion, one-on-one teaching, and comprehensive individualised assessment. Therefore, such approaches would make teaching more effective and meaningful to learners, such that learners become exposed to a variety of situations that illustrate the concepts of randomness, independence, and dependence. Moreover, at times teachers have also their personal experience views that are important and could be used as pedagogical instructional strategies in the teaching of probability concepts which are usually by the requirement of the curriculum (Eichler, 2008). This could help to make mathematics teachers develop self-competence and widen their views in the teaching of probability concepts and this should be fully supported.

Furthermore, several researchers have been using computer modeling to teach about probability concepts and address learners' and teachers' misunderstanding of the subject matter knowledge of probability. For example, for the learners to understand what a random variable is, there is a need to use a model if possible to demonstrate the understanding of the concept of randomness. In a similar case, Krishnamachari (1998) used computer workbooks to teach learners to determine and explain the independence of each toss of a coin and found that three-quarters of learners were able to determine the independence of each toss of a coin. Therefore, the use of computers facilitates learners' conceptual understanding of probability concepts by allowing learners to explore and represent the subject matter in a simplified manner (Jones, Langrall & Mooney, 2007). It is therefore important to note that the use of software technologies and computers to teach probability concepts among the learners works as a tool builder to gain a conceptual understanding of probability concepts. Furthermore, Lee and Hollebrands (2008) in their teaching of teachers on the concept of independence, used software technology which worked both as an amplifier and a reorganizer to engage teachers in tasks that simultaneously developed their understanding and provided first-hand experience of seeing the act of independence in an activity. As such, teachers of mathematics should have the ability to plan and select appropriate teaching strategies that could help to explain clearly the probability concepts.

2.6 Reviewed literature on the use of various strategies in the teaching of Probability

A study project conducted on prospective teachers' content knowledge in the mathematics education course done by Peressini et al, (2004), found that the prospective teachers' lesson demonstration during teaching practice was activity-based in which learners provided important mathematical ideas in groups and also as a whole class. The prospective teacher prepared a variety of learning activities in which learners were given opportunities to explain and justify and make conclusions over their answers. Furthermore, their teaching was characterised by asking constructive questions to elicit particular correct answers from learners using a specific procedure as they were answering the given mathematics problems. Therefore, the findings of this study project show that the prospective teachers' content knowledge of using various teaching strategies was reflected in the manner they presented their lessons which caught up the attention and allowed the learners to participate fully in the lessons.

Regarding the findings of Ball, Ball and Hill (2005); Krauss, Neubrand, Blum and Baumert (2008) and Peressini et al, (2004) contend that a deep understanding of mathematical concepts enables teachers to access a broad repertoire of strategies when explaining and representing mathematical content to their learners. This made learners to be become exposed to a variety of situations that could easily illustrate the concepts of probability which made them to easily understand the concepts. This means that teachers of mathematics had clear understanding of the subject matter which made them to easily engage learners in a class topic discussion that made them to become aware of the multiple embodiments of the mathematical concepts. Therefore, such strategies make teaching more effective and meaningful to learners, particularly when teaching abstract topics.

Furthermore, Danisman and Tanisli (2017) examined mathematics teachers' knowledge of using various instructional strategies in the teaching of probability concepts. The findings of the study showed that the mathematics teachers were able to organize visualised and enriched concrete representations materials. In this context, the teachers were able to bring different concrete representations to the classrooms which included a die made out of cardboard and made experiments, tossing of a coin, a computer, and a rolling die. However, during lesson observation mathematics teachers' content knowledge of teaching probability was reflected in their teaching process, but could not use hands-on strategies, such as cooperative teaching and learning aids strategy, group work activity, learner centeredness, and one-to-one strategy. Instead, teachers used lecturing methods as they taught and demonstrated the probability concepts.

In a related study, Bulut (2001), and Chick and Baker (2005) conducted a study using a mixed-method approach and found that the majority of teachers of mathematics did not possess the sort of repertoires of the subject matter knowledge and also lacked the knowledge to use various teaching ways in the teaching of probability concepts and other related concepts to the learners. In addition, Batanero, Canizares and Godino (2005) also observed that large numbers of teachers of mathematics had shown little knowledge of using various instructions as they taught probability concepts and this meant that they shared a variety of probability challenges with the learners. Jones and Tarr (2007) also confirm that most of the teachers of mathematics avoided teaching Probability because of a lack of the ability to explain and justify the subject matter of Probability and their pedagogy. Chick and Pierce (2008) also carried out a research on secondary school teachers of mathematics using a mixed-method approach and collected data using both open and closed-ended questions on teachers' professional knowledge. The study found that mathematics teachers lacked professional knowledge and this was evident in the manner they prepared their lesson-planning tasks in which they failed to include significant teaching strategies and concepts to the fore, despite the availability of all the opportunities that were inherent in the teaching tasks and resources. This means that majority of teachers could not recognise the concepts to be addressed through that particular teaching strategy. It is important to note that, most of the probability concepts are abstract and they demand teachers of mathematics to be highly cognitive in their subject matter knowledge and the use of multiple pedagogical strategies to easily decompress the conceptual organization of probability concepts to the learners in a comprehensible way. It is also important to argue that, clear understanding of probability concepts by learners is premised on the quality of instruction and the ability to use various pedagogical strategies during the teaching and learning process.

Shaughnessy (2003) also noted that the effectiveness of probability instruction depends on the teachers' subject content knowledge of Probability. Moreover, the understanding of the subject content knowledge is a determinant of teachers' effective way of teaching probability concepts (Ball, 1990; Halim & Meerah, 2002). In light of the above, Krauss, Neubrand, Blum and Baumert (2008) state that teachers of mathematics do not only need to understand the mathematical concepts, but they also need to know how these concepts could be best explained to the learners. For instance, teachers of mathematics should be able to engage learners in one-on-one teaching and be able to prepare comprehensive individualised assessments to investigate the intuitions that learners have about specific probability concepts. In this case, research studies contend that a deep understanding of mathematical concepts enables teachers to access a broad repertoire of strategies when explaining and representing mathematical content to their learners (Ball, Bass & Hill, 2005; Krauss, Neubrand, Blum & Baumert, 2008). For example, teachers of mathematics who have a clear understanding of probability concepts can easily engage learners into a discussion using multiple embodiments of probability concepts. In this situation, it means that learners become exposed to a variety of situations that could easily illustrate the concepts of probability so that they easily understand the concepts. Therefore, such strategies make teaching more effective and meaningful to learners, particularly when teaching abstract topics. In regard to the above assertions, Section 2.7 presents the reviewed literature on the third conceptualised component of PCK and explains on the subdivided descriptors of the study's conceptual framework for further successful teaching of abstract mathematics topics such as probability.

2.7 The pedagogical enabling prompts that could be used for successful teaching of abstract mathematics topics

As to whether the teacher's lesson is learner or teacher centered does not seem to relate in any way to performance but what is critical is when the lesson promotes understanding of the subject matter being taught, in which learners attach meaning to what is being taught through designed pedagogical enabling prompts which engage learners with simplified understanding of the subject matter (Hill, Ball & Schilling, 2008; Maja, 1998). Seah (2007) states that effective teaching is undoubtedly the most important objective in secondary school mathematics education. While, the econometric analysis suggested that some teachers are dramatically more effective than others and that these differences have lasting effects on student learning (Rivkin, Hanusahek & Kain, 2005). Likewise, Larson (2002) states that some mathematics teachers are more proficient in teaching than others, this is because they do certain things in common when providing mathematics instructions, whether they use the student-discovery or teacher-centered strategies. As such, teachers need to show additional efforts away from the use of learner engaging teaching strategies in the teaching of probability which could support learners' understanding of the subject matter as they are engaged in the teaching and learning process. In light of this, international research suggests that the use of pedagogical enabling prompts is one of the most effective way that assist learners to think about core mathematical ideas, provides an entry point to solve challenging tasks, and reduces the complexity of the given task to the level to which learners could easily understand the subject matter (Cheeseman, Dowton & Livy, 2017).

Similarly, Sullivan, Walker, Borcek and Rennie (2015) also state that the use of pedagogical enabling prompts could play a major role in making learners to easily grasp the interconnections and relationships among different aspects of the subject matter knowledge of abstract topics. In this regard, different pedagogical enabling prompts have been suggested by different scholars. Research has found that teaching with challenging mathematics problem activities could help to impart learners with clear understanding of the subject matter (Russo & Hopkins, 2019), despite this approach being pedagogically demanding for teachers (Stein, Engle, Smith & Hughes, 2008). Furthermore, the use of challenging classroom activities does help learners to develop abilities to connect different aspects of mathematical concepts to devise and explore more than one pathway to solutions in solving of the given mathematics problem activities (Sullivan et al, 2013). In addition, research has also shown that the use of different and sophisticated explanations provides different entry points into the discussion, give directionality to the learning, and support the development of flexible reasoning among learners (Cirillo, 2013). It is also important to note that, designing of an accurate and to the point explanations, written recordings and models have been shown to provide scaffolding for learners to discuss, trial and contribute to their developing awareness of themselves as legitimate creators of mathematical knowledge (Anthony & Walshaw, 2007; Attard, 2013). This means that the use of models and scaffolding alerts learners how worth knowing, exploring and synthetizing their ideas and make subject connections (Choppin, 2011; Fraivilling, Murphy & Fuson, 1999; Hunter, 2007).

Furthermore, Shulman (1986, 1987) states that engaging with learners' misconceptions is viewed as one of the strategy and component of PCK, which can also be considered as a pedagogical enabling prompt that could help to provide different entry points into the discussion and give directionality to the learning, and support the development of flexible reasoning among learners (Cirillo, 2013). In this case, learners are first provided with space to demonstrate their understanding of the subject matter knowledge and later concentrate on both their challenges and on what they know. Moreover, Thames, Ball and Bass (2008) also identified some of the mathematical tasks which teachers were using during their teaching and learning which could be considered as pedagogical enabling prompts (Cirillo, 2013), and these are; explaining mathematical ideas using mathematical language, representing the mathematical ideas, interpreting and modifying the mathematics tasks, and generating and analysing alternative solutions procedures (Nalube, 2014). This study assumes that engaging learners in probability could easily help teachers to teach learners while they are involved in knowledge construction.

Sullivan, Walker, Borcek and Rennie (2015) further state that enabling prompts in the teaching of abstract mathematics topics involves reducing of unnecessary number of steps in problem-solving, simplifying the complexity of concepts, while the use of visual cues and various forms of concrete representation of the subject matter allows learners to gain clear understanding of what a task is asking to do and help them to proceed with understanding as they solve challenging tasks. As such, Posamentier and Stepelman (1999) state that mathematics teachers need to be abreast with a broad range of specific teaching enabling prompts within themselves such as: ability to help and maintain the student students' positive attitudes towards the learning of abstract mathematics topics, being sensitive to the students' feelings, valuing every student's contribution, recognising students' needs for success, involving learners in their lessons, and by making of mathematics exciting and interesting. In this regard, Seah (2007) indicated that effective teaching and learning is a function of interactions between teachers and their students, between and amongst students, and between the class and the school environment.

Shahrill, Ismail and Mundia (2015) investigated and examined a range of factors contributing to smooth teaching and learning strategies in the upper secondary classes which included the knowledge, beliefs, understandings, and practices of mathematics teachers, their qualifications, professional developments, and relevant personal experiences, and how these impact student learning outcomes. Similarly, Ingvarson et al, (2004) identified several key factors impacting on teachers' smooth teaching of mathematics, including knowledge of the subject, PCK, the organisation and application of subject matter, teacher beliefs about teaching. Furthermore, Ingvarson et al, (2004) theorised the content knowledge described by Larson (2002) that there are four main factors that influence the effectiveness of students' learning outcomes in mathematics and these are: the school enabling conditions and these are conditions that exist in the school environment where the students spent about 90% of their time. Secondly, the teacher enabling conditions', include teachers' experience and professional development which are taking place at the school level. Thirdly, the teachers' capacity which include knowledge of the subject matter, beliefs, and understanding of teachers towards learners and the teacher practice which simply means what teachers need to do while in a classroom as a teacher.

2.8 The identified research gaps in the literature review

As it has been shown in the studies reviewed in chapter two above about the mathematics teachers' understanding of Probability, shows that the majority of the studies reviewed focused their studies on the pre-service and in-service mathematics teachers' understanding of content knowledge of abstract topics such as calculus, probability and tangent function whilst studying in their respective Universities. Furthermore, two-quarters of the reviewed literature also looked at teachers' experience in the teaching of probability and their highest levels of qualification, Bachelor's degree in education was used to determine the selection of the sample, in which the sample was investigated in terms of their knowledge of representativeness, equiprobability, and outcome orientation in the teaching of probability. However, this current study focused its study only on the experienced in-service mathematics teachers who had taught probability for ten years and above with different qualifications (See Table 4.1, Section 4.2). This study investigated mathematics teachers' PCK of teaching probability in terms of their ability to solve, explain and justify their probability reasoning. Whereas, the reviewed literature investigated the sampled mathematics teachers' profound understanding of the mathematical knowledge for teaching probability using pre-requisite knowledge of fractions, ratios, percentages, sets, and rational number concepts to represent and solve probability problems and translate tangent functions representations into correct graphical forms. The reviewed literature also further focused on mathematics teachers' understanding of the different meanings of probability terms, explanations, and justifications of their reasoning they hold in the context of Probability, Calculus and tangent functions.

This study further focused on mathematics teachers' ability to use socially agreed qualitative terms as they define probability terms as well as justifying their mathematical views. Furthermore, this study focused on mathematics teachers' ability to identify, plan, select and use multiple pedagogical strategies and enabling prompts if the teachers are to teach effectively specific concepts of probability. This study was also different from the studies that were reviewed in terms of its context and research methodology used such as research approach, and design, and instruments used to collect data. The majority of the reviewed literature used mixed-method and multi-case studies research approaches and in collecting data they used questionnaires, interviews, and focused group discussion. While, this study investigated secondary school mathematics teachers' Pedagogical Content Knowledge of Probability using a qualitative approach multiple case study designs and collected data using an SMKDT, lesson observation, and semi-interview schedules, and document analysis. Furthermore, regarding the local and international reviewed literature, this suggests that there is no study has been carried out in Zambia and internationally regarding Mathematics teachers' PCK of probability. The absence of a study that either collaborates or disagrees with the reviewed literature justifies the current study. Hence, a gap.

2.9 Summary of Chapter Two

The chapter has presented the findings in line with the studies reviewed that have shown that student and in-service mathematics teachers exhibited subject matter knowledge in terms of their ability to define, explain, interpret and identify learners' mathematical challenges such as common errors that learners make. Teachers demonstrated conceptual understanding of probability and this was evident in the manner they were selecting and using the procedures in the teaching of mathematical questions. Furthermore, the prospective and in-service teachers exhibited subject matter knowledge in the manner they used various teaching strategies which were reflected in the lessons they presented which caught up the attention and allowed the learners to participate fully in the lessons. This was evidence of elements of PCK of teachers for teaching mathematics as the focus of this study. It must be understood that PCK also includes an understanding of what makes the learning of specific topics easy or difficult.

On the other hand, the reviewed literature found that teachers showed less understanding of probability concepts and could not make clear conceptual connections with other major mathematical concepts and principles to the laws of probability irrespective of their level of education they have attained. As such, the majority of teachers avoided teaching this topic because they had less understanding of how to present the teaching and learning material such as the tossing of a coin in a realistic way to simulate probability concepts such as sample space, probability of an event, conditional probability, independence and randomness concepts. The reviewed literature indicates that teachers of mathematics exhibited insufficient knowledge in the use of various teaching strategies as they taught probability concepts. This caused mathematics teachers to teach Probability concepts using plain instruction and lecturing strategies that could not allow them to use hands-on teaching strategies such as group work activity, independent problem-solving simulation of concepts, and one-to-one teaching strategy. Therefore, the differences in subject matter knowledge for teaching probability and other topics which have been reviewed in the selected literature could have been caused as a result of teachers' lack of enabling conditions such as teachers' experience, qualifications, professional developments, subject knowledge, beliefs, understandings and practices of mathematics teachers, and relevant personal experiences. It is, therefore, important to note that probability concepts are abstract and they need to be taught to use various strategies to easily unpack its conceptual organisation to the learners in a comprehensible way. The next chapter presents the methodological aspect of the study which includes the approach and design and analysis of the study and others.

CHAPTER THREE: RESEARCH METHODOLOGY

3.1 Introduction

The previous chapter discussed the literature which was reviewed. This chapter focused on the methodological aspects of the philosophical assumptions of the study, research approach and design, the study area, target population, sample and sampling techniques, and data analysis. The chapter also presents the data collection instruments such as preliminary observation interviews, subject matter diagnostic test (SMKDT), lesson observation, interview schedules and document analysis. The chapter also looked at data analysis of the answers which were obtained from the SMKDT, lesson presentation, unstructured interview and document analysis involving the purposively selected seventeen (17) senior secondary school mathematics teachers. Thereafter a summary of the chapter was provided.

3.2 Philosophical Assumption of the Study

This study was guided by a qualitative research paradigm coined by Guba and Lincoln (1985) and Mclnnes, et al, (2017). A research paradigm is a set of beliefs system or world views that guides the investigations and provides the basis for interpreting and understanding social reality (Creswell, 2013; Cohen, Manion & Morrison, 2000; Patton, 2002). Nevertheless, this research study was studied from the context of the naturalistic view, this is because the collection of data was undertaken during normal teaching hours (Guba & Lincoln, 2005). It is also important to note that, the intention of using this qualitative naturalistic paradigm was to acquire within its context the in-depth meaning of teachers' experiences and perspectives in the teaching of probability with as little disruption to its natural settings as possible. This means that teachers' experiences and perspectives in the teaching of probability were means through which teachers' understanding of probability were explored and understood the social reality of their knowledge. It is also important to note that, the meaning of social reality in this study was also achieved by studying teachers' knowledge using their natural settings or normal organization of classes, attempting to make sense of, and interpret, phenomena in terms of the meanings people ascribe to (Guba & Lincoln, 2005). This was done without the generalisation of the findings of the study to other institutions and situations. As such, the paradigm was used to explore mathematics teachers' pedagogical content knowledge of probability in terms of the nature of the subject matter knowledge they were holding, ability to use multiple pedagogical strategies, and to design enabling prompts to support learners' understanding of probability during lesson presentations. In this context, the naturalistic paradigm postulates that the meaning of reality is understood through the participants' views and their views become shaped by the presence of the theoretical and conceptual framework of the study. This implies that this paradigm's belief in social reality is constructed from multiple realities, where the participants' ideas provide a platform for exploring and understanding the reality of the phenomena. In light of this study, the researcher's beliefs, perspectives, principles, and premises are defined within this research paradigm, shaped and acted in it, and guided the selection of the research instruments and methods (Denzin & Lincoln, 2011). In this context, Guba and Lincoln (1994) proposed four premises that link to the researcher's belief, which include ontology, epistemology, methodology, and methods. However, these assumptions played a key role in the understanding of teachers' PCK of probability. For example, the ontology and the epistemology which operate alongside each other informed the theoretical perspective of this study about what it is, and what it means to know (Crotty, 1998).

In this study, the Ontology assumption explored the nature of reality of the subject matter knowledge of probability held by the sampled teachers and to explore the knowledge, the paradigm posed questions such as what is the form and nature of reality held by the phenomenon (teachers) and what can be known about that reality from the phenomenon (teachers) (Lincoln & Guba, 1985; Ponterotto, 2005). Regarding this, this assumption states that reality is a social construct that is constructed through interaction with multiple viewpoints of participants that refute a single true reality (Lincoln & Guba, 1985). As such, the nature of reality of teachers' PCK was understood through thoughtful analysis of the experiences and responses they gave in the SMKDT test, lesson presentations, documents analysis, and through interviews which assessed their form and nature of reality they held at the time of research in probability (Creswell, 2003). As such, these different experiences and responses formed multiple meanings of the reality of the teachers' PCK of probability.

The second assumption which was used to explore the reality of teachers' knowledge was the epistemology, which is simply a way of understanding and explaining how reality is known, what is known (Crotty, 1998), and states that the dynamic interaction that existed between the researcher and participants was central to capturing and describing of the nature of reality held by participants (Ponterotto, 2005). In this study, the sampled teachers felt comfortable, and shared, and discussed their experiences of teaching probability in the follow-ups interviews as they responded to their performance in the SMKDT test and to their lesson presentation. The good relations and sense of rapport that existed between the researcher and participants made them to view this study as an opportunity for them to develop personally and professionally as teachers and the same knowledge was seen to be beneficial to learners (de Vaus, 2001). As such, through the above interaction, the teachers' reality of the nature of the subject matter knowledge of probability they held was made known to the researcher as described in the next chapter four. This means that reality could be further understood through the explorations of peoples' multiple interactive experiences within their social world. From these two perspectives, it can be seen that the paradigm emphasized the need to explore and understand the participants' nature and forms of knowledge held and their ability to transform knowledge to others using validated and objective-based study strategies (Cohen, Manion & Morrison, 2007; Patton, 2002).

The third assumption of this study's paradigm was the Methodology, which is the theoretical, and philosophical process and procedure of carrying out research (Denzin & Lincoln, 2011). In this study, the methodological aspect of this paradigm provided guidelines for the selection of appropriate research design and data collection instruments that was used to explore the real nature of knowledge held by phenomenon (Wilson & Peterson, 2006). In addition, the methodological aspect of the premise of this paradigm guided this study to use assertions, models, and notions that explain how data may be best interpreted so that new knowledge may be discovered. This further helped the researcher to plan and select data collection instruments that could be used to elaborate the subject matter in the contexts of the study's objectives, meanings, and individuals' interpretations. The fourth assumption considered was methods, which provided specific means of collecting and analysing data. However, the above assumption provided the lens to this study to explore mathematics teachers' subject matter knowledge, the effectiveness of the identified pedagogical strategies, and enabling prompts designed to support learners' understanding as they teach probability.

It is important also to note that the qualitative naturalistic paradigm is not synonymous with qualitative research methods, but the paradigm generates detailed qualitative data using data collection methods such as semi-structured interviews, observations, and document analysis (Erlandson, Hams & Allan, 1993; Guba, 1993). In light of this, Gall, Gall and Borg (2003) posit that these methods do provide detailed written and verbal information which is usually audio and video recorded to preserve the information for subsequent data analysis. Furthermore, the perspective of the assumption of the paradigm emphasizes the need to validate the reliability of the instruments prepared to collect data through the use of external validation and aligning of the instruments to the prepared objectives of the study (Guba & Lincoln, 1994). This, however, gave guidelines to the researcher on how to develop data collection instruments that need to pass through the rigorous process of validation to have trustworthy results. The paradigm also emphasised the need to analyze the results using the categories of the theoretical and conceptual framework of the study. This was meant to avoid prior outcomes and report the results as they were given without generalising them (Guba & Lincoln, 1985; Patton, 2002). It is, therefore, important to understand that the paradigm upholds multiple realities, the use of validated instruments and the non-generalisation of the results defined the focus of this research study.

The researcher explored the seventeen (17) senior secondary school mathematics teachers' subject matter knowledge of Probability found in Chongwe District, Lusaka province, and thereafter reported the findings as they were given. This was in agreement with Patton (2002) who argues that believing in a particular verified reality and truth simply means that the posture of the knower detaches from knowing the reality but believing in socially constructed multiple realities brings an understanding of the true reality. Hence, this motivated the researcher to choose this paradigm because it was in line with the researcher's view of having findings that are not influenced by one's mere thinking, but rather by the results obtained from multiple realities of the respondents. This means that no single phenomenon could suggest the nature of knowledge held by the phenomenon. In conclusion, the paradigm plays a key role in guiding this study's world views about the understanding of social reality, means of exploring and analysing the nature of the subject matter knowledge of probability held by teachers, the effectiveness of the identified pedagogical strategies, and the ability to design enabling prompts that would support learners' understanding of Probability.

3.3 Approach and Design of the Study

The study was qualitative in approach and utilized a multiple case study design. There are several types of case studies, because of different reasons these case studies could be either single or multiple case study (Gustafsson, 2017; Patton, 1990). In this study, a multiple case study design was used to explore the in-depth understanding of teachers' PCK of probability. Yin (2003) defines a multiple case study as a rich and vivid design that enables the researcher to explore differences and similarities within and between cases. Patton (1990) states that a multiple case study approach typically involves intense analyses of the findings from a large sample or population, which was the focus of this study. Thus, the choice of using a multiple case study design was a result of having a larger number of participants which the researcher purposively selected to explore in detail their pedagogical content knowledge of probability. It is also important to note that this study used a multiple case study design to allow the researcher to analyse the data both within each and across a situation (Yin, 2003).

According to Creswell (2013) a multiple case study design helps to explore multiple real-life bounded systems through detailed in-depth data collection involving multiple sources of information. Merriam (2009) further explains that a multiple case study allows the use of several data collection methods such as testing, observations, and interviews and has the potential to provide detailed descriptions of the data set collected. Nieuwenhuis (2014) also posits similar views that a multiple case study involves the use of multiple techniques when collecting data. This means that a multiple case study was the best research design that was used to conduct an in-depth investigation of mathematics teachers' pedagogical content knowledge of Probability. This is in line with the research studies that have argued that a multiple case study design generates detailed qualitative data about the phenomena (Erlandson, Hams & Allan, 1993; Guba

& Lincoln, 1994). Based on the above research studies, it can be argued that a multiple case study is seen to be appropriate for this study as it aimed at investigating in detail mathematics teachers' PCK of probability that enhances their teaching ability of abstract topics like Probability. In addition, the use of this design helped to provide a holistic and in-depth explanation to address complex issues based on the inputs and interpretative perspective of the participants about PCK of probability. Baxter and Jack (2008) further indicated that the findings generated through the use of a multiple case study are strong and reliable, which means that the design helped to generate reliable and trustworthy findings. Yin (1994) also argues that multiple cases strengthen the results by replicating the patterns thereby increasing the robustness of the findings. Although, Merriam (2009) suggests that the use of a multiple case study makes the interpretations of findings more compelling but the design helps to collaborate, qualify and extending the findings that might occur. In a multiple case study design, it is possible to work on each case that has been considered or covered in the study by dividing the cases into several sub-units (Baxter & Jack, 2008).

Furthermore, a qualitative approach in this study was considered to be appropriate as it was necessary to seek insight into the mathematics teachers' PCK of probability as they taught Probability (Merriam, 2009). In addition, Creswell (2007) explains that the qualitative research approach is best suited to address a problem where the variables are not known, which was the case of this study. To achieve this, the sampled teachers of mathematics were subjected to writing a Subject Matter Knowledge Diagnostic Test (SMKDT) and to conduct a lesson presentation on selected Probability concepts. Thereafter, the same teachers of mathematics were interviewed as a follow-up to explain and justify their answers presented in the SMKDT test and their lesson presentation schedule. This provided an opportunity to examine the sampled mathematics teachers' capacity to use multiple pedagogical strategies which simultaneously assisted in the discovery of their strengths and pedagogical challenges as they taught probability concepts.

3.4 Study Population, Sample, and Sampling Techniques

The population was composed of all senior secondary school teachers of mathematics who had taught for ten (10) years or above. The researcher used the purposive sampling technique to select seventeen (17) teachers of mathematics with teaching experience of at least ten years and above as participants in the study. In purposive sampling, researchers select the respondents in the sample based on their judgment of their typicality (Cohen, Manion & Morrison, 2000). This means that the researcher targeted the respondents who could give the required information based on their experiences. The idea of having a sample of seventeen was consistent with a qualitative multiple case study design that seeks an in-depth understanding of cases (Merriam, 2009). It is also important to note that the selected sample of seventeen was sufficient as the study is qualitative and required extensive narration of the results. Cohen, Manion and Morrison (2000) state that there is a need for the researcher to select a set of participants who can meet the specific need of a study. In addition, it was also considered an appropriate decision to select mathematics teachers who had experience in the teaching of Probability at the senior secondary school level for a long time. In line with the above, Silverman and Thompson (2008) state that experienced teachers have knowledge bases such as subject matter content, pedagogy and have the ability to contextualize which could be quickly and easily noticed as the teachers are engaged in the act of teaching.

Cole and Ormond (1996) also pointed out that experienced teachers have an accumulated wealth of in-depth conceptual knowledge of the subject matter which could be easily identified as the teachers are involved in the teaching. Thus, this made this study purposively select mathematics teachers with ten years and above teaching experience as the suitable population for this study, to be investigated if they have accumulated enough subject matter knowledge and pedagogy in the teaching of Probability.

3.5 Data Collection Instruments and Methods

In this study, a Subject Matter Diagnostic Test (SMKDT) was utilised to collect data on teachers' subject matter knowledge of Probability. Lesson observations and unstructured interviews were employed to gather information relating to the respondents' pedagogical strategies and enabling prompts used to support learners' understanding as they taught Probability. In addition, document analysis provided insight into the teachers' abilities in terms of planning skills. A preliminary observation and interviews were also conducted before carrying out the actual data collection. In the following sections, the reasons for, and purpose, and details of each aforementioned data collection instrument and methods are provided.

3.5.1 Subject Matter Knowledge Diagnostic Test (SMKDT)

The development of the SMKDT was preceded by a document analysis of the Zambian secondary school mathematics syllabus 10-12, Grade 12 mathematics past examinations question papers and mathematics textbooks. The SMKDT questions were adapted from the final Grade Twelve Examination Council of Zambia past papers from 2013 to 2018 and the recommended Grade eleven and twelve mathematics textbooks by the Ministry of General Education (MOGE). However, the questions adapted were modified to suit this study. The modification of the questions involved the rephrasing of the questions, the addition of questions that required the teachers to predict, state, explain, and justify their mathematical views on the use of certain probability concepts. In this case, the main reason for using the adapted questions from the past papers and mathematics textbooks was that the questions had already passed through the rigorous process of validation, and in them, they contained the subject matter of Probability of which this research study was looking for. The instrument (SMKDT) in nature was a diagnostic test that was designed to diagnose sampled mathematics teachers' understanding of the subject matter knowledge of Probability. Creswell (2012) posits that it is important to be clear about the type and purpose of the test instrument being developed. In this study, the researcher used the diagnostic test which was used to diagnose the consistency of mathematics teachers' subject matter knowledge of Probability. Consequently, draft test questions (See Appendix 10) were prepared for the study in line with the subject matter of Probability in the Senior Secondary School Curriculum. The questions were also developed in line with the descriptors of the study's conceptual framework in the category of the subject matter knowledge of probability (See Figure 1.1, Section 1.8.1). Before assessing mathematics teachers' subject matter knowledge of Probability, the drafted test questions were analysed question by question to ascertain if the content of Probability was included, as intended in the Zambian senior secondary school curriculum and according to the objectives of the study.

During the piloting process, two senior mathematics lecturers in the department of mathematics from Chalimbana University and three Grade twelve mathematics examiners and markers from the Rufunsa district were then consulted to comment on the suitability of the drafted test questions. This is in line with the philosophical assumption of this study that emphasises the need to judge the reliability of the instruments prepared to collect data through external validation and aligning the instruments to the prepared objectives of the study (Guba & Lincoln, 1994). Because of this, Creswell (2012) advises that participants whose characteristics are similar to the sample can ensure successful pilot results, and this was the case for this study. However, the drafted test questions were observed with inconsistencies in the subject matter knowledge of Probability about objective number one. Some questions were also poorly designed such that they were likely to cause misunderstanding of questions which could have resulted in having the invalid analysis of the results (Creswell, 2008). Furthermore, the drafted test questions did not have questions that required teachers to interpret, explain and justify their mathematical views that could have allowed teachers of mathematics to show their understanding of Probability concepts (See Appendix 9). Thereafter, the corrected draft test was piloted involving ten senior secondary school teachers of mathematics in the Rufunsa district. Hence, the piloted draft test showed that questions were clear, well-phrased, and measured the content according to the objectives (See Appendix 1). These efforts were intended to enhance the validity of the SMKDT and it provided a chance to estimate how long it would take the sampled teachers of mathematics to finish writing the test.

The final corrected SMKDT test (See Appendix 1) consisted of seventeen (17) questions with a total of 95 marks and questions had scoring marks ranging from 1 to 3. The SMKDT questions were designed in the manner that they required every given answer was to be accompanied by a well-articulated useful explanation, interpretations, and justification of their mathematical answers. This was meant to diagnose fully the sampled mathematics teachers' understanding of the subject matter knowledge of probability in terms of their ability to solve probability problems using the correct procedure, interpretations, explanations and justifications of their answers using probability reasoning as well as the correct use of probability terms and notation symbols. This was further meant to avoid including teachers' answers which were easily obtained in the analysis and claim wrongly that teachers showed their understanding of probability. Marks below 50 pass mark represented mathematics teachers' lack of consistency and inability to answer questions correctly. This includes questions that were left unanswered and others were incorrectly answered and these were all deemed as incorrect answers. This was an indication that the teachers lacked consistency in the mastery of the subject matter knowledge of Probability and the ability to answer correctly higher-order questions in the SMKDT test.

Furthermore, the researcher allocated 50 mark and above as a pass mark representing mathematics teachers' consistency and ability to answer questions correctly. The 50 pass mark and above was an indication of mathematics teachers' mastery of the subject matter knowledge of probability concepts. The 50 pass mark was more than half of the 95 total marks of the questions prepared. This was intended not to allow teachers to easily reach the pass mark. In terms of preparation of the research questions, they were consistent with Probability concepts as prescribed in the Zambian senior secondary school curriculum (See Appendix 1). As such, questions in the instrument required the teachers to define probability terms according to the socially agreed conventional meanings of probability terms, to illustrate probability concepts about real-life situations as well as to state and show conceptual connections with other mathematical ideas thereby solving Probability problems using correct procedures and identifying probability concepts. This would also make them predict and anticipate challenges that learners are likely to encounter when solving probability problems as well as to interpret, explain and justify their mathematical reasoning on specific probability ideas. What follows in Section 3.5.2 are matters of lesson observation schedule used to collect data.

3.5.2 Lesson Observation

Sidhu (2014) states that lesson observation is one of the most important research methods. This is because the method uses a natural way of collecting data and the data collected through observation is more real and true. The method also depicts exactly what transpires on the actual ground or in class as the teacher is involved in teaching which is also captured through the use of a video or audio recording. However, this makes it possible to record all the activities happening, thus representing a candid and first-hand record of the phenomenon of interest (Merriam, 2009). The use of the observation method generates detailed qualitative data which is usually recorded and preserved for subsequent data analysis (Merriam, 2009). In addition, the use of the method has been beneficial to the researchers, this is because it provides the most inclusive and least intrusive way of capturing what happens in classrooms and also helps to unfold rich knowledge, complex interactions, insights into emotions and depth understanding of concepts during the analysis of recorded data. The use of the observation method in this study was considered to be appropriate as it was necessary to seek insight into the mathematics teachers' PCK of probability as they taught probability. Wiersma and Jurs (2009) explains that the use of the observation method increases understanding of the context of teachers' method of teaching on which normal and routine interactions and their meanings become less ordinary.

It must be also noted that the researcher went into lesson observations with the prepared pre-categories of the lesson observation guide on the subject matter knowledge which were developed based on the performance of teachers in the SMKDT, and pedagogical strategies and enabling prompts from the reviewed literature. Following the analyses of the teachers' performance in the SMKDT, concepts that proved challenging and easy to the teachers became the focus of the lesson observations and interviews (see Appendix 5). This was meant to assess their insights ability in the understanding of probability concepts. In addition, the elements in the teachers' subject matter knowledge, pedagogical strategies and enabling prompts categories of the study's conceptual framework (See Figure 1.1, Section 18.1) also guided the preparation of a draft observation sheet. The developed lesson observation sheet focused on specific pre-defined probability concepts, pedagogical strategies and enabling prompts (See Appendixes 3, 4 & 5). This means that the observation sheet was developed before and refined after the lesson presentation. Before carrying out a lesson observation, the drafted concepts were analyzed concept by concept to ascertain if the content of Probability was included, as intended in the Zambian senior secondary school curriculum and according to the objectives of the study. The researcher also piloted how to record and capture relevant data during lesson presentations. Thereafter, the observation sheet was improved with the help of two senior mathematics lecturers in the Department of Mathematics from Chalimbana University and three Grade twelve mathematics examiners and markers based in Rufunsa District and the lesson observation sheet focused on teachers' subject matter knowledge, ability to plan, select and use appropriate pedagogical strategies and enabling prompts to teach probability concepts; use of identified multiple pedagogical strategies to effectively teach specific concepts of probability; handle challenges about probability concepts (subject matter knowledge) and to assess the enabling prompts that teachers used to support learners' understanding as they taught probability concepts. Thereafter, the researcher interviewed each teacher on the teaching strategies they preferred to use to teach probability concepts, and their responses are presented in Table 4.5 Section 4.5.1. Before carrying out a lesson observation on mathematics teachers' subject matter knowledge of probability, the researcher had to consider the right position on which the camera could be positioned to capture all the demonstrations which were made by teachers and learners on the blackboard. In addition, recording guidelines were also established within each class such as not walking in front of a camera and not touching the audio recorder placed in front of a teacher. This was intended to make their lesson presentation a typical and true representation of what normally happens in their classrooms.

It is important to note that all the lessons presented were video and audio recorded to have firsthand information and to avoid generalisations of the teachers' subject matter knowledge as well as to easily note what was not recorded, and correlate and compare with notes from the researcher (Merriam, 2009). The use of the observation was accompanied by taking field notes regarding the unsolicited teachers' comments on the subject matter which occurred before and after recording and the researchers' observations and reactions during lesson presentations. The emphasis for having notes was to describe how teachers taught without evaluating and avoiding making inferences and use of vague terms which teachers did not use as they taught probability. During lesson presentations, the researcher further participated by assisting in assessing learners' work in terms of making conceptual interconnections and relationships among different aspects of the subject matter knowledge of probability in the process of solving of given probability problems. This, however, made the researcher participated as a participant-observer in which he observed, marked and was involved in recording all the lessons which were presented by the sampled teachers. The use of the audio and video recording helped the researcher in the process of analysing data in which the video and audio data were always re-visited to observe how the teachers were demonstrating as they taught during lesson presentations on certain concepts. It is also important to note that the lesson observation for teachers' presentation were conducted at the time when all Grade twelves' had already learned the topic, and the lessons were conducted in their respective schools where they were purposively selected. This was meant to allow the sampled mathematics teachers to teach the same learners whom they have been teaching in order to have trustworthy results. The researcher also took time to analyse the schemes of work, lesson plan and the records of work. He also checked on the strategies and enabling prompts which teachers have been using in the teaching of probability. What follows in Section 3.5.3 are matters of the unstructured interview schedule used to collect data.

3.5.3 Unstructured Interview Schedule

The unstructured interviews were also used to collect data in terms of mathematics teachers' performance in the SMKDT, the pedagogical strategies and enabling prompts used to support learners' understanding of the subject matter during lesson observation. The rationale behind the use of interviews in this study was to have the participants reflect on their experiences in the manner they answered the SMKDT questions and the way they taught probability concepts. Thereafter, relate those experiences for the researcher and the teacher to have a mutual understanding of the meanings of the experiences or accounting of their experiences (Bryman, 2001). In addition, the interview schedule was developed based on the nature of answers that teachers of mathematics gave in the SMKDT whether correct or incorrect, strategies and enabling prompts used to engage and supporting learners to understand probability concepts. The schedule also included questions that required teachers of mathematics to give clear explanations as to why they used the strategies that arose as they taught Probability concepts, for example, the concept of independence, dependence, and mutually exclusive events. Not only that, the purpose of the interview schedule was to assess the mathematics teachers' understanding of the subject matter knowledge of Probability as a follow-up regarding what they wrote in the SMKDT, the pedagogical strategies and enabling prompts used as they taught Probability (See Appendixes 4 and 5). Creswell (2009) state that interviews are well suited for exploring and confirming ideas and provide in-depth information about particular cases of interest, as such the researcher managed to assess mathematics teachers' subject matter knowledge of Probability face to face with the researcher and they also asked questions for clarifications as sometimes questions were interpreted differently. Therefore, this kind of interaction allowed the researcher to gain an in-depth understanding of mathematics teachers' beliefs, values, reality, feelings, and experience of the phenomenon, the pedagogical content knowledge of Probability. What follows in Section 3.5.4 are matters of the document review and analysis used to collect data

3.5.4 Document Review and Analysis

Document analysis was part of data collection instruments on which the researcher carried out a cross-check on the mathematics teachers' abilities in terms of their planning skills. The following documents were gathered for analysis which included schemes of work, lesson plans, records of work and learners' mathematics books (See Appendix 8.10). The researcher checked on mathematics teachers' lesson plans, schemes of work, records of work and the teaching strategies they have been using to teach specific concepts which required the use of multiple strategies and enabling prompts to support learners' understanding as they taught probability concepts. However, the researcher remained faithful to know whether the documents contained unmitigated truth and were written for a specific reason or the audience beyond this study, contexts in which they were written, age, authenticity, and accuracy (Merriam, 2009). White (2005) states that documents such as teachers' lesson plans, records of work, schemes of work, and learners' mathematics exercise books provide useful information that could not be gathered in an interview. He further argues that when other techniques fail to provide answers to certain questions, documentary evidence does provide convincing answers. Therefore, the above-mentioned documents provided a platform for the researcher to learn about the teaching materials and strategies which teachers have been using in the teaching of probability. What follows in Section 3.6 are matters involving ethics in research during the data collection process.

3.6 Ethical Consideration

Ethical principles were paramount in planning and conducting this research, as evidenced by the approval granted by the University of Zambia faculty of Education Ethics Committee (See Appendix 13). The researcher further applied for a research authorisation permit from the Chongwe District Education Board Secretary (DEBS) Office to visit schools and select participants and collect data from sampled teachers (See Appendix 14). Ethical issues were highly considered in this study. Babbie (2007) contended that the fundamental ethical rule of social research is that it must bring no harm to research subjects. In this context, harm is defined as physical and psychological, requires diligence on behalf of the researcher to ensure participants' voluntary participation, informed consent, anonymity, and confidentiality. Habibs (2006) also stated that ''participants must be fully informed about what research is about and what participation will involve, and that they make the decision to participate without any formal or informal coercion'' (p.62). This is because freedom upholds their rights to refuse to participate or withdraw at any time and self-determination places some of the responsibility on the participant should anything go wrong (Cohen, Manion & Morrison, 2007).

In view of the above, voluntary and informed consent was sought from the participants before the commencement of data collection in the following ways. Participants were informed about the identity of a researcher, nature and the purpose of the study, participants' rights, and how they are protected. They were also informed that data was to be collected through writing a test, interviews, document analysis, and lesson observation in which a video and audio recording was used to collect data. In addition, the respondents were informed that the information gathered was purely for academic purposes. This means that participants were not forced or coerced to give information for this study and participants' views were treated with confidentiality and respect a sample of a consent form which they signed (See Appendix 12). However, the use of video and audio recording was put into consideration of the informed consent by asking participants to give their consent before the recording took place. The teachers consented to the use of their recorded voices and body images but researcher was warned of repurposing the recorded voices and body images of participants (Powell, Francisco & Maher, 2003). In light of this, the researcher was able to meet the ethical requirement of video and audio recording of data by giving sampled teachers the option of interrupting or discontinuing a recording session, to sight and consent to the actual video and audio recording if they so wished. Thereafter, the researcher promised the participants to destroy the video and audio recorded as required by the research ethics after the research passed the examination (Roschelle, 2000). In view of this, Hall (2000) warns against the ethical issue of repurposing where video and audio data can be obtained by different researchers and used for a different purpose. Furthermore, each participating class was spoken to about the purpose of the research and their rights and responsibilities. In the process of analysing the findings of the study, pseudonyms such as TM1 to TM 17 were used to protect the privacy of sampled teachers and quotations from the teachers were made necessary. In the quotations, punctuation marks were not used unless required, in terms of reflecting the context; words were not corrected, and the researcher tried to express the texts in the language of the speech. What follows in Sub-section 3.6.1 are matters of trustworthiness and the validity of the study.

3.6.1 The Validity of the Study

Kothari (2004) defines validity as the degree to which an instrument measures what it is supposed to measure. In recent years, the validity of qualitative data was addressed through honesty, depth, richness, and scope of the data achieved, the participants approached, and objectivity of the researcher (Cohen, Manion & Morrison, 2007). In this study, the researcher made sure that all the recorded information from both preliminary and post-observation interview schedules were listened to and all the answers from the SMKDT were cross-checked and aligned with the descriptors of the study's conceptual framework. A pilot study was also conducted as a way of testing the validity and reliability of the research instruments which were used in the study. What follows in Section 3.6.2 are matters involving preliminary observation interview schedules used to collect data.

3.6.2 Preliminary observation- interviews

Before conducting the SMKDT test, lesson observation and interview schedules the researcher carried out 80 minutes for preliminary observation interviews with each teacher (17) on certain specific probability concepts and the type of teaching strategies which they preferred to use as they taught Probability (See Appendix 2). The main reason for having the preliminary lesson observation-interview schedule was to cross-check on their understanding of the subject matter knowledge of probability, pedagogies and enabling prompts they would use to teach the topic before the final assessments were administered. This was also meant to prepare the sample in readiness for the final activities, which included the writing of the SMKDT test, lesson presentation and interviews. However, various questions such as 'why the teaching of probability is important to the learners in the secondary school mathematics were asked'. 'What kind of pedagogical strategies would like to use to teach probability concepts'? It is also important to note that, the preliminary interview questions were developed by the researcher in line with the descriptors of the study's conceptual framework in the category of the subject matter knowledge, pedagogical strategies and enabling prompts to examine the teachers' PCK of probability (See Figure 1.1, Section 1.8.1). To validate the questions, the drafted interview questions were analysed with the help of the experts question by question to ascertain if the content of Probability was included, as intended in the Zambian senior secondary school curriculum and according to the objectives of the study. Thus, this helped the researcher to have trustworthy questions that probed the mathematics teachers' understanding of the subject matter knowledge and provision of pedagogical strategies they preferred in the teaching of probability concepts. This, therefore, was followed by the reviewing of literature by the researcher to develop the assessment criteria to assess the sample how they use the identified strategies effectively in the teaching of probability.

A considerable number of research studies recommend that to teach probability concepts effectively, teachers need to develop a habit of engaging the learners using strategies such as topic discussion, one-on-one teaching, and comprehensive individualised assessment (Mutodi & Ngirande, 2014). Therefore, from the reviewed literatures, the researcher developed assessment criteria that focused on how the sampled teachers could effectively use the identified strategies in terms of engaging and giving a platform to the learners to demonstrate their understanding of the subject matter. However, some of their responses on the pedagogical strategies they preferred to use in the teaching of probability concepts and the assessment criteria which was developed are presented in Table 4.5, Section 4.5.1. Thereafter, the final SMKDT test, observation, and interview schedules were administered to the sample. What follows in Section 3.6.3 are matters of the data collection process used to collect data.

3.6.3 Data Collection Process

Upon upholding the ethical issues such as the acquisition of participants' informed consent was highly considered as earlier explained in Section 3.6. This was followed by conducting the preliminary observation interview schedules with the sampled teachers to prepare their mindset. Thereafter, the SMKDT test was then administered to the sampled teachers and this was followed by lesson observations, and interview schedules and document analysis which were conducted in their respective schools. All the 17 teachers of mathematics who wrote the SMKDT, were observed and interviewed respectively. The first instrument to be administered by the researcher was the SMKDT and this was done in September 2019 in their respective schools. To promote confidentiality and to avoid teachers using mathematics text to answer questions, the sampled teachers wrote as the SMKDT test at the same time in their respective schools. Following the analyses of the teachers' performance in the SMKDT, concepts that proved problematic to the teachers became the focus of the lesson observations and interviews (See Appendix 3 and 4). The idea was to investigate the pedagogical strategies and enabling prompts which the teachers would employ to teach those concepts. Furthermore, it was necessary to assess the nature of pedagogical challenges that the teachers would face when teaching concepts in which they lacked understanding. It is also important to note that, the researcher allowed the two Heads of the Mathematics Departments to participate in marking and results from analysis to avoid having biased results towards the researcher's views. Furthermore, the researcher acted as a participant-observer to check in the manner they were marking and awarding marks to the correct and incorrect answers in the SMKDT test. What follows in Section 3.7 are matters of how data from the SMKDT, preliminary observation- interviews, lesson observation and interview schedules were analysed.

3.7 Data Analysis

To acquire a comprehensive understanding of each mathematics teachers' Pedagogical Content Knowledge of Probability, the researcher undertook seven steps to analyze the data. Furthermore, the data which was obtained from the preliminary observation interviews were also analysed together using the descriptors of the study's conceptual framework. For easy identification of the answers sampled mathematics teachers gave in the SMKDT test, lesson presentation, and interview schedule, and document analysis, they were coded with numbers from 1 to 17 as TM 1 to TM17. The first step the researcher undertook was to analyse mathematics teachers' understanding of Probability in the SMKDT test and later analysed data collected from lesson presentation, preliminary and post-interview schedules, and document analysis. Thereafter, the results were combined to have one clear understanding of mathematics teachers' PCK of probability. Miles and Huberman (1994) argue that data analysis in the qualitative model comprises three levels of activities which are: data reduction, data display, and conclusion drawing (verification). Slightly different from the views of Miles and Hubermam (1994), Sjostrom and Dahlgren (2002) state that qualitative analysis may involve seven key steps which are: familiarisation, a compilation of answers from respondents, condensation, or reduction. Others are preliminary comparison or classification, the naming of categories and comparison of categories.

Although qualitative data analysis is primarily inductive (Merriam, 2009), to a larger extent, there are no specifically prescribed procedures to follow (McMillan, 2006). In this analysing study, it was seen to be prudent to use Sjostrom and Dahlgren's (2002) suggested method of qualitative data. Even though it does not refer to the analysis of the test results, the seven steps could help to analyse the mathematics teachers' Pedagogical Content Knowledge of Probability in detail as required by the descriptors of the study's conceptual framework (See Figure 1.1, Section, 1.8.1). Therefore, this study used the categories of the conceptual framework to analyse mathematics teachers' subject matter knowledge of probability. The use of the categories of the conceptual framework provided the best way in which mathematics teachers' PCK of Probability was examined thoroughly. This was in line with the philosophical assumption of this study which indicated that the use of the naturalistic paradigm implied that the analysis of the results was to be analysed using the categories of the theoretical and conceptual framework of the study. This was also intended for the researcher to avoid generalisation of the results, but to be analysed and reported as they were given (Lincoln & Guba, 1985; Patton, 2002). The first step that the researcher undertook was familiarisation: The researcher and two Heads of Mathematics Departments read through all the seventeen mathematics teachers' written SMKDT script answers to familiarise themselves and to understand the answers which were given by the sampled teachers. This stage allowed the researcher and two Heads of the Department to compare contents of Probability in the Mathematics Senior Secondary School Syllabus (10-12) and make corrections and also align the answers in the prepared rubric with the requirement of the study questions. Therefore, answers which were written contrary to the prepared rubric were treated as mathematics teachers' incorrect PCK of Probability. While correctly given answers were treated as mathematics teachers' understanding of the subject matter knowledge of Probability. Thereafter, in the second step, the researcher and two Heads of the Mathematics Department marked the SMKDT using the prepared rubric (See Appendix 11), which was prepared in line with the elements of the conceptual framework (See Figure 1.1, Section 1.8.1). After marking the scripts, the third step involved the compilation of answers from each participant's scripts. At this stage, the researcher and the two Heads of the Mathematics Department went through all the seventeen marked scripts five times to identify the most significant elements in the answers given by participants. This allowed the researcher to cross-check the given answers if they were consistent with the prepared rubric. Therefore, answers which were either correctly or incorrectly answered according to the requirements of the rubric (See Appendix 11) were coded and compiled using numbers and letters respectively. The fourth step involved preliminary comparison or classification of similar SMKDT answers. The researcher selected and grouped all the answers which were similar from all the respondents according to the categories of the descriptors in the conceptual framework listed below to identify mathematics teachers' ability in terms of solving probability problems correctly, providing clear explanation and justification on the use of certain probability concepts, correct representation of probability concepts and conditions, teachers' ability to defining probability terms using the socially agreed conventional meanings and the ability to illustrate probability events with justified examples using different representations (See Figure 1.1, Section 1.8.1).

In some instances, where the given answers were not similar to the language, principles, and content of Probability, those answers were separated and deleted. Furthermore, the fifth step involved the preliminary comparison of categories of the answers given from the SMKDT test. At this stage, the researcher carefully classified the given answers according to the categories of the descriptors in the conceptual framework which dealt with mathematics teachers' subject matter knowledge of probability and required them to provide clear interpretation, explanation, justification, representation, identification and application of probability concepts. The sixth step involved the naming of categories. At stage six, the processed answers in the SMKDT were categorised, named and the transcripts were appended into the codebook for manual coding (Creswell, 2009). On the other hand, the verbatim in the recorded interviews for some participants was identified and categorised according to the descriptors in the conceptual framework. Finally, at stage seven, the contrastive comparison of categories, the researcher analysed the categorised SMKDT answers according to each element of the subject matter category of the study's conceptual framework (See Figure 1.1, Section 1.8.1). Consequently, qualitative data collected from lesson observations, interview schedule and documents (lesson plans, learners' mathematics exercise books) on teachers' selection of effective pedagogical strategies and enabling prompts as they taught Probability concepts using multiple strategies was processed using the six steps cited above and coded, and grouped into the category of teachers' pedagogical strategies of the conceptual framework. The processed data were later analysed using the descriptors of the subject matter knowledge, pedagogical strategies and enabling prompts categories of the conceptual framework and the data were recorded in frequencies.

3.8 Summary of Chapter Three

This chapter outlined the research methodology of the study. The multiple case study design was used particularly to explore in-depth the mathematics teachers' PCK of probability. In line with the selected research study design, the researcher purposively selected seventeen (17) senior secondary school mathematics teachers from four secondary schools found in Chongwe District of Lusaka province as one unit within a similar social settling who have taught probability for ten (10) years or above. To explore teachers' PCK, the researcher developed draft test questions that were piloted in the Rufunsa district and thereafter prepared the final subject matter knowledge diagnostic test (SMKDT), developed a lesson observation sheet, and prepared unstructured interviews on selected specific Probability concepts and document analysis of lesson plans, schemes of work, a record of work and learners; mathematics exercise books. Besides, the researcher also discussed the philosophical assumption of the study, study area, target population, study sample size which was 17 respondents, and involved purposive sampling as a sampling procedure. Before conducting data collection, the researcher piloted the instruments to validate their reliability in the collection of useful data. Furthermore, the ethical issues were highly considered as in the first place, informed consent was sought from the participants before commencement of data collection. The researcher also looked at how the collected data was going to be analysed in line with the philosophical assumption of this study. Thereafter is the presentation of the findings of the study.

CHAPTER FOUR: PRESENTATION OF THE FINDINGS

4.1 Introduction

The previous chapter dealt with methodological aspects of investigation that focused on the research procedures and techniques which were used to provide answers to the research questions in chapter one. This chapter is the presentation of findings involving seventeen (17) mathematics teachers who have taught probability for ten years and above. In this regard, the study investigated mathematics teachers' subject matter knowledge of probability, the effectiveness of the identified pedagogical strategies and enabling prompts which were used to support learners' understanding as they taught Probability. This chapter also presents the findings from the SMKDT test, the identified pedagogical strategies, and enabling prompts used during lesson presentations to support learners' understanding of probability and responses from the follow-up interview schedules, and document analysis. The findings of this study were analysed using the categories of the study's conceptual framework. Therefore, the analysed findings from the SMKDT test, lesson presentation, interview schedule, and document analysis provided answers to this study's three research questions. Thereafter, a summary of the chapter is provided.

4.2 Demographic characteristics of the Sampled Teachers participated in a research

Respondents who had taken part in this study were asked to indicate their brief background of information on how long they have taught Probability and the qualifications they have. This was intended to select respondents who have taught Probability for ten years and above. A total of seventeen secondary school mathematics teachers who were sampled to write the SMKDT test, participated in lesson presentation, and interview and document analysis. In the sample, six were female while eleven were male representing gender-balanced who participated in the research. In terms of their qualifications and teaching experience, Table 4.1 provides all the details for all the seventeen respondents.

Table 4.1: Qualification and Teaching Experience of the sampled teachers

Years of teaching	Qualifications	Frequency
10 to 15	Degree	2
16 to 20	Degree	7
21 to 25	Degree	5
26 to 30	Diploma	2
31 or above	Degree	1
Total		17

As shown in Table 4.1 above, the demographic data shows that the respondents had taught Probability for ten years or above. This implies that all the respondents (teachers of mathematics) had long experience in teaching Probability. The researcher also considered the respondents' qualifications because according to the national policy on education (Educating Our Future), MOE (1996) state ''that all senior Grades (10-12) are to be taught by University graduates (Degree holders), while junior Grades (8-9) are to be taught by college graduates (Diploma holders) unless in special circumstances'' (p.111). This also helped the researcher to provide a comprehensive description of mathematics teachers who were sampled in the research and the trustworthiness of the results obtained from these teachers. However, two teachers of mathematics who are diploma holders but were seasoned teachers of mathematics and taught probability at the senior level for a long period were also selected. Therefore, this could not distort the analysed results.

4.3 Teachers' Subject Matter Knowledge of Probability

This was the first research question of the study which assessed mathematics teachers' subject matter knowledge of probability. In this case, teachers were subjected to write a SMKDT test which assessed their ability to define probability terms, calculate, interpret, explain and justify their mathematical ideas on probability concepts. The research question focused on teachers' ability to recognise, represent, formulate and identify probability concepts as well as to illustrate probability events with justified examples. The research question also looked at teachers' ability in making conceptual connections with the pre-requisite topics of probability which included sets and solving probability problems correctly. However, Table 4.2 provides a summarised explanation on the performance of teachers in the SMKDT test.

Table 4. 2: Summary of the performance of sampled mathematics teachers in the SMKDT

S/N	**Pass mark score: 50 and above**	**Number of teachers**
1	Teachers scored 50 and above pass mark	7
2	Teachers scored less than 50 pass mark	10
Total number of teachers participated		**17**

As shown above in Table 4.2, seven teachers scored 50 marks and above the pass mark, while ten (10) scored less than 50 marks in the SMKDT questions. However, Figure 4.1 presents the substantial disparities in the teachers' total scores in the SMKDT test on the bar graph.

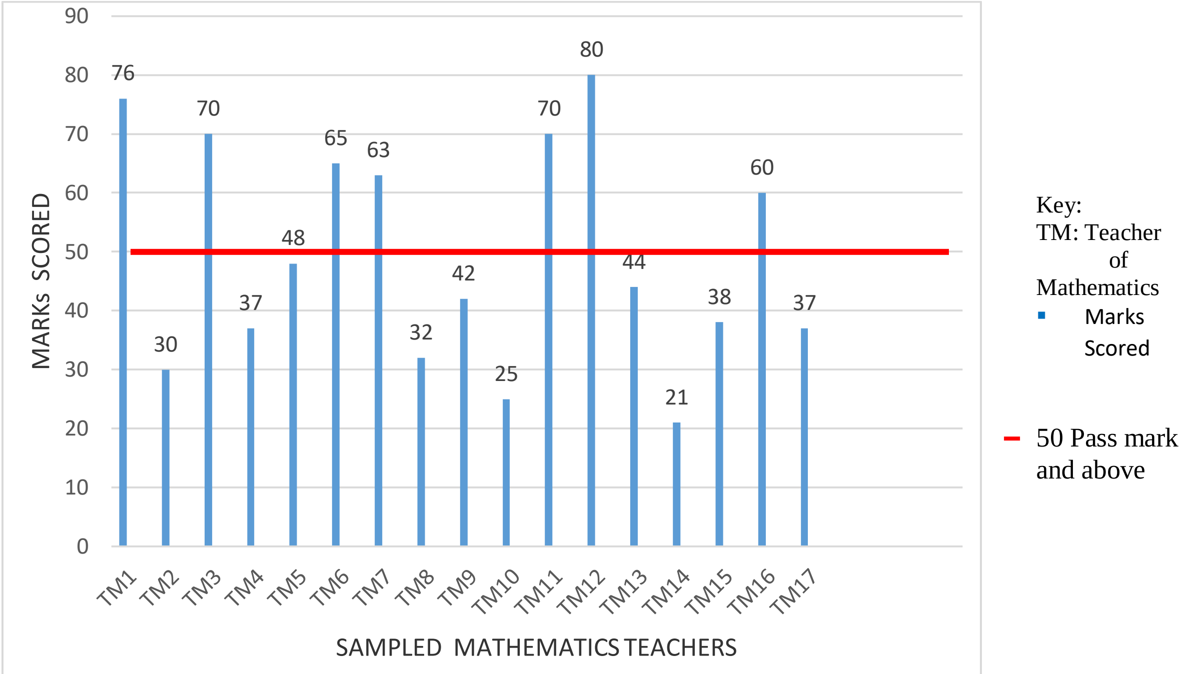


Figure 4.1: Marks scored by each mathematics teacher in the SMKDT

Figure 4.1 above shows the marks each sampled teacher scored in the SMKDT test on which teachers were investigated on their subject matter knowledge of Probability. The above Figure 4.1 also shows that the lowest mark scored was 21 and the highest mark scored was 80 out of 95 total marks. This gave a range of 59 which shows a large variation in teachers' performance in the SMKDT. Figure 4.2 presents the variations in teachers' performance in the SMKDT test using a Pie chart.

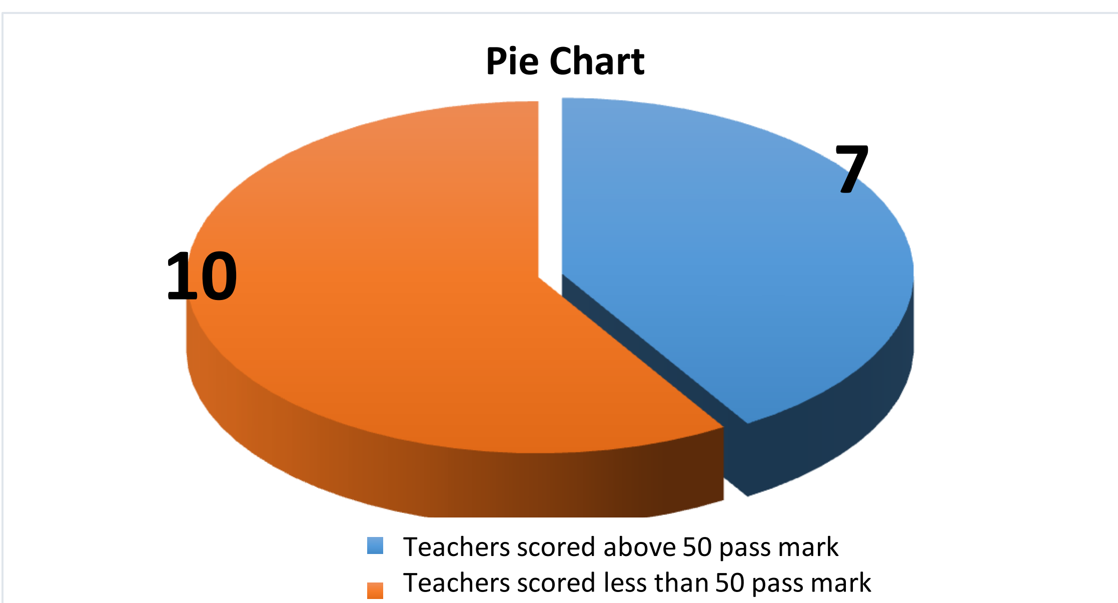


Figure 4.2: The summary of performance of sampled teachers in the SMKDT

Figure 4.2 as illustrated above, shows that the smaller part of the Pie chart represented the number of teachers who were able to apply the competences as expected while the larger part represented the number of teachers who applied the competences contrary to the objective of the research question as they were answering the SMKDT. In regard to the above explanation, Table 4.3 presents a summarised number of competences per question which mathematics teachers applied uneven to the subject matter requirement of the SMKDT questions.

Table 4. 3: Competences applied in answering the subject matter knowledge diagnostic test (SMKDT)

Question numbers	Teachers who applied the competences correctly	Teachers who applied the competences incorrectly.	competences applied correctly	competence applied incorrectly	Total competences per question
1 (a)	6	11	2	1	3
(b)	5	12	2	1	3
(c)	4	12	4	3	7
2 (a)	6	11	3	3	6
(b)	7	10	5	3	8
(c)	6	8	6	4	10
3 (a)	5	12	3	2	7
(b)	3	12	4	3	7
(c)	5	12	2	2	4
4 (a) (i)	7	10	4	3	7
(ii)	8	9	2	1	3
(b)	3	14	3	2	5
5 (a)	12	5	5	3	8
6 (a)	17	0	3	0	3
(i)	17	0	1	0	1
(ii)	12	4	2	1	3

(iii)	10	3	2	2	4
7 (a)	15	2	3	1	4
(b) (i)	13	4	3	1	5
(ii)	8	9	3	2	5
(iii)	6	11	3	2	5
8 (a)	5	12	4	3	7
(b)	4	12	2	1	3
9 (i) a	11	6	2	1	3
(b)	10	7	2	1	3
(c)	6	11	2	2	4
(d)	8	9	2	1	3
9 (ii)	4	10	3	2	5
10 (a)	17	0	2	0	2
(b)	13	4	2	1	3
(c)	4	11	2	2	4
(d)	3	10	2	1	3
11 (i) a	8	9	4	1	5
(b)	5	10	3	1	4
(c)	3	12	2	1	3
(d)	2	13	3	1	4
11 (ii)	2	13	3	2	5
12 (a)	17	0	2	0	2
b (i)	10	7	2	1	3
(ii)	11	6	3	1	4
C	9	8	3	1	4
D	14	3	3	1	4
13 (a)	4	10	2	1	3
14 (a)	5	11	3	2	5
(b)	3	13	2	1	3
15 (a)	6	10	3	2	5
(b)	3	11	3	3	6

(c)	3	13	3	3	6
16 (a)	4	13	2	1	3
(b)	4	12	2	2	4
(c)	17	0	2	0	2
17 (a)	5	12	2	1	3
(b)	4	13	2	1	3
Total	399	462	144	82	226

Table 4.3 above shows that the SMKDT questions (1 to 17) had two hundred and twenty-six (226) competencies on which the sampled mathematics were assessed in terms of their understanding of the subject matter knowledge for teaching probability. However, out of two hundred and twenty-six (226) competencies in the SMKD, one hundred and forty-four (144) were applied according to the requirement of the subject matter prepared in the rubric for the SMKDT test (See appendix 11). This represented mathematics teachers' understanding of the subject matter knowledge of probability while the eighty-two (82) competencies were applied otherwise about the subject matter and these represented mathematics teachers' lack of understanding of the subject matter knowledge of probability concepts. What follows is Section 4.3.1 with the analysis of each sampled mathematics teachers' SMKDT answers per question.

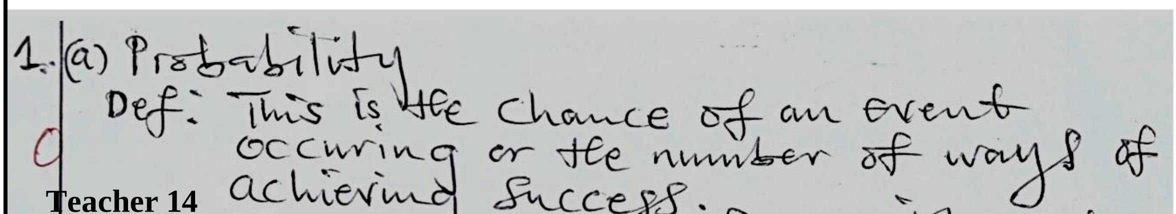
4.3.1 The analysis for each mathematics teachers' SMKDT answers per question

This section focused on how well the mathematics teachers demonstrated their understanding of the subject matter knowledge of probability in the preliminary observation interview, SMKDT test, lesson presentation, interview schedule, and document analysis about the descriptors of the conceptual framework. The analysis focused on the given answers in terms of how they defined probability terms concerning the use of socially agreed qualitative terms, interpreted, explained, and justified their mathematical ideas on Probability concepts. The answers were further analysed to find out whether teachers were able to recognize, represent and identify probability concepts as well as to illustrate probability events with justified examples. Furthermore, the answers were analyzed to investigate whether teachers were able to make clear conceptual connections of the pre-requisite topics of probability and solve probability problems correctly. Ultimately, the following questions were utilised to present the analysis of mathematics teachers' subject matter knowledge of probability in the SMKDT:

Question 1(a)

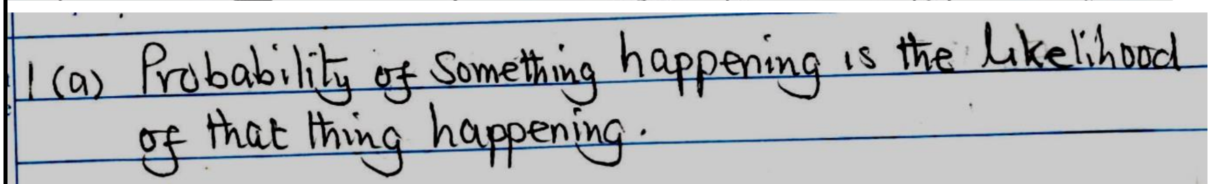
According to the prepared rubric, the first question in the SMKDT required teachers to provide a valid definition of the term probability as they would teach in class. The teachers were expected to define the term probability using the socially agreed qualitative terms such as randomness, measure, events, chance, likelihood, certain, impossible, unlikely as well as referring to other related topic concepts such as percentages, decimal and fractions. In regard to the above expectations, Figure 4.3 shows how Teachers 6 and 14 defined the term Probability.

Teacher 6



1. (a) Probability
Def: This is the chance of an event occuring or the number of ways of achieving success.

Teacher 14



1 (a) Probability of Something happening is the Likelihood of that thing happening.

Figure 2.3: Response P1

Figure 4.3 above shows that Teachers 6 and 14 definitions were not directed towards the agreed conventional meaning of the term probability. Instead, they defined probability based on one condition of certainty which was the likelihood of an event taking place regarding the probability scale. On the positive note both teachers were able to recognize and defined the term using the socially agreed conventional words used to describe probability events such as chance, likelihood, happening and event. However, their definitions did not reflect any conceptual connection with any major mathematical concepts within mathematics and other related disciplines. It can be also observed that both teachers provided recited definitions of the term probability from a mathematics learners' book 11 as they bear the same words and the almost same arrangement of grammar as it is evidenced in the later part of Teacher 6's definition where he defined probability as the total number of successful possible outcomes in a trial.

In the follow-up interviews, Teacher 6 defined Probability contrary to the definition given in the SMKDT. He defined the term as:

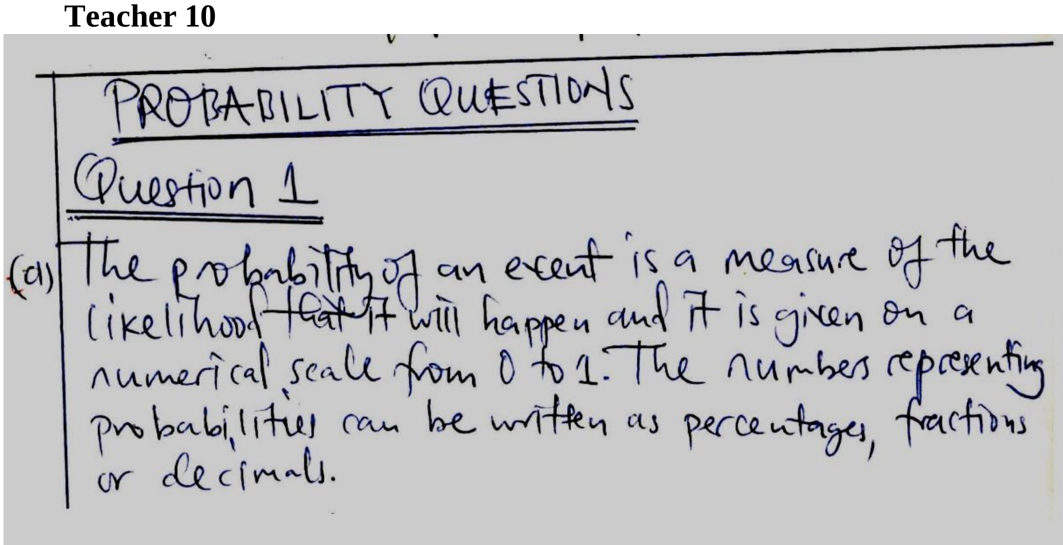
> The likelihood of an event happening or not happening that can either be expressed as percentage, fraction or decimal numbers between 0 and 1. Probability of A is denoted as P (A), given by:
>
> $$P(A) = \frac{\text{Number of favorable outcomes of A}}{\text{Total number of possible outcome in trial}}$$

In the above response of the follow-up interviews, Teacher 6's definition included both conditions of certainty (1) and impossibility (0) which he simplified as the likelihood of an event happening or not happening. In addition, the above definition shows that Teacher 6 was able to make conceptual connections with sets and representations of probability concepts with other major mathematical concepts such as decimal numbers, percentages, or fractions. In another follow-up interviews, Teacher 14 also defined the concept of Probability contrary to what he had presented earlier in the SMKDT. Instead, he defined Probability as:

> The likelihood of an event happening or not. He further stated that it is a mathematical measure that lies between 0 for uncertainty and 1 for certainty (0≤ P (E) ≤1) which can be expressed as a fraction, percentage, and decimal numbers.

The above response shows that Teacher 14's definitions included the conceptual connections of major topics concepts of probability like sets and representations of probability in terms of percentages, fractions, decimal numbers and numerical limit values used to describe probability events with probability scale. The teachers were able to define probability in the language of sets and presented it in a set builder notation form as P (0≤ P (E) ≤1). In addition, the teachers' definitions also included the use of two major words that are used to describe the occurrence of probability events and these are certainty (1) and impossibility (0) of an event taking place.
Therefore, Figure 4.4 presents the definitions that characterises Teacher 10, 11 and 17's ability in terms of defining the term Probability.

Teacher 10


PROBABILITY QUESTIONS

Question 1

(a) The probability of an event is a measure of the likelihood ~~that~~ it will happen and it is given on a numerical scale from 0 to 1. The numbers representing probabilities can be written as percentages, fractions or decimals.

Teacher 11

Figure 1.5: Sketch P

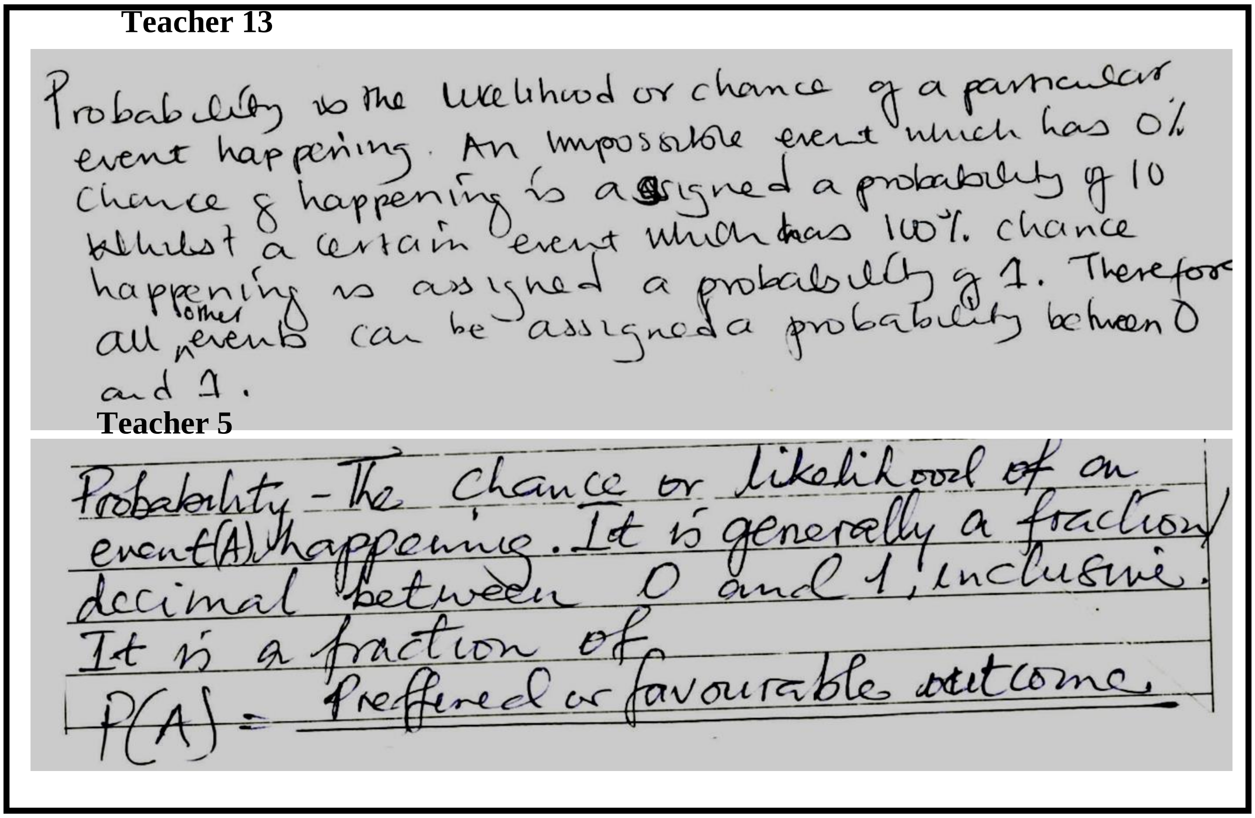
Teacher 17

Figure 4.4: Response P2

Figure 4.4 above shows that the definitions given by Teachers 10, 11, and 17 were directed towards the agreed conventional meanings of probability, and included the numerical limit values (0 to 1) which measure probability about the probability scale. Furthermore, their definitions were also directed towards the conceptual representations of probability concepts in terms of percentages, fractions and decimal numbers used to describe probability events. In addition, their definitions also included both conditions of the outcome of probability events which includes the probability of certainty (1) and impossibility (0) of an event taking place. The teachers further stated that probability lies between 0 for impossibility and 1 for certainty which means that probability measures cannot exceed one (1) and zero (0). Furthermore, Teachers 13 and 5 in their definitions of probability included both conditions of absolute

Teacher 11

certainty (1) and absolute impossibility (0) regarding probability scale and other related topic concepts. In regard to the above explanation, Figure 4.5 presents how Teacher 13 and 5 were able to define the term Probability using socially agreed qualitative terms.


Teacher 13

Probability is the likelihood or chance of a particular event happening. An impossible event which has 0% chance of happening is assigned a probability of 0 whilst a certain event which has 100% chance happening is assigned a probability of 1. Therefore all other events can be assigned a probability between 0 and 1.

Teacher 5

Probability - The chance or likelihood of an event(A) happening. It is generally a fraction/decimal between 0 and 1, inclusive. It is a fraction of.

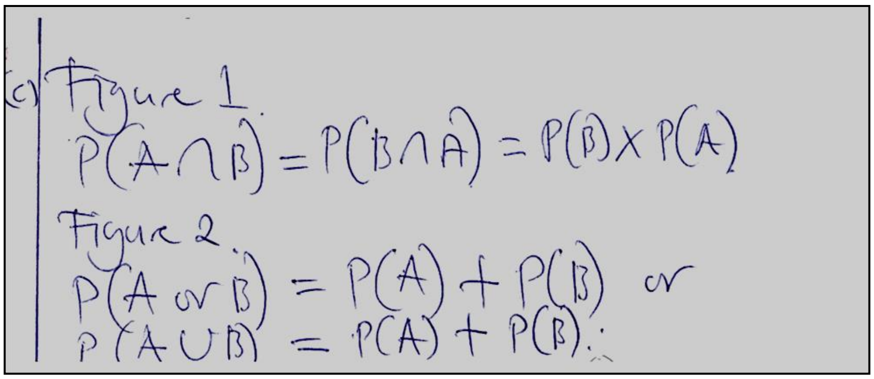
P(A) = Preffered or favourable outcome

Figure 4.5: Response P3

As shown above in Figure 4.5, both Teacher 13 and 5 also defined probability regarding the probability scale and both indicated that it lies between 0 for impossibility and 1 for certainty. Furthermore, the above-replicated definitions reflected the probability concepts which are carried in use of the socially agreed qualitative terms used to describe probability events such as chance, likelihood, event, or, certainty, uncertainty, the outcome which include the concept of limits in measuring probability (0 to 1) about probability scale, sample space and outcome of probability which can either be 0 or 1.

Question 2 (C).

In the context of conceptual connection of mathematical concepts to Probability and use of notation symbol of sets, question 2(c) assessed mathematics teachers' subject matter knowledge of using the notation symbol 'U' to formulate the probability expression using the laws of Probability and sets. The question also required mathematics teachers to present their answers in set notation using the ideas of jointed and disjointed sets while in the second part of the question, they were required to formulate the associated probabilities. Question 2(c) further assessed mathematics teachers' ability to recognize and identify the concept of addition rule for mutually exclusive events so that the respondents could represent the associated probabilities expression using the notation symbol 'U'. In addition, the question also assessed mathematics teachers' ability in the making of conceptual connection of the laws of probability and sets using the set notation symbol 'U' in Figures 1 and 2 and this was the first step that the teachers were required to understand. However, Figure 4.6 presents SMKDT answers that characterises Teacher 7's inability to use 'U' to formulate the associated probability expression.


(c) Figure 1.

$P(A \cap B) = P(B \cap A) = P(B) \times P(A)$

Figure 2.

$P(A \text{ or } B) = P(A) + P(B)$ or

$P(A \cup B) = P(A) + P(B)$.

Figure 4.6: Response P4

Figure 4.6 above shows that Teacher 7 used the notation symbol of the intersection of sets '∩' to formulate the associated probability expression for sets A and B in Figure 1 to describe the non-mutually exclusive events, which occurred as an independent event in a jointed set. This further led the Teacher to formulate the associated probability expression in the form of: P (A∩B) = P (A∩B) = P (A) P (B). On the second part of question 2(c) in Figure 2, Teacher 7 expanded the mutually exclusive events of set A and B using the expressions P (A∩B) =0 and formulated two probability expressions P (A or B) =P (A) +P (B) and P (AUB) = P (A) +P (B) using the combined knowledge of laws of probability and sets. On the same expressions, the Teacher applied the concept of additional rule for mutually exclusive events in set notation in which he expanded by adding the probabilities of the events of A and B. To justify the rule that he used to formulate the expression in Figure 2, Teacher 7 explained that in a set theory, the associated probabilities of mutually exclusive events are presented as disjointed sets and could appear as P (AUB) =P (A) +P (B). He further indicated that set A and B did not have common elements and the occurrence of one event displaced the occurrence of another.

On the same question 2(a), 2(b) and 2(c), Figure 4.7 shows how Teacher 11 provided the illustrations, explanations and justifications of mutually exclusive events with the help of examples for questions.

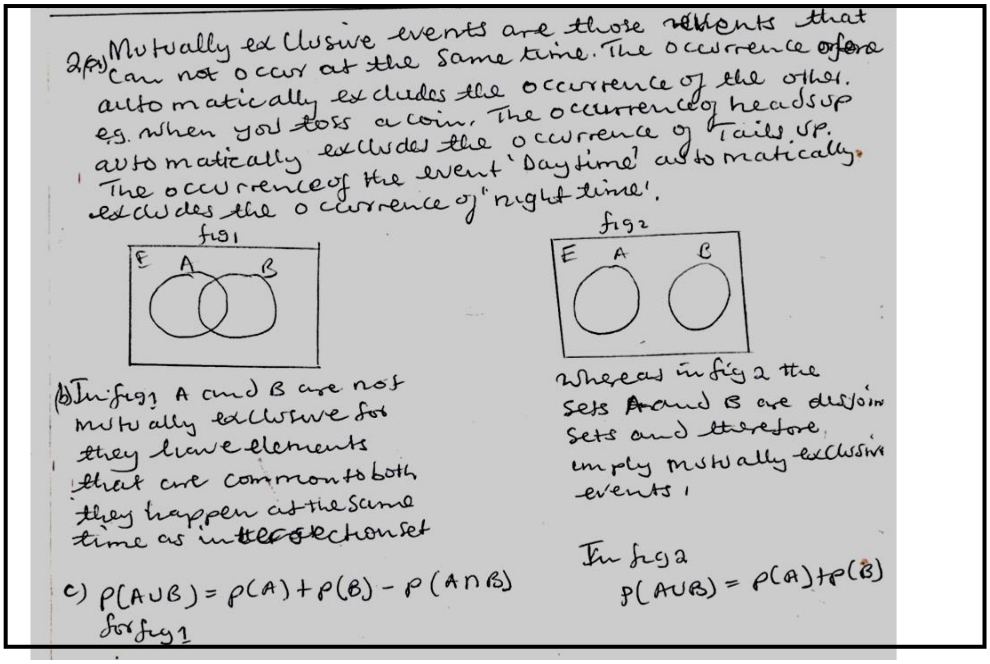
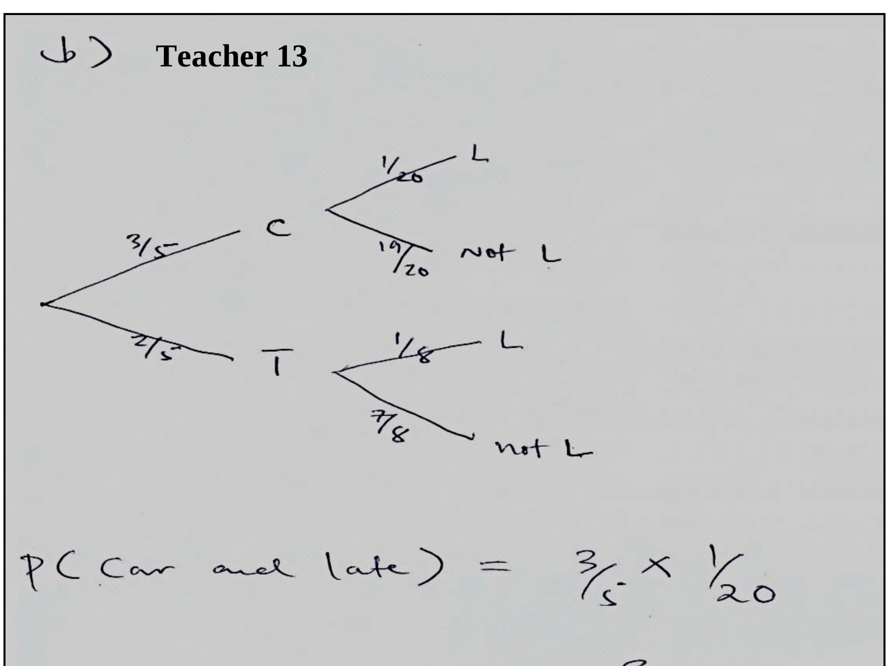


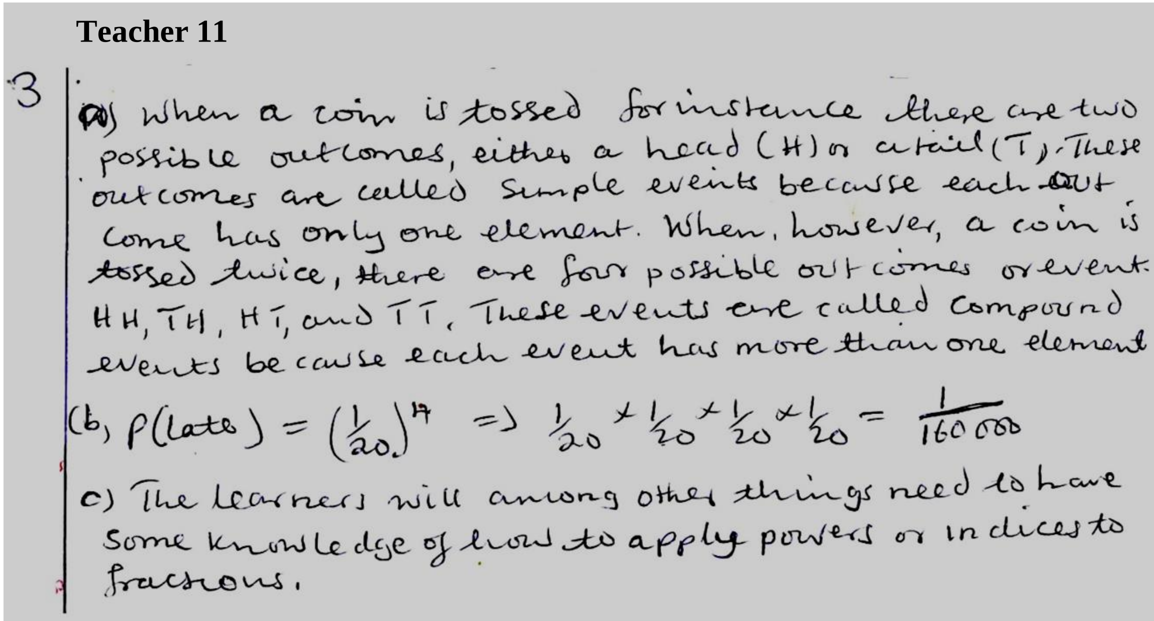
Figure 4.7: Response P5

Figure 4.7 above shows that Teacher 11 was able to describe the meaning of the mutually exclusive events using the occurrence of ‘day and night’ as events that cannot happen at the same time. He further stated that the occurrence of the event ‘day time’ automatically excludes the occurrence of the ‘night time’ as shown above in Figure 4.7. Furthermore, on question 2(b) in Figure 1, Teacher 11 stated that, set A and B were not mutually exclusive events because they had common elements (jointed set) that could have occurred at the same time, while in Figure 2, he stated that set A and B were mutually exclusive events because they had no common elements (disjointed set). Using the above understanding on question 2(a) and (b), Teacher 11used the notation symbol ‘U’ to formulate the following probability expression for non-mutually and mutually exclusive events in Figure 1 and 2 as P (AUB) =P (A) + P (B) – P (A∩B) and P (AUB) =P (A) +P (B) respectively.

Question 3(b)

Question 3(b) assessed mathematics teachers’ ability to recognize, identify, formulate probability compound spaces and calculate the probabilities of compound events. In other words, mathematics teachers were assessed on their ability to identify the concept of independence, certainty and combination of outcomes of days. This was the first step they were required to understand in order to make it easier for them to formulate probability compound spaces and solve the probability of one male reporting late for work for four consecutive days using a car. The question further required mathematics teachers to recognize that each day he traveled going for work occurred as an independent event. The question further assessed mathematics teachers' knowledge on how to formulate compound events with more than two probability spaces since each day occurred as an independent event in any of the following order: N, L, L, L; L, N, L, L; L, L, N, L and L, L, L, N. Furthermore, on question 3(c) the teachers were required to state and explain the mathematical knowledge learners were required to use to find the probability of the man reaching late for work. In this case, Figure 4.8 presents how Teacher 13 and 11 responded to the requirement of SMKDT on the question 3 (a), (b) and (c).





Teacher 11

3 (a) When a coin is tossed for instance there are two possible outcomes, either a head (H) or a tail (T). These outcomes are called simple events because each out come has only one element. When, however, a coin is tossed twice, there are four possible out comes or event. HH, TH, HT, and TT. These events are called compound events because each event has more than one element

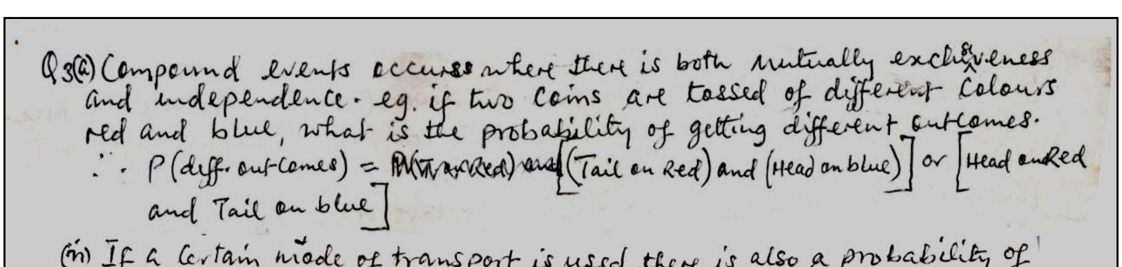
(b) $P(\text{late}) = \left(\frac{1}{20}\right)^4 \Rightarrow \frac{1}{20} \times \frac{1}{20} \times \frac{1}{20} \times \frac{1}{20} = \frac{1}{160\,000}$

c) The learners will among other things need to have some knowledge of how to apply powers or indices to fractions.

Figure 4.8: Response P6

Figure 4.8 above shows that Teacher 11 and 13 illustrated the probability compound spaces of tossing of a coin twice and both were able to obtain HH; HT; TH; TT which was a buildup for question 3(b). Concerning question 3(b), in the first step Teacher 13 generalised the formulation of the probability compound spaces as P (car and late) = P (C, L). In the second step, the Teacher subtracted the probability of reaching late (L) from 1 and found it as the probability of not reaching late (N). Furthermore, in the third step, Teacher 13 used the rule of ‘and’, then later multiplied them and found 3/100 instead of what she had calculated earlier of the fraction and as probabilities for being late (L) and not (N) respectively. Using the above understanding, Teacher 11 also formulated the probability compound spaces as P (late) and later multiplied with 1/20 four times and found 1/160000. On question 3(c), the Teacher stated that the learners required knowledge of fractions and powers of indices for them to find answers to the first question.

Besides, Teacher 11 and 13’s answers on questions 3 (a), (b) and (c), Figure 4.9 shows how Teacher 15 was able to formulate four compound spaces, provided correct explanations, calculations and justifications on the same answers.



Q3(a) Compound events occurs where there is both mutually exclusiveness and independence. eg. if two coins are tossed of different colours red and blue, what is the probability of getting different outcomes.

∴ P(diff. out-comes) = [(Tail on Red) and (Head on blue)] or [Head on Red and Tail on blue]

(ii) If a certain mode of transport is used there is also a probability of

Figure 4.8: Response P7

Figure 4.8 above shows that Teacher 15 was able to recognize, identify and differentiate the probability concepts which were attached to the SMKDT activity and formulated four probability compound spaces of the order in the form: L, L, L, N, L, L, N, L, L, N, L, L and N, L, L, L in which L represented reaching late by car and N represented not late (N). The Teacher further was able to subtract 1/20, the probability of reaching late (L) from one (1), the probability of certainty, and found 19/20 as the probability of not reaching late (N). Thereafter, the Teacher multiplied the probabilities using all the necessary steps as shown above in Figure 4.8 and found 19/16000. Not only did Teacher 15 formulate the probability compound spaces shown above, but he also stated that if a certain mode of transport was used successfully for four days, then there would be a day the man in the probability question would likely not reach late (N). On question 3(c), Teacher 15 was able to state that the learners required the mathematical understanding of concepts of certainty and the arrangement of the outcome of the combinations of the probability of reaching late (L) or not (N) as shown above in Figure 4.8. Question 4(a) (i) and (ii). This was an extension of question 3(b) which assessed mathematics teachers' ability to recognize, identify and represent probability concepts and conditions accurately. The question required teachers to recognize, identify and represent the condition of testing positive and negative as independent events and formulate the probability expression in the form of: P (P) x P (N) or P (N) x P (P) as P stood for positive and N for negative. In light of this, Figure 4.10 presents how Teacher 10 responded to the requirement of formulating the probability expression for question 4(a), (i) and (ii).

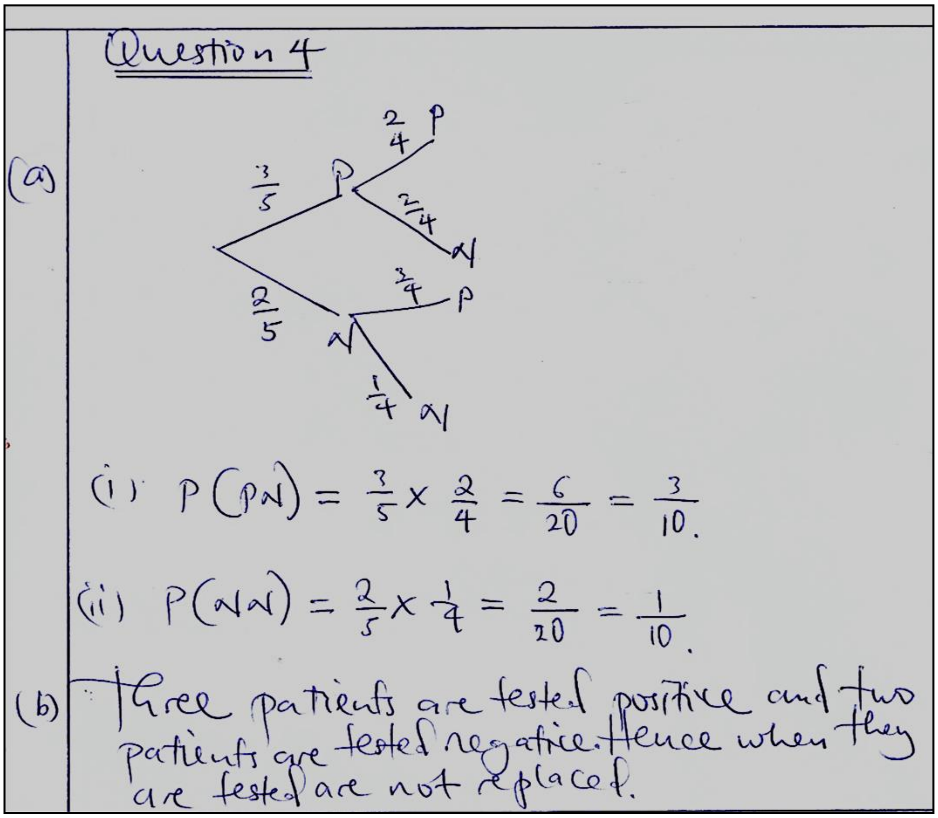
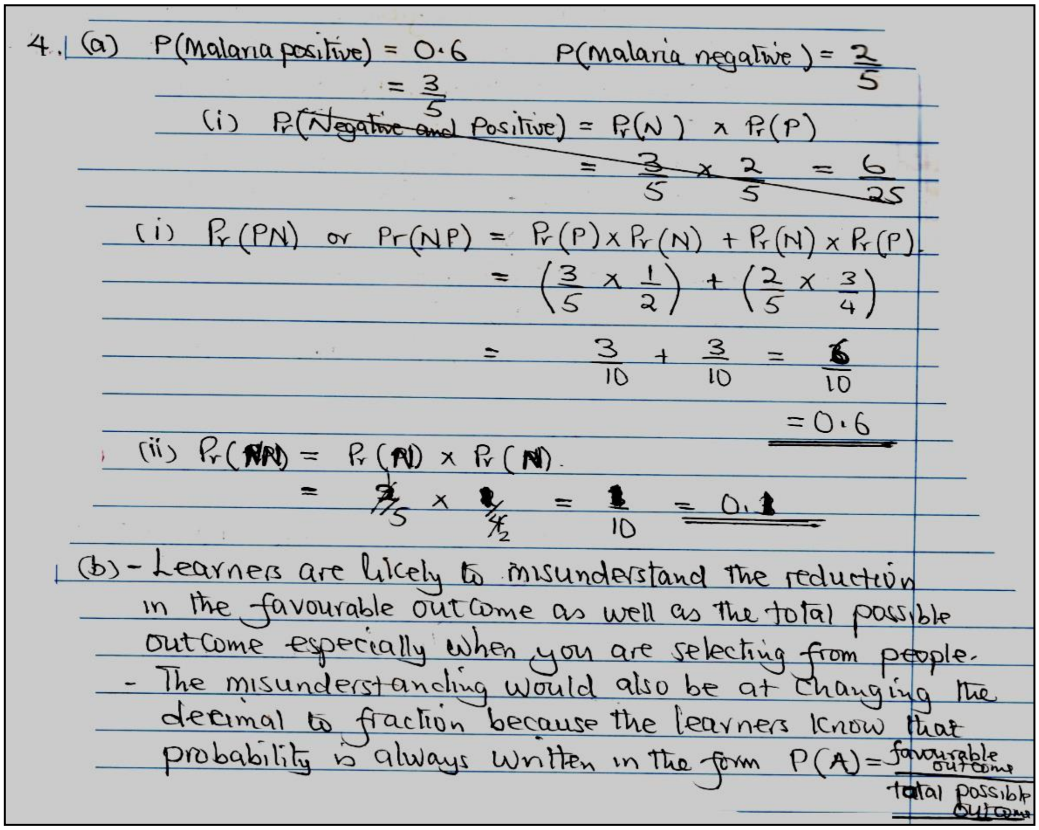


Figure 4.10: Response P8

Regarding Figure 4.10 shown above, Teacher 10 was able to identify the probability conditions of testing positive as $\frac{3}{5}$ which she later subtracted from one (1) and found the probability of testing negative as $\frac{2}{5}$. Nonetheless, on the same question, Teacher 10 formulated the probability compound expression in the form as P (P, N), where P represented testing positive while N represented testing negative. Thereafter, Teacher 10 was able to draw the tree diagram and fill in the corresponding probability of testing positive as $\frac{3}{5}$ in the first level of the tree diagram. However, on the second level, the teacher brought in the concept of dependence (not replaced) and wrote $\frac{2}{4}$ which he reduced from $\frac{3}{5}$ instead of writing $\frac{2}{5}$ as the probability of testing negative, thereby leading him to find an answer $\frac{3}{10}$. In contrast to Teacher 10's findings on question 4, Figure 4.11 presents how Teacher 14 answered question 4(a), (i) and (ii) in the following way.


4. (a) P(Malaria positive) = 0·6 = $\frac{3}{5}$ P(Malaria negative) = $\frac{2}{5}$

(i) ~~Pr(Negative and Positive) = Pr(N) x Pr(P) = $\frac{3}{5}$ x $\frac{2}{5}$ = $\frac{6}{25}$~~

(i) Pr(PN) or Pr(NP) = Pr(P) x Pr(N) + Pr(N) x Pr(P).

= $\left(\frac{3}{5} \times \frac{1}{2}\right) + \left(\frac{2}{5} \times \frac{3}{4}\right)$

= $\frac{3}{10} + \frac{3}{10} = \frac{6}{10}$

= 0·6

(ii) Pr(NN) = Pr(N) x Pr(N).

= $\frac{2}{5} \times \frac{1}{2} = \frac{1}{10}$ = 0.1

(b) - Learners are likely to misunderstand the reduction in the favourable outcome as well as the total possible outcome especially when you are selecting from people.
- The misunderstanding would also be at changing the decimal to fraction because the learners know that probability is always written in the form P(A) = $\frac{\text{favourable outcome}}{\text{total possible outcome}}$

Figure 4.11: Response P9

Figure 4.11 above shows that Teacher 14 was able to recognize, identify and illustrate the combination of outcomes of either testing positive (+ve) or negative (-ve) in the form of P (positive [+ve]) and negative (-ve) or P (negative (-ve)) and positive (+ve). Thereafter, he summarized it as presented: Pr (P, N) or Pr (N, P) and later expanded as Pr (P) Pr (N) + Pr (N) Pr (P). On the second step, Teacher 14 was able to realize that P (E) cannot exceed one and began by presenting the probability of testing positive by converting the fraction $\frac{3}{5}$ into a decimal number as (0.6), and subtracted $\frac{3}{5}$ the probability of having tested positive (+ve) from 1, the possible outcome of an event and found $\frac{2}{5}$ as a probability of having tested negative (-ve). On the third step as shown above in Figure 4.11, Teacher 14 substituted the expression and multiplied using fractions as follows $\frac{1}{2} \times \frac{3}{4}$ instead of $\frac{2}{5} \times \frac{3}{5}$ which he had earlier found as the probability of testing negative and positive. Furthermore, on the same question 4(a), (i) and (ii), Figure 4.12 presents how Teachers 8 and 12 provided flawed answers in the following way.

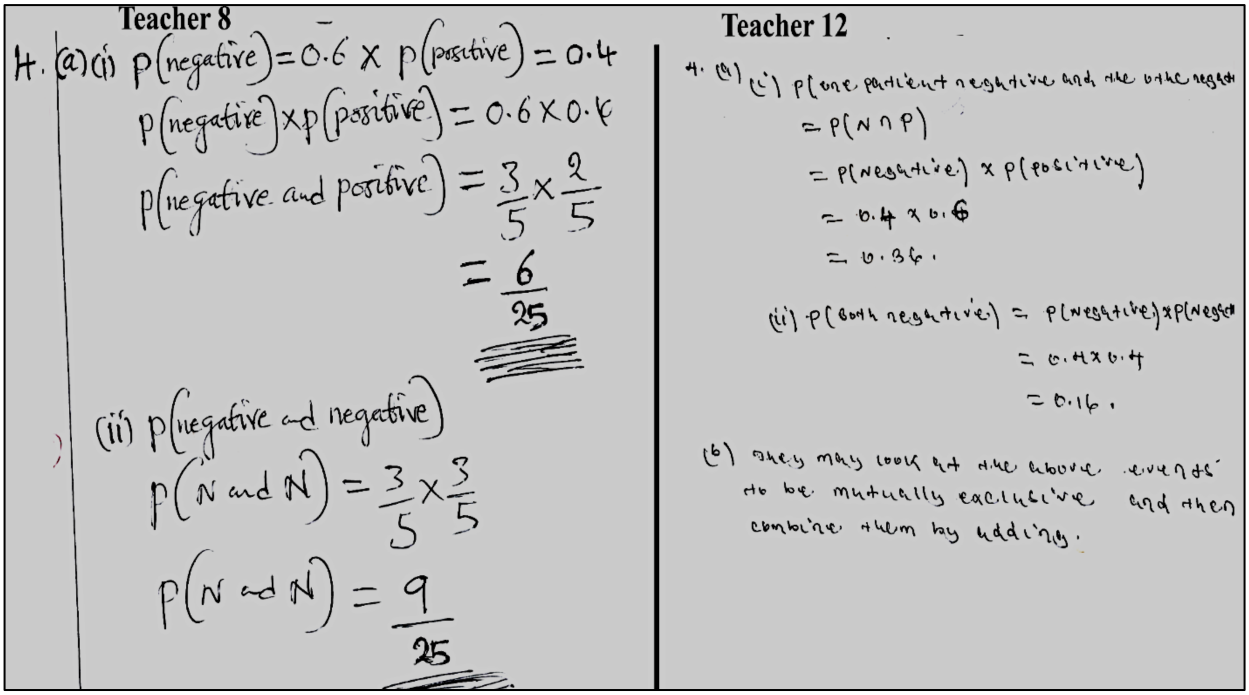


Figure 4.12: Response P10

As shown in Figure 4.12 above, Teacher 8 and 12 formulated the probability compound expressions of either testing positive or negative of malaria in the form as P (N) × P (P) and P (N∩P) respectively. In the second step, Teacher 8 identified 0.4 as the probability of testing positive and subtracted 0.4 from 1 to find 0.6 as the probability of testing negative against malaria. In the third step, the Teacher multiplied used the probability expression P (N) × P (P) and found 6/ 25 on the first question. On the second question, Teacher 8 multiplied using the probability of testing negative 3/5 using the expression P (N and N) and found 9/25. On the same question, Teacher 12 used the probability expression P (N∩P) and multiplied 0.4 × 0.6, and found 0.36. On the second question, Teacher 12 used the expression P (both negative) and multiplied using 0.4 and found 0.16. Furthermore, on question 4(b) Teacher 8 left the question unanswered while Teacher 12 indicated that learners may take the event of testing positive and negative to be mutually exclusive events and later combine and add them. In contrast to the presentation of answers on question 4(a) (i) and (ii) given by Teacher 8 and 12, Teacher 17 was able to recognize the probability condition and illustrated the combination of outcomes of either testing positive (+ve) or negative (-ve) as P (-ve, +ve) or P (+ve, -ve) which he later presented in the form of: P (P.N or N.P). In contrast to the above presented work, Figure 4.13 shows how

Teacher 17 demonstrated his ability in the construction of a tree diagram, filling of corresponding probabilities, illustration of the expression and finding of the probability of testing malaria positive and negative or both answers concerning question 4(a) and (b).

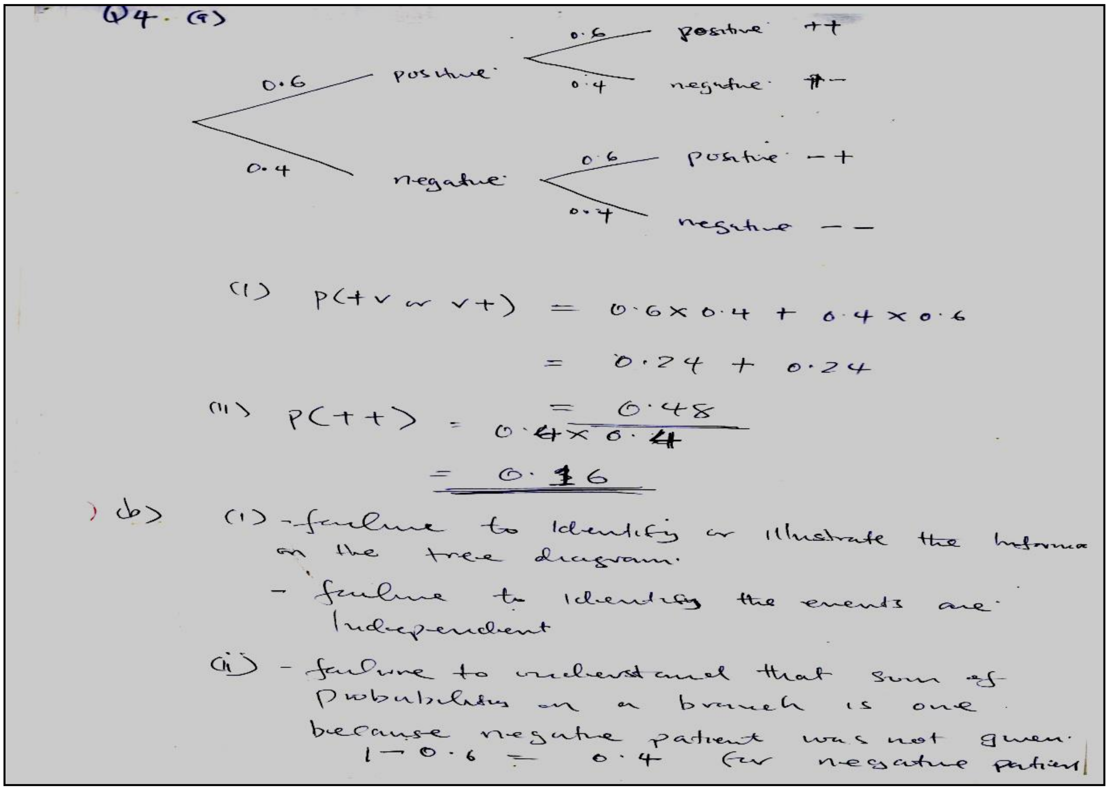


Figure 3.13: Response P11

Figure 4.13 above shows that Teacher 17 illustrated the combination of outcomes of either testing positive (+ve) or negative (-ve) in the form of: P (-ve, +ve) or P (+ve, -ve) and later presented in the form as P (P.N or N.P). In the second step, Teacher 17 presented the corresponding probabilities on the tree diagram using decimals numbers such as $\frac{3}{5}$ (0.6) and $\frac{2}{5}$ (0.4) which he later used to find the probabilities of either testing negative or positive as 0.48 as well as the probability of both patients testing negative as 0.16. The above understanding made the teachers to formulate correct expressions and solve the probability of two patients selected at random one after the other, as one tested negative and another tested positive while others tested both negative. However, Figure 4.13 above shows a good example of other answers which were found by the other four (4) teachers that similarly recognized and identified the probability condition of testing positive or negative involved the concept of independence and combination of outcomes of testing positive and negative.

Furthermore, question 4(b) required teachers to interpret and predict, explain and justify their mathematical ideas on how learners were likely to misunderstand the concepts of independence, certainty and the listing of the combination of outcome of testing positive and negative. Teachers were able to extend their understanding of the concept of independence and certainty. Similarly, to the above findings in Figure 4.13, Figure 4.14 presents how Teacher 15 was able to find correct answers, provide correct explanation and justification on how learners were likely to misunderstand the concept of mutually exclusive event.

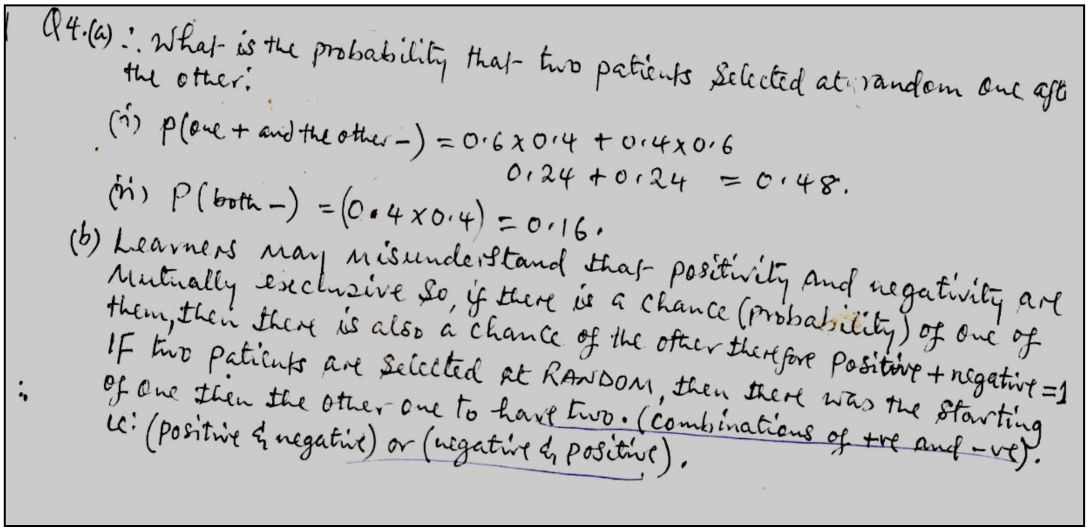

Q4.(a) ∴ What is the probability that two patients selected at random one aft the other:
(i) p(one + and the other −) = 0·6 x 0·4 + 0·4 x 0·6
0·24 + 0·24 = 0·48.
(ii) P(both −) = (0·4 x 0·4) = 0·16.
(b) Learners may misunderstand that positivity and negativity are Mutually exclusive So, if there is a chance (probability) of one of them, then there is also a chance of the other therefore positive + negative = 1
IF two patients are selected at RANDOM, then there was the starting of one then the other one to have two. (combinations of +ve and −ve).
ie: (positive & negative) or (negative & positive).

Figure 4.14: Response P12

Figure 4.14 shows that Teacher 15 was able to state that learners were likely to misunderstand that testing positive or negative was a mutually exclusive event. This means that there was chance for one, and also another one to occur as an event. He further stated this was likely to cause the learners to write an expression in the form: positive + negative =1 instead of Pr (P, N) or Pr (N, P) = Pr (P) × Pr (N) + Pr (N) × Pr (P). As shown in Figure 4.14 above, it can be stated that the Teacher was also able to state the expected expression from the given probability event in the form of: Pr (P, N) or Pr (N, P) = Pr (P) × Pr (N) + Pr (N) × Pr (P) for the two patients which were selected at random, and then there was a chance for one testing positive and another testing negative. This means that the teacher was able in another way to illustrate the combination of positive and negative expression in the form of: Pr (P) x P (N) + Pr (N) x (P). Therefore, Figure 4.14 shows a good example of explanations and justifications which were given by Teachers 1,16 and 17 as they were able to interpret and make predict of how learners could have found it difficult or easy in answering question 4(b).

On the same question 4(b), Teacher 5 stated that learners were required to first identify the concept of independence, certainty, and listing of combination of outcomes of testing positive and negative if they were to answer as the sampled teachers answered question 4(a) and (b) in the SMKD. In regard to the above requirement, Figure 4.15 presents how Teacher 5 to responded to the requirement.

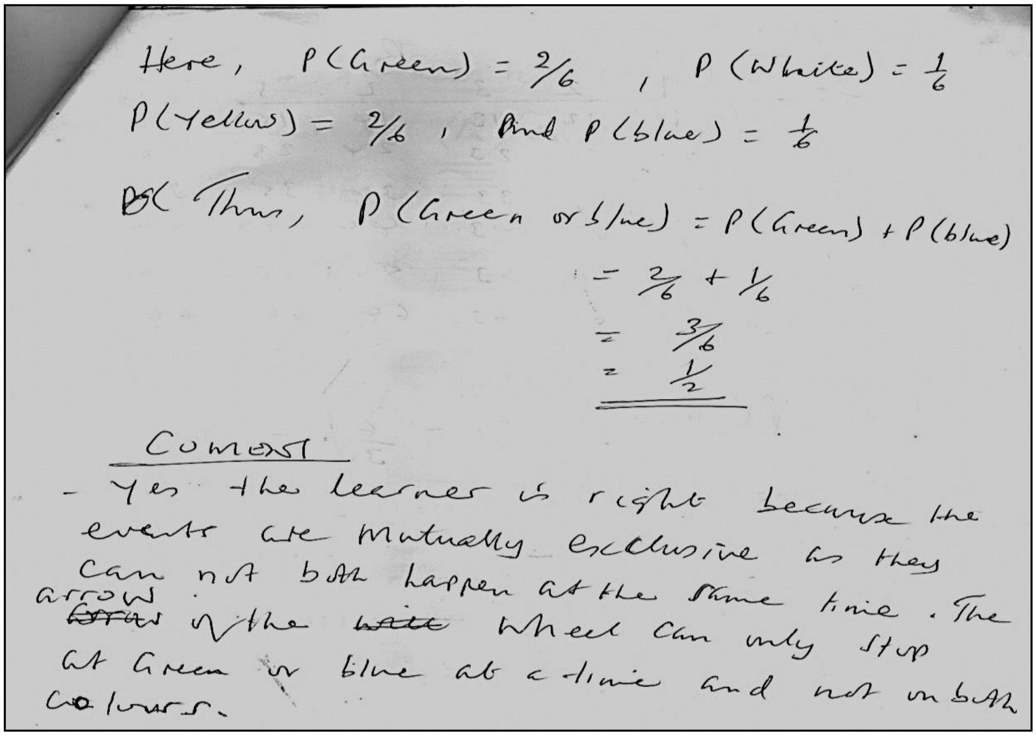
b) Assume there are 10 patients in the survey and the selection is without replacement.
It means there are only 9 patients to participate in the second selection. In (i) the events -ve then +ve and +ve then -ve are exclusive, we add where as in (ii) we multiply the two separate probabities. A tree diagram has been used to solve this question. The sum of all fractions at the end of the branches should add up to 1.

Figure 4.15: Response P13

Figure 4.15 shows that Teacher 5 responded to the SMKDT question by reciting concepts which were carried in with question 4(a) and (b) as the reasons to why learners could have misunderstood the question. The Teacher went on giving an example assuming that there were 10 patients in that survey and the selection was without replacement which had no meaning to the interpretation of the question. In addition, Teacher 5 also attempted to formulate the expression which he stated that in the event of testing malaria, the probability expression of testing malaria would appear as first negative (–ve) followed by positive (+ve) and then positive (+ve) and thereafter, negative (-ve) which he later justified his answer by stating that the event is an exclusive event, which means that there is a need to add and multiply the two separate probabilities. Therefore, Figure 4.16 shows a good example of explanations and justifications which were given by nine teachers in their effort to interpret and make predict of how learners could have found it difficult or easy in answering question 4(b).

Question 5(a).

In the context of question 5(a) which assessed mathematics teachers' ability to interpret, explain and justify their mathematical reasoning on probability concepts and teachers' ability to solve the given probability problems. The question assessed mathematics teachers' understanding of the concepts of mutually exclusive events and knowledge of the pre-requisite topics of probability which included proportions, and ratios, and rotation. In this case, Figure 4.16 presents how Teacher 4 was able to respond by providing correct calculations, explanation and justification on the learner's given answer.



Here, $P(\text{Green}) = \frac{2}{6}$, $P(\text{White}) = \frac{1}{6}$

$P(\text{Yellow}) = \frac{2}{6}$, And $P(\text{blue}) = \frac{1}{6}$

~~BC~~ Thus, $P(\text{Green or blue}) = P(\text{Green}) + P(\text{blue})$

$$= \frac{2}{6} + \frac{1}{6}$$

$$= \frac{3}{6}$$

$$= \frac{1}{2}$$

Comment

- Yes the learner is right because the events are mutually exclusive as they can not both happen at the same time. The arrow ~~arrow~~ of the ~~wheel~~ wheel can only stop at Green or blue at a time and not on both colours.

Figure 4.16: Response P14

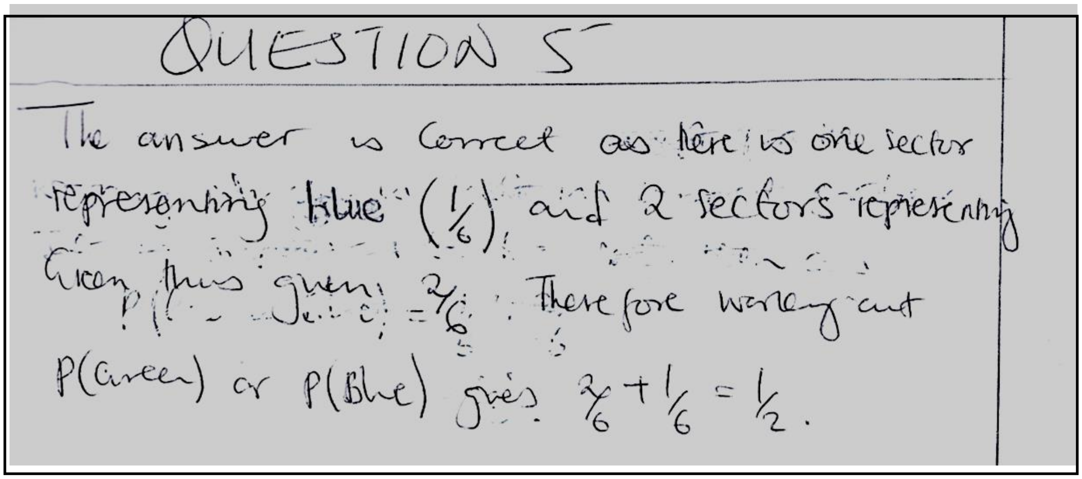
Figure 4.16 above shows that Teacher 4 used the knowledge of rotation, proportions and ratios to allocate the fractions which represented the probabilities of the colours as follows: P (Green) = $\frac{2}{6}$, P (White) = $\frac{1}{6}$, P (Yellow) = $\frac{2}{6}$ and P (Blue) = $\frac{1}{6}$. In addition, the Teacher explained that the learner's answer was correct and proved the answer by calculating the question in the following way: P (Green) or P (Blue) = $\frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$ as shown in the Figure 4.16. To justify his answer, Teacher 4 further explained that the arrow of the wheel could point at green or blue at the same time because the event was mutually exclusive. As such, the arrow was pointed at one colour after the other.

In the follow up interview, Teacher 4 explained and said that:

> Yes, the learner was right because the events were mutually exclusive as they could not both happen at the same time. The arrow of the wheel could only stop at green or blue at a time and not at both colours.

In the follow up interviews above, Teacher 4 concluded by stating that the learner was right because the event was mutually exclusive as the arrow on the wheel could only stop at green or blue at a time and not at both colours. On the same question 5(a), Teacher 13 stated that the learner’s answer was correct and provided the proof by adding the fractions which represented the probability of green and blue.

On the same question 5(a), Teacher 13 also stated that the learner’s answer was correct and provided proof by adding the fractions which represented the probability of green and blue. Figure 4.17 shows Teacher 13’s response to the question which required the sample to provide an explanation and justification on the answer given by the learner.



QUESTION 5

The answer is correct as there is one sector representing Blue (1/6) and 2 sectors representing Green thus given P(Green) = 2/6. Therefore working out P(Green) or P(Blue) gives 2/6 + 1/6 = 1/2.

Figure 4.17: Response P15

Figure 4.17 above shows that Teacher 13 was able to recall and use the knowledge of proportions, ratios and rotation to allocate the fractions as follows Blue (1/6) and Green (2/6. In addition, the Teacher indicated that the learner’s answer was correct and provided proof by calculating and got ½. However, the Teacher did not to provide a clear explanation and justification about the reasons why the learner’s answer was correct. Thereafter, a follow-up interview was conducted to understand why he stated that the learner’s answer was correct since he did not indicate in the SMKDT. In response, Teacher 13 said and explained that;

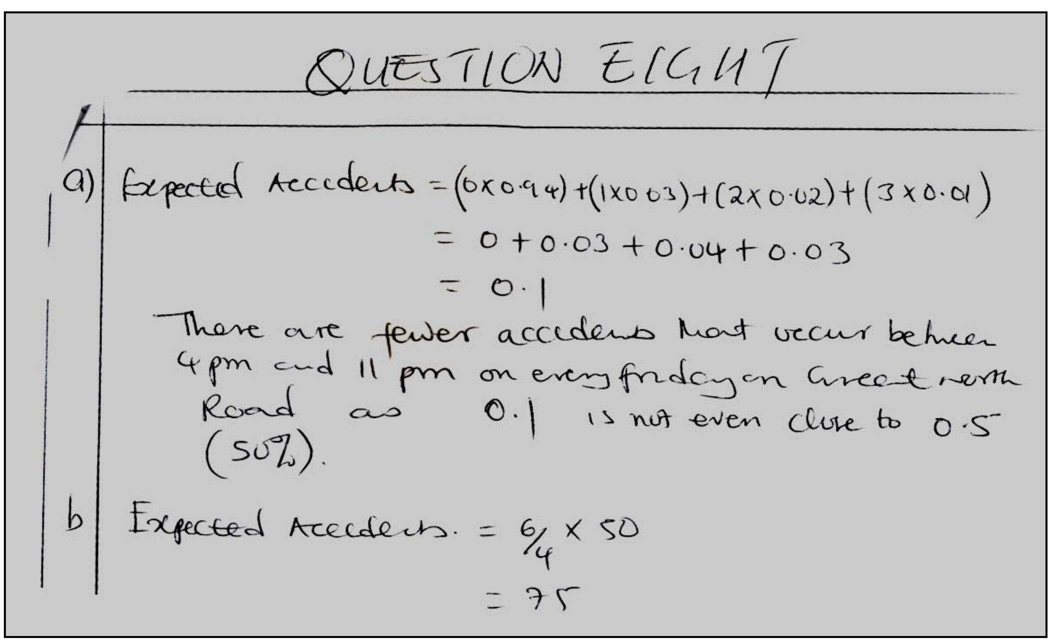
> The learner found the correct answer on question 5(a). If I remember it very well, it was a straight forward question that just required the learner to add the given fractions which were obtained from the rotating wheel and find a correct answer. The other part the learner did well in was that he remembered that the word 'or 'in probability means addition.

In the follow-up interviews, Teacher 13 still had not provided a clear explanation about the knowledge that the learner used to answer the question and the reasons why the answer was correct. On the other hand, on the same question 5(a) Teacher 9 stated that the learner's answer was wrong. In his response to the follow-up interviews, Teacher 9 explained that;

The learner's answer was wrong because the probability of green was not supposed to be 2/6 but 1/3 and the probability for blue was correctly written. Hence, the use of a wrong fraction made the learner get a wrong answer.

As shown in Figure 4.17 above and the follow up interviews which were conducted with Teacher 13 and 9, it shows that the two teachers' answers had no reflection of the subject matter knowledge of the theoretical and experimental probability in their explanations about reasons why the learner's answer was correct and wrong respectively. However, the responses given by the two teachers were a good example of other explanations and justifications which were given by three other sampled mathematics teachers.

Furthermore, question 8(a) and (b) assessed and explored mathematics teachers' ability to: solve, explain and interpret the meaning of the calculated answers about the probability scale which ranges from 0 to 1 and relate to the real-life situation. The SMKDT question also assessed mathematics teachers' ability in selecting appropriate procedures that they were required to use in solving probability problems: the Expected Frequency $=\frac{a}{b} \times$ N, the expected number of accidents =N P (A), and Expected probability number = EPX. However, these formulae were meant to provide procedural knowledge to the mathematics teachers to correctly calculate the expected answers 0.1 and 5. In regard to the above expectations, Figure 4.18 shows how Teacher 3 solved, interpreted and explained the meaning of the calculated answers using the probability scale.



QUESTION EIGHT

a) Expected Accidents $= (0 \times 0.94) + (1 \times 0.03) + (2 \times 0.02) + (3 \times 0.01)$

$= 0 + 0.03 + 0.04 + 0.03$

$= 0.1$

There are fewer accidents that occur between 4 pm and 11 pm on every friday on Great north Road as 0.1 is not even close to 0.5 (50%).

b Expected Accidents. $= 6/4 \times 50$

$= 75$

Figure 4.18: Response P16

Figure 4.18 above shows that in the first step, Teacher 3 was able to recognize and identify the procedure to be used to calculate the expected answer 0.1. In the second step, the Teacher multiplied and added the number of accidents which occurred with corresponding probabilities of the events as follows $(0 \times 0.94) + (1 \times 0.03) + (2 \times 0.02) + (3 \times 0.01)$ and found the expected answer 0.1. In the third step, the teacher was able to interpret the meaning of 0.1 using the probability scale, by stating that there were fewer road carnages that occurred between 4 pm and 11 pm every Friday on Great North Road and 0.1 was not even close to 0.5 or 50% of the likelihood of an event to be considered significant. Furthermore, in relation to question 8(b) which was a buildup question of question 8(a), the Teacher was able to identify the correct procedure, the Expected Frequency $= \frac{a}{b} \times$ N but multiplied with an incorrect fraction 4/6 and found the answer to be 75. During the follow-up interview, Teacher 3 gave the following interpretation and justification of his answer about the probability scale. Teachers 3 said that:

> This was only $\frac{1}{10}$ representing road carnages which were being recorded in the late hours along the Great North Road. This means that the probability of road carnages occurring once every week were very few.

As shown in the above follow-up interviews, Teacher 3's explanation and justification was not from the one he gave in Figure 4.18 above. Therefore, Teacher 3's response in the follow-up interview was a good example of other seven (7) teachers who found 0.1 as the answer on the first part of the question (8a) and left the second part about the probability scale unanswered.

Besides Teacher 3's exhibition of his ability and skills of solving and interpreting the calculated answer on question 8 (a) about the probability scale, Figure 4.19 shows how Teacher 5 and 14 exhibited their understanding and challenges to respond to the expected requirement of the SMKDT respectively.

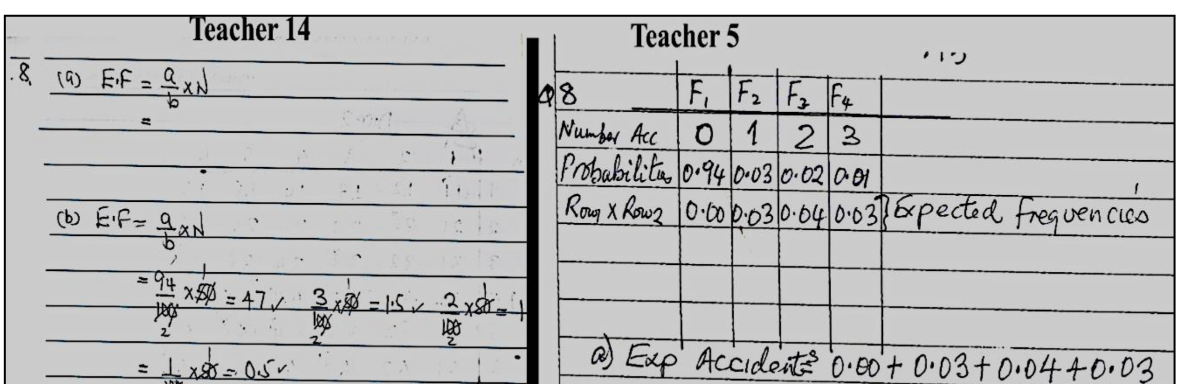
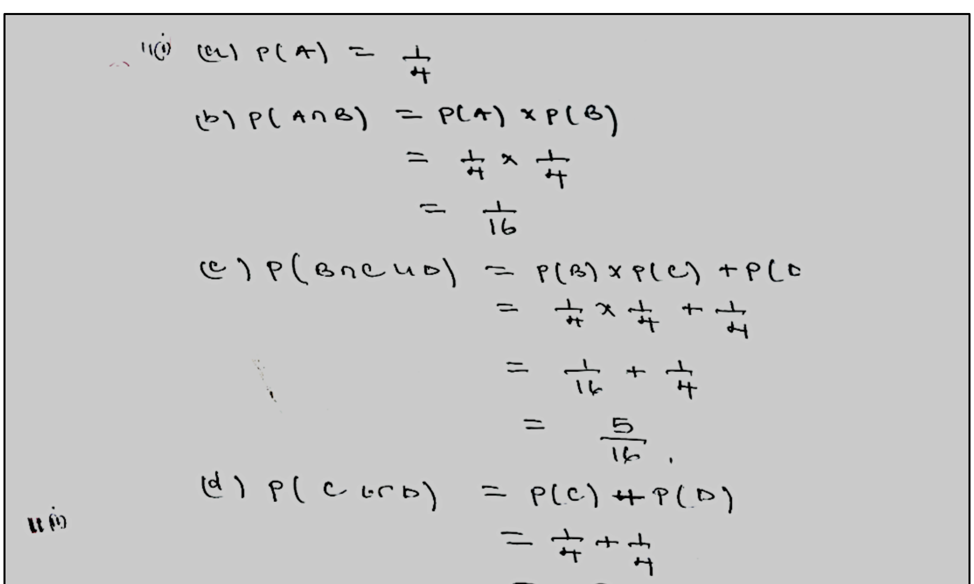


Figure 4.19: Response P17

Figure 4.19 above shows that Teacher 14 was able to use the correct procedure to calculate the expected answer 0.1, while Teacher 5 was only able to multiply and arrange the decimal numbers for addition in the following manner 0.00+0.03+0.04+0.03 and found 000.1. On the second part of question 8(a) which required teachers to relate their calculated answer 000.1 to the probability scale, both teachers left the question unanswered. Furthermore, on question 8(b) Teacher 14 merely wrote the formulae as the Expected Frequency $= \frac{a}{b} \times$ N which he had introduced in the first place with question 8(a). While, Teacher 14 introduced the formula at the right stage but could not use it to calculate the expected answer (See Figure 4.19 above). On the other hand, Teacher 5 found the correct answer 5 without indicating the procedure he used. However, this pattern of leaving questions unanswered was also observed with questions: 3(c), 4(b), 5(a), 9(ii), 11(ii), and 14 (a) and (b). Therefore, Figure 4.19 above is a good example of answers that were provided by 8 other mathematics teachers that wrote the SMKDT test.

Furthermore, question 11 was one of the most answered question in the SMKDT which assessed mathematics teachers' ability to use the concepts of sets and quadrants to solve probability problems using the two categories of teachers' ability to: solve, explain and justify their mathematical reasoning on the probability concepts. In view of the above, Figure 4.20 shows how Teacher 8 was able to reflect his ability to interpret and solve the probability questions in the SMKDT.


11(i) (a) $P(A) = \frac{1}{4}$

(b) $P(A \cap B) = P(A) \times P(B)$

$= \frac{1}{4} \times \frac{1}{4}$

$= \frac{1}{16}$

(c) $P(B \cap C \cup D) = P(B) \times P(C) + P(D$

$= \frac{1}{4} \times \frac{1}{4} + \frac{1}{4}$

$= \frac{1}{16} + \frac{1}{4}$

$= \frac{5}{16}$.

11(ii)

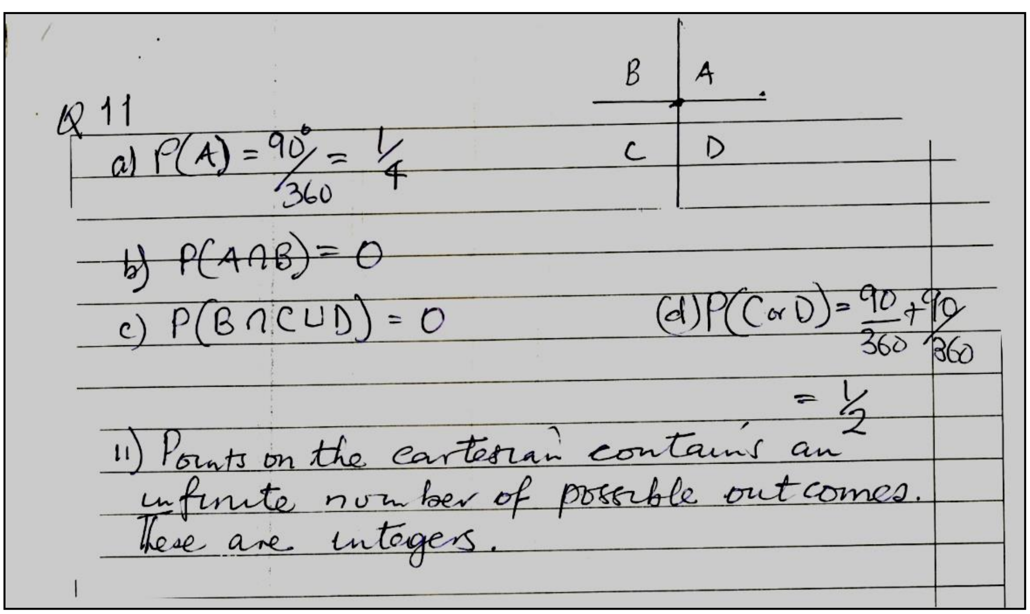
(d) $P(C \cup D) = P(C) + P(D)$

$= \frac{1}{4} + \frac{1}{4}$

Figure 4.20: Response P18

Regarding Figure 4.20 replicated above shows that Teacher 8 was able to identify that the Cartesian plane had four quadrants which added up to 360^0 and each quadrant was adding up to 90^0. Using this understanding, Teacher 8 divided $\frac{90}{360}$ and found $\frac{1}{4}$. In addition, Teacher 8 was also able to recognize, interpret and expanded the sets in the following way : P (A ∩ B) as P (A) x P (B) while P(B∩C∪D) as P(B) × P(C) +P (D). Teacher 8 further expanded P (C or D) as P(C) + P (D) and found answers on each question (See Figure 4.20 above). On the second part of question 11(b) (ii) which required teachers to provide a clear explanation and justification as to why all the sets of points on the Cartesian plane are continuous sample space, the Teacher left the question unanswered. However, this similar pattern of either answering the first part or the second part of the question as well as leaving some questions unanswered was observed with questions: 3(c), 4(b), 5(a), 9(ii), 10(ii) and 14 (a) and (b).

On the same question, Teacher 5 was only able to answer the first part of question 11(i) and left questions 11(b) and (c) unanswered which were buildup questions for question11 (ii). The following is Teacher 5's presentation of SMKDT answers in the first and second part of the questions 11 shown in Figure 4.21.


Q 11

B | A

C | D

a) $P(A) = \frac{90^0}{360} = \frac{1}{4}$

~~b) P(A∩B) = 0~~

c) $P(B \cap C \cup D) = 0$

(d) $P(C \text{ or } D) = \frac{90}{360} + \frac{90}{360}$

$= \frac{1}{2}$

ii) Points on the cartesian contains an infinite number of possible outcomes. These are integers.

Figure 4.21: Response P19

Figure 4.21 shows that Teacher 5 was only able to answer the first part of question 11(i) and left questions 11(b) and (c) unanswered which were build-up question for question11 (ii). Furthermore, Teacher 5 was able to answer question 11(ii) which required teachers to explain why all sets of the Cartesian plane are continuous sample and explained that all the points on the Cartesian plane are continuous sample because they contain infinite numbers of possible outcomes. The Teacher further justified that those numbers were integers. Similarly, Teacher 14 explained that all the points on the Cartesian plane had four quadrants and contained an infinite number of outcomes. Therefore, Figure 4.21 above is a good example of other exhibitions of teachers' subject matter which were provided by nine teachers in the SMKDT test. On this same question, five teachers were able to provide the explanation and justification for the first and second part of the question 11a (i) and 11b (ii), while three teachers provided answers only to the first part of the question.

Question 12 was also the second most answered after question 11 which assessed mathematics teachers' ability to use basic concepts of sets to solve probability using the two categories of teachers' ability to: solve, explain and justify the conceptual procedures they used to solve the questions. The following is Teacher 2's presentation of answers as shown in Figure 4.22:

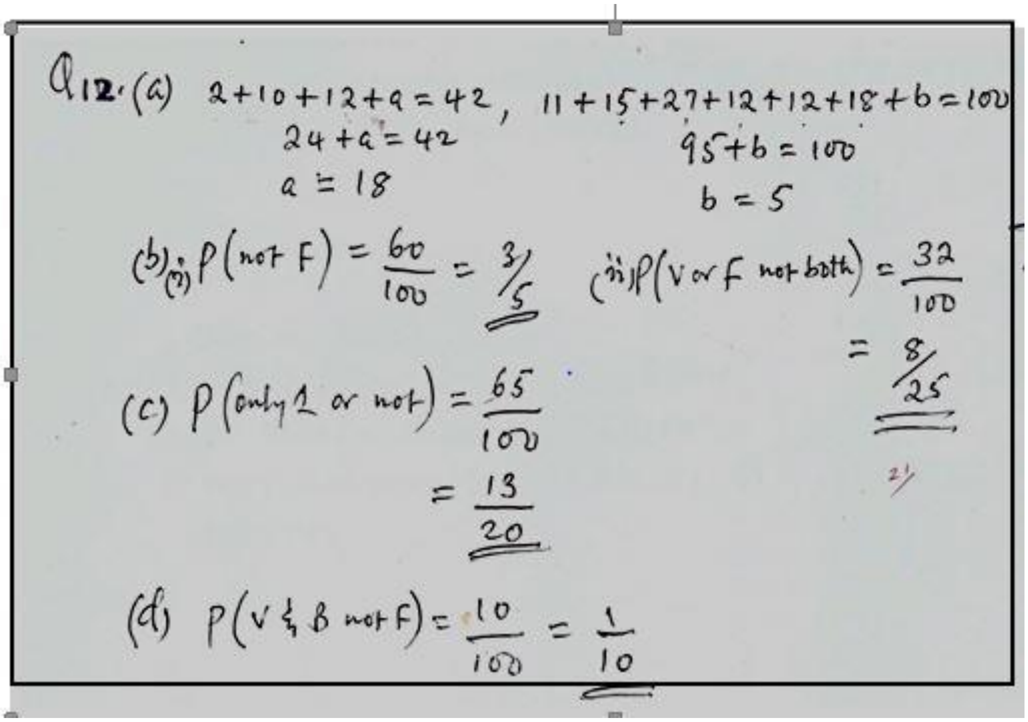


Figure 4.22: Response P20

Figure 4.22 above shows that Teacher 2 was able to display his understanding of the basic concepts of sets in solving probability problems and this was seen in his procedural fluency and the manner he calculated the answers. This understanding was also evident in the manner he presented his work and the solutions he found as shown above in Figure 4.22. In the follow-up interviews, the Teacher was required to explain the mathematical knowledge which he used to answer question 12. In response to the follow-up interviews, Teacher 2 said;

> I used mathematical knowledge of sets and laws of probability. For example, the word '**or**' in probability symbolizes the operation addition **(+)** which means to add, while the word **'and'** symbolizes the operation multiplication **(x).** I also had no challenges in solving the question because I have taught probability and sets for a long time.

Furthermore, similar answers as given in the follow up interviews above were observed with questions 6 (a), 7(a) & (b), 9(i) & (ii) and 10 (a) & (b) in which teachers left some questions unanswered.

Table 4. 4: Summary of competences mathematics teachers left unanswered, provided explanation and justification in the SMKDT

Questions numbers	No explanation and justification given by any teacher	Explanation given by teachers.	Justification given.	Total number of teachers
1c	1	12	4	17
2a	0	11	6	17
3c	3	9	5	17
4b	3	11	3	17
5a	0	12	5	17
9 (ii)	3	10	4	17
11 (ii)	2	13	2	17
14a	1	11	5	17
14b	1	13	3	17
Total	14	95	44	153

Table 4.4 above shows that a total of fourteen (14) concepts were provided with no explanations and justifications and in some situation, they were left unanswered, whereas ninety-five (95) concepts were provided with uneven explanations and justifications and forty-four concepts were also provided with even explanations and justifications by the sampled teachers of mathematics respectively.

4.4 Summary of the findings for research question one

According to the findings of the research question one, it was noted that the sampled mathematics teachers were able to exhibit the subject matter knowledge of probability as evidenced in the manner they answered the SMKDT questions. In this regard, mathematics teachers were able to define the probability terms using the socially agreed qualitative terms while others were merely reciting their definitions as they are presented in the Mathematics Grade 11 textbooks (See Figure 4.3, Section 4.3.1). In a related situation, teachers were also able to formulate the probability compound spaces with more than two compounds in form of L, L, N, L and Pr (P) Pr (N) + Pr (N)Pr (P) and found answers such as 19/60000, 0.48 and 0.16 respectively (See Figures 4.9, 4.13 & 4.14, Section 4.3.1). This was a result of teachers' ability to recognize and identify that the four consecutive days and the testing of positive and negative for malaria occurred as independent events. This understanding helped them to formulate the probability compound spaces and found the expected answers. However, majority of teachers (10) struggled to formulate the expected compound spaces in the form of P(Late) and P (Car and Late) which they used to find answers such as 1/60000 and 3/10 respectively (See Figures 4.8, 4.10 & 4.11, Section 4.3.1). Furthermore, teachers were able to differentiate between the non-mutually and mutually exclusive events. In this regard, teachers were able to extend their understanding of non-mutually and mutually exclusive events and formulated the associated probability expressions in the form of P (AUB) =P (A) + P (B) – P (A∩B and P (AUB) =P (A) + P (B) for Figure 1 and 2 using the notation symbol 'U' respectively (See Figure 4.7, Section 4.3.1). On the other hand, some teachers used the notation symbol '∩' instead of 'U' and formulated the associated probability expression in the form of P (A∩B) = P (A∩B) = P (A) P (B) (See Figure 4.8, Section 4.3.1). This further caused the teachers to fail to provide clear explanations and justifications using probability reasoning on how learners were likely to misunderstand the concepts the teachers used to answer the questions. It is also important to note that mathematics teachers demonstrated their understanding of the subject matter knowledge in the manner they recognized and identified the procedures which were required to use and calculate to find the expected answers such as 0.1 and 5, while majority of teachers found answers such as 0.001. However, the above understanding led majority of teachers to provide different interpretation of the meaning of the calculated answers concerning the probability scale while some teachers left the same questions unanswered. In some cases, mathematics teachers were able to solve the first part of the question and, provide explanation and justification while the second part of the same question was usually left unanswered and vice versa (See Figures 4.18, 4.19, 4.20 & 4.21, Section 4.3.1).

4.5 Pedagogical strategies mathematics teachers used to teach Probability Concepts

This was the second research question which focused on the two (2) aspects of the teachers' pedagogical strategies which included; mathematics teachers' ability to identify appropriate (learner-centered) pedagogical strategies to teach probability concepts as well as the ability to use multiple pedagogical strategies to teach on specific concepts of probability such as the concepts of independence, dependence and the mutually exclusive events. In regard to the above, Section 4.5.1 presents the identified pedagogical strategies which are explained in details in the Table 4.5 and Excerpts 4.1 to 4.7 on how the sampled teachers used the pedagogical strategies to teach probability concepts during lesson presentations.

4.5.1 The identified pedagogical strategies used in the teaching of probability during lesson presentations and reflected in the excerpts

Before conducting lesson observations, the researcher conducted preliminary observation interviews to assess the teaching methods and strategies they preferred to use in teaching probability concepts in a comprehensive way to the learners. It is important to note that, the development of the teaching strategies listed in Table 4.5 was not only provided by the sampled teachers, but they were also developed through document analysis of the 2013 Zambian Senior Secondary School Mathematics Syllabus 10-12 and the review of the literature of this study. This was intended to assess if teachers were aware of the use of teaching strategies that are recommended for use particularly when teaching abstract topics such as Probability. In this regard, the Senior Secondary Mathematics Curriculum (2013) emphasizes the use of teaching strategies that are in a position to help learners to show their potential in different learning areas (MESTVEE, 2013). In this view of this, it shows that the teachers were able to identify and select teaching strategies that are recommended to help learners show their potential in different learning areas. In addition, teachers were in line with the focus of the second research study objective, which aimed at exploring mathematics teachers' ability in planning and selecting learner-centered and use of multiple pedagogical strategies to effectively teach specific concepts of probability such as the concepts of independence, dependence and the mutually exclusive events. The following are the teaching methods and strategies which were identified by teachers and the review of the 2013 mathematics curriculum as shown in Table 4.5.

Table 4. 5: Pedagogical teaching methods and strategies which were identified by teachers before conducting lesson observations

Teaching methods	**Teaching strategies used to teach probability.**
Learner demonstration	Teachers to give learners a platform to demonstrate their understanding of the subject matter knowledge for them to develop self-confidence and concretize probability concepts.
Group work activity	Learners are to be given chance to discuss in reasonable groups and share ideas with the rest of the class after discussion to promote

	interaction among learners and also develop communication skills.
Problem solving	Learners to be subjected to any form of problem solving to allow them to develop self-confidence and ability to solve any given probability problems.
Discussion of learners' feedback	The teacher and the learners to discuss each learner's given answer whether correct or wrong to allow the learners to discover their own errors and concretize what they have understood.
Independent problem solving	Teachers to conduct one-to-one comprehensive individualized assessment to investigate learners' intuitions that they have about certain probability concepts.
Whole class topic discussion	Teachers give chance to the learners to contribute their thoughts. In this case, the teacher makes it learner centered by involving everyone in class. This to builds their knowledge in an active way
Individuals and individual education plans (paying attention to fast and slow learners).	The teacher to pay attention to both fast and slow learners. Since, learners are unique and have individual learning needs that should be taken into consideration at planning level.
Teaching and learning aids strategy	Teachers to display visible key knowledge required by the learners that has the possibility to enhance learners' understanding of the subject matter knowledge of the topic.
Role play in Games and Experiments	Teachers to teach using a role play in order to help learners to develop understanding of

	everyday language of probability.
Group work discussion	Teachers to put learners in reasonable groups to promote participation, cooperation, sharing of ideas and to express themselves freely to each other.
Discovery method	Teachers to give opportunities to the learners to find their own solutions so that they can generate their own knowledge and the teacher's role is providing support and assist learners in the discovery.
Question and answer method	Teachers to maintain the focus of learners and become active in the learning process. This can help learners to understand and retention of the procedures and probability concepts.
Teacher demonstration method	The Teachers to provide learners with useful information and learners to participate by providing answers as the teacher demonstrate.
Teacher exposition method	Under this methods, learners to provide answers as the teacher presents useful summarized subject matter knowledge.
Simulations of probability concepts.	Works as a methodological teaching aids and provide physical understanding of the concepts through demonstration in a trial using either a computer, coin or dice.

To assess how the sampled mathematics teachers were able to use the above identified pedagogical strategies, the researcher observed seventeen (17) teachers on specific concepts for one hour thirty minutes per lesson in their respective schools. In this regard, each teacher was observed once on a specific concept that they chose to represent, while others presented their lessons on the questions which they either answered correctly or not in the SMKDT test.

In the first lesson which was presented by Teacher 11, the Teacher taught by engaging learners with their observed challenges in the constructing and filling of the corresponding probabilities on a tree diagram. In the first step, the teacher began his lesson by giving an example and solved the given question with the help of learners using the tree diagram (See Appendix 8.1). In the lesson development, the Teacher used group work and paired the learners in groups, and allowed them to discuss and share their understanding of probability concepts of dependence. The teacher instructed learners to find the probability of two balls of different colours by showing all the possible outcomes on a tree diagram in which a ball was drawn and not replaced, and then a second one was drawn from a bag that contained two black balls, one red and one white ball. In responding to the learners' answers, the Teacher gave them chance to justify their answers and reasoning to assess their understanding of the subject matter. To achieve this, the Teacher taught on the concept of dependence using teaching methods such as teacher exposition, subjective learning, question and answer, and learner demonstration methods (See Appendix 8.1). The following Excerpts 4.1 to 4.7 shows how teachers used the identified and reviewed pedagogical strategies as they taught probability concepts. What follows next is Excerpt 4.1 which shows how Teacher 11 consistently engaged the learners with their challenges by giving them chance to defend their answers and other learners' reasoning.

Excerpt 4.1: Shows how Teacher 11 engaged learners with their challenges during lesson observation

In the first step, Teacher 11 explained that the other alternative way of approaching probability problems is to use the Venn diagrams.

Teacher 11: Demonstrated using a tree diagram on the concept of dependence.

Teacher 11: Asked the learners individually to define the term dependent events in the way they understand the term with a help of examples.

Mary: A dependent event is a situation which happens depending on the other thing for example, for the rains to fall, first clouds should form and them after that the rains will occur.

John: Explain to the class whether Mary's definition and the examples given are correct or not?

John: Explained to the class and said, she is right because for the rains to occur, clouds should form first, this means that the falling of rains depended on the formation of clouds first.

Mulenga: Explain whether John's and Mary's answers were correct or not?

Teacher 11: Introduced the expression P (A and B) =P (A) × P $(\frac{B}{A})$.

Teacher 11: Demonstrated using an expression on the concept of dependence.

Gilbert: What have noticed on the second corresponding probabilities shown a tree diagram and why the probability fraction was written as $\frac{4}{9}$ instead of $\frac{3}{9}$?

As shown above in Excerpt 4.1, the Teacher engaged the learners in the lesson by discussing both their challenges and understanding so that they would become proficient in the subject matter. In the follow-up interviews, Teacher 11 gave the following responses about the engagement of learners with their challenges and understanding of the subject matter they were familiar within which he gave them a chance to defend their answers and challenge other learners' reasoning as he taught on the concept of dependence. The Teacher stated that;

> The aim of engaging learners was to allow learners to debate on the subject content of the concept of dependence to make them realize their mistakes and correct them as they debated at length as a class. To encourage debate on the concept, i used a combination of question and answer, teacher exposition, class topic discussion, and group work activity methods to help me to engage learners to work in collaboration and individually. The aim of using the mentioned teaching methods was to promote a learner-centered approach which is being encouraged by the Ministry of Generation Education and other stakeholders.

In the second lesson presented by Teacher 5, the Teacher used question and answer format to guide the learners to discover the rules which were used to expand the associated probabilities of the concepts of independence and mutually exclusive events. The teacher started the lesson by explaining on the concept of independent and mutually exclusive events. Thereafter, instructed them to state the addition and multiplication of rules used to describe the independent and mutually exclusive events in groups (See Excerpt 4.2). In regard to the above, Excerpt 4.2 shows how Teacher 5 supported learners to understand the concepts of independence and mutually exclusive events through question and answer method which guided learners to discover on their own the rules of concepts of independence and mutually exclusive events.

Excerpt 4.2: Shows how Teacher 5 engaged and guided learners through question and answer format during lesson presentation.

Teacher 5: Differentiate mutually exclusive and independent events?
Monde and Joseph: Defined the terms mutually exclusive and independent events.
Teacher 5: Events A and B are mutually exclusive events if they cannot happen at the same time. While Events A and B could be independent events if the occurrence of event A does not influence the occurrence of Event B.
Teacher 5: Using real-life situations, state and give examples of independent and mutually exclusive events in groups.
Learners: Examples of mutually exclusive events Group 4, walking and cooking, Group 2, light and darkness, as the light disappears, then darkness comes in.
Learners: Examples of independent events, Group 3; light and darkness.
Teacher 5: Explained that the two concepts are differentiated with two terms 'or' and 'and' which are used to describe their addition and multiplication rules for mutually exclusive and independent events.
Teacher 5: Instructed learners to work in groups to identify and formulate the probability expression using 'or' and 'and' for mutually exclusive and independent events respectively.
Teacher 5: Use these mathematics books to identify how to formulate the probability expression.
Learners: Group 1 gave P (A or B) =P (A×B); Group 2 gave P (A and B) =P (A × B); Group 3 gave P (A or B) =P (A +B) and Group 4 gave P (A and B) =P (A+B).
Teacher 5: The following are the expression that represent the multiplication and addition rules for independent the mutually exclusive events: P (A and B) =P (A × B) and P (A or B) =P (A +B) respectively.

As shown in Excerpt 4.2 above, Teacher 5 engaged learners through question and answer which guided them to discover different answers in relation to the multiplication and addition rules used to describe the independent and mutually exclusive events. To support their response in the question and answer, the teacher was also able to provide the explanation to make clarity on their given answers with the help of the teaching and learning materials which were given to each group such as Grade eleven mathematics textbooks per group. In concluding the lesson, the

Teacher allowed learners to answers questions in a chorus form which he later wrote on the board without giving clear explanation and justification whether the given answers were correct or wrong. He wrote P (A and B) =P (A × B) and P (A or B) =P (A +B) respectively. In the follow-up interviews, the Teacher was asked the intention of using question and answer format to guide learners to discover on their own the rules which could be used to formulate the associated probabilities of the concepts of independence and mutually exclusive events. Teacher 5 explained that;

> I used question and answer format to guide and to allow the learners to reflect on the subject content so that they could discover on their own. This was intended to enhance their mathematical independence in solving probability problems in the process of discovering and to make them remain focused on they were being taught. Yes, I also gave them chance to demonstrate their understanding of the new knowledge learned before the class. This was meant to allow them to demonstrate what they understood so that I could start explaining from that point. Yes, I concluded the lesson without explaining because I wanted them to reflect and further research more on the two concepts so that when we meet next lesson, I can start by asking them questions on the two addition and multiplication rules.

Furthermore, Teacher 12 presented a lesson based on the Fundamental Counting Principles of Probability on which he allowed learners to work in collaboration to explore, discuss and share their understanding of the fundamental counting principles of probability. To support the idea of learners working in collaboration, Teacher 12 allowed learners to work in groups and later demonstrate their new knowledge learned before the class. In concluding the lesson, the Teacher was able to provide his mathematical point of view on the concept of fundamental counting principles of probability for the learners to have a clear understanding of the probability concepts. In light of the above, Excerpt 4.3 shows how Teacher 12 engaged learners to work in collaboration to explore, discuss and share their understanding of probability concepts that required them (learners) to make reflections before answering.

Excerpt 4.3: Shows how Teacher 12 engaged learners to work in collaboration to explore, discuss and solve probability problems during lesson observation

Teacher 12: Today we are going to look at Fundamental counting principles of probability.
Teacher 12: In the first step, we are going to define and explain on the terms sample space, random, probability scale, theoretical and experimental probability.
Teacher 12: Thereafter, he instructed learners to answer the following question in groups;
Teacher 12: A calculator is programmed to generate random integers from 1to 100 inclusive when the enter button is pressed.
What is the P (multiple of 10) when the enter button is pressed one time, interpret and explain the meaning of the answers found in relation to probability scale?
Teacher 12: Reflect on the above questions and then find the sample space.
Bright: it has 100 possible outcomes

Teacher 12: How did you find 100 as a possible outcome, explain?
Bright, I used the definition of a sample space to find 100 as the possible outcomes
Teacher12: Find the number of favourable outcomes.
Muyunda: it has 10 favourable outcomes
Teacher 12: Maurice, Explain how Muyunda found the answer to be 10 favourable outcomes.
Muyunda: I first listed all the ten possible outcome and thereafter I counted them one by one and found 10 favourable outcomes (10,20,30,40,50,60,70,80,90,100).

Teacher 12: What is P (multiple of 10)?
Jacob: the answer is 0.1.
Teacher 12: Explain and demonstrate before the class the procedures you used to find 0.1
Jacob, I used the definition for theoretical probability to find P (multiple of 10) =Number of multiple of 10/Total number of outcomes =10/100 =1/10=0.1
Teacher 12: Complete the question given as P (green then blue) = $\overline{13} \times \overline{12}$ if a bag contains 5 green and 8 blue marbles. What is the probability of picking a green marble, not replacing it, and then picking a blue marble?

As shown in Excerpt 4.3 above, Teacher 12 allowed learners to work in collaboration to answer questions that required them to reflect and analyze before answering in groups. The Teacher also gave the learners more time to explore, discuss, and share their understanding to the class. In conclusion, the Teacher was also able to provide his mathematical point of view on the Fundamental Counting Principles of Probability to consolidate old knowledge and enhance learners' understanding of probability concepts.

Based on the planned and selected teaching strategies used to teach on the fundamental counting principles of probability, follow-up interviews were conducted to understand reasons why he allowed learners to work in collaboration, provided space during lesson presentation for learners to explore, discuss, and share ideas, and also was able to provide feedback to their answers. In response to the interview, Teacher 12 explained that;

> I allowed learners to work in collaboration so that they could share ideas of fundamental counting principles of probability to enhance their mathematical independence in solving probability problems. Now, as teachers of mathematics, we need to understand that learners are supposed to be given opportunities to find answers on their own so that they can develop the spirit of perseverance rather than always being provided with free answers. To present my lesson, I used much of question and answer, learner demonstration, group work activity, one-on-one, and discovery methods and chart display which allowed learners to work in collaboration and to question the reasoning behind each answer which was given by their fellow learners whether wrong or correct.

In the lesson presented by Teacher 15, the Teacher provided limited time and opportunities for learners to participate in the teaching and learning process, as the Teacher dominated the whole lesson, despite providing teaching aids and prepared a lesson plan. The Teacher did not give learners maximum time to demonstrate how they understood the common terms used to describe probability events such as trials, events, experiments, and others. In this case, learners became spectators and passive as the Teacher became the only source of information to explain on the concepts of probability. On the other hand, it was observed that the Teacher was able to provide clarity and accuracy in his explanation in a systematic manner. In line with the above explanations, Figure 4.23 shows how learners remained passive while the lesson was progressing during lesson presentation with Teacher 15.

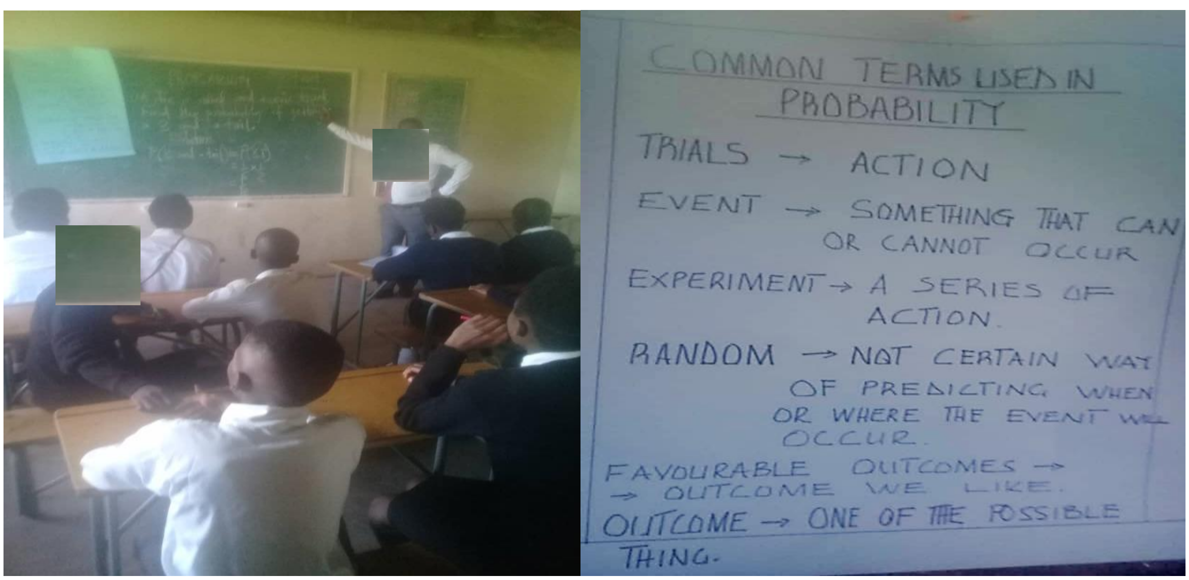


Figure 4.23: Response P21

The following is an excerpt which illustrates how Teacher15 taught on the new words (common terms) used to describe probability events.

Excerpt 4.4: Shows how Teacher 15 allowed limited time and opportunities for learners to participate in the teaching and learning process

Teacher 15: In the last lesson, who can tell us what we looked at?
Benn: We looked at probability and the laws of probability.
Teacher 15: Correct
Teacher 15: So, today we are going to look at definitions of terms or words used to describe probability events.
Teacher 15: The following terms or words are used to describe probability events
Teacher 15: An impossible outcome means it will never happen,

Certainty outcome means the event has the possibility of occurring. It is sure to happen.
Unlikely outcomes usually happen less than half the life time and is called improbable.
A likely outcome happens more than half the time and is called probable.
Equally likely outcomes usually happen as often as each other and it is called a fifty-fifty or equal chance.
More likely outcomes usually happen more often than less likely outcomes.
Event means one particular outcome in which we are interested.

Sample space, these are total number of possible outcomes of a trial.

While, a trial refers to performing an activity. For example, carrying out an experiment

As shown in Excerpt 4.4 above, the Teacher became the only source of information and a model to scaffold learners' understanding of the common terms in which he shared his mathematical know-how. This caused some learners to become attentive while others passive as it was observed in a recorded video. After lesson observations, a follow-up interview was conducted with Teacher 15 to understand the reasons to why he taught learners without giving chance for the learners to demonstrate and explain their new knowledge learnt on common terms used to describe probability events. Teacher 15 said that;

> It is not every time you expect learners to participate. In this lesson I wanted learners to listen attentively. Yes, they could be taught using a variety of teaching strategies but learners need to listen and observe passively whatever the teacher is teaching in some topics so that they can easily put it into practice. It is important to know that, when teaching abstract topics like Probability, it is important for learners to listen because this topic has a lot of confusing terms which are similar to the words used in English. Words like probably, sure, both and certain, have different meanings of the words when they are used in Probability.

Furthermore, Teacher 8 in her lesson presentation used the SMKDT question which assessed learners on the conceptual relationship between laws of probability and a theory of sets. In her teaching, it was observed that the Teacher taught the learners based on their interpretation ideas of disjointed and jointed sets in connections with the law of probability and sets. The teacher was also able to give chance to the learners to discuss questions in groups and as a whole class, even though she was uneven in her presentation. To complement her selected strategies, Teacher 8 planned her lesson and taught using question and answer, guided and discovery, whole-class discussion, and teacher and learner demonstration methods (See Appendix 8.2). Therefore, the following is an excerpt showing how Teacher 8 engaged learners as he taught on non-mutually and mutually exclusive events.

Excerpt 4.5: Shows how Teacher 8 was able to engage learners through their interpretation of shaded, disjointed and jointed sets.

Teacher 8: In the last lesson, who can tell us what we looked at?
Bupe: We looked at additions rules of non-mutually and mutually exclusive events.
Gertrude: Explained that the additions and multiplication rules of non-mutually and mutually exclusive events are described using words such as ‘or’, ‘both’ and ‘and’
Teacher 8: So today we are going to look at non-mutually and mutually exclusive events in relation to the ideas of disjoint and joint sets.
Teacher 8: State the real-life situations that describe the non-mutually and mutually exclusive events.
Matimba: Explained and gave examples of mutually exclusive events such as the rising of a sun and the dawn of the sun in the evening.
Chanda: Explained and gave examples of non-mutually exclusive events as the day cannot come until the night goes away or vice versa.
Teacher 8: Used the occurrence of light and darkness, and daytime and night time to explain the learners’ answers on the concept of non-mutually and mutually exclusive events which learners were familiar with and were able to differentiate.
Teacher 8: Instructed learners to answer the question in groups which read as, suppose you want to find the probability that the sum of scores of two rugby teams were either 7 or 9.
What set notation symbol would you use to formulate the associated probability expression?
Melody: mentioned of an ‘U’ and Alexander mentioned of ‘∩’
Teacher 8: Used the two notation symbols ‘U’ and ‘∩’ and explained by demonstrating the conceptual relationship between the disjointed sets ‘U’ with the laws of probability and interpreted that sets A and B were mutually exclusive events because of $A \cap B = \emptyset$.
Teacher 8: Formulated the associated probability expression with the help of learners using the mathematical notation symbol ‘U’ as follows $P(AUB) = P(A) + P(B)$.

As shown above in Excerpt 4.5, Teacher 8 used learners’ ideas to explain the concepts of non-mutually and mutually exclusive events using their partial knowledge of the subject matter of probability. The teacher also allowed learners to work in collaboration and discuss and answer the question in groups. In conclusion, the Teacher was also able to summarize the lesson through question and answer format. In the follow-up interviews which were conducted to understand the reasons why the Teacher used learners' ideas to explain the concepts of non-mutually and mutually exclusive events and the set notation symbols **'U'** and '∩'. In response to the interview, Teacher 8 said:

> I taught them based on their interpretation ideas of disjoint and joint sets to build up their understanding of probability concepts. You also need to know that in that class I was teaching, some learners were gifted differently, so they needed to learn in different ways so that they could understand the conceptual relationship between the theory of sets and laws of probability. I taught like that because I wanted learners to express what they know to the whole class since learners are not to be treated like empty vessels. To use effectively the selected teaching strategies, I supported my lesson using question and answer, teacher demonstration, guided discovery, worked with their answers and lastly, I allowed learners to discuss the answers they found in groups and as a whole class.

Teacher 7 also presented and taught a lesson on how to simulate and determine the actual independence of a single toss. In the first step, the Teacher organized and provided teaching and learning materials such as coins, dice, a computer, and the table to allow learners to facilitate smooth carrying out a demonstration. However, Teacher 7 did not only provide the teaching and learning materials but also demonstrated by tossing the coin, and rolling of a dice, and recorded the outcomes as follows HT, TH, HH, TT to assist and give direction to the learners in what they needed to do. Thereafter, the teacher instructed learners to simulate and determine the actual independence of a single toss of a fair coin as well as to formulate the probability compound spaces as the coin was tossed twice in groups. The Teacher was also able to move from one group to another to provide guidance and thereafter allowed learners to demonstrate how they found their answers to the whole class. Teacher 7 also gave learners time to demonstrate in which they successful conducted the trials but his explanation in the interview was not clear enough to explain on how to determine the actual independence of a single toss in the tossed coin. In regard to the above explanations, Excerpt 4.6 explains how Teacher 7 engaged learners in the process of simulating and determining the independence of a single toss after tossing a coin twice.

Excerpt 4.6: Shows how Teacher 7 was able to instruct and demonstrate on how to simulate and determine the actual independence of a single toss during lesson presentation

So today, we are going to carry out an activity called coin tossing. We going to have a coin and smooth table. What you are going to do the two of you, Michael and Jane who have volunteered to come in front is, Michael, you are going toss a coin twice on the table and cover the side of the coin with your palm and again for the second time. Then, Jane will check the side of the coin, then record the side on the black board as compounds. Class, how many sides does the coin has? Yes, Craters. Correct, there are two sides. What are those two sides? Yes, Purity, these are the Head as H and the Tail as T, correct. This activity is intended to determine the actual independence of a single coin toss and later form a compound of two outcomes which should look like this HT, TH, HH, and TT. Thereafter, the teacher demonstrated how to hold and spin the coin and gave learners to demonstrate. Then the rest of the class you need to contribute as the two are demonstrating.

The teacher gave chance to two learners to demonstrate, Michael to start and Jane to check. Therefore, learners were allowed to demonstrate in front of others as shown in Figure 4.24

Figure 4.24: Response P22

After the lesson presentation, a follow-up interview was conducted with Teacher 7 to explain why he used different teaching and learning materials to simulate and determine the actual independence of a single toss such as a fair coin, dice, and cards and thereafter formulated the probability compound spaces as they were tossed the coin twice. In response to the interview, Teacher 7 said that:

> To determine the actual independence of a single toss the learners were to understand that when the coin is tossed either the Head (H) or a Tail (T) should land first on top. When the head (H) or the Tail (T) lands first on top it means that it has made a complete toss without any influence from the occurrence Tail (T) or the Head (H). To engage learners in the experiment as well as to check on their understanding using question and answer, I used question and answer, learner and teacher demonstration, group work activity, teaching aid, and guided discovery methods.

In a lesson presented by Teacher 14, it was observed that his teaching was characterized by problem-solving questions. To engage learners in his lesson, the Teacher used the strategy of scaffolding through question and answer format which guided the learners to use specific procedures to elicit particular answers, and thereafter allowed them to solve independently the given probability problems. In this case, the Teacher instructed learners to find the probability of picking a green marble which has not been replaced and then pick a blue marble in a bag which contains 4 green and 5 marbles using the expression P (A and B) =P (A) $\times$ P ($\frac{B}{A}$) and solve the probability problem. However, Teacher 14's and learners' presentation of the concept of dependence is shown in the following Figure 4.25.

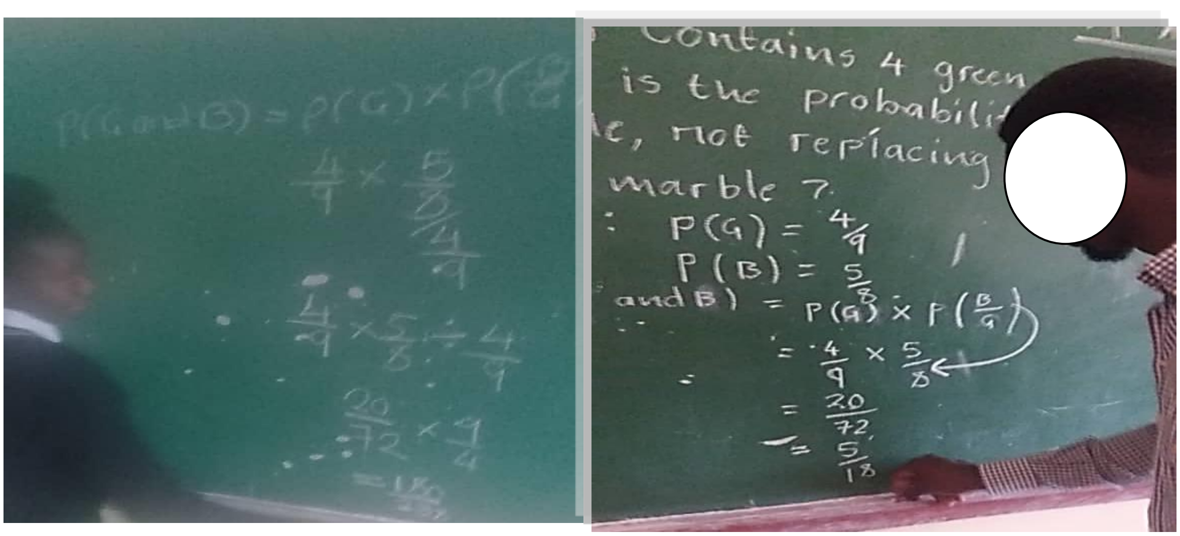


Figure 4.25: Response P23

As shown above in Figure 4.25, the Teacher allowed the learners to demonstrate and share their understanding of the subject matter knowledge of the concept of dependence, in which the teacher had instructed them to determine whether sets A and B were dependent or independent events and calculate its probability, in which the event A contained 4 green marbles and B contained 5 blue marbles. However, the Teacher's response was unable to provide accurate explanation on the meaning P ($\frac{B}{A}$) in the expression P (A and B) =P (A) × P ($\frac{B}{A}$). In the follow-up interviews, Teacher 14 pointed out the following:

> Learners need to be taught probability using various teaching strategies to meet their styles of learning. I allowed learners to discuss dependents events as a class, I also gave work in groups as group work activity and I allowed learners to demonstrate their understanding which means that I was there guiding them so that they could discover the concepts of dependents events on their own. I did this to give chance to the learners so that they could demonstrate their understanding of Probability concepts.

Table 4. 6: Summary of strategies that teachers mostly used during lesson presentation

S/N	**Teaching strategy used**	**Number of teachers Used**	**Number of teachers never used**	**Total number**
1	Engaging learners with their challenges in solving of probability problems.	6	11	17
2	Created chance for the learners to demonstrate their new knowledge learnt to the class.	4	13	17
3	Scaffolding and asking constructive questions to illicit particular correct answers and use of specific procedures.	3	14	17
4	Use of visual cues to support relevant mathematical thinking. For example, teaching aids acids, diagrams (tree)	5	12	17
5	Working with learners' ideas in	11	6	17

	solving of probability problems.			
6	Provided space during lesson presentation for learners to explore, explain and discuss on each probability concept to enhance their mathematical independence in solving probability problems	13	4	17
7	Providing of teaching and learning materials to facilitate their smooth demonstration of their understanding such as coins, dice and cards.	7	10	17
8	Teacher gave learners questions which required them to discuss and make reflections before answering.	8	9	17
9	Use of guided and discovery strategies	5	12	17
10	Provided explanation and justification of probability concepts.	9	8	17
11	Whole class topic discussion	13	4	17
12	Group discussion	13	4	17
13	Concept simulation using fair coin and dice.	3	14	17
14	One-on-one method	4	13	17

Table 4.6 above shows that, eleven (13) teachers gave chance during lesson presentation for learners to explore, explain and discuss each probability concept to enhance their mathematical independence in solving probability problems, eleven (11) worked with learners' ideas in solving probability problems, while seven(7) teachers were able to provide teaching and learning materials such as coins, dice, and cards to facilitate the smooth demonstration of their understanding of probability concepts. On the other hand, eleven (11) teachers could not engage learners with their challenges in solving probability problems and twelve (12) exhibited less understanding in the use of visual cues to support relevant mathematical thinking. For example, teaching aids, diagrams (tree). It was also further observed that during lesson presentation, the

majority (9) of mathematics teachers mostly used the plain lecturing method in their lesson presentation.

4.6 Document Analysis

Thereafter, conducting lesson observations, the researcher carried out document analysis which included schemes of work, lesson plans, records of work, and the learners' mathematics exercise books. The main intention of document analysis was to check on which teaching strategies and subject matter mathematics teachers included in their planning of what content of probability to teach. The researcher checked on their 2016, 2017, 2018, and 2019 schemes of work, lesson plans, and records of work. In their document analysis, Teachers 1, 3, 4, 6, 9 10, 14, 15 and 17 were found that they used guided discovery, they paired the learners in groups, question and answer, demonstration, teacher exposition, and discussion methods. While, teachers 2, 5, 7 and 11 used group work, discussion, and learner-centered methods and teachers 8, 13, 16 and 12 used Socratic, discovery, demonstration, and cooperative methods. However, it is important to note that the teachers used the above teaching methods to support their selected learner engagement strategies which they employed to teach different probability concepts (See Appendix 8.9).

4.7 Summary of Research findings for question two.

In the lessons presented and analysis of academic documents such as lesson plans, records of work, and schemes of work found that mathematics teachers used different teaching strategies that allowed learners to demonstrate, explore, explain and discuss each probability concept to enhance their mathematical independence in solving probability problems. Teachers were able to work with learners' ideas in solving probability problems, while others were able to provide teaching and learning materials such as coins, dice, and cards to facilitate the smooth demonstration of their understanding of probability concepts. Teachers interacted with learners using question and answer format and their interaction was largely formal and educative. In other ways, teachers were familiar and able to transform the subject matter knowledge of probability which was reflected in the manner they presented their lessons during observation to the learners. Furthermore, teachers were observed using scaffolding strategies which encouraged the use of constructive questions to elicit particular correct answers and the use of specific procedures to solve certain probability concepts. In this context, only a few teachers (8) were able to identify and select learner engaging strategies that promoted conceptual understanding of probability. However, the majority (9) of teachers could not engage learners with their challenges in solving probability problems, while others exhibited less understanding in the use of visual aids to support relevant mathematical thinking. It was also further observed that during lesson presentation mathematics teachers mostly used the plain lecturing method in their lesson presentation of question and answer teacher and learner demonstration, guided discovery, group work activity, one-to-one, and whole-class discussion strategies. Nevertheless, these selected strategies made it possible for the researcher to assess whether the planned and selected pedagogical strategies supported understanding of the subject matter being taught, did they help learners to attach meaning to what was being taught, and more importantly, where the learners able to be engaged with the lesson using the above-identified pedagogical strategies. Therefore, it is important to note that majority of teachers (9) lacked the ability to plan, select learner engaging strategies and use of multiple strategies which have a strong bearing on the learners' understanding and retention of probability concepts. It is therefore important for mathematics teachers to have the ability to plan and select a variety of teaching strategies that provide could opportunities to engage learners in the construction of probability knowledge.

4.8 The pedagogical enabling prompts that mathematics teachers used to support learners' understanding as they taught Probability concepts

This was the third research question which focused on the two aspects which are teachers' subject matter knowledge and means of supporting learners to understand the subject matter knowledge of probability through the identified teaching strategies. This research question investigated mathematics teachers' subject matter knowledge of probability in terms of their ability to design the pedagogical enabling prompts as they taught probability. The following Sections 4.8.1, 4.8.2, 4.8.3, 4.8.4, 4.8.5, 4.8.6 and 4.8.7 presents the pedagogical enabling prompts which were observed during lesson presentation and interviews:

4.8.1 Use of familiar notations or events to consolidate with learners' new knowledge of Probability concepts

In the lesson presentations made by sampled teachers, they used different types of representations that learners were familiar with as they were explaining on the subject matter knowledge of mutually exclusive events. For example, Teacher 11 used the occurrence of daytime and nighttime to explain on the concept of mutually exclusive events which learners were familiar with and were able to differentiate the occurrence of daytime and nighttime (See Figure 4.7, Section 4.3.1). In a similar context, Teacher 13 used the flipping of a coin to explain on the concept of mutually exclusive events that the Head and the Tail cannot land at the same time (See Appendix 8.5). Teacher 16 also used the ideas of withdrawing money to explain on non-mutually exclusive events that one has a possibility of drawing money at an Automated Machine (ATM) using two forms of withdrawing, that one can either use a card less through a phone or inserting a card to withdraw money and later used this information to formulate the probability compound spaces (See Appendix 8.6). Teacher 17 used the shaded Venn diagrams of two jointed and disjoint sets of S and T to explain on mutually and non-mutually exclusive events and how to use the notation symbol 'U' to formulate the probability expression as P (SUT) =P (S) +P (T) - P (S∩T) and P (SUT) = P (S) +P (T) respectively (See Appendix 8.14). In a similar context, Teacher 2 also used the shaded Venn diagrams of two disjoint sets and explained by demonstrating the conceptual relationship between the disjointed sets using the notation symbol 'U' and interpreted that sets S and T were mutually exclusive events because of A∩B = Ø (See Appendix 8.13). Thereafter, the four teachers were able to formulate the probability expression with the help of learners using the mathematical notation symbol 'U' in the form P (AUB) =P (A) +P (B) and P (SUT) =P(S) +P (T)-P (S∩T) (See Appendixes 8.13 & 8.14) respectively. Therefore, the teachers' representation on the concept of mutually and non-mutually exclusive events was a good example of other lessons which were taught by six other mathematics teachers who managed to explain the concepts of mutually exclusive events.

4.8.2 Simulation of probability concepts using various teaching and learning materials

In the second lesson presentation, teachers were able to provide teaching and learning materials. In this case, Teachers 7, 13 and 14 made available and provided teaching and learning materials to learners such as a coin, dice, table, and a projector and computer which were used as a model to simulate, determine the actual independence of a single toss (See Figure 4.24, Section 4.5.1). Not only did the teachers provided the teaching and learning materials but also demonstrated to the learners by tossing the coin, and used the projector and a computer to determine and explain the actual independence of a single toss and recorded the outcomes as follows HT, TH, HH, TT (See Excerpt 4.5, Section 4.5.1). Thereafter, the Teacher allowed learners to simulate and determine the actual independence of a single toss in groups and later demonstrated before the class each group (See Figure 4.24, Section 4.5.1).

4.8.3 Simplifying and reducing steps in solving probability problems

In the third cohort of lesson presentations, the teachers used the strategy of scaffolding and simplified the tasks which looked difficult to the learners' level of their understanding of probability and later gave to complete the work on their own. In light of the above, Teacher 7 worked with the help of learners to identify and list all the concepts which included concepts of independence and certainty which were carried with the probability event in the occurrence of four consecutive days (See Appendix 8.12). The teacher further extended the help by demonstrating how to formulate the combination of outcomes of the compound spaces, and stated that each day occurred as an independent event. However, based on the explanation given by the teacher, learners were able to independently list the combination of the outcome of the occurrences of days of reaching late (L) or not (N) using the knowledge of independent events and formulated the compound spaces in the form: L, L, L, N, L, L, N, L, L, N, L, L and N, L, L, L on which L represented reaching late and N represented not late (N) (See Appendix 8.12). This understanding further led the learners to easily subtracted 1/20, the probability of reaching late (L) from one (1) the probability of certainty, and found 19/20 as the probability of not reaching late (N). Thereafter, the Teacher multiplied the probabilities using only necessary steps and found 19/40000 (See Appendix 8.12).

4.8.4 Giving opportunities to learners to demonstrate their understanding of probability concepts

In the fourth cohort of lesson presentations, teachers allowed learners to demonstrate their understanding of the new knowledge learned with the help of their fellow learners in answering the given probability problems. In this case, Teacher 14 gave questions that required learners to reflect and discuss with each other in order to arrive at the final answer. The learners were allowed to solve, explain and demonstrate their understanding of the expression P (A and B) =P (A) $\times$ P ($\frac{B}{A}$) to the whole class (See Excerpt 4.1 & Figure 4.25, Section 4.5.1). In a similar situation, Teacher 7 also gave learners time to demonstrate and they successful conducted the trials in which they were simulating and determining the actual independence of a single toss after tossing a coin twice (See Figure 4.24 & Excerpt 4.6, Section 4.5.1). Furthermore, Teacher 8 in her lesson presentation on the conceptual relationship between laws of probability and a theories of sets also gave chance to the learners to discuss questions in groups and they presented their findings before the whole class, even though she was not consistent in allowing learners to express their understanding of the subject matter (See Excerpt 4.5, Section 4.5.1). Teacher 12 also in his lesson presentation on the Fundamental Counting Principles of Probability allowed learners to work in collaboration to explore, discuss and share their understanding of the fundamental counting principles of probability and demonstrate their findings before the class (See Excerpt 4.3, Section 4.5.1).

4.8.5 Teacher-learner interaction during lesson presentation

In their lesson presentation, teachers also encouraged teacher-learner interaction through question and answer format and also allowed learner to learner interaction to share their understanding of the subject matter. For example, Teacher 15, instructed learners work in pairs to complete the given work (See Excerpt 4.3, Section 4.5.1 & Appendix 8.1). Furthermore, the Teacher also conducted a one-to-one session to explain to the slow learners in order to actively engage them in the construction of the knowledge of probability concepts. The Teacher did not only provide an independent problem-solving activity but also guided through question and answer in the process of solving of the probability problem. While, Teachers 6 and 8 taught by giving chance first to the learners individually to define the probability terms in which included probability, experiment, sample space, certainty and impossible with reference to probability scale (See Figure 4.23, Section 4.5.1). This was followed by the whole-class discussion with the teachers which focused on learners' challenges they encountered in defining the common terms used to describe probability events. Therefore, the above instructional practices which were used by Teachers 15, 6 and 8 were a good example of other presentations that the other four teachers used to teach on probability concepts. However, the majority of teachers (9) used questions that could not guide learners to understand probability concepts and provoke their mathematical thinking.

4.8.6 Use of various forms of presentations in the teaching and learning of probability concepts

During lesson presentations teachers used various forms of presentations of the subject matter knowledge of probability concepts to the learners. For example, Teachers 15, 11 and 16 used teaching aids, group discussion and role-play activity respectively to present their subject matter knowledge of the concept of dependence. In a similar context, Teacher 4 used the tree diagram to explain on the outcome of the possible favourable and total number of possible outcome after one element was randomly picked without replacement and then the second (See Excerpt 4.1, Section 4.5.1).While, other teachers used the shaded Venn diagram of two jointed union sets to illustrate the associated probability expression using the notation symbol 'U' involving the non-mutually and mutually exclusive events (See Figures Appendixes 8.13 & 8.14).

4.8.7 Use of multiple pedagogical strategies in explaining the probability concepts

In this category, teachers used various instructional strategies in order to make it clear the conceptual connections of the laws of probability with other major mathematical concepts such as sets and statistics. For example, Teacher 8 was able teach learners based on their interpretation of ideas of disjoint and joint sets in relation to the probability of the law and also allowed them to work in collaboration and discuss and answer the question in groups. The Teacher was also able to summarize the lessons through question and answer format (See Excerpt 4.5, Section 4.5.1). However, these instructional strategies were also supported by learner and teacher demonstration, teaching and learning aids, one-on-one, and group work activity as she taught on the conceptual relationship between sets and laws of probability (See Appendix 8.2). Similarly, Teacher 7 also taught on probability terms used to describe probability events using various teaching strategies. In this case, the Teacher started his lesson by asking learners questions on terms that are used to describe probability events. Thereafter, the Teacher explained by consolidating on learners' answers and gave examples of the terms such as likelihood, events, certainty, and impossibility. This was followed by dividing the learners into groups of five per group and assigned them to define the following probability terms such as experiment, sample space, event, and outcome to work in collaboration and share ideas with each other. The Teacher later discussed each of the answers given by the group to provide feedback to both slow and fast learners. The Teacher also encouraged learners to solve probability problems independently to gain confidence in solving abstract topics. Therefore, the above use of multiple engaging teaching strategies was a good example of other lessons which were presented by other seven (7) sampled mathematics teachers. Furthermore, in the follow-up interview, Teacher 7 was asked to explain the intention of using various teaching instructional strategies. In his response, Teacher 7 said:

> To teach probability concepts effectively, I think it has more to do with teaching strategies that are learner engaging, but when you do not understand how to use the same teaching strategies, you can’t teach effectively. So you need to select different teaching strategies that could help learners to understand challenging topics like Probability. To answer your second question on the selection of teaching and learning materials according to my experience, the use of a coin alone, to determine and explain the actual independence of a single toss of a coin could be the best material to use rather than the use of dice and others, because learners

are familiar with it and it is mostly used in the senior mathematics syllabus and mathematics textbooks.

In view of the above conversation, Teacher 7's response shows that he planned and selected teaching strategies that could support learners to engage in active experiencing the initial goal of the questions given in order to promote a comprehensive understanding of the subject matter knowledge of the concept of dependence. However, in the second response, the Teacher could not meet the objective of this that requires the use of multiple teaching and learning instructional materials to easily enhanced learners' understanding of the probability concepts. In this case, the Teacher did not realize that the use of a coin as the only teaching and learning material provides different results each time the trial is performed and makes learners vary in their understanding of the subject matter.

Table 4.7: Summary of enabling prompts used support learners' understanding of probability concepts

S/N	**Pedagogical enabling prompts used to support learners' understanding of probability.**	**Purpose of the pedagogical strategies**
1	Simulating of the concept of independence and dependence using a coin and a dice.	To capture learners' understanding of the probability concepts as the coin and a dice are used as conceptual representations.
2	Use of common expressions to consolidate learners' existing and new knowledge of Probability concepts	To sort out the differences between learners' existing understanding and the new probability concept.
3	Simplified and reduced the steps in solving of probability problems	To scaffold and simplify the tasks which looks difficulty to the learners' level of performance and later leave them to think on their own.
4	Provision of a platform for learners to demonstrate their understanding of probability concepts	To engage and to demonstrate their understanding of the concept of probability such as concept of dependence.
5	Teacher-learner and learner to learner interaction	To promote interaction and collaboration in solving of the probability problems with more one solution.

6	Use of various forms of presentations in the teaching of probability concepts	To teach learners with different learning styles so as to make them understand major mathematical concepts.
7	Use of multiple pedagogical strategies in the teaching of probability concepts**.**	To make clear conceptual connections of laws of probability with other major mathematical concepts to the learners such as sets and statistics

As shown above, in Table 4.7, the teachers used different pedagogical enabling prompts which assisted to engage learners in the learning of probability concepts during lesson presentation. All in all, the 17 sampled mathematics teachers were able to use appropriate pedagogical enabling prompts and multiple teaching strategies as well as to vary their teaching and instructional materials as they taught on the concept of probability such as mutually exclusive events, independence, and dependence.

4.9 Summary of research findings for question three

In conclusion, the use of pedagogical enabling prompts assisted learners to make conceptual connections of familiar mathematics concepts and processes with the new knowledge of probability concepts and processes. Furthermore, the simulating of the concept of probability concepts, use of common events to consolidate learners' existing, provision of feedback to learners' answers, and learner platform to demonstrate their understanding of probability concepts and use of various forms of presentations during lesson presentation did not only make probability concepts and processes to be learned explicitly, but also made the purpose for learning the probability concepts explicit. This is because the strategies were able to support learners' understanding of probability concepts. Learners were able to learn constructively and took control of the process of developing new knowledge of probability such as formulation of probability expression using the laws of sets and probability. Finally, the use of pedagogical enabling prompts strategies assisted teachers to clearly explain the subject matter knowledge of probability to meet different needs of learners. For example, learners were able to make conceptual connections of laws of probability with other major mathematical concepts like sets and statistics. However, some teachers were able to make the learning of probability be explicit learned but did not meet the objective of learning as the teachers had no capacity to design pedagogical enabling prompts that could be used to engage learners in the teaching and learning of probability concepts.

CHAPTER FIVE: DISCUSSION OF FINDINGS

5.1 Introduction

The purpose of this study was to explore mathematics teachers' pedagogical content knowledge of Probability. The mathematics teachers were investigated in terms of their understanding of the subject matter knowledge of probability, ability to effectively use the identified pedagogical strategies and pedagogical enabling prompts which teachers used to support learners as they taught probability concepts. The study was guided by the following research questions.

(a) What subject matter knowledge of Probability do the mathematics teachers have?

(b) What pedagogical strategies do mathematics teachers use to teach Probability concepts?

(c) What pedagogical enabling prompts that mathematics teachers use to support learners' understanding as they teach probability?

In this section, the findings of this study were discussed using the above-mentioned research study questions. In view of this, the researcher first discussed the teachers' subject matter knowledge they exhibited in the SMKDT. This was followed by a discussion which focused on pedagogical strategies and enabling prompts that mathematics teachers used to support learners' understanding of the subject matter as they taught probability concepts.

5.2 Discussion of the findings

The findings discussed in this study are based on the results of the present study and are not meant to be overgeneralized. The discussion of the findings focused on how the three research questions were addressed. Below is the discussion and interpretation of the findings on teachers' pedagogical content knowledge of probability.

5.2.1 The teachers' subject matter knowledge of probability in the SMKDT

In a quest to assess mathematics teachers' PCK of probability, the sample was assessed based on the requirement of the descriptors in the category of the subject matter knowledge, pedagogical strategies, and enabling prompts of the study's conceptual framework (See Figure 1.1, Section 1.8.1). To address the first research question which focused on teachers' subject matter knowledge of probability and to understand the insight of mathematics teachers' understanding of the subject matter knowledge of probability, preliminary observation-interviews were conducted (See Appendix 2) and later the sample was subjected to the writing of a subject matter knowledge diagnostic test (SMKDT), lesson observation and follow up interviews. In the SMKDT, teachers scored marks that ranged from the lowest 21 to the highest mark 80 (See Figure 4.1, Section 4.3). However, the difference between the highest mark 80 and lowest mark 21 gave a large difference in the range of 59 and this suggested the presence of outliers in the marks which has an effect on the mean to use it to determine the outcome of teachers' understanding of probability. This implied that the mean was not the best central tendency to use to assess mathematics teachers' PCK of probability. Thus, this could have not given a clear picture about the nature of mathematics teachers' understanding of the subject matter knowledge of probability. Below is a discussion of the questions and interpretation of the findings of teachers' understanding of the subject matter knowledge of probability.

The analysis of the findings of teachers' ability to identify and apply the competencies in the process of solving the SMKDT probability problems shows that the sampled mathematics teachers were able to apply 144 out 226 competencies in relation to the initial objective of the questions (See Table 4.3, Section 4.3). On the other hand, eighty-two (82) competencies were not applied in accordance with the goals of the questions in their process of answering the SMKDT probability problems. This suggests that the majority of the competencies were applied in line with the goals of the SMKDT questions. In this regard, only four (4) sampled teachers were able to correctly identify and apply the competencies in line with the objective of the

SMKDT questions which was a demonstration of teachers' understanding of the subject matter knowledge of probability, which could be viewed as components of teachers' PCK of probability. While, the analysis of the thirteen (13) teachers' answers on the application of the competencies in the SMKDT questions showed that they were only able to use their basic knowledge of probability concepts to solve probability problems which required teachers to critically analyze, identify, interpret, and explain and differentiate probability concepts and conditions attached to the questions (See Figures 4.20 & 4. 21, Section 4.3.1). This suggests that the majority (13) of the sampled teachers exhibited less understanding of how to apply the expected competencies as they were answering the SMKDT questions.

Based on the analysis of the conceptual relationship between the laws of probability and theory of sets using the mathematical notation symbol 'U', the study found that mathematics teachers were able to identify, interpret and differentiate that the disjointed and jointed sets of A and B in Figures 1 and 2 in Appendix 1 question number 2 were mutually exclusive and non-mutually exclusive events respectively. In the context of set A and B being non-mutually exclusive, the teachers further interpreted that the intersecting sets of A and B in Figure 1 were not mutually exclusive events but inclusive events that occurred as independent events in a jointed set of A and B (See Appendix 1, Question 2b). This means that they have been counted twice and the mathematical notation symbol 'U' was appropriate to be used to formulate the probability expression, which is a shorthand way of writing 'or' in relation to the theory of sets and the second general law of addition of probability which is presented in the form $P(E1 \cup E2) = P(E1) + P(E2) - P(E1 \cap E2)$ where E1 and E2 are non-mutually exclusive events. This understanding led sampled teachers to formulate the probability expression for Figure 1 for question 2 Appendix 1 in the form: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (See Figure 4.7, Section 4.3.1).

Furthermore, in Figure 2 (Appendix 1), the teachers were able to extend their understanding of the subject matter that the probability of the intersection set of the disjointed sets were an empty sets, that is $P(A \cap B) = 0$, which means that sets A and B were mutually exclusive events and did not have common elements since the occurrence of one displaced the occurrence of another and the mathematical notation symbol 'U' was again appropriate to use to formulate the expression in the form $P(A \cup B) = P(A) + P(B)$. It is important to note that the teachers' ability to interpret and use the mathematical notation symbol 'U' in the formulation of both probability expression which satisfied the first and the second general additional law of probability in terms of conceptual connections of laws of probability with sets in the form P (E1 U E2) =P (E1) +P (E2) and P (E1 U E2) =P (E1) +P (E2)-P (E1 ∩ E2) was an exhibition of teachers' PCK of probability in terms of making conceptual connections with major concepts of probability within the branches of mathematics as conceptualized in this study's conceptual framework. In this case, the teachers were able to reflect the subject matter knowledge of laws of probability and made connections with ideas of sets which resulted into correct formulation of the probability expressions for jointed and disjointed sets of A and B in the form P(AUB)=P(A)+P(B)–P(A∩B) and P(AUB)=P(A)+P(B) (See Figure 4.7, Section 4.3.1). With regards to this, only five (5) mathematics teachers were able to use the mathematical notation symbol 'U' to consistently formulate the probability expressions for non-mutually and mutually exclusive events in the form P (AUB) =P (A) +P (B) – P (A∩B) and P (AUB) =P (A) +P (B) respectively (See Figure 4.7, Section 4.3.1). Regardless of the above findings, majority of teachers (12) used the mathematical notation symbol for intersection sets '∩' to formulate the associated probability expressions. In the analysis of the SMKDT findings in relation to the non-mutually exclusive events in Figure 4.6, Section 4.3.1 show that seven teachers used the mathematical notation symbol '∩' and interpreted the pairing of the two intersecting sets in relation to the laws of probability in the form P (A∩B) =P (B∩A) = P (B) x P (A) instead of using the form P (E1 U E2) =P (E1) +P(E2) - P (E1 ∩ E2) which is used to describe the occurrence of independence events in a joint set. Furthermore, three teachers used two similar laws of probability 'or' and 'U' to formulate mutually exclusive events and presented in the form P (A or B) =P (A) +P (B) or P (AUB) =P (A) +P (B). On the other hand, two teachers used two mathematical notation symbol '∩' and 'U' and interpreted the pairing of the two intersecting sets which were mutually exclusive events and presented the expression in the form P (A∩B) =P (A) +P (B) or P (AUB) =P (A) x P (B) (See Figure 4.6, Section 4.3.1). In this case, teachers were required to give one answer illustrating using the set notation symbol 'U' in the form of: P (AUB) =P (A) +P (B). This implies that teachers were holding their own informal understanding of the subject matter knowledge of non-mutually and mutually exclusive events which caused them to formulate the probability expression which did not correspond to the organization of the subject matter knowledge of probability. This means that twelve (12) teachers exhibited less understanding of the subject matter with regard to their interpretation of the first and second general additional rules of non-mutually and mutually exclusive events in the use of the mathematical notation symbols 'U' to formulate and to make the conceptual connection of the laws of probability and sets. However, the assumption of this study is that, the ability to interpret and to use the correct notation symbol in making conceptual connection of probability concepts with other major mathematical concepts of the same level like sets was viewed as the components of PCK that could help teachers to teach probability concepts in a comprehensive way (Shulman, 1986, 1987). In this regard, research scholars have pointed out that teaching of Probability for conceptual understanding requires teachers to have a major shift in emphasis of simply providing the formula, rules and procedures for calculations in addressing learners' erroneous intuitions and preconceptions of probability concepts to the level where teachers of mathematics show their full understanding of the conceptual connection of probability concepts with other major mathematical concepts in the field of mathematics and other disciplines (Garfield, 1995; Konald, 1995; Khazanor & Gourgey, 2009; Sharma, 2006). This has the potential to influence teachers to teach effectively in a comprehensive way to the learners the concept of Probability with clear understanding.

Furthermore, in the analysis of teachers' ability to interpret and formulate probability compound spaces with more than two elements, this study's findings show that teachers were able to identify the concept of independence, certainty, and listing of the combination of outcomes of the probability event that occurred for four consecutive days. In light of this, teachers were able to recognize that each day occurred as an independent event for four successive days. Moreover, this understanding made teachers easily formulate the probability compound spaces in the order; L, L, L, N; L, L, N, L; L, N, L, L, and N, L, L, L (See Figure 4.9, Section 4.3.1). Nevertheless, their understanding of the nature of the concepts of independence and certainty and the process of listing the combination of outcomes in a probability event was extended and helped them to interpret and predict that the mathematical knowledge which learners were required to use to solve SMKDT probability problems, was first to identify the concept of independence, certainty and also to list the combination of the outcomes of the occurrence of days which was carried in a probability event as either in the form L, L, L, N. These results suggest that mathematics teachers were able to demonstrate their understanding of the subject matter knowledge of probability as reflected in the manner they identified, interpreted and formulated the probability compound spaces of any order: L, N, L, and L. However, this understanding further helped them to find the fractional numerical probability of a man reaching late for work to be 19/16000.

Nonetheless, only eight teachers were able to demonstrate their understanding of the subject matter knowledge of probability concepts by their ability to recognize and identify the concept of independence, certainty and ability to list the combination of the outcomes of the occurrence of days which was carried in the event of the man reaching for work late or not for four successive days. However, in the analysis of the findings nine (9) teachers could not recognize and predict that learners were supposed to first identify the concepts of independence and certainty and listing of the combination of outcomes and thereafter formulate the probability expressions of the man reaching for work on time. This indicates that teachers' PCK was not clearly exhibited as evident in the manner they presented their responses to state the concepts which learners were required to use to answer the question. However, this exhibition of less understanding of the subject matter knowledge of probability concepts was first clearly exhibited in the manner they generalized the compound spaces in the form P (car and late) = P (C, L) and P (late) (See Figures 4.8, Section 4.3.1 & Appendix 8.6) which was a buildup knowledge for answering the next question which required teachers to predict challenges that learners were likely to encounter in answering the same question

In a similar case to the above, teachers were able to extend their understanding of probability concepts involving the testing of either positive or negative for malaria. The findings of this study, however, shows that teachers exhibited their understanding of the subject matter knowledge of probability in the manner they formulated the outcomes of testing either positive (+ve) or negative (-ve) in the probability compound space of the form Pr (P) x Pr (N) +Pr (N) x Pr (P) (See Figure 4.11, Section 4.3.1). However, their understanding of the subject matter knowledge of probability concepts allowed the formulation of the probability expression involving testing either positive or negative in the form: $P(P) \times P(N) + P(N) \times P(P)$ which they used to find the expected probability of the two patients selected at random one after the other. Furthermore, the teachers' understanding of probability concepts also led them to exhibit their ability to recognize, predict and explain that testing either positive or negative was likely to cause learners to think that it was a mutually exclusive condition, which means that there is a likelihood of the two patients to test positive and negative at the same time (See Figure 4.14, Section 4.3.1). In this context, only seven teachers were able to predict and justified that testing of either positive or negative was likely to cause learners to think that it was a mutually exclusive condition when not. However, this could be argued that teachers were able to exhibit the conceptualised components of PCK of probability as outlined in this study's conceptual framework (See Figure 1.1, Section 1.8.1), this is because PCK includes an understanding of what makes the learning of specific topics easy or difficult (Shulman, 1986, 1987). In light of this, it can be stated that teachers were able to reflect the subject matter knowledge of probability in terms of their ability to identify, interpret, predict and formulate the probability compound spaces using their understanding of the concept of independence, certainty, and ability to list the combination of outcomes.

In as much as it shows that teachers were able to recognize and identify concepts embedded in the probability event, predict and formulate the probability expression of the combination of outcomes of either testing positive or negative, a large number of sampled teachers could not predict and failed to provide a clear explanation with justified reasons after working out the question on how learners could have misunderstood that testing positive or negative were acts of mutually exclusive events. In addition, some calculated answers show that teachers did not have the ability to use the right fractions which represented the probability of testing positive or negative (See Figure 4.11, Section 4.3.1). However, these results were not consistent with the study findings of Kazima and Adler (2006) that, in the teaching of relatively a new topic, teachers found it difficult to predict in advance what ideas learners were likely to use or how they were to interpret the Probability task without solving it. These findings suggest that the majority (10) of mathematics teachers lacked the ability to identify, interpret and formulate the probability compound spaces which in turn made them further fail to explain and justify how learners were likely to misunderstand the concept of independence in the testing of positive and negative. Hill, Ball and Schilling (2008) pointed out that teachers must be able to examine and interpret the mathematics behind learners' errors prior to involving how learners went astray. This, however, could only be achieved when teachers possess pedagogical content knowledge which allows to understand the subject matter and how to present it to the learners comprehensively using various ways (Shulman, 1987). In addition, Wilson and Peterson (2006) argued that teachers should not only understand the subject matter, but they also need to be kept abreast with a deep understanding of how to illustrate probability events using real-life situations that could enhance the learners' understanding and retention of probability concepts. This would also allow mathematics teachers to develop PCK of probability as they spend time contemplating what learners need to learn, what they know and the means of how to present the subject matter knowledge of probability concepts to the learners. Ball, Ball and Hill (2004) also posit that teachers need to be able to interpret and make a mathematical and pedagogical judgment about the learners' ideas and also to respond productively to them which is part of teachers' PCK. This means that mathematics teachers, however, must have the ability to unpack and solve any conceptual organization of probability concepts as well as to predict and state what makes certain probability concepts difficult or easy to identify and use by the learners as they solve any given probability problems (Shulman,1986).

Furthermore, in the context of defining the term probability and other related terms, the study established that some mathematics teachers' definitions of probability terms were reflecting the subject matter knowledge of probability according to the socially agreed conventional meanings of the terms. Not only that, sampled teachers were also able to provide definitions that included the conceptual connections of major topics concepts such as sets, presentation of probability concepts in terms of percentages, fractions, and decimal numbers and numerical limit measures associated with probability about probability scale (See Figures 4.4 & 4.5, Section 4.3.1). In addition, teachers' definitions were also able to indicate the limits over the measures of probability which included the use of two major wordings which are used to describe the occurrence of probability events, the impossibility (0) and the certainty (1) of an event taking place with reference to the probability scale. This means that teachers were able to explain and show that probability lies between 0 for impossibility and 1 for certainty, and was also able to present it in a set builder notation of the form P $(0\leq P(E)\leq 1)$, at the same time satisfying the theory of sets and general conditions of the laws of probability that when P(E) cannot possibly occur then $n=0$ which means that $P(E)=0$ and when P(E) can possibly occur, then $n=N$, this means that $P(E)=1$. This implies that probability measures cannot exceed one (1) and zero (0) (See Figure 4.4, Section 4.3.1). It must be also noted that teachers' subject matter knowledge of probability was first exhibited in their ability to recognize and identify the socially agreed qualitative words which they used to define probability terms such as likelihood, event, or, certainty, uncertainty (See Figure 4.5, Section 4.3.1). This means that teachers were able to display their understanding of the subject matter knowledge of probability in terms of words they used which directed their definitions towards the agreed conventional meanings of probability terms with reference to the probability scale and other related concepts topics within and outside the branches of mathematics. Therefore, this study's findings of the definitions of probability terms suggest that the given definitions of probability terms in the SMKDT test reflected the conceptualized components of PCK of this study's conceptual framework in the category of subject matter knowledge for comprehensive teaching of probability to the learners (See Figure 1.1, Section 1.8.1). In this regard, only nine teachers were able to display their subject matter knowledge of probability in relation to the conceptualized components of PCK of probability.

However, five given definitions of probability focused on one condition of certainty (1) of an event from taking place misrepresenting the conceptual connections and the subject matter carried out in the definitions of probability and other related terms (See Figure 4.3, Section 4.3.1). As reflected from their definitions, teachers were unable to recognize and define probability using both conditions of absolute certain and absolute impossible, whereas other definitions lacked the complete use of socially agreed qualitative words which they were required to describe probability according to the agreed conventional meanings of probability terms such as certain, never, impossible, probably and so on (Baer, 2008; Mutodi & Ngirande, 2014).

Furthermore, other definitions which were given were merely recited from the Grade eleven mathematics secondary school learners' book and success in examination, mathematics syllabus D (See Figure 4.3, Section 4.3.1). This was also observed with questions 1(a), 2(a), 14, and 17(a). Therefore, these results suggest that nine sampled mathematics teachers were holding incomplete understanding of the socially agreed conventional meanings of probability terms as reflected in their definitions of probability terms. However, the complete definition of the term probability could be debated that, it should include both conditions of absolute certainty (1) and absolute impossibility (0) with reference to other related major topic concepts. This is because when the event P (E) can possibly occur, then P (E) =1, and when it cannot possibly occur, then P (E) =0. It is also important that mathematics teachers must be aware that probability concepts are mostly carried in the qualitative terms which are used to describe Probability events (Wilson & Peterson, 2006). In this regard, Shulman (1987) points out that the incomplete and trivial definitions (mere recited) in teaching misrepresent the correct conceptual connection and the subject matter carried in the definitions of probability terms. This, however, has the potential to provide learners with the incorrect conceptual organization of probability concepts. Wilson and Peterson (2006) also pointed out that teachers are required to give opportunities to the learners in order to understand the mathematical operations (definitions) and concepts that they encounter in ways that go beyond the mere recitation of definitions, rules, procedures, or algorithms. Moreover, teachers must have the ability to define Probability terms using the socially agreed conventional meanings of terms since the terms bear the concepts in them and how the terms relate within the branches of mathematics and other disciplines. In other words, clear definitions always carry important conceptual connections of the subject matter which teachers require to use to make meaning of probability terms to develop in learners (DoF, 2001). As such, teachers and learners need to have a clear understanding of the mathematical operations (definitions) of each concept rather than merely reciting definitions, rules, procedures, or algorithms without a clear understanding.

In the category of solving probability problems, the sampled mathematics teachers were required to use their basic knowledge of the subject matter knowledge of probability to solve the probability problems using subject matter knowledge of the concept of dependence. In the analysis of the findings, it was found that teachers had mastered the procedural formula for the probability expression and were only able to use to replace variables with the given numbers but had challenges with explaining on the interpretation and meaning of the expression P $\left(\frac{B}{A}\right)$ obtained from the expression P (A and B) =P (A) $\times$ P $\left(\frac{B}{A}\right)$ (See Figure 4.25, Section 4.5.1). In this case, teachers gave different explanations and justifications which were not clear to the learners' understanding of the subject matter knowledge of probability and the conceptual organisation of the subject matter of the concept of dependence. It is important to note that like any other concepts, the probability concepts are carried in their probability language and terms, and each term has a specific meaning which may or not resonate with the everyday use of these terms (Pimm, 1987). This required the mathematics teachers to be in a position to use the probability terms correctly, to be familiar with the probability language and meanings of terms as well as the subject matter knowledge of the concept of dependence. This could have assisted in explaining the interpretation and meaning of the expression P $\left(\frac{B}{A}\right)$ to the learners.

Similarly, on question 5(a) in the SMKDT test, it was found that mathematics teachers were only able to point out that the learner's answer was either correct or wrong without explaining the rationale of the procedure used and why the answer was correct or not. In the follow-up interviews, teachers were unable to explain how the given answer by the learner was correct or incorrect as well as to suggest possible reasons for the procedure which the learner used to arrive at the answer as he added the two fractions (See Figure 4.17, Section 4.3.1). In addition, teachers who had earlier indicated that the answer was wrong could not further suggest possible procedures a learner could have used to solve and their positions why the learner found an incorrect answer. However, the teachers were required to explain the reasons why the learner used the above fractions and added them rather than just to confirm the answer. This could be attributed to the fact that most of the time teachers of mathematics have a habit of reciting the procedures rather than understanding the reasons for using them and why they make sense. It was a clear indication that mathematics teachers were more comfortable with questions which required them just to use basic knowledge of concepts in solving of probability problems rather than those which required them to analyse and interpret the meaning of the content in the question before answering.

The above outcome has shown that mathematics teachers lacked the ability to explain and justify how the given answers were correct or wrong as well as to provide the representations of fractions used in finding the answer. Mathematics teachers could not point at how correct and incorrect the given answers and the errors which were made in the selection of probability fractions given in the event of spinning the wheel and reasons for adding the fractions. International researchers have conceptualised PCK as the ability to spot the correct and incorrect answers of any given subject matter (Shulman, 1986). It is also important to understand that PCK as a construct has been conceptualised as an amalgamation of content and teaching which has been broadened to include any package of teacher knowledge and beliefs (Ball, Thames & Phelps, 2008). Nevertheless, as shown in Figure 4.17, Section 4.3.1 the majority of teachers (14) could not exhibit PCK of probability to spot correct and incorrect answers and also to provide clear explanations and justifications as to reasons why the answers given by the learners were correct or not.

Furthermore, the analysis of the SMKDT shows that question 11 was one of the most answered questions in the SMKDT test which assessed teachers' ability to apply the concepts of sets and quadrants to solve probability problems using the two categories of teachers' ability to: solve, explain and justify their mathematical reasoning on the probability concepts. In this regard, the study found that the sampled mathematics teachers were able to demonstrate their understanding of the subject matter knowledge associated with sets and trigonometric functions in relation to the quadrants. The teachers were also able to recognize and identify that the Cartesian plane has four quadrants that add up to 360^0 and each quadrant was adding up to 90^0. Using this understanding, the teachers were able to find $\frac{1}{4}$ which they found by dividing $\frac{90}{360}$. This means that each quadrant in fraction form was one-quarter ($\frac{1}{4}$) (See Figures 4.20 & 4.21, Section 4.3.1). Furthermore, teachers were able to interpret and apply the correct operations which defined the associated probabilities; P (A ∩ B) as P (A) x P (B) which means to multiply their separate probabilities to obtain the probability of both happenings, while P (B∩C∪D) was simplified as

P(B) × P(C) +P (D) which means to multiply and then add their separate probabilities In addition, teachers were also able to interpret that P(C or D) meant to add their separate probabilities to get the probability of both happening or not and they expanded it in the form P(C) + P (D) (See Figure 4.20, Section 4.3.1). However, other teachers exhibited inability to interpret and apply the correct operations on the associated probabilities. On the other hand, some were able to explain that all points on the Cartesian are integers and contain infinite number of outcomes (See Figure 4.21, Section 4.3.1). Thus, the teachers' clear understanding of the subject matter knowledge of the concepts of probability in relation to sets, Cartesian plane, and the quadrants assisted them to solve the first part of the SMKDT question on probability (See Figure 4.20, Section 4.3.1), while some teachers had an unclear understanding of the subject matter knowledge of sets in relation to the quadrants could not help to find the expected answers in the use of associated probabilities to solve the SMKDT probability problems. However, on the second part of question 11(b) (ii) which required teachers to provide a clear explanation and justification as to why all the sets of points on the Cartesian plane are continuous sample space, some teachers left the questions unanswered, while others were able to state that all points on the Cartesian are integers and contain an infinite number of outcomes (See Figures 4.20 & 4. 21, Section 4.3.1).

Similarly, question 12 was also one of the questions which was correctly answered in the SMKDT test administered to the mathematics teachers. The question assessed mathematics teachers' ability to use basic concepts of sets to solve probability using the two categories of teachers' ability to: solve, explain and justify the conceptual procedures they used to solve the questions. In this context, the mathematics teachers were able to display their understanding of the basic concepts of sets in solving of probability problems and this was seen in their procedures that they used in solving of the combined subject matter of sets and laws of probability. The teachers' understanding of the subject matter of sets and probability was also evident in the manner they presented their work and the solutions they found (See Figure 4.22, Section 4.3.1). Although, the SMKDT findings show that all the seventeen teachers of mathematics exhibited their insight understanding of the subject matter and ability to solve all the SMKDT probability problems in question 12, this study's findings show that 13 teachers of mathematics lacked clear exhibition of their comprehensive understanding of the conceptual procedures they used to solve probability problems. The teachers did not have the ability to explain and justify the procedures they had used to solve the questions and also had difficulties in stating what the questions required them to assess. However, teachers were only able to show their subject matter knowledge by solving probability problems given in the SMKDT. This simply means that majority of teachers of mathematics to found it not easy to use their understanding of Probability to explain the procedures they used solve the given probability problems. However, similar challenges were observed with questions 6 (a), 7(a) and (b), 9(i) and (ii), and 10 (a) and (b) (See Appendix 8). In these questions, teachers of mathematics were only able to apply the basic knowledge of constructing a tree diagram and grid table with its corresponding probabilities and applied the concept of dependence (without replacement) on questions 7 (a) and (b). This means that majority of teachers were unable to explain and justify the methods and procedures they used to solve the given probability problems.

In the analysis of the findings of teacher's ability to solve, relate and interpret, and explain the calculated answers in relation to the probability scale, the findings of this study showed that teachers were able to use their understanding of multiplication and addition of decimal numbers to find the expected answer 0.1 (See Figure 4.18, Section 4.3.1). In this case, teachers were able to extend their understanding of the first question to relate and interpret the meaning of the calculated answer 0.1 using the probability scale that the road carnages recorded were less likely to occur in large numbers this is because the probability of approaching of 0.1on a probability scale simply means that there are less likely chances of the event occurring in large numbers. On the other hand, the majority of teachers (10) were only able calculate the expected answer but could not relate and interpret the calculated meaning of 0.1 using real-life situation about the probability scale despite finding the expected answer. In addition, other teachers were also able to multiply and to regroup the probabilities but never paid much attention to the addition of decimal numbers which required them to consider first the position of a decimal point as they were regrouping the decimal numbers. Teachers incorrectly placed the position of the decimal place value system which resulted into finding answers such as 0.001, 50 and so on (See Figure 4.19, Section 4.3.1). This challenge further led teachers to leave the questions unanswered in terms of relating and interpreting the meaning of their calculated answers in relation to the probability scale. However, this challenge was extended to the next question which required teachers to find the expected number (5) of road carnage during 50 such periods (See Figures 4.18 & 4.19, Section 4.3.1). These results, however, suggests that the majority (10) of mathematics teachers' subject matter knowledge of probability was put to test as they were unable to recognize that there are infinite numbers between 0 and 1. Hence, this lack of understanding caused the teachers to leave the given answers 0.1 and 0.001 without interpretation of their meanings about the probability scale. This limitation in the subject matter knowledge of probability had a negative impact on assessing the mathematics teachers' PCK of probability in terms of their ability to interpret the meaning of the calculated answers. This means that the teachers lacked the essential part of the components of the conceptualised PCK of Probability listed in this study's conceptual framework which investigated on teachers' subject matter knowledge of teaching probability. In light of this, Shulman (1986) argues that knowing a subject for teaching requires more than knowing its facts, concepts and principles. This means that teachers must understand the organising principles, rules and structures to establish what is legitimate to do in a subject. Therefore, teachers' deficiencies in the interpretation of probability events have the potential to affect learners' opportunities to understand and grasp probability concepts and be able to use them in a real-life situation. This could be the reason to why Shulman (1987) argues that teachers of mathematics who possess inaccurate information or lack deep conceptual knowledge in their subject matter are likely to pass on the same incorrect information to the learners and generations to come. This means that there is a likelihood of continuous poor performance by the learners in abstract topics like probability. Wilson and Peterson (2006) also pointed out that teachers as professionals must be well vested with strong subject matter knowledge (content knowledge) in the subjects they teach.

5.2.2 The pedagogical strategies teachers use in the teaching of Probability concepts

This was the second question of this study which explored teachers' ability to identify, plan and select learner engaging strategies as they taught probability. The following teaching strategies were observed in their lesson presentation during lesson observation included engaging learners in a lesson with their observed challenges they encountered in the process of solving probability problems, creating opportunities for learners to demonstrate their new knowledge learned in the class, scaffolding and asking constructive questions to guide learners to use specific procedures which could illicit particular answers, use of visual cues to support relevant mathematical thinking, for example, teaching aids and diagrams (tree). Furthermore, teachers were able to work with learners' ideas in solving probability problems, provided chances during lesson presentations for learners to explore, explain and discuss each probability concept to enhance their mathematical independence in solving probability problems. Teachers were also able to provide teaching and learning materials to facilitate the smooth demonstration of their understanding of the subject matter such as computers, coins, dice, and cards. They also encouraged subjective and independent problem solving, one-on-one, whole-class discussion, individual classwork, group work discussion, and teacher, and learner demonstration methods and group work activity (See Excerpts 4.1 to 4.7, Section 4.5.1).

However, the use of the above mentioned pedagogical strategies showed how teachers were able to realise that the use of multiple learner engagement strategies was central and an effective way of teaching mathematics, in particular Probability. Teachers were also able to realize that learners were not empty vessels or blank slates or passive observers, as such they provided chances to in their lesson planning to support learner participation by demonstrating their understanding of probability concepts at the beginning, in the course and at the end of a lesson. Furthermore, the findings of this study found that teachers were able to provide feedback by engaging learners in discussing their answers through whole-class discussions, one-on-one, and also provided a platform for them (learners) to demonstrate their understanding of the subject matter knowledge of probability and reasons why some answers were correct or not (See Excerpt 4.3, Section 4.5.1). Wilson and Petersen (2006) stated that for learners to gain insight into their learning and understanding of the subject matter, there is a need to provide feedback to every learner's given answer. In addition, mathematics teachers also gave chance for the learners to discuss in groups, individually to think about and explain the operation of probability concepts and justify their reasoning. They further provided guided discovery in which the teachers began by demonstrating on how to solve probability problem which looked challenging to the learners and later allowed them to complete the question in groups, individually and as a whole class (See Excerpt 4.2, Section 4.5.1). Teachers were also able to work with learners' ideas both in groups, one-on-one, and as a whole class discussion as they taught probability concepts such as simulating the concept of independence (See Excerpt 4.5, Section 4.5.1).

Teachers further encouraged independent and subject problem solving, individual classwork, and group work activity as a process of engaging learners in the teaching and learning of probability concepts (See Figure 4.25, Section 4.5.1). In this regard, this study established that eight (8) mathematics teachers were able to identify, plan and select multiple learner engaging strategies as they taught probability. However, the use of various strategies to explain on a single concept during lesson presentation suggested that the sampled teachers were able to exhibit their insight ability in planning and selecting of appropriate and use of multiple strategies that supported learner engagement in the teaching and learning of probability concepts (See Excerpt 4.6, Section 4.5.1). Moreover, the teachers' ability in planning and selecting various learner engaging teaching strategies suggests that teachers were able to exhibit components of PCK. In this case, it could be argued that teachers were able to exhibit conceptualised components of PCK indicated this study's conceptual framework, in particular, the pedagogical strategies which are centered on making the subject matter comprehensible to the learners and thus creating an understanding of what looked difficult became easy to the learners (Shulman, 1986). Shulman (1987) states that teachers should have knowledge of pedagogical strategies that could help them in re-organising the understanding of learners. In this respect, it suffices to state that the teachers' thought of having in their mind of teaching probability concepts using learner-centered and multiple teaching strategies were in line with the dictates of modern ways of teaching mathematics and Science of engaging learners in a lesson so that they could grasp the concepts quickly. This was at variance with the constructivist thought which encourages the active involvement of learners in the pursuit of knowledge (Schmidt, McKnight & Raizen, 2007). However, the idea of involving learners in the construction of knowledge is also grounded in the theories of learning and teaching as established by Wilson and Peterson (2006) that, learners are not empty vessels or blank slates or passive observers, but they are active agents of their own knowledge construction, as such teachers need to provide opportunities to the learners to demonstrate their understanding of probability concepts in their planning and selecting of strategies that support learner participation.

In addition, the findings of this study also found that teachers were able to organize and prepare teaching lessons that required the use of visualised and enriched concrete representations of materials such as computers, coins, dice, and cardboard (See Excerpt 4.6, Section 4.5.1). Teachers were able to organize learners in groups and used the computers to simulate and determine the actual independence of a single toss. Furthermore, learners were allowed to observe and become involved in the demonstration of the trials using the computers. Thereafter, the teachers were able to allow learners from each group to demonstrate and explain before the audience of learners, which brought excitement and understanding despite other learners were not competent enough in operating the computers but they were physically able to observe the processes of simulating and determining the actual independence of a single toss using a model (See Figure 4.24, Section 4.5.1). However, the use of computer modeling provided first-hand experience to the learners as they were able to see the act of simulating and determining the actual independence of a single toss as the computer modeled the activities. In terms of explaining to the slow learners the use of the computer allowed the slowing of the process of simulating and determining the single toss, and this facilitated learners' conceptual understanding of the concepts of independence by allowing learners to observe all the processes involved in determining a single toss (Jones & Tarr, 2007). It is therefore important to note that the use of software technologies and computers in teaching of probability concepts work as a tool builder to facilitate learners' understanding of probability concepts. In addition, the software technology also works as an amplifier and a reorganizer to engage learners in tasks that would simultaneously develop their understanding and provide first-hand experience of seeing the act of independence in an activity (Lee & Hollebrands, 2008). As such, teachers of mathematics should have the ability to plan and select appropriate teaching and learning materials that could help to clearly explain the probability concepts. In this regard, Shulman (1986) argues that teachers with well-established subject content knowledge know what kind of prerequisite knowledge, appropriate examples, teaching materials, and illustrations that are necessary to use to teach the subject. However, in this context, it can be argued that teachers' ability to use the appropriate teaching and learning material such as the projector and the computer it means that they were able to exhibit PCK necessarily for teaching probability which other researchers have conceptualized as part of PCK (Ball, Bass & Schilling, 2008; Shulman, 1986).

Furthermore, during lesson observation, mathematics teachers' subject matter knowledge of probability was reflected in the manner they taught, but they could not use hands-on- strategies such as group work activities, learner demonstration and one-on-one strategies despite indicating these strategies in their lesson planning (See Excerpt 4.4 & Figure 4.23, Section 4.5.1). However, their teaching was dominated with plain lecturing strategies, in some instances where they remembered to engage learners, they formed large groups which could not allow all learners to participate and contribute freely their answers. Other teachers taught the concepts of probability focusing on the correct and wrong answers without necessarily providing a feedback to the learners on how the answers became correct or wrong (See Figure 4.22, Section 4.3.1). Teachers also used unproductive question and answer methods at the beginning of a lesson to assess learners' understanding of the previous topic and some used question and answer methods at the end of the lesson to summarize the lesson. This hardly gave a clear understanding and retention of probability concepts among learners which in turn led to them giving unexpected answers in their exercises. This suggests that the majority of mathematics teachers (9) were able to plan and select appropriate teaching strategies but they could not use the planned strategies as expected.

This led teachers to teach using a plain lecturing method which did not allow them to engage learners in their lesson presentations. The above findings are consistent with the findings of Tembo (2013) where his study showed that the majority of teachers of mathematics have a habit of using the plain lecturing method in the teaching of abstract topics like Earth Geometry and Probability to avoid questions. Hess (1999) also found that the use of the plain lecturing method is a norm in regular schools nowadays. He further stated that this could be due to teaching large classes, limited time to attend to slow and fast learners, the reduced number of periods, and insufficient teaching and learning materials in schools. However, the frequent use of the plain lecturing method caused majority (85%) of learners to encounter challenges interpreting and differentiating the concept of independence, dependence, and mutually exclusive events. However, the study findings also suggest that only eight mathematics teachers were able to promote learning among learners by providing chances to the learners to interactively and collaboratively solve the probability problems. This allowed learners to be fully engaged in their lessons, and help them to improve their understanding of probability concepts.

In view of the analysis of teachers' ability in use of multiple pedagogical strategies in their lesson presentations, the study revealed that mathematics teachers were able to identify and select various teaching strategies but they could not use all the strategies to explain on a single given probability problem (See Excerpt 4.2, Section 4.5.1). For example, teachers could not use different teaching strategies and representations of materials to explain on how to determine the actual independence of a single toss of the coin to the learners, instead, they opted to teach probability concepts using a single strategy which could not allow giving chance to the learners to demonstrate their understanding (See Figure 4.24 & Excerpt 4.4, Section 4.5.1). Ball, Bass and Hill (2005) stated that having an understanding of mathematical concepts may enable teachers to access a broad selection (repertoire) of strategies for explaining and representing mathematical content to the learners in a comprehensive way.

Wilson and Peterson (2006) also argued that learners are needed to be given opportunities to learn using multiples ways so that they could understand the subject matter taught in a comprehensible way. In this regard, only six (6) mathematics teachers were able to employ multiple strategies to solve and explain on a single given probability problem as they taught probability concepts. Teachers were able to exhibit the conceptualized components of the pedagogical content knowledge (PCK) which allowed them to teach probability concepts using multiple strategies to meet different learning needs of the learners. However, this can be argued that teachers were able to exhibit components of PCK necessarily for teaching probability according to Shulman's (1986) definition that PCK comprises understanding of the most useful forms of representations of those (mathematics) ideas, analogies, illustrations, examples, explanations and demonstrations that teachers designed to make the subject matter comprehensive to the learners. Shulman (1987) also argued that pedagogical content knowledge of a subject is important for teachers to transform their content knowledge in pedagogically appropriate ways which fit different learners' needs. This means that the majority of teachers (11) used plain lecturing strategy during their lesson presentation which did not assist learners to become critical in their thinking and understand whatever they were taught. However, the use of this strategy was only advantageous to the teachers for they were able to finish whatever they had schemed. This made learners to become passive and affect their participation in the learning of probability concepts (See Figure 4.23, Section 4.5.1).

In the analysis of teachers' responses from the follow-up interviews as why they allowed limited time and opportunities to teach probability concepts, mathematics teachers argued that learners sometimes needed to pay particular attention so that they are able to practice whatever they have been taught (See Excerpt 4.4, Section 4.5.1). For example, seven out of the sampled seventeen teachers also indicated that they were aware that learners needed to learn probability concepts through collaboration with teachers and learners, interacting with classmates through group activities as well as giving learners a platform to express their views on the topic at hand as a class and demonstrate what they knew on the chalkboard. Thus, these findings suggest that the majority (9) of mathematics teachers used plain lecturing strategies which did not allow learners to interact and collaborate in solving of probability problems. In this context, Ball, Thames and Phelps (2008) conceptualised PCK as the domain of teachers' knowledge that combines the subject matter knowledge and knowledge of pedagogy. This case implies that the teachers lacked the PCK which could have helped them to think beyond the knowledge of the subject matter per se to the dimension of the subject matter knowledge for teaching and develop an extraordinary ability to transfer the subject matter using learner engaging teaching strategies that could have made learners comprehend the subject matter with less difficulties despite aiming at finishing the schemed work. Wilson and Peterson (2006) argue that teachers must have the ability to plan and select multiple strategies that could promote learning among learners. Furthermore, they emphasize that teachers should have the thoughtful use of selected strategies in their approach and should be refined over time through reflection. Therefore, teachers should have the ability to carefully plan, select appropriate, and use multiple teaching strategies that would clearly present the concepts of probability to the learners with different learning needs.

5.2.3 The pedagogical enabling prompt measures that mathematics teachers used to support learners' understanding of probability as they taught probability during lesson presentation.

In their first step of planning and preparation of their lesson presentations on probability concepts, teachers went into class with prepared lesson plans on which they based their teaching of the selected specific probability concepts (See Appendixes 8.1, 8.2, 8.14 & 8.15). This means that teachers went to class with planned and prepared work which is one of the conceptualised components of PCK in this study's conceptual framework. It is also important to note that much of the subject matter that was presented by the sampled teachers were supported with well-designed pedagogical enabling prompts that supported learners' understanding of probability. The following enabling prompts were used to support learners' understanding of probability concepts and these are; the use of common events or notation symbols to consolidate learners' new knowledge of probability concepts, simplifying and reducing of complicated steps in solving probability, the use of various representations and giving of a chance to the learners to demonstrate their understanding of probability. The use of the pedagogical enabling prompts was more effective and informative to the teaching and learning of probability concepts to both teachers and learners. Basing on Shulman's (1986) notion of PCK, this could be argued that the teachers were able to exhibit the aspect of the conceptualized components of PCK which assisted teachers in explaining and learners in understanding of the probability concepts, as teachers were able to use the most useful forms of representations in formulating the subject matter knowledge of probability in a comprehensible way to the learners. In a similar context, teachers were also able to interact and assist learners in the solving probability problems to build on the new knowledge they were acquiring from the already existing the subject matter knowledge of probability (See Excerpt 4.3, Section 4.5.1). Furthermore, teachers were able to work with learners' contributions and explored other means of solving similar probability problems to support learners' understanding of probability concepts. It must be also noted that teachers did not only make the concepts and process to be learned explicit but also made the purpose of learning explicit through the use of various forms of representations such as the use of a coin, and computers in simulating the concept of independence as they taught probability to support learners' understanding of probability concepts (See Excerpt 4.2 & Figure 4.24, Section 4.5.1). The use of a coin and computers in the process of simulating the concept of independence helped to capture learners' attention and concentration and was able to meet the diverse learning needs of learners. Therefore, the majority of teachers (9) made the learning of probability become explicit but did not meet the objective of learning on the concept of independence to the learners. As such, mathematics teachers were only able to simulate the concept but had difficulties explaining on how to identify and determine the actual independence of a single toss comprehensively.

Teachers were also observed using group work discussion, role-play activities, whole-class discussion, and teaching aids on which they presented various definitions of probability terms (See Appendix 8.2). However, the use of different representations of the subject matter in their lesson presentation provided a chance for learners to capture and broaden their understanding of the subject matter knowledge of probability. However, the use of multiple forms of representations and mathematics imagery is argued to be useful in that, it assists learners with different learning styles to understand essential mathematical concepts (Shulman, 1987). Furthermore, the teachers were observed using common events to explain on non-mutually and mutually exclusive events which learners were familiar with to assist them understand the new concepts and processes of formulating the associated probability expression. For example, teachers used the occurrence of daytime and night time, flipping of the Head and Tail of a coin and shading of Venn diagram to explain on the concept of non-mutually and mutually exclusive events which later helped learners to formulate the associated probability expression of set A and B using the jointed and disjointed sets which the learners were familiar with in the form $P(SUT) = P(S) + P(T) - P(S \cap T)$ and $P(SUT)=P(S)+P(T)$ (See Figures 4.7, Section 4.3.1 & Appendixes 8.5, 8.13 & 8.14).

It is also important to note that teachers ensured that learners were given chance to interact and collaborate in the solving of challenging probability problems which required learners to analyze the mathematical situations and critically examine, explain, justify their own and others learners' mathematics thinking in relation to the probability scale in groups. This, however, provided learners with opportunities to experience multiple understanding of the subject matter knowledge of probability. Ball, Bass and Hill (2004) pointed out that working in collaboration with learners and their ideas in restructuring tasks and constructing probability knowledge is central and an effective way of teaching mathematics, in particular probabilities to the learners. This means that the teachers' use of pedagogical enabling prompts in their teaching of probability provided a platform to broaden learners' conceptual understanding of abstract topics like probability and also to reduce poor learner performance in mathematics. However, the implication of teachers using pedagogical enabling prompts in the teaching of specific probability concepts could be argued that, it provided teachers a platform to exhibit their PCK of teaching probability. This is because PCK includes an understanding of what makes the learning of specific topics easy or difficult (Shulman, 1986). Wilson and Peterson (2006) state that every teacher needs a repertoire of instructional strategies that range from methods of direct instruction to cooperative and small group work to one–on–one work. This is because there is no single strategy that could clearly explain the probability concepts in a class with different learning needs since learners are individually unique in their learning needs.

The study further found that in their lesson planning, teachers had suggested and prepared the use of various teaching materials such a computer, dice, spinners, marbles, but the majority of teachers (10) used the common coin and a few computers to simulate and explain on the actual independence of a single toss. The teachers did not realise that rolling dice with a number 6 and tossing of a coin with the outcome either become the Head (H) or the (Tail) uppermost could easily help to explain on and justify clearly the actual independence of a single toss as compared to the use of a coin. In this regard, Batanero, Godino and Roa (2004) state that the use of a coin alone provides different results each time the trial is performed and at the same, the trial cannot be reversed in a situation a learner has not understood. In contrast to the above, seven teachers were able to use a coin, while two teachers used a computer and a projector to simulate and explain the actual independence of a single toss which captured learners' understanding of the concept of independence (See Figure 4.24, Section 4.5.1). This made learners to find it easy to understand the structure behind the experiment which made them to easily identify the actual independence of a single toss. Based on the manner they presented their lessons and responses in the follow-up interviews, the sampled teachers were able to use what Shulman (1986) called the most useful forms of subject matter representations of those mathematical ideas, illustrations, examples, explanations, and demonstrations. This means that teachers were able to use the most useful forms of representing the subject matter knowledge of probability concepts in a comprehensible way to the learners (Shulman, 1986). Therefore, the research findings of this study are consistent with the findings of Carson (1988), Grossman (1990), Marks (1990), Wilson, Shulman and Richert, (1987), and Wilson (1988) that the claim for PCK was observed among effective teachers in the manner they represented key ideas using metaphors, diagrams and explanations that were at once attuned to students' learning and to the integrity of the subject matter.

5.2.4. Pedagogical Challenges Teachers Encountered in the Teaching of Probability

Although the findings of this study showed that mathematics teachers exhibited the subject matter knowledge of probability in terms of the procedures and methods, and the ability to design pedagogical enabling prompts and strategies which assisted in engaging learners in the teaching and learning of probability, the majority (9) of teachers taught using learner engaging strategies, familiar events, and representations that were technically correct but could not provide a platform for learners to demonstrate their understanding of the new subject matter of probability. In addition, teachers were also able to select learner engaging teaching strategies, but teachers could to design pedagogical enabling prompts which could have assisted to explain on learners' misconceptions (See Appendixes 8.3 & 8.7). In this case, research studies have revealed that the majority of teachers of mathematics in secondary schools have challenges of using various teaching strategies and instructions in the teaching of abstract topics (Batanero et al, 2016; George & Mc Erwin, 1999).

In follow-up interviews, the teachers also struggled to explain how they could use other instructional teaching materials rather than the use of the coin and the computer that they used to explain on the concept of independence (See Figure 4.24, Section 4.5.1). This means that mathematics teachers showed lack of knowledge on how to pair the teaching materials such as the use of dice and a coin to simulate and explain the actual independence of a single toss of a coin. This could be the reason to why Steinberg (1990) argued that the concept of simulation is not just methodological teaching aid but rather an essential means of imparting knowledge and conceptual understanding of probability concepts. In light of this, Shulman (1986) also argued that a teacher with PCK of the subject matter would know how to pair the teaching and learning materials and formulate questions which could promote critical thinking despite handling a large class of learners. Stohl (2005) also pointed out that majority of teachers of mathematics despite having a major in teaching subject being mathematics have knowledge gaps and consistently lack experience in designing investigation or simulation of probability concepts for the learners using either coin tossing or spinners or dice.

The study also established that mathematics teachers had challenges with the algorithm of adding decimal numbers which did not require teachers to use any special knowledge of PCK to multiply and add the decimal numbers, and interpret to provide the meaning of 0.1 in relation to the probability scale (0 to 1). However, it is at this stage that sampled mathematics teachers' common content knowledge (CCK) was put to test. As such teachers could not recognize that there are an infinite number of fractions between 0 and 0.5 as well as between 0.5 and 1. In this regard, mathematics teachers were only able to multiply and group the decimal numbers for addition but had challenges with placing a decimal point in their process of adding which made them find answers such as 0.001 (Appendix 8.15). However, a large number of mathematics teachers (12) were able to find the expected answer 0.1 but could not to relate and interpret the meaning of the answer they found in relation to the probability scale. In this case, teachers could not realise that as the probabilities approaches zero (0), there is a less likely of an event becoming significant while as it approaches one (1), there is more likely that an event might become significant. Furthermore, teachers found it difficult to explain how learners could have found it easy or difficult to use the concept of independence and certainty in the question to formulate the probability expression, despite being given chance first to solve the question. In this case, teachers could not state errors that learners were likely to be committed to or mathematical knowledge which they were required to use to answer the questions to make adjustments on questions that looked difficult or less challenging. However, mathematics teachers' inability to interpret and stating of the mathematical and pedagogical judgment on learners' challenges could be viewed that it shows that teachers lacked PCK of teaching probability which has the potential to deny learners' opportunities to understand probability concepts, and this could be further speculated that there is a likelihood of continuous of poor performance of learners in abstract topics such as Probability.

The study also revealed that some questions that mathematics teachers were asking failed to guide the learners to the next step they were required to take especially when learners were given the opportunity to demonstrate their understanding. For example, during lesson presentation teachers asked questions that did not give guidance to the learners to move to the next step in their process of solving probability problems and in making conceptual connections of laws of sets and probability concepts. In other ways, the teachers were unable to construct and ask questions that were relevant to the subject matter of knowledge of probability (See Excerpts 4.5, Section 4.5.1). In addition, the exercise and homework questions which were used to assess learners' understanding of probability concepts were book lifted from the Grade eleven mathematics learners' text book which did not make learners appreciate the subject matter knowledge of probability (See Appendix 8). Hence, this made learners to think that probability was difficult to understand which resulted into reducing their confidence in the topic. In this regard, the National Council of Teachers of Mathematics (NCTM) (1989) indicated that, although teachers are well trained in methodology courses at Colleges and Universities, there were still significant pedagogical challenges in the construction of questions that could scaffold and guide to what they needed to understand and retain probability concepts. Borovenik (2012) stated that probability's abstractness and formal nature of it often causes learners to develop a negative attitude towards probability concepts if the subject matter is not well taught to the learners. This could be the reason to why Avong (2013) has argued that the presence of ill-prepared teachers in secondary schools, negative attitude of teachers and their lack of readiness to teach appropriately have the capacity to continue affecting learners' performance in mathematics. It is important that mathematics teachers should always be in a position to ask useful and guiding questions which are capable of making learners construct knowledge of probability concepts. Therefore, mathematics teachers with a clear understanding of the subject matter and accurate information about the topic increases the teachers' effectiveness in teaching such that a teacher is able to choose the right illustration, appropriate examples and provide correct pre-requisite knowledge of the topic being taught (Shulman 1986) which in turn could improve learners' performance in probability.

Furthermore, in the category of defining probability terms, the study established that only few of the sampled mathematics teachers (8) were able to define probability terms according to the socially agreed conventional meanings terms with reference to the probability scale and other related topics within and outside the branches of mathematics. Not only that, the majority of them provided incomplete definitions that lacked conceptual connection with other related topic concepts such as percentages, fractions and decimal numbers as replicated. This, therefore made mathematics teachers provide recited definitions from the Grade eleven (11) mathematics textbook (See Figure 4.3, Section 4.3.1). This was the case with questions 1(a), 2(a), 14, and 17(a). Thus, these results suggest that the majority (9) of the sampled mathematics teachers exhibited less understanding of the subject matter knowledge in terms of defining probability using the agreed qualitative terms such as certain, never, impossible, probably, and so on (Baer, 2008). Instead, they were merely citing definitions from the Grade eleven mathematics learners' textbook. It is important that mathematics teachers be aware that probability concepts are mostly carried in the qualitative terms which are used to define Probability (Wilson & Peterson, 2006). As such, teachers and learners need to have a clear understanding of the mathematical operations (definitions) of each concept rather than merely reciting definitions, rules, procedures, or algorithms without a clear understanding.

CHAPTER SIX: CONCLUSION AND RECOMMENDATION

6.1 Conclusion

This was a qualitative multiple case study that investigated mathematics teachers' Pedagogical Content Knowledge of Probability in Chongwe District of Lusaka Province, Zambia. The study collected data using subject matter knowledge diagnostic test (SMKDT), lesson observation, interview schedule, and document analysis. In respect of data collected from the SMKDT, lesson observations and interview schedules, the elements of the subject matter knowledge, pedagogical strategies, and enabling prompts categories of the study's conceptual framework were employed for the analysis of mathematics teachers' ability to solve, select and use multiple teaching strategies. Based on the analysis of the findings of teachers' ability to recognize, identify and apply the competencies which were embedded in the SMKDT questions, the study found that one hundred and forty-four (144) of the total two hundred and twenty-six (226) competencies were applied according to the initial objective of the questions, which was a demonstration of teachers' understanding of the subject matter knowledge of probability that has been conceptualized as indicators of PCK necessary for teaching of probability. However, the eighty-two competencies were incorrectly used in solving and explaining on the subject matter knowledge of probability in the process of answering the questions. This demonstrated teachers' limited understanding of the subject matter knowledge of probability as teachers lacked clear understanding of the conceptualized components of PCK described in each category of the conceptual framework of this study.

In the category of relational conceptual connections of probability concepts with other major mathematical concepts, the study found that the teachers' subject matter knowledge of probability reflected in the manner they solved, made relational conceptual connections, interpreted, explained, and justified the mathematical concepts which were embedded in the SMKDT questions. In light of this, mathematics teachers were able to demonstrate their understanding of the subject matter knowledge of probability in which they were able to formulate the expected probability expressions using the mathematical notation symbol 'U' in relation to the conceptual organization of the non-mutually and mutually exclusive events with reference to the jointed and disjointed sets respectively. Besides this, teachers were also able to recognize, identify the concepts of independence and certainty in the probability event as well as list the combination of outcomes of reaching late (L) or not (N) for four consecutive days in which each day occurred as independent event. However, using the above understanding, the teachers were able to formulate the compound spaces with more than two compounds of any of this order listed as L, N, L, L; L, L, N, L; N, L, L, L and L, L, L, N. Furthermore, mathematics teachers were able to extend their understanding of the subject matter knowledge of probability in terms of predicting and explaining how learners were likely to misunderstand the concepts which were used to solve the probability problems given in the SMKDT such as the concepts of conditional probability, certainty, and independence. In this regard, their ability to formulate compound spaces with more than two compounds and to predict, was a demonstration of teachers' understanding of the subject matter knowledge of probability which was conceptualised as the most useful component of PCK in the teaching of abstract topics such as probability as conceptualised in this study's conceptual framework. Regardless of the above findings, it was also found that mathematics experienced lack of the ability to recognise, identify and interpret the concepts of independence and certainty in the probability event as well as the ability to list the combination of outcomes of reaching late (L) or not (N). Thus, this caused teachers to generalise the formulation of the probability compound spaces and further faced a challenge in predicting the mathematical concepts which were required to be understood by the learners in their attempt to solve the probability problems given in the SMKDT questions.

Regarding the interpretation of calculated answers, the study found that majority of teachers were only able to solve the given probability problems but had no skill to interpret the answers in relation to the probability scale. In this case, teachers were only able to express their basic skill of solving probability concepts. As such, they could not extend their understanding of the subject matter knowledge of probability to answer questions which required to reflect, analyse, interpret and make conceptual connections before answering. However, this limitation was considered as lack of the PCK necessary for teaching probability comprehensively to the learners.

In the context of unpacking and decompressing of the subject matter knowledge of Probability concepts to the learners during lesson presentation and in the follow-up interviews, the study established that mathematics teachers were able to design authentic probability problems which helped to increase the learners' drive to engage in mathematics thinking and conceptual connections of probability concepts with other major topics. Similarly, teachers were also able to exhibit the subject matter knowledge of probability in the manner they defined probability terms away from the ordinary word usage of every day English. Instead, they defined the terms using the socially agreed conventional meanings of probability terms with reference to the other related probability concepts such as decimal numbers, fractions and percentages. However, in the follow-up interviews, it was found that some mathematics teachers' subject matter knowledge of probability was limited as they could not realize that the words they used to define probability terms had specific mathematical meaning(s) that could either resonate or not with the everyday uses of the terms. As such, some mathematics teachers were forced to merely recite the memorized definitions from the Grade eleven mathematics textbook. This is because they could not differentiate probability words from the ordinary English meanings of words they used in the teaching of mathematics. However, this inability to define probability according to the socially agreed conventional meanings of probability terms revealed the nature of knowledge teachers had in Probability.

On the other hand, mathematics teachers were able to exhibit the ability to employ multiple strategies as they taught probability concepts and engaging the learners in an appropriate problem solving tasks to develop mathematical reasoning and this was seen as a critical step toward exhibiting PCK necessary for teaching probability. Teachers were also able to create a conducive learning environment for their learners by allowing them to work in collaboration, working with learners' ideas both in groups, one-on-one, and as a whole class discussion, and also by engaging them using their mathematical challenges they encountered in a given question and also by giving them chance to demonstrate their new knowledge learned. Furthermore, teachers were also able to explain and clarify on certain probability concepts that looked to be beyond the learners' level of understanding. Therefore, the use of multiple strategies during lesson presentation suggested that the sampled teachers exhibited their insight PCK necessary for teaching probability as conceptualized in this study's conceptual framework. While, other teachers were only able to identify and plan but demonstrated inability to use multiple strategies as they taught probability concepts, some taught using plain lecturing methods despite their effort made to identify and indicate it in their lesson planning. Thus, the use of plain lecturing could not allow learners to participate in the teaching and learning of probability concepts. This further led teachers to fail to listen, hear and reconnect learners disconnects in the definition of mathematical terms and pronunciation of probability terms which could have helped learners to improve in the use of probability language and mastering of probability concepts. Regarding the use of different teaching and learning materials, teachers were able to identify and organise teaching and learning materials such as coin, dice, spinners, projector and computers but could only use the coin to simulate and explain the concept of independence. The study also revealed that mathematics teachers' interactive lesson questions could not build enough confidence in learners to subject them to critical thinking and provide clear direction of the next step they were required to take in making conceptual connections of probability concepts and solve probability problems. This was a clear indication that mathematics teachers lacked the ability to facilitate learners' clear conceptual understanding of Probability as they taught probability concepts.

In relation to the third research question, the findings suggest that the teachers were able to recognize the importance of using pedagogical enabling prompts to support learners in the teaching and learning of probability concepts. In this regard, teachers were able to design pedagogical enabling prompts which assisted in bringing the learners closer to the understanding of the subject matter of knowledge of probability concepts and further helped them to vary their teaching strategies as they were observing learners' reasoning, strategic thinking, and mathematical communication through learner demonstration, group work activity and individual presentation. In view of this, the teachers used multiple forms of mathematical imagery, abstractions, representations and examples from learners' real-life experiences to demonstrate the subject matter knowledge of probability to learners. As such, teachers were able to provide learners with opportunities to demonstrate their understanding of the subject matter knowledge of probability concepts learned. The teachers also emphasised on conceptual understanding of the subject matter rather than mastering procedures which cannot be attained through subjective, independent, group work and individual solving of probability problems. Teachers were also able to assist learners to develop their conceptual understanding of probability concepts to higher levels of independent proficiency by which learners were provided with the starting point of how to solve the probability problem and a platform to demonstrate their understanding of the subject matter knowledge of probability. For example, teachers gave complex work and identified ways of how to solve the probability problems. This allowed learners to become engaged with the lesson and become independent in solving probability problems. However, much as it has been indicated in this study that mathematics teachers were able to exhibit conceptualised components of PCK of this study's conceptual framework and provided learners with opportunities to express their understanding of probability concepts, used multiple forms of mathematical representations, illustrations, examples and demonstrations, but teachers failed to extend their understanding of probability beyond their existing knowledge by interpreting, relating the findings about probability scale, anticipating, learners' responses to the given tasks based on what they had noticed after solving the probability problems. In this regard, only eight

(8) mathematics teachers were able to exhibit the subject matter knowledge of probability, design enabling prompts that did not only support learners' understanding of probability concepts but helped to vary their teaching strategies as they were observing about learners' reasoning and mathematical thinking as they taught probability. However, majority of mathematics teachers (9) lacked the ability and understanding of the subject matter knowledge of probability and the skill to relate and interpret the results in relation to the probability scale. In addition, other mathematics teachers also exhibited limited ability to use both the enabling prompts and multiple strategies to unpack and decompress the subject matter knowledge of probability in relation to real-life situation that could engage and support learners' understanding of probability concepts. Therefore, the implication of the above findings is that having taught probability for ten years and above does not guarantee them the development of PCK necessary for teaching probability concepts. These findings, though limited to probability, reinforce the view that teachers' experience of teaching probability for ten years and above, although it is an important factor in the development of PCK, has not guaranteed teachers' development of PCK for teaching probability. Therefore, the above research findings are consistent with the findings of Krauss, Neubrand, Blum and Baumert (2008), and Nilsson (2015) discussed in Chapter two, Section 2.4.1 of this study that, they found no positive correlation between mathematics teachers' knowledge in the teaching of Probability and their years of teaching experience. The findings of the two studies further indicate that majority of teachers of mathematics had low confidence and ability to explain and decompress the subject matter knowledge of Probability to the learners in a comprehensible way. However, these inadequacies reflected in their subject matter knowledge such as the inability to use multiple pedagogical strategies and to design enabling prompts that could be used to support learners' understanding of probability could be probably the cause of learners' poor performance in Probability at Grade 12 Mathematics final examinations.

6.2. Recommendations

Following the findings of this study, it is hereby recommended that

- Mathematics teachers must be provided with opportunities and support under the courses of methodology to adequately train them how to plan, select appropriate and use multiple pedagogical strategies in the teaching of Probability and other mathematics topics.

- Design pedagogical enabling prompts that would support learners in the teaching and learning of probability during their training.

- Issues of teachers' challenges in interpreting, explaining and justifying their mathematical reasoning should be considered during their CPD meetings.

- Help mathematics teachers to develop the ability to interpret, analyze, explain and justify their mathematics reasoning in relation to the topics' subject matter knowledge.

- More research is still needed to learn on other essential components of the PCK that could enhance mathematics teachers' pedagogy and content knowledge to effectively enhance mathematics teachers' ability to teach probability concepts comprehensively to the learners.

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APPENDIXES

DATA COLLECTION INSTRUMENTS

APPENDIX 1: Mathematics Subject Matter Knowledge Diagnostic Test (SMKDT)

RESEARCH TITLE: Teachers' Pedagogical Content Knowledge of Probability: The case of Teachers of Mathematics in Chongwe District, Zambia.

Dear Respondent,

The researcher for this study is a Postgraduate student at the University of Zambia pursuing a Master's degree in Mathematics Education.

You are therefore kindly requested to participate in this research by answering the following questions.

Be assured that the information you will provide will be confidential and will be used for academic purposes only. Kindly do not write your name on this paper, thanking you in advance

SECTION A.

1. Teacher of Mathematics (TM)………………………………………………….

2. Date………………………………….……………………………………..
3. Sex: Male ☐ Female ☐ tick in the box
4. Years in the service 10 to 12 ☐, 13 to 15 ☐, 16 to 20 ☐ tick in the box.

SECTION B

INFORMATION

This test instrument has only one section.

The questions in this instrument are based on Probability as prescribed in the revised Zambian Secondary School Mathematics Curricular.

Answer all the 12 questions on the sheets provided.

You have a maximum of 3 hours to answer the questions in the test instrument.

You are allowed to use a calculator to answer the questions.

Show all the necessary calculations on the answer sheets and number your answers correctly.

Total marks: 95 Marks

PROBABILITY QUESTIONS

1. (a) Define the term Probability as you could to the Secondary School learners [2]

   (b) What is the Probability of getting a tail and 6 if a coin and a dice are tossed once at the same time? [3]

   (c) Explain how you understand the concept of independent and dependent events with help of examples. [3]

2. (a) Give and explain an example of a mutually exclusive event [3]

   (b) Consider the figures 1 and 2 below. For each of the figures state whether set A and B are mutually exclusive or not. Justify your answers. [3]

Figure 1

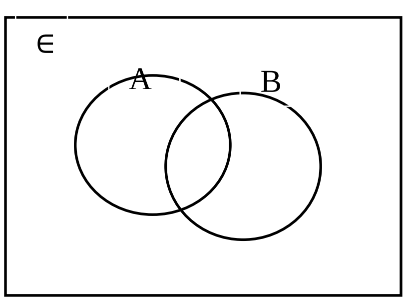
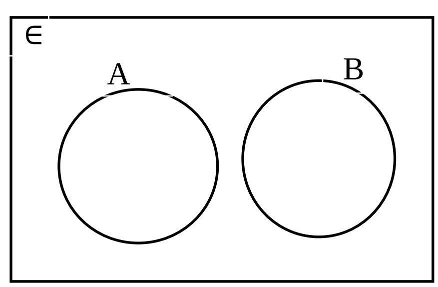


Figure 2



c) Hence, state their associated probabilities for figure 1 and 2. [3]

3. (a) Give an illustration of a compound event as you could teach the learners [2]

(b) A man goes to work either by car or by train. On any particular day the probability that he goes by car is 3/5 and the probability that he goes by train is 2/5. If he goes by car the probability that he will be late is 1/20 and the probability that he will be late if he goes by train is 1/8. If he goes by car on four (4) successive days, what is the probability that he will be late? [3]

(c) In your understanding of probability, what mathematical knowledge would the learners use to answer question 3b correctly? [3]

---

4. (a) A survey which was carried out at a certain hospital indicated that the probability that a patient tested positive for malaria was 0.6. What is the probability that the two patients selected at random one after the other?

   i. One tested negative and the other one tested positive. [2]

   ii. Both patients tested negative. [3]

(b) Explain how learners are likely to misunderstand the concepts you have used to solve questions 4a (i) and (ii). [3]

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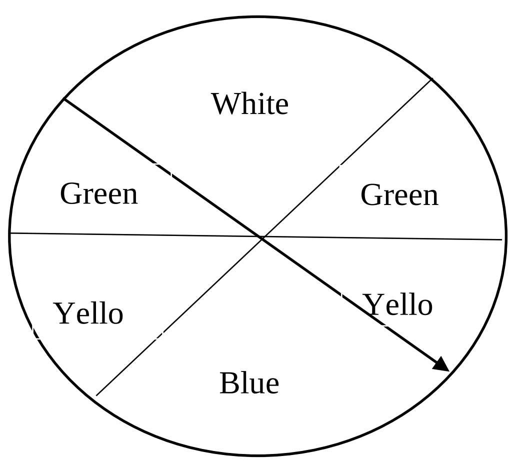
5. A wheel divided into six sectors of the same size colored differently is spun with an arrow pinned at the center as shown in the diagram below. When the wheel stops, what is the probability that the arrow will point at green or blue?



A secondary school pupil gave the answer for P (Green) or P (Blue) as 2/6 + 1/6 = ½ after answering the question above. Is the learner right or wrong? Explain and justify your position?

[3]

---

6. A red dice and blue dice are rolled at the same time;
   a) Construct a probability grid to find the probability of getting;
      i. The same score on both dices. [1]
      ii. A sum of 9 if the results are added together. [1]
      iii. A product of 36 if the results are multiplied together. [1]

---

7. There are 4 ripe and 6 unripe identical oranges in a basket. Vitumbiko picked two oranges from the basket one after the other without replacement.
   a) Draw a tree diagram to show all the outcomes and their probabilities.
   b) Use your tree diagram to find the probability that:
      i. Both were unripe? [1]
      ii. Only one was ripe? [1]
      iii. At least one was ripe? [1]

8. The number of road carnages occurred along the Great North Road high way between 4p.m and 11p.m on every Fridays in a certain month were estimated at 0,1,2,3 and were corresponding to the probability of 0.94, 0.03, 0.02 and 0.01.

(a) Find the number of expected road carnages occurred during that period and explain how that value relates to the occurrence of the accidents. [2]

(b)Calculate the expected accidents during 50 such periods. [1]

---

9. (i) An organization has a group of 24 persons of which 16 are males and 8 are females. Of the 16 males, 10 are above 45 years and the rest are below 45 years. Of the 8 females, 4 are above 45 years and 4 are below. If a person is picked at random, what is the probability that the person is:
   a) Above 45 years old? [1]
   b) A female over 45 years old? [1]
   c) Over 45 years given that she is a female? [1]
   d) Over 45 years given that he is a male? [1]

(ii) What condition(s) is associated with question 9(c) and (d) that the learner need to know in order to answer the questions correctly? [2]

10. Given that S ={1,3,4,,7,9,11,13,16,19}

A = {1, 4, 11} and

B = {3, 9, 11, 13}

If a number X is chosen at random from S, what is the probability that;

(a) X ∩ A (b) X ∈ B (c) X ∈ (A∩B) (d) X ∈ (A∪B) [4]

11. (i) A point is plotted at random on the Cartesian plane. Assume that the point does not fall on either side of the axes. If

A. = {all points lie in the 1st quadrant}

B. = {all points lie in the 2rd quadrant}

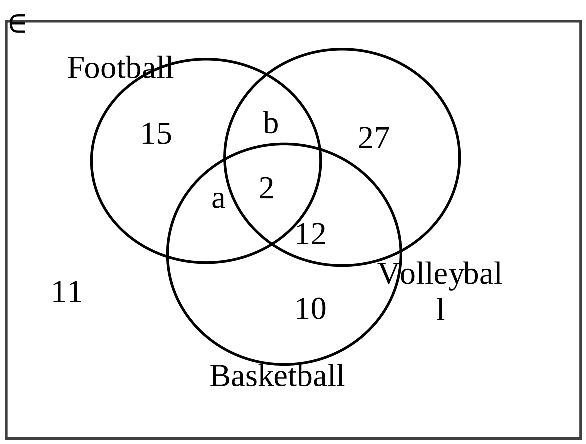
C. = {all points lie in the 3rd quadrant} and

D. = {all points lie in the 4th quadrant}

Find (a) P (A) (b) P (A∩B) (c) P (B∩C∪D) (d) P (C or D) [4]

(ii) Explain why all the sets of points on the Cartesian plane are continuous sample space? [2]

A survey was carried out on 100 pupils at Maya secondary school to determine their participation in sports. The results are show in the van diagram below.



(a) Given that 42 pupils play volleyball, find the values of a and b [2]

(b) If a pupil is selected at random from the group, what is the probability that the pupil:

(i) Does not play football. [2]

(ii) Plays volleyball or football but not both. [1]

(c) What is the probability that the pupils either participates in only one of the sports or not involved at all? [3]

(d) What is the probability that a pupil who plays volleyball also plays basketball but does not play football? [3]

A tennis knockout is entered by **n** people. How many matches are needed to produce a winner? [2]

Two fair dice are thrown and events A, B and C are defined as follows.

The sum of the two scores is odd.

At least one of the two scores is greater than 4;

The two scores are equal.

Showing your reasoning clearly in each case, which two of these events are mutually exclusive events [1] (b) Independent events

The events A and B are not independent and P (A) = 0.5, P (B) = 0.7. It is also given that P (A∩B) = 0.85. Find (a) P (A∩B) [3]

(b) P (B/A)

(c) P (AUB')

---

15. Two events A and B are such that P(A) = 0.4, P(B) = 0.5 and P (A and B) = 0.1, find

(a) P (A or B) (b) P (A and not B) [3]

(c) Shade A∩B'

16. (a) What is the sample space for choosing a letter from the word probability? [2]

(b) What is the probability of getting at least one even number after rolling a single 6 sided dice?

Adapted from, ECZ 2015; 2016; 2017; 2018 and Malambo (2015)

APPENDIX 2: Preliminary Observation-interview Schedule.

RESEARCH TITLE: Teachers' Pedagogical Content Knowledge of probability: The case of Teachers of Mathematics in Chongwe District, Zambia.

Dear Respondent,

The researcher for this study is a Postgraduate Student at the University of Zambia pursuing a Master's degree in Mathematics Education. You are therefore kindly requested to participate in the preliminary interview questions. Be assured that the information you will provide will be confidential and will be used for academic purposes only. Kindly, do not write your name on this paper. Thanking you in advance.

SECTION A

Demographic information

i. Teacher™ ……………………..……………………….………...

ii. Date ……………………………………………….……………..

iii. Sex: Male female tick in the box.

iv. Years in the service…………………………………………………

1. In your on view, how do you perceive the concept of probability?
2. In your on view, why is the teaching of probability important to be learned by secondary school learners?

3. Which aspects of probability concepts do you like most teaching and explain why?

4. What do you think could be learners' prior mathematics knowledge in the teaching, and learning and Explain why such mathematical knowledge is required?

5. Is there any relation between the concepts of probability and fractions, sets, ratios and proportions?

Briefly, what is your understanding of the term probability in relation to the probability scale?

Differentiate the terms non- and mutually exclusive events with the help of a Venn diagram.

Explain to me how learners could arrange the occurrence of probability compound spaces after the tossing the coin twice.

What would you recommend as the best strategies to teach learners probability concepts?

What play activities do you use to teach probability and explain why that role play?

Explain to me the concept of certainty and independence

Could you tell me what you know about the points on the quadrants?

Explain why P (A and B) and P (A or B) are simplified to P (A) x P (B) and P (A) + P (B) respectively?

Briefly share with me how you understood question 8 (a) and (b).

In the mathematics test on question 8 (a) explain why you concluded that there was almost no accident after getting 0.1?

Explain clearly to me what you meant in the mathematics test that all the sets of the points on the Cartesian plane are continuous sample space? Share with me a specific method that can be used to answer question 10 (a), (b), (c) and (d).

Tell me from what you have planned in your lesson plan. Is there any teaching strategy that you have used to introduce and develop your lesson plan?

As you sure. In your lesson plan, I have seen a statement which has appeared in the column for methods for five times which reads as follows: Ask questions and take notes. Would mind to explain clearly to me what you mean by that?

Do you think covering whatever you have planned can help to improve learner performance in probability?

In the SMKDT test, on question5 (a). Explain to me why you indicated that the learners were correct?

Was that a best way you planned to teach the learners to compare your answer with their answers without engaging them in any form of discussion to find the answer?

APPENDIX 3: Lesson Observation Schedule for Teachers of Mathematics

RESEARCH TITLE: Teachers' Pedagogical Content Knowledge of probability: The case of Teachers of Mathematics in Chongwe District, Zambia.

Dear Respondent,

The researcher for this study is a Postgraduate Student at the University of Zambia pursuing a Master's degree in Mathematics Education.

You are kindly requested to participate in this research by presenting a lesson on one of the probability concepts of your choice.

Be assured that the information you will provide will be confidential and will be used for academic purposes only. Kindly, do not write your name on this paper. Thanking you in advance.

SECTION A

Demographic information

i. Pseudo name for a Presenter ………………………..……………………..

ii. Date …………………………………………………………..

iii. Sex: Male ☐ female ☐ tick in the box

iv. Years in the service…………………………………………………

v. Topic ………………………………………...…………………..

vi. Sub topic …………………………………..……………………..

SECTION B

The aspects of pedagogical strategies and enabling prompts which teachers designed to support learners' understanding of probability concepts.

Below show the questions which were used to investigate the subject matter, pedagogical teaching strategies and the enabling prompts which teachers used to support learners 'understanding of probability concepts during their lesson presentation

The aspects of enabling prompts in terms of teachers' subject matter knowledge of probability concepts.

1. Did the teacher had ability to provide clear explanation and justification on probability concepts and conditions?

2. Does the teacher manage to recognize, identify, interpret and apply the competences asked in the probability problem given in the SMKDT?

3. Does the teacher show ability to explain on probability concepts and conditions comprehensively to the learners?

4. Was the teacher able to anticipate and predict challenges or less challenges learners were likely encounter in a given probability problem

5.Was the teacher able to define probability terms using socially agreed qualitative terms such as likelihood, probably, sure, certain, impossible, likely, unlikely, probability and so on.

6. Was the teacher able to define probability terms according to the socially agreed conventional meanings of probability terms?

7.Was the teacher able to list the outcomes of compound spaces L, L, N, L (H, H, H, H); (T, T, T, T) ;(T, H, H, T) & (T, H, T, T)?

8. Did the teacher manage to calculate the probabilities of compound events in different ways and found different answers?

9. Did the teacher manage to interpret, illustrate and calculate compound events with four outcomes?

10. Did the teachers manage to identify and differentiate the concepts of independence, dependence, certainty and listed the combination of outcomes in different way

11.Did the teacher manage to make conceptual connections of jointed and disjointed sets in relation to the laws of Probability using the union notation symbol 'U' and intersection '∩' in a form

12. Did the teacher manage to make conceptual connections of probability concepts with other major mathematical concepts?

13. Did the teacher able to show ability to recognize correct and wrong procedure in solving a given task

14. Was the teacher able to explain to the learners the probability terms they needed to know in order to discuss and think about probability concepts?

The aspects of enabling prompts in terms of pedagogical teaching strategies.

16. Does the teacher provide chance to the learners to demonstrate their new knowledge learnt?

17 Does the teacher use various learner engaging teaching strategies and simulating of probability concepts?

18. Does the teacher use common events to consolidate learners' new knowledge of probability concepts?

19. Does the teacher use different media of instruction in the teaching of certain specific probability concepts?

20. Does the teacher use various learner engaging teaching strategies and simulating of probability concepts?

21. Does the teacher manage to identify and use multiple pedagogical strategies as she/he taught probability?

22. Does the teacher manage to pair different teaching and learning materials to determine and explain on the actual independence of a single toss?

23. Does the teacher motivation of learners.

24.Does the teacher managed to use a strategy of scaffolding to assist learners to understand the subject matter which looked challenging before allowing them answer on their own?

25. Does the teacher manage to teach probability concepts using interpretative problem solving activities

26. Does the teacher manage to teach probability concepts using visualized and enriched concrete representations materials such as computers, coins, and dice and made up cardboards?

27. Does the teacher able to interpret and design an accurate explanation that clearly explains on probability concepts?

28. Does the teacher able to engage learners in his/her lesson using their misconceptions or challenges of probability concepts.

29. Does the teacher able to identify and use teaching strategies according to learners ‘level of thinking as they taught?

30. Does the teacher able to subject learners to a comprehensive individualized assessment?

31. Does the teacher able to engage leaners in a lesson by working with learners ‘ideas in solving of given probability problems?

32. Does the teacher manage to use various instructional pedagogical strategies as they taught certain concepts?

Was the teacher able to teach using familiar expressions or events to relate probability concepts to practical situations?

The aspects of pedagogical enabling prompts.

1 What teaching and learning materials did the teachers use to simulate and determine actual independence of a single toss?

2 How did the teacher consolidate the learners’ existing and new knowledge of probability learned?

3 What kind of assistance did the teachers use to engage learners with and interpret probability concepts which looked challenging to them?

4 Did the teachers provide learners with chance to express their existing and current understanding of probability concepts?

5. How did the teacher make the process of solving of probability problems implicit to the learners?

6 Was the teacher able to use various forms of representations to explain the probability concepts to the learners?

7. If yes, what are those various forms of representation they used to explain?

8 Does the teacher provide chance to the learners to demonstrate their understanding of the new knowledge of probability concepts?

9 Was the teacher able to provide feedback to the learners' answers?

10 Did the teachers provide learners with mathematical text questions which required learners to analyze and interpret before answering?

11 Was the teacher able to assess learners on the previous lesson taught before introducing the new concepts?

13 Did teachers use familiar representations to explain and support learners' understanding of probability concepts?

If yes,

14. What are those representations that the teacher used to explain and support learners' understanding of probability concepts?

15. Was the teacher able to simplify and reduce the cumbersome steps in the solving probability problems?

APPENDIX 4: Pedagogical Strategies which were used during Lesson Presentations

S/N	Teaching strategies	Observation made by the researcher.
1	Teaching of probability based on the interpretative probability problem activities.	
2	Use of visualized and enriched concrete representations materials such as computers, coins, dice and made up cardboards.	

3	Engaging learners in a lesson with their misconceptions or challenges.	
4	Habit of engaging learners in the whole topic discussion and one-on-one.	
5	Providing of comprehensive individualized learner assessment.	
6	Use of computers to simulate probability concepts and to facilitate conceptual understanding.	
7	Activity-based teaching and learning of probability.	
8	Designing of accurate explanation, interpretation, and modifying mathematics tasks.	
9	Prepared teaching strategies according to learners' level of thinking as they taught.	
10	Working with learners' ideas in solving of probability problems.	
11	Use of familiar expressions or events to relate probability concepts to practical situations.	
12	Teaching of probability based on the interpretative probability problem activities.	
13	Use of visualized and enriched concrete representations	

	materials such as computers, coins, dice and made up cardboards.	
14	Engaging learners in a lesson with their misconceptions or challenges.	
15	Habit of engaging learners in the whole topic discussion and one-on-one.	
16	Providing of comprehensive individualized learner assessment.	
17	Use of computers to simulate probability concepts and to facilitate conceptual understanding.	
18	Activity-based teaching and learning of probability.	
19	Designing of accurate explanation, interpretation, and modifying mathematics tasks.	
20	Prepared teaching strategies according to learners' level of thinking as they taught.	
21	Working with learners' ideas in solving of probability problems.	
22	Use of familiar expressions or events to relate probability concepts to practical situations.	

APPENDIX 5: Teachers' Assessment Performance during Lesson Observation

ASPECT OF PEDAGOGICAL CONTENT KNOWLEDGE (PCK)	TEACHERS' SUBJECT MATTER KNOWLEDGE OF PROBABILITY CONCEPTS.	COMPETENCES APPLIED, PEDAGOGICAL STRATEGIES USED, SUBJECT MATTER KNOWLEDGE OF PROBABILITY WHICH TEACHERS EXHIBITED DURING LESSON PRESENTATION.	COMMENTS
Subject Matter Knowledge	Conceptual connections of probability concepts with major mathematical concepts.	Teachers were able to make different conceptual connections of jointed and disjointed sets in relation to the laws of Probability using the union notation symbol 'U' and intersection '∩' in a form P(AUB)=P(A) +P(B)-P(A∩B); P(AUB)=P(A)+P(B) and P(A∩B) = P(A∩B)= P(A)+P(B)	
	Solve probability problems and interpret the calculated answers in relation to the probability scale	Interpreted, shaded and solved the probability problems involving the concepts of sets and laws of probability in different ways as shown below. P (A and not B) P (A or B) =P (A+B). P (A) +P (B) +P(C) =1. P(A∩B), P(B/A), Shade A∩B'	
	To identify and differentiate probability concepts.	Teachers were able to identify and differentiate the concepts of independence, dependence, certainty and listed the combination of outcomes in different ways.	

	To illustrate probability events with justified examples	Teachers were able to interpret, illustrate and calculate compound events with four outcomes	
	Illustrate probability events into compound events. To recognize and represent probability concepts and conditions accurately	Teachers were able to list compound events outcomes as shown below; L, L, N, L (H, H, H, H) ;(T, T, T, T); (T, H, H, T) & (T, H, T, T). Teachers were able to calculate probabilities compound events in different ways and found different answers.	
		Teachers were able to use tree diagrams to explain on the probability concepts Teachers were able to recognize and differentiate probability concepts of replacement and non-replacement events on both favourable and possible outcomes when filling in corresponding probabilities on a tree diagram.	

	Defining of Probability terms Use of Probability terms and notations	Teachers were able to define probability terms using various socially agreed qualitative terms such as likelihood, probably, sure, certain, impossible, likely, unlikely, probability and so on. While, others recited the definitions found in the Grade eleven mathematics text book Teachers were able to define probability terms according to the socially agreed conventional meanings of probability terms and notations. Teachers provided different definitions and meaning of probability concepts	
	Anticipating and predicting challenges or less challenges learners may encounter in a given probability problem.	Exhibited subject matter knowledge to anticipate and predict challenges or less challenges learners may encounter in a given probability problem.	

	Probability concepts and conditions	Knowledge of conditional events and continuous sample space. Ability to provide clear explanation and justification of their views on probability concepts and conditions correctly. Ability to explain probability concepts and conditions comprehensively to the learners.	
	Use of correct procedures and conceptual connections.	Teachers showed ability to recognize wrong procedures and answers. Teachers were able to simplify the probability problems which were beyond learners' levels of understanding Teachers were able to make challenging mathematics clear to the learners	
Pedagogical Strategies	**Knowledge of applying different pedagogical strategies**	Teachers were able motivate learners	
		Teachers used a strategy of scaffolding to assist learners to understand the subject matter knowledge of probability. Used various instructional pedagogical strategies as they taught certain concepts.	
		Teachers identified and used multiple pedagogical strategies as they taught probability.	

		Teachers were able to select learner engaging teaching strategies in teaching of particular concepts.	
		Teachers were able to show ability to recognize correct and wrong procedure in solving a given task	
		Used different media of instruction in teaching of specific concept	
		Paired different teaching and learning materials, while others used only used single materials.	
		Showed ability to introduce teaching and learning materials	
Pedagogical enabling prompts	**Pedagogical Knowledge of presenting probability concepts to support learners' understanding of probability concepts during teaching and Learning process.**	Used common events to consolidate learners' new knowledge of probability concepts.	
		Used various forms of representations in teaching and learning of probability.	
		Used various learner engaging teaching strategies and simulating of probability concepts.	
		Simplified and reduced cumbersome steps in solving probability problems. Provided chance to the learners to demonstrate the new knowledge learnt.	

APPENDIX 6: Unstructured Interviews on Probability

RESEARCH TITLE: Teachers' Pedagogical Content Knowledge of probability: The case of Teachers of Mathematics in Chongwe District, Zambia.

Dear Respondent,

The researcher for this study is a Postgraduate Student at the University of Zambia pursuing a Master's degree in Mathematics Education. You are therefore kindly requested to participate in this interview. Be assured that the information you will provide will be confidential and will be used for academic purposes only. Kindly, do not write your name on this paper. Thanking you in advance.

SECTION A

Demographic information

i Teacher (TM) ………………………..……………………………….………...

ii Date ………………………………………………………….……………..

iii Sex: Male ☐ female ☐ tick in the box

iv Years in the service…………………………………………………

INTERVIEWS

1. In your on view, is the teaching of probability concepts to the computer aged learners important or not?

2. Which aspects of probability do you like most teaching learners and explain why?

3. In your introduction of a lesson on probability to the learners you started by asking them questions on the previous lessons. Explain why you did so?

4. Is there any relation between the concepts of probability and fractions, sets, ratios and proportions?

5. In the mathematics test, you defined probability "as a numerical value of 1, the likelihood of an event to happen and a numerical value of 0, the impossibility of an event to happen". Briefly explain to me what you meant by this definition.

6. In the lesson you taught in class, you mentioned of independent and dependent events. What conditions are needed to qualify an event to be independent or dependent?

7. Explain to me how learners can easily arrange the occurrence of compound events.

8. What kind of knowledge do figure 1 and figure 2 on question 2b are assessing on the learners?

9. Other than the method you used in the mathematics test to answer questions 8 (a) and (b). How else could you have found the answer to these questions?

10. Explain to me what you meant in the mathematics test by the concept of certainty and combination?

11. Could you tell me what you know about the points on the quadrants?

12. Explain why P (A and B) and P (A or B) you simplified them in a form P (A) x P (B) and P (A) + P (B) respectively?

13. Briefly share with me how you understood question 8 (a) and (b).

14. In the mathematics test on question 8 (a) explain why you concluded that there was almost no accident after getting 0.1?

15. Explain clearly to me what you meant in the mathematics test that all the sets of the points on the Cartesian plane are continuous sample space? Share with me a specific method that can be used to answer question 10 (a), (b), (c), (d).

16. Tell me from what you have planned in your lesson plan. Is there any teaching strategy that you have used to introduce and develop your lesson plan?

17. As you sure. In your lesson plan, I have seen a statement which has appeared in the column for methods for five times which reads as follows: Ask questions and take notes. Would mind to explain clearly to me what you mean by that?

18. Do you think covering whatever you have planned can help to improve learner performance in probability?

19. In the SMKDT test, on question5 (a). Explain to me why you indicated that the learners was correct?

20. Was that a best way you planned to teach the learners to compare your answer with their answers without engaging them in any form of discussion to find the answer?

21. Don’t you think that, this kind of a strategy where you focused on the wrong and correct answer encourages learners to memorize the procedures and rules?

22. Do you think that use of teaching strategies that do engage learners in the lessons could affect learners’ performance in probability?

23. In your lesson, I saw learners were fully involved in the lesson. Is that the way you planned your lesson?

24. Would you share with me the teaching strategies you used in your introduction and in the lesson development as you taught on probability terms?

25. So, would mind to give me reasons why you planned your lesson in the manner you taught it without fully involvement of learners?

26. Don't you think that when teaching abstract topics like probability and other topics requires teachers to use variety of teaching strategies that allows learners to participate fully in lesson and this was a best chance to do that?

27. What was in your mind as you were preparing this lesson you have just presented?

28. Why did you plan your lesson in such a way that, it included teaching methods like guided discovery, whole class discussion and group work and you used them interchangeably?

29. Okay, okay in your presentation you allowed the whole class to discuss on how to construct a Probability scale after they defined term together with you. What did you want to achieve by doing that?

APPENDIX 7: Document Analysis Sheet

RESEARCH TITLE: Teachers' Pedagogical Content Knowledge of Probability: The case of Teachers of Mathematics in Chongwe District, Zambia.

Dear Respondent,

The researcher for this study is a Postgraduate Student at the University of Zambia pursuing a Master's degree in Mathematics Education. You are therefore kindly requested to participate in data collection by providing the teaching documents such as lesson plans, records of work and the schemes of work and also the current learners' mathematics exercise books.

Be assured that the information you will provide will be confidential and will be used for academic purposes only. Kindly, do not write your name on this paper. Thanking you in advance.

SECTION A

Demographic information

i. Teacher number ……………………..………………………..

ii. Date …………………………………………………………..

iii. Sex: Male ☐ female ☐ tick in the box

iv. Years in the service…………………………………………………………….

Lesson plans from 2017 to 2020: ……………………………………………………………

Schemes of work from 2017 to 2020 ………………………………………………………

Records of work: 2017 to 2020…………………………………………………………………..

Current learner's mathematics exercises books: ………………………………………………….

Comments:

……………………………………………………………..………………………………

APPENDIXES 8: Lesson plans and responses given during lesson presentations and answering of SMKDT by teachers

APPENDIX 8.1: LESSON PLAN

SCHOOL 1 SECONDARY SCHOOL

DEPARTMENT OF MATHEMATICS **LESSON PLAN FORM**

NAME OF TEACHER TEACHER 11 **TS No** 581420

SUBJECT MATHS **TOPIC** PROBABILITY **DATE**

TIME 0700 – 0820 **SUB-TOPIC** DEPENDENT EVENTS

DURATION 80min **CLASS** 11B

RATINALE: This lesson is the fifth in the series and helps pupils acquire the ability to differentiate dependent from independent events. I will use teacher exposition, Question and answer, group work and pair work to teach this lesson

TERMINAL OUTCOMES: At the end of the lesson PSBAT:
- Solve questions based on dependent events correctly.

REFERENCES Grade 11 Maths PS book 11 Pg 317 – 318

TEACHING METHODS Teacher Exposition, Question and Answer, Demonstration

LESSON PROGRESS

ITEMS	TEACHER ACTIVITY	PUPIL ACTIVITY	DURATION
INTRODUCTION Review independent events	Tr asks ps to work in pairs and come up with an example on independent events and the formula in general	Ps work out the examples in pairs and report to class their examples and the general formula for independent events	5 minutes
DEVELOPMENT Step 1	Tr explains to pupils what dependent events are: If the subsequent choices are affected by earlier events then these events are said to be dependent events depending on the previous ones. Tr explains that the fact that the first item is not replaced is explicitly stated in some questions and implicit in some others so that we have to infer the fact.	Ps listen attentively. Ps ask questions for clarification. Ps note down important points in their exercise books	5min
Step 2.	① Tr gives an example: A bag contains 5 red, 3 blue 2 green sweets, A child picks two sweets and eats them!	Ps copy the examples in their exercise books	

Figure 27: Response P 24

APPENDIX 8.2: LESSON PLAN

SCHOOL 1 SECONDARY SCHOOL

DEPARTMENT OF MATHEMATICS AND ICT

LESSON PLAN

TEACHER'S NAME: TEACHER 8 **DATE:** 06.08.19

SUBJECT: MATHEMATICS **DURATION:** 80 MINUTES

TOPIC: PROBABILITY **GRADE:** 12B

NUMBER OF PUPILS: 50

SUBTOPIC: LAWS OF PROBABILITY **NO OF BOYS:** 24

NO OF GIRLS: 26

STAGE/ TIME	TEACHING / LEARNING ACTIVITIES	METHOD/ STRATEGIES (OR TECHNIQUES)	PUPIL ACTIVITY
INTRODUCTION 5 MINUTES	Teacher writes the following question on the chalk board and ask learners to work out: A card is drawn from a well-shuffled pack of cards. What is the probability that it is a red card.	Question and answer.	Learners to work out on chalk board and the following is the expected correct answer. $p(red) = \frac{26}{52} = \frac{1}{2}$.
LESSON DEVELOPMENT 40 MINUTES	- Teacher to define the following terms and write down probability laws. - Probability is the measure of chance of obtaining certain results from an experiment - An experiment is a situation with different outcomes - A sample space (denoted S) is the set of all possible outcomes - An event is the result we are interest in getting from an experiment - The following are the probability values; i. $0 \leq P(E) \leq 1$ ii. $P(E) = 0 \Rightarrow$ the event will not happen.	Question and answer. Guided discovery. Demonstration.	Learners to define some terms and state probability laws.

Figure 28: Response P 25.

APPENDIX 8.3: LESSON PLAN

SCHOOL 3 Girls Secondary School

Department of Mathematics

Lesson plan Form

Teacher's Name : TEACHER 12

Subject : Mathematics

Date : 06/08/19 : Time : 08:20 – 09:40

Topic : Probability

Duration : 80 mins

Class : 12E Number of pupils : 60

Development	Teacher's Activity	Pupil's activity	Duration
Mutually exclusive events.	Def : These are events that can not happen at the same time. For example: If two events A and B are mutually exclusive : $p(A \text{ or } B) = p(A) + p(B)$ This rule is called addition rule for mutually exclusive.	To copy and compare their response with what they are given by the teacher	45m
	Examples ① In a class test the probability that Mubita will be first in class is $\frac{3}{14}$ and Nkosi will be first is $\frac{1}{5}$ find the probability that Mubita or Nkosi will be first without a tie. Expected Answer $P(M) = \frac{3}{14}$ or $P(N) = \frac{1}{5}$ $p(M \text{ or } N) = \frac{3}{14} + \frac{1}{5} = \frac{29}{70}$	Group discussion Chart display	

Figure 29: Response P 26.

APPENDIX 8.4: LESSON PLAN

DEPARTMENT OF MATHEMATICS AND ICT

LESSON PLAN

TEACHER: TEACHER 14 **TS NUMBER:** ……84315

SUBJECT: …MATHEMATICS….**TOPIC:** .PROBABILITY. **DATE:** ………19/08/2019…

CLASS: ..12B………… **SUB- TOPIC:** TREE DIAGRAMS. **DURATION:** ……40...MINUTES….

NUMBER OF PUPILS: ……45……. **BOYS:** ……25…………………………**GIRLS:** ……20……………

RATIONALE: Estimating probabilities is basis for many financial industries,banking, mortgage lending, production and marketing etc. It is also useful for working out risks before engaging in an activity

OUTCOME BASED OBJECITIVES: At the end of the PSBAT:

(1) Apply the **OR** (Addition)rule and the **AND** (Multiplication) rule to calculate probabilities

(2) Solve problems involving Tree diagrams

TEACHING AIDS: Chalk, Chalk board, Manilla paper, Calculator.

REFERENCES: …Progress in Mathematics (PIM) Bk 11 PP147- 1156

GCSE Mathematics revision and practice (David Rayner 5th Edition) PP445-480

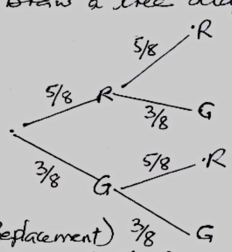
CONTENT	**METHODS**		**DURATION**
	TEACHER ACTIVITY	**PUPIL ACTIVITY**	
INTRODUCTION 1. Apply the addition and Multiplication rule 2. Tree diagrams	Tr. a) For exclusive events A and B $P(A \text{ or } B) = P(A) + P(B)$ b) For independent events A and B $P(A \text{ and } B) = P(A) \times P(B)$	Ask questions and take notes	5
LESSON DEVELOPMENT Example 1. A bag contains 5 red balls and 3 green balls. A ball is drawn at random and then replaced. Another balls drawn. What is the probability that both balls are green? Answers Draw a tree diagram  Note (Replacement) Multiply the fractions on the two branches. $\therefore P(\text{both green ball(s)}) = P(G) \times P(G)$ $= \frac{3}{8} \times \frac{3}{8}$ $= \frac{9}{64}$	Give two examples to the learners and able them to participate in finding the solution	Ask questions and take notes	10

Figure 30: Response P 27.

APPENDIX 8.5: Response in the SMKDT AN

Teacher 13

QUESTION TWO

a) Mutually exclusive events are events which can never occur at the same time. Addition law is applied in combining two or more events that are mutually exclusive i.e If A and B are mutually exclusive, then P(A) and P(B) is given by P(A) + P(B). Example of mutually exclusive events is flipping a coin, it can not land both heads and tails simultaneously.

b) Figure 1
Set A and B are not mutually exclusive because of the intersection of A and B. In definition of mutually exclusive events, two events can never occur at the same time. But in figure 1 we are seeing A and B occuring at the same time.

Figure 2.
A and B are mutually exclusive because $A \cap B = \emptyset$ i.e A and B are disjoint sets.

) Figure 1

$$P(A \cap B) = P(A) \times P(B)$$

Figure 2

$$P(A \cup B) = P(A) + P(B).$$

Figure 31: Response P 28.

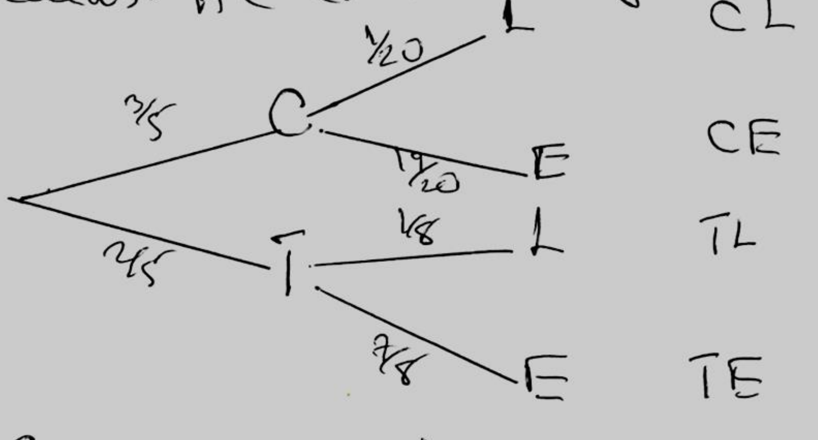
APPENDIX 8.6: Response in the SMKDT
Teacher 16

7(a) Compound events are those that can happen at the same time.

For instance Kayombo wants to draw money at the ATM machine. He can use either cardless through phone or insert the card.

He can either draw using cardless and succeeds or use the card and succeeds. He can still fail to draw.

(b)



$P(CL \times CL \times CL \times CL)$

$= P(CL) \times P(CL \times P(CL) \times P(CL)$

$= \left(\frac{3}{5} \times \frac{1}{20}\right) \times \left(\frac{3}{5} \times \frac{1}{20}\right)\left(\frac{3}{5} \times \frac{1}{20}\right)\left(\frac{3}{5} \times \frac{1}{20}\right)$

$= \frac{3}{100} \times \frac{3}{100} \times \frac{3}{100} \times \frac{3}{100}$

$= \frac{81}{100000000}$

Figure 32: Response 29

APPENDIX 8.7: Response in the SMKDT
Teacher 4

4(b) A child may begin relating the results of one patient to the other
But in the real sence, the results on one patient have no effect on the second patient being tested.

Figure 33: Response P 30.

APPENDIX 8.8: Response in the SMKDT
Teacher 1

3 (c) mutually exclusive events are combined using addition while independents events are combined using multiplication.

Figure 34: Response P 31.

APPENDIX 8.9: Response in the SMKDT
Teacher 17

Dependent events are events that influence one another. The outcome of one influences the other, and these events are mutually exclusive.

Example: If you roll a die, you cannot get a 5 and a 3 at the same time. For any two events that are dependent A and B, then
$P(A \text{ or } B) = P(A) + P(B)$.

Figure 35: Response P 32.

APPENDIX 8.10: RECORDS OF WORK
Teachers 11, 15 and 8

MINISTRY OF EDUCATION, SCIENCE, VOCATIONAL TRAINING AND EARLY CHILDHOOD EDUCATION

SECONDARY SCHOOL

SCHEME OF WORK- MATHEMATICS DEPARTMENT

TEACHER: FOUR (4) SUBJECT: MATHEMATICS GRADE: 12D YEAR: 2017

WEEK BEGINNING	TOPIC/CONTENT	METHODOLOGY AND TEACHING AIDS	REFERENCE	H.O.D'S COMMENTS	HEAD TEACHER'S COMMENT
To 10/02/17	- Volume Volume of Prisms - Solving Problems Involving area and volume · Review Test	· Models of solids · Charts · Nets of solids	· Maths 11 PSBK Pg 134-172 · Success 12 Exam Pg 14-18		
13/02/17 To 3/03/17	PROBABILITY · Laws of Probability · Tree Diagrams and grid · Probability of sets · Review Test.	· Demonstration · Guided discovery · Pair group · Coin, dice · Charts of tree diagrams	· IGCSE Pg 404-417 · Maths 11 Pg 307-320 · Progress in maths 11 Pg 147-158		

RECORDS OF WORK

TEACHER 15
SUBJECT Mathematics TERM III YEAR 2019 CLASS 11A

DATE	WORK DONE	EVALUATION	REFERENCES	TEACHER'S COMMENTS
14/10/19 to 18/10/19	Probability Combined events - Exclusive events	- Class exercise - Group work - Home work	- Success in Examination Mathematics Syllabus D JF Talbert Page 143-150	- 100% of the learners did the class exercise correctly - Good participation by learners during lesson development
	- Independent events	- Group work - Class exercise - Home work		

HOD COMMENTS

HEADTEACHER COMMENTS

SECONDARY SCHOOL

LESSON PLAN FORM

DEPARTMENT: Mathematics
NAME: TEACHER: 11 CLASS: 11A
SUBJECT: Mathematics DATE: 16/10/2019
CLASS SIZE: 68 DURATION: 80 minutes
NO OF PUPILS PRESENT: 64
TOPIC: Probability Period 3 & 4
LESSON OUT COME(S): Calculate probabilities which are exclusive event in nature
PRE-REQUISITE KNOWLEDGE: - Presentation of single probabilities
- Addition and Subtraction of fractions
KNOWLEDGE: - Proper fractions, decimals
LIFE SKILLS: - Identify, Calculate, Investigate, determine
VALUES: logical thinking, Abstract thinking, problem solving

Duration/ Time	Part	LESSON CONTENT	Methodology	Learners' Activity	Reference T/L Aid
5min	Introduction 1	- Recap on the definition of probability - $P(A) = \frac{\text{required outcome}}{\text{total possible outcomes}}$ and can be presented as a decimal or a fraction (in its lowest term).	- Question and Answer - Learner Centred	- Respond to the Question	
	lesson development	Exclusive events: If a coin is tossed once, is it possible for the coin to land on the edge? Expected answer: NO	Demonstration	A learner tosses a coin and other observe	
30min	2	If a coin is tossed once, is it possible for the coin to land on both sides? Expected answer: NO	Demonstration		

Figure 36: Response P33

APPENDIX 8.11: LESSON PLAN

Teacher 15

BOARDING SECONDARY SCHOOL

MATHEMATICS DEPARTMENT

LESSON PLAN

NAME OF THE TEACHER:

DATE:

SUBJECT: MATHEMATICS

DURATION: 80 min

TOPIC: PROBABILITY

LESSON: LAWS OF PROBABILITY

CLASS: 12 M2

NUMBER OF PUPILS: G.... B.....

REFERENCES: PROGRESS IN MATHEMATICS

TEACHING MATERIALS: PIECES OF CHALK, BLACK BOARD, DUSTER AND CHARTS

VALUES: *curiosity* in using laws of probabilities andcritical and logical thinking

PRE-REQUISITE KNOWLEDGE: ADDITION AND MULTIPLICATION LAWS

SPECIFIC OUTCOMES: DISCUSS VARIOUS TYPES OF COMPUTERS

LEARNING PROCESS

STAGE/ DURATION	TEACHING/LEARNING ACTIVITIES	METHOD/ STRATEGIES	PUPIL ACTIVITY
INTRODUCTION 10 MIN	- Probability is the likelihood of any outcome - Trial is one performance of an experiment - The total number of possible outcome of a trial form a sample space - Outcome is a result of an experiment - Event is one particular outcome in which we are interested - Probability is always expressed as a number from 0 to 1	Discussion Question and Answer	Discuss with the teacher
DEVELOPMENT 30 MIN	- $If\ A\ is\ one\ possible\ outcome\ of\ a\ trial,\ then\ the\ probability\ of\ A\ is\ denoted\ by\ P(A) and\ is\ given\ by:$ $P(A) = \frac{Number\ of\ outcome\ favourable\ to\ A}{Total\ number\ of\ possible\ outcomes\ in\ in\ the\ trial}$ $If\ A\ and\ B\ are\ the\ only\ possible\ outcomes\ of\ a\ trial\ then\ P(A) + P(B) = 1$	Discussion Question and Answer	Respond to the teacher as he asks and ask questions where not clear

Figure 37: Response P34

APPENDIX 8.12: LESSON PLAN

Teacher 7

STAGE/ DURATION	TEACHING/LEARNING ACTIVITIES	METHOD/ STRATEGIES (Or Techniques)	PUPIL ACTIVITY
	Independent Events		
INTRODUCTION The teacher wrote a question which learners were supposed to answer.	1. A man goes to work either by car or by train. On any particular day the probability that he goes by car is $3/5$ while the probability that he goes by train is $2/5$. If he goes by the car the probability that he will be late is $\frac{1}{20}$ while the probability that he will be late if he goes by train is $\frac{1}{8}$. If he goes by car on four (4) successive days, what is the probability that he will be late?		The learners to help to find the answers.
DEVELOPMENT The teacher with the learners to identify and list the concepts involved in this question	Identification of concepts in the question: - Concept of independence - Concept of certainty - The days occurred independently to each other - which means that each day occurred independently without any influence from the other. - The day he reached late (L) - The day he did not reach late (N) - The teacher was to go for work four (4) consecutive. - Is day he was going to reach late or not known?	• From the • First day: the probability could be L or N $\therefore$ L, L, L, N; L, L, N, L; L, N, L, L; N, L, L, L Concept of certainty $P(A)+P(B)=1$ $P(L)+P(N)=1$ $\frac{1}{20}+P(N)=1$ $P(N)=1-\frac{1}{20}=\frac{19}{20}$ $\therefore$ L, L, L, N $\frac{1}{20}\times\frac{1}{20}\times\frac{1}{20}\times\frac{19}{20}=\frac{19}{160000}$ $4\times\frac{19}{160000}=\frac{19}{40000}$	The teacher to assist learners to identify and list the concepts attached to the probability events. The teacher asked learners to give the fractions that they could use to find the probability of reaching late at the place of work?

Figure 38: Sketch 35

APPENDIX 8.13: LESSON PLAN

Teacher 2

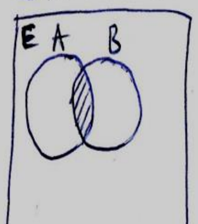
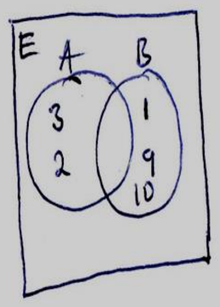
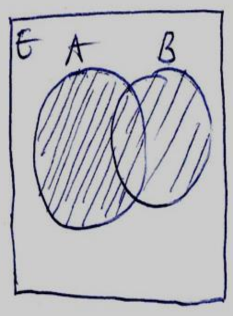
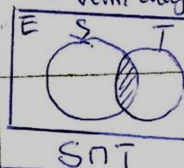
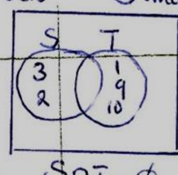
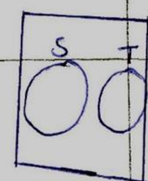
<table>
<tr><th>STAGE/ DURATION</th><th>TEACHING/LEARNING ACTIVITIES
MUTUALLY EXCLUSIVE EVENTS</th><th>METHOD/ STRATEGIES (Or Techniques)</th><th>PUPIL ACTIVITY</th></tr>
<tr><td>INTRODUCTION
The teacher asked learners to use the symbols ∪ and ∩ to explain on non and mutually</td><td>SHADING VENN DIAGRAMS

A∩B</td><td>Teacher exposition
Elements that are found in set A are also found in set B. This is a joint set.</td><td>Pupils to give answers</td></tr>
<tr><td>DEVELOPMENT
The teacher to explain using the venn diagrams</td><td>
A={2, 3}
B={1, 9, 10}
A∩B=∅

A∪B</td><td>P(A∪B)=P(A)+P(B)
∴ When choosing a elements which are either a member of set A or B involves different events which cannot happen together, then such cases requires adding of seperate probabilities to have the combined probability of a disjoint sets of A and B.
⇒ However, the symbol "∪" can be used to describe the expression
P(A∪B)=P(A)+P(B)</td><td>Learners to listen attentively as the teacher explains</td></tr>
</table>

Figure 39: Response P36

APPENDIX 8.14: LESSON PLAN
Teacher 17

STAGE/ DURATION	TEACHING/LEARNING ACTIVITIES	METHOD/ STRATEGIES (Or Techniques)	PUPIL ACTIVITY
INTRODUCTION The teacher to explain how to use the symbols ∪ and ∩.	INDEPENDENT EVENTS Product law and Shading of Venn diagrams. 1. If events A, B, C --- are independent, the probability of A and B and C and --- happening is the product of their individual probabilities $p(A) \times p(B) \times p(C) \times ---$ The Venn diagram shows sets S and T		Learners to provide answers.
DEVELOPMENT The teacher to allow learners provide answers as the lesson progress.	 $S \cap T$ $S \cap T$ This means that sets S and T are non-mutually exclusive events, i.e. jointed sets, then $P(S \cap T) = \{a, b\}$. ∴ For any pair of intersecting sets $\frac{n(S \cup T)}{n(\mathcal{E})} = \frac{n(S)}{n(\mathcal{E})} + \frac{n(T)}{n(\mathcal{E})} - \frac{n(S \cap T)}{n(\mathcal{E})}$ $\Rightarrow P(S \cup T) = P(S) + P(T) - P(S \cap T)$ ∴ For any pair of non-intersecting sets. - The task of choosing a letter which is either a member of S or T involves seperate events which cannot happen together, which excludes the other and these are mutually exclusive events. In such cases the seperate probabilities are added to give a combined probability of mutually exclusive or disjoint sets.	 or  $S \cap T = \emptyset$ $S \cap T = \emptyset$ ∴ This means sets S and T are mutually exclusive events, ie disjoint, $S \cap T = \emptyset$ and $P(S \cap T) = 0$	A learner to demonstrate what she/he understands about joint and disjointed sets. The Teacher to explain on the concept of Mutually and non-Mutually exclusive events.

$\Rightarrow P(S) + P(T) = P(S \cup T)$

Figure 40: Response P37

APPENDIX 8.15: Response in the SMKDT

15

Q8	F_1	F_2	F_3	F_4	
Number Acc	0	1	2	3	
Probability	0.94	0.03	0.02	0.01	
$Row_1 \times Row_2$	0.00	0.03	0.04	0.03	} Expected frequencies

a) i) Exp Accidents = 0.00 + 0.03 + 0.04 + 0.03

= 000.1

ii

c) Accidents in 50 periods = 0.1 × 50 = 5

Figure 41: Response P 38

APPENDIX 9: Suggested corrections made during piloting of the drafted SMKDT

ANSWERS

Probability is the numerical value that is assigned to the likelyhood of an event happening or occuring with a certainity (obvious) event having a numerical value (probability) of 1 and an impossibility of having the numerical value (Probability) of 0 i.e. $0 \leq P(A) \leq 1$ for any event A.

(b) $P(T \& 6)$ or $(6 \& T)$ Possible way of asking: What is the probability of getting a tail and 6 if a coin and a dice are tossed at the same time?

$\left(\frac{1}{2} \times \frac{1}{6}\right) + \left(\frac{1}{6} \times \frac{1}{2}\right)$

$\frac{1}{12} + \frac{1}{12} = \frac{2}{12}$.

$\frac{1}{6}$

(c) A independent event is an event that can occur or not occur without considering other situations around it while dependent events can happen (occur) or not happen (occur) as others situations around it are considered.

Q2(a) An Example of mutually exclusive events could be; light and darkness

(b) In figure 1 A and B are not mutually exclusive because it is possible for an element to belong to both at the same time while in figure 2 A and B are mutually exclusive because if an element belongs A, it cannot also belong to B as the two sets are disjoint.

i.e. In figure 1 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ while
" 2 $P(A \cup B) = P(A) + P(B)$

(c) Figure 1, its dependant probability while in figure 2 its mutually exclusive.

Q3(a) Compound events occurs where there is both mutually exclusiveness and independence. eg. if two coins are tossed of different colours red and blue, what is the probability of getting different outcomes.

$\therefore$ P(diff. out-comes) = [(Tail on Red) and (Head on blue)] or [Head on Red and Tail on blue]

(b) (BEST WAY): A man goes to work either by car or by train. On any particular day the probability that he goes by car is $\frac{3}{5}$ while the probability that he goes by train is $\frac{2}{5}$. If he goes by car the probability that he will be late is $\frac{1}{20}$ while the probability that he will be late if he goes by train is $\frac{1}{8}$. If he goes by car on four (4) successive days, what is the probability that he will be late?

(Q) (i) Learners would find it difficult if not well explained I the mode of transport to be used is supposed to be subjected t which will attract a probability of it not being picked to be used o particular day.

(ii) If a certain mode of transport is used there is also a probability of being late ie for a car it is $\frac{19}{20}$ and for a train it is $\frac{7}{8}$.

(iii) If a certain mode of transport is used successivelly for 4 days then; on any 3 days, the day when he will not be late is not specified so, combinations should be considered ie.

(L - late, N - Not late) LLLN; LLNL; LNLL; NLLL — 4 combinations whose probabilities are just the same.

Therefore a learner is suppose to apply the concept of combinations and also concept of certainity which gives the probability equal to 1.
ie. Late + not late = 1

Q4.(a) ∴ What is the probability that two patients selected at random one after the other:

(i) $p(\text{one} + \text{and the other} -) = 0{\cdot}6 \times 0{\cdot}4 + 0{\cdot}4 \times 0{\cdot}6$
$0{\cdot}24 + 0{\cdot}24 = 0{\cdot}48$.

(ii) $P(\text{both} -) = (0{\cdot}4 \times 0{\cdot}4) = 0{\cdot}16$.

(b) Learners may misunderstand that positivity and negativity are mutually exclusive so, if there is a chance (probability) of one of them, then there is also a chance of the other therefore positive + negative = 1
If two patients are selected at RANDOM, then there was the starting of one then the other one to have two. (combinations of +ve and −ve).
ie: (positive & negative) or (negative & positive).

Q5. NOTE: The stopping of the arrow will depend on the ~~Size of the angle~~ size of the angle in the sector ie. A sector with a bigger angle will have a higher probability.

(Suggestion) A wheel divided into six sectors of the same size coloured differently is spun, with an arrow pinned at the centre as shown in the diagram below; when the wheel stops, what is the probability that the arrow will point at Green or Blue?.

ANS: $P(G \text{ or } B) = \frac{1}{6} + \frac{1}{6} = \underline{\underline{\frac{1}{3}}}$

* $\frac{2}{6} + \frac{1}{6} = \frac{1}{2}$ is wrong because the pupil is assuming that the sector coloured Green is larger than the one coloured Blue which is not specified in the question (meaning; $P(G) = \frac{1}{3}$ and $P(B) = \frac{1}{6}$)

APPENDIX 10: Drafted Subject Matter Knowledge Diagnostic Test (SMKDT)

APPENDIX 3:

MATHEMATICS SUBJECT MATTER KNOWLEDGE ASSESSMENT TEST (SMKAT)

RESEARCH TITLE: Teachers' Pedagogical Content Knowledge of Probability: The case of teachers of Mathematics in Chongwe District, Zambia.

Dear Respondent,

The researcher for this study is a Post graduate student at the University of Zambia pursing a Master's degree in Mathematics Education.

You are therefore kindly requested to participate in this research by answering the following questions.

Be assured that the information you will provide will be confidential and will be used for academic purposes only. Kindly do not write your name on this paper, thanking you in advance.

SECTION A

1. Date.................................
2. Name of School.....................................
3. Sex: Male { }, Female { } tick in the box
4. Years in the service
   in the box.

   10 [ ], 11 to 15 [ ], 16 to 20 [ ] tick

SECTION B

INFORMATION

This test instrument has only one section.

The questions in the instrument are based on Probability as prescribed in the revised Zambian Secondary School Mathematics Curricular.

Answer all the 12 questions on the sheets provided.

You have a maximum of 3 hours to answer the questions in the test instrument.

You are allowed to use a calculator to answer the questions.

Show all the necessary calculations on the answer sheets and number your answers correctly.

Total marks: 68 Marks

PROBABILITY QUESTIONS

1. (a) Define the term probability as you could to the secondary school pupils. [2]

(b) Find the probability of getting a tail and a 6, if a coin is tossed once and a dice is rolled once. [3]

(c) Explain how you understand the concept of independent and dependent events with the help of examples [3]

2. (a) Give an example of a mutually exclusive event. [3]

(b) Consider the figures 1 and 2 below. For each of the figures state whether set A and B are mutually exclusive or not. Justify your answers. [3]

Figure 1

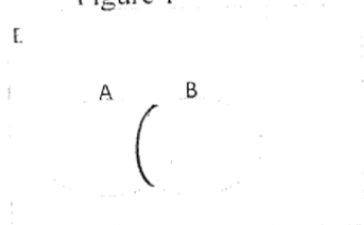
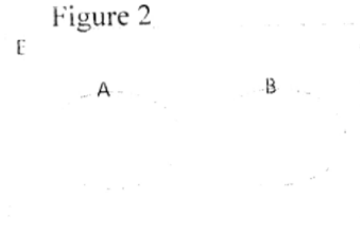


Figure 2



(c) Hence or otherwise state their associated probabilities for figure 1 and 2 [3]

3 (a) Give an illustration of compound events as you could teach learners in a lesson. [2]

(b) A man goes to work either by car or by train, If he goes by car, the probability that he will be late is 1/20, while if he goes by train, the probability is 1/8. If he travels by car for 4 successive days, what is the probability that he will be late

i. Every day...? [1]
ii. On any three of the days? [1]

(c) In your own view, what Mathematical concepts that the learners might use to answer the above questions 3b (i) and (ii) [3]

4 (a) A survey which was carried out at a certain hospital indicates that the probability that a patient tested positive for malaria was 0.6.

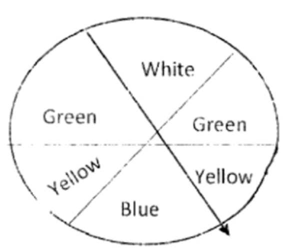
What was the probability that the two patients selected at random

i. One tested negative and the other one tested positive [2]

ii. Both patients tested negative. [3]

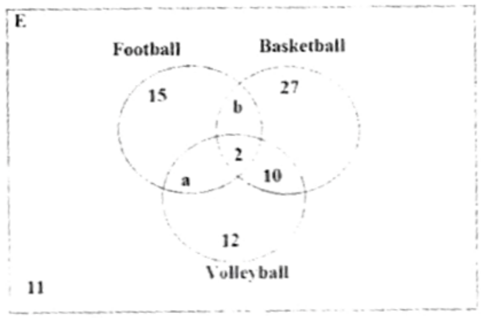
(b) Explain how learners are likely to misunderstand the concepts you used to solve question 4a (i) and (ii). [3]

5. A wheel of different colors is spun. When the wheel stops what is the probability that the arrow will point at Green or blue.



3

10. A survey was carried out on 100 pupils at Mwimba high school to determine their participation in sports. The results are shown in the venn diagram below.



(a) Given that 42 pupils play volley ball, find the values of **a** and **b. [2]**

(b) If a pupil is selected at random from the group, what is the probability that the pupil

i. Does not play football [2]

ii. Plays volleyball or football but not both [2]

(c) What is the probability that the pupils either participates in only one of the sports or not involved at all? [3]

(d) What is the probability that a pupil who plays volleyball also plays basketball but does not play football? [3]

Adapted from ECZ, 2016, 2017, 2018 final Grade 12 question papers and Malambo 2015.

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APPENDIX 11. Prepared Rubric for the Subject Matter Knowledge Diagnostic Test (SMKDT)

Test questions which assessed mathematics teachers' subject matter knowledge of probability and their ability to use multiple pedagogical strategies related concepts.

Key concepts being assessed on teachers of mathematics: Teachers' ability to	Question No. in a test	Marks
Provide a valid definition of probability using socially agreed conventional meanings of Probability terms such as likelihood, certain, impossible, likely, unlikely.	1(a)	2
To use correct concepts and procedure to solve probability problems correctly.	1(b)	3
Clear explanation of how they understand the concept of an independence and dependence events with justified examples	1(c)	3
To give and explain the meaning of mutually exclusive events with justified examples.	2(a)	3
State, explain and justify their views on whether set A and B are mutually exclusive or not	2(b)	3
Use of basic knowledge concepts of joint and disjoint sets and laws of probability to formulate the associated probabilities	2(c)	3
To use real life situation to illustrate (Presentation of) a compound events in a comprehensive way to the learners	3(a)	2
Identification of the concept of combination and also the concept of certainty which gives the probability equal to 1 and solve the probability correctly	3(b)	3
To anticipate (predict) common errors that are likely to be committed by learners. Understanding of the concepts that can be used to solve the given question correctly learners are likely to committee.	3(c)	3
Correct presentation of concepts of independence and dependence on the tree diagram to solve combined events	4(a) (i) (ii)	2, 3
Knowledge of probability challenges learners may encounter	4 (a) (i) (ii)	3
To provide clear explanation and justification of their views on the given answer	5(a)	3
Computation of probability using the probability grid. Solve probability related concepts using a probability grid	6(a) (i) (ii) (iii)	3
To Present the given information on a tree diagram and solve the following	7 (a)	1

questions correctly.		
Correct Interpretation of tree diagrams to solve compound probabilities/ events.	7 (b) (i) (ii) (iii)	3
Calculate and interpret the occurrences of accidents in relation to the probability scale which ranges from the 0 and 1	8(a)	2
Calculation of the expected probabilistic accidents using E.F (A) = $\frac{a}{b}$ x N	8(b)	1
Identification and computation of non-conditional combined concepts (events), independent and dependent events.	9(i)	4
Recognition of the dependence condition (dependent probability)	9(ii)	2
Use of basic concepts of sets to solve probability concepts and other related concepts: P (AUB) = P (A) + P (B) and P (AUB) = P (A) + P (B) – P(∩B)	10 (a), (b), (c) and (d)	4
Use of the addition rule for not and mutually exclusive events: P (A or B) = P (A) + P (B) and P (AUB) = P (A) + P (B) – P (A∩B). Use of multiplication rule for independent events: P (A∩B) = P (A) x P (B)	11 (a) (i)	4
To provide a clear explanation and justification of their views on the mutually exclusive events in relation to the concepts of sets.	11 (ii)	2
Use of basic concepts of sets to calculate probability related concepts of independence, dependent, compound mutually exclusive and conditional events	12	11
Expressing probability terms into a algebraic expression (1-n)	13	2
Recognizing differences between mutually events and independent events	14	2
Use of basic knowledge of sets and laws of probability to solve probability problems of mutually exclusive events: P (A or B); = P (A) + P (B) using the addition law of probability.	16	3
To provide correct definition of the term sample space in the process of choosing letter B from the word PROBABILITY	17(a)	3
To construct a Probability Grids and find a pair of even numbers	17(b)	2

APPENDIX: 12; Consent Form

I……………………………………………………….. (Your name), agree/do not agree (delete what is not applicable) to participate in a research entitled: **Teachers' Pedagogical Content Knowledge of Probability: The case of teachers of mathematics in Chongwe district, Zambia**. I understand that I will be writing a mathematics test based on Probability a secondary school topic for a maximum of 3 hours at a convenient venue and time within Chongwe district resource center. I understand that I may be asked to present a specific concept in Probability, a document analysis on my lesson plans, scheme of work and record of work and participate in a semi structured interview, which will be audio recorded and transcribed for analysis. The observation lesson will take place in a classroom and no one else but the learners who take part in the discussion and myself will be present during this discussion. The entire discussion will be tape-recorded, but no-one will be identified by name on the tape. The tape will be kept away from others. The information recorded is confidential, and no one else except myself and my supervisor **Dr. Malambo, P**. will have access to the tapes. The tapes will be destroyed after eight weeks. I also understand that my results in the said mathematics test, lesson observation and subsequent interviews will only be used for purposes of this research and they will in no way affect my performance in my teaching profession. I further understand that the researcher will not provide any information, during and after data collection, which will directly identify the results with me.

I also understand that the research subscribes to the principles of:

1. **Voluntary participation in research**; which implies that participants have a right to withdraw from research at any time.
2. **Informed consent**; which entails that participants must be fully informed at all times in respect of the purpose of the study and must give consent to their participation in research.
3. **Safety in participation**; which means that respondents should not be put at risk or harm of any kind.
4. **Privacy**; which implies that confidentiality and anonymity of the respondents should be protected at all times.
5. **Trust**; This means that respondents should not be subjected to any act of deception or betrayal in the research process or its published outcomes.

Signature…………………………………………………Date…………………………

APPENDIX 13. Ethical Clearance – The University of Zambia (UNZA)

THE UNIVERSITY OF ZAMBIA

DIRECTORATE OF RESEARCH AND GRADUATE STUDIES

Great East Road | P.O. Box 32379 | Lusaka 10101 | **Tel:** +260-211-290 258/291 777
Fax: +260-1-290 258/253 952 | **Email:** ...za.zm | **Website:** www.unza.zm

Approval of Study

2nd September, 2019

REF NO. HSSREC: 2019-MAY-035

Mr. Sinzala Gift Muzamaya
The University of Zambia
School of Education
Department of Mathematics and Science Education
Box 32379
LUSAKA

Dear Mr. Sinzala,

RE: "TEACHERS'S PEDAGOGICAL CONTENT KNOWLEDGE OF PROBABILITY: THE CASE OF TEACHERS OF MATHEMATICS IN CHONGWE DISTRICT, ZAMBIA"

Reference is made to your resubmission. The University of Zambia Humanities and Social Sciences Research Ethics Committee IRB resolved to approve this study and your participation as Principal Investigator for a period of one year.

Review Type	**Ordinary /Expedited Review**	**Approval No. REF No. HSSREC: 2019-MAY-035**
Approval and Expiry Date	Approval Date: 2nd September, 2019	Expiry Date: 1st September, 2020
Protocol Version and Date	Version-Nil	1st September, 2020
Information Sheet, Consent Forms and Dates	• English.	To be provided
Consent form ID and Date	Version	To be provided
Recruitment Materials	Nil	Nil

There are specific conditions that will apply to this approval. As Principal Investigator it is your responsibility to ensure that the contents of this letter are adhered to. If these are not adhered to, the approval may be suspended. Should the study be suspended, study sponsors and other regulatory authorities will be informed.

Conditions of Approval

- No participant may be involved in any study procedure prior to the study approval or after the expiration date.
- All unanticipated or Serious Adverse Events (SAEs) must be reported to the IRB within 5 days.
- All protocol modifications must be IRB approved by an application for an amendment prior to implementation unless they are intended to reduce risk (but must still be reported for approval). Modifications will include any change of investigator/s or site address or methodology and methods. Many modifications entail minimal risk adjustments to a protocol and/or consent form and can be made on an Expedited basis (via the IRB Chair). Some examples are: format changes, correcting spelling errors, adding key personnel, minor changes to questionnaires, recruiting and changes,

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THE UNIVERSITY OF ZAMBIA

DIRECTORATE OF RESEARCH AND GRADUATE STUDIES

Great East Road | P.O. Box 32379 | Lusaka 10101 | Tel: +260-211-290 258/291 777
Fax: +260-1-290 258/253 952 | Email: director@drgs.unza.zm | Website: www.unza.zm

and so forth. Other, more substantive changes, especially those that may alter the risk/benefit ratio may require Full Board review and approval. In all cases, except where noted above regarding subject safety, any changes to any protocol document or procedure must first be approved by the IRB before they can be implemented.

- All protocol deviations must be reported to the IRB within 5 working days.
- All recruitment materials must be approved by the IRB prior to being used.
- Principal investigators are responsible for initiating Continuing Review proceedings. Documents must be received by the IRB at least 30 days before the expiry date. This is for the purpose of facilitating the review process. Any documents received less than 30 days before expiry will be labelled "late submissions" and will incur a penalty.
- Every 6 (six) months a progress report form supplied by The University of Zambia Humanities and Social Sciences Research Ethics Committee IRB must be filled in and submitted to us. There is a penalty of K500.00 for failure to submit the report.
- The University of Zambia Humanities and Social Sciences Research Ethics Committee IRB does not "stamp" approval letters, consent forms or study documents unless requested for in writing. This is because the approval letter clearly indicates the documents approved by the IRB as well as other elements and conditions of approval.

Should you have any questions regarding anything indicated in this letter, please do not hesitate to get in touch with us at the above indicated address.

On behalf of The University of Zambia Humanities and Social Sciences Research Ethics Committee (IRB), we would like to wish you all the success as you carry out your study.

Yours faithfully,

Dr. Jason Mwanza
BA, MSoc, Sc., PhD
CHAIRPERSON
THE UNIVERSITY OF ZAMBIA HUMANITIES AND SOCIAL SCIENCES RESEARCH ETHICS COMMITTEE IRB

CC Director – DRGS
Assistant Director - DRGS

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APPENDIX 14: Permission Letter from Debs Office

All Communications Should Be Addressed
To The District Education Board Secretary
Telephone: +260 211 620 111
Fax: +260 211 620 111

in reply quote
no……………

REPUBLIC OF ZAMBIA
MINISTRY OF GENERAL EDUCATION

DISTRICT EDUCATION BOARD
P.O. BOX 33
CHONGWE

6th September, 2019

TO WHOM IT MAY CONCERN

RE: CONDUCTING RESEARCH: SINZALA GIFT MUZAMAYA-2017014395

The bearer of this letter is student perusing Masters in Mathematics Education at the University of Zambia.

The researcher has the permission to conduct his research in your school. This is a partial fulfilment of his course of study and is for academic purposes.

Kindly render to him all the support you may wish to provide.

Ruth C. M. Phiri (Mrs)
DISTRICT EDUCATION BOARD SECRETARY.
CHONGWE DISTRICT