NUMBER SENSE LEVELS OF IN-SERVICE PRIMARY SCHOOL TEACHERS IN CHIKANKATA DISTRICT, ZAMBIA

BY

EMADY HAMBALE

A dissertation submitted to the University of Zambia in partial fulfilment of the requirement for the award of the degree of Master of Education in Mathematics Education.

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DECLARATION

I, **Emady Hambale**, do hereby declare that this study entitled, '**Number sense levels of in-service primary school teachers in Chikankata District, Zambia**,' is my own work and that the sources I have used or quoted have been indicated and acknowledged by means of complete references.

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APPROVAL

This dissertation by Emady Hambale is approved as fulfilling part of the requirements for the award of the degree of Master of Education in Mathematics Education by the University of Zambia.

Examiner 1	Signature	Date
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Examiner 2	Signature	Date
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ABSTRACT

This study sought to assess number sense comprehension levels of In-service Primary School Teachers (IPST). This was a mixed method study where quantitative data was collected through a number sense assessment questionnaire. Out of 82 questionnaires, 69 were successfully completed and returned representing a return rate of 84%.Qualitative data was provided by six purposively selected teachers who were interviewed using questions based on number sense. The data from the questionnaire was analysed using Statistical Package for Social Sciences (SPSS) to come up with tables and charts and ideas of previous researchers on number sense were utilised to categorise the performance of IPST according to the four levels of number sense. One way ANOVA was used to determine if a significant difference in performance according to teaching experience period existed. To identify the strategies, each response from the interviews whether correct or wrong was coded as either number sensible or rule based. The theory of problem solving proficiency guided the study. The overall results of this study showed that IPST had low number sense with most of them scoring below the basic level when tested on number sense and no significant difference was observed on performance. Further, very few number sense based strategies were utilised and IPST were confident in answering questions that involved application of a rule. This study showed to what extent IPST possess the knowledge and skills that they are supposed to use for proficient teaching of numeracy. Based on the findings of this study, teacher preparatory institutions at primary school level to consider training teachers in ways that involves developing their number sense and that of learners.

Keywords*: Number Sense, In-service teachers, Numeracy*

DEDICATION

I dedicate this work to my late father Mr Cosmas Hambale and two brothers Cosmas junior Hambale and Julius Hambale.

To my wife Christine. P.M Hambale, daughter Patricia Hambale, My mother and siblings for the love and support I received throughout the study.

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ABREVIATIONS AND ACRONYMS

7th NDP	Seventh National Development Plan
CPD	Continuing Professional Development
DEBS	District Educational Board Secretary
ECZ	Examinations Council of Zambia
IPST	In-service Primary School Teachers
MOE	Ministry of Education
NAS	National Assessment Survey Report
NCTM	National Council for Teachers of Mathematics

CHAPTER 1

INTRODUCTION

1.1 Introduction

The main aim of this mixed method study was to assess in-service primary school teacher's competence levels in performing number sense activities. The study sought to determine in-service primary school teacher's proficiency levels in performing number sense activities as well as whether there were significant differences in performance on number sense activities according to teaching experience. The qualitative part explored the strategies used to solve number sense activities. Number sense is the foundation and the driver on which numeracy is built and developed and teacher knowledge of number sense is key to delivering quality numeracy programs (Courtney-Clark & Wessels, 2014; Naukushu, 2016).

1.2 Background

The government has been committed to developing literacy and numeracy skills in learners through investing more in primary education as compared to secondary and tertiary education (Ministry of Education (MOE), 2000). However, one major challenge that the education system faces is the low quality of education as evidenced by low performance results in the National Assessment Surveys. In 2012 for instance, learning achievement levels were low across all subjects with the poorest performance being in mathematics and science at both primary and secondary school level (NAS, 2012). According to Examinations Council of Zambia (ECZ) (2017) out of 305 563 candidates who sat for the examination at junior secondary School level, 657(0.22%) got zero in paper 1 and 23 967 (7.9%) got zero in paper 2. These poor learning achievement results suggest that pupils are not thinking mathematically and that they lack number sense.

According to Reys (1994) number sense is having an intuitive feeling for numbers and their various uses and interpretations, an appreciation for various levels of accuracy when computing, the ability to detect arithmetic errors and common sense approach to using numbers. Numeracy is much wider than number sense as it includes everyday mathematics that is needed to understand consumer finance, home management including a wide range of issues associated with mathematical and statistical analysis (Westerford, 2008). It is the ability to understand and use numbers, compute a range of numbers and performing the four basic operations of division, multiplication, addition and subtraction, understand the correct use of fractions and percentages and apply all skills in typical everyday situations in the home and in commercial contexts (MOE, 2013). Number sense is therefore the foundation and the driver on which numeracy is built and developed as it "encompasses important concepts, skills and attitudes that learners are supposed to acquire at primary school level as a foundation for further studies in mathematics and the development of quantitative literacy" (Courtney-Clark & Wessels, 2014, p.2).

The Ministry of Education (2013) has thus identified numeracy as one of the key competences that learners need to acquire at primary school level in order for them to be able to perform basic mathematical operations that are needed in school and everyday life. For children to be competent and apply the mathematics they learn in school, they need to experience it in a meaningful and worthwhile way as competence in mathematics is heavily dependent on appropriate and effective instruction and on opportunities to learn (Berch, 2005). Therefore, the teacher's role especially at the foundation phase cannot be underestimated since learners at this stage develop love for the subject or a fear of it (Ensor, Hoadley, Jacklin, Kuhne, Schmitt & Heuvel-Panhuizen, 2009). Mwanamonga (2016) explored strategies for teaching numeracy in primary schools in Zambia. The study revealed that teachers in primary school predominantly used teacher centred strategies such as demonstrations and exposition through drilling pupils towards mathematics solutions. Unfortunately number sense of pupils cannot be developed in this way but can only be developed if teachers believe that it is more important for pupils to make sense of the mathematics they learn than to master rules and algorithms (Reys, 1994). The overemphasis on standard written algorithms not only discourage pupils from developing number sense but also hinders the development of thinking and reasoning.

Research conducted by Hill (2008) on mathematical knowledge for teaching and the mathematical quality of instruction concluded that there is a powerful relationship between what a teacher knows, how it is known and what the teacher can do with it in terms of teaching. Further, learner performance is linked to teacher subject matter knowledge and that the lack of a sound foundation in the domain of numbers by teachers may be the root cause of low standards of learner performance in mathematics (Courtney-Clark & Wessels, 2014). Teacher knowledge of or awareness of number sense is key to delivering quality numeracy programs and has implications for the quality of learning that students achieve (Doriney, 2016).Therefore, teachers should be confident in their knowledge and understanding of how the number system works, use this understanding in flexible ways to make mathematical judgements and have various strategies for handling numbers and operations (National Council of Teachers of Mathematics, 1991). This means that teachers should have numbers sense themselves if they are to develop the number sense of their learners because one cannot develop number sense in others if one does not possess a certain measure of number sense oneself (Courtney-Clark & Wessels, 2014). If one is to act as an effective guide in an environment, it is imperative that the guide should be a comfortable resident of that environment (Greeno, 1991). It is against this background that this study seeks to assess number sense competence of in-service primary school teachers to ascertain if primary school teachers possess this important variable that is needed for proficient teaching of numeracy.

1.3 Statement of the problem

A strong conception of number and the quantity it represents is a critical part of all areas of daily life (Berch, 2005). Despite this, mathematical difficulties are widespread in Zambia. Performance of pupils in mathematics has remained unsatisfactory (ECZ, 2017; MOE, 2013). According to NAS (2012) Learning achievement levels were low across all subjects and the poorest performance was in mathematics and science at both primary and secondary school levels. These poor learning achievement results suggest that learners lack number sense which is a foundation on which numeracy is built and developed. Teacher knowledge of number sense is key to delivering quality numeracy programs and has implications for the quality of learning they achieve (Doriney, 2016).

Research has shown that learner performance is linked to teacher subject matter knowledge and that the lack of a sound foundation in the domain of numbers by teachers may be the root cause of low standards of learner performance in mathematics (Courtney-Clark & Wessels, 2014). However, it is not known whether Zambian primary school in-service teachers possess number sense which is required for proficient teaching of numeracy. Naukushu (2016) who assessed number sense of pre-service mathematics teachers noted that “it would be interesting to carry out a study to investigate the levels of number sense comprehension of practicing teachers” (p.271).

1.4 Purpose of the study

The purpose of the study was to assess in-service primary school teachers’ competence levels in performing number sense activities.

1.5 Objectives of the study

The study sought to achieve the following objectives:

1. To establish in-service primary school teacher's proficiency levels in performing number sense activities.
2. To determine if there is a difference in in-service primary school teacher's performance on number sense activities according to teaching experience.
3. To explore in-service primary school teachers' strategies when solving number sense activities.

1.6 Research questions

The research was guided by the following research questions:

1. How proficient are primary school teachers in performing number sense activities?
2. Is there a difference in primary school teacher's performance on number sense activities according to teaching experience?
3. What strategies do primary school teachers use to solve number sense activities?

1.7 Significance of the study

The study may help provide information to practicing teachers on the importance of incorporating number sense activities in the teaching and learning of numeracy in Primary schools in Zambia. The results of this study might also help in-rich literature by providing data on the number sense comprehension levels of practicing teachers.

Further, the information gathered through this study may provide teacher educators and curriculum developers with information on alternative ways of improving the teaching and learning of numeracy in Zambia and the results of this study might be useful as reference material to future researchers on number sense.

1.8 Limitations of the study

The limitation faced was that research participants were hesitant to take part in the study for fear that the number sense assessment results would be used against them by educational authorities. To mitigate this limitation, the researcher was patient with the participants and clearly explained to them that the study was only for academic purposes and all ethical issues were going to be observed.

1.9 Theoretical Framework

The study was guided by Schoenfeld (1985) theory of problem solving proficiency. Schoenfeld argued that in order to understand the success or failure of a problem solving attempt, one needs to know about the individual's: Knowledge base, problem solving strategies, metacognitive actions as well as their beliefs and practices (Schoenfeld &Kilpatrick, 2008). According to Schoenfeld (1985) "if one is interested in someone's mathematical proficiency, that is, what someone knows, can do and is disposed to do mathematically, then it is essential to consider all the four aspects of mathematical proficiency. Knowledge plays a central role, as a must but an individual's ability to employ problem solving strategies , the individual's ability to make good use of what he or she knows , and his or her beliefs and dispositions are also important" (p.71).

Knowledge base includes aspects of mathematical knowledge and ability, such as conceptual understanding and procedural fluency, that teachers need themselves and that they seek to foster in their students (Schoenfeld & Kilpatrick, 2008). The mathematical proficiency teachers need, however, goes well beyond what one might find in students. The students' development of mathematical proficiency usually depends heavily on how well developed the teacher's proficiency is.

Strategies involve one's ability to formulate, represent and solve mathematical problems flexibly (strategic competence). Knowing mathematics goes beyond being able to produce facts and definitions and executing procedures on command as this is not enough. But it is also important for one to be able to use the mathematical knowledge that they possess (Schoenfeld, 2007).

The third part of mathematical proficiency is metacognition (using what you know effectively). Reflecting on progress while engaged in problem solving, and acting accordingly (monitoring and self-regulation) is one aspect of what is known as metacognition. Broadly it about taking one's thinking as an object of enquiry (Schoenfeld, 2007). Effective problem solvers monitor how well they are making progress and persevere or change direction accordingly. Unsuccessful problem solvers tend to choose a solution path quickly and then persevere at it despite making little or no progress (Schoenfeld, 2013). What one does with the facts at his or her disposal determines to a large extent the success or failure of a problem solving attempt (Schoenfeld, 1985). Control deals with the way individuals use information potentially at their disposal, it focuses on major decisions about what to do in solving a problem, behaviours of interest include making plans and monitoring and assessing solutions as they involve and revisiting or abandoning plans when the assessment indicates that such actions should be taken (Schoenfeld, 1985).

The fourth part of mathematical proficiency is one's beliefs and dispositions. This is the productive disposition and habitual inclination to see mathematics as sensible, useful and worthwhile coupled with a belief in diligence and one's own efficacy (Schoenfeld, 2007).

In this study, the following constructs from the theory of problem solving proficiency were measured. Knowledge base, the teachers were assessed on proficiency in performing number sense activities by asking them questions based in the following number sense domains (Knowledge and facility with numbers, Knowledge of and facility with operations, applying knowledge of and facility with numbers and operations to computational settings). Strategic competence was assessed by exploring the kind strategies teachers used to solve number sense activities. Further, the theory helped to develop categories in analysing data, categories like control and confidence were drawn from the theory.

1.10 Conceptual Framework

Figure 1 shows the framework for assessing basic number sense.

1. Knowledge and facility with numbers	1.1 Sense of orderliness of numbers 1.2 Multiple representations for numbers 1.3 Sense of relative and absolute magnitude of number 1.4 System of benchmarks
2. Knowledge of and facility with operations	2.1 Understanding the effects of operations 2.2 Understanding mathematical properties 2.3 Understanding the relationship between operations
3. Applying knowledge of and facility with numbers and operations to computational settings.	3.1 Understanding the relationship between problem context and necessary computation 3.2 Awareness that multiple strategies exist
4. Beliefs and Dispositions	4.1 Confidence

Number Sense Proficiency

Figure 1: Shows the framework for assessing basic number sense.

Source: Adapted from McIntosh, Reys & Reys, 1992.

According to MacIntosh et al (1992) the framework in table1 articulates a structure which clarifies, organises and interrelates some of the general agreed upon components of basic number sense. The framework in divided into three number sense domains namely knowledge of and facility with numbers, knowledge of and facility with operations and applying knowledge of and facility with numbers and operations to computational settings. In each number sense domain, different number sense components are described.

1. Knowledge of and facility with numbers includes;

   Sense of orderliness of numbers. This requires the ability to understand how the Hindu-Arabic number system is organised, the system of place value including its application to whole and decimal numbers. Understanding rational numbers including how they are represented.

   Multiple representation of Numbers. This component entails an understanding of different contexts in which numbers may be represented such as symbols or graphical representations. Number sense encompasses the recognition that numbers take many forms and can be manipulated in many ways to benefit a particular purpose. The knowledge that numbers can be represented in many different ways as well as recognising that some representations are more useful than others in certain problem solving situations is variable and essential for developing the mathematical power(proficiency).

   Sense of relative and absolute magnitude. The ability to recognise the relative value of a number or quantity in relation to another number and the ability to sense the general size of a given number or amount is behaviour that develops with mathematical maturation and experience.

   System of benchmarks. This refers to the use of common anchors in our system which are often helpful in making judgements. Benchmarks may also evolve from personal attributes or encounters, for example a person who weighing 50kg may use this information in estimating the weight of another person.

2. Knowledge with and facility with operations. Key components of knowledge with and facility with operations include;

   Understanding the effect of operations. This implies understanding the effect of operations on various numbers including whole and rational numbers.

Understanding mathematical properties. This entails understanding commutative, associative and distributive laws.

Understanding the relationship between operations. Connections between operations provide more ways to think about and solve problems. In order to understand the relationship between operations it is essential to first understand each operation.

3. Applying knowledge of and facility with numbers to computational settings. This number sense domain includes the following components;

Understanding the relationship between problem context and necessary computation. This provides clues not only for appropriate operation but also for the numbers to be used in these operations, and whether an exact or approximate solution is appropriate.

Awareness that multiple strategies exist. Number sense involves recognising that different solution strategies often exist for a given problem. When an initial strategy appears unproductive, formulating and applying alternative strategy is an appropriate response. A child or adult with little number sense often uses a difficult method of calculation. This can be attributed to among other reasons a lack of confidence in alternative methods of calculation or a productive disposition to see mathematics as sensible or a lack of knowledge of other solution strategies.

Inclination to review data and result. When a solution is produced, people with good number sense examine their answer in light of the original problem to determine if the answer makes sense. This reflection is generally done quickly, naturally and becomes an integral part of the problem solving process. This metacognitive review of the problem context involves a reflection of the strategies that might have been used as well as an evaluation of the particular strategy selected, and finally a check to determine if the answer produced was sensible.

The in-service Primary school teachers were assessed in the three number sense domains to determine their level of number sense. An individual faced with a mathematical problem solving activity will have to draw on his or her knowledge base to come up with appropriate strategies to solve the problem, be able to monitor and regulate themselves while solving the problem to detect any arithmetic errors (metacognition). Further they also need the disposition to believe that mathematics makes sense and that it can be figured out while working on a mathematical problem as failure to do so would mean they lack number sense but success would entail that an individual is proficient in carrying out number sense activities.

According to Schoenfeld (1985) the success or failure of a problem solving attempt depends on how well the problem solver manages the resources at their disposal as this is a fundamental factor and if one wants to explain problem solving performance or teach it one must deal with: whatever mathematical information problem solvers understand or misunderstand and might bring to bear on the problem, techniques they have (or lack) for making progress when things look bleak, the way they use or fail to use the information at their disposal and their mathematical worldview which determines the way they use their knowledge.

All these aspects of mathematical proficiency will determine the success or failure in working on number sense activities described by McIntosh et al (1992). In fact McIntosh et al (1992) acknowledged the connection between number sense and problem solving proficiency and noted that interconnections within number sense components suggest a monitoring process which links number sense to metacognition as a person with good number sense thinks and reflects on results being produced. Number sense and mathematical proficiency develop together and as such, Number sense is important to the development of mathematical proficiency (Naukushu, 2016).The relationship between number sense and mathematical proficiency is that one cannot develop without the other (Griffins, 2003).

1.11 Definition of terms

- *Primary school teacher*: Refers to a qualified teacher who was trained to teach pupils in primary schools in Zambia.
- *Mental calculations* refers to calculations that are performed without the use of pencil and paper or calculator and do not involve the use of algorithms and are performed using known facts, derived facts and a variety of strategies (Courtney-Clark & Wessels, 2012)
- *Number sense* is an awareness and understanding about what numbers are, their relationships, their magnitude, the relative effect of operating on numbers, including the use of mental mathematics and estimation (Fennell & Landis ,1994)
- *Number sense proficiency* is the ability of individuals to understand the number sense concepts (Farrell, 2007).

- *Mathematical competence* comprises having knowledge of , understanding, doing, using and having an opinion about mathematics and mathematical activity in a variety of contexts where mathematics plays or can play a role (Niss & Hojgaard,2011)
- *Numeracy*: Everyday mathematics that is needed to understand consumer finance, home management, civic issues, employment conditions and benefits and media reports on a wide range of issues that arise from mathematical and statistical analysis (Westerford,2008).

1.12 Organisation of the Dissertation

Chapter 1 outlines the background of the study, the statement of the problem, the aim of the study, the objectives and research questions, significance of the study, limitations and delimitations, the theoretical framework and conceptual framework, and definition of terms used in this study.

Chapter 2 reviews relevant literature. The concept of number sense is described in detail followed by a description of characteristics that can be observed in individuals who possess number sense. Then the role of teachers in promoting number sense is described followed by a review of literature on studies that focused on number sense of teachers and brief description of the research gap for study is offered at the end

Chapter 3 presents the methodology used to collect and analyse data for this study.

Chapter 4 presents the study results on number sense levels of in-service primary school teachers. It begins with the demographic characteristics of the teachers. The study findings are presented according to research questions to make it easy for readers to follow.

Chapter 5 discusses the findings of this mixed method study. Firstly, a brief discussion of the performance of teachers in the three number sense domains is given followed by a discussion of results in each research question.

Chapter 6 provides the conclusion and recommendations based on the findings of this study.

CHAPTER 2

REVIEW OF RELATED LITERATURE

2.1 Introduction

This chapter reviews relevant literature on number sense. The first part provides a detailed description of the notion of number sense where different definitions of what is regarded as number sense are given together with the characteristics that can be observed in individuals who possess number sense. The second part brings out the teacher's role in promoting number sense followed by literature on teacher proficiency in number sense. A brief description of the research gap of this study is given at the end.

2.2 The notion of number sense

The term number sense is an elusive concept to define. Due to the complexity of the concept, there is lack of consensus defining number sense (Yang, Li & Li, 2008; Berch, 2005). Naukushu (2016) observed that no universally accepted definition of number sense exists but research in the area of number sense in the last two decades has refined and defined what can be understood as number sense. Pilmer (2008) argues that due to extensive work that has been conducted on number sense, many researchers on number sense have worked together on numerous projects, articles and teacher resources and thus it is not surprising that there are similarities in how different researchers define number sense. Pilmer (2008) notes that definitions only vary slightly but each serves to complement the definition proposed by other researchers. For instance, Hope(1989) described number sense as ability to make reasonable estimations about the various uses of numbers, being able to recognise arithmetic errors, being able to select the most effective computing method and being able to notice number patterns. Similarly, Number sense can also be defined as "a person's general understanding of number and operations, along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful and efficient strategies for managing numerical situations" (Mclntosh et al, 1992, p.61). Fennell & Landis (1994) on the other hand defined number sense as an awareness and understanding about what numbers are, their relationships, their magnitude, the relative effect of operating on numbers, including the use of mental mathematics and estimation. From the definitions given thus far, all the definitions emphasise a strong understanding of what numbers are together with the ability to use this understanding effectively in different numeric situations.

Other researchers defined number sense in terms behavioural characteristics that can be observed in individuals who possess number sense. Nickerson & Whitacre (2010) observed that individuals with good number sense tend to exhibit the following characteristics when computing mentally; sense making approach, planning and control, flexibility and appropriateness sense of reasonableness when computing. Reys (1994) argued that number sense like common sense can best be described by observing specific behavioural characteristics of those who value and use it. If a person has number sense he/she will; Look at a problem holistically before confronting details for example in adding $\frac{2}{3}+\frac{3}{4}+\frac{1}{3}$ a student might mentally re-order $\frac{2}{3}+\frac{1}{3}+\frac{3}{4}$ to take advantage of the compatible addends ($\frac{2}{3}+\frac{1}{3}$),look for relationships among numbers and operations and will consider the context in which a question is posed for example in buying 4 note books priced at k13 each, a person with k100 might reason that she or he has enough money to buy the 4 note books since 13 is less than 20 which goes five times into 100. Choose to invent a method that takes advantage of his or her own understanding of the relationships between numbers and operations and will seek the most efficient representation for a given task. For example, suppose that 75 percent of the class of 30 students needs to agree on a plan for a school trip before it can be finalised. A student might reason that 75% is 50% plus 25%, or half plus half of that so 15 + 8, or 23 students must agree, Use benchmarks to judge number magnitude. For example, $\frac{2}{5}$ of 49 is less than half of 49, recognise unreasonable results for calculations in the normal process of reflecting on answers. For example 3.2 x 4.8 cannot possibly be 153.3, since the answer must be about 3X5, or 15, so an error in decimal point placement must have been made (p.115).

Number sense is not a new topic that must be introduced in the curriculum for teachers to include in their already overcrowded mathematics program. Rather it is an important perspective from which to view learning mathematics (Reys, 1994).

Through studying and reflecting on literature associated with number sense, McIntosh , Reys & Reys (1992) came up with a framework that can be used to assess basic number sense (see appendix 3).The framework articulates a structure which clarifies, organises and interrelates some of the general agreed upon components of basic number sense (Reys & McIntosh, 1999).The framework contains three number sense domains which are; Knowledge of and facility with numbers, Knowledge of and facility with operations and applying knowledge of and facility with numbers and operations to computational settings. The domain of knowledge of and facility with numbers include a sense of orderliness of numbers, multiple representation of numbers, a sense of relative magnitude of numbers and a system of benchmarks. Knowledge of and facility with operations includes an understanding of the effects of operations on whole numbers, fractions and decimals. The ability to understand mathematical properties such as commutability, associativity, identity. It also includes the ability to understand the relationship between operations. The third number sense domain of application of knowledge of and facility with number and operations to computational settings includes an understanding of the relationship between problem context and necessary operation, the awareness that multiple strategies exist as well as the ability to review data and results for sensibility.

Researchers on number sense in the past proposed that number sense proficiency can be quantified by categorising scores on a number sense test in four categories that depict different numeric abilities of individuals (Reys & McIntosh, 2004; McIntosh et al ,1992).This idea of categorising scores was refined by Markovits and Sowder(2004) as follows;

- Advanced(75% and above)
- Proficient (60 – 74%)
- Basic (50 - 59%)
- Below basic (0 -49%)

An individual within the advanced level in terms of number sense possesses a very good / excellent understanding of the meaning and size of numbers, holds a very good/excellent ability to recognise forms of representing numbers, demonstrates an outstanding grasp of the meaning effect of operations with minimum or no problems, The individual is able to make very reasonable and accurate estimations and identifies very reasonable and accurate benchmark as a referent point of estimation. A person within below basic level generally straggles in most of the aspects of number sense described above (see appendix 4 for the rubric for interpreting levels of number sense).

In studying number sense proficiency of students aged 8 and 14 years in Austria, Sweden, United states and Taiwan, Reys & McIntosh (1999) identified six major components in which number sense play a key role and these include; understanding of the meaning and size of numbers, understanding and use of equivalent representations of numbers, understanding the meaning and the effect of operations, understanding and use of equivalent expressions, flexible computing and counting strategies and measurement benchmarks. The in-service teachers in this study were assessed in these six major components of number sense to determine their level of number sense proficiency.

2.3 The role of teachers in promoting number sense

Teachers play an important role in building number sense because the development of number sense depends on the type of classroom environment teachers create in the teaching practices they employ and in the activities they select (Tsao & Lin, 2011). Research has shown that what teachers know and believe about mathematics is directly connected to the instructional choices and procedures they follow in teaching (National Council of Teachers of Mathematics, 1991). Primary school teachers affect both the achievement and attitude of students in mathematics because of the important role they play in in the early mathematical environment for students (Tsao, 2005). Coutney-Clark & Wessels (2014) argued that learner performance is linked to teacher subject matter knowledge and that teacher's confidence in doing and teaching mathematics influences the way they teach. Therefore, the effectiveness of an education system depends heavily on the quality of its teachers as key persons in meeting the goals of the education system (Clement & Satama, 2007).

According to Reys (1994) number sense can only be valued by students only if teachers believe that it is more important for students to make sense of the mathematics they learn than to master rules and algorithms that are normally poorly understood. Number sense develops as a result of exploring numbers, visualising them in a variety of contexts and relating them in ways not limited by traditional algorithms (Pilmer, 2008). However, Mwanamonga (2016) investigated strategies for teaching numeracy skills in primary schools in Zambia and found that even though teachers made efforts to engage pupils through strategies like question and answer, group discussions to mention but a few, teachers mostly utilised teacher centred methods such as demonstrations and exposition by drilling pupils towards mathematical solutions. This raises questions about primary school teacher's ability to develop number sense of learners as the use of such methods makes it difficult for children to attain conceptual understanding once they have learnt rote procedures (Baroody, 2006). Once pupils have learnt rote procedures, these procedures become the most cherished methods of solving mathematical problems since they can be executed without thinking(McIntosh et al,1992) The characterisation of number sense imply that learners are better not taught computational techniques (such as how to add or subtract multi-digit numbers or a multitude of algorithmic acronyms of long division) as these reduce to algorithms of heuristic value that are normally poorly understood by pupils. Rather, meaningful use of numeric information within an authentic context is the essence of number sense and must be a significant consideration in the teacher's practice (Maclellan, 2012). This suggests that traditional teaching practices are not always conducive to developing number sense. Number sense is a by-product of teaching for understanding. Therefore, teachers should strive to ensure that students experience number rather than simply work with it from a peripheral point, they gain insight and meaning and make connections through the classroom environment and the activities they select (Pilmer, 2008)

In studying teacher proficiency in mental calculations for double-digit addition and subtraction in Zambia, Tabakamulamu (2010) conducted a quasi- experimental study to assess the extent to which teachers in early primary mathematics could adopt the use of strategies for mental calculations for double-digit addition and subtraction. Tabakamulamu (2010) found that the use of mental strategies changed teacher's existing beliefs about mathematics teaching and learning and to some extent their classroom practices to support the use of strategies for mental calculations. The study also revealed that pupils performed better in post-test compared to pupils in experimental schools. This shows the important role teachers plays in developing number sense of pupils in primary school mathematics. Teachers therefore, are required to help students strengthen their sense of number moving from the initial development of basic counting techniques to more sophisticated understandings of the size of numbers, number relationships, patterns, operations, and place values in order to strengthen their number sense (Sztajn ,2002).

2.4 Teacher's number sense proficiency

Content knowledge of teachers has implications for all students and the quality of learning they achieve (Evans, Wong & Newman, 2012). Teachers therefore must possess in-depth knowledge of the specific mathematics they teach as well as the mathematics that their learners will learn in the future since the teacher's subject matter has an indirect impact on learner achievement. Ball (1990) observed that for teachers to effectively teach mathematics at primary school level, they must possess in depth knowledge of primary school mathematics. Research conducted in the past on teacher knowledge of mathematics has shown that teachers do not possess adequate knowledge in the domain of numbers.

Courtney-Clark & Wessels (2014) explored the number sense of 47 final year pre-service teachers. This was a mixed-method study in which data was collected through a number sense questionnaire, a written computation and mental calculation questionnaire and the McAllen confidence in mathematics and mathematics teaching survey. Courtney-Clark & Wessels (2014) found a strong relationship between number sense and mental calculation; between number sense and confidence in both the ability to do and to teach mathematics and between mental and written calculations. Further, the overall results showed that pre-service teachers had limited number sense and therefore, they were not equipped to promote number sense in learners. Courtney-Clark & Wessels (2014) contended that the low number sense of teachers was one of the reasons for the low standards of performance of Namibian learners and the lack of improvement for the last decades. The study recommended that more studies in number sense should be conducted in developing countries facing similar challenges of low learner performance in mathematics and that mental calculations and computational estimation should become a focus of primary school education.

Similarly, Doriney (2016) conducted a mixed method study on pre-service primary school teacher's understanding of number sense which combined an error analysis using a chis-square test to assess association between error type and question type as well as type of error and cognitive domain. The study revealed that pre-service primary school teacher's number sense was low and that pre-service teachers made errors such as number selection, missing step, computation, operation, random and omission and computation errors. However, the study did not determine whether there were differences in performance among teachers in terms of teaching experience period. Akkaya (2015) found that number sense performance of secondary school students in Turkey differed significantly according to grade level and noted that there were differences in research findings on performance on number sense with regard to different grade levels by different researchers. Akkaya (2015) stressed that due to the differences in research findings, there was need to conduct more extensive research to explore whether number sense performance differs based on grade level (experience). This study explored whether there was a significant difference in performance according to teaching experience period to determine if experience in teaching would influence performance on number sense activities.

Sengul (2013) also found that number sense of pre-service teachers was low and when their solution methods were analysed, it was found that pre-service teachers preferred rule-based methods instead of number sense strategies such as estimation strategies, computation estimation strategies , rounding and mental computation which are among the fundamental components of number sense. Surprisingly Sengul (2013) found that pre-service teachers were using solution strategies similar to those that were used by students who lacked number sense. Sengul (2013) observed that by instructing pre-service teachers on the low levels of number sense possessed by students, pre-service teachers might be provided with experiences related to how students number sense abilities can be improved, how lesson plans can be prepared and the kind of activities that should be used to stimulate the development of number sense of learners. The study recommended that there was need for proper training programs for pre-service teachers in which mental calculation and estimation could be improved. Segul(2013) also indicated the need for studies on teachers metacognitive levels and their abilities to use number sense as well as classroom teacher's level of mathematical attitudes and concerns and their ability to use number sense components.

Yang, Reys & Reys (2007) conducted a study on strategies used by pre-service teachers in Taiwan. The study examined pre-service teacher's misconceptions and number sense strategies using a sample of 280 participants. Results of the study were that only one fifth of pre-service teachers were able to apply number sense based strategies such as using benchmarks appropriately or recognising number magnitudes. Majority of the pre-service elementary teachers relied on rule based strategies. Yang, Reys &Reys (2007) suggested that teacher education programs should put more emphasis on helping pre-service teachers break away from the inclination to using rule based written computation and broaden their ability to include number sense as an integral part of their teaching. Hinton (2011) investigated pre-service teacher's computational knowledge, efficacy and number sense. Data on pre-service teacher's understanding of numeric operations and their relationships was collected using an open ended questionnaire. The findings revealed that most participants relied on rule based strategies or could not give an explanation for the strategy they chose and only a few candidates relied on number sense strategies to solve mathematical problems. According to Hinton (2011) programs that prepared pre-service teachers with courses that target difficulties teachers experience in teaching and teach content that entail understanding numbers and number relationships were required.

Yaman (2015) examined the number sense performance of teacher candidates who were taking mathematics I and II courses. The study determined whether there would be a change in number sense performance of teacher candidates following Mathematics I and II courses. This was a mixed method study in which quantitative data showed that there was a significant difference in performance of teacher candidates on number sense tasks. The qualitative data indicated that prior to the two courses, pre-service teachers considered mathematics as a course involving operations. The teacher candidates were prone to using routine rules and algorithms and they lacked conceptual understanding of some of the mathematical rules and phrases they used and could not use number sense skills. The study recommended that lessons for teachers candidates should be planned in a way that makes pre-service teachers understand the conceptual aspects if educators want to prevent students from viewing mathematics as subject about procedures. The study also recommended the inclusion of number sense activities in mathematics I and II courses. Similarly, Naukushu (2016) conducted a study titled a critical theory in the development of number sense in Nambian first year pre-service Secondary mathematics teachers using a convenient sample of 60 pre-service teachers. The study established that the number sense of pre-service secondary school teachers was low before the intervention. The study also found that the critical theory had an impact as there were statistically significant differences in performance in the pre-test and post-test. Naukushu (2016) recommended the integration of number sense components in the school curriculum in Namibia at both Secondary and Primary school level to help learners understand mathematics better.

2.5 Research Gap

The literature review shows that most of the studies concentrated on assessing number sense of pre-service teachers, Naukushu(2016) noted that " it would be interesting to carry out a study to investigate the levels of number sense comprehension of practicing teachers" (p.271). In an attempt to fill this knowledge gap, this study assessed number sense of in-service primary school teachers in Chikankata District of Zambia.

CHAPTER 3

METHODOLOGY

3.1 Introduction

This chapter outlines the philosophical underpinnings of the study followed by the research design, research site, study population, sampling techniques that were used, data collection procedures, how data was analysed and how ethical issues were taken care of in this study and finally a summary of the methodology in form of a diagram is given at the end.

3.2 Philosophical underpinnings

Pragmatism guided this research since the researcher drew from both quantitative and qualitative approaches to collect and analyse data. Pragmatists look to many approaches to collect and analyse data and investigators use both quantitative and qualitative data because they work to provide the best understanding of the research problem (Creswell, 2003).

3.3 Research design

This was a mixed method study where an explanatory sequential design was used in which quantitative data was collected followed by qualitative data. "The rationale behind the approach is that quantitative data provides a general picture of the research problem; more analysis, specifically through collecting qualitative data is needed to refine, extend or explain the general picture"(Creswell,2012,p.542). In this design, weight is given to the quantitative data and mixing of data occurs when the initial quantitative results informs the secondary qualitative data collection (Creswell & Creswell, 2017). Researchers on number sense such as Yang (2008) and McIntosh et al (1992) argued that in assessing number sense, measurement should involve both qualitative and quantitative data. In this study, quantitative data was collected first on teacher proficiency levels on number sense followed by qualitative data on strategies teachers used to solve number sense activities. Quantitative data provided the general picture of level of number sense possessed by in-service primary school teachers while qualitative data helped explain the performance in the quantitative data. Integration refers to the stage or stages in the research process where mixing or integration of the quantitative and qualitative methods occurs (Cameron, 2009). For this study, mixing of quantitative and qualitative was done in the discussion of findings as well as at the initial stage where both quantitative and qualitative research questions were asked. For explanatory sequential designs, possibilities for mixing range from the beginning stage of the study while formulating its purpose and introducing both quantitative and qualitative research questions to the integration of the quantitative and qualitative findings at the interpretation stage of the study (Cameron, 2009).

3.4 Research site

The study was conducted in Chikankata District one of the rural Districts in Southern province of Zambia. According to Southern and Eastern Africa Consortium for Monitoring Education Quality (SACMEQ) (2010) pupils from Urban areas had higher scores than pupils from rural areas when they were tested on reading abilities and maths skills at grade 6 level.

3.5 Target population

According to Crotty (1990) a target population is an entire group of individuals, events or objects with observable characteristics. The target population for this study comprised of all primary school teachers in Chikankata District.

3.6 Sample Size

A subset of the population that researchers typically study is called a sample **(**Marczyk et al, 2005). Out of a population of 103 primary school teachers in Chikankata District, a sample of 82 teachers was used at $\alpha = 0.05$ significance level . According to Israel (1992) nearly the entire population would have to be sampled in small populations to achieve desirable levels of precision. For the qualitative part, 6 teachers were sampled from the 82 teachers who took part in the quantitative phase. Qualitative research values in-depth and detailed exploration and sample sizes are usually small (Bogdan & Biklen, 1982).

3.7 Sampling procedure

Simple random sampling is one in which each and every member of the population has an equal and independent chance of being selected (Fraenkel & Wallen, 2006) and it was used to sample teachers that answered the number sense questionnaire. In this study, the fishbowl draw method was used to randomly select the teachers. According to Kumar (2019) if the total population is small, an easy procedure is to number each element, put the slips into a box and then pick them one by one without looking until the number of slips selected equals the sample size decided upon. Purposive sampling (Homogeneous sampling to be specific) was used to select 6 teachers that were interviewed. Homogeneous Sampling is a purposive sampling technique that aims to achieve a sample whose units share the same or similar characteristics or traits **(**Marczyk et al, 2005).In this study, teachers were selected based on their scores on the number sense assessment test where two interviewees` were selected from each of the following categories (proficient, basic and below basic) to explore the kind of strategies used to solve number sense activities.

3.8 Research instruments

A test written as number sense questionnaire in free response mode adapted from Doriney (2016) was used to collect data on number sense proficiency levels in performing number sense activities. The number sense test consisted of 20 items belonging to one the following number sense domains; Knowledge of and facility with numbers, knowledge of and facility with operations and application to computational settings as described by the framework for considering number sense. The distribution of the questions in each of the three number sense domains was as shown in table 1.

Table 1: Distribution of the questions in each of the three number sense domains

Number sense Domain	Question number
Knowledge of and facility with numbers	2,8,10,11, 15,16,17,18,19,20
Knowledge of and facility with operations	1,5,6,7,9,12
Application to computational settings	3,4,13,14,

An interview guide adapted from Courtney-Clark & Wessels (2014) with 7 number sense questions based on the framework by McIntosh et al (1992) was used to collect qualitative data on the strategies in-service primary school teachers used to solve number sense activities. The distribution of the interview items in each number sense domain was as shown in table 2.

Table 2: Distribution of the interview items in each number sense domain

Number sense domain	Item number
Knowledge of and facility with numbers	1
Knowledge of and facility with numbers	2
Knowledge of and facility with operations	3
Knowledge of and facility with operations	4
Application to computational settings	5
Application to computational settings	6
Knowledge of and facility with numbers	7

3.9 Data collection procedures

Data were collected in two phases, the first phase involved collecting quantitative data on number sense proficiency levels of in-service primary school teachers. The number sense assessment questionnaires were taken physically to each of the respondents at the schools were they taught. The respondents were asked to complete the test in the presence of the researcher. Out of 82 questionnaires distributed, 69 questionnaires were successfully completed and returned representing a return rate of 84%. The second phase involved collecting qualitative data to explore the strategies in-service primary school teachers used to solve number sense activities. The interviews were conducted in quiet places (location) chosen by the interviewees where there were minimum disturbances. The interview items were displayed one by one using power point to ensure clarity on questions. The interviews were recorded using a recorder to ensure that information was not lost and to aid the researcher in analysis.

3.10 Validity and Reliability

Validity determines whether the research truly measures that which it was intended to measure (Joppe, 2000). This research study aimed at assessing the number sense levels of in-service primary school teachers, the instruments that were used in this study were adapted from previous experienced researchers on number sense. Further, a mixed method design was used to ensure that most of the measures of number sense were included in the study. The number sense questionnaire was piloted with 10 in-service primary school teachers and these were eliminated from the main study. The purpose of the pilot study was to detect difficult sentences, concepts, wordings including challenges in data collection. Through the pilot study, minor modifications were made for example, the researcher was able to adjust the length of the questionnaire from 30 questions to 20 and some of the questions which were not suitable to the Zambian context were removed and replaced with items from other researchers on number sense like Yang (2005), McIntosh et al (1999) for example question 15(See appendix 1).

3.11 Ethical Considerations

To ensure confidentiality, no names were used and informed concert was sought from the teachers. Also, their right to opt out of the research at any time was respected. Further, the researcher obtained clearance from the University of Zambia Ethical Committee and the researcher asked for permission from the educational authorities at the research site.

3.12 Data Analysis

The data was analysed using both quantitative and qualitative methods. The data from the questionnaire was analysed using Statistical Package for Social Sciences (SPSS) to come up with tables and charts, the responses were coded and summarised into cross tabulations tables to produce charts and percentages using SPSS. To determine the proficiency levels of in-service primary school teachers, their performance on the number sense assessment was grouped according to one of the following categories as proposed by McIntosh et al (1992), Reys & McIntosh(2002) and refined by Markovits and Sowder(2004) as follows:

- Advanced(75% and above)
- Proficient (60 – 74%)
- Basic (50 - 59%)
- Below basic (0 -49%)

One way ANOVA was used to determine if there was a significant difference on number sense performance according to teaching experience. Teachers were categorised according to years of experience in teaching as follows; 1-5 years, 6-10 years and 11 years and above. Qualitative data; to identify the strategies in-service primary school teachers used to solve number sense activities from the interviews, each response whether correct or wrong was coded according to one of the following categories:

- Number sensible- these strategies utilised aspects of estimation, using benchmarks, basic meaning of numbers, number magnitude, relative effects of operations on numbers and ability to judge reasonableness as described by the framework on number sense. For example in attempting the question: A minibus can transport 15 people. How many minibuses would you need to transport 170 people? A number sense based approach would involve mental decomposition and re-composition of numbers for example 170 = 150 + 20 = 10 x 15 + 1 x 15 + 15. Therefore, 170 ÷15 = 11 remainder 5. Considering the context of the question, 12 buses would be required.

- Rule based- these strategies utilise standard rules of standard written algorithms. For example in attempting the question: A minibus can transport 15 people. How many minibuses would you need to transport 170 people? A rule based strategy involves long division or any other paper and pencil algorithm.
- Control- this category was introduced to explain the way the participants monitored and regulated themselves during the solution process such as abandoning strategies that would not lead to a correct answer and choosing new solution paths so that they could get a correct solution. This is included the ability to review data and results for sensibility.
- Confidence category was used to identify the teacher's beliefs and practices in their own ability to solve mathematical problems.

Figure 2 shows a summary of the methodology used for this study

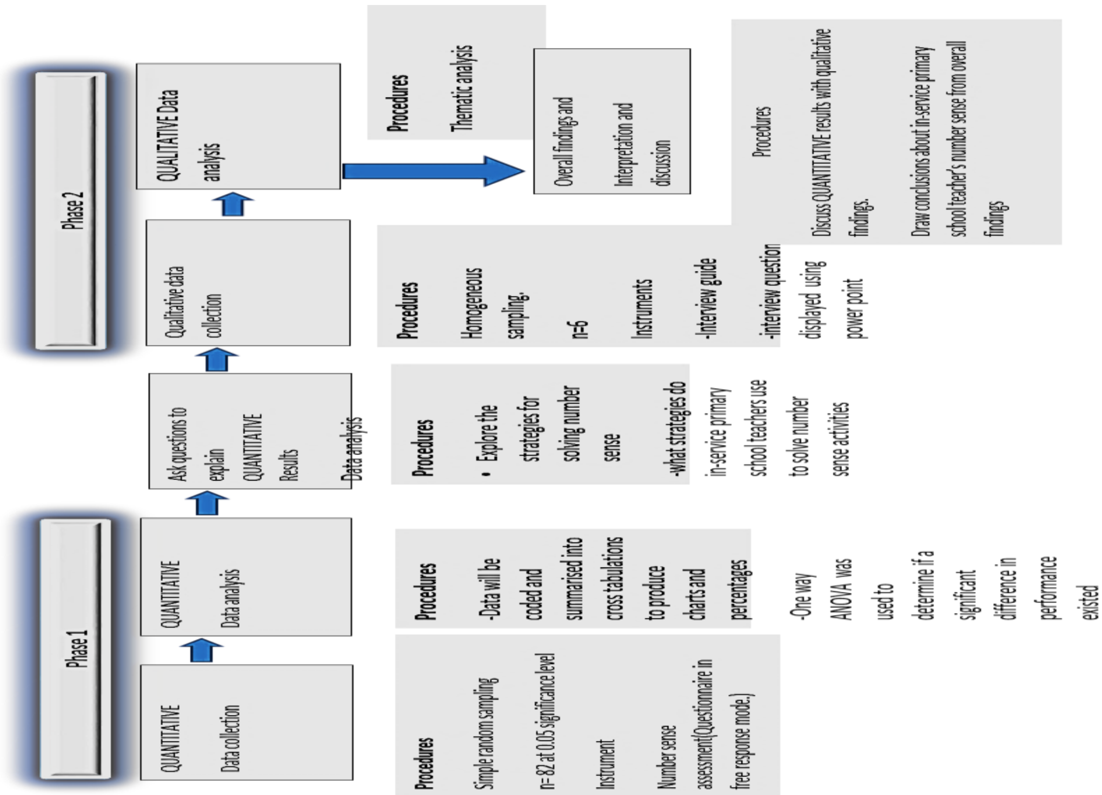


Figure 2: Diagrammatic presentation of the research methodology for the study

CHAPTER 4

FINDINGS

4.1 Introduction

This chapter presents the findings of the study. The research findings are presented according to research questions .The first part contains the demographic characteristics of the respondents followed by findings on each research question under the following sub-headings; Number sense proficiency of in-service primary school teachers, performance on the number sense assessment based on teaching experience and strategies used by in-service primary school teachers to solve number sense activities**.**

4.2 Demographic characteristics of the respondents

Out of 82 questionnaires that were distributed, 69 questionnaires were successfully completed and returned representing a return rate of 84%. Table 3 shows the distribution of teachers according to their gender.

Table 3: Distribution of teachers according to gender

	Frequency	Percent
M	**37**	53.6
F	**32**	46.4
Total	**69**	100

Table 3 shows that among the in-service primary school teachers who answered the number sense assessment questionnaire, 37 were male representing 53.6% while 32 were female representing 46.4%.

4.3 Qualifications and teaching experience of the respondents

4.3.1 Qualifications of the respondents

The qualifications of the teachers were as follows: 30 respondents representing 43.5% had primary teachers' certificates and 37 teachers representing 53.6% had primary teachers' diploma and 2 teachers were university degree holders (Primary degree). This shows that majority of the in-service primary school teachers who took part in the study were diploma holders.

4.3.2 Teaching experience of the respondents

Table 4 shows the distribution of in-service primary school teachers according to teaching experience.

Table 4: Teaching experience of respondents

Years	Frequency	Percentage
1-5	29	42
6-10	22	32
11 years and above	18	26
Total	69	100

As seen from table 4, 29 teachers representing 42% had a working experience of between 1-5 years while 22 teachers representing 32% had 6-10 years of work experience and 18 teachers (26%) had working experience of 11 years and above.

4.4 Number sense proficiency of in-service primary school teachers

This section presents findings on in-service primary school teacher's proficiency levels in performing number sense activities. Figure 4 shows the performance of the in-service primary school teachers on the number sense assessment questionnaire.

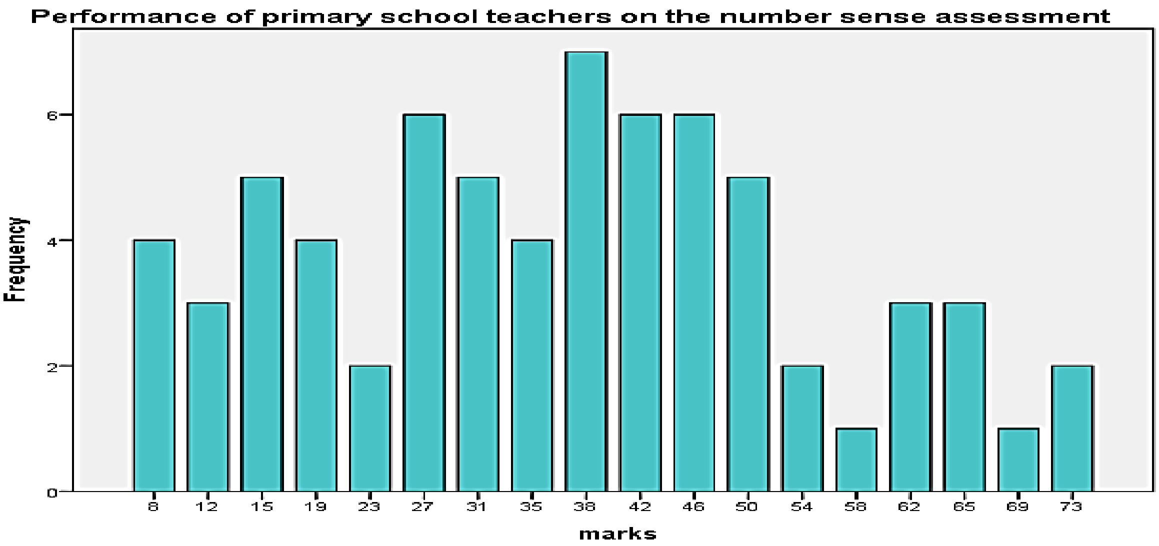


Figure 3: Performance of teachers on the number sense assessment

As seen from the bar chart in figure 3, the highest mark obtained was 73 and the lowest mark was 8%.The performance of teachers can be summarised using descriptive statistics as shown in table 5

Table 5: Descriptive statistics on the number sense assessment

Descriptive statistics on the number sense assessment

N=69	Mode	Minimum	Maximum	Mean	Std. Deviation
	38	8	73	36.64	17.356

As seen from the table 5, the mean performance was 36.6% and the standard deviation was 17.356 with a maximum and minimum mark of 8% and 73% respectively. These results suggest that in-service primary school teachers were not proficient in performing number sense activities.

To establish the proficiency levels of in-service primary school teachers in performing number sense activities, their performance on the number sense assessment was grouped according to one of the following categories as proposed by McIntosh et al (1992), Reys & McIntosh (2002) and refined by Markovits and Sowder (2004) as follows:

- Advanced (75% and above)
- Proficient (60 – 74%)
- Basic (50 - 59%)
- Below basic (0 -49%)

Figure 4 shows the distributions of primary school teachers in each of the four levels of number sense according to their performance on the number sense assessment.

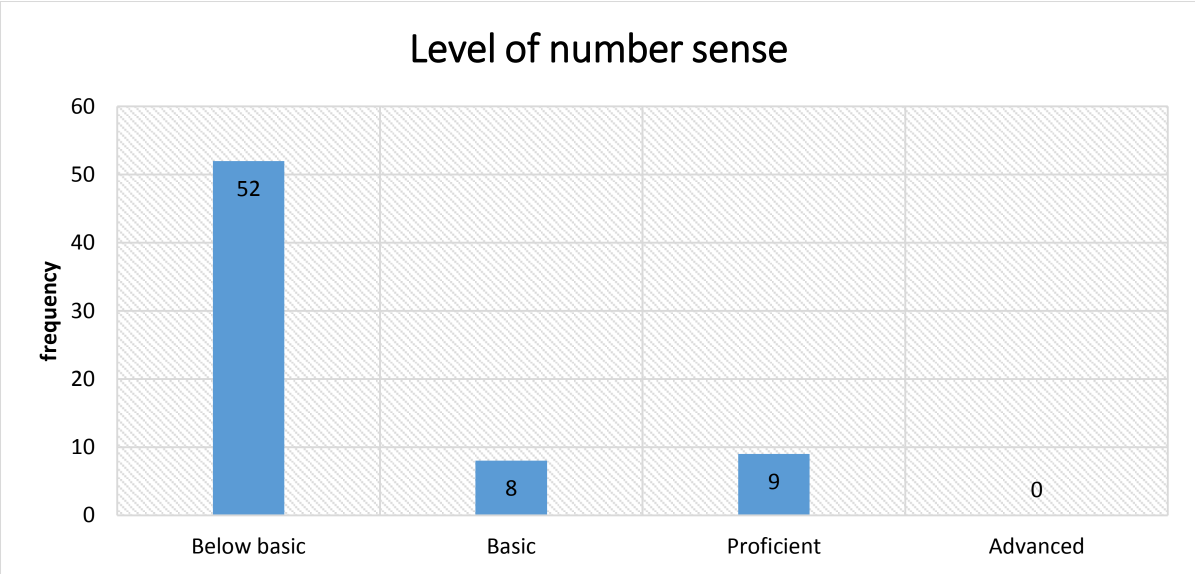
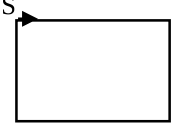


Figure 4: Distribution of teachers according to the four levels of number sense

Figure 4 shows that 52(75.4%) participants scored below the basic level, while 8 (11.6%) participants had number sense scores within the basic level. Only 9 participants representing (13%) had scores within the proficiency level and no participant scored a mark within the advanced level. This shows that majority of the in-service primary school teachers had low number sense with most of them scoring below the basic level when tested on number sense activities. Table 6 shows the performance of teachers on items that received the highest number of correct and incorrect responses from the number sense questionnaire.

Table 6 : Performance of teachers on items which received the highest number of correct and incorrect responses from the number sense assessment

Number sense domain	Item	Correct (%)	Incorrect (%)
Knowledge of and facility with numbers	10. Write these fractions in order from least to greatest. Explain your reasoning. $\frac{1}{2}$ $\frac{3}{5}$ $\frac{3}{10}$, $\frac{3}{8}$, $\frac{3}{6}$	4 (6%)	65(94%)
	17. Put two of the numbers 4, 9, 12 in the boxes to make a fraction as close as possible to $\frac{1}{2}$ $\frac{\square}{\square}$	24(35%)	45(65%)
	16. You are going to walk once around a square-shaped field. You start at the corner marked S and move in the direction shown by the arrow. Mark with an X where you will be after $\frac{1}{3}$ of your walk. S 	13(19%)	56(81%)
Knowledge of and facility with operations	5. In a soccer tournament, teams get: 3 points for a win 1 point for a tie 0 points for a loss. Zedland has 11 points. What is the smallest number of games Zedland could have played?	39(57%)	30(43%)
	12. Place the four digits 3, 5, 7, and 9 into the boxes below in the positions that would give the greatest result when the two numbers are multiplied . X $\square\square$ $\square\square$	15(22%)	54(78%)
Applying knowledge of and facility with numbers and operations to computational settings	13. Kim is packing eggs into boxes. Each box holds 6 eggs. She has 94 eggs. What is the smallest number of boxes she needs to pack all the eggs?	43(62%)	26(38%)
	14. Six hundred books have to be packed into boxes that hold 15 books each. Write an expression that could be used to find the number of boxes needed. Explain your reasoning.	56(81%)	13(19%)

According to table 6, respondents had difficulties in answering item 16 and 17, however, question 10 which was about arranging fractions in order from the least to greatest proved to be the most challenging to the respondents as it only received 4 correct responses. In arranging the fractions, in- service primary school teachers could not recognise that $\frac{3}{6}$ is equivalent to $\frac{1}{2}$ in ordering the fractions.

In the second number sense domain of knowledge of and facility with operations, Question 12 on the number sense questionnaire tested in-service primary school teacher's understanding on effects of operations on numbers. Teachers were required to place the four digits 3, 5, 7, and 9 into the boxes below in the positions that would give the greatest result when the two numbers are multiplied.

$$\begin{array}{rr} & \square\,\square \\ \mathrm{X} & \square\,\square \end{array}$$

Only 15 responses out of 69 responses were correct representing 22%. The most common answer given by the respondents was 97 x 53. Item 5 received 57% correct responses which is average in terms of performance showing that in-service primary school teachers had a certain measure of difficulties in answering items in this number sense domain.

From table 6, it is clear that in-service primary school teachers answered item 13 and 14 with relative easy as they received the highest number of correct responses 62% and 81% respectively. These questions involved applying a rule to find the solution.

4.5 Performance on the number sense assessment based on teaching experience

This section presents results on whether a significant difference in performance existed among in-service primary school teachers performance based on their teaching experience. Table 7 shows the mean performance and standard deviation for each of the three groups.

Table 7: Number sense marks according to teaching experience period

number sense marks according teaching experience period

Years of experience in teaching	Mean	N	Std. Deviation
1-5	34.17	29	15.748
6-10	36.91	22	16.767
11 Years and above	40.28	18	20.608
Total	36.64	69	17.356

According to table 7, teachers with 11 years and above experience had the highest mean performance of 40.28% while teachers with 1-5 years of experience had the lowest mean performance of 36.91%. This was not enough to draw conclusions thus, to determine if a significant difference in performance on number sense activities according to teaching experience period existed among primary school teachers, a one-way ANOVA test was computed and table 8 illustrates the test results.

Table 8: ANOVA test results

ANOVA

number sense marks

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	416.375	2	208.187	.685	.508
Within Groups	20067.567	66	304.054		
Total	20483.942	68			

No significant difference was observed on in-service primary school teacher's performance on number sense activities based on teaching experience period ($F(2, 66) = 3.14$, $p = 0.508$).

In order to explore in-service primary school teacher's strategies, interviews were conducted to explore the kind of strategies they used to answer number sense activities. The next section presents the findings.

4.6 Strategies used by primary school teachers to solve number sense activities.

Teachers were interviewed using 7 number sense questions based on the framework described by McIntosh et al (1992). The first part presents an item by item analysis of the performance of in-service primary school teachers on each number sense item they were interviewed on followed by an analysis of the overall performance of teachers on all the components.

4.6.1 Interview Item1

Number sense domain**: Knowledge of and facility with numbers**

Number sense component: Recognising the relative size of numbers

Researcher: Is $\frac{1}{2}$ closest to $\frac{3}{8}$ or $\frac{7}{13}$?

All respondents gave an incorrect initial response to this question and only one respondent out of six used a number sensible strategy and no rule based strategy was used. They all chose $\frac{3}{8}$ as the fraction that was closest to half. When asked why they chose this answer, 3 respondents gave the following explanations:

P5: *Because or just by looking, if we say 2 into 8 it goes in 4 times but in 13 it goes 6 times and it leaves a remainder.*

P6: *Because the denominator is 8, it's when you divide by 2 it will not give you a remainder and the numerator when you divide it unlike this one when you divide it by 2 it will give you a remainder.*

P4: *Okay simply because, for such fractions you have firstly to change the fraction or in short you look for the lowest common multiple of the fractions which is 3, the lowest common multiple of 8 and 13 and 8 is closest.*

However one respondent was able to exercise control and gave the following number sensible response after probing:

P3: $\frac{7}{13}$ *is half when it is six and half over thirteen or when it is seven over fourteen then* $\frac{3}{8}$ *is half when it is four over eight. Now this one to make it half the 13 needs to be enlarged to fourteen as it is above half, this one is lacking one over eight to make it* *half, this one to make it half the denominator needs to be enlarged by 1. But again if we reduce the numerator by point something, this by one over eight this 0.5 so this is closer (three over eight). This one you need to enlarge it by 1 this one reduce it by 0.5 therefore $\frac{7}{13}$ is closer.*

Researcher: *You keep on changing, maybe this one or this one which one is your final answer.*

P3: *The numerator should be the deciding factor because we are saying over this. What changes are the parts out of the whole so the denominators will be maintained, the numerators can decide what part of the whole has been taken, so in that sense $\frac{7}{13}$ is closer because what we reduce is less than what we add here.*

The lack of confidence was apparent among some of the respondents in the way they changed their choice of solution or failed to explain their solutions;

P3 *maybe this one….. Again this one*

P2 *I can't explain but if you had asked me to find the middle fraction i would have done it.*

The analysis on interview item 1 can be summarised as follows:

1. Total number of respondents
   - N=6
2. Initial answers
   - Correct- none
   - Incorrect- 6
3. Strategies used
   - Rule based - None
   - Number sensible-The residual strategy
4. Control
   - Managed 1
   - Failed 5
5. Confidence
   - The lack of confidence was observed from inability explain solutions and guessing.

4.6.2 Interview Item 2

Number sense domain**: Knowledge of and facility with numbers**

To further understand the in-service teacher's knowledge and facility with numbers, the following question was asked

Researcher: How many decimal numbers are there between 8.3 and 8.4?

Two respondents out of six responded that there were decimals between 8.3 and 8.4, the rest responded that there were no decimals between the two numbers and below is a summary of their responses;

P1: *there is nothing because we go 8.1, 8.2, 8.3 then 8.4*

P2: *there nothing I am very sure…*

P5: *Because you move from 8.3 then 8.4*

Surprisingly P6 who was among the two respondents that said there are decimal numbers between the two numbers gave this explanation:

P6: *It should be 1*

Researcher: *which one is it?*

P6: *I was thinking of the inter-quartile range…..laughs…. so that I find if there is an exact decimal place between those values, okay its 8.05*

Researcher: *you feel 8.05 is between 8.3 and 8.4*

P6: *Yeah…pause…. yes*

All 5 respondents failed to exercise control except for P3 who gave a correct response as follows;

P3: *They are 9 but if we keep on adding the number of decimal places they can be more*

Researcher: give me examples

P3*: 8.311, 8.312 etc….*

However the lack of confidence was observed from the initial answers

P4: *I think it's a tricky question sir….*

P6 *I was thinking of the interquartile range…..laughs*

The analysis on interview item2 can be summarised as follows:

1. Total number of respondents
   - ❖ N=6
2. Initial answers
   - ❖ Correct- 1
   - ❖ Incorrect- 5
3. Strategies used
   - ❖ Rule based - None
   - ❖ Number sensible-One
4. Control
   - ❖ Managed 1
   - ❖ Failed 5
5. Confidence
   - ❖ Observed from initial answers and statements like it's a tricky question sir

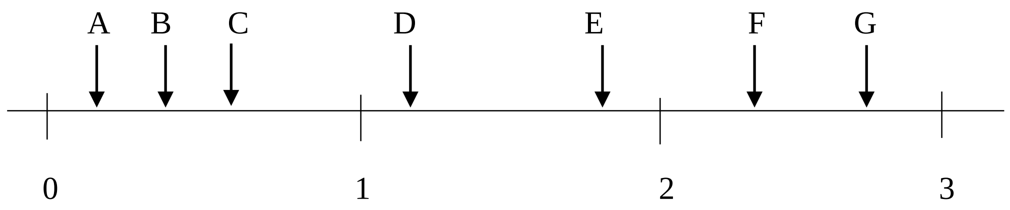
4.6.3 Interview item 3

Number sense domain**: Knowledge of and facility with numbers**

Number sense component: Understanding multiple representations

Researcher:

> Which letter in the number line names a fraction where the numerator is slightly more than the denominator?



5 out of 6 respondents had difficulties with this question and only two respondent gave a correct initial response that is P3 and P4 but guessing was evident in P4's response;

> P4 *I think D represents the fraction,*
>
> Researcher. *Why do you say D?*
>
> P4 *Because 1 is before D and I am taking it that 1 is the numerator and D is the denominator.*

However, P3 used a number sensible strategy by offering the following solution;

> P4: *It's D*
>
> Researcher: *How did you do it?*
>
> P4: *Here we are looking for the smallest improper fraction because of the slightly smaller than in the question. All these before 1 are not improper fractions.*

All other respondents failed to monitor and regulate themselves and showed lack of confidence despite probing by the researcher, they kept guessing as follows;

> P1: *its D……pause….. Laughs and says these numbers…. No I think its G*
>
> Researcher. *What if you think of them as improper fractions?*
>
> P1: *Improper fractions, let me see …. Pause….*
>
> P6: *Is it not C?*
>
> Researcher: *How did you do it?*

P6: *Because we are increasing going to the right side so the denominator there will be the number above over 0.*

Researcher. *You mean C represents the number above over 0?*

P6: *I cannot see a fraction above over Zero. I will just use general knowledge it's G.*

The analysis on interview item3 can be summarised as follows:

1. Total number of respondents
   - N=6
2. Initial answers
   - Correct- 2
   - Incorrect- 4
3. Strategies used
   - Rule based - None
   - Number sensible-flexibility with multiple representations

4. Control
   - Managed -1
   - Failed - 5
5. Confidence
   - Observed from the pauses and laughter and guessing

4.6.4 Interview item 4

Number sense domain: **knowledge with and Facility with operations**

Component: understanding the relationship between operations

Researcher: Which expression represents a larger amount?

2452 x 4

2541 + 2457 + 2460 + 2465

Three respondents used number sensible strategies and choose the second expression as the largest expression. When asked to justify their reasoning they gave the following responses:

P3*: I looked at what is above the thousands in the second expression…*

P5*: because 2452 is smaller than the rest of the added numbers*

P2*: by looking at the place values*

One respondent used a rule based strategy and gave the following justification when probed;

P2*: because the answer that you get here is different with what you get after multiplying 2452X4*

Three respondents gave confused explanations when asked to justify their reasoning reflecting their lack of control as follows:

> P4*: there is no much difference between the two because in the first one we are multiplying*
>
> P6*: Because it is doubled and multiplication is the shortcut of addition*

Lack of confidence to do the question was observed in their requests

> P2*: Can I do it using a calculator*
>
> *Researcher can you be able to do it in any other way*
>
> P2*: No*

The analysis on interview item 4 can be summarised as follows:

1. Total number of respondents
    - ❖ N=6
2. Initial answers
    - ❖ Correct- 3
    - ❖ Incorrect- 3
3. Strategies used
    - ❖ Rule based - 1
    - ❖ Number sensible-3
4. Control
    - ❖ Managed 3
    - ❖ Failed 3
5. Confidence
    - ❖ Observed from their requests to use mathematical tool like calculators or a pen and paper.

4.6.5 Interview item 5

<u>Number sense domain: **knowledge with and Facility with operations**</u>

<u>Number sense component: Understanding the effects of operations</u>

> Researcher:
>
> Place the decimal in the answer to the following
>
> 534.6 x 0.545 = 291 357

All respondents except P5 gave an incorrect initial answer and no number sense based strategy was observed among the participants. All of them used the rule based strategy of counting the number of decimal places in the two multiplicands which would not work unless the respondent realised that there should have been a zero at the end in the solution. Their use of the rule based strategy can be summarised by P3's response.

> P3*: Here you just count the number of decimal places, here they are 1 and there they are 3 so together 4 so the decimal point will be between 9 and 1.*

Despite probing, no respondent could regulate and monitor their solution path .P5 who gave a correct initial answer: when asked to justify the solution gave the following confused explanation;

> P5: *Because of the place value, this one will just be on top of 5 so when we start multiplying we count these three after the point.*

All participants were confident with the rule based strategy

The analysis on interview item5 can be summarised as follows:

1. Total number of respondents
    - ❖ N=6
2. Initial answers
    - ❖ Correct- 1
    - ❖ Incorrect- 5
3. Strategies used
    - ❖ Rule based – 6 (counting the number of decimal places)
    - ❖ Number sensible-none
4. Control
    - ❖ Managed- none
    - ❖ Failed - 6
5. Confidence
    - ❖ All were confident with the rule based strategy of counting the number of decimal places.

4.6.6 Interview item 6

Number sense domain: **Application to computational settings**

Number sense component: Recognise operation and select efficient strategy (Using benchmarks)

Researcher: 60 x 40 is an estimate of 63 x 37. Is the exact answer less than or equal to or more than 2 400. Why, please explain your answer

Only one respondent out of 6 gave a correct initial answer, the rest responded that the answer is equal 2400, when probed they justified their answers as follows;

> P1: *if you take away the three from 63 and add to 37 it will be 40 so they will be the same.*
>
> P5: *there is 60 here, here there is 63 which is the same as 40, 37 less three from 40. So it is the same number*

P2 and P3 used number sensible strategies though P2 failed to regulate and monitor himself in order to effectively use their strategy as follows:

> P2: *Here I am concerned about the number that we are going to multiply with the first number….60, 40 times and 63, 37 times, so I think the answers will be the same.*

P3 was able to use benchmarks and gave this response:

> P3: *the exact answer will be less than because of 6×4 which will be less than 24 but it is 18. Also generally if you round off a number and multiply the solution is likely to be bigger.*

The lack of confidence was apparent in their effort to change the solution when probed and attempts to use mathematical tools like calculators.

> P1 *it will be the same:*
>
> Researcher: *Are you sure*.
>
> P1: *let me see again….i need to use a pen and a paper*
>
> P3. *This question is tricky, it needs a calculator:*

Two respondents when probed further changed their solution, the following is the summary

> P4 *it will be more than*
>
> Researcher: *How did you do it*?
>
> P4 *Because of the second number that we have been given which is 63 and 37. If we multiply 3 and 7 we are going to have a round figure which is 21 but if we multiply zero in 60X40, zero by zero will give us zero hence….. It a tricky question.*

Below is a summary of the analysis on this component:

1. Total number of respondents
   - N=6
2. Initial answers
   - Correct- 1
   - Incorrect- 5
3. Strategies used
   - Rule based – One

- ❖ Number sensible-One(estimating)

4. Control
   - ❖ Managed- 1
   - ❖ Failed - 5
5. Confidence
   - ❖ Observed from attempts to use mathematical tools and from statements

4.6.7 Interview item7

<u>Number sense domain: **Application to computational settings**</u>

<u>Component: Recognise operation and choose efficient strategy</u>

<u>Researcher</u>

A minibus can transport 15 people. How many minibuses would you need to transport 170 people?

5 out of 6 respondents gave a correct initial response except for P6. All respondents were confident with the rule based strategy of dividing 15 into 170. Only P5 used a number sensible strategy as follows:

> P5: *here I used initiative, 10 X15 is 150 remainder 20, and 15 into 1 its 1 remainder 5 hence we need 12 buses.*
>
> Researcher: *can you do it in any other way*
>
> P5: yes*. I can divide 15 into 170.*

However, P6 failed to monitor and regulate himself and gave the following confused response:

> P6: *7 minibuses*
>
> Researcher: *How did you do it?*
>
> P6: *Here there will be a remainder even if we are to divide 15 into 170 it will land us to 6 remainder 10…..*

The analysis on this component can be summarised as follows:

1. Total number of respondents
   - ❖ N=6
2. Initial answers
   - ❖ Correct- 5

- ❖ Incorrect- 1

3. Strategies used
   - ❖ Rule based – 5(Dividing 15 into 170)
   - ❖ Number sensible-One

4. Control
   - ❖ Managed- 5
   - ❖ Failed - 1
5. Confidence
   - ❖ All were confident with the rule based strategy.

The study established that in-service primary school teachers were casual in using mathematical language during the interviews (This was a sign of lack of confidence in the knowledge/content or having fuzzy knowledge about the content) and table 9 presents a summary of some of the statements they made during the interviews.

Table 9: Summary of statements by in-service primary school teachers that reflect casual mathematical language use during the interviews.

Code	Summary statement
P1	I think G…….meaning the denominator is somehow small, if you look at this one, this is 2 and this is 3, there **must be small lines that are supposed to be ten between** 2 and 3.
P3	This one is 2452, even if I am to use repeated addition, **this one is ahead of this one, this one is ahead of this and this is above this one**.
P4	Looking at the decimal points, **the place values that we are multiplying which is 534.6× 0.545**, so the first one we are multiplying has one place value.
	If we multiply 3×7 we are going to have a **round figure**, we are going to have **other numbers on top** of which is 21 but zero by zero in 60 ×40 is zero.
P6	I was thinking of the interquartile range….laughs…so that I find **if there is an exact decimal place between those values**.

As shown in table 9, teachers were casual in mathematical language use during the interviews, for instance, P1 referred to divisions on the number line as ten small lines between 1 and 2 while P3 in comparing numbers used ahead instead of greater than. P4 on the other hand called the numbers that were being multiplied as place values. All this can be related to poor conceptual knowledge in the domain of numbers as it is a sign of lack of confidence in the knowledge/content or having fuzzy knowledge about the content.

4.7 Analyses of the overall performance of teachers on all the interview items

This part presents an analysis of the overall performance of teachers on all the number sense items they were interviewed on. Table 10 shows the findings on strategies and the overall performance of teachers.

Table 10: Summary of findings on strategies

Item number	**Method**		**Responses**	
	Number sensible	**Rule based**	**Correct**	**Incorrect**
1	2 respondents -Residue strategy -Converting to decimals	-None	1	5
3	3 respondents -Estimation	-None	3	3
4	-None	6 respondents -Counting the number of decimal places in the two multiplicands	0	6
5	1 respondent -Estimation	-None	1	5
6	1 respondent -Decomposing numbers	6 respondents -Dividing 15 into 170	5	1
Totals	**7/30**	**12/30**	**10/30**	**20/30**
	Knowing			
2	-Knowledge of multiple representation of numbers		1	5
7	Knowledge of fractions		1	5
Totals			**2/12**	**10/12**

According to table 10 very few number sensible strategies were utilised by in-service primary school teachers. Only 7 out 30 possible strategies representing 23% were utilised. For instance, in attempting interview item1 only one respondent was able to use a number sensible strategy (residual strategy. Further, only 12 out of 42 responses were correct representing 29%. In attempting interview item2 which involved fractions for example, 5 in-service primary school teachers out of 6 gave an incorrect response that there are no decimal numbers between 8.3 and 8.4. Similarly, when tested on multiple representation numbers, only one correct response was given.

Most of the correct responses were given on questions that involved application of a rule through rule based strategies. Interview item 6 received the highest number of correct responses as most of the teachers were confident with the rule based strategy of dividing 15 into 170. The overall performance was that 30 out of 42 responses representing 71 % were incorrect meaning majority of the answers given were incorrect.

4.8 Summary

The results of this study showed that majority of the in-service primary school teachers who took part in the study were below the basic level when tested on number sense indicating that their number sense was low and no significant difference was observed on in-service primary school teacher's performance on number sense activities based on teaching experience.

From the item by item analyses in the qualitative results, most of the responses were incorrect and very few number sense strategies were used by the in-service primary school teachers who took part in the interviews. In-service primary school teachers were confident in answering questions that involved application of a rule through rule-based strategies. Further, the study established that in-service primary school teachers were casual in mathematical language use during the interviews and this showed lack of confidence in the knowledge/content or having fuzzy knowledge about the content. The next chapter discusses the findings of this study.

CHAPTER 5

DISCUSSION OF FINDINGS

5.1 Introduction

This chapter discusses the findings of this mixed-method study which sought to investigate number sense among in-service primary school teachers. The first part offers a brief discussion of the performance of in-service primary school teachers in the three number sense domains were performance of teachers both on the number sense assessment questionnaire and the interviews is discussed followed by a discussion of the results in each sub research question of the study.

5.2 Performance of in-service primary teachers in the three number sense domains

5.2.1 Primary school in-service teacher's Knowledge of and facility with numbers

From the number sense test, question number 17 required teachers to put two numbers 4,6,12 in the boxes (─) to make a fraction as close as possible to $\frac{1}{2}$. Only 24/69 responses were correct representing 35%. The performance was similar to the results obtained during the interviews, Item one from the interviews required teachers to recognise the relative size of numbers by stating whether $\frac{3}{8}$ or $\frac{7}{13}$ is closest to half. All respondents gave an incorrect initial response as they all chose $\frac{3}{8}$ as the fraction that is closest to $\frac{1}{2}$. Only one respondent gave a correct response after probing that $\frac{7}{13}$ is closer to $\frac{1}{2}$.This is consistent with Courtney-Clark & Wessels (2014) who found that pre-service teachers had challenges establishing the relative magnitude of two fractions using $\frac{1}{2}$ as a referent point. Teacher candidates could not use number sensible or rule-based strategies such as subtracting the two fractions by applying the standard algorithm. However, the results differ from Sengul (2013) who found that 77.4 %(103 out of 133) pre-service teachers were able to answer this question correctly through either number sensible strategies, partial-number sensible or rule-based strategies. Even though pre-service teachers performed well, Sengul (2013) acknowledged that 20 participants answered this question using paper and pencil methods based on operations while 27 participants despite giving correct responses could not give explanations or their explanations could not be understood. The pre-service teachers who gave incorrect responses preferred to find the answers by changing the numbers into a more complex structure like decimals where they ended up making calculation errors (Sengul, 2013).

When tested on sense of orderliness of numbers. The fractions, $\frac{1}{2}$, $\frac{3}{5}$, $\frac{3}{10}$, $\frac{3}{8}$, $\frac{3}{6}$ were supposed to be ordered from the least to greatest. Only 4 correct responses were given representing 6% on the number sense questionnaire. In-service primary school teachers could not recognise that $\frac{3}{6}$ is equivalent to $\frac{1}{2}$ in ordering the fractions. This finding is in-line with the findings of a study by Mohamed & Johnny (2009) who found that despite a high level of competency in performing algorithms in the classroom, students were generally weak in understanding the relative size of numbers, composing numbers and recognising the effect of operations on numbers. Mohamed & Johnny (2009) noted that “failure to master the concept of fractions and decimals leads to difficulties in understanding the concepts of percentages as well as multiplication and division of fractions and decimals” (p.323)

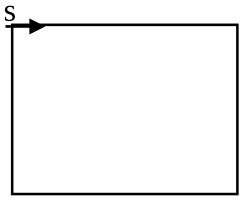
The study established that in-service teachers who took part in the interviews had limited knowledge on decimals. When asked how many decimals are there between 8.3 and 8.4, only one respondent (P3) out of 6 respondents had knowledge that there are decimal numbers between 8.3 and 8.4 and was able to give examples. Other respondents were confident just as the 6th grade students in Yang (2005)’s study that there were no decimal numbers between the two numbers. “When students were faced with non-routine questions or problems they had not come across in mathematics class, they usually lacked confidence and felt uncomfortable “(Yang, 2005, p.330). This study has shown that even practicing teachers lacked confidence and equally faced challenges on decimal numbers. The finding that in-service teachers who took part in the interviews had limited knowledge on decimals disagrees with the findings by (Reys & Yang, 1998) who observed that high level performing students either in the sixth or eighth grade had no difficulties explaining that there are infinitely many decimal numbers between 1.42 and 1.43 and were able to support their responses with at least one example.

When tested on multiple representations of numbers, teachers showed limited knowledge on this number sense component. From the interviews, item 3 required teachers to identify the letter that represents a fraction were the numerator is slightly higher than the denominator, respondents found this question challenging and guessing was observable in their answers in the way they kept changing their choices and this can be summarised by P1’s response below:

P1: *its D……pause….. Laughs and says these numbers…. No I think its G*

This explains why in-service primary school teachers performed poorly on the number sense questionnaire on question 16 where teachers were required to mark with an X where they would be after $\frac{1}{3}$ of the journey. The specific question was:

You are going to walk once around a square-shaped field. You start at the corner marked S and move in the direction shown by the arrow. Mark with an X where you will be after $\frac{1}{3}$ of your walk.



Only 13 correct responses representing 13% were given by in-service primary school teachers and this shows poorly developed sense of multiple representation of numbers. The finding that in-service teachers had challenges with questions involving multiple representation of numbers contradicts the findings by Yaman(2015) who found that some teacher candidates could answer questions involving multiple representation using number sense approaches before undergoing mathematics courses while other candidates could only answer after the mathematics courses. However, the finding is in agreement with the findings by Reys &Reys (1999) who found that students had challenges with this question and noted that most of the incorrect responses were due to the misconception of using the vertices as third markers by thinking of standing at one corner of the square and by so doing, what was seen is three corners which were viewed incorrectly as representing a third of a square. Being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes is a significant indicator of conceptual understanding (National Research Council, 2001).

5.2.2 Primary school in-service teacher's Knowledge of and facility with operations.

Item three from the interviews tested primary school teacher's knowledge in the number sense component of understanding the relationship between operations. Teachers were required to choose an expression which would produce a larger amount in the two expressions below.

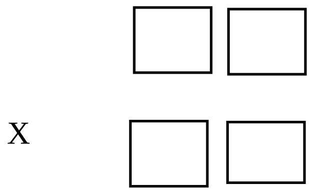
2452 x 4

2541 + 2457 + 2460 + 2465

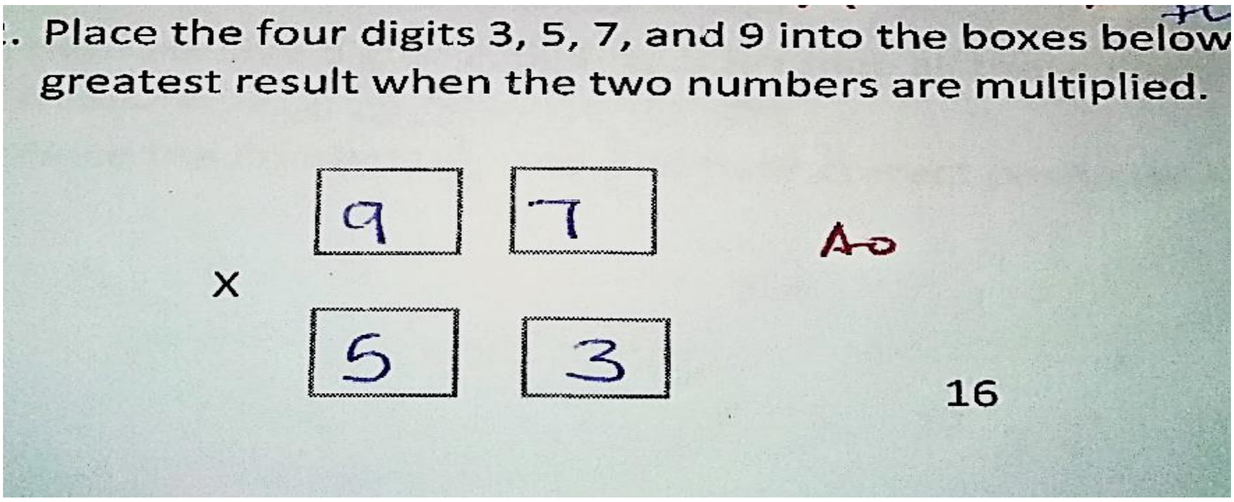
Three responses were correct out of six.

In-service teachers showed lack of confidence in answering this question as P6 even requested to use a calculator to solve this question.

Question 12 on the number sense questionnaire tested in-service primary school teacher's understanding on effects of operation. Teachers were required to place the four digits 3, 5, 7, and 9 into the boxes below in the positions that would give the greatest result when the two numbers are multiplied.



Only 15 responses out of 69 responses were correct representing 22%. The performance did not change on this component in the interviews where teachers were required to Place the decimal in the answer to the following 534.6 x 0.545 = 291 357. Only one correct response was given out of six as most of the teachers applied the rule of counting the number of decimal places which would not work unless they recognised that there should have been a zero at the end in the solution. The finding is in line with (Doriney, 2016; Courtney-Clark & Wessels, 2014). Doriney (2016) found that pre-service teachers made conceptual errors in placing the digits 3,5,7 and 9, they could not choose 93 x 75 as the correct answer to this problem. In this study, in-service teacher's most common answer was as shown below.



The response by the in-service primary school teacher showed lack of conceptual understanding of operations on numbers. Such limitations in knowledge needed to be addressed before teachers could become effective teachers as “this demonstrates limited mathematical knowledge that many researchers such as Hill (2008), Ball (2008), and Shulman (1986) deemed necessary for effective teaching” (Doriney, 2016.p.61). In placing the decimal comma to 534.6 x 0.545 = 291 357 primary school in-service teachers just like pre-service teachers in Courtney-Clark & Wessels (2014)’s study could not utilise number sensible approaches such as noticing through estimation that for example 550 ×0.5 is more than 250 but less than 300 and therefore 534.6 x 0.545 = 291. 357.

5.2.3 Primary school in-service teacher’s ability to apply knowledge of and facility with numbers and operations to computational settings

Item 5 from the interviews required teachers to recognise operation and select an efficient strategy (Using benchmarks). The specific question was: 60 x 40 is an estimate of 63 x 37. Is the exact answer less than or equal to or more than 2 400. Why, please explain your answer. Respondents found this question challenging and only one correct response was given, the rest responded that the answers will be same.

These in-service teachers exhibited poor knowledge of the relative effect of operations on numbers and did not understand that the result of multiplying two numbers depends on both numbers being multiplied. This finding supports the statement that “the use of standard written algorithms limits thinking and reasoning” (Yang, 2005, p.326). Participants lacked confidence in working out this problem involving estimation as some admitted that it was a tricky question and that they needed a pen and a paper or a calculator to solve it. This was in-line with Markovits & Sowder (1994) argument that finding the solution to questions involving estimation is quite difficult without well-developed number sense. Estimation was viewed as a significant manifestation of number sense and that most children and adults lacked the basic skills of estimation because of limited exposure to estimation in schools (Courtney-Clark & Wessels, 2012). Knowledge of ways of how to estimate results is important for mathematical proficiency (National Research Council, 2001).Following the poor performance of in-service teachers on estimation there was need for Zambian Education institutions to ensure that estimation was given the adequate attention since the emphasis on procedural rules as is the case in the Zambian education system negatively impacts the development of number sense in both teachers and learners (Reys, 2007).

In-service teachers were confident in answering questions that involved application of rule. For example, in attempting the question:

A minibus can transport 15 people. How many minibuses would you need to transport 170 people?

Only one incorrect response was given out of six. In-service primary School teachers performed well on this item, only one incorrect response was given out of six. Teachers were confident with the rule based strategy of dividing 15 into 170. Performance did not change either on the number sense questionnaire on similar items. This differs with the findings by Courtney-Clark & Wessels (2014) that final year pre-service teachers had challenges applying the rule based strategy correctly but supports the finding that very few note worth number sensible strategies were used by pre-service teachers. In this study, only one respondent was able to decompose and recompose 170 to 15×10 +20 as an efficient strategy in solving this question. The results are also in-line with the findings by Yaman (2015) who observed that teacher candidates were prone to using rules and algorithms and perceived mathematics as a class about operations. Yaman (2015) recommended that lessons for teacher candidates should be structured and planned in ways that would enable them understand the conceptual aspects of mathematics.

5.3 Number sense proficiency of in-service primary school teachers

Any problem solving performance is built on a foundation of basic mathematical knowledge available to the individual (Schoefield, 1985).The study established that majority of the in-service primary school teachers who took part in the study were below the basic level when tested on number sense indicating that their number sense was low and were therefore not proficient in performing number sense activities. This is in-line with the findings of Courtney-Clark & Wessels (2014) who found that pre-service teachers demonstrated limited number sense and possessed very few of the indicators of number sense. The study exposed one reason for the low standards of performance of Namibian learners.

That practicing teachers had limited number sense themselves in this study was worrying because the levels of content knowledge teachers have has implications for all students and the quality of learning they achieve (Doriney, 2016). The 7th National Development Plan (2017) notes that one major challenge our education system faces was the low quality of education as evidenced by low performance results in assessment surveys. In order to improve quality, 7th NDP (2017) indicates that focus should be on enhancing literacy and numeracy skills in learners especially at primary school level through implementing the revised curriculum. In-service teachers who were supposed to champion the agenda of the 7th NDP in developing numeracy skills in learners have been found to have limited knowledge themselves. One cannot develop number sense in others if one does not possess a certain measure of number oneself and Greeno (1991) argued that for one to serve as an effective guide to new comers in an environment, it was essential that the guide himself or herself should be a comfortable resident of that environment. If content knowledge of these in-service primary school teachers is not addressed, then Zambia's efforts in improving the quality of education with regards to numeracy as elaborated in the 7th NDP may not be attained.

5.4 Difference in primary school in-service teacher's performance on number sense activities based on teaching experience.

In testing whether there was a significant difference in in-service primary school teacher's performance on number sense activities according to teaching experience , no significant difference was observed on primary teacher's performance on number sense activities according to teaching experience period ($F (2, 66) = 3.14, p=0.508$). This supports the findings of Zubeyde (2016) who observed no significant differences in elementary school teacher's sense of number test. The pre-school teacher's sense of number was low and this profile did not change with regard to gender and experience in teaching. This study has demonstrated that experience in teaching did not influence performance on number sense activities and even experienced in-service teachers found challenges in solving number sense activities. This can be explained by Naukushu (2016) three stage vicious circle of lack of number sense. According to Naukushu (2016) the lack of number sense is a spiral process which occurs in three stages. The author argued that in stage one, learners left high school with inadequate number sense, in stage2 learners now became student teachers but still without number sense and in the third stage the teachers graduate and became teachers (still without number sense) and the cycle would go on and on if no proper interventions were made. Naukushu (2016) further warned that mathematical deficiencies held by teachers could be passed on to students. The finding that there was no significant difference in performance does not agree with Akkaya (2005) who found that number sense performance of secondary school students differed significantly with regard to grade level. The differences in performance were attributed to two reasons. The first reason was that due to the experiences teachers had with number sense and components increased in accordance with grade level and this positively affected the number sense performance of students. The other reason attributed to the differences in performance was that since calculation methods based on rules and memorisation and paper and pencil as opposed to number sense based methods are used more frequently as the grade level increases, the level of performance decreases as some students embraced such methods.

5.5 Strategies primary school teachers use to solve number sense activities

The study established that in-service primary school teachers used very few number sense based strategies and they were confident in answering questions that involved using a rule through rule based strategies. These findings are consistent with Yang (2015) findings that 6th grade students could not use identifiable components of number sense such as using benchmarks, understanding number magnitude or estimation because of their lack of number sense. The in-service primary school teachers in this study just like the pre-service teachers described by Sengul (2013) preferred using rule-based methods instead of number sense strategies and used similar solution strategies to those used by students in the past. They lacked strategic competence which enables one to formulate and represent problems mathematically and devise useful strategies for solving them. They failed to use concepts and procedures appropriately and could not monitor and regulate themselves by reflecting on their solutions when solving number sense activities. What one does with the facts at his or her disposal is the major determinant of the success or failure of a problem solving attempt (Schoenfield, 1985). There is some truth in the statement that "Mathematics pre-service teachers are a product of how and what they were taught in schools" (Courtney-Clark and Wessels, 2014.P.116). In-service teachers relied heavily in recalling previous learned procedures to the extent of trying to use the knowledge on questions were it was not applicable. Belief systems ; one's mathematical word view which is the set of (not necessarily conscious) determinants of an individual's behaviour about self, about the environment, about the topic and about mathematics shapes the way one does mathematics (Schoenfield, 1985). The lack of conceptual knowledge and weak number sense of teachers does not happen in a vacuum, many pre-service teachers enter teacher education programs with pre-conceived notions they formed as primary school students about the teaching and learning of mathematics. Based upon their success as students, many pre-service teachers see themselves as teaching mathematics as they were taught mostly through verbal transmissions of knowledge and pre-existing methods and procedures (Knoell, Strawhecker, Montgomery & Ding, 2015.p.28). That the in-service teachers in this study heavily relied on previously learnt procedures suggests that they perceived mathematics as comprising a set of procedures which needed to be followed .This could be one of the reasons why teachers are still finding it difficult to teach in a constructivist way as demonstrated by Mwanamonga's (2016) study.

The study established that in-service primary school teachers were casual in mathematical language use during the interviews and this was a sign of lack of confidence in the knowledge/content or having fuzzy knowledge about the content. This is in-line with the findings by Kapembwa (2014) who found that teachers used everyday language to substitute mathematical terminology such as the number on top in referring to numerator of a fraction or transfer the terms to the right for the additive inverse. In this study teachers used terms like small lines that supposed to be ten in referring to divisions on the number line or ahead instead of greater than in comparing the size of numbers. Kapembwa (2014) observed that teachers believed that substituting mathematical terminologies with easier every day words for concepts would resolve the challenges posed to pupils to understand mathematical words. He however, advised that teachers should desist from substituting mathematical terminology with easier words for the concept. Ball (1991) acknowledged that mathematical terminologies were linked to the teacher's ability to understand mathematical concepts. That the in-service primary school teachers were un-able to use correct mathematical language in this study reflected that they did not have in-depth knowledge of the mathematics they are supposed to teach.

5.6 Summary

This chapter discussed the findings of the mixed method study which sought to investigate number sense among in-service primary school teachers. The first part discussed the performance of in-service primary school teachers in the three number sense domains followed by discussions of the specific research questions. The next chapter presents the conclusion and recommendations based on the findings of this study.

CHAPTER 6

CONCLUSION AND RECOMMENDATIONS

6.1 Introduction

This study sought to investigate number sense levels of in-service primary school teachers. Data relating to the level of number sense possessed by in-service primary school teachers was collected through both qualitative and quantitative means. The aim of this chapter is to summarise the main findings of the study. Thus the rest of this chapter is structured as follows,6.1 presents the conclusions based on the findings of the study, section 6.2 provides recommendations of study and 6.3 brings out suggestions of possible areas of future research arising from the study.

6.2 Conclusion

The overall results of this study showed that in-service primary school teachers had low number sense with most of them scoring below the basic level when tested on number sense tasks. No significant difference was observed on in-service primary school teacher's performance on number sense activities based on teaching experience. Very few number sense strategies were utilised and in-service primary school teachers were confident in answering questions that involved application of a rule through rule based strategies.

This study has shown to what extent in-service primary school teachers possess the knowledge and skills that they are supposed to use during the teaching process for proficient teaching of numeracy in primary schools in Zambia. That practicing teachers had limited number sense themselves was worrying because content knowledge of teachers has implications all students and the quality of learning they achieve. What teachers know and believe about mathematics is directly connected to the instructional choices and procedures they follow in teaching (National Council of Teachers of Mathematics, 1991).

The in-service primary school teachers in this study heavily relied on previous learnt procedures to the extent of trying to use the knowledge on instances where it would not apply, at times they even resorted to guessing meaning they were not proficient in solving number sense activities. They failed to monitor and regulate themselves when working on number sense tasks. Effective problem solvers monitor how well they are making progress and persevere or change direction accordingly. Unsuccessful problem solvers tend to choose a path quickly and then persevere at it despite making little or no progress (Schenfield, 2013).

The study established that in-service primary school teachers were casual in using mathematical language during the interviews and this reflected lack of confidence in the knowledge/content or having fuzzy knowledge about the content. In-service primary school teachers should strive to ensure that they use correct mathematical language especially in classrooms during teaching.

6.3 Recommendations

Based on the findings of this study the following are recommended:

- Teacher training institutions at primary school level to consider training teachers in ways that involves developing their number sense and that of learners
- There is need to continue engaging in-service primary school teachers in continued professional development programs in order to improve their content knowledge in mathematics.
- Activities involving mental computation should be given adequate attention in primary schools in Zambia.

6.4 Possible areas of future research

Based on the findings of this study:

- Future studies can explore in-service primary school teacher’s mathematical language use.
- Considering that teacher qualification was not scrutinised in this study and that it could be a big factor in teacher effectiveness, future studies can assess number sense of in-service teachers considering this factor.
- Studies that would explore how number sense activities can be in-cooperated in the Zambian education curriculum especially at primary school level would be interesting.

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APPENDICES

Appendix 1: Questionnaire for teachers

Introduction

Dear respondent,

I am a student at the University of Zambia pursuing Masters of Education in Mathematics Education. I am conducting a research study called number sense comprehension levels of in-service primary school teachers as part of my academic requirements.

I am kindly requesting you to participate in this study by completing this questionnaire. Tick where applicable .The questionnaire has two sections. The information gathered is purely for academic purposes and will be treated confidentially and there is no need to indicate your name.

SECTION A

First we would like to ask a few questions about you

1. What is your gender: Male [ ] Female [ ]
2. What is your age as at last birthday …………
3. What grades do you teach………………
4. How long have you been teaching……………………….
5. What is your qualification………………………………………………………………………

SECTION B

Number sense assessment

This is not a formal assessment, it's purely for academic purposes. Solve the questions based on number sense below. Show your working

1. Georgia wants to send letters to 12 of her friends. Half of the letters will need 1 page each and the other half will need 2 pages each. How many pages will be needed altogether?

2. Three thousand tickets for a basketball game are numbered 1 to 3,000. People with ticket numbers ending with 112 receive a prize. Write down all the prize-winning numbers.

3.

Ingredients	
Eggs	4
Flour	8 cups
Milk	$\frac{1}{2}$ cup

The above ingredients are used to make a recipe for 6 people. Sam wants to make this recipe for only 3 people. What does Sam need to make the recipe for 3 people?

4. Tom ate ½ of a cake, and Jane ate ¼ of the cake. How much of the cake did they eat altogether? Justify your answer.

5. In a soccer tournament, teams get:

   3 points for a win 1 point for a tie 0 points for a loss

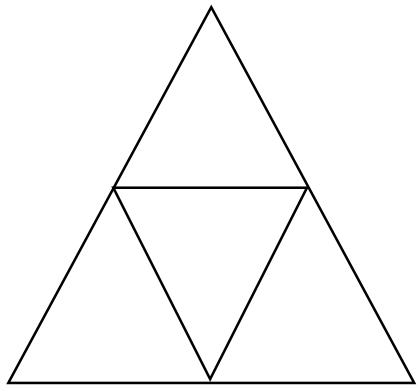
   Zedland has 11 points.

   What is the smallest number of games Zedland could have played?

6. E stands for the number of pencils Pete had. Kim gave Pete 3 more pencils. How many pencils does Pete have now?

7. Multiply the following expression. Round your answer to the nearest hundred. 9 x 22

8. Shade ½ of the large triangle. Justify your reasoning.



9. Joan had 12 apples. She ate some apples, and there were 9 left. Write a number sentence to describe what happened?

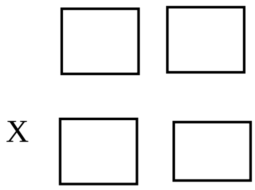
10. Write these fractions in order from least to greatest. Explain your reasoning.

$\frac{1}{2}$ $\frac{3}{5}$ $\frac{3}{10}$, $\frac{3}{8}$, $\frac{3}{6}$

11. Anna has these cards with numbers on them.

What is the smallest three-digit number she can show with the cards? She may use each card only once. Justify your answer.

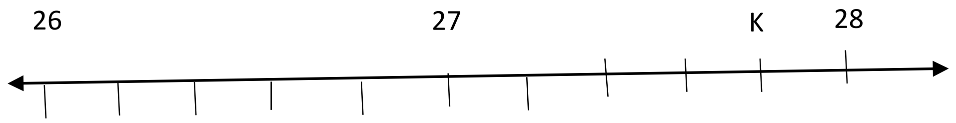
12. Place the four digits 3, 5, 7, and 9 into the boxes below in the positions that would give the greatest result when the two numbers are multiplied.



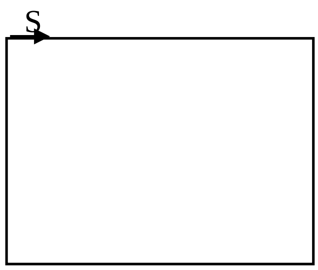
13. Kim is packing eggs into boxes. Each box holds 6 eggs. She has 94 eggs. What is the smallest number of boxes she needs to pack all the eggs?

14. Six hundred books have to be packed into boxes that hold 15 books each. Write an expression that could be used to find the number of boxes needed. Explain your reasoning.

15. What number does K represent on this number line? Justify your reasoning.



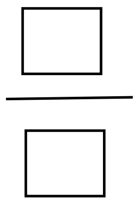
16. You are going to walk once around a square-shaped field. You start at the corner marked S and move in the direction shown by the arrow. Mark with an X where you will be after $\frac{1}{3}$ of your walk.



17. Put two of the numbers

    4, 9, 12

    in the boxes to make a fraction as close as possible to $\frac{1}{2}$



18. How much of this box is shaded? Give your answer as a fraction.

19. Here are five digits: 2, 6, 3, 5, 1. Arrange all these digits to make the smallest number possible.

20. Place the numbers $\frac{1}{10}$ and $\frac{4}{5}$ in their correct positions on the number line below

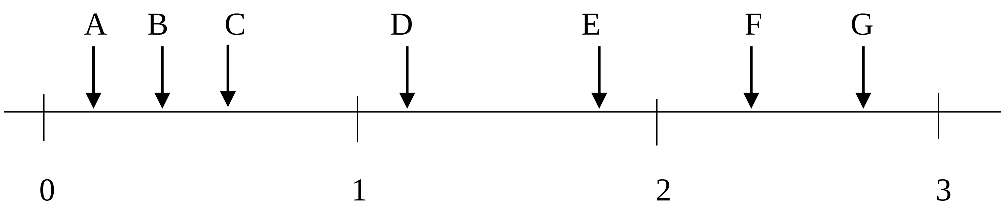
Appendix 2: Interview guide

Introduction

- We will look at 7 questions based in domain of numbers
- The questions will be displayed one by one using PowerPoint.
- I will ask you to answer the question and explain to me how you can work out the answer.
- If you cannot work out the answer, I would like you to tell me why.
- I hope it is okay with you that I record our conversation? You are welcome to listen to the conversation at the end of the interview.

1. Is $\frac{1}{2}$ closest to $\frac{3}{8}$ or $\frac{7}{13}$?

2. Which letter in the number line names a fraction where the numerator is slightly more than the denominator?



3. Which expression represents a larger amount

   2452 x 4

   2541 + 2457 + 2460 + 2465

4. Place the decimal in the answer to the following

   534.6 x 0.545 = 291 357

5. 60 x 40 is an estimate of 63 x 37. Is the exact answer less than or equal to or more than 2 400. Why , please explain your answer

6. A minibus can transport 15 people. How many minibuses would you need to transport 170 people?

7. How many different decimal numbers are there between 8.3 and 8.4?

We have reached the end of the interview, thank you taking part and God bless you.

Appendix 3: A Framework for considering basic number sense

1. Knowledge of and facility with numbers	1.1Sense of orderliness with numbers 1.2 Multiple representation of numbers. 1.3 Sense of relative and absolute magnitude of numbers. 1.4 System of benchmarks	1.1.1 Place value 1.1.2 Relationship between number types 1.1.3 Ordering numbers within and among number types 1.2.1 Graphical/symbolical 1.2.2 Equivalent numerical forms (Including decomposition/recomposition) 1.2.3 Comparison to benchmarks 1.3.1 Comparing to physical referent 1.3.2 Comparing to mathematical referent 1.4.1 Mathematical 1.4.2 Personal
2. Knowledge of and facility with operations.	2.1 Understanding the effects of operations 2.2Understanding mathematical properties 2.3Understanding the relationship between operations	2.1.1 Operating on whole numbers 2.1.2 Operating on fractions / decimals 2.2.1 Commutativity 2.2.2 Associativity 2.2.3 Distributivity 2.2.4 Identities 2.2.5 Inverses 2.3.1 Addition/multiplication 2.3.2 Subtraction/division 2.3.3 Addition/subtraction 2.3.4 Multiplication/division
3. Applying knowledge of and facility with numbers and operations to computational settings	3.1 Understanding the relationship between problem context and necessary computation. 3.2 Awareness that multiple strategies exist 3.3 Inclination to utilize an efficient representation and/or method 3.4 Inclination to review data and result for sensibility	3.1.1 Recognise data as exact or approximate 3.1.2 Awareness that solution may be exact or approximate 3.2.1 Ability to create and/or invent strategies 3.2.2 Ability to apply different strategies 3.2.3 Ability to select an efficient strategy 3.3.1 Facility with various methods (mental, calculator, paper/pencil) 3.3.2 Facility choosing efficient numbers 3.4.1 Recognise reasonableness of results 3.4.2 Recognise reasonableness of calculation

Source: McIntosh et al, 1992, p.4

Appendix 4: The rubric for interpreting the levels of number sense

Number sense Domain	**Below Basic** (0- 49%)	**Basic** (50- 59%)	**Proficient** (60- 74%)	**Advanced** (75% +)
The meaning and size of numbers both rational and irrational numbers	Possess no or minimum understanding of the meaning and size of numbers.	Possesses minimum understanding of the meaning and size of numbers.	Possesses a good/reasonable understanding of the meaning and size of numbers.	Possesses a very good/exceptional understanding of the meaning and size of numbers.
Equivalence of numbers both rational and irrational numbers	Experiences difficulties and has very poor ability to and cannot recognise different forms of representing numbers.	Experiences difficulties or struggles to recognise different forms of representing numbers	Experiences minimum or no difficulties to recognise numbers when presented in different forms	Holds very good/excellent ability to recognise different forms of representing numbers.
Meaning and effects of operations	Demonstrates very minimum understanding and struggles to recognise and understand the meaning and effect of operations	Demonstrates limited grasp and often struggles to recognise and understand the meaning and effect of operations	Demonstrates a reasonable grasp and often recognises and apprehends the meaning and effect of operations.	Demonstrates an outstanding grasp and regularly apprehends the meaning and effect of operations with minimum or no problems.
Counting and computational strategies	Holds little or lack of knowledge in counting and computing tactics and relies on a calculator heavily to most if not all the sums	Holds little competence in counting and computational strategies and sometimes relied on a calculator to do many of the sums	Holds reasonable competence in counting and computational strategies and sometimes used a calculator only when necessary.	Holds counts proficiently, is fast paced and totally in command of the count.
Estimation using relevant benchmark without calculating	Cannot make any reasonable estimation and cannot identify a benchmark as a referent point	Finds it difficult to make reasonable estimations and does not find it easy to identify a reasonable benchmark as a referent point on estimation.	Can often make reasonable estimations and does not find it difficult to identify a reasonable benchmark as a referent point of estimation	Makes very reasonable and accurate estimations and identifies very reasonable and accurate benchmark as a referent point of estimation.