The University of Zambia

Final examination papers

School of natural sciences

2012– 2013

1. BIO 5102 Biosystematics of tropical animal taxa theory paper
2. BIO 5122 Biodiversity assessment and management theory paper
3. BIO 5452 Insect- plant host and insect- animal- host relationship theory paper
4. BIO 5165 Ecology and management of tropical wetlands theory paper
5. BIO 5492 Forest/ woodland pest management theory paper II
6. BIO 5502 invertebrate systematic theory paper I
7. BIO 5522 fresh water entomology theory paper I
8. BIO 5532 Taxonomic methods theory paper II
9. C 5722 medical chemistry II
10. CHE 5011 General chemical techniques
11. CHE 5222 electrochemical and chromatographic methods
12. CHE 5612 thermo-electrodynamics of solutions
13. CHE 5635 introduction to statistical thermodynamics
14. GEO 5702 Advance quantitative geography
15. GEO 5802 the philosophy and methodology of geography
16. MAT 5111 ordinary Differential equation and integral equations
17. MAT 5122 Partial differential equations
18. MAT 5141 Tropics in mathematics methods
19. MAT 5311 Lebesgue measure and Lebesgue integration
20. MAT 3331 Functional Analysis
21. MAT 5352 Tropics in Analysis
22. MAT 5611 statistical inference
23. MAT 5632 Design and analysis of experiments

24.	MATS 5642	statistical methods in epidemiology
25.	MATS 56 62	theory of Non-parametric statistics
26.	MAT 5911	mathematics and statistics
27.	PHY 5022	mathematical methods for physics
28.	PHY 5222	condensed matter physics II
29.	PHY 5822	solar energy materials
30.	PHY 5911	computational physics and modelling I
31.	PHY 5922	computational physics modelling II

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2009-2010 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

BIO5102: BIOSYSTEMATICS OF TROPICAL ANIMAL TAXA
THEORY PAPER

TIME: THREE (3) HOURS

INSTRUCTIONS: ANSWER **FIVE** QUESTIONS. ILLUSTRATE YOUR ANSWERS WHERE NECESSARY.

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1. Discuss determinants of the most primitive metazoan and name the most likely candidate of such an organism amongst the extant invertebrates. In your answer also explain the relationship of the named invertebrate group to choanoflagellate protozoa.

2. Discuss the traditional classification of the Protozoa and explain the relationship of this animal grouping to the invertebrates.

3. Discuss the views regarding the monophyletic and polyphyletic origins of the Arthropods.In your view indicate which is likely to be true concerning the origin of this heterogenous group among the invertebrates.

4. Discuss the purpose of taxonomy, and describe rules contained in the International Code of Zoological Nomenclature (ICZN).

5. Discuss difficulties of the species concept in the classification of birds and amphibians.

6. Briefly describe the following fish morphological structures and organs in relation to the identification and description of fish species:
   (a) Trunk
   (b) Caudal fin
   (c) Mouth
   (d) Barbels
   (e) Scales

7. Describe discuss and the following:
   (a) Main characteristics of the family Cichlidae;
   (b) Sub-groups of the family Cichlidae
   (c) Indicate sub-group of the family Cichlidae used in fish farming and give reasons for use of the subgroup in small scale aquaculture

TURN OVER

8. Discuss the characteristic and significance of the family Latidae in fisheries of Zambia. Provide some names of fish species of this family to demonstrate knowledge of the group.

9. Discuss key characteristics of the family Cyprinidae and describe key features that are used to identify species within this family. Provide species names some fish species as a way of demonstrating familiarity of the family.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2009-2010 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

BIO BIO5122 BIODIVERSITY ASSESSMENT AND MANAGEMENT
THEORY PAPER

TIME: THREE (3) HOURS

INSTRUCTIONS: ANSWER **FIVE** QUESTIONS. ILLUSTRATE YOUR ANSWERS WHERE NECESSARY.

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1. The simplest measure of species diversity is to count the number of species in a given area. Discuss the main limitations and assumptions of this approach, and in you discussion, explain the main sources of experimental error.

2. Compare and contrast *Alpha* diversity and *Beta* diversity of species in a given area.

3. Discuss the main advantages in using the Shannon Index instead of simply a population count to determine diversity.

4. Explain why a species is the acceptable unit of measurement of biodiversity. Discuss difficulties associated with this approach.

5. Wallace A.R. (1878) recognized that there were more species in the tropics than other parts of the world. Discuss hypotheses that refute or corroborate with this claim.

6. Discuss the significance of biodiversity in Zambia, and explain why some species are rapidly disappearing.

7. Describe the Biodiversity Convention and discuss its limitations in the conservation of species.

8. Discuss the gene and ecosystem levels of measuring biodiversity and discuss limitations and advantages associated with measuring each of these categories.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2009-2010 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

BIO 5452: INSECT-PLANT HOST AND INSECT-ANIMAL-HOST RELATIONSHIPS
THEORY PAPER 1

TIME: THREE HOURS

INSTRUCTIONS: ANSWER **FIVE** QUESTIONS. TWO QUESTIONS FROM SECTION A, TWO QUESTIONS FROM SECTION B AND THE FITH QUESTION FROM EITHER SECTION.

SECTION A

1. Discuss the process of host finding in a named polyphagous lepidopteran insect.

2. Summarise **three** of the following concepts:

   (a) Host plant resistance.
   (b) Lotka-Voltera equations.
   (c) Ant-acacia mutualism.
   (d) Alkaloids in plant defence.

3. Plant defence against herbivory describes a range of adaptations evolved by plants. Citing examples, discuss the various strategies (physical and chemical) used by plants to defend themselves against herbivores.

4. Discuss the theory of mutualism with examples.

SECTION B

5. Critically discuss the prospects and constraints of integrating host plant resistance (HPR) in integrated pest management (IPM) for a resource poor farmer.

6. Describe insect semiochemicals, outlining their possible utilisation in An a agroecosytem.

7. Describe a standardized host-plant selection sequence, terminating in the insects departure from the host plant.

8. Discuss the factors affecting the expression of plant resistance to arthropods.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
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2009-2010 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

BIO 5165: ECOLOGY AND MANAGEMENT OF TROPICAL WETLANDS
THEORY PAPER

TIME: THREE (3) HOURS

INSTRUCTIONS: ANSWER **FIVE** QUESTIONS. ILLUSTRATE YOUR ANSWERS WHERE NECESSARY.

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1. Compare and contrast the main differences between the Riverine and Palustrine wetland habitat systems.

2. Explain the nutrient cycling function of a fresh water flood plain in an arid environment in Southern Africa and discuss problems that may be associated with management of such wetlands.

3. Discuss major threats of any **TWO** of the following wetlands of Southern Africa:
   (a) Rufigi delta in Tanzania
   (b) Bangweulu swamps in Zambia
   (c) Cuanza in Angola
   (d) Shire marsh in Malawi

4. Discuss features which would indicate that a *dambo* wetland system was being overexploited, and prescribe measures you would recommend in the restoration of such a wetland.

5. Describe the economic values of wetland ecosystems and possible impact of downstream dam development on the Kafue Flats.

6. Construct and discuss the wetland hydrological model of a lacustrine environment in Southern Africa

7. Discuss the concept of *wise use* of wetlands as it relates to the conservation of wetlands in Zambia.

8. Discuss the main functions of wetlands and give reasons why most wetlands are being degraded.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
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2009-2010 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

BIO 5492: FOREST/WOODLAND INSECT PEST MANAGEMENT
THEORY PAPER II

TIME: THREE HOURS

INSTRUCTIONS: ANSWER **FIVE** QUESTIONS. TWO QUESTIONS FROM EACH SECTION AND THE FITH QUESTION FROM EITHER SECTION. GIVE EXAMPLES OR ILLUSTRATIONS WHERE NECESSARY.

SECTION A

1. Discuss some methods used in the prevention of forest insect outbreaks and their applicability in Zambia

2. Summarise **each** of the following concepts:
   a) Individual tree factors
   b) Stand factors
   c) Forest farming
   d) Functional diversity

3. Outline the principles of pest control in Forestry management.

4. Give an account of the conditions of the beetles, *Phoracantha. Semipunctata* (Fabricius) and *P. recurva* Newman, which led to their outbreaks in Zambia during 1980-1983.

5. Outline some sivicultural practices, which a Silviculturist should use in an an attempt to maintain vigorous growth of trees and minimize insect damage.

SECTION B

6. Evaluate the procedures used in risk assessment of forest insects and their relevancy to the Zambian situation.

7. The white lerp *(Cardiaspina albitextura* Taylor) is a serious forest insect pest in southeastern Australia.
   (a) Describe the life cycles of this insect.
   (b) Describe the outbreak dynamics of this insect.

8. Classify and describe the life cycles of some named insect pests, commonly found in Zambian tree plantations and comment on their pest statuses.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
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2009-2010 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

BIO 5502: INVERTEBRATE SYSTEMATICS
THEORY PAPER I

TIME: THREE HOURS

INSTRUCTIONS: ANSWER **FIVE** QUESTIONS, TWO QUESTIONS FROM EACH SECTION AND THE FIFTH QUESTION FROM EITHER SECTION

SECTION A: Overview of the Science of Systematics

1. You are provided with table 1 below showing characters and character states of taxa taken from a hypothetical invertebrate family called ALPHABETIDAE. Using Wagner's method, construct a cladogram showing the relationships of the taxa.

Table 1. Character states of members of the Alphabetidae.

TAXON	**CHARACTER**									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Outgroup	0	0	0	0	0	0	0	0	0	0
A	1	0	0	0	1	1	0	0	0	1
B	1	0	0	0	1	0	0	0	0	1
C	0	0	0	0	0	0	0	0	1	1
D	0	1	1	0	0	0	1	1	0	1
E	0	1	1	1	0	0	0	1	0	1

2. Compare and contrast four modern concepts of a species and explain the difficulties encountered when applying them.

3. Using the Hennig argumentation method, construct a cladogram to show the phylogenetic relationships of the taxa shown in table 2 below.

Table 2. Character states of hypothetical taxa

TAXON	**CHARACTER**									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Outgroup	0	0	0	0	0	0	0	0	0	0
Alpha (α)	0	0	0	1	1	1	0	0	0	0
Beta (β)	0	0	0	1	0	1	0	0	0	0
Gamma (γ)	0	0	0	0	0	0	1	0	1	0
Delta (δ)	1	1	1	0	0	0	1	1	1	0
Epsilon (ε)	1	1	1	0	0	0	1	0	1	1
Zeta (ξ)	1	1	0	0	0	0	1	0	1	1
Theta (θ)	1	0	0	0	0	0	1	0	1	1

TURN OVER

4. Compare and contrast views of the traditional, phenetical and cladistical schools of *thought on invertebrate systematics, indicating which of the schools presently has greater* following and why.

SECTION B: Invertebrate Evolution

5. Discuss evidences that exist in support Wegener's (1912) theory of continental drift and explain how this phenomenon helps explain the phylogeny of invertebrates.

6. Distinguish between syncytial and colonial theories on the origin of the Metazoa and in your answer explain which of the theories have greater backing and why.

7. Discuss the phylogenetic positioning of the following groups among the invertebrates:
   (a) Arthropoda
   (b) Mollusca
   (c) Lophophorates

8. Discuss determinants of the major branches of the animal kingdom and show the state of these determinants in the invertebrate phyla Annelida and the Echinodermata.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2009-2010 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

BIO 5522: FRESHWATER ENTOMOLOGY
THEORY PAPER I

TIME: THREE HOURS

INSTRUCTIONS: ANSWER **FIVE** QUESTIONS, TWO QUESTIONS FROM EACH SECTION AND THE FIFTH QUESTION FROM EITHER SECTION

SECTION A: Freshwater Habitat Diversity and Characteristics

1. Describe the biologically relevant physical properties of freshwater and show how aeropneustic freshwater insects obtain O_2.

2. Discuss the ecological classification of freshwater habitats, their life zones and the insect groups associated with them.

3. Describe the aquatic nitrogen cycle highlighting the reactions involved in the cycle and the roles of named freshwater insect groups.

4. Discuss anthropogenic disturbances of streams, rivers, dams and lakes and their impacts on freshwater insect fauna and in your discussion indicate which insect groups are important bio-indicators of freshwater ecosystem conditions

SECTION B: Insect Adaptations to Freshwater Habitats

5. Discuss the basic morphological features common to all freshwater insect taxa and related to movement, feeding and dealing with the water currents.

6. Summarise the following physiological insect adaptations to freshwater living:
   (a) Plastron respiration
   (b) Respiratory siphons
   (c) Physical gill
   (d) Nitrogenous wastes excretion
   (e) Cryoprotectants secretion

7. Explain the phylogenetic relationships of freshwater insects with other freshwater Arthropods and Invertebrate groups.

TURN OVER

8. Discuss physical and physiological features that permit insects to colonize a diversity of freshwater environments.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
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2009-2010 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

BIO 5532: TAXONOMIC METHODS
THEORY PAPER II

TIME: THREE HOURS

INSTRUCTIONS: ANSWER **FIVE** QUESTIONS, TWO QUESTIONS FROM EACH SECTION AND THE FIFTH QUESTION FROM EITHER SECTION

SECTION A: Phenetic Taxonomic Methods

1. Discuss the basis and the founding principles of phenetic taxonomy and indicate its shortcomings when classifying taxa.

2. You are provided below, a Euclidean distance matrix comparing 6 taxa. Using the Unweighted Pair Group Method with Arithmetic Mean (UPGMA), construct a phenogram to show the phenetic relationships of the taxa.

	A	B	C	D	E
B	2				
C	4	4			
D	6	6	6		
E	6	6	6	4	
F	8	8	8	8	8

3. Discuss the different types of hierarchical clustering algorithms used in classifying organisms.

4. Discuss the advantages and disadvantages of using numerical taxonomy in organismal classification.

SECTION B: Cladistic Taxonomic Methods

5. Explain the following principles that relate to Cladistics:
   a. Hennig's auxiliary principle.
   b. Grouping rule.
   c. Inclusion/exclusion rule.
   d. Rule of parsimony.

TURN OVER

6. You are provided with the following data matrix involving Manhattan distances between OTU pairs. Using neighbour joining method, create a cladogram showing the relationship of the five OTUs.

	A	B	C	D	E
A	-				
B	5	-			
C	2	3	-		
D	4	3	6	-	
E	3	2	1	5	-

7. Discuss the limitations of cladistics analysis.

8. Discuss under what circumstances classification systems change and why despite its limitations the phenetic classification system still finds its uses in contemporary cladistics.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
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2009-2010 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

BIO 5532: TAXONOMIC METHODS
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END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2009-2010 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

BIO 5532: TAXONOMIC METHODS
THEORY PAPER II

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B	5	-			
C	2	3	-		
D	4	3	6	-	
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END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2010 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

C5722: MEDICINAL CHEMISTRY II

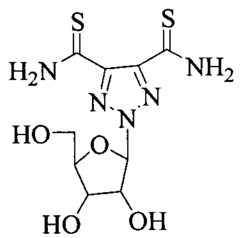
TIME: THREE HOURS

INSTRUCTIONS:
1. Answer any four questions.
2. Marks allocation for questions is shown (x)

Question 1

(a) Provide a rationale for the clinical application of dihydrofolate antagonosists in the chemotherapy of human neoplastic diseases. **(7 marks)**

(b) Suggest a synthesis of the anticancer drug **A**, shown below, from the indicated starting material,**B**, and any readily available reagents/ chemicals. **(14 marks)**

A: 

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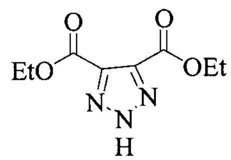
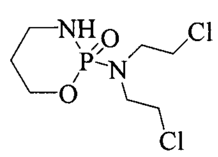
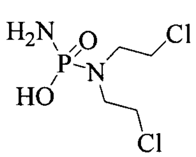
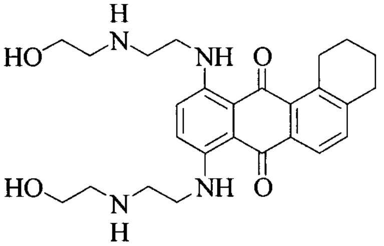
B: 

(c) Propose a plausible sequence of reactions for the biotransformation of the anticancer drug **C** into the bioactive metabolite **D**, structures shown below. **(5 marks)**

C: 

D: 

(d) State the mode of anticancer action of the drug **E**, shown below. **(4 marks)**

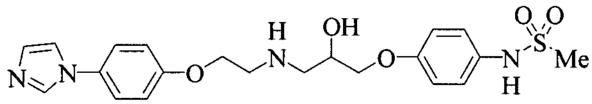
E: 

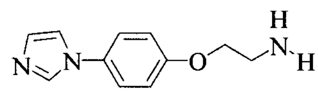
Question 2

(a) Give the structures of two naturally occurring cardiac glycosides currently used for the management of congestive heart failure(CHF). **(4 marks)**

(b) Anti-arrhythmic drugs with both class II and class III anti-arrhythmic activities reduce the incidence of sudden cardiac death in arrhythmia patients as compared to drugs with only class II or only class III anti-arrhythmic activity. Explain. **(6 marks)**

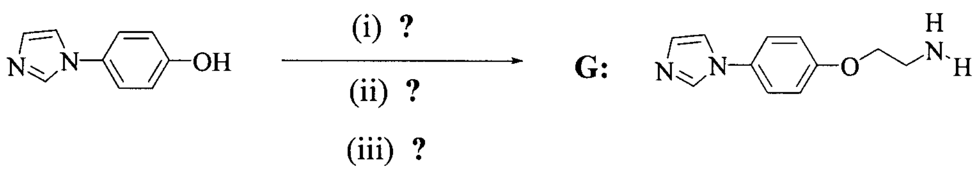
(c) The anti-arrhythmic agent **F**, shown below, exhibits both class II and class III anti-arrhythmic activities.

F: 

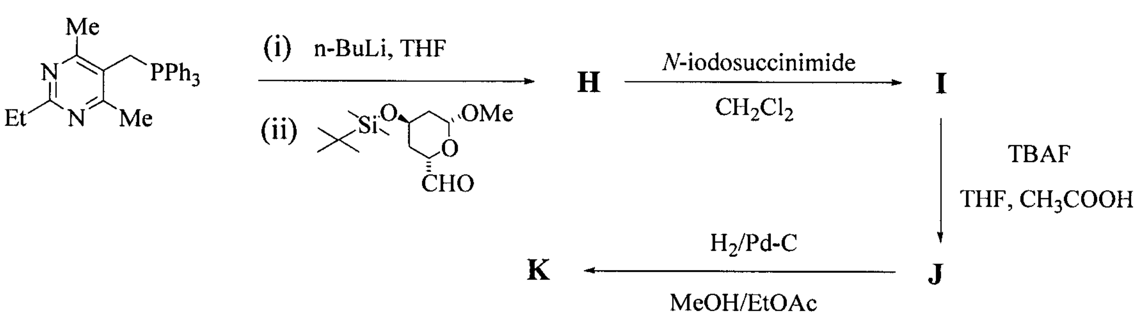
G: 

(i) Devise a synthesis of **F** from the key intermediate **G**, shown above. **(9 marks)**

(ii) Provide the missing reagents for the following synthesis of the key intermediate **G**. **(3 marks)**

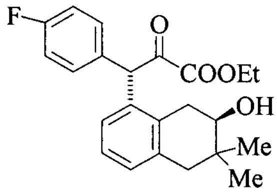


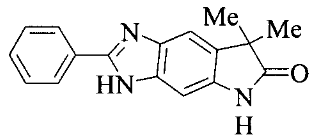
(d) Deduce the structures of the intermediates **H-J** and that of the cardiovascular drug (vasodilator) **K** from the following synthesis. **(8 marks)**

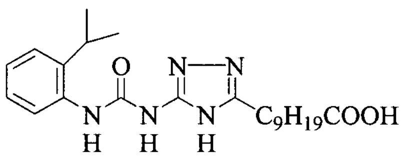


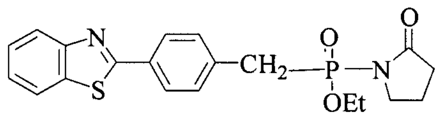
Question 3

(a) Consider the structures of the four drug molecules shown below:

L: 

M: 

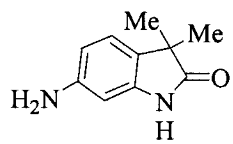
N: 

O: 

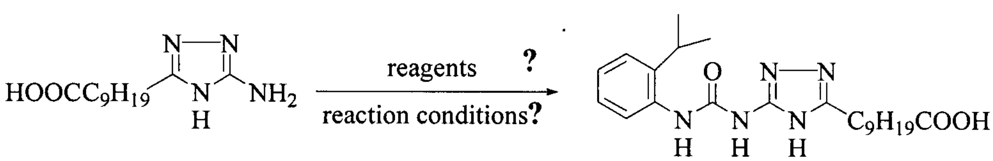
(i) State the principal usage of the drugs **L**, **M**, **N** and **O**. **(1 mark each)**

(ii) Briefly explain the mode of action of **L**. **(4 marks)**

(iii) Devise a synthesis of **M** from the following starting material and any readily available reagents/chemicals. **(12 marks)**

M from 

(iv) Provide missing intermediates and reagents in the following synthesis of **N**: **(4 marks)**



N

(c) Explain the rationale for the use of acyl CoA:cholesterol O-acyltransferase, ACAT, inhibitors in the treatment of atherosclerosis. **(6 marks)**

Question 4

(a) Mention six factors that affect blood pressure. **(3 marks)**

(b) Calcium channel blockers are often used for the treatment of hypertension.

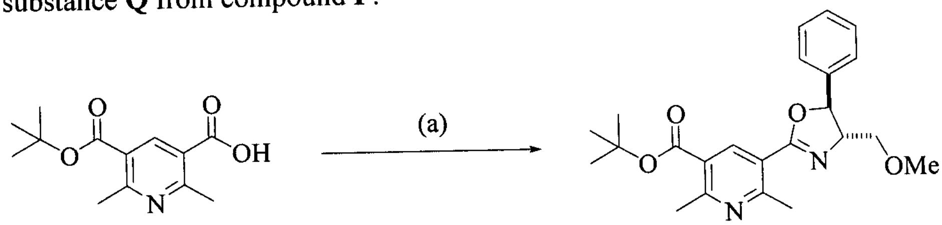
(i) Give a classification for calcium channel blockers.

(ii) Explain the rationale for their use as hypotensive drugs.

(7 marks)

(c) State five major classes of diuretics and write short notes on any two of them. **(10 marks)**

(d) A substance **Q**, structure shown below, is the key intermediate in an asymmetric synthesis of chiral dihydropyridine calcium channel blockers. It is prepared from the compound **P** by the following reaction. Provide a mechanism for the formation of the substance **Q** from compound **P**.



Compound P **Substance Q**

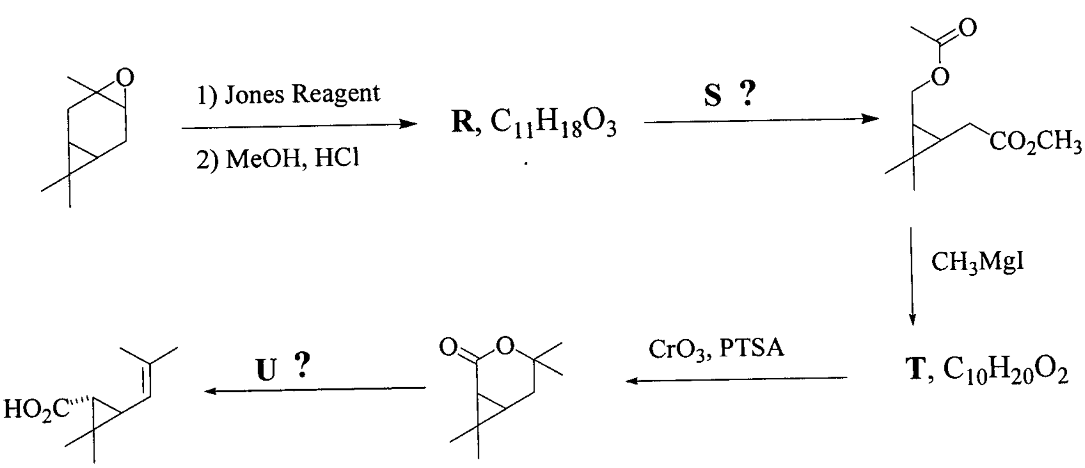
(a) PPh_3, CCl_4 /NEt_2 /2-amono-3-methoxy-1-phenyl-1-propaanol/DEAD, CH_3CN /pyridine (1:1)

(10 marks)

Question 5

(a) Give three principal classes of insecticides based on chemical structures and write short notes on the mechanisms of actions of **any two** of them. **(9 marks)**

(b) (i) Provide the structures of the missing intermediates and reagents in the following synthesis of the acid moiety **V** of pyrethrin I, a natural product with potent insecticidal activity.



ACID MOIETY : V

(ii) Give the mechanism for the last step. **(15 marks)**

(c) Discuss the structure-activity relationships in the N-(2-aminoethyl)-5-isoquinoline sulfonamide coronary vasodilators. **(6 marks)**

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
2010 ACADEMIC YEAR FIRST SEMESTER FINAL EXAMINATION
CHE 5011 GENERAL CHEMICAL TECHNIQUES

Duration: Three (3) Hour

Instructions:

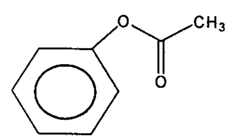
Answer all questions
Answer each question in a separate booklet

Question 1

(a) (i) What is meant by *chiral shift reagents*?
(ii) Briefly explain how chiral shift reagents can be used for determining enantiomeric excesses in a mixture of enatiomers.

(b) Briefly explain what is meant by 'NOE' difference spectrum' and state its applications.

(c) Fries rearrangement of the o-acetyl phenol



gave a compound X, $C_8H_8O_2$. The 13 CNMR spectra (a), (b) and (c) (attached) were recorded in a mixed solvent, $CDCl_3$: $(CD3)Cl_3$ (1:1) at 25 °C, 20 MHz.

Spectrum (a) : ^1H broadband decoupled spectrum
Spectrum (b) : NOE enhanced coupled spectrum (gated decoupling)
Spectrum (c) : Expanded spectrum (b)

(i) Analyse the spectra and identify compound X.
(ii) From the spectral data, what can you say about the regioselectivity of migration in Fries rearrangement.

Question 2

The capillary viscometers are used for the determination of viscosity and molecular mass of polymers.

(a) Describe how to determine relative viscosity using the Ostwald's viscometer and write down the equation for calculation.

(b) What are the disadvantages of the capillary viscometers used for the relative viscosity measurements?

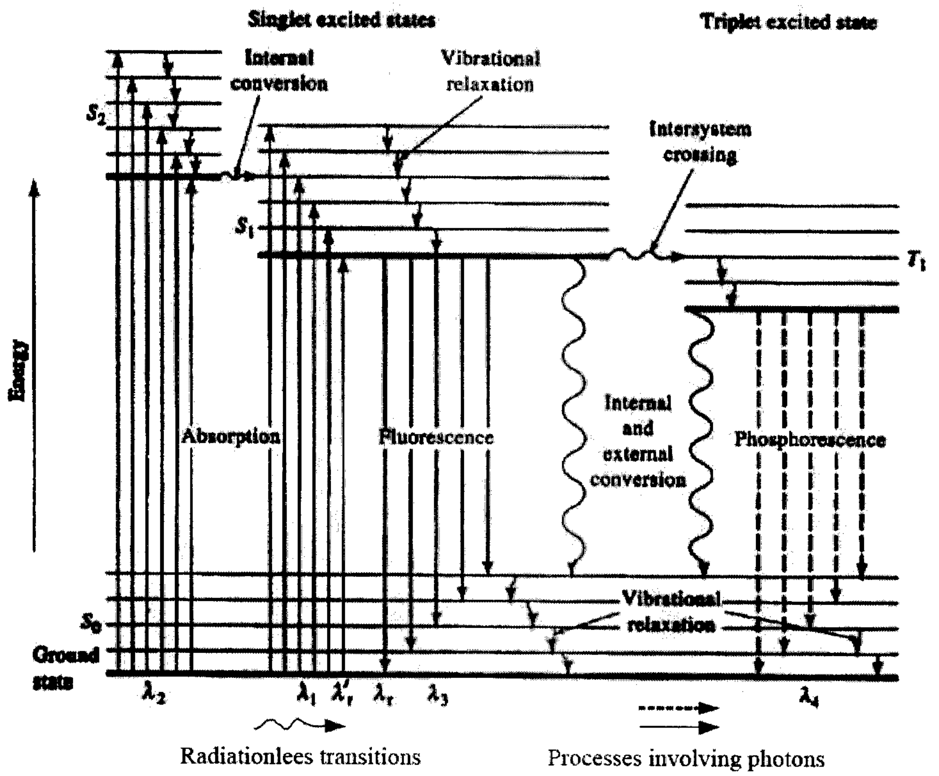
(c) Outline the rotational viscometers and their practical application.

(d) Describe the following properties of light: (i) speed of light in vacuum and air (ii) difference between natural and artificial light (iii) monochromatic light.

(e) What do you understand by the term light 'scattering effect'.

Question 3

The diagram below shows a partial energy diagram for a photoluminescence system (Jablonski diagram).



(a) Use the diagram to explain the various phenomenon indicated in the diagram.

(b) Explain how phosphorescence can be used to determine the bond energies of a particular molecule.

— **END OF EXAMINATION** —

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2011 ACADEMIC YEAR SECOND SEMESTER

FINAL EXAMINATIONS

CHE 5111 Micro- & Macromolecular Biochemistry

INSTRUCTIONS:

Answer any **FOUR (4)** questions

Time: **THREE (3)** hours

All questions carry **EQUAL** marks

Question 1

Write short notes on the following explaining how they are used in DNA manipulating technology:

a) Restriction enzyme
b) Deoxyribonucelase
c) DNA ligase
d) DNA polymerase III

Question 2

Suppose you live in a country where genetic engineering research is not permitted. **Write** an essay that should convince your government to reconsider its decision on this matter. In your essay, **give** clear scientific reasons why this kind of research must be conducted. Also **allay** any fears that your government might have.

PLEASE TURNOVER THE PAGE

Question 3

i) Palmitic acid synthesis from acetyl CoA occurs in the cytoplasm. However, acetyl CoA production occurs in the mitochondrial matrix. Acetyl CoA cannot tranverse the mitochondrial membrane system. **Explain** in detail how the fatty acid synthesizing machinery obtains its supply of acetyl CoA.

ii) A person is on a fat-free diet and yet becomes obese. **What** might be the explanation?

iii) **Explain** in detail the role of fatty acids in membrane architecture.

Question 4

Humans tend to suffer from gout.

i) **What** biochemical process is altered in persons that suffer from gout and how might this condition be managed?

ii) It known that most animals do not suffer from gout. **What** might be the explanation?

Question 5

Explain how prokaryotes like *E. coli* response to changes in the environment using the *lac operon.* **Can** this kind of response take place in a eukaryotic cell? If not, why not?

Question 6

Explain in detail protein synthesis. **Highlight** the source of energy in this process.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2009/10 ACADEMIC YEAR SECOND SEMESTER

FINAL EXAMINATIONS

CHE 5222: ELECTROCHEMICAL AND CHROMATOGRAPHIC METHODS

TIME: THREE HOURS

ANSWER ALL QUESTIONS

Question 1 Describe process instruments and automated analysis.

Question 2 Explain principle automatic analyzers in a modern laboratory.

Question 3 Write down the basic principle replacement of human effort and senses.

Question 4 Describe the basic principle of laboratory robots.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

DEPARTMENT OF CHEMISTRY

UNIVERSITY SEMESTER II SESSIONAL EXAMINATIONS

30 APRIL 2010

CHE 5612: THERMO–ELECTRODYNAMICS OF SOLUTION

TIME: THREE HOURS

ANSWER ANY FIVE OF THE SIX QUESTIONS.

DATA

F = 96485 C. mol^{-1}; For solvent water, A = 0.509 mol$^{-1/2}$.kg$^{-1/2}$; R = 8.314 J. K^{-1}.mol^{-1}; Density of 5% Catsle Lager Beer = 991.2 kg.m^{-3}; g = 9.81 m.s^{-2}; 1Pa = 1 N.m^{-2}; 1 N = m. kg.s^{-2}

QUESTION 1

a). Write the equation for the partial molar quantity, $\overline{F_i}$ for an arbitrary thermodynamic function F. Is F an extensive or an intensive function?

b). The addition of one mole of $H_2O(\ell)$ to a very large quantity of a solution of composition 0.4 mole fraction $CH_3OH(\ell)$ and $H_2O(\ell)$ increases the volume of solution by 17.35 cm^3. The addition of one mole $CH_3OH(\ell)$ to the same solution increases the volume by 39.01 cm^3. Calculate the volume of a solution made of 0.4 moles $CH_3OH(\ell)$ and 0.6 moles $H_2O(\ell)$. Define all symbols used in the equation(s).

QUESTION 2

Consider the following change in state at T = 298.15 K and p = 1.00 bar:

$$n\, H_2O(\ell) \;+\; H_2SO_4(\ell) \;\leftrightarrow\; (H_2SO_4.nH_2O)(\text{solution})$$

The enthalpy change is experimentally determined to be:

$$\Delta H = -\frac{78659}{n+1.8} \quad \text{J for } n \leq 20$$

For $n \rightarrow \infty$, ΔH = - 92,048 J

a). Write an expression for the integral heat of solution in terms of partial molar enthalpies and enthalpies of the pure substances.

b). For the following changes in state define the type of heat involved and calculate ΔH:

(i). $H_2SO_4(\ell) + 10\, H_2O(\ell) \leftrightarrow (H_2SO_4.10H_2O)(\text{solution})$

(ii). $(H_2SO_4.10H_2O)(\text{solution}) + H_2SO_4(\ell) \leftrightarrow (H_2SO_4.10H_2O)(\text{solution})$

(iii). $(H_2SO_4.10H_2O)(\text{solution}) + H_2O(\ell) \leftrightarrow (H_2SO_4.11H_2O)(\text{solution})$

QUESTION 3

Consider the Daniel cell at 25 °C and p = *1.00 bar.*

$$Zn(s)|Zn^{2+}(aq, m = 0.003)||Cu^{2+}(aq, m = 0.001)|Cu(s)$$

Under these conditions ΔG^o = - 212.7 *k*J. mol^{-1}. Calculate

a). the ionic strengths of the solutions,

b). the mean ionic coefficients in the half cells,

c). the reaction quotient, Q,

d). the standard cell Emf, and

f). the cell Emf.

Take $\gamma_+ = \gamma_- = \gamma_\pm$

QUESTION 6

The chemical potential of an electrolyte is given by the equation

$$\mu_{el.} = \mu^{o}_{el.} + \nu RT\ln m_{\pm} + \nu RT\ln\gamma_{\pm}$$

a). Identify all the terms in the equation.

b). Briefly explain why $\mu_{el.} - \mu_{ideal}$ is negative.

c). Write the Debye-Hückel Limiting law for a 1:1 electrolyte and define all the terms in the equation. State any two assumptions made in its derivation. Over what range of concentrations is the law valid?

d). Emf measurements in water and in methanol at 298 K for an electrolyte $M^{+}X^{-}$(ac gave the following results for the values of the activity coefficient of HCℓ:

m	$\gamma_{\pm}$	
	$H_2O(\ell)$	$CH_3OH(\ell)$
0.002	0.95	0.81
0.005	0.92	0.72
0.008	0.90	0.66

Show that the Debye-Hückel Limiting law is valid in both water and methanol at the given molalities.

END OF CHE 5612 EXAMINATION

QUESTTION 4

a). Define contact angle θ by drawing a diagram to indicate how it is measured. Label all the relevant parameters.

b). The surface tension of Castle Lager Beer, which is 5 % ethanol in water, is 54.2×10^{-3} N. m^{-1} at 30 °C. If the change in pressure of a drop of Castle Lager Beer is 125.2 *k*Pa at this temperature, calculate the radius of the drop.

c). The contact angle for Castle Lager Beer was measured to be 30°. Calculate the height to which Castle Lager Beer would rise in a capillary tube whose radius is 2.5 mm. Assume that the beer does not foam.

QUESTION 5

a). Derive the Kelvin equation. In your derivation clearly state any assumptions you make.

b). What are the major differences in the Kelvin equation derived for a drop of liquid and for a bubble?

c). Explain the major difference between the Laplace and the Kelvin equations.

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2010 ACADEMIC YEAR FIRST SEMESTER

FINAL EXAMINATIONS

CHE 5635: INTRODUCTION TO STATISTICAL THERMODYNAMICS

TIME: THREE HOURS

INSTRUCTIONS: ANSWER QUESTION ONE AND ANY *THREE* OTHERS

DATA

$\mathbf{h = 6.626 \times 10^{-34}\ J\ s;\ c = 3.00 \times 10^{8}\ m\ s^{-1};\ }k_B\mathbf{ = 1.38 \times 10^{-23}\ J\ K^{-1};\ }N_A\mathbf{ = 6.022 \times 10^{23}\ mol^{-1};}$

$\mathbf{O = 16.00;\ }u\mathbf{ = 1.661 \times 10^{-27} kg;\ }V_m\mathbf{(298\ K) = 22.4 \times 10^{-3}\ m^3\ mol^{-1};}$

$$q_{vibrational} = \sum_1^2 e^{-\frac{\varepsilon_n}{kT}} = 1 + e^{-\frac{hc\sigma}{kT}} + e^{-\frac{2hc\sigma}{kT}}\ ;\ P_i = \frac{g_i e^{-\frac{\varepsilon_i}{kT}}}{\sum g_i e^{-\frac{\varepsilon_i}{kT}}}\ ;$$

$$q_{translational} = \left(\frac{2\pi mkT}{h^2}\right)^{\frac{3}{2}} V$$

QUESTION 1 (*40 Marks*)

a). For CH_4 (g) at 298 K, the spectroscopic (statistical) entropy $S_{spec.}$ agrees within experimental error with the calorimetric entropy S_{cal}. For CH_3D (g) at 298 K, the value $(S_{spec.} - S_{cal.})$ is 11.59 ± 0.21 J K^{-1} mol^{-1}. Give a quantitative explanation of these data in terms of possible disorder in the solid methane at 0 K.

b). The vibrational frequency of I_2 is $\sigma = 208$ cm^{-1}. Calculate

(i) the vibrational partition function of I_2 with energy levels n = 1 and 2.

(ii) the probability of I_2 populating the n = 2 vibrational energy level at 298 K.

c). (i). State the condition necessary for the applicability of Boltzmann statistics to a system of particles.

(ii). Apply this condition to show whether or not Boltzmann statistics are applicable to a system of one mole of oxygen molecules at 298 K.

d). Distinguish between the number-average, viscosity-average, weight-average and Z-average molar masses. Discuss what experimental techniques can measure each of these.

e). Sections of solid fuel rocket boosters of the space shuttle Challenger were sealed together with O-ring rubber seals of circumference 1 m. The seals failed at 10 °C, a temperature well above the crystallization temperature of rubber. Speculate on why the failure occurred.

QUESTION 2 (*20 Marks*)

Consider a microcanonical ensemble system that has a non-degenerate ground state energy and a single excited state energy ε with degeneracy g.

a). Derive an expression for

(i) the probability of occupation of the ground state;

(ii) the probability of occupation of the excited state.

b). Derive an expression for the Helmholtz free energy.

QUESTION 3 (*20 Marks*)

a). On assumption that the tension required to keep a sample at constant length is proportional to temperature (t = aT, $p \propto T$). Show that the tension can be ascribed to the dependence of the entropy on the length of the sample. Account for this result in terms of molecular nature of the sample.

b). Answer **either** question (i) **or** (ii)

(i) Consider the thermodynamic description of stretching rubber. The observables are tension, *t*, and length *l* (the analogues of p and *V* for gases). Because $dw = tdl$, the basic equation is $dE = TdS + tdl$. (The term pdV is supposed to be negligible throughout). If $G = E - TS - tl$, find the expression for dG and dA and deduce the following Maxwell relations:

$$\left(\frac{\partial S}{\partial l}\right)_T = -\left(\frac{\partial t}{\partial T}\right)_l ; \quad \left(\frac{\partial S}{\partial t}\right)_T = -\left(\frac{\partial t}{\partial T}\right)_l$$

Go on to deduce the equation of state for rubber:

$$\left(\frac{\partial E}{\partial l}\right)_T = t - \left(\frac{\partial t}{\partial T}\right)_l$$

(ii) The viscosities of solutions of polyisobutylene in benzene were measured at 24 °C with the following results:

$c/(g/10^2 cm^3)$	0	0.2	0.4	0.6	0.8	1.0
$\eta/10^{-3} kg\ m^{-1} s^{-1}$	0.647	0.690	0.733	0.777	0.821	0.865

Deduce the molar mass of the polymer given that $K = 8.3 \times 10^{-2}\ cm^3\ g^{-2}$ and $\alpha = 0.50$.

QUESTION 4 (*20 Marks*)

Consider the equation $\overline{n_i} = \dfrac{1}{e^{(\varepsilon_i - \mu)/kT} \pm 1}$

a). Define all the symbols in the above equation.

b). Distinguish between fermions and bosons, giving one example of each. In your distinction, cite two properties that distinguish one from the other.

c). (i) Write the equation which is applicable to Fermi-Dirac (FD) statistics.

(ii) Write the equation which is applicable to Bose-Einstein (BE) statistics.

d). Sketch a labeled diagram of $\overline{n_i}$ as a function of $(\varepsilon_i - \mu)/kT$ for each of FD and BE statistics. On the same diagram sketch $\overline{n_i}$ as a function of $(\varepsilon_i - \mu)/kT$ for the limiting case of the Boltzmann statistics.

e). Using the sketch diagram in part d), discuss the behavior of $\overline{n_i}$ in the limit of

(i) $T \rightarrow 0\ K$

(ii) $T \rightarrow \infty\ K$

In your discussion do include the Fermi energy and the Bose-Einstein condensate.

QUESTION 5 (*20 Marks*)

The concept of "*ensemble*" is critical to the statistical thermodynamic treatment of very large ($\sim 10^{23}$) number of particles. Two of the three most useful of these "collections" are the *canonical* and the *microcanonical ensembles.*

a). Discuss these two ensembles in terms of, among others, applicable variables and the distribution functions P(E) as a function of the energy E. What conclusion(s) can you draw from this distribution?

b). In the canonical ensemble write the partition function Q in terms of the molecular partition function q for

(i) distinguishable particles;

(ii) indistinguishable particles.

c). For the microcanonical partition function, derive the pressure p of the ensemble.

END OF CHE 5635 EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2009 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATION
GEO 5702: ADVANCED QUANTITATIVE TECHNIQUES IN GEOGRAPHY 1I

TIME : **TRHEE HOURS**

INSTRUCTIONS: **Answer any THREE questions.**
All questions carry equal marks.
Use of an approved calculator is allowed.

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1. Initially, the investigator was somewhat surprised by the negative correlation between reading activity and age, since it was felt that people should tend to read relatively more, when they are older. However, as expected, people with high acuity do tend to read more, as shown by r_{13}, and visual acuity does decline with age, as reflected by r_{23}. What would be the relationship between reading activity and age if the variability due to visual activity were removed? You might find some of the following coefficients useful in your analysis.

   $r_{12} = -0.10$, $r_{13} = 0.60$, $r_{23} = -0.40$ $n = 15$

2. Assuming that x_1= suicide rate; x_2 = Average age; x_3 = Percentage male and x_4 = Business failure rate. The raw data are presented in Figure 1.

   i) What value would you get for $R_{1.234}$?
   ii) Interpret the $R_{1.234}$ value?

3. Table 1 illustrate production and change in production and price from preceding year.

Table 1: Production and change in production and price from preceding year.

Year	Production (million tones)	Price per tone (in dollars)	Production Change in Percentages	Price change in percentages	Production (in million Tones)	Price (in US $
1932	49.8					...
1933	53.1					101
1934	59.1					92
1935	63.6					101
1936	65.6					96
1937	66.6					96
1938	60.4					122
1939	66.3					112
1940	71.6					78
1941	68.8					116
1942	62.9					116
1943	52.1					119
1944	69.1					81
1945	62.3					106
1946	65.8					

i) Compute the percentage production of the preceding year.
ii) Calculate the percentage price of the preceding year.
iii) Comment on the observed changes in both production and price changes from the preceding year.

4. Use the appropriate method to fill in the empty columns with missing values.

Table 2: Measuring distribution of population in relation to area

Area (in squire Kilometres)	Cumulative Area (in Squire Kilometres)	Percentage of area	Population (in ‘000)	Cumulative Polation (in `000	Percentage of population
21			299		
276			3157		
562			1031		
183			582		
120			694		
214			749		
246			622		
207			138		
47			239		
88			745		
276			1824		
683			608		
248			841		
370			616		
272			668		
340			351		
176			511		
269			883		
475			521		
291			330		

(a) Draw the line of best fit.
(b) Draw the Lorenz curve.
(c) Interpret the graph.

5. A GEO 5702 student assumes the a number of variables create causal relationships among aforestation (in aggregate number of trees per year) and a number of independent variables which include number of trees planted per day, number of forestry extension officers, amount of pesticides. Data are given in Table 3.

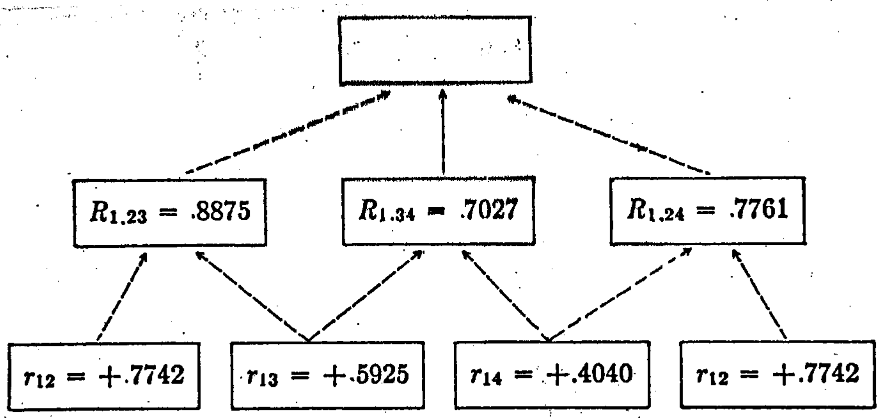
    i) Use the forward inclusion to assess whether the variables have equal influence in a number of trees planted in an area.
    ii) Plot the required lines of best fit.
    iii) Define each lines of best fit.
    iv) Indentify the strongest independent variable.

Table 3: Number of trees grown per day, number of forest extension officers, amount of pesticides (in litres) applied per hactre

Aggregate No.trees planted per per year	Trees planted Per day	No. of forest officers	amount of pesticides (in litres per hactre)
1500	50	15	10
1000	40	10	8
800	35	12	5
1200	45	20	12
500	30	08	4
600	32	06	5
450	25	11	6
1010	36	38	9
750	33	7	6
650	31	5	4.5

END OF EXAMINATION

Figure 1.



$R_{1.23}$ = .8875
$R_{1.34}$ = .7027
$R_{1.24}$ = .7761
r_{12} = +.7742
r_{13} = +.5925
r_{14} = +.4040
r_{12} = +.7742

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2009 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS

GEO 5802 : THE PHILOSOPHY AND METHODOLOGY OF GEOGRAPHY

TIME : Three Hours

INSTRUCTIONS : Answer Any FOUR Questions. Candidates are advised to make use of illustrations and examples wherever appropriate.

1. Why is Positivism referred to as a 'hidden' philosophy in geography?

2 Compare and contrast 'Nature-as-Usufruct' and 'Nature-as-Nurture' in the new environmentalism?

3 Explain the meaning of 'the philosophy of geography' and show why it is needed in a university situation.

4 In what ways has Marxist geography been affected by the ending of the cold war, especially in terms of achieving its central objective?

5 Describe the major themes in an Existential approach to the study of geographic thought.

6 Explain the contrasts between Idealism and Realism in the study of the philosophy of geography.

END OF EXAMINATION

The University of Zambia

School of Natural Sciences

Department of Mathematics & Statistics

First Semester Examinations - October/November 2010

MAT5111 - Ordinary Differential Equations & Integral Equations

Time allowed : 1 hour 30 mins

Instructions: • Answer **any two (2)** questions. All questions carry equal marks.

• **Full credit** will only be given when **detailed work** is shown.

• Indicate your **computer number** on all answer booklets.

This paper consists of 3 pages of questions.

1\. a) Given the Sturm-Lioville system defined by $u'' + \lambda u = 0$ and the boundary conditions

(i) $u'(0) = u(\pi) = 0$ (ii) $u'(0) = u'(\pi) = 0$

Show that the eigenvalues are $\left(k + \frac{1}{2}\right)^2$ and k^2. respectively. What are the eigenfunctions?

b) Show that a function $\phi(x_1, \ldots, x_n)$ of class C^1 is an integral of the system

$$\frac{dx_i}{dt} = X_i(x_1, \ldots, x_n), \qquad i = 1, \ldots, n$$

if and only if it satisfies the partial DE

$$X_1 \frac{\partial \phi}{\partial x_1} + \ldots + X_n \frac{\partial \phi}{\partial x_n} = 0 .$$

c) Show that the functions xyz and $x^2 + y^2$ are integrals of the system

$$\frac{dx}{dt} = xy^2, \qquad \frac{dy}{dt} = -x^2 y, \qquad \frac{dz}{dt} = z(x^2 - y^2) .$$

Describe the loci $xyz =$ constant, and sketch typical integral curves.

2. a) Consider the Volterra equation of the second kind

$$f(t) = g(t) + \int_0^t K(t, s, f(s))ds, \qquad 0 \le t \le T . \tag{1}$$

(i) Show that an initial-value problem for a first-order ordinary differential equation can be written in this form.

(ii) Show also that an integro-differential equation of the form

$$f'(\tau) = \int_0^\tau m(\tau, s)f(s)ds + r(\tau), \qquad f(0) = f_0 , \quad 0 \le s \le t \le \tau ,$$

can, by integrating both sides over the interval $[0, t]$, be written in the form of equation (1).

b) Solve the equation

$$f(t) = 1 + \int_0^t (\lambda + \mu(t - s))f(s)ds \qquad t \ge 0$$

by reducing it to an equivalent initial-value ODE problem. Hence show that the equation is stable (solutions tend to zero) if $\lambda, \mu < 0$.

c) The renewal equation is given by

$$h(t) = \rho(t) + \int_0^t h(s)\rho(t - s)ds .$$

Solve the renewal equation when $\rho(t) = te^{-t}$.

3. a) Given a linear autonomous system

$$\frac{dx}{dt} = ax + by , \qquad \frac{dy}{dt} = cx + dy , \tag{2}$$

where a, b, c and d are constants

(i) Define the secular equation of (2).

(ii) Prove that if $(x(t), y(t))$ is any solution of the plane autonomous system (2), then $x(t)$ and $y(t)$ are solutions of the secular equation of (2).

b) Show that linearly equivalent plane autonomous systems have the same secular equation.

c) Given a Volterra equation of the second kind

$$f(t) + \int_0^t K(t,s)f(s)ds = g(t), \qquad 0 \le t \le T,$$

where $K(t,s) = P(t)Q(s)$, show that the equation can be written as an initial-value problem.

END!

The University of Zambia

School of Natural Sciences

Department of Mathematics & Statistics

Second Semester Examinations - May 2011

MAT5122 - Partial Differential Equations

Time allowed : Three (3) hours

Instructions:
- Attempt **any five (5)** questions. All questions carry equal marks.
- **Full credit** will only be given when **necessary work** is shown.
- Indicate your **computer number** on all answer booklets.

This paper consists of 3 pages of questions.

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1. a) Laplace's equation in two dimensions is given by

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 .$$

Show that Laplace's equation in two dimensions is satisfied by the functions $u(x, y) = \ln((x-a)^2 + (y-b)^2)$.

b) (i) Show that the wave equation can be reduced to two first order partial differential equations.

(ii) Derive the method of characteristics using the wave equation and show that a general solution of the equation is $w(x,t) = P(x - ct)$.

c) Hence or otherwise solve the differential equation

$$\frac{\partial w}{\partial t} + c\frac{\partial w}{\partial x} = e^{2x} , \qquad w(x, 0) = f(x) .$$

2. a) Use the method of separation of variables to reduce the wave equation

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

to two ordinary linear differential equations.

b) Investigate the three possible solutions that may be considered from the ODEs in part a, to obtain a general solution of the wave equation and show that the frequency of vibration is $\frac{\lambda}{2\pi}$.

c) For a vibrating rod, given that both ends are clamped (fixed), find the value of λ in part b.

3. a) Show that under the substitution $u(x,y) = w(x,y)e^{-(bx+ay)}$, the equation

$$\frac{\partial^2 u}{\partial x \partial y} + a\frac{\partial y}{\partial x} + b\frac{\partial u}{\partial y} + cu = 0$$

becomes

$$\frac{\partial^2 w}{\partial x \partial y} + (c - ab)w = 0 \ .$$

b) Given the differential equation

$$(y+u)\frac{\partial u}{\partial x} - (x+u)\frac{\partial u}{\partial y} + (x-y) = 0 \ , \ u(s,2s) = s$$

show that the equation $x^2 + y^2 + u^2 = F(x+y-u)$ is a general solution.

c) Hence or otherwise, solve the differential equation in b).

4. a) Find the general solution, and then solve using the given data.

$$\frac{3}{x-y}\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right) = 2 \ , \ u(s,0) = -s^2$$

b) A semi-infinite string is initially at rest in a position that coincides with the positive half of the x-axis. At $t = 0$, the left end of the string begins to move along the y-axis in a manner described by $y(0,t) = f(t)$, where $f(t)$ is a known function. Use laplace transforms to find the displacement $y(x,t)$ of the string at any point at any subsequent time.

c) Solve the quasilinear equation

$$\frac{\partial \rho}{\partial t} - \rho^2\frac{\partial \rho}{\partial x} = 3\rho \ , \ \rho(x,0) = f(x) \ .$$

5. a) Solve the equation $u_{xy} = -u_x$.

b) Discuss the existence of integral surface of $2p + 3q + 8z = 0$ which contains the curve

(i) $\Gamma \ : \ z = 1 - 3x$ in the xy-plane.

(ii) $\Gamma \ : \ z = x^2$ on the line $2y = 1 + 3x$.

(iii) $\Gamma \; : \; z = e^{-4x}$ on the line $2y = 3x$.

What can you conclude from the solutions of (i)-(iii)?

c) Find the integral surface of $xp+yq = z$ which contains $\Gamma \; : \; x_0 = s^2 \, , \; y_0 = s+1 \, , \; z_0 = s$.

6. a) Derive D'Alembert's solution for the wave equation

$$u_{tt} - c^2 u_{xx} = 0 \, , \quad -\infty < x < \infty, \; t > 0 \, , \quad u(x,0) = f(x), \quad u_t(x,0) = g(x) \; .$$

b) Use D'Alembert's solution to solve the equation

$$u_{tt} - c^2 u_{xx} = 0 \, , \quad -\infty < x < \infty, \; t > 0 \, , \quad u(x,0) = x, \quad u_t(x,0) = x^2 \; .$$

c) Solve the differential equation

$$u_x + 2u_y = u^2 \, , \; u(x,0) = h(x) \; .$$

END!

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

MATHEMATICS AND STATISTICS DEPARTMENT

MAT5141: Topics in Mathematical Methods

First Semester Final Examinations November, 2010

Time Allowed: Three (3) hours

Instructions:
- Answer **any** four (4) Questions
- Full Credit will only be given where **all** **necessary** work is shown.

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1. a) Define the following:

   (i) Infinite integral of type 2

   (ii) Convergence of infinite integral of type 2

   (iii) Cauchy principle value of infinite integral type 2

   b) Test for convergence of the following:

   (i) $\int_1^\infty \frac{1}{t(t+1)}\,dt$ (ii) $\int_2^3 \frac{1}{\sqrt{t-2}}\,dt$

   c) (i) State Hermite's differential equation.

   (ii) The generating function for Hermite polynomials is

$$e^{2tx-t^2} = \sum_{n=0}^{\infty} H_n(x)\frac{t^n}{n!}$$

   Prove the recursion formula

   $H_{n+1}(x) = 2x\, H_n(x) - 2n\, H_{n-1}(x)$

   (iii) Prove that for Hermite polynomials

$$\int_{-\infty}^{\infty} e^{-x^2} H_m(x)\, H_n(x)dx = \begin{cases} 0 & if\ m \neq n \\ 2^n\, n!\,\sqrt{\pi} & if\ m = n \end{cases}$$

2) a) Define the following:

(i) Absolute convergence of an infinite integral.

(ii) Family of cylinder functions.

(ii) Bessel's equation of order v.

b) *Given that* $J_v(z) = \left(\frac{z}{2}\right)^{v} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!\,\Gamma(v+k+1)} \left(\frac{z}{2}\right)^{2k}$

(i) Prove that $\frac{d}{d^z}\ z^{v} J_v(z)) = Z^{v} J_{v-1}(z)$

(ii) Prove that $\frac{d}{d^z}(z^{-v} J_v(z)) = Z^{-v} J_{v+1}(z)$

(iii) Using the results in (i) and (ii) above, prove that

$$J_v'(z) = \frac{1}{2}\left[J_{v-1}(z) - J_{v+1}(z)\right]$$

c) (i) Give Legendre's differential equation

(ii) The generating function for Legendre polynomials is

$$\frac{1}{\sqrt{1-2xt+t^2}} = \sum_{n=0}^{\infty} P_n(x) t^n$$

Prove the recurrence formula for Legendre polynomials

$$P_{n+1}(x) = \frac{2n+1}{n+1} \times P_n(x) - \frac{n}{n+1} P_{n-1}(x)$$

(iii) Prove the orthogonality of Legendre polynomials

3) a) Define the following:

(i) The general Gamma function.

(ii) Integral representation of Gamma function.

(iii) The beta function.

b) (i) Let $\Gamma(z)$ be the integral representation of gamma function. Prove that

$$\Gamma(z) = \lim_{n \to \infty} \frac{n^z\, n!}{z(z+1)(z+2)\ldots\ldots(z+n)}$$

(ii) Using the result in (i) above prove that $\Gamma(z+1) = z\,\Gamma(z)$

(iii) Prove that $B(p, q) = \dfrac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$

c) (i) *Given that* $\displaystyle\int_0^\infty \frac{x^{P-1}}{1+x}\,dx = \frac{\pi}{\sin(P\pi)}$, *show that* $\Gamma(P)\Gamma(1-P) = \dfrac{\pi}{\sin(P\pi)}$

(ii) *Using the result (i)above evaluate* $\displaystyle\int_0^\infty \frac{1}{1+x^4}\,dx$

(iii) *Evaluate* $\displaystyle\int_0^2 x\sqrt[3]{8-x^3}\,dx$

4. a) (i) State Laguerre's differential equation.

(ii) State Rodrigues formula for Laguerre polynomids.

(iii) Using Rodrigue's formula for Laguerre polynomials, determine Laguerre polynomials $L_n(x)$, for n = 0, 1, 2, 3.

b) (i) Prove the orthogonality of Laguerre polynomials.

(ii) Prove that $\int_0^\pi \sin(z\cos\Theta)\, sin^{2n}(\Theta)\, d\Theta = \dfrac{\Gamma(n+1)\sqrt{\pi}}{\left(\frac{z}{2}\right)^n} J_n(z)$

hint: $\displaystyle \text{Sin}\,(z\cos\Theta) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!}\, z^{2k+1}\, cos^{2k+1}(\Theta)$

c) (i) Using the generating function for Legendre polynomials,
Derive the expression for P_n (-1)

(ii) Prove that for any cylinder function $c_\nu(z)$, $c_{-n}(z) = (-1)^n c_n(z)$
for any positive integer n.

5. a) Legendre's association differential equation is given by

$$(1 - x^2)\, y'' - 2xy' + (n\,(n + 1) - \frac{m^2}{1-x^2})\, y = 0.$$

(i) Give the general solution to Legendre's associated differential equation.

(ii) Give the Legendre associated functions in terms of ordinary Legendre functions.

b) (i) Define type 1 Lommel Integral.

(ii) Using the identities in 2(b) (i) and (ii) Prove that

$$I_{m-1}\,(r_1\,, s_1) = \frac{x^m}{m - r_1 - s_1} \quad (J_{r_1}\,(\mathrm{x})\, J_{s_1}\,(\mathrm{x}) \;+\; J_{r_2}(\mathrm{x})\, J_{s_2}\,(\mathrm{x}))$$

Where m + v_2 + s_2 = 0, r_2 = r_1-1, and s_2 = s_1 - 1

(iii) Identify values of m for which

$\int x^m J_v$ (x) J_3 (x) dx can be computed using the result in (ii) above.

c) Prove that $\frac{d}{dx}\left[x\, J_v\,(x) J_{v+1}\,(x\,)\right] = \mathrm{x}\left[J_v{}^2\,(x) - J^2{}_{v+1}\,(\mathrm{x})\right]$

(ii) Prove that $\frac{\Gamma(z+n)}{\Gamma\,(z)}$ = z (z + 1)(z + 2) ……. (z + n – 1)

END

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

Department of Mathematics & Statistics

FIRST SEMESTER FINAL EXAMINATIONS

MAT5311—LEBESGUE MEASURE AND LEBESGUE INTEGRATION

November, 01, 2010

Time allowed : THREE(3) HOURS

Instructions : There are seven(7) questions. Answer **ANY FIVE (5)** questions. All questions carry equal marks. Show all your working to earn full marks.

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1. (a)(i) Let $\{A_n\}_{n=1}^{\infty}$ be a sequence of sets. Prove that $\chi_{[\inf_{n\geq k} A_n]} = \inf_{n\geq k}[\chi_{A_n}]$.

   (ii) Define the outer measure μ^* of a subset E of $\mathbb{R}$.

   (iii) Show that the outer measure of a singleton set is zero.

   (b)(i) When is a subset E of $\mathbb{R}$ said to be measurable?

   (ii) If two subsets A and B of $\mathbb{R}$ are measurable, prove that $A \cap B$ is also measurable.

2. (a)(i) Define the Lebesgue measure μ of a subset E of $\mathbb{R}$ in relation to the outer measure.

   (ii) If A is such that $\mu^*(A) < \infty$ and there is a measurable subset $B \subseteq A$ with $\mu(B) = \mu^*(A)$, show that A is measurable.

   (b)(i) Let $f : E \to \mathbb{R}_\infty$ be a function, where E is a measurable subset of $\mathbb{R}$. When is f said to be a measurable function?

   (ii) If f is a measurable function on E, prove that f^2 is also measurable.

(c) Let $f, g : \mathbb{R} \to \mathbb{R}_\infty$ with f measurable and g continuous. Prove that $g \circ f$ is a measurable function.

3\. (a)(i) Define a simple function f on a measurable subset E of $\mathbb{R}$.

(ii) Let $S(E)$ denote the set of all simple functions $s : E \to \mathbb{R}$, with $\mu(E) < \infty$. Define $I_E : S(E) \to \mathbb{R}$ by $I_E(s) = \sum_{k=1}^{n} c_k \mu(D_k)$ where $D_k = s^{-1}(c_k)$, $k = 1, 2, \ldots\ldots, n$. If $E_1, E_2, \ldots\ldots, E_m$ are disjoint measrable subsets of E with $\mu(E) < \infty$, show that $s = \sum_{k=1}^{m} a_k \chi_{E_k}$ with real coefficients $a_1, a_2, \ldots\ldots, a_m$ is a simple function and $I_E(s) = \sum_{k=1}^{m} a_k \mu(E_k)$.

(b) Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of measurable functions with $f_n : E \to \mathbb{R}_\infty$ for each n. Prove that:

(i) $p = \inf_n f_n$ is measurable.

(ii) $k = \sup_n f_n$ is measurable.

(c) State and prove Fatou's lemma.

4\. (a) Let

$$f_n(x) = \begin{cases} -n^2, & x \in (0, \frac{1}{n}) \\ 0, & \text{otherwise.} \end{cases}$$

(i) Show that

$$\lim_{n\to\infty} \int_{\mathbb{R}} f_n(x)dx \neq \int_{\mathbb{R}} \lim_{n\to\infty} f_n(x)dx.$$

(ii) Explain why this does not contradict Fatou's lemma.

(b) State and prove the Dominated Convergence theorem.

(c) For $n \geq 1$, let

$$f_n(x) = \begin{cases} 2n^2, & \frac{1}{2n} \leq x \leq \frac{1}{n} \\ 0, & x \in (0, \frac{1}{2n}) \cup (\frac{1}{n}, 1). \end{cases}$$

(i) Verify that the Dominated Convergence theorem does not hold for the sequence $\{f_n\}$.

(ii) Explain why the Dominated Convergence theorem can not work.

5. (a) Let $f(x) = \frac{1}{x^r}$, $0 < x \leq 1$. Show that:

(i) $f \in L^1([0,1])$ if and only if $r < 1$.

(ii) $L^p([0,1]) \not\subseteq L^q([0,1])$ for $1 \leq p < q$..

(b) If $1 \leq p < q < \infty$ and $\mu(E) < \infty$, prove that

$$L^q(E) \subset L^p(E).$$

(c) If $\mu(E) < \infty$, f is measurable and $f \in L^2(E)$, prove that f is integrable by directly applying Holder's inequality.

6. (a) If $0 < p < q < r \leq \infty$, prove that $L^q \subset L^p + L^r$.

(b) State and prove Holder's inequality. You may use the fact that if $p > 0$ and a and b are any nonnegative reals, $ab \leq \frac{a^p}{p} + \frac{b^q}{q}$.

(c) Use Holder's inequality to show that if $\mu(E) = K$ for some constant K, then if $1 < p < \infty$ and $f \in L^p$, we have that $f \in L^1$ and $\|f\|_1 \leq \|f\|_p K^{\frac{1}{q}}$ where p and q are conjugate indices.

7. (a) Prove that given a sequence $\{A_n\}_{n=1}^{\infty}$ of sets,

$$\chi_{[\sup_{n \geq k} A_n]} = \sup_{n \geq k} \chi_{A_n}.$$

(b) Use the definition of outer measure to prove that every interval is uncountable.

(c)(i) State and prove Minkowski's inequality.

END.

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

Department of Mathematics & Statistics

FIRST SEMESTER FINAL EXAMINATIONS

MAT5331—FUNCTIONAL ANALYSIS

October, 27, 2010

Time allowed : THREE(3) HOURS

Instructions : There are seven(7) questions. Answer **ANY FIVE (5)** questions. All questions carry equal marks. Show all your working to earn full marks.

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1. (a) Define the following:

   (i) A semi inner product on a vector space $\mathbb{X}$ over a field $\mathbb{F}$.

   (ii) An inner product on a vector space $\mathbb{X}$ over a field $\mathbb{F}$.

   (b) Let $\mathbb{H}$ be a collection of all sequences $\{\alpha_n : n \geq 1\}$ of scalars $\alpha_n \in \mathbb{F}$ such that $\alpha_n = 0$ for all but a finite number of values of n. Define addition and scalar multiplication in the usual pointwise sense. Let $\langle x, y\rangle = \sum_{n=0}^{\infty} \alpha_{2n}\overline{\beta_{2n}}$, where $x, y \in \mathbb{H}$. show that $\langle ., .\rangle$ is a semi inner product that is not an inner product.

   (c) State and prove the Cauchy-Bunyakowsky-Schwarz inequality.

2. (a) If $\mathbb{H}$ is a Hilbert space and $f, g \in \mathbb{H}$, show that

$$\|f+g\|^2 + \|f-g\|^2 = 2(\|f\|^2 + \|g\|^2).$$

   (b) Prove that any norm that satisfies the parallelogram law is derived from an inner product.

   (c) If $\mathcal{A}$ is a subset of a Hilbert space $\mathbb{H}$, prove that $\mathbb{A}^{\perp}$ is a closed linear subspace of $\mathbb{H}$.

3. (a) Let $\{e_n : n \in \mathbb{N}\}$ be an orthonormal basis for a Hilbert space $\mathbb{H}$, and put

$$\mathcal{C} = \{x \in \mathbb{H} : \sum (1 + \frac{1}{n})^2 |\langle x, e_n\rangle|^2 \leq 1\}.$$

Define $Tx = \sum(1+\frac{1}{n})\langle x, e_n\rangle e_n$.

(i) Show that T is a bounded linear operator on $\mathbb{H}$.

(ii) Show that $\mathcal{C} = \{x \in \mathbb{H} : \|Tx\| \leq 1\}$.

(iii) Prove that $\mathcal{C}$ is convex.

(b) Let $\mathcal{M}$ be a closed linear subspace of a Hilbert space $\mathbb{H}$ and $h \in \mathbb{H}$. Let Ph be the unique point in $\mathcal{M}$ such that $h - Ph \perp \mathcal{M}$. Prove that $KerP = \mathcal{M}^{\perp}$.

(c) If P is the projection defined in part (b) above, prove that $P^2 = P$.

4. (a) If $\{e_n : n \in \mathbb{N}\}$ is a orthonormal set and h is an element of a Hilbert space $\mathbb{H}$, prove that

$$\sum_{n=1}^{\infty} |\langle h, e_n\rangle|^2 \leq \|h\|^2.$$

(b) If E is an orthonormal set in a Hilbert space $\mathbb{H}$, and if $h \in \mathbb{H}$ is such that

$$\|h\|^2 = \sum\{|\langle h, e\rangle|^2 : e \in E\},$$

prove that E is a basis for $\mathbb{H}$.

(c) Let $X = \mathbb{C}^3$, the set of all 3-tuples of complex numbers, with inner product

$$\langle x, y\rangle = \sum_{i=1}^{3} \alpha_i \overline{\beta_i},$$

where $x = (\alpha_1, \alpha_2, \alpha_3)$ and $y = (\beta_1, \beta_2, \beta_3)$. Apply the Gram-schmidt orthogonalisation process to the vectors $x_1 = (1, 0, 1)$, $x_2 = (0, 1, 1)$ and $x_3(1, 1, 0)$ to get an orthonormal basis for X.

5. (a) Define the following:

(i) A seminorm on a vector space X over a field $\mathbb{F}$.

(ii) A Banach space.

(b) Let X be any Hausdorff space, and let $C_b(X)$ be the space of all continuous functions $f : X \to \mathbb{F}$, with the sup-norm and usual addition and scalar multiplication. Prove that $C_b(X)$ is a Banach space.

(c) Let $C_0(X)$ be the space of all continuous functions $f : X \to \mathbb{F}$ such that for all

$\epsilon > 0$, the set $\{x \in X : |f(x)| \geq \epsilon\}$ is compact. Prove that $C_0(X)$ is a subspace of $C_b(X)$.

6. (a) State the Hahn-Banach theorem for a complex normed vector space X.

   (b) Let X be a normed space and $x_0 \in X$, with $x_0 \neq 0$ fixed. Show that there is a continuous linear functional λ on X with $\|\lambda\| = 1$ such that

$$\lambda(x_0) = \|x_0\|.$$

   (c) Let l be a linear functional on the linear space X. Show that Ker$l$ is a linear subspace of X of codimension one.

7. (a) Define the following:

   (i) An interior point of a set A.

   (ii) A nowhere dense set A.

   (b) State, without proof, Baire Category theorem.

   (c)(i) State, without proof, the Open Mapping theorem.

   (ii) If $\|T^{-1}y\| < 1$ whenever $\|y\| < r$, show that $\|T^{-1}\| \leq \frac{1}{r}$ using the fact that for any $\epsilon > 0$, $\|r(\|y\| + \epsilon)^{-1}y\| < r$.

   (iii) The Inverse Mapping theorem states that any one to one and onto bounded linear mapping between Banach spaces has a bounded inverse. Use the Open mapping theorem to prove this result.

END.

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

Department of Mathematics & Statistics

SECOND SEMESTER FINAL EXAMINATIONS

MAT5352—TOPICS IN ANALYSIS

May 6, 2011

Time allowed : THREE(3) HOURS

Instructions : There are seven(7) questions. Answer **ANY FIVE (5)** questions. All questions carry equal marks. Show all your working to earn full marks.

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1. (a) Define $\mathcal{S}$, where $\mathcal{S}$ is a Schwartz space .

   (b) Define the set $\mathcal{S}'$ of tempered distributions.

   (c) Prove that a linear functional T on $\mathcal{S}$ is a tempered distribution if and only if there exists a $C > 0$ and some $k, m \in \mathbb{Z}^+$ such that

$$|T(f)| \leq C\|f\|_{(k,m)}$$

   for all $f \in \mathcal{S}$.

   (d) Let $g \in L^2(\mathbb{R}^n)$. Show that the linear map

$$T_g : f \to \int g(x)f(x)dx$$

   on $\mathcal{S}$ defines a tempered distribution.

2. Let $\mathcal{S}$ be the Schwartz space, and $\mathcal{S}'$ the space of tempered distributions.

(a) Prove that the functional

$$P(\frac{1}{x}) : f \to \lim_{\epsilon \downarrow 0} \int_{|x| \geq \epsilon} \frac{1}{x} f(x) dx$$

belongs to $\mathcal{S}'$.

(b) Let (ϕ_n) be a sequence of functions on $\mathbb{R}$ such that $\phi_n(x) \geq 0$ for all $x \in \mathbb{R}$, $\int \phi_n(x) dx = 1$ for all n, and for any $a > 0$, $\int_{|x| \geq a} \phi_n(x) dx \to 0$ as $n \to \infty$. Show that $\phi_n \to \delta$ in $\mathcal{S}'(\mathbb{R})$, where δ is the dirac delta function.

(c) Deduce from part (b) that for any $\epsilon > 0$, if we let

$$g_\epsilon = \frac{\epsilon}{(x - x_0)^2 + \epsilon^2},$$

then $g_\epsilon \to \pi \delta_{x_0}$ in $\mathcal{S}'$.

3. Let $\mathcal{S}$ be the Schwartz space, and $\mathcal{S}'$ the space of tempered distributions.

(a) Define the weak derivative $\partial^\alpha T$ for $T \in \mathcal{S}'$ and $\alpha \in \mathbb{Z}_+^n$.

(b) For any $\alpha \in \mathbb{Z}_+^n$, show that

$$\partial^\alpha : \mathcal{S} \to \mathcal{S}$$

is continuous.

(c) For any $T \in \mathcal{S}'$, show that $\partial^\alpha T \in \mathcal{S}'$.

4. (a) Define a distribution.

(b) Suppose u is a locally integrable function. Show that the map

$$T_u : \phi \to \int_{\mathbb{R}^n} u(x) \phi(x) dx$$

is a distribution.

(c) Prove that every tempered distribution is a distribution.

5. Let $\mathcal{S}$ be the Schwartz space, and $\mathcal{S}'$ the space of tempered distributions.

(a) Define the Fourier transform $\hat{f} = \mathcal{F}(f)$ of the function $f \in \mathcal{S}$.

(b) If $f \in \mathcal{S}$,and $\alpha, \beta \in \mathbb{Z}_+^n$, show that

$$((i\lambda)^\alpha \partial^\beta \hat{f})(\lambda) = \mathcal{F}[\partial^\alpha((-ix)^\beta f(x))](\lambda).$$

(c) If $f \in L^1$, $g \in C^k$, and $\partial^\alpha g$ is bounded for $|\alpha| \leq k$, prove that

$$f * g \in C^k$$

and

$$\partial^\alpha (f * g) = f * (\partial^\alpha g)$$

for all $|\alpha| \leq k$.

6. (a) Prove Chebyshev's Inequality, which states that if $f \in L^p$, $0 < p < \infty$, then for any $\alpha > 0$,

$$\mu(\{x : |f(x)| > \alpha\}) \leq [\frac{||f||_p}{\alpha}]^p.$$

(b) Suppose K is a compact subset of the normed space $\mathcal{H}$, $\epsilon > 0$, and A is a finite subset of K such that $K \subseteq \cup\{B(a;\epsilon) : a \in A\}$. Define $\phi_A : K \to \mathcal{H}$ by

$$\phi_A(x) = \frac{\sum\{m_a(x)a : a \in A\}}{\sum\{m_a(x) : a \in A\}},$$

where

$$m_a(x) = \begin{cases} 0, & ||x-a|| \geq \epsilon \\ \epsilon - ||x-a||, & ||x-a|| < \epsilon \end{cases}.$$

Assuming the continuity of ϕ_A, show that

$$||\phi_A(x) - x|| < \epsilon$$

for all $x \in K$.

(c) Let E be a closed bounded convex subset of a normed space $\mathcal{H}$. Let K be a compact subset of E. Explain why we can find a finite subset A_n of K such that

$$K \subseteq \cup\{\mathcal{B}(a; \frac{1}{n}) : a \in A_n\}.$$

Hence, following the definition of ϕ_A in part (b), Explain why

$$\phi_{A_n}(K) \subseteq co(K) \subseteq E.$$

7. (a) If K is a non empty compact convex subset of a finite-dimensional normed space $\mathcal{H}$ and $f : K \to K$ is a continuous function, prove that there is a point x in K such that $f(x) = x$, assuming Brouwer's Theorem.

(b) State, without proof, the Schauder Fixed-Point Theorem.

(c) Consider the following boundary value problem:

$$f''(x) + \lambda\psi[x, f(x)] = g(x), \quad f(0) = f(1) = 0.$$

Suppose that $\psi : [0, 1] \times \mathbb{C} \to \mathbb{C}$ is continuous and bounded, and g is continuous. By assuming a equivalent integral operator of the form

$$Af(x) = \lambda \int_0^1 k(x, y)\psi[y, f(y)]dy + h(x),$$

where k is the Green's function and h is a known continuous function, apply the Schauder Fixed-Point Theorem to conclude that the above boundary value problem has a solution $f \in C^2([0, 1])$ for any $\lambda \in \mathbb{C}$.

END.

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS

2010 ACADEMIC YEAR
FIRST SEMESTER EXAMINATIONS

MAT5611 : STATISTICAL INFERENCE

TIME ALLOWED: Three (3) Hours

INSTRUCTIONS:
1. Answer any **Four** (4) Questions
2. Show All Essential Working

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1. (a) Define the following:
   (i) complete statistic T.
   (ii) uniformly most powerful test.

   (b) Let $X_1,\ldots,X_n$ be a random sample from a GEO(θ) distribution with probability function $f_\theta(x)=\theta(1-\theta)^x$, $x=0,1,2,\ldots$; $0<\theta<1$.
   Given that $T=\sum_{i=1}^{n} X_i$,
   (i) find the distribution of $f(x_1,\ldots,x_n \mid T=t)$. What do you conclude about T?
   (ii) show that $T=\sum_{i=1}^{n} X_i$ is complete.
   (iii) find the UMVUE of $\tau(\theta)=\dfrac{1-\theta}{\theta}$.

   (c) Let $X_1,\ldots,X_n$ be a random sample from the EXP(θ) distribution with p.d.f. $f_\theta(x)=\dfrac{e^{-\frac{x}{\theta}}}{\theta}$, $x>0$, $\theta>0$.
   (i) Find the most powerful test of size α for testing $H_0:\theta=\theta_0$ against $H_1:\theta=\theta_1$ where $\theta_1<\theta_0$.
   (ii) Find the uniformly most powerful test of size α for testing $H_0:\theta=\theta_0$ against $H_0:\theta<\theta_0$.

2. (a) (i) State Lehmann – Scheffe's theorem.

(ii) Define an exponential family distribution.

(b) Let $X_1, \ldots, X_n$ be a random sample from a POI(θ) distribution with probability function $f_\theta(x) = \dfrac{e^{-\theta}\theta^x}{x!}$. Let $\tau(\theta) = \theta\, e^{-\theta} = P(X_1 = 1)$,

(i) Show that $T = \sum_{i=1}^{n} X_i$ is a minimal sufficient statistic for θ.

(ii) Find the UMVUE of $\tau(\theta) = \theta\, e^{-\theta}$.

(c) Suppose $X_1, \ldots, X_n$ is a random sample from the $GAM(\alpha, \beta)$ distribution with p.d.f.

$$f_\theta(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha - 1} e^{\left(-\frac{x}{\beta}\right)} \quad , \quad x > 0 \;\; ; \;\; \alpha > 0 \;\; , \;\; \beta > 0$$

where $\theta = (\alpha, \beta)$.

(i) Show that X has a regular exponential family distribution.

(ii) Find a complete sufficient statistic for this model.

3. (a) State and prove Basu's theorem.

(b) Suppose $X_1, \ldots, X_n$ is a random sample from the distribution with p.d.f.

$f_\theta(x) = \theta^2 x\, e^{-\theta x} \quad , \quad x > 0 \;\; ; \;\; \theta > 0$.

(i) Find the maximum likelihood estimator of θ.

(ii) Find the maximum likelihood estimator of $\tau(\theta) = \dfrac{2}{\theta}$.

(iii) Find the Cramer – Rao lower bound for estimating $\tau(\theta) = \dfrac{2}{\theta}$.

(iv) Does the maximum likelihood of $\tau(\theta) = \dfrac{2}{\theta}$ attain the Cramer – Rao lower bound?

4. (a) Define the following:

(i) location invariant family.

(ii) scale invariant estimator $\tilde{\theta}^k$.

(b) Let $X_1, \ldots, X_n$ be a random sample from a distribution with

$E(X_i) = \theta$ and $Var(X_i) = \theta$, $i = 1, \ldots, n$.

Consider the estimating function

$$\psi(\theta, X) = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2 - \theta$$

(i) Show that $\psi(\theta, X)$ is an unbiased estimating function.

(ii) Find an estimator $\hat{\theta}$ that satisfies $\psi(\hat{\theta}, X) = 0$.

(iii) Find an approximate 95% confidence interval for θ.

(c) Let $X = (X_1, \ldots, X_n)$ be a random sample from a regular model $\{ f_\theta(x) : \theta \in \Omega \}$ and let $S(\theta, X)$ the score function. Show that

(i) $E_\theta[S(\theta, X)] = 0$

(ii) $\operatorname{var}_\theta[S(\theta, X)] = E_\theta\{(S(\theta, X))^2\}$

5. (a) Define the following:

(i) locally most powerful test.

(ii) level of significance of a test.

(b) Suppose that X and Y are independent binomial random variables with $X \sim BIN(n, \theta_1)$ and $Y \sim BIN(n, \theta_2)$.

(i) Find the likelihood ratio test for testing $H_0 : \theta_1 = \theta_2$ against $H_1 : \theta_1 \neq \theta_2$.

(ii) Find an asymptotic distribution of the test in (i).

(c) Suppose $X_1, \ldots, X_n$ is a random sample from the distribution with p.d.f.

$$f_\theta(x) = \frac{2\theta^2}{x^3} \quad , \quad x > \theta \quad ; \quad \theta > 0 \ .$$

(i) Find the maximum likelihood estimator $\hat{\theta}_n$ of θ.

(ii) Show that $\hat{\theta}_n$ is a consistent estimator of θ.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS

2011 ACADEMIC YEAR
SECOND SEMESTER SUPPLEMENTARY EXAMINATIONS

MAT5632 : DESIGN AND ANALYSIS OF EXPERIMENTS

TIME ALLOWED: Three (3) Hours

INSTRUCTIONS:
1. Answer any **Four** (4) Questions
2. Show All Essential Working

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1. (a) Given a one factor fixed effects model

$$Y_{ij} = \mu + \tau_i + \varepsilon_{ij} \quad , \quad i = 1, 2, ..., t \quad ; \quad j = 1, 2, ..., r$$

where

$$\varepsilon_{ij} \sim N(0, \sigma^2),$$

Prove that $E(MSTrt) = \frac{r}{t-1}\sum_{i=1}(\tau_i - \bar{\tau})+\sigma^2$, where $MSTrt$ is the mean treatment sum of squares.

(b) An experiment was run to find the best conditions for a process. The process has three factors A, B and C, which are varied according to a 2^3 factorial design and the response variable is denoted by y. The results from the experiment are given in the following table.

A	B	C	y
−	−	−	10
+	−	−	16
−	+	−	18
+	+	−	26
−	−	+	22
+	−	+	16
−	+	+	32
+	+	+	20

(i) Estimate the interaction effects AB, AC and ABC.

(ii) Estimates of other effects are:

Factor	Estimate
A	− 1
B	8
C	5
BC	− 1

Plot all the effects on a normal probability paper and identify the effects that appear significant.

(iii) Prepare an ANOVA table based on the factors identified in (ii) assuming all the other factors are negligible. Test for the significance of the factors.

(iv) Write down a regression equation of the significant factors identified in (iii).

2. (a) Consider a 2^4 factorial design with one replication.

(i) Suppose the experiment is run in four blocks with ABC and BCD confounded with the blocks. Write down the block arrangement for the experiment clearly showing the observations in each block. What other effect(s) are confounded with the blocks?

(ii) Obtain a quarter fraction of the design using ABC and BCD as defining contrasts.

(iii) Write down a complete alias structure of the design in (ii).

(iv) Given that the actual experiment conducted gave the following data:

$(1) = 19$	$d = 13$
$a = 15$	$ad = 18$
$b = 15$	$bd = 18$
$ab = 18$	$abd = 18$
$c = 15$	$cd = 18$
$ac = 18$	$acd = 19$
$bc = 17$	$bcd = 18$
$abc = 19$	$abcd = 20$

Using your quarter fraction in (ii) above, estimate the main effects A, B and D.

(b) In an agricultural experiment, it was proposed to study three factors A, B and C, each at two levels. Since it is difficult to get homogeneous blocks of size 8, the following treatment layout was proposed:

Run	Block	A	B	C	Response
1	1	+	+	−	y_1
2	1	+	−	+	y_2
3	1	−	−	−	y_3
4	1	−	+	+	y_4
5	2	+	−	−	y_5
6	2	+	+	+	y_6
7	2	−	−	+	y_7
8	2	−	+	−	y_8

(i) Find the treatment effect confounded with blocks in this design. Show your steps clearly.

(ii) Explain whether this design allows estimation of the main effect A and the BC interaction effect. If these effects can be estimated, give formulas for their estimates in terms of the response values y_1, y_2, … , y_8.

(iii)

Run	Block	A	B	C	Response
9	3	+	−	−	y_9
10	3	−	+	+	y_{10}
11	3	−	−	−	y_{11}
12	3	+	+	+	y_{12}
13	4	+	−	+	y_{13}
14	4	+	+	−	y_{14}
15	4	−	−	+	y_{15}
16	4	−	+	−	y_{16}

Find the treatment effect confounded with blocks in this design. Show your steps clearly.

(iv) Consider the data in Blocks 3 and 4 only. Explain whether this design allows estimation of the main effect A and the BC interaction effect. If these effects can be estimated, give formulas for their estimates in terms of y_9, y_{10}, … , y_{16}.

3. (a) (i) Explain the difference between a random effects model and a fixed effects model.

(ii) Define a two stage nested design.

(b) To study the effect of operators and filters in a filtration process, a company measures the percentage of material lost in coffee for 12 experimental conditions with 3 runs on each condition. The company has 4 operators and many different types of filters. Three filters were selected at random for the experiment resulting in the following measurements:

		Operator				
		1	2	3	4	Total
Filter	1	16.2 16.8 17.1	15.9 15.1 14.5	15.6 15.9 16.1	14.9 15.2 14.9	188.2
	2	16.6 16.9 16.8	16.0 16.3 16.5	16.1 16.0 17.2	15.4 14.6 15.9	194.3
	3	16.7 16.9 17.1	16.5 16.9 16.8	16.4 17.4 16.9	16.1 15.4 15.6	198.7
	Total	151.1	144.5	147.6	138	581.2

(i) Write down a model for the above experiment. Explain all the terms in the model and state all the assumptions.

(ii) Find the expected mean squares of all the factors in the model.

(iii) Copy and complete the following ANOVA table.

Source	SS	df	MS	F
Filter				
Operator				
Filter-operator interaction	1.657			
Error	4.440			
Total	21.049			

(iv) Test for the effects of filter, operator and filter–operator interaction on the filtration process. State your hypotheses and conclusions clearly.

(v) Estimate the components of variance due to filters, filter – operator interaction, and experimental error.

4. (a) Define the following:
   (i) orthogonal contrasts.
   (ii) factorial experiment.

(b) An experiment is conducted in which the surface finish of metal parts in a certain factory is being studied. Four machines are randomly selected for the experiment. Each machine is run by three different operators and two specimens from each operator are collected and tested. Because of the location of the machines, different operators are used on each machine and the operators are chosen at random. The data collected from the experiment are shown in the following table:

	Machine 1			Machine 2			Machine 3			Machine 4			
Operator	1	2	3	1	2	3	1	2	3	1	2	3	Total
	79	94	46	92	85	76	88	53	46	36	40	62	
	62	74	57	99	79	68	75	56	57	53	56	47	
Sub-total	141	168	103	191	164	144	163	109	103	89	96	109	
Total	412			499			375			294			1580

(i) What design is used in the experiment? Explain.
(ii) Write down a model for the experiment. Explain all the terms in the model and state all the assumptions.
(iii) Find the expected mean squares of the factors in the model.
(iv) Copy and complete the following ANOVA table.

Source	SS	df	MS	F
Machine				
Operator(Machine)	2817.6667			
Error	1014			
Total				

(v) Test for the effect of machines and operators. State your hypotheses and conclusions clearly.
(vi) Estimate the components of variance due to machines, operators and experimental error.

5. (a) (i) State and define the three principals of experimental design.
   (ii) State one reason why each principal of experimental design is important.

(b) An experiment is performed to determine the effect of temperature and heat treatment time on the strength of steel. Two temperatures and three times are selected. The experiment is performed by heating the oven to a selected temperature and inserting three specimens. After 10 minutes one specimen is removed, after 20 minutes the second is removed and after 30 minutes the final specimen is removed. Then the temperature is changed to the other level and the process is repeated. The experiment was done in four shifts. The data obtained are given below. Assume that the shift and temperature are random factors while time is fixed.

Shift	Time (minutes)	Temperature (°C) 800	900	Total
1	10	63	89	420
	20	54	91	
	30	61	62	
2	10	50	80	382
	20	52	72	
	30	59	69	
3	10	48	73	416
	20	74	81	
	30	71	69	
4	10	54	88	405
	20	48	92	
	30	59	64	
Total		693	930	1623

(i) What design is used in the experiment? Explain.

(ii) Write down a model for the experiment assuming that the three way interaction is negligible. Explain all the terms in the model and state all the assumptions.

(iii) Find the expected mean squares of the factors in the model in (ii).

(iv) Copy and complete the following ANOVA table.

Source	SS	df	MS	F
Shift				
Temp.				
Shift-temp interaction	240.458			
Time				
Shift-time interaction	478.417			
Temp-time interaction	795.250			
Error	244.417			
Total	4403.625			

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2010 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS

MAT 5642: STATISTICAL METHODS IN EPIDEMIOLOGY

INSTRUCTIONS:

1. Answer any FIVE (5) questions.
2. Calculators are allowed.
3. You may use statistical tables or formulae provided if necessary.
4. Show all your work to earn full marks.

TIME: THREE (3) Hours

Q[1] (a) List the four purposes of epidemiology.

(b) Describe the usual five-step pattern of reasoning in epidemiology.

(c) (i) Contrast the design features of prospective and case-control studies.

(ii) Discuss the strengths and limitations of case-control studies.

Q[2] (a) (i) What is meant by person-time in a cohort study?

(ii) Why is person-time a desirable unit of measurement in the estimation of rates in a cohort design ?

(b) "A stone crusher of Kamushanga compound in Kabwe district was instantly killed yesterday when the pit he was digging collapsed on him."
http://www.lusakatimes.com/2009/07/29/stone-crusher-crushed-to-death/
This is a quote from the Website of Lusaka Times dated 29/07/2009 describing some of the risks associated with stone crushing, a common enterprise in a number of cities in Zambia.

(i) State two other possible risks that could be associated with stone crushing.

(ii) State sufficiently, a study design you could use to test the strengths of association between one of the risk factors mentioned in (i) and stone crushing.

(c) The following data arose from a density-type of cohort study which focused on the possible association between elevated serum lipid levels and all-causes mortality.

	Elevated Levels	Normal Levels
Death during 1970-1975	150	150
Person-years of follow-up during 1970-1975	2,500	3,056

(i) Obtain an estimate of incidence density ratio (IDR)

(ii) Test the hypothesis H_0: IDR = 1 versus H_A: IDR > 1 using a large sample test at 5% level of protection.

(iii) Construct a test-based 95% confidence interval for the true IDR.

Q[3] (a) Suppose that in a case-control study of the association between exposure and disease the data are displayed in a table such as shown below.

	DISEASE STATUS		
EXPOSURE	D+	D–	TOTAL
E+	a	b	a + b
E–	c	d	c + d
TOTAL	a + c	b + d	n

Where D+ indicates the subject had the disease and E+ indicates presence of exposure, a – indicates the absence of the condition.

Let $p_0 = Pr(E+|D-)$ $p_1 = Pr(E+|D+)$ $f = Pr(E+)$ be the prevalence of exposure for individuals without disease and RR be the relative risk.

(i) Write down an expression for the relative risk of disease (RR).

(ii) Starting with p_1 show that: $p_1 = \frac{RR \times f}{1 + f \times (RR - 1)}$

A case-control study is being designed to investigate the association between primary bladder cancer and use of artificial sweeteners. The investigators have made an initial attempt at determining an appropriate sample size on the basis of the following specifications:

- A two-sided test with $\alpha = 0.05$, $1\text{-}\beta = 0.80$, RR = 4
- The prevalence of exposure to artificial sweeteners in individuals without disease $p_0 = 0.05$
- An equal number of individuals in the two study groups.

(b) (i) What is their initial sample size?

(ii) In which direction will the sample size go if they want to be able to detect a RR of 3 rather than 4?

(c) After all of the alternatives have been considered the investigators design their case-control study to include 200 cases and 200 controls. The number of cases and controls from their study that have used sweeteners are displayed in the following table, along with a classification of their smoking status.

CHARACTERISTICS	Cases	Controls
Any use of sweeteners	32	10
Smoking status		
Current	63	45
Former	22	30
Never	115	125

(i) Write a descriptive sentence that characterizes the frequency of current smokers in the two study groups.

(ii) Determine the odds of sweetener exposure for the bladder cancer cases.

(iii) Determine the odds of sweetener exposure for controls.

(iv) Determine the odds ratio and interpret.

Q[4] (a) Suppose that $\hat{\theta}$ is an estimate of θ and that $E(\hat{\theta}) = \theta_0$

(i) Write down an expression for the bias of $\hat{\theta}$.

(ii) Show that $E(\hat{\theta} - \theta)^2 = Var(\hat{\theta}) + \left(Bias(\hat{\theta})\right)^2$

(iii) You wish to test the hypothesis H_0: $\theta = \theta_1$ and you construct a statistic $\frac{\hat{\theta}-\theta_1}{\sqrt{Var(\hat{\theta})}}$ for the purpose. In what way will the bias affect your test?

(b) (i) Describe briefly what is meant by exposure misclassification.

(ii) State briefly why exposure misclassification is of concern to epidemiologists.

(c) Below are two tables showing a classification of disease and exposure for a target population which was followed over time. At the end of follow-up another table was constructed but resulted in disease misclassification.

TARGET POPULATION:

	Disease status		
Exposure	D+	D-	TOTAL
E+	300	700	1000
E–	200	800	1000

MISCLASSFIFIED POPULATION:

Exposure	Disease status D+	D–	TOTAL
E+	325	675	1000
E–	250	750	1000

(i) Obtain the estimates of the cumulative incidence ratio (CIR) for the target and the misclassified populations.

(ii) Obtain the estimate of the relative bias of CIR and state its direction.

(iii) Obtain a 95% Taylor series confidence interval of the true CIR using the misclassified population.

Q[5] A 1985 study identified a group of 518 cancer cases ages 15-59 and a group of 518 age and sex-matched controls. The main purpose of the study was to look at the effect of passive smoking on cancer risk. In the study, passive smoking was defined as exposure to the cigarette smoke of a spouse who smoked at least one cigarette per day for at least 6 months. Some spouses were smokers and others were non-smokers. The data for smokers and non-smokers are given below. We would like to determine whether we can combine the two tables.

Non-smoking Spouses:

Passive smoker	Case-control status Case	Control	TOTAL
Yes	120	80	200
No	111	155	266
TOTAL	231	235	466

Smoking spouse

Passive smoker	Case-control status Case	Control	TOTAL
Yes	161	130	291
No	117	124	241
TOTAL	278	254	532

(a) (i) Obtain the separate estimates of the log of odds ratio for the two tables, i.e., $\mathrm{Ln}\,\hat{\psi}_i$

(ii) Obtain the estimates of the standard errors.

(iii) Obtain the value of the combined Chi-square statistic, χ^2_{TOTAL}.

(b) (i) Obtain the pooled estimator of the log of odds ratio.

(ii) Obtain the estimate of its standard error

(iii) Obtain the Chi-square statistic for the overall association, $\chi^2_{\text{ASSOCIATION}}$.

(c) (i) Obtain the value of the Chi-square statistic for homogeneity, $\chi^2_{HOMOGENEITY}$, by using (a) and (b).

(ii) Test the hypothesis H$_0$: No Overall Association, at 1% level of significance.

(iii) Does it make sense to combine the estimates for the two tables?

Q[6] The following data were extracted from the Zambia Demographic Survey Report of 2007. One of its summary tables gives percentage distribution of women and men aged 15-49 by current marital status, according to age. For the questions that follow use the data below which are for women only.

		MARITAL STATUS		
		Divorced/ Separated/ Widowed	Married/ Living together	
AGE INTERVAL	MIDPOINT	D+	D-	TOTAL
15 – 19	17	25	280	305
20 – 24	22	121	889	1010
25 – 29	27	154	1054	1208
30 – 34	32	182	827	1009
35 – 39	37	144	590	734
40 – 44	42	142	411	553
45 – 49	47	118	293	411

Let CI_i be the incidence of losing a spouse in age interval i, i = 1 (15 -19), i = 2 (20-29) and so on.

(a) Copy and complete the table below where Risk = CIR = $CI_j/CI_{15\text{-}19}$ and $2 \leq j$.

Midpoint age interval	17	22	27	32	37	42	47
Risk	1						

(b) (i) Plot Risk versus Age (mid-value of each interval)

(ii) What is the nature of the relationship between age and risk of loosing a spouse through divorce or separation or death?

(iii) What is the drawback of defining a case as "divorced/separated/widowed"?

(c) (i) Collapse the data into a 2×2 table by creating two age categories; 15-34 and 35-49, i.e., the young and the older.

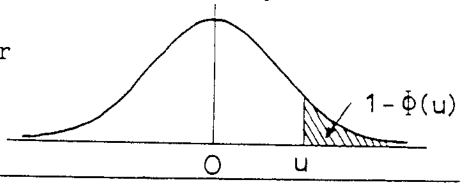
(ii) Test the significance of the Odds Ratio for the two age categories using Mantel-Haenszel Chi-Square statistic at 5% of significance.

END OF EXAMINATION

Table 3

AREAS IN TAIL OF THE NORMAL DISTRIBUTION

The function tabulated is $1 - \Phi(u)$ where $\Phi(u)$ is the cumulative distribution function of a standardised Normal variable u. Thus $1 - \Phi(u) = \frac{1}{\sqrt{2\pi}} \int_u^\infty e^{-u^2/2}\, du$ is the probability that a standardised Normal variable selected at random will be greater than a value of u $\left(= \frac{x-\mu}{\sigma}\right)$



$\frac{(x-\mu)}{\sigma}$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
3.0	.00135									
3.1	.00097									
3.2	.00069									
3.3	.00048									
3.4	.00034									
3.5	.00023									
3.6	.00016									
3.7	.00011									
3.8	.00007									
3.9	.00005									
4.0	.00003									

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS & STATISTICS
2010 ACADEMIC YEAR

SECOND SEMESTER FINAL EXAMINATIONS

MAT5662 : Theory of Non-Parametric Statistics

TIME ALLOWED: Three (3) Hours

INSTRUCTIONS: There are four (4) Questions. Full credit will only be given when all the necessary work is shown.

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1. (a) Define the following:

   (i) Equal in distribution

   (ii) Distribution – free statistic

   (iii) Non – parametric distribution-free statistic

   (b) Prove the following where $X_1, X_2, \dots, X_n$ is a random sample from a continuous distribution.

   (i) If $\psi_i = \psi(X_i - \theta_o)$, $i = 1, 2, \dots, n$ then the $\psi_1, \psi_2, \dots, \psi_n$ are *iid* Bernoulli and any statistic, $T(\psi_1, \psi_2, \dots, \psi_n)$ is distribution – free non-parametric statistics.

   (ii) If $R = (R_1, R_2, \dots, R_n)$ is a vector of ranks, where R_i is the rank of X_i among $X_1, X_2, \dots, X_n$, then R is uniformly distributed over
   $B = \{r : r \ is \ a \ permutation \ of \ integers \ 1,2,3, \dots, n\}$.

   (c) Given a random sample $X_1, X_2, \dots, X_n$ from uniform on $(0, \theta)$, find the following:

   (i) Mean and variance of $X_{(r)}$, $r = 1, 2, \dots, n$.

   (ii) Product moment of $X_{(1)}$ and $X_{(n)}$,

   (iii) Covariance of $X_{(1)}$ and $X_{(n)}$,

2. (a) (i) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a continuous distribution whose pdf $p(x)$ is symmetrical about the mean μ. Show that the pdf for $X_{(r)}$ and $X_{(n-r+1)}$ are mirror images of each other i.e. $f_r(\mu + x) = f_{n-r+1}(\mu - x)$.

(ii) The midpoint range in a random sample of size n is defined as $M = \frac{1}{2}(X_{(1)} + X_{(n)})$. If the random sample is from a continuous distribution with cdf $P_{(x)}$, show that the cdf of M is given by $F_M(m) = n\int_{-\infty}^{m}(P(2m-x) - P(x))^{n-1}\, p(x)\, dx$.

(b) Prove the following where $U(X_1, X_2, \dots, X_n)$ is a U-statistic for an estimable parameter γ with a symmetric kernel $h(X_1, X_2, \dots, X_K)$ and $E\{h^2(X_1, X_2, \dots, X_K)\} < \infty$, then

(i) $\lim_{n\to\infty} n\, var\, \{U(X_1, X_2, \dots, X_n)\} = K^2 \xi_1$.

(ii) $U(X_1, X_2, \dots, X_n)$ converges in quadratic mean to γ.

(c) Given $X_1, X_2, \dots, X_n$ a random sample of size n from an exponential with mean λ i.e. $f(x) = \frac{1}{\lambda} e^{-x/\lambda}, x > 0$. Derive the pdf of

$W = X_{(S)} - X_{(r)}$ *where* $r < S$.

3. (a) (i) Derive a general expression for the variance of a U – statistic for the estimable parameter γ.

(ii) Suppose that $X_1, X_2, \dots, X_n$ is a random sample from a continuous distribution that is symmetric about the unknown median θ. If θ_o is known and $H_o: \theta = \theta_o$ *vs* $H_a: \theta > \theta_o$, define $z_i = X_i - \theta_o, i = 1, 2, \dots, n,$ $\psi_i = \psi(z_i)$ *and let*

$W^+ = \sum_{i=1}^{n} \psi_i R_i$, be the Wilcoxon signed rank statistic.

Then prove under H_o that the distribution of W^+ is symmetric about its mean $\mu = \frac{n(n+1)}{4}$.

(b) Prove the following:

(i) If the sequence of random variables $\{V_n\}$ has asymptotic distribution with cdf $F(\cdot)$ and $\{W_n\}$ is a sequence of random variables such that $\{W_n - V_n\}$ converges in probability to zero, then the limiting distribution of $\{W_n\}$ is the same as that of $\{V_n\}$.

(ii) If $W = W(X_1, X_2, \ldots, X_n)$ treats n *iid* random variables

$X_1, X_2, \ldots, X_n$ symmetrically and $E(W) = 0$. If V^* denotes the projection of W onto ϑ, then for any $V \in \vartheta$

$$E\{(W - V^*)^2\} \leq E\{(W - V)^2\}$$

(c) (i) State Hoeffding's one-sample U-statistic theorem.

(ii) Prove Hoeffding's one-sample U-statistic theorem.

4. (a) Define the following:

(i) Signed rank.

(ii) Estimable parameter

(iii) U-statistic for estimable parameter.

(b) Given a random sample $X_1, X_2, \ldots, X_n$ from a continuous distribution with pdf $p(x)$ and cdf $P(x)$, derive the following:

(i) pdf of $X_{(r)}$

(ii) Joint pdf of $X_{(r)}$ and $X_{(s)}$ with $r < S$

(iii) pdf for the range $R = X_{(n)} - X_{(1)}$.

(c) Given a random sample $X_1, X_2, \dots, X_n$ of size n from

$$P(x) = \begin{cases} \frac{\lambda}{a}\left(\frac{x}{a}\right)^{\lambda-1}, & 0 < x < a,\ \lambda > 0 \\ 0 & otherwise. \end{cases}$$

Find the following using the results found in (b) above.

(i) Pdf of the r^{th} order statistic

(ii) Joint pdf of the r^{th} and s^{th} order statistic, with $r < S$

(iii) Pdf for the range $R = X_{(n)} - X_{(1)}$.

5. (a) Define the following:

(i) Convergence in probability of sequence of random variables to a constant.

(ii) Convergence in quadratic mean.

(iii) Projection of a U-statistic over some space.

(b) (i) State Slutsky's theorem.

(ii) Use Slutsky's theorem and other results to prove that, if

$X_1, X_2, \dots, X_n$ denote a random sample from a population with mean μ and variance σ^2, and $\zeta^2 = E(X_i - \mu)^2 - \sigma^2, i = 1,2,\dots,n$ satisfying $0 < \zeta^2 < \infty$ Then, $W_n = \sqrt{n}\left\{\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2 - \sigma^2\right\}$ has a limiting normal distribution with mean zero and variance ζ^2.

(c) (i) Derive the U-statistic estimator $U_2(X_1, X_2, \dots, X_n)$ of the variance with symmetric kernel $h(X_1, X_2) = (X_1 - X_2)^2$.

(ii) Find the projection of $U_2(X_1, X_2, \dots, X_n) - \gamma$ on the set

$$\vartheta = \{V : V = \textstyle\sum_{i=1}^{n} K(x_i),\ where\ K(\cdot)\ is\ some\ real\ valued\ function\}$$

END OF EXAM

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

MATHEMATICS AND STATISTICS DEPARTMENT

MAT 5911

First Semester Final Examinations November, 2010

Time Allowed: Three (3) hours

Instructions:
- Answer **any four (4)** Questions
- Full Credit will only be given where **all detailed** work is shown.

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1 a. (i) Define a discrete time Markov Chain.

(ii) Define a discrete time homogeneous Markov Chain.

(iii) Let $p_{ij}{}^{(n)}$ denote the n – step transition probability from state i to state j for a homogeneous Markov Chain $\{X_n, n = 0, 1, 2, \ldots\}$.

Show that

$$p_{ij}{}^{(n+m)} = \sum_{k=0}^{\infty} p_{ik}{}^{(n)}\; p_{kj}{}^{(m)}$$

(iv) Show that $P^{(2)} = P^2$ where $P^{(2)}$ is 2 – step transition probabilities matrix and P is one step transition probabilities matrix.

b. Consider a large population of individuals in which each individual possesses a particular pair of genes which are formed from gene types A and/or a. Hence each individual has one of the three pair types AA, Aa or aa.

Assume that the proportion of individuals in the present generation whose gene pairs are AA, aa or Aa are respectively p_0, q_0 and r_0 ($p_0 + q_0 + r_o = 1$). When two individuals mate, each contributes one of his or her genes, chosen at random to the resultant offspring. Assuming that the mating occurs at random.

(i) Show that the probability of a randomly chosen gene from the present generation being type A is $p_0 + \frac{r_0}{2}$

(ii) Show that the probability of a randomly chosen gene from the present generation being type a is $q_0 + \frac{r_0}{2}$

(iii) Assume each individual has exactly one offspring. Let X_n denote the genetic state (AA, aa, Aa) of an offspring in the nth generation. Find the transition matrix of the Markov Chain $\{X_n$, n = 0, 1, 2, ...}

c. Coin one comes up head with probability 0.6 and coin two with probability 0.5. A coin is continually flipped until it comes up tails at which time that coin is put aside and we start flipping the other one. If we start with coin one, Find the probability that coin two is used on the fifth flip.

2. a. Consider a discrete time Markov Chain with state space { 0, 1, 2, ...}.
Define the following terms

(i) State i communicates with state j

(ii) Irreducible Markov Chain

(iii) A recurrent state

(iv) An aperiodic state

(v) A positive recurrent state

(vi) An ergodic state

b. (i) Using the proposition that state i is recurrent if

$$\sum_{n=1}^{\infty} p_{ii}{}^{n} = \infty,$$

Prove that if state i is recurrent and state i communicates with state j then state j is recurrent.

(ii) The transition matrix of a Markov Chain with state space. {1, 2, 3} is given below. Let f_{11} denote the probability that starting from state 1, the chain returns to state 1 first time. Show that f_{11} has value one and find the mean recurrence time of state 1.

$$P = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

c. Two containers A and B are placed adjacent to each other. A total of four molecules is distributed between the two containers. At each time step, one molecule is picked at random, removed from its container and placed in the other container. Let X_n denote the number of molecules in container A after n time steps have passed.

Write the transition matrix of the Markov Chain $\{X_n,\ n = 1, 2, \ldots\}$ and find the proposition of time when all four molecules will be in container A.

3 a. Consider a one dimensional random walk in which on each transition, the process is equally likely to move one step to the right or one step to the left. Let X_n denote the position at the nth transition.

(i) Write the state space of the Markov Chain $\{X_n$, n = 0, 1, 2,.....}

(ii) Write with explanation what should be $p_{00}{}^{2n-1}$, n = 1, 2,.....

(iii) Find $p_{00}{}^{2n}$

(iv) Using sterling approximation $n! \sim n^{n+\frac{1}{2}}\, e^{-n} \sqrt{2\pi}$,

$$Show\ that \sum_{n=1}^{\infty} p_{00}{}^{2n} = \infty$$

(v) State what the result in part (iv) implies.

b. A gambler at each play of the game has probability p of winning one unit and probability 1 – p of losing one unit. Assume that the successive plays of the game are independent and that the gambler starts with i units. The gambler continues betting until either his total fortune is N units or he lost all his i units. Let X_n denote the gambler's fortune at the nth play.

(i) Find the probability that the gambler's fortune is N units.

(ii) Deduce from above that for $p > \frac{1}{2}$, there is a positive probability that the gambler's fortune will increase indefinitely.

(iii) Classify the states of the Markov Chain $\{X_n$, n = 0, 1, 2, ...}

c. (i) Define a branching process

(ii) Let $p_k = qp^k$, $k \geq 0$, where p + q = 1, be the probability that each individual has produced k new individuals. Assume number of offsprings of individuals are independent. Let the size of the zeroth generation be 1 i.e $X_0 = 1$. Find the expected size of the nth generation.

(iii) Let π_0 denote the probability that the population will eventually die out, where

$$\pi_0 \ satisfies\ the\ equation\ \pi_0 = \sum_j \pi_0{}^j \ \ p_j$$

Show that $\pi_0 = 1$ for $p = \frac{1}{2}$.

4 a. Consider a population modeled as a pure birth process in which birth rates are given by $\lambda j = j\lambda$, j = 0, 1, 2, ……

Let N(t) denote the size of the population at time t. Assume N(0) = 1 and let probability (N (t) = j) = p_j (t)

Show that p_j (t) = $\bar{e}^{\lambda t}$ (1 − $\bar{e}^{\lambda t}$, $^{j-1}$

You may use the result that the moment generating function of a geometric random variable is $\frac{pe^t}{1-q\ et}$.

b. (i) Flies and wasps enter a room in the manner of independent Poisson processes with respective intensities λ and μ. Show that the arrival of flying insects form a Poisson process with intensity $\lambda + \mu$.

(ii) The number of environmental shocks N(t) experienced by a component in the interval (0, t] is a Poisson process with rate λt. With probability p, the component will continue to function inspite of the shock and with probability 1 − p, the shock is fatal. Show that the number of fatal shocks is a Poisson process with rate (1 − p) λ.

c. A group of N animals graze in a field. The animals graze independently. Periods of grazing and resting for any one animal alternate, having exponential distributions with parameters μ and λ respectively.

Let X(t) denote the number of animals grazing at time t and M (Θ, t) be the moment generating function of X(t). Find the partial differential equation in M(Θ, t).

5. a (i) Customers arrive at a single server service station in accordance with a Poisson process having rate λ. Each customer, upon arrival, goes directly into service if the server is free and if not, the customer joins the queue. When the server finishes serving a customer, the customer leaves the system and the next customer in line, if there is any, enters service. The successive service times are assumed to be independent exponential random variables having mean $\frac{1}{\mu}$.
Let X(t) denote the number of customers in the system at time t. Find the stationery distribution of X(t).

(ii) The manager of a supermarket can hire Mary to serve on a till . Mary gives service at an exponential rate of 20 customers per hour. The manager estimates that on the average, each customer's time is worth K1 million, per hour. If customers arrive at a Poisson rate of 10 per hour, find the average per hour cost of the time in service.

b. Define the following terms for a stochastic process $\{X(t), t \geq 0\}$ whose parameter set T and state space S both are sets of real numbers.

(i) Independent increments

(ii) Stationery increments

(iii) A Weiner process

c. Suppose that the stochastic process $\{Y(t), 0 \leq t \leq 1\}$ is a Weiner process with variance parameter σ^2.

Compute

(i) $P(Y(1) > 0 \mid Y(\frac{1}{2}) = \sigma)$

(ii) $P(Y(\frac{1}{2}) > 0 \mid Y(1) = \sigma)$

You can use the following result: For a standard Wiener process X(t), the conditional distribution of X (s), given that X(t) = B, for $s < t$ is normal with mean $E(X \mid s) \mid X(t) = B) = \frac{s}{t} B$, $Var(X(s) \mid X(t) = B) = \frac{s}{t}(t - s)$

UNIVERSITY OF ZAMBIA

DEPARTMENT OF PHYSICS

M.Sc.

2011 SECOND SEMESTER UNIVERSITY EXAMINATIONS

PHY5022

MATHEMATICAL METHODS FOR PHYSICS

DURATION: Three hours.

INSTRUCTIONS: Answer any four questions from the six given. *Each question carries 25 marks with marks indicated in parenthesis.*

MAXIMUM MARKS: 100

DATE: ~~Tuesday 3th~~ May 2011.
Thursday 5th

Formulae that may be needed:

1.

$$[lm, n] = \frac{1}{2}\left(\frac{\partial g_{ln}}{\partial x^m} + \frac{\partial g_{mn}}{\partial x^l} - \frac{\partial g_{lm}}{\partial x^n}\right)$$

$$\left\{ \begin{array}{c} s \\ lm \end{array} \right\} = g^{sn}[lm, n]$$

2. Definition of the covariant derivative

$$A^{p_1 \dots p_m}_{r_1 \dots r_n ; q} = \frac{\partial A^{p_1 \dots p_m}_{r_1 \dots r_n}}{\partial x^q} - \left\{ \begin{array}{c} s \\ r_1 q \end{array} \right\} A^{p_1 \dots p_m}_{s r_2 \dots r_n} - \left\{ \begin{array}{c} s \\ r_2 q \end{array} \right\} A^{p_1 \dots p_m}_{r_1 s r_r 3 \dots r_n} - \dots\dots - \left\{ \begin{array}{c} s \\ r_n q \end{array} \right\} A^{p_1 \dots p_m}_{r_1 \dots r_{n-1} s}$$
$$+ \left\{ \begin{array}{c} p_1 \\ qs \end{array} \right\} A^{s p_2 \dots p_m}_{r_1 \dots r_n} + \left\{ \begin{array}{c} p_2 \\ qs \end{array} \right\} A^{p_1 s p_3 \dots p_m}_{r_1 \dots r_n} + \dots\dots + \left\{ \begin{array}{c} p_m \\ qs \end{array} \right\} A^{p_1 \dots p_{m-1} s}_{r_1 \dots r_1 s}$$

3.

$$\frac{\partial^2 x^r}{\partial \overline{x}^j \partial \overline{x}^k} = \overline{\left\{ \begin{array}{c} n \\ jk \end{array} \right\}} \frac{\partial x^r}{\partial \overline{x}^n} - \frac{\partial x^i}{\partial \overline{x}^j} \frac{\partial x^l}{\partial \overline{x}^k} \left\{ \begin{array}{c} r \\ il \end{array} \right\}$$

4.

$$g^{\mu\mu} = \frac{1}{g^{\mu\mu}}$$

5. For orthogonal systems $g_{\mu\nu} = 0$ for $\mu \neq \nu$

6.

$$g_{11} = 1, \quad g_{22} = r^2, \quad g_{33} = r^2 \sin^2\theta$$

7.

$$\sum_{i=1}^{i=s} n_i^2 = h$$

8.

$$\frac{1}{h}\sum_{k=1}^{k=s} p_k\ \chi^{(i)}(c_k)^* \chi^{(j)}(c_k) = \delta_{ij}$$

9.

$$\chi(c_k) = \sum_{k=1}^{k=s} c_i \chi^{(i)}(c_k)$$

10.

$$c_i = \frac{1}{h}\sum_{k=1}^{k=s} p_k\ \chi^{(i)}(c_k)^* \chi(c_k)$$

11. For the regular representation we have:

$$\chi(g) = \begin{cases} h & \text{if } g = I \\ 0 & \text{otherwise,} \end{cases}$$

12.

$$\chi(g) = n_i(\sqrt[m]{1})$$

13.

$$\sqrt[n]{z} = \sqrt[n]{r}\left(\cos\frac{\theta + 2k\pi}{n} + i\ \sin\frac{\theta + 2k\pi}{n}\right), \qquad k = 0, 1, 2, 3, \ldots, (n-1),$$

where θ is the angle the complex number z makes with the real axis.

14. The coefficients of the of the two variable series in terms of cosine and associated Legendre polynomials

$$f(\theta)g(\phi) = \sum_n c_n P_n^m(\theta)\cos n\phi \sin\theta$$

are

$$c_0 = -\frac{1}{2\pi}\int_0^\pi\int_0^\pi f(\theta)g(\phi)\ d\theta d\phi, \quad c_n = N_{nm}\int_0^\pi\int_0^\pi f(\theta)g(\phi)P_n^m \cos n\phi\ d\phi d\theta,$$

where

$$N_{nm} = \frac{(2n+1)}{\pi}\frac{(n-m)!}{(n+m)!}.$$

QUESTION 1

Determine the Christoffel symbols of the first and second kind in spherical coordinates.

(25 marks)

QUESTION 2

(a) Draw three diagrams to show how the Euler angles achieve an arbitrary rotation of axes. Then write down the three matrices corresponding to these rotations. From these obtain the matrix in terms of Euler angles corresponding to an arbitrary rotation.

(12 marks)

(b) Write down the definition for covariant differentiation of a general mixed tensor.

(6 marks)

(c) Express in matrix notation the transformation equations for (i) a covariant vector, (ii) a contravariant tensor of rank two, assuming $N = 3$.

(7 marks)

QUESTION 3

Consider the Helmholtz equation describing some field inside and outside a sphere of radius $r = a$, with boundary condition $u(r = a) = 0$:

$$\nabla^2 u + k^2 u = 0.$$

(a) Write down Green's function $G(\mathbf{x}, \mathbf{x}')$ for the Helmholtz equation, and give its physical interpretation. Take the disturbance to occur at $r' = |\mathbf{x}'|$.

(4 marks)

(b) The solution of the equation for the Green's function of part (a) is found by transforming the equation into spherical polar coordinates. The solutions before and after r' are

$$\begin{aligned} G &= \sum_n A_n j_n(r) P_n^m(\omega) \cos n\phi, \quad r < a,\ n \geq 0,\ m \geq 0, \\ G &= \sum_n B_n [j_n(r) n_n(a) - n_n(r) j_n(a)] P_n^m(\omega) \cos n\phi, \quad r > a,\ n \geq 0,\ m \geq 0, \end{aligned}$$

where $\omega = \cos\theta$. State the condition on $G(\mathbf{x}, \mathbf{x}')$, and derive the condition on its derivative $G'(\mathbf{x}, \mathbf{x}')$ needed to solve A_n and B_n. (Hint: Do not derive the coefficients of the double variable series that arises in the derivation. These are given in formula 14 of the question paper).

(21 marks)

QUESTION 4

(a) Solve the equation

$$\psi(x) = x + \frac{1}{2} \int_{-1}^{1} (t + x)\psi(t)\, dt$$

by (i) the algebraic method, and (ii) by Neumann's method up to third order.

(18 marks)

(b) Check that the exact solution by the algebraic method satisfies the integral equation.

(4 marks)

(c) By comparing with your exact answer from part (a)(i), calculate the error in the Neumann third-order approximation of part (a)(ii).

(3 marks)

QUESTION 5

(a) Define a subgroup and a class. (3 marks)

(b) Consider the permutation of group S_4 of four elements

(i) Determine the number of elements of the group. (2 marks)

(ii) Write down the elements of the group. (6 marks)

(iii) Write each element in cyclic notation and hence divide the group into classes.

(8 marks)

(iv) Find the period of each class. (6 marks)

QUESTION 6

Find the character table for the symmetry group S_6 of the equilateral triangle, which consists of the three classes: class C_1, containing the identity element, class C_2, containing three reflection elements, and class C_3 containing two rotation elements of 120°. (25 marks)

END

THE UNIVERSITY OF ZAMBIA

DEPARTMENT OF PHYSICS
SECOND SEMESTER EXAMINATION 2009

PHY5222: CONDENSED MATTER PHYSICS II

TIME: 3 HOURS
INSTRUCTIONS: ANSWER ANY **FOUR** QUESTIONS
TOTAL MARKS 100
ALL QUESTIONS CARRY EQUAL MARKS

Fermion operator anticommutation relations
$$b_i^+ b_j + b_j b_i^+ = \delta_{ij}$$
$$b_i^+ b_j^+ + b_j^+ b_i^+ = 0$$
$$b_i b_j + b_j b_i = 0$$

Spherical polar coordinates $\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \phi^2}$

Electrodynamics wave equation $\frac{\partial \ell}{\partial A_\lambda} - \sum_{m=1}^{4} \frac{\partial}{\partial x_\mu}\left\{\frac{\partial \ell}{\partial(\partial A_\lambda / \partial x_\mu)}\right\} = 0, \quad \lambda = 1,2,3,4$

Q1. (a) Demonstrate Pauli's exclusion principle using the fermion operators b and b^+. [3]

(b) (i) Find matrix representations for the operators a^+, a, p and q. [8]

(ii) Show that the boson operators a^+ and a are not Hermitian, but are adjoints of one another. [2]

(iii) Show that the matrices representing p and q are Hermitian. Why is this result to be expected? [2]

(c) An anharmonic oscillator has a Hamiltonian of the form

$$H = \frac{1}{2}\left(p^2 + \omega^2 q^2\right) + \varepsilon q^4 .$$

Use the first-order perturbation theory to calculate the level-shift produced by the small anharmonic term. [10]

Q2. (a) Show that the operator $b_j^+ b_j$ has eigenfunctions corresponding to the occupation number ν_j of a Slater determinant. [4]

(b) Show that the state $\prod_i (b_i^+)^{\nu_i} |0\rangle$ is orthogonal to the state $\prod_i (b_i^+)^{\nu_i'} |0\rangle$ unless $\nu_i = \nu_i'$ for all i. [8]

(c) Consider a system of N harmonic oscillators with respective eigenfunctions $|n_1\rangle, \ldots\ldots |n_k\rangle, \ldots\ldots. |n_N\rangle$. In condensed matter physics it is convenient to represent this system by a single eigenfunction $|n_1, \ldots\ldots n_k, \ldots\ldots n_N\rangle$.

Let $|0\rangle = |0, \ldots\ldots 0, \ldots\ldots 0\rangle$ represent the ground state of the system. It was shown in the lecturers that

$$|n_1, n_2, \ldots, n_k, \ldots\rangle = \prod_k \frac{(a_k^+)^{n_k}}{\sqrt{n_k!}} |0\rangle .$$

Now show that the conjugate state of the above eigenfunction is

$$\langle n_1, n_2, \ldots, n_k, \ldots| = \langle 0| \prod_k \frac{(a_k)^{n_k}}{\sqrt{n_k!}} .$$ [13]

Q3. (a) What is the Lagrangian density of a system? [3]

(b) A continuous system can be described by a number of fields $\phi_1, \phi_2, \ldots\ldots \phi_n$ which are continous functions of space and time. Consider a system whose Lagrangian density is given by

$$\ell = \sqrt{g}\left(\sum_{\mu,\nu=1}^{4} g^{\mu\nu} \frac{\partial\phi}{\partial x_\mu}\frac{\partial\phi}{\partial x_\nu} + \kappa^2\phi^2\right).$$

If

$$g^{\mu\nu} = \begin{pmatrix} 1\ldots0\ldots\ldots.0\ldots\ldots..0 \\ 0\ldots\frac{1}{r^2}\ldots.0\ldots\ldots..0 \\ 0\ldots0.\frac{1}{r^2\sin^2\theta}.0 \\ 0\ldots0\ldots\ldots.0\ldots\ldots..1 \end{pmatrix}$$

and $\sqrt{g} = r^2\sin\theta$. What are the equations of motion expressed in terms of the coordinates $x_1 = r$, $x_2 = \theta$, $x_3 = \phi$, and $x_4 = ict$? [10]

(c) Show that the variational principle leads us to the wave equation

$$\frac{\partial\ell}{\partial\phi_i} - \sum_{\mu=1}^{4}\frac{\partial}{\partial x_\mu}\left[\frac{\partial\ell}{\partial(\partial\phi_i/\partial x_\mu)}\right] = 0$$

Q4. (a) Write down the three steps used in quantum electrodynamics to quantise the electromagnetic potentials. [6]

(b) The vector and scalar electromagnetic potentials can be combined into a 4-vector

$A_\lambda = \{\vec{A}, i\phi\}$, $\lambda = 1,2,3,4.$

Show that when substituted into the electromagnetic wave equation, the Lagrangian density given in terms of electromagnetic potentials by

$$\ell = -\frac{1}{8\pi}\sum_{\lambda,\mu=1}^{4}\left(\frac{\partial A_\lambda}{\partial x_\mu}\right)^2 + \frac{1}{c}\sum_{\lambda=1}^{4} A_\lambda j_\lambda$$

leads to the common wave equation

$$\Box^2 A_\lambda = -\frac{4\pi}{c} j_\lambda$$

where $\Box^2 = \Delta^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ and j_λ is the current density. [8]

(c) If we represent the Hamiltonian density as

$$H = \sum_{\lambda=1}^{4} \frac{\partial A_\lambda}{\partial t}\left\{\frac{\partial \ell}{\partial(\partial A_\lambda / \partial t}\right\} - \ell :$$

(i) By using

$$\ell = -\frac{1}{8\pi}\sum_{\lambda,\mu=1}^{4}\left(\frac{\partial A_\lambda}{\partial x_\mu}\right)^2 + \frac{1}{c}\sum_{\lambda=1}^{4} A_\lambda j_\lambda$$

divide the Hamiltonian density into an interaction term, H_i and a radiation term, H_r. Recall that $x_4 = ict$; [7]

(ii) Give the physical interpretations of H_i and H_r in (i). [4]

Q5. (a) Why is the application of quantum field theory well suited to the study of condensed matter? [3]

(b) Consider the transition of a system from an initial state $|A\rangle$ to a final state $|B\rangle$. Given that $b_i^+ b_i |A\rangle = v_i |A\rangle$ where v_j is the occupation number of a Slater determinant and that $|A\rangle$ is properly normalised, show that $|B\rangle$ is also properly normalised for the following transitions:

(i) $|B\rangle = \frac{1}{\sqrt{n_{\vec{k}\lambda}+1}} b_j^+ b_i a_{\vec{k}\lambda}^+ |A\rangle$; [7]

(ii) $|B\rangle = \frac{1}{\sqrt{n_{\vec{k}\lambda}}} b_j^+ b_i a_{\vec{k}\lambda} |A\rangle$. [7]

Here $n_{\vec{k}\lambda}$ is the number of particles of wave number $\vec{k}$ and polarization λ.

(c) Give a detailed physical example to interpret each of the two transitions in (b). [6]

(d) What particular particles are represented by $n_{\vec{k}\lambda}$ in your physical examples of (c). [2]

Q6. (a) What do Feynman diagrams in general and the number of vertices on each diagram in particular represent? [4]

(b) Consider the transition from a state $|A\rangle$ to a final state $|B\rangle$ given by

$$|B\rangle = \frac{1}{\sqrt{n_{\vec{k}\lambda}}} a^{+}_{\vec{k}'\lambda'} a_{\vec{k}\lambda} |A\rangle .$$

Show that if $|A\rangle$ is properly normalised then $|B\rangle$ is also properly normalised. [7]

(c) Assuming the interaction Hamiltonian H_i is known, distinguish between first and second-order transitions of a system from an initial state $|A\rangle$ to a final state $|B\rangle$. Use the first and second-class intermediate transitions

$|C_1\rangle = \frac{1}{\sqrt{n_{\vec{k}\lambda}}} b^{+}_{j} b_i a_{\vec{k}\lambda} |A\rangle$ and $|C_2\rangle = b^{+}_{j} b_i a^{+}_{\vec{k}'\lambda'} |A\rangle$ respectively to do this. [6]

(d) (i) Draw Feynman diagrams showing the first and second-class intermediate transitions given in (c). [4]

(ii) What single general physical process is represented by the diagrams in (i). [2]

(iii) Write an expression for the total probability of a transition from the state $|A\rangle$ to the state $|B\rangle$ per unit time. [2]

END OF PHY5222 EXAMINATION

UNIVERSITY OF ZAMBIA
DEPARTMENT OF PHYSICS

2011 SECOND SEMESTER UNIVERSITY EXAMINATIONS

PHY 5822
SOLAR ENERGY MATERIALS

DURATION:	Three (3) hours
INSTRUCTIONS:	Answer **any four (4)** questions. The marks for each question are given in square brackets.
MAXIMUM MARKS:	100
DATE:	Monday May 16, 2011

Some useful identities and formulae are given below:

$h = 6.63x10^{-34}\,J.S$ $\quad k = 1.38x10^{-23}\,J/K$ $\quad \hbar = 1.055x10^{-34}\,J.s$ $\quad \eta = \dfrac{I_{sc}V_{oc}}{I_N A}FF$

$n = N_c \exp\left[-\dfrac{E_c - E_F}{kT}\right]$ $\quad p = N_v \exp\left[-\dfrac{E_F - E_v}{kT}\right]$ $\quad n = n_i \exp\left[-\dfrac{E_i - E_F}{kT}\right]$ $\quad N = n + ik$

$i\sin\delta = \dfrac{e^{i\delta} - e^{-i\delta}}{2}$ $\quad \cos\delta = \dfrac{e^{i\delta} + e^{-i\delta}}{2}$ $\quad \delta = \dfrac{2\pi nd}{\lambda}$ $\quad I = I_o\left[e^{\frac{qV}{kT}} - 1\right]$ $\quad R = rr^*$

$\nabla^2 = i\dfrac{\partial^2}{\partial x^2} + j\dfrac{\partial^2}{\partial y^2} + k\dfrac{\partial^2}{\partial z^2}$ $\quad \alpha = \dfrac{4\pi k}{\lambda}$ $\quad q = \dfrac{\omega}{c}N$ $\quad T = \dfrac{T_1T_2\exp(-\alpha d)}{1 - R_1R_2\exp(-2\alpha d)}$

$R = R_1 + \dfrac{R_1(1 - R_1^2)\exp(-2\alpha d)}{1 - R_1^2\exp(-2\alpha d)}$ $\quad R = \dfrac{(1-n)^2 + k^2}{(1+n)^2 + k^2}$ $\quad I = I_o e^{-\alpha x}$ $\quad \alpha = \dfrac{4\pi k}{\lambda}$ $\quad T = tt^*$

$T_\perp = \dfrac{N_2}{N_1}\left|\dfrac{2N_1}{N_2 + N_1}\right|^2$ $\quad R_\perp = \left|\dfrac{N_1 - N_2}{N_1 + N_2}\right|^2$ $\quad n_i^2 = np$ $\quad c = \dfrac{1}{\sqrt{\mu_o\varepsilon_o}}$ $\quad \varepsilon = \varepsilon_1 + i\varepsilon_2$

Q1. (a) Aluminium on glass is often used as a mirror. Aluminium has refractive index $(N = 1 + i7)$ in the visible. Calculate the reflectance on both sides of such a mirror. The glass is non-absorbing (extinction coefficient k = 0) and the aluminium film is opaque. The refractive index of glass is 1.5. (Illustrate your answer by a suitable diagram).

[13]

(b) The total amplitude transmittance for a thin film of thickness *"d"* is given by

$t = \dfrac{t_1 t_2 e^{-i\delta}}{1 + r_1 r_2 e^{-2i\delta}}$, where $\delta = \dfrac{2\pi n d}{\lambda}$. Show that the total transmittance T for the thin film may be expressed as $T = \dfrac{1 + r_1^2 r_2^2 - (r_1^2 + r_2^2)}{1 + 2r_1 r_2 \cos 2\delta + r_1^2 r_2^2}$. Note that $t_1 = 1 - r_1$ and $t_2 = 1 - r_2$. [12]

Q2. (a) One of Maxwell's equations for a plane wave propagating in an energy absorbing medium is given by

$$\nabla^2 E = \frac{\varepsilon\mu}{c^2}\frac{\partial^2 E}{\partial t^2} + \frac{4\pi\sigma\mu}{c^2}\frac{\partial E}{\partial t}.$$

The solution to this equation for a plane wave propagating along the x-axis is of the form

$$E(x,t) = E_o \exp i(qx - \omega t).$$

Where E_o is the amplitude at x = 0 and is perpendicular to the wave vector $q = \dfrac{N\omega}{c}$, ω is the angular frequency, N is the complex refractive index and c is the speed of the plane wave in vacuum. μ is the magnetic permeability and σ is the optical conductivity.

(i) Show that the following relation holds.

$$q^2 = \mu\frac{\omega^2}{c^2}\left(\varepsilon + i\frac{4\pi\sigma}{\omega}\right).$$ [8]

(ii) Use the result in (i) above to show that $n^2 - k^2 = \varepsilon$ and $2nk = \varepsilon_2$ for non-magnetic material; where $\varepsilon_2 = \dfrac{4\pi\sigma}{\omega}$. [8]

(b) (i) What is ellipsometry? (ii) An ellipsometer has a light source, a polarizer, a compensator, an analyzer and a detector. Draw a labeled diagram of an ellipsometer. [9]

Q3. **(a)** The dark saturation current density of homo-junction solar cells at room temperature (27°C) for material 1 is $10^{-8} A/m^2$ and the corresponding value for material 2 is the $10^{-11} A/m^2$. Which of these materials would you expect to have the smaller band gap? Show your working clearly. (Hint: $I_o = 1.5x10^5 \exp(-E_g/kT) A/cm^{-2}$) [12]

(b) A silicon ingot is doped with 10^{22} phosphorous atoms per m^3. Find

(i) the carrier concentrations (n and p) and

(ii) the Fermi level at room temperature (300K). The values $n_i = 1.45x10^{16} m^{-3}$ and $n_c = 2.8x10^{25} m^{-3}$ for silicon are at room temperature and complete ionization is assumed. [13]

Q4. **(a)** The photovoltaic effect phenomenon may be expressed as

$$I = I_o\left[\exp\left(\frac{eV}{kT}\right) - 1\right] - I_L,$$

where the symbols have their usual meaning. (i) Derive an expression for the voltage when the overall cell current is zero. (ii) Hence, calculate the voltage for a silicon solar cell with the following parameters at 40°C; $I_L = 4.2A$, $I_o = 1.5x10^5 \exp(-E_g/kT)$ and $E_g(S_i) = 1.11eV$. [14]

(b) The standard 4mm float glass transmits 90% in the middle of the visible spectrum ($\lambda = 0.5\mu m$). Calculate the value of the extinction coefficient k at this wavelength. [11]

Q5. **(a)** The reflectance of an anti-reflecting coating is generally given as $R = \left(\frac{n_3 - n_2^2}{n_3 + n_2^2}\right)^2$.

Where n_3 is the refractive index of the substrate, and n_2 is the refractive index of the thin film (coating). A thin semiconductor film has been deposited on quartz with refractive index 1.5. If the reflectance of the film is 0.2, calculate the

(i) thickness of the film

(ii) refractive index of the film

(iii) band gap energy when the cut-off wavelength is 0.43μm. [15]

(b) In a certain experiment, the total reflectance of silicon was found to be 30% at zero photon energy i.e $\omega = 0$. Use this information to calculate the static dielectric constant, $\varepsilon_1(0)$ of silicon. Hint: Determine the refractive index of silicon. [10]

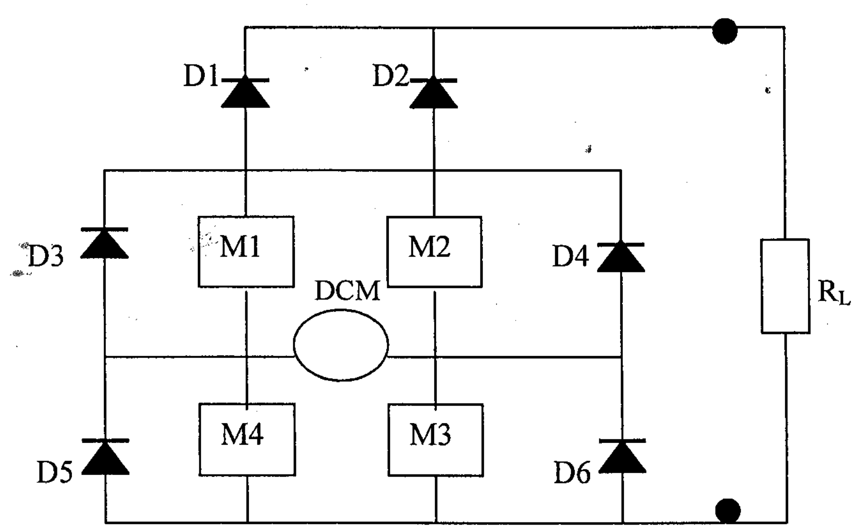
Q6. (a) (i) Draw an equivalent circuit of a solar cell and label all the essential parts.

(ii) Draw a labeled diagram of a typical solar cell showing all the essential features.

[5+7]

(b) In the figure below, the modules (M1 – M4) are connected in a bridge circuit where DCM is a DC motor and R_L is the load resistance.

(i) Identify the diodes (D1- D6) whether they are blocking diodes or by-pass diodes.



(ii) Explain the functions of each diode. [13]

--------------End of Examination--------------

The University of Zambia

Department of Physics

University Examination

2010/11 Academic Year

PHY 5911

Computational Physics and Modeling I

Instructions **Max. Marks 100**

- ***Time allowed:*** *Three (3) Hours.*
- ***All questions carry equal marks.***
- *Marks for each question are shown in the square brackets [ ].*
- *Whenever necessary, use the information given in the* ***appendix***.
- ***Answer any four (4) questions.***

Q.1 **(a)** Using the quadratic Lagrange interpolating polynomial $P_2(x)$ to the three distinct equally spaced points x_0, x_1, and x_2, with $x_1 = x_0 + h$ and $x_2 = x_0 + 2h$ for smaller values of h, derive;

i) the three-point central difference formula for finding the approximation of the first derivative of the function $f(x)$ at the given point x_1, **[4 Marks]**

ii) the three-point forward difference formula for finding the approximation of the first derivative of the function $f(x)$ at the given point x_0, **[5 Marks]**

iii) the three-point backward difference formula for finding the approximation of the first derivative of the function $f(x)$ at the given point x_2. **[5 Marks]**

(b) Consider the points

$$\begin{aligned}(x_0, y_0) &= (1, 2)\\ (x_1, y_1) &= (2, 4)\\ (x_2, y_2) &= (3, 8)\\ (x_3, y_3) &= (4, 16)\\ (x_4, y_4) &= (5, 32)\end{aligned}$$

Estimate $f'(x_2) = f'(3)$, using;

i) three-point central difference formula, **[2 Marks]**

ii) three-point forward difference formula, **[2 Marks]**

iii) three-point backward difference formula. **[2 Marks]**

(c) Use the third-order Taylor's expansion to derive the three-point central difference formula for the approximation of the *second derivative* of the function $f(x)$ at a given point x_i given data at three equally spaced nodes x_{i-1}, x_i and x_{i+1}. **[5 Marks]**

Q.2 **(a)** Explain the difference between quadrature and the Newton-cotes methods of integration and give an example of each. **[5 Marks]**

(b) Assuming that the function values are given at $x_0 = -h$, $x_1 = 0$ and $x_2 = h$ as y_0, y_1 and y_2 respectively and the polynomial that interpolates these points as

$$p(x) = ax^2 + bx + c$$

Show that integrating this polynomial from $-h$ to h gives the basic Simpson's rule

$$\int_{-h}^{h} (ax^2 + bx + c)dx = \frac{h}{3}(y_0 + 4y_1 + y_2)$$

[10 Marks]

(c) Use the Romberg method to approximate $\displaystyle\int_0^1 \frac{x^2}{1 + x^3}dx$ up to $R_{2,2}$ **[10 Marks]**

Q.3 A car traveling along a straight road is clocked at a number of points. The data from the observations are given in the following table, where the time is in seconds, the distance in meters, and the speed is in meters per second.

Time	0	3	5	8	13
Distance	0	68.58	116.74	189.90	302.67
Speed	22.86	23.47	24.38	22.56	21.95

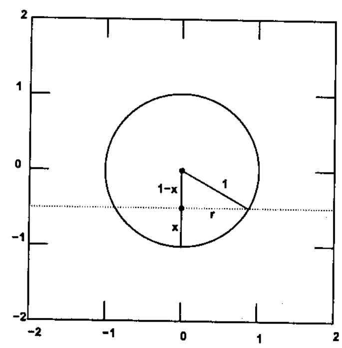
i) Use a Hermite polynomial to predict the position of the car and its speed when $t = 10$ s. **[10 Marks]**

ii) Use the derivative of the Hermite polynomial to determine whether the car ever exceeds a 90 km/h speed limit on the road. If so, when is the first time the car exceeds this limit? **[10 Marks]**

iii) What is the predicted maximum speed of the car? **[5 Marks]**

Q.4 (a) Consider the problem of finding the floating depth for a cork ball of radius 1 whose specific gravity is ρ (see figure below). The volume of the submerged segment of the sphere is

$$V = \frac{\pi}{6}x(3r^2 + x^2)$$



To find the depth at which the ball floats, solve the equation which states that the volume of the submerged segment is ρ times the volume of the entire sphere.

i) Show that the equation for the depth simplifies to $x^3 - 3x^2 + 4\rho = 0$. **[5 Marks]**

ii) If the specific gravity of cork is $\rho = 0.25$, use *three* steps of the secant method to estimate the floating depth of the ball. *Hint*: The zero that is of physical interest is between 0 and 2 since the ball is of unit radius and x is measured up from the bottom of the ball. **[15 Marks]**

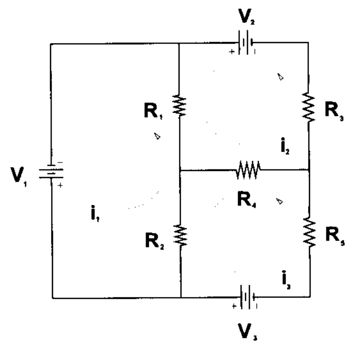
(b) Use the fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^3 + 4x^2 - 10 = 0$ on $[1, 2]$. Use $p_0 = 1.5$. **[5 Marks]**

Q.5 (a) Using the tabulated data for viscosity at several different temperatures, find the Lagrange interpolating polynomial and estimate the viscosity at $T = 25$ and $T = 40$.

$T(C)$	5	20	30	50	55
$m(N - sec/m^2)$	0.08	0.015	0.009	0.006	0.0055

[10 Marks]

(b) The figure below shows an electric circuit. The problem is to find the currents in different parts of the circuit.



Given that $\mathbf{R_1} = 20$, $\mathbf{R_2} = 10$, $\mathbf{R_3} = 25$, $\mathbf{R_4} = 10$, $\mathbf{R_5} = 30$, $\mathbf{V_1} = 0$, $\mathbf{V_2} = 0$ and $\mathbf{V_3} = 200$,

i) Show that the equations for the three loops simplify to

$$30i_1 - 20i_2 - 10i_3 = V_1$$

$$-20i_1 + 55i_2 - 10i_3 = V_2$$

$$-10i_1 - 10i_2 + 50i_3 = V_3$$

[5 Marks]

ii) Use the appropriate LU factorization method to find the currents in the above circuit. **[10 Marks]**

Q.6 (a) Given the matrix

$$\mathbf{H} = \begin{bmatrix} 11 & -6 & 4 & -2 \\ 4 & 1 & 0 & 0 \\ -9 & 9 & -6 & 5 \\ -6 & 6 & -6 & 7 \end{bmatrix}$$

i) Use the basic power method to find the dominant eigenvalue, **[5 Marks]**

ii) Use the inverse power method to find the eigenvalue of smallest magnitude.

[5 Marks]

(b) Approximate the integral

$$\int_0^1 x \sin(\pi x) dx$$

using the Gaussian quadrature method. **[5 Marks]**

(c) Find the trigonometric polynomial $y(x)$, of degree two (2), for the five data points shown in the following table:

n	0	1	2	3	4
x	0	$2\pi/5$	$4\pi/5$	$6\pi/5$	$8\pi/5$
y	0	3	2	0	-1

[10 Marks]

⋆⋆⋆⋆⋆⋆⋆ **End of Examination** ⋆⋆⋆⋆⋆⋆⋆

Appendix

Lagrange Coefficient Polynomial:

$$L_k(x) = \prod_{\substack{i=0 \\ i\neq k}}^{n} \frac{(x-x_i)}{(x_k-x_i)} = \frac{(x-x_0)(x-x_1)\dots(x-x_{k-1})(x-x_{k+1})\dots(x-x_n)}{(x_k-x_0)(x_k-x_1)\dots(x_k-x_{k-1})(x_k-x_{k+1})\dots(x_k-x_n)}$$

for each $k = 0, 1, 2, \cdots, n$ and n is number of data points.

Taylor's Polynomials:

$$f(x+h) \approx f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x) + \frac{h^4}{4!}f^{(4)}(\eta_1)$$

$$f(x-h) \approx f(x) - hf'(x) + \frac{h^2}{2!}f''(x) - \frac{h^3}{3!}f'''(x) + \frac{h^4}{4!}f^{(4)}(\eta_2)$$

where $x \leq \eta_1 \leq x+h$ and $x-h \leq \eta_2 \leq x$

Composite Trapezoidal Rule:

$$R_{k,1} = \frac{1}{2}\left[R_{k-1,1} + h_{k-1}\sum_{i=1}^{2^{k-2}} f(a+(2i-1)h_k)\right]$$

for each $k = 2, 3, \cdots, n$ and $R_{1,1} = \frac{(b-a)}{2}[f(a)+f(b)]$. a and b are lower and upper limits of intergration respectively. The step size, $h_k = (b-a)/m_k = (b-a)/2^{k-1}$. m_k is the number of trapezoids. n is a positive integer.

Romberg Integration:

$$R_{k,j} = R_{k,j-1} + \frac{R_{k,j-1} - R_{k-1,j-1}}{4^{j-1}-1}$$

for each $k = 2, 3, \cdots, n$ and $j = 2, \cdots, k$ and n is a positive integer.

Hermite Polynomial:

$$P(x) = \sum_{i=0}^{n} f(x_i)H_i(x) + \sum_{i=0}^{n} f'(x_i)\hat{H}_i(x)$$

where

$$H_i(x) = \left[1 - 2(x-x_i)L_i'(x_i)\right] L_i^2(x)$$

and

$$\hat{H}_i(x) = (x-x_i)L_i^2(x)$$

$L_i(x)$ is the i^{th} Lagrange coefficient polynomial.

Gaussian Quadrature:

n	x_i	c_i
2	± 0.57735	1.00000
3	0	0.8889
	± 0.77459	0.55556
4	± 0.33998	0.65215
	± 0.86114	0.34785
5	0	0.56889
	± 0.53847	0.47863
	± 0.90618	0.23693
6	± 0.23862	0.46791
	± 0.66121	0.36076
	± 0.93247	0.17132

Secant Method:

$$x_{k+1} = x_k - \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})} f(x_k)$$

Doolittle:

$L = [l_{ij}]$, $U = [u_{ij}]$ where

$$\begin{aligned}
u_{1j} &= a_{1j} && j = 1, \cdots, n \\
l_{i1} &= a_{i1}/u_{11} && i = 2, \cdots, n \\
u_{ij} &= a_{ij} - \sum_{s=1}^{i-1} l_{is} u_{sj} && j = i, \cdots, n \quad i \geqq 2 \\
l_{ij} &= \frac{1}{u_{jj}}(a_{ij} - \sum_{s=1}^{j-1} l_{is} u_{sj}) && i = j+1, \cdots, n \quad j \geqq 2
\end{aligned}$$

Cholesky:

$L = [l_{jk}]$, where

$$\begin{aligned}
l_{11} &= \sqrt{a_{11}} && \\
l_{j1} &= a_{j1}/l_{11} && j = 2, \cdots, n \\
l_{jj} &= a_{jj} - \sum_{s=1}^{j-1} l_{js}^2 && j = 2, \cdots, n \\
l_{pj} &= \frac{1}{l_{jj}}(a_{pj} - \sum_{s=1}^{j-1} l_{js} l_{ps}) && p = j+1, \cdots, n \quad j \geqq 2
\end{aligned}$$

Three-point Difference Formulas:

$$
\begin{aligned}
f'(x_i) &\approx \frac{f(x_{i+1}) - f(x_{i-1})}{x_{i+1} - x_{i-1}} \\
f'(x_i) &\approx \frac{4f(x_{i+1}) - 3f(x_i) - f(x_{i+2})}{x_{i+2} - x_i} \\
f'(x_i) &\approx \frac{3f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{x_i - x_{i-2}} \\
f''(x_i) &\approx \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2}
\end{aligned}
$$

The University of Zambia

Department of Physics

University Second Semester Examination-2010/2011

PHY 5922

Computational Physics and Modelling II

Instructions **Max. Marks 100**

- ***Time allowed:*** *Three (3) Hours.*
- ***All questions carry equal marks.***
- *Marks for each question are shown in the square brackets [ ].*
- *Whenever necessary, use the information given in the* ***appendix.***
- ***Answer any four (4) questions.***

Q.1 A simple model of the spread of disease gives

$$P' = kP(C - P)$$

where $P(t)$ represents the number of individuals in the population who are infected, and C is the constant size of the total population. The solution of this differential equation is the logistic function. Suppose now that the parameter k fluctuates (e.g., perhaps because of the population is more susceptible during certain seasons). Solve the modified problem,

$$P' = (k + 0.1\sin(t))P(C - P)$$

at $t = 5$ using the second order Taylor series expansion and a step size of five years. Compare the result to that of the original model. Take $P(0) = 10$, $k = 2$, $C = 2000$.

[25 Marks]

Q.2 The approximation y_{n+1} of $y(x_{n+1})$ is given by

$$y_{n+1} = y_n + \int_{x_n}^{x_{n+1}} P(x)dx.$$

Given the points (x_{n-1}, y_{n-1}) and (x_n, y_n), choose an appropriate polynomial of first degree (i.e $P_2(x)$) and derive the second order Adams-Bashforth formula;

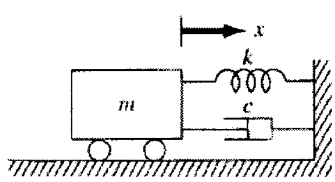
$$y_{n+1} = y_n + \frac{3}{2}hf(x_n, y_n) - \frac{h}{2}f(x_{n-1}, y_{n-1}).$$

[25 Marks]

Q.3 The motion of a damped spring-mass system (see figure below) is described by the following ordinary differential equation

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0.$$

where x = displacement from equilibrium position (m), t = time (s), m = 20 kg mass, and c = the damping coefficient (N.s/m).



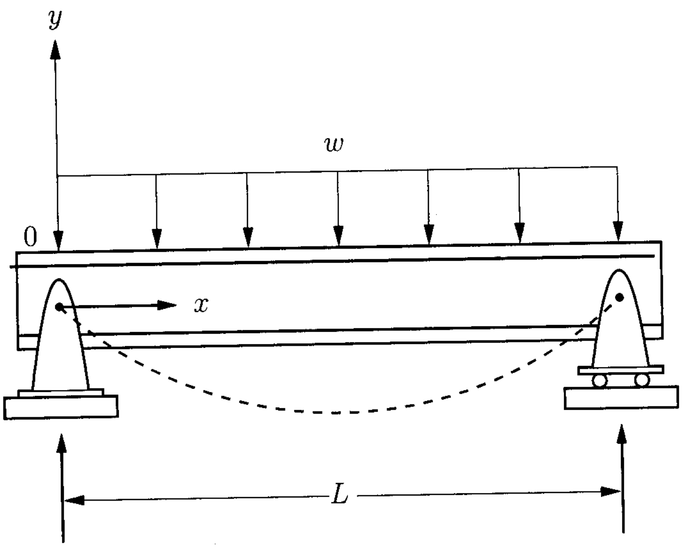
If the damping coefficient c has a value of 5 (underdamped), the spring constant $k = 20$ N/m, the initial velocity is zero, and the initial displacement $x = 1$ m, solve this equation using Euler's method over the time period $0 \le t \le 6$ using a time-step of 2s.

[25 Marks]

Q.4 The basic differential equation of the elastic curve for a simply supported, uniformly loaded beam (see figure below) is given as

$$EI\frac{d^2y}{dx^2} = \frac{wLx}{2} - \frac{wx^2}{2}$$

where E = the modulus of elasticity, and I = the moment of inertia.



The boundary conditions are $y(0) = y(L) = 0$. Solve for the deflection of the beam using the shooting method. The following parameter values apply; E = 200 GPa, $I = 30,000\text{cm}^4$, $w = 15\text{kN/m}$, and $L = 3\text{m}$. Use $\Delta x = 0.6\text{m}$.

[25 Marks]

Q.5 The temperature in a rod of unit length is given by partial differential equation;

$$\frac{\partial u}{\partial t}u(t,x) - \frac{\partial^2 u}{\partial x^2}(t,x) = 0, \qquad \text{for} \qquad 0 < x < 1, \qquad 0 < t.$$

The initial temperature of the rod is

$$u(x,0) = x^4, \qquad \text{for} \qquad 0 < x < 1$$

and the temperatures at $x = 0$ and $x = 1$ are respectively

$$u(0,t) = 0, \qquad \text{and} \qquad u(1,t) = 1 \qquad \text{for} \qquad 0 < t.$$

Taking a coarse mesh with $h = \Delta x = 0.2$ and a time step $k = \Delta t = 0.02$, use an explicit method to find the temperature at all the mesh points of the rod for $0 \leq t \leq 0.1$.

[25 Marks]

Q.6 Consider the motion of a vibrating string of unit length with both ends held fixed and an initial displacement described by the partial differential equation;

$$\frac{\partial^2 u}{\partial t^2}u(x,t) - \frac{\partial^2 u}{\partial x^2}(x,t) = 0, \qquad \text{for} \qquad 0 < x < 1, \qquad 0 < t.$$

The initial conditions are

$$u(x,0) = x(1-x), \qquad \frac{\partial u}{\partial t}(x,0) = 0 \qquad \text{for} \qquad 0 < x < 1$$

with boundary conditions of

$$u(0,t) = 0, \qquad u(1,t) = 0 \qquad \text{for} \qquad 0 < t$$

at $x = 0$ and $x = 1$ repectively. Use $h = \Delta x = 0.2$ and $k = \Delta t = 0.2$ to determine the position of all the mesh points on the string for $t = 0.0$s to $t = 1.0$s.

[25 Marks]

⋆⋆⋆⋆⋆⋆⋆ **End of Examination** ⋆⋆⋆⋆⋆⋆⋆

Appendix

Taylor series:

$$y(x+h) = y(x) + hf(x,y) + \frac{h^2}{2} f'(x,y) + \cdots + \frac{h^n}{n!} f^{(n-1)}(x,y) + \frac{h^{n+1}}{(n+1)!} f^{(n)}(\xi, y(\xi))$$

where ξ is some number between x and $x+h$

Parabolic Partial Differential Equation:

The heat equation

$$\frac{\partial u}{\partial t}(x,t) = \alpha^2 \frac{\partial^2 u}{\partial x^2}(x,t), \qquad 0 < x < l, \qquad , t > 0$$

subject to the conditions

$$u(0,t) = u(l,t) = 0, \qquad t > 0,$$

and

$$u(x,0) = f(x), \qquad 0 \leq x \leq l,$$

α is a constant and taking a step-size $h = l/m$ (for any positive integer m) and time-step k. By the explicit method, the approximate value of $u(x,t)$ at the point (x_i, t_i) is denoted as $u_{i,j}$ and is given by;

$$u_{i,j+1} = (1 - 2\lambda) u_{i,j} + \lambda (u_{i+1,j} + u_{i-1,j})$$

where $\lambda = \alpha^2 k/h$.