

**EXPLORING THE TEACHING OF GEOMETRIC AND SPATIAL REASONING IN
EARLY CHILDHOOD EDUCATION IN SELECTED PRIMARY SCHOOLS IN
SHIBUYUNJI DISTRICT**

BY

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**A Dissertation submitted to the University of Zambia in partial fulfilment of the
requirements for the award of the Degree of Masters of Education in Mathematics
Education (M. Ed-Mathematics Education)**

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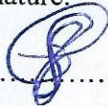
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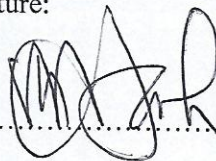
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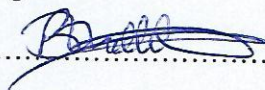
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ABSTRACT

The Ministry of General Education (MOGE, 2017) made Early Childhood Education (ECE) compulsory to all Primary Schools. Early Childhood Education (ECE) is the foundation of a child's school education and progress at a later stage in life. Early Childhood education encompasses areas such as numeracy, literacy, expressive arts, technology and social studies. Numeracy and later Mathematics is one of the areas that learners find challenging especially in later school life. Meanwhile the development of spatial sense and geometrical reasoning is an essential tool for mathematical thinking. The acquisition of spatial reasoning and geometrical reasoning by learners, however, is highly attributed to how teachers teach them. The study therefore explored the teaching of geometric and spatial reasoning at early childhood Education in selected primary schools of Shibuyunji district of Zambia. The van Hiele level theory and Piaget developmental stages were used. The approach was qualitative and used descriptive research design. The sample size was seven ECE teachers who were selected using purposive sampling. Data was collected using semi structured interview guides, lesson observation and document analysis. Data was analysed thematically according to emerging themes. The findings were that ECE teachers used a number of strategies such as exposing learners to real objects, practical activities and demonstrations, while discussion was not effective because of the age and the level of reasoning of the learners at ECE. In view of these findings, the study recommended that teachers should use suitable methods which should depend on the age and level of understanding of the learners. Seminars on the teaching of geometry for ECE teachers should be conducted in order to enhance the in-depth knowledge on the topic to enable them teach it with confidence and use appropriate materials and methods.

Keywords: *Strategy, teaching, Spatial reasoning, Geometric, and materials.*

DEDICATION

To Joseph Nyarota Tembo and Judith Nguzu

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ABBREVIATIONS AND ACRONYMS

CDC	Curriculum Development Centre
CPD	Continuing Professional Development
DEBS	District Education Board Secretary
ECZ	Examinations Council of Zambia
MESVTEE	Ministry of Education, Science, Vocational Training and Early Education
MOE	Ministry of Education
MoGE	Ministry of General Education
NCTM	National Council of Teachers of Mathematics
NRC	National Research Council
UNZA	University of Zambia

DEFINITION OF TERMS

Early Child Education: This is early education for age ranges 0-6 years, where 0-2 is day care, 3-4 years nursery and 5 – 6years reception (MESVTEE, 2013).

Geometric reasoning: This is the use of critical thinking, logical argument and spatial reasoning to solve problems and find new relationships of different shape and ideas.

Spatial reasoning: This is being able to accurately describe the location of objects in the surroundings and of shapes in an abstract plane.

Strategies: Are sets of plan or activities which help in the delivering of a lesson to make the learners understand.

Teaching: This is the engagement of teachers with learners to enable their understanding and application of knowledge, concepts and processes indifferent activities.

Teaching: this is the engagement of teachers with learners to enable their understanding and application of knowledge, concepts and processes in different activities.

Topology is the mathematical study of the properties that are preserved through deformations, twisting and stretching of objects.

Topology: is the mathematical study of the properties that are preserved through deformations, twisting and stretching objects.

CHAPTER ONE

INTRODUCTION

1.1 Overview

This chapter explains the background, statement of the problem and the purpose of the research. It also states the objectives, research questions, significance of the study, its limitations, delimitations theoretical framework and conceptual framework, operational definitions of key terms used in the study are given. And finally the organization of the study.

1.2 Background

Mathematics is one of the most useful tools for communication we have as human beings without which, we could not come up with the thoughts of trade or transactions we express to others, nor could we engage in meaningful activities that commonly take place every day in the society we build ourselves (Cockcroft, 1982). Also Martinez (2003) points out that individual performance reward are often discouraging. It can be argued that any person who is ignorant in Mathematics would be at the mercy of others and would easily be cheated. Despite the efforts made by government in a bid to improve performance of learners in Mathematics, the performance of learners in Mathematics examinations results of grade five, seven, nine and twelve has continued to decline as from examination council of Zambia reports (2013).

The Ministry of General Education (MOGE), attaches great importance to Mathematics and they have made it compulsory from Early Childhood Education (ECE) to grade twelve. The provision of early childhood education in government schools was started in 2014, Monitoring and Research Centre (PMRC, 2014); Zambia started the provision of early childhood education in Government schools. The teachers for this group of learners were trained at Chalimbana College of Education.

The development of mathematical proficiency begins in the preschool years, and individuals become increasingly mathematically proficient over their years in educational settings. This

implies that educators in the range of early childhood settings need to develop effective teaching methods practices that engage learners in high-quality mathematics experiences.

Early Childhood Care, Development and Education (ECCDE) (2017); states that Early Childhood Care, Development and Education (ECCDE) is one of the major factors that determines a child's school performance and progress at a later stage in life, and Mathematics is not left out. The ECE syllabus of Pre-Mathematics in Zambia consists of a number of Geometry related topics, for example plane, shapes and measurements as according to MOGE (2013) ECE syllabus of Zambia. The performance by most of the learners in this topic has not been impressive. These topics are not done well by the learners at different levels of educational examinations where Mathematics is compulsory as observed in grades five, seven, nine and twelve that is according to examination analysis. Table 1 shows 2014 examination results analysis for grade five released by the examination council of Zambia, of the national assessments at grade five (5) levels which the Ministry successfully conducted on the years that the examinations were did and these were 1999, 2001, 2003 2008, 2012 and 2014.

Table 1: Shows the statistics on the performance of grade 5 Learners in Mathematics for 1999 – to date as no grade five examinations have been done.

1999	2001	2003	2008	2012	2014
34.3	35.7	38.5	39.3	38.3	34.9

Source: Results for grade five released by ECZ 2014.

Teachers of ECE are key stakeholders in making the learners at this level acquire the Geometrical and spatial reasoning needed in Mathematics, as we can see from the table that even with the efforts from the ECE teachers still the performance is not good. Sarama & Clements (2009) but still, Geometry and Spatial thinking are often ignored or minimised in both early education and in the professional development of early childhood teachers. Ginsburg et al (2006) how the teachers teach their learners is what matters most. It is important that ECE teachers use a variety of teaching methods and techniques in order to cater for the range of learning needs taking into account of the available local resources (Ministry of Education, Science, Vocational Training and Early Education (MOESVTEE), 2013). MOESVTEE went on to say that ECE

teachers should as much as possible, use methods that promote active learner participation and interaction.

1.3 Statement of the Problem

Despite an increase in the number of teachers who graduate from many Colleges and Universities that the country has year after year. Even the training of Mathematics and science teachers through fast track, distribution of teaching and learning materials and improved teaching and learning methods through Continuing Professional Development, the performance of the learners at all level of learning including the examination classes in their Leaving Examinations in Mathematics has not been impressive especially in the geometry topic. As seen in Table 1 which showed the statistics on the performance of grade 5 Learners in Mathematics for 1999 – 2014 to date. The researchers on children's learning in the first six years of life demonstrated the importance of early experiences of geometric and spatial reasoning in mathematics and saw teachers of ECE as key people in teaching of the experiences in ECE learners. The teachers' engaging and encouraging climate for children's early encounters with Geometry in Mathematics develops their confidence in their ability to understand and use Geometry and Spatial reasoning. The Teaching at ECE should develop positive experiences in Geometry and Spatial reasoning which may help children to develop dispositions such as curiosity, imagination, flexibility, inventiveness, and persistence, which contribute to their future success in and out of school. Sarama and Clements (2009) teaching should be that which guide children in seeing connections of ideas within Geometry as well as developing their mathematical knowledge throughout the day so as to build a good foundation. Unfortunately, the teaching of geometric and spatial reasoning are often ignored or minimised in early education. In view of this, this study explored how the teaching of Geometric and Spatial reasoning in ECE is done in selected primary schools of Shibuyunji district.

1.4 Purpose of the study

The purpose of the study was to explore the teaching of geometric and spatial reasoning in ECE in Shibuyunji District Primary Schools.

1.5 Objectives of the Study

The study was guided by the following objectives;

1. to establish the strategies teachers use' when teaching geometric and spatial reasoning at ECE
2. to examine the methods teachers use when teaching geometric and spatial reasoning at ECE
3. to explore the material used when teaching geometric and spatial reasoning at ECE.

1.6 Research questions

The following research questions guided the study;

- What strategies do teachers use to teach geometric and spatial reasoning in ECE?
- To examine the methods teachers use when teaching geometric and spatial reasoning in ECE?
- What are the teaching and learning materials used when teaching geometric and spatial reasoning in ECE?

1.7 Significance of the study

It is hoped that the findings of the study might lay the basis for the teaching of geometric and spatial reasoning in mathematics at ECE. The study might help school managers and teachers of Early Childhood education on how to plan and teach geometric and spatial reasoning in ECE. The study might also contribute to the existing Mathematics literature base.

1.8 Delimitations

According to Creswell (1994), delimitations are used to indicate how the study is narrowed in scope. This study was restricted to three Schools that had ECE section and trained ECE teachers of Shibuyunji District of Central province in Zambia. The three schools were used in order to have an in-depth understanding of the teaching of geometric and spatial reasoning at ECE.

1.9 Limitations of the Study.

Limitations are those conditions which are beyond the control of the researcher and may also place restrictions on the conclusions of the study (Best and Kahn, 2009). The limitations of the study were the negative attitudes of some ECE teachers who started by showing less interest but how ever with time they took part well willing. The other was that other participants had no lesson plans and record of work which affected the data collected from document analysis instrument. The other was that the ECE learners are young to give their ideas, hence only observed.

1.10 Theoretical framework

This study was located on the theoretical background of the four stages of development of Piaget and the five levels of development of Hiele. The work of Piaget, Van Hiele and Del Grande offers guidance. The Van Hiele model of geometric understanding was proposed by the Dutch mathematician Diana Van Hiele Geldof and her husband Pierre Marie Van Hiele. Crowley (1987) described five levels of developmental thinking and reasoning in geometry. This progression was based on experience and education not age. They stated that students had trouble with high school geometry because their early training had not allowed them to pass through five developmental stages. Teachers of young children are concerned with levels 0 through 2, which establish a foundation on which to build in future years. Whitney (2017) wrote that, there are five ordered levels in the van Hiele Model of Geometric Thinking: (0) visualization, (i) analysis, (2) informal deduction, (3) formal deduction, and (4) rigor. At Level 0, children can name and recognize shapes, but specific properties may not be identified. They might recognize characteristics, but these characteristics are not used for recognizing the shapes. At Level 1, children start to use vocabulary relating to properties. Size and orientation become irrelevant as they begin to focus on specific properties of a shape. At Level 2, children recognize relationships between shapes and are able to reason about relationships. At Level 3, children begin to understand deduction, postulates, theorize and make proofs. At Level 4, children begin to understand how to work with axiomatic systems. Since the Van Hiele progression in geometric thinking is not age-based, developing a geometric grade-based curriculum using the van Hiele model can be challenging, but some attempts have been made. For example, learners

in primary grades are assumed to be at Level 1 and many pre-schoolers are at Level 0, which is described that Level 0 children learn to recognize geometric figures such as squares and circles by their holistic physical appearance.

Level 0: visualization-recognizing and naming figures

Level 1: Analysing-describing the attributes

Level 2: informal deduction- classifying and generalizing by attributes

Level 3; Deduction-developing proofs using axioms and definition

Level 4: Rigor- working in various geometrical systems (Crowley, 1987)

Clements and Battista (1992) stated that people may be on different Van Hiele levels for different topics. For example, a person may operate on level 1 when working with two dimensional figures, but on level 0 with three- dimension figures. But the teaching of ECE is seen to fall on level zero of Hiele.

Piaget (1964) learning is provoked by external situations and he also talked about the four stage of development which are sensory motor which is lower than the level 0 of Hiele and it caters for the ages from birth to two years, the second stage is the pre- operational which is two to seven years and this is when the level 0 of Van Hiele starts going into operation then follows the concrete operational stage which includes ages seven to eleven. This stage corresponds with level 1 of Van Hiele theory of analysing and describing the attributes. The formal operational which caters for the ages twelve and above is the last stage and it corresponds with levels 2, 3 and 4 of the Van Hiele. This will inform the current study on what the learners need to know at what stage. Piaget and Inhelder (1956, 1960) suggested that early spatial conceptions are topological in nature and their ideas are very general and inclusive, and give an infant a very broad understanding of his or her world. Table 2 shows the five levels of Hiele and four stages of Piaget's cognitive stages of development.

Table 2: The Five Levels of Hiele and Four Stages of Piaget's Cognitive Stages of Development

Van HIELE LEVELS AND PIAGET'S COGNITIVE STAGES OF DEVELOPMENT					
Level	Characteristics	Stage	Age	Characteristics	Similarities'
Level 0	Visualization/reorganizing /naming of figures.	Sensorimotor	Birth to 18–24 months old	Motor activity without use of symbols. All things learned are based on experiences, or trial and error.	Object permanence
Level 1	Analyzing	Preoperational	2 to 7 years old	Development of language, memory, and imagination. Intelligence is both egocentric and intuitive.	Symbolic thought
level2	Informal deduction/classifying/generalizing by attributes.	Concrete operational	7 to 11 years old	More logical and methodical manipulation	Operational thought

				of symbols. Less egocentric, and more aware of the outside world and events.	
level3	Deduction-developing proofs and definition	Formal operational	Adolescen ce to adulthood	Use of symbols to relate to abstract concepts. Able to make hypotheses and grasp abstract concepts and relationships .	Abstract concepts
level4	Rigor – working in various geometrical systems	-	-	-	-

The Application of the Theories above to the Study

The theories have been used in this study because the study was exploring the teaching of geometrical and spatial reasoning to a child hence the teacher needs to understand the level of child development. Crowley (1987) wrote that students had trouble with high school geometry because their early training had not allowed them to pass through five developmental stages.

Hiele theory has critical implications on the current study. This theory acknowledges that children's learning has some stages or levels they have to pass through for learning to take place. It also recognizes that learners must undergo all the stages so as to learn and have good background.

1.10 Conceptual Framework

Punch (2005) defines a conceptual framework as a representation, either graphically or in narrative form, of the main concepts or variables, and their presumed relationship with each other. It is best shown as a diagram. In this conceptual framework I have two variables. This variables are independent and dependent which will be interdependent on each, were the predictors variables are influenced by mediating variables and then mediating influences the performance variables.

Figure 1 shows model for explaining the teaching of geometrical and spatial reasoning.

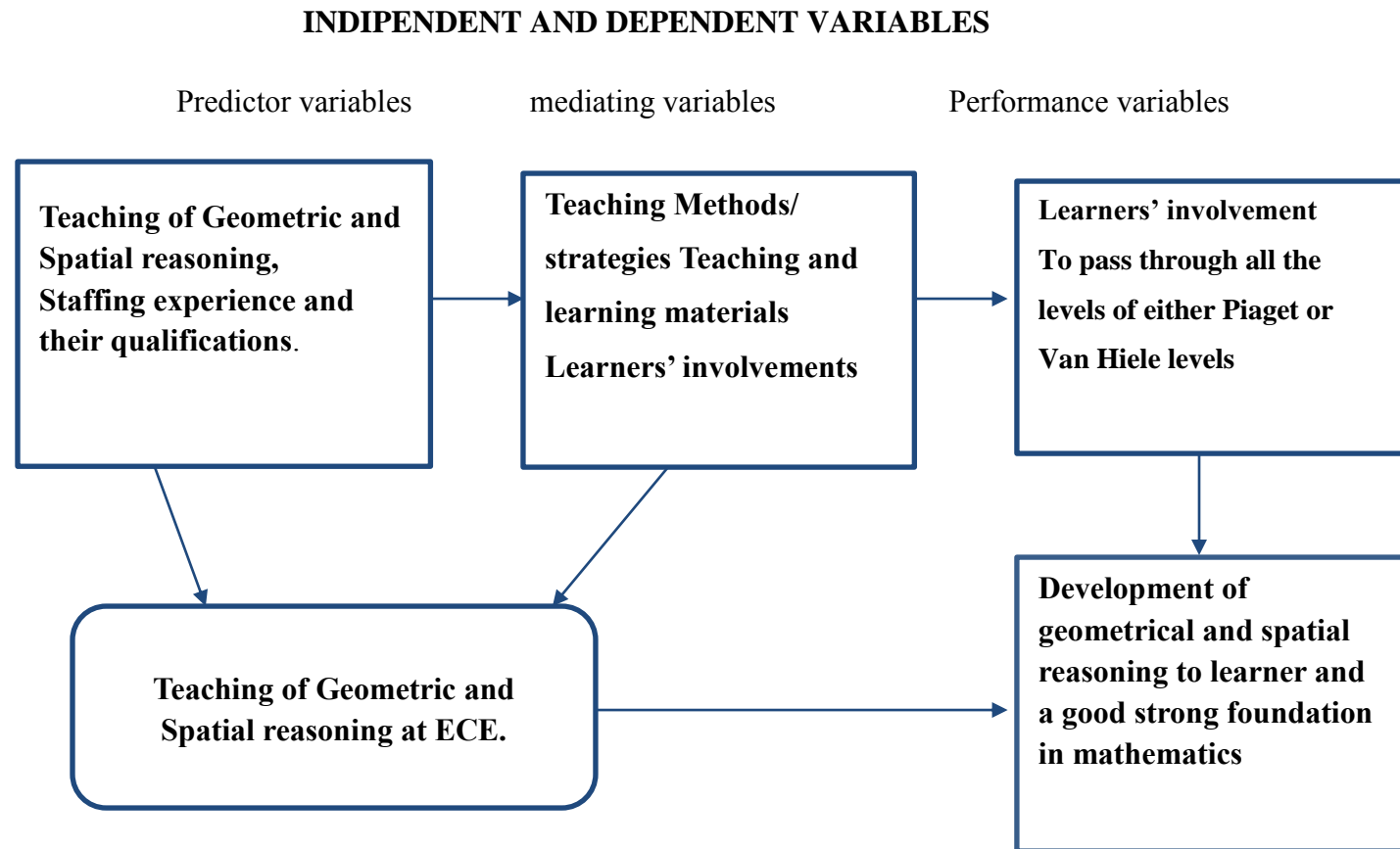


Figure 1: Model for Explaining the teaching of geometrical and spatial reasoning.

From Figure1 Model above, the teaching of geometric and spatial reasoning is influenced by the staff experience and their qualifications. Teacher`s experience predicts the teaching methods, the strategies and materials that are suitable in the teaching of geometric and spatial reasoning at ECE, which can only be achieved if the learners are well involved in the lesson, which can help in the good foundation of mathematics, which can help the learners in higher level mathematics.

1.11 Organization of the study

Chapter 1 introduced the study by giving the background of the teaching of geometric and spatial reasoning at ECE in Shibuyunji district of Zambia. It also outlined some key items such as the statement of the problem, purpose of the study, objectives with their research questions, and the significance of the study, delimitation and limitation, the theoretical and conceptual framework and finally the operational definitions.

Chapter 2 is a review of literature related to the problem under investigation. Literature was presented under the following sub-headings: The teaching of geometric, strategies used when teaching geometric at ECE and the materials used. The chapter concludes with different studies on the teaching of geometric and spatial reasoning.

Chapter 3 gave the methodology which included the research design, approaches, methods,

Chapter 4 presented an analysis of qualitative findings from Lesson observation schedule, semi structured interviews and document analysis. The chapter ends with a summary just after analysis of findings regarding constraints and measures.

Chapter 5 provided the discussion of the findings presented in chapter four in the light of the research objectives. The findings were further discussed in view of the literature reviewed and the theoretical foundations that mirrored the study.

Chapter 6 provided the conclusion and recommendations based on the findings. The conclusion summarises the study while recommendations provide more suggestions to inform the ECE teachers and the head teachers.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter gives an outline of other researchers' works on geometry that are related to my study. Tromp and Kombo, (2006) defines literature in the context of review as the works the researcher consulted in order to understand and investigate the research problem. They further indicated that literature review is an account of what has been published on a topic by accredited scholars and researchers and gives the researcher insight into what has already been done in the selected field identifying its strengths and weaknesses. The literature in this study was thematically reviewed under the following objectives. To examine the methods used to teach geometric and spatial reasoning in ECE, establish the strategies teachers use' when teaching geometric and spatial reasoning in ECE and explore teaching and learning material used by ECE when teaching geometric and spatial reasoning in ECE.

2.2 Strategies used to teach geometric and spatial reasoning in ECE

Duru (2010) an experimental teaching in some of topics in geometry. The aim of the study was to compare experimental teaching method (ETM) with the teacher centred traditional teaching method based on students' success. This study was conducted on 54 students, randomly divided into two groups; an experimental group and a control group. Experimental teaching method was used for the experiment group and traditional teaching method was used for control group. The test was applied to both groups in two different times. The first test was applied before and the second test was applied after the teaching. The test was used to compare the two groups and the level of significance was measured as $p < 0.005$. According to the research results, the founding were that the experimental teaching method was more effective than teacher-cantered or traditional teaching method in the knowledge and comprehension level. Experimental method of teaching is more effective than the teacher cantered.

The lack of attention to geometry reveals itself in prospective teachers' responses to tasks (Fujita and Jones, 2006). An example was given by Clement and Sarama (2011) where approximately 13% of prospective teachers in Scotland identified a square as a rectangle and approximately 18% realised that a parallelogram is a trapezium. The author noted that such results contrasts with Kawasaki's (1992) findings that 73% of Japanese prospective teachers defined a trapezium correctly, as cited in Fujita and Jones (2006). Although almost all prospective teachers could draw a square, almost 2/3 could not define it correctly, leaving out any mention of angles or other constraining properties.

Clement and Sarama (2011) researched with hundreds of pre-school teachers and it suggested that most early childhood teachers also have not attained adequate levels of geometric knowledge. This is consistent with the finding that teachers of young children are provided with very limited professional development in mathematics (Ginsburg et al. 2006). 70% of prospective elementary teachers were below Van Hiele level 3, at which people understand relationships between classes of figures. Several were at the pre-recognition level (level 0) and almost 2/3 was at the visual level. Recent research points to early childhood teachers to have not attained adequate levels of geometric knowledge which can affect the strategies in the teaching of geometrical and spatial reasoning.

Marchis (2012) aimed at presenting a research on how pre-service primary school teachers' mastered elementary Geometry notions and properties related with some basic shapes and solids. The research showed that there were students, who couldn't recognize basic geometrical shapes or solids. Two third of the students couldn't define correctly basic geometrical shapes as they don't know the correct properties of the shapes; they knew the properties of the shapes, but they repeated some properties in the definition or they missed some properties from the definition. As regarding geometrical solids, more than one third of the students couldn't draw the correct two-dimensional representation and one third didn't know how to draw the net of them. Recent research points to students failing to define correctly basic geometrical shapes as they don't know the correct properties of the shapes;

2.3 Methods on teaching of geometry and spatial reasoning

It is impossible to think about good mathematics teaching methods for children aged 3–8 years without acknowledging that much early mathematical learning occurs in the context of children's play NCCA (2014). In which the learners needed to understand their environment and the things inside it. Educators need to understand how mathematics learning is promoted by young children's engagement in helping them acquire the spatial reasoning. How best they can support that learning. Learning through play is seen as fundamental to good mathematics teaching methods in early childhood. It assumes varying degrees of emphasis depending on the age of the child. Recent research points to a number of other important principles; for example knowing what the learners know and what they should be taught and how they should be taught, which underpin good mathematics teaching methods for children aged 3–8 years. For example Anthony & Walshaw, 2009a; NRC, (2005), said, both the principles and the features of pedagogy are consistent with the aim of helping children to develop mathematical proficiency, Geometry and Spatial thinking, Algebraic thinking, and data and chance should be given appropriate attention.

The momentum for reform in mathematics education began in the early 1980s, Walle (2004), in response to a “back to basics” call to address community concerns about the state of mathematics education. Although “basic” means different things to different people, Cavenagh (2006) states that, it typically consists primarily of geometric, arithmetic or computation based on drill and practice and is the mathematics that parents and legislators recognise as the subject they were taught in schools. Reform-oriented approaches, on the other hand, are based on the recommendations made by the NCTM (1989) and involve a range of spatial processes, such as problem solving, reasoning and proof, communication and reflection (Bobis & Anderson, 2006). Reform advocates wanted students to value mathematics and be confident in their ability to do mathematics Walle (1999). Consistent with the reform approach, is an increased emphasis on the need for students to develop a spatial and conceptual understanding of important mathematical ideas and an ability to connect these ideas, in order to build up a network or foundation on which to base future learning.

According to Walle (1999), reform and basics are not opposite ends of the same continuum; rather, the basics tend to be about content whereas reform is much more about how children learn. The implications of establishing working partnerships between home and schools in the context of the reform movement mean that careful consideration needs to be given to the ways in which reform practices are communicated to parents and the extent to which teachers can expect parents to follow, implement and support reform practices and should see that the learners are active participant in their learning and other activities.

Edward & Warin (1999) found, for example, that there were considerable discrepancies between approaches recommended volunteers and their actual practice. In addition, they found that teachers were often unaware of the demanding nature of the tasks they were expecting parents to do and tended to under estimate the professional expertise of teachers. It is also important to acknowledge the tension that exists between how mathematics is taught today compared with how it was learned by parents Marshall and Swan (2010) and Peressini (1998). Many parents tend to value their own forms of doing mathematics over school mathematics while many children value school form of knowledge over the parents knowledge, hence demonstrating the potential tensions that may arise when engaging in mathematical tasks and assignments at home. The learning of geometric and spatial reasoning was to be practice also at home. Civil (2006) states that project expressed concerns that they were not familiar with the homework tasks set and therefore unsure about the best ways in which to help their children even when they had practical examples in their homes, hence leaving the teaching of geometry and spatial reasoning to teachers in school only. This simply shows the importance of geometric and spatial reasoning and also that it can be learnt from home activities. The current study informed this study as it look at how the learners should be actively involved in the learning of geometric and spatial reasoning.

Handi (2018) Perspectives on the Teaching of Geometry: Teaching and Learning Methods. Pittalis & Christou (2010) Geometry was an important branch of Mathematics and it has a place in education for the development of critical thinking and problem solving, furthermore. They added that geometrical shapes are parts of our lives as they appear almost everywhere, they went on and said that geometry is utilized in science and art as well. Recent research defines geometry

teaching and puts forth why it has been given an important place in teaching mathematics. The major issue deals with facilitating teaching Geometry through employing some useful approaches such as construction of the 3D. It also states that Geometry is basically divided into two categories and these are conceptual part and graphical part. Teaching these two categories requires different approaches.

The conceptual parts must be transformed into perception by visualization; that is to say by the graphical parts. The aim was to describe and analyse the structure of 3D geometry thinking by identifying different types of reasoning and to examine their relation with spatial ability. To achieve this goal, two tests were administered to students in grades 5 to 9. The results of the study showed that 3D geometry thinking could be described by four distinct types of reasoning which refer to the representation of 3D objects, spatial structuring, conceptualisation of mathematical properties and measurement. The analysis of the study also showed that 3D geometry types of reasoning and spatial abilities should be modelled as different constructs. Student's spatial abilities, which consist of spatial visualisation, spatial orientation and spatial relations factors, are a strong predictive factor of the four types of reasoning in 3D geometry thinking. Recent research points to the importance of geometry and the method used but did not consider the materials used in the teaching of geometry, in the analysis the study showed the type of reasoning and the spatial abilities that can be developed by exposing learners to 3D geometry. This study informed my study in that it guides on how spatial reasoning can be acquired or taught.

Taylor & Francis (2009) did a study on Differences in learning geometry among high and low spatial ability pre-service mathematics teachers. The objective of this study was to investigate and characterize the geometric thinking and understanding of four pre-service middle and secondary mathematics teachers while considering their spatial ability levels. To investigate the differences, if any, that existed among these pre-service middle and secondary teachers with different spatial ability levels and understanding geometry, pre- and post-test designs were employed using Mayberry's model and investigation of Van Hiele levels of geometric thought in undergraduate pre-service teachers. Four contrasting cases in terms of spatial ability scores were examined using the van Hiele model to provide a description of geometric understanding. In

their study, participants were chosen using the Purdue visualization of rotations test results which was supportive of previous research in this area. Learners with low spatial abilities were more challenged geometry lessons. While learners identified with midrange spatial abilities showed the most change in van Hiele levels after instruction, the low spatial ability students showed the least amount of change. The students with high spatial ability showed some change after instruction. It was identified that instructional activities that afford opportunities for fostering spatial abilities must be included in pre-service programmes so that future teachers may have a mathematical foundation from which to teach geometry. This study used the same theoretical frame work as the current study and also looked at how spatial ability can help the learners acquire or respond to instructions.

Marchis (2012) pre-service primary school teachers' elementary geometry knowledge geometrical notions and properties occur in real-life problems, meaning Geometry has an important place in school Mathematics curricula. Primary school curricula builds the foundation of Geometrical knowledge, pupils learn Geometry notions and properties by exploring their environment. It is very important that every primary school teacher has a good base in elementary Geometry. The aim of the study was to present a research on how pre-service primary school teachers' master elementary Geometry notions and properties related with some basics shapes and solids. The research showed that there were students, who could not recognize basic geometrical shapes or solids. Two thirds of the students can define correctly basic geometrical shapes: they did not know the correct properties of the shapes; they know the properties of the shapes, but they repeat some properties in the definition or they miss some properties from the definition. As regarding geometrical solids, more than one third of the students couldn't draw the correct two-dimensional representation and one third didn't know how to draw the net of them.

Schools in Malaysian, for example, geometry were usually taught using mainly the textbooks, chalkboard and sometimes the compass and protractor (Mullis: 2000). Sometimes the teachers used geometric kits to show the different geometric solids mentioned in the syllabus. Sadly this teaching approach did not seem to help many learners as evidenced by poor geometry performance at Form Two and Form Five (Malaysia: Ministry of Education, 1996). When

compared to that in high performing countries, the mathematics curriculum in Malaysia lacked emphasis on teaching approaches that encouraged thinking and reasoning skills such as communicating mathematically, solving non-routine problems, deriving proofs and projects. Though this represents an overall picture of mathematics instruction, it could be inferred to the teaching of geometry. Noraini (1999) insisted that geometry instruction needed to encourage more non-routine problem solving activities such as geometric puzzles and problems based on real-life situations to enhance geometric thinking skills. Geometry instruction also needed to be designed to encourage more interactions between teachers and learners to enhance mathematical communications. According to Van Hiele (1999) optimal geometry learning was achieved when students developed their thinking and reasoning skills. The reviewed study discourages the use of activities that do not allow learners to interact with the teacher, but advocates for interactions between teacher and learners.

Chang, Sung and Lin (2007) conducted a study on developing geometric thinking through multimedia learning activities. In their study, a multimedia learning software program named GeoCAL was described. It was based on van Hiele's geometric thinking level theory, which consists of four levels: recognition, visual association, description/analysis, and abstraction/relation. In addition to presenting the software design, the study also explored the learning effects of GeoCAL on each of the geometric thinking levels and on overall geometric thinking. The subjects of this study were second in elementary school students of an average age of eight. The experimental results indicated that, with the exception of recognition ability, GeoCAL produces significant learning effects on visual association, description/analysis and abstraction/relation as well as overall geometric thinking. The viewed study focused on teaching of geometric and spatial reasoning using GeoCAL it also found that experimental was also used and result that GeoCAL produces significant learning effects on visual association, description/analysis and relation and overall geometric thinking in accordance with the Hiele levels.

Lampert (1988) an attempt to draw on the thinking of a group of secondary school geometry teachers who are participants in the Laboratory Sites Study of the Educational Technology Centre (ETC) at Harvard University. The purpose of the Lab Sites Study was to understand the

process of implementing technology-enhanced guided exploration in school classrooms. The data analysed in the paper was collected as a sub study of the Lab Sites project, which looked at comprehensive questions of implementation in relation to materials produced at ETC for teaching, mathematics. The sub study reported here was concerned with teachers' points of view about using one piece of educational technology the Geometric supposer to substantially change the way they teach geometry. The Supposer was designed to fundamentally change the way instruction was delivered in classrooms by enabling learners to engage directly in the exploration of subject matter. What was reported was the teacher users' thinking about that broader change in the way they do their work, as well as their thoughts about the technology. The study informed my study in that it also pointed out on learners involvement in class.

Edward (2017) Kindergarten students benefit from strong spatial skills and geometry. Education was done on three to six years children and the findings were that the students were able to perform some sophisticated spatial skills such as mental rotation symmetry, perspective skills and basic map making in which the researcher was just starting to understand the place of spatial reasoning and geometry in their learning of geometry and spatial reasoning in class. The reviewed study pointed that students were able to perform some sophisticated spatial skills such as mental rotation symmetry, perspective skills and basic map making in line with what the current study which was finding out the methods used to acquire them.

Fujita and Jones (2006) reported on the geometric knowledge of Scottish pre-service primary teachers and the ways that these pre-service teachers defined and classified quadrilaterals which are geometrical shapes. Based on the ideas of concept definition and concept image introduced by Tall and Vinner (1981), and of figural concept initiated by Fischbein (1993), Fujita and Jones (p. 130) distinguished what they called the individuals' "personal figural concept" (coming from personal experiences) from the "formal figural concept" (as defined in geometry). Almost 160 pre-service primary teachers in the first year of their studies were examined in questions related to quadrilateral properties, and 124 pre-service teachers in the third year of their studies were examined about quadrilateral relationships. Analysis of the first group's answers showed that there was a gap between figural concepts and definitions provided. Similarly, the analysis of the answers of the second group indicated a weak understanding of the hierarchical

relationship of quadrilaterals. The reviewed study pointed on in service teachers lacking of the understanding figural concepts and definitions of some geometrical shapes which in this case the quadrilateral and my study was finding out the methods that are used to teach geometrical and spatial reasoning which are those same concepts.

Tatsis and Moutsios-Rentzos (2013) their focus was the capability of pre-service primary school teachers to interpret and evaluate verbal information related to two dimension geometrical objects. The researchers found, in contrast with their conjecture, that the pre-service teachers mostly showed a stronger positive evaluation of the geometrical descriptions, followed by weaker positive evaluations of the topological descriptions. Researchers were accompanied by relatively negative evaluations for everyday descriptions. While the above studies focus on pre-service elementary teachers, but used topological to the two dimension and my study was finding out the methods that are used to teach geometrical and spatial reasoning which are those same concepts.

Silfverberg and Joutsenlahti (2014) studied pre-service elementary and secondary teachers' notions of angles in a plane. They found that some of their respondents "interpreted an angle as a line consisting of two line segments, some consisting of two rays, and some as a region defined by these elements" (p. 190). What is more, interpretations differed as to "whether an angle continues outside the part shown in the drawing in the direction determined by the angle. The reviewed study pointed out about the weakness while my study was finding out the methods that can be used sought out the weakness in geometry.

Although there are exceptions, teachers in many countries, including the UK (Jones, 2000) and South Africa (Van der Sandt, 2007) were not always provided with enough preparation in geometry and the teaching and learning of geometry. Of all mathematics topics like geometry was one prospective teacher claimed to have learned the least and so they were least prepared to teach (Jones et al. 2002). From this statement we see that it cause or made a chain of less knowledge being transferred as the teachers have less knowledge.

A study by Azerem (2012) aimed at finding the weaknesses of secondary school students at geometry questions of measures, angles and shapes, transformations and construction and 3-D

shapes. The grade seven curriculums contained four geometry topics out of 17 mathematics topics. In addition to this, the study was aimed at finding out the mistakes the 7th grade students made in the last 4 exams including two midterms and two final exams. The results of this study revealed that 7th grade secondary school students have a number of misconceptions, lack of background knowledge, reasoning and basic operation mistakes at the topics mentioned above. The reviewed study pointed out about the weakness while my study was finding out the methods that can be used to point out the weakness in geometry.

Luneta (2015) conducted a research on understanding students' misconceptions, analysing final grade 12 examination questions in geometry. The investigation emanated from the realisation that grade 12 at school final mathematics examination performed poorly in geometry in South Africa. The aim of the research was to establish errors students made when solving coordinate geometry problems in the final Grade 12 examinations. The outcome of the investigation revealed that the students in grade 12 mathematics geometry question operated at level 2 of Van Hiele's hierarchy instead of level 3 and 4. The students' errors were classified as conceptual and procedural errors. This included the inappropriate use of formulae, application error, which are those that learners made if they were so prompted. These results posed a challenge to the teachers of mathematics showing the areas they need to work extra hard in the teaching of geometry. This study is different from mine in that my study is focusing on the challenges teachers face when teaching geometrical and spatial reasoning at ECE, but it used the same theoretical framework as my study as it also used the Van Hiele theoretical framework.

Fatima (2015) Geometry content knowledge of elementary pre-service teachers. The purpose of the research was to examine pre-service elementary school teacher's geometry learning as investigated by both qualitative and quantitative methods. For the qualitative investigation, narrative analysis and thematic analysis methods were used. The findings of narrative analysis indicated two main kinds of stories: as a learner and as a beginning teacher. The thematic analysis findings yielded three themes: history of learning geometry, perceptions about geometry, effective geometry instructional practices. The findings informed the quantitative investigation on geometry content knowledge for the case of quadrilaterals. During the second phase of the study, 102 participants who had enrolled in the methods course completed pre and post-test of

teachers' geometry content knowledge. A treatment group participants (n=54) received a number of activities (geometry activities and student work analysis) focusing on quadrilaterals, and the control group participants (n=48) received traditional instruction. Repeated measures ANOVA results showed a significant change in the treatment group participants' geometry content knowledge. The mixed ANOVA results indicated a significant main effect of knowledge but no significant interaction between geometry content knowledge and grouping. Even though treatment group participants' geometry content knowledge growth was significant, the difference between treatment group and control group participants' growth in geometry content knowledge was not significant. This study informs mathematics teacher education in three important areas; limited knowledge of pre-service teachers' geometry content knowledge, integrating mathematics content and the context of teaching into methods course, and use of student work with pre-service teachers. This study is different from main as it was focusing on the teacher's knowledge, while main looked at the teaching of geometric and spatial reasoning, but, though it used both qualitative and quantitative research the use of the thematic was the same way analysed my data. The reviewed study also pointed out about the history of learning geometry and my study was finding out the methods used to teach geometry.

In many geometry classrooms today, teachers merely introduced learners to facts about geometry and then drill them with concepts in deductive reasoning (Mullis, 2000). Learners were seldom given the opportunity to discover and conceptualise geometry on their own. Hoyles and Jones (1998) argue that although the deductive method is central to Mathematics and intimately involved in the development of geometry, providing a meaningful experience for learners at school appeared to be difficult. Research shows that learners fail to see a need to distinguish forms of mathematical reasoning such as explanation, argument, verification and proof (Jones, 2002). Another reason advanced for learners' poor performance was because of the teaching methods which concentrate much on calculation rather than problem solving and proof which encouraged critical thinking (Yeo, 2000). This paper was looking at the deductive reasoning of the learners which is not the focus of my paper, as my study was on the teaching of geometric and spatial reasoning at ECE. The reviewed study pointed at deductive method is central to Mathematics and intimately involved in the development of geometry, providing a meaningful

experience for learners at school appeared to be difficult hence my study was finding out the best methods used when teaching geometry.

Douglas (2017) teaching and learning geometry at early foundations of young children's abilities to engage in geometric thought. Spatial reasoning can support their overall mathematical and cognitive development. Geometry was not always addressed in early childhood curriculum and, even if included, was not explored in ways recommended by research. In this paper, they presented three studies that examined the teaching and learning of geometry with related math research and discuss curricular and instructional implications. In the first study, that was done they examined the effects of a geometry curriculum that synthesized the visual cognition elements of the program. The second study they explored the impact of an early math curriculum, based on learning trajectories. The third study examined teachers' math talk and its impact on children's overall math concept acquisition. We conclude that geometry curriculum for the young child is most effective when it includes a broad array of tasks that are based on learning trajectories with varied examples and non-examples, nurtures visual cognition with progression towards analytical thinking, and integrates rich and diverse math talk. This study looked at the geometry curriculum, while my study looked at the teaching of geometry which is the implementation stage of the curriculum. It also informed my study as it looked at the impact of the importance of early mathematics in the learning of mathematics and the importance of spatial reasoning.

2.4 Teaching and learning Materials used to teaching geometric and spatial reasoning at ECE.

Schroeter, (2017) Exploring the role of spatial reasoning and geometry for young learners. The Kindergarten students mostly benefit from strong spatial skills and geometry education. It is simple act of documenting the mathematics learning in kindergarten students' block and puzzle play that led educators to some revelations about young learner of three-, four, five and six year-old students. They are capable of relatively advanced spatial skills such as mental rotation, symmetry, perspective-taking, use of scale, navigation skills, and basic map-making. Their just began to understand the place of spatial reasoning and geometry in their learning. This study

only looked at the materials but did not consider the methods used, hence makes it different from main. The study pointed out blocks and puzzle which are some of the materials used.

Carment (2016) geometry and spatial adaptations, writes that Geometry and Spatial sense is best, taught through concrete hands on experiences when instructing students who are blind or visually impaired. This is beneficial not only for students with visual impairments but for students with normal vision as well. Have students assemble puzzles, and sort shapes and objects related to the topic. Play “Treasure Hunt” games. Have students follow verbal or written spatial directions given to: Locate objects in the classroom, school or community.

Encourage students to locate shapes within the environment when transitioning through the school or on community outings. Intentionally discuss positional concepts (on, in, on top of, underneath, thick, thin, rough, smooth, hard, soft, etc.) and incorporate counting and patterning activities and pairing numbers with groups of objects. Encourage students to follow directions and use low vision devices to read directions and discriminate between cards. Make math fun by incorporating graphs, charts, card and board games. Incidentally embedded math patterning and positional concepts into activities of daily living as well as waiting games during transitions. The write up was teaching the use of some teaching materials while main explores the materials used when teaching geometry.

Kamla-Raj (2015), in his paper report on an exploration of grade nine learners’ experiences in the design and construction of double-story art facts project at a secondary school in Kwa Zulu Natal, South Africa. The project used a process of drawing and construction of art facts in a technology education classroom to enhance and inform the teaching of geometry and to allow learners to both reflect and use the Geometry they know as a springboard for further study of Euclidean Geometry. It was a qualitative study in which data was collected through observations of artefacts and semi-structured interviews with a purposefully selected sample of five learners.

The analysis of data revealed that Geometry taught in a free environment allows learners to reflect and share their experiences for better understanding of mathematics concepts. This study is different from my study as main was exploring the materials used in the teaching of geometry and spatial reasoning. Although there are exceptions, teachers in many countries, including the

UK (Jones, 2000) and South Africa (Van der Sandt, 2007) were not always provided with enough preparation in geometry and the teaching and learning of geometry. Of all mathematics topics like geometry was one prospective teacher claimed to have learned the least and so they were least prepared to teach (Jones et al. 2002). From this statement we see that it cause or made a chain of less knowledge being transferred as the teachers have less knowledge.

Douglas & Clements the core of Building Blocks: research-based learning trajectories.

All components of the resulting Building Blocks curriculum are based on learning trajectories for each core topic. First, each topic was examined to ensure it was appropriate and important mathematically and generative of future learning. Secondly, empirically based models of children's thinking and learning are synthesized to create a developmental progression of levels of thinking in the goal domain according to Clements & Sarama (2004

Third, sets of activities were designed to engender those mental processes or actions hypothesized to move children through a developmental progression. In geometry, researches such as that reviewed were seen supported the importance of the topic of geometric shape and spatial reasoning. It also revealed distinct levels of geometric thinking Clements (1992) Hiele (1986) synthesis of this corpus produced a developmental progression the core of a learning trajectory for young children's learning of two-dimensional geometric figures.

Clements (1992) the progression for knowledge of geometric figures moves from increasingly sophisticated comparing (matching) through levels of recognizing and naming, identification of the components of figures. Understanding of properties of shapes, and uses of those properties when classifying and analysing sets of geometric figures. Instruction was designed to help gain competencies at each level. To provide an illustration, consider the related topic of shape composition. Composing of two-dimensional geometric figures was determined to be significant for children in two ways. First, it is a basic geometric competence, growing from pre-schooler's building with shapes to sophisticated interpretation and analysis of geometric situations in high school mathematics and above. Second, the concepts and actions of creating and then iterating units and higher-order units in the context of constructing patterns, measuring, and computing are established bases for mathematical understanding and analysis (Clements, Battista, Sarama,

& Swaminathan, 1997). This study used the same theories as the ones I used and it guide my study on the uses of the Van Hiele and Piagetian stage of development.

Ddokuz (2013) the effect of learning geometry topics of 7th grade in primary education. Dynamic geometer's sketch pad geometry software to success and retention. The aim of this study was to investigate the effect of learning geometry topics of 7th grade in primary education with dynamic geometer's sketchpad geometry software to student's success and retention. The experimental research design with The Posttest-Only Control Group was used in this study. In the experimental group, dynamic geometer's sketchpad geometry software adapted to Computer assisted instruction; and in the control group, traditional teaching method was used. Quantitative research approaches were adopted in the study. Data was collected through 6th grade SFBS (state free boarding and scholarship) 2005 test, achievement test and worksheets. Mann Whitney U test and Wilcoxon signed-rank test were used to analyse the quantitative data of the study. As a result of this study, it was found that there was a significant difference between achievement test scores of experimental group learning geometry with GSP dynamic geometry software and control group learning through traditional method in favour of experimental group. The reviewed study pointed out the using of dynamic geometry software which was the material used in the teaching of geometry and in line with my third objective.

2.5. Summary

In a nutshell, the related literature to the study was based on the themes drawn from the study objectives which are as follows; To examine the teaching of geometric and spatial reasoning at ECE, to establish the strategies teachers use' when teaching geometric and spatial reasoning at ECE and to explore the material used by ECE teachers when teaching geometric and spatial reasoning at ECE. The following chapter discussed the methodology that was used in the study.

CHAPTER THREE

METHODOLOGY

3.1 Introduction

This section focussed on the different procedures used to execute the study. The researcher first described the research paradigm, research design, population, sampling procedures and research instruments, data collection and data analysis methods, credibility and trustworthiness then finally ethical considerations.

3.2 Research Paradigm

According to Creswell (2007), a paradigm or a worldview is a basic set of beliefs that guide the action or a study to be conducted. A qualitative study includes paradigms such as philosophical assumption, epistemologies and ontologies conceived research methodologies and alternative knowledge claims. Creswell (2007) mentioned that paradigms used by qualitative researchers vary with the set of beliefs they bring to research and the types have continually evolved over time. This study used epistemology as the researcher want to dealt with the study of knowledge and how knowledge was acquired. This study explored the teaching of geometric and spatial reasoning in ECE.

3.3 Research Design

Research design is the process that involves the overall assumptions of research up to the method of data collection and analysis (Creswell, 2009). The research design that was used in this study was descriptive one, which according to Denzin and Lincoln (2005) is a situated activity that locates the observer in the world consisting of a set of interpretive, material practices that make the world visible and turn it into a series of representations, including filed notes, interviews, conversations, photographs, recording and memos. Jackson (2009) also revealed that descriptive research design is informative because it gives a rich description of a particular situation.

Therefore, this study focussed on exploring the teaching of geometric and spatial reasoning. This study used a qualitative approach. Cohen, Manion and Morrison (2007, p.168) defined

qualitative research as “an investigative process where the researcher is the main instrument conducting research in a natural setting”. Berg (2007) described qualitative kind of research by saying: “Qualitative research properly seeks answers to questions by examining various social settings and the individuals who inhabit these settings, qualitative procedures provide a means of accessing unquantifiable facts about the actual people researchers observe and talk to. Qualitative research methods are useful in answering different kinds of questions,” (p.8). Also Lester (1985) advocated for qualitative methods of conducting research in problem solving instruction. He stated that, “Adopting a holistic view of problem solving and problem solving instruction necessitates the use of naturalistic [inquiry] rather than traditional scientific research paradigms” (p. 52). By naturalistic inquiry he was referring to qualitative research done in a natural setting such as a classroom. The qualitative approach for my study will be descriptive in nature. Patton (2002) defined descriptive qualitative research design as research which designates phenomena as they exist, and that it is used to identify and obtain information on the characteristics of a particular problem.

3.4 Research Site Location of the Study

The study site was undertaken in Shibuyunji district of Central province, Zambia. According to Msabila and Nalaila (2013) there are many motivating factors that could influence the researcher’s choice of the study site, such as; the nature and incidence of the problem, research time frame, and data accessibility, clients’ interest and instructions, resource availability, performance in a particular field, goals and objectives of the study. Therefore, I selected Central province as the study site for my study because the performance in Mathematics from 2015 to 2018 has been below 40%. The reason for the choice of the study site is backed by the Examination Council of Zambia, 2018 general performance analysis that reported that, “the average performance of mathematics in Shibuyunji district from 2015 to 2018 has been below 40 percent (36.53%)” (ECZ, 2018, p. 24, ECZ, p.2).

The researcher collected data from three schools which were named as School Sa, Sb and Sc. Sa represented the first site that the researcher visited, Sb represented the second site the researcher visited and Sc represented the third site the researcher visited.

The School site Sa was a combined school for both boys and girls which had classes running from ECE to Grade Eleven (11). The school had a total number of 922 pupils. Out of 922, 84 were in ECE class. That centre is located in a low income rural area. The main occupation of most of the parents of the children in that school is subsistence farming.

The School Sb, was the second site visited by the researcher. It was also a school for both boys and girls situated in rural area of the district. The school runs from ECE to grade twelve (12) with the total enrolment of nine hundred and fifty-four (954), from that number 68 were at ECE.

The school site Sc was the third site visited by the researcher which was co-education as well. The enrolment for that school was 631 while 51 of them were in ECE.

The ECE teachers were named as Sa1, Sa2 and Sa3 for teachers from School Sa, the teachers from school Sb were named as Sb1 and Sb2 and the teachers from the third school were named as Sc1 and Sc2. The coding was done for easy identification. However, the data analysis procedure in this study was specifically guided by the following research questions:

- (a). what Method do teachers use to teach geometric and spatial reasoning at ECE?
- (b).What strategies do teachers use to teach geometric and spatial reasoning at ECE?
- ©. What teaching and learning materials are used when teaching geometric and spatial reasoning at ECE?

3.5 Target Population

Target group of individuals that have common characteristics that are of interest to a researcher (Best and Khan, 1993). Another scholar Bryman (2012) defines population as a group of elements or causes whether individuals, objects or events that conform to specific criteria and to which the research intends to generalise its results. In this study, the population included all the early childhood teachers from the all primary schools that's has ECE Selected Shibuyunji district of Zambia.

3.6 Sample Size

Creswell (2014) qualitative research is context bound and uses a small sample size. In this study, the sample comprised of seven ECE teachers in Shibuyunji district. The distributions of the teachers were two from the first site, three from the second site and the last two were from the third and last site.

3.7 Sampling Techniques

Purposive sampling was used to select early childhood teachers in Shibuyunji district. This study employed the homogeneous type of purposive sampling. This is so because the study only had the early childhood teachers as participants and this made them have a shared characteristic or set of characteristics which often was considered when the research questions being addressed are specific to the characteristics of the particular group of interest, which is subsequently examined in details (Cresswell, 2014).

3.8 Data Collection Instruments

Sahaya and selvam (2017) instruments dependence on research designs to be used. The instruments described in the main text are the ones to be added in the appendix. This study was qualitative in nature and only qualitative data collection methods were used. For this study, three research methods were used to collect the required information from the respondents and these included: Lesson observations, document analysis and Semi-structured interview. Therefore, the following sub-sections provide more details about the data collection methods and procedures regarding the same.

3.8.1 Semi-structured interviews

With semi-structured interviews (see appendix 3) the researcher had a clear list of matters that was to be addressed and questions to be answered. However, with the semi-structured interview the researcher was ready to be flexible in terms of the order in which the topics were well-thought-out, and, perhaps more importantly, to let the participants develop ideas and speak more

extensively on the issues that were raised by the researcher. The answers were open-ended, and there was more emphasis on the participants explaining points of interest (Descombe, 2005).

3.8.2 Lesson observations

Literature shows that lesson observation (see appendix 4) is also a methodology that is well suited for the collection of qualitative data (Cohen et al., 2007). It occurs in natural settings that already exist and not “in contrived settings” (Litchman, 2010, p. 162). Joskin, (2013, p. 113) defined observation as “the process of examining, and recording the environment and interactions describing teachers and students’ activities from the classrooms”. The researcher observed lessons and this enabled her obtain data from naturally occurring settings on the strategies teachers of mathematics used to teach geometrical and spatial reasoning. Morrison (1993) noted that observations enabled data collection by researchers from four areas: the physical setting, the human setting, the interactional setting, and the programme setting.

3.8.3 Document analysis

Bowen (2009) describes document analysis as a systematic approach for reviewing or evaluating documents. Documents can include both printed and electronic materials. In this study official document (see Appendix 5) like the lesson plans, records of work and assessment papers or learner books for the early childhood teachers who were observed were analysed as according to the check list in Appendix 5.

3.9 Data collection procedure

The researcher first got clearance from ethics committee at the University of Zambia, then the researcher got permission from the District Board secretary (DEBS) to visit the targeted schools (see Appendix 1). The researcher also got permissions from head teachers of the targeted schools (see Appendix 2). The researcher collected primary data during the second term of the school calendar, specifically, between May and July 2019. Using the instruments stated above. The researcher first observed lessons on pre mathematics topics that had some geometry in them and then researcher observed all the seven teachers. After lesson observations the researcher conducted semi-structured interviews with the seven teachers that were observed and finally the

researcher did the analysis of their documents. The order of data collection was to collect true information and also developing provoking questions before the respondents knows what the researcher wanted.

3.9.1 Primary data

According to Beck (2000), primary data is a type of information that is obtained directly from first-hand sources by means of surveys, observation, focus groups, interviews or experimentation, visual and visual-audio materials. In this study, collection of primary data was through observations of the lessons, interviews with teachers in Shibuyunji district.

3.9.2 Secondary data

Secondary data is information that has already been collected and is usually available in published or electronic form (Sleeper, 2001). In this study, secondary data collection was from the past lesson plans of the writing of other researchers on the similar topic selected, teacher's lesson plans and record of works of the seven teachers.

3.10 Data Analysis

The collected data was analysed thematically. Data analysis was described by Bogdan and Biklen (2007, p. 159) “the process of systematically searching and arranging the interview transcripts, field notes and other materials to increase your own understanding of them and to enable you to present what you have discovered to others”. The data from semi-structured interviews, lesson observations and document analysis was analysed thematically, it was systematically searched and arranged to increase its understanding and for easy presentation (Bogdan and Biklen, 2007). It was put into categories of related topics and major themes were identified to provide rich deep description of the phenomenon under study (Creswell, 2014). This can also be said that data analysis was simplifying of collected information to themes that can easily be understood by other people to be informed. Creswell (2014) asserted that thematic analysis categorizes related topics, and major themes are identified to provide rich deep description of the phenomena under study.

3.11 Ethical Considerations

The researcher sought permission from the Ethics Committee at the University of Zambia. The researcher ensured that respondents were respected and their rights were not trampled upon during the course of the study. Creswell (2009) argues that ethical issues need to be anticipated and effectively dealt with by the researcher in the research process. Therefore, it was important to emphasis on the need to take into serious consideration all ethical issues whenever a research was being conducted, be it with human or animal subjects. Researchers need to take responsibility to secure the actual permission, interests and rights of people in the study as well as their privacy and sensitivity. Hence the current study took into consideration all the possible ethical issues. All participants were given informed consent. They were told that participation in the study were completely on a voluntary basis. With regards to privacy, the participants were informed that they were free to avert some information which they did not feel comfortable disclosing and not ready to share with anyone including the researcher. The participants were assured that they would not be punished for keeping to themselves some information they did not feel comfortable disclosing and not willing to share with anyone. Informed consent was sought and the participants were assured that the information provided was purely for research and academic purposes and will be kept in a locked place accessed only by researcher and the supervisors. Participants were also given an assurance on the anonymity of their names. The principle of anonymity as indicated by Trochim (2006) as meaning that the participant will remain anonymous throughout the study, even to the researchers themselves in some cases as anonymity standard itself was a strong guarantee of privacy and very necessary.

The withholding of Participants identities ensured their safety as the results generally did not reflect the views of particular individuals but the community as a whole. Great efforts were made to protect the participants' privacy. Prior to conducting any observations or interviews, all participants were asked to sign a letter of informed consent. The form will clearly outline what would be happening in the study, and inform them that they have the right to refuse to participate. To avoid identifying participants by their names, codes were used instead of their actual names. The researcher had set questions which did not cause any psychological harm to the respondents. Before commencement of the study, the researcher obtained clearance from the

Ethics Committee of the University of Zambia. The study findings were shared with the four participants as the other three were not interested

3.12 Credibility and Trustworthiness

It is the process where investigators first establish the preliminary themes in a study and then search through the data for evidence that is consistent with or disconfirms these themes. The other procedure is member checking. The participants in the study are also checked for their credibility. To ensure trustworthiness in my study, participants own words and vignettes (pictures) of pupils' were used in the presentation of findings. The themes after data analysis were subjected to expert review to see whether they were in line with recordings and recognizable (Merriam, 1998; Adler, 1996).

3.13. Summary of the chapter

In summary, this chapter presented the description of the methodology used in the study. The researcher first described the research paradigm, research design, population, sampling procedures and research instruments, data collection and data analysis methods, credibility and trustworthiness then finally ethical considerations. The next chapter present the study findings according to the themes that emerged.

CHAPTER FOUR

PRESENTATION OF THE FINDINGS

4.1 Introduction

The previous chapter described the research methodology, which was employed in the study to come up with the results which are presented in this chapter. This chapter presented the findings of the study which aimed at exploring the teaching of geometry and spatial reasoning in ECE level in Shibuyunji district of central province in Zambia. The chapter briefly describes the three sites visited by the researcher during data collection, analysis and demographic. The themes that are presented in this chapter emerged from the data collected from lesson observations, document analysis and semi structured interviews. The research questions were analysed based on the themes that emerged from lesson observations, document analysis and semi structured interviews. However, triangulation of data was also achieved by comparing what the teachers said in the semi structured interview with the observations made during the lesson observation.

4.2. Strategies that teachers use to teach geometric and spatial reasoning at ECE

Research question number one was on the strategies used to teach geometric and spatial reasoning at ECE. The study established that the teachers largely used tracing, drawing and exposing/exploring as strategies for teaching geometric and spatial reasoning.

4.2.1. Tracing

The finding from the lesson observations show that tracing was used as four teachers out of the seven (4 out of 7) were seen practicing it and it was also substantiated by teachers' responses from semi structured interviews as all the seven said they use tracing as a strategy. Tracing also came out from the document analysis of the three teachers who had all the documents, as only three documents were analysed because the other teacher did not have the lesson plans and the records of work but only had the schemes of work. ECE teachers during semi structured interviews expressed that they use tracing as a strategies when teaching at ECE as evidenced by

the following excerpts in form of verbatim responses by teachers (Sa1 & Sc2) as typical examples: The teachers' views were similar. The following verbatim responses illustrates

At School Sa and teacher: Sa1 said *"I teach them using tracing"*

Teacher Sc2 said *"I make the learners trace and cut cardboards of different shapes and.....ok I think that is all"*

The researcher wanted to get what teacher Sc2 meant,

How do you expose learners to rich informal mathematical activities?

Teacher Sc2 said *"because most of mathematical problems involve calculation of perimeters, area, volume and others which the learners will be involved in higher mathematics."*

The lesson observations and document analysis also showed that tracing was used. A picture of learners captured by the researcher in class during an observation session is shown in Figure 2(a) and 2(b) shows traced work.

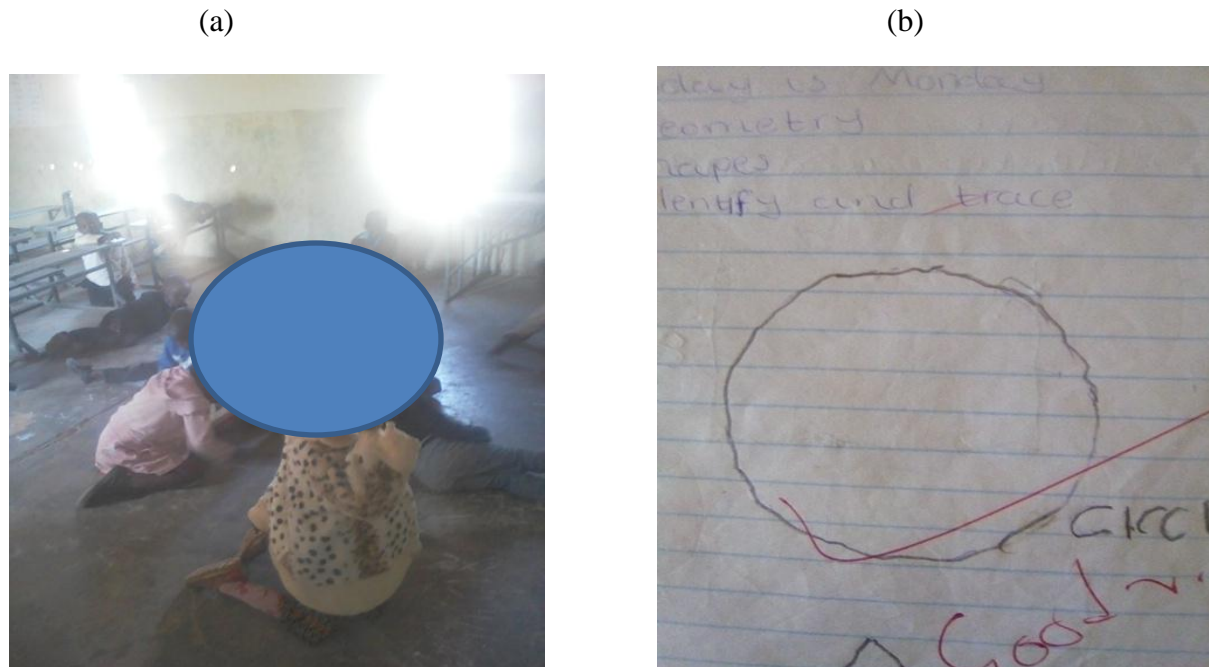


Figure 2: Showing picture (a) learners tracing in group, picture (b) traced work from a learner

4.2.2 Drawing

The ECE teachers (5 out of 7) representing used drawing as a strategy. In nearly most of the times when teaching at ECE, the teachers during the semi structured interviews expressed that they continued to use drawing to help the learners at ECE acquire geometric and spatial reasoning, as evidenced by the following verbatim responses expressed by teachers (Sa1, Sb1, Sb2 & Sc1):

Teacher Sa1 said: *“I teach them using repeatedly drawing the same object for example I make them write numbers from shapes”*

Teacher Sb1 said: *“I teach them using drawing some lines either vertical or horizontal and ask the learners to draw or come up with some shape”*

Teacher Sc1 said: *“I make the learners draw and I also dismantle some patterns and make the learners arrange them own their own”*

Teacher Sb2 said: *“I cut different shapes or draw some lines either vertical or horizontal and ask the learners to draw or come up with some shape”*

The findings from the document analysis of the three teachers who had the document showed that the teachers in their planning planned and also showed that they drew some shapes on the lesson plan shown in Figure 3(a) lesson plan that was used by one of the teachers. Also on the picture Figure 3(b) was a learner holding the string that was used to draw a circle as the teacher had no other instruments to use, while Figure 3(c) shows drawn work.

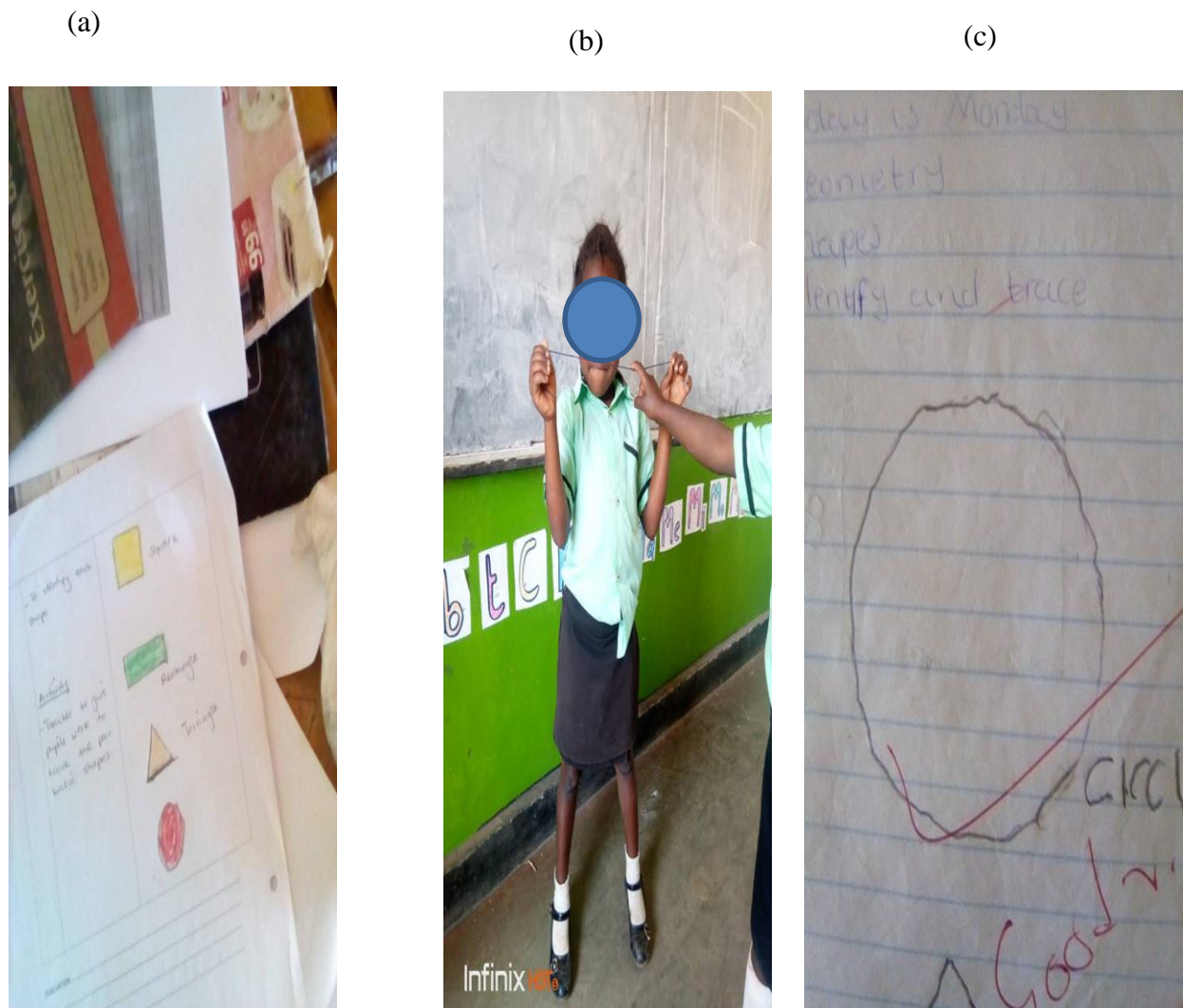


Figure 3: Showing Picture (a) Drawing Lesson Plan, picture (b) String used for Drawing, picture (c) Work Drawn

4.2.3 Exposing/exploring

Findings from lesson observations shows that only five teachers out of the seven (5 out of 7) ECE teachers exposed or made learners explore with the real objects, findings from the semi structured interviews showed that all the teachers explored or exposed their learners to real objects. The following verbatim responses expressed by teachers (Sa 2 & Sb1) act as typical examples:

Teacher sb1 said “I expose them to mathematical activities and expose learners to rich informal mathematical activities”

The researcher asked from teacher Sb1 how learners were exposed to mathematical activities, and the conversation was as follows;

Researcher *“How do you expose learner to mathematical activities?”*

Teacher sb1 said *“Depending on what I have on the plan” for example I make learner play with their environment by playing with sandy or make shapes out of clay soil.*

Teacher Sa 2 said *“By telling them, those names of real object”*

Teacher Sa 2 showed an example of a ball made by a pupil using local materials shown in Figure 4(a), 4(b) shows a learner measuring water which is a local material and Figure 4(c)

the learner identified a shape.

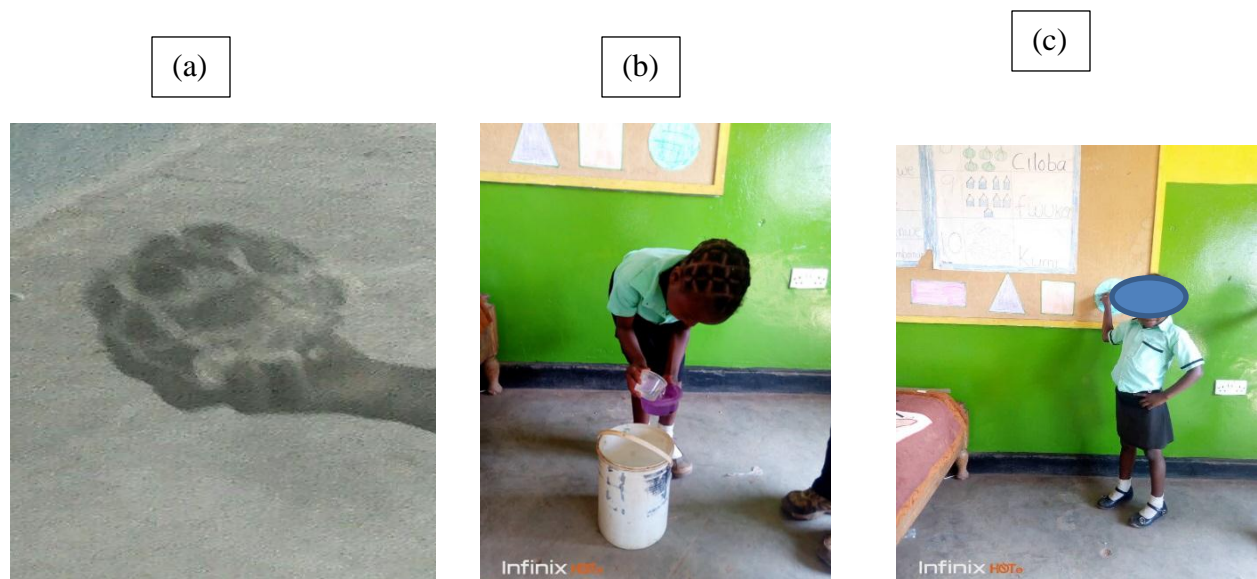


Figure 4: Showing picture (a) ball made from local materials, (b) learner measuring water and picture (c) a child identifying a shape

The findings show that two teachers out of the seven did not understand or differentiate a method from a strategy, as evidenced by the following verbatim response expressed by teacher (Sa3);

Teacher Sa 3 said *“is strategy the same as method?”...I think I use learner centred and I also expose them to dolls from clay.*

Teacher sb1 said “*I expose them to mathematical activities*” in the lesson observation the teacher Sb1 was observed exposing learners to games made of lines and circles. The lines made a triangle as shown in Figure 5.

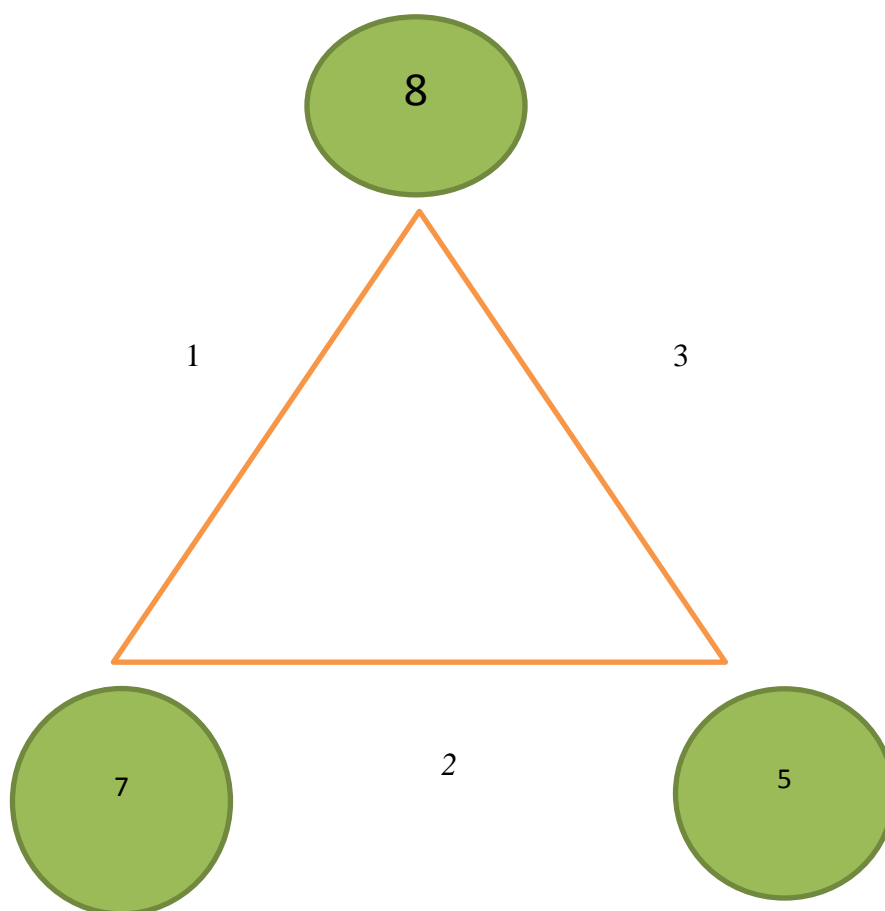


Figure 5: Mathematical Game

$$7+1=8$$

$$8-1=7$$

$$5+2=7$$

$$7-2=5$$

$$8-3=5$$

$$3+5=8$$

Strategies of the findings shown in Table 3.

Table 3: Shows findings on the strategies

Strategy	Semi structured interview	Lesson observations	Document analysis
Tracing	7	4	5
Drawing	7	5	3
Exposing/exploring	7	5	3

4.3 Methods of teaching geometric and spatial reasoning at ECE

The previous section provided findings on the strategies used when teaching at ECE. Research question two sought to identify the method used by ECE teachers when teaching at ECE. This question was answered through lesson observation, document analysis and semi-structured interviews with the teachers. Vignettes were used to supplement the findings. The document analyses were done on three lesson plans and three records of work. The methods that were predominantly stated in the findings were discussion, demonstration and experimental.

4.3.1. Discussion method of teaching

The findings showed that four out of the seven (4 out of 7) teachers of the ECE teachers used discussion method when teaching geometric and spatial reasoning in their class, From the document analysis three teachers' documents showed that discussion was used. The following verbatim responses were expressed by teachers (Sb1 & Sc1);

Teacher Sb1 said “I use discussion and explaining method”

Teacher Sc1 said “I use demonstration, group work, question and answer and discussion”

Similarly, results from lesson observations showed that teachers used discussion method as evidenced by the following Figure 6 as typical examples, where learners were discussing on the identifications of the shapes.

Teacher Sc1 said: from those shapes choose triangles you put them in one group, (mwevela? Mutenge matriangles muaike eka namaseko eka hai.)

Below in the Figure 6(a) and 6(b) photo shot got on a group discussion that was done in the class.

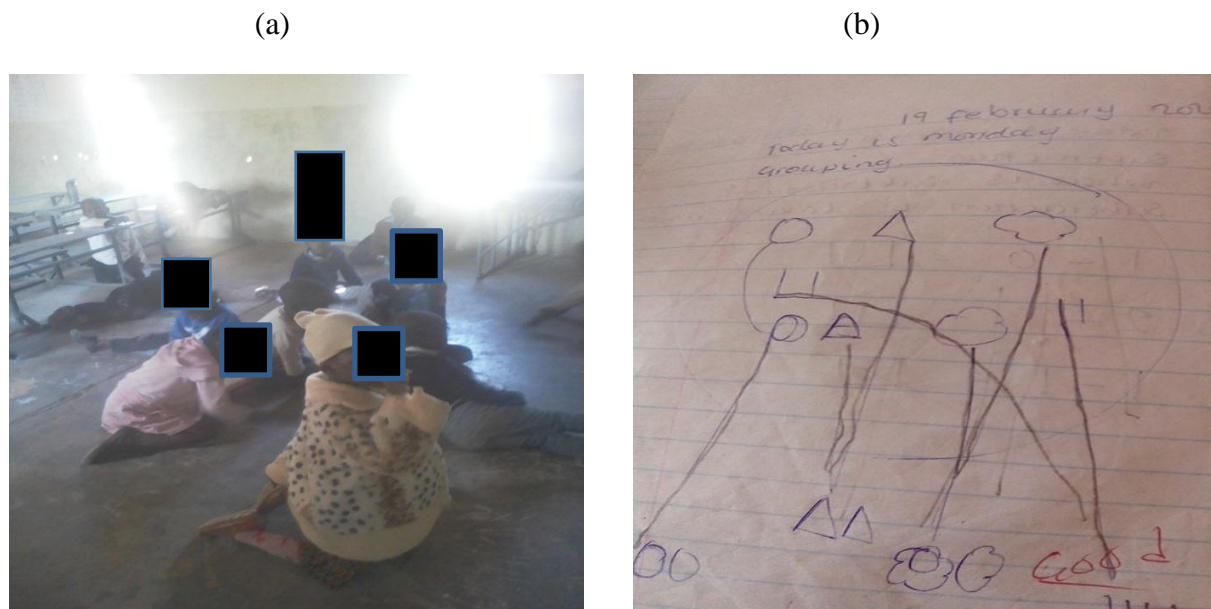


Figure 6: Showing picture 6(a) learners in group discussion, 6(b) work came from group discussion

4.3.2. Demonstration Method of teaching

The findings showed that all the seven teachers used demonstration, and this was also observed during lesson observations where three (3) teachers out of the seven teachers observed were seen using demonstration, which was also reflected in analysed documents.

The following statements by teachers (Sa2, Sa3, Sb2, and Sc1 & Sc2) act as typical examples;

Teacher Sa2 said: *“I use demonstration, tracing, dancing-shape dancing and read and repeat”* Teacher Sa3 said: *“I think I use learner cantered as I already said.”*

Teacher Sb2 said: *“I use demonstration and look and say method”*

Teacher Sc1 said: *“I use demonstration, group work, question and answer and discussion”*

Teacher Sc2 said: *“I use play methods, group work, demonstration as well as gaming.”*

From lesson observation

Teacher Sa3: class bring half cup of sand:

Sc2 said: Chipu show as shape –dancing.

Below in the Figure 7(a) and 7(b) are some photo shots that were gotten on a demonstration lesson observation that was done in the class, it was going to make more meaningfully if it was a video.



Figure 7: Showing picture (a) demonstration on half, (b) full cup of sand

In Figure 7 the learner was demonstrating the measuring of water showing that also that the same quantity of water could reach different levels in different size of containers, while in figure 14 the learner was showing the shape dancing.

4.3.3 Experimental Method of teaching.

The findings showed that three teachers out of the seven teachers were doing experiments, although through interviews, five out of the seven teachers said they used experiments. The following verbatim responses expressed by teachers (Sa1, Sb1 & Sc2) act as typical examples of what was said;

Teacher Sb1 said *“I use demonstration, experimental and look and say method”*

Teacher Sa1 said *“I use demonstration, experimental, group work, question and answer and discussion”* Teacher Sc2 said *“I use play methods, group work, experimental, demonstration as well as gaming.”*

With regards to teacher’s actions when teaching geometric and spatial reasoning in the lesson observations the actions were as shown in the figure 8(a) and 8(b) below. , it was going to make more meaningfully if it was a video.

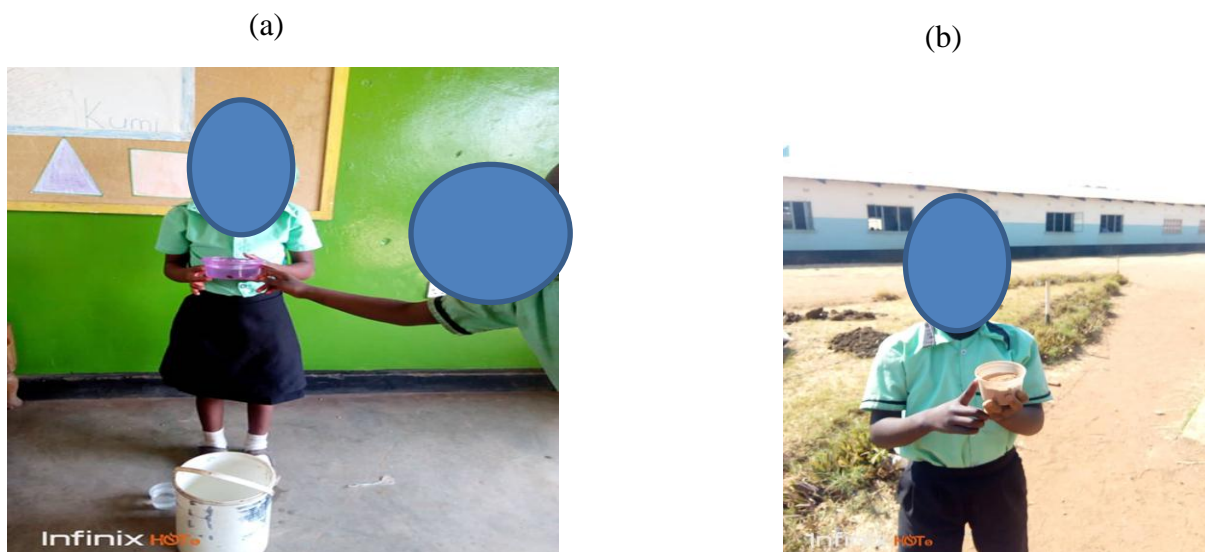


Figure 8: Showing picture 8(a) demonstrating Measure of water, 8(b) demonstrating Measure of Sand

Frequency of use of methods by respondents on the teaching of geometric and spatial reasoning shown in Table 4.

Table 4: Shows frequency of use of methods by respondents on the teaching of geometric and spatial reasoning

Method	Lesson observations	Semi structured interview	Document analysed
Discussion	4	7	3
Demonstration	4	6	3
Experimental	5	6	3

4.4 Materials used when teaching geometric and spatial reasoning at ECE

The previous section provided findings on the teaching of geometric and spatial reasoning at ECE as indicated by the second research question. Research question three sought to identify the materials used by ECE teachers when teaching at ECE. To find out the materials used, the researcher carried some semi structured interview, lesson observation and some document analysis.

The findings showed that cards, charts, real objects, natural settings were some of the materials used when teaching geometric and spatial reasoning at ECE.

4.4.1 Cards/charts

Findings showed that all the ECE teachers that were observed used charts and also results from the semi structured interviews showed that all the teachers interviewed used charts and cards to teach geometric and spatial reasoning at ECE. From semi structured interviews the following verbatim responses were expressed by teachers (Sa2, Sa3, Sb2, and Sc1 & Sc2);

Teacher Sa2 said “*Materials are a challenge hence I use cards to make different shapes.*”

Teacher Sc1 said” *I some time get old calendars to draw different shapes*”.

From lesson observation

Teacher Sc1: who can identify a rectangle?

Learners: Me! Me!

Teacher Sc1: Ok Irene

Similarly Figure 9 shows the photo that was gotten during lesson observations which show the charts and cards that were drawn on the notice board.



Figure 9: Learner Identifying a Rectangle made of a Card

4.4.2 Real objects/Natural settings

The lesson observations showed that five of the ECE teachers used real objects in natural settings and also results from the semi structured interviews showed that all the teachers

interviewed used real objects in natural settings The following statements by teachers (Sa2, Sa3, Sb2, Sc1 & Sc2) act as typical examples;

Teacher sb1 said “I expose them to mathematical activities” expose learners to rich informal mathematical activities

The researcher asked how learners were exposed to mathematical activities

Teacher sb1 said depending on what I have on the plan.

Teacher Sa1 said “materials are a problem. I just have to improvise”

The other respondents said they used real objects to teach learners that real materials could be used to make different geometrical shapes. For example one teacher made a ball out of plastics. Below are some of the photos of a child playing with sand in Figure 10.

(a)



(b)



Figure 10: Showing picture 10(a) and 10(b) learner playing in the Local Environment

Summery for research question 3 on the materials used when teaching geometric and spatial reasoning at ECE shown in Table 5.

Table 5: Summery for research question 3 on the materials used when teaching geometric and spatial reasoning at ECE.

materials	Findings
Cards and charts	Both observations and semi structured interview showed that cards and charts were used when teaching at ECE,
Real objects natural environments	The findings were that real objects and natural environment were used as five out of the seven teachers said so.
Beds, dolls and toils	The findings from all the three sources showed that beds, dolls & toils as materials

This chapter presented the findings of the study which aimed at exploring the teaching of geometry and spatial reasoning in ECE level in Shibuyunji district of central province in Zambia. The chapter briefly describes the three sites visited by the researcher during data collection, analysis and demographic. The themes that are presented in this chapter emerged from the data collected from lesson observations, document analysis and semi structured interviews. The research questions were analysed based on the themes that emerged from lesson observations, document analysis and semi structured interviews. However, triangulation of data was also achieved by comparing what the teachers said in the semi structured interview with the observations made during the lesson observation. The following chapter discussed the findings and issues that rose from the study.

CHAPTER FIVE

DISCUSSION OF FINDINGS

5.1 Introduction

This chapter presents the discussion, of the research findings. In this research, the researcher explored the teaching of geometry and spatial reasoning at ECE in Shibuyunji district of Central province in Zambia. The findings presented in the previous chapter were discussed. The discussions of the findings were according to the research questions and some of the strength and errors that were made were analysed according to the two theories of Van Hiele and Jean Piaget.

5.2. Strategies used to teach geometric and spatial reasoning at ECE

The research findings were that a number of strategies were used by ECE teachers to introduce concepts of geometric and spatial reasoning.

5.2.1 Tracing

Tracing was one of the strategies that was used or that came up in the findings. Tracing or repeatedly tracing of one shape also help the learners to improve the psycho motor skill, which is advocated by Piaget first stage of development which is from zero to two years and it also help in the teaching of geometric and spatial reasoning as learners are made to trace geometrical shapes. For child to trace he/ she must have visualised the shape or number that is to be traced, which is in line with Hiele level 0 of development. The findings were in line with the writing of Shelley (2007) who wrote that the three-year-olds get practice recognizing and making letters and shapes by tracing. You can create your own tracing activities such as geometrical shapes that allow you to customize the letters or strokes you teach. Children can trace the shapes with fingers or place a piece of paper over and scribble with a crayon to reveal the shape and this can help them master the geometrical shapes and develop spatial reasoning. The writing of a 3-year-old isn't recognizable as real words. More often, the writings consist of random scribbles that don't even look like real letters or numbers. However, those early activities help the learners to develop geometric and spatial reasoning as they were able to trace and come up with the desired shapes.

And the Prewriting activities also help children learn that the symbols on the paper have meaning which could be a sign of learning geometric and spatial reasoning.

5.2.2 Drawing

The findings showed that drawing the same object or same shape for example teacher Sa1 made the learners to write a number five (5) using repeatedly drawing of a semi-circle which was line with Kamla though it was at higher level. Kamla-Raj (2015), in his project used a process of drawing and construction of art facts in a technology education classroom to enhance and inform the teaching of geometry and to allow learners to both reflect and use the Geometry they know as a springboard for further study of Euclidean Geometry and Mathematics as subject. Through drawing and tracing the children also acquire or improve their motor skills and coordination. From the findings we can conclude that the children are being made to do some manipulative activities which will help them to develop orientation on mental transformations and spatial awareness hence developing geometric and spatial reasoning. Also if the child can start drawing geometric shapes at early stage they will not have challenges to know the properties of the shapes and even solving related questions will not be difficult at a later stage, Nile (2012) wrote that the perceptual viewpoint of drawing is an intuitive one, because it matches the phenomenological experience of drawing. As regarding geometrical solids, more than one third of the students couldn't draw the correct two-dimensional representation and one third didn't know how to draw the net of the solids. Through drawing the learners develop symbolic thoughts as the preoperational stage of Piaget cognitive stage of development and visualization of shapes as level 0 of Van Hiele levels of development.

5.2.3 Exposing and Exploring

The curriculum implementation roadmap of the revised curriculum that is the stage of development and implementation stage 2013, one of its vision is that it should make a real difference to learners in both schools and in their real life hence it advocate for play based teaching where learners learn by doing. Hence exposing or making of the learners explore their environments help the learners to learn by doing and then develop the geometrical and spatial reasoning, which is in line with Hiele level 1 as this help them to analyse. Learners were seen

making their own real objects for example the ball that the learner made using the local materials, which shows that the learner is able to appreciate their world. As stated by Shaw & Blake (1998) that to make sense of the world, it is important to understand the space, shape and patterns. Geometry for young children is seen to help the children interpret, understand, and appreciate their world. Teacher Sa2 also showed an example of a ball made by a grade four pupil using local materials which was previously shown in figure 8 of chapter four. The respondent also allowed the learners to play with sand which is good and help children to develop navigating skills as stated by Taylor & Francis (2009), that when children have ample opportunities to explore their environments, results in the gain of greater fine and gross motor control, they learn to navigate more skilfully.

Teacher Sa3 said “Is strategy the same as method?” Aaa! I think I use learner cantered.” This is a method that may include a number of strategies. An essential factor for a learner cantered approach is placing the learning characteristics of all learners under the microscope with specific emphasis on low-performing learners. McCombs (1997) explained that the locus in a learner-cantered approach is on individual learners' heredity, experiences, perspectives, backgrounds, talents, interests, capacities, and needs. The teacher did not know what a strategy is or he did not know the difference between strategy and method. Teacher sb1 said “I expose them to mathematical activities” in the lesson observation the teacher Sb1 was observed exposing learners to games made of lines and circles. This game was for that reception grade those who will be going to grade one. The game was seen to make the learning of mathematics interesting and helped learners learn a number of Mathematics concepts including the spatial sense using the same game, though it was seen to be used to a low level grade that was according to my observation. The same game also helped the learners learn how to calculate, count and to learn the concept of shapes. This was a very good model but it was not used to the correct stage of Piaget stages of development. It was going to be more beneficial if it was used to the learners of 7 to 12 years which is the Concrete operational stage or the third stage of Piaget were the learners tainted to develop more logical and methodical manipulation of symbols. According to the model of Van Hiele levels the game used by the teacher was supposed to be for the second level which is the level were learners can analyse issues, as this game needs to be analysed.

The researcher asked how learners were exposed to mathematical activities. The explanation was that the respondent uses some games to teach geometric and spatial reasoning at ECE. This teacher, from the observation, it can be said that the teacher skipped some stage of development from both those of Piaget and Van Hiele level, which may affect the performance of learners in future mathematics. The research says so, because the activities that were seen in the observation of some ECE teachers skipped the first level, as learners were not seen doing the recognition level, were learners are made to identify the real object and name it, instead just rushed into identifying on a chart without seeing the real object.

From the findings we can conclude that the children were made to do some manipulative activities which can help them to develop orientation on mental transformations and spatial awareness and also when children have ample opportunities to explore their environments, resulting in the gain of greater fine and gross motor control, they learn to navigate more skilfully and also be able to pass through the first level of Van Hiele, Crowley (1987) wrote that students had trouble with high school geometry because their early training had not allowed them to pass through five developmental levels of Van Hiele. It was seen that to conceptualise children's formation of geometrical concepts, Piaget (e.g.1956; 1960) took a cognitive developmental stand. That is, geometric and spatial reasoning thought develops in stages following an experiential order which does not necessarily reflect the historical development of geometry. At the first stage, a child uses sensory-motor activities to explore space, constructing representations of topological concepts such as interior and exterior, without size or shape but still leading to the development of geometric and spatial reasoning. At the second stage, the child develops concepts of projective geometry such as a straight line or a right angle. At the third and last stage, children discriminate location in two- and three-dimensional space succeeding with measurement and higher level tasks (Piaget, Inhelder, & Szeminska, 1960). At this stage, the child is ready to study notions of Euclidean geometry such as angularity and parallelism. In general, Piaget differentiated between topological and Euclidean figures and conceived of geometry as the study of space.

5.3 Methods used to teach geometric and spatial reasoning at ECE.

5.3.1 Discussion method

According to the first theme the study revealed that most of the ECE teachers used the Demonstrations methods in the teaching Geometric and spatial reasoning at ECE which was followed by the discussion. The study further showed that demonstration was the most suitable method in the teaching Geometric and spatial reasoning, while Discussion was not a good method of teaching geometric and spatial reasoning for that age as the reasoning of giving each other chance to talk or present their views is still less or not yet developed, hence it caused confusion in class. The age of three to six may not even have the ideas of what is to be discussed. Despite discussion being a very good method in the teaching of mathematics at upper and other higher grades as stated by Kateri (2015), that Teacher centred has had its day. Effective teachers are increasingly using a learner centred approach. Cooperative learning sparks engagement in classrooms as it encourages interaction among the learners themselves, which is mostly done at higher grades and not at ECE levels. The teacher should be putting students in groups rather than calling on one student at a time. This makes the students to work just as hard as the teachers. Teaching and learning is no longer considered a one-man show, the teacher's role becomes that of a facilitator instead. But discussion is not good for ECE learners. Geddes and Fortunato (1993) claimed that quality of instruction was one of the greatest insights of the learners' acquisition of geometrical knowledge. Strutchens (2001) advised that instruction in geometry should practice hands-on explorations, developing geometric thinking and spatial reasoning.

5.3.2 Demonstration Method of Teaching,

The finding showed that demonstration was one of the methods that were used by ECE teachers to teach at ECE and the researcher observed that demonstration was seen to be one of the best methods that were used at ECE. The researchers observation was in line with Battista and Clements, (2000), who suggested that geometry at early levels should be taught using demonstration in the study of objects, motions and relationships in a spatial environment and they also noted that poor performance in geometry was due, to the elementary school geometry which focused on recognizing and naming of geometrical shapes instead of the teacher to

demonstrate or involve the learners in demonstration. The learners' first experiences with geometry should emphasize the informal study of physical shapes and their properties and have teachers who set their primary goal of developing learner's geometrical and spatial reasoning intuition and knowledge about their spatial environment which can be done through demonstration (see 4.2.4.) Subsequently experiences should involve analysing and abstracting geometric concepts and relationships in increasing formal settings through demonstration. Although the Van Hiele (1999) model was not a perfect one but based on other research, it seemed to model the progress of geometrical thinking through demonstration.

The researcher noted during observation that it could be easier for learners to learn geometry at a high level if they encountered informal "proofs" in earlier school years where they were required to justify their statements and reasoning which they should learn through demonstration. Chakerian (1972) who said that, of course would not be on such formal level as it was at a higher level but simply a mind set for learning and teaching geometry. Mathematical statements and truths were, justified through demonstrations. It was noticed that a lot of geometry that was taught during the field work may not help the learners at higher level of geometrical thinking as even the demonstrations that were seen provided less help to the learners. It may result in the learners experiencing a lot of difficulties at a high level. The use of demonstration if well planned exposes learners to real objects and this was supported also by Piaget who said that Parents and teachers can help build a child's various schemas to promote learning and development throughout the stages. This should be achieved by giving children plenty of exposure to the outside world. Being exposed to a variety of learning by doing experiences by demonstration from a young age may help build up geometric and spatial reasoning and build a

The researcher saw the teachers during lesson observation asking the learners to identify the shapes that were put on the notice board as seen in chapter on 4. The method was more learner centred, with the teacher as a facilitator of learning. Students were more involved in the construction of knowledge through active involvement which helped them develop geometric and spatial reasoning. The learners were made to identify the shapes as the teachers were trying to inquire if the learners knew the name and the shape. The method could also be used to identify the properties of the shapes which were not done by any of the teachers. The use of inquiry

method was also in line with the writings of Comb & Carr, (2010) who wrote that early childhood education that makes use of inquiry-based learning where the teacher asks a learner to point at something or asks learners what they know gives children more opportunities to express themselves, to experiment with topics and methods and to try out new technologies than they would receive in a traditional classroom setting. When teachers lead young students through guided inquiry, the children feel more like they are at play than they would in a teacher-focused lesson. Further, the openness of the inquiry based classroom gives students a sense of freedom that leads to deeper engagement with instructional material. Though play Based methods is known to be the main methods of teaching at ECE it was not dominated or mostly used during the time data was being collected. Play method is a method where learners are asked to search or find something as part of play but with inherent learning points. For example, when a teacher asks learners to find or identify ‘things/shapes’ which are round in a given room and to bring them to her/him and to see who brings many, there is a fun element but also learning points. Basically the idea was to use concrete or hands-on experiences assembling, sorting shapes or objects in their environment being able to follow directions, locating objects in class as well as in their environment verbalising what they are doing thereby laying foundation for geometrical and spatial reasoning, including its vocabulary.

5.3.3 Experimental method of teaching

Although most of the teachers indicated that they used the experimental approach in teaching of Geometric and spatial reasoning this was not corroborated by the findings in the lesson observations. This indicated that most of the teachers depended on unplanned activities which was seen yielding poor results as some of the activities did not show the development of geometric and spatial reasoning, which is not in line with Duru (2010) , the experimental teaching method was more effective than the teacher-centred or traditional teaching method in the knowledge and comprehension level. But the experimental was done and it was well guided the development of geometric and spatial reasoning were seen developing. Iuliana and Marchis (2012) The Geometrical notions and properties occur in real-life problems hence should be taught in a practical way using methods such as experimental method of teaching. Geometry has an important place in school Mathematics curricula. Primary school curricula builds the

foundation of Geometry knowledge, pupils learn geometry notions and properties by exploring their environment mostly through experimental method. Clements, Swaminathan, Hannibal, & Sarama, (1999). For example, learners may be able to separate triangles from quadrilaterals, noting the difference between the number of sides the polygons have, but not be able to distinguish between different quadrilaterals. As a result, they hardly understand concepts which can be taught by teachers, when they inquire the learners only recalled their geometry experience, many of them recalled it not only as unpleasant but they often also recall the difficulties that they experienced in learning geometry. Strutchens, Harris and Martin (2001) stated that learners learnt geometry by memorizing geometric properties rather than by exploring and discovering the underlying properties. They further argued that geometry knowledge learnt in this way was limited and superficial. Eventually the learners found it difficult to apply the limited geometry knowledge in problem solving. This lack of understanding often discouraged learners, thus leading to poor performance in geometry. Hence the use of experimental method can help the learners explore and discover the properties of geometrical shapes.

5.4 Materials used when teaching geometric and spatial reasoning at ECE

Teaching materials can refer to a number of teacher resources; however, the term usually refers to concrete examples, such as worksheets or manipulative teaching aids. Joyce and Weil (1985) ,Teaching materials are different from teaching "resources," the latter includes more theoretical and intangible elements, such as essays or support from other educators, or places to find teaching materials. To find out the materials used when teaching geometric and spatial reasoning at ECE, the researcher did some semi structured interviews, lesson observation and some document analysis. The document analysis was done on lesson plans and records of work of only the three teachers out of the seven teachers who were observed and interviewed. The findings showed that teaching materials were used to support student learning and increase student success in acquiring geometric and spatial reasoning. The teaching materials were adapted to the content in which they were being used and to the age of learners in whose class they were used, and the teacher teaching in that class. Teaching materials were in many shapes and sizes, such as charts, dolls, locally made ball and sand, just to mention a few, but more teaching materials needed to be used to help the ECE learners develop geometric and spatial reasoning. In modern

teaching, geometry is supported by technology besides making use of some parts of the traditional method, but the teachers who were observed did not use anything, if they did then it was less than expected that was related to technology. One would want to know some examples that are used in the teaching of mathematics or geometry to be specific, some of these are, invariance, which was put forward by the Mathematician Klein (1872) who described geometry “as the study of the properties of a configuration that are invariant under a set of transformations that can be illustrated in all angle theorems like Thales’ theorem and triangles. But this may be hindered by the financial state of the school or the location of the school and the age of the learners too. The material used at ECE should be that which helps learners to develop the pre-operation stage of Piaget Stages which say the child has to go through the development of language, memory, and imagination which should help them develop geometric and spatial reasoning. The child’s intelligence is both egocentric and intuitive and the thoughts are symbolic, meaning the child has difficulties in thinking outside of their own viewpoints and of the environment.

5.4.1 Real objects

Many materials can be used to make cards and shapes so as to help children’s understanding of geometric and spatial reasoning. Perhaps most important are familiar classroom objects and situations to draw children’s attention to the use of geometric reasoning. The findings showed that real objects act as motivators to the learners. The other respondents said they use real objects to teach learners that real materials can be used to make different geometrical shapes. For example one teacher made a ball out of plastics as seen in figure 19. The making of the ball or showing the ball can help the learners to have a picture of what a real ball is and can also help them to develop the mind of critical thinking and hence explore the environment they live in and the things that are found in it. This is in line with Hiele level 0 as learners can visualises the real objects. Strutchens (2001) advised that instruction in geometry should emphasis on hands-on explorations were learners are asked to make things using the local materials such as clay, plastics and others to help them develop geometric thinking and reasoning, making conjectures and carrying out geometric projects.

5.4.2 Charts and cards

The learners were made to identify the shapes on the charts and they were able to name and identify the shapes, which according Van Hiele was level 0 and it was noted to be a good way of giving instruction in the teaching of geometric and spatial reasoning. Geddes and Fortunato (1993) claimed that quality of instruction was one of the greatest influences of the learners' acquisition of geometry knowledge and also as already stated by Strutchens (2001) that instruction in geometry should emphasise hands on explorations that help the learners in the development of geometrical thinking and spatial reasoning. Hence learners should also be involved in the making of the charts. It was also observed that teachers did not have the will of improving, as it was noted that materials to make cards and charts were not easily available. Teacher Sa1 said, *"Materials are problem, I just have to improvise"*

5.5. Summary of the discussion

The study established that teachers mostly used the inquiry, experimental, demonstration and discussion methods when teaching Geometric and Spatial reasoning at ECE. The study further found that demonstration was better to be used than discussion method of teaching at ECE, though some teachers used discussion method. The findings also showed that materials such as charts sandy water were used which was not enough. It was established that teachers did not get adequate support in the area of geometric and spatial reasoning as they had no adequate or enough teaching materials. It was also noted that the tendency by most teachers to teach geometry by informing learners of the properties of shapes and then completing exercises contributed to the poor performance in geometry but use of real objects was good

Findings showed that although the ECE teachers used some strategies such as exploring, exposing, and drawing and others, more strategies needed to be used. The teaching of geometric and spatial reasoning at ECE involved a number of methods among them; discussion which was observed not to be a good method to use at an early stage. The learners at ECE were observed doing different activities from what they were assigned to do under the discussion method, therefore more methods needs to be implored. In an area like Shibuyunji district, where materials were found to be a challenge at ECE. The ECE teachers need to be very resourceful and creative

to be able to use locally available materials. To address the constraints established from the study, participants suggested the need for an adequate supply of ECE materials, comprehensive teachers teaching approaches, pre-requisite knowledge in pre-geometric and spatial reasoning of ECE topics.

CHAPTER SIX

CONCLUSION AND RECOMMENDATIONS

6.1. Introduction

This chapter concludes the study and makes some recommendations based on the findings of the study.

6.2 Conclusion

This study was aimed at exploring the teaching of geometrical and spatial reasoning in ECE In Shibuyunji district of Zambia. The objective of the study were examine the methods used to teach geometric and spatial reasoning in ECE, establish the strategies teachers use when teaching geometric and spatial reasoning in ECE and explore teaching and learning material used by ECE when teaching geometric and spatial reasoning in ECE. The study took a qualitative approach and descriptive design in the teaching of geometrical and spatial reasoning. The study was guided by the theories of Van Hiele and Piaget cognitive development. The literature in this study was thematically reviewed under each objective.

Different procedures were used to execute the study. The researcher first described the research paradigm, research design, population, sampling procedures and research instruments, data collection and data analysis methods, credibility and trustworthiness and ethical considerations.

Data was presented according to the themes that rose. The study established that teachers mostly used the inquiry, experimental, demonstration and discussion methods when teaching Geometric and Spatial reasoning at ECE. The study further found that demonstration was better to be used than discussion method of teaching at ECE, though some teachers used discussion method. The findings also showed that materials such as charts sandy water were used which was not enough. It was established that teachers did not get adequate support in the area of geometric and spatial reasoning as they had no adequate or enough teaching materials. It was also noted that the tendency by most teachers to teach geometry by informing learners of the properties of shapes and then completing exercises contributed to the poor performance in geometry but use of real objects was good.

Findings showed that although the ECE teachers used some strategies such as exploring, exposing, and drawing and others, more strategies needed to be used. The teaching of geometric and spatial reasoning at ECE involved a number of methods among them; discussion which was observed not to be a good method to use at an early stage. The learners at ECE were observed doing different activities from what they were assigned to do under the discussion method, therefore more methods needs to be implored.

6.2.1 Contributions

This study has made a contribution to the body of knowledge regarding the teaching of geometrical and spatial reasoning at ECE. It proposes that teachers should consider the age and level of understanding of the learners when choosing the methods of teaching. The teaching and learning materials should be mostly real objects that learners are familiar with to help learners develop geometric and spatial reasoning that are visual

6.3 Recommendations

In view of the findings and conclusions, the following recommendations are proposed

1. The study recommended that teachers should use suitable methods which should depend on the age and level of understanding of the learners.
2. Seminars on the teaching of geometry for ECE teachers should be conducted in order to enhance the in-depth knowledge on the topic to enable them teach it with confidence and use the correct materials and method.
3. Teachers should prepare to teach using experimental method.

6.3.1 Areas of future research

1. From the findings of this research, this researcher proposes that future research be focused on exploring the challenges faced by ECE teachers when teaching geometric and spatial reasoning.
2. Exploring the play based method of teaching geometric and spatial reasoning at ECE,

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APPENDICES

Appendix 1: Letter to the District Education Board Secretary

THE UNIVERSITY OF ZAMBIA,
SCHOOL OF EDUCATION,
P.O BOX 32379,
LUSAKA.

April, 2019.

THE DISTRICT EDUCATION BOARD SECRETARY,
SHIBIYUNJI DISTRICT,
SHIBUYUNJI,
ZAMBIA,

Dear Sir/Madam

Ref: Permission to conduct research in your district.

I am a student at the University of Zambia pursuing a Master of Education in mathematics education. I am seeking permission to conduct my research in three primary schools in your district. The topic under research is EXPLORING THE TEACHING OF GEOMETRIC AND SPATIAL REASONING IN EARLY CHILDHOOD EDUCATION IN SELECTED PRIMARY SCHOOLS IN SHIBUYUNJI DISTRICT. My study is targeting 6 teachers of Early Childhood Education, two teachers from each school. This research is purely for academic purposes.

Your consideration will be highly considered.

Yours sincerely

Chingala Catherine

Masters Student

Computer #: 2017014387

Appendix 2: Letter to the School Headteacher

The University Of Zambia,
School Of Education,
P.O Box 32379,
Lusaka,
April, 2019.

The Headteacher,
Mamvule Primary School,
Shibuyunji,
ZAMBIA,

Dear Sir/Madam

Ref: Permission to conduct research at your school

I am a student at the University of Zambia pursuing a Master of Education degree in Mathematics Education. I am seeking permission to conduct a research in your school on the topic *The teaching of geometric and spatial reasoning in early childhood education in selected primary schools in shibuyunji district*.

The research procedures will involve data collection through classroom observations, semi-structured interviews with the teachers and document analysis.

Before data collection begins, I will first come to your school to explain the research and outline the roles of participants to the participants. Participants have the right to withdraw from the research activities any time.

Thank you in advance for your consideration

Yours sincerely

Chingala Catherine

Computer #: 2017014387

Appendix 3: Semi-structured Interview guide for ECE teachers

1. What is your professional qualification?
2. How long have you been teaching Early Childhood Education (ECE)?
3. How do you teach geometry in ECE?
4. What are the strategies that are used in the teaching of geometric and spatial reasoning?
5. Do you think geometry is important at ECE?
 1. *If yes, how is it important?*
2. What are the methods used in the teaching of geometric and spatial reasoning at ECE?
3. Do learners show that they have learnt the concept of geometric and spatial reasoning?
 1. *If the answer is yes, how do you know that the learners have acquired the concept?*
2. Do you think the teaching of geometry help children develop geometric and spatial reasoning?
 3. *If yes, then how?*
4. Do you think the teaching of geometric and spatial reasoning has an impact in learning of mathematics?
 5. *If yes, then how?*
6. What are the materials that you use when teaching geometric and spatial reasoning at ECE?
7. Do you face any challenges when teaching geometric and spatial reasoning at ECZ?
8. *If yes what are some of the challenges that you face?*
9. What possible solutions do you think can address the challenges that you face?

Appendix 4: Lesson observation schedule

School code.....

Teacher's code.....

Time.....

Number of learners: Boys....., Girls.....

Date.....

Activity	Observation	Comment
Is the teacher giving learners shapes or diagrams to identify?		
Is the teacher asking learners to draw the shapes on their own?		
Is the teacher guiding learners to make shapes using local viable materials?		
Is the teacher giving examples of some real objects on the stated name of the shape?		
Does the teacher seem to have enough knowledge on geometry?		

GENERAL

OBSERVATION.....

.....

.....

Appendix 5: Document analysis checklist

Document	Specific items	Comment
Lesson plans	<ol style="list-style-type: none">1. Check if the teacher planned what they were teaching in class.2. Check if their objectives were meant to develop learners' geometrical and spatial reasoning.3. Check the examples if they are developing geometrical and spatial reasoning in learners.	
Records of work	<ol style="list-style-type: none">1. Check if they met their objectives in the topics involving geometrical and spatial reasoning.2. Check if the learners were given any assessments to assess their geometrical and spatial reasoning	

Appendix 6: Proposed Research Budget

S/N	DESCRIPTION	QUANTITY	UNITY PRICE (ZMW)	TOTAL COST (ZMW)
1	Stationery			
	- Reams of paper	02	55	110
	- Box of pens	01	25	25
	- Flash disk 4GB	01	70	70
2	Research equipment			
	-MP 3 Recorder	01	800	800
3	Research equipment			
	-Typing proposal and report	08	300	2400
	-Printing: copies of proposal and report.	08	300	2400
	- Binding: copies of proposal and report	08	100	800
	- Communication processes (air time)	-	600	600
4	Research cost			
	1. Transport to and from Shibuyunji.	4	440	1760
	2. Lodging per night	14	200	2800

	3. Feeding per day	14	100	1400
5	4. Ethical clearance	01	500	500
	5. Poster	01	300	300
	6. Editing	01	2000	2000
5	Miscellaneous			1000
6	GRAND TOTAL			K16,965

Appendix 7: Proposed Work Plan and Time line

S/N	ACTIVITY DETAILS	DATES	DURATION
1	Preparation of research proposal	September - October, 2018	Eight weeks
2	Development piloting recasting of data collection instruments	December, 2018	Two weeks
3	Data collection processing and analysis	April-June 2019	Twelve weeks
4	Report writing, typing and editing	June-July, 2019	Six weeks
5	Proof reading, production & submission of first draft	August 2019	Four weeks
6	Refining and submission of second draft report	September, 2019	Four weeks
7	Refining and presentation of final draft	October, 2019	Four weeks