THE UNIVERSITY OF ZAMBIAA SCHOOL OF NATURAL

SCIENCES

2012/13 SECOND SEMESTER

1. BIO1022- MOLECULER BIOLOGY AND GENETICS
2. BIO2032- BASIC MICROBIOLOGY
3. BIO2062- DIVERSITY OF ANIMALS
4. BIO3182- GENETICS
5. BIO4332- ECOLOGY AND MANAGEMENT OF FISHERIES
6. C102- INTRODUCTION TO CHEMISTRY
7. C212- INTRODUCTION TO BIOCHEMISTRY
8. C252- ORGANIC CHEMISTRY
9. C322- ANALYTICAL CHEMISTRY
10. C362- COLLOIDS AND ELECTROCHEMISTRY
11. C482- INORGANIC INDUSTRIAL CHEMISTRY
12. C492- ORGANIC INDUSTRIAL CHEMISTRY
13. CST3252- ELECTRONICS FOR COMPUTING
14. CST4012- ADVANCED OPERATING SYSTEMS AND DISTRIBUTED SYSTEMS
15. EM312- ENGINEERING MATHMATICS
16. GEO112- INTRODUCTION TO HUMAN GEOGRAPHY
17. GEO155- INTRODUCTION TO PHYSICAL GEOGRAPHY
18. GEO175- INTRODUCTION TO MAPPING TECHNIQUES IN GEOGRAPHY
19. GEO211- THE GEOGRAPHY OF AFRICA
20. GEO271- QUANTITATIVE TECHNIQUES IN GEOGRAPHY
21. GEO381-ENVIRONMENT AND DEVELOPMENT
22. GEO912- GEOGRAPHY OF MIGRATION AND REFUGEES
23. GEO922- GEOGRAPHY OF REGIONAL PLANNING AND DEVELOPMENT
24. GEO952- GEOGRAPHY HYDROLOGY

25.GEO962- BIOGEOGRAPHY
26.GEO995- ENVIRONMENTAL AND NATURAL RESOURCES MANAGEMENT
27.GEO2502- FUNDAMENTALS OF NATURAL RESOURCES ECONOMICS
28.GEO5595- ENVIRONMENT AND NATURAL RESOURCES ECONOMICS
29.M111- MATHMATICAL METHODS
30.M112- MATHMATICAL METHODS
31.M162- INTRODUCTION TO MATHMATICS, PROBABILITY AND STATISTICS
32.M212- MATHMATICAL METHODS
33.M221- LINER ALGEBRA
34.M292- INTRODUCTION TO PROBABILITY
35.M325- GROUP AND RING THEORY
36.M432- REAL ANALYSIS
37.M462- BAYESIAN INFERENCEAND DISCRETE ANALYSIS
38.M911- MATHMATICAL METHODS
39.P192- INTRODUCTORY PHYSICS
40.P212- MAGNETISM IN MATTER AND ATOMIC PHYSICS
41.P252- CLASSICAL MECHANICS
42.P312- COMPUTATIONAL PHYSICS
43.P412- NUCLEAR PHYSICS
44.P422- SOLID STATE PHYSICS
45.P455- QUANTUM MECHANICS
46.PHY4815- PHYSICS OF RENEWABLE ENERGY RESOURCES AND ENVIRONMENT

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

BIO 1022 MOLECULAR BIOLOGY AND GENETICS
THEORY PAPER

TIME: THREE HOURS

INSTRUCTIONS:

1. Use the answer sheet provided to answer the questions
2. Answer all questions.
3. Choose the best answer.
4. Each correct answer carries 4 marks.
5. Each wrong answer carries (-1) marks.
6. A blank space carries (-1) marks.
7. I don't know carries 0 marks.
8. The table of the genetic code is given on the last page of the examination.

---

1. The bond between a nitrogenous base and pentose sugar is a(n) ....

1. hydrogen bond
2. N-glucosidic bond
3. phosphoester bond
4. peptide bond
5. None of the above
6. I do not know

2. Molecules consisting of a pentose sugar, a nitrogenous base and a phosphate are called …

1. deoxyribose
2. ribose
3. nucleotides
4. nucleosides
5. bases
6. I do not know

3. DNA is made up of … different chemical compounds.

1. 3
2. 4
3. 5
4. 6
5. 7
6. I do not know

4. Name two bases which form a double bond between them in the formation of DNA.

1. Adenine and Thymine
2. Uracil and Thymine
3. Guanine and Adenine
4. Thymine and Guanine
5. Guanine and Cytosine
6. I do not know

5. DNA is ......

1. a molecule that does not carry a charge.
2. both positively and negatively charged.
3. neutral.
4. positively charged
5. negatively charged.
6. I do not know

6. Given the following DNA template AAA TTT GGG CC , Choose the correct answer:

	Complementary DNA sequence	**mRNA sequence**	**Number of associated tRNAs**
1	UUU AAA CCC GG	AAA UUU GGG CC	3
2	CC GGG TTT AAA	GGG AAA CCC CC	4
3	TTT AAA CCC GG	UUU AAA CCC GG	3
4	CC GGG UUU AAA	GGG AAA CCC CC	4
5	AAA UUUGGG CC	AAA TTT GGG CC	4
6	I do not know		

7. In which of the following processes does DNA expose its bases?

1. Translation
2. Transcription
3. Replication
4. Translation and replication
5. Transcription and replication
6. I do not know

8. Complementary base pairing is important for ...

1. RNA transcription
2. DNA replication
3. RNA translation
4. tRNA function
5. All the statements above are correct
6. I do not know

9. Transcription occurs ...

1. in the cytoplasm of eukaryotes
2. in the nucleus of eukaryotes
3. in the nucleus of prokaryotes
4. at ribosomes in all cells
5. as a last step of protein synthesis
6. I do not know

10. During transcription ...

1. RNA polymerase binds to the enzyme helicase.
2. DNA polymerase binds to the promoter site of DNA.
3. RNA polymerase binds to the promoter site of RNA.
4. the enzyme helicase breaks hydrogen bonds in DNA.
5. DNA polymerase breaks hydrogen bonds in DNA
6. I do not know

11. Transporting amino acids to ribosomes for assembly into proteins is the function of ...

1. Amino acyl synthetase
2. DNA
3. tRNA
4. rRNA
5. mRNA
6. I do not know

12. In eukaryotes, introns are ... regions and exons are ... regions of mRNA.

1. translation, transcription
2. coding, non-coding
3. transcription, translation
4. protein, nucleoside
5. non-coding, coding
6. I do not know

13. During initiation of m-RNA transcription in eukaryotes …

1. the third base is usually methionine.
2. the third deoxy ribonucleotide is usually adenine.
3. the third nitrogenous base is usually adenine.
4. the third ribonucleotide is usually thymine.
5. the third base is usually guanine.
6. I do not know

14. The stages of transcription arranged in reverse order are …

1. initiation, termination, elongation
2. termination, initiation, elongation
3. elongation, initiation, termination
4. elongation, termination. initiation
5. termination, elongation, initiation
6. I do not know

15. During elongation of translation in prokaryotes…

1. formyl methionine binds to the A site of the 70S ribosomal complex.
2. the 70S ribosomal subunit binds an m-RNA.
3. a new amino acid binds to the A site of the 70S ribosomal complex.
4. formyl methionine binds to the P site of the 70S ribosomal complex.
5. a new amino acid binds to the P site of the 70S ribosomal complex.
6. I do not know.

16. Codons are...

1. made up of two nucleotide sequences on mRNA
2. located on tRNA molecules
3. sequences of proteins on DNA
4. made up of three nucleotide base sequences on mRNA
5. sequences of polypeptides on chromosomes
6. I do not know

17. The number of codons in the genetic code is governed by the number of available nitrogenous bases and the size of each codon. From the table below, choose the correct combination.

	Number of nitrogenous bases	**Number of bases in each codon**	**Number of codons**
1	3	3	9
2	4	2	16
3	3	4	64
4	5	3	75
5	5	4	20
6	I do not know		

18. Which molecule is degraded after translation?

1. DNA
2. tRNA
3. rRNA
4. proteins
5. mRNA
6. I do not know

19. During protein synthesis, the first tRNA attaches to ...

1. glycine
2. leucine
3. tryptophan
4. methionine
5. cysteine
6. I do not know

20. Which one of the following nucleotide triplets belongs to the genetic code?

1. TCU GGG TTT GCA
2. AUA GGC AUG CAC
3. GGT CCC GCC GTG
4. CAA UGA TCA CCT
5. AGC GGA TUC UAT
6. I do not know.

21. All of the following are directly involved in translation, except...

1. DNA
2. ribosome
3. tRNA
4. start codon
5. stop codon
6. I do not know

22. Given the two complementary strands of DNA below, determine which of the two strands would be transcribed into mRNA and the base sequence of that mRNA.

5'....ATCGGACTTCG...3' DNA strand 1
3'....TAGCCTGAAGC...5' DNA strand 2

1. Strand 2; mRNA 5'...AUCGGACUUCG...3'
2. Strand 1; mRNA 5'...TAGCCTGAAGC...3'
3. Strand 1; mRNA 3'...AUCGGACUUCG...5'
4. Strand 2; mRNA 3'...TAGCCTGAAGC...5'
5. Strand 2; mRNA 5'...ATCGGACTTCG...3'
6. I do not know.

23. Identify a termination codon from the following:

1. UUU
2. UGU
3. AUA
4. UAG
5. CAA
6. I do not know.

24. With the aid of the genetic code, identify the amino acid(s) with a second degree degenerate codon(s) among the following: Leucine , Arginine and Methionine.

1. Leucine
2. Leucine and Arginine
3. Arginine and Methionine
4. Arginine
5. Methionine
6. I do not know.

25. A cell has 20 amino acids. How many molecules of tRNA are in the cell?

1. Four
2. Three
3. More than 20
4. At least twenty
5. Fifteen
6. I do not know.

26. Choose the statement which is directly linked to the Wobble Hypothesis.

1. The genetic code is comaless.
2. tRNA can match with more than one codon.
3. The genetic code is universal.
4. All of the above statements are linked to the Wobble Hypothesis.
5. None of the above statements is linked to the Wobble Hypothesis.
6. I do not know

27. An anticodon is found on..... and it is used to ...... on.....

1. tRNA; read the base sequence; mRNA
2. mRNA; read the base sequence; tRNA
3. ribosomes; decode messages; mRNA
4. tRNA; read the base sequence; rRNA
5. mRNA; assemble proteins on; the ribosomes.
6. I do not know.

28. The following is a segment of a DNA nucleotide sequence:
5'...ACTCCTGAATGCAAA...3'

Identify which one of the following nucleotide sequences will be the tRNA anticodons.

1. UUUGCAUUCAGGAGU
2. UGAGGACUUACGUUU
3. AAACGUAAGUCCUCA
4. ACUCCUGAAUGCAAA
5. UCAUCCAAGCGUAAA
6. I do not know.

29. Given the following sequence of nucleotide bases of mRNA strand:

5'...UAGCCGAUGUA...3'. Determine which sets of bases would be read by tRNA.

1. 3'...UAGC CGAU GUA...5'
2. 3'...UA GC CG AU GU A...5'
3. 5'...UAG CCG AUG UA...3'
4. 3'...AUG UAG CCG AU...5'
5. 5'...AUC GG CUA CAU...3'
6. I do not know

30. The operator of the lac operon is kept in the 'off' position by a small protein called a ...

1. histone
2. regulator
3. repressor
4. promoter
5. inducer
6. I do not know

31. Glucose is a (an)... for the lac operon.

1. operator
2. regulator
3. promoter
4. inducer
5. repressor
6. I do not know

32. Which of the following statements about gene expression is **true**?

1. RNA polymerase is responsible for transcribing all the structural genes to mRNA.
2. More than one ribosome is involved in translation.
3. A nonsense codon normally stops translation.
4. Some amino acids are coded for by more than one codon.
5. All the above statements are true.
6. I do not know.

33. What is an operon?

1. Genes that occupy different locations on the *E.coli* chromosome.
2. A group of genes whose function is to inhibit the intake of a substrate.
3. An assemblage of genes that work together to achieve a common purpose.
4. A group of genes, where one gene suppresses other genes in order to metabolise a substrate.
5. Genes located on different chromosomes but work together to make a common product.
6. I do not know.

34. In the *Lac* operon, ... is a substrate while ... is a product.

1. galactose, lactose
2. glucose, galactose
3. lactose, glucose.
4. fructose, lactose
5. galactose, fructose
6. I do not know.

35. A repressor protein is coded for by a (an)…

1. structural gene
2. regulator gene
3. promoter gene
4. operator
5. inducer
6. I do not know

36. The control of gene expression in an operon enables organisms to…

1. reproduce more quickly
2. avoid mutations from taking place in DNA
3. produce proteins only when needed
4. form new combinations of genes when needed
5. provide maximum growth for a cell
6. I do not know.

37.Gene mutation can either be... or ...

1. spontaneous, induced
2. spontaneous, due to mutagens
3. transcription, translation
4. 1 and 3 are correct
5. 1 and 2 are correct
6. I do not know

38. Depurination in the DNA molecule may be due to ...

1. ultra violet radiation
2. deamination
3. aflatoxin
4. methylation
5. nitrosamine
6. I do not know

39. During protein synthesis, the newly synthesised peptide chain is covalently bound to ...

1. DNA
2. the 5' end of mRNA
3. one end of tRNA
4. the small subunit of a ribosome
5. the large subunit of the a ribosome
6. I do not know

40. State the kind of mutation shown below:

DNA sequence before mutation: ACGGCTTACCG
DNA sequence after mutation: ACGGGTTACCG

1. Deletion
2. Insertion
3. Duplication
4. Thymine dimerization
5. Substitution
6. I do not know.

41. Deamination in the DNA molecule may be due to ...

1. ultra violet radiation
2. viral mutagen
3. aflatoxin
4. methylation
5. nitrosamine
6. I do not know

42. The primary effect of ultraviolet radiation is the production of…

1. breaks in DNA.
2. base deletions in DNA.
3. pyrimidine dimers.
4. insertion of bases.
5. substitution of bases.
6. I do not know.

43. Which of the following human diseases is caused by UV radiation?

1. Lung cancer.
2. Liver cancer.
3. Skin cancer
4. Stomach cancer.
5. Pancreatic cancer.
6. I do not know

44. A point mutation can result from …

1. addition of a base to a DNA sequence.
2. replacement of one pyrimidine with another pyrimidine.
3. loss of a base from a DNA sequence.
4. replacement of one purine with another purine.
5. All of the above are correct.
6. I do not know.

45. Nonsense mutation occurs when…

1. a triplet codes for a different amino acid which is not functional
2. a triplet codes for a different amino acid whose function is similar to the normal amino acid
3. a triplet codes for chain termination
4. Both 1 and 2 are correct
5. Both 1 and 3 are correct
6. I do not know.

46. Formation of thymine dimers in the DNA molecule…

1. will modify two bases so that they can no longer pair with their complementary bases
2. is caused by exposure to aflatoxin leading to skin cancer
3. will replace a base in the DNA molecule leading to skin cancer
4. is caused by exposure to viruses leading to HIV.
5. is caused by exposure to nitrosamine.
6. I do not know.

47. Sickle-cell anaemia is an inherited condition that results from a …

1. addition of a base to a DNA sequence.
2. replacement of one base with another base.
3. loss of a base from a DNA sequence.
4. breaks in the DNA molecule.
5. All of the above are correct.
6. I do not know.

48. The substitution of a purine nucleotide with a pyrimidine nucleotide or the other way around is called …

1. transition.
2. insertion.
3. transversion.
4. positive frame-shift.
5. negative frame-shift.
6. I do not know.

49. Identify the mutation that is correctly represented.

1. ACGATTACG → ACTTACG insertion
2. TTUCGATAA → TTUUGATAA deamination
3. GGACTACGG → GGACCTACGG deletion
4. AATAGACCG → AATAGACCG insertion
5. CCAATTGCA → CCATTTGCA deletion
6. I do not know.

50. Choose the statement which is true about biological mutagens.

1. They are usually viruses.
2. They are deaminating agents.
3. They cause pyrimidine dimerization.
4. All the above statements are correct.
5. Only statements 1 and 2 are correct.
6. I do not know.

51. A somatic cell always has … of chromosomes.

1. a haploid number
2. a diploid number
3. 23 pairs
4. 2 pairs
5. pairs
6. I do not know.

52. The primary growth phase of a cell is the ….

1. G_1 phase.
2. G_2 phase.
3. S phase.
4. M phase.
5. 2 and 3 are correct
6. I don't know

53. Choose the statement which is true about the G_2 phase of the cell cycle.

1. The phase in which DNA replicates.
2. The phase in which the cell grows before DNA replication.
3. The phase after DNA replication.
4. The same phase as prophase.
5. None of the above
6. I do not know

54. Interphase in the cell cycle is …

1. the time interval between meiosis I and meiosis II.
2. composed of G_1, G_2, M and S.
3. the stage when chromosomes are visible.
4. the time of cell growth and duplication of cellular components.
5. the division of a cell.
6. I don't know.

55. Crossing-over is identified by…

1. recombinant homologous chromosomes.
2. chiasma(ta) between sister chromatids.
3. recombinant non-homologous chromosomes
4. chiasma(ta) between non-sister chromatids.
5. 3 and 4 together
6. I do not know.

56. Choose the statement which is **true** about the human reproductive cycle.

1. The mother cells from which gametes are produced are diploid.
2. Meiosis is important because it is a source of genetic variation.
3. The male reproductive organs are called testes.
4. All the above statements are true
5. All the above statements are false
6. I do not know

57. Which of the following features of cell division are very different for animal and plant cells?

1. Prophase
2. Metaphase
3. Anaphase
4. Telophase
5. Cytokinesis
6. I do not know

58. During … synapsis occurs in the phase called …

1. mitosis, telophase
2. meiosis, prophase
3. mitosis, anaphase
4. meiosis, metaphase
5. None of the above.
6. I do not know.

59. Choose the correct statement…

1. The cell plate is formed in animal cells during cytokinesis.
2. Chromosomes align themselves along the equatorial plane during anaphase.
3. Centrioles are present in both animal and plant cells.
4. DNA replicates during the M-phase.
5. None of the above.
6. I do not know.

60. Choose the genetic term which matches with the corresponding description.

	Genetic term	**Description**
1	Haploid	Any chromosome other than the sex chromosome
2	Meiosis	Reduction division
3	Cytokinesis	Number of chromosomes in the gamete
4	Autosomes	Site on the chromosome to which spindle fibres attach
5	Centromere	Division of the cytoplasm
6	I do not know	

61. … most closely resembles events of mitosis except that the cells are…

1. Interphase, diploid
2. Meiosis II, haploid
3. Interphase, haploid
4. Prophase, haploid
5. Meiosis II, diploid
6. I do not know

62. During interphase of the cell cycle, chromosomes are…

1. not condensed, genetically active and visible
2. condensed, not genetically active and not visible
3. not condensed, genetically active and are not visible
4. condensed, genetically active and visible
5. not condensed, not genetically active and not visible
6. I do not know.

63. The centromere divides during…

1. mitosis and meiosis II
2. mitosis and meiosis I
3. interphase of mitosis and meiosis
4. telophase of mitosis and meiosis
5. prophase of mitosis and meiosis
6. I do not know.

64. Which of the following is unique to mitosis and not a part of meiosis?

1. Homologous chromosomes pair forming bivalents
2. Homologous chromosomes cross over
3. Chromatids are separated during anaphase
4. Homologous chromosomes synapse
5. Chromosomes exchange segments
6. I do not know.

65. During what phase of meiosis do sister chromatids separate?

1. Prophase I
2. Telophase I
3. Prophase II
4. Anaphase II
5. Telophase II
6. I do not know

66. Choose the statement which is true.

1. Sister chromatids separate from each other during the first meiotic anaphase.
2. Homologous chromosomes separate from each other during first meiotic anaphase.
3. Crossing over takes place between sister chromatids during prophase I.
4. Crossing over takes place between non-sister chromatids during prophase II.
5. All the above statements are false.
6. I do not know.

67. Mendel's use of garden peas was a good choice for genetic studies because...

1. They produce many offspring
2. They are easy to grow
3. They are capable of self-pollination
4. They produce sexually
5. All of the above are correct
6. I do not know

68. Which one of the following statements is **true** about Mendel's experiments on garden peas?

1. Mendel was able to observe both the genotypic and phenotypic ratios.
2. Mendel was only able to observe the genotypic ratios.
3. Mendel was only able to observe the phenotypic ratios.
4. All of the above statements are true.
5. None of the above statements is true.
6. I do not know.

69. Two tall pea plants were crossed producing 105 tall plants and 32 short plants. The genotypes of the tall parent plants are ...

1. TT and Tt
2. Tt and tt
3. tt and tt
4. TT and TT
5. Tt and Tt
6. I do not know

70. A test cross was carried out to determine whether a parent fruitfly was either homozygous (WW) or heterozygous (Ww) for long wings. Fifty percent of the offspring had long wings and the other 50% had short wings. Which of the following is the test parent's genotype?

1. WW
2. ww
3. WwW
4. Ww
5. www
6. I do not know

71. How many types of gametes can be produced by an individual with three genes and each gene having two alleles?

1. 9
2. 6
3. 8
4. 27
5. 3
6. I do not know

72.Which of the following gametes represent(s) independently assorting genes?

1. ABCD
2. abcd
3. AbcD
4. 1 and 2 are correct
5. 2 and 3 are correct
6. I do not know

73. A boy has brown hair and blue eyes, while his brother has black hair and blue eyes. The different combinations of traits in the two is best explained by the concept known as...

1. gene interaction
2. multiple alleles
3. complete linkage
4. co dominance
5. independent assortment
6. I do not know

74. Codominant alleles are pairs of alleles that both affect the phenotype when present in a …

1. dominant homozygote
2. codominant heterozygote
3. recessive heterozygote
4. codominant homozygote
5. dominant heterozygote
6. I do not know.

75. Choose the statement which explains why some genes are always inherited together.

1. Mutations keep them together on the chromosome.
2. They are very close to each other on the chromosome.
3. They are not linked.
4. They are far apart on the chromosome.
5. They assort independently.
6. I do not know.

76. Genetic variability …

1. is produced through reshuffling of genes on chromosomes in meiosis
2. is typical of organisms produced by sexual reproduction
3. helps populations of organisms to survive environmental changes
4. is increased by crossing over
5. All of the above are correct
6. I do not know

77. Chromosome(s) … and contain genetic information.

1. is another name for alleles
2. are made of spindle fibres and proteins
3. are made of DNA and proteins
4. are made of DNA and RNA
5. are doubled in daughter cells after mitosis
6. I do not know

78. A phenotypic ratio of 9:3:3:1 in the offspring of a mating of two organisms heterozygous for two traits is expected when…

1. the genes reside on the same chromosome
2. each gene contains two mutations
3. the gene pairs assort independently during meiosis
4. only recessive traits express themselves
5. only dominant phenotypes appear in the offspring
6. I do not know

79. A student crossed wrinkled-seeded (rr) pea plants with round seeded (RR) pea plants. Only round seeds were produced by the resulting plants. This illustrates the principle of...

1. independent assortment.
2. lethal alleles .
3. dominance.
4. incomplete dominance.
5. codominance.
6. I do not know.

80. Albinism is a condition in which the affected individual lacks skin and eye pigmentation due to a recessive allele. If an albino male is married to a heterozygous normal female, the children produced will be...

1.100% normal homozygous.
2.100% normal heterozygous.
3.50% albino: 50% heterozygous normal.
4. 50% normal homozygous: 50% heterozygous normal.
5. None of the above.
6. I do not know.

81. Given that black hairs of guinea pigs is due to a dominant allele *B*; white is due to the alternative recessive allele *b*. Several black guinea pigs were crossed among themselves and they produced 52 black and 48 white offspring. Predict the genotypes of the parents.

1. BB x BB
2. BB x Bb
3. Bb x bb
4. Bb x Bb
5. 1 and 2 together
6. I do not know

82. Mendel's second law states that...

1. Alleles of different genes assort independently.
2. Independently assorting alleles occur in unequal numbers.
3. There is linkage among different alleles.
4. Alleles of each gene segregate from each other in equal numbers.
5. 1 and 4 are both correct.
6. I do not know.

83. Which one of the following crosses would give the same F_2 phenotypic ratio as the cross AABB x aabb?

1. AaBb x AABB
2. AaBb x AaBb
3. AABB x AABB
4. AaBb x AAbb
5. AAbb x aaBB
6. I do not know

84. In a cross between two individuals differing in two genes, predict the number of phenotypes and genotypes that will result in the F_2 generation due to recombination in the gametes of the F_1.

1. 3 genotypes and 6 phenotypes
2. 4 genotypes and 4 phenotypes
3. 27 genotypes and 8 phenotype
4. 4 phenotypes and 9 genotypes
5. None of the above
6. I do not know

85. In tomatoes, red fruit (R) is dominant over yellow fruit (r) and tallness (T) is dominant over dwarfness (t). A plant that is Rr TT is crossed with a plant that is rrtt. What are the chances of an offspring being homozygous for both traits.

1. 0.
2. 0.15.
3. 0.25.
4. 0.50.
5. 0.75.
6. I don't know.

86. A cross was carried out between two plants; YyRr (yellow, round seeds) x yyrr (green, wrinkled seeds). Out of 120 progeny, the following results were obtained:

Yellow round	31
Yellow wrinkled	29
Green round	32
Green wrinkled	28

A chi-squared test was then carried out to test the results of the progeny using Table 1

Table 1. Tabulated (critical) chi-squared values for up to four degrees of freedom

		Probability							
		0.01	**0.05**	**0.10**	**0.20**	**0.5**	**0.7**	0.8	0.9
Degrees of freedom	**1**	6.6	3.8	2.7	1.6	0.5	0.2	0.06	0.02
	2	9.2	6.0	4.6	3.2	1.4	0.7	0.5	0.2
	3	16.3	7.8	6.3	4.6	2.4	1.4	1.0	0.6
	4	13.3	9.5	9.2	7.3	4.4	3.0	2.3	1.6

The calculated chi-squared value is...

1. 0.165
2. 0.333.
3. 5%.
4. 95%.
5. 0.666.
6. I do not know.

87. The tabulated (critical) chi-squared value in Question 86 is...

1. 9.5.
2. 3.8.
3. 7.8.
4. 6.0.
5. 2.4.
6. I do not know.

88. The results of the test in Question 86 suggest that...

1. The gene for seed colour and the gene for seed shape are linked.
2. The two genes are on the same chromosome.
3. The gene for seed colour and the gene for seed shape are not linked.
4. Seed colour and seed shapes are controlled by multiple alleles.
5. Seed colour and seed shapes are controlled by polygenes.
6. I do not know.

89. Mendel did not deal with ...

1. multiple alleles
2. independent assortment.
3. polygenic inheritance.
4. segregation.
5. Both 1 and 3 are correct.
6. I don’t know.

90. Which alleles are responsible for the AB blood type?

1. Dominant alleles.
2. Recessive alleles.
3. Incompletely dominant alleles.
4. Lethal alleles.
5. Codominant alleles.
6. I do not know.

91. A child was found to have blood group A. Determine all possible genotypes of the mother if the father was blood group O.

1. I^AI^A and I^AI^O
2. I^0I^0
3. I^0I^A and I^0I^B
4. I^0I^0, I^0I^A, I^0I^B
5. I^0I^B
6. I do not know

92. An organism with a diploid number of ten chromosomes, would contain… tetrads during the prophase of meiosis I.

1. 4
2. 5
3. 10
4. 20
5. 40
6. I do not know

93. A couple whose blood groups were AB and A respectively had the children together. Which of the following children was not a product of their marriage?

1. The child with blood group O
2. The child with blood group A
3. The child with blood group AB
4. The child with blood group B
5. All the children were products of their marriage
6. I do not know.

94. A broad range of phenotypes for a given gene is usually the result of…

1. incomplete dominance
2. multiple allelic inheritance
3. codominance
4. lethal autosomal dominance
5. polygenic inheritance
6. I do not know

95. Consider the following cross CcDd x Ccdd, where alelle C is dominant to c and D is dominant to d. What phenotypic ratio would you expect from this cross?

1. 3:1:3:1
2. 9:3:3:1
3. 9:3:3:0
4. 1:2:2:1
5. 3:1:2:1
6. I do not know

96. Epistasis is...

1. an interaction between two genes.
2. an expression of co-dominance between two genes.
3. the fatal effect of a recessive allele.
4. the same as dominance.
5. the same as incomplete dominance
6. I do not know

97. A cross between two animals produced 37 yellow offspring and 19 grey ones. The experimenter expected a ratio of 3:1 yellow offspring to grey ones. Determine the type of phenomenon in genetics that is responsible for this kind of result.

1. Action of lethal alleles
2. Codominant alleles are involved
3. A cross between a homozygous recessive and a heterozygtes.
4. A cross between two homozygous dominant parents
5. Such an occurrence does not happen in genetics
6. I do not know

98. Red colour in wheat is produced by the genotype R_B_ , white by the double recessive genotype rrbb. The genotypes R_ bb and rrB_ produce brown kernels. A homozygous red variety is crossed to a white variety. What phenotypic results are expected in the F_2?

1. 13 red: 3 brown
2. 9 red: 4 brown: 1 white
3. 12 red: 3 brown: 1 white
4. 9 red: 6 brown: 1 white
5. 9 red: 3 brown: 4 white
6. I do not know.

99. In a certain variety of cats, when the allele C is present, the animals are pigmented; cc individuals are albino. Another pair of alleles, B and b determines the difference between black coat colour (B_) and brown coat colour (bb) in pigmented animals. When a black cat was crossed with an albino, the ratios of the progeny were: 1 black: 1 brown: 2 albino. The probable genotypes of the parents were...

1. CCBB x ccbb
2. CcBb x ccbb
3. CcBb x ccBb
4. CCBB x ccBb
5. CcBb x ccBB
6. I do not know

100. A white flowered plant was crossed with a red flowered plant. If all the offspring were pink flowered, what phenomenon does this show?

1. Complete dominance
2. Co dominance
3. Incomplete dominance
4. Epistasis
5. Recessiveness.
6. I do not know

Table of the genetic code

		Second Letter					
		U	C	A	G		
First Letter	U	UUU UUC } Phe UUA UUG } Leu	UCU UCC UCA UCG } Ser	UAU UAC } Tyr UAA UAG } Nonsense	UGU UGC } Cys UGA Nonsense UGG Trp	U C A G	Third Letter
	C	CUU CUC CUA CUG } Leu	CCU CCC CCA CCG } Pro	CAU CAC } His CAA CAG } Gln	CGU CGC CGA CGG } Arg	U C A G	
	A	AUU AUC AUA } Ile AUG Met	ACU ACC ACA ACG } Thr	AAU AAC } Asn AAA AAG } Lys	AGU AGC } Ser AGA AGG } Arg	U C A G	
	G	GUU GUC GUA GUG } Val	GCU GCC GCA GCG } Ala	GAU GAC } Asp GAA GAG } Glu	GGU GGC GGA GGG } Gly	U C A G	

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

BIO 2032: BASIC MICROBIOLOGY
THEORY PAPER

TIME: THREE HOURS

INSTRUCTIONS: ANSWER **FIVE** QUESTIONS. ANSWER THREE QUESTIONS FROM SECTION A AND TWO FROM SECTION B. USE SEPARATE ANSWER BOOKLETS FOR EACH SECTION. USE ILLUSTRATIONS WHERE NECESSARY

SECTION A

1. (a) Distinguish cell wall structure and chemistry in Gram positive and Gram negative bacteria.
   (b) Describe the Gram staining technique for classifying bacteria as Gram positive or Gram negative.

2. (a) Describe how bacteria obtain new genetic information.
   (b) Compare and contrast conjugation and transformation in bacteria.

3. (a) Distinguish between glycocalyx and slime layer in prokaryotes.
   (b) Describe the structure and functions of glycocalyx and slime layer in bacteria.

4. (a) Describe virion morphology.
   (b) Discuss virus infection and assembly with reference to plant viruses.

5. Discuss the physiology of plants infected by viruses.

SECTION B

6. Describe the classification of viruses based on genome structure and nucleic acid type and state an example of a virus belonging to each class whose genome you have described.

TURN OVER

7. Summarise each of the following:
   (a) Antiviral chemotherapy.
   (b) Cap-snatching.
   (c) Virus attachment to host cells.
   (d) Reasons why the development of a viable vaccine against the influenza virus has not been very successful.

8. Explain four different reasons that can be attributed to virus genome structure and three reasons attributed to virus biology why viruses are able to thrive in nature in spite of their relatively small genome sizes and dependence on hosts.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

BIO 2062: DIVERSITY OF ANIMALS
THEORY PAPER

TIME: THREE HOURS

INSTRUCTIONS: ANSWER **FIVE** QUESTIONS. **TWO** QUESTIONS FROM EACH SECTION AND THE **FIFTH** QUESTION FROM EITHER SECTION. USE ILLUSTRATIONS WHERE NECESSARY. USE **SEPARATE ANSWER BOOKLETS** FOR EACH SECTION.

SECTION A: Invertebrates

1. Describe the similarities and differences in morphological features of members of the Class Oligochaeta and Class Hirudinea.

2. Describe with examples the characteristic features of Phylum Mollusca.

3. Describe the similarities and differences in the body forms of members of the Phylum Porifera with reference to feeding.

4. Describe the features responsible for the success and diversity of the Subphylum Mandibulata.

SECTION B: Vertebrates

5. Describe the distinctive characteristics that set chordates apart from all other phyla.

6. Describe the features and characteristics of the members of Dipnoi.

7. Discuss the main adaptive features that have made the Aves one of the most successful vertebrate groups.

8. Describe the similarities and differences in reproduction in the mammals and reptiles.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

BIO 3182 GENETICS

THEORY PAPER

TIME: THREE HOURS

INSTRUCTIONS: ANSWER FIVE QUESTIONS, TWO FROM SECTION A, TWO FROM SECTION B AND THE FIFTH QUESTION FROM EITHER SECTION. USE DIAGRAMS AND OTHER ILLUSTRATIONS WHEREVER POSSIBLE.

SECTION A

1. (a) Explain the important landmarks used in the identification of chromosomes and in the diagnosis of chromosomal structural abnormalities.
   (b) Explain one positive effect of duplication within a genome using a simple example.
   (c) Describe how a diploid plant can be produced from a pollen grain.

2. (a) Using appropriate symbols, predict the response to selection in a breeding experiment based on a quantitative trait.
   (b) List the factors which affect the response to selection in a breeding experiment based on a quantitative trait and briefly explain the effect of each factor.

3. (a) Explain three strategies employed by organisms to promote out crossing.
   (b) Explain four factors that promote inbreeding in named organisms.
   (c) Using appropriate symbols, derive the equation to estimate the allele migration rate *m* from one population into another population

4. Summarise the experiments on genetic exchange in bacteria carried out by each of the following scientists:

   (a) Frederick Griffiths (1928)

   (b) Joshua Lederberg and Edward Tatum (1946)

   (c) Joshua Lederberg and Norton Zinder (1951)

TURN OVER

SECTION B

5. (a) Design an experiment to map genes in a bacterial strain based on conjugation.

(b) Design an experiment to demonstrate genetic exchange in bacteriophages.

6. Estimation of variances in body mass in a pig population gave results as shown in Table 1.

Table 1. Variances in body mass in pigs

Parameter	**Value**
Total genetic variance	117
Variance due to dominance effects	27
Variance due to epistatic effects	13
Total environmental effects	211

(a) Calculate the narrow- and broad-sense heritabilities from these variance estimates.

(b) Describe how the raw data above could have been obtained.

7. A plant was found to be heterozygous for an inversion with breakpoints in one chromosome as shown in Figure 1.

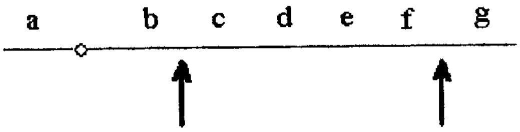


Figure 1. First inversion in a plant chromosome

PROCEED TO NEXT PAGE

A heterozygous descendant of this plant underwent a further inversion within the limits of the original one with breakpoints shown in Figure 2.

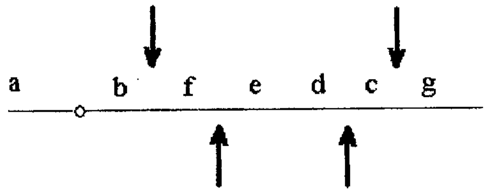


Figure 2. Second inversion in a plant chromosome

(a) Explain the type of inversion in both Figures 1 and 2.

(b) Show by means of diagrams the pachytene pairing configurations you would expect to see in double inversion heterozygotes (Figure 2).

(c) Work out the possible types of gametes from the plant in Figure 1.

8. (a) Three genes of maize (*Zea mays*) X, Y and Z lie on chromosome 9. The map of the three genes using map units is:

X-----------10-----------Y---------20-----------Z

If these results are based on 200 progeny of a trihybrid testcross of the plant (XxYyZz) arising from true breeding XXYYZZ and xxyyzz parents;

(i) Estimate the number of X_yyZ_ plants that would be expected if there was no interference.

(ii) If the coincidence was 0.3, estimate the number of the plant type in (i) above.

(b) A chromosome segment ABC has map distances *A-B* = 10 and *B-C* = 20. In addition, 1.6% double crossovers are observed in a testcross experiment.

Calculate the number of individuals expected to have double crossovers if 800 individuals were used in the study.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

BIO 4332: ECOLOGY AND MANAGEMENT OF FISHERIES
THEORY PAPER

TIME: THREE HOURS

INSTRUCTIONS: ANSWER **FIVE** QUESTIONS. ANSWER QUESTIONS **1, 2** AND **3** AND ANY OTHER **TWO** QUESTIONS. USE ILLUSTRATIONS WHERE NECESSARY

---

1. (a) Explain the rationale for the use of hard body parts such as: otoliths and scales, for determining growth curves of fish stocks.
   (b) Compare and contrast the Rosa Lee (1920) method and tagging methods as applied in determining fish growth curves.
   (c) Explain why the Petersen method is preferred for computing growth curves by fisheries biologist working in tropical fisheries.

2. Summarise the following as applied to the management of fish stocks:
   (a) Input controls.
   (b) Minimum mesh size.
   (c) Adaptive fisheries management.

3. Defend the view that control of fishing effort, as used in artisanal fisheries of developing countries, is a better method of managing fisheries in comparison to the catch quota system employed for the management of industrial fisheries in developed countries.

4. (a) Describe a method for fish stock size estimation using egg and larval surveys.
   (b) Justify the reluctance among fisheries biologists working in tropical fisheries to develop fish stock assessment methods based on egg and larval surveys.

5. (a) Describe two surplus production models commonly used for estimating the Maximum Sustainable Yield (MSY).
   (b) Describe the type of data needed for each model in (a) above.
   (c) Discuss assumptions that are made when applying surplus production models for estimating Maximum Sustainable Yield.

TURN OVER

6. (a) Discuss the rationale for using the age-based catch curves for estimating total mortality coefficient of fish stocks.
   (b) Summarise the reasons for not using data of young age groups, in age-based catch curves, when estimating total mortality coefficients.

7. (a) Describe two models commonly used for demonstrating relationships between parent stock sizes and number of recruits in fish stocks.
   (b) Summarise characteristics of fish stocks associated with each model.

8. (a) Describe the advantages and disadvantages of fisheries management objectives that aim at exploiting fish stocks at the Maximum Economic Yield (MEY).
   (b) Summarise the advantages of a fisheries management objective that aims at exploiting fish stocks at the economic break-even.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

C102: INTRODUCTION TO CHEMISTRY II

TIME: THREE (3) HOURS

INSTRUCTIONS TO CANDIDATES:

1. Indicate your **student ID number** (computer number) and **TG number** on **ALL** your answer booklets.

2. This examination paper consists of two **(2)** sections: **A** and **B**

3. Section **A** has ten **(10)** short answer questions (Total marks = **40**).

   ANSWER ALL QUESTIONS. Questions carry equal marks.

4. Section **B** has five **(5)** long answer questions. (Total marks = **60**). **ANSWER QUESTION B1 and ANY THREE QUESTIONS, EACH IN A SEPARATE ANSWER BOOKLET.** Questions carry equal marks.

5. **YOU ARE REMINDED OF THE NEED TO ORGANISE AND PRESENT YOUR WORKING CLEARLY AND LOGICALLY.**

Information to Candidates:

1. Useful data is on page 7
2. Periodic Table of Elements is printed on the last page of this question paper.

SECTION A ANSWER ALL QUESTIONS

Question A1.

Calculate K_p for the reaction at 700 K

$$CO(g) + 2H_2(g) \rightleftharpoons CH_3OH(g) \qquad \Delta G^o_{700} = - 13.5 \text{ kJ/mol}$$

Question A2.

Phosphine decomposes as shown below.

$$4PH_3(g) \rightarrow P_4(g) + 6H_2(g)$$

This reaction takes place at 100° C and the rate constant is 0.372 min^{-1}.What is the rate constant at 70° C if the activation energy is 73.2 kJ mol^{-1}?

Question A3.

Calculate the solubility of CaF_2 in a solution containing 0.0100 M $Ca(NO_3)_2$ at 298.15 K. The K_{sp} of CaF_2 is 3.9×10^{-11}at 298.15 K.

Question A4.

A basic buffer containing 0.1000 NH_3 solution and 0.100 M NH_4Cl solution was prepared. K_b of ammonia is1.8×10^{-5} at 25 °C.

(a) Give the Henderson-Hasselbalch equation.

(b) Determine the pOH of the buffer solution.

Question A5.

What types of intermolecular forces are present in the compound H_2S? Briefly explain in two lines..

Question A6.

A solution was made up by dissolving 3.75 g of a pure hydrocarbon in 95.0 g of acetone. The boiling point of pure acetone is 55.95 °C and that of solution was 56.50 °C. If the molal boiling point constant of acetone is 1.71°C.kg/mol, what is the molecular mass of the hydrocarbon?

Question A7.

A solution containing 250.0 mL of water and 48.0 g of sucrose,(molar mass = 342.34 g mol^{-1}) was prepared at 300 K. Pure water was separated from the solution by means of semi-permeable membrane. What pressure must be applied above the solution in order to just prevent osmosis?

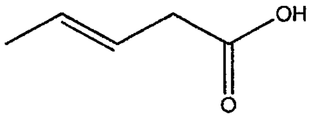
Question A8.

(a) Write line formula for ethyl 2-aminopropanoate.

(b) What are the classifications of the carbons in 2-butene?

Question A9.

(a) Name the following compound by IUPAC rules.

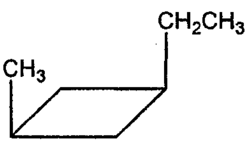


(b) Give skeletal formula for m-Dinitrobenzene.

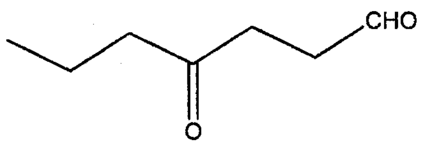
Question A10

Give the IUPAC name the following compounds

(i)



(ii)



SECTION B **ANSWER B1; AND OTHER THREE QUESTIONS**

Question B1.

In Laboratory Experiment 5 you determined the acid dissociation constant, K_a, of a weak dibasic acid.

(a) One of the students wanted to determine the pK_a value of the **weak monobasic acid.** He transferred 25.00 cm^3 of 0.1000 M solution of the acid into a conical flask and titrated against standard 0.09980 M sodium hydroxide solution using an indicator. The pH of the solution when **half of the acid** is neutralized was 5.9.

(i) Calculate mass of the acid required to prepare 100.00 $cm^3$of 0.100 M solution of this acid. (Molar mass of the acid = 150 g mol^{-1}) **(3 marks)**

(ii) Calculate the volume of sodium hydroxide required for complete neutralization. **(5 marks)**

(iii) What is the pK_a of the acid? Calculate K_a of this acid? **(4 marks)**

(b) Another student used the procedure given in Laboratory Manual to measure the volume required for neutralization of 25.0 cm^3 of acid by 0.100 M sodium hydroxide using an indicator.

The pH range for the two indicators are given below.

Indicator	**pH range for the colour change**
Methyl red	4.4 – 6.0
phenolphthalien	8.4 – 10.0

Justify, using the information given in the table above and part (a) why phenolphthalein is a suitable indicator for this titration. **(3 marks)**

Question B2

Hydrogen sulfide, a colorless gas with a foul odor, diṣsociates on heating: as shown below.

$$2H_2S(g) \rightleftharpoons 2H_2(g) + S_2(g)$$

When H_2S at an initial pressure of 1.168 bar was put into a 10.00 Lvessel and heated to 1405.15 K, it gave an equilibrium mixture containing H_2 of 0.3330 bar partial pressure.

(a) Make an ICE Table of the reaction, using pressure units, based on the data above **(5 marks)**

(b) Calculate the value of K_p at that temperature. **(5 marks)**

(c) Determine the value of K_c at that temperature **(5 marks)**

Question B3

(a) Identify and describe the intermolecular forces between solution components in an ` aqueous sodium chloride solution. (4 marks)

(b) The compounds carbon tetrachloride, CCl_4, and carbon tetraiodide, CI_4, have similar intermolecular forces. One of the compounds is a liquid and the other solid under standard conditions.

(i) What types of intermolecular forces are present? (3 marks)

(ii) Identify the liquid and solid compound justifying your answer. (3marks)

(c) When 30.0 g of non volatile solute having the empirical formula CH_2O is dissolved in 800.0 g of water, the solution freezes at -1.16 °C. What is the molecular formula of the solute? K_f of water is 1.86°C.kg/mol. (5 marks)

Question B4

(a) Natural gas, a widely used energy resource, is a combustible mixture of hydrocarbons, including largely methane and a small proportion of propane. Give a balanced equation for complete combustion of propane in excess oxygen. **(3 marks)**

(b) Give two differences between σ and π bonds. **(2 marks)**

(c) Several constitutional isomers can be written for the molecular formula $C_5H_{10}O$. Give line formulas for the isomers with the following descriptions and name them.

(i) A cyclic secondary alcohol with **no** primary carbon. **(2 marks)**

(ii) A ketone with one methyl group as a substituent **(2 marks)**

(d) *X-ray diffraction studies show that all six carbon-carbon bonds in benzene are of* equal length, 139 pm.

(i) Write a Kekule structure for benzene. **(1 mark)**

(ii) Assuming that benzene was made up of single and double bonds, what bond lengths would you expect for the carbon-carbon bonds in your Kekule structure? **(2 marks)**

(iii) Show resonance structures to account for the observed bond length (139 pm). **(3 marks)**

Useful Data, Bond lengths:
C-C single bond (sp^3-sp^3) = 154 pm; C=C double bond (sp^2-sp^2) = 133 pm

Question B5

(a) Calculate the IHD for $C_6H_{10}NO_2Br$ and state all the possible interpretations.**(4 marks)**

(b) There are a number of compounds with the molecular formula $C_6H_{10}NO_2Br$. Draw:

(i) a five- membered ring structure that is a carboxylic acid, **(2 marks)**
(ii) a six-membered ring that has a nitro group. **(2 marks)**

(c) Draw the molecular orbitals of ethene **(3 marks)**

(d) Draw the structures for: **(2 marks each)**

(i) Trans-3-bromocyclopentanecarbaldehyde

ii) Pentanamide

================END OF EXAMINATION ===================

USEFUL DATA

$N_A = 6.022 \times 10^{23}$ mol^{-1}

Universal Gas constant R

8.3145 J mol^{-1} K^{-1}
0.083145 L bar mol^{-1} K^{-1}
0.08206 L atm mol^{-1} K^{-1}
62.364 L torr mol^{-1} K^{-1}
62.364 L mmHg mol^{-1} K^{-1}

Pressure conversion

1 atm = 1.01325 x 10^5 Pa
= 1.01325 x 10^{5}N m^{-2}
= 760 torr
= 760 mmHg
= 1.01325 bar

1 bar = 1.00000 x 10^5 Pa
= 1.00000 x 10^5 N m^{-2}

Acceleration due to gravity
g = 9.80665 m s^{-2}

Density of water

ρ = 1.00 g cm^{-3} = 1.00 g (mL)$^{-1}$=1.00 x10^3 kg m^{-3}

Mass conversion

1L of water = 1 kg of water

1 kg = 1000 g

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER

FINAL EXAMINATION

C212: **INTRODUCTION TO BIOCHEMISTRY**

TIME: **THREE HOURS**

1. Answer any **five (5)** questions
2. There are **FOUR (4)** printed pages in this examination paper
3. Each question carries **20 marks**

QUESTION 1

A student is provided with 25 cm^3 of 0.1 M ethanoic acid and desires to carry out a titration experiment using 0.1 M sodium hydroxide. (pK_a ethanoic acid = 4.76).

a) **Write** a balanced chemical equation for the reaction between ethanoic acid and sodium hydroxide. **[4 marks]**

b) **Calculate**:

(i) the pH of the solution before addition of sodium hydroxide. **[8 marks]**

(ii) the pH of the solution after addition of 10 cm^3 of sodium hydroxide. **[8 marks]**

QUESTION 2

(a) Below is a table of some amino acids.

Amino acid	pK$_1$	pK$_2$	pK$_R$
Aspartic acid	2.10	9.80	3.86
Cysteine	2.05	10.25	
Glycine	2.35	9.78	
4-Hydroxyproline	1.82	9.65	
Leucine	2.33	9.74	
Lysine	2.20	9.00	10.50
Proline	2.00	10.60	

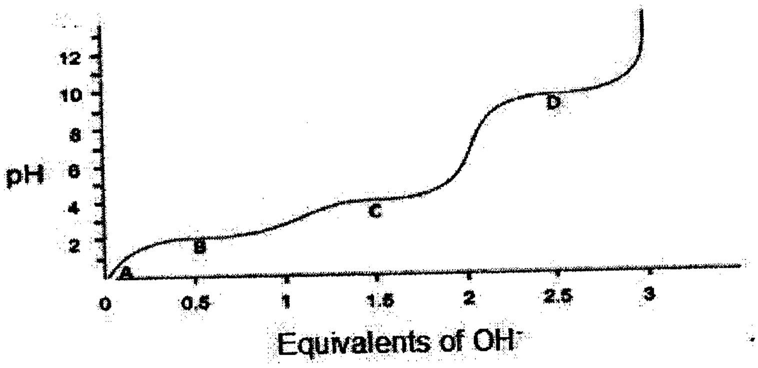
Which of these amino acids;

(i) will bind to a cation resin in ion exchange chromatography at pH 4.5
(ii) may not promote α-helix formation
(iii) may be involved in formation of disulphide bonds
(iv) is likely to be found in the inside of a globular protein
(v) not coded for by DNA **[10 marks]**

(b) A 60 amino acid-residue segment of a protein folds into a two-stranded antiparallel β structure of equal length with a 6 amino acid residue hairpin between them. Given that the distance between each amino acid is 3.5 Å, **calculate** the length of each antiparallel strand.

[5 marks]

(c) A 0.1 M solution of NaOH is titrated against 0.1 M solution of aspartic acid. Below is the titration curve for this reaction.



(i) **Draw** the species present at points A, B, C and D

(ii) **Calculate** the isoelectric point of aspartic acid. **[5 marks]**

QUESTION 3

(a) In **which** form is glucose stored in animal cells? Briefly **explain** why this form of storage is more beneficial to the cell than storage of actual glucose.

[2 marks]

(b) **Name** the test(s) you would use to distinguish:
- (i) a monosaccharide from a disaccharide
- (ii) an aldose from a ketose

[2 marks]

(c) **Define** the following terms in relation to carbohydrates:
- (i) anomers
- (ii) epimers
- (iii) enantiomers

[6 marks]

(d) An unknown disaccharide was purified from bacteria. Equal amounts of D-glucopyranose and D-galactopyranose were obtained after acid hydrolysis of the disaccharide. The sugars were found to be linked by an α-glycosidic linkage. Exhaustive methylation of the disaccharide produced equal amounts of 2,3,4,6-tetramethylgalactopyranose and 2,4,6-trimethyglucopyranose. **Draw** the structure of the disaccharide. **[10 marks]**

QUESTION 4

(a) Decolourisation of Bromine solution can be used as a simple test for unsaturation in lipids. If two atoms of bromine are required to fully saturate one double bond, **calculate** the number of atoms of bromine that will be required to fully saturate the following lipids;
- (i) 1 molecule of Linolenate
- (ii) 2 molecules of Trioleate

[10 marks]

(b) A synthetic compound (X) was completely hydrolysed and subjected to various techniques. Mass spectrometry and chromatography revealed the presence of glycerol, two fatty acids of the same type (cis 16:Δ^9) and a phosphate group. Given that this compound was optically active, **draw** the structure of compound X.
[10 marks]

QUESTION 5

(a) **Calculate** the free energy needed to move 1 mole of sodium ions from the plasma where $[Na^+]$ = 150 mM, into the red blood cell where $[Na^+]$ = 5 mM assuming a membrane potential of -70 mV at 37^0C (R = 1.98 $cal.mol^{-1}.K^{-1}$ and F = 23062 $cal.V^{-1}.mol^{-1}$). **[12 marks]**

(b) If this process is 41% efficient, **how many moles** of ATP can be synthesized when coupled to such an influx of ions? (Hydrolysis of 1 mole of ATP releases 7.3 kilo calories of energy). **[8 marks]**

QUESTION 6

(a) Briefly **explain** the following terms:

(i) transition state energy
(ii) cofactor
(iii) thermo labile

[6 marks]

(b) For an enzyme that follows Michaelis-Menten kinetics, **calculate** the ratio of [S] to K_m when velocity is 80 % of V_{max}.

(c) Consider the following data:

[S] mM	v mmol/min	
	Without Y	With Y
3.0	4.58	3.66
5.0	6.40	5.12
7.0	7.72	6.18
9.0	8.72	6.98
11.0	9.50	7.60

Using the double reciprocal plot of this data, **determine** K_m, and V_{max} in presence and absence of Y.

[8 marks]

Does Y combine with E, ES or with both? Briefly **explain**.

[6 marks]

`END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

C252 – ORGANIC CHEMISTRY II

TIME: THREE (03) HOURS.

INSTRUCTIONS:

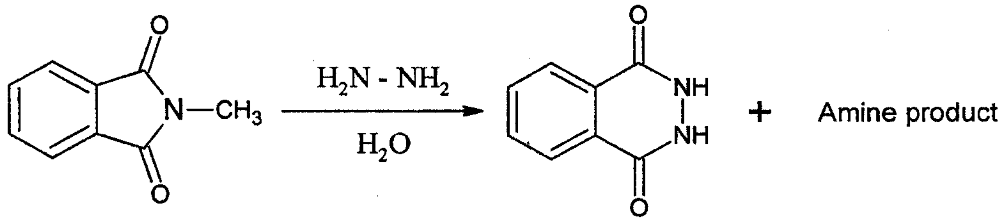
1. **This paper has six (6) questions. Answer four (04) questions.**
2. **Marks for each question are shown.** **Total marks 120.**

QUESTION ONE.

a) Propose a synthesis of ethylamine by the Gabriel method. Clearly show the starting material(s) and all reagents required. **The mechanism is not needed.**

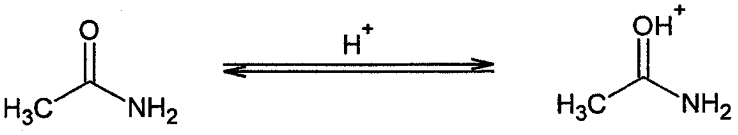
[6 marks]

b) An alternative to Gabriel procedure for amine synthesis is the reaction of N-alkyl phthalimide with hydrazine as shown below. Suggest a mechanism for this reaction and provide the structural formula and the common name of the amine product.



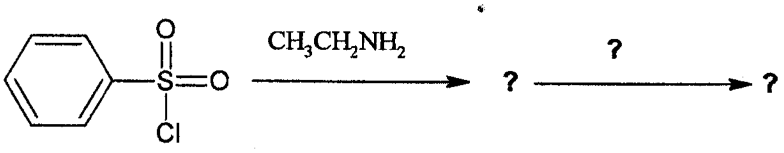
[7 marks]

c) (i) Protonation of acetamide occurs on oxygen rather than on nitrogen. Suggest a reason for this behaviour.



[3 marks]

(ii) Provide the missing products in the reaction below. What are the name and the laboratory application of this reaction?



[8 marks]

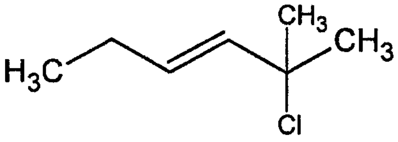
d) Draw a flow chart to show how n-decylamine, $CH_3(CH_2)_9NH_2$, contaminated with dodecane, $CH_3(CH_2)_{10}CH_3$, can be purified in a laboratory.

[6 marks]

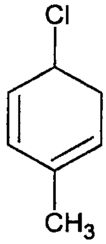
QUESTION TWO.

a) Give the constitutional isomers you would expect to obtain if the compounds shown below were hydrolysed (reacted with water) via an S_N1 mechanism.

i)



ii)



[5 marks]

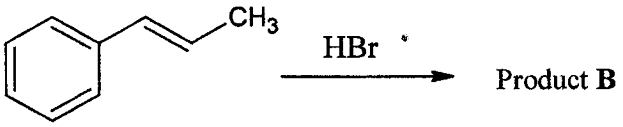
b) The table below shows the heats of hydrogenation of cyclic alkenes

Reactant	**Product**	ΔH^0 Hydrogenation ***(kcal/mol)***
Cyclohexene	Cyclohexane	28.6
1, 3- Cyclohexadiene	Cyclohexane	55.4
1, 3, 5-Cyclohexatriene ***(This is an hypothetical name)***	Cyclohexane	49.8

From the table, the energy of hydrogenation of the three double bonds of 1, 3, 5-cyclohexatriene is 49.8 kcal/mol and yet the energy of hydrogenation of one double bond is 28.6 kcal/mol. Explain fully the loss of the energy of hydrogenation of 1, 3, 5-cyclohexatriene. To what family of compounds does the 1, 3, 5-cyclohexatriene belong.

[6 marks]

c) The reaction below yields only one product, product **B.**



(i) Give the structure of product **B** and explain why other product(s) are not formed.

(ii) Suggest a mechanism for the reaction. **[7 marks]**

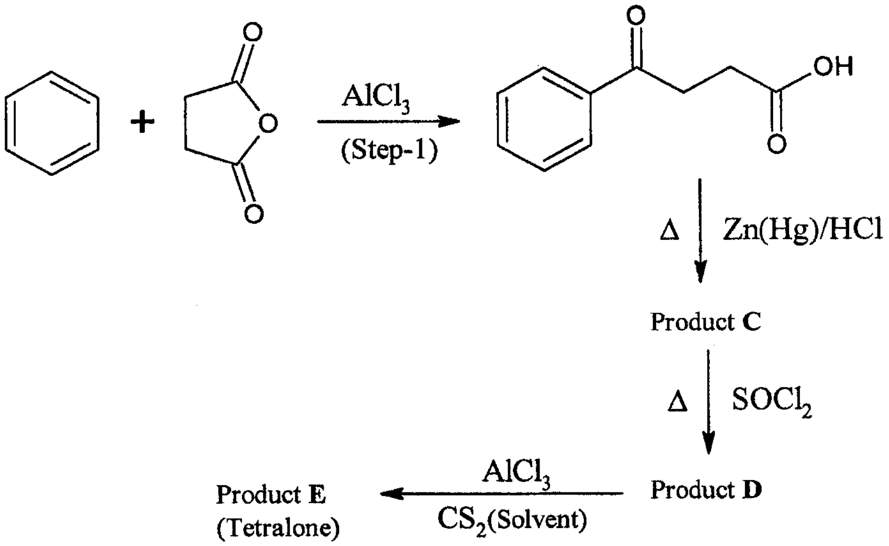
d) Experiments have shown that bromination of methoxybenzene (anisole) preferentially yields o- and p-isomers, while bromination of benzaldehyde yields a m-substituted compound. Rationalize these observations with reference to resonance forms of the intermediates for these reactions. **[12 marks]**

QUESTION THREE.

a) Experiments have shown that trimethylamine (bp 3^0C) boils lower than dimethylamine (bp 7^0C), even though trimethylamine has a higher molecular weight. Explain.

[2 marks]

b) Intramolecular Friedel-Craft's reactions are important in organic syntheses. In this connection provide the missing products and reagents for the following multi-step synthesis. Provide the mechanism for **step-1.**



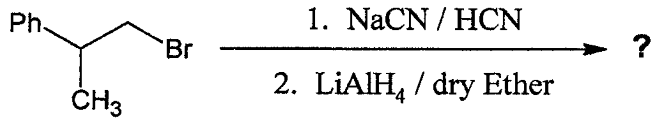
[10 marks]

c) Reactions of monosubstituted benzenes take place either in the ring or on the substituent (the side chain). Basing on this fact, when a mixture of toluene (methylbenzene) and bromine was treated with hydrogen peroxide a number of products were obtained. Show in detail all steps of the most likely mechanism of this reaction.

[12 marks]

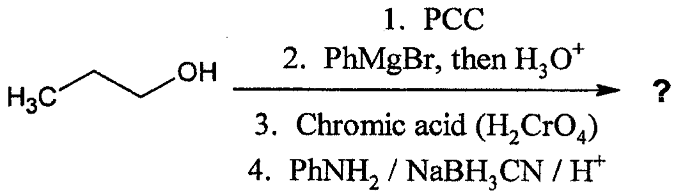
d) Predict the products of the following reactions. **Reaction mechanisms are not required.**

i)



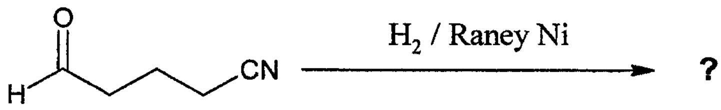
[1 mark]

ii)



[4 marks]

iii)

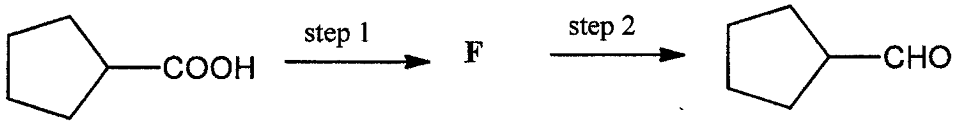


[1 mark]

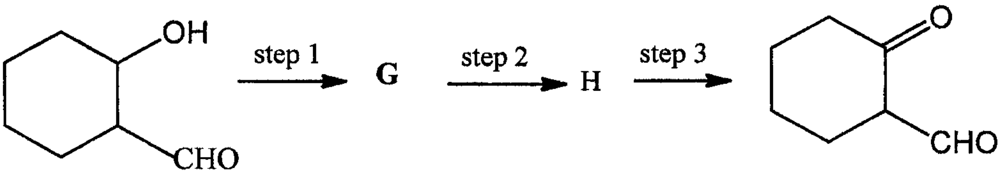
QUESTION FOUR.

a) How would you carry out the following transformations? Clearly show the reagents and intermediates for each step. **No reaction mechanisms please.**

(i)

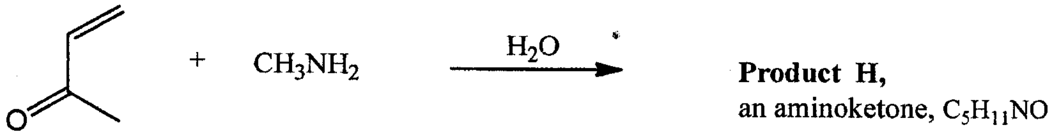


(ii)



[4 marks each]

b) For the reaction shown below:

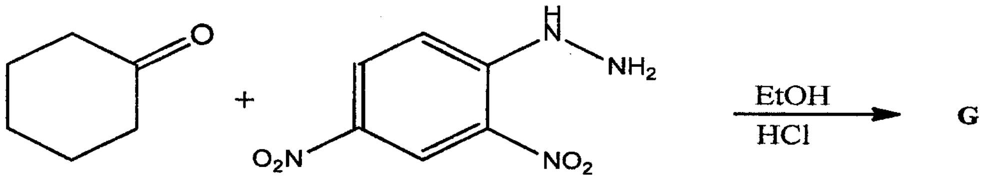


(i) Provide a plausible reaction mechanism. **[7 marks]**

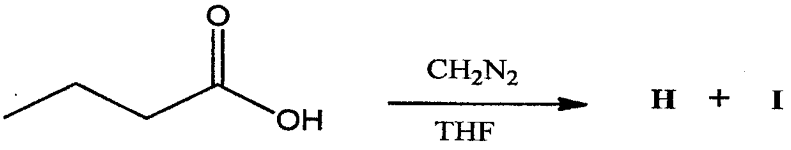
(ii) Name the type of reaction. **[1 mark]**

c) Provide the missing reagents, starting materials, and products for the following reactions:

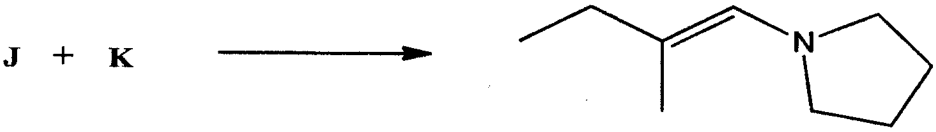
(i)



(ii)



(iii)

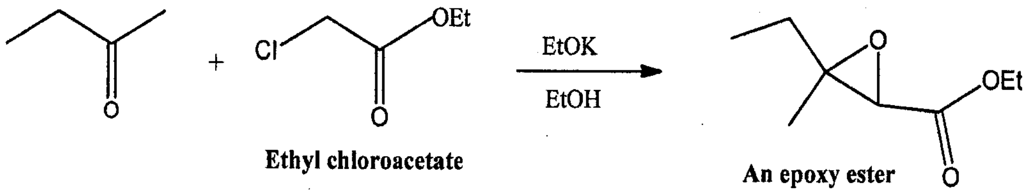


[2 marks each]

d) i) The α- hydrogens of which of the two compounds, propanone and ethyl 2-fluoro ethanoate (FCH_2COOEt), are more acidic and why?

[2 marks]

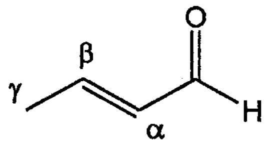
ii) The Darzens reaction, shown below, involves two step base-catalyzed condensation of ethyl chloroacetate with a ketone to give an epoxy ester. The first step is a nucleophilic addition to the ketone and the second step is an S_N2 reaction. Formulate the complete reaction mechanism.



[6 marks]

QUESTION FIVE.

a) The hydrogen atoms of the γ–carbon of crotonaldehyde are acidic. Give resonance structures to account for this observation.

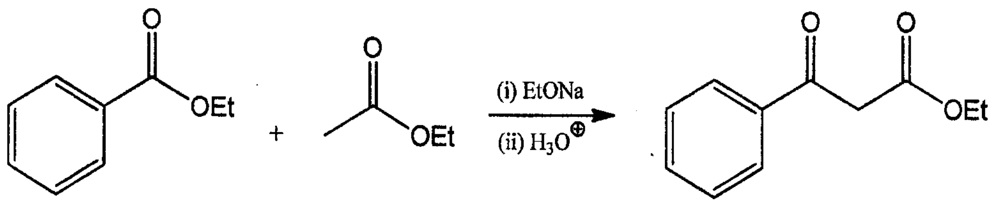


Crotonaldehyde

[4 marks]

b) Provide the mechanism for the following reaction:

(i)



[9 marks]

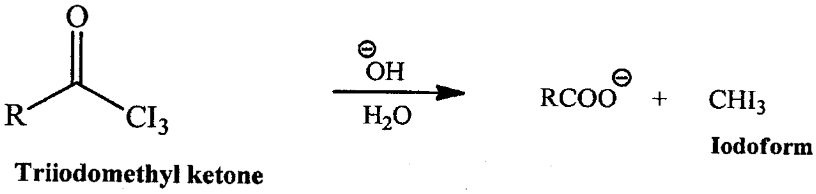
c) Pentanoic acid can be prepared from 1-bromobutane via the Grignard reagent. Alternatively, it can be obtained from 1-bromobutane via a nitrile. Clearly show all steps for the synthesis of pentanoic acid from 1-bromobutane by both methods. **Reaction mechanisms are not needed.**

[8 marks]

d) i) It is difficult to stop the base promoted halogenation of ketones at the monohalogen product. Briefly explain why.

[2 marks]

ii) In the iodoform reaction, a triiodomethylketone reacts with aqueous base to yield a carboxylate ion and a compound, iodoform. Suggest a mechanism for this step:



[5 marks]

iii) Which of the following compounds would give a positive iodoform test? Propanal, isopropanol, propanone and propanoic acid.

[2 marks]

QUESTION SIX.

a) Arrange the following compounds in order of increasing reactivity towards nucleophilic acyl substitution.

i) CH_3COOH ii) CH_3CONH_2 iii) CH_3COCl iv) CH_3COOCH_3

[4 marks]

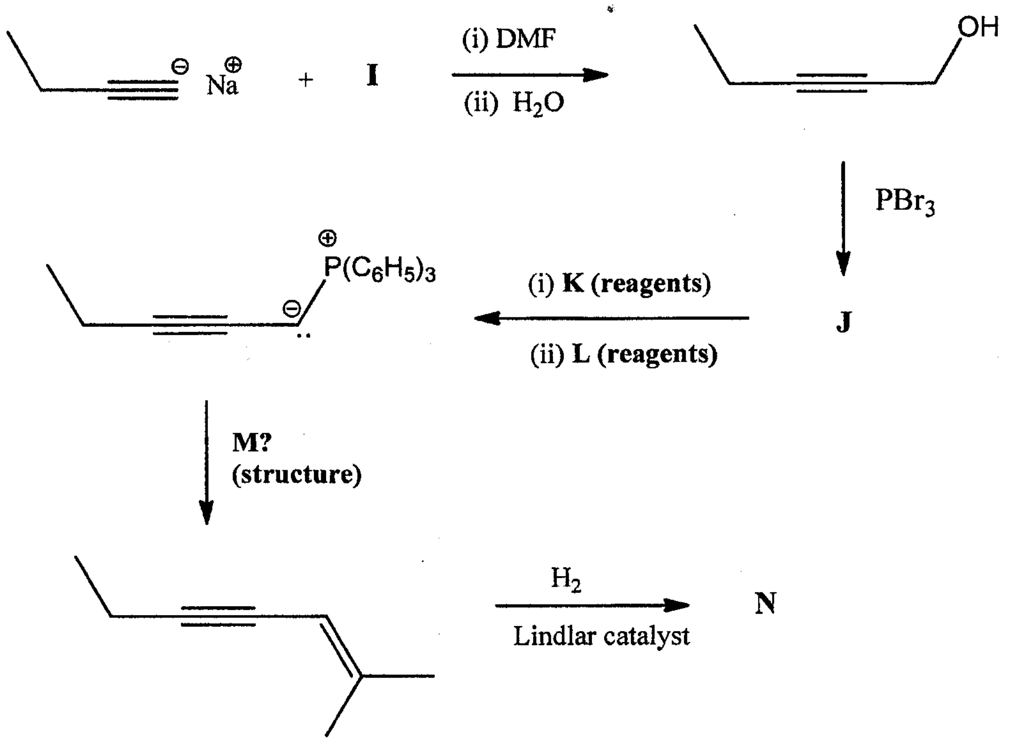
b) Clearly show synthetic routes to the following compounds:

i) From hexanoic acid to 1-aminopentane.

ii) From propanal to 2-hydroxybutanoic acid.

[4 marks each]

c) Provide the missing reactants, reagents, solvents, intermediates and the final product for the following synthetic sequence:

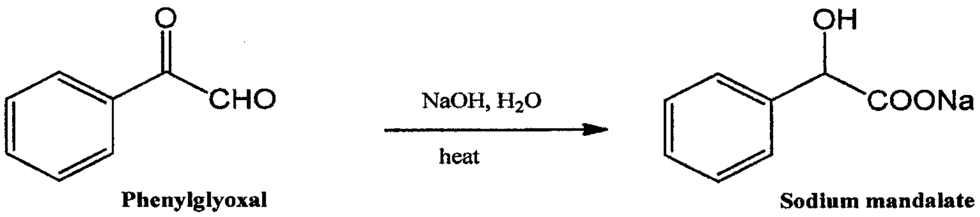


[10 marks]

d) i) Ketones are less reactive towards nucleophiles than aldehydes. Explain.

[2 marks]

ii) Phenylglyoxal is converted by aqueous sodium hydroxide into sodium mandalate as shown below. Suggest a likely mechanism for this reaction.



[6 marks]

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER EXAMINATIONS

C322 ANALYTICAL CHEMISTRY III

TIME: THREE HOURS

INSTRUCTIONS

1. There are five questions in this examination paper.
2. Answer any **four** questions.
3. Questions carry equal marks.

Question 1

(a) (i) Distinguish between concentration polarization and kinetic polarization.

(ii) What is Ohmic potential? How is it related to cell potential and applied potential.

(b) Describe three mechanisms responsible for the transport of dissolved species to and from an electrode surface

(c) A solution contains 1.0 g of iodine dissolved in 20.0 cm^3 of potassium iodide solution. If we shake this solution with 20.0 cm^3 of tetrachloromethane (an organic liquid), how much iodine will be transferred into the tetrachloromethane? For this system, $K_D = 85$

(d) A 250 mg hydrated sample of Na_2HPO_4 decreases to a mass of 145.7 mg after heating to 15 ° C. What is the number of water hydration in Na_2HPO_4.

Question 2

(a) A mixture of $CaCO_3$ and CaO is analysed using TGA technique. TG curve of the sample indicates that there is a mass change from 145.3 mg to 115.4 mg between 500–900 °C. Calculate the percentage of $CaCO_3$ in the sample.

(b) A C 322 student carried out a chromatographic analysis for substances A and B yielding the following data: retention times (A) = 16.4 minutes; (B) = 17.63 minutes, while an unretained species passed through the column in 1.3 minutes. The peak widths at base for A and B were 1.11 minutes and 1.21 minutes respectively. Calculate the column resolution, R_s, for the substances.

(c) A 9.14 mg sample of $R(NO_2)_3$, M.W. 229, was subjected to a controlled potential coulometric reduction in a strong acid solution and the coulometer registered 65.7 coulombs of electricity at the completion of electrolysis. Calculate the number of electrons involved in the reduction of $R(NO_2)_3$.

(d) (i) How can you remove oxygen from the polarographic cell and why?

(ii) Name two indication electrodes and two reference electrodes

Question 3

(a) During a chromatographic experiment, substance X registered a retention time of 13.5 minutes while an unretained species in the same sample was held for only 35 seconds. Calculate the actual the time substance X was held on the stationary phase.

(b) Voltammetry was used to determine the zinc content of a breakfast cereal. A 2.314g sample was digested in boiling concentrated nitric acid. After the sample dissolved, it was diluted to 100ml. A 5.00ml portion of this solution was analyzed by differential pulse polarography, giving a current of 2.31μA. When 50.0μL of 100ppm zinc standard was added to this solution, the current was 2.99μA. What is the concentration of zinc in the cereal?

(c) The diffusion current of lead in an unknown solution is 5.60μA. one milliliter of a 1.00×10^{-3}M lead solution is added to 10.0ml of the unknown solution and the diffusion current of the lead is increased to 12.2μA. what is the concentration of lead in the unknown solution?

(d) A calcium ISE is immersed in a food sample at 25°C and the potential is measured. Theoretically, what should happen to the signal if the sample were diluted exactly 20-fold?

Question 4

(a) Show how a metallic indicator electrode may be used to respond to the concentration of anions with which its cations form sparingly soluble precipitates. Show how the concentration of the analyte anion may be expressed in terms of the potentials present in the cell.

(b) The resistance of 0.1 M solution of a salt occupying a volume between two platinum electrodes 1.80 cm apart and 5.4 cm^2 in area was found to be 32 ohms. Calculate the molar conductivity of the solution.

(c) The resistance of a conductivity cell was 702 ohms when filled with 0.1 M KCl when filled with 0.1 M KCl solution ($K = 0.14807\ ohm^{-1}\ m^{-1}$) and 6920 ohm when filled with 0.01M acetic acid solution. Calculate the cell constant and molar conductance for the acid solution.

(d) (i) Explain what happens in a separation if the retention factor is equal to 1; and also if the retention factor is less than 1.

(ii) Ethanol and methanol are separated on a capillary GC column with retention times of 370s and 385s respectively and peak base widths of 16.0 s and 17.0 s while an tm for the same was equal to 10.0 s. Determine the resolution, $R_{s,}$ for the two compounds.

Question 5

(a) Ion selective electrode and reference electrode pair was placed in exactly 100ml of the sample; a reading of 21.6mV was obtained. After the addition of exactly 10ml of a standard solution with a concentration of 100µg/ml, the electrode pair reading gave a reading of 43.7mV. The response slope of the indicator electrode was previously determined to be 57.8mV. What is the sample concentration?

(b) (i) Nonpolar aromatic compounds were separated by HPLC using a bonded phase containing octadecyl groups [--$(CH_2)_{17}CH_3$] covalently attached to silica particles. The eluent was 65% (vol/vol) methanol in water. How would the retention times be affected if 90% methanol were used instead?

(ii) What types of species can be separated by high-performance liquid chromatography but not by gas-liquid chromatography

(c) (i) Using clearly labeled axes, draw diagrams to illustrate the following:

(1) Fronting peak

(2) Tailing peak

(3) Good; and poorly resolved pairs of peaks

(ii) Give two characteristics which differentiate partition and ion-exchange chromatography equilibration processes.

(d) Explain how you would determine the molar conductivity at infinite dilution for a strong and weak electrolyte in a food sample.

END OF EXAMINATION

PERIODIC TABLE OF THE ELEMENTS

KEY
Atomic number
X
Atomic mass
Name of the element
X

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 **H** 1.01 Hydrogen																1 **H** 1.01 Hydrogen	2 **He** 4.00 Helium
3 **Li** 6.94 Lithium	4 **Be** 9.01 Beryllium											5 **B** 10.81 Boron	6 **C** 12.01 Carbon	7 **N** 14.01 Nitrogen	8 **O** 16.00 Oxygen	9 **F** 19.00 Fluorine	10 **Ne** 20.18 Neon
11 **Na** 23.00 Sodium	12 **Mg** 24.31 magnesium											13 **Al** 27.99 Aluminium	14 **Si** 28.09 Silicon	15 **P** 30.99 Phosphorus	16 **S** 32.07 Sulphur	17 **Cl** 35.45 Chlorine	18 **Ar** 39.95 Argon
19 **K** 39.10 Potassium	20 **Ca** 40.08 Calcium	21 **Sc** 44.96 Scandium	22 **Ti** 47.88 Titanium	23 **V** 50.94 Variadium	24 **Cr** 52.00 Chromium	25 **Mn** 54.94 Manganese	26 **Fe** 55.85 Iron	27 **Co** 58.93 Cobalt	28 **Ni** 58.69 Nickel	29 **Cu** 63.65 Copper	30 **Zn** 65.39 Zinc	31 **Ga** 69.72 Gallium	32 **Ge** 71.61 Germanium	33 **As** 74.92 Arsenic	34 **Se** 78.96 selenium	35 **Br** 79.90 Bromine	36 **Kr** 83.80 Krypton
37 **Rb** 85.47 Rubidium	38 **Sr** 87.62 Strontium	39 **Y** 88.91 Yttrium	40 **Zr** 91.22 Zirconium	41 **Nb** 92.91 Niobium	42 **Mo** 95.94 Molybdenum	43 **Tc** 97.91 Technetium	44 **Ru** 101.07 Ruthenium	45 **Rh** 102.91 Rhodium	46 **Pd** 106.42 Palladium	47 **Ag** 107.87 Silver	48 **Cd** 112.41 Cadmin m	49 **In** 114.82 Inidum	50 **Sn** 118.71 Tin	51 **Sb** 121.76 Antimony	52 **Te** 127.60 tellurium	53 **I** 126.90 Iodine	54 **Xe** 131.29 Xenon
55 **Cs** 132.91 Caesium	56 **Ba** 137.33 Barium	57 – 71	72 **Hf** 178.49 Hafnium	73 **Ta** 180.95 Tantalum	74 **W** 183.84 Tungsten	75 **Re** 186.21 Rhenium	76 **Os** 190.23 Osmium	77 **Ir** 192.22 Iridium	78 **Pt** 195.08 Platinum	79 **Au** 196.97 Gold	80 **Hg** 200.59 Mercury	81 **Tl** 204.38 Thallium	82 **Pb** 207.2 Lead	83 **Bi** 208.98 Bismuth	84 **Po** 208.98 Polonium	85 **At** 209.99 Astatine	86 **Rn** 222.02 Radon
87 **Fr** (223.02) Francium	88 **Ra** 226.03 Radium	89 - 103	104 **Unq** 261.11	105 **Unp** 262.11	106 **unh** 263.12	107 **uns** 262.12	108 **Uno** 265.00	109 **Une** 265									

57 **La** 138.91 Lanthanu m	58 **Ce** 140.12 Cerium	59 **Pr** 140.91 Praseodymium	60 **Nd** 144.24 Neodymium	61 **Pm** 144.91 Promethium	62 **Sm** 150.36 Samarium	63 **Eu** 151.97 Europium	64 **Gd** 157.25 Gadolinium	65 **Tb** 158.93 Terbium	66 **Dy** 162.50 Dysprosium	67 **Ho** 164.93 Holmium	68 **Er** 167.26 Erbium	69 **Tm** 168.93 Thulium	70 **Yb** 173.04 Ytterbium	71 **Lu** 174.97 Lutetium
89 **Ac** 227.03 Actinium	90 **Th** 232.04 Thorium	91 **Pa** 231.04 Protactirium	92 **U** 238.03 Uranium	93 **Np** 237.05 Neptunium	94 **Pu** 244.0 Plutonium	95 **Am** 243.06 Americium	96 **Cm** 247.07 Curium	97 **Bk** 247.07 Berbelium	98 **Cf** 251.08 Californium	99 **Es** 252.08 Einstienium	100 **Fm** 257.10 Fermium	101 **Md** 260 Mendelevium	102 **No** 259.10 Nobelium	103 **Lr** 262.11 Lawrencium

Department of Chemistry-UNZA

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF CHEMISTRY

ACADEMIC YEAR 2012 UNIVERSITY SESSIONAL EXAMINATIONS SEMESTER II

19 AUGUST 2013

C 362: COLLOIDS AND ELECTROCHEMISTRY

TIME: THREE HOURS

INSTRUCTIONS: ANSWER ANY *FIVE* OF SIX QUESTIONS.

DATA

Physical constants and other data you may wish to use are given on the *ATTACHMENT*.

QUESTION 1 (*20 MARKS*)

a) (*5 marks*)

Explain the difference between *kinetic stability* and *thermodynamic stability* of a colloid.

b) (*5 marks*)

The equilibrium S(R, s, p, T) $\rightleftharpoons$ S (l, p, T) $\rightleftharpoons$ S(g, p, T), which occurs at T = 387 K, is one of the many possible phase equilibria of the one component system sulfur. R stands for rhombic sulfur which is the stable form of sulfur at ordinary temperatures.

(i) What type of equilibrium is this?

(ii) Draw a labeled diagram to show the variation of the chemical potential, μ, of rhombic sulfur as a function of temperature, T, in the neighbourhood of T = 387 K.

c) (*5 marks*)

The system acetone and carbon disulfide, CS_2 exhibits positive deviations from Raoult's law. Some acetone (solute) is dissolved in the solvent carbon disulfide.

Draw a labeled pressure, *p* versus X_{CS_2} diagram and, on the same diagram, indicate the behaviour of Raoult's and Henry's laws.

d) (*5 marks*)

(i) What is a *micelle*?

(ii) What is *cmc* and how can it be detected?

QUESTION 2 (*20 MARKS*)

a) (*5 marks*)

At 25 °C, the ionization constant of propanoic acid is 1.4×10^{-5} and the molar concuctance at infinite dilution is 386.6 $S.cm^2.mol^{-1}$. Calculate the molar conductance of 0.05 *M* propanoic acid solution at 25 °C?

b) (*11 marks*)

A solution containing 0.00739 g of $AgNO_3$ per gram of water was electrolyzed between silver electrodes. During the experiment 0.0078 g of silver was deposited in a silver coulometer placed in series. At the end of the experiment, the anodic solution contained 23.14 g of water and 0.236 g of $AgNO_3$. What are the transport number of Ag^+ and NO_3^- ions?

c) (*4 marks*)

The ionic mobility of the acetate ion ($CH_3CO_2^-$) is 4.24×10^{-8} $m^2.s^{-1}.V^{-1}$ at infinite dilution and the value of the limiting ionic conductivity of the proton is 349.6 $S.m^2.mol^{-1}$. Use this information to calculate the molar conductivity of acetic acid at infinite dilution.

QUESTION 3 (*20 MARKS*)

The following cooling curves are measurements from Jean Timmermans: *Physico-chemical Constants of Binary Systems in Concentrated Solutions*; Interscience, 1959, Volume 1, 577 on the binary system naphthalene (N), $C_{10}H_8$ and diphenylamine (DPA), $C_{12}H_{11}N$.

Wt.% N	100	89.0	75.9	65.3	55.8	45.0	36.0	30.0	24.5	18.0	11.2	5.0
Freezing temp.(°C)	79.5	77.5	68.0	61.5	54.7	45.6	36.5	32.0	35.1	39.6	45.2	49.8

a) (*8 marks*)

Draw the phase diagram of the naphthalene (N) – diphenylamine (DPA) system.

b) (6 *marks*)

Determine the eutectic temperature and the mole fraction eutectic composition. What is the melting point of diphenylamine?

QUESTION 3 CONTINUES TO THE NEXT PAGE

c) (*3 marks*)

A mixture of overall composition 20 wt % diphenylamine is heated to a temperature 323.15 K. What phases are present and what is the percentage of each phase in the mixture at that temperature?

d) (*3 marks*)

A mixture of overall composition 90 wt % diphenylamine is cooled from 60 °C to 20 °C. Draw a labeled temperature – time cooling curve for the 90 wt % isopleth.

QUESTION 4 (*20 MARKS*)

a) (*4 marks*)

Write the change in state for nitric oxide at its normal boiling point of 121 K.

b) (*9 marks*)

Derive the Clausius-Clapeyron equation for nitric oxide at its normal boiling point. Clearly state any assumption(s) that you make in the derivation.

c) (*7 marks*)

For uranium hexafluoride the vapour pressure (in Pa) for the solid and liquid are given by

$$\ln p_s = 29.411 - \frac{5895.5}{T}$$

$$\ln p_l = 22.254 - \frac{3479.9}{T}$$

Calculate the temperature and pressure of the triple point.

QUESTION 5 (*20 MARKS*)

a) (*2 marks*)

A Tafel plot is prepared for an electrode process that has an exchange current density of 35 μA cm^{-2}. What is the numerical value of the intercept on this Tafel plot?

b) (*9 marks*)

An electrode which has an equilibrium potential of 0.47 V is at an applied potential of 0.54 V. The electrode process taking place has an exchange current density of 28 μA cm^{-2} and the temperature is 25 °C. What is the total current that is passed at the electrode under these conditions? (Transfer coefficient, $\alpha = 0.5$)

QUESTION 5 CONTINUES TO THE NEXT PAGE

c) (*9 marks*)

What is the effective resistance at 25 °C of an electrode interface when the overpotential is small? Calculate the effective resistance for 1.0 cm^2

(i) Pt, $H_2|H^+$ ($J_o = 7.9 \times 10^{-4}$ A cm^2)

(ii) Hg, $H_2|H^+$ ($J_o = 7.9 \times 10^{-13}$ A cm^2)

QUESTION 6 (*20 MARKS*)

a) (*10 marks*)

The following table gives the number of moles and molecular mass of fractions of a polymer sample

Mole	0.003	0.007	0.015	0.024	0.040	0.032	0.010	0.005
Molecular mass	30,000	35,00	40,000	45,000	50,000	55,000	60,000	65,000

(i) Calculate the *number-average* molecular mass, $\bar{M}_n$ and the *mass-average* molecular mass $\bar{M}_m$.

(ii) Calculate the *polydispersity index* of the polymer.

b) (*10 marks*)

A mixture of toluene, $C_6H_5CH_3$ and benzene, C_6H_6 contains 30 % by weight of toluene. At 30 °C the vapour pressure of pure toluene is 4893 Pa while that of pure benzene is 15758 Pa. Many measurements of the vapour pressure [e.g. Bell, T. and Wright, R. *J. Phys. Chem.* **31** (1927) 1884] show that solutions of this system obey Raoult's law. Calculate the total pressure and the partial pressure of each component above the solution at 30 °C.

END OF C 362 SESSIONAL EXAMINATION SEMESTER II

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF CHEMISTRY

ACADEMIC YEAR 2013 UNIVERSITY SESSIONAL EXAMINATIONS SEMESTER II

19 AUGUST 2013

C 362: COLLOIDS AND ELECTROCHEMISTRY

ATTACHMENT

DATA

$R = 8.314\ J\ K^{-1}mol^{-1} = 0.08314\ dm^3\ bar\ K^{-1}\ mol^{-1}$; $F = 96{,}485\ C\ mol^{-1}$,

$1\ Pa = 1\ J\ m^{-3}$; $1\ bar = 100\ kPa$; $T = 273.15 + t\ (^oC)$

$C = 12.01$; $H = 1.01$; $N = 14.01$; $O = 16.00$; $Ag = 107.87$
atomic mass constant, $m_u = 1.661 \times 10^{-27}kg$

$d\bar{G} = d\mu = -\bar{S}dT + \bar{V}dp$; $\Delta G = \Delta H - T\Delta S$; $\bar{M}_n = \frac{\sum_i n_i M_i}{\sum_i n_i}$; $\bar{M}_m = \frac{\sum_i n_i M_i^2}{\sum_i n_i M_i}$

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF.CHEMISTRY

C 482: INORGANIC INDUSTRIAL CHEMISTRY

TIME : 3 HOURS **EXAMINATION – 21 AUGUST 2013**

ANSWER : ANSWER ANY FIVE QUESTIONS.
: ALL QUESTIONS HAVE EQUAL MARKS.

QUESTION 1

a) The HRN unit is a very essential component of the Ammonium Nitrate Plant. Describe the operation process and the uses of the heat recovered from the heat transfer processes.

b) Describe the basic functions and the principle of the two series evaporator system in the manufacture process of Ammonium nitrate.

c) With the aid of a flow Diagram describe the manufacturing process of Ammonium Nitrate.

QUESTION 2

a) The manufacturing process of Superphosphate depends on reacting phosphate rock with sulfuric acid and the fertilizer contains about (16 - 20 %) P_2O_5. The net reaction proceeds as follows:

$$2Ca_5F(PO_4)_3 + 7H_2SO_4 + 3H_2O \rightarrow 3Ca(H_2PO_4)_2 \,.\, H_2O + 7CaSO_4 + 2HF$$

The process can be divided into two stages, explain briefly what each stage represents.

b) With the aid of a flow Diagram describe the manufacturing process of Single superphosphate Fertilizer.

c) Describe the process recovery of Fluorine fumes in the manufacture of Superphosphate.

QUESTION 3

a) Briefly describe the Production process of Synthetic Ammonia.

b) Write the reaction equations of the production process of Synthetic Ammonia process, Temperature and Concentration of raw materials and products.

c) With the aid of a flow Diagram, describe the manufacturing process of Synthetic Ammonia.

QUESTION 4

a) Describe the Manufacturing Process of Sulphuric Acid by double adsorption process or any other process.

b) Briefly describe the oxidation of SO_2 to SO_3 process (Indicate Kindling and other temperatures if Vanadium catalyst is used).

c) Describe the absorption process of SO3 and explain why a second absorption stage is necessary.

QUESTION 5

a) Briefly describe the manufacturing process of nitric acid – 60% strong. Write the reactions involved in the Production process.

b) What is the influence of Temperature on the NH_3 oxidation process?

c) The **AK-72** process for the Manufacture of Nitric acid is based on a closed – cycle power cogeneration scheme which includes two ammonia oxidation steps. With reference to the flow sheet of the **AK-72** nitric acid plant, briefly outline the production process

QUESTION 6

In the production of Sulphuric acid Iron pyrite and Sulphur are usually used.

a) What are the advantages and disadvantages associated with the use of these raw materials?

b) State the properties of 98.3% Sulphuric acid and why this acid is used for absorption of SO_3 containing gas?

c) Outline the production process of concentrated Sulphuric acid.

Wish you well, The End

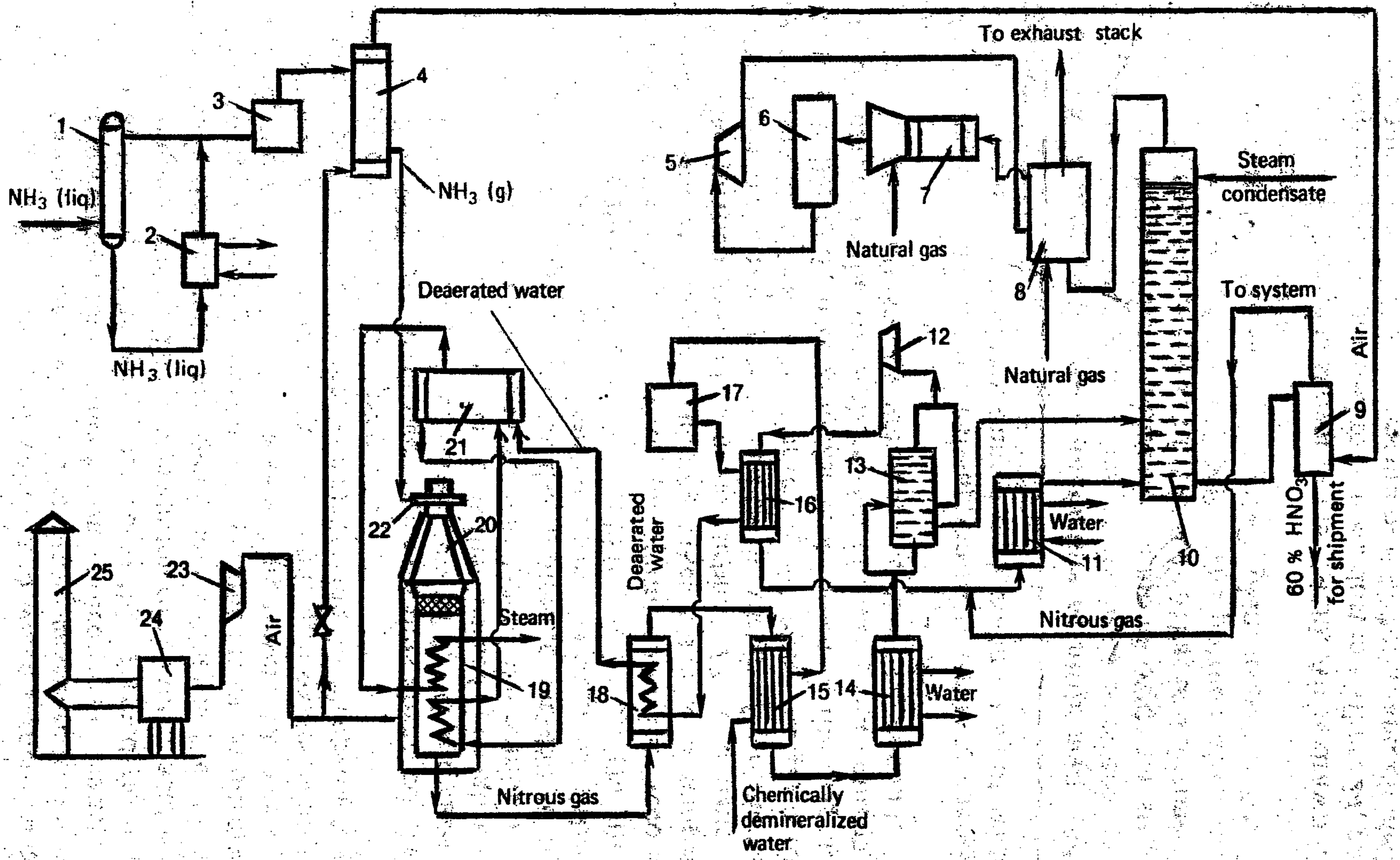


Fig. Process flowsheet of the AK-72 nitric acid plant:

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
2012 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

C492: ORGANIC INDUSTRIAL CHEMISTRY II
TIME: THREE (3) HOURS
INSTRUCTIONS: Answer any four (4) questions.

QUESTION 1

(a) Polymers may be classified as thermoplastic or thermosetting.

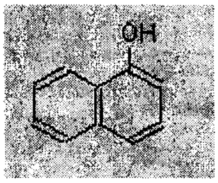
(i) Differentiate between the two types of polymers.
(ii) For each type of polymer described above, give one example together with structure and name.

(b) (i) Define and explain in brief the term glass transition as it relates to semi-crystalline polymers
(ii) Draw a sketch diagram that shows the change in specific volume as function of temperature for three different materials: glass, semicrystalline solid (polymer) and crystalline. Indicate all major transitions on the diagram.

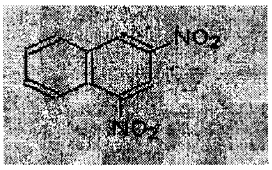
(c) Describe in brief the process of manufacturing polyethylene plastic bags using the process of blown film extrusion.

QUESTION 2

(a) Consider the structure shown below. Explain the fact that 1-napththanol is colourless while 2, 4 -dinitronaphthalene is pale yellow.



1-naphthanol



2, 4-dinitronaphthalene

(b) (i) Describe the laboratory synthesis of indigo. Include relevant chemical reactions and structures.
(ii) Use a flow chart to show the process that you follow to dye a pair of jeans with indigo dye.
(c) Dyes and pigments are used to impart colour to various substances. Compare and contrast the two types of substances.

QUESTION 3

(a) What are the major characteristics that must be inherent in an explosive substance?
(b) To which class of explosives does trinitrotoluene belong? Describe in brief its manufacturing process.
(c) (i) What is an initiatory explosive?
(ii) Outline characteristics of a typical initiatory explosive.

QUESTION 4

(a) Define/explain the following terms: (i) hide (ii) leather
(b) Pickling is one of the pre-tanning processes in leather manufacturing.
    (i) What is the purpose of pickling?
    (ii) Describe the pickling process.
(c) What class of dyes are typically used to impart colour to leather. Describe the key characteristics of this class of dyes.

QUESTION 5

(a) With the aid of a flow diagram describe the cane sugar production processes.
(b) What is the role of the clarification process in sugar production? Explain in brief.
(c) How is raw sugar recovered from the molasses?
(d) Explain the significance of multiple effect evaporators in the sugar refinery process.

— END OF EXAMINATION —

THE UNIVERSITY OF ZAMBIA
School of Natural Sciences
Department of Computer Studies

2012/2013 SECOND SEMESTER
FINAL EXAMINATION

ELECTRONICS FOR COMPUTING II
CST 3252

Date:	28th AUGUST 2013
Time:	09:00hrs – 12:00hrs
Duration:	3 Hours
Venue:	New Dining Hall

Instructions

1. There are six (6) questions and **two (2) sections** in this paper.
2. Each question carries **20 marks,** *you are required to answer a total of **Five (5) Questions***
3. *Answer all the questions in Section A* and choose *any three (3) questions from Section B*

SECTION A

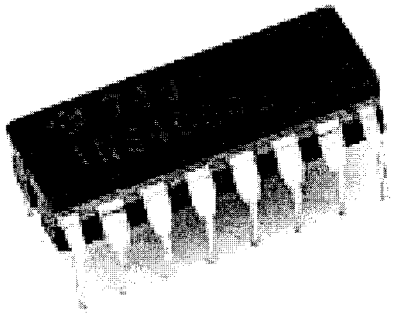
This section has Two Question. Answer all the questions

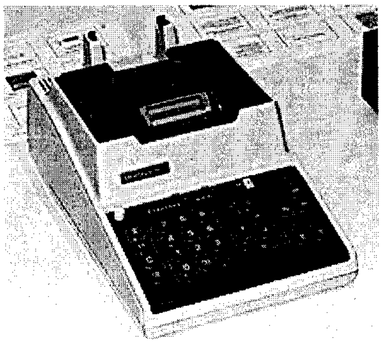
Question One (1)

a) Give a brief description for each of the following terms [2]
   i. Counters
   ii. Controllers

b) The main thing you will see with the n-bit processor is the amount of RAM they can use. By showing your calculations, determine how much RAM can be used by the following processors with the following bit address [10];
   i. 4 – bit
   ii. 8 – bits
   iii. 16 – bit
   iv. 32 – bit
   v. 64 – bits

c) The first microprocessor that was widely used in digital devices was the Intel 4004 which was 4 bits wide. It was originally intended for desk calculator applications shown below;





   i. Explain how the bit width affects the speed of the CPU [4]
   ii. Give two (2) examples of the 32 bit and 64 bit CPU with the corresponding typical speed (in Hz). The example for each should come from Intel and AMD respectively [4]

Question Two (2)

a) Computer memory is one of the most valuable resources used by computers and other electronic devices. With the aid of a diagram, draw a basic architecture of [6]
   i. SRAM memory unit
   ii. DRAM memory unit

b) Explain how each (SRAM and DRAM) is used to store information (a bit) and how information is read and written to and from the memory [10]

c) Give an explanation, how SRAM and DRAM are packed and how they work as modules used in modern computers [4]

SECTION B

This section has Four (4) Questions. Choose any three (3) questions

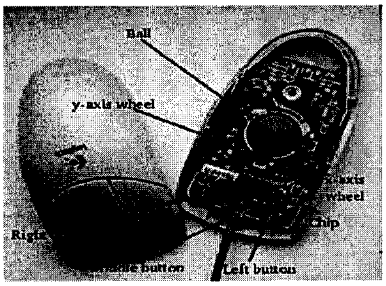
Question Three (3)

a) Give a brief description for each of the following terms [2]
   i. VDUS
   ii. Timers

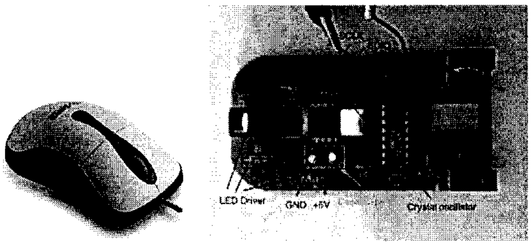
b) List and explain three ways used by CLOCK SOFTWARE to maintain the time of the day in the computer [6]

c) The mouse is a pointing device which helps us to operate the computer. Two common types of MICE used are the optical mouse and the roller/ball mouse as shown the diagrams below. Briefly explain how [12]

   i. The roller/ball Mouse works
   ii. The Optical Mouse works
   iii. Compare and contrast the operation of the two types of the mice by giving the advantages and disadvantages



Trackball Mouse



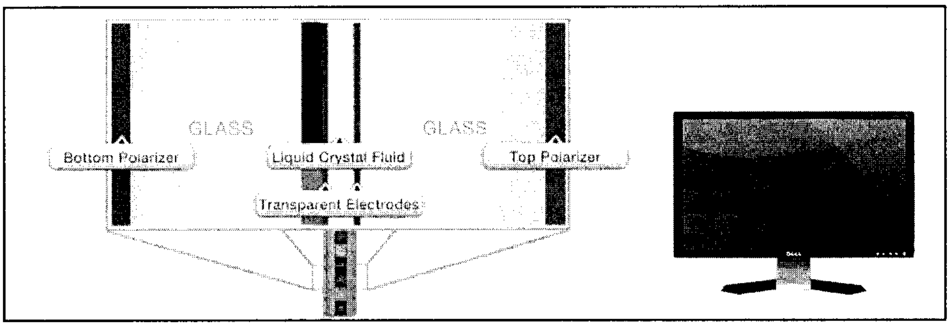
Optical Mouse

Question Four (4)

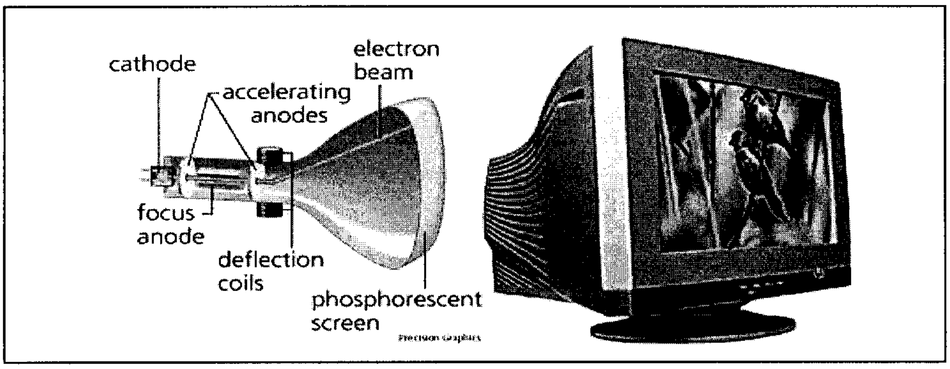
a. Briefly explain how the clock is used to control the operation of the computer [4]

b. Give a brief description for each of the following types of scanners and where they are largely used [6]
   i. Drum Scanners
   ii. Handheld Scanners
   iii. Sheet-fed Scanners

c. The diagrams below show the CRT Monitor and the LCD monitor. With the aid of the diagram, briefly explain how the [10]
   i. The LCD Monitor works
   ii. CRT Monitor works
   iii. Compare and contrast the operation of the two types of monitors



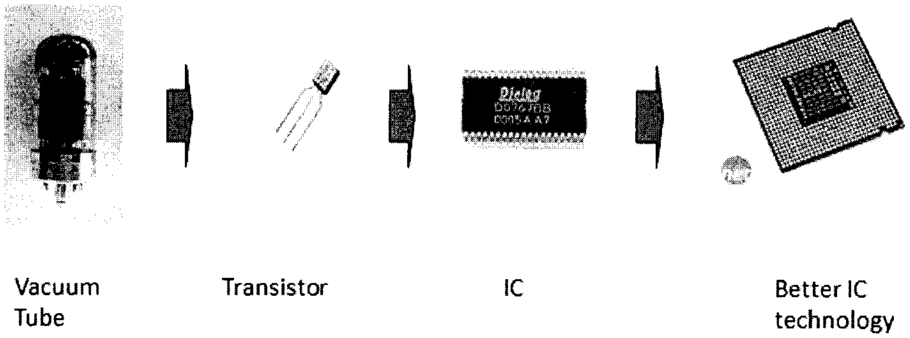
(a) LCD Monitor



(b) CRT Monitor

Question Five (5)

a. Digital electronics have evolved over the years. The diagram below shows the Vacuum Tube, Transistor, Integrated Circuit and CPU (Microprocessor).



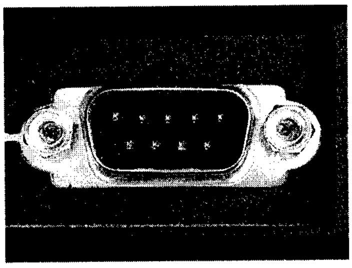
Give a brief description of the major changes in technology of the four components shown above and how they differ from each other. Highlight the advantages of using one component over the others [6]

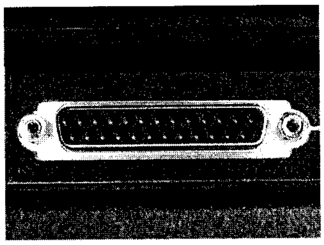
b. With the aid of the diagram, explain how each of the following works [8]
   i. Interrupts
   ii. Watchdog Monitor

c. Give a brief description for each of the following terms [6]
   i. Soft Timers
   ii. Mechanical Timers
   iii. Electronic Timers

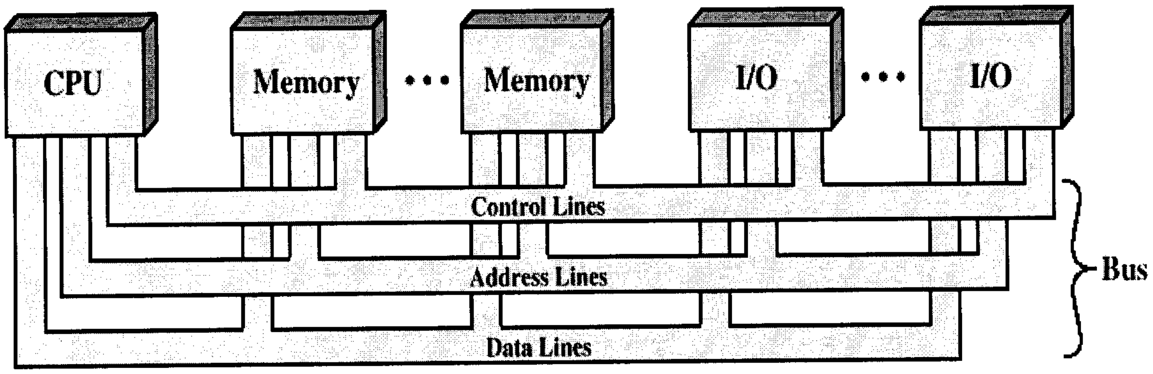
Question Six (6)

a. The diagram below shows the serial and parallel ports. For each of these ports give a brief explanation, advantages and disadvantages. Explain how the serial port uses the UART [8]





b. The diagram below shows the buses which consist of data lines, control lines, and address lines. Briefly describe the function of each of the three bus lines [6]



c. With the aid of the diagram, give a brief description for each of the following terms [6]

i. Numeric Co-Processors;
ii. Graphics and I/O Accelerators
iii. Disk Drive Controllers

THE UNIVERSITY OF ZAMBIA
School of Natural Sciences

Department of Computer Studies

2012/2013 SECOND SEMESTER
FINAL EXAMINATION

ADVANCED OPERATING SYSTEMS AND DISTRIBUTED SYSTEMS
CST 4012

Date:	3rd SEPTEMBER, 2013
Time:	09:00hrs – 12:00hrs
Duration:	3 Hours
Venue:	Library Basement

Instructions

1. *Answer all the questions from Section A*
2. *Answer any three (3) questions from Section B*

Section A: 40 Marks

Answer all questions

1. Operating systems provide an environment for execution of programs and services to programs and users. State at least three services provided by an operating system and explain how each provides convenience to the users. Explain also in which cases it would be impossible for user-level programs to provide these services. [10]
2. What is the main difficulty that a programmer must overcome in writing an operating system for a real-time environment? [6]
3. Why would an application programmer prefer programming according to an API rather than invoking actual system calls? [8]
4. Some distributed systems can simply not scale, no matter which techniques are applied. Give an example of such a system and explain why scalability is (close to) impossible. [6]
5. State four important design issues that need to be taken into account when implementing a general purpose security services. [4]
6. Consider the following communication scenario;

Alice and Bob share a secret key K_{AB}

1. *Alice uses* K_{AB} *and an agreed encryption function* $E(K_{AB}, M)$ *to encrypt and send any number of messages* $\{ M_i \}_{K_{AB}}$ *to Bob.*
2. *Bob reads the encrypted messages using the corresponding decryption function* $D(K_{AB}, M)$.

Discuss two issues (weaknesses) with the following communication scenario. [6]

Section B: 60 Marks

Answer ANY THREE Questions from this section

1.

a) The appearance of running several separate Linux environments on a single physical machine can be realised in a number of different ways:

   i. A low-level virtual machine monitor exporting interfaces which look identical to the underlying hardware (e.g. VMWare)

   ii. A low-level `paravirtualizing' machine monitor which exports modified interfaces (e.g. Denali, Xen)

   iii. Modifications to the operating system to group processes into `virtual machines' all running over the same OS kernel (e.g. Linux VServers).

What are the advantages and disadvantages of each approach? [15]

b) Among the benefits of implementing virtualisation include; server and application consolidation. Highlight some instances in which server and application consolidation is challenging to realise. [5]

2. Given this very simplistic RSA encryption which uses public key (n=33,e=3) and encodes text one character at a time using the following numeric codes for each character:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t

21	22	23	24	25	26	27	28	29	30	31	32
u	v	w	x	y	z	.	,	;	!	@	?

Show enough working so as to verify your calculations.

a) What is the RSA private key? [4]

b) Encrypt *cs.unza.zm* [10]

c) Decrypt *@iez* [6]

3.

a) Explain why transient synchronous communication has inherent scalability problems, and how these could be solved.[6]

b) State the difference between unstructured communication and structured communication. Explain why both require low-level communication support. [8]

c) Why are transport-level communication services often inappropriate for building distributed applications? [6]

4.

a) Would you consider a URL such as **http://www.acme.org/index.html** to be location independent? What about **http://www.acme.nl/index.html**? [4]

b) A special form of locating an entity is called anycasting, by which a service is identified by means of an IP address. For instance, sending a request to an anycast address, returns a response from a server implementing the service identified by that anycast address. Outline the implementation of an anycast service based on the hierarchical location service. [6]

c) Describe a microkernel architecture in terms of how user programs and system services interact? What are the advantages and disadvantages of using the microkernel approach? [10]

5.

a) Is it always a good idea to aim at implementing the highest degree of transparency possible in distribute systems? State reasons for your answer [6]

b) Discuss two models for code migration. [8]

c) Describe how connectionless communication between a client and a server proceeds when using sockets.[6]

GOOD LUCKY!

UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SESSIONAL EXAMINATIONS FOR ACADEMIC YEAR 2012

SECOND SEMESTER

EM312-ENGINEERING MATHEMATICS IV

MAXIMUM TIME ALLOWED: THREE (3) HOURS

INSTRUCTIONS:

THERE ARE TWO SECTIONS IN THIS EXAMINATION PAPER. YOU ARE REQUIRED TO ANSWER AT LEAST THREE QUESTIONS FROM SECTION A AND AT LEAST ONE QUESTION FROM SECTION B. All TOGATHER ANSWER A TOTAL OF FIVE QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

SECTION A

1 a. (i) Write the equation of a right circular cylinder of radius R around the y- axis.

(II) Find an equation of the plane through the point (4,-3,5) and normal to the vector (4,-8,3).

(iii) Show that the planes x+2y-4z=0 and -3x-6y+12z=8 are parallel.

(iv) Find parametric equations for the straight line passing through the point (2,0,3) and parallel to the line x=4+t, y=2, z=6-2t

b. Find the area of the portion of the sphere $x^2+y^2+z^2=2$ which lies inside the cone $z = \sqrt{x^2 + y^2}$

c. (i) Find values for constants a, b and c which will make the vector field

$$\vec{F}(x,y,z) = (x^2 + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$$

irrotational.

(ii) Use vector field $\vec{F}(x,y,z)$ from part (i) and find a scalar function f(x,y,z) such that $\nabla f = \vec{F}$.

2 a. Find a unit tangent vector to the Curve C: $x = \sin t, y = 2\cos\ t, z = \frac{2t}{\pi}, t \geq 0$ at the point (0,-2,1)

b. (i) Sketch the cone $z = \sqrt{x^2 + y^2}$ given that its height is one unit.

(ii) Find the surface area of the cone of part (i) given that it is opened from the top.

c. (i) State Green's theorem.

(ii) Use Green's theorem to find the area enclosed by the curve $\frac{x^2}{9} + y^2 = 1$

3. a. (i) Find the equation of the tangent plane to the surface $xyz^3+yz^2=4$ at the point (1,2,1).

(ii) The rate of change of a function f(x,y) at a point (x_0,y_0) in direction i+2j is 3 and in direction -2i-j is -1. Find the rate of change of f at (x_0,y_0) in direction 2i+3j.

b. Find the volume in the first octant cut from the cylinder $x^2+z^2=4$ by the plane y+z=6.

c. Evaluate $\int_c \frac{z}{y} dx + (x^2 + y^2 + z^2)dz$ where the curve C lies in the first octant and is the intersection of surfaces $x^2+y^2=1$ and z=2x+4.

4 a. Evaluate $\int_0^2 \int_y^2 e^{x^2}\, dx\, dy$

b. Evaluate $\iint_S z^2\, dS$ where S is the sphere $x^2+y^2+z^2=4$.

c. (i) Find the area of the triangle with vertices at points (0,0,1), (0,1,0) and (2,0,0).

(ii) State the Stokes's theorem.

(iii) Use Stokes's theorem to evaluate

$\oint_C 2xy^3\, dx + 3x^2y^2 dy + (2z + x)dz$ where C is the triangle described in part (i).

SECTION B

5. a The following data represents the average SO_2 (sulfur dioxide) emission rates from industrial boilers (lb/million Btu) at copper mining sites in Zambia.

2.3	2.7	1.5	1.5	0.3	0.6	4.2	1.3	1.2	0.4	0.5
2.2	4.5	3.8	1.2	0.2	1.0	0.7	0.2	1.4	0.7	3.6
1.0	0.7	1.7	0.5	0.1	0.6	2.5	2.7	1.5	1.4	2.9
1.0	3.4	2.1	0.9	1.9	1.0	1.7	1.8	0.6	1.5	2.4

(i) Group this data in to class intervals of equal width starting with the first class interval as 0 -< 0.5.

(ii) Compute the cumulative frequencies for the grouped data of part (i)

(iii) Give an approximate percent of mining sites having SO_2 emission rates at least 20 lb/million Btu.

b Zambia Breweries classifies each of its employee in one of the six classes namely clerical, manager, technical, professional, sales, unskilled and assigns points 1,2,3,4,5,6 respectively to these classes for compact coding. Discuss if you can regard these assigned data points as quantitative data and would be interested in mean, variance, median etc or you will analyze such a data by qualitative methods. Explain how will you display this kind of data in a chart.

C In a study investigating the effect of car speed on accident severity, 5000 cases of fatal automobile accidents were recorded and vehicle speed at impact was estimated in some way. It was determined that average speed was 42 mph and standard deviation was 15 mph. In addition, a histogram revealed that vehicle speed at impact could be described by a bell shaped curve. Determine approximately what proportion of accidents exceeded speed 57 mph.

6. a Parts produced on a machine were checked for diameter of the part and condition of its edge. The following table summarizes the data collected on 200 parts.

Edge Condition		**Diameter**	
		Above target	Within target
	Coarse	15	10
	Moderate	25	20
	Smooth	60	70

A part is selected at random from the lot of 200 parts. Find the probability that

(i) the part selected has a moderate edge condition and diameter within target.

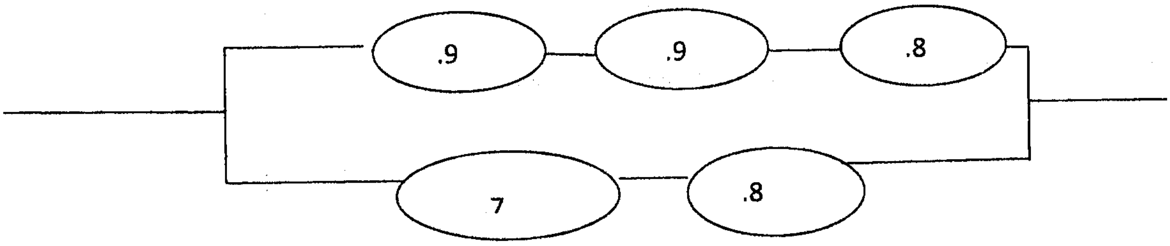
(ii) the part selected has a moderate edge condition or diameter within target.

(iii) the part selected does not have a moderate edge condition or does not have diameter within target.

b. An assembler of electric fans uses motors from two sources. Company A supplies 90% of the motors and company B supplies the other 10% of the motors. Suppose it is known that 5% of the motors supplied by company A are defective and 3% Of the motors supplied by company B are defective. An assembled fan is found to have a defective motor. Find the probability that this motor was supplied by company B.

Please turn over for part C

c.. The circuit below operates if and only if there is a path of functional devices from left to right. Assume that devices operate independently and probability of each device operating is shown in the circuit diagram. Find the probability that the circuit operates.



THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS

GEO 112: INTRODUCTION TO HUMAN GEOGRAPHY II

TIME: **Three hours.**

INSTRUCTIONS: **Answer any four questions. Candidates are advised to make use of illustrations wherever appropriate. Use of a University Atlas is allowed.**

---

1. Define land tenure and explain the arguments for and against the promotion of communal or individual/private land tenure systems in Sub-Saharan Africa.

2. " Knowledge is truly the mother of all other resources"(Zimmermann,1964 :12). Comment on this statement with regard to what African people should do to achieve sustainable socio-economic development.

3. Define Industrialization and explain the evolution of industrial procedures from the simple to the complex during the Industrial Revolution in England.

4. Describe Rostow's Five Stages of Economic Growth and comment on whether African countries such as Zambia, have achieved economic take off into sustained growth.

5. Define Modernization and suggest ways in which Africans may achieve this process without losing all their indigenous culture.

6. Define culture and describe its components with special emphasis on world views in a changing global environment.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

INSTITUTE OF DISTANCE EDUCATION BOARD

2012 ACADEMIC YEAR DISTANCE EDUCATION FINAL EXAMINATIONS

GEO 155: INTRODUCTION TO PHYSICAL GEOGRAPHY

TIME: **Three Hours**

INSTRUCTIONS: **Answer any four questions. All questions carry equal marks.**

---

1. Explain how human activities influence the distribution of vegetation.
2. Outline and describe the main processes that influence the formation of soils.
3. With the aid of examples describe the main processes responsible for the weathering of rocks.
4. Explain the theory of plate tectonics.
5. With the aid of a diagram, describe the structure of the earth's atmosphere.
6. Explain the theories of precipitation formation.

END OF EXAMINATIONS

THE UNIVERSITY OF ZAMBIA
INSTITUTE OF DISTANCE EDUCATION

2012 ACADEMIC YEAR DISTANCE EDUCATION FINAL EXAMINATIONS
GEO 175: INTRODUCTION TO MAPPING TECHNIQUES IN GEOGRAPHY

PAPER II
CLASSIFICATION OF NUMERICAL DATA, CONSTRUCTION OF TABLES, STATISTICAL MAPS AND DIAGRAMS

TIME: Three Hours

INSTRUCTIONS: Answer any four questions.
The use of a Philip's University Atlas and a certified calculator is allowed. Candidates are encouraged to make use of illustrations wherever appropriate.

---

1. Write short explanatory notes on **all** of the following:

   (a) Statistical maps [5 marks]
   (b) Nominal scale of measurement [5 marks]
   (c) Three entry tables [5 marks]
   (d) Age and sex graphs [5 marks]
   (e) Kinds of variables [5 marks]

2. Study the data given in Table 1 on the Journey to work by a sample of urban population and then answer the questions that follow.

Table 1: Journey to work
(Sample of Urban Population)

Type of Transport	Frequency
Foot	50
Cycle	20
Bus	170
Train	246
Car	1, 060
Total	**1, 546**

Source: Hypothetical

(a) Use the most suitable statistical diagram to show the data in Table 1. [15 marks]

(b) What are the merits and demerits of the technique that you have used in part (a) [5 marks]

(c) Comment on the major highlights of your diagram. [5 marks]

3. Study the data given in Table 2 showing the distribution of population of the Departments of Aquitaine together with their respective sizes and then answer the questions that follow.

Table 2: Population of the Aquitaine Departments in 1962

Department	**Size (Km²)**	**Population Size**
Gironde	10, 724	935, 448
Landes	9, 364	260, 495
Bessel-Pyrénées	7, 712	446, 038
Dordogne	9, 224	375, 455
Lot et Garonne	5, 385	275, 028
Aquitaine	**42, 411**	**2, 312, 464**

Source: Adapted from Truran H.C. (1975:57) *A Practical Guide to Statistical Maps & Diagrams*, London, Heinemann.

(a) Use the most appropriate statistical mapping technique to show the data in Table 2 on the outline map of the Aquitaine departments (Figure 1) provided. [15 marks]

(b) What are the merits and demerits of the technique that you have used in part (a)? [5 marks]

(c) What other information in addition to that given in Table 2 would you require if you were asked to draw a population distribution map [5 marks]

4. Examine the data given in Table 3 showing a frequency distribution with classes specified in terms of their class limits.

Table 3: Frequency Distribution

Sno.	**Class Limit**	**Class Frequency (f)**
1	≥ 10 and < 14	5
2	≥ 14 and < 18	7
3	≥ 18 and < 22	10
4	≥ 22 and < 26	15
5	≥ 26 and < 30	21
6	≥ 30 and < 34	10
7	≥ 34 and < 38	11
8	≥ 38 and < 42	16
9	≥ 42 and < 46	3
10	≥ 46 and < 50	2
	Total	**100**

Source: Hypothetical

(a) Specify the classes in Table 3 in terms of their class boundaries. [5 marks]

(b) Calculate the class marks of the ten classes in Table 3. [5 marks]

(c) Calculate the cumulative distributions. [10 marks]

(d) How should the first class interval be written so that it becomes an open class? [2 marks]

(e) What is the relative frequency of the eighth class? [3 marks]

5. Examine Figure 2, an outline map with spot heights indicated and then answer the questions that follow.

   (a) On the outline map (Figure 2) provided, interpolate contours using a contour interval of 100metres and begin with the 200metre contour. [15 marks]

   (b) What are the advantages of the mapping technique that you have used in (a). [4 marks]

   (c) Describe any two methods that can be used in selecting the contour interval if it is not given. [6 marks]

6. Study the climatic data given in Table 4 and then answer the questions that follow.

Table 4: Climatic Data for Station X

Month	**Jan**	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
Temp. (° C)	04	04	16	08	12	15	17	17	14	10	07	04
Rainfall (mm)	48	43	46	38	46	51	61	56	46	66	61	61

Source: Hypothetical

(a) Use the most suitable statistical diagram to show the data in Table 4. [15 marks]

(b) Calculate the range of temperatures for Station X. [2 marks]

(c) Calculate the total annual rainfall for Station X. [2 marks]

(d) What is the basic difference between the annual mean temperature and annual temperature range? [3 marks]

(e) Using your diagram, comment on the relationship between temperature and rainfall at Station X. [3 marks]

END OF EXAMINATION

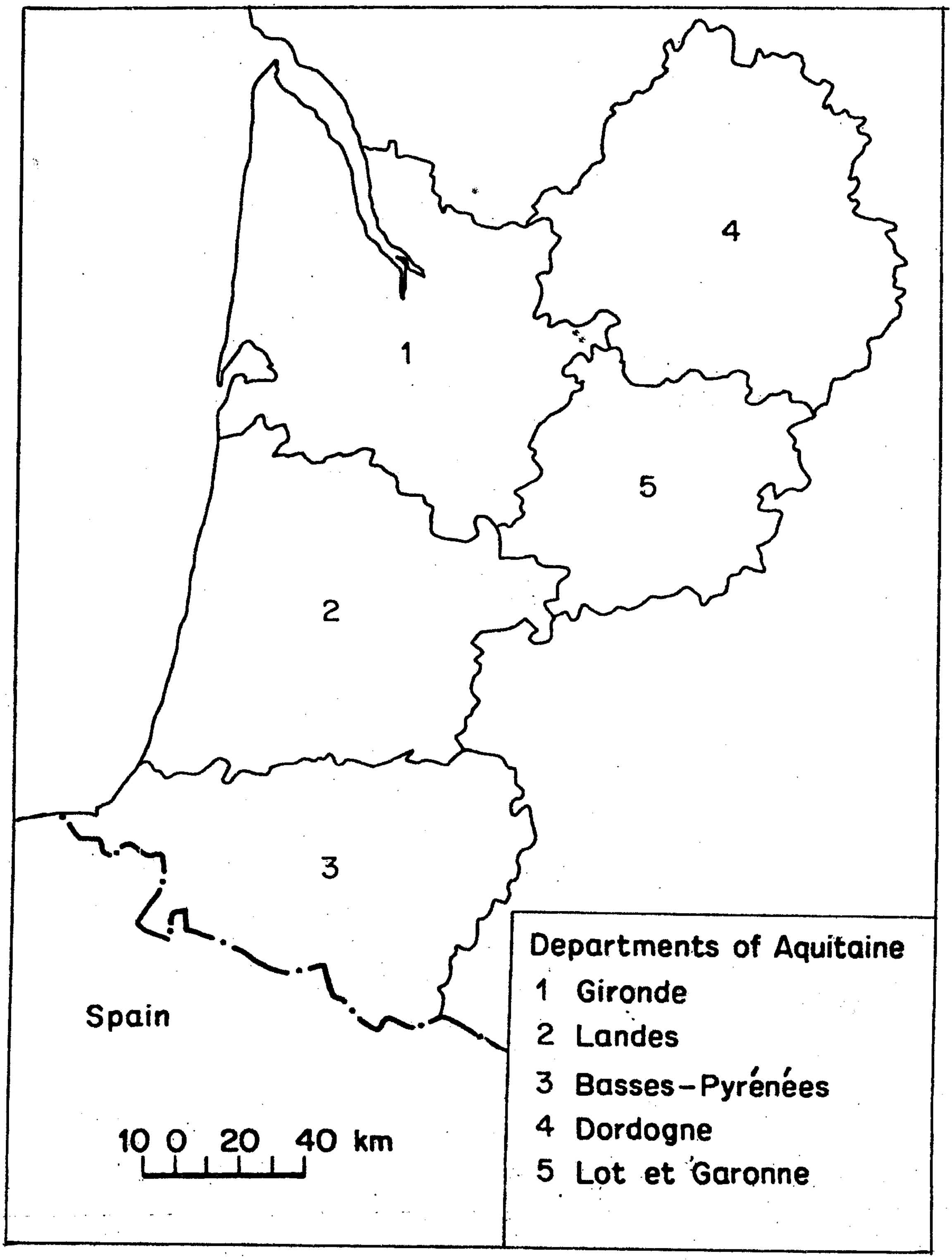

4
1
5
2
3
Spain
10 0 20 40 km
Departments of Aquitaine
1 Gironde
2 Landes
3 Basses-Pyrénées
4 Dordogne
5 Lot et Garonne

Figure 1

Computer No.------------------

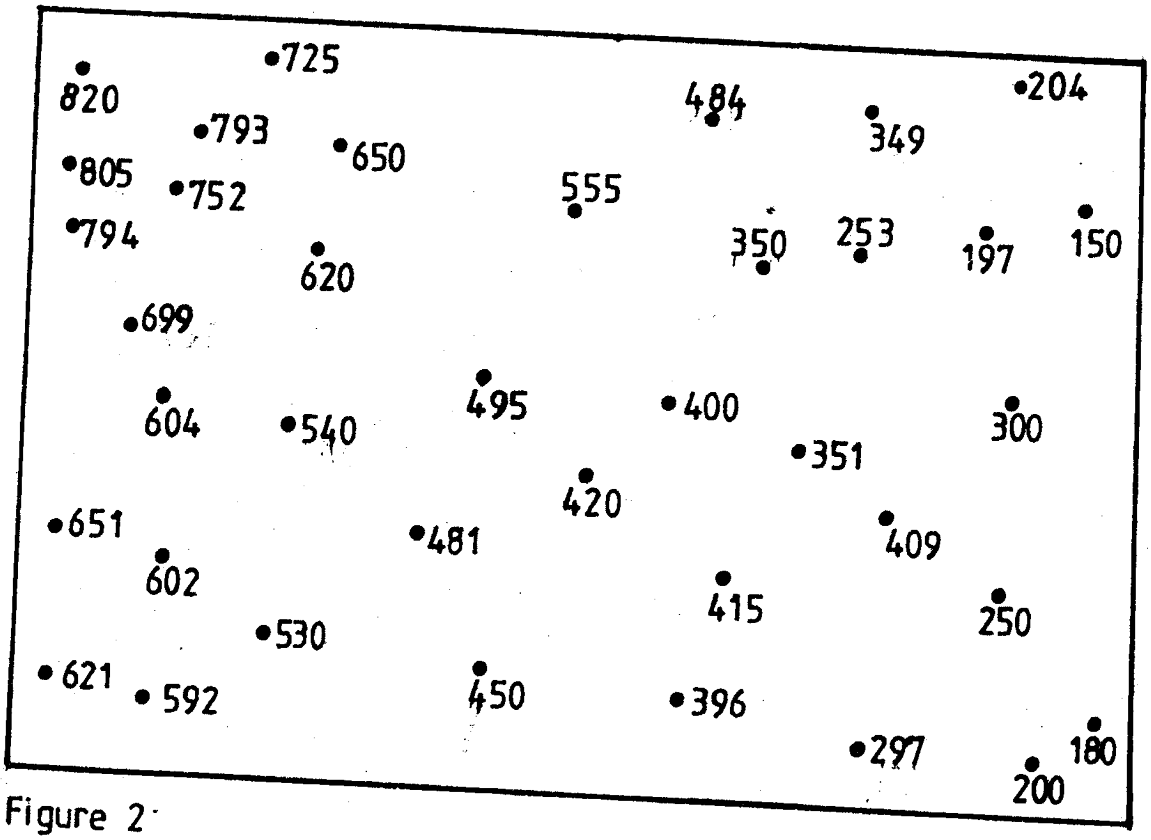


Figure 2

THE UNIVERSITY OF ZAMBIA

INSTITUTE OF DISTANCE EDUCATION

2012 ACADEMIC YEAR DISTANCE EDUCATION FINAL EXAMINATIONS

GEO 211 : THE GEOGRAPHY OF AFRICA

TIME:	**Three hours**
INSTRUCTIONS:	**Answer question One (1) and any Three other questions. Candidates are advised to make us of illustrations wherever appropriate. Use of a University Atlas is allowed.**

---

1. Discuss the ecological benefits of forests in Africa and suggest ways in which such forests can be sustained.

2. Explain the importance of water bodies found in Africa to socio-economic development.

3. How can Africa's abundant natural resource potential be a basis for rapid industrialization in this era of globalization?

4. Define the Neolithic Revolution in Africa, and describe the contributions by the Mande people of West Africa and the ancient Egyptians.

5. Compare and contrast the development paths that Kenya and Tanzania adopted after the attainment of their political independence.

6. Suggest ways in which unity in diversity can be promoted in Africa, despite the existence of many different races, languages and ethnic groups.

END OF EXAMINATION.

THE UNIVERSITY OF ZAMBIA

INSTITUTE OF DISTANCE EDUCATION

2012 ACADEMIC YEAR FINAL EXAMINATIONS

GEO 271: QUANTITATIVE TECHNIQUES IN GEOGRAPHY I

TIME: THREE (3) HOURS

INSTRUCTIONS: ANSWER QUESTIONS ONE (1) AND THREE (3) OTHERS. CREDIT WILL BE GIVEN FOR USE OF RELEVANT ILLUSTRATIONS. QUESTION ONE CARRIES 40 MARKS

---

1. Solid waste disposal is one of the most disturbing environmental issues faced by land-use planners in growing urban centres. You decide to conduct a study on the problems of solid waste management in Kalingalinga compound in Lusaka,
   (a) Formulate a relevant statement of the problem
   (b) Formulate a relevant aim and three (3) objectives of the study
   (c) Construct a questionnaire for collecting such data.
   (40 Marks)

2. Explain the strengths and weaknesses of any three (3) qualitative research methods. *(20 Marks)*

3. Write short explanatory notes on ALL of the following: *(20 Marks)*
   (a) Naturalistic inquiry
   (b) Inductive analysis
   (c) Snowball sampling
   (d) Quasi-experimental design
   (e) Applied versus Fundamental research

4. Provide reasons for conducting a thorough literature review as part of a research process *(20 Marks).*

5. Discuss different technical models for selecting a research problem. *(20 Marks).*
6. Compare and contrast non-scientific and scientific methods of inquiry in research *(20 Marks).*

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

INSTITUTE OF DISTANCE EDUCATION

2012 ACADEMIC YEAR DISTANCE EDUCATION FINAL EXAMINATIONS

GEO 381: ENVIRONMENT AND DEVELOPMENT I

Time: Three hours.

Instructions: Answer any four questions. All questions carry equal marks. Candidates are encouraged to make use of illustrations wherever possible.

---

1. Explain four conditions that are necessary for the achievement of sustainable development.
2. With the aid of a diagram, explain the interactions between the biotic and abiotic components of the environment.
3. Discuss four major characteristics of contemporary environmental problems.
4. With the aid of examples, explain how both regulatory and economic instruments can be used to achieve environmental protection.
5. Discuss the evolution of the concept of sustainable development since the late 1960s.
6. With the help of examples, discuss three direct and two underlying causes of deforestation and forest degradation.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS

GEO 912: GEOGRAPHY OF MIGRATION AND REFUGEES

TIME : Three hours
INSTRUCTIONS : Answer any FOUR questions.
All questions carry equal marks.

---

1. Distinguish between a refugee camp from a refugee settlement.'

2. Explain why some people migrate from one urban area to another urban area.

3. Examine any six mandates of the United Nations High commission of Refugees.

4. 'The principle of 'burden sharing' is what has helped some displacees and refugees to survive"(Rogge, 1989:95). Elucidate this statement with reference with any country you are familiar with except Zambia.

5. 'Refugees can either be sources of development or problems in the country of asylum.' Discuss.

6. "Both the United States of America and Australia have not been very co-operative in giving assistance and asylum to refugees from many neighboring islands"(UN,2000: 57). Discuss.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS

GEO 922: GEOGRAPHY OF REGIONAL PLANNING AND DEVELOPMENT

TIME: **Three Hours**

INSTRUCTIONS: **Answer any FOUR questions. All questions carry equal marks.**

---

1. Discuss the usefulness of the core-periphery approach in the classification of regional economies.

2. To what extent is the assertion that 'economic growth within a region is dependent on competing producers, and demand in outside markets' valid?

3 Discuss the accuracy of per capita measures in determining the development status of a region.

4 Critically examine Christaller's theorization of 'central places' and explain its applicability to regional planning.

5 'While decentralization is a term that is very common and often used without question, in development planning, it has several conceptualizations'. Discuss.

6 Explain the notion of a growth pole in regional planning and discuss its potential to produce a 'spread effect' in a developing country context.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS

GEO 952: GEOGRAPHICAL HYDROLOGY

TIME: **Three Hours**

INSTRUCTIONS: **Answer question one and any other three. Use of appropriate scientific calculators is allowed.**

---

1. Table1 shows the river X discharge data measured over a period of ten (10) years. Study the data in table 1 and answer the following questions:

Table 1: Discharge data for river X

Year	Maximum discharge ($m^3.sec-^1$)
1967	61.9
1968	98.9
1969	78.1
1970	82.3
1971	37.2
1972	48.6
1973	63.4
1974	50.7
1975	78.8
1976	79.7

Source: hypothetical

a) Rank the discharges.

b) Give the Recurrence Interval equation.

c) Calculate the Recurrence Interval for the discharges.

d) Give a brief explanation of the implication of the recurrence interval for the highest discharge.

e) Give a brief explanation for the implication of the recurrence interval for the lowest discharge.

2. Write short explanatory notes on **ALL** of the following:

   a. Groundwater flow.

   b. Darcy's Law as regards the movement of water through a porous medium.

   c. Contrast between the Bergeron and the Collision-Coalescence processes.

   d. Importance of evaporation is in water resources studies.

   e. Three types of discharge measuring structures.

3. Outline and explain the three factors that influence soil moisture infiltration.

4. As a newly appointed manager for the Kafue Catchment Council, explain in detail how you would ensure that the river is managed in a sustainable manner but at the same time improving the social and economic welfare of the communities in the catchment.

5. With the aid of a diagram, explain the different components of the Hydrological Cycle.

6. 'Water is life.' Discuss how the tretrahedral structure of water qualifies the statement when water temperature is at less than 0°C, 25°C and at 100°C.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
2012 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS
GEO 962: BIOGEOGRAPHY

Time: Three hours

Instructions: Answer any four questions. All questions carry equal marks. Use of the Philips University Atlas is allowed. Candidates are encouraged to make use of illustrations wherever appropriate.

---

1. Write short explanatory notes on ALL of the following
    a) Cladistic vicariance
    b) Principal marine biomes
    c) Arthropoda
    d) Dispersal
    e) Ecological tolerance
2. 'The theory of evolution is not of merely academic interest.' Discuss.
3. Describe the effects of the past climatic changes on the distribution pattern of species during the Pleistocene.
4. "To understand why a particular group of organism is found in a specific area requires knowledge of the organism's ecological relationships…." (Cox and Moore, 1985: vii). Discuss.
5. Imagine you are a planner in charge of the establishment of a conservation area in the tropical savanna which occasionally experience bush fires. The conservation area should take into consideration the human needs and conservation of the biodiversity of the organisms.
    a) Describe the criteria you will use to select and delimit the conservation area.
    b) Outline the appropriate management strategies for the conservation area.
6. Compare and contrast the distribution patterns of mammalian and flowering plants in the distant past and today in the world.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS

GEO 995: ENVIRONMENTAL AND NATURAL RESOURCES MANAGEMENT I

Time: Three hours

Instructions: Answer any four questions. All questions carry equal marks. Candidates are encouraged to make use of illustrations wherever possible.

---

1. Explain four benefits of conservation agriculture.

2. 'Without energy flow and nutrient cycling, ecosystems would practically ground to a halt'. Discuss.

3. Explain the key features that distinguish forests from other biomes and show how forest biomes are themselves subdivided into different types.

4. Elucidate the main challenges that have characterized programmes aimed at introducing community based forest management in Zambia.

5. Discuss the role of property regimes in the management of common pool resources.

6. Discuss the concept of species diversity in the context of the following: (a) measures of species diversity; (b) Species endemism; (c) importance of keystone species and; (d) species conservation measures.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS

GES 2502: FUNDAMENTALS OF NATURAL RESOURCES ECONOMICS

TIME:	**Three Hours**
INSTRUCTIONS:	**Answer any four questions**
MATERIALS PROVIDED:	**A4 sized metric graph papers**

---

1. Write short explanatory notes of **all** of the following:
    a) Positive and normative statements
    b) Nominal and real Gross Domestic Product
    c) Types of unemployment
    d) Factors that determine the quantity demanded of a good
    e) Reasons why we study economics

2. Outline and explain the main factors that have hindered economic growth in some natural resource rich developing countries.

3. Explain the mechanisms that firms operating in imperfect markets employ to prevent potential competitors from entering their markets.

4. Student A's consumption of doughnuts is shown in Table 1.

Table 1.Student A's consumption of doughnuts and utility

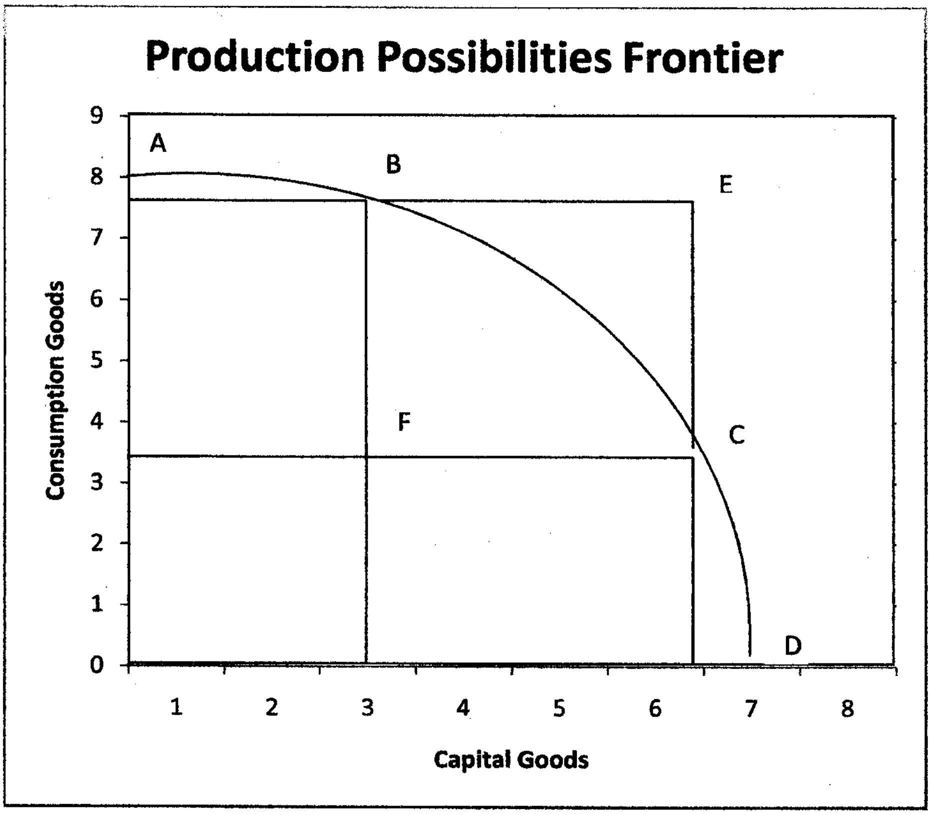
Quantity of Doughnuts Consumed	**Total Utility**	**Marginal Utility**
1	0	
2	10	
3	18	
4	24	
5	28	
6	30	
7	29	

Source: hypothetical

a. Copy Table 1 into your answer booklet and complete it by filling in the marginal utility column.
b. Using an A4 sized graph paper, plot the quantity of doughnuts consumed against total utility (total utility on y axis) using the top half of the graph paper.
c. On the same graph paper as for (b) above but using the bottom half, plot quantity of doughnuts consumed against marginal utility (marginal utility on y axis). Use the same scale for the y-axis as in (b) above.
d. On the total utility graph, mark with an X, the point at which marginal utility is 0.
e. On the marginal utility graph, mark the point below which the consumer experiences disutility.
f. What is happening to marginal utility as total utility increases?
g. What economic law is illustrated by the two graphs?

5. Figure 2 shows the production possibilities frontier (PPF) for an economy producing consumer and capital goods. Study the diagram carefully and answer the questions that follow

Figure 2. Production Possibility Frontier



(a) Which point on the PPF Curve shows a "productively efficient level" of output?
(b) What are the implications of a country's economy operating at F?
(c) Explain four factors which can lead to the rightward shift of the PPF curve?

(d) Imagine that the economy is currently operating at B. The government decides to concentrate on producing capital goods and proposes to increase its production of capital goods by operating at C. However, to do this, citizens are asked to reduce their consumption to C. This is done by the government removing food subsidies and channelling the extra income to capital goods. The citizens are not happy and decide to protest against the removed subsidies by continuing to consume at B. Meanwhile the government continues to operate at C by increasing its production of capital goods. At which point on the PPF is the economy now operating?

(e) Explain what will happen to the economy if it continues operating at the point identified in part (d).

(f) What changes would you notice to the PPF curve and the opportunity cost if the economy was producing cars in place of consumer goods and tractors in place of capital goods?

(g)

6. Table 2 shows the market demand schedule for good X. Carefully study the table and answer the questions below.

Table 2 Demand schedule for good X

Price (K)	**Quantity Demanded (Units)**	**Quantity Supplied (Units)**	**Impact on Price**
10	3	3	
8	9	9	
6	18	18	
4	25	25	
2	32	32	

(a) Copy Table 2 into your answer booklet and complete it by filling in the impact on price column.

(b) Graphically show the equilibrium price.

(c) Explain the factors which influence the amounts demanded as well as supplied at each price.

(d) Suppose a consumer's income increases from K30, 000 to K36, 000. As a result his consumption of good X increases from 25 to 30 units. What is the income elasticity of demand for good X?

(e) How would you classify good X based on your calculation of the income elasticity of demand in (c) above?

END OF EXAMINATIONS

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS

GES 5595: ENVIRONMENT AND NATURAL RESOURCES ECONOMICS

TIME: **Three Hours**

INSTRUCTIONS: **Answer any four questions. Use of an approved calculator is allowed.**

---

1. 'Rectification of nearly all problems of resource mismanagement and degradation requires improved institutions'. Discuss.

2. Discuss the pros and cons of carbon trading.

3. (a) Distinguish between static efficiency and dynamic efficiency.

   (b) Assume that you are the head of a project aiming at preserving the source of the river Zambezi. Using the information presented in Table 1, calculate the efficient number of kilometres of the river that has to be preserved from the source.
   Assumptions:
   1. Demand: P = 1000 – Q, where P is the price (cost) in Zambian kwacha.
   2. Marginal costs: P = Q, where Q is the number of kilometres.

   Table 1. Price and Number of Kilometres

P	100	200	300	400	500	600
Q	100	200	300	400	500	600

   Source: hypothetical

   (c) Explain one limitation of using the above economic approach to environmental management.

4. (a) Explain the four types of crises in fisheries management identified in Cochrane's (2000) paper on reconciling sustainability, economic efficiency and equity in fisheries.

   (b) Briefly discuss any four of Cochrane's eight principles of fisheries management.

5. Using either forest or fisheries resources for illustration, discuss the use of quotas in natural resources management.

6. Zambezi Mining Resources plans to spend US$ 474 million to open a copper mine in Lower Zambezi National Park. It is projected that 8 million tonnes of copper will be produced annually and the mine will have a life span of 25 years. Decommissioning and closure of mine is projected to cost US$ 260 000. A group of environmentalists present a petition to government contending that the environmental impact assessment for the project ignored its adverse impact on the ecosystem and the communities. The petitioners argue that the communities are currently benefiting from tourism activities in the Lower Zambezi National Park valued at US$ 10,000 annually, which would be lost if the project is allowed to proceed. The mining activities will also pollute the Zambezi, Luangwa, and Kafue rivers, and cause boreholes in the area to dry up.

   (a) Using a discount rate of 10%, and copper price of USD 7000 per tonne, calculate the Net Present Value of the proposed copper mining project.
   (b) How would you go about estimating the monetary value of the pollution to the rivers and the drying up of the boreholes?
   (c) Discuss the use of discounting in natural resources management.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

INSTITUTE OF DISTANCE EDUCATION

2012/2013 Academic Year

Semester I

M111 Mathematical Methods I

FINAL EXAMINATION

Time Allowed: Three (3) Hours

Instructions:

1. You must write your **Computer Number** on each answer booklet you have used.
2. There are Six (6) questions in this paper, Attempt **Any Five (5)** questions. All questions carry equal marks
3. Calculators are **Not** allowed.

1. (a) (i) Express $0\cdot\overline{45}$ in the form $\frac{a}{b}$ where a and b are integers.

(ii) Express $\frac{1+3\sqrt{2}}{3-\sqrt{2}}$ in the form $a+b\sqrt{2}$ where a and b are rational numbers.

(iii) Express $\frac{2+i\sqrt{3}}{\sqrt{3}-i}$ in the form $a+ib$ where a and b are real numbers.

(b) Let $P(x)=x^3-x^2+ax+b$. Given that $x-2$ is a factor of $P(x)$ and that the remainder when $P(x)$ is divided by $x-3$ is 10, find the values of a and b.

(c) Let $A=[0,\infty)$ and $B=(-7,12]$ be subsets of the universal set **R**, the set of real numbers, find the following sets and display them on a number line:

(i) B'

(ii) $B-A$

(iii) $(A-B)\cup(B-A)$

2. (a) Define a binary operation $*$ on the set of real numbers by $a*b=2^{a^2-b}$.

(i) Is $*$ commutative? Give reason for your answer.

(ii) Evaluate $(-1*-1)*4$

(b) Let $f(x)=\frac{5x}{x+1}$ and $g(x)=x^2-1$ be real valued functions.

(i) Find $f^{-1}(x)$, the inverse of $f(x)$

(ii) Solve the equation $(fog)(x)=1$

(c) A quadratic function $f(x)=ax^2+bx+c$ passes through the points $(0,3)$ and $\left(\frac{1}{2},0\right)$. The function is symmetric about the line $x=\frac{7}{4}$.

(i) Find the values of a, b and c.

(ii) Sketch the graph of the function $f(x)$.

(iii) Find the gradient of the tangent to the curve of $f(x)$ at $x=\frac{13}{8}$.

3. (a) Solve the inequalities:

(i) $3x+1 \geq x-2$

(ii) $-\frac{1}{3} \leq \frac{x}{x+2} < \frac{1}{4}$

(b) Given that A and B are sets, simplify as far as possible $A \cap [(A \cup B)' \cup B]'$.

(c) (i) Sketch the graph of $f(x) = |\, 2x+1 \,|$.

(ii) On the same diagram sketch also the graph of $g(x) = \sqrt{1-2x}$.

(iii) Hence find all values of x such that $\sqrt{1-2x} > |\, 2x+1 \,|$.

4. (a) Differentiate from the First Principle:

(i) $f(x) = \sqrt{x}$

(ii) $g(x) = \frac{1}{x^2}$

(b) Find the general solution to the equation $2\cos 2x + 1 = 0$.

(c) The function $f(x) = 2 - \sqrt{ax+1}$ crosses the x - axis at the point $(1, 0)$

(i) Find the value of a.

(ii) State the **domain** and the **range** of $f(x)$

(iii) Sketch the graph of $f(x)$.

5. (a) Solve each of the equations below:

(i) $\sqrt{5x-1} = \sqrt{x} + 1$

(ii) $\frac{1}{x-3} + \frac{1}{x-2} = \frac{3x-8}{(x-3)(x-2)}$.

(b) Prove the identity $\frac{\sin 2x + \sin x}{1 + \cos 2x + \cos x} = \tan x$

(c) The function $h(x)$ is given by $h(x) = \frac{1-2x}{3x+2}$.

(i) Find $h\left(\frac{2}{3}\right)$

(ii) Show that $h(x)$ is a one – to – one function.

(iii) Find $h'(x)$

6. (a) (i) Evaluate the following limit:

$$\lim_{x \to -\infty} \frac{2-3x-5x^2}{1+2x^2}$$

(ii) Given that $f(x) = x^2 \cos x$, find $f'\left(\frac{\pi}{4}\right)$

(b) (i) Let α and β be roots of the equation $2x^2 - 5x + 3 = 0$. Find an equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

(ii) Find $\frac{dy}{dx}$ if $y = (3x^3 - 1)^{4/3}$

(c) Let $f(x) = 1 - 2\cos 2x$ be a function.

(i) Determine the **amplitude** and the **period** of the function.

(ii) Sketch the graph of $f(x)$ in the interval $-\pi \le x \le 2\pi$.

(iii) Find all solutions to the equation $f(x) = 3$ in the interval $-\pi \le x \le 2\pi$.

End of Exam

THE UNIVERSITY OF ZAMBIA

INSTITUTE OF DISTANCE EDUCATION

Department of Mathematics and Statistics

2012 Academic Year

Semester II

M112 Mathematical Methods II

DEFERRED FINAL EXAMINATION

Time Allowed: Three (3) Hours **September, 2013.**

Instructions:

1. You must write your **Computer Number**, on each answer booklet you have used.
2. There are Six (6) questions in this paper, Attempt **Any Five (5)** questions. All questions carry equal marks
3. Calculators are **Not** allowed.

---

1. (a) (i) Let $S_n = 1 + 2 + 3 + 4 + \ldots\ldots\ldots\ldots + n$. Show that $2S_n = n(n + 1)$

(ii) Find the fourth term and the term independent of x in the expansion of $\left(x^2 - \frac{2}{x^2}\right)^8$

(b) Let $f(x) = 3x^3 + 4x^2 - x + 2$

(i) Find the extreme points of $f(x)$

(ii) Determine the intervals where $f(x)$ is increasing and where it is decreasing.

(iii) Sketch the graph of $f(x)$.

(iv) Find the equation of the tangent to the curve at the point (0 , 2)

2. (a) Let $A = \begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} -2 & 1 & 7 \\ -2 & 5 & -1 \end{pmatrix}$ and $C = \begin{pmatrix} -2 & -14 \\ -3 & 19 \\ 15 & 25 \end{pmatrix}$. Find

(i) $C^T - 3B$

(ii) CA^{-1}

(iii) The relationship between the matrices A , B and C

(b) (i) Express the complex number $\dfrac{\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^5}{\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)^2}$ in the form $a + ib$ where a and b are rational numbers.

(ii) Find the area bounded by the curve $y = 5\sin 2x$ and the x – axis between the lines $x = \frac{-\pi}{4}$ and $x = \frac{\pi}{2}$

3. (a) (i) The second term in the expansion of $(1 - x)(1 + 2x)^n$ is $19x$. Find the value of n

(ii) Expand $\left(\frac{p+1}{p}\right)^{-3}$ as a series of ascending powers of $\frac{1}{p}$ as far as and including the term containing $\frac{1}{p^3}$. State the range of values of p for which the expansion is valid.

(b) Let $f(x) = x^3 + ax^2 + bx + c$ where a , b and c are constants.

(i) If the graph of $f(x)$ passes through the point (3 , 4) and has a turning point at $x = 2$, express b and c in terms of a

(ii) If further the point of inflection of $f(x)$ is at $x = 1$ find the values of a , b and c.

(iii) Hence sketch the graph of $f(x)$.

4. (a) Evaluate the following integrals

(i) $\int_0^2 f(x)dx$ where $f(x) = \begin{cases} 2x^2 & if \quad 0 \le x \le 1 \\ 3 - \frac{1}{x} & if \quad 1 < x \le 2 \end{cases}$

(ii) $\int_0^{\frac{\pi}{2}} (x^2 + \cos x)dx$

(iii) $\int_{\frac{1}{2}}^1 \left(1 + \frac{1}{u^2}\right)^2 du$

(b) (i) Use Cramer's rule to solve the system of equations

$x + 2y + 3z = 6$

$2x + y + z = 5$

$3x + y - 2z = 1$

(ii) Find the center and the radius of the circle $2x^2 + 2y^2 + 3y = 0$

5. (a) Let $\boldsymbol{a} = 10\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$, $\boldsymbol{b} = 2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$ and $\boldsymbol{c} = \mathbf{i} + 10\mathbf{j} - 2\mathbf{k}$. Find

(i) $\boldsymbol{a} \bullet (\boldsymbol{b} + \boldsymbol{c})$

(ii) $\boldsymbol{a} \bullet (\boldsymbol{b} \times \boldsymbol{c})$

(iii) the cosine of the acute angle between $\boldsymbol{b}$ and $\boldsymbol{c}$

(b) (i) Find the inverse of the matrix

$$M = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & 1 \\ 1 & 3 & 0 \end{pmatrix}$$

(ii) Calculate the distance from the point Q(4 , −1) to the line $2x - y + 5 = 0$

6. (a) Find the following integrals

(i) $\int \frac{x^4 + 1}{x^2} dx$

(ii) $\int \left(1 - x^{\frac{3}{2}}\right)^4 \sqrt{x}dx$

(iii) $\int x \sin 2x dx$

(b) (i) Express $\frac{5x + 2}{(2x - 1)(x + 1)}$ into partial fractions and hence expand the expression as a series in ascending powers of x giving the first four terms.

(ii) Find the coordinates of a point P which divides the line segment joining the points A(2 , 5) and B(−3 , 8) internally in the ration 3:4

End of Exam

The University of Zambia

School of Natural Sciences

Department of Mathematics and Statistics

2012 Academic year

Second Semester Examinations

M112 Mathematical Methods II

Time Allowed: Three (3) Hours **19th August 2013**

Instructions:

1. You must write your **Computer Number**, Your **TG Number** on each answer booklet used.
2. Indicate the number of each question attempted in the first column on the right side of the main answer booklet
3. There are **seven (7)** questions in this paper. You must answer any **five (5)** questions. All questions carry equal marks.
4. No **Calculators** to be used

---

1. (a) Solve the following equations;

(i) $2^{2x} - 5(2^{x+1}) + 16 = 0$

(ii) $\log_2 x + 4\log_x 2 = 5$

(b) (i) Find the term independent of x in the expansion of

$$\left(\frac{1}{3x} - \frac{3x^2}{2}\right)^9$$

(ii) Given that $y = e^x + x^2$. Prove that $\frac{d^2y}{dx^2} = y + 2 - x^2$

(c) Show that $\int_1^{\sqrt{3}} \frac{x+3}{\sqrt{4-x^2}} dx = \frac{\pi}{2} + \sqrt{3} - 1$

2. (a) (i) Express $\frac{x^2+x+1}{(x+1)(x^2+1)}$, in partial fractions

(ii) Show that $\int_0^1 \frac{x^2+x+1}{(x+1)(x^2+1)} dx = \frac{3}{4}\ln 2 + \frac{\pi}{8}$

(b) The point (6,3) divides the line segment PQ where P(4,5) and Q(x,y) in the ratio 2:5 internally. Find

(i) the coordinates of Q

(ii) the coordinates of the midpoint of PQ

(c) Use Mathematical induction to prove that $x^{2n} - y^{2n}$ is divisible by x – y for every positive integer values of n.

3. (a) (i) Given that $A = \begin{pmatrix} 1 & 0 & 2 \\ t & 3 & 1 \\ -2 & -1 & 1 \end{pmatrix}$, find the value of t if the matrix A is singular.

(ii) Given that $B = \begin{pmatrix} 0 & -1 & 1 \\ 2 & -1 & 2 \\ 1 & 0 & 1 \end{pmatrix}$. Show that $B^3 = I_3$

(iii) Use (ii) above to find B^{-1}

(b) (i) By using De Moivrés' Theorem, or otherwise , show that $\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$

(ii) Find the roots of the equation $z^5 - 1 = 0$ in the form $r(\cos\theta + i\sin\theta)$ and show them on an argand diagram

(c) (i) Find in Cartesian form an equation of an equation of the circle which passes through the points (2,0), (8,0), and (10,4).

(ii) Prove that the y-axis is a tangent to the circle in (c)(i) above.

(iii) State the coordinates of the point of contact of the tangent in (ii) to the circle in (i) above.

4. (a) For the curve $y = \dfrac{x^2 + 9}{x^2 - 1}$, find

(i) The turning point(s) of the curve y

(ii) The vertical asymptotes

(iii) The Horizontal asymptotes

(iv) Sketch the curve .

(b) (i) The angle between the vector i + j and $2i + j + \lambda k$ is $\frac{\pi}{4}$, find the possible values of λ.

(ii) Solve the equation $3\sinh^2 x - 2\cosh x - 2 = 0$

(c) (i) Use Binomial theorem to expand $(3 + 10x)^4$, giving each coefficient as an integer.

(ii) Use your expansion in (i) above with appropriate value of x, to find the exact value of $(1003)^4$

5. (a) The curve C is given by $y = \frac{15 - 2x - x^2}{4}$. The line with equation x +2y =3 Meets the curve C at the points P and Q. Denote the region bounded by C and the line PQ by R.

(i) Determine the coordinates of P and Q

(ii) Sketch the curve C and the line PQ

(iii) Find the area of region R.

(b) Find (i) $\int \cos^3 x dx$

(ii) $\int x(3x+1)^{-\frac{1}{2}} dx$

(iii) $\int \frac{dx}{\sqrt{x}(3+\sqrt{x})^3}$

(c) Using Mathematical Induction prove that

$1^2 + 3^2 + 5^2 + \ldots\ldots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$, for all positive values of integer n.

6. (a) Let $A = \begin{pmatrix} 1 & 0 & 1 \\ 3 & 1 & 1 \\ 4 & 2 & 7 \end{pmatrix}$

(i) Find A^{-1}

(ii) Hence, solve the system of equations

$$x + z = 2$$
$$3x + y + z = 3$$
$$4x + 2y + 7z = 1$$

(b) Find the equation of the circle whose centre is at (-2,-5) and passes though the Point (6,1).

(c) Given that $y = x^3 - x^2 - x + 1$, find

(i) The stationary points

(ii) State the nature of the stationary points

(iv) Determine the intervals where the curve y is increasing and decreasing.

(v) Sketch the curve of y

7. (a) The variable point P(x,y) moves in such a way that $AP^2 = 4BP^2$ where A is the point (1,3) and B is the point (4,-3)

(i) Show that P lies on the circle C with equation

$x^2 + y^2 - 10x + 10y + 30 = 0$

(ii) The line OT is a tangent to the circle C, where O is the origin and T lies on the circle. Calculate the length OT.

(b) (i) Find the modulus and argument of the complex number $\frac{5+i}{3-2i}$

(ii) Solve the equation $Z^4 - \frac{5+i}{3-2i} = 0$, giving your answer in the form $r(\cos\theta + i\sin\theta)$.

(a) (c) (i) USING CRAMER'S RULE.

$$x + y + 2z - 2 = 0$$
$$-3x + y + 2z - 1 = 0$$
$$6x + 2y + z + 4 = 0$$

(ii) Given that **a**= –i + 2j +-5k and **b** = 5i – 2j + k , find a unit vector which is perpendicular to both **a** and **b**

END of EXAMINATION

UNIVERSITY OF ZAMBIA

DEPARTMENT OF MATHEMATICS AND STATISTICS

SESSIONAL EXAMINATIONS, SECOND SEMESTER 2012

M162 - INTRODUCTION TO MATHEMATICS, PROBABILITY AND STATISTICS II

INSTRUCTIONS

Total Time allowed is three (3) Hours. THERE ARE **SEVEN** QUESTIONS IN THIS PAPER WHICH ARE DIVIDED INTO TWO SECTIONS. ANSWER ANY **FIVE** QUESTIONS IN TOTAL WITH ATLEAST **THREE** QUESTIONS FROM SECTION **A** AND AT LEAST **ONE** QUESTION FROM SECTION **B.**

SECTION A

1 a. Find the equation of the tangent line to the curve $y^2 = \frac{x^3}{4-x}$ at point (2,2).

b. The population of Zambia (in millions) from year 1980 through 2010 is described by the function $P(t) = 0.007t^2 + 0.78t + 3.6$ where t = 0 represents the year 1980.

(i) Find P(0) and state what it means.

(ii) Determine the rate of change of population at t = 1 and at t = 2 and draw a conclusion from your values.

c. Find the following limits

(i) $\lim_{x \to 0} \frac{\sqrt{x+1}-1}{x}$

(ii) $\lim_{x \to -3} \frac{x^2+x-6}{x+3}$

2 a. Let a function f(x) be defined as

$$f(x) = \begin{cases} 18, & 0 < x \leq 1 \\ 20, & 1 < x \leq 2 \\ 22, & 2 < x \leq 3 \end{cases}$$

(i) Sketch the graph of f(x)

(ii) Evaluate $\lim_{x \to 2} f(x)$

(iii) State points of discontinuity if there are any

b. In a population of bacteria, the number of bacteria at time t is given by $P(t) = 500(1 + \frac{4t}{50+t^2})$, where time t represents hours. Find the rate of change of population at t = 1. You may leave your answer unsimplified.

c. (i) Find y given that x= lny

(ii) Show that $\ln e = 1$

(iii) Given that $\ln(e^y) = \ln\left(\sqrt{\frac{x^2+1}{x^2-1}}\right)$

show that $y = \frac{1}{2}[\ln(x^2 + 1) - \ln(x^2 - 1)]$

(iv) Use y from part (iii) to find dy/dx

3 a. Given the curve $f(x) = x^3 - 3x^2 + 3x$

(i) Find the points at which the curve f(x) crosses the x-axis

(ii) Show that f(x) is increasing on entire real line

(iii) Determine the intervals on which curve f(x) is concave up and concave down

(iv) Sketch the curve f(x)

b. Use the first derivative test to find all relative extrema of the function $f(x) = x^4 - x^3$

c. Evaluate the following integrals:

(i) $\int x\sqrt{x^2+2}$ dx

(ii) $\int \frac{2x}{x^2+3} dx$

(iii) $\int_{-1}^{2} e^{2t} dt$

(iv) $\int \frac{1}{xlnx} dx$

4 a. Given the function $g(x) = (x-2)(x+1)^2$

(i) By applying the first and second derivative test, find all points of relative extrema and inflection

(ii) Sketch the graph of f(x)

b. (i) Evaluate $\int_0^1 (2t+1)^3 \, dt$

(ii) Sketch the region R bounded by the x-axis and $f(x) = x^2 + 1$, $2 \leq x \leq 3$

(iii) Find the area of R

c. Find the equation of the tangent line to the curve y = ln(x^3) at the point (1,0)

5 a. Given the function $F(x) = \begin{cases} x^2 + 1, & x \le 0 \\ x, & x > 0 \end{cases}$

(i) Sketch the graph of F(x)

(ii) Find $\lim_{x \to 0} F(x)$

(iii) Determine if F(x) is continuous at x=0

b. A farmer has 750 meters of fencing material. He wants to enclose a rectangular area by fencing it and then divide the fenced area in to four equal size pens using fencing material. Find the dimensions of a pen such that the area enclosed in it is maximum.

c. Find the derivative of the following functions:

(i) $y = (2x^2 + 3x - 1)^2$

(ii) $y = \frac{1}{x^2+1}$

(iii) $y = \ln\left(\frac{x^3}{2x+1}\right)$

(iv) $y = e^{x^2} + \frac{1}{e^x}$

SECTION B

6 a. The following data is on the amount of phosphate (measured in grams) found in one kilo gram soil at each of the twenty five mining sites in Zambia included in a study.

0.74 6.47 1.90 2.69 0.75 0.32 9.99 1.76 2.41 1.96 1.66 0.7

2.42 0.54 3.36 3.59 0.37 1.09 8.32 4.06 4.55 0.76 2.03 5.7

12.48

(i) Group the data into class intervals of equal width with the first class interval as " 0 - < 2"

(ii) Draw a relative frequency histogram for these data. You may use calculation $\frac{1}{25} = 0.4$

(iii) Find what percent values are smaller than 2

b. Use the data of part (a)

(i) Find the mean

(ii) Find the median

c. A set of 340 examination scores exhibiting a bell shaped relative frequency distribution has mean 72 points and standard deviation 8 points. Determine approximately what percentage of the scores would you expect to fall in the interval (56, 88).

7. a.(i) What do you understand by mutually exclusive events? Explain by giving an example.

(ii) Suppose you hold 20 out of a total 500 tickets sold for a lottery. The grand prize winner is determined by the random selection of one of the 500 tickets. Find the probability that you win the grand prize.

b. Two balanced dice are rolled. Find the probability that

(i) the number three shows up at least on one die

(ii) different numbers show up on two dice

c. A soap manufacturer decides to market two new brands. After an analysis of current market conditions, the manufacturer assesses all possible combinations of success (S) and failure (F) of each brand within the first year after launch and comes up with the following probabilities:

Outcome	S S	S F	F S	F F
Probability	0.09	0.21	0.21	0.49

Define the following events:

A: Both new brands are successful in the first year

B: At least one new brand is successful in the first year

Find

(i) P(A)

(ii) P(B)

(III) P(A∩B)

(iv) P(A/B)

The University of Zambia

Department of Mathematics & Statistics

2012 Academic Year
Semester 2 Examinations

M212 - Mathematical Methods - IV

Time allowed : Three (3) hrs Full marks : 100

Instructions:

1. Attempt **any five (5)** questions out of the **six (6)** questions.
2. All questions carry equal marks.
3. **Full credit** will only be given when **necessary work** is shown.
4. Indicate your **computer number** on all answer booklets.

This paper consists of 4 pages of questions.

---

1. **(a)** **(i)** Let P be a point not on the line L that passes through the points Q and R. Show that the distance d from the point P to the line L is:

$$d = \frac{|\overrightarrow{QR} \times \overrightarrow{QP}|}{|\overrightarrow{QR}|}.$$

Hence,

(ii) find the distance from the point $P(0, 6, 8)$ to the line through $Q(1, 1, 1)$ and $R(-1, 4, 7)$.

(b) **(i)** Given the lines:

$L_1 :\ x = 1{+}t,\ y = -2{+}3t,\ z = 4{-}t \quad L_2 :\ x = 2s,\ y = 3{+}s,\ z = -3{+}4s.$

Show that the lines L_1 and L_2 are skew.

(ii) Find symmetric equations of the line which passes through the point $(-1, 1, 2)$ and whose direction vector is orthogonal to the direction vectors of the lines:

$$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-1}{3} \quad \text{and} \quad \frac{x-1}{5} = \frac{y-2}{2} = \frac{z+1}{-3}.$$

(c) (i) If $P(x_0, y_0, z_0)$ is a point in space and D is the distance from P to the plane $ax + by + cz + d = 0$, use scalar projection to prove that

$$D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}.$$

(ii) Find the distance between the planes

$$10x + 2y - 2z = 5, \quad \text{and} \quad 5x + y - z = 1$$

2. **(a) (i)** Find a vector function that represents the curve of intersection of the cylinder $y^2 + z^2 = 1$ and the plane $x + z = 2$.

(ii) Suppose $\mathbf{u}$ is a differentiable vector function and f is a real-valued function, prove that

$$\frac{d}{dt}[f(t)\mathbf{u}(t)] = f'(t)\mathbf{u}(t) + f(t)\mathbf{u}'(t).$$

(b) Suppose C is a curve given by the vector equation

$$\mathbf{r}(t) = \cos t\mathbf{i} + t\mathbf{j} + \sin t\mathbf{k}; \; 0 \leq t \leq 2\pi.$$

(i) Find the equation of the normal plane of the curve at the point $(0, \frac{\pi}{2}, 1)$.

(ii) Sketch the curve C, indicating the direction of motion of a particle moving on it.

(c) (i) Show that, if $|\mathbf{r}(t)| = c$ (a constant), then $\mathbf{r}'(t)$ is orthogonal to $\mathbf{r}(t)$ for all t.

(ii) Given that the curvature of the curve $\mathbf{r}(t)$ is given by

$$K(t) = \frac{|\mathbf{T}'|}{|\mathbf{r}'|},$$

where $\mathbf{T}$ is the unit tangent vector of the vector $\mathbf{r}$, prove that

$$K(t) = \frac{|\mathbf{r}' \times \mathbf{r}''|}{|\mathbf{r}'|^3}.$$

3. **(a)** A moving particle starts at an initial position $\mathbf{r}(0) = \langle 0, 1, 0\rangle$, with initial velocity $\mathbf{v}(0) = \langle -1, 1, 1\rangle$, and its acceleration is $\mathbf{a}(t) = \langle 6t, 4t, 1\rangle$. Find its velocity and position at time t.

   (b) **(i)** Find parametric equations for the tangent line to the helix with parametric equations

$$x = 2\cos t,\ y = \sin t,\ z = t$$

   at the point $P(0, 1, \frac{\pi}{2})$.

   (ii) Find the length of the arc of the helix given by

$$x = \cos t,\ y = \sin t,\ z = t$$

   from the point $(1, 0, 0)$ to $(1, 0, 2\pi)$.

   (c) **(i)** find the curvature of the twisted cubic

$$\mathbf{r} = \langle t, t^2, t^3\rangle$$

   at the point $(0, 0, 0)$.

   (ii) Show that the function $f(x, y) = x^4 + 2x^3y - 3x^2y^2 + xy^3 - 4y^4$ satisfies Euler's theorem of homogeneous functions.

4. **(a)** Show that the equation

$$xy^2z + z^2 = 4 + xe^z$$

   defines z implicitly as a function of x and y, say, $z = f(x, y)$. Hence, find $\frac{\partial f}{\partial x}$ at the point $(0, e, 2)$.

   (b) Find the extrema of the function

$$f(x, y) = x^2y + 9x - 3y^2 - 6.$$

   and hence, determine their nature.

   (c) **(i)** Show that the function

$$f(x, y) = \begin{cases} \frac{x^2+y^2}{\sin(x^2+y^2)}, & \text{if}(x, y) \neq (0, 0), \\ 1, & \text{if}(x, y) = (0, 0) \end{cases}$$

   is continuous at $(0, 0)$.

   (ii) Given that f is a differentiable function with $f(5, 2) = 6$, $f_x(5, 2) = -1$, and $f_y(5, 2) = -1$, find the approximate value of $f(5.1, 1.9)$.

5. **(a)** Find the absolute minimum and maximum of

$$f(x, y) = x^2 + y^2 + 2y - 1$$

on

$$D = \{(x, y)|x^2 + \frac{y^2}{4} \leq 1\}.$$

(b) Assume that f and g have continuous second order partial derivatives, show that any function of the form

$$z = f(x + at) + g(x - at)$$

is a solution of the wave equation

$$\frac{\partial^2 z}{\partial t^2} = a^2 \frac{\partial^2 z}{\partial x^2}.$$

(c) Use the method of variation of parameters to solve the equation

$$y'' - y = e^{-2x}.$$

6. **(a)** Solve the following equations:

i) $y' + y \tan x = \sin 2x; \quad y(0) = 1$

ii) $y'' + 3y' + 2y = 1 + 3x + x^2; \quad y(0) = 0, \quad y'(0) = 1.$

(b) i) Verify that $y = e^x$ is a solution of the homogeneous equation

$$(x - 1)y'' - xy' + y = 0.$$

ii) Find the second solution and find the general solution of $(x-1)y'' - xy' + y = 0$.

(c) Consider the differential equation

$$x^2 \frac{d^2 y}{dx^2} - x\frac{dy}{dx} + y = \ln x.$$

By the substitution $x = e^t$, or $t = \ln x$, use the chain rule of differentiation and show that the equation reduces to

$$\frac{d^2 y}{dt^2} - 2\frac{dy}{dt} + y = t.$$

END OF EXAM!

THE UNIVERSITY OF ZAMBIA

INSTITUTE OF DISTANCE EDUCATION

2012/2013 Academic Year

Semester I

M221 Linear Algebra I

FINAL EXAMINATIONS

Time Allowed: Three (3) Hours

Instructions:

1. You must write your **Computer Number**, on each answer booklet you have used.
2. There are Six (6) questions in this paper, Attempt **Any Five (5)** questions. All questions carry equal marks

1. (a) (i) Express the matrix $\begin{pmatrix} 1 & 1 & 0 & -1 \\ 2 & -1 & 2 & 0 \\ -1 & 4 & 1 & 1 \end{pmatrix}$ in its reduced echelon form.

(ii) Factorize the determinant:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

(b) Determine the value of k so that the system

$$x + y + kz = 1$$
$$x + ky + z = 1$$
$$kx + y + z = -2$$

(i) has a unique solution

(ii) has no solution

(iii) has infinitely many solutions.

2. (a) Let V and W be vector spaces over the field F.

(i) Define a subspace U of a vector space V

(ii) Define a linear transformation $T : V \rightarrow W$.

(iii) Let $T : V \rightarrow W$ and $S : W \rightarrow U$ be linear transformations. Let $ST : V \rightarrow U$ be defined by $(ST)(v) = S(T(v))$ for all $v \in V$. Show that ST is a linear transformation.

(b) (i) Let U and W be subspaces of a vector space V, prove that $U \cap W$ is a subspace of V.

(ii) Use elementary row operations to find the inverse of the matrix:

$$M = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}$$

3. (a) Define the following:

(i) A basis of a vector space $V_n(\mathbf{R})$

(ii) The kernel of a linear transformation $T : V \rightarrow W$.

(iii) The image of a linear transformation $T : V \rightarrow W$

(b) (i) Let $T : V_4(\mathbf{R}) \rightarrow V_3(\mathbf{R})$ be a linear transformation defined by:

$$T(\alpha_1 , \alpha_2 , \alpha_3 , \alpha_4) = (\alpha_1 - \alpha_2 + \alpha_3 + \alpha_4 , \alpha_1 + 2\alpha_2 - \alpha_3 + \alpha_4 , 3\alpha_2 - 2\alpha_3)$$

Find the bases for $\ker T$ and $\operatorname{Im} T$.

(ii) Let S be a subspace of $V_n(\mathbf{R})$. Given that $B = \{u_1 , u_2 , \ldots\ldots , u_r\}$ and $C = \{v_1 , v_2 , \ldots\ldots , v_t\}$ are two bases for S, prove that B and C contain the same number of vectors.

4. (a) Let $S=\{(1,0,-1,1),(2,-1,0,1),(1,1,2,1)\}$

(i) Show that $x=(1,3,3,2)\in$ span S and $y=(0,1,1,-1)\notin$ span S.

(ii) Find a basis for Span S which contains the vector x.

(iii) Find a basis for $V_4(\mathbf{R})$ which contains the vector y.

(b) Determine the dimension of the vector subspace;

(i) spanned by the matrices:

$$A=\begin{pmatrix}1 & 0\\ 2 & 1\end{pmatrix},\ B=\begin{pmatrix}1 & -1\\ 0 & 0\end{pmatrix} \text{ and } C=\begin{pmatrix}-1 & 0\\ 1 & 2\end{pmatrix}$$

(ii) defined by $S=\{(x,y,z):x-2y-2z=0\}$.

5. (a) Let $v_1=(1,0,1),v_2=(2,-1,1)$ and $v_3=(4,1,1)$

(i) Show that these vectors are linearly independent.

(ii) Find the coordinates of the vector $u=(1,-1,1)$ relative to these vectors.

(b) Let $\{u_1,u_2\}$ be a basis for $V_2(\mathbf{R})$ and let $\{v_1,v_2,v_3\}$ be a basis for $V_3(\mathbf{R})$. A linear transformation $T:V_2(\mathbf{R})\rightarrow V_3(\mathbf{R})$ is defined by:

$$T(u_1)=v_1+2v_2-v_3$$
$$T(u_2)=v_1-v_2$$

(i) Find the matrix of T relative to these bases.

(ii) Find also the matrix of T relative to the bases $\{-u_1+u_2,2u_1-u_2\}$ and $\{v_1,v_1+v_2,v_1+v_2+v_3\}$ for $V_2(\mathbf{R})$ and $V_3(\mathbf{R})$ respectively.

(iii) What is the relationship between these two matrices?

6. (a) (i) Let A be an $m\times n$ matrix and let

$$N_A=\{x\in\mathbf{R}_n:Ax=0\}.$$

Show that N_A is a subspace of $\mathbf{R}_n$

(ii) Find the nullity of the matrix;

$$A=\begin{pmatrix}1 & 3 & -2\\ -3 & -9 & 6\\ 2 & 6 & -4\end{pmatrix}$$

(b) Let T be a linear transformation $T:V_2(\mathbf{R})\rightarrow V_2(\mathbf{R})$ defined by:

$$\begin{matrix}T(u_1)=u_1-u_2\\ T(u_2)=u_1\end{matrix}, \text{ where } \{u_1,u_2\} \text{ is a basis for } V_2(\mathbf{R}).$$

(i) Find the matrix A of T relative to the basis $\{u_1,u_2\}$.

(ii) Find the matrix B of T relative to the new basis $\{v_1,v_2\}$, where

$$v_1=3u_1-u_2$$
$$v_2=u_1+u_2$$

(iii) Find an invertible matrix S such that $S^{-1}AS=B$

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR
SECOND SEMESTER EXAMINATIONS

M292: INTRODUCTION TO PROBABILITY

Time Allowed: Three (3) Hours

Instructions:
1. Answer any **Five (5)** Questions
2. Show All Essential Working
3. Calculators are Allowed
4. All fractions must be expressed in lowest terms

---

1. (a) Define the following:
   - (i) mutually exclusive events $A_1, A_2, \ldots, A_k$.
   - (ii) a permutation.

   (b) Three manufacturers of DVDs A, B and C, produce 15%, 25% and 60% respectively of all DVDs made. Manufacturer A produces 5% defective DVDs, manufacturer B produces 7% defective DVDs and manufacturer C produces 4% defective DVDs. A DVD is chosen at random.
   - (i) Find the probability that it is defective.
   - (ii) If it is defective, what is the probability that it came from manufacturer B?
   - (iii) If it is not defective, what is the probability that it came from manufacturer A?

   (c) A discrete random variable X has probability density function

x	-2	-1	0	1	2
$P(X = x)$	0.1	0.1	k	0.4	0.1

   Find
   - (i) the value of k.
   - (ii) the cumulative distribution function of X.
   - (iii) E(X)
   - (iv) Var(X)

2. (a) State and prove Baye's Theorem.

(b) A and B are two events such that $P(A)=\frac{8}{15}$, $P(B)=\frac{2}{3}$ and $P(A \cap B)=\frac{1}{5}$. Find

(i) $P(B|A)$
(ii) $P(A'|B)$
(iii) $P(A \cup B')$

(c) Suppose that X and Y are continuous random variables with joint probability function

$$f(x,y)=k(x^2+y^2) \quad , \quad -1<x<1 \quad ; \quad -1<y<1 ,$$

(i) Show that $k=\frac{3}{8}$.

(ii) Find the marginal probability function $f_X(x)$.

3. (a) State four properties of a cumulative distribution function F(x).

(b) If all the letters of the word BIOLOGY are to be arranged in a line, find the

(i) number of distinct letter arrangements possible.
(ii) probability that the arrangement starts and ends with the letter O.
(iii) probability that the two O's are together.

(c) A continuous random variable X has probability function

$$f(x)=\begin{cases} \frac{k}{3}x & , \quad 0 \le x < 3 \\ k & , \quad 3 \le x \le 4 \\ 0 & , \quad otherwise \end{cases}$$

(i) Show that $k=\frac{2}{5}$.

(ii) Find Var(X).
(iii) Find the cumulative distribution function of X.

4. (a) Given the Geometric probability function

$$P(X = x) = p(1-p)^{x-1}, \quad x = 1, 2, \ldots$$

(i) Show that the moment generating function of X is given by

$$M_X(t) = \frac{pe^t}{1-qe^t} \quad , \quad t < \ln\left(\frac{1}{q}\right)$$

(ii) Use the moment generating function in (i) to find $E(X)$.

(b) A biased coin that shows heads with probability 0.3 is tossed. Find the probability that

(i) at least 2 heads are obtained in 8 tosses.
(ii) the first head occurs on the 4th toss.
(iii) the second head occurs on the 4th toss.

(c) Given that X follows a Beta distribution with probability density function

$$f(x) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1} \quad , \quad 0 < x < 1 \quad , \quad \alpha > 0 \quad , \quad \beta > 0$$

Find

(i) E(X)
(ii) Var(X)

5. (a) Define the following:

(i) correlation coefficient of two random variables X and Y.
(ii) joint moment generating function of $X_1, X_2, \ldots, X_n$

(b) Prove the following:

(i) If X and Y are jointly distributed random variables then $E[E(X|Y)] = E(X)$.
(ii) For any two random variables X and Y $Cov(X-Y, X+Y) = Var(X) - Var(Y)$.

(c) Suppose X and Y have joint probability function

		x	
$f(x,y)$		0	1
	0	0.1	0.15
y	1	0.2	0.3
	2	0.1	0.15

Find

(i) the marginal probability functions $f_X(x)$ and $f_Y(y)$.
(ii) $f(x|y)$
(iii) $E(X|y)$
(iv) $\rho(X,Y)$

6. (a) Riots at a certain college occur at a rate of 3 per year and are known to follow a poisson distribution. Find the probability that there are

(i) 4 riots in a year.
(ii) 8 riots in 4 years.

(b) Let X be an exponential random variable with probability density function $f(x) = \lambda e^{-\lambda x},\ x > 0$.

(i) Find the cumulative distribution function of X.
(ii) Find the moment generating function of X.
(iii) Use the moment generating function in (ii) to find Var(X).

(c) From a group of 6 men and 8 women, 5 people are chosen at random. Find the probability that

(i) 2 men and 3 women are chosen.
(ii) More men than women are chosen.

END OF EXAMINATION

The University of Zambia
School of Natural Sciences
Department of Mathematics and Statistics
End of Second Semester Examinations
M 325 Group and Ring Theory

Duration: Three (3) hours

Instructions:

(i) There are six questions in this paper
(ii) Answer any five
(iii) All questions carry equal marks

---

1. (a) Define the following terms:

(i) The greatest common divisor of two non-zero integers a and b.
(ii) An equivalence relation on a set S.

(b) Prove the following:

(i) If a and b are any non-zero integers, then there exist integers s and t such that $\gcd(a,b) = as + bt$, and $\gcd(a,b)$ is the smallest positive integer of the form $as + bt$.

(ii) The equivalence classes of an equivalence relation on a set S constitute a partition of S.

(c) (i) State the Euclidean algorithm.

(ii) Apply the Euclidean algorithm to find $\gcd(3274, 286)$.

2. (a) Give the definition of each of the following:

(i) A group G.
(ii) The order of an element in a group.

(b) Suppose that G is a group and $g \in G$. Prove that

(i) if $|g| = \infty$, then all distinct powers of g are distinct group elements. If $|g| = n < \infty$, then $\langle g \rangle = \{e, g, g^2, ..., g^{n-1}\}$ and $g^i = g^j$ if and only if n divides $i - j$.

(ii) If $G = \langle g \rangle$ is cyclic and $|G| = n$, then $\langle g^k \rangle = G$ if and only if $\gcd(k, n) = 1$.

(c) Let G be an Abelian group generated by the elements α and β, where $\alpha^3 = e = \beta^2$.

(i) Construct the Cayley table for G.

(ii) Find the order of each element of G.

(iii) Find all the subgroups of G.

3. (a) Define

(i) the index of a subgroup H of a group G,
(ii) the external direct product of groups $G_1, G_2, ..., G_n$.

(b) (i) State and prove Lagrange's Theorem.

(ii) Prove that if $G_1, G_2, ..., G_n$ are groups and $(g_1, g_2, ..., g_n)$ is in $G_1 \oplus G_2 \oplus \cdots \oplus G_n$, then $|(g_1, g_2, ..., g_n)| = \operatorname{lcm}(|g_1|, |g_2|, ..., |g_n|)$.

(c) (i) Let G be a group and $g \in G$. If $|g| = 15$, find all the left cosets of $\langle g^5 \rangle$ in $\langle g \rangle$.

(ii) Let G be an Abelian group generated by the elements α and β, where $\alpha^2 = e = \beta^2$. List the elements of $G \oplus S_3$.

(iii) Show that $G \oplus S_3$ in (ii) above has a subgroup of order 3 and a subgroup of order 8.

4. (a) Give the definition of each of the following:

(i) A Sylow p-subgroup.
(ii) A group homomorphism.

(b) (i) Prove Sylow's First Theorem.

(ii) State and prove the First Isomorphism Theorem.

(c) (i) Find all the Sylow 2-subgroups and Sylow 3-subgroups of the symmetric group S_3.

(ii) Let G be a group of permutations. For each $\sigma \in G$, define

$$\operatorname{sgn}(\sigma) = \begin{cases} +1 & if\ \sigma \text{ is an even permutation} \\ -1 & if\ \sigma \text{ is an odd permutation} \end{cases}$$

Show that this is a homomorphism from G to the multiplicative group $\{+1, -1\}$. What is the kernel?

5. (a) Define the following:

(i) A ring R.
(ii) An integral domain D.

(b) Let a, b and c be elements from a ring and let m and n be integers. Prove the following:

(i) $a(-b) = (-a)b = -(ab)$,

(ii) $a(b-c) = ab - ac$,

(iii) $(ma)(nb) = (mn)ab$,

(iv) $n(-a) = -(na)$,

(v) $m(ab) = (ma)b = a(mb)$.

(c) (i) Prove that every finite integral domain is a field.

(ii) Show that $R = \{0, 2, 4, 6, 8\}$ under addition and multiplication modulo 10 is a field.

6. (a) What is meant by the following:

(i) An ideal I in a ring R.
(ii) A homomorphism $\phi : R \to S$ between rings.

(b) (i) Let R be a ring and I a subring of R. Prove that $R/I = \{r + I : r \in R\}$ is a ring under the operations $(s+I)+(t+I) = s+t+I$ and $(s+I)(t+I) = st+I$ if and only if I is an ideal of R.

(ii) Let $\phi : R \to S$ be a ring homomorphism. Show that if A is a subring of R, then $\phi(A) = \{\phi(a) : a \in A\}$ is a subring of S.

(iii) If $\phi : R \to S$ is a ring homomorphism, prove that Ker $(\phi) = \{r \in R : \phi(r) = 0\}$ is an ideal of R.

(c). Let $R = \left\{ \begin{pmatrix} a & b \\ b & a \end{pmatrix} : a, b \in \mathbb{Z} \right\}$ and let $\phi : R \to \mathbb{Z}$ be the map defined by $\phi\left(\begin{pmatrix} a & b \\ b & a \end{pmatrix} \right) = a - b$.

(i) Show that ϕ is a homomorphism.

(ii) Determine the kernel of ϕ.

(iii) Determine $R/\text{Ker}\ (\phi)$.

END OF EXAMINATION

The University of Zambia
Department of Mathematics & Statistics
2012 Academic Year Second Semester Final Examinations
M432 - Real Analysis VI

Time allowed : **Three (3) hours** Full marks : 100

Instructions:
- Attempt **any five (5)** questions. **All** questions carry **equal** marks.
- Indicate your **computer number** on all answer booklets used.
- No help materials allowed.

This paper consists of 3 pages of questions.

---

1. (a) i. Define a Cauchy sequence.

   ii. When is a metric space said to be complete?

   (b) Show that a continuous function need not map Cauchy sequences to Cauchy sequences.

   (c) Prove that If $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ are cauchy sequences in a metric space X, then the sequence $\{d(x_n, y_n)\}_{n=1}^{\infty}$ converges in $\mathbb{R}$ with the usual metric, $d(x, y) = |x - y|$.

   (d) Let (X, d) be a complete metric space. Given a nested sequence of closed balls $B[x_n, r_n]$ with $r_n \to 0$, show that

   $$\bigcap_{n \in \mathbb{N}} B[x_n, r_n]$$

   is non-empty.

   Hint: Nested means that $B[x_{n+1}, r_{n+1}] \subset B[x_n, r_n]$.

2. (a) Let $f : X \to X$ be a function on a metric space (X, d).

   i. What is meant by a fixed point of f.

   ii. When is f called a contraction on X?

   (b) Suppose $\mathbb{R}^2$ is given the usual metric. Prove that the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by

   $$f(x, y) = \left(\frac{1}{2}\cos y, \frac{1}{2}\sin x + 1\right)$$

   has a unique fixed point.

(c) Let f and g both be contractions of the metric space (X, d).

   i. Show that the compostion $f \circ g$ is also a contraction of (X, d).

   ii. Show also that if x is a fixed point of $f \circ g$ then $g(f(g(x))) = g(x)$.

   iii. Hence find a fixed point of $g \circ f$.

(d) Define the function f on the interval $[1, \infty)$ by

$$f(x) = e^{-\tan^{-1} x}.$$

Prove that this is a contraction.

3. (a) What do you understand by a norm on a vector space.

(b) Let $p \in (0, 1)$. For $x \in \mathbb{R}^n$, define

$$||x||_p = \left(\sum_{i=1}^{n} |x_i|^p\right)^{\frac{1}{p}}.$$

Show that $|| \cdot ||_p$ is not a norm on $\mathbb{R}^n$ if $n \geq 2$.

(c) For $x \in \mathbb{R}^n$ and $p \geq 1$, define

$$||x||_p = \left(\sum_{i=1}^{n} |x_i|^p\right)^{\frac{1}{p}} \quad \text{and} \quad ||x||_\infty = \max_{1 \leq i \leq n} |x_i|.$$

Show that $\lim_{p \to \infty} ||x||_p = ||x||_\infty$.

(d) The space $C[0, 1]$ of real-valued continuous functions on the closed unit interval with

$$||f|| = \max_{0 \leq x \leq 1} |f(x)|$$

is an normed linear space. Prove that this is, in fact, a Banach space.

4. (a) Define the following:

   i. Bounded linear transformation.

   ii. Dual space of a normed linear space.

(b) Let X and Y be normed linear spaces and $T \in \mathcal{L}'(X, Y)$. Show that if T is continuous on X, then it is bounded.

(c) Let $(C[-1, 1], || \cdot ||_\infty)$ be the vector space of continuous functions on the interval $[-1, 1]$, with $||f||_\infty = \sup_{0 \leq x \leq 1} |f(x)|$. Consider

$$(Tf)(x) = \int_{-1}^{0} f(x)\, dx - \int_{0}^{1} f(x)\, dx.$$

Prove that T is linear, continuous and bounded.

(d) Let X be a normed linear space, A a subspace of X, and suppose $x \in X$. Prove that if $d = \inf_{y \in A} ||y - x||$, then there exists a linear functional $\Lambda : X \to \mathbb{R}$ such that

$$||\Lambda|| \leq 1, \quad \Lambda(x) = d \text{ and } \quad \Lambda(y) = 0$$

for all $y \in A$.

5. (a) Let V be an inner product space, $A \subset V$ and $x, y \in V$. When is
   i. A called convex?
   ii. x said to be orthogonal to y?

   (b) Let H be a Hilbert space, A a convex subset of H, and $\{x_n\}$ a sequence in A with

$$||x_n|| \to \inf_{x \in A} ||x||.$$

   Show that $\{x_n\}$ converges in H.

   (c) Let M be a linear subspace of an inner product space V. Prove that $M^\perp$ is a closed linear subspace of V.

   (d) Let $\{x_n\}_{n=1}^N$ be an orthonormal set in an inner product space V. Prove that for $x \in V$,

$$\left|\left| x - \sum_{n=1}^{N} c_n x_n \right|\right|$$

   is minimised by choosing $c_n = (x_n, x)$.

6. (a) Define the following:
   i. Complete sequence in a Hilbert space.
   ii. Orthonormal basis.

   (b) Let $\{x_n\}_{n=0}^\infty$ be an orthonormal basis in a Hilbert space H. Then for each $y \in H$,

$$y = \sum_{n=0}^{\infty} (y, x_n) x_n \quad \text{and} \quad ||y||^2 = \sum_{n=0}^{\infty} |(y, x_n)|^2.$$

   Prove this theorem.

   (c) Let $P_3(\mathbb{R})$ denote the vector space of real polynomials of degree at most 3. Define an inner product on $P_3(\mathbb{R})$ by

$$(f, g) = \int_0^1 f(x) g(x) \, dx.$$

   Produce an orthonormal basis for $U = \text{span}\{x^2, x^3\}$.

   (d) Find $p \in U$ that makes

$$\int_0^1 |2 + 3x - p(x)|^2 \, dx$$

   as small as possible. Here, U and the inner product are as described in 6(c).

END OF PAPER

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS

M462: BAYESIAN INFERENCE AND DISCRETE ANALYSIS

INSTRUCTIONS:
1. Answer any FIVE (5) questions.
2. Calculators are allowed.
3. You may use statistical tables or formulae when provided.
4. Show all your work to earn full marks.

TIME: THREE (3) Hours

Q1 (a) Let H_1 denote a hypothesis and H_2 denote an alternative hypothesis. Further, let D denote data supporting either H_1 or H_2.

(i) Define prior <u>odds</u> for H_1 against H_2

(ii) Define posterior <u>odds</u> for H_1 against H_2

(iii) Define the support of D for H_1 against H_2 in terms of log-odds.

(iv) State the Baye's updating rule from before to after the observation of D for inference about H_1 against H_2, in terms of log-odds and support.

(b) It is known that a large batch of items are either all of type I or all of type II. It is also known that type I items have exponentially distributed lifetimes with mean 1 whereas type II items have Gam(2,2) distributed lifetimes. A sample of n items is selected at random and their lifetimes are measured. Let H_1 denote that items are of type I and H_2 denote that items are of type II.

(i) Determine, in log-odds form, the Baye's updating rule for the inference about H_1 versus H_2.

(ii) State the condition for which the updating rule supports H_1 as opposed to H_2.

(iii) Suppose 8 lifetimes are available as shown below.

0.15 0.56 0.95 0.34 0.17 1.2 0.43 0.59

Do the data support H_1?

Q2 The number of trials, Y, required to observe r successes in independent Bernoulli experiments is said to have a negative binomial distribution with the probability density function given below.

$$f(y) = \binom{y-1}{r-1} p^r (1-p)^{y-r} \quad y = r, r+1, r+2, . . .$$

(a) Assume that the prior distribution of p is a Beta(α,β) $= \frac{1}{B(\alpha,\beta)} p^{\alpha-1}(1-p)^{\beta-1}$, where $B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$.

(i) Obtain the posterior density assuming negative binomial sampling with a beta prior.

(ii) What is the updating rule for parameters?

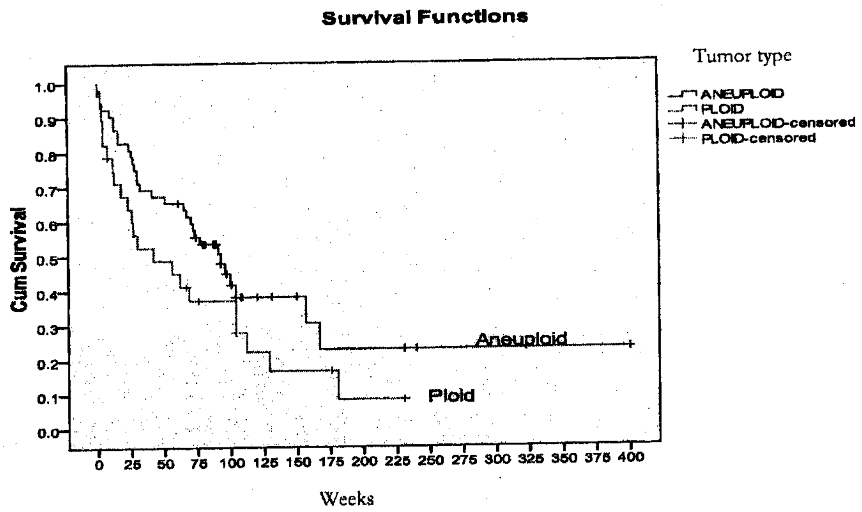
(iii) Is this beta with negative binomial sampling conjugate?

(b) (i) Show that f(y) = $\exp\{\lambda(y\theta - b(\theta)) + c(y, \lambda)\}$, i.e., f(y) belongs to the family of exponential dispersion models, where $\lambda = \frac{\omega}{\phi}$

(ii) Show that $E(Y) = b'(\theta) = \frac{r}{p}$

(iii) Show that $\text{Var}(Y) = \frac{\phi}{\omega} b''(\theta) = \frac{r(1-p)}{p^2}$

Q3 A study was conducted on the effects of ploid on the prognosis of patients with cancers of the mouth. Patients were selected who had a paraffin-embedded sample of the cancerous tissue taken at the time of surgery. Follow-up survival data (weeks) was obtained on each patient. The tissue samples were examined using a flow cytometer to determine if the tumor had an aneuploid (abnormal) or ploid (normal) DNA profile using some technique. Summary graphs and statistics are given below.



(a) (i) Briefly discuss the survival of the two groups based on the graph.

(ii) Estimate the median survival times of the two groups from the graph.

(iii) The logrank statistic yielded the value shown below, test at 5% level of significance whether survival is different between the two groups.

Overall Comparisons

	Chi-Square	df
Log Rank (Mantel-Cox)	2.790	1

(b) Below is survival information for 15-week intervals for the group with aneuploid DNA profile.

Time intervals	Number alive	Number dying	Number Censored	Survival Estimate
[0, 15)	52	7	0	
[15, 30)	45	6	0	
[30, 45)	39	4	0	
[45, 60)	35	1	0	
[60, 75)	34	5	2	
[75, 90)	27	1	7	
[90, 105)	19	5	4	
[105, 120)	10	0	2	
[120, +∞)	8	2	6	

(i) Copy the table and compute the survival function using the actuarial method.

(ii) What is the probability of surviving the first 30 weeks?

Q4 (a) Let Y have a Poisson distribution with parameter λ. Suppose the prior density of λ is a Gamma: $f(\lambda) = \frac{\theta^k \lambda^{k-1}}{\Gamma(k)} e^{-\theta\lambda}$, $\lambda, \theta > 0$.

(i) Derive the prior mean.

(ii) Obtain the posterior distribution for λ, given a random sample of size n from a POI(λ).

(iii) Suppose a sample of size 5 yields values of y as: 3, 4, 7, 2, 3 and k = 3, determine the posterior mean for λ.

(b) Suppose that the failure time X, of an organism is thought to have a uniform distribution U(a, b), where b – a > 1

(i) Derive the survival function S(t).

(ii) Derive the hazard function $\lambda(t)$.

(iii) In one Plot, sketch the graphs of U(a, b), S(t) and $\lambda(t)$, identify all intersecting points, if any.

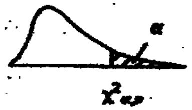
Q5 (a) Suppose $\pi(x)$ = Prob(Y =1/X) and 1-$\pi(x)$ = Prob(Y =0/X) and Y_i ~indep Bin(n_i, π_i). Here, Y_i is a sum of Bernoulli variables and X is a covariate.

(i) Show that the Bin(n_i, π_i) distribution is a member of the exponential family of

Table 8

PERCENTAGE POINTS OF THE χ^2 DISTRIBUTION

Table of $\chi^2_{\alpha;\nu}$ — the 100 α percentage point of the χ^2 distribution for ν degrees of freedom



α =	.995	.99	.98	.975	.95	.90	.80	.75	.70	.50	.30	.25	.20	.10	.05	.025	.02	.01	.005	.001	= α
ν = 1	$.0^4393$	$.0^3157$	$.0^3628$	$.0^3982$	.00393	.0158	.0642	.102	.148	.455	1.074	1.323	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.827	ν = 1
2	.0100	.0201	.0404	.0506	.103	.211	.446	.575	.713	1.386	2.408	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.815	2
3	.0717	.115	.185	.216	.352	.584	1.005	1.213	1.424	2.366	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.268	3
4	.207	.297	.429	.484	.711	1.064	1.649	1.923	2.195	3.357	4.878	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.465	4
5	.412	.554	.752	.831	1.145	1.610	2.343	2.675	3.000	4.351	6.064	6.626	7.289	9.236	11.070	12.832	13.388	15.086	16.750	20.517	5
6	.676	.872	1.134	1.237	1.635	2.204	3.070	3.455	3.828	5.348	7.231	7.841	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.457	6
7	.989	1.239	1.564	1.690	2.167	2.833	3.822	4.255	4.671	6.346	8.383	9.037	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.322	7
8	1.344	1.646	2.032	2.180	2.733	3.490	4.594	5.071	5.527	7.344	9.524	10.219	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.125	8
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	5.899	6.393	8.343	10.656	11.389	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877	9
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	6.737	7.267	9.342	11.781	12.549	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588	10
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	7.584	8.148	10.341	12.899	13.701	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264	11
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	8.438	9.034	11.340	14.011	14.845	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909	12
13	3.565	4.107	4.765	5.009	5.892	7.042	8.634	9.299	9.926	12.340	15.119	15.984	16.985	19.812	22.362	24.736	25.472	27.688	29.819	34.528	13
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	10.165	10.821	13.339	16.222	17.117	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.123	14
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	11.036	11.721	14.339	17.322	18.245	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.697	15
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	11.912	12.624	15.338	18.418	19.369	20.465	23.542	26.296	28.845	29.633	32.000	34.267	39.252	16
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	12.792	13.531	16.338	19.511	20.489	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.790	17
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	13.675	14.440	17.338	20.601	21.605	22.760	25.989	28.869	31.526	32.346	34.805	37.156	42.312	18
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	14.562	15.352	18.338	21.689	22.718	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.820	19
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	15.452	16.266	19.337	22.775	23.828	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.315	20
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	16.344	17.182	20.337	23.858	24.935	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.797	21
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	17.240	18.101	21.337	24.939	26.039	27.301	30.813	33.924	36.781	37.659	40.289	42.796	48.268	22
23	9.260	10.196	11.293	11.688	13.091	14.848	17.187	18.137	19.021	22.337	26.018	27.141	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728	23
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	19.037	19.943	23.337	27.096	28.241	29.553	33.196	36.415	39.364	40.270	42.980	45.558	51.179	24
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	19.939	20.867	24.337	28.172	29.339	30.675	34.382	37.652	40.646	41.566	44.314	46.928	52.620	25
26	11.160	12.198	13.409	13.844	15.379	17.292	19.820	20.843	21.792	25.336	29.246	30.434	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.052	26
27	11.808	12.879	14.125	14.573	16.151	18.114	20.703	21.749	22.719	26.336	30.319	31.528	32.912	36.741	40.113	43.194	44.140	46.963	49.645	55.476	27
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	22.657	23.647	27.336	31.391	32.620	34.027	37.916	41.337	44.461	45.419	48.278	50.993	56.893	28
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	23.567	24.577	28.336	32.461	33.711	35.139	39.087	42.557	45.722	46.693	49.588	52.336	58.302	29
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	24.478	25.508	29.336	33.530	34.800	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.703	30
40	20.706	22.164	23.838	24.433	26.509	29.051	32.345	33.660	34.872	39.335	44.165	45.616	47.269	51.805	55.759	59.342	60.436	63.691	66.766	73.402	40
50	27.991	29.707	31.664	32.357	34.764	37.689	41.449	42.942	44.313	49.335	54.723	56.334	58.164	63.167	67.505	71.420	72.613	76.154	79.490	86.661	50
60	35.535	37.485	39.699	40.482	43.188	46.459	50.641	52.294	53.809	59.335	65.227	66.981	68.972	74.397	79.082	83.298	84.580	88.379	91.952	99.607	60
70	43.275	45.442	47.893	48.758	51.739	55.329	59.898	61.698	63.346	69.334	75.689	77.577	79.715	85.527	90.531	95.023	96.388	100.425	104.215	112.317	70
80	51.171	53.539	56.213	57.153	60.391	64.278	69.207	71.145	72.915	79.334	86.120	88.130	90.405	96.578	101.880	106.629	108.069	112.329	116.321	124.839	80
90	59.196	61.754	64.634	65.646	69.126	73.291	78.558	80.625	82.511	89.334	96.524	98.650	101.054	107.565	113.145	118.136	119.648	124.116	128.299	137.208	90
100	67.327	70.065	73.142	74.222	77.929	82.358	87.945	90.133	92.129	99.334	106.906	109.141	111.667	118.498	124.342	129.561	131.142	135.807	140.170	149.449	100

For values of $\nu > 30$, approximate values for χ^2 may be obtained from the expression $\nu\left[1-\frac{2}{9\nu}\pm\frac{x}{\sigma}\sqrt{\frac{2}{9\nu}}\right]^3$, where $\frac{x}{\sigma}$ is the normal deviate cutting off the corresponding tails of a normal distribution. If $\frac{x}{\sigma}$ is taken at the 0.02 level, so that 0.01 of the normal distribution is in each tail, the expression yields χ^2 at the 0.99 and 0.01 points. For very large values of ν it is sufficiently accurate to compute $\sqrt{2\chi^2}$, the distribution of which is approximately normal around a mean of $\sqrt{2\nu-1}$ and with a standard deviation of 1. This table is taken by consent from Statistical Tables for Biological, Agricultural, and Medical Research, by R. A. Fisher and F. Yates, published by Oliver and Boyd, Edinburgh, and from Table 8 of Biometrika Tables for Statisticians, Vol. I, by permission of the Biometrika Trustees.

distributions.

(ii) Identify the link function in this case.

(iii) Write down the generalized linear model that links π with X, a covariate.

(b) Suppose that a failure in a certain electronic system can occur because of either a minor defect or a major defect. Suppose also that 80 percent of the failures are caused by minor defects and 20 percent of the failures are caused by major defects. When a failure occurs, n independent soundings $X_1, X_2, \ldots, X_n$ are made on the system. If the failure was caused by a minor defect, these soundings form a random sample from a Poisson distribution for which the mean is 3. If the failure was caused by a major defect, these soundings form a random sample from a Poisson distribution for which the mean is 7. The cost of deciding that the failure was caused by a major defect when it was actually caused by a minor defect is $400. The cost of deciding that the failure was caused by a minor defect when it was actually caused by major defect is $2500. The cost of choosing a correct decision is 0.

Let:
d_0 denote decision for minor defects
d_1 denote decision for major defects
$\theta_0 = 0.80$
$\theta_1 = 0.20$
$r(\delta)$ be the expected cost with a test procedure δ

(i) State the Bayes test procedure δ which minimizes the expected cost $r(\delta)$

(ii) Copy the 2X2 table and fill in the appropriate loss values.

	d_0	d_1
θ_0		
θ_1		

(iii) For a given set of observed values of $X_1, X_2, \ldots, X_n$ determine the Bayes procedure which minimizes the expected cost.

Q6 (a) Assume that failure times of a certain type of organism have a Weibull distribution given below. A sample of size n yields both failure times and censored observation as indicated below: $\{ (T_1, \delta_1), (T_2, \delta_2), \ldots, (T_n, \delta_n)\}$, where δ_i is a censoring indicator; $\delta_i = 1$ (failure) and $\delta_i = 0$ (censored).

(i) If the Weibull distribution is given by $f(t) = \alpha\lambda t^{\alpha-1}e^{-\lambda t^{\alpha}}$, where $\alpha, \lambda > 0$ $t \geq 0$ and α is known but λ is unknown, obtain the MLE for λ with the censored observations in mind.

(ii) Obtain an estimate of λ if $\alpha = 2$ and a random sample yields the following data:

+1.16 1.65 +1.95 1.05 4.93 2.04 5.6 8.12

a plus indicates censored observations.

(b) (i) Assume that the variables $X_1, X_2, \ldots, X_n$ form a random sample from a distribution $f(x/\theta)$ and that there are two possible values of θ to be decided upon as stated in the two hypotheses:

$H_0: \theta = \theta_0$

$H_1: \theta = \theta_1$

Let $f_0(\underline{x}/\theta_0)$ and $f_1(\underline{x}/\theta_1)$ be the likelihoods under H_0 and H_1, respectively. Suppose that a and b are specified positive constants, state an optimal test procedure δ such that $a\alpha(\delta) + b\beta(\delta)$ will be a minimum, where $\alpha(\delta)$ and $\beta(\delta)$ are the probabilities of type I and II errors, respectively.

(ii) Suppose that $X_1, X_2, \ldots, X_n$ form a random sample from a Bernoulli distribution for which the value of the parameter p is unknown. Let p_0 and p_1 be specified values such that $0 < p_1 < p_0 < 1$, and suppose that it is desired to test the following simple hypotheses:

$H_0: p = p_0$

$H_1: p = p_1$

Show that a test procedure for which $\alpha(\delta) + \beta(\delta)$ is minimum rejects H_0 when $\bar{X} < c$

(iii) In (ii) above, find the value of c if $p_1 = 0.3$ and $p_0 = 0.6$.

END OF EXAMINATION

The University of Zambia

Department of Mathematics & Statistics

2012 Academic Year, Distance Examinations

M911 - Mathematical Methods - V

Time allowed : Three (3) hrs   Full marks : 100

Instructions:

1. Attempt **any five (5)** questions out of the **six (6)** QUESTIONS.
2. All questions carry equal marks.
3. **Full credit** will only be given when **necessary work** is shown.
4. Indicate your **computer number** on all answer booklets.

This paper consists of 3 pages of questions.

---

1. **(a)** Use the $(\epsilon - \delta)$ definition of the limit to prove that:

$$\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^2+y^2} = 0.$$

(b) Consider the function

$$f(x,y) = \begin{cases} \frac{3x^2y}{x^2+y^2}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0) \end{cases}$$

Show that $f(x,y)$ is continuous at $(0,0)$.

(c) A rectangular box without a lid is to be made from $12\,m^2$ of cardboard. Use the method of Lagrange multipliers to find the maximum volume of such a box.

2. **(a)** The distribution of voltage on a metal plate is given by

$$v = 50 - x^2 - 4y^2.$$

Describe the path of a particle that starts at $(1, -2)$ and moves in the direction of greatest voltage increase.

(b) Find the first four terms in the tailor series of the function

$$f(x, y) = e^x \cos y,$$

about the point (a, π).

(c) Show that the following functions are functionally dependent and find a relation connecting them:

$$f(x, y) = e^{(x+y)}$$

$$g(x, y) = (x + y)^2 + 2(x + y).$$

3. **(a)** Show that the equation

$$xy^2z + z^2 = 4 + xe^z$$

defines z implicitly as a function of x and y, say, $z = f(x, y)$. Hence, find $\frac{\partial f}{\partial x}$ at the point $(0, e, 2)$.

(b) Find the local maximum, minimum, and saddle points of the function

$$f(x, y) = x^2y + 9x - 3y^2 - 6.$$

(c) Given that S is a surface given by the function $F(x, y, z) = x^2y^2z^2$.

(i) Find the tangent plane to S at the point $(1, 2, 3)$.

(ii) Find the directional derivative of F at the point $(1, 2, 3)$ in the direction of $g(1)$, where, $g(t) = (2t - 1)\mathbf{i} + (2t)\mathbf{j} + 2\mathbf{k}$.

4. **(a)** Given a function $\varphi(x, y, z)$ and a vector field

$$\mathrm{F}(x, y, z) = u(x, y, z)\mathbf{i} + v(x, y, z)\mathbf{j} + w(x, y, z)\mathbf{k}$$

prove that

$$\mathrm{div}(\varphi \mathrm{F}) = \varphi(\mathrm{div}\, \mathrm{F}) + (\nabla\varphi) \cdot \mathrm{F}.$$

(b) Let $\mathbf{V} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $r = |\mathbf{V}|$. Verify that

$$\mathrm{div}\,(r^n\mathbf{V}) = (n + 3)r^n.$$

(c) Given the vector field $\mathrm{F} = e^y(\cos x\mathbf{i} + \sin x\mathbf{j}) + z\mathbf{k}$, show that F is irrotational, hence find functions f such that $\nabla f = \mathrm{F}$.

5. **(a)** Given the integral

$$I = \int_0^4 \int_{\sqrt{x}}^2 x \sin y^5 dydx.$$

(i) Sketch the region of integration

(ii) Evaluate I.

(b) Let M be the region bounded by the square with vertices $(0,1)$, $(1,2)$, $(2,1)$ and $(1,0)$. Evaluate

$$\int\int_M (x+y)^2 \sin^2(x-y)dA$$

(c) Use a triple integral to find the volume which lies below the paraboloid $z = 1 - x^2 - y^2$ and above the $xy-$plane.

6. **(a)** Given that Q is the region given by $0 \leq x \leq 1$, $0 \leq y \leq 1$, evaluate

$$\int\int_Q (1 - \frac{1}{2}x^2 - \frac{1}{2}y^2)dA.$$

(b) Given the transformation

$$u = x^2 - y^2, \quad v = 2xy,$$

mapping the region D given by

$$0 \leq x \leq 2, \quad 0 \leq y \leq 2$$

onto the region R and is one-to-one on D.

(i) Find the Jacobian of the transformation and hence find the area of the region R by integrating over D.

(ii) Find the region R and compute $\int\int_R vdudv$ directly.

(c) Use spherical coordinates to find the volume of the solid that lies above the cone

$$z = \sqrt{x^2 + y^2}$$

and below the sphere

$$x^2 + y^2 + z^2 = z.$$

END OF EXAM!

The University of Zambia
Physics Department
University Examinations 2012
P-192: Introductory Physics- II

All questions carry equal marks. The marks are shown in brackets. Question 1 is compulsory. Attempt four more questions. Clearly indicate on the answer script left column on the cover page the questions you have answered.

Time : Three hours. **Maximum marks = 100.**

Do not forget to write your computer number clearly on the answer book as well as on the answer sheet for Question 1. Tie them together.

==

Wherever necessary use :

$g = 9.8 \text{m/s}^2$

1 metric ton = 1000 kg

$P_A = 1.01 \times 10^5 \text{ N/m}^2$

1 cal. = 4.18 J

$\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$

$c = 3 \times 10^8 \text{ m/s}$

$h = 6.63 \times 10^{-34}$ J-s

1 pascal =1 N/m^2

$1\text{eV} = 1.6 \times 10^{-19}$ J

$e = 1.6 \times 10^{-19}$ C

$m_e = 9.11 \times 10^{-31}$ kg

$k = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$

$\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$

$\rho_{water} = 1000 \text{ kg/m}^3$

$G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$

The University of Zambia
Physics Department
University Examinations 2012/2
P-192: Introductory Physics - II

Answer sheet for Question 1

Computer Number ....

Q1. Put a cross (×) or tick mark (√) in the appropriate box. If it is on the dividing line, it will not be counted.

	a	b	c	d
A				
B				
C				
D				
E				
F				
G				
H				
I				
J				

Do NOT write here.
For official use only :

	Number of parts N	Factor f	Marks f × N
Correct		2	
Wrong		–(0.67)	
	Net Marks :		

Attach this sheet firmly with the main answer book. If you lose this sheet, you will lose the marks for Question 1 !!

Question 1 : For each correct answer, 2 marks will be given. For each wrong answer, 0.67 will be deducted. For no answer, zero mark will be given. The minimum total mark for Question 1 is zero.]

(A) If 10 J of work is done in moving a 2 C charge from one point to another in an electric field, the potential difference between the two points is:

a) 20 V
b) 5 V
c) 10 V
d) 12 V

(B) An ideal gas is heated at constant volume from 75°C to 150°C. If the original pressure was 1.5 atm, the new pressure will be:

a) doubled
b) halved
c) less than doubled
d) the same

(C) In a stretched string with constant tension T, as the frequency of the wave increases, the wavelength:

a) decreases
b) increases
c) remains the same
d) increases and then decreases

(D) The amplitude of a simple harmonic oscillator is doubled. Which of the following is also doubled?

a) its total energy
b) its period
c) its maximum speed
d) its frequency

(E) Coulomb's law concerns itself with:

a) the force exerted by one mass on another
b) the interaction between two charges
c) conservation of energy
d) conservation of charges

(F) To double the period of a simple pendulum, the length must be:

a) decreased by one fourth
b) decreased by half
c) increased twofold
d) increased fourfold

(G) A Carnot engine has the same efficiency between 100 K and 500 K as between X K and 1000 K, X being the lower temperature (cold reservoir). The value of X is:

a) 100 K
b) 500 K
c) 200 K
d) 300 K

(H) Ohm's law states that in a circuit at constant temperature:

a) The current through a resistor is inversely proportional to the applied voltage.
b) The voltage across a resistor is not directly proportional to the current passing through.
c) Resistance is not the constant of proportionality between voltage and current.
d) The current through a resistor is directly proportional to the applied voltage.

(I) The pitch of a sound depends on its:

a) wave amplitude
b) wave speed
c) harmonic content
d) frequency

(J) For capacitors connected in series:

a) The charge on each capacitor is the same.
b) The potential difference is the same for all.
c) The resultant capacitance is greater than the sum of the individual capacitances.
d) The resultant capacitance is equal to the sum of the individual capacitances.

Attempt any four questions from the following:

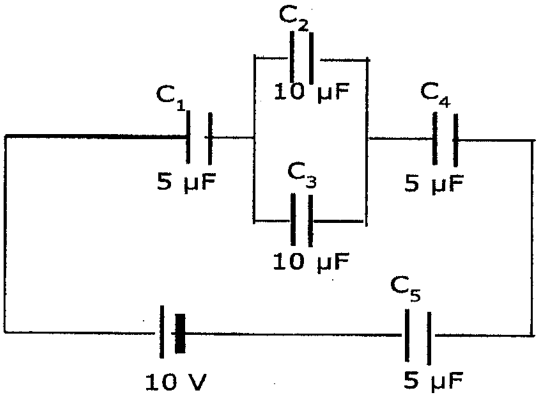
Q 2 (a) A quantity of air is compressed isothermally from a pressure of 1.5 atm to a pressure of 3 atm. The volume and temperature are 15 litres and 20 °C respectively. Next the air is expanded back adiabatically to its original pressure. ($\gamma = 1.4$ for air)

i) Find the final volume and temperature, and [12]
ii) Sketch the PV diagram for the two processes. [2]

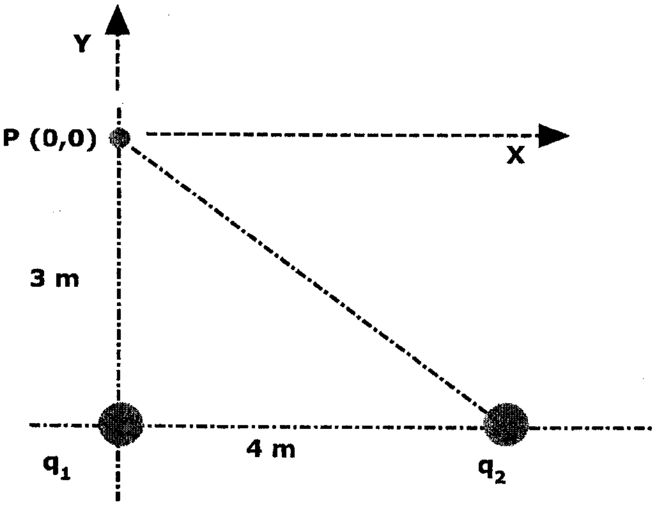
(b) A speaker rated at 0.5 W of acoustic power emits a sound. Assume it behaves as a point source radiating sound uniformly in all directions. At what distance from the speaker does the sound intensity level fall to 65 dB? [8]

Q.3 (a) Five capacitors are connected as shown. Find:

i) the equivalent capacitance, and
ii) the charge and potential difference across the capacitor C_1. [9]



(b) Find the electric field at a point P at (0,0) due to two charges q_1 = 8 μC placed at (0m, -3m) and q_2 = -6 μC at (4m, -3m) as shown below.



Q.4(a) What is the minimum amount of ice at -14 °C that must be added to 620 grams of water at 19 °C in order to bring the temperature of the water down to 0 °C? (c_{ice} = 2.09 kJ/kg..°C, H_f = 335 kJ/kg) [10]

(b) A young boy blows a whistle with a note of 800 Hz while running away from a stationary man towards a wall with a speed of 1.7 ms^{-1}. What is the beat frequency heard by the stationary man. (v_{sound} = 340 ms^{-1}) [10]

Q.5 (a) A resistor carries a current of 0.2 A when connected to 220 V source. What will the current carried by the resistor be if:

i) the operating voltage is reduced to 190 V, and
ii) the operating voltage is increased to 220 V?
iii) If this resistor was made of a wire 4.0 cm long and a diameter 0.1 mm. Calculate the resistivity of this wire? [9]

(b) The efficiency of a Carnot engine is 0.4 if the temperature of the cold reservoir is lowered by 45 °C the efficiency becomes 0.50. Find the temperatures of the hot and the cold reservoirs. [11]

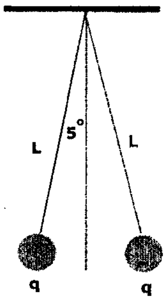
Q.6 (a) Two identical guitar strings under 230 N of tension are emitting fundamental tones with frequencies of 601 Hz. One of the strings loosens and its tension drops to 226 N. How many beats per second are heard when the strings are struck at the same time so as to emit fundamental tones? [8]

(b) A Carnot refrigerator absorbs heat from a freezer chamber at a temperature of -15 °C and exhausts it into a room at 26 °C.

i) How much work must the refrigerator compressor do to change 500 gram of water at 26 °C into ice at -15 °C?
ii) If the compressor output is 230 W, what is the minimum time needed to accomplish this task? [9]

(c) Name three processes involved in heat transfer and give a brief explanation of each. [3]

Q.7 (a) Two balls made of aluminium foil each with a mass of 3 x 10^{-2} kg hang in equilibrium as shown in the figure below. The length of each string is 15 cm and each string makes an angle of 5° with respect to the vertical. Find the magnitude of the charge on each ball, based on the fact they have identical charges. [11]



(b) A horizontal spring with a force constant *k* of 1300 Nm^{-1} attached to a wall is connected to a 3 kg mass resting on frictionless surface. The mass is pulled a distance of 2.5 cm and released. Calculate:

i) The frequency of oscillation
ii) The maximum speed
iii) The maximum acceleration
iv) The speed of the mass when the spring is compressed by 1 cm

[9]

Q.8 (a) A brass kettle has a base area of 0.1 m^2 and a thickness of 9 mm. It boils off water at the rate of 500 g/min when placed on a gas stove. Estimate the temperature of the part of the flame in contact with the boiler. (K_{brass} = 109 J/m.K.s, H_v = 2260 kJ/kg) [7]

(b) A brass plug is to be placed in a ring made of iron. At 20°C, the diameter of the plug is 8.753 cm and that of the inside ring 8.743 cm. To what common temperature must they be brought in order for the plug to fit? (α_{Fe} = 12 x 10^{-6}/°C, α_{Br} = 19 x 10^{-6}/°C) [9]

(c) A parallel plate capacitor is constructed from two square plates of side d 2.5 cm and separated by a material whose dielectric constant is 5. The dielectric material is 1.5 mm thick. What is the value of this capacitor in μF? [4]

END OF P 192 EXAMINATION

Some useful equations

Thermal properties of matter:

$Q/t = e\sigma AT^4$ $Q/\Delta t = (kA\Delta T)/\Delta L$ $\Delta Q = mc\Delta T = nC\Delta T$ $\Delta L = \alpha L\Delta T$

$\Delta V = \gamma V\Delta T$ $\Delta W = P\Delta V$ $\Delta W = nRT.\ln(V_f/V_i)$ $C_v = C_p - R$

$PV = nRT$ $P_1V_1^{\gamma} = P_2V_2^{\gamma}$

Thermodynamics:

$Q = \Delta U + W$ Carnot engine, $e = 1 - T_2/T_1 = \frac{\text{work done}}{\text{input heat at high temperature}}$

$S = k\ln\Omega$ $\Delta S = Q/T$ Efficiency $= W/Q_h$ $COP_{fridge} = Q_c/W_{in}$

$COP_{heat\ pump} = Q_h/W_{in}$ $COP_{max\text{-}fridge} = T_c/(T_h\text{-}T_c)$ $COP_{max\ h.\ pump} :T_h/(T_h\text{-}T_c)$

Waves and vibrations:

$F = -kx$ $\omega = 2\pi f$ $f = (1/2\pi)\surd(g/L$ $a_c = \omega^2x_0$

$P.E. = (1/2)kx^2$ $1/2)kx^2 + (1/2)mv^2 = (1/2)kx_0^2$ $\omega = \surd(k/m$

$f = (1/2\pi)\surd(k/m)$ $v = f\lambda$ $f_n = v/\lambda_n = n(v/2L)$ $L = n(\lambda_n/2)$

$v = \sqrt{T}/\mu$ $f = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$

Sound waves:

$v = \surd(Y/\rho)$ $v = \surd(B/\rho)$ $I_o = 10^{-12}\ W/m^2$ $I(dB) = 10\log(I/I_o)$

$I(r) = P/4nr^2$ $f' = f\left(\frac{v+v_l}{v-v_s}\right)$ (moving source and moving listener)

Electric forces and fields, electric potential:

$\mathbf{F} = q\mathbf{E}$ $\mathbf{E} = kQ/r^2$ $\mathbf{F} = (k\,q_1\,q_2)/r^2$ $V_{AB} = Ed$

$V = kq/r$ $\Delta PE = qEd$ $C = (\varepsilon_0\,A)/d$ $W = qV_{AB}$

$C = \kappa(\varepsilon_0\,A)/d$

DC electricity:

$I = \frac{\Delta q}{\Delta t}$ $R = \frac{V}{I}$ $R = \frac{\rho L}{A}$ $P = IV = I^2R$

The University of Zambia

School of Natural Sciences

Department of Physics

2012-13 Academic Year Second Semester

Final Examinations

P-212: Magnetism in Matter and Atomic Physics

Attempt any five questions. All questions carry equal marks. The marks are shown in brackets.

Time: Three hours.

Maximum marks = 100.

Write clearly your computer number on the answer book.

===

Below is a table of constants that might be useful.

Acceleration due to gravity $g = 9.8$ m/s^2	Avogadro's number $N_{Av.} = 6.023 \times 10^{23}$ per mole
electron charge $e = -1.6 \times 10^{19}$C	Electron volt $1eV = 1.6 \times 10^{-19}$J
Permeability of free space $\mu_0 = 4\pi \times 10^7$	$\epsilon_0 = 8.85 \times 10^{12}$C^2/N.m^2
Electron mass $m_e = 9.1 \times 10^{-31}$kg	Neutron mass $m_n = 1.00897$amu
$1amu = 1.66 \times 10^{-27}$kg $= 931$ MeV	Planck's constant $h = 6.63 \times 10^{-34}$ J.s
Electron mass $m_e = 9.11 \times 10^{-31}$kg $= 0.511$MeV	Proton mass $m_p = 1.00758$amu
Wien's constant $b = 2.9 \times 10^{-3}$m.K	Stefans constant $\sigma = 5.67 \times 10^{-8}$Wm^{-2}K^{-4}
Boltzmann constant $\kappa = 1.38 \times 10^{-23}$J.K^{-1}	$1\mathring{A} = 10^{-10}$ m. $1nm = 10^{-9}$ m.
Rydberg constant $R = 1.0974 \times 10^7$m^{-1}	speed of light $c = 3 \times 10^8$m/s
Atomic number Cobalt $Z = 27$	converting year to second 1yr $= 31557600 \approx 3.16 \times 10^7$s

Below is a list of formulas that might be helpful.

Decay law: $N = N_0 e^{-\lambda t}$. The decay rate or activity $A = \lambda N$

Stefans law $E = \sigma A T^4$

Absorbed intensity $I = I_0 e^{-\mu x}$

Rydberg equation:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Planck's formula:

$$\Psi_\lambda d\lambda = \frac{8\pi c h \lambda^{-5}}{e^{ch/\lambda\kappa T} - 1} d\lambda$$

Photoelectric equation: $\frac{1}{2}mv^2 = h\nu - \varphi$

Compton scattering equation: $\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta)$

Moseley's law: $\sqrt{\nu} = a(Z - b)$

Wien's displacement law: $\lambda_{max} T = 0.2898 \times 10^{-2}$ m.K

De Broglie wavelength: $\lambda = \frac{h}{mv}$

Q1 (a) Briefly discuss the implication of the equation $\oint \mathbf{B}.d\mathbf{S} = 0$, Gausss law for magnetic flux. [2]

(b) The magnetic field strength in a piece of Fe_2O_3 is 10^6 Amp/m. Given that the susceptibility, χ_m of Fe_2O_3 at room temperature is 1.4×10^{-3}, find the

(i) flux density B, [3]

(ii) magnetization M, and [3]

(iii) the relative permeability μ/μ_0 of the material. [2]

(c) A closely spaced thin copper coil carrying a current of 2 Amps is wound over a toroid of magnetic specimen of average radius 5 cm. The toroidal specimen, whose susceptibility is $\chi_m = -0.086 \times 10^{-6}$ cm^3/g, has 3000 turns. Find

(i) the solenoid current density j^s_{free} [2]

(ii) the induced magnetization in the material and [3]

(iii) the relative permeability of the material. [2]

(d) Differentiate between paramagnetic and diamagnetic materials. [3]

Q2 (a) The temperature of your skin is approximately 35°C. Using Wien's displacement law, determine the least frequency of the radiation it emits. [3]

(b) A sodium surface is illuminated with light of wavelength 300 nm. The work function for sodium metal is 2.46 eV. Find:

(i) the maximum kinetic energy of the ejected photoelectrons [3]

(ii) the cutoff wavelength for sodium [3]

(iii) the maximum speed of the photoelectrons. [4]

(c) X-rays of wavelength $\lambda_0 = 0.20$ nm are scattered in a Compton scattering event from a block of material. The scattered X-rays are observed at an angle of 45° to the incident beam.

(i) Calculate the wavelength of the X-rays scattered at this angle. [3]

(ii) Find the fraction of energy lost by the photon in this collision. [4]

(iii) Calculate the energy of the X-rays scattered at this angle. [3]

Q3 (a) A beam of electrons bombards a sample of hydrogen. Through what potential difference must the electrons be accelerated if the first line of the Balmer series is to be emitted? [4]

Note: The Balmer series occurs when there is a transition to state $n = 2$ from any state $n > 2$

(b) Electrons in a television tube are accelerated through a potential difference of 25,000 volts. Calculate the speed of the electrons [4]

(c) One of Bohrs postulate states that in the one-electron atom, angular momentum is quantized, i.e.,

$$L = mvr = \frac{nh}{2}$$

where $n = 1, 2, 3, \cdots$, and m is the electron mass. Show from this that the circumference of the electron orbits is an integral number of de Broglie wavelengths. [5]

(d) (i) A magnetic material has a magnetization of 230Amp/m and produces a flux density (B) of 0.00314 Wbm^{-2}. Calculate the magnetization field H, and the relative permeability of the material. [4]

(ii) Diamagnetic Al_2O_3 is subjected to an external magnetic field of 10^5 Amp/m. Evaluate the magnetization, M, and the magnetic flux density B in Al_2O_3. [3]

Q4 (a) (i) State the de Broglie hypothesis [2]

(ii) Calculate the de Broglie wavelength associated with an electron which has been accelerated through a potential difference of 150 volts [4]

(iii) An alpha particle is ejected from the nucleus of a radium atom with 5.78MeV of kinetic energy. What is de Broglie wavelength of this particle? [4]

(b) (i) What are X-rays? [2]

(ii) When X-rays transverse a material of a certain thickness, the transmitted intensity can be computed using the relation $I = I_0 e^{-\mu x}$. Define all the symbols appearing in this equation. [3]

(iii) What percentage of the X-ray beam is absorbed when it passes through 0.1mm of liquid mercury for which $\mu = 0.22$ cm^{-1} [5]

Q5 (a) The X-rays from a certain cobalt target tube are composed of the strong K-series of cobalt and the weak K lines due to impurities. The wavelengths of the K_α lines are $1.785\mathring{A}$ for cobalt and $2.285\mathring{A}$ and $1.537\mathring{A}$ for impurities. Using Moseley's law, calculate the atomic number of each of the two impurities, (For the K-series, $b = 1$ in Moseley's law). [6]

(b) The wavelength of the K absorption edge of tungsten is $0.178\mathring{A}$ and the average wavelength of the K-series lines are $K_\alpha = 0.210\mathring{A}$, $K_\beta = 0.184\mathring{A}$, and $K_\gamma = 0.179\mathring{A}$.

(i) Construct the X-ray energy level diagram of tungsten. [5]

(ii) What is the least energy required to excite the $L-$series? [3]

(iii) What is the wavelength of the L_α line? [3]

(v) What is the shortest tungsten wavelength that could be emitted? [3]

Q6 (a) Differentiate between natural radioactivity and artificial radioactivity. [3]

(b) (i) The activity of a certain radio-nuclide decreases by 15% of its original value in 10 days. Find its disintegration constant and its half-life. [5]

(ii) The half-life of the radio-active substance is 15 years. Calculate the period in which 2.5% of the initial quantity will have decayed. [5]

(c) The decay law is stated as $N = N_0 e^{-\lambda t}$. Explain the meaning of all symbols appearing in this equation. Hence show that the activity, A, can be written as $A = \lambda N$ [7]

Q7 (a) (i) Define half life in radioactivity. [2]

(ii) The half life of the radioactive nucleus $^{226}_{86}Ra$ is 1.6×10^3 years. If the sample contains 3×10^{16} such nuclei, determine the activity at this time. [3]

(iii) The ^{226}Ra nucleus undergoes alpha decay according to the equation

$$^{226}_{88}Ra \rightarrow ^{222}_{86} Rn + ^{4}_{2} He$$

Calculate the Q value (the disintegration energy) for this process. Take the mass of ^{226}Ra to be 226.025406 u, the mass of ^{222}Ra to be 222.017574 u, and the mass of ^{4}He to be 4.002603 u. [2]

The disintegration energy is $Q = (M_X - M_Y - M_\alpha) \times 931.50$ MeV/u

(b) The radioactive sample contains 3.50 μg of pure $^{11}_{6}C$, which has a half life of 20.4 min

(i) Determine the number of nuclei present initially. [2]

(ii) What is the activity of sample initially and after 8 hours? [5]

(c) Differentiate between negative and positive beta decay taking into account the following:

(i) The charge and mass of the emitted particle. [3]

(ii) The nuclear charge and mass of the resulting daughter. [3]

==End of P-212 Examination==

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF PHYSICS
2012/13 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATION

P-252: CLASSICAL MECHANICS II AND SPECIAL RELATIVITY

Time allowed: **3 Hours**

Instructions

- **This examination paper contains 7 questions. Each question carries 20 marks. Attempt any 5 questions out of the 7 questions given.**
- **This paper has a total of 100 marks. All questions carry equal marks**
- **Show all your work clearly. Omission of essential work will result in loss of marks**
- **Write your computer number clearly on the answer sheet**

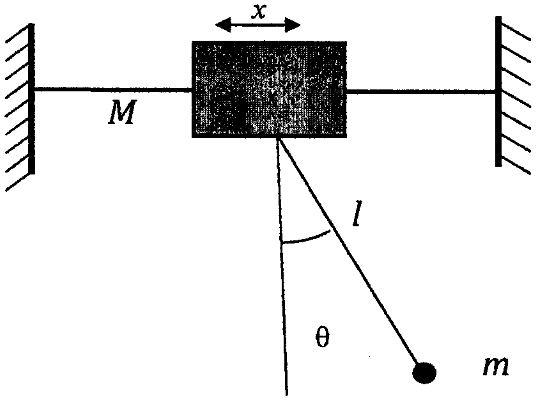
Where necessary, you may use the following:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right)-\frac{\partial L}{\partial q_k}=0,\quad \dot{q}_k=\frac{\partial H}{\partial p_k},\quad \frac{\partial H}{\partial t}=-\frac{\partial L}{\partial t},\qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right)-\frac{\partial L}{\partial q_k}=Q_k,\quad -\dot{p}_k=\frac{\partial H}{\partial q_k}$$

$$x'=\gamma(x-vt),\qquad t'=\gamma\left(t-\frac{Vx}{c^2}\right),\quad y'=y,\quad z'=z,\qquad \gamma=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{\partial^2 u(x,t)}{\partial t^2}=\frac{T}{\mu}\frac{\partial^2 u(x,t)}{\partial x^2},\quad \gamma L=L_0,\quad T=mc^2-m_0c^2,\ t=\gamma t_0$$

Q1 A mass M is free to slide along a frictionless rail. A pendulum of length l and mass m hangs from M as shown in the figure below



(a) Find the equations of motion using the Lagrangian approach. **[15 marks]**

(b) Show that for small oscillations and retaining only terms that are first order (linear) in $\theta, \dot{\theta}$ and $\ddot{\theta}$, the equations reduce to

$$(M+m)\ddot{x}+ml\ddot{\theta}=0 \text{ and}$$

$$\ddot{x}+l\ddot{\theta}+g\theta=0$$ **[2 marks]**

(c) The first equation in (b) above expresses momentum conservation. Show that its solution in general is

$$x=-\left(\frac{ml}{M+m}\right)\theta+At+B=0$$ **[3 marks]**

Q2(a). A string of length L and linear mass density μ lies horizontally and is fixed at both ends. If the string is slightly pulled upward in the middle and allowed to vibrate, show that the total energy in the string at any time is given by

$$E=\frac{\mu}{2}\left[\int_0^L\left\{\left(\frac{dy}{dt}\right)^2+v^2\left(\frac{dy}{dx}\right)^2\right\}\right]dx$$ **[12 marks]**

(b) The relativistic expression for the kinetic energy of a moving particle is given by

$$T=mc^2-m_0c^2$$

Show that for velocities v much lower than the speed of light c, this expression reduces to the Newtonian one. **[4 marks]**

(c) Given that

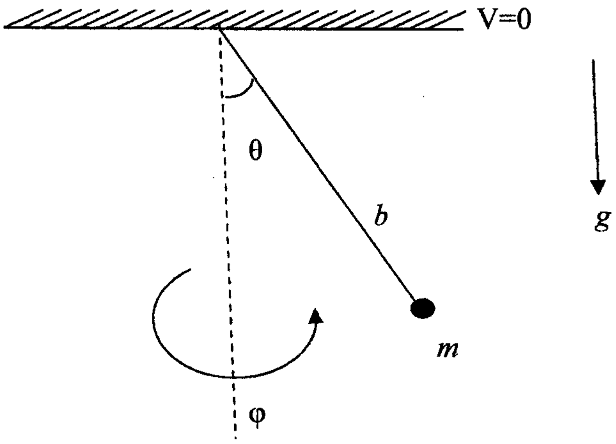
$$x' = \gamma(x - vt), \quad t' = \gamma\left(t - \frac{Vx}{c^2}\right), \quad y' = y, \quad z' = z$$

where the symbols have their usual meanings in the theory of special relativity, show that the composition of velocity in the laboratory frame for the x – component is

$$u_x = \frac{v'_x + V}{1 + \frac{V}{c^2} u'_x}$$

[4 marks]

Q3(a) Use the Hamiltonian method to find the equations of motion for a spherical pendulum of mass m and length b depicted in the figure below. **[14 marks]**



(b) One of a pair of identical twins sets off on a voyage at the age of 20 years and his ship travels at 90% the speed of light. He comes back 3 years later. How old will he appear to the brother he left on Earth. **[2 marks]**

(c) In the laboratory the life time of a particle moving with speed 2.8×10^{10} cm/s is found to be 2.5×10^{-7} s. Calculate the proper life time of the particle. (Take $c = 3.0 \times 10^{10}$ cm/s).

[4 marks]

Q4(a) Show that the sum of the waves $u_1(x,t) = f(x-vt) = A\sin(kx-\omega t)$ and $u_2(x,t) = g(x+vt) = A\sin(kx+\omega t)$ travelling in a medium is

$$u(x,t) = 2A\sin(kx)\cos(\omega t)$$ **[3 marks]**

(b) Find the expression for the power (P) transported along a vibrating string for a wave travelling to the right given by $u = f(x-vt) = f(\xi)$. **[4 marks]**

(c) If the energy were moving from right to left, what would the power delivered be?

[4 marks]

(d) For $u = f(\xi) = A\cos(kx-\omega t)$, evaluate the power delivered by using

$$P = F_y\dot{u} = \left(-T\frac{\partial u}{\partial x}\right)\left(\frac{\partial u}{\partial t}\right)$$ **[4 marks]**

(e) Prove that Galilean invariance does not hold for spherical electromagnetic waves propagating with constant speed c. **[5 marks]**

Q5(a) The solution for the motion of a particle undergoing simple harmonic motion is given as $x = A\sin(\omega_0 t+\delta)$. The particle has a velocity v_1 when the displacement is x_1 and a velocity v_2 when the displacement is x_2. Find the

(i) angular frequency and **[10 marks]**

(ii) amplitude of the motion in terms of the given quantities. **[6 marks]**

(b) The equation of motion of a damped harmonic oscillator is given as $\ddot{x}+k\dot{x}+\omega^2 x = 0$. Assuming a solution of the form $x = e^{\lambda t}$, show that the general solution may be written as $x = Ae^{\frac{-k+\sqrt{k^2-4\omega^2}}{2}t} + Be^{\frac{-k-\sqrt{k^2-4\omega^2}}{2}t}$ where A and B are arbitrary constants. **[4 marks]**

Q6(a) The displacement of a string is given by

$$y = 0.01\cos(3x - 7t)\text{metres}$$

Obtain

(i) the velocity of the point at $x = 3.5\text{m}$ when $t = 3\text{s}$, **[2 marks]**

(ii) the wavelength of the waves on the string, **[4marks]**

(iii) the period of the wave motion. **[2 marks]**

(b) Using the Galilean transformations

$$x_1' = x_1 - vt, \quad x_2' = x_2, \quad x_3' = x_3, \quad t' = t,$$

show that Newton's second law of motion for a particle of mass m is invariant in a stationary inertial frame S and an inertial frame S' moving with a constant velocity v with respect to S. **[4 marks]**

(c) A meter rule travels at 90% of the speed of light along its length. What is its length as seen by an observer in the laboratory frame? **[4 marks]**

(d) Show that $y = \sin(2x)\cos(vt)$ is not a solution to the wave equation

$$\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 u(x,t)}{\partial t^2} = 0$$ **[4 marks]**

Q7(a) Relative to the inertial frame S two events are observed to occur simultaneously on the x-axis. Relative to the inertial frame S' (in standard configuration with S), the two events occur with a time interval of 0.1s, a distance 10^8m apart. Calculate the speed of S' relative to S. **[6 marks]**

(b) A particle is moving parallel to the x-axis of the inertial frame S with a velocity v and acceleration a. The S frame is in standard configuration with the S' frame. Obtain the expression for the acceleration a' of the particle relative to the S' frame. **[14 marks]**

************END OF P252 EXAMINATION**********

The University of Zambia

Department of Physics

Computational Physics I

(P302)

University Second Semester Examination

2012/2013 Academic Year

Instructions **Total Marks 100**

- **Time allowed:** Three (3) Hours.
- **All questions carry equal marks.**
- Marks for each question are shown in the square brackets [ ].
- Whenever necessary, use the information given in the **appendix**
- **Answer:**
    - i) Question one (1).
    - ii) Any three (3) questions from 2, 3, 4, 5 and 6.

Q.1 **(a)** The following figure shows a single-precision 32-bit floating-point binary number representation. If the fraction part is normalized and the exponent is short real-biased (Excess-127), what is the real decimal equivalent of the following binary number?

1	10000101	11101111000000000000000

[6]

(b) The following variable names are encountered in declaration statements of a C program;

i) `keyword` ii) `void` iii) `6thname`

Determine which ones are valid and invalid variable names. If invalid, explain why.

[6]

(c) In an attempt to print all values for x given by the expression $x = j \div u + w$ for integer values $u = 3$, $w = 4$ and $1 \leq j \leq 10$, a student writes the following C program.

```
#include<stdio.h>

int main()
{
  float x;
  int j,u=3,w=4;

  for(j=1;j<=10;j++)
      x=j/u+w;
      printf("%f\n",x);

  return 0;
}
```

The program builds and compiles without any errors; however, it produces undesirable results when executed. Why is this so and how would you rectify the problem?

[6]

(d) Describe the output that will be generated by the following program.

```
#include<stdio.h>

int main()
{
  int i=0, x=0;
do  {
    if(i % 5 == 0) {
        x++;
        printf("%d\n",x);
       }
    ++i;
  } while(i < 20);
 printf("\nX=%d\n",x);
 return 0;
}
```

[7]

Q.2 The following is the composite Simpson's rule algorithm for approximating an integral

$$I = \int_a^b f(x)dx$$

```
INPUT  endpoints $a, b$; even positive integer $n$.
OUTPUT  approximation $XI$ to $I$.
Step 1  Set $h = (b - a)/n$.
Step 2  Set $XI0 = f(a) + f(b)$;
            $XI1 = 0$;   (Summation of $f(x_{2i-1})$.)
            $XI2 = 0$.   (Summation of $f(x_{2i})$.)
Step 3  For $i = 1, \dots, n - 1$ do Steps 4 and 5.
    Step 4  Set $X = a + ih$.
    Step 5  If $i$ is even then set $XI2 = XI2 + f(X)$
                else set $XI1 = XI1 + f(X)$.
Step 6  Set $XI = h(XI0 + 2 \cdot XI2 + 4 \cdot XI1)/3$.
Step 7  OUTPUT ($XI$);
        STOP.
```

Using the above algorithm and the function declaration and definition, write a C program that approximates the following integral.

$$I = \int_3^5 \frac{1}{\sqrt{x^2 - 4}} dx$$

Take $n = 20$.

[25]

Q.3 (a) The solid of revolution obtained by rotating the region under a curve $y = f(x)$, where $a \leq x \leq b$. about the $x - axis$ has surface area given by

$$\text{area} = 2\pi \int_a^b f(x)\sqrt{1 + (f'(x))^2}dx$$

Approximate the surface area using the composite trapezoidal rule with $n = 5$ for

$$f(x) = x^3$$

[10]

(b) A projectile is fired at an angle of elevation $b_0 = 45°$, with $v_y = v_x = 50$ m/sec. Taking into account air resistance, the equations of motion become

$$y = f(t) = 3800(1 - e^{-t/10}) - 320t$$

and

$$x = r(t) = 350(1 - e^{-t/10})$$

Using the Newton-Raphson method, find the elapsed time until impact and find the range. Use the initial guess $t_0 = 8.0$.

[15]

Q.4 **(a)** Translate the following expressions to C expressions

i) $v = u \ln \left(\frac{m+u}{m-qt} \right)$ ii) $u = \frac{x^a + y^b}{z^{1/2}}$ iii) $y = a + \frac{b^{1/2}}{e^{-x}}$

[6]

(b) An electrical circuit consists of a capacitor $C = 1.1F$ in series with a resistor $R_0 = 2.1\Omega$. A voltage $E(t) = 110\sin(t)$ is applied at time $t = 0$. When the resistor heats up, the resistance becomes a function of current i. The differential equation for the current i is given by

$$\left(1 + \frac{2k}{R_0} i\right) \frac{di}{dt} + \frac{1}{R_0 C} i = \frac{dE}{dt}$$

and the current-dependent resistance is

$$R(i) = R_0 + ki$$

where $k = 0.9$ and current $i = 0$ at $t = 0$. Using function definition and declaration and taking $dt = 2$ sec and $0 \leq t \leq 60$ write a C program that will print the current i and resistance R at time t to the standard output device. What are the values of the current i and resistance R at $t = 2$ and $t = 4$ sec?

[19]

Q.5 **(a)** Consider the data presented in the table below

x_i	0.00	0.20	0.80	1.00	1.20	1.90	2.00	2.10	2.95	3.00
y_i	0.01	0.22	0.76	1.03	1.18	1.94	2.01	2.08	2.90	2.95

Find, by the method of least squares, the line $y = ax + b$ approximating these data. Write a C program that will determine the values of a and b for the given data.

[15]

(b) Given the following matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ -1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 6 & 2 & 3 \\ 0 & 2 & 4 & 5 & 6 & 7 \\ 0 & 0 & -1 & -1 & -4 & 0 \\ -2 & 8 & 12 & 18 & 23 & 21 \end{pmatrix},$$

write a C program, with the above matrix initialized and using *nested* `for` statements, that will calculate the sum of the *squares* of the main diagonal elements.

[10]

Q.6 A text data file, `sample.dat`, contains the following sample data for two independent variables t and x respectively, of a two variable function

$$y = f(t, x) = 0.5\sin(3t - 3\pi x)$$

0.0	0.78
0.2	0.71
0.4	0.20
0.6	0.70
0.8	0.18
1.0	0.13
1.2	0.54
1.4	0.51
1.6	0.68
1.8	0.87
2.0	0.22

Write a C program that will do the following

i) open the file `sample.dat` and check if it has been opened successfully,

ii) read the values t and x from the file, `sample.dat`,

iii) calculate the value of y from the read values of t and x ,

iv) print the values of t, x and y to the file `oscillation.dat`.

[25]

⋆⋆⋆⋆⋆⋆⋆ **End of Examination** ⋆⋆⋆⋆⋆⋆⋆

Appendix

Commonly Used Library Funcitons:

Function	Type	Purpose
`abs(i)`	`int`	`Returns the absolute value of i`
`cos(d)`	`double`	`Returns the cosine of d`
`exp(d)`	`double`	`Raises` e `to the power d` ($e = 2.7182818\ldots$)
`fabs(d)`	`double`	`Returns the absolute value of d`
`log(d)`	`double`	`Returns the natural logarithm of d`
`log10(d)`	`double`	`Returns the logarithm (base 10) of d`
`pow(d1,d2)`	`double`	`Returns d1 raised to the d2 power`
`sin(d)`	`double`	`Returns sine of d`
`sqrt(d)`	`double`	`Returns the square root of d`
`tan(d)`	`double`	`Returns the tangent of d`

Newton-Raphson:

Given an equation $f(x) = 0$, the root can be obtained iteratively using the Newton-Raphson method:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Trapezoidal Method:

The integral I of a function $f(x)$,

$$I = \int_a^b f(x)dx$$

can be approximated numerically using the Trapezoidal rule:

$$I = \frac{h}{2}\left[f(a) + f(b) + 2\sum_{i=1}^{N-1} f(x_i)\right]$$

where N is the number of strips (Trapezoids) in the interval $[a, b]$, $h = (b-a)/N$ is the step-size. And $x_i = a + ih$.

Simpson's Method:

The integral I of a function $f(x)$,

$$I = \int_a^b f(x)dx$$

can be approximated numerically using Simpson's method using:

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a) + 2\sum_{i=2j}^{N-2} f(x_i) + 4\sum_{i=2j-1}^{N-1} f(x_i) + f(b)\right]$$

where N is an even number of strips in the interval $[a, b]$, $h = (b - a)/N$ is the step-size.

Euler's Method:

The Euler's method for finding the approximate solution of a first-order ODE, $y' = f(x, y)$, with initial condition$f(x_0) = y_0$, on an interval $[x_0, x_n]$ with step size $h = (x_n - x_0)/n$, is given by

$$y_{i+1} = y_i + hf(x_i, y_i)$$

where $h = (x_n - x_0)/n$ and n is the number of division between x_0 and x_n.

Least squares fit to a straight line:

$$y = ax + b$$

$$a = \frac{ns_{xy} - s_x s_y}{ns_{xx} - s_x s_x}$$

$$b = \frac{s_{xx}s_y - s_{xy}s_x}{ns_{xx} - s_x s_x}$$

where $s_x = \sum_{i=1}^{n} x_i, \quad s_{xx} = \sum_{i=1}^{n} x_i^2, \quad s_y = \sum_{i=1}^{n} y_i, \quad s_{xy} = \sum_{i=1}^{n} x_i y_i$

The University of Zambia
School of Natural Sciences
Physics Department
University Examinations 2013
Second Semester
P-412: Nuclear Physics

Attempt <u>any four</u> questions. All questions carry equal marks. The marks are shown in brackets. Clearly indicate on the answer script cover page which questions you have attempted.

Time: Three hours. **Maximum marks = 100.**

Write your computer number clearly on the answer book.

Wherever necessary use:

ε_0 = 8.85 × 10^{-12} $C^2/N.m^2$	$m_{hydrogen\ atom}$ = 1.007825 a.m.u.
$m_{neutron}$ = 1.008665 a.m.u. = 939.551 MeV	m_{alpha} = 4.002603 a.m.u.
1 a.m.u. = 931.5 MeV = 1.6604 × 10^{-27} kg	m_p = 1.67 × 10^{-27} kg = 938.28 MeV
c = 3 × 10^8 m/s	m_e = 9.11 × 10^{-31} kg = 0.511 MeV
h = 6.63 × 10^{-34} J-s	e = 1.6 × 10^{-19} C
$\hbar$ = 6.58 × 10^{-22} MeV-s = 1.05 × 10^{-34} J-s	1eV = 1.6 × 10^{-19} J
1 fermi = 10^{-15} m	1 barn = 10^{-28} m^2
Avogadro's constant = 6 x 10^{23} per mole	Velocity of light = 3 x 10^8 m.sec^{-1}.
$\left(e^2/4\pi\varepsilon_0\right)$ = 1.44 MeV-fermi	$m = \left(m_0c^2/c^2\right) \equiv \left(MeV/c^2\right)$
$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$	$e^2/\hbar c = (1/137)$ $\hbar c$ = 197.33 MeV-fermi

$(1s_{1/2})^2$, $(1p_{3/2})^4$, $(1p_{1/2})^2$, $(1d_{5/2})^6$, $(2s_{1/2})^2$, $(1d_{3/2})^4$, $(1f_{7/2})^8$, $(2p_{3/2})^4$, $(1f_{5/2})^6$, $(2p_{1/2})^2$,

$(1g_{9/2})^{10}$, [50]. $E = \dfrac{\hbar^2}{2\Im}\left[J(J+1) - BJ^2(J+1)^2\right]$. $\Delta E_C = \dfrac{3}{5}\dfrac{e^2}{R}\left[Z^2 - (Z+1)^2\right]$

Q1(a) (i) Name the four basic interactions known in nature and give a number characterizing the strength of each interaction. **[4]**

(ii) Discuss the range of each of these interactions and explain how each one is believed to arise. **[3]**

(iii) List a few important processes for which each one of these interactions is essential. **[3]**

(b) The electrostatic energy of a charge q uniformly distributed throughout a sphere of radius R is $U = \frac{3}{5}\frac{e^2}{R}\frac{1}{4\pi\varepsilon_0}$, $[W_{Coul.} = \frac{3}{5}\frac{q^2}{R}]$.
Using this in the case of positron beta decay, derive an expression for the decrease in Coulomb energy. **[5]**

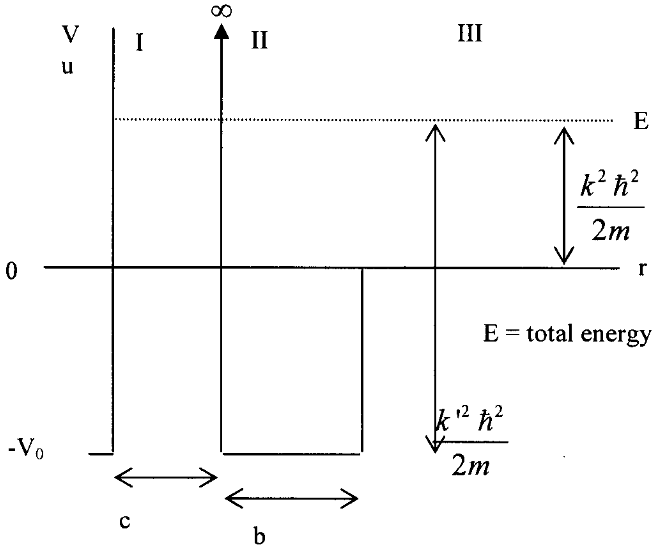
(c) Show that an alpha particle with total energy E_0 incident on a potential barrier of energy V ($V > E_0$) and of thickness b has a quantum probability of penetrating it. **[8]**

Sketch the energy diagram showing the incident and transmitted waves of the particle. [**2**]

Q2(a) A particle of mass m and energy E is scattered by a square-well potential of depth V_0 and radius R. Show that the s-wave phase shift for $k \rightarrow 0$ is given by

$$\tan\delta_0 = \frac{k}{k'}\tan k'R - \tan kR$$

where $$k^2 = \frac{2mE}{\hbar^2} \text{ and } (k')^2 = \frac{2m(E+V_0)}{\hbar^2}$$ [**15**]



In region II, $\frac{d^2u}{dr^2}+\frac{2m}{\hbar^2}(V_0+E)u=0$. In region III $\frac{d^2u}{dr^2}+\frac{2m}{\hbar^2}Eu=0$

(b) Explain the meaning of the term *scattering length.* Explain in which case it is positive, and when it is negative.

What do you understand by *effective range* approximation in nucleon-nucleon scattering? When is the approximation valid? [5]

(c) (i) What general piece of information does the quadrupole moment of a nucleus give us? [2]

(ii) The deuteron is supposed to be in a state of angular momentum l = 0 and yet it has a quadrupole moment. What does this tell us about the nature of the force between a proton and a neutron? [3]

Q3 (a) Consider the alpha-particle decay $^{230}_{90}Th \rightarrow {}^{226}_{88}Ra+\alpha$ and use the following expression to calculate the values of the binding energy B for the two heavy nuclei involved in the process.

$$B=a_V A-a_S A^{2/3}-a_C\frac{Z(Z-1)}{A^{1/3}}-a_A\frac{(N-Z)^2}{A}-a_P\frac{1}{A^{3/4}}$$

where $a_V=15.5\,\text{MeV}$, $a_S=16.8\,\text{MeV}$, $a_C=0.72\,\text{MeV}$, $a_A=23.0\,\text{MeV}$, and $a_P=-34.5\,\text{MeV}$.

Given that the total binding energy of the alpha particle is 28.3 MeV, find the energy Q released in the decay. [7]

(b). The tabulated masses of $^{13}_{5}B$, $^{13}_{6}C$, and $^{13}_{7}N$ are 13.0178, 13.0034, and 13.0057 a.m.u. respectively. Calculate in MeV the values of the Coulomb and asymmetry coefficients in the semi-empirical mass formula: [18]

$$M(A,Z) = Z M_H + N M_n - \alpha A+\beta A^{2/3} + \frac{\gamma Z^2}{A^{1/3}}+\frac{\delta(A-2Z)^2}{A}+ \lambda\,(A,Z)$$

Given M_H = 1.00783 amu, M_n = 1.00867 amu and 1 amu = 931.5 MeV.

Q4.(a) (i) Describe briefly the basic assumptions concerning the features of the forces involved in the single-particle shell model of the nucleus. **[4]**

(ii) Write down the rules for determination of the angular momenta and parities of nuclear ground states as obtained from the shell model. **[4]**

(b) (i) How does the inclusion of a strong spin-orbit coupling in the single-particle shell model of the nucleus lead to the splitting of a state of given l ?

(ii) Show that the splitting is proportional to $(2l + 1)$. **[8]**

(c) The ground state of ${}^{39}_{20}Ca$ has spin-parity value of $\frac{3}{2}^+$. [9]

The next higher (excited) states have spin-parity values of $\frac{1}{2}^+, \frac{7}{2}^-$ and $\frac{3}{2}^-$.

Interpret these values on the basis of the single-particle shell-model.

Q5(a) In the experimentally determined ground state rotational band of ${}^{238}_{92}U$, the first excited state with $J^\pi = 2^+$ has an excitation energy of 44 keV, and a higher-lying state with $J^\pi = 8^+$ is found to have an excitation energy of 500 keV.

Obtain the angular momenta, parity, and the expected excitation energies of the second and third excited states of this rotational band. **[14]**

(b) In a certain nucleus the ground and first excited states have respectively the following values of

$$J^\pi : \frac{1}{2}^+, \frac{11}{2}^-, \frac{5}{2}^+ \text{ and } \frac{7}{2}^+ .$$

Identify the *multipolarities* of the following gamma-transitions:

$$\frac{7}{2}^+ \to \frac{5}{2}^+ , \quad \frac{7}{2}^+ \to \frac{11}{2}^- , \quad \frac{5}{2}^+ \to \frac{1}{2}^+ , \quad \frac{11}{2}^- \to \frac{1}{2}^+$$ **[8]**

(c) Describe how the electric and magnetic *multipole* radiations are emitted by an excited nucleus. **[3]**

Q6(a) Give short explanations of the terms *allowed, super allowed, first forbidden*, and *second forbidden* used in beta transitions in terms of the nuclear matrix element $|M_{if}|$, log fT values and the nuclear shell model. **[12]**

(b) Distinguish between the *Fermi* and the *Gamow-Teller* selection rules in beta decay of nuclei. **[7]**

(c) For the following beta transitions deduce the *degree of forbiddenness* of the transitions.

$$^{39}_{18}Ar \rightarrow \, ^{39}_{19}K \qquad ^{7}_{3}Be \rightarrow \, ^{7}_{4}Li$$

Explain the transitions in terms of the single-particle shell model. **[3+3]**

==End of P-412 Exam==

THE UNIVERSITY OF ZAMBIA
PHYSICS DEPARTMENT
UNIVERSITY OF ZAMBIA
SECOND SEMESTER EXAMINATIONS 2013
P422 SOLID STATE PHYSICS II

TIME: THREE HOURS
ANSWER: ANY FOUR QUESTIONS
MAXIMUM MARKS: 100

The reciprocal-lattice primitive vectors are given by

$$\mathbf{b}_1 = 2\pi\frac{\mathbf{a}_2 \times \mathbf{a}_3}{\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3)}, \; \mathbf{b}_2 = 2\pi\frac{\mathbf{a}_3 \times \mathbf{a}_1}{\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3)} \text{ and } \mathbf{b}_3 = 2\pi\frac{\mathbf{a}_1 \times \mathbf{a}_2}{\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3)}$$

$$\int_0^\infty x^{1/2}e^{-x}dx = \left(\frac{\pi}{4}\right)^{1/2}$$

---

1. **(a)** **(i)** What is the distinction between the direct and the reciprocal lattice? [3 marks]

(ii) Explain the Wigner-Seitz prescription for constructing the unit cell and state its connection to the Brillouin zone. [2 marks]

(b) One choice of the primitive vectors for the hexagonal space lattice is

$$\mathbf{a}_1 = \frac{a}{2}\hat{\mathbf{i}} + \frac{a\sqrt{3}}{2}\hat{\mathbf{j}}, \ \mathbf{a}_2 = -\frac{a}{2}\hat{\mathbf{i}} + \frac{a\sqrt{3}}{2}\hat{\mathbf{j}} \text{ and } \mathbf{a}_3 = c\hat{\mathbf{k}}$$

(i) Determine the primitive vectors in the reciprocal lattice and comment.[6 marks]

(ii) Calculate the volumes of the primitive cells in both cases. [4 marks]

(c) The Laue condition for X-ray diffraction by a crystal is

$$\mathbf{R} \cdot (\mathbf{k} - \mathbf{k}') = 2\pi m, \quad m = 0, 1, 2, 3, ...$$

(i) Explain the connection between this formula and the reciprocal lattice. [5 marks]

(ii) Show that it is equivalent to the Bragg diffraction condition

$$2d \sin\theta = n\lambda$$

[5 marks]

2. **(a)** Give one reason for the importance of crystal planes. [2 marks]

(b) In a certain simple-cubic crystal, first-order Bragg reflection corresponds to an angle of 25.2° when X rays of wavelength 1.37 Å are used. It is known that a family of lattice planes with the Miller indices (122) is responsible for the diffraction. Calculate the lattice constant. [4 marks]

(c) A paramagnetic substance is composed of N particles per unit volume, each with a spin of $s = 1/2$. If the magnetic moment of each particle is μ_B,

(i) Show that the magnetisation of the substance is

$$M = N\mu_B \tanh\left(\frac{\mu_B H}{k_B T}\right)$$

where H is the applied magnetic field. [12 marks]

(ii) Obtain the limiting values of M for $T \to 0$ and $T \to \infty$ and comment. [4 marks]

(iii) Obtain the values of the magnetic susceptibility in the limit $T \to 0$ and $T \to \infty$. [3 marks]

3. **(a)** **(i)** Interpret and prove Bloch's theorem

$$\psi_{\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u_{\mathbf{k}}(\mathbf{r})$$

[15 marks]

(ii) Show that in one dimension, $u_{\mathbf{k}}(\mathbf{r}) = u_k(x)$ satisfies the Schroedinger equation

$$-\frac{\hbar^2}{2m}\left(\frac{d^2}{dx^2} + 2ik\frac{d}{dx} - k^2\right)u_k(x) + V(x)u_k(x) = Eu_k(x)$$

[4 marks]

(b) In the Kronig-Penney model, the energy quantisation condition is

$$\cos kl = \begin{cases} \cos\alpha c\cosh\beta b - \frac{\alpha^2-\beta^2}{2\alpha\beta}\sin\alpha c\sinh\beta b, & E < 0 \\ \cancel{\cos}\cos\alpha c\cos\lambda b - \frac{\alpha^2+\lambda^2}{2\alpha\lambda}\sin\alpha c\sin\lambda b, & E > 0 \end{cases}$$

where $\alpha = \alpha(E)$ and b, λ and β are constants.

Use this to show that

(i) the energy levels occur in bands. [4 marks]

(ii) the energy is periodic in k. [2 marks]

4. **(a)** **(i)** Explain the difference between an intrinsic and an extrinsic semiconductor. [4 marks]

(ii) Explain how to obtain n-type and p-type semiconductors from an intrinsic semiconductor. [4 marks]

(iii) Justify the observed dependence of the conductivity of a semiconductor on temperature. [3 marks]

(b) The cyclic boundary conditions

$$\psi(x) = \psi(x + L)$$

are often used in band theory.

(i) Show that when used with the Bloch wave

$$\psi_k(x) = e^{ikx}u_k(x)$$

they lead to quantisation of the values of k. [4 marks]

(ii) Use this information to prove that the number of states in a band in a crystal equals the number of unit cells. [4 marks]

(iii) Calculate the number of electrons in the valence band of a crystal of volume 1 mm^3 whose primitive lattice vectors are

$$\mathbf{a} = \frac{a}{2}(\widehat{\mathbf{j}} + \widehat{\mathbf{k}}), \quad \mathbf{b} = \frac{a}{2}(\widehat{\mathbf{i}} + \widehat{\mathbf{k}}) \ , \mathbf{c} = \frac{a}{2}(\widehat{\mathbf{i}} + \widehat{\mathbf{j}})$$

where $a = 1$Å. [6 marks]

5. Given the Fermi-Dirac distribution function

$$f(E) = \frac{1}{e^{(E-E_F)/k_BT} + 1}$$

and the density of electronic states in the energy range dE as

$$D(E) = \frac{4\pi}{\hbar^3}(2m)^{3/2}E^{1/2}$$

(**i**) Show that the number of holes per unit volume in the valence band of a semiconductor is

$$p = 2\left(\frac{2\pi m_h^* k_B T}{\hbar^2}\right)^{3/2} \exp\left[-\left(\frac{E_F - E_V}{k_B T}\right)\right] = N_V \exp\left[-\left(\frac{E_F - E_V}{k_B T}\right)\right]$$

You may assume that

$$\frac{|E - E_F|}{k_B T} \gg 1$$

Note that m_p^* is the effective mass of the hole, h is Planck's constant, T is the absolute temperature, E_F is the Fermi energy, k_B is Boltzmann's constant and E_v is the valence-band edge. [15 marks]

(**ii**) Given also that the number of electron carriers per unit volume in the conduction band is

$$n = 2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2} \exp\left[-\left(\frac{E_C - E_F}{k_B T}\right)\right] = N_C \exp\left[-\left(\frac{E_C - E_F}{k_B T}\right)\right]$$

and that $N_C = N_V$, show that the Fermi level for an intrinsic semiconductor is in the middle of the band gap when $T = 0$ or when m_p^*, the hole effective mass, is equal to the electron effective mass m_e^*. [4 marks]

(**iii**) Show that the carrier concentration can be written as

$$n = 2\left(\frac{2\pi k_B T}{h^2}\right)^{3/2} \left(m_e^* m_p^*\right)^{3/4} \exp\left(-\frac{E_g}{2k_B T}\right)$$

and explain how this can be used to measure the band gap E_g. [6 marks]

6. (**a**) Use band theory to explain the distinction between insulators, conductors and semiconductors. [4 marks]

(**b**) (**i**) Prove that the effective mass of an electron is given by

$$m^* = \frac{\hbar^2}{d^2E/dk^2}$$

where E is the energy and $\mathbf{k}$ is the wave number. [8 marks]

(**ii**) Explain why the effective mass may differ from the actual mass and may in fact sometimes be negative. [2 marks]

(**c**). According to the so-called tight-binding approximation, the energy of an electron in a band of a simple-cubic crystal of lattice parameter a is given by

$$E(k) = E_0 - \alpha - 8\gamma \cos k_x a \cos k_y a \cos k_z a$$

where E_0, α and γ are constants

$$k = \left(k_x^2 + k_y^2 + k_z^2\right)^{1/2}$$

If in general the velocity of an electron in a band is given by

$$v = \frac{1}{\hbar}\frac{dE}{dk}$$

(**i**) Show that the velocity of an electron in this band is

$$\begin{aligned} v &= \frac{8\alpha k\gamma}{\hbar}\left(\frac{1}{k_x}\sin k_x a\cos k_y a\cos k_z a + \frac{1}{k_y}\cos k_x a\sin k_y a\cos k_z a\right. \\ &\left.+\frac{1}{k_z}\cos k_x a\cos k_y a\sin k_z a\right) \end{aligned}$$

[5 marks]

(**ii**) Determine the lower and upper edges of this band. [4 marks]

(**iii**) Determine the width of the band. [2 marks]

******END OF EXAMINATION*****

UNIVERSITY OF ZAMBIA
DEPARTMENT OF PHYSICS
2013 SECOND SEMESTER UNIVERSITY EXAMINATIONS

P455
QUANTUM MECHANICS II

DURATION: Three hours.

INSTRUCTIONS: Answer four questions from the six given. *Each question carries 25 marks with marks indicated in brackets.*

MAXIMUM MARKS: 100

DATE: 22nd August 2013

Formulae that may be needed

All symbols used have their usual meaning.

1.

$$\begin{aligned}\left(H_0 - W^{(0)}\right) v^{(0)} &= 0,\\ \left(H_0 - W^{(0)}\right) v^{(1)} + \left(H' - W^{(1)}\right) v^{(0)} &= 0,\\ \left(H_0 - W^{(0)}\right) v^{(2)} + \left(H' - W^{(1)}\right) v^{(1)} - W^{(2)} v^{(0)} &= 0\end{aligned}$$

2.

$$[\hat{a}, \hat{a}^{\dagger}] = 1$$

3.

$$\begin{aligned}\hat{a}^{\dagger}\hat{a}|n\rangle &= n|n\rangle\\ \hat{a}\hat{a}^{\dagger}|n\rangle &= (n+1)|n\rangle\\ \hat{a}|n\rangle &= \sqrt{n}|n-1\rangle\\ \hat{a}^{\dagger}|n\rangle &= \sqrt{n+1}|n+1\rangle\end{aligned}$$

4.

$$|u_n\rangle = \frac{\hat{a}^{(\dagger)^n}}{\sqrt{n!}}\,|u_0\rangle$$

5.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

6. Parity formula for hydrogen atom energy eigenfunctions

$$\hat{P}\phi_{nlm}(r) = \phi_{nlm}(-r) = (-1)^l \phi_{nlm}(r)$$

7. Hydrogen atom ground state wave function

$$\phi_{100} = \frac{1}{\sqrt{\pi}} \left(\frac{Z}{a_\mu}\right)^{3/2} e^{-Zr/a_\mu}$$

8.

$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{me^2}, \quad a_\mu = \frac{m}{\mu}a_0, \quad \alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} = \frac{1}{137}$$

QUESTION 1

(a) Determine the commutators $[\hat{a}^\dagger\hat{a}, \hat{a}]$ and $[\hat{a}^\dagger\hat{a}, \hat{a}^\dagger]$. (4 marks)

(b) Derive the spacing of the eigenvalues of $\hat{a}^\dagger\hat{a}$. Then, show clearly how the eigenvalue spectrum can be fixed, hence obtaining the eigenvalues of $\hat{a}^\dagger\hat{a}$. (16 marks)

(c) Determine the matrix for the raising operator $\hat{a}^\dagger$ in the harmonic oscillator energy representation. (5 marks)

QUESTION 2

(a) An electron in the Coulomb field of a proton is in the state described by the wave function

$$\psi(r) = \left(\frac{\alpha}{\sqrt{\pi}}\right)^{3/2} e^{-\alpha^2 r^2/2}$$

Determine the integral which gives the probability of finding the electron in the ground state of the hydrogen atom, but DO NOT evaluate the integral. (7 marks)

(b) Consider a hydrogen atom whose wave function at $t = 0$ is the following superposition of energy eigenfunction $\psi_{nlm}(r)$:

$$\psi(r, t = 0) = \frac{1}{\sqrt{14}} \left[2\psi_{100}(r) - 3\psi_{200}(r) + \psi_{322}(r)\right]$$

(i) Is this wave function an eigenfunction of the parity operator? (4 marks)

(ii) What is the probability of finding the system in the ground state (100), in the state (200) in the state (322), or in any other energy state? Show that your answers are consistent with probability theory. (8 marks)

(iii) What is the expectation value of $\hat{L}_z$? (6 marks)

QUESTION 3

(a) State Born's probability rule. (4 marks)

(b) Define the hermitian conjugate of an operator $\hat{A}$ and prove that it can act forward or backward in a scalar product expression. Use Dirac notation. (5 marks)

(c) If U is a unitary operator show that if $\langle\psi|\psi\rangle = 1$ then $\langle U\psi|U\psi\rangle = 1$ (3 marks)

(d) State the expansion postulate. What is meant by a representation? By finding the expectation value of an operator using the expansion postulate, give the meaning of the expansion coefficients in the expansion postulate. (13 marks)

QUESTION 4

Consider a system which is in a two-fold degenerate eigenstate of the Hamiltonian H_0. A time-independent perturbation H' acts on the system. All symbols that follow have their usual meaning.

(a) Using the first order equation of time-independent perturbation theory derive the equation

$$\sum_n{}' \left(E_n - E_m\right) c_n^{(1)} u_n + \left(H' - W^{(1)}\right)\left(a_1^{(0)} u_{m1} + a_2^{(0)} u_{m2}\right) = 0$$

(9 marks)

(b) From your result in part (a) derive the equation

$$\left(h_{11} - W^{(1)}\right) a_1^{(0)} + h_{12} a_2^{(0)} = 0$$

(4 marks)

(c) From your result in part (a) derive the equation

$$h_{21} a_1^{(0)} + \left(h_{22} - W^{(1)}\right) a_2^{(0)} = 0$$

(4 marks)

(d) Solve the two simultaneous equations of part (b) and (c) to obtain $W^{(1)}$. Give the physical interpretation of your result for $W^{(1)}$. (8 marks)

QUESTION 5

Consider a system in an eigenstate $\hat{H}_0$. The eigenvalue equation of $\hat{H}_0$ is

$$\hat{H}_0 u_n = E_n u_n$$

A time-independent perturbation $\hat{H}'$ acts from time 0 to t. All symbols that follow have their usual meaning.

(a) Beginning with the time-dependent Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = \left(\hat{H}_0 + \lambda\hat{H}'\right)\psi$$

derive the equation

$$\frac{da_f}{dt} = \frac{1}{i\hbar}\sum_n \lambda a_n e^{i\omega_{fn}t}H'_{fn}$$

State the meaning of $|a_f(t)|$. (12 marks)

(b) The solutions for zeroth, first and second order equations are

$$\begin{aligned}
a_f^{(0)}(t) &= \delta_{ij} \\
a_f^{(1)}(t) &= \frac{1}{i\hbar}\int_0^t e^{i\omega_{fi}t'}H'_{fi}(t')\,dt', \\
a_f^{(2)}(t) &= \frac{1}{(i\hbar)^2}\sum_n\int_0^t e^{i\omega_{fn}t''}H'_{fn}(t'')\,dt''\int_0^{t''} e^{i\omega_{ni}t'}H'_{ni}(t')\,dt',\ t'' > t'
\end{aligned}$$

What interpretation of $|a_f^{(i)}(t)|$ does the initial condition $a_f^{(0)}(t=0)$ allow? Interpret $a_f^{(0)}(t)$ from the solution above. By first re-ordering the solution for $a_f^{(1)}(t)$, give its interpretation. Illustrate your answer for $a_f^{(1)}(t)$ with a diagram. Give the interpretation of the second and third order terms with a diagram and a brief explanation in words only. Hence give the interpretation of $a_f^{(r)}(t)$ in words only. Finally, express the total $a_f(t)$ as a series and give its interpretation in terms of your interpretation of the various orders $a_f^{(i)}(t)$. (13 marks)

QUESTION 6

(a) Consider the variation method.

(i) In what year and by who was this method developed? What problems was the method originally applied to and to which quantum mechanical problems is it most useful?

(3 marks)

(ii) Prove that

$$W = \int \psi^* \hat{H} \psi \, dx \geq E_0,$$

where E_0 is the ground state energy, i.e., the lowest eigenvalue of the Hamiltonian operator $\hat{H}$, and ψ is an arbitrary wave function.

(7 marks)

(iii) On what idea is the variation method based? Describe in general terms how the method is used to obtain an approximation. (7 marks)

(b) By substituting $\psi(x) = Ae^{iS(x)/\hbar}$ into the one-dimensional time-independent Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

derive the zeroth and first order differential equation of the WKB approximation.

(8 marks)

END

THE UNIVERSITY OF ZAMBIA
PHYSICS DEPARTMENT
UNIVERSITY EXAMINATIONS –SECOND SEMESTER 2013
PHY4815 - PHYSICS OF RENEWABLE ENERGY RESOURCES AND ENVIRONMENT

TIME: 3 HOURS MAX MARKS: 100

ATTEMPT ANY **FOUR** (4) QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS. THE MARKS ARE SHOWN IN SQUARE BRACKETS.

You may use the following information:

Boltzmann constant $k = 1.38 \times 10^{-23}$ JK^{-1}
Gas constant R = 8314 J/kmol.K
1 electron volt = 1.6×10^{-19} J
Stefan's constant σ = 5.67×10^{-8} W/m^{2}K^4
Sun's radius R_s = 6.96×10^8m
Mean Earth-Sun distance r_0 = 1.496×10^{11}m
Solar constant I_{sc} = 1367 Wm^{-2}
Earth's radius R_e = 6.37×10^6m
Planck's constant h = 6.6×10^{-34}J.s
Speed of light c = 3×10^8m.s^{-1}

In the usual notation

$$E_0 = \left(\frac{r_0}{r}\right)^2 = 1 + 0.033\cos\left(\frac{360\, d_n}{365}\right)$$

$$\delta = 23.45^\circ \sin\left[\frac{360}{365}(d_n + 284)\right]$$

$$\cos\theta_z = \sin\delta\sin\phi + \cos\delta\cos\phi\cos\omega$$

$$\tan\psi = \frac{\cos\delta\sin\omega}{\cos\delta\sin\phi\cos\omega - \sin\delta\cos\phi}$$

$$\cos\psi = \frac{\sin\alpha\ \sin\varphi - \sin\delta}{\cos\alpha\ \cos\varphi}$$

$$\cos\theta = (\sin\varphi\cos\beta - \cos\varphi\sin\beta\cos\gamma)\sin\delta$$
$$+ (\cos\varphi\cos\beta + \sin\varphi\sin\beta\cos\gamma)\cos\delta\cos\omega$$
$$+ \cos\delta\sin\beta\sin\gamma\sin\omega$$

$$\omega = 15^\circ(12 - t); \quad \omega_s = \cos^{-1}(-\tan\phi\tan\delta)$$

Solar time = clock time + $4(L_l - L_s)$ min + EOT

Wien's Law: $\lambda_{max}T = 2898$ µm.K

The emissive power of a black body $B_\lambda(T)$ (in W/m^2 per unit wavelength range) is

$$B_\lambda(T) = \frac{2\pi hc^2}{\lambda^5\left(e^{\frac{hc}{\lambda kT}} - 1\right)}$$

Direct flux on an inclined surface

$$F^{dir} = I_{sc}\cos\theta\exp\left(-\frac{\tau}{\cos\theta_z}\right)$$

In a single heat exchanger the exit temperature is

$$T_{f,e} = T_B - (T_B - T_{f,i})\exp\left(-\frac{U_L L}{\dot{m}C_f}\right),$$

and the heat extraction rate is

$$\dot{Q} = \dot{m}C_f(T_B - T_{f,i})\left[1 - \exp\left(-\frac{U_L L}{\dot{m}C_f}\right)\right].$$

Fresnel's equations

$$r_{\|} = \left[\frac{n_r^2 \cos\theta_i - n_i \sqrt{n_r^2 - n_i^2 \sin^2\theta_i}}{n_r^2 \cos\theta_i + n_i \sqrt{n_r^2 - n_i^2 \sin^2\theta_i}} \right]^2$$

$$r_{\perp} = \left[\frac{n_i \cos\theta_i - \sqrt{n_r^2 - n_i^2 \sin^2\theta_i}}{n_i \cos\theta_i + \sqrt{n_r^2 - n_i^2 \sin^2\theta_i}} \right]^2$$

Overall reflectance and transmittance of a single glazing are

$$R = r\left[1 + \frac{\alpha^2(1-r)^2}{1-\alpha^2 r^2}\right]$$

$$T = \frac{\alpha(1-r)^2}{1-\alpha^2 r^2}$$

The carrier concentration in an intrinsic semiconductor is

$$n_i = p_i = AT^{3/2} exp\left(-\frac{\varepsilon_g}{2kT}\right)$$

The resistivity of an extrinsic material is

$$\rho = \frac{1}{e(n\mu_n + p\mu_p)}$$

The reverse saturation current density is

$$J_0 = DT^3 exp\left(-\frac{\varepsilon_g}{kT}\right)$$

The forward current density is

$$J = J_0\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

The J-V characteristic equation for a single cell is

$$J = \overline{K} F - J_0\left(e^{\frac{eV}{kT}} - 1\right)$$

Yearly variation of the equation of time

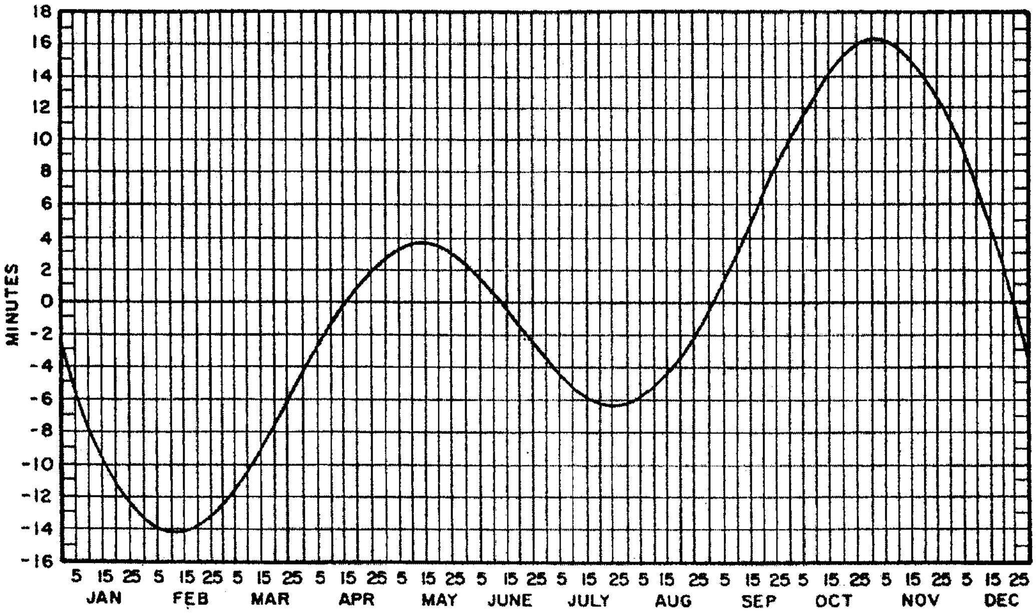


TABLE *The function $f(x)$*

x(μm-K)	$f(x)$	x(μm-K)	$f(x)$	x(μm-K)	$f(x)$
1100	0.001	4600	0.580	8100	0.860
1200	0.002	4700	0.594	8200	0.864
1300	0.004	4800	0.608	8300	0.868
1400	0.008	4900	0.621	8400	0.871
1500	0.013	5000	0.634	8500	0.875
1600	0.020	5100	0.646	8600	0.878
1700	0.029	5200	0.658	8700	0.881
1800	0.040	5300	0.669	8800	0.884
1900	0.052	5400	0.680	8900	0.887
2000	0.067	5500	0.691	9000	0.890
2100	0.083	5600	0.701	9100	0.893
2200	0.101	5700	0.711	9200	0.895
2300	0.120	5800	0.720	9300	0.898
2400	0.140	5900	0.730	9400	0.901
2500	0.161	6000	0.738	9500	0.903
2600	0.183	6100	0.746	9600	0.905
2700	0.205	6200	0.754	9700	0.908
2800	0.228	6300	0.762	9800	0.910
2900	0.251	6400	0.770	9900	0.912
3000	0.273	6500	0.776	10000	0.914
3100	0.296	6600	0.783	11000	0.932
3200	0.318	6700	0.790	12000	0.945
3300	0.340	6800	0.796	13000	0.955
3400	0.362	6900	0.802	14000	0.963
3500	0.383	7000	0.808	15000	0.969
3600	0.404	7100	0.814	16000	0.974
3700	0.424	7200	0.819	17000	0.978
3800	0.443	7300	0.824	18000	0.981
3900	0.462	7400	0.830	19000	0.983
4000	0.483	7500	0.834	20000	0.986
4100	0.499	7600	0.840	30000	0.995
4200	0.516	7700	0.844	40000	0.998
4300	0.533	7800	0.848	50000	0.999
4400	0.549	7900	0.852		
4500	0.564	8000	0.856		

Q.1.

(a) Assuming the following different hypothetical scenarios, state for each of them what difference it would make for the Earth if

(i) The Earth only rotated about its axis but did not revolve round the Sun.
(ii) The Earth only revolved round the Sun but did not rotate about its axis.
(iii) The axis of the Earth's rotation were perpendicular to the ecliptic plane.

[6]

(b) A photovoltaic panel with an efficiency of 12% and surface area of $2m^2$ is placed horizontally in Livingstone (latitude 18°S and longitude 26°E). Calculate the power produced by the solar panel on 20 August at 10.00 a.m. clock time if the optical thickness of the atmosphere is $r = 0.2$. The standard longitude for Livingstone is 30° East of the Greenwich meridian. Assume there is no diffuse flux.

[19]

Q.2 (a)

(i) Write down the equations of heat transfer from plate to glazing and from glazing to surroundings for a flat-plate collector. [4]

(ii) Using the equations in (i) above and assuming the heat transfer coefficients to be constants, obtain an expression for the overall heat transfer coefficient of the collector and explain its significance with the help of a network of resistors.

[8]

(b) A flat-plate solar heating panel contains two glazings. If T_p=80°C and T_{sky}=T_a=20°C, find

(i) the overall transfer coefficient for the system, [6]
(ii) the flux loss from the absorber, and [2]
(iii) the temperature of each glazing. [5]

The coefficients for heat transfer from the plate to the inner glazings are $U_{d,1}^{(c)}$= 2.5 W/m^2-°C and $U_{d,1}^{(r)}$= 5 W/m^2-°C. Those for heat transfer from one glazing to the other are $U_{d,2}^{(c)}$= 2.3 W/m^2-°C and $U_{d,2}^{(r)}$= 6 W/m^2-°C. The coefficients for heat transfer from the outer glazing are $U_{\infty}^{(c)}$= 8 W/m^2-°C and $U_{\infty}^{(r)}$= 7 W/m^2-°C.

Q.3. (a) Clearly but briefly distinguish between

(i) surface reflectance and overall reflectance of a glazing, [2]
(ii) surface transmittance and overall transmittance of a glazing. [2]

(b) Show that the overall reflectance *R* and the overall transmittance *T* for a single glazing are given in terms of the surface reflectance *r* and the bulk transmittivity α by

$$T = \frac{\alpha(1-r)^2}{1-\alpha^2 r^2}.$$

$$R = r\left[1 + \frac{\alpha^2(1-r)^2}{1-\alpha^2 r^2}\right].$$

Under what condition is R+T=1?

[9]

(c) Direct solar radiation is incident normally on a glazing of thickness 1.0cm. The refractive index of the glazing material is 1.60, the extinction coefficient $k=0.03\text{cm}^{-1}$ and the intercepted flux is $F_{inc}=1000\text{W/m}^2$. Find the

(i) surface reflection coefficient [2]
(ii) bulk transmittivity [2]
(iii) overall transmittance [2]
(iv) flux leaving the lower surface of the glazing [2]
(v) overall reflectance [2]
(vi) flux reflected backwards. [2]

Q.4 (a) (i) Define optical efficiency, thermal efficiency and overall efficiency of a heating panel. [2]

(ii) Express the overall efficiency in terms of the optical and thermal efficiency. [2]

(iii) What considerations should be given to construct an efficient solar heating panel? [2]

(b) (i) What do you understand by "stagnant conditions" as referred to a collector? [2]

(ii) Write down the equation of energy balance under non-steady conditions of such a collector and solve it to show

$$T_p - T_a = \frac{F_{abs}}{U_c}\left[1 - \exp\left(-\frac{U_c t}{C_A}\right)\right],$$

[6]

(iii) What will be the value of T_p - T_a under steady-state stagnant conditions? [1]

(iv) What is the time constant of the collector and what is its physical significance? [2]

(c) An absorber plate of a collector is made of copper (C = 389 J/kg-°C) and has an area of 3 m^2 and a mass of 30 kg. The overall heat transfer coefficient to the surroundings is U_c = 8 W/m^2.

(i) Find the time constant of the collector. [3]
(ii) If the insolation suddenly changes from zero to some constant value, how much time will it take for (T_p-T_a) to reach 80% of the stagnant limit? [5]

Q.5 (a) Assuming air behaves as an ideal gas and is in hydrostatic equilibrium

(i) Clearly stating the assumptions made, obtain an expression for the density profile of the atmosphere. [6]

(ii) Assuming that the temperature does not change with height, show that the density profile of an exponential atmosphere is given by [2]

$$\rho = \rho_0 \exp\left(-\frac{z}{H}\right)$$

(iii) What do you understand by the scale height of an exponential atmosphere? [2]

(iv) Show that if the entire atmosphere were redistributed with a uniform density equal to its sea level value, it would extend only as high as the scale height. [5]

(b)

A single solar heating panel uses water (C_f = 4186 J.kg^{-1}.^{o}C^{-1}) as the transfer fluid. The water is flowing at a rate $\dot{m}$ = 0.005 kg.s^{-1} while it enters the panel at 20^{o}C and leaves at 50^{o}C. The fluid is carried to a storage tank by an exterior pipe 10 m long whose overall heat transfer coefficient per unit length is $\overline{U}_L$ =0.2 W.m^{-1}.^{o}C^{-1} The ambient temperature is T_a = 15 ^{o}C. Find the temperature of the water entering the storage tank and the percentage of the heat produced by the panel that is lost by the pipe. [10]

Q. 6 (a) Consider a semiconductor with band gap ε_g = 0.75 eV.

(i) Find the cut off wavelength λ_c for the semiconductor. [3]

(ii) Find the maximum fraction of the solar spectrum (6000K) that can possibly be harnessed by the semiconductor photovoltaic. [3]

(iii) If the spectral responsivity is equal to K_λ = 0.1 A/W for 0<λ<1 μm and K_λ = 0.3 A/W for 1 μm < λ < λ_c, find the average responsivity for the solar spectrum and calculate the photocurrent density at a flux of 1 Sun. [6]

(b) A PV array has 200 circular cells each with diameter of 10 cm. The array has 10 parallel strings each with 20 cells in series. Given that $\overline{K}$ =25mA.cm^{-2}.Sun^{-1}, J_0 = 5x10^{-10} mA.cm^{-2} and T = 300 K:

(i) Using the *J-V* characteristic equation for a single cell, obtain the *I-V* characteristic equation for the array [6]

(ii) Find the open circuit voltage of the array for a radiation of 1 Sun. [5]

(iii) Find the short circuit current for a radiation of 1 Sun. [2]

------------------- END OF THE EXAM --------------------