Article

The Development of an Assessment Instrument for Numeracy Competence and its Application to Selected Primary Schools in Zambia

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Abstract

Low achievement in numeracy by primary school students is one of the biggest concerns of Zambia. Since it aims at developing the nation, development of the human resources is an inevitable issue to tackle. This research is to develop an assessment instrument to identify the students' level of numeracy competence. It pays attention to *structure* through the relevant previous researches such as Mulligan & Mitchelmore (2014) and Roberts (2015). As a result of the survey, the developed instrument identified distribution of different levels of competence, prevalence of counting-all strategy, and possible solutions to facilitate students' level of competence.

Keywords: Numeracy, Assessment, Structure

1. Background

The Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ) consists of 16 countries that cooperate in undertaking integrated research and training activities (SACMEQ, 2019). It has conducted four assessments of mathematics achievement in its member countries, the number of participating countries rising from 7 in SACMEQ I in 1999, 14 in SACMEQ II in 2004, and 15 in SACMEQ III in 2011 and SACMEQ IV in 2015.

Zambia has participated in SACMEQ since 1999 and has consistently recorded poor performance. For example, 67.3% of Zambian 6th grade students did not even reach the category "Basic Numeracy" in SACMEQ II (Hungi et al., 2010). Since numeracy is the foundation for further mathematics and science, it forms an essential component of the human resources needed for the middle- and long-term development of society. Therefore, the Ministry of General Education (MoGE) in Zambia gives priority to numeracy as well as literacy (Ministry of Education, 1996; Ministry of Education, Science, Vocational Training and Early Education, 2013; Ministry of National Development Planning, 2017)

Uchida (2012) studied Zambian primary students' calculation abilities and identified widespread use of a "counting-all" strategy without use of grouping by tens. He developed a diagnostic method to investigate their strategies more closely using a Newmann approach (Newmann 1977; Clements 1980) that combined usage of concrete materials with oral questioning in the local language, Nyanja. His most significant finding was that even poor performers showed some understanding of basic mathematics. For example, some of them was not able to read the word problem aloud but was able to solve it when it was read by the interviewer.

Roberts (2015) conducted a similar investigation in South Africa. Referring to Dowling (1998), he identified three modes of representations in Grade 2 student's responses: iconic, indexical and symbolic. He also found that students' most common calculation strategy was counting-all. Slow learners especially employed counting-all strategies in counting objects and also in addition and subtraction of up to two-digit numbers. He stated that the strategy was very time consuming, even for relatively simple calculations, and students often made errors due to mistakes in counting.

The above studies have recognized that students acquire competence related to numeracy by different degrees, achieving a variety of strategies such as counting all, counting on, counting by groups and representational methods. They, however, did not pay much attention to develop a tool to measure students' numeracy competence in a wider scope and to analyze the competence qualitatively and quantitatively.

Therefore, the objective of the present research was to develop an assessment instrument to identify Zambian students' numeracy competence. Our target group is lower and intermediate graders (Grades 1 to 4). The outcomes of this study could provide both valuable information on the causes of the observed low level of numeracy competence and suggest a more effective teaching strategy.

2. Previous studies

Clements & Sarama (2013) described children's development of understanding by employing the idea of learning trajectories. Table 1 shows their learning trajectory for counting. They stated that "the learning trajectory has a goal, a developmental progression and instructional activities. To attain a certain mathematical competence within a given domain (goal), children typically learn each successive level of thinking (the developmental progression), aided by activities (instructional tasks)" (Clements & Sarama, 2013, p.122). The development does not occur in the same way for all students, so teaching needs to pay attention to students' present condition and development of each student. The value of a learning trajectory is in "structuring the activities in accordance with theoretically and empirically based methods models of children's thinking." (Clements & Sarama, 2013, p.137)

Table 1 Learning trajectory (Clements & Sarama, 2010, pp.3-4)

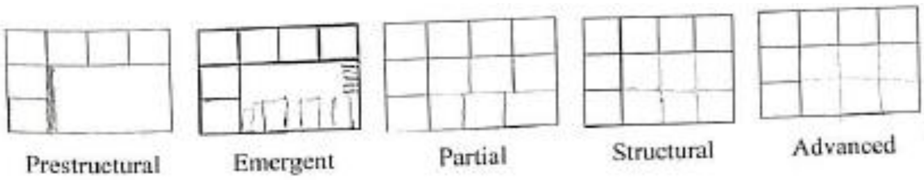
Age	Development progression	Instructional tasks
1	[Pre-counter] *Verbal*, No verbal counting [Chanter] *Verbal*, Chants "sing-song" or sometimes-indistinguishable number words.	Associate number words with quantities and as components of the counting sequence. Repeated experience with the counting sequence in varied contexts.
2	[Reciter] *Verbal*, Verbally counts with separate words, not necessarily in the correct order.	Provide repeated, frequent experience with the counting sequence in varied contexts. *Count and Race* Children verbally count along with the computer (up to 50) by adding cars to a

		racetrack one at a time.
3	[Reciter (10)] *Verbal*, Verbally counts to ten, with some correspondence with objects.	*Count and Move* Have all children count from 1-10 or an appropriate number, making motions with each count. For example, say, "one" [touch head], "two" [touch shoulders], "three" [touch head], and so forth.
	[Corresponder] Keeps one-to-one correspondence between counting words and objects (one word for each object), at least for small groups of objects laid in a line.	*Kitchen Counter* At the computer, children click on objects one at a time while the numbers from one to ten are counted aloud. For example, they click on pieces of food and a bite is taken out of each as it is counted.
4	[Counter (Small Numbers)] Accurately counts objects in a line to 5 and answers the "how many" question with the last number counted.	*Cubes in the Box* Have the child count a small set of cubes. Put them in the box and close the lid. Then ask the child how many cubes you are hiding. If the child is ready, have him/her write the numeral. Dump them out and count together to check. *Pizza Pizzazz 2* Children count items up to 5, putting toppings on a pizza to match a target amount.
	[Producer (Small Numbers)] Counts out objects to 5. Recognizes that counting is relevant to situations in which a certain number must be placed.	*Count Motions* While waiting during transitions, have children count how many times you jump or clap, or some other motion. Then have them do those motions the same number of times. Initially, count the actions with children. *Pizza Pizzazz 3* Children add toppings to a pretend pizza (up to 5), to match target numerals.
5	[Counter and Producer (10+)] Counts and counts out objects accurately to 10, then beyond (to about 30). Has explicit understanding of cardinality (how numbers tell how many).Keeps track of	*Counting Towers (Beyond 10)* To allow children to count to 20 and beyond, have them make towers with other objects such as coins. Children build a tower as high as they can, placing more coins, but not straightening coins already in the tower. The goal is to estimate and then count to find out how many coins are in your tallest tower. *Dino Shop 2* Children add dinosaurs to a box to match target

	objects that have and have not been counted, even in different arrangements.	numerals.

Mulligan & Mitchelmore (2009; 2013) have studied early mathematical development from the perspective of pattern and structure. They developed the Pattern and Structure Assessment (PASA) to investigate the mathematical awareness of primary school students. They found that children's representations showed various levels of structural understanding (see Fig. 1). They stated that children's awareness of structure was closely related to their mathematical ability. In other words, the degree of mathematical ability can be understood in terms of the

Fig. 1 Levels of structure (Mulligan et al. 2013)



degree of structuralization of representation:

> We formed the hypothesis that the more a student's internal representational system has developed structurally, the more coherent, well organized, and stable in its structural aspects will be their external representations and the more mathematically competent the student will be. (Mulligan et al. 2011, p.555)

Low achievers are characterized by low levels of structuralization:

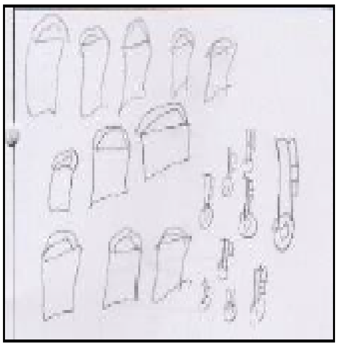
> We found that low achievers ... were more likely to produce poorly organized representations and were only able to replicate models of grouped, arrays or patterns that had been produced by others. They tended to use unitary counting exclusively, and appeared unable to visualize part-whole relations. Moreover, they made little progress between Grade 2 and Grade 5. (Mulligan & Mitchelmore, 2013, p.33)

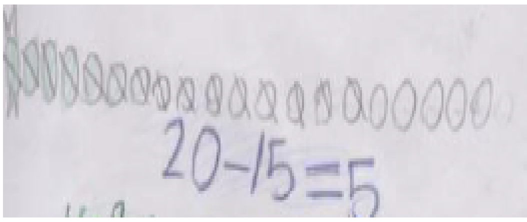
In other words, the low level of structuralization (including the use of the counting-all strategy), can hinder primary students from moving onto a higher level of mathematical competence.

Roberts (2015) also pointed out the weakness of the counting-all strategy: "There is growing

Fig. 2 Iconic, indexical and syntactical representations (Roberts, 2015)

consensus in South Africa that one of the major factors inhibiting learners' mathematical progression is continued using counting in ones strategies for Mathematical calculation." (p.243). Here the counting in ones is equivalent of counting all. By adding concrete and symbolic representations into the three representations previously mentioned, he classified students' representations into five levels: concrete, iconic, indexical, symbolic, and syntactical. Figure 2 shows examples for some of these representations. Iconic representation shows diagrams of real objects, while indexical one represents objects with semi-abstract diagrams of circles. Both of them work based on one-to-one correspondence. On the other hand, syntactical representation shows numerical symbols is assigned to some group of objects such as 10. In these previous studies except Clements and Sarama (2010), they pay attention to such key characteristics as structure, levels, and representation at certain ages.





These characteristics appear in the activity and reflect their level of understanding. Mulligan and Mitchelmore (2009; 2013) emphasized that structure and representation had different levels of organization. Some representations are more organized in terms of structure and others are less organized; the most advanced level of structure shows how it is possible to extend the visible pattern to cases that are currently invisible. In other words, the children's understanding can be evaluated by assessing the structural organization in their representations. For example, in the arrangement of numbers and circles in Figure 3, the latter are visually structured; they are aligned horizontally and vertically. Their arrangement helps us to recognize the pattern in

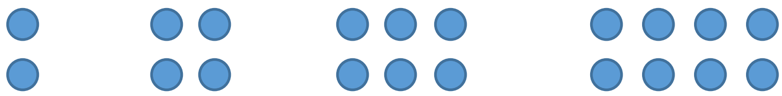


Fig. 3 Arrangement of numbers and circles

Other related studies (Lesh, 1981; Nakahara, 1995) paid attention to representation that it is closely related to conceptual understanding. In their studies, such external representations can play an important role in explaining their own ideas to others and thus even to promote students' understanding. Activities such as interpreting drawn objects and arranging objects for oneself promote structural understanding. From these, we can consider that each level of counting would show a specific level of understanding and that change in the levels shows development of understanding, however slowly this occurs. We further may make the assumption that it would be possible to facilitate the development of students' understanding as a whole, by properly addressing the early stages of development. In this sense, the learning trajectory (Clements & Sarama, 2010) provides a clue for extending this relation to the lower end since it starts at the very young age, which shows a very fundamental stage.

3. Research method and assessment instrument

3.1. Research project

The research project on which this paper is based is to investigate possible sources of the low mathematical performance of Zambian students. This project, which is being conducted between March 2018 and December 2020, has as its ultimate aim, the development of an assessment instrument for diagnosing students' level of numeracy competence, and developing a teaching strategy to facilitate it. Both of them are developed from the perspective of seeing structurally.

3.2. Research sample and process

Due to Zambian students' weakness and cultural characteristics in seeing structures in numbers, the existing instruments such as TIMSS and PISA for assessing their numeracy competence may not function well and a new one needs to be developed further to measure students' level of competence and way of seeing structure. However, the new instrument needs to have a secure connection to existing instrument so that good performers can also be assessed according to them (Uchida, 2012). For example, PISA for Development is connected to the main PISA study. The chosen research method is therefore exploratory, so that an initial instrument is developed based on the literature review and later may be adjusted according to the data collected in the study.

The sample size was limited by logistic factors. The final sample will be consisted of students from ten schools in Lusaka district, Lusaka Province, selected after consultation with the MoGE. Two schools were selected from each of the five socioeconomic zones within the district. In each school, two high performers and two medium performers were selected by class teachers in each of Grade 1 to 4 classes. We are planning six field surveys, as shown in the Table 2. The first four surveys are preliminary ones to explore a new instrument to assess the Zambian students more appropriately, confirm the validity of this instrument. The last two surveys are main ones which will measure the impact of intervention between pre- and post-surveys.

After each survey, the results were analyzed to examine whether each interview item for the task and response categories adequately described students' level of understanding and the instrument as whole was revised wherever necessary. So far, we have finished the third survey.

Table 2: Field survey with sample sizes

Time	2018		2019		2020	
	1st survey	2nd survey	3rd survey	4th survey	Main survey (Pre)	Main survey (Post)
	Mar.	Sep.	Mar.	Sep.	Mar.	Aug.
No. of Schools	2	2	4	4	10	10
No. of 1st Graders	8	8	16	16	20	20

No. of 2nd Graders	8	8	16	16	20	20
No. of 3rd Graders	8	8	16	16	20	20
No. of 4th Graders	8	8	16	16	20	20

Development proceeded in three stages as follows:

Stage 1: confirm the practicality of the chosen approach.

Stage 2: develop an assessment instrument (with tasks arranged vertically and response levels arranged horizontally; See Annex 2), construct appropriate tasks, and assess its validity, the time required and the prevalence of judgement errors.

Stage 3: shorten the time taken by reducing the number of tasks, and increase the validity of the interview by reducing the number of judgement errors.

Since it was important to collect data on students' responses using the above instrument that are as detailed as possible, we adapted the interview method. A video record of each interview was made, accompanied by a qualitative description made by observers. Each interview team consisted of at least three persons: interviewer, observer and camera man, with the camera man acting as the second observer. Each interview for each child took 30 to 60 minutes.

3.4. Numerical competence and assessment instrument

The most recent school mathematics syllabus in Zambia (Ministry of Education, Science, Vocational Training and Early Education, 2013) was analyzed to define numeracy competence. In the Grade 1 to Grade 4 syllabus, there are key terms such as count, count in tens, read, write, use ten as a unit, order numbers, add, subtract, multiply, divide and so on. Thus "Numeracy Competence" to be measured in this study was defined as a combination of sub-competences as follows.

1. Counting objects one by one and by groups, counting forward and backward
2. Recognizing patterns and structure of numbers
3. Composing and decomposing numbers
4. Seeing numbers in terms of unit and relative size of numbers
5. Understanding the base ten numeration system
6. Performing the four basic arithmetical operations

In developing the assessment instrument, the Pattern and Structure Assessment (Mulligan & Mitchelmore, 2013) and the concept of levels of representation in Robert (2015) were the important contributions. The identified characteristics are structure, representation, and activity as previously mentioned. Based on these, the first draft of assessment instrument, the ten-frame (Fig. 4) was developed to grasp the numeracy competence spatially and structurally and "Bottle tops" are used to ensure the activity by students. The students are encouraged to fill these tops into the frame. So, this set of ten-frame and bottle tops has the following important characteristics.

i. Flexibility: To move bottle tops freely
ii. Expression and internalization: To correspond how to express and how to understand
iii. Structural understanding: To understand place value system with base ten
iv. Sustainability: Made with the available materials which are cost-friendly

The instrument consisted of a number of tasks (of which were changing in different survey for the better tasks for students) using the above ten frames and bottle tops, with a five-level classification of responses for each task. Originally there were more tasks for this competence and they are reduced to the present form.

Figure 4: Ten frame

3.5. Developing assessment tasks

Each of the sub-competences was assessed by several tasks. All the instructions and questions by the interviewer are given in Nyanja, one of the local languages but the responses could be either in English or in the local language. For example, the numbers were commonly answered in English by students. The interview tasks are as follows:

1.1 The interviewer places 20 bottle tops randomly in front of the student and says "Count these and tell me the number". When s/he counts one by one correctly, says "Count in 2s".

1.2 The interviewer asks the child to "Count up to 20". If this task is too difficult, the interviewer says "You may use the bottle tops for counting." For students who reach the fourth response category, the interviewer asks "Count by 2s and 5s up to 20".

1.3 The interviewer asks "Count down from 20". If the task is too difficult, the interviewer says "You may use the bottle tops for counting". For students who reach the fourth response category, the interviews asks "Count down from 20 by 5s and 10s."

2.1 The interviewer places 10 each of white and red bottle tops (altogether 20) randomly and says "Show patterns using red and white bottle tops on a line."

2.2 The interviewer places 7 bottle tops on a ten-frame and asks "How many bottle tops are there?"

2.3 The interviewer places 18 bottle tops on two ten-frames and asks “How many bottle tops are there?”

2.4 The interviewer places 20 bottle tops randomly in front of the student and says “Suppose there are ten-frames and arrange the bottle tops in the imaginary ten-frames.”

3.1 The interviewer places 3 bottle tops on one ten-frame, and 9 bottle tops on the other, and says “How many bottle tops are there altogether? You may move the bottle tops”.

3.2 The interviewer places 13 bottle tops on one ten-frame, and 19 bottle tops on the other, and says “How many bottle tops are there altogether? You may move the bottle tops”.

3.3 The interviewer places 12 bottle tops on two ten-frames, and says “How many bottle tops do you need to fill up to 20?”

3.4 The interviewer places 27 bottle tops on four ten-frames, and says “How many bottle tops do you need to fill up to 40?”

4.1 The interviewer places 49 bottle tops on the vertically arranged five ten-frames, and says “How many bottle tops are there?”

5.1 The interviewer says "how many dots are there in the following dot diagram." (As for the tasks 5.1 to 5.3, see Annex 1.)

5.2a The interviewer says "Point the number 13 on the number line.

5.2b The interviewer says “Circle the amount of 13 marbles on the dotted marble sheet."

5.3a The interviewer says "Indicate the number 76 on the number line 1.”

5.3b The interviewer says "Indicate the number 76 on the number line 2.”

5.3c The interviewer says “Circle the amount of 76 marbles on the dotted marble sheet 1.”

5.3d The interviewer says “Circle the amount of 76 marbles on the dotted marble sheet 2.”

6.a1 The interviewer shows the expression “7+8” on a card, and says “Arrange the bottle tops, and tell the sum.”

6.a2 The interviewer shows the expression “11 + 13” on a card, and says “Arrange the bottle tops, and tell the sum.” Then, he/she says “Write the math sentence in numerals on a sheet of paper.”

6s1 The interviewer shows the expression “15－8” on a card, and says “Arrange the bottle tops, and tell the answer”. Then, he/she says “Write the math sentence in numerals on a sheet of paper.”

6.s2 The interviewer shows the expression “25－12” on a card, and says “Arrange the bottle tops, and tell the answer.” Then he/she says “Write the math sentence in numerals on a sheet of paper.”

6.m1 The interviewer shows the expression “2x3” on a card, and says “Arrange the bottle tops, and tell the answer.” Then he/she says “Write the math sentence in numerals on a sheet of paper.”

6.m2 The interviewer shows the expression “12x3” on a card, and says “Arrange the bottle tops, and tell the answer.” Then he/she says “Write the math sentence in numerals on a sheet of paper.”

6.d1 The interviewer shows the expression “8÷2” on a card, and says “Arrange the bottle tops, and tell the answer.” Then he/she says “Write the math sentence in numerals on a sheet of paper.”

6.d2 The interviewer shows the expression "36÷3" on a card, and says "Arrange the bottle tops, and tell the answer." Then he/she says "Write the math sentence in numerals on a sheet of paper."

3.6. Scoring the assessment tasks

Responses to each of the assessment tasks were classified into five levels with reference to correctness and the degree of organization of structure in their responses. We regard that even behind a counting-all strategy (level 3), the students have at least grasped the objects to count as a whole and constructed a one to one correspondence with the counting numbers. In order to grasp Zambian students' characteristics clearly, it was necessary to extend the instrument to the lower end by differentiating "Not at all" (Level 1) and some partial order (Level 2). Table 3 shows an example of how this was done. The entire set of response categories are listed as Annex 2.

Table 3: Response categories for Task 2.2

Task	1 No at all	2 Partly implicit	3 Implicit	4 Structural	5 Advanced
The interviewer places 7 bottle tops on a ten-frame and asks "How many bottle top are there?"	S/he tries to count, but cannot do it completely	S/he makes a mistake in counting	S/he can identify the number (7) by counting one by one.	S/he can identify the number (7) using any groups or counting on from a certain number.	Besides level 4, s/he can explain verbally.

4. Results

Against the original plan in the Table 2, the table 4 indicates the allocation of different interview tasks in different grades in the third field survey. According to the time limitation, each grader cannot take all tasks. Each task takes four students per class and there are four classes per grade. However, due to some accidents and adjustment, 2nd graders takes some tasks and don't in other tasks.

Table 4: Interview tasks were answered in different grades

	Com.1	Com. 2	Com.3	Com.4	Com.5	Com.6
1st Graders	4x4	4x4	4x4			
2nd Graders	4x1+5x1	4x3+5x1	4x3+5x1	4x3+5x1	4x2	
3rd Graders			4x4	4x4	4x4	4x4
4th Graders			4x4	4x4	4x4	4x4

Total number	25	33	65	49	40	32

Note: The number in the cell shows "(no of students per class) x (no of classes)".

The results so far are shown below. Because of the small sample sizes and the apparent lack of variation, numbers have been combined across grades.

(1) Sub-competence 1 Counting objects one by one, by groups, count forward and backward

Table 5: Result of Sub-competence 1

Task	L1	L2	L3	L4	L5	Total
1.1	0	1	0	9	15	25
1.2	0	0	1	9	15	25
1,3	6	4	0	10	5	25

The result of tasks 1.1 and 1.2 revealed that all students except one counted up to 20 and also did with 20 objects, with most of them reaching a structural or advanced level. However, 40% of students (L1 and L2) had difficulty in counting down from 20 in Task 1.3.

(2) Sub-competence 2 Recognizing patterns and structure of numbers

Table 6: Result of Sub-competence 2

Task	L1	L2	L3	L4	L5	Total
2.1	2	18	6	2	5	33
2.2	0	0	23	2	8	33
2.3	0	2	20	3	8	33
2.4	1	4	14	6	8	33

The result of task 2.1 showed that more than the half of the students could not make a meaningful pattern (L1 & L2). The result of task 2.2.and 2.3 revealed that more than the half of them counted 7 and 18 bottle tops one by one (L3) but eight students consistently used grouping tens (L5). Task 2.4 asked students to think of an imaginary frame in mind. The result showed that a third of students used some kind of pattern.

(3) Sub-competence 3 Composing and decomposing numbers

Table 7: Result of Sub-competence 3

	L1	L2	L3	L4	L5	Total
3.1	0	0	28	20	17	65
3.2	3	4	25	11	22	65

3.3	2	2	37	9	15	65
3.4	1	1	37	9	17	65

The result of task 3.1 and 3.2 reveals that about half of the students used grouping in combining numbers of bottle tops (L4 & L5). However, a large number of students counted all (L3). The results of task 3.3 and 3.4 show similar but stronger tendency in that direction, more than half of them using the counting-all strategy. A quarter to a third of the students, however, used a grouping in tens strategy (L5).

(4) Sub-competence 4 Seeing numbers in terms of a unit

Table 8: Result of Sub-competence 4

	L1	L2	L3	L4	L5	Total
4.1	2	5	15	2	25	49

The result of task 4.1 revealed that about half of the students used a grouping by tens strategy (L5). This may look contradictory to the previous results, suggesting that the smaller the number, the more frequently they use ‘count all’ strategy. This may indicate the students’ potential capability to use a grouping by tens strategy.

(5) Sub-competence 5 Understanding the base ten system

Table 9: Result of Sub-competence 5

	L1	L2	L3	L4	L5	Total
5.1	0	2	20	2	15	39
5.2a	8	9	4	5	14	40
5.2b	3	4	19	5	9	40
5.3a	2	11	2	6	19	40
5.3b	14	22	0	1	3	40
5.3c	6	11	18	2	3	40
5.3d	3	16	9	4	8	40

The result of task 5.1 showed about half of the students used a counting-all strategy (L3), which was consistent with the above results. The tasks 5.2a, 5.3a, and 5.3b were to locate the number along the number line. The result revealed that the task 5.3b was more difficult than the other two tasks, because the students were expected to plot the point not on the markers but between the markers proportionally. The tasks 5.2b, 5.3c, and 5.3d were to encircle the expected number of dots. The result revealed that task 5.3c was more difficult than the other two, because the arrangement is four 5x5 matrixes and seemed unfamiliar to them.

(6) Sub-competence 6 Significance, procedure and proficiency of calculation

Table 10: Result of Sub-competence 6

	L1	L2	L3	L4	L5	Total
6.a1	0	1	17	8	6	32
6.a2	1	0	17	7	7	32
6.s1	3	1	22	5	1	32
6.s2	2	2	22	3	3	32
6.m1	2	1	7	7	15	32
6.m2	2	2	12	9	7	32
6.d1	10	1	2	7	11	31
6.d2	11	2	2	9	8	32

The result of the tasks 6.a1 and 6.a2 showed that all students except one could solve addition problems but more than half of them (17 students) employed counting all strategies. This tendency become strong for subtraction according to the result of task 6.s1 and 6.s2, where 22 students employed counting-all strategies. Four students were even wrong (L1 and L2) and only a few students employed some grouping strategies (L4 and L5).

As for the tasks of multiplication such as 6.m1 and 6.m2, the result reveals that most of them could find the correct answers (L3, L4 and L5). However, the majority of the students arranged the bottle tops in groups but still counted them all. In the division tasks, many students could not make sense of the tasks (L1).

5. Discussion of results

From the above results, we can discuss three points. Firstly, the assessment instrument and tasks have revealed Zambian students' varying levels of competence. The majority of them obtained correct answers but use a counting-all strategy (L3), which helps them get an answer but might serve as hindrance to further development. Roberts (2015) has revealed that in South Africa, there are also many students who have such a tendency of counting. On the other hand, the result showed also that there were several students who employed some kind of grouping and can explain their strategies verbally (L4 & L5).Thus, it seems possible that the future intervention and teaching strategy may improve Zambian students' strategies.

Secondly, the analysis revealed that there was some consistency and inconsistency among achievement levels of tasks. In other words, some students employed some grouping strategies in composition of numbers but could not employ grouping in calculation. Those students tended to go back to a counting-all strategy when asked to a give reason. Such a swing back phenomenon may show the influence of their teachers, or it may be a characteristic of the tasks used. The relation among tasks is to be further investigated in further analysis.

Thirdly, the distribution of levels prompted us to consider the teaching strategies to promote students' understanding to the upper level. This strategy should be different to facilitate some students, who are at level 1, to level 2 and also those, who are at level 3, to level 4. For example, students at level 3 can be facilitated to level 4 by some grouping activities during the lesson. During the interviews, we observed that some students seemed to implicitly use a ten-frame, suggesting that it may be possible to teach them use it explicitly.

6. Conclusion

All of the above three discussion points are related not only to Zambian students' poor level of numeracy competence but also to the Zambian culture of teaching. We have observed that, when students have a slight difficulty in a problem, the teachers seem to encourage students to use a counting-all strategy. It would be helpful to encourage teachers to use a teaching strategy that promotes an intentional shift from one stage to another so that students can overcome such slight difficulties without reverting to counting all. In addition, many students we interviewed appeared to be unaccustomed to being asked to give reasons for their answers or an explanation for their procedures. This phenomenon is probably also a result of the prevailing classroom culture. We believe that students' meta-cognitive activity would be stimulated if Zambian teachers would more often request explanations and discussion of the students' calculation strategies. However, the classroom culture has not been clarified in this paper yet. This is an issue for future research.

Note 1

1. [1]To address students' poor performance in terms of "numeracy competence", there are three purposes:
2. To review the previous studies (theoretical and practical) and define "Numeracy Competence".
3. To identify the status of students' numeracy competence and the challenges, and to formulate a draft prescription through field surveys.
4. To polish a draft prescription to the Education Package, and to propose it to the respective authorities regarding mathematics education in future. Here the Education Package represents a set of instrument, materials and prescription.

Note 2

This paper reports a part of the research project entrusted by JICA (Japan International Cooperation Agency). In conducting the project, the team presented the research instrument and methodology to MoGE, which supplied the list of project schools. The team, however, is fully responsible for implementation of the research project, the analysis of its results and the interpretation of the analysis.

Note 3

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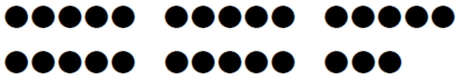
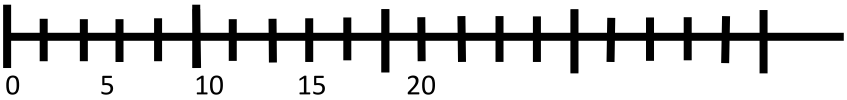
ANNEX 1

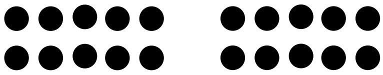
5-1



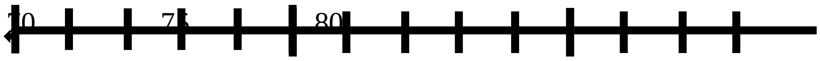
5-2(a) Number line



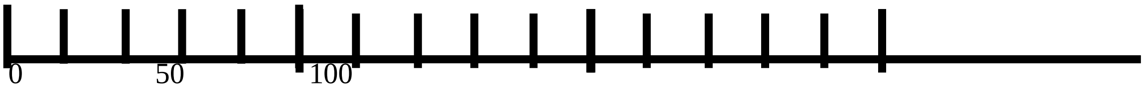
5-2 (b) Representation with dots



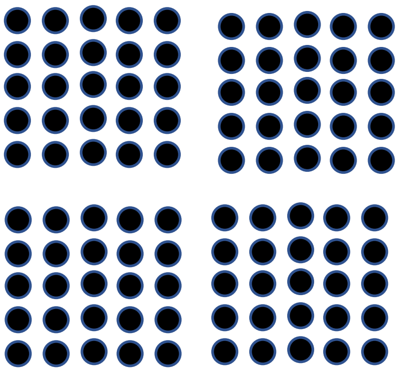
5-3 (a) Number line (1)



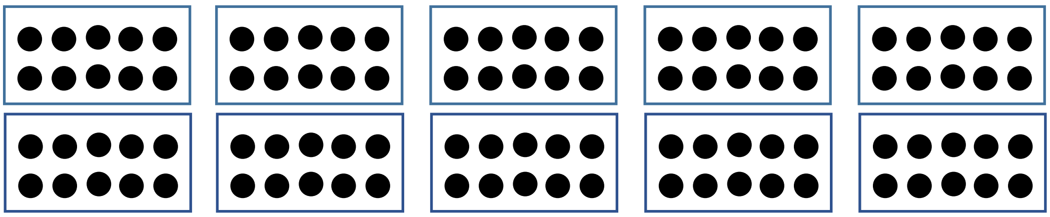
5-3 (b) Number line (2)



5-3 (c) Representation with dots (1)



5-3 (d) Representation with dots (2)



ANNEX 2

	L1	L2	L3	L4	L5
1.1	S/he makes a mistake in counting, 1, 2, 3, 4, or 5.	S/he can counts correctly up to 5.	S/he can count one by one up to 20	S/he can count up to 20 by some grouping	Besides level4, S/he can explain it.
1.2	S/he cannot count numbers up to 10.	S/he tries to count numbers but failed to count up to 20.	S/he can count numbers upward one by one with a tool or without tool.	S/he can count numbers upward up to 20 by some grouping says 2s, 5s, and 10s.	Besides level 4, S/he can explain it.
1.3	S/he cannot count numbers downward to 10.	S/he tries to count numbers but failed to count downward to 1.	S/he can count numbers downward one by one with a tool without tool.	S/he can count numbers downward to 1 says 5s and 10s.	Besides level 4, s/he can explain it.
2.1	S/he cannot make any patterns.	S/he makes some shapes without any pattern.	S/he can make a line with one by one patterns.	S/he can make a line with original patterns.	Besides level 4, S/he can explain it
2.2	S/he tries to count, but cannot do it completely in some reasons	S/he makes a mistake in counting	S/he can identify the number (7) by counting one by one.	S/he can identify the number (7) using any groups or counting on from a certain number.	Besides level 4, s/he can explain verbally.
2.3	S/he tries to count, but cannot do it completely in some reasons	S/he makes a mistake in counting.	S/he can identify the number (18) by counting one by one from 1	S/he can identify the number (18) using any groups or counting on from a certain number.	Besides level 4, s/he can explain verbally it.
2.4	S/he cannot arrange the bottle tops	S/he can place 20 bottle tops but they are not placed structurally (place randomly)	S/he can place 20 bottle tops structurally but not 5 x 2	S/he can place 20 bottle tops correctly considering frame of 10 (5x2).	Besides level 4, s/he can explain verbally what s/he has done.
3.1	S/he tells a wrong answer that is beyond our expectations	S/he tells the incorrect answers which are closed to the right answer, e.g. 10,	S/he can find an answer (12) by counting mentally or physically one by	S/he can find an answer (12) by moving bottle tops or by using any groups.	Besides level 4, s/he also can explain with mathematical expressions.

		12	one from 1.		
3.2	S/he tells a wrong answer that is beyond our expectations	S/he tells the incorrect answers which are closed to the right answer, e.g. 30, 31, 33 or 34.	S/he can find an answer (32) by counting mentally or physically one by one from 1	S/he can find an answer (32) by moving bottle tops or by using any groups.	"Besides level 4, s/he also can explain with mathematical expressions.
3.3	S/he tells a wrong answer that is beyond our expectations	S/he tells incorrect answers which are close to numbers such as 7 or 9.	S/he can find the answer (8) by counting mentally or physically one by one from 1.	S/he can find an answer (8) by counting on or by using any groups.	"Besides level 4, s/he also can explain with mathematical expressions.
3.4	S/he tells a wrong answer that is beyond our expectations	S/he tells incorrect answers which are close to numbers such as 11, 12 or 14.	S/he can find an answer (13) by counting mentally or physically one by one from 1.	S/he can find an answer (13) by counting on or by using base 10.	"Besides level 4, s/he also can explain with mathematical expressions by using base 10.
4.1	S/he tells a wrong answer that is beyond our expectations	S/he tells incorrect answers such as 48 or 50 which is near 49 by counting.	"S/he can tell 49 by counting one by one from 1.	S/he can tell 49 quickly by using base 10 or counting on from a certain number or using groups.	Besides level 4 s/he can explain verbally using base 10.
5.1	S/he tells a wrong answer that is beyond our expectations	S/he count the number one by one, however, miscounted in the middle of counting.	S/he can count the number one by one from 1.	S/he can find an answer by counting on from a certain number or using groups.	Besides level 4 s/he can explain verbally.
5.2a	S/he tells a wrong answer that is beyond our expectations	S/he cannot indicate correctly, however the answer is close to 13.	S/he can indicate the number 13 by counting one by one from 1.	S/he can indicate the number 13 at glance or counting on from a certain number.	Besides level 4 s/he can explain verbally.
5.2b	S/he tells a wrong answer that is beyond our expectations.	S/he cannot count and circle the amount of 13 marbles, however the answer is close to 13.	S/he can circle the amount of 13 marbles by counting one by one from 1.	S/he can circle the amount of 13 marbles by counting on from a certain number or using groups.	Besides level 4 s/he can explain verbally.
5.3a	S/he tells a wrong answer that is beyond our	S/he cannot indicate correctly, however the answer is close to	S/he can indicate the number 76 by counting one by	S/he can indicate the number 76 at glance or	Besides level4, s/he can explain verbally.

	expectations	76.	one from 70.	counting from 75	
5.3b	S/he tells a wrong answer that is beyond our expectations	S/he cannot indicate correctly, however the answer is close to 76.	S/he can indicate the number 76 by counting one by one from 70.	S/he can indicate the number 76 at glance or counting from 75	Besides level4, s/he can explain verbally.
5.3c	S/he tells a wrong answer that is beyond our expectations	S/he cannot count and circle correctly, however the answer is close to 76.	S/he can indicate the number 76 by counting one by one from 1.	S/he can circle the number 76 at glance or counting from 50	Besides level4, s/he can explain verbally.
5.3d	S/he can count from 1 by corresponding to the bottle tops, however s/he cannot answer correctly.	S/he cannot count and circle the amount of 76 marbles, however the answer is close to 76.	S/he can circle the amount of 76 marbles by counting one by one from 1.	S/he can circle the amount of 76 marbles by counting on from a certain number or using groups.	Besides level 4 s/he can explain verbally
6.a1	S/he cannot place bottle tops correctly.	S/he can place 7 and 8 bottle tops correctly but cannot answer correctly.	S/he can place 7 and 8 bottle tops and tell the sum by counting all.	S/he can place 7 and 8 bottle tops and tell the sum by manipulating bottle tops considering the groups.	Besides level 4, s/he can explain using base 10 by words and/or gesture.
6.a2	S/he used bottle tops, however could not answer correctly.	S/he can place 11 and 13 bottle tops correctly but S/he cannot answer correctly.	S/he can place 11 and 13 bottle tops and tell the sum by counting all bottle tops.	S/he can place 11 and 13 bottle tops and tell the sum by manipulating bottle tops considering the groups.	Besides level 4, s/he can explain using base 10 by words and/or gesture.
6.s1	S/he used bottle tops, however could not answer correctly.	S/he can place the necessary number of bottle tops but S/he cannot answer correctly. Wrong answer/counting all.	"S/he can place 15 bottle tops and remove 8 from them and counting all bottle tops,	S/he can place 15 bottle tops and remove 8 from them, tell the answer verbally by considering the groups.	Besides level 4, s/he can explain using base 10 by words and/or gesture.

6.s2	S/he used bottle tops, however could not answer correctly.	S/he can place the necessary number of bottle tops but S/he cannot answer correctly.	S/he can place 25 bottle tops and remove 12 from them and counting all bottle tops,	S/he can place 25 bottle tops and remove 12 from them, tell the answer verbally by considering the groups and place value	Besides level 4, s/he can explain using base 10 by words and/or gesture.
6.m1	S/he used bottle tops, however could not answer correctly.	S/he can place 2 bottle tops in 3 rows but S/he cannot answer correctly.	S/he can place 6 bottle tops and tell the answer verbally by counting.	S/he can place 2 bottle tops in 3 rows, and tell the answer verbally by considering the groups.	Besides level 4, s/he can explain using base 10 by words and/or gesture.
6.m2	S/he used bottle tops, however could not answer correctly.	S/he can place 12 bottle tops in 3 rows, but S/he cannot answer correctly.	S/he can place 32 bottle tops and tell the answer verbally by counting.	S/he can place 12 bottle tops in 3 rows, and tell the answer verbally by the groups and place value.	Besides level 4, s/he can explain using base 10 by words and/or gesture.
6.d1	S/he used bottle tops, however could not answer correctly.	S/he can place 8 bottle tops.	S/he can place 8 bottle tops and find the answer by dividing one by one.	S/he can place 8 bottle tops 2 by 4 in order, and tell the answer verbally.	Besides level 4, s/he can explain using base 10 by words and/or gesture.
6.d2	S/he used bottle tops, however could not answer correctly.	S/he can place 36 bottle tops.	S/he can place 36 bottle tops and find the answer by dividing one by one.	S/he can place 8 bottle tops 2 by 4 in order, and tell the answer verbally.	Besides level 4, s/he can explain using base 10 by words and/or gesture.