THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2011 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS

GEO 5800 : SELECTED RESEARCH TOPICS AND TECHNIQUES IN GEOGRAPHY
THE DEVELOPMENT OF AGRICULTURE AND LAND USAGE IN ZAMBIA

TIME: **THREE HOURS**

INSTRUCTIONS : **Answer any four questions.**
Candidates are advised to make use of illustrations and examples wherever appropriate.

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1. Show how some of the traditional agricultural practices that were documented by Trapnell and Clothier in Northern Rhodesia (Zambia) in the 1930's could be considered as being sustainable.

2. Explain the meaning of land carrying capacity as suggested by Allan (1949), with respect to African land usage in Northern Rhodesia (Zambia).

3. Describe the evolution of traditional land use systems in Zambia according to Schultz (1976).

4. Make a review of opportunities and challenges that were experienced by small-holder farmers in Northern, Western, and Southern Provinces as a result of the colonial interface with indigenous land use practices.

5. Outline and evaluate the various agricultural/rural development schemes that were implemented in Zambia from the First to the Third Republics.

6. 'Agricultural geography is concerned with the distribution of crops, animals and types of rural economy and the challenges associated with them'. Discuss.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2011 ACADEMIC YEAR FIRST SEMESTER FINAL EXAMINATIONS

GEO 5801: HISTORY AND DEVELOPMENT OF GEOGRAPHY

TIME: Three Hours

INSTRUCTIONS: Answer any four questions. All questions carry equal marks. The use of a Philips University Atlas is allowed. Candidates are encouraged to make use of illustrations wherever appropriate.

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1. Write short explanatory notes on **all** of the following:

   (a) Jean Brunhes' world maps and their importance
   (b) Al-Idrisi book entitled 'Amusement for him who desires to travel around the world' published in 1154
   (c) Ptolemy's Guide to Geography
   (d) Classical Geography
   (e) Germany geography during the pre-World War I period (1905-1914)

2. "The labours of Humboldt, of Ritter, of Guyot, and their followers have given the science of geography a more philosophical, and, at the same time, a more imaginative character than it had received from the hands of their predecessors" (Marsh 1864:8). Elucidate.

3. With the help of specific examples, outline and discuss the distinctive characteristics of post- modern British Geography.

4. The Modern Period (1859-1960s) in the history of geography was marked by the emergence of 'New geography'. Briefly, explain what 'New Geography' was and comment on what was new about 'new geography'.

5. "In 1949 mainland China was taken over by the communists and became the People's Republic of China. This meant that the study of geography was reformed to fit the Soviet model" (James and Martin, 1981:273). Discuss.

6. How did scholars who gathered together in monasteries in Christian Europe study the earth and explain the major challenges that they faced.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
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2011 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS

GEO 5802: PHILOSOPHY AND METHODOLOGY OF GEOGRAPHY

TIME: Three hours

INSTRUCTIONS: Answer any **four** Questions. All questions carry equal marks. Use of illustrations were appropriate is highly encouraged

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1. Define philosophy and the philosophy of Geography and comment on what you perceive is the philosophy of Geography at the University of Zambia.

2. How can students of Geography reconcile the assertion by ideal positivists that researchers should use the value free scientific method, while the philosophy of Geography contends that all research is guided by a set of philosophical beliefs?

3. Evaluate the view that participatory methodologies should be considered as the new pragmatism in Geographical research.

4. Using a hypothetical research problem of your choice, explain the value of an eclectic research methodological approach in geographical research.

5. Discuss the assertion that both realists and constructivists are opposed to positivist research methods.

6. Explain the following concepts

   (a) Induction and deduction in scientific research
   (b) Structure and Agency in realist research

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR FIRST SEMESTER FINAL EXAMINATIONS

GES 2001: HUMANS AND THE ENVIRONMENT

TIME : **THREE HOURS**

INSTRUCTIONS : **ANSWER QUESTION ONE AND ANY OTHER THREE QUESTIONS.**
ALL QUESTIONS CARRY EQUAL MARKS.

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1. Write short explanatory notes on ALL of the following:
   (a) Ecocentrism.
   (b) Naturalisation of humans.
   (c) Composition and structure of geographic objects or phenomena.
   (d) Environmental Possibilism.
   (e) Natural resource renewability and exhaustibility.

2. 'Human beings can conquer nature by obeying it'. Discuss

3. How can Torsten Hagerstrand's ideas of spatial diffusion be used to explain the spread of an environmental disease such as cholera.

4. 'Resources are not; they become' (Zimmermann, 1964:24). Discuss.

5. Throughout its development as a discipline, Geography has been characterised by at least four major themes. Explain.

6. With the aid of examples
(i) Explain why Geographers construct models for analysis of geographical processes?
(ii) Outline and explain the three stages in model building.

END OF EXAMINATION

CANDIDATE'S COMPUTER NUMBER: ....

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR FIRST SEMESTER FINAL EXAMINATIONS
GES 2801: MAPPING AND FIELD TECHNIQUES IN GEOGRAPHY

PAPER 1I
MAP READING AND MAP INTERPRETATION

TIME: Three Hours

INSTRUCTIONS: Answer **all** the questions. The use of a Philip's University Atlas and a certified calculator is allowed. Candidates are encouraged to make use of illustrations wherever appropriate.

MATERIALS PROVIDED: Topographic Map Sheet 1627 C4
A4 Metric Graph Paper
A4 Tracing Paper

FOR USE BY EXAMINER

Question	Examiner's Mark	Moderator's Mark
Q1		
Q2		
Q3		
Q4		
Total		

IMPORTANT

Please read the instructions before attempting any question on this examination paper.
Failure to read and follow instructions will lead to automatic loss of marks.

SECTION A: MAP SHEET 1627 C4 BASED QUESTIONS

Using Map Sheet 1627 C4 provided, answer all the questions in the spaces provided on this question paper

1 (a) When was map sheet 1627 C4 first published and by whom? [2 marks]

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(b) In which direction does the Hazibolo River generally flow? [2 marks]

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(c) Using map evidence only, explain how you could read a Four-Figure Grid Reference on map sheet 1627 C4. [4 marks]

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(d) Using any one named method of your choice, calculate the approximate size of the land above 1000 metres of Nkala Hill in square kilometres. [2 marks]

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(e) Determine the direction of Kauba School at grid reference point 463360 from the trigonometrical station in grid square 4833 firstly, as a compass direction and secondly, as a bearing from true north. [2 marks]

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(f) Determine the distance along the D776 regularly maintained road from the road junction in Grid square 2931 to Choma in Kilometres. [2 marks]

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(g) Assuming that a straight line profile is drawn from the spot height on Nkala Hill in grid square 4623 to the trigonomertrical station on the summit of Kauba Hill in grid square 4833, would the two points on the profile be intervisible? [2 marks]

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(h) Calculate the average gradient along a straight line between the spot height on the summit of Nkala Hill in grid square 4623 and grid reference point 460223 firstly, as a ratio and secondly, as an angle in degrees. [3 marks]

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(i) If you were travelling southwards to Sinazongwe, what other map sheet would you require? [1 marks]

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(j) What evidence is there on map sheet 1627 C4 to show that Simwami Muzuma National Forest No. 180 is experiencing human encroachment? [2 marks]

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(k) What drainage pattern does the Njongola River system generally exhibit? [2 marks]

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(l) Using map evidence only, why is there no cultivation in grid square 4622. [2 marks]

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(m) What drainage features are associated with the Njongola River between eastings 49 and 51? [2 marks]

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(n) On the sheet of tracing paper provided and assuming that the river is half a centimetre wide between the two stated points, draw the Njongola River from eastings 44 to 46 at:

(i) half the original size

(ii) the scale of 1:25,000. [12 marks]

SECTION B: MAP DIAGRAM QUESTIONS

All questions in this section do not require the use of the 1627 C4 topographic map. Answer all questions in this section in the spaces provided on this question paper.

2. (a) With the help of an annotated diagram, describe a ***dendritic*** drainage pattern and briefly explain the characteristics of the topography of the area on which it develops. [5 marks]

(b) Using the contour method, draw an annotated diagram to show an ***escarpment*** with a river flowing on the scarp slope with its source near the summit. [5 marks]

(c) With the help of diagrams, explain how a ***map reference number*** such as 1627 C4 is derived. [5 marks]

(d) With the help of a diagram, explain what you understand by the term ***auxiliary contour*** and show how such a contour can be distinguished from the standard contours on a map? [5 marks]

SECTION C: MAP SCALES

Answer all questions in this section in the spaces provided on this question paper as directed (Show all the steps of your work wherever appropriate)

3. (a) Express the scale 1: 1,000,000 as a scale in words. [5 marks]

(b) Express 2.5 kilometres to a centimetre as a scale in figures. [5 marks]

(c) Using a scale of 1: 250,000, calculate the dimensions to scale of a regular polygon measuring 10 kilometres by 6 kilometres (10 Km x 6 Km) [5 marks]

(d) What is the total ground area represented by a polygon measuring 1 cm by 1 cm (1 cm x 1 cm) on a 1:10,000 map? [5 marks]

(e) Draw a line scale in metric units representing a scale of 1:30,000, given that the maximum space available for the scale is 17 centimetres. [5 marks]

SECTION D: EXPLANATORY QUESTIONS

Answer all the questions in this section in the spaces provided on this question paper

4. Write short explanatory notes on **ALL** of the following:

   (a) The development of a river basin. [4 marks]

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   (b) Characteristics of a good map symbol. [4 marks]

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(c) Map projections [4 marks]

(d) Grid references [4 marks]

(e) Vertical exaggeration on a profile and its importance. [4 marks]

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END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR FIRST SEMESTER FINAL EXAMINATIONS

GEO 155: INTRODUCTION TO PHYSICAL GEOGRAPHY

TIME: **THREE (3) HOURS**

INSTRUCTIONS: Answer any FOUR questions. Use of an approved atlas is allowed. All questions carry equal marks

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1. Write short explanatory notes on ALL of the following:

    a. The Nebulae Hypothesis

    b. Soil forming factors

    c. Structure of the earth

    d. Wet and Dry Adiabatic Lapse Rate

    e. Biomes

2. With the aid of a diagram, describe the structure of the atmosphere.

3. Discuss the hydrological cycle.

4. With aid of relevant diagrams, explain the theories of precipitation formation.

5. Using relevant examples show how anthropogenic activities influence the distribution of vegetation in Zambia.

6. Discuss the processes that are involved in soil formation.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

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2012 ACADEMIC FIRST SEMESTER FINAL EXAMINATIONS

GEO 271: QUANTITATIVE TECHNIQUES IN GEOGRAPHY I

TIME: THREE (3) HOURS

INSTRUCTIONS: ANSWER QUESTION ONE (1) AND THREE (3) OTHERS. CREDIT WILL BE GIVEN FOR USE OF RELEVANT EXAMPLES AND ILLUSTRATIONS

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1. Professor Mwenzi conducted a study on 'Geo 271 students' awareness of the degradation of the environment and their perceived capacity to take action to improve the environment'. Quantitative data was collected using a questionnaire while qualitative data was collected through interviews and observation. A week later, Dr. Nchimunya conducted the same research whose sample was collected from the same population as Professor Mwenzi's. Surprisingly the results of the two researchers were significantly different.
   Explain what could have led to the differences in the research findings of the two researchers *(40 Marks).*

2. What are the benefits of a longitudinal design over a cross-sectional design in a non experimental study? *(20 Marks)*

3. Write short explanatory notes on ALL of the following:
   (a) Extraneous variables
   (b) Unit of analysis
   (c) Snowball sampling
   (d) Quasi-experimental design
   (e) Applied vs. Fundamental research *(20 Marks).*

4. Provide reasons for conducting a thorough literature review as part of a research process *(20 Marks).*

5. Discuss the merits and de-merits of using a questionnaire as a data collection instrument in research *(20 Marks).*

6. Show how employing direct observation method becomes important in studying environmental problems *(20 Marks).*

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
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2012 ACADEMIC YEAR FIRST SEMESTER FINAL EXAMINATIONS
GEO 481: ENVIRONMENT AND DEVELOPMENT II

TIME: **Three hours**

INSTRUCTIONS: **Answer any FOUR questions**
All questions carry equal marks

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1. Agriculture and industry exert mounting pressure on both the quantity and quality of water resources. Discuss.

2. With local examples, discuss the negative impacts of the informal sector on the urban environment and propose possible solutions.

3. Explain how a non-point source pollution rich in nitrogen and phosphorus can affect a receiving body of water.

4. 'In the face of global concerns about the sustainability of fossil fuels, the nuclear energy option has evoked both enthusiastic and pessimistic views from environmentalists and the public'. Discuss.

5. Discuss the consequences of tropical deforestation and suggest measures to prevent and/or reduce it.

6. Explain at least four key agricultural issues in African that must be addressed in order to contribute towards the creation of a sustainable future.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOLOF NATURAL SCIENCES

2012 ACADEMIC YEAR FIRST SEMESTER FINAL EXAMINATIONS

GEO 495: ENVIRONMENTAL HAZARDS AND DISASTERS

TIME: Three Hours

INSTRUCTIONS: Answer any **FOUR** questions. All questions carry equal marks. The use of a Philip's University Atlas and a certified calculator is allowed. Candidates are encouraged to make use of illustrations wherever appropriate.

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1. Write short explanatory notes on **ALL** of the following:
   (a) Disaster management
   (b) Voluntary risks
   (c) Dissonant perception
   (d) Classification of hazards according to origin
   (e) Restoration of normality in an area following a disaster strike.
2. In response to a hazard, human beings either adjust or adapt. Explain.
3. With the help of a labeled diagram, explain what a descriptive event tree technique is and briefly comment on its importance.
4. 'The increased awareness of environmental hazards has brought fundamental differences in interpretation and different schools of thought'. Discuss.
5. In terms of ascertaining the scale of disaster occurrence, what are the limitations that one needs to keep in mind?
6. With the help of an annotated diagram, explain the contention that environmental hazards exist at the interface between the natural events and the human use systems and show how human responses to hazards can modify both the natural events in, and the human use of, the environment.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

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2012 ACADEMIC YEAR FIRST SEMESTER FINAL EXAMINATIONS

GEO 911: POPULATION GEOGRAPHY

TIME: Three hours

INSTRUCTIONS: Answer Question 1 and any other three. Question 1 carries 40 Marks of this paper while the rest carry 20 marks each. Use of a certified calculator is allowed.

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1. With the aid of a diagram, explain the Demographic Transition Theory (DTT).

   [40 Marks]

2. `Children have both non-economic and economic values to their parents.' Discuss.

   [20 Marks]

3. "In the study of the family with the exception of census material , the national data of Africa (including those of Zambia) are of outstandingly ~~of~~ poor quality." (Goody, 1990: 119). Elucidate.

   [20 Marks].

4. Examine the validity of Trawartha's (1953) triangle in relation to geographical studies.

   [20 Marks]

5. Use examples to discuss Chamie's (1977) religious hypotheses related to human fertility.

   [20 marks].

6. `Both men and women should be active partners in using Natural Family Planning (NFL) methods ." Discuss.

   [20 Marks]

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
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2012 ACADEMIC YEAR FIRST SEMESTER FINAL EXAMINATIONS

GEO 921: ECONOMIC GEOGRAPHY

TIME: **Three Hours**

INSTRUCTIONS: **Answer any FOUR questions. All questions carry equal marks.**

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1. Compare and contrast Weber's Industrial Location Theory and Smith's Variable Cost Model.

2. Critically evaluate Wood (1969) criticisms of Weber's Theory of Industrial Location.

3. Outline and discuss the key elements of the 'Space Economy'.

4. 'International trade can take place between countries with identical production possibilities'. Discuss.

5. In what ways and to what extent can agriculture contribute towards the diversification of Zambia's economy?

6. With reference to Zambia, outline and discuss the challenges the locally manufactured goods face in penetrating foreign markets.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR FIRST SEMESTER FINAL EXAMINATIONS

GEO 931: RURAL GEOGRAPHY

TIME: **THREE HOURS**

INSTRUCTIONS: **Answer any four questions. All questions carry equal marks. Candidates are advised to make use of illustrations and examples wherever appropriate. Use of a Philips University Atlas is allowed.**

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1. Describe the Participatory Rural Appraisal (PRA) methodology in rural research and comment on why it became a popular approach in the 1990s.

2. 'Land, farm-inputs and markets are essential factors that can contribute to the development of rural economies in Africa if they are adequately accessed'. Discuss.

3. With the use of a diagram, critically analyse Von Thunen's model of land use with emphasis on the Crop and Intensity theories.

4. Explain the factors that led to the emergence of the small-holder commercial African agriculture in Zambia during the colonial period.

5. Show how rural women in Sub-Saharan Africa are important contributors to both household food security and environmental management.

6. In what ways is the slogan'water is life' applicable to rural settings with respect to the provision of water and sanitation in Zambia?

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR FIRST SEMESTER FINAL EXAMINATIONS

GEO 951: CLIMATOLOGY

TIME: **Three hours**

INSTRUCTIONS: Answer any **FOUR** questions.

All questions carry equal marks. Candidates are advised to make use of illustrations and examples wherever appropriate.

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1. Write short explanatory notes on ALL of the following:

   a) Pyranometer and Porometer
   b) The Penman equation and its components
   c) Ozone destruction by nitrogen oxide
   d) Drivers of climate change
   e) Modification of air masses

2. Discuss the effects of various plumes on dispersal of air pollutants in the course of a day in an area characterized by one industrial stack.

3. Account for the dispersal of 100 units of solar radiation entering the earth's atmosphere and explain why the Earth-Atmosphere system is in radiative equilibrium while sub-systems are not.

4. With the use of diagrams, describe the forces causing atmospheric motion and how they influence the general circulation system on the earth's surface.

5. Discuss the merits and demerits of increased UV radiation reaching the earth's surface in the context of the ozone dilemma.

6. Using specific example related to Zambia, compare and contrast the concepts of 'coping' and 'adapation' in the study of climate variability and change.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2011 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS

GEO 952: GEOGRAPHICAL HYDROLOGY

TIME: **Three hours**

INSTRUCTIONS: Answer any **FOUR** questions.

All questions carry equal marks. Candidates are advised to make use of illustrations and examples wherever appropriate.

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1. Write short explanatory notes on **ALL** of the following:

    a) Flashy floods
    b) Interception
    c) Groundwater balance
    d) Hydrometric network
    e) Baseflow separation methods

2. With the aid of a diagram, describe the relationship between unconfined and confined aquifer.

3. What are the merits and demerits of using empirical formulae in hydrology to evaluate components of the hydrological cycle.

4. Diagrammatically describe runoff pathways and explain how some components can be measured and evaluated.

5. Describe five methods than can be used to estimate evaporation of the Goma Lakes taking into account any merits and limitations.

6. Trace the development of evaporation modeling and outline the salient features of the Penman equation.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

Department of Mathematics and Statistics

2012 Academic Year

Semester I

M111 Mathematical Methods I

FINAL EXAMINATION

Time Allowed: Three (3) Hours **25th February, 2013.**

Instructions:

1. Write your **Computer Number**, and your **TG Number** on each answer booklet you have used.
2. There are Seven (7) questions in this paper. Attempt **Any Five (5)** questions. All questions carry equal marks
3. Calculators are **Not** allowed.
4. Should you have any problem or if you need more answer booklet, put up your hand, an invigilator will come to attend to you.

1. (a) (i) Express $3.\overline{14}$ in the form $\frac{a}{b}$ where a and b are integers and $\frac{a}{b}$ is in its lowest terms.

(ii) Express $\frac{\sqrt{5}-1}{2\sqrt{5}}+\sqrt{5}+3$ in the form $a+b\sqrt{5}$ where a and b are rational numbers.

(iii) Express $\frac{3-i}{1+i}+\frac{1}{i}$ in the form $a+ib$ where a and b are rational numbers.

(b) A function $f(x)=\frac{x+a}{x-b}$ is such that $f(0)=\frac{2}{3}$ and $f(-1)=-1$.

(i) Find the values of a and b.
(ii) Find the domain and the range of $f(x)$
(iii) Find the inverse $f^{-1}(x)$ of $f(x)$.

(c) A binary operation $*$ is defined on the set of real numbers as follows:

$$a*b=(a-b)(2b+a), \qquad a\,,\,b \in \mathbf{R}$$

(i) Is the operation $*$ commutative? If not give a counterexample.
(ii) Find the value of $(3*2)*4$

2. (a) Consider the function $f(x)=\frac{2}{x-1}+x$.

(i) Find $f(-2)$
(ii) Find also $f'(x)$
(iii) Given that $g(x)=\frac{1}{x}$, find in its simplest form $(gof)(x)$.

(b) Let **R**, the set of real numbers be the universal set. If $A=[-7\,,8)$ and $B=[0\,,\infty]$, find the following sets and display them on the numberline:

(i) A'.
(ii) $A \cap B$.
(iii) $(A \cup B)'$.

(c) Let $g(x)=|\,x+3\,|-|\,2x+3\,|$ be a function.

(i) Sketch the graph of the function $g(x)$
(ii) Find the set of values of x for which $g(x) \geq 0$.

3. (a) Let $f(x)=\sqrt{x+1}$ be a function.

(i) Find $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

(ii) Sketch the graph of $f(x)$.

(b) The remainder when the polynomial $P(x)=x^3-px+q$ is divided by x^2-3x+2 is $4x-1$.

(i) Find the constants p and q.

(ii) Use synthetic division to find the remainder when $P(x)$ is divided by $2x-1$.

(iii) Hence express the polynomial $P(x)$ in the form $P(x)=(2x-1)Q(x)+R$ where $Q(x)$ is the quotient and R is the remainder when $P(x)$ is divided by $2x-1$.

(c) (i) Solve the inequality $\frac{x+1}{x-2}>\frac{1}{3}$

(ii) Solve for x and y given that $\frac{x}{1-3i}+\frac{y}{1+i}=2i$

4. (a) Solve the following equations for all values of x in the interval $[0, 360^0]$

(i) $\sin 2x-\cos x=0$.

(ii) $\sin^2 x+5\cos^2 x=3$.

(b) Given that the roots of the equation $4x^2+4x-3=0$ are α and β, find

(i) $\alpha^2+\beta^2$.

(ii) $\frac{1}{\alpha}+\frac{1}{\beta}$.

(iii) an equation whose roots are $\frac{1}{\beta}$ and $\frac{1}{\alpha}$.

(c) (i) Differentiate $f(x)=\frac{1}{x^2}$ from **first principle.**

(ii) Given that $x-k$ is a factor of $P(x)=kx^3-3x^2-5kx-9$ where $k\in\mathbf{R}$, find possible values of k.

5. (a) Given the quadratic function $f(x) = 2 - x - 6x^2$

(i) Find the y – intercept and the x – intercept.

(ii) Determine the maximum or the minimum point of the function.

(iii) Sketch the graph of $f(x)$.

(b) The equation $x^2 + y^2 - 4x + 6y + 11 = 0$ represents a curve.

(i) Find an expression for $\frac{dy}{dx}$.

(ii) Find the equation of the tangent to the curve at $(1, -2)$.

(c) Let $f(x) = -2\sin 2x$ be a function.

(i) State the amplitude and the period of the function $f(x)$.

(ii) Sketch the graph of $f(x)$ for values of x in the interval $-\pi \leq x \leq \pi$.

(iii) Solve the equation $f(x) = 1$ for $-\pi \leq x \leq \pi$.

6. (a) Evaluate the following limits

(i) $\lim_{x \to 3} \frac{3 - x}{2 - \sqrt{x^2 - 5}}$.

(ii) $\lim_{x \to \infty} \frac{3x^2 - 2x + 1}{5 - 8x^2}$.

(b) (i) Find the complex number Z such that $\frac{Z}{2+i} = 1 - Z$.

(ii) Given that A and B are disjoint sets, express $A \cap (A \cap B)' \cup [A \cap (A' \cup B)]$ in its simplest form.

(iii) Solve the inequality $|x + 2| \leq 3$.

(c) (i) Prove the identity $\cos 2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta}$.

(ii) Find the general solution of the equation $\frac{1 - \tan^2\theta}{1 + \tan^2\theta} = \frac{1}{2}$

(iii) Show that $\frac{1 - \cos 2A}{3 + \cos 2A} = \frac{\sin^2 A}{1 + \cos^2 A}$.

7. (a) Solve the following equations for real values of x.

(i) $|2x+5|=1$.

(ii) $-2+\sqrt{1-x}=\frac{x}{2}+5$.

(iii) $2x^2+3x-2=0$

(b) (i) Given that $x=1\cdot\overline{21}$ and $y=0\cdot3\overline{24}$ are two rational numbers, express $x+y$ in the form $\frac{a}{b}$ where a and b are integers.

(ii) The curve $g(x)=\frac{1}{x}$ passes through the point $P(a\,,\,g(a))$. Find in terms of a the equation of the tangent to the curve at P.

(c) Find $\frac{dy}{dx}$ given that

(i) $y=(3x^3+1)^{\frac{2}{9}}$.

(ii) $y=x^2\tan x$.

(iii) $y=e^{-3x^4}$.

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
2012 ACADEMIC YEAR
FIRST SEMESTER FINAL EXAMINATIONS

M161- INTRODUCTION TO MATHEMATICS, PROBABILITY AND STATISTICS I

INSTRUCTIONS:
1. There are **seven** [7] questions in this paper.
2. Answer any **five (5)** questions.
3. All questions carry equal marks.
4. Show all the necessary work to earn full marks.
5. Write down the questions attempted in the **first column** on the front page of the main booklet.
6. Use of calculators **is not** allowed.

TIME ALLOWED: Three (3) hours.

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1. [a] [i] Consider the subsets $X = [-2, 2)$, $Y = (-4, 5]$ and $Z = [-1, 0)$
Find the set $X' \cup (Y \cap Z)$ if the universal set is $(-5, 5]$

[ii] Let A and B be any two sets. Show that:
Show that $(A' \cup B)' \cap (B \cup A)' = \phi$

[b] Given that $f(x) = \dfrac{x}{x+2}$ and $g(x) = \dfrac{1}{2-x}$,

Solve the equation $f[g(x)] = 2$

[c] Find the values of x which satisfy the following inequation:

$$x^3 - x^2 < 12x$$

2. [a] Given that α and β are roots of the equation $x^2 + 2x + 3 = 0$,

[i] Evaluate $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$.

[ii] Find an equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$

[b] Solve the following:

[i] $\frac{1}{x} > 1$

[ii] $\left| \frac{x+5}{x-3} \right| = 1$

[c] Expand and simplify: $\left(x + \frac{1}{\sqrt{x}}\right)^3 + \left(x - \frac{1}{\sqrt{x}}\right)^3$.

Hence find the value of the above expansion when $x = 2$

3. [a] When $f(x) = 2x^3 - 3x^2 - 11x + k$ is divided by $x - 2$, the remainder is -12.

[i] Find the value of k.

[ii] With this value of k found in [i], solve $f(x) = 0$.

[b] [i] Find a, b and c if

$$a + b\sqrt{c} = \frac{5\sqrt{3} + 2}{3\sqrt{3} - 1}$$

[ii] Find the constant p given that

$4\sqrt{27} - 3\sqrt{3} + 2\sqrt{48} = P\sqrt{3}$

[c] Express the following number as a fraction:

1.121 121 121.................

4. [a] For the curve $f(x) = 5x^2 - 4x - 1$,

[i] Find the x and y intercepts.

[ii] Find a , b and c given that $f(x) = a(x-b)^2 + c$.

[iii] Find the turning point of $f(x)$.

[iv] Sketch the graph of $f(x)$

[v] Using the same axes as in [iv], sketch the graph of $|f(x)|$

[b] [i] Given that $\sqrt{3} \approx 1.732$, without using a calculator,
find the value of $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$.

[ii] Given that $z = 2 + 3i$, express $z + \dfrac{1}{\bar{z}}$ in the form $x + iy$

[c] Resolve the following into partial fractions:

$$\frac{x+2}{x^2 + 7x + 12}$$

5. [a] Solve the following for $x \in \Re$:

$$5^{2x} - 6\left(5^x\right) + 5 = 0$$

[b] For the polynomial $f(x) = 6x^4 - x^3 + ax^2 - 6x + b$,
find a and b given that $x+1$ and $x-2$ are factors of $f(x)$.

[c] [i] Solve for x and y given that
$x + iy = \dfrac{1}{i^{99}}$

[ii] Solve the simultaneous equation
$\log_{10} x - 3\log_{10} y = 1$, $xy = 160$

6. [a] Find the coefficient of the term containing x^3y^4 in the expansion of:

$$(2x+y)^7$$

[b] Find the constant k given that .

[i] $$\binom{5}{2}+\binom{4}{0}=k$$

[ii] the function $f(x)=kx^2+(1+k)x+1$ has maximum value 1. .

[c] Solve the following system of equations:

$$2x-4y=5$$
$$xy=12$$

7. [a] Show that $\dfrac{\cos\theta}{1+\sin\theta}+\dfrac{\cos\theta}{1-\sin\theta}=2\sec\theta$

[b] [i] Find the exact value of $\cos 75^0$

[ii] Sketch the graph of $y=\cos x.$, $0^0 \le x \le 360^0$

On the same axes, sketch the graph of $|\cos x|$

[c] Solve the equation

$\sin^2 x+\cos 2x-\cos x=0$ if $0^0 \le x \le 360^0$

END OF EXAMINATION

The University of Zambia

School of Natural Sciences

Department of Mathematics & Statistics

2012 Academic Year First Semester Final Examinations

M 211 - Mathematical Methods II

Time allowed : **Three (3) hours** Full marks : 100

Instructions:
- There are **six (6)** questions in this paper.
- Attempt **any five (5)** questions. **All** questions carry **equal** marks.
- **Full credit** will only be given when **necessary work** is shown.
- Indicate your **computer number** on all answer booklets.
- **Calculators** are **not** allowed.

This paper consists of 3 pages of questions.

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1. Given the equation of a conic section below

$$6xy + 8y^2 - 12x - 26y + 11 = 0,$$

a) transform the equation into the standard form. Hence, identify the conic.

b) find the center, focus (or foci) and vertex (or vertices).

c) sketch the curve, clearly showing the new axes and all the characteristics listed in (b) above.

2. **a)** Given the equation below,

$$r = \frac{20}{5 + 2\cos\theta}.$$

i. Identify the conic.

ii. Find the intercepts.

iii. Sketch the graph of the conic showing all of its characteristics.

b) Show that the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

can be written as

$$r^2 = \frac{b^2}{1 - e^2 \cos^2 \theta}$$

where e is the eccentricity.

c) A comet moves in a parabolic orbit with the sun at the focus. When the comet is 4×10^7 miles from the sun, the line from the sun to it makes an angle of 60 degrees with the axis of the orbit (drawn in the direction in which the orbit opens). Suppose that the sun is at the origin,

i. sketch the orbit of the comet as described above.

ii. how near does the comet come to the sun?

3. a) Define the limit of $f(x)$ as x approaches a.
Use the definition to show that

$$\lim_{x \to 4} \frac{1}{x+5} = \frac{1}{9}.$$

b) Define continuity of a function $f(x)$ at a point $x = c$.
Hence determine whether or not the function $f(x) = \begin{cases} x, & \text{if } x < 1 \\ x^2 + 4, & \text{if } x \geq 1 \end{cases}$ is continuous at $x = 1$.

c) Given the function $f(x) = \ln(1 + x)$.

i. Obtain the Taylor series expansion of f in powers of x.

ii. Hence find the approximate value of $\ln(1.1)$ correct to 2 decimal places.

4. a) Find the following:

i. $\lim_{x \to 0} \frac{2^x + 5^x - 2}{4x}$.

ii. $\lim_{x \to \infty} \left(\frac{x}{x-4}\right)^x$.

b) State Rolle's theorem.
Hence, show that

$$f(x) = \frac{x^3 - x}{x + 3},$$

satisfies Rolle's theorem on the interval $[-1, 1]$.

c) i. Expand $f(x) = \cos x$ by Maclaurin's theorem.

ii. Hence, evaluate

$$\lim_{x \to 0} \frac{\cos x - 1}{x}.$$

5. a) State the Mean Value theorem. Hence,

   i. use it to approximate $\sqrt{15}$.

   ii. find $c \in (1, \frac{5}{2})$ which satisfies the conclusion of the Mean Value Theorem, if

$$f(x) = (x-1)(x-2)(x-3),\ x \in \left[1, \frac{5}{2}\right].$$

b) Let

$$f(x) = 2x^3 + 7x^2 + x - 6.$$

   i. Find a formula for the 4th order Taylor polynomial of f in powers of $(x+3)$.

   ii. Hence calculate $p_4(-3)$.

c) Evaluate

$$\int \frac{dx}{\sin x - \cos x - 1}\cancel{dx}.$$

6. a) Evaluate the following integrals

   i. $\int \frac{x^3}{\sqrt{x^2+1}} dx$,

   ii. $\int \frac{dx}{(4-x^2)^{\frac{3}{2}}}$.

b) Given the curves $y = \frac{3+x}{3}$ and $y = 1 + \sqrt{x}$,

   i. sketch the two curves on the same axes.

   ii. calculate the area of the region enclosed by the two curves.

c) Find the volume of the solid of revolution formed by revolving the area enclosed by the curves $x\sqrt{y} = 2,\ y = 2,\ y = 3$ about the y-axis.

END OF EXAMINATION!

The University of Zambia

School of Natural Sciences

Department of Mathematics & Statistics

2012 Academic Year Second Semester Final Examinations

M221 - Linear Algebra I

Time allowed : **Three (3) hours** Full marks : 100

Instructions:
- Attempt **any five (5)** questions. **All** questions carry **equal** marks.
- Indicate your **computer number** on all answer booklets.
- **Calculators** are **not** allowed
- Unless specified otherwise, V and W denote vector spaces over a field $\mathbb{F}$

---

1. (a) Define a **homogeneous system** of equations. [2 marks]

   (b) i. Find the reduced echelon matrix of

$$A = \begin{pmatrix} 1 & 0 & -1 & -1 \\ 2 & 1 & -1 & 1 \\ -1 & 2 & 0 & 4 \end{pmatrix}. \text{ [7 marks]}$$

   ii. Hence or otherwise, find all the solutions to the system

$$\begin{aligned} x_1 - x_3 - x_4 &= 0 \\ 2x_1 + x_2 - x_3 + x_4 &= 0 \\ -x_1 + 2x_2 + x_4 &= 0. \text{ [3 marks]} \end{aligned}$$

   (c) Let $A = \begin{pmatrix} 7 & 4 \\ -9 & -5 \end{pmatrix}$, then prove that, for any positive integer k,

$$A^k = \begin{pmatrix} 1+6k & 4k \\ -9k & 1-6k \end{pmatrix}. \text{ [8 marks]}$$

2. (a) What do you understand by the adjoint of a matrix? [2 marks]

(b) i. Let $B = \begin{pmatrix} -2 & 7 & 5 \\ 4 & 1 & -1 \\ 12 & -3 & 3 \end{pmatrix}$. Find the adjoint of B. [6 marks]

ii. Is the matrix B invertible? Justify your answer. [3 marks]

(c) Let $C = \begin{pmatrix} 1 & x & y & 1 \\ 1 & x & x & x \\ x & 1 & xy & y \\ x & x & xy & 1 \end{pmatrix}$.

i. Show that $\det C = (x-1)(x-y)(1-xy)$. [7 marks]

ii. What conditions should x and y satisfy for C to be invertible. [2 marks]

3. (a) Use elementary row operations to find the inverse of $D = \begin{pmatrix} 2 & 2 & 5 \\ 6 & 1 & 5 \\ 2 & 0 & 2 \end{pmatrix}$. [8 marks]

(b) Find the condition(s) on a for the system

$$\begin{aligned} 5x + 2y - z &= 1 \\ 2x + 3y + 4z + &= 7 \\ 4x - 5y + az &= a - 5 \end{aligned}$$

to be consistent. [6 marks]

4. (a) i. Name the three conditions that are necessary and sufficient for a subset U of V to be a subspace of V. [3 marks]

ii. Prove that if $c \in \mathbb{F}$, then

$$\{(x_1, x_2, x_3) \in \mathbb{F}^3 : x_1 + 2x_2 = -3x_3 + c\}$$

is a subspace of $\mathbb{F}^3$ if and only if $c = 0$. [7 marks]

(b) Prove or give a counterexample: if U_1, U_2, W are subspaces of V such that

$$U_1 + W = U_2 + W,$$

then $U_1 = U_2$. [5 marks]

(c) Suppose U is the subspace of $\mathcal{P}(\mathbb{F})$ consisting of all polynomials p of the form

$$p(z) = az^4 + bz^7,$$

where $a, b \in \mathbb{F}$. Find a subspace W of $\mathcal{P}(\mathbb{F})$ such that $\mathcal{P}(\mathbb{F}) = U \oplus W$. [5 marks]

5. (a) i. Name the two conditions that are necessary and sufficient for a mapping $T \in \mathcal{L}(V, W)$ to be a linear transformation. [2 marks]

ii. Give an example of a function $f : \mathbb{R}^2 \to \mathbb{R}$ such that

$$f(av) = af(v)$$

for all $a \in \mathbb{R}$ and all $v \in \mathbb{R}^2$ but f is not linear. [4 marks]

(b) Prove or give a counterexample: if T is a linear transformation from $\mathbb{F}^4$ to $\mathbb{F}^2$ such that

$$\operatorname{null} T = \{(x_1, x_2, x_3, x_4) \in \mathbb{F}^4 : x_1 = 4x_2 \text{ and } x_3 = 6x_4\},$$

then T is surjective. [6 marks]

(c) Let $\{x_1, x_2\}$ and $\{y_1, y_2, y_3\}$ be $\mathbb{R}$-bases for $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively, and define $T \in \mathcal{L}(\mathbb{R}^2, \mathbb{R}^3)$ by

$$\begin{aligned} Tx_1 &= y_1 + 2y_2 - y_3, \\ Tx_2 &= y_1 - y_2. \end{aligned}$$

Find the matrix of T relative to these bases. Hence or otherwise, find the matrix of T relative to the $\mathbb{R}$-bases $\{-x_1 + x_2, 2x_1 - x_2\}$ and $\{y_1, y_1 + y_2, y_1 + y_2 + y_3\}$ for $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively. [8 marks]

6. (a) i. When do we say a vector space is finite dimensional? [1 mark]

ii. Suppose that V is finite dimensional and $S, T \in \mathcal{L}(V)$. Prove that ST is invertible if and only if both S and T are invertible. [5 marks]

iii. Suppose U and W are both two-dimensional subspaces of $\mathbb{R}^3$. Prove that $U \cap W \neq \{0\}$. [4 marks]

(b) Prove or give a counterexample: there exists a basis $(p_0, p_1, p_2, p_3, p_4)$ of $\mathcal{P}_4(\mathbb{F})$ such that none of the polynomials p_0, p_1, p_2, p_3, p_4 has degree 3. [4 marks]

(c) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by

$$T(x, y, z) = (x + 2y - z, 2x + z, x - 2y + 2z).$$

Find $\mathbb{R}$-bases for $\operatorname{range} T$ and $\operatorname{null} T$. [6 marks]

END OF PAPER

The University of Zambia

School of Natural Sciences

Department of Mathematics & Statistics

2012 Academic Year First Semester Final Examinations

M231 - Real Analysis I

Time allowed : **Three (3) hours** Full marks : 100

Instructions: • Attempt **any five (5)** questions. **All** questions carry **equal** marks.

• Indicate your **computer number** on all answer booklets used.

• **Calculators** are **not** allowed

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1. (a) What do you understand by a totally ordered field? When is a totally ordered field said to be complete?

   (b) Let $(\mathbb{F}, +, \cdot, P)$ be a totally ordered field. Prove that if $x \in P$ then $x^{-1} \in P$, and if $0 \neq 1$ then $1 \in P$.

   (c) Let $(\mathbb{F}, +, \cdot, P)$ be a totally ordered field and let $A \subset F$. Prove that the following statements are equivalent.

      i. $u = \text{l.u.b.}A$,

      ii. u is an upper bound of A and $\forall x \in \mathbb{F}, x < u \Rightarrow \exists a \in A$ such that $x < a \leq u$,

      iii. $\forall \varepsilon > 0$,

         α) $\forall a \in A, a < u + \varepsilon$, and

         β) $\exists a \in A$ such that $u - \varepsilon < a$.

   (d) Suppose that X and Y are bounded sets of positive real numbers such that

   $$XY = \{xy | x \in X, y \in Y\}.$$

   Use 1(c) to show that $\sup(XY) = \sup X \sup Y$.

2. (a) i. Give the definition of a section of rational numbers.

 ii. Use your defintion in (i) to show that between any two different real numbers, lie infinitely many real rational numbers.

 (b) State and prove the Archimedean property of real numbers.

 (c) Suppose w is such that for sufficiently large n, $b^w < y$ implies that $b^{w+(1/n)} < y$, and $b^w > y$ implies that $b^{w-(1/n)} > y$. Use the Archimedean property to show that that there is a **unique** real x such that $b^x = y$.

 Hint: Let A be the set of all w with $b^w < y$, and show that $x = \sup A$ satisfies $b^x = y$.

 (d) Let $B \subset \mathbb{R}$ be defined by

$$B = \{n^{(-1)^n} : n \in \mathbb{N}\}.$$

 Find $\inf B$ and use the Archimedean property to justify your answer.

3. (a) Define the following:

 i. Cartesian product of two sets A and B.

 ii. Equivalence relation.

 (b) Let $A = \{1,2,3\}$ and $B = \{a,b,c\}$. Define the functions $f = \{(1,a),(2,b),(3,a)\}$ and $g = \{(a,1),(b,3),(c,2)\}$.

 i. How many possible relations are there on the set A?

 ii. Describe each of the following functions, g^{-1} and $f \circ g \circ f$, by listing its ordered pairs. Then state its range.

 (c) Let $p \in \mathbb{N}$, p a prime. Let

$$A = \{(m,n) : m,n \in \mathbb{Z} \text{ and } \frac{m-n}{p} \in \mathbb{Z}\}.$$

 Show that A is an equivalence relation.

 (d) A complex number is a number that can be expressed in the form $a + bi$, where a, b are real and $i = \sqrt{-1}$. That is, an ordered pair (a,b) of real numbers. The sum and product of the complex numbers $X = (a,b)$ and $Y = (c,d)$ are defined by

$$X + Y = (a+c, b+d) \text{ and } XY = (ac - bd, ad + bc).$$

 With these operations, the set $\mathbb{C}$ of complex numbers is a field. Show that this field fails to be totally ordered.

 Hint: Use the fact that in a totally ordered field, the following rules hold:

 i. If $x > 0$ then $-x < 0$,

 ii. If $x \neq 0$ then $x^2 > 0$. In particular, $1 > 0$.

4. (a) When is a relation from a set A to a set B called a function?

   (b) Define the function $f : \mathbb{R} \to \mathbb{R}$. The image and inverse image of a set $A \subset \mathbb{R}$ under f are respectively

   $$f(A) = \{y : y = f(x) \text{ for some } x \in A\} \text{ and } f^{-1}(A) = \{x : f(x) \in A\}.$$

   i. Show that $f(f^{-1}(A)) \subset A \subset f^{-1}(f(A))$.

   ii. Give a counter example to show that $f^{-1}(f(A)) \neq A$.

   (c) For two functions, $f : A \to B$ and $g : B \to C$, prove or give a counterexample:

   i. If $g \circ f$ is surjective, then f is surjective.

   ii. If $g \circ f$ is surjective, then g is surjective.

   (d) Suppose that A has exactly four elements and B has exactly three. How many different functions are there from A to B? How many of these are injective? How many are surjective?

5. (a) What is meant by a well ordered set?

   (b) Justify each of the following:

   i. A well ordered set is totally ordered.

   ii. If $m < n$, then the interval $(0, 1)$ is equivalent to the interval (m, n).

   (c) State and prove Cantor-Schröder-Bernstein theorem.

   (d) Show that the relation $\sim$ is a partial order if it is defined on $\mathbb{N}$ by $x \sim y$ if and only if $y = 2^n x$ for some integer $n \geq 0$.

6. (a) Let A be a subset of $\mathbb{R}$. When is A said to be

   i. closed in $\mathbb{R}$?

   ii. open in $\mathbb{R}$?

   (b) Show that a subset of $\mathbb{R}$ is closed in $\mathbb{R}$ if and only if its compliment is open in $\mathbb{R}$.

   (c) Prove that if for each α in an indexing set I, F_α is closed set in $\mathbb{R}$, then

   $$\bigcap_{\alpha \in I} F_\alpha$$

   is a closed set in $\mathbb{R}$.

   (d) Let $A \subset \mathbb{Q}$ be defined by

   $$A = \left\{ \frac{x^2}{1 + x + 2x^2} : x > 0, x \in \mathbb{Q} \right\}.$$

   Find $\sup A$ and $\inf A$. Justify your answers.

END OF PAPER

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR
FIRST SEMESTER EXAMINATIONS

M261 INTRODUCTION TO STATISTICS

Time Allowed: Three (3) Hours

Instructions:
1. Answer any **Five (5)** Questions
2. Show All Essential Working
3. Statistical Tables are provided
4. Calculators are Allowed

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1. (a) (i) State two causes of non-response bias.
   (ii) Define a proportional stratified sample.

   (b) The following data represent the number of computers sold at a certain computer shop for a sample of 26 days.

46	31	60	28	14	33	54	71	20	55	22	11	32
66	85	22	43	78	25	37	69	57	48	24	35	42

   (i) Construct an ordered stem and leaf plot for the data.
   (ii) Describe the distribution of the data.
   (iii) Construct a boxplot for the data and identify outliers, if any.

   (c) The number of hours that students spend studying at a certain college is normally distributed with a mean of 8.4 hours and a standard deviation of 2.7 hours per week.
   (i) Find the probability that a randomly selected student spends more than 11 hours studying per week.
   (ii) Find the probability that the mean time spent studying per week for a random sample of 36 students is between 7.5 and 9.5 hours.
   (iii) If 93.32% of the students study for more than c hours per week, find the value of c.

2. (a) Define the following:
   (i) a probability sample.
   (ii) power of a test.

(b) A company selected nine employees and secretly monitored their computers for one month. The times in hours spent by these employees using their computers for non-job related activities (playing games, personal communications, etc) during this month are given below.

7	12	9	8	11	4	14	1	6

   (i) Construct a 95% confidence interval for the mean number of hours the employees in this company use their computers for non-job related activities.
   (ii) Is there sufficient evidence to indicate that the standard deviation of the number of hours is less than 5? Test using $\alpha = 0.05$.
   (iii) State the assumption(s) required for (i) and (ii) to be valid.

(c) A marketing researcher wants to estimate the mean amount of money that visitors to a certain park spend per day. The amounts spent per day by visitors to this park are normally distributed with a standard deviation of K15.
   (i) How large a sample should the researcher select so that she is 95% confident that the estimate will be within K3 of the population mean?
   (ii) Is there sufficient evidence to suggest that visitors to the park spend an average of K50 per day if a random sample of 26 visitors spent an average of K65? Use $\alpha = 0.02$.

3. (a) (i) Define non-response bias.
   (ii) State the central limit theorem.

(b) A random sample of 45 customers who drive luxury cars showed that their average distance driven between oil changes was 5129 km with a standard deviation of 68.2 km. Another random sample of 40 customers who drive compact lower-priced cars resulted in an average distance of 5172 km with a standard deviation of 81.6 km.
   (i) Using a 1% level of significance, can we conclude that the mean distance between oil changes is lower for luxury cars than for compact lower-priced cars?
   (ii) What error may have been made in the conclusion in (i)? Explain.
   (iii) Find the p-value for the test in (i).

(c) The distribution of vehicle makes in a certain city in 2010 was as follows:

Vehicle make	Toyota	Nissan	Mazda	Honda
2010 percentage	42	36	8	14

A recent survey of 850 vehicles from the same city produced the following distribution:

Vehicle make	Toyota	Nissan	Mazda	Honda
Number of vehicles	377	299	61	113

Test at the 10% level of significance whether the distribution of the vehicle makes is significantly different from the 2010 distribution.

4. (a) Define the two main branches of statistical inference.

(b) A survey showed that 62% of workers in Lusaka province felt that they had job security, whereas 55% of workers in Copperbelt province felt that they had job security. These results were based on samples of 300 workers from Lusaka province and 200 workers from Copperbelt province.

(i) Construct a 97% confidence interval for the difference between the true population proportions.

(ii) Using a 2% level of significance, can we conclude that the proportion of workers in Lusaka province who feel that they have job security is higher that of the Copperbelt province by more than 5%?

(c) Four different washing solutions are being compared to study their effectiveness in retarding bacteria growth in 2.5 litre milk containers. The analysis is done in a laboratory and only four trials can be done on any day. Observations are taken for three days and the following data are obtained.

		Solution				
		A	B	C	D	Total
Day	1	9	11	3	7	30
	2	8	13	5	10	36
	3	7	12	7	4	30
Total		24	36	15	21	96

(i) What design was used in the experiment?

(ii) Write down a model for the above design. Explain all the terms in your model. State all the assumptions.

(iii) Are the washing solutions equally effective in retarding bacteria? Test at the 5% level of significance.

5. (a) Define the following:
   (i) coefficient of determination.
   (ii) type II error.

(b) A survey was conducted on the use of sports cars. Some people feel that their use should be tightly monitored, others prefer a few restrictions and others feel no changes should be made. The results of the survey for a sample of 400 people are summarised in the following table.

		More restrictions	Fewer restrictions	No Change
Age	Young	40	90	60
	Old	50	70	90

Test at the 2.5% significance level whether age and opinion in regard to the use of sports cars are independent.

(c) A consulting agency conducted a study to investigate whether business majors were better salespersons than those with other majors. A sample of 13 salespersons with a business degree sold an average of 11 insurance policies with a standard deviation of 1.8 insurance policies per week. Another sample of 16 salespersons with a degree other than business sold an average of 9 with a standard deviation of 1.35 insurance policies per week. Assume that the populations are normally distributed.
   (i) Using a 1% significance level, can we conclude that persons with a business degree are better salespersons than those who have a degree in another area? Assume the two population variances are equal.
   (ii) Construct a 95% confidence interval for the ratio of the population variances of the two groups.
   (iii) Using your confidence interval in (ii), explain whether the assumption of equal variances made in (i) was justified.

6. (a) (i) Define the linear correlation coefficient.

(ii) State three properties of the linear correlation coefficient.

(b) While browsing through a magazine rack at a bookstore, a statistician decides to examine the relationship between the price of a magazine (in Zambian Kwacha) and the percentage of the magazine space that contains advertisements. The data for a sample of 8 magazines are given in the following table.

Percentage of advertisements (x)	37	43	55	49	70	28	65	32
Price in Kwacha (y)	5.5	7	5	6	4	9	5.5	6.5

(You may use the following summary statistics:

$\sum_{i=1}^{8} x_i = 379$, $\sum_{i=1}^{8} x_i^2 = 19577$, $\sum_{i=1}^{8} y_i = 48.5$, $\sum_{i=1}^{8} y_i^2 = 309.75$, $\sum_{i=1}^{8} x_i y_i = 2171$)

(i) Estimate the simple linear regression equation.

(ii) Explain the meaning of the estimated parameters in (i).

(iii) Copy and complete the following ANOVA table

Source of variation	Sum of squares (SS)	Degrees of freedom (df)	Mean square (MS)	F*
Regression Error			 0.9705	
Total				

(iv) Compute a 95% confidence interval for the slope parameter.

(v) Calculate the coefficient of determination.

(vi) Explain the meaning of the coefficient in (vi).

(vii) Is there a significant linear relationship between the price of a magazine and the percentage of space containing advertisements? Use a 5% level of significance.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

Department of Mathematics and Statistics

FIRST SEMESTER FINAL EXAMINATIONS

M335 – TOPOLOGY

MARCH - 2013

INSTRUCTIONS:

- Answer any five questions.
- All questions carry equal marks.
- Show all necessary work to earn full marks.

TIME ALLOWED: Three (3) hours

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1. (a) (i) Define a metric on a non empty set X.

(ii) Define a bounded metric on a set X.

(b) Let d be a metric on a set X. Show that

$$d^*(x,y) = \frac{d(x,y)}{1+d(x,y)},\ x,y \in X$$

also defines a metric on X.

(c) Find $f(E)$ and $f^{-1}(E)$ given that $f(x) = x^2 - 1$ and $E = (-2, 2]$.

2. (a) Define the following:

(i) A closed ball in a metric space (X, d).

(ii) A closed set in a metric space (X, d).

(b) In any metric space (X, d), prove that a closed ball is a closed set.

(c) Let (X, d) be a metric space and $x, y \in X$, with $x \neq y$. Prove that there are disjoint open sets G_1 and G_2 such that $x \in G_1$ and $y \in G_2$.

3. (a) Let A be a subset of a metric space (X, d). Define each of the following:

(i) The closure, $\overline{A}$, of A.

(ii) The boundary, $\text{bd}(A)$, of A.

(b) If $A = \{z \in \mathbb{C} : \ Rez = 1\}$, find the boundary of A.

(c) (i) Define the interior, $\text{int}(A)$, of a subset A of a metric space (X, d).

(ii) If A is a subset of a metric space (X, d), prove that $\overline{A} = \text{int}(A) \cup \text{bd}(A)$.

4. (a) Define the following:

(i) A closed set in a topological space.

(ii) A continuous function between topological spaces.

(b) (i) Suppose that X is a topological space. Prove that the intersection of any collection of closed subsets of X is closed and the union of any finite collection of closed sets is closed.

(ii) Let X and Y be topological spaces, and $f : X \to Y$ a function. Prove that f is continuous if and only if $f^{-1}(G)$ is closed in X for every closed subset G of Y.

(c) (i) Let X, Y and Z be topological spaces and $f : X \to Y$ and $g : Y \to Z$ be continuous functions. Prove that the composition $g \circ f : X \to Z$ is continuous.

(ii) Let X and Y be topological spaces, $f : X \to Y$ a function, and let $X = F_1 \cup F_2 \cup ... \cup F_n$, where $F_1, F_2, ..., F_n$ are closed sets in X. Suppose that the restriction of f to the F_i is continuous for $i = 1, 2, ..., n$. Prove that f is continuous.

5. (a) Define the following:

(i) A basis for a topology.

(ii) A compact topological space.

(b) (i) Suppose that $\mathscr{B}$ is a basis for a non-empty set X, and let $\mathscr{T} = \{A \in 2^X : A \text{ is a union of sets from } \mathscr{B}\}$. Prove that $\mathscr{T}$ is a topology for X.

(ii) Let $f : X \to Y$ be a continuous function, where X and Y are topological spaces. If K is a compact subset of X, prove that $f(K)$ is a compact subset of Y.

(c) (i) Prove that a closed subset F of a compact topological space X is compact.

(ii) Prove the *Heine-Borel theorem.*

6. (a) Define:

(i) A connected topological space.

(ii) A path-connected topological space.

(b) (i) Prove that every path-connected topological space is connected.

(ii) Let $f : X \to Y$ be a continuous function, where X and Y are topological spaces. If X is connected, prove that $f(X)$ is connected in Y.

(c) (i) Prove the *intermediate value theorem.*

(ii) State and prove the *fixed point theorem.*

END

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR FIRST SEMESTER FINAL EXAMINATIONS

M361: MATHEMATICAL STATISTICS

INSTRUCTIONS:

1. Answer any FOUR (4) questions.
2. Show all your work to earn full marks.

TIME ALLOWED: THREE (3) Hours

Q1. (a) Suppose $X \sim N(\mu, \sigma^2)$ and $X_1, X_2, \ldots, X_n$ are independent random variables with the same normal distribution.

(i) State the moment generating function (MGF) of X.

(ii) Find the MGF of $Y = a_1X_1 + a_2X_2 + \cdots + a_nX_n$, where $a_1, a_2, \ldots, a_n$ are constants.

(iii) Hence, deduce the MGF of $\overline{X}$, the sample mean.

(b) Suppose that the time a student spends talking on the cell phone has a normal distribution with mean μ and variance σ^2. A random sample of 10 talking times, in minutes, is recorded as follows 3, 5, 4, 4, 1, 1, 4, 8, 5 and 4. Obtain the method of moments estimate for;

(i) μ
(ii) σ^2

(c) Let $X_1, X_2, \ldots, X_n$ be a random sample from $f_X(x, \theta) = \frac{3x^2}{\theta^3}$, $x \in (0, \theta)$

(i) Find the maximum likelihood estimator for θ

(ii) Show that this family of distribution given above, has monotone likelihood ratio.

(iii) Using the result in (ii) above, construct the uniformly most powerful test of size α for testing $H_0 : \theta = \theta_0$ vs $H_a : \theta > \theta_0$.

Q2. (a) Define the following:

(i) Mean – squared error of an estimator

(ii) Sufficient statistics for θ

(iii) Size of a critical region for testing $H_0 : \theta \in \Omega_0 \; vs \; H_a : \theta \in \Omega - \Omega_0$

(b) (i) State the simple likelihood ratio test.

(ii) Construct the most powerful test of size α for the testing
$H_0 : \theta = \theta_0$ vs $H_a : \theta = \theta_1$
where $\theta_0 > \theta_1$ given a random sample $X_1, X_2, .., X_n$ from
$f_X(x, \theta) = (\theta + 3)(1 - x)^{\theta+2}, 0 < x < 1, \theta > -3$

(c) The probability density function of a random variable X is given by
$f_X(x, \lambda, \beta) = \lambda e^{-\lambda(x-\beta)}, \; x \geq \beta, \lambda > 0$

(i) Show that the moment generating function of X is given by

$M_X(t) = \frac{\lambda}{(\lambda - t)} e^{\beta t}$

(ii) Using the moment generating function in (i) find the <u>mean</u> and <u>variance</u> of X.

Q3. (a) (i) <u>State</u> Neyman Pearson Lemma.

(ii) Prove Neyman Pearson Lemma.

(b) Suppose that random variables X and Y have a joint continuous distribution given as follows;

$f_{X,Y}(x, y) = 4xy, \quad 0 < x < 1, \; 0 < y < 1$

Let $Z = \frac{X}{Y}$ and $W = XY$, find the following

(i) Joint density function of Z and W

(ii) The marginal density function of Z.

(c) Given a random sample $X_1, X_2, .., X_n$ from
$f_X(x, P) = P(1 - P)^{x-1}, \; x = 1, 2, 3, ...$

(i) Derive the maximum likelihood estimator for P.

(ii) Find the most powerful test of size α for testing
$H_0 : P = \frac{1}{2}$ vs $H_a : P = \frac{1}{4}$

Q4. (a) Define the following:

(i) The power function of a test γ.

(ii) The most powerful test.

(iii) Generalized likelihood ratio.

(b) (i) Let X be a random variable with pdf $f(x) = 4x^3$, if $0 < x < 1$, and zero otherwise. Find the pdf of $Y = -\ln X$.

(iii) Let X and Y be a random variables for which the joint pdf is as follows;

$$f(x,y) = \begin{cases} \frac{1}{4}(x+y) & if \quad 0 \le y \le x \le 2 \\ 0 & otherwise \end{cases}$$

Find the pdf of $Z = X + Y$

(c) Let $X_1, X_2, .., X_n$ be a random sample from Uniform, $U(\theta - 1, \theta)$.

(i) Find the method of moments estimator (MME) for θ.

(ii) Show that a Maximum likelihood estimator for θ is $\hat{\theta}_{mle} = \frac{1+Y_1+Y_n}{2}$,

where $Y_1 = \text{minimum } \{X_1, X_2, \ldots, X_n\}$, and
$Y_n = \text{maximum } \{X_1, X_2, \ldots, X_n\}$

(iii) Find the relative efficiency of $\hat{\theta}_{mle}$ with respect to $\hat{\theta}_{MME}$

Q5. (a) Define the following:

(i) Uniformly most powerful test.

(ii) Uniformly minimum variance unbiased estimator

(iii) The Jacobian of transformation $(X, Y) \rightarrow (U, V)$ where $U = g_1(x, y)$ and $V = g_2(x, y)$

(b) (i) State Cramer-Rao lower bound theorem

(ii) Show that the pdf $f_X(x,\theta) = 3\theta x^2 e^{-\theta x^3}, x > 0$ belongs to the exponential family.

(iii) Using the result in (ii) construct the uniformly most powerful test of size α, for testing $H_0 : \theta = \theta_0$ vs $H_a : \theta > \theta_0$, given a random sample $X_1, X_2, \ldots, X_n$ from the pdf in (ii).

(c) A certain explosive device will detonate if any of n short-lived fuses last longer than 1.5 seconds. Let X_j represent the life of the j^{th} fuse. Assume that each X_j has a uniform distribution on the interval 0 to 1.8 seconds, and the X_j's are independent.

(i) If the device has 10 fuses, what is the probability that it will detonate?

(ii) How many fuses are needed if one wants to be 98% certain that the device will detonate?

END OF EXAMINATION

The University of Zambia

Department of Mathematics & Statistics

2012 Academic Year First Semester Examinations

M421 - Structure and Representations of Groups

Time allowed : Three (3) hrs Full marks : 100

Instructions:
- Attempt **any five (5)** questions. All questions carry equal marks.
- **Full credit** will only be given when **full understanding** is demonstrated.
- Indicate your **computer number** on all answer booklets.

This paper consists of 3 pages of questions.

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1. **a)** Define the following:

   i) a representation of a finite group G; [2 marks]

   ii) the character of a representation. [2 marks]

   b) Show that $\rho : G \longrightarrow GL(n, \mathbb{F})$ defined by $\rho(g) = A$ is a representation of $G = \langle a : a^k = 1 \rangle$ if and only if $A^k = I_n$. [3 marks]

   c) i) Show that if x and y are conjugate in G then x and y have the same character. [4 marks]

   ii) Let $G = \langle a, b : a^6 = b^2 = 1,\ b^{-1}ab = a^{-1} \rangle$ and let ρ be a representation of G defined by

$$\rho(a) = \begin{pmatrix} e^{\pi i/3} & 0 \\ 0 & e^{-\pi i/3} \end{pmatrix} \text{ and } \rho(a) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

   Find the character values of ρ on each element of G. [9 marks]

2. **a)** What is the meaning of the following:

   i) V is a completely reducible $\mathbb{F}G$-module; [2 marks]

   ii) W is an $\mathbb{F}G$-submodule of V. [2 marks]

   b) State and prove Maschke's Theorem. [2+8 marks]

   c) Let $G = S_3$, $V = span\{v_1, v_2, v_3\}$ where $\beta = \{v_1, v_2, v_3\}$ is the natural basis for V and let $\beta' = \{v_1 + v_2 + v_3, v_1 - v_2, v_1 - v_3\}$ be another basis for V. Find the matrix $[23]_{\beta'}$. [6 marks]

3. **a)** Define the following:

   i) an $\mathbb{F}G$-module; [3 marks]

   ii) an irreducible representation. [2 marks]

   b) Let $G = \langle a : a^2 = 1 \rangle$ and let $V = \mathbb{F}^2$. For $(x, y) \in V$, define $(x, y)1 = (x, y)$ and $(x, y)a = (y, x)$.

   i) Show that V is an $\mathbb{F}G$-module; [6 marks]

   ii) Find all the $\mathbb{F}G$-submodules of V. [3 marks]

   c) Let $G = \langle a : a^3 = 1 \rangle$ and $\rho(g) = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$; for $g \in G$.

   i) Show that ρ is a representation of G. [3 marks]

   ii) Is ρ irreducible? Justify your answer. [3 marks]

4. **a)** Define the following:

   i) a central series; [2 marks]

   ii) an upper central series. [2 marks]

   b) **i)** Show that an upper central series is also a central series. [5 marks]

   ii) Let p be a prime, show that every group of order p^n is nilpotent. [7 marks]

   c) Prove that every nilpotent group is solvable. [3 marks]

5. **a)** Give meaning to the following:

   i) a normal series of a finite group G; [2 marks]

   ii) a commutator subgroup G' of G. [2 marks]

   b) Prove that

   i) if G has a normal series then every normal subgroup of G has a normal series. [3 marks]

   ii) if G has a normal subgroup N such that G/N is abelian, then G contains the commutator subgroup G'. [2 marks]

   c) **i)** Find the commutator subgroup of S_n, the symmetric group of order n. [3 marks]

   ii) Show that every factor group of a nilpotent group is nilpotent. [8 marks]

6. **a)** Define the following:

   i) an internal direct product of two groups; [2 marks]

   ii) a solvable group. [2 marks]

   b) Show that if G is an internal direct product of N and H then

   i) $G \cong N \times H$; [7 marks]

   ii) $|G| = |N|\,|H|$. [3 marks]

   c) Prove that G is solvable if and only if the k^{th} commutator subgroup $G^{(k)}$ is identity. [6 marks]

End of Examination!

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR FIRST SEMESTER FINAL EXAMINATIONS

M461: MULTIVARIATE ANALYSIS

INSTRUCTIONS:

1. Answer any FIVE (5) questions.
2. Calculators are allowed.
3. You may use statistical tables provided if necessary.
4. Show all your work to earn full marks.

TIME: THREE (3) Hours

Q1 Three suppliers provide parts in shipments of 50 units. Random samples of six shipments from each of the three suppliers were carefully checked, and the numbers of parts not conforming to standards were recorded. These numbers are listed in the following table:

	Supplier	
A	B	C
8	2	3
7	7	8
5	8	8
9	3	3
3	7	7
4	3	7

(a) (i) Obtain the sample mean vector

(ii) Briefly comment on the defective parts shipped by the suppliers

(b) (i) Using matrix algebra only obtain the sample covariance matrix S with $n = 6$.

(ii) Comment on the variation of defective parts supplied.

(c) Let μ_1 be the true mean for the number of defectives supplied by A and μ_2, μ_3 be similarly defined for suppliers B and C, respectively. Further, let the mean vector $\underline{\mu}$ and matrix C be as defined below:

$$\underline{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix} \text{ and } C = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix}$$

(i) The null hypothesis $H_0 : C\underline{\mu} = \underline{0}$ versus H_1: $C\underline{\mu} \neq \underline{0}$ is of interest. Explain what the null hypothesis is testing.

(ii) Carry out the test in (i) using the F statistic through a critical value approach at $\alpha = 0.05$ significance level.

Q2 You are given the random vector $\underline{X}' = (X_1, X_2, X_3)$ which has a multivariate normal distribution with mean and variance-covariance matrix given below.

$$\underline{\mu} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{ and } \Sigma = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 9 \end{pmatrix},$$

(a) (i) Obtain $|\Sigma|$, the determinant of Σ

(ii) Obtain Σ^{-1}

(b) (i) Using (a) (ii) express $(\underline{x} - \underline{\mu})' \Sigma^{-1} (\underline{x} - \underline{\mu})$ in terms of X_1, X_2 and X_3.

(ii) Determine the probability, $\Pr\left[(\underline{x} - \underline{\mu})' \Sigma^{-1} (\underline{x} - \underline{\mu}) \leq 9\right]$

(c) You are given that $\underline{Y} = A\underline{X}$, where $A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

(i) Find the mean and variance of $\underline{Y}$

(ii) State the distribution of $\underline{Y}$ by stating the name and parameter values.

Q3 You are given the random vector $\underline{X}' = (X_1, X_2)$ whose variance-covariance matrix Σ has the following eigen values and vectors:

$\lambda_1 = 2$ with its corresponding eigen vector $\underline{e}_1 = \begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$ and

$\lambda_2 = 1$ with its corresponding eigen vector $\underline{e}_2 = \begin{pmatrix} -\frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix}$

(a) (i) Obtain the variance-covariance matrix Σ

(ii) Find the correlation between X_1 and X_2.

(b) (i) Define $\Sigma^{\frac{1}{2}}$ in terms of λ_1, λ_2, $\underline{e}_1$ and $\underline{e}_2$.

(ii) Show that $\Sigma^{\frac{1}{2}} \times \Sigma^{\frac{1}{2}} = \Sigma$ using the values of λ_1, λ_2, $\underline{e}_1$ and $\underline{e}_2$ given above.

(c) Suppose $\underline{X}' = (X_1, X_2)$ has a multivariate normal distribution $N_2(\underline{\mu}, \Sigma)$ and $\underline{Y} = \Sigma^{-\frac{1}{2}}(\underline{X} - \underline{\mu})$

(i) Determine the mean of $\underline{Y}$

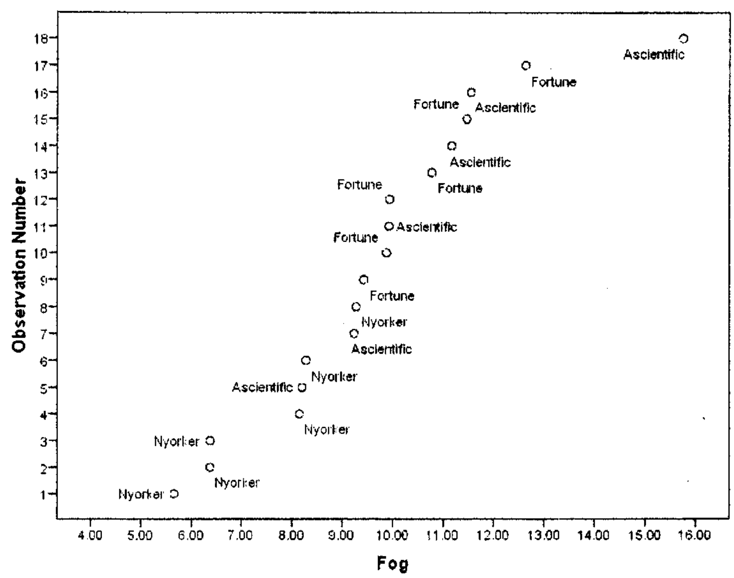
(ii) Determine the variance of $\underline{Y}$

(iii) State the distribution of $\underline{Y}$ by stating the name and parameter values.

Q4 The fog index is used to measure the reading difficulty of a written text: The higher the value of the index, the more difficult the reading level. We want to know if the reading difficulty index is different for the three magazines: Scientific American, Fortune, and the New Yorker. Independent random samples of 6 advertisements were taken from Scientific American, Fortune, and the New Yorker, and the fog indices for the 18 advertisements were measured, these are shown below.

American Scientific	(X)	15.75	11.55	11.16	9.92	9.23	8.2
Fortune	(Y)	12.63	11.46	10.77	9.93	9.87	9.42
New Yorker	(Z)	9.27	8.28	8.15	6.37	6.37	5.66

(a) (i) The 18 observations were plotted on a graph shown below, with observation number on the vertical axis and the Fog index on the horizontal axis. State the name given to such a plot.



Note that in the figure opposite:

- American Scientific = Ascientific
- New Yorker = Nyorker

(ii) What advantages does the plot have over the dot plot?

(iii) Briefly state your observations regarding the reading difficulty of the three magazines based on the plot above.

(b) (i) Obtain the sample mean vector for the reading difficulty.

(ii) Given the partial summary information obtain the sample variance-covariance matrix.

$\sum X^2$	$\sum Y^2$	$\sum Z^2$
756.85	691.60	334.10
$\sum XY$	$\sum XZ$	$\sum YZ$
718.33	500.99	479.19

(iii) Briefly state your observations regarding variation of reading difficulty in the three magazines.

(c) A one-way analysis of variance is performed yielding the results below.

One-Way Analysis of Variance

Analysis of Variance for Fog

Source	DF	SS	MS	F	P
Magazine	2	48.53	24.26	6.97	0.007
Error	15	52.22	3.48		
Total	17	100.75			

(i) State the null and alternative hypotheses associated with the results above.

(ii) State your conclusion on the null hypothesis in (i) using the results above at the 5% level of significance.

Q5 In the first phase of a study of the cost (in dollars) of transporting milk from farms to dairy plants, a survey was taken of firms engaged in milk transportation. Cost data on X_1 = fuel, X_2 = repair, and X_3 = capital, all measured on a per-kilometer basis for 16 diesel trucks, are presented in table below.

Obs	x1	x2	x3	Obs	x1	x2	x3
1	8.5	12.26	9.11	9	9.15	2.94	13.7
2	7.42	5.13	17.15	10	9.7	5.06	20.8
3	10.28	3.32	11.23	11	9.77	17.9	35.2
4	10.16	14.72	5.99	12	11.61	11.8	17
5	12.79	4.17	29.28	13	9.09	13.3	20.7
6	9.6	12.72	11	14	8.53	10.1	17.5
7	6.47	8.89	19	15	8.29	6.22	16.4
8	11.35	9.95	14.53	16	15.9	12.9	19.1

(a) (i) State briefly why these data represent a repeated measures design.

(ii) What graphical tool would you use to visualize variation in the three variables?

(b) (i) You are given the summary statistics below.

$\sum X_1$	$\sum X_2$	$\sum X_3$
158.61	151.28	277.57

What are your observations regarding average costs?

(ii) Below is the sample variance –covariance matrix S, determine a 95% Student t confidence interval for $l'\underline{\mu} = \frac{1}{3}\mu_1 + \frac{1}{3}\mu_2 + \frac{1}{3}\mu_3$, where

- μ_1 is the true mean for X_1
- μ_2 is the true mean for X_2
- μ_3 is the true mean for X_3
- $\underline{\mu}' = (\mu_1, \mu_2, \mu_3)$ is the mean vector and $l' = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$.

$$S = \begin{pmatrix} 4.71 & 1.42 & 2.60 \\ 1.42 & 19.12 & 3.05 \\ 2.60 & 3.05 & 49.15 \end{pmatrix}$$

(c) We would like to test the hypothesis $H_0: \mu_1 = \mu_2 = \mu_3$ versus H_1: Not all means are equal, at $\alpha = 0.05$ level of significance. A printout of the results on this test are given below.

Statistic	Value	F Value	Num DF	Den DF	Pr > F
Wilks' Lambda	0.45775404	8.29	2	14	------
Pillai's Trace	0.54224596	8.29	2	14	------
Hotelling-Lawley Trace	1.18457932	8.29	2	14	------
Roy's Greatest Root	1.18457932	8.29	2	14	------

(i) Calculate the critical value of the Hotelling's T statistic.

(ii) Calculate the observed value of the Hotelling's T statistic.

(iii) Using the critical value approach carry out the test using Hotelling's T statistic.

Q6. The data below show a random sample of size 10 of three assignment scores (X1, X2, X3), each graded out of 10 for students who took M461 in the 2011 academic year.

X1	X2	X3
6.0	6.1	1.4
6.5	5.7	4.8
6.5	8.3	2.9
8.0	7.8	2.6
8.0	8.3	4.0
5.0	6.1	1.4
8.0	9.6	5.2
5.5	5.7	2.6
9.0	8.7	3.1
7.5	7.8	5.2

(a) Looking at these data it is evident that students performed poorly in assignment 3 (X3). Consequently, the interest is in testing whether performance in assignments 1 and 2 was comparable, i.e., finding whether the average performance was the same for X1 and X2. Let:

- μ_1 be the true mean for assignment 1 scores
- μ_2 be the true mean for assignment 2 scores
- μ_3 be the true mean for assignment 3 scores

(i) Obtain the sample mean vector $\bar{x}' = (\bar{x}_1, \bar{x}_2, \bar{x}_3)$ to one decimal point.

(ii) The hypotheses of interest are: $H_0: \mu_1 = \mu_2$ versus $H_1: \mu_1 \neq \mu_2$.
Express the hypotheses above in the form $H_0: C\underline{\mu} = \underline{0}$ versus $H_0: C\underline{\mu} \neq \underline{0}$, where C is a 1×3 matrix and $\underline{\mu}' = (\mu_1, \mu_2, \mu_3)$.

(b) (i) Obtain the value of $C\bar{x}$

(ii) Obtain the value of $(CSC')^{-1}$, given that S is the sample variance-covariance matrix for the data and is shown below.

$$S = \begin{pmatrix} 1.67 & 1.47 & 0.97 \\ 1.47 & 1.97 & 0.92 \\ 0.97 & 0.92 & 2.04 \end{pmatrix}$$

(c) (i) Find the Hotelling's critical value for testing the hypothesis in (a) (ii) with α =0.05 as the level of significance.

(ii) Carry out the test using a critical value approach with α =0.05 as the level of significance.

END OF EXAMINATION

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2012 ACADEMIC YEAR FIRST SEMESTER FINAL EXAMINATIONS

M465: NON-PARAMETRIC METHODS

INSTRUCTIONS:

1. Answer any FIVE (5) questions.
2. Calculators are allowed.
3. You may use statistical tables provided if necessary.
4. Show all your work to earn full marks.

TIME: THREE (3) Hours

Q1 Highway crashes killed more than 75,000 occupants of passenger cars during 1986-1988 in the USA. Using this grim statistic as a starting point, researchers at the Insurance Institute for Highway Safety computed deaths rates for the 103 largest-selling vehicle series. Vehicles were categorized as large, midsize or small. Looking at the rates (deaths per 10,000 registered) the figures ranged from 1.1 to 4.1 and the ranks for the figures were as follows:

Large	4.5	9	13	17	17	17	22	29										
Midsize	1.5	4.5	4.5	4.5	9	9	9	9	13	13	17	22	22	22	26	26	29	31
	34	37	37	39	41	43	45											
Small	1.5	17	22	26	29	34	34	37	41	43	44	46	47					

(a) (i) Obtain the summary statistics of the given ranks.

(ii) Use the Kruskal-Wallis test to test whether the three populations means are equal using $\alpha = 0.05$. Show all the necessary steps.

(b) (i) Carry out all pair-wise tests on an experimentwise with $\alpha = 0.05$.State only the hypotheses and your conclusion using a critical value approach.

(ii) Were the tests in (i) necessary?

Q2 Suppose the numbers 1, 2, . . ., N are randomly subdivided in p groups of sizes $n_1, n_2, \ldots, n_p$ for group 1, group2, . . . , and group p, respectively. Hence, $N = n_1 + n + \ldots + n_p$.
Let R_{ij} be the jth number being assigned to the ith group from among the N numbers, where i = 1, 2, . . ., p and j = 1, 2, . . .,n_i.
The number R_{ij} maybe considered as the rank of Y_{ij}, the jth observation in the ith group when an individual takes p samples from some population with sample sizes $n_1, n_2, \ldots, n_p$.
Define the following:

$I_{rs} = \begin{cases} 1 & \text{if } r = i \text{ and } s = j \\ 0 & \text{if } r \neq i \text{ and } s \neq j \end{cases}$ where $r = 1, 2, \ldots, p$ and $s = 1, 2, \ldots n_r$

(a) (i) Determine the probability that any of the N numbers is assigned to the ith group.

(ii) The mean of ranks for group i maybe expressed as follows:

$$\overline{R}_{i+} = \frac{\sum_{r=1}^{p} \sum_{s=1}^{n_r} R_{rs} I_{rs}}{n_i}$$

Show that $E(\overline{R}_{i+}) = \frac{N+1}{2}$

(b) (i) Use the results in (a) (ii) to obtain an expression for the expected value of R_{i+}, the sum of the ranks in group i.

(ii) Suppose the experiment above is repeated k times and that the variance of R_{i+} is given by $Var(R_{i+}) = \frac{n_i(N+1)(N-n_i)}{12}$ in a single trial, using this and the result in (i) construct a large sample test statistic for testing the null hypothesis that the mean of the i group equals some specified value at some significance level α.

(c) Three countries, including Zambia, are competing a series of 11 independent competitions. In each competition each country has three athletes. At the end of each competition the athletes are ranked 1 up to 9, with 1 being the best performance and 9 the worst. The ranks of the three athletes for each country are totaled in each competition. At the end of the series the average for each country is obtained. A country getting an average of

- 9 points or less in the entire series of competition is given first prize overall
- 10 to 15 points is given second prize, and
- Above 15 points the third prize.
- If countries are tied in any category the actual average values are used to rank them.

Assume that the totals in the 11 competitions for Zambia are as shown below. A fan claims that Zambia is a first or second prize winner, test the claim at $\alpha = 0.05$ using the statistic in (b) (ii).

20 23 15 18 20 22 16 16 16 24 19

Q3 Students in an e-business technology course were given a written final examination as well as a project to complete as part of their final grade. For a random sample of 11 students, the scores on both the exams and project were as follows:

Student	1	2	3	4	5	6	7	8	9	10	11
Exam	81	62	74	78	93	69	72	83	90	84	80
Project	76	71	69	76	87	62	80	75	92	79	77

The examiners are interested in studying the association between performance in an exam and performance in a project.

(a) (i) What graphical tool would you use to scout for any association?

(ii) Why is this setup different from the usual regression problem?

(b) (i) Obtain the ranks within exam and also ranks within project.

(ii) Test for a positive correlation using Spearman's test with $\alpha = 0.10$, showing all necessary steps.

Q4 (a) (i) State four advantages of nonparametric tests.

(ii) State three disadvantages of nonparametric tests.

(b) Clinical data concerning the effectiveness of two drugs in treating a particular disease were collected from ten hospitals. The number of patients treated with the drugs varied from one hospital to another as well as between drugs within a given hospital. The data, in percentage recovery, are shown in the table below.

	Drug A		Drug B	
Hospital	Number in group	Percentage Recovered	Number in group	Percentage Recovered
1	84	75.0	96	85.4
2	63	69.8	83	83.1
3	56	85.7	91	80.2
4	77	74.0	47	74.5
5	29	69.0	60	70.0
6	48	83.3	27	81.5
7	61	68.9	69	75.4
8	45	77.8	72	79.2
9	79	72.2	89	85.4
10	62	77.4	46	80.4

(i) Determine whether there is sufficient evidence to indicate a higher recovery rate for one of the two drugs. Use the Wilcoxon Signed Rank test utilizing the P-value approach with $\alpha = 0.10$

(ii) Why would it be inappropriate to use a Student's t test in analyzing the data?

Q5 Three real estate agents were each asked to assess the value of 10 houses in a neighborhood. The results, in thousands of rebased kwacha, are given in the table below.

House	Agent A	B	C
1	170	168	175
2	185	180	195
3	245	240	260
4	300	290	350
5	400	420	450
6	210	218	226
7	192	190	198
8	183	187	185
9	227	223	237
10	242	240	237

(a) State the linear additive model associated with the design yielding data such as in the table

(b) Use Friedman's procedure to determine whether there is sufficient evidence to indicate a difference in assessment between the three agents using $\alpha = 0.05$

Q6. Two farmers A and B each with two animals are competing in an agricultural and pastoral show. The animals are ranked 1 to 4 based some characteristics. Let W be the sum of the ranks assigned to animals for farmer A.

(a) (i) Copy and complete the table below showing all possible permutations of the numbers 1, 2, 3 and 4 and the value of W.

Permutations Farmer A	Farmer B	W
12	34	3
12	43	3
...	...	...
43	21	7

(ii) Copy and complete the table below showing the distribution of W. Leave the probabilities in a/b form.

W	3	4	5	6	7
Pr(W = w)					

(iii) If we are interested in testing the null hypothesis that farmer A's animal are better than farmer B's, what is the smallest type I error (α) possible with the distribution in (ii)?

(b) (i) A test such as was described in (a) (iii) is planned with $\alpha = 0.35$, construct a rejection set using the distribution of W in (a) (ii).

(ii) At the end of the competition the judges assign ranks 2 and 3 to the animals of farmer A. Are farmer A animals better at $\alpha = 0.35$?

(iii) What special name is given to this permutation test?

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

School Of Natural Sciences

Department of Mathematics & Statistics

M911 Mathematical Methods V

Final Examination 2012 Academic Year

Allowed Time: Three (3) Hours

Instructions

1. There are **six (6)** questions in this examination paper, and you are required to answer **Five (5)** questions only.
2. Marks are indicated at the end of each question
3. Calculators are not allowed in this examination
4. No tables are required in this examination
5. ALL necessary working must be shown clearly in the answer booklet to avoid loss of marks.

1. (a) Let $u(x,y)=\frac{x+y}{x-y}$ and $v(x,y)=\frac{xy}{(x-y)^2}$. Show that the functions $u(x\,,y)$ and $v(x\,,y)$ are functionally dependent.

(b) The two functions $x+y=uv$ and $xy=u-v$ determine x and y implicitly as functions of u and v. Find expressions for $\frac{\partial x}{\partial u}$ and $\frac{\partial y}{\partial v}$

(c) Let $f(x\,,y)$ be a differentiable function at the point $(1\,,2)$ such that its directional derivative in the direction towards $(2\,,2)$ is -2 and its directional derivative in the direction towards $(1\,,1)$ is 2.

(i) Determine the gradient vector of f at $(1\,,2)$ i.e $\nabla f(1,2)$

(ii) Calculate also its directional derivative in the direction towards $(4\,,6)$

2. (a) Find the second order Taylor expansion of the function $h(x\,,y)=\ln(1+2x+5y)$ about the point $(0\,,0)$.

(b) Let $g(x\,,y)=x^4+y^4-4xy$.

(i) Find the stationary points of the function.

(ii) Using the method of eigenvalues determine the nature of the stationary points.

(c) The profit obtained by producing x units of product A and y units of product B is approximated by the model $P(x,y)=8x+10y-(0.001)(x^2+xy+y^2-10{,}000)$.
Find the production level that produces a maximum profit .

3. (a) Let $\mathbf{F}(x\,,y)=\left(\frac{y}{x}+3y^3\right)\mathbf{i}+\left(\ln x+9xy^2\right)\mathbf{j}$ be a vector field,

(i) Show that $\mathbf{F}(x\,,y)$ is exact.

(ii) Find the potential function $f(x\,,y)$ such that $\nabla f=\mathbf{F}$

(b) Let $z=x^y$, where $x=f(u\,,v)$, $y=g(u\,,v)$ define z as a function of u and v. Given that when $u=1$ and $v=2$ then $f(1\,,2)=2$, $g(1\,,2)=-2$, $\frac{\partial x}{\partial u}=-1$, $\frac{\partial x}{\partial v}=3$, $\frac{\partial y}{\partial u}=5$ and $\frac{\partial y}{\partial v}=0$, calculate using chain rule the value of $\frac{\partial z}{\partial u}$ at the point $(u\,,v)=(1\,,2)$.

(c) Determine the maximum and the minimum of the function $f(x\,,y\,,z)=x+y+z$ subject to the constraints $x^2+y^2=1$ and $z=2$.

4. (a) Evaluate

(i) $\int_0^1 \int_{2x}^2 \exp(y^2)\,dydx$

(ii) $\int_0^1 \int_0^{2-y} (3y - x)\,dxdy$

(b) Let the vector field $F(x,y,z) = e^x(\sin yi + \cos yj)$.

(i) Verify that $Curl\, F = 0$.

(ii) Hence, find f such that $F = \nabla f$.

(c) Let T(u,v) = (x(u,v),y(u,v)) be the mapping define by $T(u,v) = (4u, 2u + 3v)$. Let D* be a rectangle [0,1]x[1,2].

(i) Find $D = T(D^*)$ and

(ii) Hence, evaluate $\int_D xydxdy$ by making a change of variables.

5. (a) (i) Define triple integral of types 1 and 2.

(ii) Find the equation of the tangent plane to the surface $z = x^2 + y^2$ at (1,5,26).

(b) Let w be a region bounded by the planes x = 0, y = 0,, z = 3 and the surface $x^2 + y^2 - z = 0$, $x \geq 0, y \geq 0$;

(i) Sketch the solid w and region D.

(ii) Hence, compute $\int_w xdxdydz$.

(c) (i) Find the iterated integral of the function f defined by $f(x,y) = x^2 y$ over the region $D = \{(x,y) : 0 \leq x \leq 1, 2y^2 \leq y \leq 2y\}$. Sketch the region D.

(ii) Find the potential function for the vector field $F(x,y,z) = 2xyi + (x^2 + z^2)j + 2zyk$

6. (a) (i) Show that the volume of a cone is $\frac{1}{3}\pi r^2 h$ where r is the base radius and h the height of a cone, using double integral.

(ii) State and prove the mean value theorem of double integrals

(b) (i) Find the divergence at (3,2,0) for the vector field

$F(x, y, z) = e^{-xyz}(i + j + k)$

(ii) Sketch several representative vectors in the vector field

$F(x, y) = 4xi + yj$

(c) (i) Find the principal normal for the curve represented by

$r(t) = 3ti + 2t^2$ at t=1.

(ii) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sin(x^2 + y^2)dydx$

END of Examination

The University of Zambia

Department of Mathematics & Statistics

First Semester Examinations - November/December 2011

MAT5111 - Ordinary Differential Equations & Integral Equations

Time allowed : Three (3) hrs | Full marks : 100

Instructions: • There are six (6) questions in this paper. Attempt **any five (5)** questions. All questions carry equal marks.

- **Full credit** will only be given when **necessary work** is shown.
- Indicate your **computer number** on all answer booklets.
- Calculators **not** allowed.

This paper consists of 4 pages of questions.

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1. a) Bessel's differential equation is given by

$$x^2y'' + xy' + (x^2 - \nu^2)y = 0 \, .$$

(i) Use the Frobenius method to show that the indicial equation is given by

$$(r+\nu)(r-\nu) = 0 \, .$$

(ii) For the case $r = \nu$ where $\nu \geq 0$, show that the recurrence relation for the coefficients is

$$a_{2m} = \frac{(-1)^m a_0}{2^m m!(\nu+1)(\nu+2)\dots(\nu+m)} \qquad m = 1, 2, \dots$$

b) Use an appropriate substitution to obtain a particular solution $J_\nu(x)$, the Bessel function of the first kind of order ν.

c) Sketch the Bessel functions of the first kind of order 0 and order 1.

2. a) Prove that

(i) $$\frac{d}{dx}\left[x^{\nu}J_{\nu}(x)\right] = x^{\nu}J_{\nu-1}(x)$$

(ii) $$\frac{d}{dx}\left[x^{-\nu}J_{\nu}(x)\right] = -x^{-\nu}J_{\nu+1}(x)$$

b) Use results from (a) to show that

$$J_{\nu-1}(x) + J_{\nu+1}(x) = \frac{2\nu}{x}J_{\nu}(x) .$$

Hence or otherwise show that

$$J_{-\frac{3}{2}}(x) = -\sqrt{\frac{2}{\pi x}}\left(\frac{\cos x}{x} + \sin x\right)$$

c) Express $J_4(ax)$ in terms of $J_0(ax)$ and $J_1(ax)$, where a is a constant.

3. a) Given the Bessel differential equation

$$x^2y'' + xy' + (x^2 - n^2)y = 0$$

and Legendre's equation

$$(1 - x^2)y'' - 2xy' + n(n+1)y = 0 ,$$

show that both equations can be expressed as a Sturm-Liouville equation.

b) Given the equation

$$y'' + \lambda y = 0 \qquad y(0) = 0 \qquad y'(\pi) = 0$$

show that the eigenvalues and corresponding eigenvectors are given by

$$\lambda_n = \frac{2n-1}{2} , \qquad \phi_n(x) = B\sin\left(\frac{2n-1}{2}x\right) , \qquad n = 0, \pm1, \pm2, \ldots$$

respectively. B is some constant.

c) For $\lambda \in \mathbb{R}$, solve $y'' + \lambda y = 0$ with periodic boundary conditions $y(0) - y(\pi) = 0$ and $y'(0) - y'(\pi) = 0$.

4. a) Given an initial value problem

$$\frac{dy}{dx} = f(x,y) \ , \qquad a \le x \le b \ , \qquad y(a) = \alpha$$

define the following :-

(i) the Lipschitz condition in the variable y.

(ii) a well-posed problem.

b) Given the equation

$$y' = -\frac{2}{t}y + t^2e^t \qquad 1 \le t \le 2 \qquad y(1) = \sqrt{2}e$$

show directly that the initial value problem is well-posed, given that the perturbation $\delta(t)$ is proportional to t.

c) Show that, for any constants a and b, the set $D = \{(x,y)|a \le x \le b\,, -\infty \le y \le \infty\}$ is convex.

5. a) Define the following:-

(i) An orthogonal set of functions

(ii) An orthonormal set of functions

b) Show that the eigenvalues, if any, of a regular Sturm-Liouville boundary value problem are real.

c) Prove that the eigenfunctions of a regular Sturm-Liouville boundary value problem corresponding to distinct eigenvalues are orthogonal with respect to weight function r on $[a,b]$.

6. a) Use Bessel functions to write a complete solution of the equation $y'' + y = 0$.

b) Given Legendre's equation

$$(1-x^2)y'' - 2xy' + n(n+1)y = 0 \ ,$$

prove that

$$\int_{-1}^{1} P_m(x)P_n(x)dx = 0 \qquad \text{if } \ m \ne n$$

where $P_i(x)$ are Legendre's polynomials.

c) (i) Find a general solution of the homogeneous linear system

$$\begin{aligned} y_1 &= -3y_1 + y_2 \\ y_2' &= y_1 - 3y_2 \end{aligned}$$

(ii) Draw a phase diagram for the general solution in (i) and classify the node.

END!

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

Department of Mathematics & Statistics

FIRST SEMESTER FINAL EXAMINATIONS

MAT5311—LEBESGUE MEASURE AND LEBESGUE INTEGRATION

November, 21, 2011

Time allowed : THREE(3) HOURS

Instructions : There are six(6) questions. Answer **ANY FIVE (5)** questions. All questions carry equal marks. Show all your working to earn full marks.

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1. Let X be a nonempty set.

(a) Let $\mu : \mathcal{P}(X) \to [0, \infty)$ be a finitely additive set function such that $\mu(A) = 0$ or 1 for every $A \in \mathcal{P}(X)$. Let

$$\mathcal{U} = \{A \in \mathcal{P}(X) : \mu(A) = 1\}.$$

Show that $\mathcal{U}$ has the following properties:

(i) $\emptyset \notin \mathcal{U}$.

(ii) If $A \in U$ and $B \supseteq A$, then $B \in \mathcal{U}$.

(iii) If $A, B \in \mathcal{U}$, then $A \cap B \in \mathcal{U}$.

(iv) For every $A \in \mathcal{P}(X)$, either $A \in \mathcal{U}$ or $A^c \in \mathcal{U}$.

(Any $\mathcal{U} \subseteq \mathcal{P}(X)$ satisfying (i) to (iv) is called an ultrafilter in X.)

(b) Let $\mathcal{U}$ be any ultrafilter in X. Define $\mu : \mathcal{P}(X) \to [0, \infty)$ by

$$\mu(A) = \begin{cases} 1, & A \in \mathcal{U} \\ 0, & A \notin \mathcal{U} \end{cases}$$

Show that μ is finitely additive.

(c)(i) Define an algebra on a set X.

(ii) Let $\mathcal{F}$ be a collection of subsets of X. Show that $\mathcal{F}$ is an algebra if and only if the following hold:

(1) $\emptyset, X \in \mathcal{F}$.

(2) $A^c \in \mathcal{F}$ whenever $A \in \mathcal{F}$.

(3) $A \cup B \in \mathcal{F}$ whenever $A, B \in \mathcal{F}$.

2. (a)(i) Define the Lebesgue measure μ of a subset E of $\mathbb{R}$ in relation to the outer measure.

(ii) If A is such that $\mu^*(A) < \infty$ and there is a measurable subset $B \subseteq A$ with $\mu(B) = \mu^*(A)$, show that A is measurable.

(b)(i) Let $f : E \to \mathbb{R}_\infty$ be a function, where E is a measurable subset of $\mathbb{R}$. When is f said to be a measurable function?

(ii) If f and g are measurable functions on E and their sums and squares are also measurable, prove that fg is also measurable.

(c)(i) Define a σ-algebra on a set X.

(ii) Let $\mathcal{A}$ be a σ-algebra and $f : A \to \mathbb{R}$ where $A \in \mathcal{A}$.

Define $\mathcal{F} = \{B \in \mathbb{R} : f^{-1}(B) \in \mathcal{A}\}$. Prove that $\mathcal{F}$ is a σ-algebra.

3. (a)(i) Define a simple function f on a measurable subset E of $\mathbb{R}$.

(ii) Use the function $f : [a, b] \to \mathbb{R}$ defined by $f(x) = 1$ when x is rational and $f(x) = 0$ when x is irrational to show that Lebesgue integrable does not mean Riemann integrable.

(b) Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of measurable functions with $f_n : E \to \mathbb{R}_\infty$ for each n. Prove that:

(i) $p = \inf_n f_n$ is measurable.

(ii) $k = \sup_n f_n$ is measurable.

(c) State and prove Fatou's lemma.

4. (a) Let

$$f_n(x) = \begin{cases} -n^2, & x \in (0, \frac{1}{n}) \\ 0, & \text{otherwise.} \end{cases}$$

(i) Show that

$$\lim_{n\to\infty} \int_{\mathbb{R}} f_n(x)dx \neq \int_{\mathbb{R}} \lim_{n\to\infty} f_n(x)dx.$$

(ii) Explain why this does not contradict Fatou's lemma.

(b) State and prove the Dominated Convergence theorem.

(c) For $n \geq 1$, let

$$f_n(x) = \begin{cases} 2n^2, & \frac{1}{2n} \leq x \leq \frac{1}{n} \\ 0, & x \in (0, \frac{1}{2n}) \cup (\frac{1}{n}, 1). \end{cases}$$

(i) Verify that the Dominated Convergence theorem does not hold for the sequence $\{f_n\}$.

(ii) Explain why the Dominated Convergence theorem can not work.

5. (a)(i) Let

$$f(x) = \frac{1}{\sqrt[3]{x}(1 + \ln x)}.$$

Show that $f \in L^3[1, \infty)$.

(ii) Find an example to show that $L^2((1,\infty)) \not\subseteq L^1((1,\infty))$.

(b) If $1 \leq p < q < \infty$ and $\mu(E) < \infty$, prove that $L^q(E) \subset L^p(E)$.

(c) If $\mu(E) < \infty$, f is measurable and $f \in L^3(E)$, prove that f is integrable by directly applying Holder's inequality.

6. (a) If $0 < p < q < r \leq \infty$, prove that $L^q \subset L^p + L^r$.

(b) State and prove Holder's inequality. You may use the fact that if $p > 0$ and a and b are any nonnegative reals, $ab \leq \frac{a^p}{p} + \frac{b^q}{q}$.

(c) Use Holder's inequality to show that if $\mu(E) = K$ for some constant K, then if $1 < p < \infty$ and $f \in L^p$, we have that $f \in L^1$ and $\|f\|_1 \leq \|f\|_p K^{\frac{1}{q}}$ where p and q are conjugate indices.

END.

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

Department of Mathematics and Statistics

2011 Academic Year

Semester II

MAT5622: Generalized Linear Models

Final Examination

Time Allowed: Three (3) Hours **May 2012**

Instructions:

1. You must write your **Computer Number** on each answer booklet used.
2. There are Five (5) questions in this paper, Attempt any **Four (4)** questions.
3. Full credit will only be given where all the necessary reasoning is shown.

1. (a) (i) What are the three processes in Generalized linear models?
(ii) What is an important characteristic of a Genaralized linear model?

(b) (i) Show that for a random variable Y with an exponential distribution with canonical parameter θ, $E(Y)=b'(\theta)$ and $\text{Var}(Y)=b''(\theta)a(\phi)$
(ii) Find the mean and the variance for Binomial and Poisson.

(c) (i) What is Simpson's Paradox involving several 2×2 tables?
(ii) What is Mantel – Halnszel estimator of the odds ratio involving several 2×2 tables?

(iii) If $H_0:\psi=0$ is based on the sufficient statistic $T=\sum Y_{00}$ the sum of the $(0,0)$ elements in the k, 2×2 tables, what are the exact mean and variance of T when H_0 is true?

2. (a) Let Y have a probability density function:

$$f_Y(y,\theta,\phi)=\exp\left\{\frac{y\theta-b(\theta)}{a(\phi)}+c(y,\phi)\right\}$$ for some real valued functions $a(\cdot)$, $b(\cdot)$ and $c(\cdot)$.

(i) What is the exponential family distribution with canonical parameter θ ?
(ii) Write the exponential family distribution with canonical parameter θ for the Normal and Binomial.
(iii) Derive the mean and variance for Normal and Binomial.

(b) (i) What are the three choices of Link functions in models for binary responses?
(ii) Which of these link functions in *b*(i) above is preferred and why?

(c) Let $Y\sim B(n_1,\theta_1)$ and $Z\sim B(n_2,\theta_2)$ with Y and Z independent,

$$\psi=\frac{\theta_1(1-\theta_2)}{\theta_2(1-\theta_1)}$$ the odds ratio.

(i) Write down the conditional distribution of Y given $Y+Z=m$.
(ii) Using the results in (i) above find an expression for the conditional r^{th} moment of Y.
(iii) Hence or otherwise, find the conditional log – likelihood for ψ.

3. (a) (i) Describe an algorithm for fitting Generalized linear models.
(ii) Justify the fitting procedures in (i) above.

(b) (i) What is the deviance for the Generalized linear model?
(ii) Derive the deviance for the Generalized linear model for Normal and Poisson.

(c) For the Prospective study the logistic model is given by
$P(D^+ / X) = \dfrac{e^{\alpha+\beta' X}}{1+e^{\alpha+\beta' X}}$ for the probability of diseased given an arbitrary vector of covariates X.
Show that this model does apply for retrospective studies in terms of estimating the vector β of parameters.

4. (a) **Describe** the following:
(i) Pearson residual.
(ii) Deviance residual.
(iii) Anscombe residual.

(b) Find the following:
(i) Anscombe residual for Poisson distribution.
(ii) Deviance residual for Poisson distribution.

(c) The linear predictor for two covariates is given by $\eta = \beta_1 X_1 + \beta_2 X_2$.
(i) Describe the Geometry of least squares fitting.
(ii) If the points P_1, P_2 and P_{12} are perpendicular from Y onto X_1, X_2 and (X_1, X_2) plane respectively, derive the analysis of variance whose Geometrical interpretation is
$$SST = SS(X_2) + SS(X_1 - X_2) + SSR \text{, where,}$$
SST = total sum of squares,
$SS(X_2)$ = sum of squares for X_2 ignoring X_1,
$SS(X_1 - X_2)$ = sum of squares for X_1 eliminating X_2,
SSR = Residual sum of squares.

5. (a) (i) Describe the aliasing in Generalized linear models.
(ii) What are the canonical link functions for Poisson and Gamma distributions.

(b) (i) Describe the Gram – Schmidt method of direct decomposition of the model matrix.
(ii) If ψ is a vector of length k (i.e there are k 2×2 tables , ψ_i = odds ratio for the i^{th} table), write the full log – likelihood function.

(c) (i) Derive the deviance for the Binomial model i.e $Y_i \sim B(n_i, \theta_i)$, $i = 1, 2, \ldots\ldots, k$.

(ii) The data where population is classified according to having the disease (D^+) or not (D^-) and being exposed (X^+) or not being exposed (X^-) to the condition. The classification can be presented in a 2×2 table

Exposure status	Disease status D^+	D^-
X^+	θ_{11}	θ_{10}
X^-	θ_{01}	θ_{00}

Describe the analysis for prospective or retrospective studies and hence show that either can be used and get the same result.

End of Exam

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS

2011 ACADEMIC YEAR
SECOND SEMESTER EXAMINATIONS

MAT5632 : DESIGN AND ANALYSIS OF EXPERIMENTS

TIME ALLOWED: Three (3) Hours

INSTRUCTIONS: 1. Answer any **Four** (4) Questions
2. Show All Essential Working

1. (a) Given a one factor random effects model

$$Y_{ij} = \mu + \tau_i + \varepsilon_{ij} \quad , \quad i = 1, 2, \ldots, t \quad ; \quad j = 1, 2, \ldots, r$$

where

$$\varepsilon_{ij} \sim N(0, \sigma^2), \qquad \tau_i \sim N(0, \sigma_\tau^2) \ ,$$

τ_i and ε_{ij} are independent

Prove that $E(MSTrt) = r\sigma_\tau^2 + \sigma^2$, where $MSTrt$ is the mean treatment sum of squares.

(b) An experiment was run to find the best conditions for a process. The process has three factors A, B and C, which are varied according to a 2^3 factorial design and the response variable is denoted by y. The results from the experiment are given in the following table.

A	B	C	y
−	−	−	25
+	−	−	29
−	+	−	50
+	+	−	46
−	−	+	38
+	−	+	36
−	+	+	54
+	+	+	47

(i) Estimate the main effects A, B and C.

(ii) Estimates of interaction effects are:

Factor	Estimate
AB	-3.25
AC	-2.25
BC	-3.75
ABC	0.75

Plot all the effects on a normal probability paper and identify the effects that appear significant.

(iii) Prepare an ANOVA table based on the factors identified in (ii) assuming all the other factors are negligible. Test for the significance of the factors.

(iv) Write down a regression equation of the significant factors identified in (iii).

2. (a) Consider a 2^4 factorial design with one replication.

(i) Obtain a half fraction of the design using BCD as the defining contrast.

(ii) Show the effects that are aliased with the four main effects in your half fraction in (i).

(iii) Divide the half fraction in (i) into two blocks of 4 units each by confounding ABC.

(iv) Given that the actual experiment conducted gave the following data:

$(1) = 19$	$d = 13$
$a = 15$	$ad = 18$
$b = 15$	$bd = 18$
$ab = 18$	$abd = 18$
$c = 15$	$cd = 18$
$ac = 18$	$acd = 19$
$bc = 17$	$bcd = 18$
$abc = 19$	$abcd = 20$

Using your half fraction in (i) above, estimate the main effects A, B, C and D.

(b) In an agricultural experiment, it was proposed to study three factors A, B and C, each at two levels. Since it is difficult to get homogeneous blocks of size 8, the following treatment layout was proposed:

Run	Block	A	B	C	Response
1	1	+	+	–	y_1
2	1	+	–	+.	y_2
3	1	–	–	–	y_3
4	1	–	+	+	y_4
5	2	+	–	–	y_5
6	2	+	+	+	y_6
7	2	–	–	+	y_7
8	2	–	+	–	y_8

(i) Find the treatment effect confounded with blocks in this design. Show your steps clearly.

(ii) Explain whether this design allows estimation of the main effect A and the BC interaction effect. If these effects can be estimated, give formulas for their estimates in terms of the response values y_1, y_2, … , y_8.

(iii)

Run	Block	A	B	C	Response
9	3	+	–	–	y_9
10	3	–	+	+	y_{10}
11	3	–	–	–	y_{11}
12	3	+	+	+	y_{12}
13	4	+	–	+	y_{13}
14	4	+	+	–	y_{14}
15	4	–	–	+	y_{15}
16	4	–	+	–	y_{16}

Find the treatment effect confounded with blocks in this design. Show your steps clearly.

(iv) Consider the data in Blocks 3 and 4 only. Explain whether this design allows estimation of the main effect A and the BC interaction effect. If these effects can be estimated, give formulas for their estimates in terms of y_9, y_{10}, … , y_{16}.

3. (a) (i) Explain the difference between a random effects model and a mixed effects model.

(ii) Define a 2^{k-p} fractional design.

(b) To estimate the various components of variability in a filtration process, the percentage of material lost in coffee was measured for 12 experimental conditions with 3 runs on each condition. Three filters and 4 operators were selected at random for the experiment resulting in the following measurements:

		Operator				
		1	2	3	4	Total
Filter	1	16.2 16.8 17.1	15.9 15.1 14.5	15.6 15.9 16.1	14.9 15.2 14.9	188.2
	2	16.6 16.9 16.8	16.0 16.3 16.5	16.1 16.0 17.2	15.4 14.6 15.9	194.3
	3	16.7 16.9 17.1	16.5 16.9 16.8	16.4 17.4 16.9	16.1 15.4 15.6	198.7
	Total	151.1	144.5	147.6	138	581.2

(i) Write down a model for the above experiment. Explain all the terms in the model and state all the assumptions.

(ii) Find the expected mean squares of all the factors in the model.

(iii) Copy and complete the following ANOVA table.

Source	SS	df	MS	F
Filter				
Operator				
Filter-operator interaction	1.657			
Error	4.440			
Total	21.049			

(iv) Test for the effects of filter, operator and filter–operator interaction on the variability of the filtration process. State your hypotheses and conclusions clearly.

(v) Estimate the components of variance due to filters, operators, filter – operator interaction, and experimental error.

4. (a) Define the following:

(i) resolution R design.

(ii) partial confounding.

(b) An experiment is conducted in which the surface finish of metal parts made on four machines is being studied. Each machine is run by three different operators and two specimens from each operator are collected and tested. Because of the location of the machines, different operators are used on each machine and the operators are chosen at random. The data collected from the experiment are shown in the following table:

	Machine 1			Machine 2			Machine 3			Machine 4			
Operator	1	2	3	1	2	3	1	2	3	1	2	3	Total
	79	94	46	92	85	76	88	53	46	36	40	62	
	62	74	57	99	79	68	75	56	57	53	56	47	
Sub-total	141	168	103	191	164	144	163	109	103	89	96	109	
Total	412			499			375			294			1580

(i) What design is used in the experiment? Explain.

(ii) Write down a model for the experiment. Explain all the terms in the model and state all the assumptions.

(iii) Find the expected mean squares of the factors in the model.

(iv) Copy and complete the following ANOVA table.

Source	SS	df	MS	F
Machine				
Operator(Machine)	2817.6667			
Error	1014			
Total				

(v) Test for the effect of machines and operators. State your hypotheses and conclusions clearly.

(vi) Estimate the components of variance due to operators and experimental error.

5. (a) (i) State and define two principals of experimental design.

(ii) State two reasons why each principal of experimental design is important.

(b) An experiment is performed to determine the effect of temperature and heat treatment time on the strength of steel. Two temperatures and three times are selected. The experiment is performed by heating the oven to a selected temperature and inserting three specimens. After 10 minutes one specimen is removed, after 20 minutes the second is removed and after 30 minutes the final specimen is removed. Then the temperature is changed to the other level and the process is repeated. The experiment was done in four shifts. The data obtained are given below. Assume that the time and temperature are fixed factors while shifts are random.

Shift	Time (minutes)	Temperature (°C) 800	900	Total
	10	63	89	
1	20	54.	91	420
	30	61	62	
	10	50	80	
2	20	52	72	382
	30	59	69	
	10	48	73	
3	20	74	81	416
	30	71	69	
	10	54	88	
4	20	48	92	405
	30	59	64	
Total		693	930	1623

(i) What design is used in the experiment? Explain.

(ii) Write down a model for the experiment assuming that the three way interaction is negligible. Explain all the terms in the model and state all the assumptions.

(iii) Find the expected mean squares of the factors in the model in (ii).

(iv) Copy and complete the following ANOVA table.

Source	SS	df	MS	F
Shift				
Temp.				
Shift-temp interaction	240.458			
Time				
Shift-time interaction	478.417			
Temp-time interaction	795.250			
Error	244.417			
Total	4403.625			

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS

MAT 5642: STATISTICAL METHODS IN EPIDEMIOLOGY

INSTRUCTIONS:

1. Answer any FIVE (5) questions.
2. Calculators are allowed.
3. You may use statistical tables or formulae provided if necessary.
4. Show all your work to earn full marks.

TIME: THREE (3) Hours

Q1 (a) State the two fundamental assumptions of epidemiology

(b) Discuss the strengths and limitations of cohort studies.

(c) The Zambia Demographic and Health Survey report of 2007 found that of the 4473 rural children in its study, 32.9% slept under a mosquito net and of the 1774 urban children 35% were under the net a day before the survey was conducted. Determine, at 5% level of significance whether rural children were at a greater risk of going to bed without a mosquito net.

Q2 (a) The media in Zambia provided information in both verbal and pictorial form of gender-based violence which seem to suggest that females are at greater risk. Describe a study you would conduct to validate or invalidate this hypothesis.

(b) In 1985, the performance of a test for antibodies to HIV-1 was evaluated. Among 74 patients known to have AIDS with unequivocal test results, 72 (97%) had detectable antibodies. Among 261 healthy blood donors with unequivocal test results, 257 (98%) had no detectable antibodies.

(i) State the sensitivity and specificity of the test.

(ii) Construct a 2×2 table showing a cross tabulation of unequivocal results versus the screening results of the 335 persons tested.

(iii) What percentage of people had a wrong test result?

(c) Eighty very young trainee cyclists were enrolled into a training program, on the first day of training they were to cycle 20km. Some gave up after cycling some distance and other could not continue because the bicycles developed a fault (these were censored). The times to giving up and censorship are given below:

Times of cycling: 3.7, 5.2* , 6.1, 4.5, 3.8, 6.5*, 8.7, 9.2
A (*) indicates that the observation was censored.

Produce a life-time table of one unit intervals (3, 4), (4, 5), and so on, showing the necessary information including the risk of-giving up.

Q3 (a) Describe any two of the following measures; Proportion, Rate or Ratio giving specific examples as they relate to epidemiology.

(b) State any three questions an investigator must ask in order to determine the sample size required in a study for testing of hypotheses.

(c) A cohort study is being designed to investigate the association between lung cancer and use of tobacco. The investigators have made an initial attempt at determining an appropriate sample size on the basis of the following specifications:

- A two-sided test with $\alpha = 0.05$, $1\text{-}\beta = 0.80$, RR = 5
- The prevalence of lung cancer among nonsmokers is thought to be $p_0 = 0.005$
- An equal number of individuals in the two study groups.

Determine the sample size

(i) For a two-sided test

(ii) For a one-sided test.

Q4 (a) Let $\widehat{ID}_1$ be the estimate of incident rate for the exposed group to some known factor and $\widehat{ID}_0$ be the incident rate for the unexposed group. Let $\widehat{AR}=(\widehat{ID} - \widehat{ID}_0)/\widehat{ID}$, where:

$$\widehat{ID}= \hat{p}\, \widehat{ID}_1+(1-\hat{p})\, \widehat{ID}_0 \text{ and } \hat{p} = \frac{L_1}{L_1+L_0}$$

(i) State what AR attempts to measure.

(ii) Show that $\widehat{AR} = \dfrac{\hat{p}(\widehat{IDR} - 1)}{\hat{p}(\widehat{IDR} - 1)+1}$

(b) Discuss briefly the benefits of knowing AR to the health of a community.

(c) Suppose that in a certain community a section of the community does not believe in modern medical treatment due to some traditional beliefs and an outbreak of a disease threatens the community. Health officials carry out a preventive measure among the subgroup that is receptive to modern treatment and the other group is left unattended to. After a certain period of follow up 400 in the community contract the disease as shown in the table below.

	The subgroup: Not receptive to modern medicine	The subgroup: Receptive to modern medicine	Total
Number contract the disease	100	300	400
Person-time	40,000	300,000	340,000

(i) Calculate the relative risk for the group not receptive to modern medicine

(ii) Interpret the relative risk in (i).

(iii) Calculate AR as define in (a)

(iv) Interpret the value in (iii).

Q5 (a) Discuss Diagnostic testing as an application of Epidemiology, also include the role of sensitivity and specificity in your discussion.

(b) Suppose that a case-control study is conducted to measure the relationship between household access to improved drinking water and diarrhea in infants (under-one) in a population with 6349 infants. The table below shows the prevailing situation in the target population.

	Diarrhea status: Yes	Diarrhea status: No	Total
No improved drinking water	1515	1222	2737
Improved drinking water	2000	1612	3612
	3515	2834	6349

(i) Obtain the prevalence Odd Ratio

(ii) Interpret the result in (i).

(c) Suppose that samples of cases and non-cases are drawn from the hospital that serves the community and that the non-cases are infants admitted with a skin rash, a condition believed to be unrelated to diarrhea. Assume the following:

- Prevalence of unsafe drinking water to be .20
- Prevalence of diarrhea to be 13% and
- Prevalence of rash to be 0.05

(i) Construct a 2×2 relating the three conditions.

(ii) Obtain the prevalence odds ratio

(iii) How does the estimate in (ii) compare with that in (b) (i)?

Q6 (a) State three sources of bias an investigator should be aware of when drawing a sample of individuals from a population for the purpose of examining the association that may exist between a disease and an exposure.

(b) Suppose that a case-control study is conducted to evaluate the interrelationships between unemployment and taking strong drink popularly known as Tujilijili. Suppose that information on employment status and use of Tujilijili is collected from 200 young people and is as shown below.

Are you unemployed ?	Do you take Tujilijili ? Yes	No	Total
Yes	56	84	140
No	10	50	60
Total	66	134	200

(i) Obtain an estimate of the Odd Ratio

(ii) Suppose that the selection probabilities are as follows:

$\hat{\alpha} = .7$ for unemployed and take Tujilijili
$\hat{\beta} = .02$ for unemployed and do not take Tujilijili
$\hat{\gamma} = .4$ for employed and take Tujilijili
$\hat{\delta} = .02$ for employed and do not take Tujilijili

Obtain an adjusted table showing what the true picture would be in the target population.

(iii) Calculate the Odds Ratio of the corrected table.
(iv) What is the direction of bias in the observed data, if at all any?

(c) Suppose that a case-control study is conducted to evaluate the interrelationships between the amount of time a student takes to study and passing an exam. Also collected was information on whether the student had passed continuous assessment or not. Below are the data that were collected.

Status on Continuous Assessment (CA)

	Passed CA					Failed CA		
	Final Examination					Final Examination		
Hours of study/week	Failed	Passed	Total		**Hours of study/week**	Failed	Passed	Total
Less than 10	60	120	180		Less than 10	125	25	150
10 or more	80	320	400		10 or more	55	25	80
Total	140	440	580		Total	180	50	230

(i) Obtain an estimate of the Odds ratio for the association of Examination performance and the number of hours of study, i.e. independent of continuous assessment.

(ii) Interpret the estimate in (i).

(iii) Obtain the Odds ratios for each level of Continuous Assessment.

(iv) How do the estimates in (iii) compare with that in (i) ?

(v) In studying the relationship between hours of study and exam performance, would continuous assessment introduce any bias?

(vi) If yes to (v), what nature of bias would it be ?

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS & STATISTICS

2011 ACADEMIC YEAR
FIRST SEMESTER FINAL EXAMINATIONS

MAT 5911 – STOCHASTIC PROCESSES

TIME ALLOWED: THREE (3) HOURS

INSTRUCTIONS: There are four questions in this question paper with each question having four parts. Answer any three parts from each question.

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1. (a) (i) Suppose X is a Poisson random variable with mean λ. The parameter λ is itself a random variable whose distribution is exponential with mean 1. Show that $P(X = n) = \frac{1}{2^{n+1}}$.

(ii) Let T be exponential random variable with mean 5. Find $E(T \mid T > 2)$.

(b) Consider a counting process $\{N(t), t \geq 0\}$, which satisfies the following conditions:

- $N(0) = 0$
- the process has stationary and independent increments.
- $P(N(h) = 1) = \lambda h + 0(h)$
- $P(N(h) \geq 2) = 0(h)$

Show that $P(N(t) = r) = \frac{e^{-\lambda t}(\lambda t)^r}{r!}$ for a non negative integer r.

(c) Let S_n denote the time of occurrence of the n^{th} event in the counting process N(t) of part b.

(i) Show that the probability density function of S_n is given by

$$f(t) = \frac{\lambda e^{-\lambda t}(\lambda t)^{n-1}}{(n-1)!}, \quad t \geq 0$$

(ii) $E(S_n) = \frac{n}{\lambda}$

(d) (i) There are three jobs and a single worker is assigned to these jobs, who works on job 1, then on job 2 and finally on job 3. The amounts of time that he spends on each job are independent exponential random variables with mean 1. Let C_i be the time at which job i is completed, $i = 1, 2, 3$. Let $X = \sum_{i=1}^{3} C_i$. Find E(X).

(ii) Suppose that the disintegration of radio active atoms in a certain quantity of radio active material follows a Poisson process with parameter λ. Suppose, in addition, that there is a probability p of each particle emitted actually being recorded, the probabilities being independent. Show that the events actually recorded follow a Poisson process with parameter λp.

2. (a) Let Y be a Binomial random variable with parameters n and p. Find the probability generating function of Y.

(b) Suppose that X(t) represents the total number of individuals in a population at time t. The chance that any individual dies in time Δt is $\mu \Delta t$. There is no birth and the individuals die independently. Let $X(0) = a$. Find $p_n(t)$ where $p_n(t)$ = Probability (X (t) = n).

(c) (i) Suppose that customers arrive at a single server service station in accordance with a Poisson process having rate λ. Each customer upon arrival, goes directly into service if the server is free and if not, the customer joins the queue. When the server finishes serving a customer, the customer leaves the system and the next customer in line, if there is any enters service. The successive service times are assumed to be independent exponential random variables having mean $\frac{1}{\mu}$. Find the equilibrium state of the queue when $\lambda < \mu$.

(ii) Suppose that customers arrive at a Poisson rate of one per every 12 minutes and that the service time is exponential at a rate of one service per 8 minutes. Find the average number of customers in the queue system in the long run.

(d) (i) Define a Brownian motion process.

(ii) Let X(t) be a standard Brownian process. For $s < t$, show that the conditional distribution of X(s) given $X(t) = B$ is normal with the following parameters

$$E\,(X(s) \mid X(t) = B\,) = \frac{s}{t}\,B$$

$$V[X(s) \mid X(t) = B\,] = \frac{s}{t}\,(t-s)$$

(iii) Let $\{ Y(t), t \geq 0 \}$ be a Brownian motion process s . t. $V(Y(t)) = 4t$. Find $P\,(Y(1) < 2 \mid Y(2) = 4)$.

3. (a) Define the following terms:

(i) A discrete time Markov Chain
(ii) A homogeneous Markov Chain
(iii) n step transition probability P_{ij}^{n} for a homogeneous Markov Chain with discrete time parameter.

(b) (i) State and prove Chapman – Kolmogorov equations.

(ii) Use the Chapman – Kolmogorov equations to show that

$$P^{(2)} = P^2 \quad where \quad P^{(2)} = \left(P_{ij}^2\right), \ P = (P_{ij})$$

(c) A Markov Chain $\{ X_n , n \geq 0 \}$ with states 0, 1, 2 has the transition probability matrix

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ 0 & 1/3 & 2/3 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

Given that $P(X_0 = 0) = \frac{1}{4}$, $P(X_0 = 1) = \frac{1}{2}$,

(i) Determine the distribution of X_3.
(ii) Find $E(X_3)$.

(d) Three molecules are distributed among two urns. At each time step, one molecule is chosen at random and is removed from its urn and placed in the other one. Let X_n denote the number of molecules in the first urn immediately after the n units of time have passed.

(i) Find the transition probability matrix of the Markov Chain
$\{ X_n , \ n = 0, 1, 2, \dots \}$

(ii) Find the proportion of time, the first urn will be empty.

4. For a M.C. $\{ X_n , n = 0, 1, 2, \dots \}$

(a) Define the following:

(i) State j is accessible from state i
(ii) State i and j communicate
(iii) a recurrent state
(iv) an irreducible chain.

(b) (i) Prove that if state i is recurrent and state i and state j communicate then state j is also recurrent.

(ii) Show that if state i is recurrent and states i and j do not communicate then $P_{ij} = 0$.

(iii) A Markov Chain consisting of the states 0, 1, 2 and 3 has transition probabilities matrix

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & \frac{3}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Show that state E_2 is recurrent. Classify the states of the chain.

(c) (i) Define a two dimensional symmetric random walk.

(ii) Show that all the states in a two dimensional symmetric random walk are recurrent.

(d) (i) Define a branching process.

(ii) Let X_n denote the size of the n^{th} generation. Assuming $X_0 = 1$ and μ describe the mean number of off-springs of a single individual, show that $E(X_n) = \mu^n$.

(iii) Show that the population becomes extinct in the long run for $\mu < 1$.

END OF EXAMINATION.

THE UNIVERSITY OF ZAMBIA

UNIVERSITY SECONF SEMESTER EXAMINATIONS 2011/12

MCN 5132: COMMUNICATION OF INNOVATIONS

TIME: THREE HOURS

INSTRUCTIONS: ANSWER THE QUESTION IN **SECTION A**, AND THEN ANY THREE FROM **SECTION B**.

SECTION A

BEHAVIOURAL CHANGE (FICTITIOUS)

1. The University of Balunda is located in a small town in Muchinga Province where one of the major student activities on the weekend is numerous parties involving alcohol consumption. One popular game among students, for example, is Beer Pong, which often leads to Binge drinking. Campus and Zambia Police Service report a major rise in "disturbance" complaints, and arrests for public drunkenness are up 150 percent.

   Something must be done, so the University Vice Chancellor asks your graduating Communication for Development class to come up with a 'Social Behavioural Change' campaign programme that would (1) inform and educate students about the dangers of binge drinking, (2) convince students to drink more responsibly, and (3) actually lower the number of arrests for public drunkenness.

   State all you will do in coming up with the desired programme, including the factors to be considered, the kind of messages to be generated, and the communication strategies and tactics to be employed. You should be creative and use a variety of tactics to accomplish your objectives. (40 Marks)

SECTION B

2. Discuss how you can apply *item response models* to improve psychometric methods in health education and health behaviour research practice. (20 Marks)

3. Social change scholars have stated "Individual perception of the five characteristics of an innovation predicts the rate of adoption of innovations." What are these characteristics, and what does this sentiment mean? (20 Marks)

4. According the Generation of Innovations, innovation can be seen as the process that renews something that exists and not, as is commonly assumed, the introduction of something new." What does this mean? (20 Marks)

5. In the contributions and criticisms of the diffusion approach, it is argued that the central point is that change agents, given their accountability to all citizens, have a responsibility to address negative consequences. How can they do this to ensure they engineer society in a favourable manner? (20 Marks)

6. The Diffusion of Innovation Process Model can be viewed as the adoption part of the Diffusion of Innovations Model by Everett Rogers. Discuss the validity of this statement. (20 Marks)

End of Examination

UNIVERSITY OF ZAMBIA

DEPARTMENT OF PHYSICS

2013 FIRST SEMESTER UNIVERSITY EXAMINATIONS

MP415

MATHEMATICAL METHODS FOR PHYSICS

DURATION: Three hours.

INSTRUCTIONS: Answer any four questions from the six given. *Each question carries 25 marks with the marks for parts of questions indicated.*

MAXIMUM MARKS: 100

DATE: Monday 25th February 2013.

Formulae that may be needed:

1.
$$u_x = v_y, \qquad u_y = -v_x$$

2.
$$f^{(n)}(z_0) = \frac{n!}{2\pi i} \oint_C \frac{f(z)}{(z - z_0)^{n+1}}\, dz, \qquad (n = 1, 2, \ldots).$$

3.
$$\sum_{m=0}^{\infty} q^m = 1 + q + q^2 + \ldots = \frac{1}{1-q}, \qquad |q| < 1.$$

4.
$$\left|\frac{z_{n+1}}{z_n}\right| \leq q < 1, \quad \text{for } n \text{ greater than some } N.$$

5.
$$\lim_{n\to\infty} \left|\frac{z_{n+1}}{z_n}\right| = L.$$

6.
$$\sqrt[n]{|z_n|} \leq q < 1, \quad \text{for } n \text{ greater than some } N.$$

7.
$$\lim_{n\to\infty} \sqrt[n]{|z_n|} = L.$$

8.

$$
\begin{aligned}
R &= \frac{1}{L^*}, \qquad L^* = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| \\
R &= \frac{1}{\tilde{L}}, \qquad \tilde{L} = \lim_{n\to\infty}\sqrt[n]{|a_n|} \\
R &= \frac{1}{\tilde{l}}, \qquad \tilde{l} = \text{largest limit of } \lim_{n\to\infty}\sqrt[n]{|a_n|}
\end{aligned}
$$

9.

$$(\cosh z)' = \sinh z, \qquad (\sinh z)' = \cosh z.$$

10.

$$\operatorname*{Res}_{z=z_0} f(z) = \lim_{z\to z_0}(z-z_0)f(z).$$

11.

$$\operatorname*{Res}_{z=z_0} \frac{p(z)}{q(z)} = \frac{p(z_0)}{q'(z_0)}.$$

12.

$$\operatorname*{Res}_{z=z_0} f(z) = \frac{1}{(m-1)!}\lim_{z\to z_0}\left\{\frac{d^{m-1}}{dz^{m-1}}[(z-z_0)^m f(z)]\right\}.$$

13.

$$\oint_C f(z)\,dz = 2\pi i\sum_{j=1}^{k}\operatorname*{Res}_{z=z_j} f(z).$$

14. Improper integrals of rational trigonometric functions of $\sin\theta$ and $\cos\theta$ (integration taken counterclockwise)

$$\int_C f(z)\,\frac{dz}{iz} = 2\pi i\sum_{j=1}^{k}\operatorname*{Res}_{z=z_j}\left[\frac{f(z)}{iz}\right], \qquad C:\ |z|=1$$

where $f(z)$ is obtained from $f(\cos\theta, \sin\theta)$ by the substitutions

$$\cos\theta = \frac{1}{2}\left(z+\frac{1}{z}\right), \qquad \sin\theta = \frac{1}{2i}\left(z-\frac{1}{z}\right).$$

15. Improper integrals of rational functions:

$$\int_{-\infty}^{\infty} f(x)\,dx = 2\pi i\sum \mathrm{Res} f(z).$$

16.

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$$

17.

$$y = \sum_{m=0}^{\infty} a_m x^m.$$

18.

$$\cosh iz = \cos z, \quad \sinh iz = i\sin z, \quad \cos iz = \cosh z, \quad \sin iz = i\sinh z$$

QUESTION 1

(a) Give the definition for the continuity of a complex function at a point z_0.

(4 marks)

(b) Sketch the region of the complex plane represented by

$$\frac{\pi}{3} \leq \arg z \leq \frac{\pi}{2}$$

(4 marks)

(c) Use the definition of complex differentiation to find $\frac{df(z)}{dz}$ for $f(z) = (1+z)/z$.

(8 marks)

(d) Show that the function $u(x,y) = 4xy - 3x + 2$ is harmonic. Find the harmonic conjugate $v(x,y)$ of $u(x,y)$ and hence construct the corresponding analytic function.

(9 marks)

QUESTION 2

(a) Use an appropriate theorem to integrate the following function:

$$\oint_C e^z \, dz.$$

State the theorem used. (4 marks)

(b) Integrate the following integral by the method of path:

$$\oint_C (5z+2) \, dz,$$

where the contour C is the line from $-i+1$ to $3i+2$. Include a sketch of the integration path. (12 marks)

(c) Evaluate

$$\oint_C \frac{5z^2 - 3z + 2}{(z-1)^3} \, dz, \qquad C: \; |z-1| = 3,$$

using the derivative formula for integration. (9 marks)

QUESTION 3

(a) Determine if the following series is convergent or divergent:

$$S = \sum_{n=1}^{\infty} \frac{n^{2n} + i^{2n}}{n!}.$$

(4 marks)

(b) Find the center and radius of convergence of the series

$$S = \sum_{n=2}^{\infty} \left(\frac{3n^2 + n}{n^2 - 1} \right) (z + i)^n.$$

(6 marks)

(c) (i) Find all the Laurent series of $1/(z^2 - z^3)$ with center 0. (5 marks)

(ii) Find the Laurent series of $1/(z^2 - 1)$ with center z=-1. (7 marks)

(d) State the theorem that relates zeroes to poles. (3 marks)

QUESTION 4

Use residues to integrate

$$\int_0^{2\pi} \frac{\cos\theta}{1 - 2a\cos\theta + a^2}\, d\theta, \qquad 0 < a < 1.$$

Hint: $[az^2 - (a^2 + 1) + a] = (az - a^2)(z - \frac{1}{a})$ (25 marks)

QUESTION 5

(a) Find the inverse of the matrix

$$M = \begin{bmatrix} 3 & -2 & 2 \\ 1 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix}$$

by Gauss-Jordan elimination. (12 marks)

(b) Find the eigenvalues and eigenvectors of the matrix

$$N = \begin{bmatrix} 2 & 2 \\ -1 & 5 \end{bmatrix}.$$

(8 marks)

(c) Diagonalize the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

given the matrix X, whose columns are the eigenvectors of the matrix A, and its inverse X^{-1}

$$X = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 3 & -1 \\ 1 & 1 & 1 \end{bmatrix}, \quad X^{-1} = -\frac{1}{6} \begin{bmatrix} 4 & 1 & -7 \\ -2 & -2 & 2 \\ -2 & 1 & -1 \end{bmatrix}.$$

What is the significance of the diagonal elements? (5 marks)

QUESTION 6

Use the power series method to solve Hermite's differential equation

$$y'' - 2xy' + 2\alpha y = 0.$$

Calculate enough coefficients to enable you to write general formulae for the coefficients of the series solution $y(x)$. You must write the final solution using these general formulae for the coefficients. (25 marks)

END

The University of Zambia
School of Natural Sciences
Department of Physics
2012/13 Academic Year First Semester
Final Examinations
P-191: Introductory Physics - I

Question 1 is compulsory. All questions carry equal marks. The marks are shown in brackets. Attempt four more questions. Clearly indicate on the answer script in the left column of the cover page which questions you have attempted.

Time: Three hours. **Maximum marks = 100.**

Do not forget to write your computer number clearly on the answer book as well as on the answer sheet for Question 1. Tie them together!!

===

Wherever necessary use:

$g = 9.8 \text{m/s}^2$

$P_A = 1.01 \times 10^5 \text{ N/m}^2$

1 cal. = 4.18 J

$\rho_{\text{water}} = 1000 \text{kg/m}^3$

1 hp = 746W

1 Pascal =1 N/m^2

$G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$

1 metric ton = 1000 kg

Question 1: Sample answers: F (a), G (d).... etc. **DO NOT <u>guess</u>** the answer. For <u>each</u> correct answer, **2 marks** are given. For <u>each</u> wrong answer, **0.67** will be deducted. For no answer, zero mark. The minimum total mark for Question 1 is zero. [10 × 2 = 20]

(A) An aeroplane requires 20 s of uniform acceleration and 400 m of runway to become airborne, starting from rest. Its velocity when it leaves the ground is:

a) 80 m/s
b) 32 m/s
c) 20 m/s
d) 40 m/s

(B) An aeroplane travels 100 km to the north and then 200 km to the east. The displacement of the aeroplane from its starting point is approximately:

(a) 300 km
(b) 220 km
(c) 100 km
(d) 200 km

(C) What never changes when two or more objects collide?

(a) the total kinetic energy of all the objects
(b) the momentum of each one
(c) the kinetic energy of each one
(d) the total momentum of all the objects

(D) The only modulus that applies to liquids is:

(a) modulus of rigidity
(b) bulk modulus
(c) Young's modulus
(d) Shear modulus

(E) A ball is thrown is with a speed of 8.0 ms^{-1} at an angle of 40° below the horizontal. After 0.4 s the horizontal component of its velocity will be:

(a) 6.1 ms^{-1}
(b) 10.4 ms^{-1}
(c) 6.7 ms^{-1}
(d) 5.1 ms-1

(F) A 40 kg girl runs up a staircase to a floor 5.0 m higher in 7.0 s. Her power output is:

(a) 14 kW
(b) 29 W
(c) 1.4 kW
(d) 0.28 kW

(G) A motor takes 8.0 s to go from 60 rad/s to 140 rad/s at constant angular acceleration. The total angle through which its shaft turns during this time is:

(a) 800 rad
(b) 320 rad
(c) 1600 rad
(d) 640 rad

(H) A ring and a disc of the same mass and radius roll down an inclined plane. At the bottom they have the same:

(a) gravitational potential
(b) angular momentum
(c) angular speed
(d) KE of rotation

(I) In an equilibrium problem the point at which torques are calculated:

(a) must pass through the object's centre of gravity
(b) must pass through one end of the object
(c) may be located anywhere
(d) must intersect the line of action of at least one force acting on the object

(J) A car pulling a trailer is accelerating on a level road. The force the car exerts on the trailer is:

(a) equal to the force the road exerts on the trailer
(b) equal to the force the trailer exerts on the road
(c) equal to the force the trailer exerts on the car
(d) greater than the force the trailer exerts on the car

ATTEMPT ANY FOUR QUESTIONS FROM BELOW:

Q.2 (a) A football is kicked at an angle of 60 degrees above the horizontal. Show qualitatively on a diagram how the trajectories of the football would look, if:

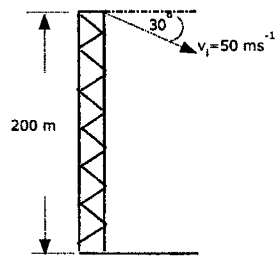
(i) there were no gravitational force **[2]**
(ii) there was normal gravitational force **[2]**
(iii) there was stronger gravitational force than normal **[2]**
(iv) the gravitational force was acting vertically upwards (and not downwards) **[2]**

(b) A bullet is fired from the top of a 200 m high tower at an angle of 30° below the horizontal with a speed of 50 ms^{-1}.

i) Find the time the bullet takes to hit the ground. **[4]**

ii) What is the direct distance between the top of the tower and the point of impact on the ground? **[4]**

iii) Find the speed of the bullet when it hits the ground. **[4]**

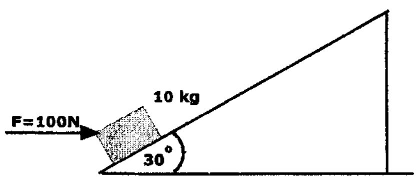


Q.3 A block of mass 10 kg on a 30° inclined plane is being pushed by a horizontal force *F* as shown in the figure below. Find:

i) the time it takes to move the block up the incline by a distance of 2 m, if the block starts from rest and the surface of the inclined plane is smooth and *F* = 100 N. **[5]**

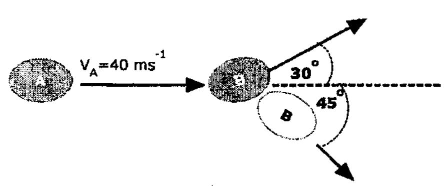
ii) the horizontal force needed to move the block 2 m up the incline in a time of 1 s, if the surface is rough with coefficient of kinetic friction of 0.2 between it and the block and the block starts from rest. **[8]**

iii) the time the block will take to reach a speed of 5 ms^{-1} starting from rest at the top when the force *F* is removed and the surface is smooth. **[7]**



Q.4 (a) Two displacements **A** and **B** lie in the x-y plane. Vector **A** is 49 cm in magnitude at θ = 42° with the *x*-axis and vector **B** is 32 cm in magnitude at θ = 115° with the *x*-axis. What is the difference **B-A**? **[9]**

(b) Two asteroids of equal mass in the asteroid belt between Mars and Jupiter collide with a glancing blow. Asteroid *A*, which is initially travelling at 40 ms^{-1}, is deflected 30° from its original direction, while asteroid *B*, which is initially at rest, travels at 45° to the original direction of *A* as shown in the figure below. Find the speed of each asteroid after the collision.



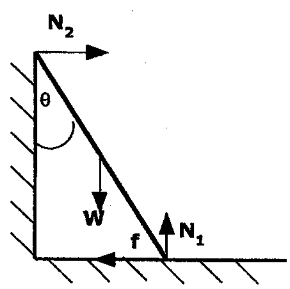
Q.5 (a) Starting from rest, a hollow sphere rolls down a frictionless inclined plane 5 m long without slipping. If it reaches the bottom 4 s later, determine the angle the plane makes with the horizontal. Take the moment of inertia of a hollow sphere to be $I = \frac{2}{3} mr^2$. **[10]**

(b) A hockey player is standing on a frozen pond when an opposing player skates past with a uniform speed of 12 ms^{-1}. After 3 seconds the stationary player makes up his mind to chase his opponent. If he accelerates uniformly at 4 ms^{-2}:

i) how long does it take him to catch his opponent, and
ii) how far has he travelled in that time? (assume player motion at constant speed) **[10]**

Q.6 (a) A uniform ladder of length 4 m and weight 100 N rests on a rough floor and against a frictionless wall as shown. The coefficient of static friction at the floor is 0.6.

i) Find the maximum angle θ to the wall such that the ladder does not slip, and
ii) the force exerted by the wall at this angle (θ). **[13]**



(b) A coin is placed 11 cm from the axis of a rotating disc of variable speed. When the speed of the disc is slowly increased, the coin remains fixed on the disc until a speed of 36 rev/min is reached and the coin slides off. What is the coefficient of static friction between the coin and the disc? **[5]**

(c) Define Young's modulus. **[2]**

Q.7 (a) A stone is thrown vertically upward with a speed of 18 ms^{-1}.

i) How fast is it moving when it reaches a height of 11 m?
ii) How long does it take to reach this height?
iii) Why are there two answers to part (ii) of this question? **[10]**

(b) A composite wire of 3 mm uniform diameter consisting of a copper wire of length 2.2 m and a steel wire of length 1.6 m stretches under a load by 0.700 mm. Calculate the load. Take copper Young's modulus for copper to be 1.1×10^{11} Nm^{-2} and that of steel to be 2.0×10^{11} Nm^{-2}. **[10]**

Q.8 (a) State Archimedes' principle. **[3]**

(b) What volume *V* of helium gas is needed if a balloon is to lift a load of 200 kg (including the weight of the balloon)? Take the density of air as 1.29 kg/m^3 and that of helium ss 0.179 kg/m^3. **[5]**

(c) A satellite of mass 5500 kg orbits the Earth (m_e=6.0 x 10^{24} kg and radius 6.38 x 10^6) and has a period of 6200 s. Find:

i) the magnitude of the Earth's gravitational force on the satellite, and
ii) the altitude of the satellite from the surface of the Earth **[12]**

END OF EXAMINATION

Equations

Uniformly accelerated motion:

$x = \bar{v}t$ $\quad \bar{v} = \frac{1}{2}(v_f + v_i)$ $\quad v_f = v_i + at$ $\quad v_f^2 = v_i^2 + 2ax$

$x = v_i t + \frac{1}{2}t^2$

Projectile motion:

$v_x = v_i \cos\theta_i$ = constant $\quad v_y = v_i \sin\theta_i - gt$ $\quad y = (v_i \sin\theta_i)t - \frac{1}{2}gt^2$

$y = (\tan\theta_i)x - \left[\frac{g}{2v_i^2(\cos^2\theta_i)}\right]x^2$ $\quad R = \frac{v^2}{g}sin2\theta$ $\quad t = \frac{2v_i \sin\theta}{g}$

Force and motion:

$F = ma$ $\quad w = mg$ $\quad F_{AB} = -F_{BA}$ $\quad F_f = \mu F_N$

Energy & Power:

$PE = wh = mgh$ $\quad KE = \frac{1}{2}mv^2$ $\quad W = Fx\cos\theta$ $\quad P = \frac{W}{t} = Fv\cos\theta$

Linear momentum:

$p = mv$ $\quad F\Delta t = \Delta(mv)$

Circular motion and gravitation:

$T = \frac{2\pi r}{v}$ $\quad a_c = \frac{v^2}{r}$ $\quad F_c = \frac{mv^2}{r}$ $\quad F_{grav} = G\frac{m_A m_B}{r^2}$

Rotational motion and angular momentum:

$\theta = \frac{s}{r} = \left(\frac{\omega_i + \omega_f}{2}\right)t$ $\quad \omega = \frac{\theta}{t}$ $\quad \theta = \omega_i t + \frac{1}{2}\alpha t^2$ $\quad \omega_f = \omega_i + \alpha t$

$v = \omega r$ $\quad \omega_f^2 = \omega_i^2 + 2\alpha\theta$ $\quad \alpha = \frac{\Delta\omega}{\Delta t} = \frac{a_T}{r}$ $\quad I = \sum mr^2$

$KE_{rot} = \frac{1}{2}I\omega^2$ $\quad \tau = FL = I\alpha$ $\quad W = \tau\theta$ $\quad P = \tau\omega$ $\quad L = I\omega$

Properties of matter:

$\rho = \frac{m}{V}$ $\quad F = -kx$ $\quad \frac{\Delta L}{L_i} = \frac{1}{Y}\frac{F}{A}$ $\quad \phi = \frac{s}{d} = \frac{1}{s}\frac{F}{A}$

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF PHYSICS

2012/13 ACADEMIC YEAR FIRST SEMESTER FINAL EXAMINATIONS

P231: PROPERTIES OF MATTER AND THERMAL PHYSICS

TIME: THREE HOURS MAXIMUM MARKS – 100

Attempt any five questions.
All questions carry equal marks.
The marks are shown in brackets.

Q1. (a) Define Poisson's ratio. [2 Marks]

(b) The moduli are not independent since any possible change in the size and shape of the body may be obtained by first changing the size, but not the shape (i.e. by a volume strain) and then changing the shape without altering the size (by means of shears). Complete the table given below and find a relationship between Young's modulus and bulk modulus. [10 Marks]

Table of stresses and resulting strains

Stresses applied along			**Strains produced along**			
x	y	z	x	y	z	
-P	0	0				
0	-P	0				
0	0	-P				
-P	-P	-P				Total volume strain $\frac{3P}{Y}(1-2\sigma)$
-P	0	0				
0	+P	0				

(c) (i) Modulus of rigidity and Poisson's ratio of the material of a wire are 2.87 x $10^{10}\frac{N}{m^2}$ and 0.379 respectively. Find the value of Young's modulus of the material of the wire. [4 Marks]

(ii) A steel wire 8m long and 4mm in diameter is fixed to two rigid supports. The coefficients of linear expansion and Young's modulus of steel are 12 x $10^{-6}/^0C$ and 2 x $10^{11}\ \frac{N}{m^2}$ respectively. Calculate the increase in tension when the temperature falls by 10^0C. [4 Marks]

Q2. (a) Define flexural rigidity of a beam. [5 Marks]

(b) Show that the expression for the period of oscillation of a cantilever with the maximum depression δ is given by

$$\delta = \frac{WL^3}{3YAk^2} \text{ where}$$

W is the load
δ is the depression
L is the length of the cantilever
Y is Young's modulus for the material of the cantilever
A is the cross-sectional area of the beam
k is the radius of gyration. [5 Marks]

(c) Show that excess pressure on the concave side of a curved membrane in equilibrium is given by

$$P_1 - P_2 = 2\Upsilon\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \text{ Where}$$ [10 Marks]

P_1 is the pressure on the concave side of the membrane
P_2 is the pressure on the convex side of the membrane
r_1 and r_2 are radii of curvature of the curved surfaces of the membrane.

Q3. (a) Define shearing strain. [2 Marks]

(b) A sphere of mass 400 g and radius of 1.2 cm is suspended from a wire of length 80 cm and radius 0.4 mm. If the period of torsional oscillations of this system is 1.21 s, calculate the modulus of rigidity of the wire given that moment of inertia of a sphere is $\frac{2}{5}mr^2$ where m is its mass and r is the radius. [8 Marks]

(c) Show that the deflection δ at the free end of a uniform loaded cantilever of length, l, clamped horizontally at one end is given by:

$$\delta = \frac{(W+\frac{3}{8}W1)l^3}{3YIg} \quad \text{where}$$

W is the load
W_1 is the weight of the cantilever
l is the length of the cantilever,
Y is Young's modulus for the material of the cantilever. [10 Marks]

Q4. (a) Define Modulus of rigidity. [2 Marks]

(b) Describe the statical method for determining the modulus of rigidity of a wire. [10 Marks]

(c) Calculate the work done in twisting a steel wire of radius 0.5 mm and length 12.5 cm through an angle of 30^0 given that the modulus of rigidity of the material of the steel wire is $\eta = 8 \times 10^{10}$ N/m^2 [8 Marks]

Q5. (a) Define strain rate. [5 Marks]

(b) Derive Poiseuille's formula for the flow of a liquid through a narrow capillary tube. [10 Marks]

(c) A spherical glass ball of mass 1.34×10^{-4} kg and diameter 4.4×10^{-3} m takes 6.4 s to fall steadily through a height of 0.381 m inside a large volume of oil of specific gravity 0.943. Calculate the coefficient of viscosity of the oil. [5 Marks]

Q6. (a) Show that using postulates of kinetic theory of gases, kinetic pressure is given by

$$P=\frac{Nm\overline{C^2}}{V}$$

where N is the number of molecules
m is mass of each molecule
$\overline{C^2}$ is the mean square velocity of the molecule
V is the volume of the cube

[10 Marks]

(b) Calculate the root mean square speed of the molecules of hydrogen at 273K. The density of hydrogen at s.t.p is $9.00 \times 10^{-2}\ \frac{kg}{m^3}$ and one standard atmosphere is 1.01×10^5 Pa. [5 Marks]

(c) Calculate the two principal molar heat capacities of oxygen if their ratio is 1.40. The density of oxygen at s.t.p is $1.43\ \frac{kg}{m^3}$ and one standard atmosphere is 1.01×10^5 Pa and the molar mass of oxygen is $32.00 \times 10^{-3}\ \frac{kg}{mol}$. [5 Marks]

Q7. (a) Show that the equation that determines the change of entropy of a gas from the knowledge of temperature and volumes is given by

$$S_2 - S_1 = C_p \ln\frac{V_2}{V_1} + C_v \ln\frac{T_2}{T_1}\frac{V_1}{V_2} \quad \text{where}$$

$S_2 - S_1$ the change of entropy,
T_1, T_2, V_1, V_2, are the state variables,
C_p the specific heat capacity at constant volume,
C_v is the specific heat capacity at constant volume
R is the universal gas constant

[8 marks]

(b) A gas whose original pressure and temperature were $3.0 \times 10^5\ \frac{N}{m^2}$ and 25° C respectively is compressed according to the law

$$PV^{1.4} = C$$

until its temperature becomes 180° C. Determine the new pressure of the gas.

[6 Marks]

(c) Show that the surface tension, γ, for liquids such as water, alcohol and chloroform whose contact angles are usually assumed zero is given as

$$\gamma = \frac{\rho g r}{2}\left(h + \frac{r}{3}\right) \text{ where}$$

ρ is the density of the liquid
g is the acceleration due to gravity
r is the radius of the capillary tube
h is the height of the capillary ascent

[6 Marks]

The University of Zambia
Department of Physics
FINAL EXAMINATION
2012/13 Academic Year
P341: Introduction to Analogue Electronics

INSTRUCTIONS: Answer four (4) questions only. All questions are of equal marks. The marks are shown in square brackets

MAXIMUM MARKS

DURATION: 3hrs

Some formulas you may find useful:

$I_C = \beta I_B$; $I_E = I_C + I_B$ $I = \left[\frac{V_S - V_Z}{R_S}\right]$; $I_L = \frac{V_Z}{R_L}$

$N = \frac{\rho}{A} N_A$; $\sigma_i = en_i(\mu_n + \mu_h)$; $\sigma_n = eN_d\mu_e$;$\sigma_p = eN_a\mu_h$ $\rho = \frac{1}{\sigma}$; $z = a + jb$;

$A' = \frac{A}{1+\beta A}$;

Q1 (a) State both Thevenin and Norton's theorems clearly explaining the important terms. [6]

(b) Using Thevenin's theorem, find the current in the 15Ω resistor, figure 1. [9]

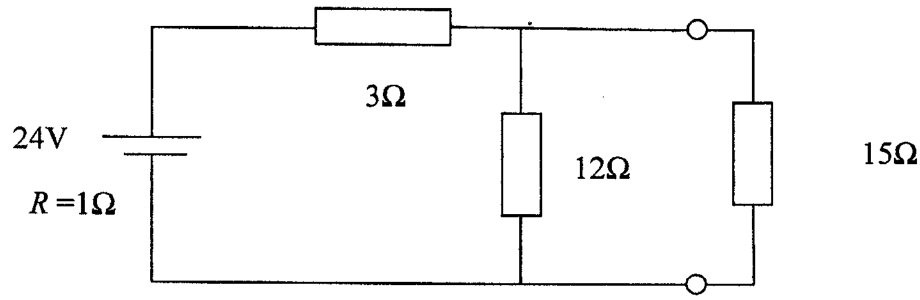


Figure 1

(c) Apply Norton's theorem to calculate the current flowing through the 5Ω Resistor of figure 2. Show all the steps in the calculation. [10]

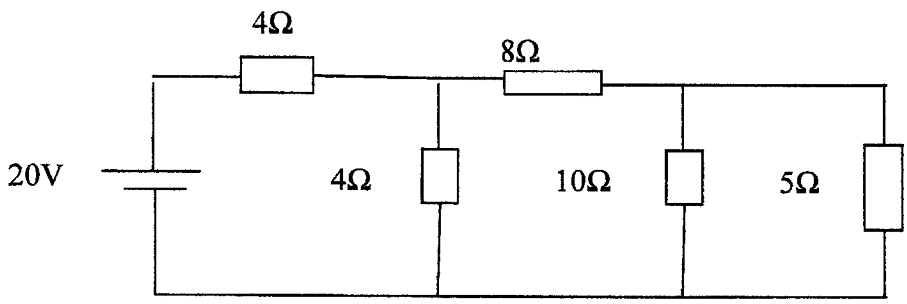


Figure 2

Q2 (a) (i) State the Maximum Power Transfer theorem. [3]

(ii) With reference to figure 3, give an account of the mathematical proof of the theorem mentioned in (i) above. [10]

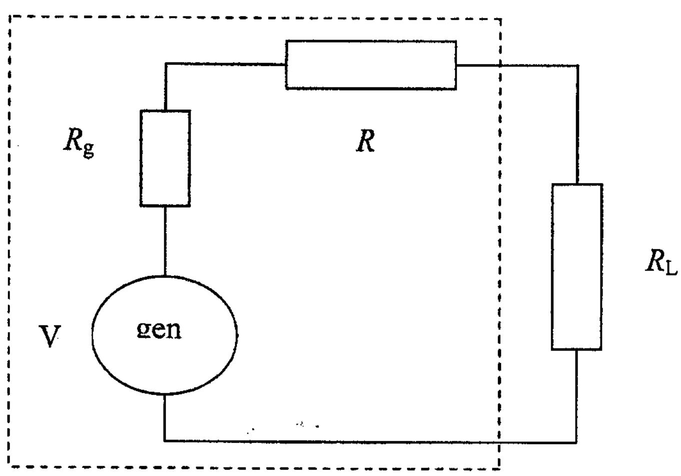


Figure 3

(c) (i) Write short notes on "doping" as understood in semiconductor physics. Hence distinguish between "acceptor" and "donor" impurities clearly giving an example of each. [4]

(ii) Compute the relative concentrations of silicon atoms and electron-hole pairs at 300K. Hence calculate the resistivity, σ of silicon.

Data: Avogadro's number $N_A = 6.02 \times 10^{23}$ atoms/g-atom

Atomic weight (Si) $= 28.1$

Density (Si) $= 2.33 \times 10^6$ gm^{-3}

Intrinsic carrier density (Si) = 1.5×10^{16} m^{-3}.

Electron mobility, $\mu_e = 0.14$ $m^2/V\text{-}s$

Hole mobility, $\mu_h = 0.05$ $m^2/V\text{-}s$. [6]

Q3 (a) Find the main current and the current in each branch of the circuit. All impedances and resistances are in ohms. [9]

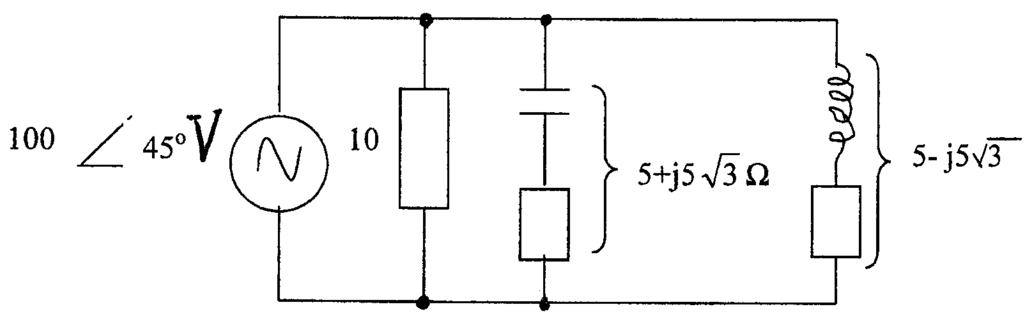


Figure 4

(b) Draw the I-V characteristics of the Zener diode. Indicate on your graph: the Zener voltage, V_Z, minimum Zener current, I_{Zmin} and I_{Zmax}. [6]

(c) Calculate the battery current I, Zener current I_Z, and the load current, I_L in figure 5. How will these values be affected if the source voltage increases to 70V? Neglect Zener resistance. [10]

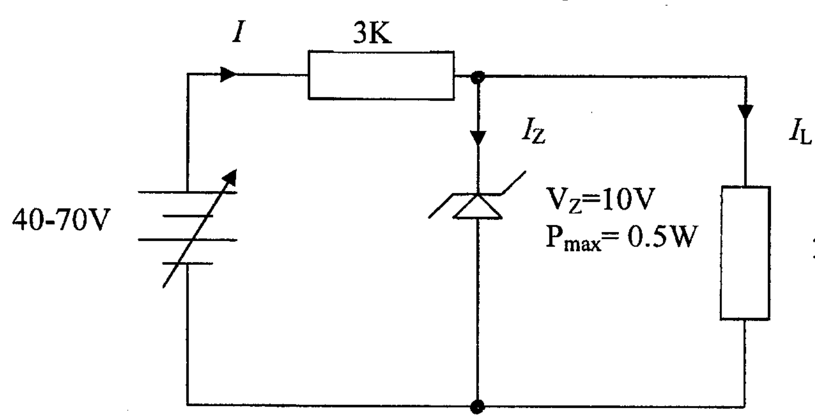


Figure 5

Q4 (a) (i) What are a.c. filters? Distinguish clearly between **high pass** and **low pass** filters. [4]

(ii) Draw a well labelled diagram of a **low-pass** filter using a resistor, R and an inductor, L. Hence draw a corresponding diagram of the filter's frequency response clearly indicating the cut-off frequency. [6]

(iii) Show that the transfer function T for the low pass filter cited above is given by [4]

$$T = \frac{V_o}{V_i} = \frac{1}{\sqrt{1+\left(\frac{\omega L}{R}\right)^2}},$$

Where V_o and V_i are the output and input voltages to the filter.

(b) An a.c. signal that has a constant amplitude of 10V but variable frequency is applied across a low pass (RL) filter circuit with a cut-off frequency of 1 kHz. Calculate

(i) the value of L given that $R = 1\,\text{k}\Omega$ [2]

(ii) the output voltage when $f = 10 f_c$ [3]

(c) (i) Show that the average value of a sinusoidal current integrated over half a cycle is given by $I_{av} = \frac{2I_m}{\pi}$, where I_m is the maximum value of the current. [3]

(ii) Repeat the above exercise and show that the same current integrated over a complete cycle gives zero. [3]

Q5 (a) A silicon *pnp* transistor is connected in the circuit shown in figure 6.

(i) Complete the drawing by putting the arrow-head on the emitter symbol and indicating the proper polarities for the batteries including directions of base, emitter and collector currents. [4]

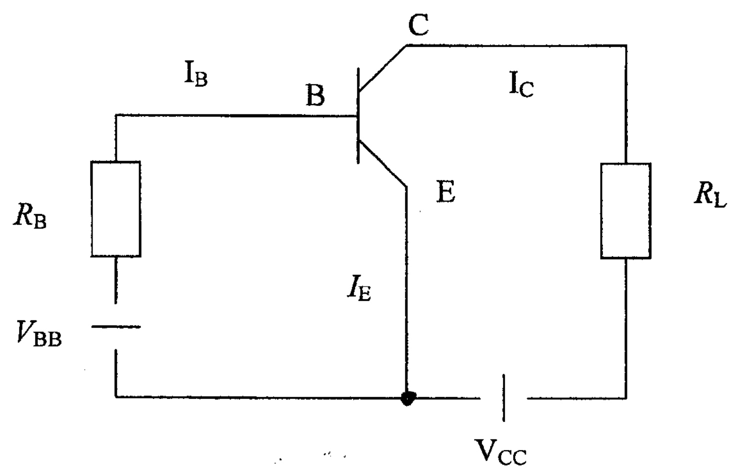


Figure 6

(ii) Assuming $|V_{BE}| = 0.7\,\text{V}$, calculate I_B for $V_{BB} = 4\text{V}$ and $R_B = 33\text{K}\Omega$. [2]

(iii) If $V_{CC} = 15\text{V}$, $R_L = 1.5\text{K}\Omega$, and $\beta = 60$, calculate values for I_C, I_E and V_{CE}. [4]

(b) For the circuit in figure 7, find

(i) collector saturation current, I_{Csat} [2]

(ii) collector current, I_C [2]

(iii) V_{CE} [3]

(iv) I_B [3]

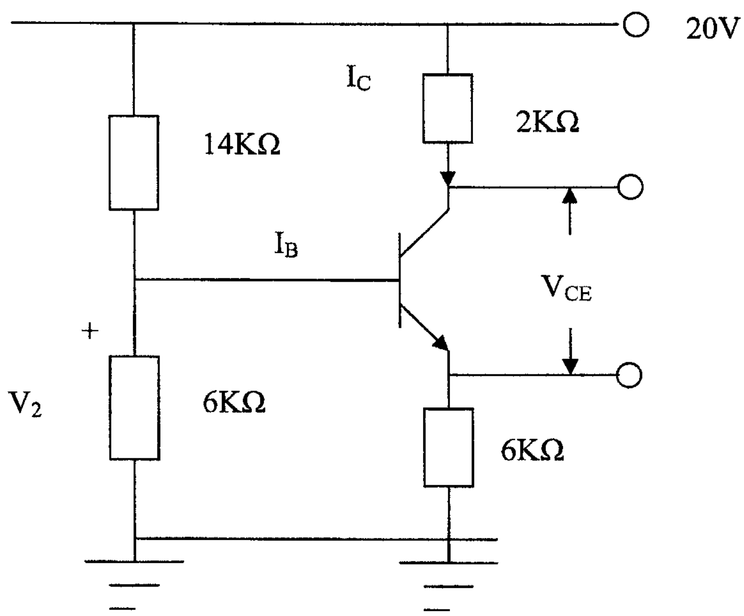


Figure 7

(c) A JFET is connected as in figure 8.

(i) Complete the diagram by indicating the proper polarities for V_{GG} and V_{DD}. What kind of channel FET is it? [4]

(ii) For $V_{GG} = -3\,\text{V}$ and $V_{DD} = 20\,\text{V}$, predict the drain current I_D and calculate the value of V_{DS}. Use the output characteristics in figure 9. [4]

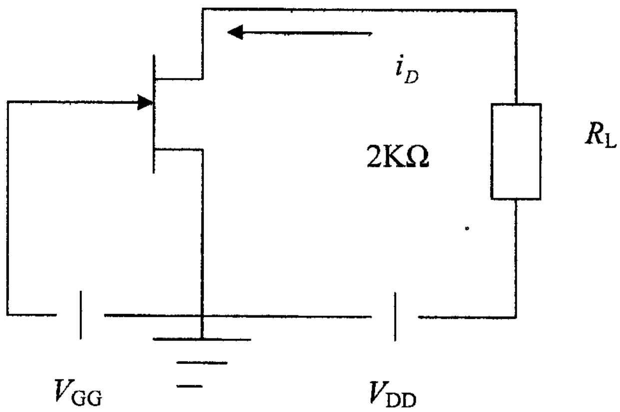


Figure 8

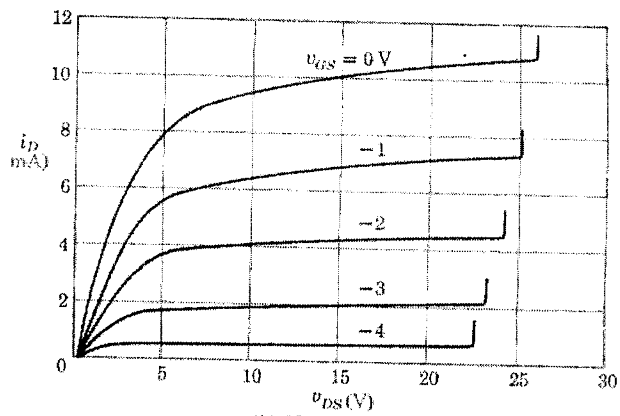


Figure 9

Q6 (a) (i) State the reasons why the JFET is called a "**unipolar**" transistor while BJT is called a "**bipolar**" junction transistor. [3]

(ii) State four differences between the JFET and the BJT [4]

(b) Draw a well labelled block diagram of a *p-channel JFET* and include its circuit symbol and sketches of its output characteristics. [8]

(c) (i) Differentiate between" **positive feed-back**" and "**negative feed-back**" in amplifiers. Mention **four** features that make negative-feedback amplifiers more preferred to positive amplifiers. [6]

(ii) For the negative feed-back amplifier shown below in figure 10 find

- The open loop gain
- The closed loop gain [4]

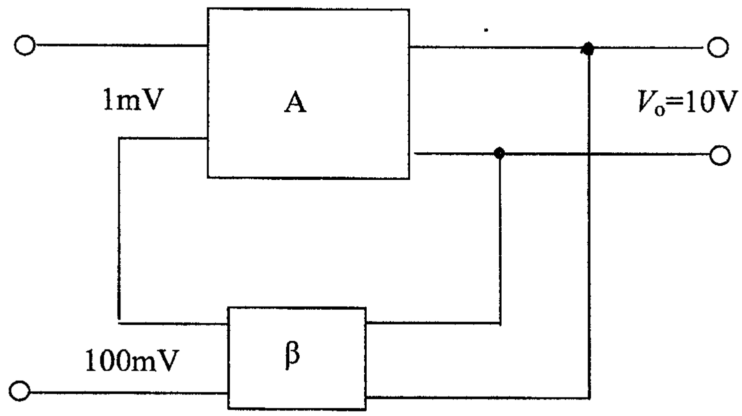


Figure 10

END OF EXAMINATION

The University of Zambia

Department of Physics

University Examinations

First Semester 2012/13

P361

Electromagnetic Theory and Waves

Duration: Three (3) Hours

Total Marks: 100

Instructions

- Answer any four (4) questions.
- Show all your working clearly. Omission of essential work will result in loss of marks.
- All questions carry equal marks. Marks allocated for each question are indicated in square brackets [ ].
- The first two pages contain vector definitions, identities and theorems that you may find useful.

Physical Constants

- Permittivity of free space, $\varepsilon_0 = 8.854\times10^{-12}\,\mathrm{Fm^{-1}}$
- Permeability of free space, $\mu_0 = 4\pi\times10^{-7}\,\mathrm{Hm^{-1}}$
- Elementary charge, $e \approx 1.602\times10^{-19}\,\mathrm{C}$
- Speed of light in vacuo, $3\times10^{8}\,\mathrm{ms^{-2}}$
- Coulomb's constant, $k = \dfrac{1}{4\pi\varepsilon_0} = 9\times10^{9}\,\mathrm{Nm^2.C^{-2}}$

Formulae You May Find Useful

$\vec{\mathbf{F}} = \hat{\mathbf{r}}\frac{1}{4\pi\varepsilon_o}\frac{qQ}{r^2}$	$\vec{\mathbf{E}} = \hat{\mathbf{r}}\frac{1}{4\pi\varepsilon_o}\int_v \frac{\rho dv}{r^2}$	$V_b - V_a = -\int_a^b \mathbf{E}\cdot\mathbf{dl}$	$V = \frac{1}{4\pi\varepsilon_o}\int_v \frac{\rho dv}{r}$
$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$	$\int_s \vec{\mathbf{E}}\cdot d\vec{\mathbf{S}} = \frac{1}{\varepsilon_o}\int_v \rho\, dv$	$\nabla\cdot\vec{\mathbf{E}} = \frac{\rho_f+\rho_p}{\varepsilon_o}$	$\nabla^2 V = \frac{\rho_f+\rho_p}{\varepsilon_o}$
$C = Q/V$	$\int_s \vec{\mathbf{J}}\cdot d\vec{\mathbf{S}} = -\frac{d}{dt}\int_v \rho\, dv$	$\rho_p = -\nabla\cdot\vec{\mathbf{P}}$	$\sigma_p = \vec{\mathbf{P}}\cdot\hat{\mathbf{n}}$
$\vec{\mathbf{P}} = \varepsilon_o\chi_e\vec{\mathbf{E}}$	$\vec{\mathbf{D}} = \varepsilon_o\vec{\mathbf{E}} + \vec{\mathbf{P}}$	$\varepsilon = \varepsilon_0\varepsilon_r$	$\nabla\cdot\vec{\mathbf{D}} = \rho$
$\int_S \vec{\mathbf{D}}\cdot d\vec{\mathbf{S}} = \int_v \rho\, dv$	$\frac{\partial\mathbf{D}}{\partial t} = \frac{\partial}{\partial t}\varepsilon_0\mathbf{E} + \frac{\partial\mathbf{P}}{\partial t}$	$\vec{\mathbf{B}} = \frac{\mu_o I}{4\pi}\oint \frac{d\vec{\mathbf{l}}\times\hat{\mathbf{r}}}{r^2}$	$\nabla\cdot\vec{\mathbf{B}} = 0$
$\vec{\mathbf{B}} = \nabla\times\vec{\mathbf{A}}$	$\vec{\mathbf{A}} = \frac{\mu_o}{4\pi}\oint \frac{Id\vec{\mathbf{l}}}{r}$	$\int_l \vec{\mathbf{A}}\cdot d\vec{\mathbf{l}} = \Phi$	$\int_l \vec{\mathbf{B}}\cdot d\vec{\mathbf{l}} = \mu_o\int_s \vec{\mathbf{J}}\cdot d\vec{\mathbf{S}}$
$\vec{\mathbf{F}} = Q\left(E + \vec{\mathbf{v}}\times\vec{\mathbf{A}}\right)$	$\oint_E \vec{\mathbf{E}}\cdot d\vec{\mathbf{l}} = -\frac{d\Phi}{dt}$	$\vec{\mathbf{E}} = -\nabla V - \frac{\partial\vec{\mathbf{A}}}{\partial t}$	$\nabla\times\vec{\mathbf{E}} + \frac{\partial\vec{\mathbf{B}}}{\partial t} = 0$
$\oint_b \vec{\mathbf{E}}\cdot d\vec{\mathbf{l}} = -M\frac{dI_a}{dt}$	$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{l}} = -L\frac{dI}{dt}$	$\vec{\mathbf{J}}_m = \nabla\times\vec{\mathbf{M}}$	$\vec{\mathbf{J}}_t = \vec{\mathbf{J}}_m + \varepsilon_o\frac{\partial\vec{\mathbf{E}}}{\partial t}$
$\mu = \mu_o\mu_r$	$\nabla\times\left(\frac{\vec{\mathbf{B}}}{\mu_o} - \vec{\mathbf{M}}\right) = \vec{\mathbf{J}}$	$\vec{\mathbf{B}} = \mu_o\left(\vec{\mathbf{H}} + \vec{\mathbf{M}}\right)$	$\nabla\times\vec{\mathbf{H}} = \vec{\mathbf{J}}$
$\int_l \vec{\mathbf{H}}\cdot d\vec{\mathbf{l}} = I$	$\vec{\mathbf{B}} = \mu_o(1+\chi_m)\vec{\mathbf{H}}$	$\nabla\times\vec{\mathbf{B}} - \varepsilon_o\mu_o\frac{\partial\vec{\mathbf{E}}}{\partial t} = \vec{\mathbf{J}}$	$\vec{\mathbf{J}}_m = \vec{\mathbf{J}}_f + \varepsilon_o\frac{\partial\vec{\mathbf{P}}}{\partial t} + \nabla\times\vec{\mathbf{M}}$

Useful integrals

$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2(x^2+a^2)^{1/2}}$	$\int \frac{dx}{\sqrt{x^2+d^2}} = \ln\left(x+\sqrt{x^2+d^2}\right)$

Vector Definitions

Divergence of a vector:

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}, \text{ where } \vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}.$$

$$\nabla \cdot \vec{A} = \frac{1}{r}\frac{\partial}{\partial r}\left(rA_r\right) + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}, \text{ where } \vec{A} = A_r\hat{r} + A_\theta\dot{\theta} + A_z\hat{z}.$$

$$\nabla \cdot \vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(A_\theta \sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial \varphi}, \text{ where } \vec{A} = A_r\hat{r} + A_\theta\hat{\theta} + A_\varphi\hat{\varphi}.$$

Curl of a vector:

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{z}, \text{ where } \vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}.$$

$$\nabla \times \vec{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right)\hat{r} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(rA_\theta\right) - \frac{\partial A_r}{\partial \theta}\right]\hat{z}, \text{ where } \vec{A} = A_r\hat{r} + A_\theta\hat{\theta} + A_z\hat{z}.$$

$$\nabla \times \vec{A} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}\left(A_\varphi \sin\theta\right) - \frac{\partial A_\theta}{\partial \varphi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{\partial}{\partial r}\left(rA_r\right)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(rA_\theta - \frac{\partial A_r}{\partial \theta}\right)\right]\hat{\varphi}$$

where $\mathbf{\vec{A}} = A_r\hat{\mathbf{r}} + A_\theta\hat{\theta} + A_\varphi\hat{\varphi}$.

Vector Identities and Theorems

1. $\vec{A} \times \left(\vec{B} \times \vec{C}\right) = \vec{B}\left(\vec{A} \cdot \vec{C}\right) - \vec{C}\left(\vec{A} \cdot \vec{B}\right)$
2. $\nabla\left(fg\right) = f\nabla g + g\nabla f$
3. $\nabla \cdot \left(f\vec{A}\right) = \left(\nabla f\right) \cdot \vec{A} + f\left(\nabla \cdot \vec{A}\right)$
4. $\nabla \cdot \left(\vec{A} \times \vec{B}\right) = \vec{B} \cdot \left(\nabla \times \vec{A}\right) - \vec{A} \cdot \left(\nabla \times \vec{B}\right)$
5. $\nabla \times \left(f\vec{A}\right) = \left(\nabla f\right) \times \vec{A} + f\left(\nabla \times \vec{A}\right)$
6. $\nabla \times \nabla \times \vec{A} = \nabla\left(\nabla \cdot \vec{A}\right) - \nabla^2 \vec{A}$
7. **Divergence theorem:**

   $\int_s \vec{A} \cdot d\vec{S} = \int_v \nabla \cdot \vec{A}\, dv$

   Stokes's theorem:

   $\oint_c \vec{A} \cdot d\vec{l} = \int_s \left(\nabla \times \vec{A}\right) \cdot d\vec{S}$

Q1. (a) State Coulomb's law and write down its mathematical form. **[2 marks]**

(b) Given the definition of the electric field intensity

$$\vec{E} = k\frac{Q}{r^2}\hat{r}.$$

Derive an expression for electric field intensity due to a charge uniformly distributed over an infinite plane with charge density σ and show that it is given by

$$E = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\sigma dS}{r^2}\hat{r}.$$

[4 marks]

(c) (i) Define the electric potential. **[1 mark]**

(ii) The work done by Coulomb's force in moving a charged particle from point a to point b is given by

$$W = \int_{r_a}^{r_b} \vec{F}\cdot d\vec{l}.$$

The associated potential energy is equal to the negative of the work done. Show that in an electric field, the potential difference between two points a and b along any given path is given by

$$V_a - V_b = -\int_a^b \vec{E}\cdot d\vec{l}.$$

[5 marks

(d) (i) A circular ring of radius a is charged uniformly with a total charge Q. Find the electric potential at a point h from the ring along its axis. **[5 marks]**

(e) (i) Describe what is meant by an electric dipole. **[1 mark]**

(ii) Show that the electric potential due to a dipole consisting of charges $+q$ and $-q$ separated by a small distance d is

$$V = \frac{qd\cos\theta}{4\pi\varepsilon_o R^2}$$

where θ is the angle which the position vector of the point from which the dipole is observed makes with the midpoint of the line joining the two charges. (Assume d is much smaller than the field point distance R). **[7 marks]**

Q2.(a) (i) State Gauss's law and, **[2 marks]**

(ii) show that its differential form is given by

$$\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}.$$

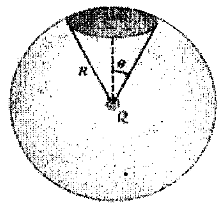
[4 marks]

(b) The electric field contained in a certain spherical geometry is found to be $\mathbf{E} = kr^3\hat{\mathbf{r}}$, where k is a positive constant, r is the radial distance from the origin and $\hat{\mathbf{r}}$ is a unit vector in the radial direction. Use the differential form of Gauss' law to show that the volume charge density is

$$\rho = 5\varepsilon_0 kr^2.$$

[6 marks]

(c) A sphere of radius R surrounds a point charge Q located at its center.



(i) Show that the differential surface element of a circular cup of half-angle θ is given by

$$dA = 2\pi R^2 \sin\theta d\theta$$

[4 marks]

(ii) Using the result in (i), show that the total area of the circular cup is

$$A = 2\pi R^2 \left(1 - \cos\theta\right)$$

[4 marks]

(iii) Use Gauss' law to show that the electric flux through the circular cup is

$$\Phi_E = \frac{Q}{2\varepsilon_0}\left(1 - \cos\theta\right).$$

[5 marks]

Q3. (a) Explain how the electric field is distributed inside and outside a conductor.

[2 marks]

(b)(i) Explain in detail the behavior of a dielectric material in an external electric field. **[3 marks]**

(ii) Define dielectric strength of a material. **[1 mark]**

(c) (i) Starting with the differential form of Gauss' law

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

in free space, show that in a dielectric material and therefore any medium, the form is modified and given as

$$\nabla \cdot \vec{D} = \rho$$

and this can expressed in integral form as

$$\oint_S \vec{D} \cdot d\vec{S} = Q$$

[8 marks]

(ii) Calculate the total charge enclosed by a cube of side 2 m, centered at the origin and with the edges parallel to the axes when the electric flux density over the cube is given by

$$\vec{D} = \frac{10x^3}{3}\hat{x}$$

[4 marks]

(d) Two conducting plates of a parallel-plate capacitor of area S are separated by a distance d and have charges $+Q$ and $-Q$ respectively. The space between the plates is filled with a dielectric material of constant permittivity ε. Assuming negligible fringing effect at the edges and given that the normal component of the electric flux density at a boundary between a dielectric medium and a conductor is given by $D = \varepsilon E_n = \sigma$, determine

(i) the potential at any point between the plates, and **[5 marks]**

(ii) the capacitance of the parallel-plate capacitor. **[2 marks]**

Q4. (a) A sphere of radius R carries a polarization

$$\hat{P} = kr\hat{r}$$

where k is a constant and $\hat{r}$ is the radial unit vector.

(i) Calculate the bound charge densities, σ_p and ρ_p due to the polarization.

[3 marks]

(ii) Find the field inside and outside the sphere. **[5 marks]**

(b) Use the Biot-Savart's law namely,

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$$

to find the magnitude of the magnetic flux density at a point P on the perpendicular bisector of a conductor of finite length L carrying a current I. Show that this simplifies to

$$B = \frac{\mu_0 I}{2\pi a}\frac{1}{\sqrt{1+\frac{4a^2}{L^2}}}$$

(Take the point P to be at a distance a from the conductor). **[9 marks]**

(c) A steady current I flows in a conductor bent in the form of a square loop of sides w. Find the magnetic flux density at the center of the current loop and show that it is given by

$$B = \frac{2\sqrt{2}w\mu_0 I}{\pi w}.$$

[4 marks]

(d) Given that the force on a wire of length L carrying a current I placed in a magnetic field $\vec{B}$ is

$$F = ILB$$

derive the expression for torque developed in a rectangular closed circuit carrying a current I in a uniform magnetic field $\vec{B}$ and show that it is simplifies to

$$\tau_{net} = wILB.$$

[4 marks]

Q5. (a) (i)) State Ampere's law and write down the corresponding equation in terms of the magnetic flux density $\vec{B}$. **[2 marks]**

(ii) Use Ampere's law to show that for an infinitely long straight current filament, the magnetic flux density is given by

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$

where $\hat{\phi}$ is a unit vector in the azimuthal direction. **[4 marks]**

(b) Show that the magnetic force per unit length between two long parallel conductors separated by a distance d carrying equal currents in the same direction is given by

$$\frac{F}{L}=\frac{\mu_0 I^2}{2\pi d}$$

[4 marks]

(c) (i) Explain magnetization in magnetic materials. **[2 marks]**

(ii) Show that when the effect of magnetization is taken into account, this leads to the following form of Ampere's law

$$\oint_l \vec{H}\cdot d\vec{l}=I .$$

[8 marks]

(iii) Show that for a magnetic material whose magnetic properties are linear and isotropic

$$\vec{B}=\mu\vec{H}$$

where μ is the absolute permeability of the medium. **[5 marks]**

Q6. (a) Show that the magnetic flux density at a point on the axis of a circular loop of radius b that carries a direct current I is given by

$$\vec{B}=\frac{\mu_o}{2}\frac{Ib^2}{(z^2+b^2)^{3/2}}\hat{z}$$

(Assume the loop sits in the xy-plane with its axis aligned with the z-axis).

[7 marks]

(b) (i) State Faraday's law. **[1 mark]**

(ii) Derive Maxwell's equation from Faraday's law in integral form and show that it is given by

$$\oint_l \vec{E}\cdot d\vec{l}=-\int\frac{\partial\vec{B}}{\partial t}\cdot d\vec{S} .$$

[4 marks]

(iii) Show that the differential form of the Maxwell's equation from Faraday's law is given by

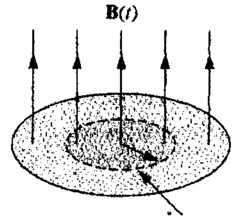
$$\nabla\times\vec{\mathbf{E}}+\frac{\partial\vec{\mathbf{B}}}{\partial t}=0.$$

[3 marks]

(iv) A uniform magnetic field $B(t)$, pointing straight up, fills the shaded circular region of the figure below. If B is changing with time, show that the induced electric is given by

$$\vec{E}=\frac{s}{2}\frac{dB}{dt}\hat{\phi}$$

[4 marks]



(c) With necessary explanation and justification, derive Ampere's law with Maxwell's correction for time varying fields and show that its differential form is given by

$$\nabla \times \vec{B} = \mu \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

where the term $\vec{J}_D = \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t}$ is the displacement current density. **[6 marks]**

THE UNIVERSITY OF ZAMBIA
PHYSICS DEPARTMENT
UNIVERSITY EXAMINATIONS – 2012/13
P351 - QUANTUM MECHANICS

TIME: 3 HOURS MAX MARKS: 100

ATTEMPT IN ALL **FOUR** QUESTIONS **INCLUDING QUESTION 1 WHICH IS COMPULSORY**. ALL QUESTIONS CARRY EQUAL MARKS.

You may use the following information:

Electron rest mass $m_0 = 9.1 \times 10^{-31}$ Kg
Boltzmann constant $k = 1.38\times10^{-23}$ J/K
Proton rest mass $M_p = 1.67 \times 10^{-27}$ Kg
Planck's constant h $= 6.6 \times 10^{-34}$ J.s
Speed of light c $= 3 \times 10^{8}$ m/s
$\hbar/2\pi$ $= 1.05 \times 10^{-34}$ Js
1 e.V. $= 1.6 \times 10^{-19}$J
Stefan's constant σ $= 5.67 \times 10^{-8}$ W/m^2.K^4

Q.1 (i) Briefly explain Planck's postulate for the derivation of the formula for the blackbody radiation and how it departed from the classical concepts. [2]

(ii) State the position-momentum uncertainty relationship. Briefly explain with reason if it of practical relevance in the case of a macroscopic object like a football. [2]

(iii) In quantum mechanics, what is the physical significance of a single slit intensity distribution pattern when an electron passes through the slit? Does the electron split into parts which go in different directions? [2]

(iv) Give the physical interpretation of the wave-function. Why does the wave-function need to be normalized? Why is it necessary for the wave-function to be finite throughout its domain? [4]

(v) What do we mean when we say that two operators A and B commute? What is the physical significance of this statement? [2]

(vi) Without calculation, write down the values of the following commutators: $[x, p_y]$, $[p_x, p_y]$, $[z, p_y]$, $[x, y]$, $[x, x^2]$, $[p_xp_y, p_z]$, $[L_x, L_y]$ and $[L^2, L_z]$. [4]

(vii) Show that the expectation value of the momentum operator p_x is real. [5]

(viii) Prove the basic commutation relation between the position and the momentum operator. [4]

Q.2 (a) For a potential barrier of width a and height V_0, the reflection and transmission coefficients for a particle with energy $E<V_0$ are given by

$$R=\left[1+\frac{4k_0^2k^2}{\left(k_0^2+k^2\right)^2\sinh^2(ka)}\right]^{-1}=\left[1+\frac{4E(V_0-E)}{V_0^2\sinh^2(ka)}\right]^{-1}$$

$$T=\left[1+\frac{\left(k_0^2+k^2\right)^2\sinh^2(ka)}{4k^2k_0^2}\right]^{-1}=\left[1+\frac{V_0^2\sinh^2(ka)}{4E(V_0-E)}\right]^{-1}$$

where $k=\sqrt{[2m(V_0-E)/\hbar^2]}$ and $k_0=\sqrt{[2mE/\hbar^2]}$.

(i) Calculate $R+T$. (5 marks)

(ii) Explain how the above results lead to the "tunnel effect". (5 marks)

(iii) Use the tunnel effect to explain two phenomena in atomic/nuclear physics. (5 marks)

(b) Calculate the reflection coefficient R and the transmission coefficient T of an electron of energy $E = 4\ eV$ for a rectangular barrier potential of height $V_0 = 5\ eV$ and width $a = 2\ \text{Å}$. (10 marks)

Q.3 (a) Define (i) a hermitian operator and (ii) an adjoint operator. [4]

(b) Prove that the eigenvalues of a hermitian operator are real. [4]

(c) If A and B are hermitian, prove that $(A+B)^n$ is hermitian. [6]

(d) For any operators A and B, prove that (i) $(AB)^+ = B^+A^+$ and (ii) $(A^+)^+ = A$, where the superscript + denotes adjoint of an operator. [3+3]

(e) Consider the hermitian operator H that has the property that $H^4=1$. What are the eigenvalues of the operator H? What are the eigenvalues if H is not restricted to being hermitian? [5]

Q.4 For a particle in a one-dimensional infinite square well potential of width $2a$, the normalized wave-function is given by

$$u(x) = C\left(\frac{1}{2}\sin\frac{2\pi x}{a} + \sin\frac{3\pi x}{a} + \cos\frac{\pi x}{2a}\right)$$

(a) Show that $C = 2/3\sqrt{a}$ (8 marks)

(b) If a measurement of the energy is made, what are the possible values that can result? (9 marks)

(c) Calculate the probability of obtaining each of the energy values and the expectation value of the energy. (8 marks)

Q.5. (a) Use the classical definition of the orbital angular momentum and the basic commutation relations between the position and momentum operators to obtain all the commutation relations among the orbital angular momentum operators L_x, L_y, L_z and L^2. Give the physical significance of these commutation relations. (13 marks)

(b) If J is the angular momentum operator and $J_\pm = J_x \pm iJ_y$, use the fundamental commutation relations for the angular momentum operators to prove that

(i) $[J^2, J_\mp] = 0$

(ii) $J_\pm J_\mp = J^2 - J_z^2 \pm \hbar J_z$

(iii) $[J_z, J_\pm] = \hbar J_\pm$

[4+4+4]

Q.6. Consider a particle of mass m confined within a rectangular box with impenetrable walls of sides L, $2L$ and $3L$.

(a) Write down the Schrödinger equation for the particle and the boundary conditions. [3]

(b) Use the method of separation of variables to solve the Schrödinger equation and obtain the eigen-functions. [10]

(c) Normalize the eigenfunctions obtained in (b) above [4]

(d) Find the allowed energy levels. [2]

(e) Obtain the allowed energy levels if the box were cubical in shape with each side being of length L. [2]

(f) Discuss the degeneracy of the first two energy levels in (d) above. [2]

(g) What happens to the spacings of the energy levels as the dimensions of the box increase? [2]

.... **END OF THE EXAMINATION** ....

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012/13 ACADEMIC YEAR FIRST SEMESTER FINAL EXAMINATIONS

P441 : Analog Electronics II

Time : Three hours. **Maximum marks = 100.**

Attempt any four questions.
All questions carry equal marks.
The marks are shown in brackets.

Wherever necessary use:

Electrical parameters of 714C operational amplifier

$A = 400,000$
$R_i = 33\ M\Omega$
$R_o = 60\Omega$
Supply voltages = $\pm$ 15V
Maximum output voltage swing = $\pm$13V
UGB = 0.6 MHz

LM307 operational amplifier

$V_{io} = 10$ mV dc
$I_B = 300$nA
$I_{io} = 70$ nA

Q1. **(a)** Derive an expression for the gain of a differential amplifier with one operational amplifier. Draw the necessary circuit diagram. [9]

(b) For an inverting amplifier, suppose that the circuit is nulled and the voltage across terminals $+V_{CC}$ and $-V_{EE}$ measures 26V dc. Because of poor regulation, the voltage across terminals $+V_{CC}$ and $-V_{EE}$ varies with time from 23V to 29V. Determine

(i) The change in the input offset voltage caused by the change in dc supply; [4]

(ii) The total output voltage V_o if V_{in}= 7 mV dc. [2]

The operational amplifier is MC1741 with R_1=100Ω, R_F=4.7kΩ and $\frac{\Delta V_{io}}{\Delta V} = 15.85 \mu V / V$.

(c) Design an input voltage compensating network for the μA715 operational amplifier for which V_{io}= 6mV maximum. The operational amplifier uses ±15V supply voltages and is to be used as an inverting amplifier. [10]

Q2. **(a)** The 714C operational amplifier is connected as a voltage follower with R_1=470Ω, R_F=4.7 kΩ, A=400,000, R_i=33 MΩ, R_o=60 Ω, f_o=5Hz, supply voltages = ± 15V and output voltage swing = ± 13 V. Compute the values of [7]

(i) voltage gain
(ii) input resistance
(iii) output resistance
(iv) bandwidth
(v) total output offset voltage.

(b) **(i)** Define common mode voltage gain and common mode rejection ratio for operational amplifiers. [3]

(ii) An amplifier has a differential gain of 300 and a CMRR of 60dB. If V_{in1}=40 mV, V_{in2}=60 mV and V_{noise}=5mV, determine the differential output and noise output. [6]

(c) **(i)** Explain the working of a non-inverting comparator with the aid of a diagram. [5]

(ii) Draw the input and output waveforms of a comparator when +4V is applied at the inverting input and a sinusoidal signal of 6V peak at the non-inverting input. [4]

Q3. (a) (i) Draw the schematic diagram of a Wien bridge oscillator. What are the two conditions for sustained oscillations? [5]

(ii) A certain Wien bridge oscillator uses $R=4.7k\Omega$ and $C=0.01\mu F$. Determine the frequency of oscillation. [2]

(b) (i) Design a narrow band pass filter so that $f_c=2$ kHz, $Q=20$ and $A_F= 10$. Assume the value of capacitor used is $0.1\mu F$. [9]

(ii) What modifications are necessary in the filter circuit to change the centre frequency f_c to 2.5 kHz, keeping the gain and bandwidth constant? [2]

(c) The peaking amplifier has the following values. $R_1=1$ kΩ, $L=100$ μH with a 3Ω internal resistance. $C=0.01$ μF, $R_F= 6.8$ kΩ and $R_L=10$ kΩ. Determine

(i) The peak frequency [2]

(ii) The gain of the amplifier at peak frequency [5]

Q4. (a) What causes the gain of an operational amplifier to roll off after a certain frequency is reached? How do we account for this in the high frequency model of the operational amplifier? Draw the circuit diagram. [9]

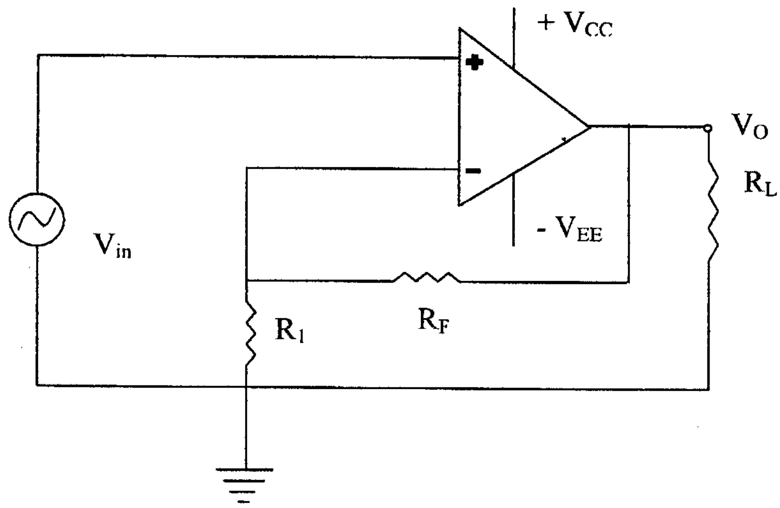
(b) The operational amplifier 741C has the following gain and break frequency. $A=200,000$ and $f_0=5$ Hz.

(i) Write the open loop gain equation for the op-amp as a function of break frequency and dc gain A. [2]

(ii) Prepare the frequency response data for the following frequencies. [9]

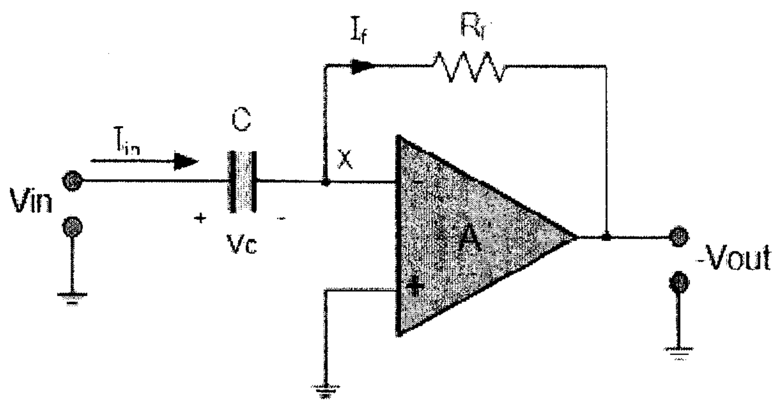
0 5 Hz 200Hz 2000Hz 1 MHz

(c) Find the closed loop gain A_F for the amplifier given below. Given $R_1=10$ kΩ and $R_F=20$ kΩ, How does A_F change if a third resistance $R_2=10$ kΩ is connected in series with R_1? In parallel with R_1? [5]



Q5. (a) Design a second order low pass Butterworth filter at a cut off frequency of 1.2 kHz. Using the frequency scaling technique, convert the above filter to a cut off frequency of 1.6kHz. [11]

(b) Identify the circuit below and derive an expression for the output voltage of the following operational amplifier circuit. [6]



(c) A Colpitt oscillator used as the local oscillator in an AM radio receiver is to produce a frequency 1 to 2 MHZ.

(i) What must be the inductance of the coil if the minimum capacitance obtainable is 43pF? [3]

(ii) What must be the maximum value of capacitance to produce the necessary frequencies? [2]

(iii) What frequency would be produced if this coil were used in a Hartley oscillator circuit with C= 100pF? [3]

Q6. **(a)** Determine the maximum possible change in output offset voltage for LH0041C operational amplifier after 6 months. The following data are given.

R_1=82Ω R_F=8.2kΩ $\frac{\Delta V_{io}}{\Delta t} = \frac{5\mu V}{week}$ $\frac{\Delta I_{io}}{\Delta t} = 2nA/week$ [4]

(b) **(i)** Derive the input resistance of a voltage shunt feedback amplifier using appropriate circuit diagrams. [6]

(ii) Calculate the following for a first order low pass Butterworth filter. [6]

(i) Cut off frequency
(ii) Pass band gain of the filter
(iii) Phase angle at the cut off frequency

Given that R_1=10kΩ, R_F=10kΩ, R=22kΩ, C=0.047μF and R_L= 10kΩ

(c) Write short notes on [9]

(i) Transient response
(ii) Inverting zero crossing detector
(iii) Piezo - electric effect

END OF EXAMINATION

UNIVERSITY OF ZAMBIA

DEPARTMENT OF PHYSICS

M.Sc.

2011 SECOND SEMESTER UNIVERSITY EXAMINATIONS

PHY5022
MATHEMATICAL METHODS FOR PHYSICS

DURATION:	Three hours.
INSTRUCTIONS:	Answer any four questions from the six given. *Each question carries 25 marks with marks indicated in parentheses*
MAXIMUM MARKS:	100
DATE:	Monday 21st May 2012

Formulae that may be needed

In this question paper the following labels will be used: s=number of classes, n_i=dimensions of an irreducible representation i, h=number of elements in the group, p_k= number of elements in class c_k, and $c_{ij,s}$=class multiplication coefficients.

1.

$$[lm, n] = \frac{1}{2}\left(\frac{\partial g_{ln}}{\partial x^m} + \frac{\partial g_{mn}}{\partial x^l} - \frac{\partial g_{lm}}{\partial x^n}\right)$$

$$\left\{ \begin{array}{c} s \\ lm \end{array} \right\} = g^{sn}[lm, n]$$

2.

$$\frac{\partial^2 x^r}{\partial \overline{x}^j \partial \overline{x}^k} = \overline{\left\{ \begin{array}{c} n \\ jk \end{array} \right\}} \frac{\partial x^r}{\partial \overline{x}^n} - \frac{\partial x^i}{\partial \overline{x}^j}\frac{\partial x^l}{\partial \overline{x}^k} \left\{ \begin{array}{c} r \\ il \end{array} \right\}$$

3.

$$g^{\mu\mu} = \frac{1}{g^{\mu\mu}}$$

4. For orthogonal systems $g_{\mu\nu} = 0$ for $\mu \neq \nu$

5.

$$g_{11} = 1, \qquad g_{22} = r^2, \qquad g_{33} = r^2 \sin^2\theta$$

6. For an inhomogeneous equation of the form

$$Lu(x) \mp \lambda u(x) = f(x),$$

the eigenvalue formula for the Green's function of this equation is

$$G(x,x') = \pm \sum_n \frac{u_n(x)u_n(x')}{\lambda_n - \lambda}$$

7.

$$\sum_{i=1}^{i=s} n_i^2 = h$$

8.

$$C_j.C_k = \sum_r c_{jk,r} C_r$$

9.

$$p_j p_k \chi_k^{(i)*} \chi_j^{(i)} = n_i \sum_{r=1}^{r=s} c_{jk,r} p_r \chi_r^{(i)}$$

10.

$$\sum_{k=1}^{k=s} p_k \ \chi_k^{(i)*} \chi_k^{(j)} = \delta_{ij} h$$

11.

$$\chi_k = \sum_{k=1}^{k=s} c_i \chi_k^{(i)}$$

12.

$$c_i = \frac{1}{h} \sum_{k=1}^{k=s} p_k \ \chi_k^{(i)*} \chi_k$$

13. For the regular representation we have:

$$\chi(g) = \begin{cases} h & \text{if } g = I \\ 0 & \text{otherwise,} \end{cases}$$

QUESTION 1

(a) Prove that

$$\text{div } A^p = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^k} (\sqrt{g} A^k)$$

given that

$$\left\{ \begin{matrix} p \\ pq \end{matrix} \right\} = \frac{\partial}{\partial x^q} (\ln \sqrt{g}).$$

(12 marks)

(b) Given that

$$\text{div.grad } \phi = \text{div} \left(g^{kr} \frac{\partial \phi}{\partial x^r} \right),$$

prove that

$$\nabla^2 \Phi = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^k} \left(\sqrt{g} \, g^{kr} \frac{\partial \Phi}{\partial x^r} \right).$$

(8 marks)

(b) Show that

$$\operatorname{curl} A_p = A_{p;q} - A_{q;p} = \frac{\partial A_p}{\partial x^q} - \frac{\partial A_q}{\partial x^p}.$$

(5 marks)

QUESTION 2

(a) The equation of a curve is given by

$$\vec{x} = \vec{\phi}(s),$$

where s is the arc length.

(i) Determine the unit vector tangent to the curve. (4 marks)

(ii) Show that it does indeed have unit magnitude. Illustrate your derivation with a diagram. (1 marks)

(b) Determine a vector perpendicular to the curve of part (a) and show how it is related to the curvature k, i.e., write down the first Frenet-Serrat formulae. Where quantities have special names, state their name. (8 marks)

(c) Determine what kind of object is the volume element

$$d^n\vec{x}$$

of an n-dimensional space. State the special name given to this kind of tensor object. (5 marks)

(d) (i) State the quotient law. (2 marks)

(ii) Let B^{jk} be the inverse tensor of A_{ij} and therefore satisfies the relation

$$A_{ij}B^{jk} = \delta_i^k.$$

Use the qoutient law to determine what kind of tensor object is B^{jk}. State clearly any special precaution used in the determination. (5 marks)

QUESTION 3

(a) Determine the eigenfunctions and eigenvalues of the homogeneous part of the one-dimensional Helmholtz equation,

$$\frac{d^2y(x)}{dx^2} + k^2y(x) = f(x),$$

in the region $0 < x < L$ with $y(x) = 0$ at $x = 0$ and at $x = L$. (8 marks)

(b) Normalize the eigenfunctions found in part (i). (7 marks)

(c) Determine the Green's function for the Helmholtz equation of part (i) using the results of part (i). (4 marks)

(d) Find $y(x)$ if $f(x) = 1$ for $\frac{L}{4} < x < \frac{3L}{4}$ and equals 0 outside this range. Hint: Use only the sine possibility for the solutions of the eigenvalue equation. Hence write down the solution $y(x)$. (6 marks)

QUESTION 4

Use Hilbert-Schmidt theory to solve the integral equation

$$y(x) = x + \lambda \int_0^1 xt\, y(t)\, dt.$$

(25 marks)

QUESTION 5

Consider the group of the symmetry operations of the equilateral triangle. The six elements of the group are: E is the identity element, $C_3(1)$ is a clockwise rotation of 120° about the center, $C_3(2)$ is a counterclockwise rotation of 120° about the center, C_2, C_2' C_2'' are reflections in the three triangle bisectors, respectively. The multiplication table for the group is:

E	$C_3(1)$	$C_3(2)$	C_2	C_2'	C_2''
$C_3(1)$	$C_3(2)$	E	C_2'	C_2''	C_2
$C_3(2)$	E	$C_3(1)$	C_2''	C_2	C_2'
C_2	C_2''	C_2'	E	$C_3(2)$	$C_3(1)$
C_2'	C_2	C_2''	$C_3(1)$	E	$C_3(2)$
C_2''	C_2'	C_2	$C_3(2)$	$C_3(1)$	E

(a) Decompose the group into classes using the definition of equivalent elements. (9 marks)

(b) Determine the regular representation of the symmetry group. (7 marks)

(c) Use the following equation

$$\frac{1}{h}\sum_{k=1}^{k=s} p_k \chi_k^* \chi_k \quad \begin{cases} = 1 & \text{IRREDUCIBLE} \\ \neq 1 & \text{REDUCIBLE,} \end{cases}$$

to test whether or not the regular representation is reducible. (4 marks)

(d) State Schur's lemma and state the test based on Schur's lemma for testing whether or not a representation is reducible. (5 marks)

QUESTION 6

The elements of the permutation group S_3 are $E = [123]$, $p_1 = [231]$, $p_2 = [312]$, $q_1 = [321]$, $q_2 = [213]$ and $q_3 = [132]$. The group divides into the following classes: $C_1 = E$, $C_2 = p_1, p_2$, $C_3 = q_1, q_2, q_3$. The multiplication table of S_3 is:

E	p_1	p_2	q_1	q_2	q_3
p_1	p_2	E	q_2	q_3	q_1
p_2	E	p_1	q_3	q_1	q_2
q_1	q_3	q_2	E	p_2	p_1
q_2	q_1	q_3	p_1	E	p_2
q_3	q_2	q_1	p_2	p_1	E

Determine the character table of S_3. (25 marks)

END

UNIVERSITY OF ZAMBIA
DEPARTMENT OF PHYSICS
MSc
2011 FIRST SEMESTER UNIVERSITY EXAMINATIONS

PHY5311
THEORETICAL PHYSICS

DURATION: Three hours.

INSTRUCTIONS: Answer four questions from the six questions given. At least one question must be answered from section A and from section B. There are four questions in section A, and two questions section B. *Each question carries 25 marks with the marks for parts of questions indicated.*

MAXIMUM MARKS: 100

DATE: Monday 28th November 2011.

Formulae that may be needed:

1.
$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t}$$

2.
$$\mathbf{B} = \nabla \times \mathbf{A}$$

3.
$$[a_k', a_k^\dagger] = \delta_{k'k}$$

4.
$$\int_V \mathbf{u}_{k'}^* \mathbf{u}_k \, dr = \delta_{k'k}, \qquad \int_V \mathbf{u}_{k'} \mathbf{u}_k \, dr = \int_V \mathbf{u}_{k'}^* \mathbf{u}_k^* \, dr = \delta_{-k'k}$$

$$\int_V (\nabla \times \mathbf{u}_{k'}^*)(\nabla \times \mathbf{u}_k) \, dr = \delta_{k'k} |\mathbf{k}|^2, \quad \int_V (\nabla \times \mathbf{u}_{k'})(\nabla \mathbf{u}_k) \, dr = \int_V (\nabla \times \mathbf{u}_{k'}^*)(\nabla \times \mathbf{u}_k^*) \, dr = \delta_{-k'k} |\mathbf{k}|^2$$

5.
$$E^{(+)} = i \sum_k \left(\frac{\hbar \omega_k}{2\epsilon_0} \right)^{1/2} a_k \mathbf{u}_k(r) e^{-i\omega_k t}$$

6.

$$\begin{pmatrix} n \\ m \end{pmatrix} = \frac{n!}{(n-m)!m!}$$

7.

$$[B, A^n] = nA^{n-1}[B, A]$$

Section A

QUESTION 1

(i) State the gauge used for quantum optics and give the mathematical conditions that define it. (3 marks)

(ii) Given that the vector potential is

$$\mathbf{A}(r) = \sum_k \left(\frac{\hbar}{2\omega_k \epsilon_0}\right)^{1/2} [a_k \mathbf{u}_k(r) e^{-i\omega_k t} + a_k^\dagger \mathbf{u}_k^*(r) e^{i\omega_k t}],$$

express $\mathbf{E}$ and $\mathbf{B}$ in terms of a_k and $a_k^\dagger$. Starting from the classical expression for the Hamiltonian of the electromagnetic field show that the Hamiltonian in the quantum theory can be written as

$$H = \sum_k \hbar\omega_k \left(a_k^\dagger a_k + \frac{1}{2}\right).$$

(22 marks)

QUESTION 2

Consider the Hanbury-Brown-Twiss experiment.

(i) Draw a diagram of the experimental set up. (3 marks)

(ii) Beginning with destruction electric field operator for two photons, one with momentum k and the other with momentum k',

$$E^{(+)}(\mathbf{r}_i, t) = \epsilon[a_k e^{i(\mathbf{k}\mathbf{r}_i - \omega t)} + a_{k'} e^{-i(\mathbf{k}'\mathbf{r}_i - \omega t)})],$$

calculate $G^{(2)}(\mathbf{r}_1, \mathbf{r}_2; t, t)$ for thermal light. Hint: In any expansion you need not write terms that are obviously zero. (22 marks)

QUESTION 3

(i) For a phototube of quantum efficiency η show that the probability of counting m photons in n detections for a state of the field with density operator ρ_{nm} is given by

$$P_m^{(n)} = \sum_{n=m} \frac{n!}{(n-m)!m!} \eta^m \eta^{n-m} \rho_{nm}$$

(11 marks)

(ii) Given that the density operator ρ_{nm} is given by

$$\rho_{nm} = \int P(\alpha^*\alpha)\frac{|\alpha|^{2n}}{n!}e^{-|\alpha|^2}\, d^2\alpha,$$

express the probability P_m in terms of the P-representation $P(\alpha^*\alpha)$. (13 marks)

QUESTION 4

If A and B are two operators whose commutator $[A, B]$ commutes with each of them, i.e.,

$$[[A, B], A] = [[A, B], B] = 0,$$

show that the following identity holds

$$e^{A+B} = e^A e^B e^{-\frac{1}{2}[A,B]}.$$

(25 marks)

Section B

QUESTION 5

(i) (a) Define a unitary operator and explain the importance of such operators in quantum mechanics. (5 marks)

(b) Show that under a unitary transformation both the eigenvalues and the expectation values of a quantum system remain unchanged. (5 marks)

(ii) The spin-$\frac{1}{2}$ system is a two-dimensional quantum system. When S_z and S^2 are simultaneously diagonalised, the complete orthonormal set χ_1 and χ_2 of eigenfunctions is such that

$$S_z\chi_1 = \frac{\hbar}{2}\chi_1, \qquad S_z\chi_2 = -\frac{\hbar}{2}\chi_2,$$

while

$$S^2\chi_i = \frac{3}{4}\hbar^2\chi_i.$$

Show that the matrix treatment of the system yields

$$[S_z] = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \ [S^2] = \frac{3}{4}\hbar^2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ [\chi_1] = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \text{and } [\chi_2] = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

(10 marks)

(c) Show that the wave-mechanical operator $A = CD$ is represented by the matrix product $[A] = [C][D]$ in matrix mechanics. (5 marks)

QUESTION 6

(i) In the Heisenberg picture of quantum mechanics, the operator A from the Schrödinger picture has the form

$$A' = e^{iHt/\hbar} A e^{-iHt/\hbar},$$

where H is the Hamiltonian of the problem.

(a) Show that if A has no explicit time dependence, its variation with time is given by

$$\frac{dA'}{dt} = \frac{i}{\hbar}[H, A'].$$

(6 marks)

(b) Show that if $A = H$, then

$$\frac{dH}{dt} = 0,$$

and explain the meaning of this. (6 marks)

(ii) Two spin-$\frac{1}{2}$ systems are combined into one. Show that the result is either a singlet state or three states forming a triplet and give the wave functions of each of these four states. (13 marks)

END

THE UNIVERSITY OF ZAMBIA
DEPARTMENT OF PHYSICS
UNIVERSITY OF ŻAMBIA
SECOND SEMESTER EXAMINATIONS 2011/2
PHY 5322 THEORETICAL PHYSICS II

TIME: THREE HOURS
ANSWER: ANY FOUR QUESTIONS
MAXIMUM MARKS: 100

Symbols: c: velocity of light; $\hbar$:Planck's constant; ϵ_0 : permittivity of free space

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Recursion relation for Legendre polynomials:

$$(2l+1)(1-w^2)^{1/2}P_l^{|m|}(w) = P_{l+1}^{|m|}(w) - P_{l-1}^{|m|}(w) where w = \cos\theta.$$

Orthogonality relation for Legendre polynomials:

$$\int_{-1}^{+1} P_l^{|m|}(w)P_{l'}^{|m|}(w)dw = \frac{2\,(l+|m|)!}{(2l+1)\,(l-|m|)!}\delta_{ll'}$$

---

1. (**a**) What is meant by homogeneity of space? [2 marks]

(**b**) In the infinitesimal continuous symmetry transformation δS corresponding to the unitary operator $U_{\delta S}(\varepsilon)$ given by

$$U_{\delta S}(\varepsilon) = I + i\varepsilon F_s$$

the quantity F_s is the generator of the infinitesimal transformation, I is the identity operator and ε is an infinitesimally small quantity.

(**i**) Show that if space is homogeneous, then the momentum operator $\mathbf{p} = -i\hbar\nabla$ is the generator of infinitesimal translations. [7 marks]

(ii) Hence show that conservation of linear momentum is a result of the invariance of the Hamiltonian with respect to translations when space is homogeneous. [6 marks]

(c) **(i)** Explain why parity is known as a discrete symmetry. [2 marks]
(ii) Describe and interpret the parity operation [3 marks]
(iii) Determine the eigenvectors of the parity operator and show that they can be used to represent any arbitrary function and are therefore complete. [5 marks]

2. **(i)** What is spontaneous emission? [2 marks]
(ii) Show that the Einstein coefficient B_{ba} for stimulated absorption of radiation is given by

$$B_{ba} = \frac{cW_{ba}}{I(\omega_{ba})}$$

where W_{ba} is the transition rate for absorption per atom and $I(\omega_{ba})$ is the intensity of radiation of frequency $\omega = \omega_{ba}$ in the enclosure containing the atoms and the radiation. Note that in the dipole approximation gives

$$W_{ba} = \frac{\pi I(\omega_{ba})}{3c\hbar^2\epsilon_0}|\mathbf{D}_{ba}|^2$$

where $\mathbf{D}_{ba}$ is the matrix element for the dipole operator, [7 marks]
(iii) Explain why the rate of transition in the direction $b \longrightarrow a$ is given by

$$\dot{N}_{ab} = A_{ab}N_b + B_{ab}N_b\rho(\omega_{ba})$$

where A_{ab} is the transition rate for spontaneous emission of photons, B_{ab} is the Einstein B coeficient, N_b is the number of atoms in the state a and $\rho(\omega_{ba})$ is the density of radiation of angular frequency ω_{ba} in the enclosure. [4 marks]
(iv) Show that

$$e^{\hbar\omega_{ba}/kT} = \frac{A_{ab} + B_{ab}\rho(\omega_{ba})}{B_{ab}\rho(\omega_{ba})}$$

where k is Boltzmann's constant and T is the absolute temperature. [4 marks]

(**v**) Given that

$$\rho(\lambda, T) = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

where λ is the wavelength, show that the transition rate for spontaneous emission is

$$W_{ba}^S = \frac{\omega_{ba}^2}{3\pi c^3 \hbar \varepsilon_0} |\mathbf{D}_{ba}|^2$$

[8 marks]

3. (**a**) (**i**) Justify the dipole approximation in the treatment of the interaction of radiation with matter. [4 marks]

(**ii**) Explain what selection rules are. [2 marks]

(**b**) Stimulated absorption and emission as well as spontaneous emission of radiation all depend on the dipole matrix element

$$\mathbf{D}_{ba} = \langle \psi_b | \mathbf{D} | \psi_a \rangle = -e \int \psi_b^*(\mathbf{r}) \mathbf{r} \psi_a(\mathbf{r}) d\mathbf{r}$$

where $\mathbf{D} = -e\mathbf{r}$ is the electric dipole operator, $\psi_b(\mathbf{r})$ is the atomic state of higher energy and $\psi_a(\mathbf{r})$ is the atomic state of lower energy. The matrix element $\mathbf{D}_{ba}$ contains the factors

$$K_x(m, m') = \int_0^{2\pi} \exp\left[i\left(m - m'\right)\phi\right] \cos\phi d\phi$$

$$K_z(m, m') = \int_0^{2\pi} \exp\left[i\left(m - m'\right)\phi\right] d\phi$$

and

$$L^+(l', l, m) = \int_0^{\pi} P_{l'}^{m+1}(\cos\theta) \sin\theta P_l^m(\cos\theta) d\theta$$

where m and m' are magnetic quantum numbers, l and l' are angular momentum quantum numbers and $P_l^m(\cos\theta)$ are the associated Legendre polynomials.

(**i**) Show that the factor $K_x(m, m')$ leads to the selection rule

$$\Delta m = \pm 1$$

for the magnetic quantum numbers. [7 marks]

(**ii**) Show that the factor $K_z(m, m')$ leads to the selection rule

$$\Delta m = 0$$

for the magnetic quantum numbers. [4 marks]

(**iii**) Show that the factor $L^+(l', l, m)$ leads to the selection rule

$$\Delta l = +1.$$

for the angular momentum quantum numbers. [8 marks]

4 (**a**) (**i**) What is meant by isotropy of space? [2 marks]

(**ii**) Show that under a rotation of ϕ about the y axis, the coordinates of a point change from (x, y, z) to

$$\begin{aligned} x' &= z\sin\phi + x\cos\phi \\ y' &= y \\ z' &= z\cos\phi - x\sin\phi \end{aligned}$$

[9 marks]

(**iii**) Show that the generator of infinitesimal rotations about the y axis is the operator

$$L_y = i\hbar\left(x\frac{\partial}{\partial z} - z\frac{\partial}{\partial x}\right)$$

for the y component of the angular momentum. [12 marks]

(**iv**) If L_y commutes with the Hamiltonian, then the values of L_y are quantized. Give these values. [2 marks]

5. (**a**) (**i**) With the aid of the heuristic rules for converting dynamical quantities into operators, use the relativistic expression for the energy

$$E^2 = p^2c^2 + m^2c^4$$

to obtain the one-particle Klein-Gordon relativistic wave equation

$$\left(-\hbar^2c^2\nabla^2 + m^2c^4\right)\Psi(\mathbf{r}, t) = -\hbar^2\frac{\partial^2}{\partial t^2}\Psi(\mathbf{r}, t)$$

Note that $\mathbf{p}$ is the momentum and m the rest mass of the particle.[4 marks]

(**ii**) Show that the continuity equation corresponding to this equation is

$$\nabla \cdot \mathbf{j} + \frac{\partial \rho}{\partial t} = 0$$

where

$$\rho = \frac{i\hbar}{2mc^2}\left(\Psi^* \frac{\partial \Psi}{\partial t} - \Psi \frac{\partial \Psi^*}{\partial t}\right)$$

and

$$\mathbf{j} = \frac{\hbar}{2im}\left(\Psi^* \nabla \Psi - \Psi \nabla \Psi^*\right)$$

[12 marks]

(**iii**) Explain why the Klein-Gordon equation interpreted as a relativistic particle equation is problematic and outline how Weisskopf and Pauli reinterpreted it.

(**b**) Use the substitution $\psi(\mathbf{r}, t) = \psi'(\mathbf{r}, t)e^{-imc^2t/\hbar}$ to show that in the limit $c \rightarrow \infty$ the Klein-Gordon equation reduces to the nonrelativistic Schroedinger equation for a free particle

$$i\hbar \frac{\partial \psi'}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 \psi'$$

[9 marks]

6. (**i**) Explain why the Klein-Gordon can give negative probability densities. [2 marks]

(**ii**) Explain why Dirac sought to linearize the relativistic equation

$$E^2 = p^2c^2 + m^2c^4$$

for the energy. [3 marks]

(**iii**) Show that the linearization procedure of Dirac

$$E = c\sum_{\mu=0}^{3} \alpha_u p_\mu$$

leads to the quantities α_μ having the properties

$$\alpha_\mu \alpha_{\mu'} + \alpha_{\mu'} \alpha_\mu = 2\delta_{\mu\mu'} \text{ and } \alpha_\mu^2 = 1.$$

[5 marks]

(**iv**) The four Dirac matrices are

$$\alpha_1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad \alpha_2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}$$

$$\alpha_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \quad \alpha_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Show that α_1 and α_2 do obey the properties in (**iii**) above. [4 marks]

(**b**) In a central potential field $V(r)$, the Hamiltonian for the Dirac equation is

$$H = c(\boldsymbol{\alpha} \cdot \mathbf{p}) + \rho_3 mc^2 + V(r)$$

where

$$\rho_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

(**i**) Show that H does not commute with the z component $L_z = (xp_y - yp_x)$ .of the orbital angular momentum. [7 marks]

(**ii**) Given that in addition H does not commute with L_x, L_z and $\mathbf{L}^2$ but does commute with the z component of

$$\mathbf{J} = \mathbf{L} + \mathbf{S} = \mathbf{L} + \frac{1}{2}\hbar\boldsymbol{\sigma}$$

and with $\mathbf{J}^2$, interpret the Dirac equation. Note that $\mathbf{J}$ is the total spin while $\mathbf{S}$ is the intrinsic spin and $\boldsymbol{\sigma}$ is the Pauli spin operator. [4 marks]

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END OF EXAMINATION

UNIVERSITY OF ZAMBIA
DEPARTMENT OF PHYSICS

2011 SECOND SEMESTER UNIVERSITY EXAMINATIONS

PHY 5822
SOLAR ENERGY MATERIALS

DURATION:	Three (3) hours
INSTRUCTIONS:	Answer **any four (4)** questions. The marks for each question are given in square brackets.
MAXIMUM MARKS:	100
DATE:	Monday May 28, 2012

Some useful identities and formulae are given below:

$h = 6.63x10^{-34}\,J.S$ $\quad k = 1.38x10^{-23}\,J/K$ $\quad \hbar = 1.055x10^{-34}\,J.s$ $\quad \eta = \dfrac{I_{sc}V_{oc}}{I_N A}FF$

$n = N_c \exp\left[-\dfrac{E_c - E_F}{kT}\right]$ $\quad p = N_v \exp\left[-\dfrac{E_F - E_v}{kT}\right]$ $\quad n = n_i \exp\left[-\dfrac{E_i - E_F}{kT}\right]$ $\quad N = n + ik$

$i\sin\delta = \dfrac{e^{i\delta} - e^{-i\delta}}{2}$ $\quad \cos\delta = \dfrac{e^{i\delta} + e^{-i\delta}}{2}$ $\quad \delta = \dfrac{2\pi nd}{\lambda}$ $\quad I = I_o\left[e^{\frac{qV}{kT}} - 1\right]$ $\quad R = rr^*$

$\nabla^2 = i\dfrac{\partial^2}{\partial x^2} + j\dfrac{\partial^2}{\partial y^2} + k\dfrac{\partial^2}{\partial z^2}$ $\quad \alpha = \dfrac{4\pi k}{\lambda}$ $\quad q = \dfrac{\omega}{c}N$ $\quad T = \dfrac{T_1 T_2 \exp(-\alpha d)}{1 - R_1 R_2 \exp(-2\alpha d)}$

$R = R_1 + \dfrac{R_1(1 - R_1^2)\exp(-2\alpha d)}{1 - R_1^2 \exp(-2\alpha d)}$ $\quad R = \dfrac{(1-n)^2 + k^2}{(1+n)^2 + k^2}$ $\quad I = I_o e^{-\alpha x}$ $\quad \alpha = \dfrac{4\pi k}{\lambda}$ $\quad T = tt^*$

$T_\perp = \dfrac{N_2}{N_1}\left|\dfrac{2N_1}{N_2 + N_1}\right|^2$ $\quad R_\perp = \left|\dfrac{N_1 - N_2}{N_1 + N_2}\right|^2$ $\quad n_i^2 = np$ $\quad c = \dfrac{1}{\sqrt{\mu_o \varepsilon_o}}$ $\quad \varepsilon = \varepsilon_1 + i\varepsilon_2$

$R_p \approx \dfrac{\left[(n^2 + k^2)\cos^2\theta - 2n\cos\theta\right] + 1}{\left[(n^2 + k^2)\cos^2\theta + 2n\cos\theta\right] + 1}$ $\quad R_s \approx \dfrac{\left[(n^2 + k^2) - 2n\cos\theta + \cos^2\theta\right]}{\left[(n^2 + k^2) + 2n\cos\theta + \cos^2\theta\right]}$

Q1. **(a)** The optical constants of austenitic stainless steel are $n = 1.9;\ k = 3.5$ at 500nm and $n = 5.4;\ k = 7.5$ at 2000nm. Calculate the reflectance at these wavelengths at

(i) normal incidence ($\theta_i = 0^o$),

(ii) glancing incidence ($\theta_i = 85^o$). [16]

(b) The total amplitude transmittance for a thin film of thickness *"d"* is given by $t = \dfrac{t_1 t_2 e^{-i\delta}}{1 + r_1 r_2 e^{-2i\delta}}$, where $\delta = \dfrac{2\pi n d}{\lambda}$. Show that the total transmittance T for the thin film may be expressed as $T = \dfrac{(1-r_1)^2(1-r_2)^2}{1 + 2r_1 r_2 \cos 2\delta + r_1^2 r_2^2}$. [9]

Q2. **(a)** One of Maxwell's equations for a plane wave propagating in an energy absorbing medium is given by

$$\nabla^2 E = \frac{\varepsilon\mu}{c^2}\frac{\partial^2 E}{\partial t^2} + \frac{4\pi\sigma\mu}{c^2}\frac{\partial E}{\partial t}.$$

The solution to this equation for a plane wave propagating along the x-axis is of the form

$E(x,t) = E_o \exp i(qx - \omega t)$. Where E_o is the amplitude at x = 0 and is perpendicular to the wave vector $q = \dfrac{N\omega}{c}$, ω is the angular frequency, N is the complex refractive index and c is the speed of the plane wave in vacuum. μ is the magnetic permeability and σ is the optical conductivity.

(i) Show that the following relation holds.

$$q^2 = \mu\frac{\omega^2}{c^2}\left(\varepsilon + i\frac{4\pi\sigma}{\omega}\right).$$ [8]

(ii) Use the result in (i) above to show that $n^2 - k^2 = \varepsilon$ and $2nk = \varepsilon_2$ for non-magnetic material; where $\varepsilon_2 = \dfrac{4\pi\sigma}{\omega}$. [9]

(b) A typical metal has $k \approx 5.0$ in the visible part of the electromagnetic spectrum. How deep into such a metal does the un-reflected part of the light wave penetrate? [8]

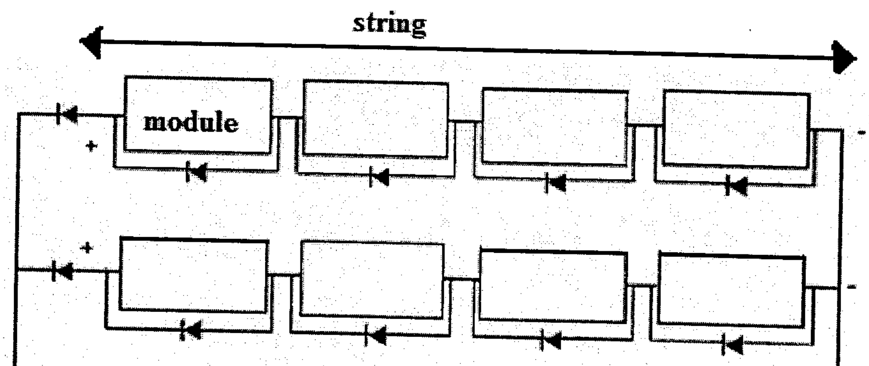
Q3. **(a)** The total reflectance at normal incidence of a single thin film of thickness d and refractive index n_1 on a substrate of known refractive index n_2 is given by

$$R = \frac{r_1^2 + r_2^2 + 2r_1 r_2 \cos 2\delta}{1 + r_1^2 r_2^2 + 2r_1 r_2 \cos 2\delta}.\ \text{Where } \delta = 2\pi n_1 d/\lambda,\ r_1 = \frac{n_o - n_1}{n_o + n_1},\ \text{and } r_2 = \frac{n_1 - n_2}{n_1 + n_2}.$$

(i) Plot a graph of R against the optical thickness $(n_1 d/\lambda)$ = 0.0, 0.2, 0.4, 0.5, 0.6, 0.8, and 1.0, for $n_o = 1.0$, $n_1 = 2.0$ and $n_2 = 1.5$. [10]

(ii) On the same graph, re-plot the reflectance R with same values of the optical thicknesses but for $n_o = 1.0$, $n_1 = 1.75$ and $n_2 = 1.5$ [9]

(b) In the figure below, the modules are arranged in series making a module string. Identify the modules in this figure and explain their functions. [6]



Modules connected in series and diodes fitted to a large array.

Q4. **(a)** The photovoltaic effect phenomenon may be expressed as

$$I = I_o\left[\exp\left(\frac{eV}{kT}\right) - 1\right] - I_L,$$

where the symbols have their usual meaning. (i) Derive an expression for the voltage when the overall cell current is zero. (ii) Hence, calculate the voltage for a silicon solar cell with the following parameters at 40°C; $I_L = 4.2A$, $I_o = 1.5x10^5 \exp(-E_g/kT)$ and $E_g(S_i) = 1.11eV$. [13]

(b) The spectral response of a solar cell is defined as the Amps generated per watt of incident light. Thus

Spectral response = $\frac{q\lambda}{hc}.QE$, where

QE is the quantum efficiency of the solar cell, q is the electronic charge and h is the Planck's constant. Calculate the short-circuit current of a solar cell of area $4cm^2$, illuminated by light of wavelength 800nm at an intensity of $200W/m^2$. Take the quantum efficiency of the cell to be 0.80. [12]

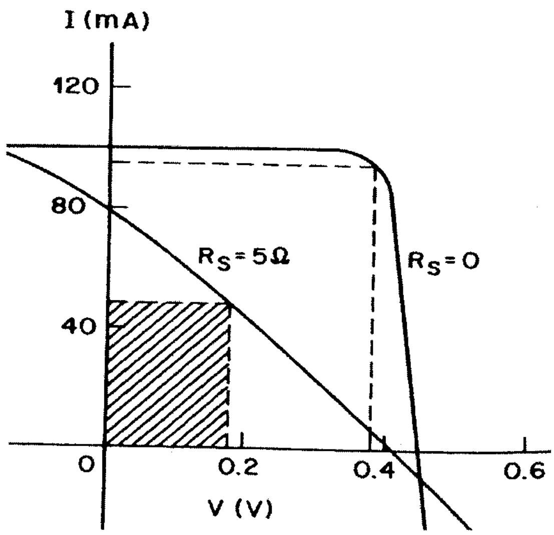
Q5. (a) In a certain experiment, the total reflectance of silicon was found to be 30% at zero photon energy i.e $\omega = 0$. Use this information to calculate the static dielectric constant, $\varepsilon_1(0)$ for silicon. Hint: Determine the refractive index of silicon. [15]

(b) Briefly outline at least four (4) limiting factors that affect the efficiency of a photovoltaic cell. [10]

Q6. (a) A gallium arsenide solar cell with a band gap of 1.42 eV at a temperature of 300 K has a short circuit current of 2.34 A under normal illumination. Calculate the corresponding open circuit voltage given that the dark saturation current is $I_o = 1.5x10^{11}\exp\left(-\frac{E_g}{kT}\right)$. [14]

(b) In the figure below, find the relative maximum power output for series resistance R_s of 0 and 5Ω. What conclusions do you draw on the effect of the series resistance?

[11]



-------------End of Examination-------------

The University of Zambia

Department of Physics

University First Semester Examination

2011/12 Academic Year

PHY 5911

Computational Physics and Modelling I

Instructions **Max. Marks 100**

- ***Time allowed:*** *Three (3) Hours.*
- ***All questions carry equal marks.***
- *Marks for each question are shown in the square brackets [ ].*
- *Whenever necessary, use the information given in the* ***Appendix***.
- ***Answer any four (4) questions.***

Q.1 **(a)** The Lagrange and Hermite approximation methods are used to find polynomials to fit to a given set of data or to find a simpler form of a complex function. What is the major difference between the two methods?

[5 Marks]

(b) The following data give viscosity at several different temperatures.

T(C)	5	20	30	50	55
m(N-sec/m^2)	0.08	0.015	0.009	0.006	0.0055

Find a Lagrange polynomial of degree 4 that intepolates this data. Use this polynomial to find an estimate for the viscosity at $T = 25$ and $T = 40$.

[10 Marks]

(c) Let

$$f(x) = 3xe^x - e^{2x}.$$

Use the Hermite interpolating polynomial of degree three to approximate $f(1.03)$ using $x_0 = 1$ and $x_1 = 1.05$.

[10 Marks]

Q.2 **(a)** In quadratic spline interpolation, a quadratic spline can be defined to have the form

$$S_i(x) = Y_i + Z_i(x - x_i) + \frac{Z_{i+1} - Z_i}{2(x_{i+1} - x_i)}(x - x_i)^2$$

on the interval $[x_i, x_{i+1}]$, where Z_i is the slope of the function $f(x)$ to be interpolated and $Y_i = f(x_i)$. Show that

$$Z_{i+1} = 2\left(\frac{Y_{i+1} - Y_i}{x_{i+1} - x_i}\right) - Z_i$$

[10 Marks]

(b) From **(a)**, if the slope Z_i is specified, other slopes can be computed sequentially. Given the following data, $x = [0,\ 1,\ 2,\ 3]$ and $Y = [0,\ 1;\ 4,\ 3]$ and that $Z_0 = 0$, find the quadratic spline representing this data.

[15 Marks]

Q.3 **(a)** Using Taylor's theorem, derive the three-point central-difference formula;

$$f''(x_1) = \frac{f(x_0) - 2f(x_1) + f(x_2)}{h^2}$$

for approximating the second derivative of a function at a point x_1 given $x_0 = x_1 - h$ and $x_2 = x_1 + h$.

[12 Marks]

(b) Let $f(x) = x \ln x + x$, and $x = [0.9,\ 1.3,\ 2.1,\ 2.5,\ 3.2]$. Use the three-point central-difference formula for the approximation of the second derivative of the function $f(x)$ to approximate value of $1/x$ at $x = 1.9$. Also compute the absolute error.

[13 Marks]

Q.4 **(a)** Given that the improved approximation of an integration or differentiation using the first-level Richardson extrapolation formula is A, show that the the more improved approximation using the second-level Richardson extrapolation is

$$A = \frac{16B_2 - B_1}{15}$$

where B_1 is the approximation to A using step sizes h and $\frac{h}{2}$ and B_2 is the approximation to A using step sizes $\frac{h}{2}$ and $\frac{h}{4}$.

[15 Marks]

(b) Apply the first-level Richardson extrapolation method to improve the value of the integral

$$\int_1^2 \frac{dx}{x}$$

using the Trapezoid rule starting with one subinterval.

[10 Marks]

Q.5 **(a)** Use three iterations of the Horner's and Newton-Raphson method to approximate the zero of the function

$$f(x) = 2x^4 - 3x^2 + 3x - 4$$

that is close to $x = 2$.

[10 Marks]

(b) Some techniques for solving the differential equation describing the deflection of a uniform beam with both ends fixed, subject to a load that is proportional to the distance from one end of the beam, require the positive roots of the function

$$f(x) = \cosh(x)\cos(x) - 1$$

In order to keep the function values within a reasonable range, it is better to consider the equivalent problem, of finding the zeros of

$$g(x) = \cos(x) - \frac{1}{\cosh(x)}$$

Using the Secant, approximate the root of $g(x)$ between 4.0 and 5.0 after two iterations.

[10 Marks]

Q.6 (a) Find the LU factorization of the the given matrix using the method of Gaussian elimination.

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 0 \\ 2 & 6 & 2 & 2 & 1 & 0 \\ 3 & 14 & 12 & 3 & 4 & 1 \\ 1 & 2 & 1 & 2 & 6 & 12 \\ 0 & 2 & 2 & -1 & -3 & -6 \\ 2 & 4 & 3 & 3 & 2 & 2 \end{bmatrix}$$

[10 Marks]

(b) Given that $\mathbf{b} = \begin{bmatrix} 4 & 12 & 23 & -21 & 15 & 3 \end{bmatrix}'$ use the *Doolittle* method to solve the system

$$\mathbf{Ax} = \mathbf{b}$$

[15 Marks]

⋆⋆⋆⋆⋆⋆⋆ **End of Examination** ⋆⋆⋆⋆⋆⋆⋆

Appendix

Lagrange Polynomial: A Lagrange polynomial of degree n passing throung $n+1$ points is given by

$$P_n = \sum_{k=0}^{n} L_k f(x_k)$$

where

$$L_k = \prod_{\substack{i=0 \\ i \neq k}}^{n} \left(\frac{x - x_i}{x_k - x_i} \right)$$

Hermite Polynomial: A Hermite polynomial of degree $2n+1$ for a set of $n+1$ points is given by

$$P_n(x) = \sum_{k=0}^{n} [H_k(x)Y_k + K_k(x)Z_k]$$

where $H_k = (x) = L_k^2(x)(1 - 2L_k'(x_k)(x - x_k))$ and $K_k(x) = L_k^2(x)(x - x_k)$

Richardson Extrapolation: The first step Richardson extrapolation is given by

$$A = \frac{4A_2 - A_1}{3}$$

where A_1 is an approximation with step size h and A_2 is an approximation with $h/2$.

The University of Zambia

Department of Physics

Computational Physics and Modeling II

(PHY 5922)

University Second Semester

Examination-2011/2012

Instructions **Max. Marks 100**

- **Time allowed:** Three (3) Hours.
- **All questions carry equal marks.**
- Marks for each question are shown in the square brackets [ ].
- Whenever necessary, use the information given in the **Appendix**
- **Answer:**

  i) Question one (1).

  ii) Any three (3) questions from 2, 3, 4, 5 and 6.

Q.1 (a) Given the initial-value problem of the following first order differential equation,

$$y' = \frac{(x-y)}{2}, \qquad y(0) = 1, \qquad h = 0.25$$

use Taylor's method of order four to find $y(0.25)$.

[14]

(b) Consider the differential equation

$$y' = f(x,y) = \begin{cases} y(-2x + 1/x) & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

on the interval $0 \leq x \leq 2$ with initial value $y(0) = 0.0$. Using $n = 10$ subdivisions on the given interval, use the classic Runge-Kutta method to determine the value of $y(0.2)$.

[6]

(c) Show that the following initial-value problem has a solution and that the solution is not unique.

$$y' = 3y^{2/3}, \qquad y(0) = 0$$

[5]

Q.2 (a) Derive the **Heun's predictor-corrector** method

$$y_{i+1} = y_i + \frac{h}{2}\left[f(x_i, y_i) + f(x_{i+1}, y^*_{i+1})\right]$$

for determing the solution of an initial value-problem

$$y' = f(x,y), \qquad y(x_0) = y_0, \qquad x_0 \leq x \leq x_n$$

where y^*_{i+1} is the predicted approximation of $y_i + 1$ from a single-step method, such as the Euler's method and $h = x_{i+1} - x_i$.

[10]

(b) Use the non-self-starting Heun method to determine $y(1)$ given the initial-value problem;

$$y' = 4e^{0.8x} - 0.5y, \qquad y(0) = 2$$

Hint: Use Euler's method as a predictor and $h = 1$.

[15]

Q.3 (a) The motion of a pendulum of length L subject to damping can be described by the angular displacement of the pendulum from vertical, θ as a function of time. If m is the mass of the pendulum, g the gravitational constant, and c the damping coefficient, then the ordinary differential equation initial-value problem describing this motion is

$$\theta'' + \frac{c}{mL}\theta' + \frac{g}{L}\sin(\theta) = 0; \qquad \theta(0) = a, \qquad \theta'(0) = b$$

Choosing $g/L = 1$ and $c/(mL) = 0.3$, $a = \pi/2$, and $b = 0$, convert the above second-order ordinary differential equation to a system of first-order ordinary differential equations. Using Euler's method for $h = 0.3$, determine the displacement θ and the angular speed θ' of the pendulum at $t = 0.6$ sec.

[15]

(b) Apply Euler's method with $n = 2$ to the system of ordinary differential equations

$$\begin{aligned} u_1' &= u_2 \\ u_2' &= u_3 \\ u_3' &= x + 2u_1 - 3u_2 + u_3 \end{aligned}$$

with initial conditions $u_1(0) = 4$, $u_2(0) = 3$, and $u_3(0) = 2$ on $x \in [0, 1]$.

[10]

Q.4 Consider the one-dimensional heat equation,

$$\frac{\partial u(x,t)}{\partial t} - \frac{\partial^2 u(x,t)}{\partial x^2} = 0, \qquad 0 < x < 1, \qquad 0 < t$$

show that it is a parabolic partial differential equation. Let $x_i = ih$ and $t_j = jk$, for $i = 0, 1, \cdots$ and $j = 0, 1, \cdots$, denote the mesh for the space and time intervals respectively and denote the solution at a grid point $u(x_i, t_j)$ as $u_{i,j}$. By approximating the partial derivatives with the difference formulas,

$$\frac{\partial u(x,t)}{\partial t} \Rightarrow \frac{1}{k}[u_{i,j+1} - u_{i,j}]$$

$$\frac{\partial^2 u(x,t)}{\partial x^2} \Rightarrow \frac{1}{h^2}[u_{i-1,j} - 2u_{i,j} + u_{i+1,j}]$$

where $k = \Delta t = 0.02$ and $h = \Delta x = 0.2$, show that

$$u_{i,j+1} = 0.5u_{i-1,j} + 0.5u_{i+1,j}$$

Given that the above heat equation represents temperature in a rod of unit length with initial temperature

$$u(x, 0) = x^4, \qquad \text{for} \qquad 0 < x < 1$$

and the temperatures at $x = 0$ and $x = 1$ are, respectively

$$u(0, t) = 0, \qquad u(1, t) = 1, \qquad \text{for} \qquad 0 < t$$

complete the following table of the solution for the equation for the first few time steps

t	$x =$ / $i =$ / j	0.0 / 0	0.2 / 1	0.4 / 2	0.6 / 3	0.8 / 4	1.0 / 5
0.00	0						
0.02	1						
0.04	2						

[25]

Q.5 Consider the one-dimensional wave equation,

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0, \qquad 0 < x < 1, \qquad 0 < t$$

show that it is a hyperbolic partial differential equation. Let $x_i = ih$ and $t_j = jk$, for $i = 0, 1, \cdots$ and $j = 0, 1, \cdots$, denote the mesh for the space and time intervals respectively and denote the solution at a grid point $u(x_i, t_j)$ as $u_{i,j}$. By approximating the partial derivatives with the difference

$$\frac{\partial^2 u(x,t)}{\partial t^2} \Rightarrow \frac{1}{k^2}[u_{i,j-1} - 2u_{i,j} + u_{i,j+1}]$$

$$\frac{\partial^2 u(x,t)}{\partial x^2} \Rightarrow \frac{1}{h^2}[u_{i-1,j} - 2u_{i,j} + u_{i+1,j}]$$

$$\frac{\partial u(x,t)}{\partial t} \Rightarrow \frac{1}{2k}[u_{i,j+1} - u_{i,j-1}]$$

where $k = \Delta t = 0.2$ and $h = \Delta x = 0.2$, show that

$$u_{i,j+1} = u_{i-1,j} + u_{i+1,j} - u_{i,j-1}$$

Given that the above heat equation represents a vibrating string of unit length with both ends held fixed and an initial condition

$$u(x,0) = x(1-x), \qquad \frac{\partial u(x,0)}{\partial t} = 0, \qquad 0 < x < 1$$

with boundary conditions

$$u(0,t) = 0, \qquad u(1,t) = 0, \qquad 0 < t$$

at $x = 0$ and $x = 1$, respectively, show that

$$u_{i,1} = 0.5u_{i-1,0} + 0.5u_{i+1,0}$$

Complete the following table of the solution of the above wave equation for the first time step.

	$x =$	0.0	0.2	0.4	0.6	0.8	1.0
	$i =$	0	1	2	3	4	5
t	j						
0.0	0						
0.2	1						

[25]

Q.6 **(a)** The deflection of a beam supported at both ends, subject to uniform loading along its length, is described by the boundary value problem for the ordinary differential equation;

$$y'' = \frac{T}{EI}y + \frac{wx(x-L)}{2EI}, \qquad 0 \le x \le L, \qquad y(0) = y(L) = 0$$

Choosing the parameters

$$L = 100, \qquad w = 100, \qquad E = 10^7, \qquad T = 500, \qquad I = 500$$

the problem is linear of the form

$$y'' = p(x)y' + q(x)y + r(x), \qquad a \le x \le b.$$

with boundary conditions $y(a) = 0$ and $y(b) = 0$. The solution can be obtained by solving the following initial-value problems over $a \le x \le b$:

$$u'' = q(x)u + r(x), \quad u(a) = 0, \quad u'(0) = 0$$

$$v'' = q(x)v, \qquad v(a) = 0, \quad v'(a) = 1$$

and is given by

$$y(x) = u(x) - \frac{u(b)}{v(b)}v(x).$$

i) Obtain the two second-order initial-value problems of the differential equation.

ii) Convert the two second-order initial-value problems in (i) to a system of four first order initial-value problem by change of variable.

iii) Using Euler's method and taking $h = \Delta x = 50$, determine the deflection of the beam at $x = 50$.

[25]

⋆⋆⋆⋆⋆⋆⋆ **End of Examination** ⋆⋆⋆⋆⋆⋆⋆

Appendix

Taylor's Expansion:

$$y_{i+1} = \sum_{k=0}^{n} \frac{y^{(k)}(x_i)h^k}{k!}$$

Euler's Method:

The Euler's method for finding the approximate solution of a first-order ODE, $y' = f(x, y)$, with initial condition$f(x_0) = y_0$, on an interval $[x_0, x_n]$ with step size $h = (x_n - x_0)/n$, is given by

$$y_{i+1} = y_i + hf(x_i, y_i)$$

where $h = (x_n - x_0)/n$ and n is the number of division between x_0 and x_n.

Classic Runge-Kutta Method:

The Runge-Kutta method for finding the approximate solution of a first-order ODE, $y' = f(x, y)$, with initial condition$f(x_0) = y_0$, on an interval $[x_0, x_n]$ with step size $h = (x_n - x_0)/n$, is given by

$$y_{i+1} = y_i + \frac{1}{6}k_1 + \frac{1}{3}k_2 + \frac{1}{3}k_3 + \frac{1}{6}k_4$$

where

$$\begin{aligned} k_1 &= hf(x_i, y_i) \\ k_2 &= hf(x_i + 0.5h, y_i + 0.5k_1) \\ k_3 &= hf(x_i + 0.5h, y_i + 0.5k_2) \\ k_4 &= hf(x_i + h, y_i + k_3) \end{aligned}$$

and $h = (x_n - x_0)/n$ and n is the number of division between x_0 and x_n.

Partial Differential Equations:

The general form of a second-order linear partial differential equation involving two independent variable x and y is:

$$a\frac{\partial^2 U}{\partial x^2} + b\frac{\partial^2 U}{\partial x \partial y} + c\frac{\partial^2 U}{\partial^2 y} + d\frac{\partial U}{\partial x} + b\frac{\partial U}{\partial y} + fU + g = 0$$

where a, b, c, d, e, f and g are constants.