# PRODUCTION OPTIMIZATION OF UNDERGROUND MINING OPERATIONS: A CASE STUDY OF THE RENCO MINE EXPANSION PROJECT.

BY

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# DECLARATION

I, CHARLES FREEMAN ALOLIBILA AKAYULI, hereby, declare that this research, or any part of it, has not been previously submitted for a degree at this or any other University.

CHARLES F A AKAYULI ....................................
signature

DATE: ....................................

## APPROVAL

This dissertation of CHARLES F A AKAYULI is approved as fulfilling part of the requirements for the award of the degree of Master of Mineral Sciences in Mining Engineering by the University of Zambia.

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## DEDICATION

This work is DEDICATED to my loving wife, OLIVIA and to the kids ANGELA, STEPHEN and CLARA who have endured three years of "ACADEMIC" separation

## ABSTRACT

This study was undertaken to develop an optimum production plan for an underground mine using Renco Mine, an underground gold Mine in Zimbabwe, as a case study.

A detailed structural analysis of the orebody's grade distribution has been done to establish technical parameters for production planning. This showed that the grade is lognormally distributed with a mean of 6.49g/t and a variance of $48.88 (g/t)^2$ and the experimental variogram can be modelled by the spherical scheme with a nugget of 30, a sill of 17, and a range of 30m.

The open-ended dynamic programming model has been used to compute the optimum production rates and cutoff grades for the project which yielded a Net Present Value(NPV) of Z$7,215,000. A computerised stoping and development generator has also been developed which calculates the number of stoping and mining blocks to maintain in order to achieve the production plan.

A single parameter sensitivity analysis has been done on the sensitive economic parameters used in executing the dynamic programme. It has been found out that the annual gold price increment rate and the annual cost escalation rate are sensitive and their changes can significantly affect the optimum production plan and the project NPV.

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## ABBREVIATIONS

| | |
|---|---|
| DCF | Discounted Cash Flow |
| NPV | Net Present Value |
| DCFROR | Discounted Cash Flow Rate Of Return |
| IRR | Internal Rate of Return |
| Z$,$ | Zimbabwe Dollar |
| US$ | United States Dollar |
| g/t | Grams per Tonne |
| Oz | Troy ounzes |
| t | metric tonne |
| LP | Linear Progrmming |
| DP | Dynamic Programming |
| IP | Integer Programming |
| RV | Random Variable |
| RF | Random Function |

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## GENERAL INTRODUCTION

## OBJECTIVE OF THIS STUDY

The management of Renco Mine, a subsidiary of RTZ, has been faced with the problem of declining gold production since the peak production of 1933kg in 1986. This decline in production is mainly due to the fall in ore grade and old age of treatment equipment. From the onset management identified two possible solutions:

(1) Improved mechanization to increase ore production and/or

(2) Expansion of the mill to handle increased tonnage.

Management chose the second option to expand the mill and new equipment has already been installed.

This option has the following advantages:

(i) The Capital expenditure utilised in the expansion project is less than that for mechanization.

(ii) Existing facilities and structures will be used to produce the necessary tonnages envisaged.

(iii) The labour does not need to be retrained.

This study is therefore about the determination of the annual production rate and grade that will maximise wealth, defined as the market value of the firm's shares, for the shareholders of the mine by maximising monetary returns in both the short-term and over the life of the mine. This is essentially done by calculating the optimum cut-off grade, determining the optimum production schedule and then sequencing the calculated optimum tonnages and grades over the remaining life of the mine.

## Structure of Work

The thesis is in five chapters:

Chapter 1 is an introductory chapter. All the essential details regarding Renco Mine are covered in this chapter.

Chapter 2 deals with orebody modelling. A grade-tonnage curve is developed for the mine by carrying out a statistical and geostatistical analysis of data from surface diamond drill boreholes and underground development sampling using a computer.

Chapter 3 gives an overview of the various aspects of mine production planning and the application of mathematical programming techniques in production planning.

Chapter 4 deals with production optimization. The first part of the chapter treats the economic aspects of mine planning. The cost and revenue structure of the mine are developed. The second section deals with production optimisation using the open-ended dynamic programming model.

Chapter 5 deals with the sensitivity analysis of the economic parameters considered sensitive to the project viability and profitability.

The last chapter is a discussion of the results obtained in the analyses,the conclusions arrived at and recommendations for improved productivity.

## INTRODUCTION TO RENCO MINE

### 1.1 Location

Renco Mine is located 75km south-east of Masvingo and 5km north of Bangala dam in the Nyajena Communal land close to the Muturukwe river at 20° 38'S and 31° 10'E. Fig. 1.1.

### 1.2 Geology

The surrounding area is characterised by a broken granite terrain covered with Brachystegia woodland and elevations of 500-800m above mean sea level (MSL). The mine is located within the northern Marginal Zone of the Limpopo Mobile Belt, near the southern gradational contact of a body of charnokite rocks mainly enderbite and is surrounded by biotite and granulite gneisses. In the mine, gold mineralization occurs in reef horizons which are present as sub parallel shear zones. The reef is a structurally complex tabular hydrothermal type orebody striking NE-SW and dipping at 30° SE. In several places various reef horizons have been identified from hangingwall to footwall with the red reef being the most consistent. Four main reef types namely, sulphide, silica and mylonite reefs and mineralised country rocks occur.

Gold occurs as very fine free gold of less than 10µm and as an intergrowth with maldorite. Bismuth occurs as free grains or in association with the gold or in contact with maldorite and gold. Sulphide minerals like pyrrhotite, chalcopyrite and pyrite occur in small amounts. Small

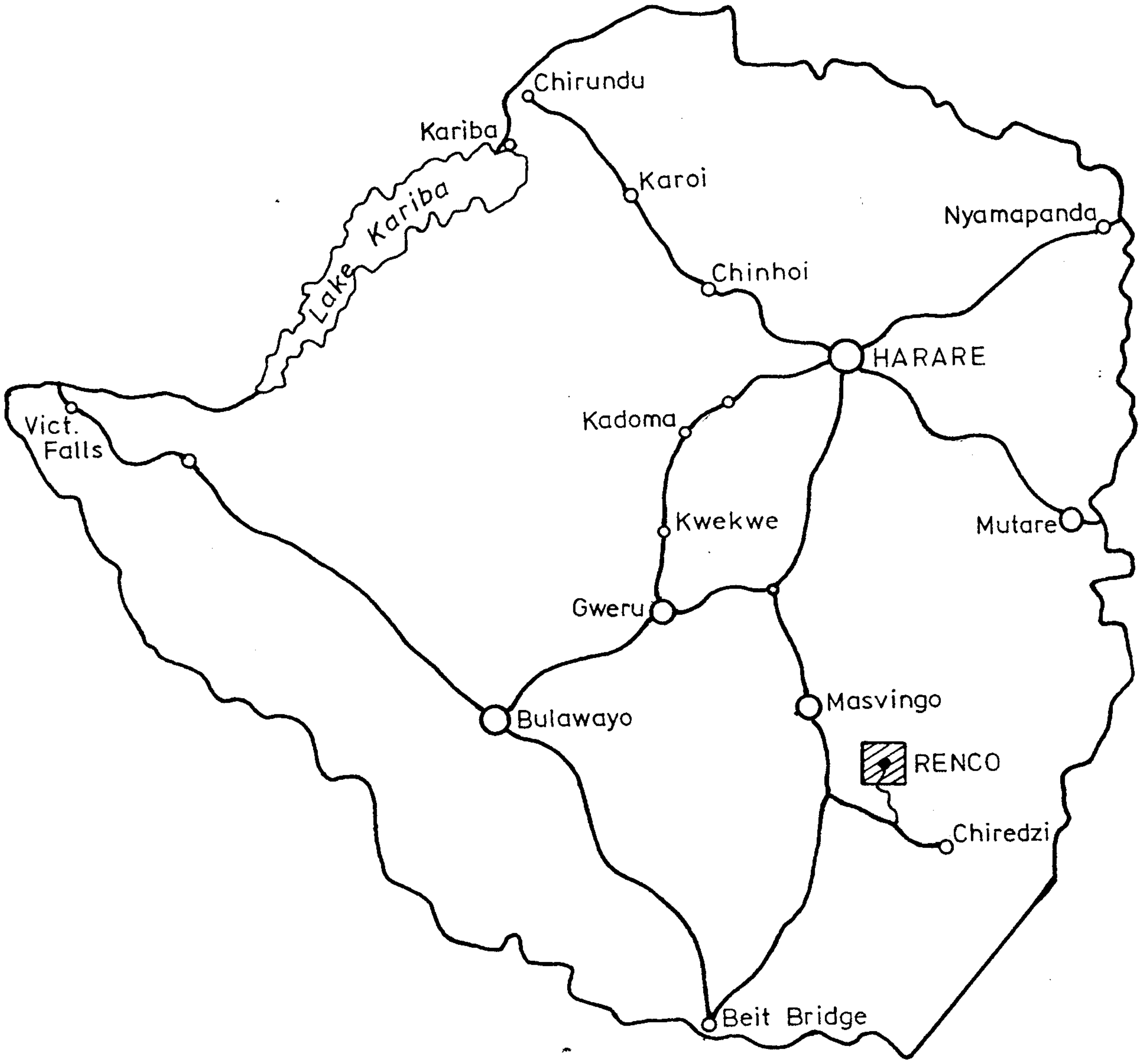



Fig. 1.1. LOCATION OF RENCO

### 1.3.3 Stoping

A down-dip underhand method of mining is practised with face scraping into gulleys and gulley scraping into boxes. The stope face is orientated perpendicular to the major joints and is usually carried at 60° to the raise. An average stope width of 1.29m is normally maintained. Support installations follow mine standards based on the joint strike directions and the brows created due to mining and the presence of dyke intrusions. In narrow stopes (≤1.5m) gum timber props spaced at 1m x 2m grid or solid matt packs of 1.5m long timber spaced at 3.9m x 3.9m centres are used. Stopes above 1.5m are supported using 2.7m x 16mm shepherd-crooked rebars at a 1.1m grid installed at the steepest angle possible. 1.8m x 16mm rebars are installed in tighter areas.

### 1.3.4 Ore Transfer and Hoisting

Tramming from boxes uses battery locos and 1.5t side tipping cars. 0.5t cocopans are used for hand tramming in sublevels. The shaft loading system in current use consists of ore and waste passes on 630, 570, 480 levels and 475 conveyor level.

### 1.3.5 Services

Ventilation of the mine is achieved by natural and mechanical means. The quarries area downcasts to all levels below 510 level. The main shaft downcasts. 4 x 50Hp x 1.2m diameter fans exhaust 52m³/s from the mine through the Normac Ventilation adit, No. 1 shaft and No. 2 shaft. Average wet and dry bulb temperatures are 24° C and 25° C respectively.

Drainage is achieved by directing all water in the mine to a 110m conical settling sump at 475m level. From here it is pumped to the surface with 3 x 125Hp multi-stage KSB 80/4 WKL pumps. 35Hp flight pumps are used at the shaft bottom.

## 1.4 Metallurgy

Due to the fine grain size of the gold particles, adequate liberation requires a grind of 100% of -200 mesh (d80 = 43µm). This is achieved through crushing with jaw and gyratory crushers and grinding in two steel lined grate-discharge ball mills in closed circuit with hydrocyclones. A final pulp density of 45% solids is achieved in a Dorr Oliver 480A thickener. After pre-aeration in flotation cells, 10-20g/t mercuric chloride and 0.746 kg/t sodium cyanide is added before leaching. After 24 hours leaching a pH of 9.5 is maintained by adding sodium hydroxide. Gold is recovered by the carbon in-pulp process. Gold bars are produced after elution and electrowinning.

## 1.5 Ore Reserves

A mine mineral inventory is developed and maintained using a geostatistical computer software developed by Rio Tinto Technical Services. Two programmes are in use. Surface borehole data of grade and thickness are kriged to show indicated ore reserves. Indicator kriging is used to check block kriging. Underground development and stope sampling data are also kriged to evaluate blocks more accurately for mining. Four categories of ore reserves are used in Renco:

1. Proved Reserves:- Computed from fully developed mining blocks.
2. Probable Reserves:- Computed from blocks which have been only partially development.
3. Reclamation reserves:- Calculated from blocks or partial blocks which are not minable at the present time but could be mined later.
4. Indicated reserves:- Calculated from surface diamond drill samples.

## 2. OREBODY MODELLING AND GRADE TONNAGE RELATIONSHIPS

### 2.1 Modelling Techniques

The selection of a specific mathematical method for modelling an orebody depends on the geological considerations, exploration or sampling method, the availability, reliability and volume of data, the specific purpose of the estimation and the requirements of accuracy[1]. The best model is the one that results in the smallest difference between the estimated and the actual values.

Essentially, there are two methods of estimating mineral reserves namely: Conventional methods based on three principles:

1. the rule of gradual changes
2. the rule of nearest point or equal influence
3. the rule of generalization

and predictive methods using statistical or geostatistical theory.

### 2.2 Conventional Methods

The rule of gradual changes or law of linear functions implies that all sample elements of a mineral body change gradually and continuously as a linear function along a straight line connecting adjacent sample points. This rule is applied in the traditional triangular method of calculating reserves. The technique involves joining all drill holes or sample points by straight lines into systems of triangles. Each triangle represents the base of an imaginary prism of some thickness.The average grade of each prism is usually calculated as the arithmetic mean of the three samples forming the corners of the triangle. Also on the basis of this rule, the cross-sectional method has been used to compute reserves when fences of drill holes extend across an irregularly shaped mineral deposit. Each mineral block is defined by two sections and each end block by a single section.The areas of mineral in each section is calculated. The value is calculated by using average values of the samples and later the volume of the block is computed. Multiplying this volume by the tonnage factor and dividing by the volume will give the volume-tonnage weighted value of the block.

The rule of nearest point or equal influence suggests that the value of any point between adjacent samples is constant and equal to the value of the nearest sample. The polygonal and rectangular methods are based on this principle.The area under study is divided into polygons or rectangles for uniformly spaced sample spacing. Each polygon is assigned the sample value enclosed within it.

The polygons are constructed by drawing perpendicular bisectors to lines connecting all samples. The size of the polygon is decided by the area of influence which is usually developed as a rule of thumb from geological and mining experience.

The rule of generalization is usually arbitrarily applied as a matter of judgement reflecting past opinions and experience.

## 1.3 The Weighted Moving Average (WMA) Method

The weighted moving average method of reserves estimation is based on the rule of gradual changes. It uses straightforward mathematics for weighting the influence of all surrounding samples upon the block being evaluated. The equation below describes the mathematical model of the weighted moving average method[2].

$$e_{xy} = \frac{\sum \dfrac{r_i}{(a^k + d^k)}}{\sum \dfrac{1}{a^k + d^k}} \qquad (1)$$

where $e_{xy}$ = estimated value of the parameter at coordinate points (x, y).

$r_i$ = value of the parameter at coordinate points (x, y).

n = number of samples

di = distance between points (x, y) and ($x_i$ and $y_i$)

a = constant, a function of local variability for the parameter.

K = constant, a function of the regional variability of the parameter.

The constants a and k are orebody and parameter dependent and require evaluation for each deposit and each parameter, using subjective or empirical methods. Note that when K = 1 and a = 0, the WMA method reduces to Inverse Distance Method (IDM)

| | | |
|---|---|---|
| K = 2 | a = 0 | the MWA is identical to Inverse Distance Squared (IDS) method. |
| K = 0 | | the WMA gives the arithmetic mean of the samples for the estimation at any point. |

## 2.4 Advantages and Disadvantages of the Conventional Analysis Techniques

The Conventional ore reserves estimation methods are cheap and simple to handle since they do not involve rigorous mathematical applications. Some of the methods like the polygonal and IDS methods are easy to computerise. Where the orebody is uniform, the conventional methods give a sufficient estimation of reserves so the additional costs incurred in using more sophisticated methods are not justified.

The triangular method has some added advantages in that it uses data from three rather than one drill hole and so some reasonable weighting is possible. The calculation of the area is easily done by geometry or by coordinates using a programmable calculator.

The major shortcomings of the conventional methods are:

(1) Generally, the area of influence given to individual drill hole assays far exceeds their actual area of influence. Most often the actual mean grade contained within such an area may be quite different or unrelated to the drill hole assays obtained.

(2) They do not provide the measure of confidence in the overall tonnage and mean grade estimates. A knowledge of the uncertainty associated with these estimates, particularly grade, is important for evaluating the economics of mine development. Use of the conventional methods precludes risk analysis and assessment of risk associated with profitability criteria. This limitation is not, however, inherent in the data but rather, it occurs because these methods fail to fully utilise the data.

## 2.5 Statistical Methods

Statistical analysis provides the framework for assessing the uncertainty or reliability of geological estimates. Classical statistical techniques are based on the assumption that observed variations are due to random fluctuations i.e. there are no trends in the data. The most important parameters of the probability distribution relevant to the deposit characteristics are the mean (expected value) and the standard deviation.

The reliability of the expected value depends on the sample size and the variation within the deposit itself.

Variability is estimated by the standard deviation of the sample standard deviation. The reliability of the mean is estimated by the standard error of the mean.

## 2.6 Geostatistical Method

### 2.6.1 The Theory of Regionalised Variables

The geostatistical method of ore reserves estimation is based on the theory of regionalised variables.[3] A regionalised variable is one that is spatially distributed i.e. parameter values at any point depend on the sample size, its location within the orebody and its distance from other samples. A regionalised variable has two characteristics as regards to variability.

(i) a local random, erratic aspect

(ii) a general (average) structural aspect which requires a certain functional representation.

These characteristics of the regionalised variable are expressed by the definition of the random function (RF).

One realization cannot provide the probability law of the random function. To make statistical inference possible, supplementary hypotheses about the RF need to be introduced in order to reduce the number of parameters on which the law depends. The assumptions about the probability law should be relaxed and reduced to the minimum.

The minimum assumption about the random function needed to solve the problem is that it is intrinsic. The minimum characteristic of the model that is necessary for a statistical inference is that of a quasi-stationary covariance function. To be acceptable the model chosen must meet the following conditions:-

(i) It must be capable of statistical inference so that it should be possible to estimate the parameters of the model.

(ii) It must be operational in that it gives an answer to the question posed.

(iii) The model must be compatible (has to fit the data).

(iv) Its prediction must be checked by experience. A model meeting these conditions is capable of resolving most of the problems of mine valuation.

### Stationary and Non-Stationary Processes

1) **Strictly Stationary Random variables (R.V)**

A random variable (R.V) is strictly stationary if the distribution of the R.V. at points $X_1$, $X_2$, $X_3$ .... $X_k$ is the same as that of the R.V. at a distance h away.

i.e. $P\{X_1, X_2 .... X_K\} = P\{X_1 + h, X_2 + h .. X_{k+h}\}$

this implies that (2)

a) The expectation of the R.V. $Z_{(x)}$ at $X_o$ is:

$$M(x_o) = E\{Z(x_o)\} = M(x_o+h) \quad (3)$$

Since Z(x) is stationary, $M(x_o)$ is constant and independent of the support point ($X_o$). This implies that this mean is the equal to the population mean (m) everywhere.

$$M = E\{z(x)\} \quad (4)$$

b) the covariance between $X_o$ and $X_o + h$ is C ($X_o$,h) and depends only on the vector h.

$$C(h) = E\{Z(x).z(x+h)\} \quad (5)$$

Since this is often not the case this assumption is inapplicable in many cases.

ii) **Second Order Stationarity**

A process is 2nd order stationary if

a) the expected value of the random variable at each point is independent for that point

$$E\{z(x)\} = m(x) \quad (6)$$

(b) the covariance of the R.V at two points depends only on the distance separating them h.

$$E\{z(x)-m, z(x+h)-m\} = C(x,x+h) = C(h) \qquad (7)$$

This process has a finite variance which is equal to the variance of Z(x) = C(o). Since this is often not the case further relaxations are necessary.

iii) <u>Processes with Stationary Increments</u>

In this case the random function (RF) need not be stationary at points x and x + h; and only the increments Z(x) - Z(x+h) are assumed stationary. The characteristics of the model are then:

$$E\{z(x+h)-z(x)\} = m(x)$$

$$Var\{z(x+h)-z(x)\} = 2 \ (h) \qquad (8)$$

The first relation implies that $m(h+h') = m(h) + m(h \ )$ m(h) is called the drift. The function (h) is called the semi-variogram. In practice m(h) is taken as zero. The following relation can also be derived

$$(h) = C(0) + C(h) \qquad (9)$$

If the RF is second order stationary it is also intrinsic but the converse is not true.

(iv) <u>Non-Stationary Process</u>

When trend is observable in a deposit, the above assumptions are insufficient. The expectation varies in the region. If x, y and x + h are taken in the region,

$$E\{z(x)\} = m(x) \text{ or } E\{z(x) - z(y)\} = m(x) + m(y) \qquad (10)$$

$$\text{with} \quad m(x) = \sum a_i f_i(x) \qquad (11)$$

When the process is non stationary, the estimation procedure is as follows:

- The functions f(x) have to be determined as to kind and number
- The $a_i$ s are unknown coefficients and have to be determined
- The drift, m(x), and its form has to be estimated.

The RF Z(x) is now considered to be the sum of the drift and the residual RF Y(x) such that the expectation of the residual function is zero.

$E\{Y(x)\} = 0$ and

$$Cov(Z(x).Z(x+h)) = E\{Z(x).Z(x+h)\} \qquad (12)$$

$$\text{or } Var(Z(x+h) - Z(x)) = E\{Y(x+h) - Y(x)\} = 2\ (h)$$

This implies that to undertake any operation in linear geostatistics the first and second moments must be known.

<u>1st Moment:</u> Mean

$E\{Z(x)\} = m$

2nd Moment:

(a) Variance, Covariance, Variogram

$Var(Z(x) = E\{[Z(x) - m(x)]^2\}$ which is a function of x.

(b) the covariance for two RVs $Z(x_1)$ and $Z(x_2)$ centred on points $x_1$ and $x_2$ is defined as:

$$C(x_1, x_2) = E\{[Z(x_1 - m(x_1)][Z(x_2 - m(x_2)]\} \tag{13}$$

(c) the variogram is defined notationally

$$2\ (x_1, x_2) = Var([Z(x_1) - Z(x_2)]) \tag{14}$$

Properties of the covariance and the Variogram

(i). Positive definite conditions

If Y is any finite linear combination of the RF Z(x) of the type $Y = \sum \lambda_i z(x_i)$ for any weights $\lambda_i$,

this linear combination is a RV and its variance is never negative var $\{Y\} \geq 0$ This is written as:

$$Var\ \{Y\} = \sum_i \sum_j \lambda_i \lambda_j C(x_i - x_j) \geq 0 \tag{16}$$

The covariance function should be such that the Var {Y} is never zero. Hence the function C(h), the covariance of the RF Z(x) is said to be positive definite.

The variance can be written in terms of the variogram as:

$$\mathrm{Var}\ \{Y\} = C(0)\sum_i \lambda_i \sum_j \lambda_j - \sum_i \sum_j \ (x_i - x_j) \qquad (17)$$

Where the variance C(0), does not exist, only the intrinsic hypothesis is assumed and the variance is written as:

$$\mathrm{Var}\ \{Y\} = - \sum\sum \lambda_i \lambda_j \ (x_i - x_j) \qquad (18)$$

on condition that $\sum \lambda_i = 0$

By definition $-\ (h)$ is said to be a conditional positive definite function.

(ii) Authorised Linear Consideration

When only the intrinsic hypothesis is assumed, the only linear combination that have a finite variance are those verifying the weight condition $\sum \lambda_i = 0$. In linear geostatistics, only those authorised linear combinations are considered and there is no need to calculate the variance or the expectation of the covariance, the variogram alone is sufficient.

(iii) Properties of the covariance

$C(0) = \mathrm{Var}\ \{Z(x)\} \geq 0$ an apriori variance cannot be negative

$C(h) = C(-h)$ the convariance is an even function

$C(h) \leq C(o)$ Scharz's inequality {19}

(iv) Properties of the variogram

$(0) = 0 \quad (h) = (-h) \geq 0$

The following properties are also obtainable.

$C(h)$ 0 when h and in practice we can put $C(h) = 0$ once $h \geq a$.

The distance a beyond which $C(h)$ can be considered equal to zero is called the range beyond which there is absence of spatial correlation.

$() = \text{var} \{Z(x)\} = C(o)$

This is called the sill of the RF.

2.6.2 Variogram Modelling

Definition of a Variogram

A variogram expresses the similarity or dissimilarity which exists between any regionalised variable at one point and another some distance away. This is done by considering the squared difference between values at all possible points x and x + h distance h apart. It condenses them in the relationship:

$$2 \ (h) = \text{average } [Z(x+h) - Z(x)]^2$$

(a) The variogram can be used to estimate the errors of estimation.

(b) It can be used to interpret the geological features.

(i) The continuity present in the deposit

(ii) The zone of influence. The zone of influence in the case of the transition schemes extends up to the distance called the 'range'

(iii) Where second order stationarity of increments is obtained, the range is the distance beyond which samples are not correlated and the semi-variogram reaches the sill value.

(iv) Anisotropies: Variograms may show differences in behavior in different directions by producing different values of variograms in some directions.

Methods of Estimating the Variogram

The theoretical variogram is defined as:

$$(h) = \frac{1}{2} E\{[Z(x) - Z(x+h)]\}$$

$$= \frac{1}{2} \int_{v} [Z(x) \times Z(x+h)]^2 \, dx \qquad (21)$$

Which is an integral obtainable only with an infinite number of pairs and an infinite field. The infinite field is unavailable and only the local variogram in the finite field L over which samples are taken is obtainable. This is:

$$(h)^1 = \frac{1}{2(L-h)} \int_{0}^{L-h} [Z(x_i) - Z(x_i+h)]^2 \, dx \qquad (22)$$

This function involves infinite number of pairs. With a finite number of pairs only an experiment model can be calculated as:

$$(h)^* = \frac{1}{2N(h)} \sum_{i} [Z(x_i) - Z(x_i + h)]^2 \qquad (23)$$

where:

N(h) is the number of pairs of [Z (xi) - Z (xi + h)] available at distance xi

The experimental variogram is an adequate estimator of the local variogram since the expectations of the local and experimental variograms are equal to the theoretical variogram.

$$E\{{}^*(h)\} = E\{{}^1(h)\} = (h) \qquad (24)$$

Variograms can be drawn in two or three dimensions and algorithms exist to handle these. In this study, the data are irregularly distributed. Two dimensional variograms were estimated.

<u>Estimating the Experimental Variogram of Irregularly Distributed Data in two Dimensions.</u>

In this case the pairs of samples are grouped into classes of distance and direction. The function to estimate is:

$$^*(\theta, h_i) = \frac{1}{\Delta L \, \Delta \alpha} \int_{-\frac{\Delta L}{2}}^{\frac{\Delta L}{2}} \int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} [h + x, \theta + \phi] \, dx \, d\phi \qquad (25)$$

This is a regularised function to which a model can be fitted.

A complete computer software Geo-EAS developed by the United States Environmental Protection Agency in conjunction with the Centre of Geostatistics, Fontainbleau was used in the calculation of variograms. All pairs between points $x_1$ and $x_2$ are used so that for each lag interval $hi_i$:

$$(hi) = \frac{\sum_{n_i} (x_1 x_2)}{n_i} \qquad (26)$$

Where $n_i$ is the number of pairs $x_1 x_2$ grouped in the same distance class. What is obtained by this algorithm is the set of one dimensional smoothed variograms in several selected directions making an angular sampling of a two dimensional array.

### 2.6.3 Theoretical Models of the Variogram

Several intrinsic schemes or mathematical models have been proposed to represent a probability function that describes the behaviour of the experimental variogram. Examples of such models are the linear, parabolic, De Wisjian, Spherical and exponential models. The spherical model is the most commonly employed model in Mining and is completely defined by the following equation:-

$$\begin{aligned} (h) &= Co + C \; \frac{3}{2}\frac{h}{a} - \frac{1}{2}\frac{h^3}{a^3} \qquad \text{for } h \leq a \\ &= Co + C \qquad\qquad\qquad \text{for } h > a \end{aligned} \qquad (27)$$

(h) is the variogram at distance h,

C(o) is the nugget effect,

a zone of influence or range, and

C0 + C the sill, which refers to the variance of the samples.

Figure 2.1 is an example of spherical model.

### 2.6.4 Geostatistical Variances

The variogram function, the shape and size of the mineralised blocks are used to calculate several variances employed in geostatistical studies of mineralisation. The most important of these are Extension Variance, Estimation Variance, and the Kriging Variance. These variances are defined notationally as:

$$\sigma^2_{ex} = E\,[(z(v) - Z(v))^2]$$

$$= \frac{1}{v^2}\int_v\int_v c(x-y)\,dx\,dy + \frac{1}{(v^1)^2}\int_{v1}\int_{v1} c(x-y)\,dxdy$$

$$- \frac{2}{vv^1}\int_v\int_{v1} c(x-y)\,dxdy \qquad (28)$$

and using the relationship of equation (4)

$$\sigma^2_{ex} = \frac{2}{vv^1}\int_v\int_{v1} (x-y)\,dxdy - \frac{1}{v^2}\int_v\int_v (x-y)\,dx\,dy$$

$$- \frac{1}{(v^1)^2}\int_{v1}\int_{v1} (x-y)\,dxdy \qquad (29)$$

where $\sigma^2_{ex}$ is the extension variance

The estimation variance is the variance of error produced when the expected value Y(V) is estimated by the mean of the sample values $Y^*(V)$

$$\sigma^2_{ex} = \frac{2}{nv} \int_v \sum_{i=1}^{n} (x-x_i)\,dx - \frac{1}{v^2} \int_v \int_v (x-y)\,dxdy$$

$$- \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} (x_i - x_j) \qquad (30)$$

The right hand size can be rewritten using the mean values of the variograms as:

$$\sigma^2_{ex} = 2 \quad (V,v) - \quad (V,V) - \quad (v,v) \qquad (31)$$

Where $\sigma^2_{ex}$ is the estimation variance

(V,v) represents the mean value of (h) when one extremity of the vector h describes the domain V(x) and the other extremity independently describes the domain V(x')

2.6.5 Kriging

The true unknown grade for a block V can be estimated by using samples of known grades Zi (i = 1,2,....n). The estimate $Z^*$ is a linear combination of weighted grades obtained from the n samples. Kriging provides the Best Linear Unbiased Estimator of the unknown grade.

$$Z^* = \sum_{i=1} \lambda_i Z_i \qquad (32)$$

From the intrinsic hypothesis, E[Zi] = m hence $\sum_i \lambda i = 1$ (33)

Where $\lambda i$ is the weight of the sample

The best estimator is that which minimises the variance of the error term, Var (Z*-Z)

By definition:

$$\sigma^2_e = \mathrm{Var}[Z^x - Z] = E\,[(\sum \lambda i\ Zi - Z)^2]$$

$$= \sum_i \sum_j \lambda_i \lambda_j \sigma_{ij} - 2 \sum_i \lambda_{iz} + \sigma^2_z \qquad (34)$$

Where $\sigma_{ij}$ = covariance between the grades of the samples

$\sigma_{iz}$ = covariance between the grades of the samples grade of the block

$\sigma^2_z$ = variance of the true grades in the block

The optimization problem is therefore a constrained ore of the form

Minimize $\sigma^2_e$

Subject to $\sum_i \lambda_i = 1$

Using Lagrange multiplier method to solve the contrained minimisation problem, we produce unconstrained minimization problem of the form

$$\sum_i \lambda_j \sigma_{ij} + \mu = \sigma_{iz} \quad \text{for} \quad i = 1, n \qquad (35)$$

$$\sum \lambda_i = 1$$

where $\mu$ is half the Lagrange multiplier and is the new unknown. The problem is solved in matrix notation as AX =B where A, B and X are given below as

$$
A = \begin{matrix} \sigma_{11} & \sigma_{12} & .. & \sigma_{1n} & 1 \\ \sigma_{21} & \sigma_{22} & .. & \sigma_{2n} & 1 \\ .. & .. & .. & .. & . \\ .. & .. & .. & .. & . \\ \sigma_{n1} & \sigma_{n2} & .. & \sigma_{nn} & . \\ 1 & 1 & .. & 1 & 0 \end{matrix}, \quad X = \begin{matrix} \lambda_1 \\ \lambda_2 \\ . \\ . \\ \lambda_n \\ \mu \end{matrix}, \quad B = \begin{matrix} \sigma_{12} \\ \sigma_{22} \\ .. \\ .. \\ \sigma_{nz} \\ 1 \end{matrix}
$$

The weight $\lambda_i$ is determined and the average grade of the block Z* is computed from equation (31)

The Kriging variance is then computed as

$$\sigma_k^2 = \sigma_z^2 - \mu - \sum_{i}^{n} \lambda_i \sigma_{iz} \tag{36}$$

## 2.7 Data Analysis of Renco Mine Using Geo-EAS Software

### 2.7.1 The Geo-EAS Software

Geo-EAS (Geostatistical Environmental Assessment Software) is a collection of interactive software tools for performing two-dimensional geostatistical analyses of spatially distributed data. Programs are provided for data file management, data transformations, univariate statistics, variogram analysis, cross validation, Kriging, Contour mapping, post plots and line/scatter graphs - features such as hierachical menus, informative messages full-screen data entry, parameter files, and graphical displays are used to provide a high degree of interactivity, and an intimate review of results.

## THE SYSTEM MENU

The programmes comprising the Geo-EAS system can be run on floppy diskettes. However, a programme has been provided on the distribution diskettes which allows access to all programs from a common menu, called the System Menu. The Geo-EAS menu system comprises of the following programmes.

| | | | |
|---|---|---|---|
| 1. | Data prep | 8. | Krige |
| 2. | Trans | 9. | Conrec |
| 3. | Stat 1 | 10. | Xygraph |
| 4. | Scatter | 11. | Postplot |
| 5. | Prevar | 12. | Hpplot |
| 6. | Vario | 13. | View |
| 7. | X Valid | 14. | Quit |

A brief description of the System Menu is as follows:

1. DATAPREP

Dataprep provides utilities for Geo-EAS data files. It has two divisions, the DOS utilities and the file operators. The DOS utilities include utilities to manipulate Geo-EAS data files.

2. TRANS

This programme is designed to create, delete or modify Geo-EAS data file variables. The operations may be unary, binary, or an indicator transform operation. The results generated by the specified operation may replace the contents of an existing variable or a new variable may be created.

3. STAT1

Stat 1 is an interactive programme which computes basic univariate statistics and displays histograms or probability plots for variables in a Geo-EAS data set. Options are available for calculating statistics on natural logs of the selected variable for specifying a variable used as a "weighting factor," and performing calculations on subsets of the input data through the use of upper and lower limits.

4. SCATTER

Scatter produce plots of variable pairs in a Geo-EAS data file. Options allow for log and semi-log plots and for a regression line to be calculated.

5. PREVAR

Prevar is a preprocessor programme for the programme vario. All variogram calculations use the distance and relative direction between pairs of points in the sampled area. Prevar computes these so that variogram parameters may be changed and variograms recalculated quickly in vario.

6. VARIO

Vario is a two-dimensional variogram analysis and modelling programme Vario uses a pair comparison file (PCF) produced by prevar to calculate variogram values and other statistics for a specified set of pair distance intervals called lags. Tolerances may be specified for pair directions and lag distance intervals.

7. *X VALID*

This is the cross-validation programme. It involves estimating values at each sample location in an area by kriging with the neighbouring sample values (excluding the value of the point being estimated). The estimates are compared to the original observations in order to test if the hypothetical variogram model and neighbourhood search parameters will accurately reproduce the spatial variability of the sampled observations.

8. KRIGE

Krige is an interactive programme for performing two dimensional kriging. A rectangular grid of kriged estimates is created and stored in a Geo-EAS data file. Contour plots may be generated from these gridded estimates with the programme Concrec. Options are provided to control the type of kriging, the neighbourhood search area, the grid spacing and extents, and the variogram model for each variable kriged.

9. POSTPLOT

Postplot produces a plot of (2D) sample locations and values for a variable in a Geo-EAS data file. Sample locations may be marked with a symbol, value, or both.

10. XYGRAPH

XY Graph produces line and/or scatter plots for up to six variables in a Geo-EAS data file. Plots of up to six dependent variables with one independent-variable can be obtained.

11. CONREC

Concrec produces contour maps of variables with gridded coordinates in a Geo-EAS data file. The data must form a complete grid. A grid is a set of values with equally spaced X and Y coordinates which form a rectangle. A file called a "metacode" file is created for redisplay or producing a hard copy.

12. VIEW

View displays on the screen the graphs contained in metacode files.

13. HPPLOT

Hpplot translates the device independent plotting instructions in metacode files into a file of HPGL plotting commands. The output file can be routed to a HP plotter by setting up the serial port (COM) by running a batch file HPSETUP.Bat and then using the DOS PRINT command to send the file to the plotter.

GEO-EAS DATA FILES

All the programmes in the system use a common format for data files. Data files are simple ASCII text files which may be created with any text editor.

### 2.7.2 Input Data Description

Two sets of data were used for structural analysis. These are the underground development sampling data from the North-Western section of the mine and surface diamond drill borehole logs from the South-East section where underground development and surface diamond drilling were taking place concurrently. A total of 257 underground data and 167 surface data were used in this study. (Appendices 1 and 2)

### 2.7.3 Coordinate System

Two coordinate systems are used in Renco Mine: the mine coordinates and the geostatistics grid. The major axes of the geostatistical grid are parallel to and at right angle to the regional strike of the Red reef. The origin of the geostatistical grid has the following parameters.

| | MINE COORDINATES | GEOSTATISTICAL GRID |
|---|---|---|
| Origin X | + 82200 | 0 |
| Origin Y | - 17800 | 0 |

### 2.7.4 Definition of Input Variables

The diamond drill data for this study was extracted from Renco Mine geological database. Each borehole is defined by the borehole number and coordinates (X,Y,Z,Y2) the variables of interest are the grade in grams per tonne (g/t) and the stoping width or thickness in centimetres.

Development data are also extracted from the database. They represent underground sampling of drives and raises. In all cases, a minimum thickness of 100cm is allowed as to conform to the minimum stoping width of 100cm.

### 2.7.5 Statistical Analysis

(i) <u>Basic Statistical Analysis of Surface Diamond Drill Data</u>

The program STAT1 within the Geo-EAS software was used to perform univariate statistical analysis of the data. Such a statistical study is necessary for every variographic study so as to enable comparison between geological hypothesis. (Journel and Huijbregts, 1978)[4]

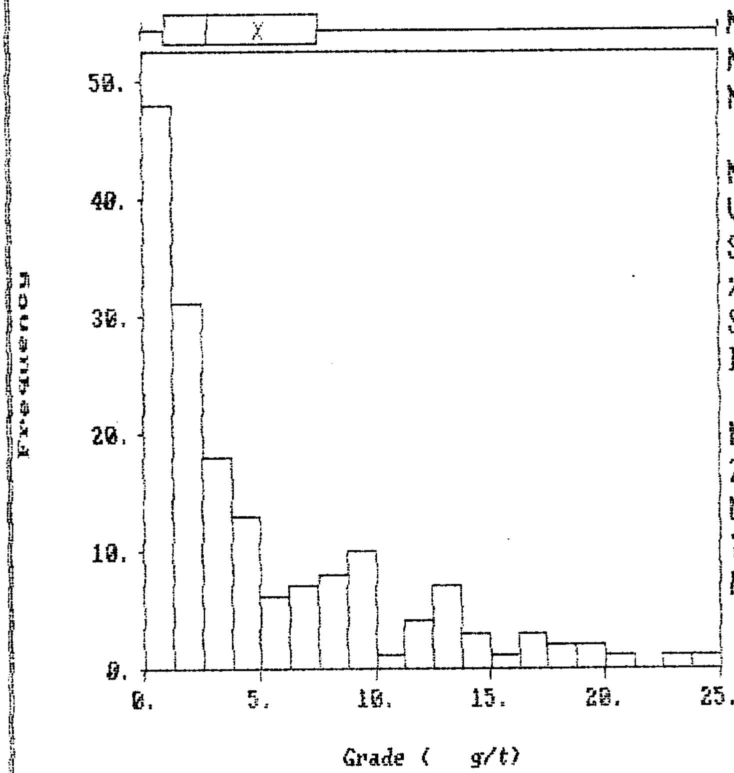
Analysis of the raw data gave means of 5.09g/t and 144cm and variance of 36.74 and 5749.80 for the grade and stope width respectively. The data was trimmed using a lower cut off of 0.7g/t and an upper cut off of 25g/t. Table 2.1 shows the results obtained. The lower variances realised is justification for the trimming. Figures. 2.2 and 2.3 show the histograms of the raw data and figures 2.4 and 2.5 show the histogram and probability plot respectively of the trimmed grade values.The histograms show a strong positive skew suggesting possibly a log normal distribution.

TABLE 2.1 PARAMETERS OF TRIMMED DATA

| Variable | No of Samples | Mean | Variance | Coeff of Variance % | Coeff of Skewness |
|---|---|---|---|---|---|
| Grade(g/t) | 132 | 6.224 | 29.661 | 87.510 | 1.281 |
| Stope W(cm) | 132 | 149.26 | 6211.58 | 52.81 | 2.37 |

A natural log transformation of the trimmed grade values was done and the resulting histograms showed a more or less normal distribution with a coefficient of skewness of 0.0079. The probability plot gave a near straight line plot (figures 2.6 and 2.7) These suggest that the grade is lognormally distributed. The stope width, however, still showed some degree of skewness but at a reduced level after trimming.This is

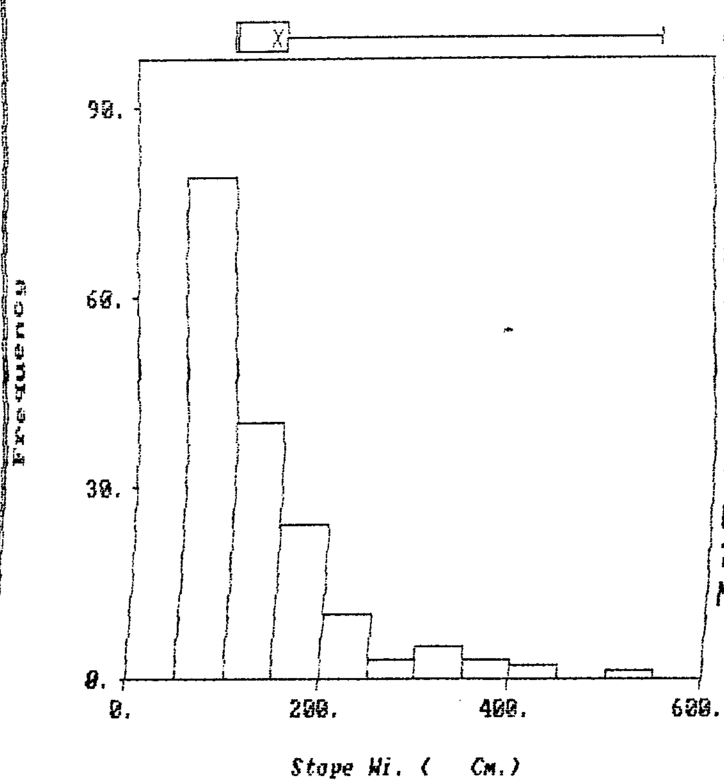
Figure 2.2: Histogram of Grade: Surface Borehole Data
Data file: renco.dat




Statistics

| | |
|---|---|
| N Total : | 167 |
| N Miss : | 0 |
| N Used : | 167 |
| Mean : | 4.968 |
| Variance: | 29.394 |
| Std. Dev: | 5.422 |
| % C.V. : | 109.127 |
| Skewness: | 1.447 |
| Kurtosis: | 4.607 |
| Minimum : | .000 |
| 25th % : | .938 |
| Median : | 2.770 |
| 75th % : | 7.590 |
| Maximum : | 25.000 |

Figure 2.3: Histogam of Thickness:Surface Borehole Data
Data file: renco.dat




Statistics

| | |
|---|---|
| N Total : | 167 |
| N Miss : | 0 |
| N Used : | 167 |
| Mean : | 144.144 |
| Variance: | 5749.798 |
| Std. Dev: | 75.827 |
| % C.V. : | 52.605 |
| Skewness: | 2.449 |
| Kurtosis: | 9.704 |
| Minimum : | 100.000 |
| 25th % : | 100.000 |
| Median : | 104.000 |
| 75th % : | 155.000 |
| Maximum : | 547.000 |

Figure 2.4: Histogram of Trimmed Grade Values:Surface Data
Data file: ren1.dat

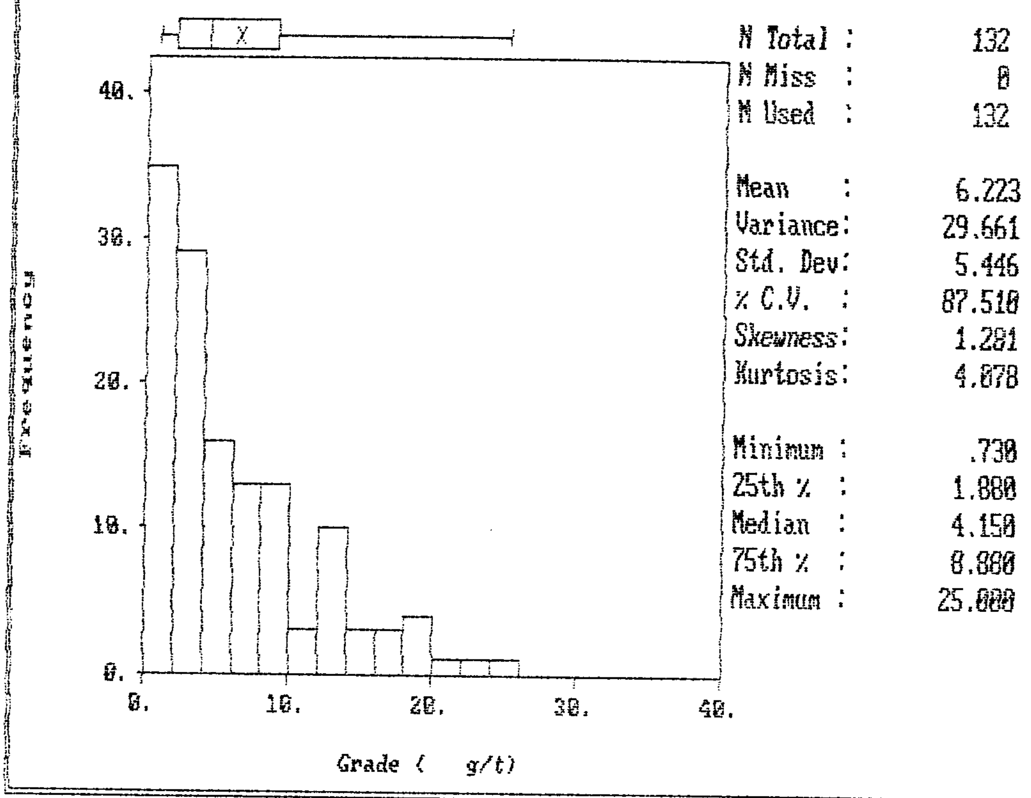



Fig. 2.5 Normal Probability Plot for Grade
Data file: ren1.dat

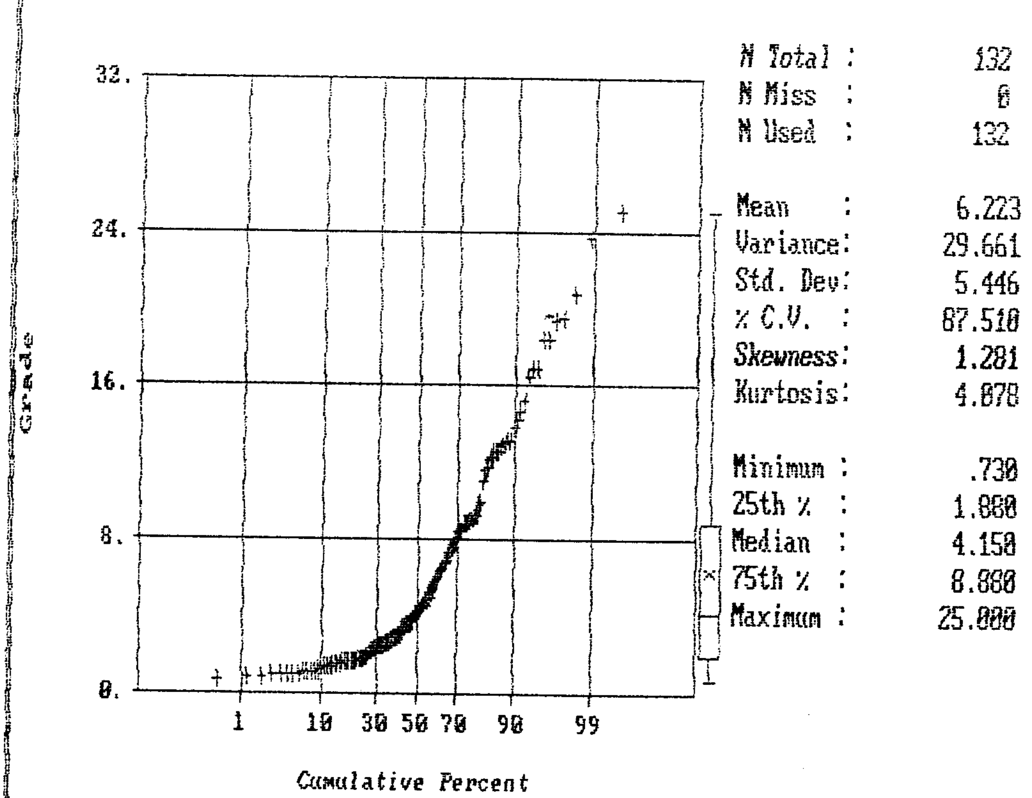


Figure 2.6: Histogram of Transformed Trimmed Surface Data
Data file: reni.dat

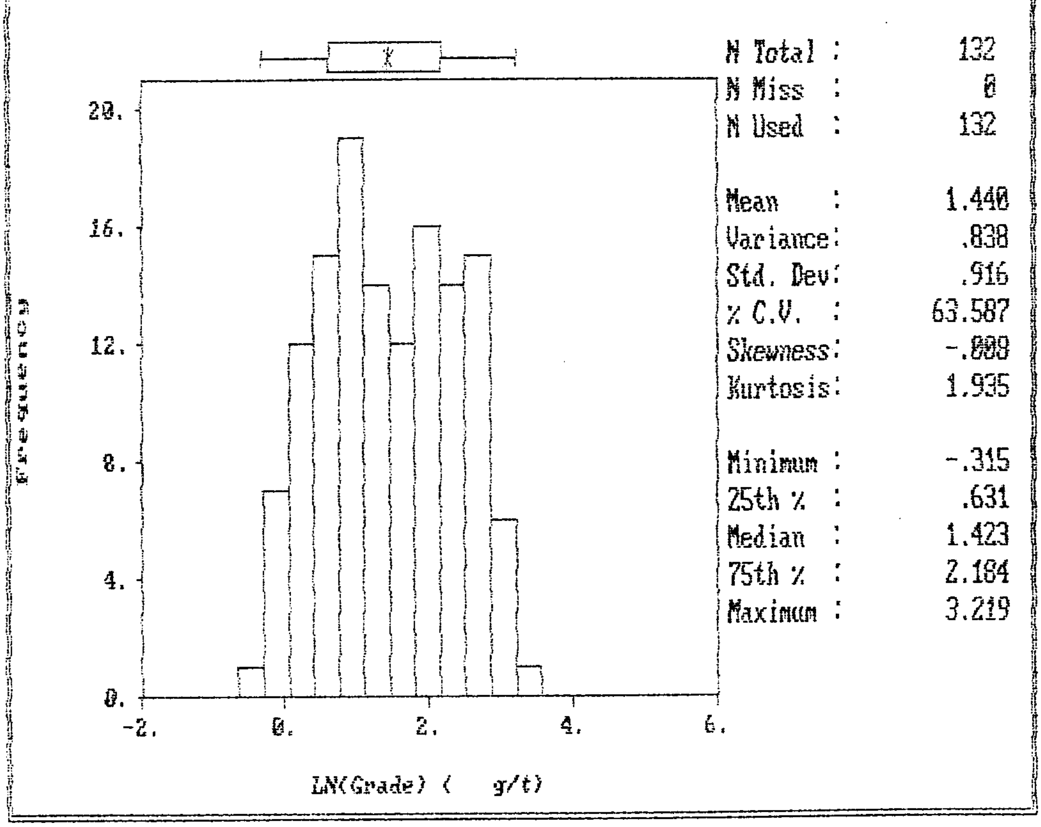
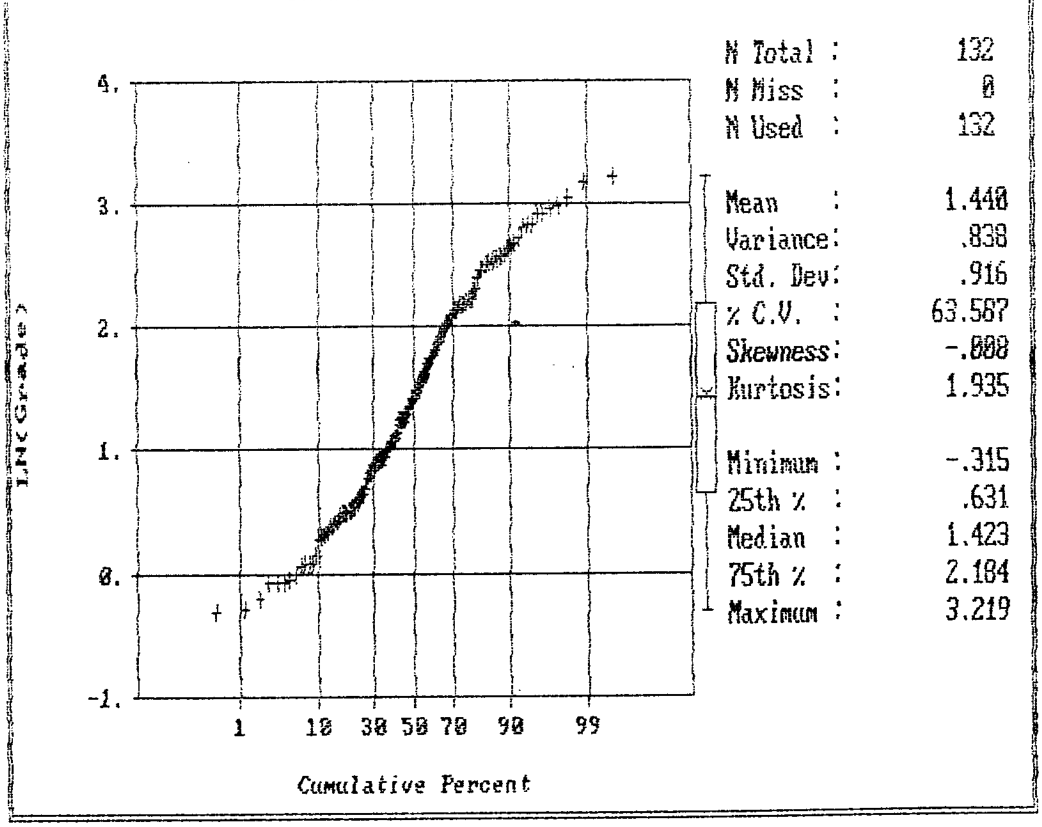



Fig. 2.7 Normal Probability Plot for LN(Grade)
Data file: reni.dat



probably due to the fact that about 50% of the area has a stope width of about 100cm. The results of the transformed data are given in table 2.2

Table 2.2 Parameters of transformed data

| Variable | No of Samples | Mean | Variance | Coeff of Variance % | Coeff of Skewness |
|---|---|---|---|---|---|
| Grade(g/t) | 132 | 1.44 | 0.8384 | 63.587 | 0.0079 |
| Stope W(cm) | 132 | 4.912 | 0.30 | 8.09 | 1.367 |

(ii) Basic statistics of underground development data:

Similar analyses were carried out on the underground development sampling data. Minimum and maximum cut off grades of 0.9g/t and 25g/t respectively were used to trim the data. The parameters computed are listed in tables 2.3 and 2.4. The histograms and probability plots are shown in figures 2.8 to 2.13.

Table: 2.3 parameters of trimmed Development Data

| Variable | No of Samples | Mean | Variance | Coeff of Variance % | Coeff of Skewness |
|---|---|---|---|---|---|
| Grade(g/t) | 199 | 6.157 | 24.062 | 79.661 | 1.303 |
| StopeW(cm) | 199 | 113.77 | 772.09 | 24.4 | 3.56 |

Figure 2.8: Histogram of Grade: Underground Dev. Data
Data file: mine.dat

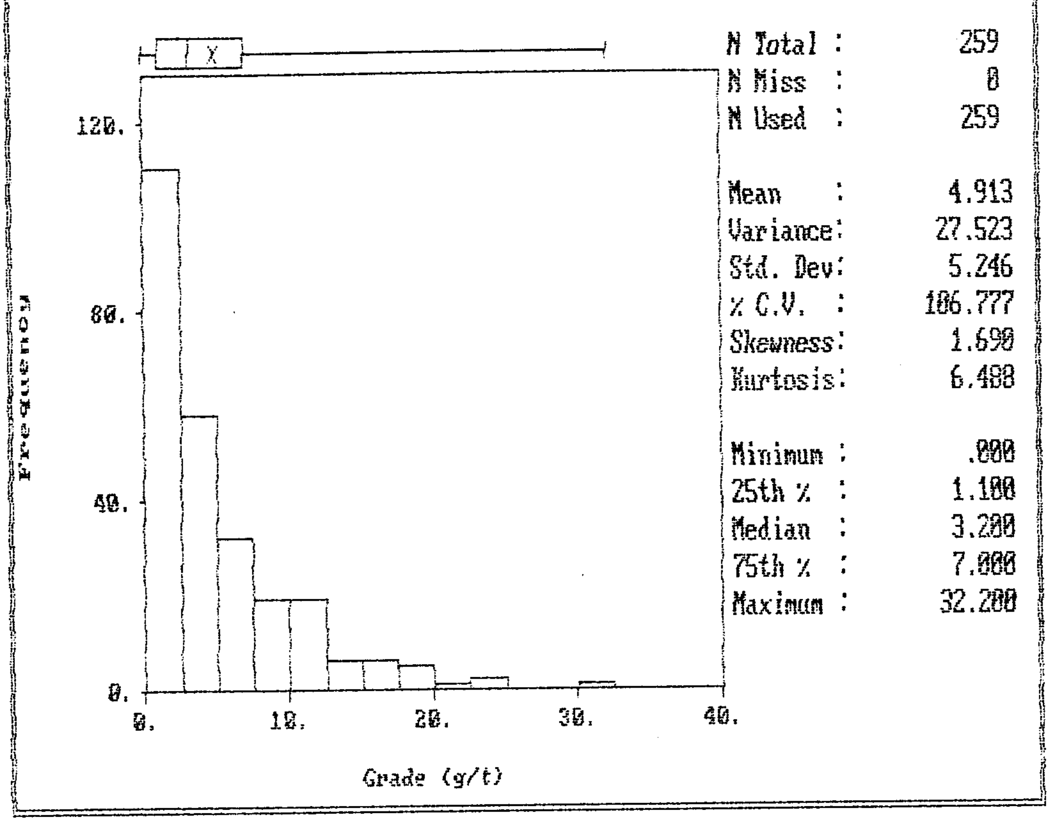



Figure 2.9: Histogram of Thickness: Underground Dev. Data
Data file: mine.dat

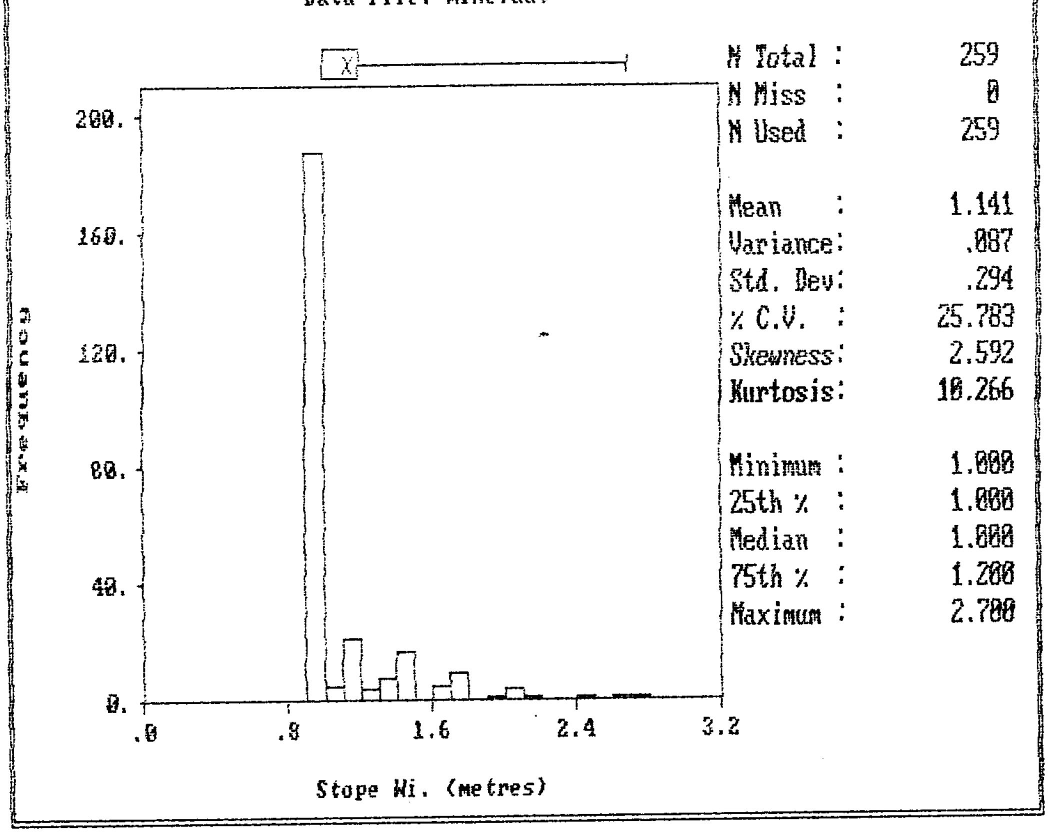


Fig 2.10: Histogram of Grade Values: U/G Dev. Data
Data file: min.dat

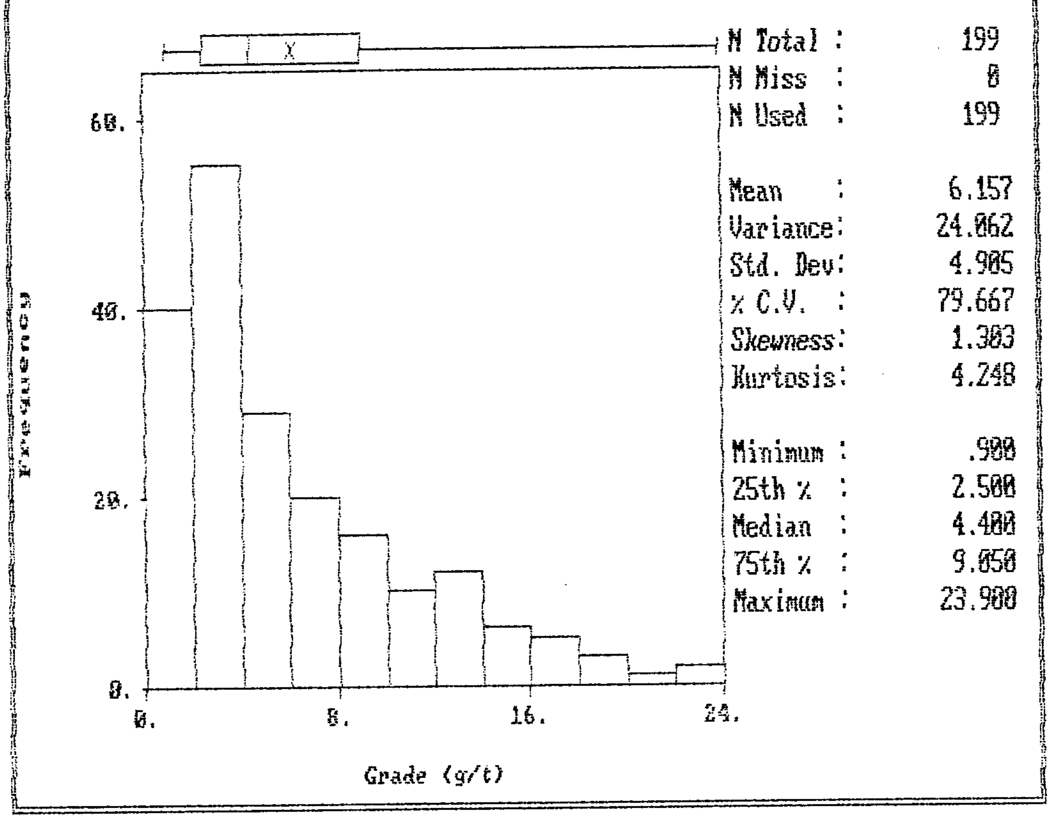



Fig. 2.11 Normal Probability Plot for Grade
Data file: min.dat

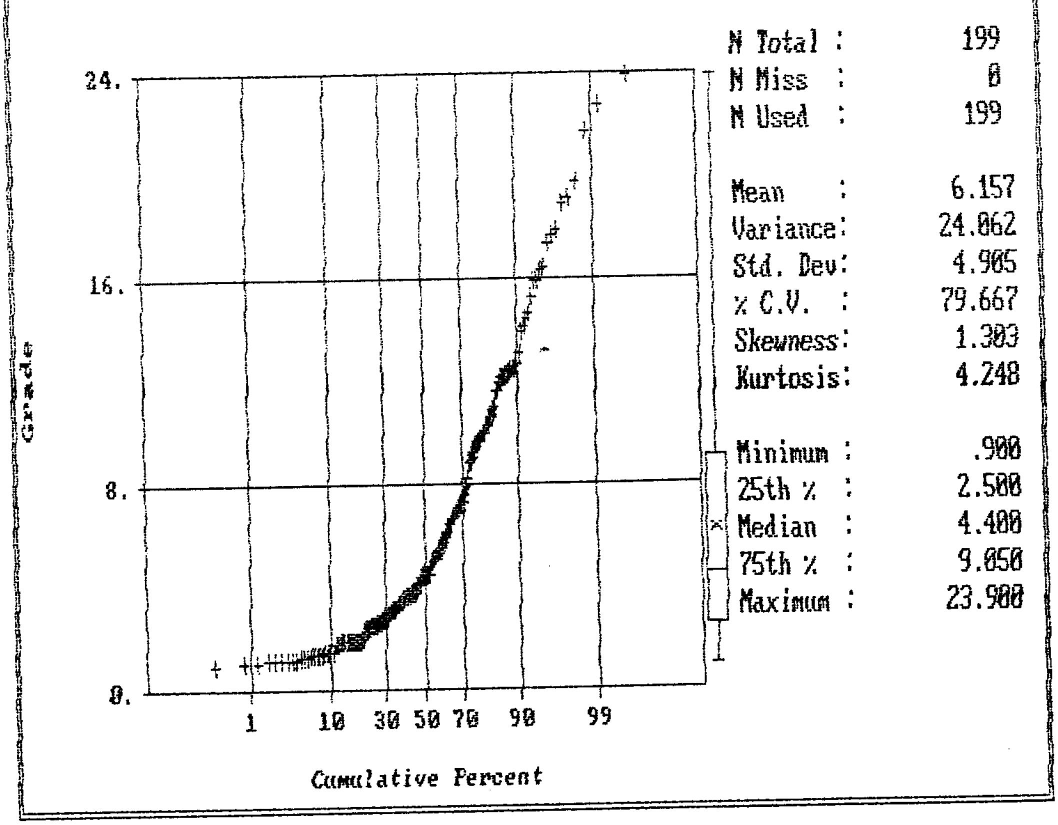
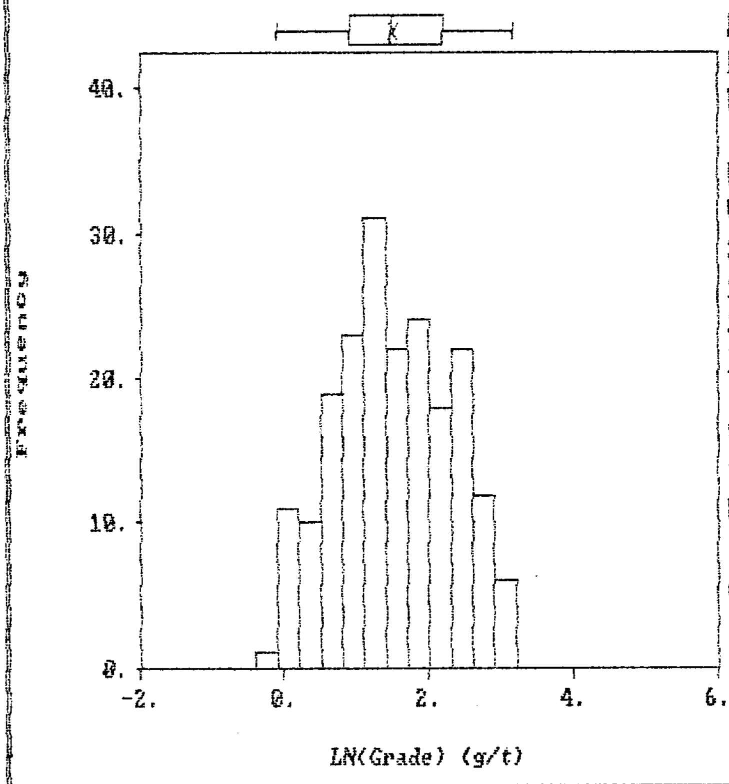


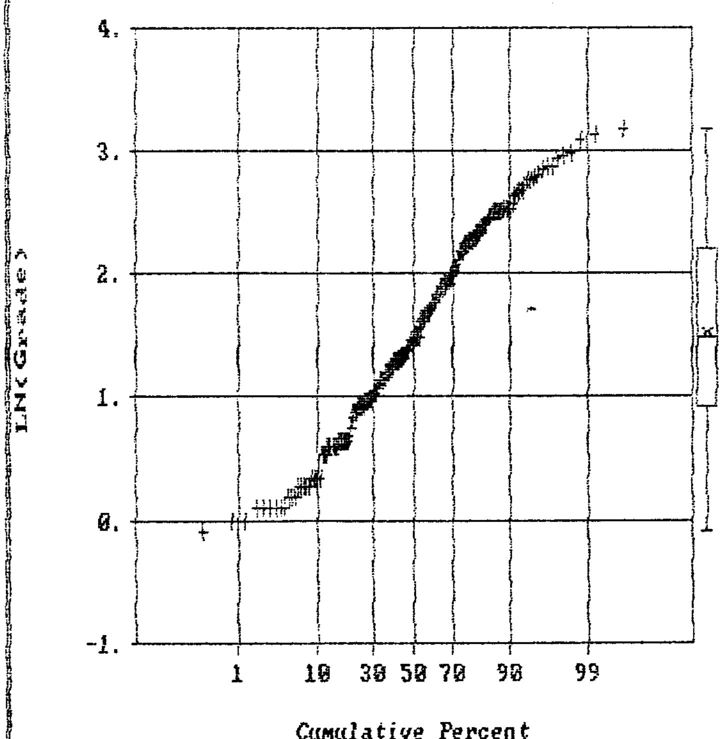
Fig. 2.12 Histogram of Transformed Grade Values: U/G Data
Data file: min.dat




Statistics

| | |
|---|---|
| N Total : | 199 |
| N Miss : | 0 |
| N Used : | 199 |
| Mean : | 1.510 |
| Variance: | .648 |
| Std. Dev: | .805 |
| % C.V. : | 53.326 |
| Skewness: | -.001 |
| Kurtosis: | 2.094 |
| Minimum : | -.105 |
| 25th % : | .916 |
| Median : | 1.482 |
| 75th % : | 2.203 |
| Maximum : | 3.174 |

Fig,2.13 Normal Probability Plot for LN(Grade)
Data file: min.dat




Statistics

| | |
|---|---|
| N Total : | 199 |
| N Miss : | 0 |
| N Used : | 199 |
| Mean : | 1.510 |
| Variance: | .648 |
| Std. Dev: | .805 |
| % C.V. : | 53.326 |
| Skewness: | -.001 |
| Kurtosis: | 2.094 |
| Minimum : | -.105 |
| 25th % : | .916 |
| Median : | 1.482 |
| 75th % : | 2.203 |
| Maximum : | 3.174 |

Table: 2.4 Parameters of transformed Data

| Variable | No of Samples | Mean | Variance | Coeff of Variance % | Coeff of Skewness |
| --- | --- | --- | --- | --- | --- |
| Grade(g/t) | 199 | 1.510 | 0.648 | 53.326 | -0.0015 |
| StopeW(cm) | 49 | 4.71 | 0.040 | 4.22 | 1.97 |

(iii) Comparison of Results

Table 2.5 below is a comparison of the statistics of Renco mine and the statistics obtained from the foregoing analyses using the GEO-EAS Software.

Table 2.5: Comparison of Results between Renco and Geo-EAS System.

| | Underground Development Data | | | |
| --- | --- | --- | --- | --- |
| | RENCO System(Untrimmed) | | GEA-EAS (Trimmed) | |
| | Grade (g/t) | Stoping W(cm) | Grade (g/t) | Stop. w (cm) |
| No of Values | 4371 | 4366 | 199 | 199 |
| Mean | 10.0 | 122 | 6.157 | 113.77 |
| Variance | 370.1 | 1545.8 | 24.062 | 772.09 |
| Range of Values | 0-380 | 80-400 | 0.7-25 | 100cm-410 |
| Standard Error | 0.085 | 0.354 | 0.121 | 0.572 |

| Surface Diamond Drill Data | | | | |
|---|---|---|---|---|
| | Renco System(Untrimmed) | | Geo-EAS (Trimmed) | |
| | Grade (g/t) | Stoping W(cm) | Grade (g/t) | Stope w (cm) |
| No of Values | 152 | 152 | 132 | 132 |
| Mean | 6.58 | 163.76 | 6.224 | 149.258 |
| Variance | 68.98 | - | 29.661 | 6211.826 |
| Range of Values | 0-584 | 100-547 | 0.7-25 | 100 -410 |
| Standard Error | 0.454 | 1.077 | 0.225 | 47.06 |

It can be concluded from the comparison that the parameters obtained by Renco mine cannot be used in determining the population parameters due to the following

1. Large variances in Renco analysis
2. The data used include unreasonably high and low values which could have been the results of sampling errors. The data could have been trimmed.
3. The data used for Renco analysis is from high grade areas and are therefore unrepresentative of the total deposit.

(iv) Population Parameters

Table 2.6: Computation by lognormal theory of m, a statistic to estimate the mean and a statistic to estimate the variance for Gold grade at Renco Mine

| Calculation of M | | | Calculation of $V^2$ | | |
|---|---|---|---|---|---|
| | U/G Data | Surface DD Data | | U/G.Dev Data | Surface DD Data |
| $\bar{U}$ | 1.5097 | 1.44 | $\bar{U}$ | 1.5097 | 1.44 |
| $S_U{}^2$ | 0.6481 | 0.8384 | $S_U{}^2$ | 0.6481 | 0.8384 |
| $e^u$ | 4.5254 | 4.2207 | $e^{2u}$ | 20.479 | 17.814 |
| n | 0199 | 132 | n | 199 | 132 |
| $\frac{1}{2}S_u{}^2$ | 0.32405 | 0.4192 | $\phi_n(S_u{}^2)$ | 2.387 | 3.637 |
| $\psi_n(\frac{1}{2}S_u{}^2)$ | 1.43315 | 1.5599 | $V^2$ | 48.88 | 64.7899 |
| M | 6.49 | 6.58 | V | 6.99 | 8.049 |

Koch and Link [5] gave the following formula for computing the unbiased efficient estimates of the mean and variance of a population which has a lognormally distributed phenomenon.

$$\text{Mean } M = e^u\, \psi_n\left(\tfrac{1}{2}S^2{}_u\right) \qquad (37)$$

$$\text{Variance } V^2 = e^{2u}\left[\psi_n(2S_u{}^2) - \psi_n\left(\frac{n-2}{n-1} S_u{}^2\right)\right]$$

$$= e^{2u}\, \phi_n(S_u{}^2) \qquad (38)$$

Where:-

The values of $\psi_n$ are read from tables

$S_u{}^2$ is the variance of the natural log transformed data u is the mean of the natural log transformed data

Using the Geo EAS parameters of tables 2.5 the population mean and variance are computed using the Koch and Link relationships and tabulated above in table 2.6.

### 2.7.6 Application of Geostatistics Using The Geo-Eas Software:

The programme PREVAR, a preprocessor for the programme VARIO was used to create pair comparison files of the trimmed data. Each file contains all possible pairs of the data set within a minimum and maximum lag spacing for all variables in the set. These pair comparison files are used for calculating and plotting variograms.

1. Drift Analysis

The intrinsic hypothesis implies that

$$E\,[Z\,(x + h) - Z(x)] = 0$$

A geostatistical study is possible when this condition exists. When the expected value is dependent on the location of the samples then there exists drift or trend. The Drift called Modogram in this software was plotted against the lag. A horizontal line with no significant deviations, indicates that the intrinsic hypothesis can be assumed. Figures 2.14 and 2.15.

Fig. 2.14: Graph of drift of Surface Borehole Data

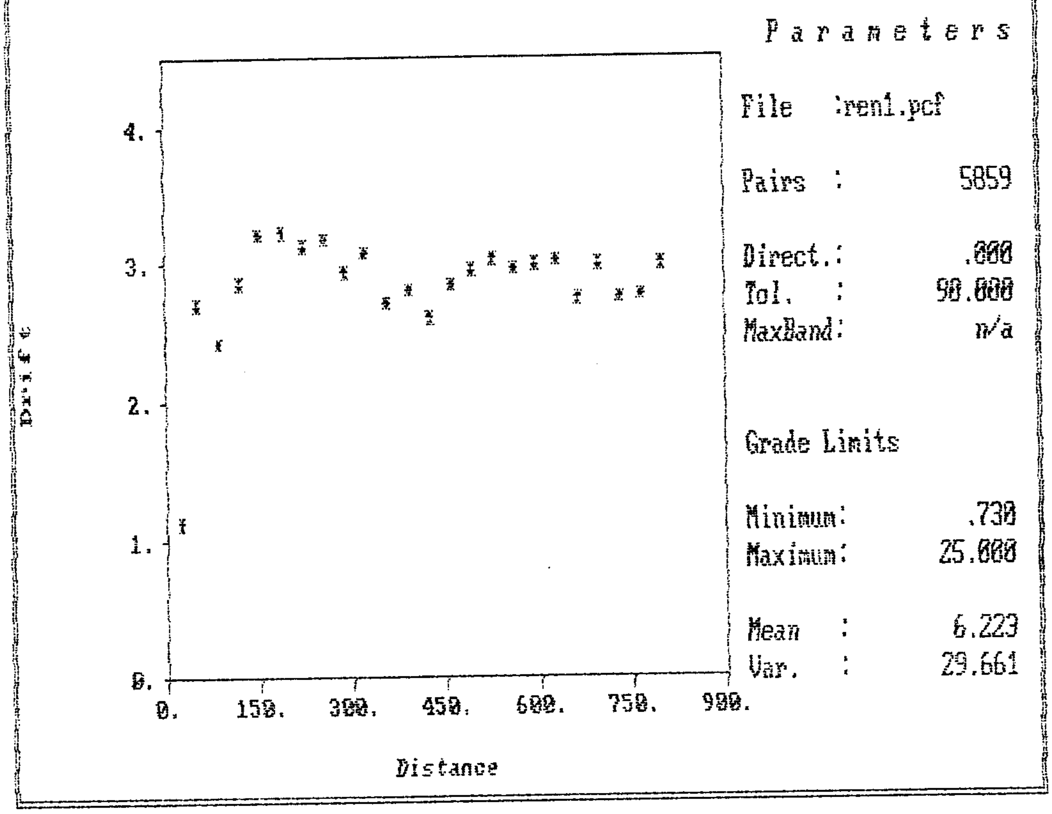
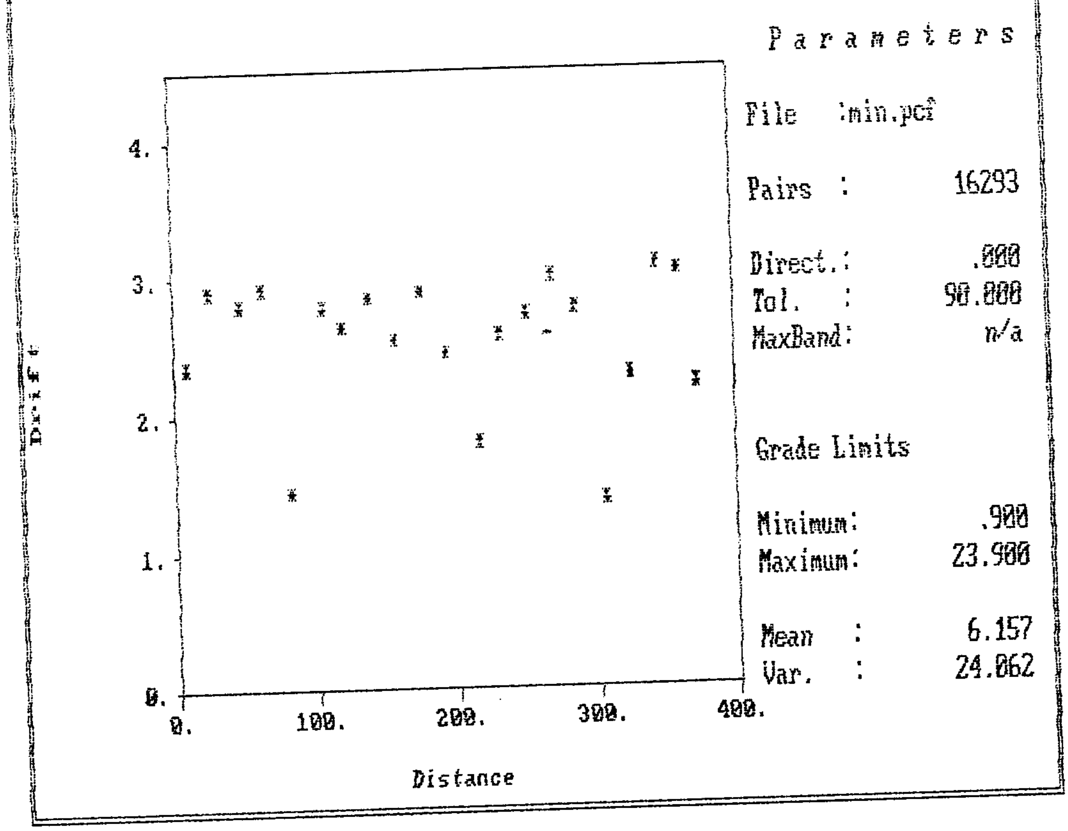



Fig. 2.15: Graph of Drift of Underground Dev. Data



## 2. Variographic Modelling

The experimental variogram defined notationally as:

$$2 \quad (h) \quad = \quad \frac{1}{N(h)} \sum_{i=1}^{N(h)} \quad [Z(xi + h) + Z(xi)]^2$$

Where N(h) the number of pairs of the variable was computed and plotted using the programme VARIO. Average variograms at $0^0$ direction and $90^0$ angular tolerance were computed and plotted. Directional variograms at $0^0$ 45 $^0$ $90^0$ at $135^0$ and at $22.5^0$ angular tolerances were computed and plotted. The lag spacing used to compute directional variograms were significantly larger than those for average variograms so as to reduce "Noise" in the variograms. The angular tolerance of $22.5^0$ ensures that all pairs in all directions are covered. The reference direction is the horizontal axis designated $0^0$. The spherical scheme fits the experimental variograms and the parameters for each data group are tabulated in tables 2.7 and 2.8. Figures 2.16 to 2.19 and 2.20 to 2.23 show the directional and average variograms with their fitted models for surface borehole and underground development trimmed grade values respectively.The range a, is computed as A =3/2R where R is the distance at which the variogram assumes a constant value.

Fig.2.16: Average Variogram for Grade: Surface Data
0 degrees direction and 90 degrees tolerance

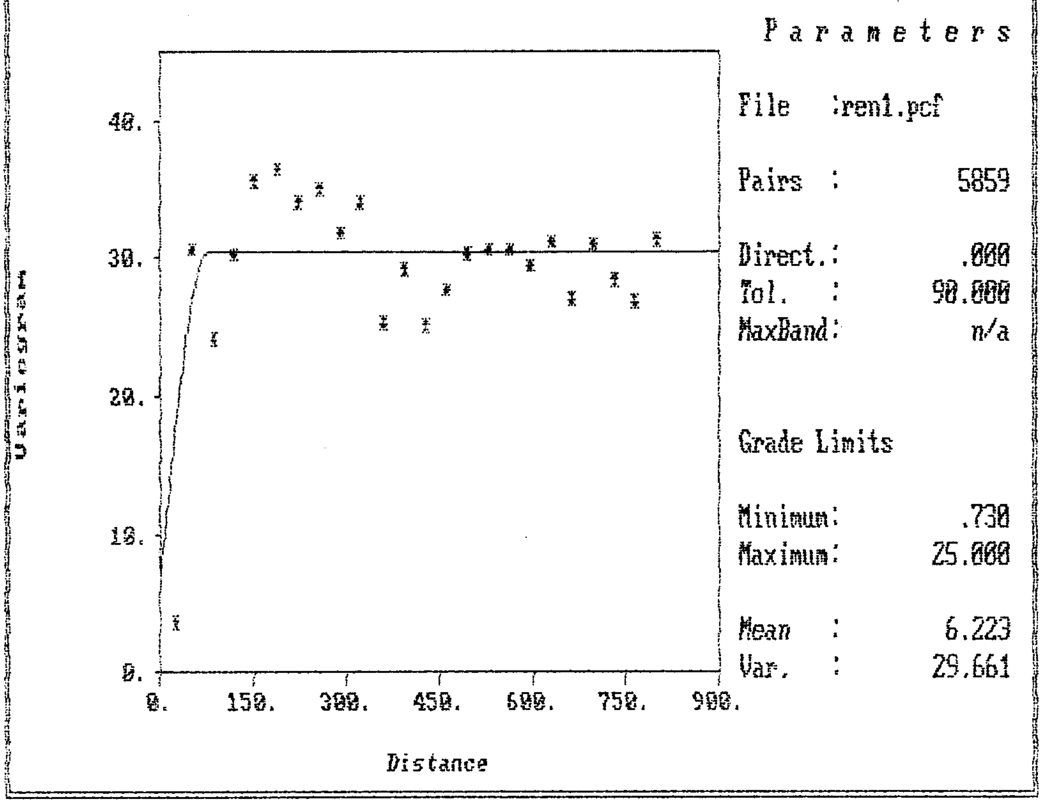



Fig. 2.17: Directional Variogram for grade: Surface Data
0 degrees Direction.

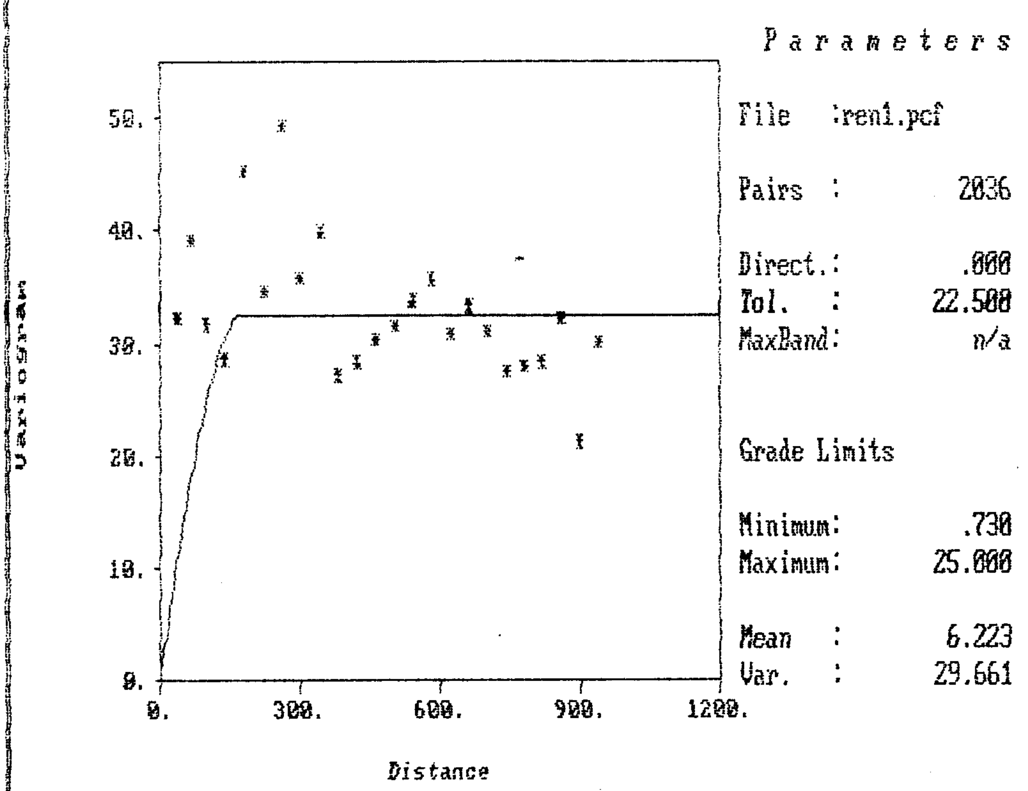


Fig. 2.18: Directional Variogram for Grade: Surface Data
45 degrees Direction.

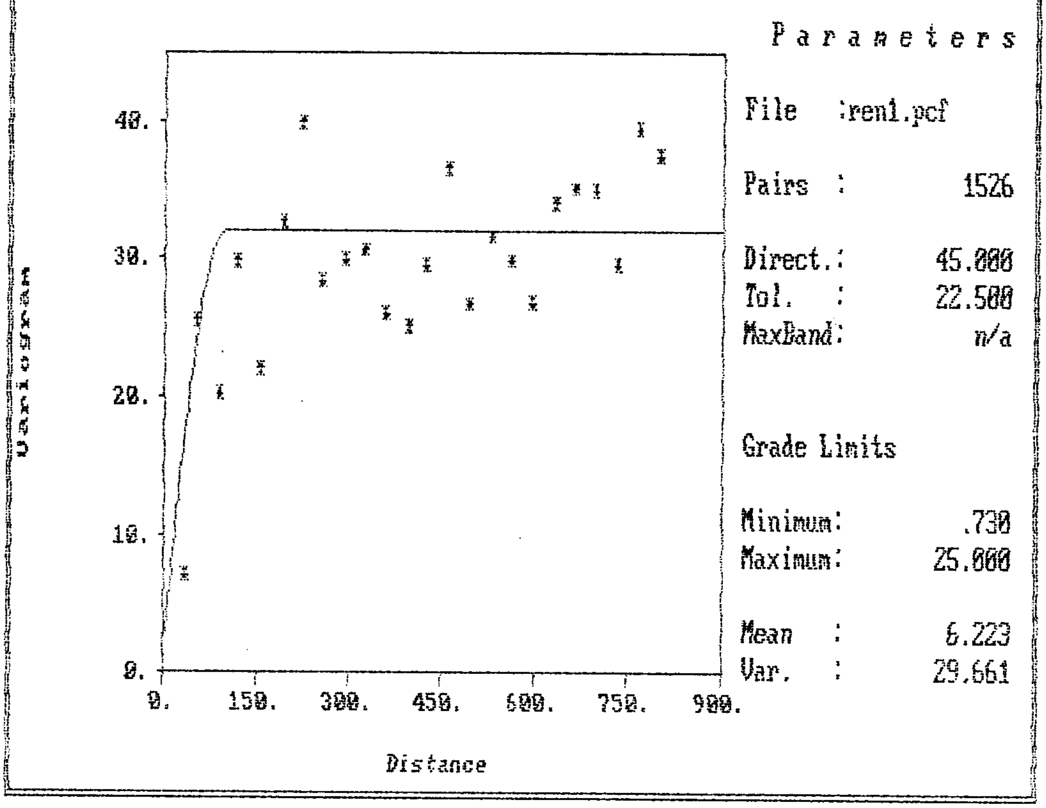



Fig. 2.19: Directional Variogram for Grade: Surface Data
135 degrees Direction.

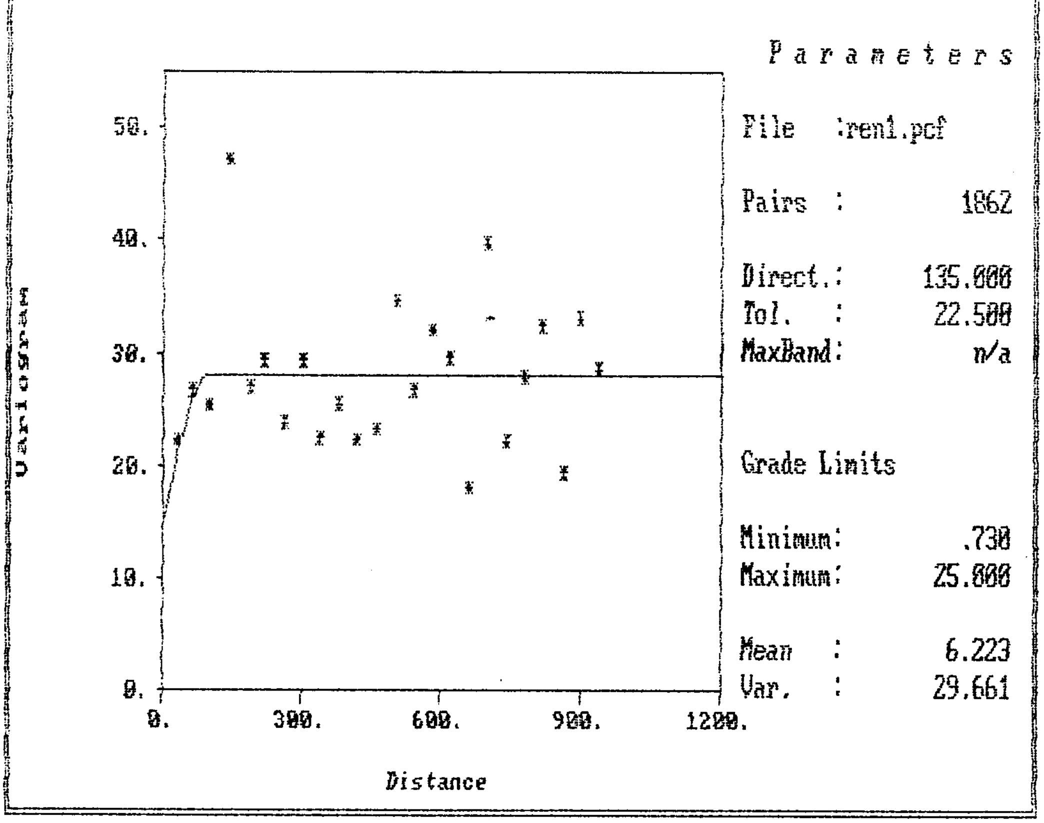


Fig. 2.20: Average variogram for Grade: U/G Dev. Data
0 degrees direction and 90 degrees tolerance

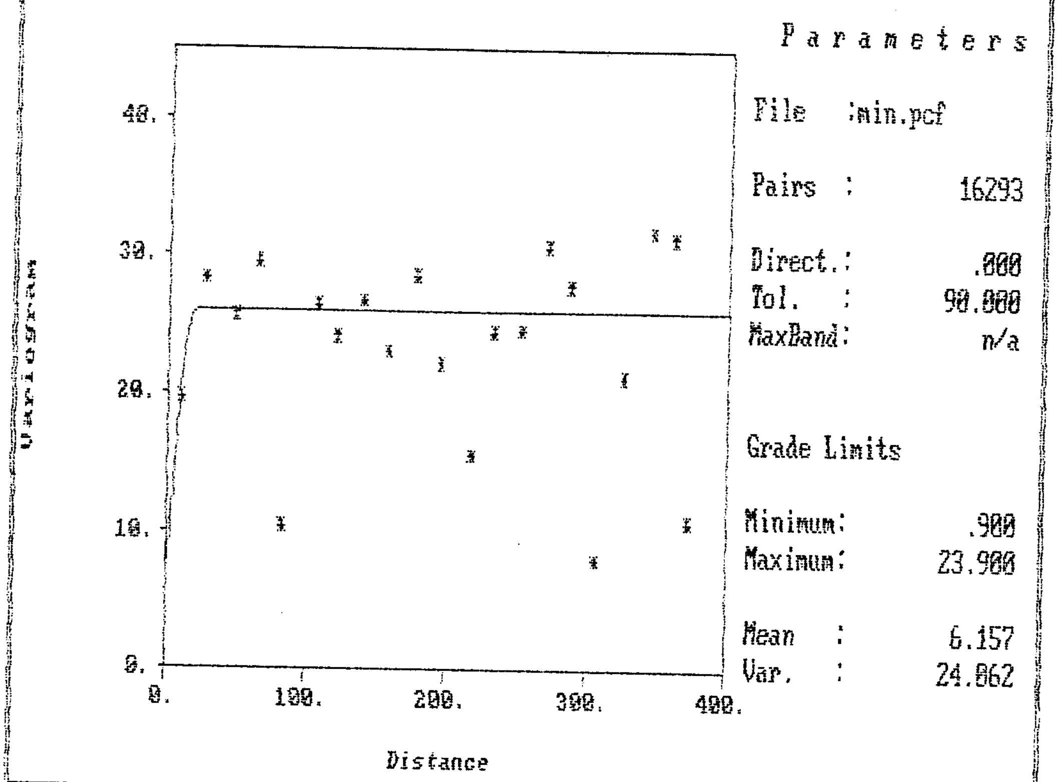



Fig. 2.21: Directional Variogram for Grade: U/G Dev. Data
0 degrees Direction

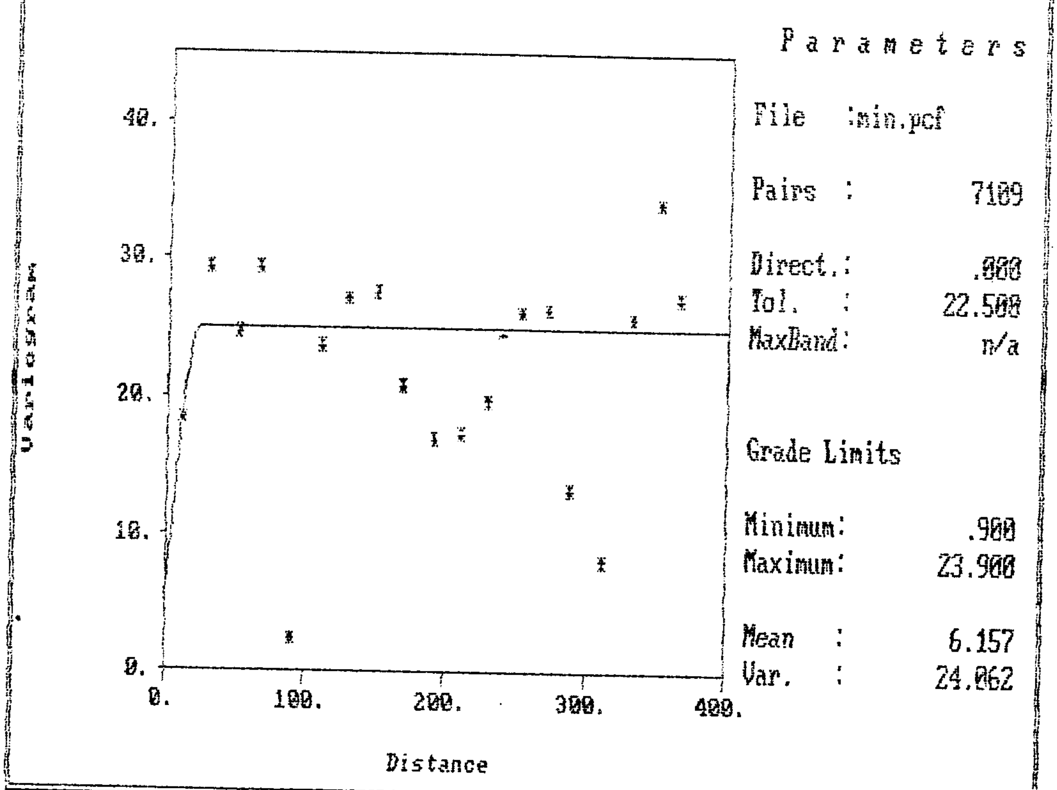


Fig. 2.22: Directional Variogram for Grade; U/G dev. Data
45 degrees Direction

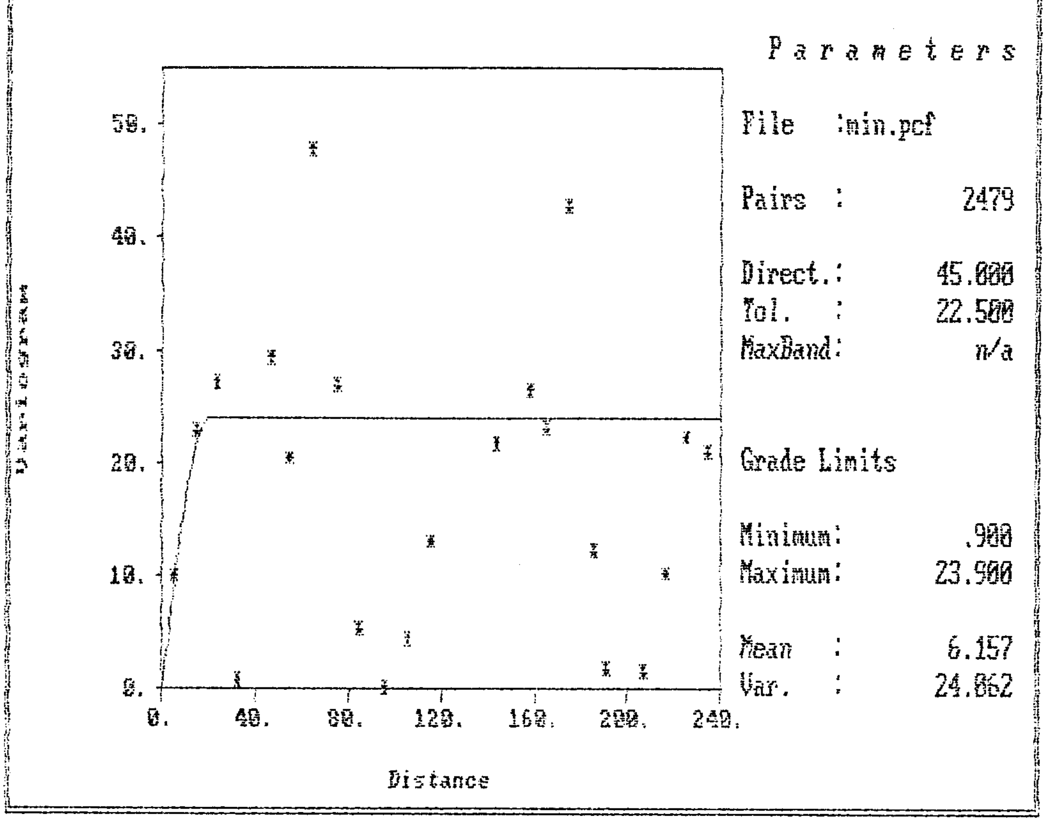



Fig. 2.23: directional Variogram for Grade: U/G dev. Data
90 degrees Direction

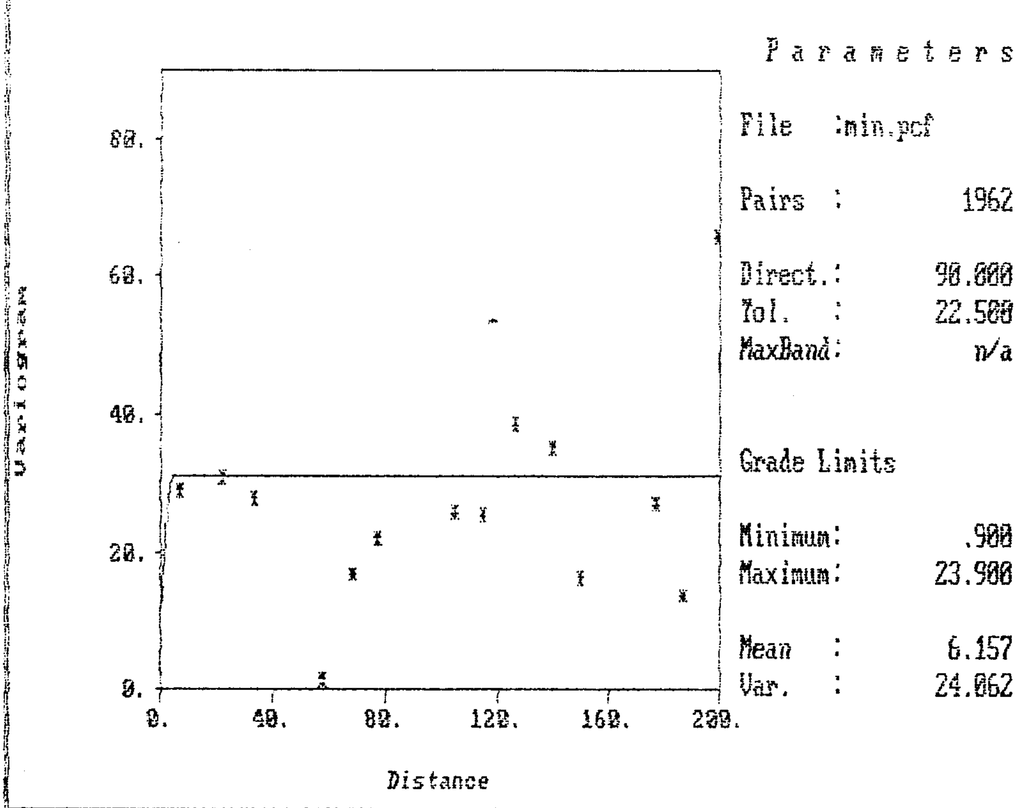


Table 2.7 Variogram Parameters for Underground Development Data

| Variable | Direction(degs) | Type | Co | C | R | a |
|---|---|---|---|---|---|---|
| | Average | Spherical | 7 | 19 | 18 | 27 |
| | $0^0$ | " | 6 | 19 | 22 | 33 |
| GRADE | $45^0$ | " | 0 | 24 | 20 | 30 |
| | $90^0$ | " | 11 | 20 | 3.5 | 5.25 |
| | $135^0$ | " | 3 | 18 | 5 | 7.5 |

Table 2.8 Variogram Parameters for Surface Diamond Drill data

| Variable | Direction(degs) | Type | Co | C | R | a |
|---|---|---|---|---|---|---|
| | AVERAGE | Spherical | 7 | 23.5 | 78 | 117 |
| | $0^0$ | " | 1 | 31.5 | 175 | 262.5 |
| | $45^0$ | " | 2 | 30 | 95 | 142.5 |
| GRADE | $90^0$ | " | 7 | 190 | 120 | 180 |
| | $135^0$ | " | 15 | 13 | 97 | 145.5 |

3. Comparison of Parameters

Working on total data of over 4,000 samples, Rio Tinto (Zimbabwe) Technical services carried out variographic modelling of the Renco orebody and developed global parameters which are used for Kriging. After further studies, they concluded that global variogram parameters be used for Kriging instead of the local variogram parameters.

However, this is an over simplification of the issue since the Renco orebody is very complex and each ore type exhibits a different distribution of gold and should be treated as different populations for both statistical and variographic analysis.(Armitage, 1990)[6]

Table 2.9 is a comparison between the Renco, global parameters and the parameters obtained in this study.

Table 2.9 Comparison of Variographic Parameters.

| | | Underground Dev. Data | | Surface DD Data | |
|---|---|---|---|---|---|
| Variable | P/meter | RENCO | GEOEAS | RENCO | GEOEAS |
| | Co | 30 | 7.0 | 11.5 | 7.0 |
| GRADE | C | 17 | 19.0 | 8.5 | 23.5 |
| | a | 30m | 27.m | 200m | 117m |

2.8 Conclusion

The local parameters developed are close to the Renco global parameters. Some slight anisotropy was observed in some of the local variograms but is not significant and is therefore ignored. Despite the shortcomings in these parameters, the Renco global variogram parameters and the population statistics developed in previous sections are used for further work on this project.

## 2.9 Grade - Tonnage Relationship

The grade distribution of most mineral depostis tend to follow the lognormal distribution.[7] Even though this is not absolutely true, most authors agree that lognormality is often an adequate and helpful approximation for orebodies other than those physically most rich. The relationship between grade and tonnage or the proportion of tonnage above cut off grade is an important tool in pre-feasibility studies and production monitoring and control. Since first proposed by Lasky[8], the grade - tonnage curve has become a very popular tool for mine planning.

### 2.9.1 Lasky's Model

The model is based on the annual statistics of a producing mine. At a time t. the orebody will be characterised by the following values:-

- The total tonnage mined until t: T
- The corresponding metal content: Q
- The corresponding grade: M

For the copper deposit he studied, Lasky observed a linear relationship between the mean grade and the logarithm of the tonnage.

$$M = \alpha - \beta \log T \quad (39)$$

Which is known as Lasky's Law

$\alpha$ and $\beta$ are constants

Guibal and Touffait[9] showed that Lasky's model gives the distribution of grades to be exponential and depends only on one parameter, and the mean grade as a function of the cut off grade. Since this behaviour is not practical, they concluded that Lasky's model is completely unsuitable for solving problems of grade-tonnage relationships. They proposed the geostatistical technique.

## 2.9.2 Geostatistical techniques

Geostatistical methods of grade-tonnage determination takes into account the support, the available information on blocks and on the constraints imposed by the mining method. The distribution of ore grade is determined from structural analysis of the data from which geostatistical parameters are also obtained. A grade tonnage relationship can therefore be developed. The tonnage of ore above cut off grade is given by the general geostatistical expression as follows:

$$T_+ = T \int_{gc}^{\infty} f(x)dx \tag{40}$$

$$Q = T \int_{gc}^{\infty} xf(x)dx \tag{41}$$

$$U_+ = \frac{Q}{T_+} = \frac{\int_{gc}^{\infty} xf(x)\ dx}{\int_{gc}^{\infty} f(x)\ dx)} \tag{42}$$

where:-

$T_+$ is tonnage above cut off grade

$T$ is total tonnage in the deposit

$g_c$ is the cut of grade

$U_+$ is the average grade above cut off grade

$Q$ is the metal content above cut off grade

Clark [10] demonstrated a simplified procedure for developing grade-tonnage relationships from sampling data as follows:

The proportion of ore above cut off grade P is given as:-

$$P = 1 - \Phi(z) \tag{43}$$

Where $z = \dfrac{c - g}{s}$

$c$ = the cut off grade

$g$ = average sample grade

$s$ = sample standard deviation

$\Phi(z)$ is read from standard normal table.

The average grade above cut off grade $g_c$ for normally distributed deposit is

$$g_c = g + \frac{s}{p} + \phi(z) \tag{44}$$

where $\phi(z) = \dfrac{1}{\sqrt{(2\pi)}} e^{-z^2/2}$

The block standard deviations $S_v$ can be computed from the values of F(L,b) read from tables, So that the Z-value is computed as:

$$z = \frac{c - g}{S_v}$$

For a lognormal distribution, and using the same notations as above,

$$S^2{}_y = \text{Ln} \left(\frac{S^2}{g^2} + 1\right)$$

where y is the log of the grad and $S^2{}_y$ is the block variance.

$$y = \text{Ln } g - 0.5S^2{}_y$$

$$z = \frac{\text{Ln } C - y}{S_y}$$

$$P = 1 - \Phi(z) \qquad (45)$$

$$g_c = \frac{Q}{P} \qquad (46)$$

where $Q = 1 - \Phi(z - sy)$

## 2.10 Grade-tonnage Relationship for Renco orebody

Using the statistical and the variographic parameters derived in previous sections of this chapter, the grade tonnage relationship can be developed as follows:-

•

The mean grade 6.49g/t, variance = 48.88

Block size = 50m x 50m

Range a = 30

Sill c = 17.0

Nugget Co = 30.0

The block standard deviation is calculated from the variogram parameters F(50 x 50)

when a = 30 is given from tables as 0.839955

with C = 17.0 the value becomes 0.839955 x 17 = 14.28

since there is a nugget effect we add it to this to give

14.28 + 30 = 44.28

The block variance = 48.88 - 44.28 = 4.6

$$y = \text{Ln } g - 0.5S^2_y$$

$$S^2_y = \text{Ln } \left(\frac{s^2_v}{g^2} + 1\right)$$

$$= \text{Ln } \left(\frac{4.60}{(6.49)^2} + 1\right) = 0.104$$

$$= Sy = 0.322$$

$$y = \text{Ln } 6.49 - 0.5 \times 1.04$$

$$= 1.818$$

$g = 6.49$ $\qquad$ $Sv = 2.145$

$y = 1.818$ $\qquad$ $Sy = 0.322$

Table 10 shows the proportion of the deposit above cut off grade and the average grade above cut off grade for the different cut off grades. Figure 2.24 shows the Grade- Tonnage model for the orebody.

TABLE 3.10: GRADE - TONNAGE CALCULATION.

| C | LnC | Z | Z _ Sy | Q | P | Gc |
|---|---|---|---|---|---|---|
| 0.50 | -0.6931 | -7.799 | -8.121 | | | |
| 1.00 | 0.0000 | -5.646 | -5.968 | | | |
| 1.50 | 0.4055 | -4.387 | -4.709 | | | |
| 2.00 | 0.6931 | -3.493 | -3.815 | 0.9998 | 0.9998 | 6.49 |
| 2.50 | 0.9163 | -2.800 | -3.122 | 0.9991 | 0.9974 | 6.50 |
| 3.00 | 1.0986 | -2.234 | -2.556 | 0.9948 | 0.9871 | 6.54 |
| 3.50 | 1.2528 | -1.755 | -2.077 | 0.9812 | 0.9608 | 6.63 |
| 4.00 | 1.3863 | -1.341 | -1.663 | 0.9515 | 0.9099 | 6.79 |
| 4.50 | 1.5041 | -0.975 | -1.297 | 0.9032 | 0.8365 | 7.01 |
| 5.00 | 1.6094 | -0.648 | -0.970 | 0.834 | 0.7422 | 7.29 |
| 5.50 | 1.7047 | -0.352 | -0.674 | 0.7486 | 0.6368 | 7.63 |
| 6.00 | 1.7918 | -0.081 | -0.403 | 0.6554 | 0.5319 | 8.00 |
| 6.50 | 2.0149 | 0.612 | 0.290 | 0.5596 | 0.4325 | 8.40 |
| 7.00 | 1.9459 | 0.397 | 0.075 | 0.4681 | 0.3446 | 8.82 |
| 7.50 | 2.0149 | 0.612 | 0.290 | 0.3859 | 0.2709 | 9.25 |
| 8.00 | 2.0794 | 0.812 | 0.490 | 0.3121 | 0.209 | 9.69 |
| 8.50 | 2.1401 | 1.000 | 0.678 | 0.2483 | 0.1587 | 10.15 |
| 9.00 | 2.1972 | 1.178 | 0.856 | 0.1949 | 0.119 | 10.63 |
| 9.50 | 2.2513 | 1.346 | 1.024 | 0.1539 | 0.0885 | 11.29 |
| 10.00 | 2.3026 | 1.505 | 1.183 | 0.119 | 0.0655 | 11.79 |
| 10.50 | 2.3514 | 1.656 | 1.334 | 0.0918 | 0.0495 | 12.04 |
| 11.00 | 2.3979 | 1.801 | 1.479 | 0.0694 | 0.0359 | 12.55 |
| 12.00 | 2.4849 | 2.071 | 1.749 | 0.0401 | 0.0192 | 13.55 |
| 13.00 | 2.5649 | 2.320 | 1.998 | 0.0228 | 0.0102 | 14.51 |
| 14.00 | 2.6391 | 2.550 | 2.228 | 0.0129 | 0.0054 | 15.50 |
| 15.00 | 2.7081 | 2.764 | 2.442 | 0.0073 | 0.0029 | 16.34 |
| 20.00 | 2.9957 | 3.658 | 3.336 | 0.0004 | 0.0001 | 25.96 |

## 2.11 Conclusion

It can be concluded from the above analyses that the Renco Orebody is lognormally distributed with a population mean of 6.58g/t. From the Grade - Tonnage curve of figure 2.24 a cut-off grade of 6.58g/t will give an average mine grade of 8.25g/t.At a cut off grade of about 2.0g/t the entire orebody will lie above cut off grade with an average grade of about 6.49g/t. This does not take into account dilution and ore losses.

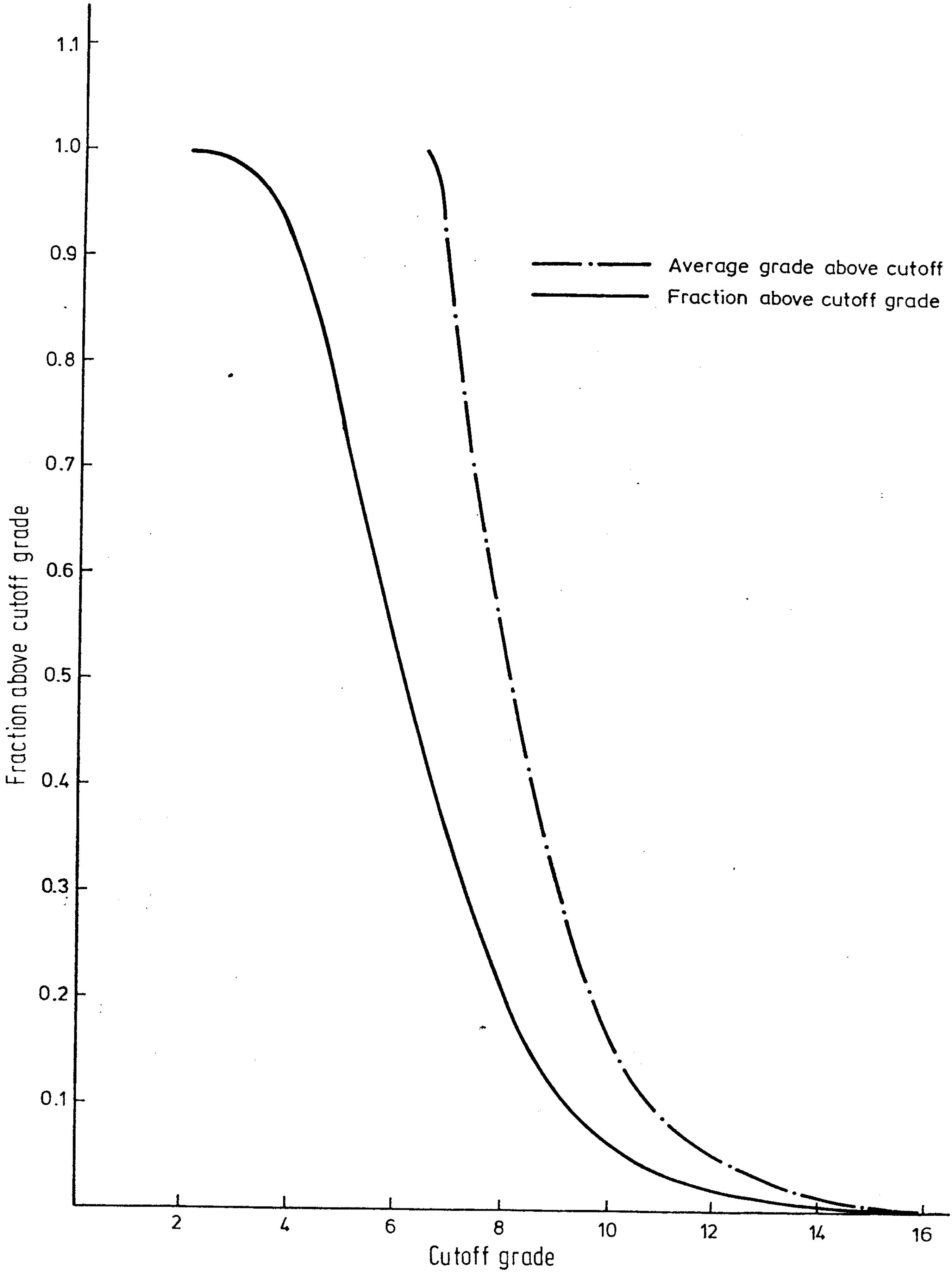



Fig. 2.24 GRADE - TONNAGE CURVE FOR RENCO MINE OREBODY

# 3. PRODUCTION SCHEDULING

The determination of the production rate, quality (grade), and the scheduling of production in the long and short term forms an important component of the entire mine planning process. In long range planning production rate is determined with the view to maintaining the viability of the whole operation. Sequencing and timing of working places are done to achieve the desired production tonnage and grade in space and time.

Short range production planning is the tactical outlays and use of resources to achieve production rates that satisfy the objectives set up in the long term plans. Scheduling in the short term involves making specific operational decisions to assign equipment and personnel in an optimum manner and studying a situation to determine the most effective general operating techniques.

## 3.1 Application of Operations Research in Production Scheduling.

Production scheduling is the last task in the mine planning hierarchy both in the long and short term phases.Most computer methods used in underground mine planning tend to focus on production simulation, scheduling of tonnage hauled, ventilation, pumping and ground conditions mainly for long range production scheduling. The desire for short range scheduling to provide solutions that satisfy both the long range plan and the day to day operational details of the mine requires incorporating sophisticated operations research methods and computer capabilities to help obtain optimum answers in time.

Among the frequently utilised techniques can be cited: Linear, Integer and Dynamic programming, Graph and Network theory, Simulation and Heuristic Methods.

### 3.1.1 Dynamic Programming (D.P.) Models

Dynamic programming is the mathematical technique of dealing with the optimization of multistage processes. It is most widely used in production scheduling. Hauck [11] made use of a dynamic programming function to solve a truck assignment problem. Roman [12] used a D.P. model to determine optimum mine-mill production schedules. Other applications of D.P. models in production optimization are found in the works of Elbrond and Elbrond et al [13], Dôwd[14]. Since this method is going to be used to determine optimum production in this study, a more detailed treatment of Dynamic programming is done at a later stage in this chapter.

## 3.1.2 Linear Programming (LP) Models

The linear programming model consists of a linear objective function of the form:

$$C_1 X_1 + C_2 X_2 + \ldots\ldots. C_n X_n$$

plus a set of general linear constraints of the form.

$$a_{11}\ x_1 + a_{12}\ x_2 + \ldots\ldots.\ a_{1n}\ x_n \leq b_1$$
$$a_{21}\ x_1 + a_{22}\ x_2 + \ldots\ldots.\ a_{2n}\ x_n \leq b_2$$
$$a_{m1}\ x_1 + a_{m2}\ x_2 + \ldots\ldots.\ a_{mn}\ x_n \leq b_m$$

and a set non-negativity constraint.

$$x_1,\ x_2\ \ldots\ldots.\ x_n \geq 0.$$

Where:

$x_1, \ldots, x_n$ are the decision variables

$a_{11}, \ldots, a_{mn}$ and $b_1, \ldots, b_m$ are constraints whose values are determined by the nature of the problem.

The objective function is the mathematical expression to be optimised by properly choosing the decision variables $x_1$, $x_2$, .... $x_n$. The solution to the optimisation problem is done graphically or by an iterative procedure called the Simplex algorithm.

/Linear programming modes have been used in the mining industry for production optimization, and most of the early work tended to be directed towards meeting specific grades and production requirements given face production, haulage, geological and geometric constraints. Spilane et al [15] Burne, [16], Wilke and Reimer [17] used L. P. models for open pit optimization. Gershon's [18] model is a typical example of the application of L. P. models in blending problems. The models have also been applied to open pit trucks scheduling (Dessureaulf and Galibois, [19]) and production scheduling (Gershon,[18]).

### 3.1.3 Integer Programming (IP) Models

Integer planning is an extension of Linear programming that places a further constraint to cater for anomalies of fractional values of decision variables such as 4.5 shovels or trucks. If all the decision variables are limited to non-negative integers the problem is said to be one of pure integer programming. In the case where only some of the decision variables are restricted to non-negative integers it is called mixed integer programming. Most IP applications to production scheduling seem to be oriented towards truck and shovel assignment (Lambert and Mutmansky, [20])Gangwar [21], used a chance constrained binary programme to optimise pit production scheduling.

### 3.1.4 Graph and Network Theory

Most mining production problems are formulated as network formats. A network is a graph (consisting of nodes and branches connected together) associated with some flow. The most common network formats are Critical Path Method (CPM) and Project Evaluation Review Technique (PERT). Network problems are used in project control and occassionally in production scheduling. Collins [22] used Network theory to find a smooth schedule of work activities in minimising resources necessary to complete the work.

### 3.1.5 Simulation

Simulation is the use of a model to experiment with any given system. Simulations in mineral operations are written in computer languages for use on digital computers to ease computation. The most common simulation languages are GPSS, SIMSCRIPT and GASP. General purpose languages such as FORTRAN and PL1 are also suited to the task but are less efficient.

The application of simulation to traditional production scheduling problems have come about in a rather haphazard manner,even though in the mine planning process, simulation has been used very extensively. Manula et al [23] developed a simulator for production scheduling oriented towards the entire system and Dowd and Pariseau [24] developed a hybrid model in GPSS to solve Integer programming problems.

### 3.1.6 Heuristic Methods

Heuristic methods are procedures not mathematically proven or centred but are based on practical and/or logical operating procedures. The procedures used are either rule of thumb or iterative. Heuristic models are common in practice but due to their subjective nature they have not been documented sufficiently in the literature.

## 3.2 Production Planning and Scheduling at Renco Mine

The procedure for determining mine production and development rate and Run of Mine (ROM) grade at Renco mine is a heuristic method developed out of logical operational procedures and mine performance statistics. The annual production rate in terms of tonnage and grade are determined from the mining blocks available and their grades. The allocation of resources such as labour, materials etc. depends on the "call" for the section. There are three sections each under the control of a Mine Captain. The sectional production depends on the number of blocks available in that particular section. Total mine production is the sum of the productions from all the sections.

During the planning process, an effort is made to keep development ahead of stoping to ensure that enough mining blocks are made ready for production to replace those being depleted and to meet production targets. This planning procedure ensures optimum recovery but neglects the economic aspects of production planning and so does not ensure that the wealth maximization goal of planning is achieved.

A planning procedure that enables the maximization of the Net Cash flow at every period will in the long run maximise the market value of the firm's shares - it's wealth. A procedure that ensures an optimum production schedule was developed by Elbrond et al [25] using the open ended dynamic programming model. The next section therefore overviews the theory of Dynamic programming in general and the open ended Dynamic programming model in particular.

## 3.3 Dynamic Programming in Production Planning

The multiperiod production planning problem can often be decomposed and formulated as a dynamic programming problem. Dynamic programming if properly applied cuts across all fields of mathematical programming and provides solutions to multistage problems in all fields of engineering.

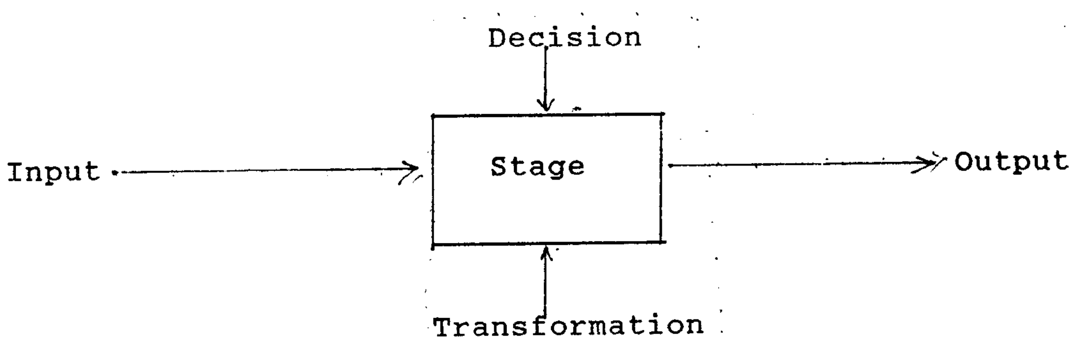
### 3.3.1 The General Dynamic Programming Model - Theoretical Review.

The basic concept of dynamic programming is contained within the principle of optimality stated as:

'The optimal set of decisions in a multistage decision process has the property that whatever the initial state, stage, and decisions are the remaining decisions must constitute an optimal sequence of decisions for the remaining problem with the state and stage resulting from the first decision (or occuring naturally) considered as initial conditions'. Dynamic programming makes it possible for multivariable optimization problems to be solved sequentially.

<u>Mathematical Description</u>

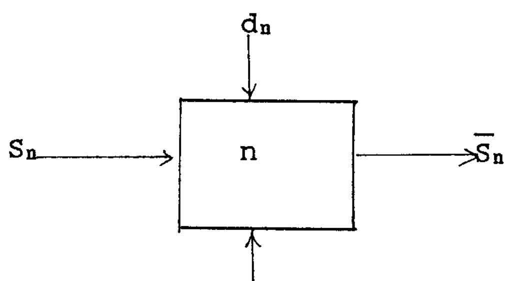
Assume a problem in which there are known input parameters and it is desired that these parameters are used in an optimal fashion (either maximise or minimise). The point at which a decision is made is denoted as a stage and the input parameters denoted as state. The decision made is governed by a rule or equation called a transformation. Pictorially the problem is represented as:




In the D.P. problem, a decision has to be made at each stage and each decision made has a relative benefit or worth reflected by a decision benefit equation.

Let this equation be represented as a return function. This return function will, in general, depend on both the state variable and the decision made at stage n. An optimal decision at stage n will be that decision which yields a maximum (or minimum) return for a given value of the state variables, Sn.

Functionally, for a single stage we have:




$g_n = {}_n(S_n , d_n)$

where $S_n$ = Input state $\overline{S}_n$ = Output state

n = Stage number

$d_n$ = decision $g_n$ = return function

$= {}_n(S_n , d_n)$.

If we are faced with a number of decision points (stages) related in some manner by a tansition function, then:

$$\overline{S}_n = S_n * d_n \qquad (1)$$

(Output of stage n) = (input to stage n) * (decision made at stage n).

* is an operand representing +, −, ÷, x etc. The

units of $S_n$, $d_n$ and $\bar{S}_n$ must be homogeneous and are determined by the particular problem being solved.

## The General N-Stage Multistage System

Suppose there are exactly N stages at which a decision is to be made. These N stages are all linked by the tansition function or functions of the form of equation (1).

Since a state variable is both the output from one stage and an input to another, it is sometimes represented by more than one symbol, namely:

$$\bar{S}_{i+1} \equiv S_i \qquad i = 1, 2, \ldots\ldots\ldots (N - 1)$$

Note that the stages are numbered in an opposite diection to the flow of information. This is called backward recursion.

Since the problem is always solved sequentially, it is necessary to keep track of all the returns accumulated in the decision process as we proceed from stage to stage.

Let: $f_n\ (S_n,\ d_n)$ = the accumulated total return calculated over n stages, given a particular state variable $S_n$.

$f^*_n(S_n)$ = optimal n-stage total return for a particular input state $S_n$.

That means a particular value of $S_n$ might give rise to many possible decisions, $d_n$, among which is a decision $d_n{}^*$ which gives rise to an optimal n-stage return $[f_n{}^*(S_n)]$.

Since $f_n{}^*(S_n)$ consists of accumulated optimal returns, then it can be written as

$$f_n{}^*(S_n) = \underset{d_n,\ d_{n-1},\ \dots\dots\ d_1}{\text{opt}} \{g_n * g_{n-1} * \dots\dots\dots * g_1\}$$

$$= \underset{d_n,\ d_{n-1},\dots\dots\dots\ d_1}{\text{opt}} \{r_n(d_n,\ S_n) \quad {}_{n-1},\ S_{n-1}) * \quad {}_1(d_1,\ S_1)\} \dots (2)$$

## 3.3.2 Developing an Optimal Decision Policy

Suppose we want to develop an optimal decision policy for a multistage problem similar to the one above. The following characteristics are noticed:

1. There are exactly N points at which a decision must be made.

2. Starting from stage 1, nothing affects an optimal decision except the knowledge of the state of the system at stage 1 and the choice of the decision variable.

3. Stage 2 only affects the decision at stage 1; the choice made at stage 2 is governed only by the state of the system at stage 2 and the restriction on the decision variable.

4. And so on to stage N.

The one-stage return function is given by:

$$g_1 = r_1(S_1, d_1)$$

the optimal one-stage return is found by searching over all possible decision variables (defined by a particular state variable) Hence,

$$f_1^*(s_1) = \underset{d_1}{\text{opt}} \{ r_1(S_1, d_1) \}$$

the range of $d_1$ is determined by $s_1$, but $s_1$ is determined by what has happened in the previous stage. Specifically define $S_1 = S_2 - d_2$ for illustrative purpose.

The optimal two-stage return is:

$$f_2^*(S_2) = \underset{d_2}{\text{opt}} \{ r_2(S_2, d_2 + f_1^*(S_1) \}$$

or $$f_2^*(S_2) = \text{opt} \{ r_2(S_2, d_2) + f_1^*(S_2 - d_2) \} \quad \text{.........(3)}$$

Since $S_1 = S_2 - d_2$

It can be noted that $f_2^*(S_2)$ is only a function of $S_2$ and $d_2$ provided $f_1^*(S_1)$ is known for all possible values of $S_1$. For a general N-stage system, one could write

$$f_1^*(S_N) = \underset{d_N}{\text{opt}} \{ r_N(d_N S_N) + f_{N-1}(S_N - 1) \}$$

Notice from equation (3) that once the optimal policy is determined at stage 2, for any incoming state $S_2$, then $S_1$ is known. This process can be repeated until the Nth stage has been reached. In particular the relationship is given by

$$f_n^*(S_n) = opt\ \{r_n(S_n;\ d_n) + f_{n-1}(S_n - d_n) \quad ......(4)$$

$n = 1,\ 2,\ ....\ N$ (where $f_0^*(So) \equiv 0$).

In general, the recursive relationship is given by

$$f_n^*\ (S_n) = opt\ \{r_n(S_n,\ d_n) * f_{n-1}(S_n\ -1)\}........(5)$$

$$S_{n-1} = S_n * d_n \quad .....................(6)$$

### 3.3.4 The Forward Recursion Model

In the general DP problem, developed previously, the recursive relationships for the N-stage system started from stage 1 on the right and flowing against the direction of the material flow from stage N(left) to stage 1 (right) This is called backward recursion. In many problems, the solution procedure can be developed from stage N (left) to stage 1 (right). This is called forward recursion and generally involves a different definition of the state variables and the transition function than those used in backward recursion.

The same procedure is involved in developing an optimal policy and the problem is stated as:

$$f^*_{n+1}(S_{nH}) \quad = \quad opt\{r(S_{n+1},\ d_{n+1}) \quad + \quad f^*_{n+2}(S_{n+r}) \quad +$$

$$\ldots\ldots\ldots\ldots + f^*_N(S_N)\}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

## The Open-ended dynamic Programming Model for Production Planning.

Bellman's[26] principle of optimality as stated above is restated as a reversed optimality when applied to mine planning problems. Roman [26] restated the principle as 'the highest return possible must be achieved with any given quantity mined up to date (state) so as to determine the highest return possible with the entire deposit (depleted or not)'.

The objective of the open-ended dynamic programming optimization is to find the sequence of mining rates that renders the highest discounted cash flow generated by mining rate decisions, the process in any one of the permitted states. The model allows optimization of the sequence of annual mining rates and cut off grades discounted with the running rates.

By using the reverse resolution as above, it has been found that a predetermined state is no longer an implicit constraint in the process. This particular dynamic programme offers a greater similarity to mining since the goal is not of mining a fixed amount of rock at maximum benefit but in fact to obtain maximum benefit from exploitation of the deposit, mined out or not.

The reversed principle is used in solving the dynamic programme by determining first the value (sum of discounted cash flows) of all possible states at all possible steps and is expressed as:

$$f_n(P) = \max_{s} \{R_{n-1}(P,q) + f_{n-1}(T_{(n-1)}, P,q))\} \ldots\ldots (8)$$

in which:

$f_n(P)$= the total value of a process at state P after n decisions.

s = the set of permitted states

$R_{n-1}(P,q)$ = the value of decision q at stage n-1 placing the process in state p at stage n.

$T_{(n-1,\ p,q)}$ = the state at stage n-1 from which the process is taken by decision q to obtain state p at stage n.

After computing the values for all states, the best mining sequence is obtained by following the marked path of optimal decisions back to the initial state of the process. The end constraint is relaxed so as to obtain the best N-state stage solution even though it might not be optimal.

It is also possible to introduce into the model constraints in the form of mining rates for some years to meet management's objectives. This concept is called constrained optimization and can either be in the initial year or even in a restrictive final year.

Figure 3.1 illustrates how mining can be represented as a dynamic programming network. There are three possible mining rates for 6 years. The best cut off grade to associate with each mining rate is selected from 3 possible grades (in this example). The number of states is obtained by dividing the total ore reserves by the increment in possible mining rates.

The values used for computing the discounted cash flows are broken into:

Stage dependent Variable

Fixed and variable mining cost, concentration costs: smelter and concentrate transport charges, prices and deductions.

State dependent Values

Fixed and variable depth costs: incremental investments to open new levels and increasing costs as mining gradually proceeds downwards can be accounted for by these costs.

Determining the Stage-State Optimal Value

Suppose we want to determine the maximum cash flow at stage n and state m $f_n(m)$ using the algorithm of equation (7)

Assume that there are three routes to point (n, m). i.e. three decisions will have to be made in order to

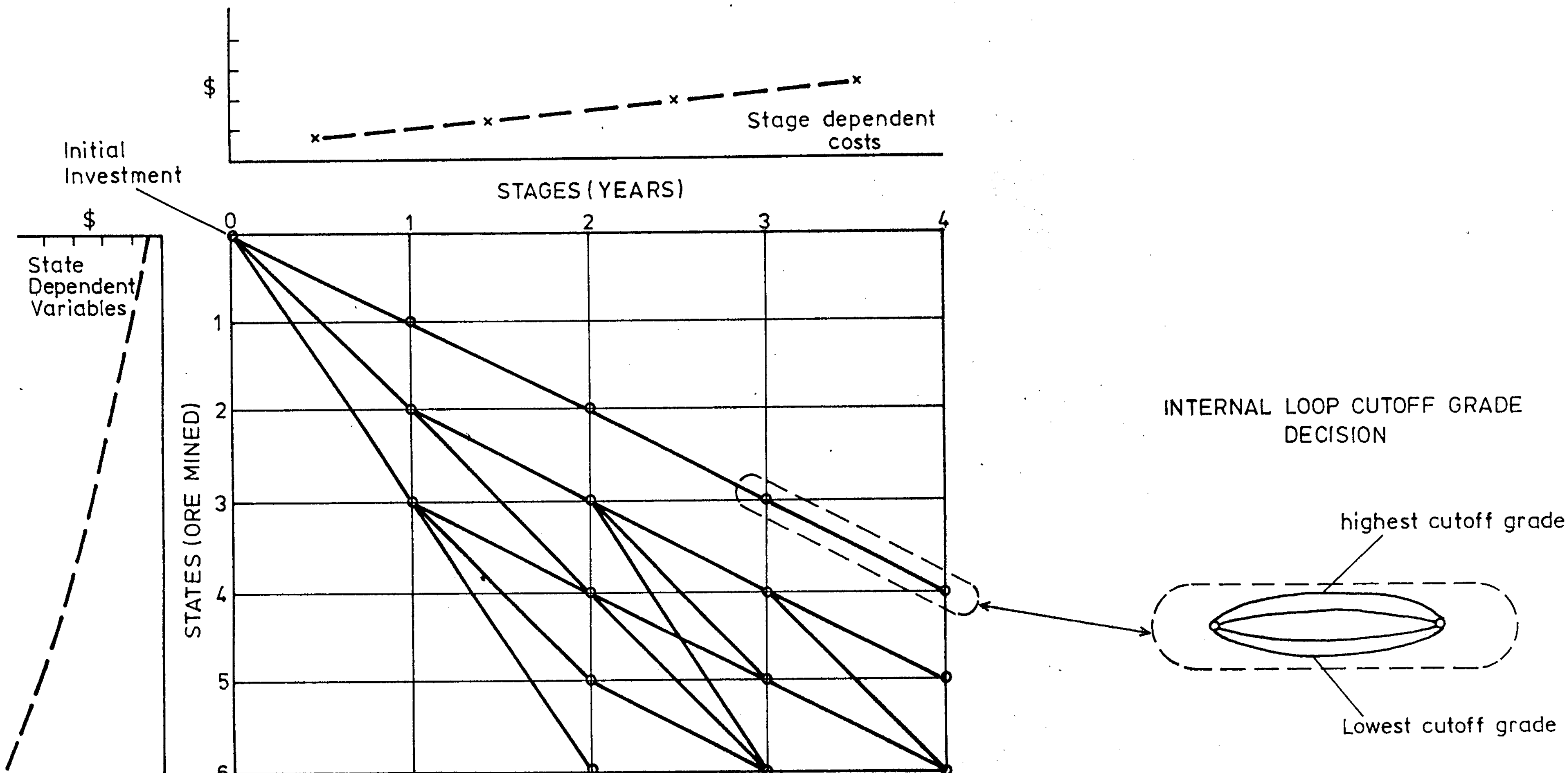



Fig. 3.1. MINING AS A DYNAMIC PROGRAMMING NETWORK

determine $f_n(m)$; $f_n(m)$ is defined therefore as:

$$f_n(m) = \max_{s} \begin{Bmatrix} r_{n-1}(m,3) + f_{n-1}(m-3) \\ r_{n-1}(m,2) + f_{n-1}(m-2) \\ r_{n-1}(m,1) + f_{n-1}(m-1) \end{Bmatrix}$$

The maximum among the three alternatives is taken as the value of $f_n(m)$.

3.4 Scheduling Using the Mine Potentialities Concept.

The number of stopes to put under production and the number of development ends to maintain to meet the targeted production rate and to ensure that enough mining blocks are developed to replace depleted stopes can be determined from the mine Potentialities Concept.[32]

Basically the concept requires that the mine capacity be established in accordance with the existing mine potential. The general relation in this concept is:

$$A = A_{st} + A_d \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Where : A is the total annual production t/yr.

$A_{st}$ is the stope production t/yr.

$A_d$ is the development production.

The total tonnage A is computed as:

$$A = \frac{12 \times P_o \times n_o}{K_o \ \mu} \quad \ldots\ldots\ldots\ldots\ldots (10)$$

Where: $P_o$ is the monthly stope production

$n_o$ is the number of stopes in production

$\mu$ is the assurance factor and lies between (1.2 - 1.3). It is the reserve coefficient to take care of production abnormalities.

$$K_o = \frac{1}{1 + e} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

e is defined as $\frac{A_d}{A_{st}}$

The total number of blocks in both production and development, n, is found from the relationship:

$$n = n_o + n_d \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

Where: $n_d$ is the number of blocks being developed

If the total time to work out a block, $t_{st}$, and the time required to develop a block ready for mining, $t_d$, are known, the following relation also exists:

$$\frac{n_o}{n_d} = \frac{t_{st}}{t_d} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

From the relations developed in this section it is possible to schedule the annual stoping and development rates to meet planned production targets.

# 4. PRODUCTION OPTIMIZATION

## 4.1 Economic Concepts

The primary objective of a firm is to maximise value or wealth of its owners.[27] Wealth refers to the total current market value of the firm's assets. Wealth maximization deals with the current and future prospective earnings of the firm. The amount or quality of these earnings depends on management's decisions on financing, investment and dividend policies. Capital investment decisions ultimately relate to how resources like labour, land and capital are allocated and utilised. For mining projects, the most important resource is the deposit itself and how it is exploited is important in achieving the firm's wealth maximization goal. Benefits are accrued by converting the deposit into monetary value during exploitation. The quantity and timing of these benefits, so as to maximise wealth, is achieved in the mine planning process, and production optimization ensures that the deposit is utilised in an optimal manner.

A mining project is defined by three parameters:

- the tonnage of the extracted material
- the tonnage of the selected material sent to the mill
- the metal content.

The most important of these parameters is the metal content since mine revenue and profit formulae are functions of this. The metal content of the selected material sent to the mill depends on the cut-off grade of the mine. In mine production planning and scheduling several tonnages and cut off grades are possible. Generally, when the cut-off grade is low, the tendency is to mine more tonnage at cheaper cost to achieve higher metal recovery. In most mines, the break even grade is used as the cut-off grade for the mine. This decreases when the price of recovered mineral increases or the production costs decrease.

Several production combinations are possible during planning. Each production combination yields a particular cash flow to the mine. The optimum production schedule is that production combination that yields a maximum, total discounted cash flow (or Net present palue)at the end of the project or the planning period.

Therefore the goal of production optimization is to develop a production schedule for the mining project that results in maximum economic benefits at the end of the project. These benefits are quantified as the Net Present Value of the resulting cash flows.

## 4.2 Cash Flows

Cash flows are frequently calculated on annual basis for evaluation purposes and cash flow analysis. Cash flow analysis may be made for any investments which has income and expense associated with it. The table below illustrates the basic calculation procedure for determining annual cash flows for a mining property.

Table 4.1: Components of an Annual Cash Flow

| Calculation | Component |
| --- | --- |
| | Revenue |
| Less | Royalties |
| Equal | Gross Income from Mining |
| Less | Operating Costs |
| Equal | Net Operating Income |
| Less | Depreciation and Amortization Allowance. |
| Equal | Net Income after Depreciation and Amortization. |
| Less | Depletion Allowance |
| Equal | Net Taxable Income |
| Less | State Income Tax |
| Equal | Net Profit After Tax |
| Add | Depreciation and Amortization Allowance. |
| Add | Depletion Allowance |
| Equal | Operating Cash Flow |
| Less | Capital Expenditures |
| Less | Working Capital |
| Equal | Net Annual Cash Flow |

Table 4.2 lists some of the main important factors relating

to pre-production production and post production mining activities which need to be considered in cash flow analysis.

Table 4.2: Factors for Consideration in Cash Flow Analysis of a Mining Property.

Preproduction Period

Exploration expenses
Water rights
Mine and Plant Capital requirement
Sunk costs
Working Capital

Land and Mineral rights
Environmental costs
Development costs
Financial Structure
Administration

Production Period

Price

Processing Costs
Recovery
Post concentrate Costs
Reserves and percent removable
Grade
Investment Tax Credit
State taxes

Federal taxes

Depletion rate

Depreciation

Capital Investment - Replacement and Expansions.
Royalty
Mining Costs
Development costs

Exploration cost
General and administration
Insurance
Production rate in tons per year.

Financial year production begins.

Percent production not sent to processing plant.

Operating days per year.

Post production

Salvage

Contractural and reclamation expenditures.

## 4.3 MINE REVENUE ESTIMATION

### 4.3.1 Components of Revenue

Annual mine revenue is calculated by multiplying the number of units produced and sold during the year by the sales price per unit. The determination of the components of revenue is critical and more difficult especially the unit price of saleable material.

The number of units produced depends on the tonnage and grade of the ore mined. The quantity of ore mined is determined by the mining schedule which is a function of the deposit's geological characteristics, mining method and other factors.

In-situ grade must be adjusted for dilution and mining extraction. The total saleable unit is then calculated by multiplying the adjusted tonnage by the adjusted grade and further adjusted by the Recovery factor to cater for losses in the reduction plant.

The second major component in mine revenue calculation is the price of the mineral produced. In investment analysis, it is necessary to project this price into the future in order to determine future benefits accrued to the project. The projection of mineral prices is a difficult task with a high degree of error but it is necessary for the above reason to be able to predict future prices with some accuracy.

Minerals are marketed in commodity markets e.g. the London Metal Exchange (LME) or as Purchase contracts, Smelter Contracts, or as Administered prices. No matter the method of sale of the mineral adopted by individual firms or the industry as a whole, the accurate projection of mineral prices into the future is very critical in evaluating capital investment projects.

### 4.3.2 Projecting Mineral Prices

Mineral price forecasts or projections are highly unreliable due to the influence of world political events on the supply and demand of minerals. Since it is not possible to quantify these events by mathematical equations, an attempt to use complicated mathematical models to forecast prices is not only unnecessary but costly. Nonetheless numerous methodologies, some mathematical have been developed for forecastng mineral prices.
It must be stressed that no single method is fool proof and the best method is that which works.

(i) Naive Methods

These methods are based on the assumption that historic prices alone determine the level of future prices. Under these methods we have the No Change Model under which the spot price at any given time is assumed to be as good as any other single point in the future; and the Same Change Model which assumes that the future price is determined by extrapolating a historical price trend into the future using standard regression techniques.

Advantages : Its major advantage is its simplicity.

Disadvantages: The methods are unsatisfactory due to the complex nature of minerals supply and demand so past prices alone cannot simply determine future prices no matter the mathematical complexity of the model used.

(ii) Econometric Modelling

This involves constructing quantitative models for various minerals and relating the price as the endogenous variable to a variety of lagged and unlagged exogenous variables. These statistical models can provide important quantitative

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(ii) Econometric Modelling

This involves constructing quantitative models for various minerals and relating the price as the endogenous variable to a variety of lagged and unlagged exogenous variables. These statistical models can provide important quantitative information about historical relationships between price and other variables. The modelling process can also aid in prioritizing exogenous variables for further study by the analyst. The disadvantages of this method is that the model does not by itself yield credible long-run mineral forecasts and like the naive methods discussed above, only historical data are used to forecast future prices.

(iii) Rational Pricing

This involves estimating a long-run 'normalised' price of the mineral concerned based on assumptions that:

(i) rational investors will not invest in new mines unless the investment promises to deliver some minimum acceptable return and

(ii) new productive capacity will be needed on continuing basis to offset rising demand and for depletion of present mines.

The advantage is that, the normalised price tends to be more stable than the spot price and represents the longer range equilibrium of supply and demand. The normalised price is that price which will cover, in the long-run, Cost of production including some minimum return on the capital invested in the project.

The disadvantages associated with the method are:

1) High sensitivity of the rationalised price to the type of deposit and not to the whole industry.

2) Response to market signals is different for different firms hence a rationalised price cannot be assumed for the whole industry.

3) Severe shifts in demand may make rationalised prices unrealistic.

4) Fluctuating exchange rates especially in developing countries seriously affect projected prices.

(iv) Supply and Demand Schedules

This method is an attempt to take into account the demand for the minerals in projecting future prices. This is done based on production cost schedules, short and long-run supply curves, and demand schedules. Even though this method projects future prices based on calculated production costs as well as the total supply and demand of the mineral concerned, some disadvantages are also associated with the method. These are:

1. Difficulty of forecasting demand of minerals since minerals demand depend on some unquantifiable world events e.g. wars.

2. There is difficulty in estimating true variable costs since in the long run all costs are variable.

## .4 ESTIMATING RENCO MINE REVENUE

Renco Mine revenue is realised mainly from the sale of gold bullion even though other sources like sale of scrap equipment,old timber, etc also generate some revenue. The total production in the month is sold to the Bank of Zimbabwe at the prevailing market price of gold and converted to Zimbabwe dollars. The government, however, has a guaranteed price of Z$750 per oz. (as at 1990) which is paid to producers when the gold price falls below this limit.

For planning purposes, Renco mine revenue is calculated by multiplying the total annual production by a forecasted average price of gold. Costs associated with the sale of the gold as well as smelter charges are treated as divisional costs which are paid as fixed rates to headquarters for each year.

Gold prices are forecasted using the Equal Change Model. Future prices are forecasted by compounding the current price of gold in US$ at 4% annually. The exchange rate of the Zimbabwe dollar to the US dollar is also forecasted to fall annually at 10%. These parameters are developed by the R.T.Z. Headquarters in Harare for all their subsidiaries in Zimbabwe.

For this project this method is adopted in computing project revenues. The base case is the 1990 Renco mine Cost and Revenue budget and the three-year Rio Tinto Zimbabwe group budget. The method offers the following advantages:

1. It is simple since no complicated mathematical forecasting algorithms are involved.
2. It meets the planning needs of the company since it takes account of the production cost schedules of the company.

Table 4.4: GOLD PRICE FORECASTS BASED ON THE EQUAL CHANGE MODEL: YEAR 0 = 1990

| Year | Price in US$ per oz. | Exchange Rate Z$/US$ | Price in Z$ per oz. | Price: Z$ per kg*. |
|---|---|---|---|---|
| 0 | 375 | 0.40 | 938 | 30167 |
| 1 | 390 | 0.36 | 1083 | 34830 |
| 2 | 406 | 0.33 | 1230 | 39558 |
| 3 | 421 | 0.29 | 1452 | 46697 |
| 4 | 439 | 0.26 | 1689 | 54320 |
| 5 | 456 | 0.24 | 1900 | 61105 |
| 6 | 474 | 0.21 | 2257 | 72586 |
| 7 | 493 | 0.19 | 2595 | 83456 |
| 8 | 513 | 0.17 | 3018 | 97060 |
| 9 | 533 | 0.15 | 3553 | 114266 |

1 kg = 32.16 oz

## 4.5 CAPITAL COST ESTIMATION

Feasibility studies form a crucial part of investment analysis. During each level of feasibility studies, it is essential that capital costs are estimated, the accuracy of which depends on the level or stage of the feasibility study. For a detailed feasibility study an accuracy of -2 to +10% is expected. The cost information needed to develop detailed cost estimates are from complete engineering drawings, specifications and site surveys. Several methods are used in developing cost estimates for capital investment analysis. Among these are;

1. Conference Method
2. Unit Cost Method
3. Turn-over Method
4. Exponential Capacity Method
5. Cost Ratio Method
6. Component Cost Ratio Method
7. Module Method
8. Detailed Cost Estimates
9. Cost Indices.

In this project, cost estimates are detailed based on completed engineering drawings, site surveys and plans for the expansion project.

## Detailed Project Capital Cost Estimates

Detailed cost estimates are the final and most accurate type of cost estimates and are based on detailed engineering drawings, layouts,flowsheets and equipment lists showing model, numbers, specifications etc. At the time of data collection a total of Z$2m was spent to finish off the final contractual work on the expansion project. This information is included in the capital expenditure budget of the mine for 1990.

### 4.6.1 <u>Capital Expenditure and Replacement Cost Estimation</u>

Assumptions made:

1. Capital expenditure at year zero (=1990) of the project is the total book value of Renco assets plus all expenditure incurred on the expansion project.

2. Capital equipment are retired and replaced at the end of the periods stipulated in the depreciation policy.

3. The ages of initial capital equipment at year zero are computed from the original cost, the book value and the annual depreciation allowance.

4. All assets in a particular asset class are assumed to have been purchased in the same year.