The University of Zambia

Final examination papers

School of natural sciences

2009 – 2013

1. BIO 5011 research statistical method paper 1
2. BIO 5011 research statistical method paper 2
3. BIO 5101 Biosystematics of tropical animal taxa theory paper
4. BIO 5101 Biosystematics of tropical animal taxa practical paper
5. BIO 5102 Biosystematics of tropical animal taxa theory paper
6. BIO 5122 Biodiversity assessment and management theory paper
7. BIO 5122 Biodiversity assessment and management practical
8. BIO 5145 ecology and management of world life population theory paper
9. BIO 5145 ecology and management of world life population practical paper
10. BIO 5155 Aquatic ecology and fish population theory paper
11. BIO 5155 Aquatic ecology and fish population practical paper
12. BIO 5165 Ecology and management of tropical wetlands theory paper
13. BIO 5165 Ecology and management of tropical wetlands practical paper
14. BIO 5452 Insect- plant host and insect- animal- host relationship theory paper
15. CHE 5011 General chemical techniques
16. CHE 5222 electrochemical and chromatographic methods
17. CHE 5411 Applied inorganic Chemistry
18. CHE 5435 Further Bio-Inorganic Chemistry
19. CHE 5522 Natural product chemistry
20. CHE 5522 Natural products Chemistry
21. CHE 5522 The Chemistry and Biosynthesis of natural products
22. CHE 5635 introduction to statistical thermodynamics
23. CHE 5711 Medicine Chemistry I

24. GEO 5892 Geographic information systems and remote sensing
25. GES 5592 Natural Resources economics
26. GES 5601 principles of environmental and natural resources management
27. GES 5624 Environmental law
28. GES 5645 sustainable land management and food security
29. GES 5665 Forest and world life management
30. GES 5881 Research methods
31. GES 5881 Research methodology
32. GES 5892 Geographic information systems and remote sensing
33. MAT 5111 ordinary Differential equation and integral equations
34. MAT 5122 Partial differential equations
35. MAT 5141 Tropics in mathematics methods
36. MAT 5311 Lebesgue measure and lebesgue integration
37. MTA 5342 operator theory
38. MAT 5611 statistical inference
39. MAT 5632 Design and analysis of experiments
40. MATS 5642 statistical methods in epidemiology
41. MATS 5662 theory of Non-parametric statistics
42. MAT 5911 stochastic processes
43. PHY 5021 mathematical methods for physics
44. PHY 5022 condensed matter physics
45. PHY 5032 computational and modelling II
46. PHY 5311 theoretical physics
47. PHY 5322 theoretical physics II
48. PHY 5502 theoretical physics II
49. PHY 5822 solar energy materials

50. PHY 5911 computational physics and modelling

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR FIRST SEMESTER
FINAL EXAMINATIONS

BIO 5011: RESEARCH STATISTICAL METHODS
THEORY PAPER II

TIME: THREE HOURS

INSTRUCTIONS: ANSWER **ALL** QUESTIONS USING THE APPROPRIATE STATISTICAL ANALYSIS SOFTWARE PROVIDED. USE THE STATISTICAL OUTPUT ILLUSTRATIONS WHERE NECESSARY.

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1. Table 1 contains data on bird species richness from three habitats during different sampling periods of an ecological monitoring programme in Sioma Ngwezi National Park (SNNP) in Sesheke District.
   Determine whether there are significant differences in bird species richness:
   (a) Among different habitats.
   (b) During different sampling periods.

   Table 1. Bird species richness data from three habitats in Sioma Ngwezi National Park in Sesheke District.

Sampling period	Riverine woodland	Kalahari Sand woodland	Mkusi woodland
Baseline	24	28	22
1st spray	14	22	25
2nd spray	33	27	40
3rd spray	40	20	52
4th spray	41	31	33
5th spray	37	32	45
Post spray	41	34	47
Hot and dry season	62	49	59
Rainy season	65	48	69
Cool and dry season	62	45	63

TURN OVER

2. Table 2 shows the results of a controlled green house experiment whose objective was to investigate the growth (height) of 16 plants of *Bidens pilosa* in relation to the rate of transpiration and photosynthesis when the growth was constant and increasing.
Determine the relationship between photosynthesis and transpiration when growth is:
(a) Constant.
(b) Increasing.

Table 2. Rates of transpiration and photosynthesis of *Bidens pilosa* under constant and increasing growth.

	Rate of transpiration (cm^3 of water/hour)		Rate of photosynthesis (grams of glucose/hour)	
Plant	Constant	Increasing	Constant	Increasing
1	9.1	10.8	65.4	72.9
2	5.6	5.9	73.7	94.4
3	6.7	7.2	37.4	43.3
4	8.1	7.9	26.3	29
5	16.2	17	65	66.4
6	11.5	11.6	35.2	36.4
7	7.9	8.4	24.7	27.7
8	7.2	10	23	27.5
9	17.7	22.3	133.2	178.2
10	10.5	11.1	38.4	39.3
11	9.5	11.1	29.2	31.8
12	13.7	11.7	28.3	26.9
13	9.7	9	46.6	45
14	10.5	9.9	61.5	58.2
15	6.9	6.3	25.7	25.7
16	18.1	13.9	48.7	42.3

3 The data provided in the Excel© spreadsheet is presence and abundance data of bird species in Sioma Ngwezi National Park in Sesheke District.
(a) Group the bird species based on their similarity in distribution and abundance using Agglomerative clustering and the Ward's Method.
(b) Determine the number of clusters at 75% similarity and discuss membership to the clusters.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR
FINAL EXAMINATIONS

BIO 5011: RESEARCH STATISTICAL METHODS
THEORY PAPER I

TIME: THREE HOURS

INSTRUCTIONS: ANSWER **FIVE** QUESTIONS.

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1. Four hundred and forty-five (445) university students were classified according to both their frequency of marijuana use and their parental use of alcohol and psychoactive drugs. The data collected are summarised in Table 1 below:

Table 1. Classification of university students on the basis of marijuana and alcohol uses.

Student use of marijuana	**Parental use of alcohol and psychoactive drugs**		
	Both parents	**Neither parent**	**One parent**
Never	17	141	68
Occasionally	11	54	44
Regularly	19	40	51

Test the Null hypothesis that there is no association between parental use of alcohol and psychoactive drugs and the use of marijuana by their children in universities.

2. Table 2 shows mean numbers of egg pods of the Armoured Ground Cricket that were collected from soils with different concentrations of phosphorous element (mg/kg);

Table 2. Egg pods of the Armoured Ground Cricket collected from in soils with different Phosphorous concentrations.

Mean Egg Number per Pod: (X)	4.80	9.40	4.20	4.80	5.00	0	1.60	0.60
	4.60	6.80	1.80	2.20	1.80	1.60	1.80	0.60
Phosphorous (mg/kg) (Y)	5.81	30.94	8.26	14.11	2.42	3.50	2.14	12.43
	0	6.76	17.54	4.34	9.70	9.42	3.50	2.77

TURN OVER

Calculate the correlation between number of egg pods and soil phosphorous content and test the Ho that rho equals zero.

3. In a study on species diversity in four African lakes, data presented in Table 3 below were collected on a number of different species caught in six catches from each lake:

Table 3. Number of different fish species caught in four African Lakes.

Catches	LAKE			
	Tanganyika	**Victoria**	**Malawi**	**Chirwa**
1.	64	78	75	55
2.	72	91	93	66
3.	68	97	78	49
4.	77	82	71	64
5.	56	85	63	70
6.	95	77	76	68

(a) Conduct an ANOVA to test the Ho that the four lakes have the same species diversity.
(b) Using Least Significant Difference (LSD), separate the sample means to identify the lake that has the highest fish diversity.

4. In an automobile exhaust emission study, four cars and four drivers were used to test the possible differences among four petrol additives (A, B, C & D) in reducing the amount of oxides of nitrogen emitted in exhaust gases. A Latin Square Design (LSD) was employed in the experimental setup and Table 4 below gives the results of this study:

Table 4. Results of a study conducted to determine nitrogen emission in car exhaust gases

Driver	CAR			
	1	**2**	**3**	**4**
1	A = 21	B = 26	D = 20	C = 25
2	D = 23	C = 26	A = 20	B = 27
3	B = 15	D = 13	C = 16	A = 16
4	C = 17	A = 15	B = 20	D = 20

(a) Conduct an ANOVA to test the Ho that there were no significant differences in the emission levels of nitrous oxides among the additives.
(b) If the Ho is rejected, separate the means using the LSD statistic.

CONTINUE TO THE NEXT PAGE

5. Measurements (mm) of the head capsule (x) and the compound eye diameter (y), also in millimetres, of an insect found around the University of Zambia (UNZA) Great East Road Campus provided the following data:

$\Sigma x = 67.70$	$\Sigma y = 15.30$	$n = 10$
$\Sigma x^2 = 458.71$	$\Sigma y^2 = 23.45$	$\Sigma xy = 103.60$
$(\Sigma x)^2/n = 458.329$	$(\Sigma y)^2/n = 23.409$	$(\Sigma x)(\Sigma y)/n = 103.581$
$\Sigma x^2 - (\Sigma x)^2/n = 0.381$	$\Sigma y^2 - (\Sigma y)^2/n = 0.041$	$\Sigma xy - (\Sigma x)(\Sigma y)/n = 0.019$

(a) Calculate the Product Moment Correlation Coefficient (r) of X and Y.
(b) Test the significance of the correlation coefficient of the two measurements of this UNZA insect.

6. A fertilizer trial on strawberries consists of 4 replicates of the 4 treatment combinations of nitrogen and phosphorus (Θ, n, p, np). The resulting crop yields per plot (in suitable units) are presented in Table 5 below:

Table 5. Crop yields of an experiment involving various combinations of nitrogen and phosphorus

Blocks	Treatment			
	Θ	n	p	np
I	13	24	16	27
II	12	25	14	34
III	18	24	15	32
IV	15	31	20	30

Carry out an analysis to determine the effects (if any) due to blocks, n, p and the interaction np.

7. Five fertilizer treatments, each replicated three times, were applied to a crop in a randomized block design setup. At the end of the experiment two (2) of the replications had for some reason missing yield data (**a** & **b**) as indicated in Table 6 below:

Table 4. Yield results of a fertilizer treatments experiment on a crop.

Treatment (kg fertiliser per acre)	**Replications**		
	1	**2**	**3**
1	**a**	8.00	7.93
2	8.14	8.15	7.87
3	7.76	**b**	7.74
4	1.17	7.57	7.80
5	7.46	7.68	7.21

(a) Suppose you are now told that the experimenter estimated the missing values using the appropriate equation for the design and found them to be 7.85 and 7.91, respectively. Conduct an ANOVA to test the Ho that there are no significant differences among the crop yield means.

TURN OVER

(b) If the N.H. is rejected, separate the mean using L.S.R.

8. Given the following data (Table 7) of an experiment involving a Completely Randomized Design (CRD), test the H_o that there are no significant differences among the treatment means:

Table 7. Completely Randomised Design data.

Replicate	Treatment			
	A	**B**	**C**	**D**
1	2.0	1.7	2.0	2.1
2	2.2	1.9	2.4	2.2
3	1.8	1.5	2.7	2.2
4	2.3		2.5	1.9
5	1.7		2.4	

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR
FINAL EXAMINATIONS

BIO 5011: RESEARCH STATISTICAL METHODS
THEORY PAPER II

TIME: THREE HOURS

INSTRUCTIONS: ANSWER **ALL** QUESTIONS. ILLUSTRATE YOUR ANSWERS WHERE NECESSARY. USE ANY OF THE FOLLOWING STATISTICAL PACKAGES PROVIDED TO ANSWER THE QUESTIONS; MS EXCEL©, STATISTIX© AND PC-ORD ©. USE THE DESKTOP COMPUTERS PROVIDED OR AUTHORISED LAPTOPS.

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1. You are provided with data on a flash disk and desktop computers on bird community structure in an MS Excel file named 2014 BIO 5011 EXAM. The data includes bird species richness (SPECIES), the number of bird guilds (GUILDS) and bird abundance (ABUND) obtained from 30 plots.

   (a) Group the plots according to their similarity in bird community structure at 90% similarity using hierarchical clustering.
   (b) Determine whether there are differences in bird species richness and abundance in the clusters identified in (a) above.

2. Table 1 below shows data on bird community structure as well as vegetation structure and composition i.e. total plant species richness (TOTRICH), total plant species diversity (TOTDIV), sapling density (SAPDEN), tree species richness (TRERICH) and tree density (TREDEN) obtained from 15 plots.

   (a) Determine the best subset regression models for determining bird species richness and number of bird guilds from vegetation structure and composition data.
   (b) Determine:
      (i) The linear regression models from the variables of the best subset regression models obtained from (a) above.
      (ii) Whether the regression of vegetation variables on the bird community structure variables is significant.
      (iii) Summarise the relationship between vegetation and bird community structure in the form of linear equations.

TURN OVER

Table 1. Bird community structure and vegetation data.

PLOT	SPECIES	GUILD	TOTRICH	TOTDIV	SAPDEN	TRERICH	TREDEN
PLOT 1	9	6	36	3.584	52	7	14
PLOT 2	16	7	34	3.526	20	9	21
PLOT 3	15	7	30	3.401	87	9	18
PLOT 4	10	4	39	3.664	13	7	13
PLOT 5	9	7	30	3.401	11	5	12
PLOT 6	17	11	27	3.296	25	4	7
PLOT 7	9	6	31	3.434	12	8	11
PLOT 8	11	8	30	3.401	42	9	23
PLOT 9	19	9	29	3.367	80	7	12
PLOT 10	10	7	0.794	1.1	4	0.7	9.25
PLOT 11	6	4	31	3.434	9	17	34
PLOT 12	9	6	32	3.466	25	9	23
PLOT 13	15	10	33	3.497	77	4	12
PLOT 14	10	7	32	3.466	41	3	3
PLOT 15	4	4	39	3.664	21	16	54

(Source: Lumbwe, 2010)

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR FIRST SEMESTER
FINAL EXAMINATIONS

BIO 5101: BIOSYSTEMATICS OF TROPICAL PLANT TAXA
THEORY PAPER

TIME: THREE HOURS

INSTRUCTIONS: ANSWER **QUESTION ONE** AND ANY OTHER **FOUR** QUESTIONS.

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1. Summarise any four of the following:
   (a) Leaf dimorphism in the aquatic Polypodiales.
   (b) Circinate vernation.
   (c) Androecium in Annonaceae.
   (d) Biosystematic phase of systematic knowledge.
   (e) Leptosporangiate ferns.

2. Explain briefly the concept of phytochoria and describe four floristic regions of Africa as delimited by Frank White (1971).

3. Describe the distribution, characteristic features and diversity of the family Aristolochiaceae.

4. Discuss the taxonomy, diversity and characteristic features of the Coniferophyta.

5. Describe the evolution of artificial plant classification systems into the current natural systems.

6. Describe the general and special purpose types of classification and explain why one is better than the other for use in Systematics.

7. Briefly outline the history of the international code of nomenclature for algae, fungi and plants.

8. (a) Describe the five basic categories of botanical databases.
   (b) Give five examples of online botanical databases.

END OF EXAMINATION

UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR
FINAL EXAMINATIONS

BIO 5101: BIOSYSTEMATICS OF TROPICAL PLANT TAXA
PRACTICAL PAPER

TIME: TWO HOURS

INSTRUCTIONS: ANSWER **BOTH** QUESTIONS.

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1. (a) Use the taxonomic Key provided to identify specimens **A to J.**

   (b) List diagnostic features of each specimen identified.

2. Construct a taxonomic key for the identification of specimens **K** to **O.**

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR
FINAL EXAMINATIONS

BIO 5101: BIOSYSTEMATICS OF TROPICAL PLANT TAXA
THEORY PAPER

TIME: THREE HOURS

INSTRUCTIONS: ANSWER FIVE QUESTIONS; **QUESTION ONE** AND ANY OTHER **FOUR** QUESTIONS.

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1. Summarise **any four** of the following:
   (a) Apocarpy, syncarpy and polypetally.
   (b) Consolidation phase of plant systematics knowledge.
   (c) General purpose classification.
   (d) Synthetic characters.
   (e) Paratype.

2. Compare and contrast the biological, ecological and phylogenetic species concepts.

3. Describe the distribution, characteristic features and diversity of the family Piparaceae.

4. Compare and contrast the characteristic features of the Convolvulaceae and the Cucurbitaceae.

5. Discuss the floristic regions of Africa and their provincial compositions as delimited by Frank White (1971).

6. Discuss the Principles of the international code of nomenclature for algae, fungi and plants.

7. Discuss the evolution of natural classification systems for plants.

8. (a) Describe the various chorionomic categories.
   (b) Describe the floristic realms of the world and state which ones cover parts of Africa.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

BIO 5102: BIOSYSTEMATICS OF TROPICAL ANIMAL TAXA
THEORY PAPER

TIME: THREE HOURS

INSTRUCTIONS: ANSWER **FIVE** QUESTIONS. **ONE** QUESTION FROM EACH SECTION AND THE OTHER QUESTION FROM ANY SECTION. USE ILLUSTRATIONS WHERE NECESSARY. USE SEPARATE ANSWER BOOLET FOR EACH SECTION.

SECTION A: Principles of Nomenclature

1. Discuss five basic tenets of the codes of biological nomenclature and show how these are used to ensure that names given to organisms remain unambiguous, stable and universal.

2. Distinguish between each of the following pairs of terms and concepts:

   (a) Parallel evolution and convergent evolution.
   (b) Splliter and Lumper approaches to biological classification.
   (c) Biological systematics and biological taxonomy.
   (d) Homologous characteristic and analogous characteristic.
   (e) Natural classification and artificial classification.

SECTION B: Systematics of Invertebrates

3. Compare and contrast characteristics of protostome and deuterostome invertebrate lineages and indicate which of the two lineages gave rise to the Chordata.

4. Explain the evolutionary advances shown by members of the phylum Mollusca over members of the phyla Platyhelminthes and Nematoda.

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SECTION C: Systematics of Terrestrial Vertebrates

5. (a) Compare and contrast the main taxonomic characteristics of the amphibians and reptiles.
   (b) Summarise the distinguishing taxonomic characteristics between crocodiles and alligators.

6. Summarise each of the following:
   (a) Ophidia
   (b) Pleurodira
   (c) Life cycle of amphibians
   (d) Distribution of *Python sebae*

SECTION D: Systematics of Fishes

7. Discuss the significance of the trunk in identification and description of fish.

8. (a) Describe the two lineages of the family Cichlidae
   (b) Summarise characteristics that are important in aquaculture of cichilids.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

BIO 5122: BIODIVERSITY ASSESSMENT AND MANAGEMENT
PRACTICAL PAPER

TIME: THREE HOURS

INSTRUCTIONS: ANSWER **ALL** QUESTIONS. USE ILLUSTRATIONS WHERE NECESSARY. USE COMPUTERS PROVIDED OR YOUR CERTIFIED LAPTOPS, AND THE FOLLOWING STATISTICAL PACKAGES TO ANSWERS THE QUESTIONS; MSEXCEL©, STATISTIX© AND PC-ORD©

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1. Five points along the Zambezi River in Senanga District, Western Province were sampled using the traditional funnel fish basket method for catching small cyprinids (Nchenga). The baited fish baskets were left overnight in the five sampling points and all the fish caught in the basket at each sampling point were identified to species and counted. The data on the fish catches is summarised in table 1. Use PC- ORD© to analyse the fish catches. From the PC-ORD output:

 (a) Provide statistical summaries of species richness, species diversity and abundance for each sampling point in the form of a diagram.
 (b) Determine the dominant cyprinid species along the Zambezi River in Senanga District using the frequency and abundance data.

Table 1. Fish catches along five sampling points on the Zambezi River in Senanga District.

Sampling point	**Species of Cyprinid**					
	Barbus barotseensis	*Barbus lineomaculatus*	*Barbus unitaeniatus*	*Barbus banardi*	*Barbus radiatus*	*Barbus fasciolatus*
1	5	2	1	8	0	1
2	1	4	0	0	8	12
3	3	5	6	7	0	10
4	8	0	4	0	6	0
5	3	7	0	9	11	0

2. Table 2 shows seedling occurrence data collected using the point frequency frame in a Kalahari Sand woodland in Western Province.

 (a) Determine the species richness, species diversity and equitability for each frame.
 (b) Determine the frequency of occurrence and density of each seedling species.
 (c) Determine the dominant seedling species in the area.

TURN OVER

Table 2. Seedling occurrence data in a Kalahari Sand woodland in Western Province.

Frame	Points									
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
1	*Baikiaea plurijuga*	*Baikiaea plurijuga*	*Terminalia sericea*	*Pterocarpus angolensis*	*Baikiaea plurijuga*	*Baikiaea plurijuga*	*Pterocarpus angolensis*	*Pterocarpus angolensis*	*Pterocarpus angolensis*	*Baikiaea plurijuga*
2	*Lonchocarpus capassa*	*Lonchocarpus capassa*	*Lonchocarpus capassa*	*Albizia antunesiana*	*Strychnos spinosa*	*Pterocarpus angolensis*	*Baikiaea plurijuga*	*Pterocarpus angolensis*	*Pterocarpus angolensis*	*Terminalia sericea*
3	*Lonchocarpus capassa*	*Lonchocarpus capassa*	*Terminalia sericea*	*Monotes glaber*	*Baikiaea plurijuga*	*Terminalia sericea*	*Protea angolensis*	*Acacia polyacantha*	*Protea angolensis*	*Baikiaea plurijuga*
4	*Bobgunia madagascariensis*	*Pterocarpus angolensis*	*Guibourtia coleosperma*	*Acacia polyacantha*	*Guibourtia coleosperma*	*Pterocarpus angolensis*	*Baikiaea plurijuga*	*Baikiaea plurijuga*	*Lonchocarpus capassa*	*Baikiaea plurijuga*
5	*Rothmania englerana*	*Baikiaea plurijuga*	*Pericopsis angolensis*	*Lonchocarpus capassa*	*Rothmania englerana*	*Guibourtia coleosperma*	*Acacia polyacantha*	*Lonchocarpus capassa*	*Baikiaea plurijuga*	*Uapaca nitida*
6	*Uapaca kirkiana*	*Guibourtia coleosperma*	*Acacia polyacantha*	*Uapaca nitida*	*Terminalia sericea*	*Pseudolachnostylis maproneufolia*	*Acacia polyacantha*	*Guibourtia coleosperma*	*Terminalia sericea*	*Terminalia sericea*

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR
FINAL EXAMINATIONS

BIO 5122: BIODIVERSITY ASSESSMENT AND MANAGEMENT
THEORY PAPER

TIME: THREE HOURS

INSTRUCTIONS: ANSWER **FIVE** QUESTIONS. USE ILLUSTRATIONS WHERE NECESSARY.

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1. Discuss the ecosystem-based conservation approach.
2. Discuss natural resources management with respect to the concept of co-management.
3. Discuss each of the following:
   (a) Ecological redundancy with respect to species extinction.
   (b) Captive breeding as an *ex situ* conservation technique.
4. Discuss the IUCN Protected Area System categories.
5. Discuss the concept of resistance and resilience in the context of biodiversity conservation.
6. Discuss the IUCN threat of extinction categories.
7. Summarise each of the following species-based conservation approaches:
   (a) Umbrella species conservation.
   (b) Endangered species conservation.
   (c) Biodiversity hotspot conservation.
   (d) Flagship species conservation.
8. Discuss the process of decision making in natural resources management.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR
FINAL EXAMINATIONS

BIO 5122: BIODIVERSITY ASSESSMENT AND MANAGEMENT
PRACTICAL PAPER

TIME: THREE HOURS

INSTRUCTIONS: ANSWER **ALL** QUESTIONS. USE ILLUSTRATIONS WHERE NECESSARY. USE COMPUTERS PROVIDED OR CERTIFIED LAPTOPS, AND THE FOLLOWING STATISTICAL PACKAGES TO ANSWER THE QUESTIONS; MSEXCEL©, STATISTIX© AND PC-ORD©

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1. Determine the importance value (IV) of tree species in a woodland community using the data provided in Table 1.

Table 1 Diameter at breast height (dbh) data of woody species in a woodland community in Mkushi District.

Plot 1	**Tree number**	**Species**	**dbh (cm)**
	1	*Anisophyllea boehmii*	18
	2	*Syzygium guineense*	24
	3	*Brachystegia spiciformis*	8
	4	*Brachystegia floribunda*	10
		Species	**dbh (cm)**
Plot 2	1	*Syzygium guineense*	32
	2	*Brachystegia spiciformis*	12
	3	*Syzygium guineense*	19
	4	*Anisophyllea boehmii*	12
	5	*Brachystegia floribunda*	19
	6	*Ochna pulchra*	9
	7	*Parinari curatellifolia*	19
		Species	**dbh (cm)**
Plot 3	1	*Ochna pulchra*	16
	2	*Syzygium guineense*	9
	3	*Julbernardia paniculata*	12
	4	*Parinari curatellifolia*	14
	5	*Brachystegia floribunda*	22

TURN OVER

2. Describe each of the following:
   (a) The different types of quadrats used in the assessment of biodiversity.
   (b) The type of data that can be recorded from the quadrats in (2a) above.
   (c) The type of analysis and results that can be obtained from the data in (2b) above.

3. Determine:
   (a) The species richness, species diversity and evenness of a bird community using the data in Table 2.
   (b) The dominant bird species using frequency and abundance data.

Table 2. Presence and abundance data of a bird community in Chanyanya area, Kafue District.

Plot 1	**SPECIES**	**ABUNDANCE**
	Glossy Starling	1
	Yellow-billed Hornbill	4
	Tawny-flanked Prinia	1
	Red-eyed Dove	1
	Long-tailed Shrike	4
Plot 2	**SPECIES**	**ABUNDANCE**
	Crimson-breasted Shrike	1
	White-browed Robin	1
	Tawny-flanked Prinia	4
	Grey Lourie	1
	Glossy Starling	3
	Cape Turtle Dove	2
Plot 3	**SPECIES**	**ABUNDANCE**
	Tawny-flanked Prinia	8
	White-browed Robin	1
	Crimson-breasted Shrike	2
	Glossy Starling	3
	Lilac-breasted Roller	1
	Grey Lourie	9

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR FIRST SEMESTER
FINAL EXAMINATIONS

BIO 5145: ECOLOGY AND MANAGEMENT OF WILDLIFE POPULATIONS
PRACTICAL PAPER

TIME: THREE (3) HOURS
INSTRUCTIONS: ANSWER **ALL** QUESTIONS. ILLUSTRATE YOUR ANSWERS WHERE NECESSARY.

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1. A rodent survey in the Nampundwe area used a mark-recapture method to determine the population of the Cane Rat (*Thryonomys swinderianus*, Temminck 1827). Twenty five traps were set at different points along each of the four transects in the area for five days, and each captured rat was marked and released. The following data were obtained (Table 1):

Table 1: The mark and recaptures of the Cane Rate in Nampundwe area.

Sampling occasion	# in sample	recaptures	# newly marked
Day 1	40	--	40
Day 2	44	9	35
Day 3	38	14	24
Day 4	46	24	22
Day 5	35	19	16

(a) Using the Schnabel method, a modification of the Lincoln – Petersen index method, calculate the population of the Cane rat in the area.
(b) Discuss assumptions and limitations associated with this technique in estimating animal populations.

2. Munyamadzi Farms Ltd is considering establishing a game sanctuary in Kalomo District along the Nazhila stream. Initial investigations show that the range is suitable for Impala, Zebra, Wildebeest, Kudu , Hartebeest and Buffalo. The range is relatively flat, well watered and nearly all the range is within 3.5km from water. Based on the information from the Ministry of Agriculture and Livestock in Choma, the soils are generally fersiallitic

TURNOVER

and excellent for a game sanctuary. The average rain fall is 800mm and the vegetation is dominated by Acacia-Combretum woodland and dry Miombo. In addition, results from your preliminary investigations indicate that the production of key forage species averages about 100kg/ha of dry matter per year. The proposed Sanctuary is 10,000 ha in size. Assuming that allowable use is 25% and daily dry matter intake is 2% of the animal body weight,

(a) Determine the number of 90 kg Hartebeest you would stock as your base herd in the area.
(b) Discuss the limitations of this method in estimating the stocking rate of wildlife species.

3. Biologists monitoring populations of Impala (*Aeopyceros melampus,* Lichtenstein 1812) on Chete island in lake Kariba, Sinazongwe between 1958 and 1985 gave figures as given in the Table 1 below. The island is approximately 5km^2 and is generally covered by a thicket of *Combretum* sp. Mean annual rainfall is 900mm. The island is a protected area and is regularly patrolled by Game Scouts. However, artisanal fishermen in the lake are allowed to land fish in certain parts of the island. For nearly 12 years the island was exposed to liberation war between 1968 and 1980, and part of the island was defoliated with herbicides. Impalas are polygamous and only a male breeds with a herd of females. Non breeding males form a bachelor herd.

Table 2: Impala population at Chete Island, Lake Kariba, based on transect ground counts.

Year of census	Total population	Juveniles	Males	Females
1968	136	6	54	76
1972	150	10	40	100
1975	308	40	58	210
1980	263	65	60	138
1985	232	75	65	95

Using the information and data provided,
(a) Discuss the population trends of Impala on the island.
(b) Discuss factors most significant to the population growth of this species.

PROCEED TO NEXT PAGE

4. Two hundred (200) skulls of Kafue Lechwe (*Kobus leche kafuensis*) were collected and aged as follows:

Table 3: Aged skulls of Kafue Lechwe collected in 1975 in Kafue Flats Game Management Area.

Age class	Number of skulls
0.0	95
1.0	13
2.0	15
3.0	11
4.0	13
5.0	4
6.0	5
7.0	11
8.0	14
9.0	8
10.0	6
11.0+	3

a) Construct a life table of this population
b) Assuming that m_x for the species is known to be 0.5 for all age classes except for ages 0 and 1, determine R_o and r_m for the species population.
c) Discuss limitations and assumptions for determining r_m

END OF EXAMINATION

7. Discuss the following as used in wildlife population management studies :
   (a) $N_t = N_o e^{rt}$
   (b) *Syncerus caffer*.
   (c) r- Selection strategies.
   (d) Species territory.

8. Discuss what you understand by the concept of Maximum Sustainable Yield and give reasons why this concept is regarded as an epitaph in harvesting species populations.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR
FINAL EXAMINATIONS

BIO 5145: ECOLOGY AND MANAGEMENT OF WILDLIFE POPULATIONS
PRACTICAL PAPER

TIME: THREE HOURS

INSTRUCTIONS: ANSWER **ALL** QUESTIONS. ILLUSTRATE YOUR ANSWERS WHERE NECESSARY.

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1. A bird survey of Swainson's Francolin (*Francolinus swainsonii*) in the Rufunsa area used a capture-recapture method to determine the population of the birds. Traps (nets) were set at different points along each transect in the area for 2 occasions, and each captured bird was marked and released. The data below were obtained.
   (a) Using the Lincoln-Petersen index method, calculate the populations of the birds in the area.
   (b) Discuss limitations of this technique in estimating animal populations.
   (c) Discuss assumptions associated with this technique.

   Table 1: Data of animal capture and recaptures in Rufunsa Area

TRANSECTS	**A**	**B**	**C**	**D**	**E**	**G**
Initial Capture	3	5	2	5	1	0
Second Capture	5	10	1	4	5	1
Recaptures	3	8	0	2	0	1

2. Briefly describe the following research methods applied in wildlife population management studies:
   (a) King census method.
   (b) Aerial animal counting.
   (c) Point Centered Quarter Method.
   (d) Parker 3-step method.

TURNOVER

3. Munyamadzi Farms Ltd is considering establishing a Game Ranch in the Mpika District along the Munyamadzi River. Initial investigations show that the range is suitable for Impala, Zebra, Wildebeest, Kudu and Buffalo. Ten per cent (10%) of the range is a steep hill and in addition only 60% of the range is within 6.5 km from water. Based on the information from the Ministry of Agriculture and Cooperatives in Mpika, the soils are generally suitable for game ranching. Furthermore, results from your preliminary estimates indicate that the production of key forage species averages about 450kg/ha of dry matter per year. The proposed Sanctuary is 10,000 ha in size.

   Assuming that allowable use is 25% of the total biomass and daily dry matter intake is 2% of the animal body weight,
   (a) Determine the number of 950 kg buffaloes you would stock as your base herd in the area.
   (b) Discuss limitations of this method in estimation the stocking rate of wildlife species.
   (c) Discuss assumptions associated with this method.

4. One hundred and eighty eight (188) skulls of Kafue Lechwe (*Kobus leche kafuensis*) were collected and aged as follows:

   Table 2: Aged skulls of 188 collected in 19754 in Kafue Flats Game Management Area.

Age class	Number of skulls
0.0	90
1.0	13
2.0	12
3.0	11
4.0	13
5.0	4
6.0	5
7.0	11
8.0	12
9.0	8
10.0	6
11.0+	3

   a) Construct a life table of this population.
   b) Assuming that m_x for the species is known to be 0.7 for all age classes except for ages 0 to 2, determine R_o and r_m for the species population.
   c) Discuss limitations and assumptions for determining r_{m}.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR: FIRST SEMESTER
FINAL EXAMINATIONS

BIO 5155: AQUATIC ECOLOGY AND FISH POPULATIONS
THEORY PAPER I

TIME: THREE HOURS

INSTRUCTIONS: ANSWER **FIVE** QUESTIONS: ANSWER QUESTION **NUMBER 8: ANSWER TWO QUESTIONS** FROM **EACH** SECTION AND A **FIFTH** QUESTION FROM **EITHER** SECTION

SECTION A: Aquatic Ecology

1. Summarise the following as used in the categorisation of aquatic ecosystems and organisms:
   (a) Annual minimum and maximum water temperatures.
   (b) Water currents.
   (c) Size of plankton.

2. (a) Discuss the types of ions that determine the alkalinity of freshwater bodies; and
   (b) Explain the relationship, as appropriate between alkalinity and productivity of inland aquatic ecosystems.

3. (a) Summarise the factors that determine the amount of light available at the surface of a water body.
   (b) Describe three possible fates of light as it strikes a water column.
   (c) Explain the significance of Secchi disk measurements in determining penetration of light in water bodies.

4. Discuss the effects of zooplankton populations on phytoplankton community structure in summer for a warm monomictic lake.

SECTION B: Fish Populations

5. Summarise the following in relation to management of fisheries stocks:
   (a) Age at first capture (t_c).
   (b) Total Allowable Catch (TAC).
   (c) Maximum Economic Yield (MEY).

TURN OVER

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR
FINAL EXAMINATIONS

BIO 5155: AQUATIC ECOLOGY AND FISH POPULATIONS
THEORY PAPER I

TIME: THREE HOURS

INSTRUCTIONS: ANSWER **FIVE** QUESTIONS:**ANSWER TWO QUESTIONS** FROM **EACH** SECTION AND A **FIFTH** QUESTION FROM **EITHER** SECTION

SECTION A

1. Summarise the following as applied in categorising aquatic habitats:
   (a) Littoral zone.
   (b) Pelagic area.
   (c) Profundal region.

2. (a) Discuss the types of ions that determine the alkalinity of freshwater bodies.
   (b) Explain the relationship between alkalinity and productivity of inland aquatic ecosystems.

3. (a) Summarise the factors that affect the amount of light available at different depths in a water body.
   (b) Describe three possible fates of light as it strikes a water body.
   (c) Discuss the significance of Secchi disk measurements in determining light penetration in water bodies.

4. Discuss the effects of zooplankton populations on phytoplankton community structure in summer for a warm monomictic lake.

SECTION B

5. Summarise the following in relation to sustainable management of fish stocks:
   (a) Age at maturity (t_m).
   (b) Total Allowable Catch (TAC).
   (c) Maximum Sustainable Yield (MSY).

TURN OVER

6. Describe two different models used to demonstrate relationships between parent stock size and number of recruits in fish populations.

7. Discuss the type of data and a method required for estimating the following von-Bertalanffy (1936) growth parameters using the Ford-Walford plot:
   (a) The growth coefficient (k);
   (b) Length at infinity (L_{∞}).

8. (a) Describe the type of data required for estimating the Maximum Sustainable Yield (MSY) when applying the Surplus Production Models.
   (b) Compare and contrast the Schaefer and Fox Models as applied in estimates of Maximum Sustainable Yields in fish populations.

END OF EXAMINATION

UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR
FINAL EXAMINATIONS

BIO 5155: AQUATIC ECOLOGY AND FISH POPULATION
PRACTICAL PAPER

TIME: THREE HOURS

INSTRUCTIONS: ANSWER **ALL** QUESTIONS.

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1. Figure 1 below is a simplified seasonal pattern of phytoplankton biomass in the limnetic zone for a eutrophic, dimictic lake. Note that physical conditions in the lake are indicated across the top of the figure. Numbers above portions of the graph (1-8) refer to different periods during the year. Diatoms for instance comprise a dominant proportion of the phytoplankton biomass during time periods 3 and 7

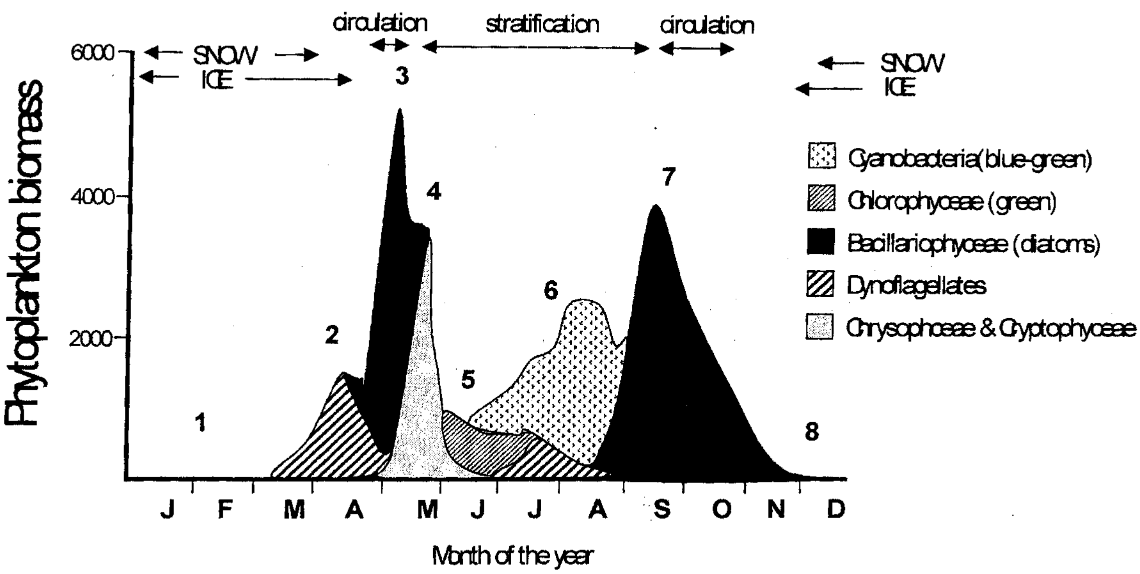

Fig 1 Annual variation of phytoplankton biomass.

(a) Describe physical and chemical condition of the lake that correspond to diatom abundance.
(b) Explain why time period 5 corresponds to the clearwater phase in the lake.
(c) Discuss the dominance of Cyanobacteria during time period 6.
(d) Describe a method that could be used to collect water samples in order to obtain results shown in Figure 1 above.
(e) Describe a method for processing water samples so as to produce a figure of results such as those shown in Figure 1 above.

TURN OVER

2. A cohort is recruited into the fishery with an estimated 100,000 fish (N_{tr}). The total mortality coefficient (Z) is 1.5 and the natural mortality coefficient is 0.5 per year, for the fishery. The number of survivors can be estimated using the formula:

$$N_t = N_o \exp[-Z(t - t_r)]:$$

(a) Estimate the number of survivors from the first to the sixth year after recruitment and construct an appropriate exponential decay curve.
(b) Approximate the fish caught and those dying due to natural causes annually and number of fish of the cohort caught annually since recruitment.
(c) Determine if the given cohort is underexploited or overexploited.

3. Table 1 on page 3 below shows the catch and effort data for the fishery based on the exploitation of *Limnothrissa miodon* (Boulenger,1906) commonly known as Kapenta on Lake Kariba. The data are based on the figures provided by operators to fishery management authorities in both Zambia and Zimbabwe from 1974 to 1999. The catch is given in tonnes and fishing effort in boat nights. The Maximum Sustainable Yield (MSY) can be estimated using the following models:

$$MSY = a^2/(4b) \text{ and}$$

$$f_{msy} = -a/(2b)$$

(a) Construct an appropriate curve that demonstrates the relationship between yield and effort for the combined catch between Zambia and Zimbabwe
(b) Describe the variation in catch in relation to fishing effort.
(c) Approximate parameters of the Schaefer model **a** and **b**.
(d) Estimate the effort needed to exploit the Kapenta fishery of the lake at Maximum Sustainable Yield.

CONTINUE TO THE NEXT PAGE

Table 1. Catch (in tonnes) and effort (boat nights) data for the Kapenta fishery of Lake Kariba

	ZAMBIA		ZIMBABWE		COMBINED		
Year	Catch in tonnes	Effort in boat nights	Catch in tonnes	Effort in boat nights	Catch in tonnes	Effort in boat nights	CPUE
1974	-	-	488	616	488	616	0.79
1975	-	-	656	1298	656	1298	0.51
1976	-	-	1050	1833	1050	1833	0.57
1977	-	-	1172	3114	1172	3114	0.38
1978	-	-	2805	5973	2805	5973	0.47
1979	-	-	5732	15108	5732	15108	0.38
1980	-	-	7952	31747	7952	31747	0.25
1981	-	-	11137	37972	11137	37972	0.29
1982	4136	18874	8450	37776	12586	56650	0.22
1983	4965	16670	8548	38865	13513	55535	0.24
1984	5959	27832	10394	41234	16353	69066	0.24
1985	7422	30304	14586	41403	22008	71707	0.31
1986	8226	32163	15747	45790	23973	77953	0.31
1987	5858	31755	15823	52414	21681	84169	0.26
1988	6319	33218	18366	53403	24685	86621	0.28
1989	7758	34897	20112	54919	27870	89816	0.31
1990	6948	37413	21758	59193	28706	96606	0.3
1991	7284	39284	19306	62208	26590	101492	0.26
1992	7692	45004	18931	71066	26623	116070	0.23
1983	8526	46416	19957	68155	28483	114571	0.25
1994	7178	36402	19232	71249	26410	107651	0.25
1995	6736	44797	15280	75443	22016	120240	0.18
1996	5728	36033	15423	73524	21151	109557	0.19
1997	5927	37170	17034	75633	22961	112803	0.2
1998	7960	43965	15288	74770	23248	118735	0.2
1999	6766	46492	11208	64091	17974	110583	0.16

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

BIO 5165: ECOLOGY AND MANAGEMENT OF TROPICAL WETLANDS
THEORY PAPER

TIME: THREE HOURS

INSTRUCTIONS: ANSWER **FIVE** QUESTIONS. ILLUSTRATE YOUR ANSWERS WHERE NECESSARY.

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1. Compare and contrast the main features of the riverine and palustrine wetland habitat systems.

2. Explain the nutrient cycling function of a fresh water flood plain in an arid environment in Southern Africa and discuss problems that may be associated with management of such wetlands.

3. Discuss major threats to any **two** of the following wetlands:
   (a) Rufigi delta in Tanzania.
   (b) Bangweulu swamps in Zambia.
   (c) Cuanza flood plain in Angola.
   (d) Shire marsh in Malawi.

4. Discuss features which would indicate that a dambo wetland system was being over-exploited, and prescribe measures you would recommend in the restoration of such a wetland.

5.
   (a) Describe values of wetland ecosystems

   (b) Discuss the possible impact of downstream dam development on wetland ecosystems.

6. Construct and discuss the wetland hydrological model of a lacustrine environment in Southern Africa.

7. Discuss the concept of "wise use" of wetlands as it relates to the conservation of wetlands in Zambia.

8. Discuss the main functions of wetlands and give reasons why most wetlands are being degraded.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

BIO 5165: ECOLOGY AND MANAGEMENT OF TROPICAL WETLANDS
PRACTICAL PAPER

TIME: THREE HOURS

INSTRUCTIONS: ANSWER ALL QUESTIONS. ILLUSTRATE YOUR ANSWERS WHERE NECESSARY.

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1. Study the maps provided (Figures**1a** and **1b)**. Using the wetland ecological character as defined by the Ramsar Convention, compare and contrast the Zambezi delta (Fig. 1 a) along the Indian Ocean and the Okavango internal delta (Fig. 1b) in the arid region of Southern Africa.

2. Draw the general map of the wetlands of Zambia and discuss the emerging threats to any **four** of the following wetlands:
   (a) Mweru –wa- ntipa marsh.
   (b) Zambezi Flood Plain.
   (c) Lukanga Swamp.
   (d) Kafue Flats.
   (e) Luangwa Flood Plain.

TURN OVER

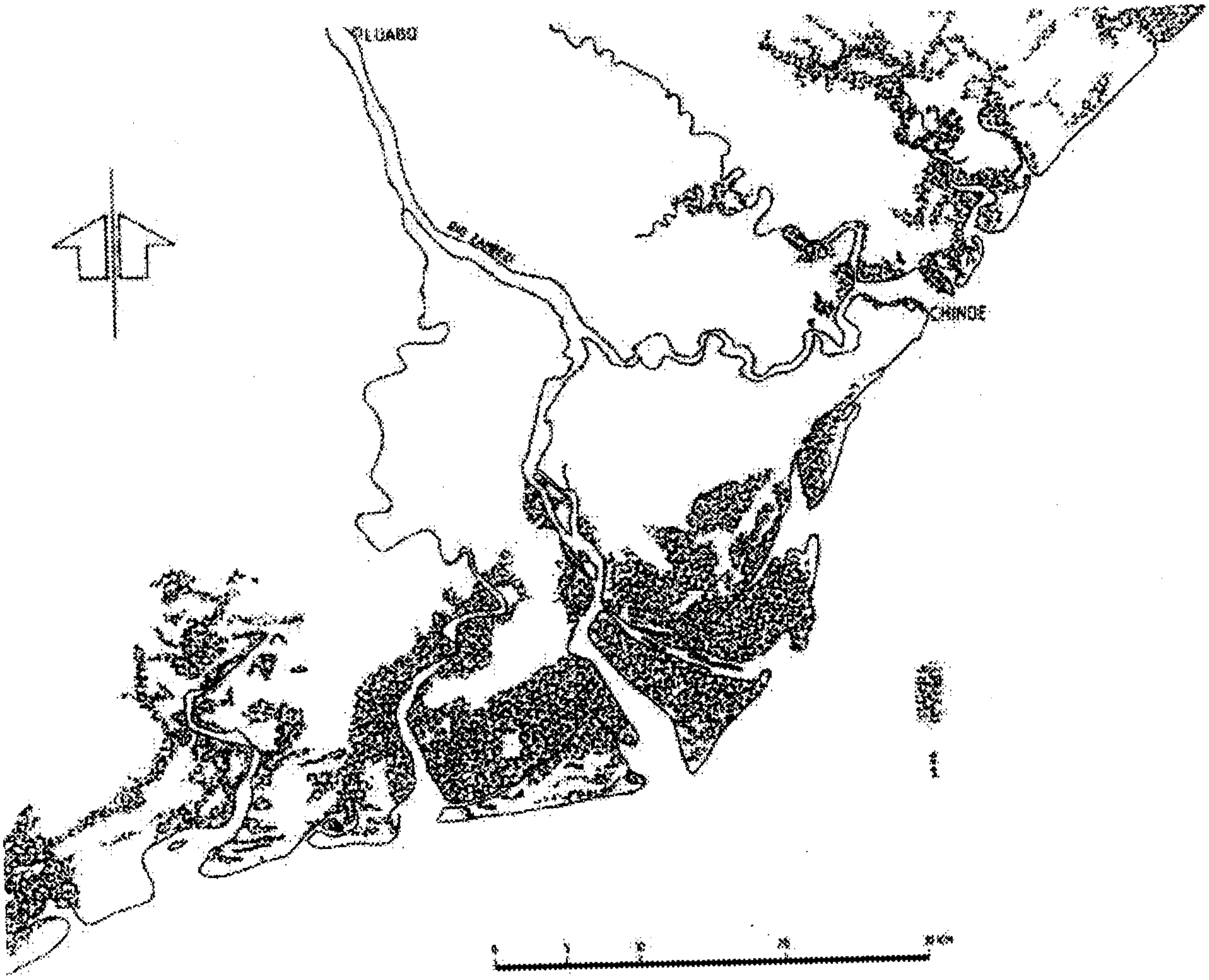

FIGURE 1a: Zambezi Delta

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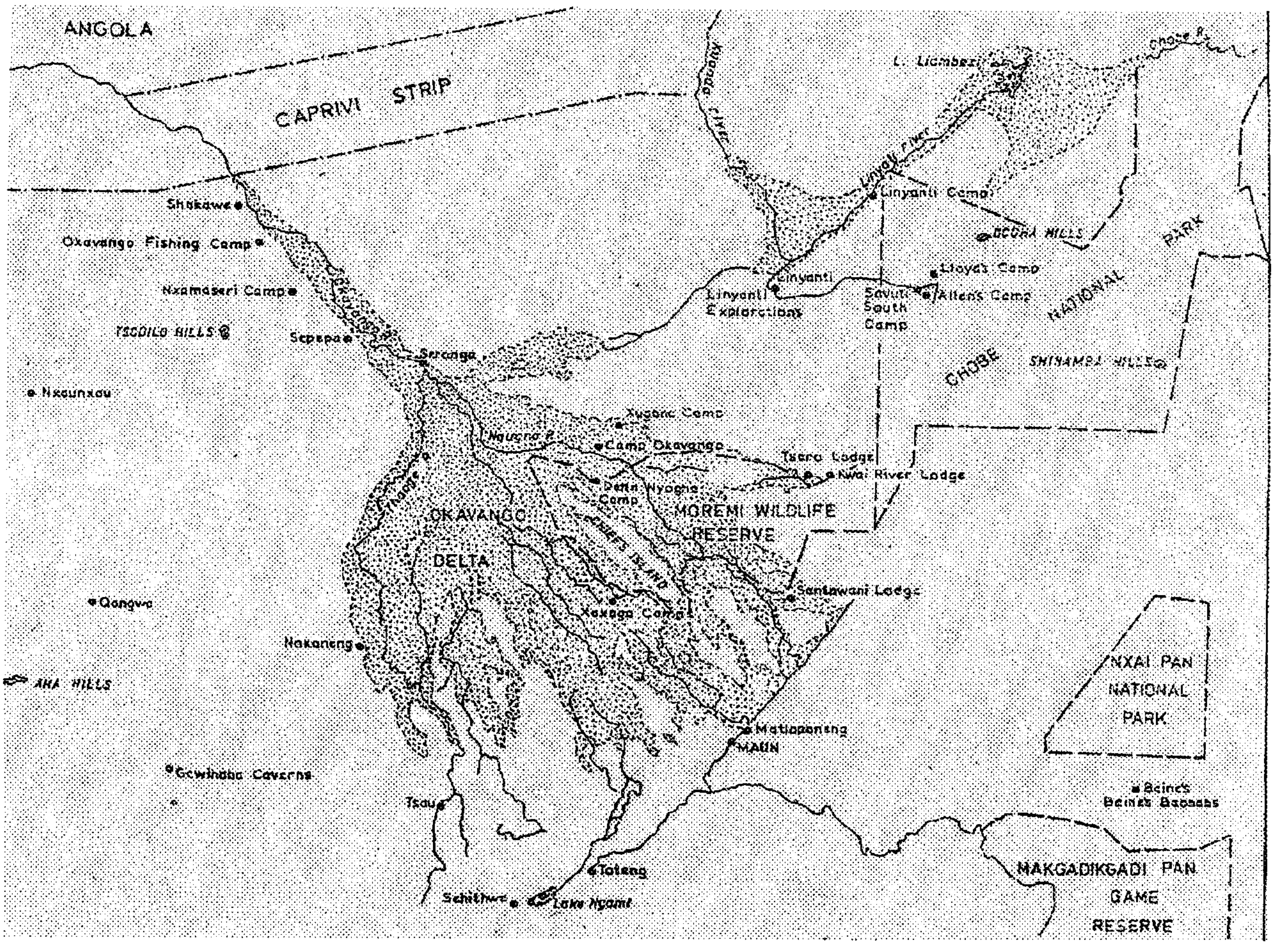

FIGURE 1b: Okavango Delta

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR
FINAL EXAMINATIONS

BIO 5165: ECOLOGY AND MANAGEMENT OF TROPICAL WETLANDS
PRACTICAL PAPER

TIME: THREE HOURS

INSTRUCTIONS: ANSWER **ALL** QUESTIONS. ILLUSTRATE YOUR ANSWERS WHERE NECESSARY.

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1. Using the wetland ecological character as defined by the Ramsar Convention, compare and contrast the Zambezi delta (Fig. 1 a) along the Indian Ocean and the Okavango internal delta (Fig. 1b) in the arid region of Southern Africa.

2. (a) Draw and label the general map of wetlands of Zambia and for each wetland, describe the wetland ecological character.

   (b) Draw and label the cross section of the Kafue Flats and describe each wetland ecological zone.

TURN OVER

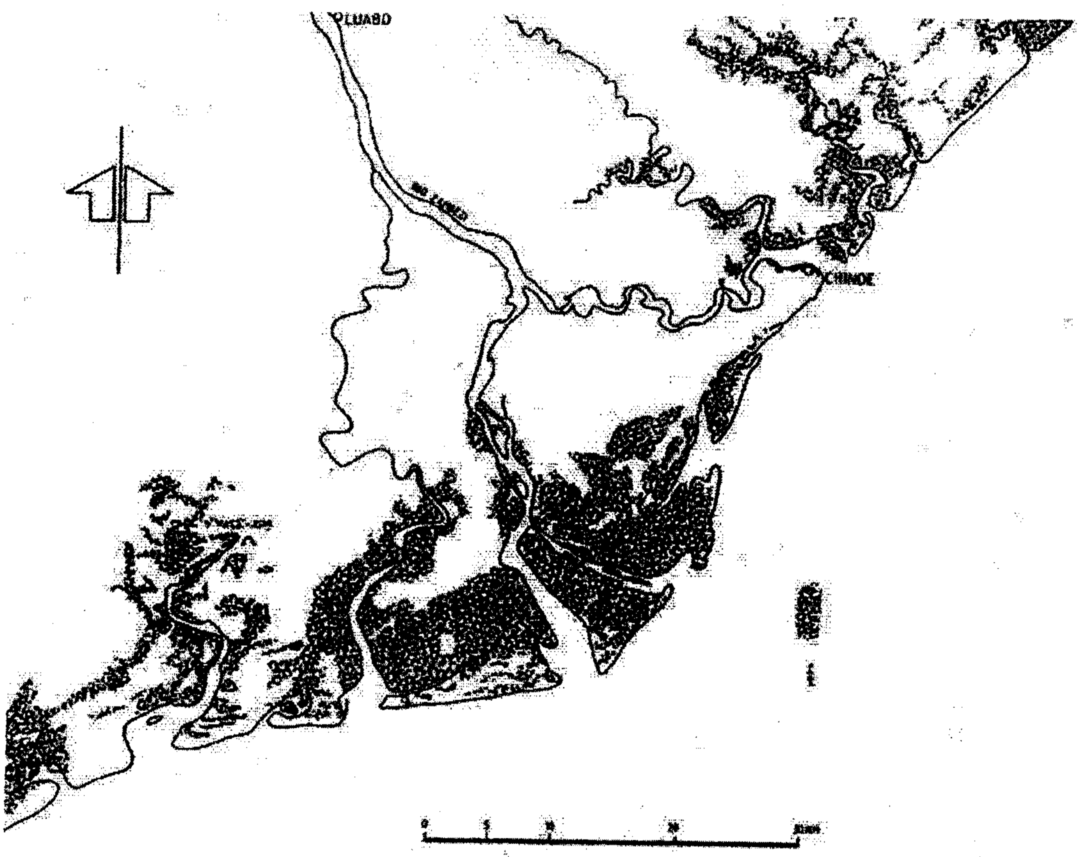

FIGURE 1a: Zambezi Delta in Mozambique showing mangrove forest (dark).

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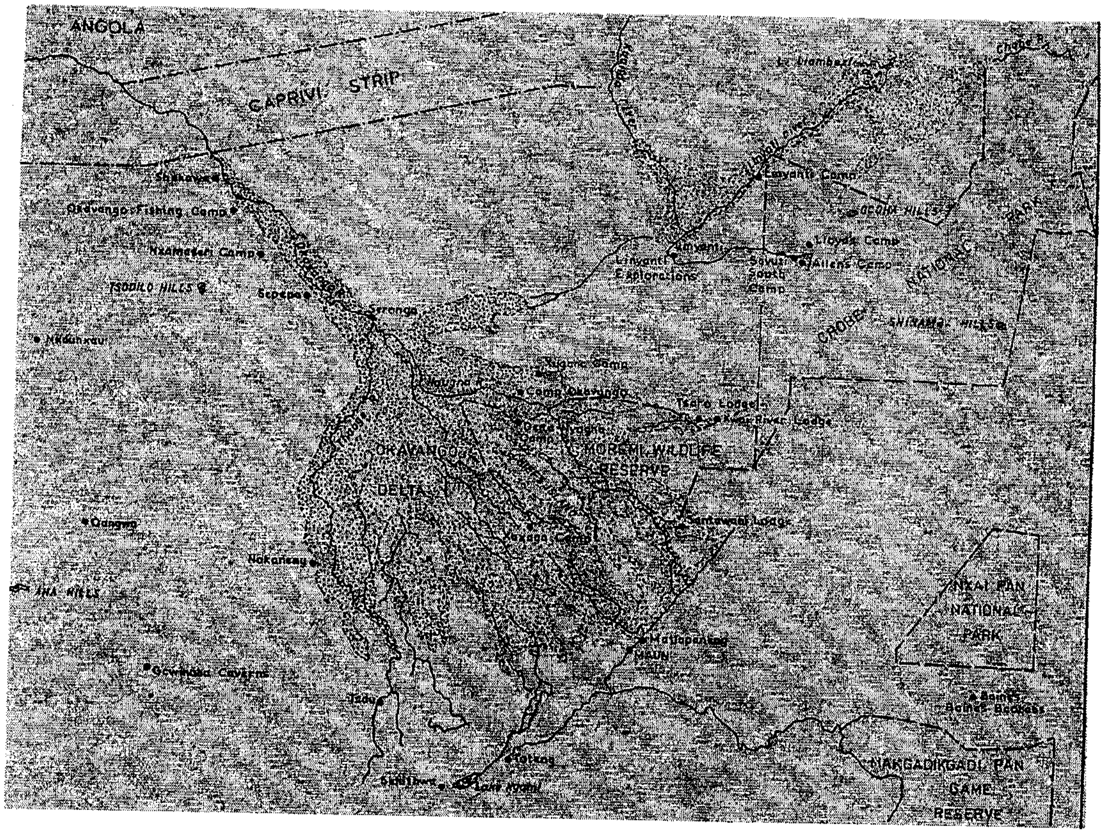


FIGURE 1b: Okavango Delta in Botswana.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER

FINAL EXAMINATIONS

C5722: MEDICINAL CHEMISTRY II

TIME: THREE HOURS

INSTRUCTIONS:

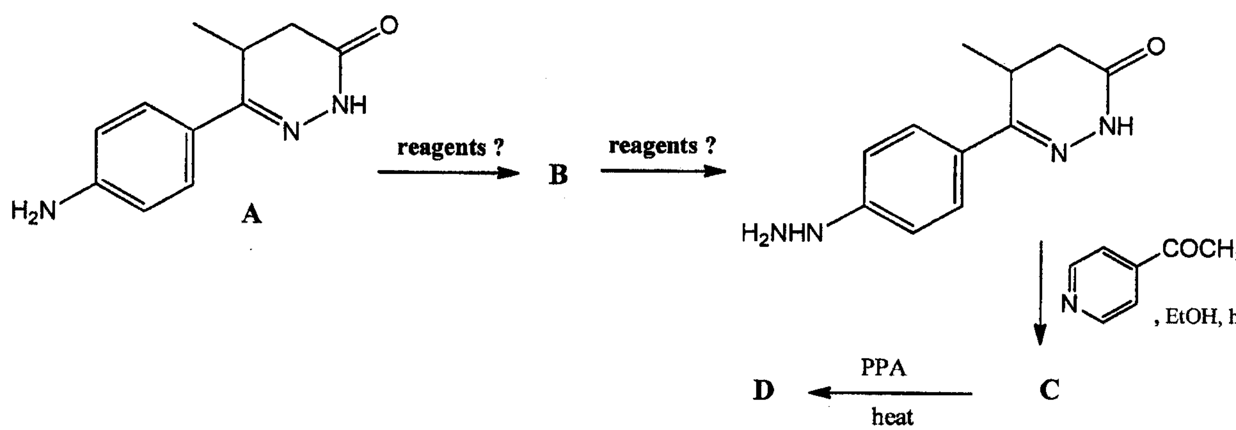
1. Answer any four questions.
2. Marks allocation for questions is shown (x) **Max. marks: 120**

Question 1:

(a) Explain the rationale for the use of positive inotropic agents for treatment of congestive heart failure (CHF). **(4 marks)**

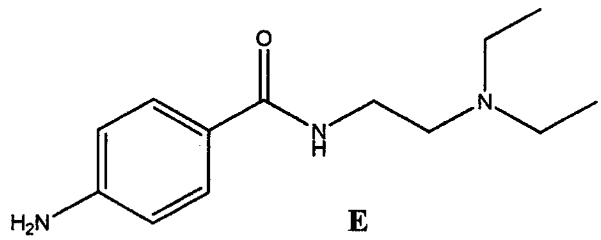
(b) Give the structure and mode of pharmacological action of digoxigenin. **(6 marks)**

(c) (i) Provide the missing reagents, solvents and conditions and identify compounds **B**, **C** and **D** for the synthesis of a cardiotonic agent **D**, shown below. **(8 marks)**



(ii) Give a synthesis for the starting material **A**, shown above. **(5 marks)**

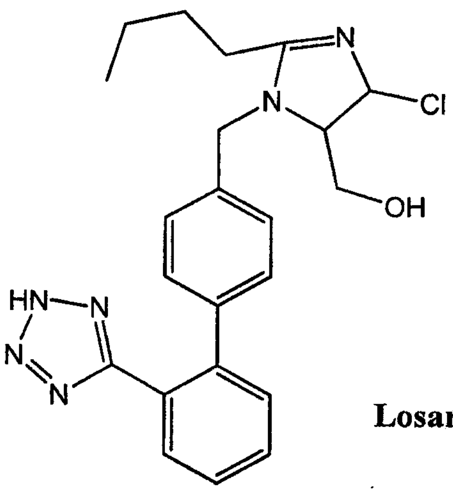
(d) State the principal medicinal uses and predict the metabolites for the drug E, shown below: **(7 marks)**



E

Question 2

(a) Structure of the vasodilator antihypertensive drug losartan is shown below:



Losartan

(i) Briefly explain the mechanism of vasodilatory and antihypertensive action of losartan. **(5 marks)**

(ii) Discuss the qualitative structure-activity relationships in losartan. **(5 marks)**

(iii) Propose an analogue of losartan based on bioisosteric replacement of its tetrazole moiety with a view to improve its potency. **(2 marks)**

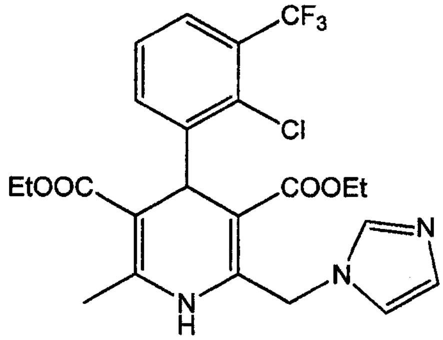
(iv) Suggest a synthesis of your proposed analogue. **(11 marks)**

(b) Give two examples, structures and names, of organic nitrates used for treatment of angina pectoris and state their mechanism of action. **(7 marks)**

Question 3

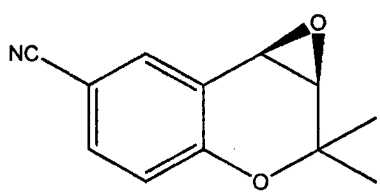
(a) Explain the mode of action of the antihypertensive drug nitredipine. **(5 marks)**

(b) Propose a synthesis for the nitredipine analogue, shown below, from 2-chloro-3-trifluoromethyl benzaldehyde, imidazole and readily available non- heterocyclic reagents. Clearly show all steps. **(9 marks)**

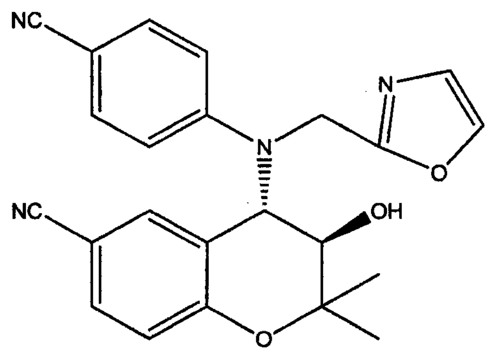


Nitredipine analogue

(c) Suggest a synthetic route to the cardiovascular agent **F** from the epoxide **E**, structures shown below. Clearly show all steps, including the intermediates. **(9 marks)**



E



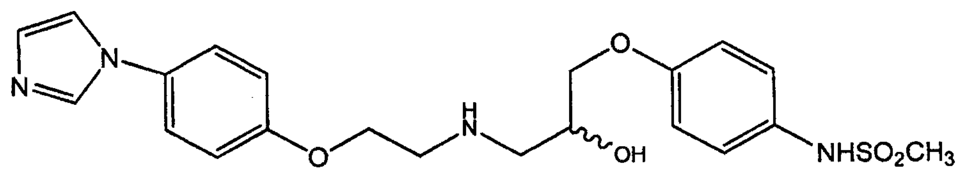
F

(c) Write short note on enzyme inhibitors as anti-atherosclerotic drugs. **(7 drugs)**

Question 4

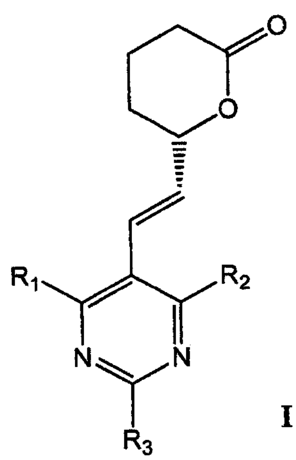
(a) Give an example, structure and name, for each of the Class II and Class III antiarrhythmic drugs and state their modes of action. **(10 marks)**

(b) Suggest a synthesis for the antiarrhythmic drug, **H**, shown below: **(10 marks)**



H

(c) Discuss the structure-activity relationships in the antiarrhythmic drug I, shown below: **(4 marks)**



I

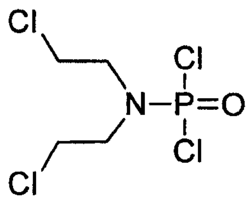
(d) Sate the principal medicinal use and the modes of action of **<u>any two</u>** of the following drugs: **(3 marks each)**

(i) Clofibrate
(ii) Warfarin
(iii) Fostedil

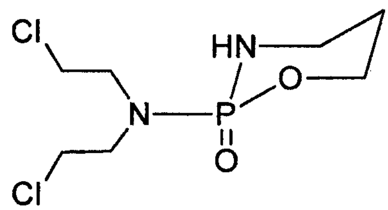
Question 5

(a) Cancer or neoplasm (Greek, *νέοζ* = new, and *πλασμα* = formation) refers not to a single disease but to over 100 related but unique forms of a disease. List the major causes of cancer and briefly describe the **two** principal groups of cancer. **(6 marks)**

(b) Suggest a synthesis for the anti-cancer agent cyclophosphamide from the starting material **A**, shown below. Assume all the necessary organic/ inorganic reagents and solvents are readily available. **(3 marks)**



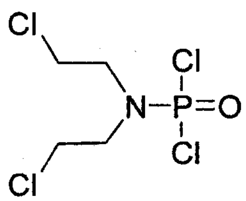
A



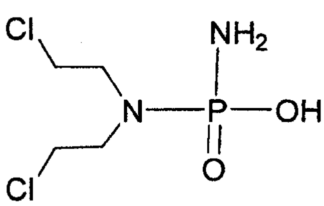
cyclophosphamide

(c) Cyclophosphamide is actually a prodrug, meaning it must be activated *in vivo* into the active species phosphoramide mustard and nor-nitrogen mustard for activity. Acrolein, $H_2C{=}CH{-}CHO$, is a by-product of this bio-activation process, and is less active as an antitumor agent but appears to be responsible for a major side effect of cyclophosphamide, that is, haemorrhagic cystitis.

How has the toxicity problem been circumvented clinically? Provide a plausible mechanism to explain. **(6 marks)**



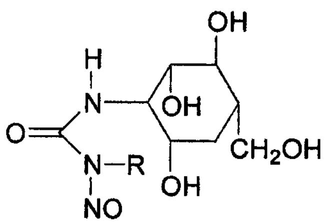
Nornitrogen mustard



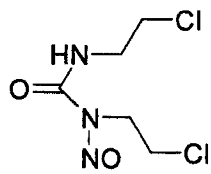
Phosphoramide mustard

(d) Provide the mode of action of the following anti-neoplastic agents, depicted below:

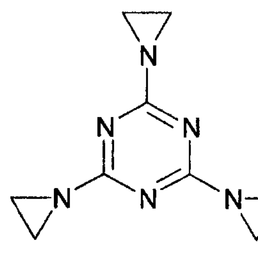
(15 marks)



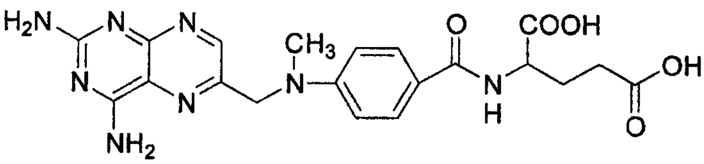
streptozotocin $R = CH_3$
chlorozotocin $R = CH_2CH_2Cl$



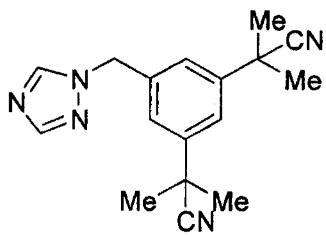
carmustine



TEM
Triethylenemelamir



Methotrexate



Anastrazole

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR MID – YEAR EXAMINATIONS

CHE 5011 GENERAL CHEMICAL TECHNIQUES

TIME: **THREE HOURS**

INSTRUCTIONS

1. There are **six** questions in this Examination Paper.
2. Answer **Question 1** and any other **Four** questions.
3. Question 1 carries 30 marks and the remaining questions carry equal marks.
4. Each question should be answered in a separate answer booklet.
5. Essential information and data are provided for this paper

---

Question 1

(a) Briefly describe a procedure for microscale distillation. Illustrate your answer with appropriate diagram (s).

(b) Use the 1D-NMR (^1H and ^{13}C) of propanol as given in the table below to sketch both H-H COSY and H-C COSY

Proton			**Carbon-13**	
δ	**multiplicity**	**area**	**δ**	**Carbon position**
0.8	triplet	3	10.0	3
1.5	sextet	2	30.2	2
3.0	Singlet and exchangeable	1		
3.5	triplet	2	62.7	1

(c) Describe, with the aid of a diagram how an X-ray tube source works?

(d) Describe some advantages and disadvantages of the electron capture detector (ECD) as a GC detection system.

(e) Draw a sketch of primary photochemical processes. In the diagram indicate the main singlet and triplet manifold, the absorption (excitation) radiation, fluorescence, (F), inter system crossing, (ISC) and phosphorescence (P) transitions

Question 2

(a) Classify the capillary electroseparation methods. **(4 marks)**

(b) What is meant by electrophortic mobility? Give the mathematical equation for electrophoretic mobility and define its terms. **(5 marks)**

(c) Write short notes on: **(7 marks each)**

(i) Capillary column electrophoresis
(ii) Gel electrophoresis

(d) State the advantages and disadvantages of capillary column electrophoresis and HPLC in terms of flow, peak capacity and complexity of instrumentation. **(3 marks)**

(e) Define the terms in the following mathematical expression pertaining to electrophoresis:

$$R_s = \frac{1}{4}\left(\frac{\Delta\mu_p\sqrt{N}}{\mu_p + \mu_a}\right)$$

(4 marks)

Question 3

Deduce the structure of the compound whose spectra are given as problem 307. Use all the data. **(30 marks)**

Question 4

(a) Fully define or explain each of the following terms or statements, giving examples where appropriate: **(12 marks)**

(i) gradient elution

(ii) Kovats Retention Index

(iii) theoretical plate

(b) Why is the rotary valve injection system used on HPLC instruments instead of the rubber septum sealed port as used commonly on GC instruments? **(8 marks)**

(c) What is the short-wavelength limit, in angstroms, to the continuum radiation produced by a X-ray tube with a molybdenum target that is impacted by electrons driven by a 50 kV potential? **(10 marks)**

The Duane-Hunt equation is: $eV_0 = \frac{hc}{\lambda_{min}} = h\nu_{max}$

Note: 1 angstrom (Å) is 10^{-10} meters.

Question 5

(a) (i) Explain the difference between capacity factor and selectivity factor as used in chromatography. **(4marks)**

(ii) What is the Van Deemter equation? Explain the significance of each term. **(6 marks)**

(b) One of the components of the Van Deemter equation is a mass transport related term. Explain how this is affected by variations in HPLC. Specifically answer the question as to whether band broadening increases or decreases with each of the following:

(i) decreasing stationary phase depth

(ii) decreasing mobile phase flow rate **(8 marks)**

(c) (i) Name two different detectors used in gas chromatography and tell how they work (A mass Spectrometer doesn't count). **(2 marks)**

(ii) An XPS experiment detected a photoelectrons with kinetic energy of 1073.5 eV when a Mg Kα source was used (λ = 9.8900 A, which corresponds to a source energy of 1253.6 eV). The spectrometer was found to have a workfunction of 11.2 eV. Calculate the binding energy for the photoelectrons using the following equation: **(5 marks)**

$$E_{binding} = E_{source} - E_k - w$$

(iii) Powder X-ray diffraction (PXRD/XRPD) and single-crystal X-ray diffraction (SCXRD) make use of very similar instrumentation, with the primary differences being with the goniometer used in SCXRD to reorient the crystal. However, the chemical and analytical information that can be obtained from these techniques is very different. Describe this difference. In other words, what can one learn from SCXRD that cannot usually be learned from PXRD, and what can PXRD do that SCXRD cannot? **(5 marks)**

Question 6

(a) The molar absorption coefficient for a compound is 3×10^4 L mol^{-1} cm^{-1} at 313 nm where the compound undergoes a photochemical bleaching reaction. The product does not absorb at 313 nm.
A 1.5×10^{-3} M solution of the compound in a 1.00 cm^3 cell of 1.00 cm^2 cross-section was placed in the path of a laser beam of 1.00×10^{-5} W cm^{-2} at 313 nm.

(i) Calculate the path length of the cell. **(1 mark)**

(ii) What fraction of the incident intensity is absorbed? **(3 marks)**

(iii) **Calculate the** absorbed intensity, I_a, in (photons) cm^{-3} s^{-1}. **(6 marks)**

(b) Photodissociation of CH_3CO using ultraviolet radiation of wavelength of 313.0 nm and intensity of 3.08×10^{-2} W produced CO and CH_3 radical with a quantum yield of 2.00 during an exposure of 1.00 hour. Assume that all of the intensity of the incident radiation was absorbed in the cell.

(i) Determine the number of moles of CH_3CO absorbed in the reaction.**(5 marks)**

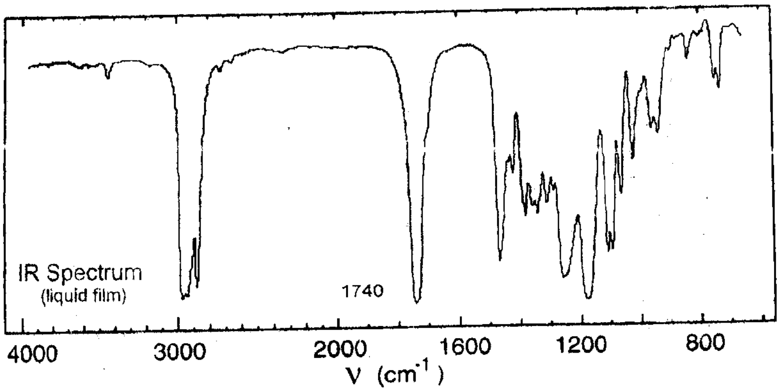
(ii) Write an expression for quantum yield. **(2 marks)**

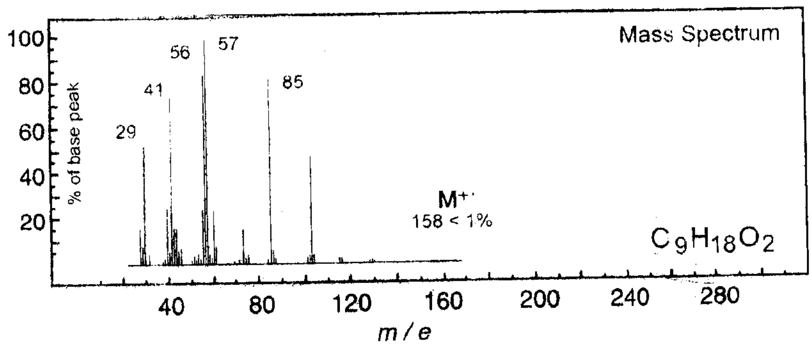
(iii) Calculate the number of moles of CO produced in the cell. **(3 marks)**

END OF EXAMINATION

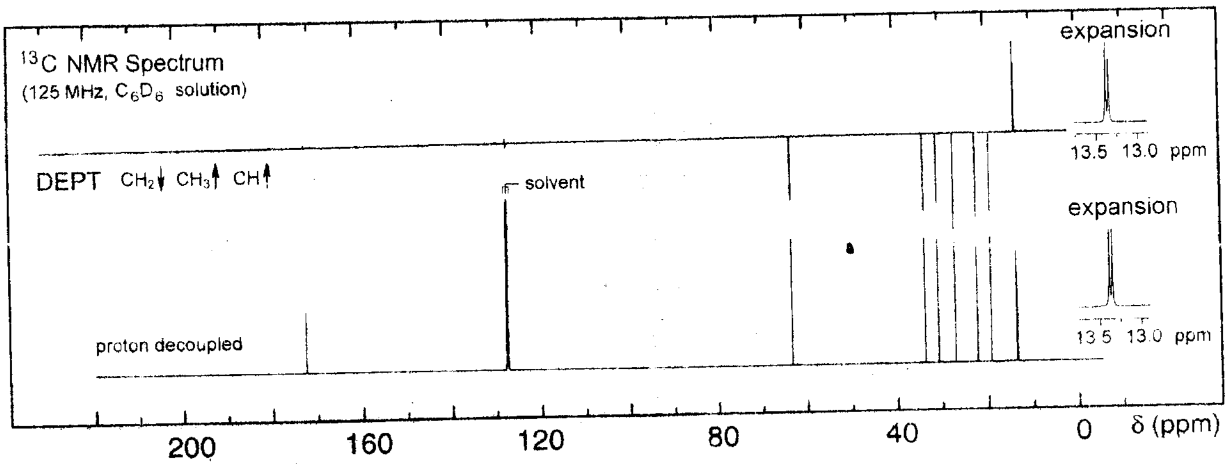
Problem 307

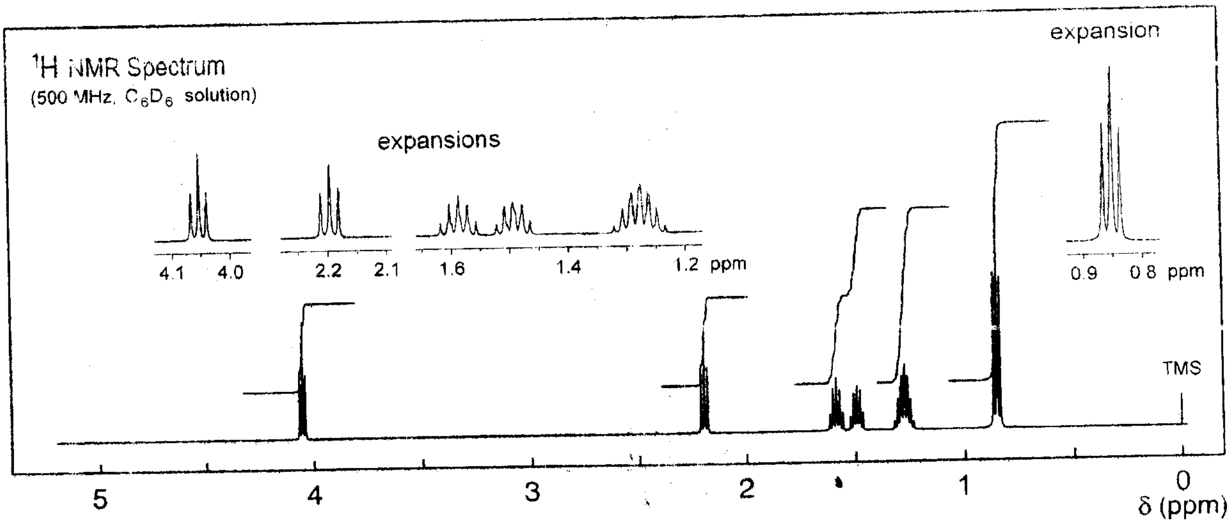
Use the basic spectral data plus the COSY and C-H correlation spectra on the facing page to deduce the structure of this compound.



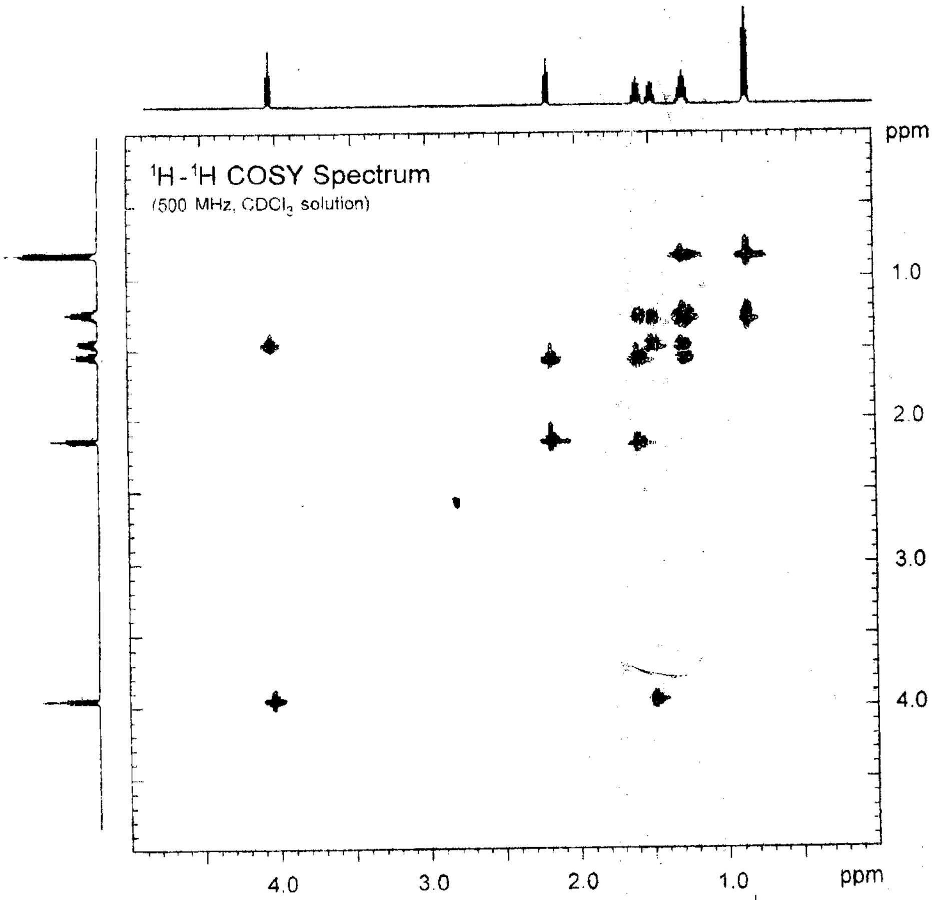


No significant UV absorption above 220 nm

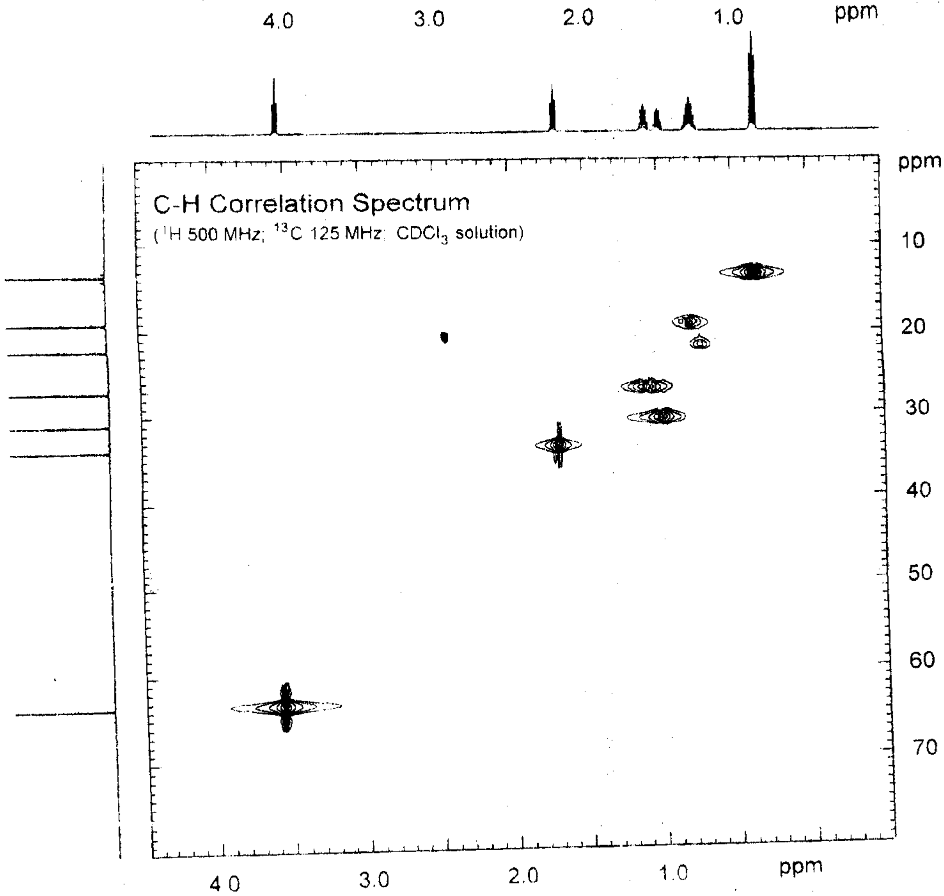




Problem 307



^1H-^1H COSY Spectrum
(500 MHz, $CDCl_3$ solution)
ppm
1.0
2.0
3.0
4.0
4.0
3.0
2.0
1.0
ppm



C-H Correlation Spectrum
(^1H 500 MHz; ^{13}C 125 MHz; $CDCl_3$ solution)
ppm
10
20
30
40
50
60
70
4.0
3.0
2.0
1.0
ppm

The Periodic Table of Elements

Key

relative atomic mass
atomic symbol
name
atomic (proton) number

1	2											3	4	5	6	7	0 (8)
						1.0 **H** hydrogen 1											(18)
																	4.0 **He** helium 2
(1)	(2)											(13)	(14)	(15)	(16)	(17)	
6.9 **Li** lithium 3	9.0 **Be** beryllium 4											10.8 **B** boron 5	12.0 **C** carbon 6	14.0 **N** nitrogen 7	16.0 **O** oxygen 8	19.0 **F** fluorine 9	20.2 **Ne** neon 10
23.0 **Na** sodium 11	24.3 **Mg** magnesium 12	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	27.0 **Al** aluminium 13	28.1 **Si** silicon 14	31.0 **P** phosphorus 15	32.1 **S** sulfur 16	35.5 **Cl** chlorine 17	39.9 **Ar** argon 18
39.1 **K** potassium 19	40.1 **Ca** calcium 20	45.0 **Sc** scandium 21	47.9 **Ti** titanium 22	50.9 **V** vanadium 23	52.0 **Cr** chromium 24	54.9 **Mn** manganese 25	55.8 **Fe** iron 26	58.9 **Co** cobalt 27	58.7 **Ni** nickel 28	63.5 **Cu** copper 29	65.4 **Zn** zinc 30	69.7 **Ga** gallium 31	72.6 **Ge** germanium 32	74.9 **As** arsenic 33	79.0 **Se** selenium 34	79.9 **Br** bromine 35	83.8 **Kr** krypton 36
85.5 **Rb** rubidium 37	87.6 **Sr** strontium 38	88.9 **Y** yttrium 39	91.2 **Zr** zirconium 40	92.9 **Nb** niobium 41	95.9 **Mo** molybdenum 42	[98] **Tc** technetium 43	101.1 **Ru** ruthenium 44	102.9 **Rh** rhodium 45	106.4 **Pd** palladium 46	107.9 **Ag** silver 47	112.4 **Cd** cadmium 48	114.8 **In** indium 49	118.7 **Sn** tin 50	121.8 **Sb** antimony 51	127.6 **Te** tellurium 52	126.9 **I** iodine 53	131.3 **Xe** xenon 54
132.9 **Cs** caesium 55	137.3 **Ba** barium 56	138.9 **La*** lanthanum 57	178.5 **Hf** hafnium 72	180.9 **Ta** tantalum 73	183.8 **W** tungsten 74	186.2 **Re** rhenium 75	190.2 **Os** osmium 76	192.2 **Ir** iridium 77	195.1 **Pt** platinum 78	197.0 **Au** gold 79	200.6 **Hg** mercury 80	204.4 **Tl** thallium 81	207.2 **Pb** lead 82	209.0 **Bi** bismuth 83	[209] **Po** polonium 84	[210] **At** astatine 85	[222] **Rn** radon 86
[223] **Fr** francium 87	[226] **Ra** radium 88	[227] **Ac*** actinium 89	[261] **Rf** rutherfordium 104	[262] **Db** dubnium 105	[266] **Sg** seaborgium 106	[264] **Bh** bohrium 107	[277] **Hs** hassium 108	[268] **Mt** meitnerium 109	[271] **Ds** darmstadtium 110	[272] **Rg** roentgenium 111							

Elements with atomic numbers 112-116 have been reported but not fully authenticated

*Lanthanide series	140 **Ce** cerium 58	141 **Pr** praseodymium 59	144 **Nd** neodymium 60	[147] **Pm** promethium 61	150 **Sm** samarium 62	152 **Eu** europium 63	157 **Gd** gadolinium 64	159 **Tb** terbium 65	163 **Dy** dysprosium 66	165 **Ho** holmium 67	167 **Er** erbium 68	169 **Tm** thulium 69	173 **Yb** ytterbium 70	175 **Lu** lutetium 71
*Actinide series	232 **Th** thorium 90	[231] **Pa** protactinium 91	238 **U** uranium 92	[237] **Np** neptunium 93	[242] **Pu** plutonium 94	[243] **Am** americium 95	[247] **Cm** curium 96	[245] **Bk** berkelium 97	[251] **Cf** californium 98	[254] **Es** einsteinium 99	[253] **Fm** fermium 100	[256] **Md** mendelevium 101	[254] **No** nobelium 102	[257] **Lr** lawrencium 103

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR END OF YEAR EXAMINATIONS

CHE 5222 ELECTROCHEMICAL AND CHROMATOGRAPHIC METHODS

TIME: **THREE HOURS**

INSTRUCTIONS

1. There are **six** questions in this Examination Paper.
2. Answer **Question 1** and any other **Four** questions.
3. Question 1 carries 20 marks and the remaining questions carry equal marks.
4. **Each question** should be answered in a **separate** answer **booklet.**
5. Essential information and data are provided for this paper

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Question 1

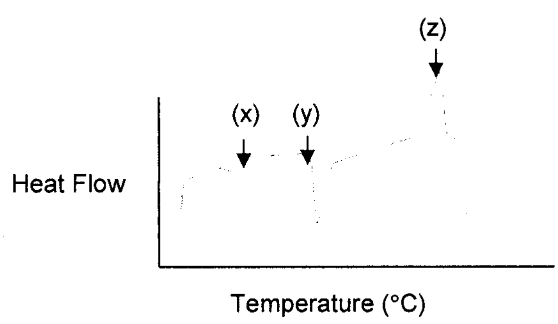
(a) Explain the pros and cons of the refractive index detectors in point form.

(b) Explain and relate the following terms: thermal noise, shot noise and flicker noise.

(c) Explain why differential pulsed polarography (DPP) gives superior detection limits to sampled-dc polarography.

(d) Calibration is important aspect towards obtaining good data in DSC. Draw a sketch to illustrate a typical DSC calibration curve. What substance is typically used?

(e) The diagram below shows a typical DSC curve for PET



(i) Identify the key features labelled x, y and z
(ii) What is the physical significance of x, y and z.

Question 2

A reversed-phase HPLC column of dimensions 4.6 mm diameter by 150 mm length is used at a liquid flow rate of 1.0 mL/min. An unretained compound elutes after 1.12 min. A compound with a K_{ow} of 31 takes 2.63 min. to elute when using a mobile phase of 90% water/10% acetonitrile.

(a) Calculate the mobile phase volume [10 marks]

(b) Assuming the K_C is equal to the K_{ow}, calculate the stationary phase volume. [10 marks]

Question 3

(a) Compare and contrast the differences in theory of operation and equipment needed for potetiometric and voltammetric measurements. List any theoretical expressions that relate activity or concentration of an analyte to a measured electrochemical signal. [10 marks]

(c) Describe how you can use cyclic voltammetry to obtain the diffusion coefficient D_o of reactant O. Describe what CV experiments are required and specifically how the data would be analyzed. [10 marks]

Question 4

(a) Anodic Stripping Voltammetry (ASV) has the best detection limits of any voltammetric methods available today. Briefly outline the steps involved in an ASV analysis and indicate why it has detection limits superior to the other Electrochemical methods we covered. [10marks]

(b) Explain using examples why electrochemistry is well suited for chemical and biochemical sensing applications. What types of sensors have been developed? What are their advantages and special features? [10 marks]

Question 5

(a) Describe the basic principle behind differential thermal analysis **[6]**

(b) How are the following factors likely to affect results in differential thermal analysis?

(i) Sample packing **[3]**

(ii) Heating rate **[3]**

(iii) ~~Heating rate~~ Chamber atmosphere **[3]**

(c) Draw fully labelled sketch diagram of a thermometric set-up **[5]**

Question 6

(a) The potential of a sulfate specific ion electrode (SCE reference electrode) was measured as -213 mV in a solution that contained 1.00×10^{-5} M sulfate ion, while the potential was -156 mV in a 1.00×10^{-4} M sulfate ion solution. A solution containing an unknown concentration of sulfate ion was measured with a potential of -187 mV. What is the concentration of sulfate ion in the unknown solution? [10marks]

(b) White noise occurs at all frequencies and can often be reduced by simply reducing the frequency bandwidth of the measurement or recording device used. Usually, simple filtering can be used to reduce the frequency bandwidth to about 1 Hz. What would happen to the S/N if you were to reduce the frequency bandwidth to, say, 0.01 Hz from 1 Hz. Be sure to clearly state your assumptions! [10marks]

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
School of Natural Sciences
CHE5411 Examination, 2014

Applied Inorganic Chemistry

Time: 3 Hours **February 2014**

Instructions:

Answer any Four (4) Questions.
Periodic Table is provided
Show clarity in your answering.

Question 1

(i) **Derive the ground state term for a d^2 configuration defined as $^{2S+1}L_J$.**

(ii) **Each term is split in magnetic field by spin-orbital coupling into states specified by the value of J differing by unity ranging from J value $|L-S|$ to $|L+S|$.**

The energy separation between a state specified by J and the one specified by J+1 is

$$\Delta E_{J,\,J+1} = \lambda(J+1)$$

For d^2 transition metal ion sketch the splitting of the ground state free ion term showing energy value.[For a shell less than half full λ is positive and the lowest J value lies lowest in energy]

Question 2

(a) Given that $\mu_{eff} = \mu^{s.o}\left(1 - \frac{\alpha\lambda}{10D_q}\right)$**; where 10Dq =** $fg \times 10^3 cm^{-1}$.

(i) **Define and explain $\mu^{s.o.}$**

(ii) **In the $Mn(acac)_3$ complex, what is the value of $\mu^{s.o.}$.**

(iii) **Given that f= 1.2 for acac ligand and g = 21 for Mn^{3+}; determine 10Dq for the complex.**

(iv) **The spin-orbital coupling parameter, ζ, for Mn^{3+} is $355cm^{-1}$ and** $\lambda = \pm\frac{\zeta}{2S}$;

(a) **Determine λ and α**

(b) **Hence, or otherwise find the μ_{eff} for the complex at 80K**

(v) **How does this value in (iv)b compare with $\mu^{s.o.}$?**

(vi) **Given that at 300K μ_{eff} is 4.86B.M, discuss the effect of temperature on μ_{eff} of the complex.**

Question 3

(a) **Sketch the structure of the ZSM-5 and outline some of its industrial uses.**

(b) **Sketch the framework structure of a zeolite and interpret this general feature and its relevance.**

(c) **Explain how zeolites have found application in**

(i) **Aqua-culture**
(ii) **Catalysis**

Question 4

(a) **Work out the RS terms associated with a d^4 configuration.**

(b) **As the ligand field strength increases the ground state also changes. Discuss these changes in an octahedral environment and its impact on the paramagnetism.**

Question 5

(a) **Consider the lanthanide complex, $Nd_2(SO_4)_3.8H_2O$;**

(i) **Determine the GS term for the Nd^{3+} ion.**

(ii) **Given that the experimental μ_{eff}^{300K} is 3.50B.M, calculate μ_{eff} and discuss the nature of magnetism being exhibited by the complex.**

(b) **What are the relevant features that make magnetism important subject in chemistry?**

PERIODIC TABLE OF THE ELEMENTS

KEY

Atomic number **X** Atomic mass Name of the element X

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 **H** 1.01 Hydrogen																1 **H** 1.01 Hydrogen	2 **He** 4.00 Helium
3 **Li** 6.94 Lithium	4 **Be** 9.01 Beryllium											5 **B** 10.81 Boron	6 **C** 12.01 Carbon	7 **N** 14.01 Nitrogen	8 **O** 16.00 Oxygen	9 **F** 19.00 Fluorine	10 **Ne** 20.18 Neon
11 **Na** 23.00 Sodium	12 **Mg** 24.31 magnesium											13 **Al** 26.98 Aluminium	14 **Si** 28.09 Silicon	15 **P** 30.99 Phosphorus	16 **S** 32.07 Sulphur	17 **Cl** 35.45 Chlorine	18 **Ar** 39.95 Argon
19 **K** 39.10 Potassium	20 **Ca** 40.08 Calcium	21 **Sc** 44.96 Scandium	22 **Ti** 47.88 Titanium	23 **V** 50.94 Vanadium	24 **Cr** 52.00 Chromium	25 **Mn** 54.94 Manganese	26 **Fe** 55.85 Iron	27 **Co** 58.93 Cobalt	28 **Ni** 58.69 Nickel	29 **Cu** 63.55 Copper	30 **Zn** 65.39 Zinc	31 **Ga** 69.72 Gallium	32 **Ge** 71.61 Germanium	33 **As** 74.92 Arsenic	34 **Se** 78.96 Selenium	35 **Br** 79.90 Bromine	36 **Kr** 83.80 Krypton
37 **Rb** 85.47 Rubidium	38 **Sr** 87.62 Strontium	39 **Y** 88.91 Yttrium	40 **Zr** 91.22 Zirconium	41 **Nb** 92.91 Niobium	42 **Mo** 95.94 Molybdenum	43 **Tc** 97.91 Technetium	44 **Ru** 101.07 Ruthenium	45 **Rh** 102.91 Rhodium	46 **Pd** 106.42 Palladium	47 **Ag** 107.87 Silver	48 **Cd** 112.41 Cadmium	49 **In** 114.82 Inidum	50 **Sn** 118.71 Tin	51 **Sb** 121.76 Antimony	52 **Te** 127.60 Tellurium	53 **I** 126.90 Iodine	54 **Xe** 131.29 Xenon
55 **Cs** 132.91 Caesium	56 **Ba** 137.33 Barium	57 – 71	72 **Hf** 178.49 Hafnium	73 **Ta** 180.95 Tantalum	74 **W** 183.84 Tungsten	75 **Re** 186.21 Rhenium	76 **Os** 190.23 Osmium	77 **Ir** 192.22 Iridium	78 **Pt** 195.08 Platinum	79 **Au** 196.97 Gold	80 **Hg** 200.59 Mercury	81 **Tl** 204.38 Thallium	82 **Pb** 207.2 Lead	83 **Bi** 208.98 Bismuth	84 **Po** 208.98 Polonium	85 **At** 209.99 Astatine	86 **Rn** 222.02 Radon
87 **Fr** (223.02) Francium	88 **Ra** 226.03 Radium	89 - 103	104 **Unq** 261.11	105 **Unp** 262.11	106 **Unh** 263.12	107 **Uns** 262.12	108 **Uno** 265.00	109 **Une** 265									

57 **La** 138.91 Lanthanum	58 **Ce** 140.12 Cerium	59 **Pr** 140.91 Praseodymium	60 **Nd** 144.24 Neodymium	61 **Pm** 144.91 Promethium	62 **Sm** 150.36 Samarium	63 **Eu** 151.97 Europium	64 **Gd** 157.25 Gadolinium	65 **Tb** 158.93 Terbium	66 **Dy** 162.50 Dysprosium	67 **Ho** 164.93 Holmium	68 **Er** 167.26 Erbium	69 **Tm** 168.93 Thulium	70 **Yb** 173.04 Ytterbium	71 **Lu** 174.97 Lutetium
89 **Ac** 227.03 Actinium	90 **Th** 232.04 Thorium	91 **Pa** 231.04 Protactinium	92 **U** 238.03 Uranium	93 **Np** 237.05 Neptunium	94 **Pu** 244.0 Plutonium	95 **Am** 243.06 Americium	96 **Cm** 247.07 Curium	97 **Bk** 247.07 Berkelium	98 **Cf** 251.08 Californium	99 **Es** 252.08 Einsteinium	100 **Fm** 257.10 Fermium	101 **Md** 260 Mendelevium	102 **No** 259.10 Nobelium	103 **Lr** 262.11 Lawrencium

UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF CHEMISTRY

2013-2014 ACADEMIC YEAR HALF YEAR SESSIONAL EXAMINATIONS

CHE 5635: STATISTICAL THERMODYNAMICS.

ANSWER ANY *FIVE* OF SIX QUESTIONS

TIME: THREE HOURS

DATA YOU MAY WISH TO USE IS PROVIDED ON THE *ATTACHMENT*

QUESTION 1 (*20 MARKS*)

Consider a system of carbon monoxide $^{12}C^{16}O$ gas molecules, assumed to be independent, non-interacting and indistinguishable at 300 K.

a) (*3 marks*)

Write a general expression for the total canonical partition function of the system in terms of the molecular partition function of carbon monoxide.

b) (*8 marks*)

Derive a general expression for the chemical potential, μ, of carbon monoxide using the canonical partition function you wrote in Question 1a).

c) (*3 marks*)

The infra-red vibration frequency of $^{12}C^{16}O$ is 2170.21 cm^{-1}. Calculate the vibration temperature of carbon monoxide.

d) (*6 marks*)

The vibration canonical partition function of carbon monoxide, in the high temperature limit has been shown to be $q_{vib} = \frac{\theta_{vib}}{T}$.

(i) Calculate the fraction of carbon monoxide molecules that are in the excited vibration states (n > 0) at 300 K

ii) Comment on your result with reference to the contribution of the vibration degree of freedom to the total energy of carbon monoxide gas system.

QUESTION 3 (*20 MARKS*)

The rotational energy levels of a heteronuclear diatomic molecules are given by $E_J = BhcJ(J + 1)$.

a) (*4 marks*)

The rotational constant of $^{12}C^{1}H$ is 14.457 cm^{-1}. What is its characteristic rotational temperature, θ_r?

b) (*8 marks*)

Write down a general expression for the molecular partition function of a heteronuclear diatomic molecule. Transform the sum to a suitable integral and use the Euler-Maclaurin formula to obtain q_{rot} in the high temperature limit.

c) (*8 marks*)

To a first approximation, the intensity of a rotational absorption transition from $J \rightarrow J'$ is determined by, $f(J)$ the fraction of molecules found in the rotational energy level J:

$$f(J) = \frac{(2J+1)}{q_{rot}} e^{-BhcJ(J+1)/kT}$$

Show that the line with maximum spectral intensity is given by $J_{max} = \sqrt{\frac{T}{2\theta}} - \frac{1}{2}$

QUESTION 4 (*20 MARKS*)

The molecule NO has a nearly degenerate electronic ground state: $^2\pi_{3/2}$ and $^2\pi_{1/2}$ with energy difference, ε. The state $^2\pi_{3/2}$ has lower energy.

a) (*12 marks*)

(i) Derive an equation for the contribution of these electronic states to the total energy.

(ii) What is the electronic energy in the temperature limits $T \rightarrow 0$ and $T \rightarrow \infty$?

b) (*8 marks*)

(i) Derive an equation for the contribution of these states to the heat capacity of NO.

(ii) What is the electronic heat capacity in the temperature limits $T \rightarrow 0$ and $T \rightarrow \infty$?

QUESTION 5 (*20 MARKS*)

a) (*3 marks*)

Distinguish, briefly between the *micronanonical* and the *canonical* ensembles?

b) (*6 marks*)

(i) Write the probability distribution function P(E) for canonical and for microcanonical ensembles.

(ii) Sketch a labeled graph to show the distribution function P(E) as a function of the energy E for each ensemble.

c) (*11 marks*)

State the condition necessary for the applicability of Boltzmann statistics to system of particles. Apply this condition to decide whether or not Boltzmann statistics are applicable to one mole of oxygen molecules at 273 K and pressure of 1 bar. Show all work.

QUESTION 6 (*20 MARKS*)

The translational molecular partition function for indistinguishable particles is given in the Data in an Attachment to this question paper.

a) (*3 marks*)

Calculate the de Broglie thermal wavelength of neon at 300 K.

b) (*9* marks)

(i) Derive an equation for the translational contribution to the internal energy, E of a canonical ensemble of one mole of neon molecules.

(ii) Calculate the translational contribution to the internal energy, E of 3.01×10^{23} molecules o neon at 300K.

c) (*8 marks*)

Derive an equation for the translational contribution to pressure.

END OF CHE 5632 EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF CHEMISTRY

CHE 5635: INTRODUCTION TO STATISTICAL THERMODYNAMICS (2013/2014)

ATTACHMENT

DATA SHEET

$N_A = 6.02 \times 10^{23}$ mol^{-1}; $k = 1.38 \times 10^{-23}$ J K^{-1}; $h = 6.63 \times 10^{-34}$ J s;

R = 8.314 J K^{-1} mol^{-1}; c = 3.00×10^{8} m s^{-1}; 1eV = 1.602×10^{-19} J;

1J = 1kg m^2 s^{-2}; $ln\ e = 1$; $\bar{V} = 22.4 \times 10^{-3}$ m^3 at 273 K

$\frac{\partial \ln f(x)}{\partial x} = \frac{1}{f(x)} \frac{\partial f(x)}{\partial x}$; $\frac{d(e^{f(x)})}{dx} = e^{f(x)} \frac{df(x)}{dx}$; $\frac{d(\ln x)}{dx} = \frac{1}{x}$;

$\frac{d(f(x)/g(x))}{dx} = \frac{1}{g(x)} \frac{df(x)}{dx} - \frac{f(x)}{g(x)^2} \frac{dg(x)}{dx}$; $\frac{d(f(x)g(x))}{dx} = f(x) \frac{dg(x)}{dx} + g(x) \frac{f(x)}{dx}$

$\sum_{J=m}^{n} f(J) = \int_m^n f(J) dJ + \frac{1}{2} [f(m) + f(n)]$ + residue ; $\int_0^\infty e^{-ax} dx = \frac{1}{a}$

$ln\ N! = NlnN - N$, for $N >> 1$ $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + ... = \frac{1}{1-x}$, $|x|<1$

$e^x = 1 + x + \frac{x^2}{2!} + \ldots, x << 1$ $e^{-x} = 1 - x + \frac{x^2}{2!} +- \ldots, x << 1$

1 amu = 1.661×10^{-27} kg; C = 12.01; N = 14.01; O = 16.00; Ne = 20.18;

$q_{trans} = \left[\frac{2\pi mkT}{h^2}\right]^{3/2} V$; $q_{rot} = \frac{kT}{\sigma hcB_e}$; $q_{vib} = \frac{1}{1-e^{-hc\sigma/kT}}$; $\Lambda = \left[\frac{h^2}{2\pi mkT}\right]^{1/2}$;

$P_i = \frac{g_i e^{-\varepsilon_i/kT}}{\sum_i g_i e^{-\varepsilon_i/kT}}$; $B_e = \frac{h}{8\pi^2 cI} = \frac{h}{8\pi^2 c\mu r_e^2}$ cm^{-1}; $I = \mu r_e^2$

$\varepsilon_{trans} = \frac{h^2}{8ma^2} n^2$; $\varepsilon_{vib} = \left(n + \frac{1}{2}\right) hc\sigma$; $\varepsilon_{rot} = J(J+1) hcB_e$

$E = kT^2 \left(\frac{\partial lnQ}{\partial T}\right)_{N,V}$; $A = -kTlnQ$; $S = kT\left(\frac{\partial lnQ}{\partial T}\right) + klnQ$;

$C_V = \left[\frac{\partial E}{\partial T}\right]_V$; $\frac{S_{trans}}{R} = \frac{5}{2} lnT + \frac{3}{2} ln\ m + ln\ p - 1.15$, p(bar)

$dA = -SdT - pdV + \mu dN$;

THE UNIVERSITY OF ZAMBIA
School of Natural Sciences
Semester I Examination, 2012

CHE5435 Further Bio-Inorganic Chemistry

Time: 3 Hours **November 2012**

Instructions:

Answer any Four (4) Questions.
Show clarity in your answering.

Question 1

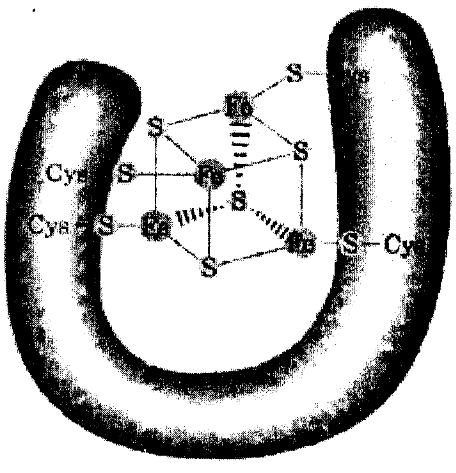
(i) A student prepared a compound with empirical formula $Pt(NH_3)Cl_2$, which showed potency as anti-tumour agent for regression of both fast and slow-moving tumours.

(a) Show the geometrical isomers of the compound
(b) Explain how the compound acts towards cancer growth.

(ii) Choosing an appropriate ligand type for chelation therapy requires a thorough understanding of what metal site the ligand should bind. Outline steps essential to achieving this.

Question 2

(i) What is depicted in Fig (c) and explain its function? What other types of such proteins are known?

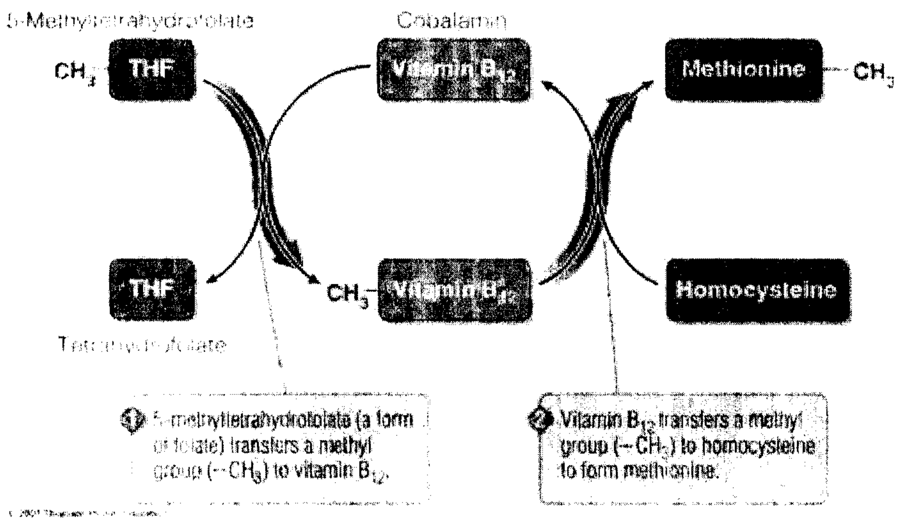


(c)

(ii) Indicate four types of reactions in which these proteins are involved in.

Question 3

Study the diagram below. Methylcobalamin is a cofactor or coenzyme in Methionine synthase(MS).



(a) Write briefly on vitamin B12.
(b) Explain the term coenzyme, and the structure and name of the other vitamin B12 coenzyme.
(c) What is vitamin B12 deficiency aneamia?
(d) What are the effects of vitamin B12 deficiency?

Question 4

(i) **Write down reactions catalysed by the following**

(a) Cytochrome *c* oxidase
(b) Catalase
(c) Superoxide dismutase
(d) peroxidase

(ii) **Demonstrate mutase activity in cobalamin-dependent enzyme (coenzyme B12) from the following:-**

(i) $HO_2CCH(NH_2)CH_2CH_2COOH \Longrightarrow$?
(ii) $CH_3CH(OH)CH_2OH \Longrightarrow ? \Longrightarrow ?$
(iii) $H_2NCH_2CH_2OH \Longrightarrow$?
(iv) $H_3C-\overset{\displaystyle C(O)\text{-}S\text{-}CoA}{\overset{|}{C}}HCH_3 \Longrightarrow$?

(iii) **Show the mechanism for reactions in (ii).**

Question 5

(i) Using the equation, [Fe-complex] + $O_2 \Longrightarrow$ [Fe-complex].O_2, and by making relevant assumptions show how you can derive the Hill Equation.

(ii) Compare and contrast the Weiss and Puling explanation of dioxygen binding with blood.

(iii) Describe the functions of the Type I blue copper protein.

END OF EXAM

UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

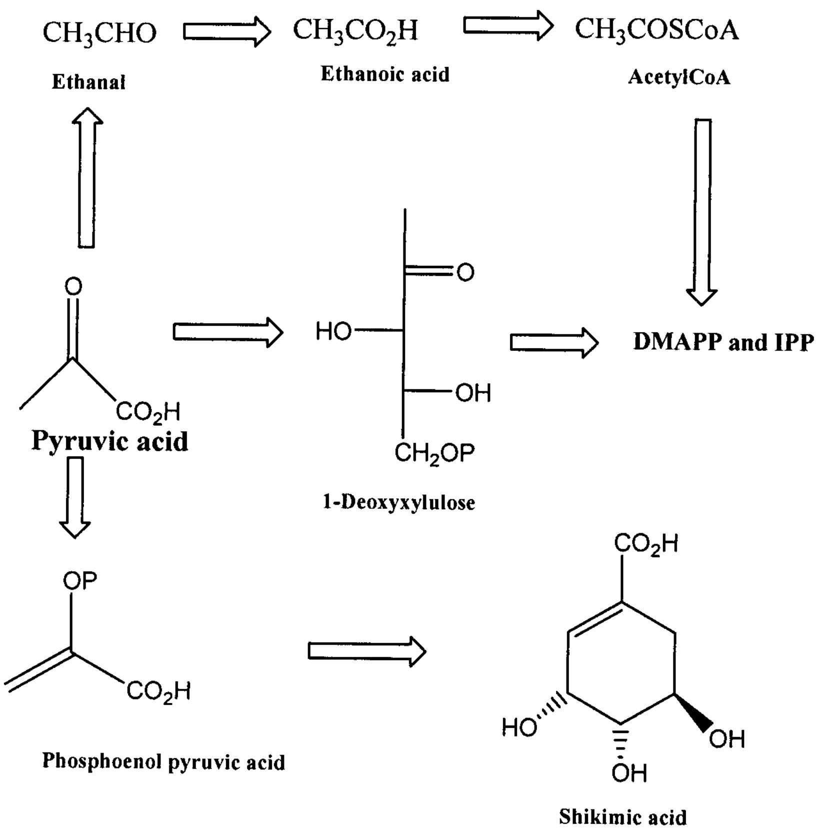
CHE 5522: NATURAL PRODUCTS CHEMISTRY
TIME: THREE HOURS

INSTRUCTIONS
Answer any <u>four</u> questions

Total marks 120

Question one

Pyruvic acid is a necessary compound in Natural Products Chemistry. It is the spring board for all the biosynthetic pathways as shown below.



(a) What do the letters DMAPP and IPP stand for? **(4 marks)**

(b) Provide reagents and mechanisms for conversion of pyruvic acid to ethanal. **(7 marks)**

(c) Describe the steps and mechanisms for the biosynthesis of shikimic acid. **(10 marks)**

(d) Provide a detailed account for the transformation of acetylCoA to IPP. **(9 marks)**

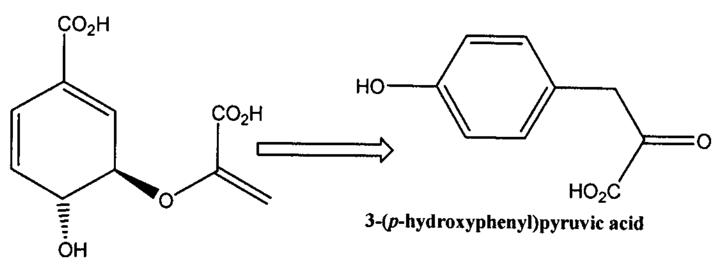
Question Two

The shikimate pathway uses shikimic acid as the starting material for several natural products.

(a) How is shikimic acid converted to p-hydroxybezoic acid? **(9 marks)**

(b) Describe the preparation of salicylic acid from shikimic acid. **(11 marks)**

(c) Show the steps for the transformation of chorismic acid to the 3-(*p*-hydroxyphenyl)pyruvic acid. **(10 marks)**



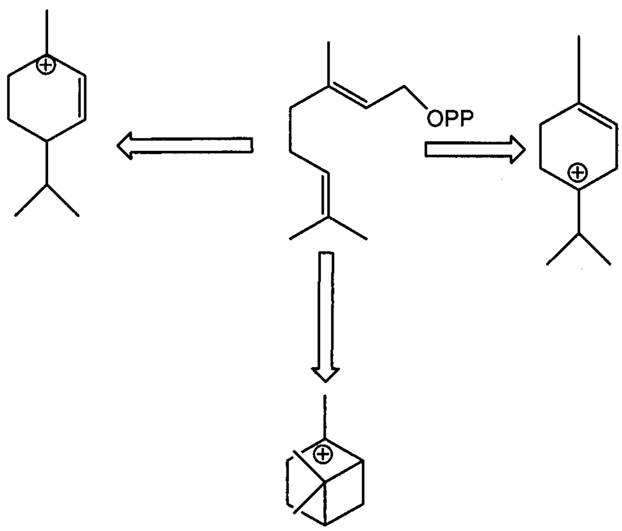
Chorismic acid **3-(*p*-hydroxyphenyl)pyruvic acid**

Question Three

(a) The terpene carbon skeletons are largely biosynthesized from DMAPP and IPP. Write mechanisms for the changes from these two moieties into GPP to FPP to GGPP. **(10 marks)**

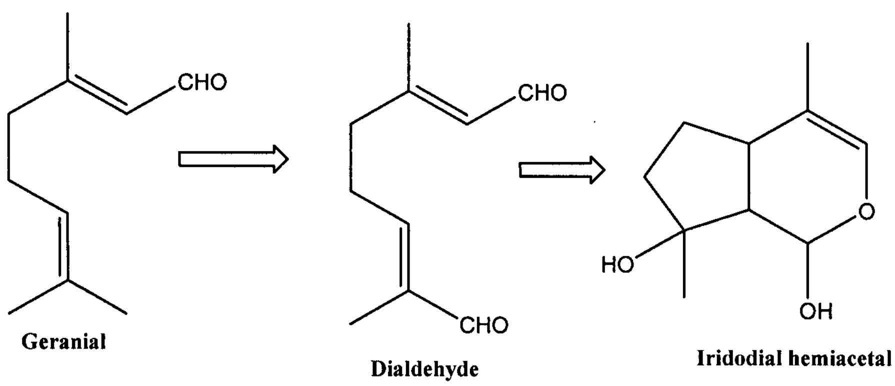
(b) Ionization of GPP gives a cation which on rearrangement leads to several other cations. Give the stepwise rearrangements from ionization of GPP to the given cations below.

(10 marks)



(c) Enzyemic oxidation of geranial gives the α, β-unsaturated dialdehyde which when exposed to other enzymes, water and acid gives the iridodial hemiacetal. Advise on the mechanism transforming the dialdehyde to the hemiacetal using acid and water.

(10 marks)



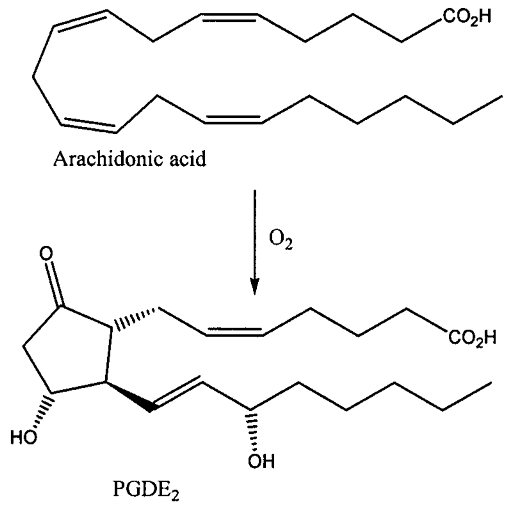
Question Four

a) Write a biosynthetic procedure for hexanoic acid using acetate derivatives.

(10 marks)

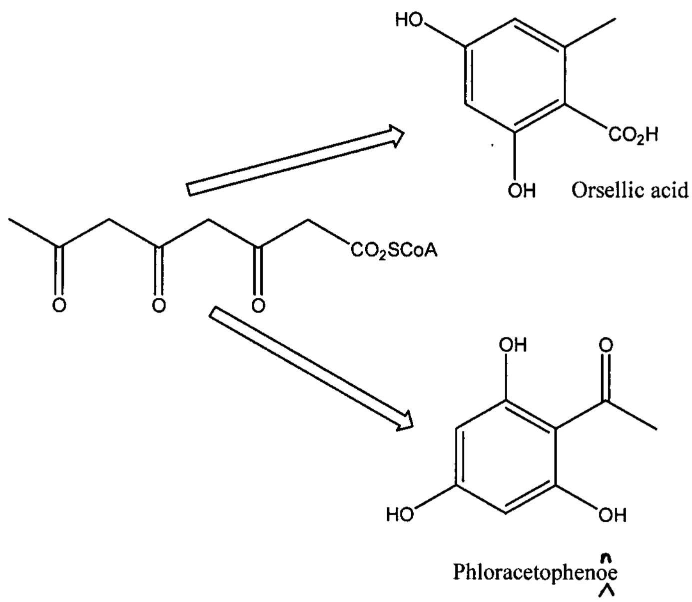
b) It is the unsaturated fatty acids that easily transform to prostaglandins in the presence of radical oxygen. Give the mechanism for the conversion arachidonic acid to $PGDE_2$.

(10 marks)



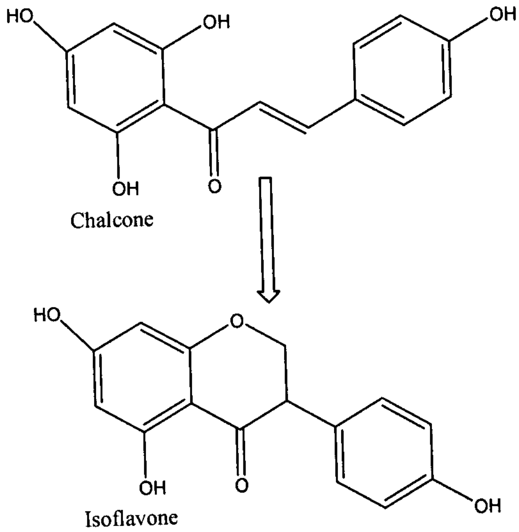
c) Provide the steps and mechanisms involved in converting the tetrapolyketide to orsellic acid and phloracetophenone.

(10 marks)



Question Five

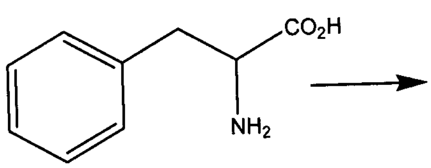
a) Several reactions of phenols go through radicals; some examples are formation of lignins and isoflavones. Show all intermediates in the transformation of ~~a~~ the chalcone to the isoflavone. **(10 marks)**



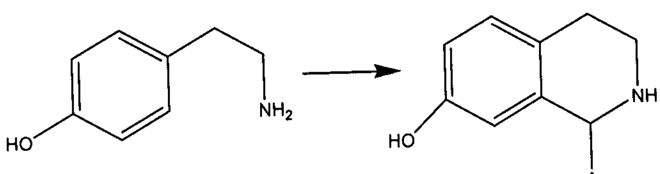
Chalcone

Isoflavone

b) Provide reagents or cofactors for the reactions given below and a mechanism for the step. **(10 marks)**



c) What class of compounds may one use to react with tyramine to form tetrahydroisoquinoline alkaloids? Write a mechanism for the formation of tetrahydroisoquinolines from tyramine. **(10 marks)**



Tyramine

Tetrahydroisoquinolines

END OF EXAMINATION

UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR SECOND HALF
FINAL EXAMINATIONS

CHE 5522: THE CHEMISTRY AND BIOSYNTHESIS OF NATURAL PRODUCTS
TIME: THREE HOURS

INSTRUCTIONS
Answer any <u>four</u> questions

Total marks 120

Question one

Select five cofactors from given list (see attachment) and

(a) Clearly indicate their active sites (centers that undergo change). **(5 marks)**

(b) Briefly describe with examples how each of the selected cofactor above is used during biosynthesis of secondary metabolites. **(25 marks)**

Question Two

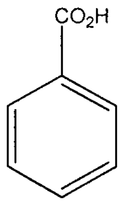
(a) Describe in detail how acetyl-CoA is converted to D-deoxyxylulose and mevalonic acid. **(20 marks)**

(b) Shikimic acid is biosynthesized from pyruvic acid and a tetraose. Give the steps that lead to shikimic acid. **(10 marks)**

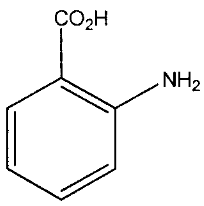
Question Three

(a) Show how shikimic acid may be converted to benzoic acid and anthranilic acid.

(20 marks)

(i) 

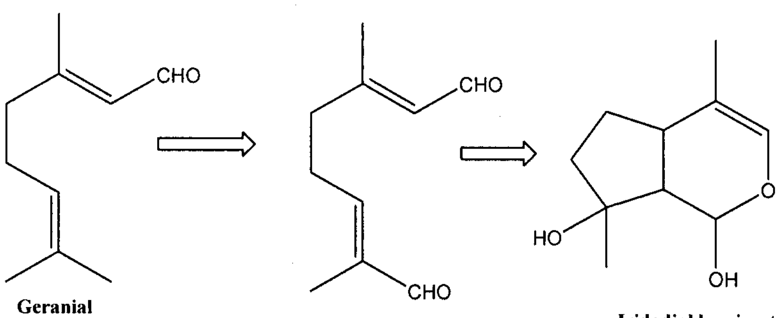
Benzoic acid

(ii) 

Anthranilic acid

(b) Conversion of geranial by enzymatic oxidation gives the α, β-unsaturated dialdehyde which, in turn, undergoes an intramolecular cyclisation into the iridodial hemiacetal. Advise on the mechanism transforming the dialdehyde to the hemiacetal in the presence of acid and water.

(10 marks)



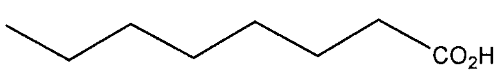
Geranial **Dialdehyde** **Iridodial hemiacetal**

Question Four

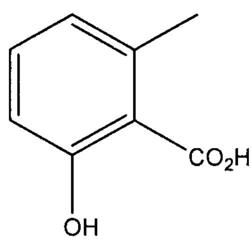
(a) The biosynthesis of fatty acids and polyketides use acetyl-CoA as the starting material. Starting with Acetyl-CoA, describe the biosynthesis of the two moieties with eight carbons namely:

(i) caproic acid,

(ii) 6-methylsalicylic acid.



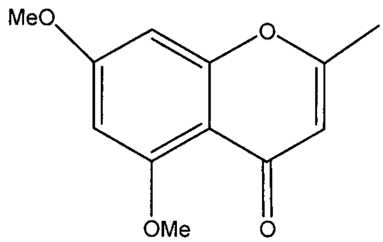
Caproic acid



6-Methylsalicylic acid

(20 marks)

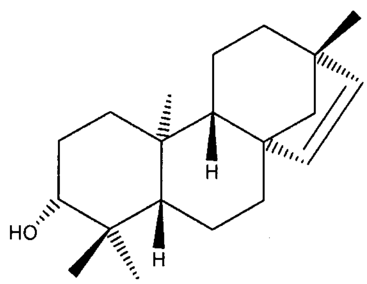
(b) How might you decide whether eugenin is a metabolite of shikimic acid or from the acetate pathway? **(10 marks)**



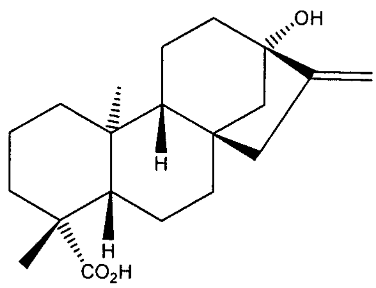
Eugenin

Question Five

(a) Both 3-stachenol and steviol are referred to as *ent*-diterpenoids and are found in plants.



3-Stachenol

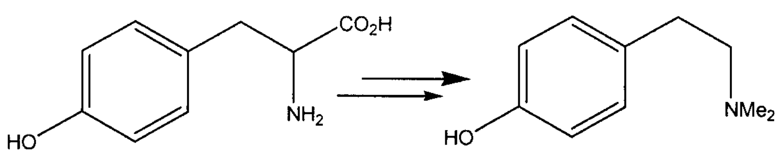


Steviol

(i) Draw the chair conformation of each compound. **(10 marks)**

(ii) Show how the carbon skeleton of one of them may be constructed from DMAPP and IPP. **(10 marks)**

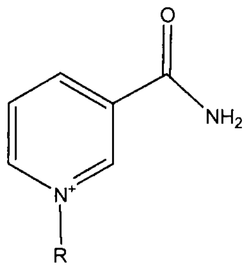
(b) Hordenine, a germination inhibitory alkaloid from barley, is biosynthesized from tyrosine as shown below. Provide reagents or cofactors for the transformation and give a mechanism for the first step. **(10 marks)**



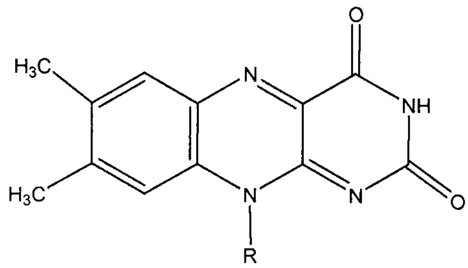
Hordenine
(A germination inhibitory alkaloid)

END OF EXAMINATION

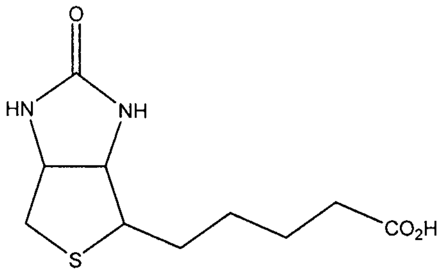
Cofactors



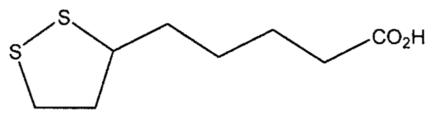
NAD(P)$^+$



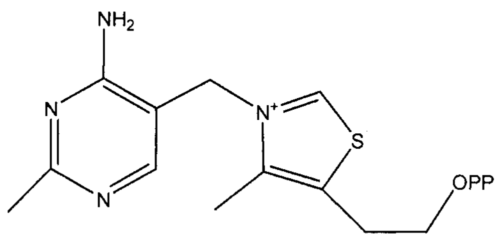
FMN and FAD



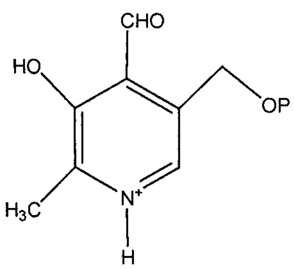
Biotin



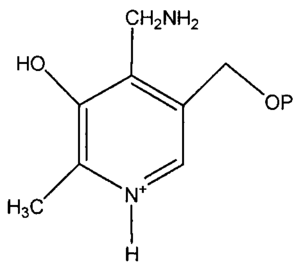
Lipoic aid



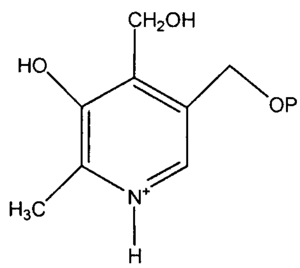
ThiaminPP
VitaminB$_1$PP



Pyridoxal phosphate
(PLP)

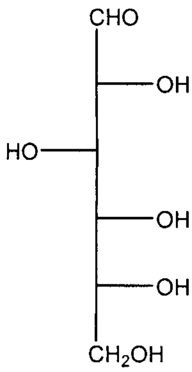


Pyridoxamine Phosphate
(PMP)

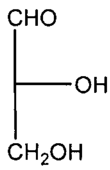


Pyridoxine Phosphate
(PIP)

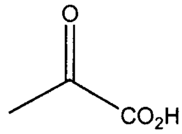
Sugar moieties



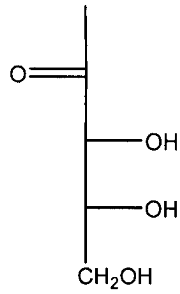
D-Glucose



D-Glyceraldehyde



Pyruvic acid

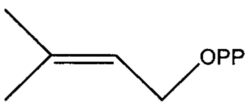


D-Deoxyxylulose

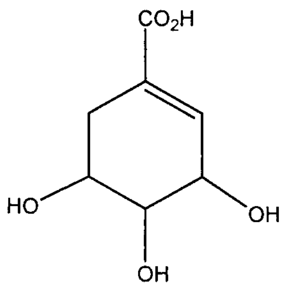
Moiety pathways

H_3C—COSCoA

Acetyl-CoA



DMAPP



Shikimic acid

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SIENCES

DEPARTMENT OF CHEMISTRY

2012 SESSIONAL EXAMINATIONS SEMESTER I

22 NOVEMBER 2012

CHE 5635: INTRODUCTION TO STATISTICAL THERMODYNAMICS

TIME: THREE HOURS

INSTRUCTIONS: ANSWER: *QUESTION 1* AND *ANY THREE* OTHERS.

DATA

Physical constants and other data you may wish to use are given on the ***ATTACHMENT.***

QUESTION 1*(40 MARKS)*

a) Consider a system of nitrogen gas, assumed to be independent (i.e. non-interacting) and indistinguishable at room temperature. The canonical partition function for such a system is $Q(N, V, T)$.

(i) Write the canonical partition function of the system in terms of the molecular partition function, q.

(ii) Show that for this system the entropy $S = NkT\frac{\partial lnq}{\partial T} + Nkln\frac{qe}{N}$

(iii) Calculate the translational temperature of a system of nitrogen gas contained in a cubic box of length 0.1732 m.

(iv) How many translational energy levels contribute to the internal energy, E, of the system at 300 K? (Hint*: this question can be answered without calculation*).

b) (i) What is the *symmetry number* σ and what is its importance ?

(ii) What is the *symmetry number* for nitrous oxide, N_2O if its structure were linear NNO and if it were linear NON?

c) Identify the following equation and all the symbols in it:

$$K_p = e^{-\Delta D_0^o/RT}\frac{\left[\left(\frac{q^*}{N}\right)^c\left(\frac{q^*}{N}\right)^d\right]}{\left[\left(\frac{q^*}{N}\right)^a\left(\frac{q^*}{N}\right)^b\right]}.$$

Question 1 continues to the next page.

d) The translational entropy, S_{trans}, of a monatomic gas is normally called the Sackur-Tetrode equation.

(i) Show that at constant volume and temperature, the molar entropy of a system of a monatomic gas can be given as $\frac{S_M^o}{R} = \alpha + \frac{3}{2} ln\ m$ where α is a constant independent of the chemical identity of the gas and m is the atomic or molar mass of the gas.

(ii) Calculate the approximate difference in the molar entropy S_M^o(Ne) - S_M^o(He) of neon and heli

QUESTION 2 (*20 MARKS*)

Nitric oxide, NO (g), has two electronic energy levels at 300 K. In the ground state electronic level J = $\frac{3}{2}$ and in the excited state J = $\frac{1}{2}$. The low-lying excited electronic state level is 121.1 cm^{-1} above the ground state. The moment of inertia and the vibrational frequency do not depend appreciably on the electronic state of the molecule. Therefore, the electronic contribution to the thermodynamic properties of NO (g) can be calculated separately. The degeneracy of an electronic state is 2J + 1.

a) Write the electronic partition function, q_{el} of nitric oxide.

b) Calculate the electronic temperature, θ_{el} of nitric oxide.

c) Calculate the probability of occupation of (or the fraction of molecules in) the ground state.

d) Calculate the probability of occupation of (or the fraction of molecules in) the excited state.

e) Calculate the Helmholtz free energy per mole of nitric oxide.

QUESTION 3 (*20 MARKS*)

The following questions relate to a system of vibrating heteronuclear diatomic molecules.

a) Discuss, briefly, the following partition functions:

$$q_{vib}^* = \frac{1}{1-e^{-hc\sigma/kT}}\text{; and } \quad q_{vib} = \frac{e^{-hc\sigma/2kT}}{1-e^{-hc\sigma/kT}}.$$

b) (i) What is the value of q_{vib}^* at T = 0 K?

(ii) What is the value of q_{vib}^* in the limit T → ∞?

(iii) Hence draw a labeled sketch diagram to show the variation of q_{vib}^* with temperature.

c) (i) Derive an expression for the internal energy, E_{vib}^* for one mole of vibrating molecules.

(ii) What is the value of E_{vib}^* in the high temperature limit T → ∞?

QUESTION 4 (*20 MARKS*)

K. Clusius and R. Riccoboni, *Z. Phys. Chem.*; **B38**, 81 (1937), determined the entropy of xenon from calorimetric data and found the value S^o_{298} = 170.29 ± 1.2 J K^{-1} mol^{-1}.

a) Using the methods of statistical thermodynamics, calculate the entropy of xenon at 298 K and p = 1 bar. Compare it with the experimental value.

b) Calculate, also, the values of the thermodynamic functions

(i) C_p,

(ii) H/T; and

(iii) G/T.

QUESTION 5 (*20 MARKS*)

A molecule of carbon monoxide, CO, is placed in a volume of 1.500 cm^3 at 300 K. The vibrational frequency, σ = 2169. 8 cm^{-1}, the rotational constant, B_e = 1.9313 cm^{-1} and the dissociation energy, D_o = 1070.2 kJ mol^{-1}. *Note: the energy zero is taken to be the bottom of the well of the potential energy.*

a) Calculate for the carbon monoxide molecule

(i) the equilibrium internuclear distance, r_e,

(ii) the rotational temperature, θ_{rot}; and

(iii) the vibrational temperature, θ_{vib} .

b) Calculate the molecular partition function, $q = q_{tr}q_{vib}q_{rot}q_{el}$ of carbon monoxide.

c) Calculate the Helmholtz free energy, A, per mole of carbon monoxide molecules. Assume that the molecules are distinguishable and do not interact.

END OF CHE 5635 EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF CHEMISTRY

CHE 5635: INTRODUCTION TO STATISTICAL THERMODYNAMICS

ATTACHMENT

DATA SHEET

$N_A = 6.02 \times 10^{23}$ mol^{-1}; $k = 1.38 \times 10^{-23}$ J K^{-1} ; $h = 6.63 \times 10^{-34}$ J s ;

R = 8.314 J K^{-1} mol^{-1}; c = 3.00 x 10^{8} m s^{-1} ; 1eV = 1.602 x 10^{-19} J;

1 amu =1.6601 x 10^{-27} kg; 1J = 1kg m^{2} s^{-2} ; $ln\ e = 1$;

$\frac{\partial \ln f(x)}{\partial x} = \frac{1}{f(x)} \frac{\partial f(x)}{\partial x}$; $\frac{d(e^{f(x)})}{dx} = e^{f(x)} \frac{df(x)}{dx}$; $\frac{d(\ln x)}{dx} = \frac{1}{x}$

$ln\ N! = NlnN - N$, for $N >> 1$ $\quad \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \ldots = \frac{1}{1-x}$, $|x| < 1$

$e^x = 1 + x + \frac{x^2}{2!} + \ldots, x << 1$ $\quad e^{-x} = 1 - x + \frac{x^2}{2!} +- \ldots$, x<<1

He = 4.00; C = 12.01; N = 14.01; O = 16.00; Ne = 20.18; Xe = 131.29

$q_{trans} = \left[\frac{2\pi mkT}{h^2}\right]^{3/2} V$; $q_{rot} = \frac{kT}{\sigma hcB_e}$; $q_{vib} = \frac{1}{1-e^{-hc\sigma/kT}}$; $\Lambda = \left[\frac{h^2}{2\pi mkT}\right]^{1/2}$;

$P_i = \frac{g_i e^{-\varepsilon_i/kT}}{\sum_i g_i e^{-\varepsilon_i/kT}}$; $B_e = \frac{h}{8\pi^2 cI} = \frac{h}{8\pi^2 c\mu r_e^2}$ cm^{-1}; $I = \mu r_e^2$

$\varepsilon_{trans} = \frac{h^2}{8ma^2} n^2$; $\varepsilon_{vib} = \left(n + \frac{1}{2}\right) hc\sigma$; $\varepsilon_{rot} = J(J+1)\, hcB_e$

$E = kT^2\left(\frac{\partial lnQ}{\partial T}\right)_{N,V}$; $A = -kTlnQ$; $S = kT\left(\frac{\partial lnQ}{\partial T}\right) + klnQ$;

$C_V = \left[\frac{\partial E}{\partial T}\right]_V$; $\frac{S_{trans}}{R} = \frac{5}{2}lnT + \frac{3}{2}ln\ m + ln\ p - 1.15$, p(bar)

$A = E - TS$; $H = E + PV = kT^2\left(\frac{\partial \ln Q}{\partial T}\right)_V + V\left(\frac{\partial \ln Q}{\partial V}\right)_T$;

$G = A + PV = -kT\ ln\ Q + kTV\left(\frac{\partial \ln Q}{\partial V}\right)_T$; $PV = NkT$

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR FIRST SEMESTER
FINAL EXAMINATIONS

CHE5711: MEDICINAL CHEMISTRY I

TIME: THREE HOURS

INSTRUCTIONS:

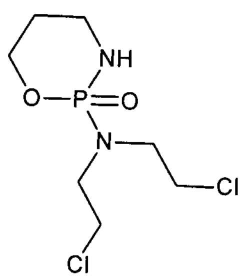
Answer any four questions.

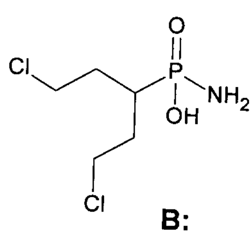
Max. Marks: 120

Question One

(a) (i) Briefly explain what is meant by 'Pro-drug'. **(2 marks)**

(ii) The prodrug **A** was developed to overcome the problem of toxicity of a potent carcinoma drug **B**. It has been shown that the prodrug **A** is converted in several steps into the active drug **B** *in vivo* in man. Propose a plausible sequence or reactions for the biotransformation of **A** into **B**.
(7 marks)


A

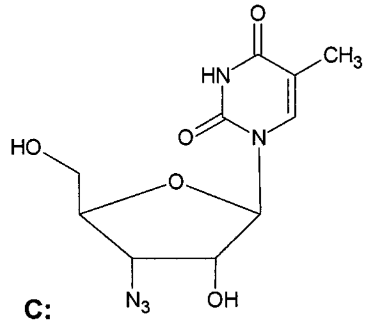

B:

(b) Deficiency of a neurotransmitter DOPAMINE, 3,4-dihydroxyphenylethylamine in brain has been implicated in the aeteology of Parkinson's syndrome/disease. Briefly explain the following:
(2 marks each)

(i) Can DOPAMINE be used for the treatment of Parkinson's syndrome?

(ii) Can L-DOPA (3,4-dihydroxyphenylalanine) be used for treatment of Parkinson's syndrome?

(c) (i) Propose a synthesis for the antiviral drug **C**, currently used for the treatment of HIV-AIDS. **(12 marks)**



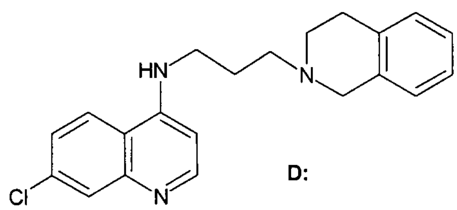
C:

(ii) Briefly explain the mode of antiviral action of the drug **C**. **(5 marks)**

Question Two

(a) Give the structure and mode of action of the currentl first choice antimalarial drug artemesinine. **(7 marks)**

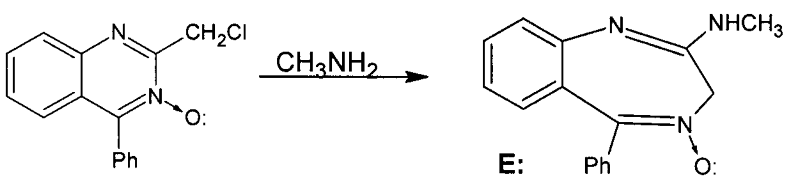
(b) Propose a synthesis for a potent antimalarial agent **D**, reported to be four times more potent than chloroquin. **(13 marks)**



D:

(c) Cycloguanil palmoate is a long acting antimalarial drug ,but cycloguanil chloride has only short duration of action. Explain. **(3 marks)**

(d) (i) Suggest the reaction mechanisms for the following synthesis of a biologically active compound **E**. **(5 marks)**

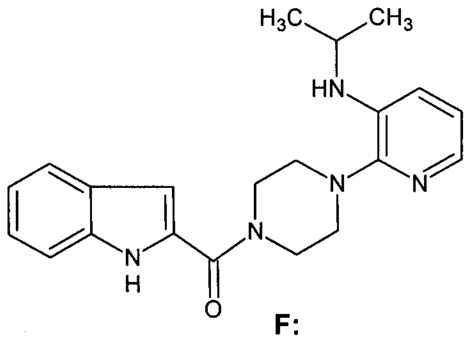


(ii) State the principal medicinal usage of **E**. **(2 marks)**

Question Three

(a) Gamma aminobutyric acid (GABA) has been implicated in the aeteology of epileptic seizures. On this basis explain the rationale for the use of GABA agonists for treatment of epilepsy.
(6 marks)

(b) Devise a synthesis for the anticonvulsant drug **E** from readily available non-heterocyclic starting materials. State the reagents and reaction conditions for each step of your proposed synthesis.
(13 marks)



F:

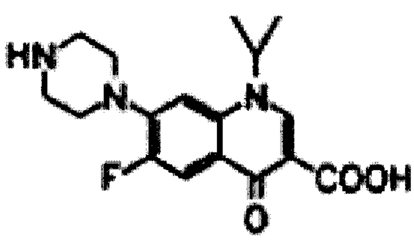
(c) State the principal pharmacological effect(s) and briefly explain the mode of action of the following drugs: **(3 marks each)**

(i) Diazepam
(ii) Carbamazepine (an iminostilbene)

(d) Briefly discuss the structure activity relationships in □–phenylethylamine CNS stimulants.
(5 marks)

Question Four

(a) Give a synthesis of the third generation quinolone antibiotic ciprofloxacin and briefly explain its mode of antimicrobial action. **(17 marks)**



Ciprofloxacin

(b) (i) Naturally occurring □–lactam antibiotics, penicillins/cephalosporins, are orally ineffective for treatment of microbial infections. Briefly explain why. **(4 marks)**

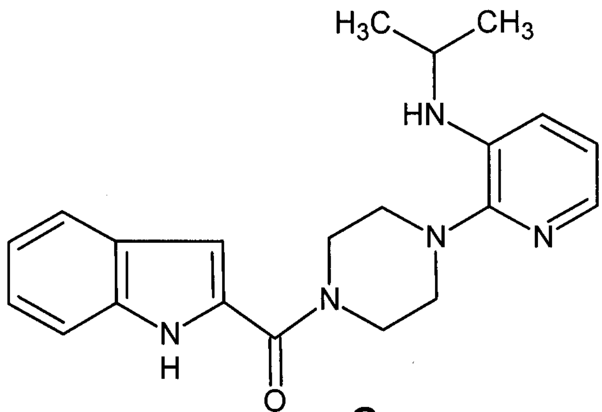
(ii) Suggest an analogue of Pen V (phenoxymethyl penicillin)) with a view to improve its oral bioavailability. **(3 marks)**

(iii) Give a synthetic route for <u>partial synthesis</u> of the analogue you suggested in (ii) above from any naturally occurring penicillin. **(6 marks)**

Question Five

(a) Briefly discuss the structure activity relationships in □–phenylethylamine CNS stimulants. **(5 marks)**

(b) (i) Suggest a synthesis of the potent anti-retroviral drug **F**, shown below. Assume that piperazine is available. **(12 marks)**

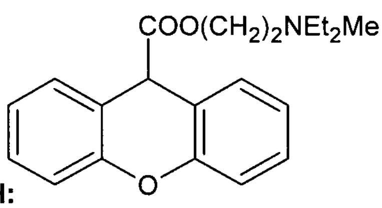


G

(ii) Give the mechanism of antiviral action of the drug **F**. **(5 marks)**

(c) Give a synthesis for the molecule **H**, shown below, and state its principal medicinal usages. **(8 marks)**

H:



END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS

GEO 5892: GEOGRAPHIC INFORMATION SYSTEMS AND REMOTE SENSING

TIME: Three hours

INSTRUCTIONS: Answer any four questions.

All questions carry equal marks.

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1. Medium and coarse spatial resolution remote sensing systems are invaluable and the data from such systems have become widespread among the land cover/land use research scientists. Compare and contrast between the two systems using one example from each.

2. Explain remotely sensed data radiometric pre-processing and its relevance.

3. Explain the theoretical basis of the Normalized Difference Vegetation Index (NDVI) and its application in crop phenology characterization.

4. Define a data model and describe the two most commonly used data models.

5. Using an example of your choice, explain why a geographic information system (GIS) is considered as a process to aid decision making.

6. Choma, being a new capital of Southern Province, is expected to experience rapid infrastructure development. An investor who wants to use GIS to find a suitable location for a shopping mall, has a roads layer with the following data attributes: (i) Geographic Coordinate System: GCS_WGS_1984 and (ii) Datum: D_WGS_1984. A block of twelve analogue aerial photographs are available for use in preparing a current land cover/land use (LC/LU) map. Explain the steps you would undertake in producing a digital LC/LU map of Choma to be used in the GIS analysis.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR FINAL EXAMINATIONS

GES 5595: NATURAL RESOURCES ECONOMICS

TIME: Three hours

Instructions: Answer **ONE** and any other **THREE** questions. All questions carry equal marks. Use of approved calculator is allowed.

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1. Conventional ploughing causes soil erosion with onsite effects such as loss of soil health and off-site effects such as siltation in the nearby lake. A terracing project is proposed to minimise the soil erosion. This project will have the following economic effects:
   - Initial investment at 350,000 ZMK
   - Annual maintenance cost at 5,000 ZMK
   - Improved crop yield at 40,000 ZMK per year
   - In the event that the project is not carried out, removal of silt from the lake will have to be done in year 10 and again in year 20, at a cost of 250,000 ZMK each time. This cost will be avoided if the project is implemented.

   a) Calculate the net present value of the project with 10% discount rate and 20 years project lifetime. Should the project continue? Explain.
   b) Calculate the net present value of the project with 20% discount rate and 20 years project lifetime. Should the project continue? Explain.
   c) What are the implications of discount rates on management decisions?

2. Define Hartwicks' rule and assess its implications in the development pathway of countries like Zambia.

3. Discuss socio-economic implications of deforestation and suggest mitigation measures for the negative effects.

4. 'Payments for Ecosystem Services (PES) is an agenda for the developed countries to control the development process of developing countries'. Discuss.

5. Explain problems associated with efficient allocation of water resources in Zambia and suggest ways of minimizing these problems.

6. Explain how the following modify human and firms' behaviour in the utilization of natural resources:
   a) Taxes
   b) Seasonal bans
   c) Subsidies
   d) Pedagogic measures
   e) Tradable permits.

END OF EXAMINATIONS

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR FINAL EXAMINATIONS
GES 5625: ENVIRONMENTAL LAW

TIME: **Three hours**

INSTRUCTIONS: **Answer any FOUR questions**
All questions carry equal marks

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1. Two days after your graduation from a short course on environmental law in Zambia, you decide to go to Soweto market for the usual morning bargains. While there, you overhear a group of vendors arguing about who is responsible for maintaining the unsightly market surroundings. One of the vendors argues that he pays a daily fee of K5 which he has been told is surrendered to the local authority for sanitation. Another vendor emphatically argues that he cannot be responsible for the maintenance of the surroundings because he is aware that the newly enacted Environmental Management Act, No. 12 of 2011 provides that "every person living in Zambia has the right to a clean, safe and healthy environment" which includes "the right of access to the various elements of the environment for recreational, education, health, spiritual, cultural and economic purposes." Prepare short notes on how you would participate in this discussion.

2. The term "environmental law" is defined in several ways in different publications and legislation.

   (a) Clearly outline what "law" and "environment" encompass (10 marks).
   (b) What is the purpose of environmental law? (5 marks).
   (c) In your view, what should be the most important considerations in defining what international and national environmental law should include? (10 marks).

3. Write short explanatory notes on how environmental law in Zambia relates to ALL of the following:

   a) Sustainable development (5 marks)
   b) Polluter pays (5 marks)

c) Access to environmental information (5 marks)
d) Precautionary principle (5 marks)
e) Community participation and involvement in natural resources management and sharing of benefits. (5 marks)

4. Section 3 of the Environmental Management Act No. 12 of 2011 provides as follows:

> Subject to the Constitution, where there is any inconsistency between the provisions of this Act and the provisions of any other written law relating to environmental protection and management, which is not a specific subject related to law on a particular environmental element, the provisions of this Act shall prevail to the extent of the inconsistency.

(a) Explain what this section means in relation to other sectors of Zambia's economy. (10 marks)
(b) In your view, what challenges are there in realizing the provisions of Section 3 of the Environmental Management Act? (15 marks)

5. A recent research argues that:

> It is well settled in modern times that securing environmental well-being ranks highly on the global political agenda. In this vein, environmental law and policy have been identified as 'basic conceptual tools for environmental management'. Law is a cross-disciplinary topic with an increasingly important role to play in environmental sustainability. Science and technology are therefore important underlying considerations in ensuring global environmental sustainability.

Discuss whether you agree or disagree that science and technology should influence the present day environmental law making process.

6. Outline the sources of environmental law, clearly relating them to the development of environmental law in Zambia.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

GES 5645: SUSTAINABLE LAND MANAGEMENT AND FOOD SECURITY

2013 ACADEMIC YEAR FINAL EXAMINATIONS

TIME: Three hours

Instructions: Answer any four questions

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1. With the use of examples, show how small scale farmers in sub-Saharan Africa were able to devise indigenous food production systems that could be designated as being based on sustainable land management.

2. Explain any four factors that contribute to the unsustainable use of land resources in Zambia and suggest measures to address the challenges.

3. Write short explanatory notes on **all** of the following approaches to addressing food security
   a. Market based approach
   b. Production based approach
   c. Human Right based approach
   d. Social welfare approach

4. 'Some indigenous food production systems can be a basis for the promotion of adaptive approaches to climate change in sub-Saharan Africa.' Explain

5. 'Food insecurity will never be reduced because it supports a global big business networks' Discuss

6. 'Food security is assessed at various levels of social organisation'. Using examples, explain the linkage between the choice of food security indicators and the levels of food security assessment in society?

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

GES 5665: FORESTS AND WILDLIFE MANAGEMENT

2013 ACADEMIC YEAR FINAL EXAMINATIONS

TIME: Three hours

INSTRUCTIONS: Answer any four questions

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1. Critically discuss the concept of ecological succession and explain how it has historically influenced forest conservation.

2. 'The Payments for Ecosystem Services (PES) conservation paradigm has the potential to mitigate the deficiencies of both fortress and community based conservation'. Discuss.

3. Elucidate how poverty, globalization of trade and westernization of diets threaten wildlife resource sustainability.

4. Explain the main causes of human wildlife conflicts around protected areas in Zambia, and suggest ways in which such conflicts could be minimized.

5. "If local people do not support protected areas, then protected areas cannot last" (Ramphal, 1993:56). Discuss this assertion with reference to wildlife resources in Southern Africa.

6. Discuss the role of traditional institutions in conservation of biodiversity in rural Africa.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR FINAL EXAMINATIONS
GES 5715: CLIMATE CHANGE AND SUSTAINABLE DEVELOPMENT

TIME: **Three hours**

INSTRUCTIONS: **Answer any FOUR questions**
All questions carry equal marks

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1. Write short explanatory notes on ALL of the following.
   a) The ALARP Model
   b) The concepts of resilience and adaptation
   c) Radiative forcing
   d) The earth's elliptical orbit and its implications on earth surface temperatures
   e) Albedo and clouds

2. 'Climate change, sustainable development and poverty reduction are mutually reinforcing and structurally linked such that failure in one is most likely to undermine the other two'. Discuss.

3. The earth's present mean temperature is a result of interactions between the sun, the atmosphere, and the earth's surface. Explain with the aid of a diagram

4. Suggest climate change mitigation measures in any three economic sectors that developing countries can undertake in their efforts to transition into low carbon economies.

5. 'Throughout the 4.5 billion years of the earth's existence, its climate has been in constant evolution'. Discuss this statement and describe the natural factors that account for this evolution.

6. Developed countries have used a disproportionate share of the 'carbon space' (also known as development space) which developing countries now badly need to meet their primary goal of poverty alleviation. Explain how this reality will pose a challenge to agreeing on a legally binding framework in 2015 that places heavy carbon cuts on both developed and developing countries.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC FIRST YEAR FINAL EXAMINATIONS

GES 5881: RESEARCH METHODS

TIME: Three hours

INSTRUCTIONS: Answer any four questions. All questions carry equal marks.

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1. Using an example, examine the steps that a researcher would follow in an experimental research design

2. a) When reviewing literature, what is being searched for?

   b) Based on a research topic of your own choice, create a literature 'map' and provide a justification for it.

3. Explain how critical realism and interpretivism attempt to mediate against the deficiencies of the positivist approach in environmental research.

4. Using any research topic of your choice:

   (a) State the aim of your study.

   (b) State four specific objectives related to your topic and explain why each of the objectives is suitable and study.

   (c) Justify your study.

5. 'Quantitative research design strategies differ in many ways such as in purpose, process and data collection'. Discuss on the basis of the three mentioned characteristics, any three design strategies.

6. Examine and contrast ethnographic and survey research designs.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR FINAL EXAMINATIONS

GES 5881: RESEARCH METHODOLOGY

TIME: Three hours

Instructions: Answer question **ONE** and any other **THREE** questions. All questions carry equal marks. Use of your research proposal is allowed.

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1. Study the statistical analysis output from Minitab version 14.0 done by, Sililo Sililo, a postgraduate student to determine factors influencing the size of land owned in Kabangwe Ward of Lusaka.

Regression Analysis:

```
The regression equation is
Area of land owned = - 0.494 - 0.037 gender + 0.0150 age + 0.0403 education
          + 0.0979 monthly income - 0.0290 workshops attended + 0.316 loan
          - 0.0511 household size
410 cases used, 230 cases contain missing values
```

Predictor	Coef	SE Coef	T	P
Constant	-0.40942	0.05815	-0.85	0.396
gender	-0.00367	0.03038	-0.12	0.904
age	0.15006	0.06531	2.30	0.022
education (years)	0.04033	0.03627	1.11	0.267
monthly income(zmk)	0.09792	0.03827	2.56	0.011
workshops attended	-0.02904	0.05823	-0.50	0.618
loan accessed(zmk)	0.31561	0.04914	6.42	0.000
household size	-0.05107	0.01535	3.33	0.001

S = 1.83236   R-Sq = 26.0%   R-Sq(adj) = 24.4%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	9	472.258	52.473	15.63	0.000
Residual Error	400	1343.014	3.358		
Total	409	1815.272			

a) Suggest a hypothesis that the student was testing
b) Explain how the 410 sample could have been chosen and give a reason for your answer
c) Interpret the results
d) Based on your hypothesis in (a) and your interpretation in (c) what conclusions can you make?
e) How different is this method of analysis from content analysis?

2. Explain the concepts of epistemology and ontology and show how the two concepts influenced your choice of research methods in the development of your proposal.

3. Explain practical challenges that you faced in the process of research proposal writing and suggest how you plan to or have addressed them.

4. Drawing on the literature review section of your proposal, explain:
   a) How the literature review process led you to the identification of the research problem.
   b) Two core concepts underpinning your study and the role literature review process played in the conceptualisation of your study.
   c) Two research methods most often used in studying similar research problems and the limitations of such methods.

5. Based on your research proposal:
   a) State one objective and a respective hypothesis or research question
   b) Develop a sample of data collection sheet for the objective in (a).
   c) Outline how you would proceed from sampling to conclusions for your findings on the objective in (a).
   d) What are the possible limitations associated with the processes in (c).
   e) State three ethical obligations you will have to observe in your research process.

6. Evaluate the appropriateness of web-based sources (such as Wikipedia), academic books, and academic journals in terms of:
   a) Reliability of information
   b) Linkage to theoretical debates
   c) Breadth of coverage
   d) Coverage of current events.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR FINAL EXAMINATIONS

GES 5892: GEOGRAPHIC INFORMATION SYSTEMS & REMOTE SENSING

TIME: Three hours

INSTRUCTIONS: Answer any FOUR questions
All questions carry equal marks

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1. Discuss the significance of Normalized Difference Vegetation Index (NDVI) in combating climate change.

2. You have been asked to perform a time-series change detection analysis on Lusaka Province between 1980 and 2014. Describe important sensor considerations you must take into account in terms of spatial, spectral and temporal resolutions and costs.

3. Explain using practical examples how a GIS practitioner working for the Disaster Management Unit (DMMU) in the office of the Vice President can use GIS in conducting a risk assessment on flash floods in Chembe district.

4. Provide five examples of how remote sensing and GIS data can be integrated when solving environmental problems.

5. Describe how you would perform supervised and unsupervised classifications on remote sensing imagery and assess the accuracy.

6. Explain the significance of geo-referencing data in GIS and describe how you can geo-reference a topographic map in ArcGIS.

END OF EXAMINATION

The University of Zambia

School of Natural Sciences

Department of Mathematics & Statistics

First Semester Examinations - November 2012

MAT5111 - Ordinary Differential Equations & Integral Equations

Time allowed : 3hrs

Instructions: • Attempt **all five (5)** questions. All questions carry equal marks.

• **Full credit** will only be given when **necessary work** is shown.

• Indicate your **computer number** on all answer booklets.

This paper consists of 3 pages of questions.

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1. a) Given an initial value problem

$$y'(t) = f(t, y(t)) \ , \qquad a \le t \le b \ , \qquad y(a) = \alpha \ ,$$

define the following :-

i) a Lipschitz condition.

ii) a well-posed problem

iii) a non-autonomous differential equation

b) Let A be a real $n \times n$ matrix and consider the equation $x' = Ax$. Show that the function

i) $t \mapsto e^{\lambda t}v$ is a real solution if and only if $\lambda \in \mathbb{R}$, $v \in \mathbb{R}^n$ and $Av = \lambda v$.

ii) If $v \neq 0$ is an eigenvector for A with eigenvalue $\lambda = \alpha + i\beta$ such that $\beta \neq 0$, then the imaginary part of ν is not zero and that if $\nu = u + iw \in \mathbb{C}^n$, then there are two real solutions

$$t \to e^{\alpha t}[(\cos \beta\, t)u - (\sin \beta\, t)w]$$

$$t \to e^{\alpha t}[(\sin \beta\, t)u + (\cos \beta\, t)w]$$

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR FINAL EXAMINATIONS
GES 5892: GEOGRAPHIC INFORMATION SYSTEMS & REMOTE SENSING

TIME: **Three hours**

INSTRUCTIONS: **Answer any FOUR questions**
All questions carry equal marks

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1. Discuss the significance of Normalized Difference Vegetation Index (NDVI) in combating climate change.

2. You have been asked to perform a time-series change detection analysis on Lusaka Province between 1980 and 2014. Describe important sensor considerations you must take into account in terms of spatial, spectral and temporal resolutions and costs.

3. Explain using practical examples how a GIS practitioner working for the Disaster Management Unit (DMMU) in the office of the Vice President can use GIS in conducting a risk assessment on flash floods in Chembe district.

4. Provide five examples of how remote sensing and GIS data can be integrated when solving environmental problems.

5. Describe how you would perform supervised and unsupervised classifications on remote sensing imagery and assess the accuracy.

6. Explain the significance of geo-referencing data in GIS and describe how you can geo-reference a topographic map in ArcGIS.

END OF EXAMINATION

iii) Show that the solutions in (ii) are linearly independent.

c) Given the initial value problem

$$y^{'}(t) = 1 + t\sin(ty) \ , \qquad 0 \le t \le 2 \ , \qquad y(0) = 0 \ ,$$

show that a unique solution exists to the above initial value problem.

2. a) State and prove Growall's inequality.

b) Let ϕ and ψ be two solutions of the initial value problem, $x^{'} = f(t,x)$, $x(t_0) = x_0$, $|t - t_0| \le \alpha$. Use Gronwall's inequality to show that $\phi = \psi$.

c) Solve the equation

$$f(t) = 1 + \int_0^t (\lambda + \mu(t-s))f(s)ds \qquad t \ge 0$$

by reducing it to an equivalent initial-value ODE problem. Hence show that the equation is stable (solutions tend to zero) if $\lambda, \mu < 0$.

3. a) Given a linear system $x^{'} = A(t)x(t)$, define the following:-

i) a fundamental set of solutions.

ii) a fundamental matrix solution.

iii) a resolvent matrix

b) Let ϕ and ψ be two fundamental matrix solutions to the linear system $x^{'} = A(t)x(t)$ (where $A \in M_n(\mathbb{K})$). Show that there exists a constant non-singular matrix $C \in M_n(\mathbb{K})$ such that $\psi(t) = \phi(t)C$ for all $t \in I$.

c) Show that the resolvent satisfies the identity

$$\frac{\partial}{\partial s} R(t,s) = -R(t,s)A(s) \ .$$

4. a) Given the initial value problem $y^{'} + y = 0$, $y(0) = c$, $c \in \mathbb{R}$. Use Picard's successive approximation to find a solution to the IVP.

b) Let f be continuous and locally Lipschitz on a domain R defined by

$$R = \{(t,x)\} : |t - t_0| \le a \ , |x - x_0| \le b$$

for $a, b \ge 0$. Let

$$M = \sup_{(t,x)\in R} \parallel f(t,x) \parallel < \infty \qquad \text{and} \qquad \alpha = \min\left(a, \frac{b}{M}\right) \ .$$

Show that the sequence defined by Picard's successive approximation method converges uniformly on the interval $|t - t_0| \leq \alpha$.

c) Show that if the sequence of functions $\phi_n(t)$ in b) converges uniformly and are continuous then the sequence leads to the integral form of the solution for the initial value problem.

5. a) Consider the Volterra equation of the second kind

$$f(t) = g(t) + \int_0^t K(t, s, f(s))ds, \qquad 0 \leq t \leq T\,. \tag{1}$$

i) Show that a second-order differential equation $y'' = f(x, y)$ can be written in this form.

ii) Show that an integro-differential equation of the form

$$f'(\tau) = \int_0^\tau m(\tau, s)f(s)ds + r(\tau), \qquad f(0) = 0$$

can, by integrating both sides over the interval $[0, t]$, be written in the form of equation (1).

b) Given $K(t, s)$ is continuous on $0 \leq s \leq t \leq T$ and $g(t)$ is continuous in $0 \leq t \leq T$, show that the linear Volterra equation of the second kind has a unique continuous solution in $0 \leq t \leq T$.

END!

The University of Zambia

School of Natural Sciences

Department of Mathematics & Statistics

Second Semester Examinations - August/September 2013

MAT5122 - Partial Differential Equations

Time allowed : 2 hours 30 mins | Full marks : 100

Instructions: • Attempt **any (5) five** questions. All questions carry equal marks.

- **Full credit** will only be given when **necessary work** is shown.
- Indicate your **computer number** on all answer booklets.

This paper consists of 3 pages of questions.

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1. a) Let $V(x, y, z) = (P(x, y, z), Q(x, y, z), R(x, y, z))$ be a vector field in some domain Ω of R^3 that is non-vanishing in Ω, and $P, Q, R \in C^1(\Omega)$. Define the following:-

   (i) an integral curve of the vector field V.

   (ii) a first integral of the vector field V.

   b) Solve the system
$$\frac{dx}{y+z} = \frac{dy}{y} = \frac{dz}{x-y},$$
   and show that the resulting first integrals are functionally independent.

2. a) (i) Find the general integral of
$$xz_x + yz_y = z .$$

   (ii) Use the general integral obtained in (i) to find corresponding solutions to the differential equation when the arbitrary function is given as $F(u_1, u_2) = u_1 - u_2$ and $F(u_1, u_2) = u_1 - u_2^2$

   b) Given $z = u(x, y)$, find a general solution to the partial differential equation
$$uu_x + yu_y = x .$$

3. a) Use the method of separation of variables to transform the wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

into two ordinary differential equations.

b) Hence or otherwise solve the above PDE on the interval $x \in [0, L]$ subject to the conditions that

$$\begin{aligned} u(0,t) &= 0\,, \qquad u(L,t) = 0 \\ u(x,0) &= \sin(\pi x/L) + \sin(2\pi x/L) \\ u_t(x,0) &= 0 \end{aligned}$$

4. a) Derive D'Alembert's solution for the wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \qquad (-\infty < x < \infty\,, t > 0)\,.$$

b) Use the result obtained in part a) to find a solution to the wave equation

$$\frac{\partial^2 u}{\partial x^2} = 4\frac{\partial^2 u}{\partial t^2} \qquad (-\infty < x < \infty\,, t > 0)\,.$$

subject to

$$\begin{aligned} u(x,0) &= e^{-x^2} \qquad (-\infty < x < \infty) \\ \frac{\partial u}{\partial t}(x,0) &= xe^{-x^2} \qquad (-\infty < x < \infty) \end{aligned}$$

5. a) Find the general solution of the quasilinear equation

$$[y(x+y)+z]\frac{\partial z}{\partial x} + [x(x+y)-z]\frac{\partial z}{\partial y} = z(x+y)$$

b) Solve the non-linear unidirectional wave equation

$$\frac{\partial z}{\partial t} + \left(\frac{\partial z}{\partial x}\right)^2 = 0$$

subject to $z(x,0) = xd$ where d is a constant.

6. a) Find the integral surface of $V = (x, y, z)$ containing the curve C given by

$$y = x + 1 , \qquad z = x^2 , \qquad x > 0 .$$

b) Show that the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

with initial condition $u = 0$ for $0 \leq x \leq \frac{1}{2}$ at $t = 0$ and with

$$\frac{\partial u}{\partial x} = 0 \qquad \text{at } x = 0 \qquad \forall\, t > 0$$

and

$$\frac{\partial u}{\partial x} = 1 \qquad \text{at } x = \frac{1}{2} \qquad \forall\, t > 0$$

has the solution

$$u(x,t) = -\frac{1}{12} + x^2 - \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2\pi^2} e^{-(2\pi n)^2 t} \cos 2n\pi x .$$

END!

The University of Zambia

School of Natural Sciences

Department of Mathematics & Statistics

Mid-Year Examinations

MAT5141 - Topics in Mathematical Methods

Time allowed : Three (3) hrs | Full marks : 100

Instructions:
- Attempt **any four (4)** questions. All questions carry equal marks.
- **Full credit** will only be given when **necessary work** is shown.
- Indicate your **computer number** on all answer booklets.

This paper consists of 3 pages of questions.

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1. a) (i) State Hermite's differential equation.

   (ii) Prove that the Hermite polynomial can be expressed by

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}\left(e^{-x^2}\right) .$$

   (iii) Prove that the Hermite polynomials, satisfy the recursion formula

$$H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x) .$$

   b) (i) Establish the relation between the Gamma and Beta functions.

   (ii) Derive the duplication identity formula for the Gamma function

$$\Gamma\left(P + \frac{1}{2}\right) = \frac{\sqrt{\Pi}\,\Gamma(2P+1)}{2^{2P}\,\Gamma(P+1)} .$$

   c) Evaluate the following:-

   (i) $\Gamma\left(\frac{9}{2}\right)$

   (ii) $\Gamma\left(-\frac{9}{2}\right)$

2. a) Define the following

(i) Gamma function

(ii) Beta function

(iii) Member of the family of cylinder function

b) Prove the following

(i) $J_{\nu+1}(z) = \frac{\nu}{z} J_\nu(z) - J_\nu'(z)$, where $J_\nu(z) = \left(\frac{z}{\nu}\right)^\nu \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{z}{\nu}\right)^{2\nu}}{k!\,\Gamma(\nu+k+1)}$

(ii) $\frac{d}{dz}\left(z^{-\nu} C_\nu(z)\right) = -z^{-\nu} C_{\nu+1}(z)$, where $C_\nu(z)$ is a cylinder function.

c) Evaluate the following

(i) $\int_0^{\pi/2} \cos\theta \operatorname{cosec}^4\theta \, d\theta$

(ii) $\int_0^\infty x^{5/2} e^{-2x} \, dx$

3. a) The generating function for the Legendre polynomials is given by

$$\frac{1}{\sqrt{1-2xt+t^2}} = \sum_{n=0}^{\infty} P_n(x)\, t^n .$$

(i) Derive the recurrence formula

$$P_{n+1}(x) = \frac{2n+1}{n+1} x P_n(x) - \frac{n}{n+1} P_{n-1}(x)$$

(ii) Derive the expression for $P_4(0)$ and $P_8(0)$.

b) (i) State the Legendre's associated differential equation.

(ii) Write the Legendre associated function in terms of ordinary Legendre functions.

(iii) Prove that Laguerre polynomials satisfy the recursion formula

$$L_{n+1}(x) = (2n+1-x)\, L_n(x) - n^2 L_{n-1}(x) .$$

c) Verify that Legendre associated functions of first kind are orthogonal by evaluating

$$\int_{-1}^{1} P_2^2(x) P_3^2(x)\, dx ,$$

given that

$P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = \frac{1}{2}(3x^2 - 1)$ and $P_3(x) = \frac{1}{2}(5x^3 - 3x)$.

4. a) (i) State Laguerre's differential equation

(ii) Write the expression of Laguerre polynomial as a solution to Laguerre's differential equation.

(iii) Prove the orthogonality of Laguerre polynomials $L_n(x)$ and $L_m(x)$, when $n \neq m$.

b) (i) State the Dirichlet conditions.

(ii) Show that the coefficients for the Legendre series expansion are given by

$$A_k = \frac{2k+1}{2} \int_{-1}^{1} f(x)\, P_k(x)\, dx$$

(iii) If

$$f(x) = \begin{cases} 1 & \text{if} \quad x \in (-1, 0) , \\ 0 & \text{if} \quad x \in [0, 1) . \end{cases}$$

$P_0(x) = -1$, $P_1(x) = -x$, write down the first four terms of the Legendre series expansion of $f(x)$.

c) Evaluate the following

(i) $\displaystyle\int x^5\, J_4(x)\, J_2(x)\, dx$

(ii) $\displaystyle\int x^3\, J_{-4}(x)\, J_6(x)\, dx$

5. a) (i) Let $\Gamma(z)$ be the integral representation of the Gamma function. Prove that it can be expressed as

$$\Gamma(z) = \lim_{n \to \infty} \frac{n!\, n^z}{z(z+1)(z+2)\ldots(z+n)} .$$

(ii) Prove that the Legendre polynomial can be written as

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} \left(x^2 - 1\right)^n .$$

b) (i) Prove that the Legendre functions are orthogonal in the interval $(-1, 1)$.

(ii) Show that

$$\int_{-1}^{1} P_n^2(x)\, dx = \frac{2}{2n+1} .$$

c) Show the following

$$\int_{-\infty}^{\infty} e^{-x^2} H_n^2(x)\, dx = n! 2^n \sqrt{\pi}$$

END!

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

Department of Mathematics & Statistics

FIRST SEMESTER FINAL EXAMINATIONS

MAT5311—LEBESGUE MEASURE AND LEBESGUE INTEGRATION

November, 12, 2012

Time allowed : THREE(3) HOURS

Instructions : There are six(6) questions. Answer **ANY FIVE (5)** questions. All questions carry equal marks. Show all your working to earn full marks.

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1. (a)(i) Define an algebra on a non empty set X.

   (ii) Define a σ-algebra on a non empty set X.

   (iii) Let $\mathcal{A} = \{A \subseteq \mathbb{R} : \text{ Either } A \text{ or } A^c \text{ is countable}\}$. Prove that $\mathcal{A}$ is a σ-algebra.

   (b) Let X be a set. Prove that the intersection of an arbitrary non empty collection of σ-algebras on X is a σ-algebra on X.

   (c)(i) Let $\mathcal{A}$ be a σ-algebra of subsets of X and let $Y \subseteq X$. Show that

$$\mathcal{A} \cap Y = \{E \cap Y : E \in \mathcal{A}\}$$

   is a σ-algebra of subsets of Y.

   (ii) Let X be a set, and let $\mathcal{A}$ be an algebra on X. Prove that $\mathcal{A}$ is a σ-algebra if $\mathcal{A}$ is closed under the formation of unions of increasing sequences of sets.

2. (a)(i) Define the outer measure μ^* of a subset E of $\mathbb{R}$.

   (ii) Define the Lebesgue measure μ of a subset E of $\mathbb{R}$ in relation to the outer measure.

   (iii) If A is such that $\mu^*(A) < \infty$ and there is a measurable subset $B \subseteq A$ with $\mu(B) = \mu^*(A)$, show that A is measurable.

(b)(i) Let $f: E \to \mathbb{R}_\infty$ be a function, where E is a measurable subset of $\mathbb{R}$. When is f said to be a measurable function?

(ii) If f and g are measurable functions on E and their sums and squares are also measurable, prove that fg is also measurable.

(iii) Show that if f is measurable, then the set $\{x : a < f(x) < b\}$ is measurable for each pair of real numbers a and b.

(c) Prove that μ^* is countably additive on the collection of all measurable subsets of $\mathbb{R}$.

3. (a) Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of measurable functions with $f_n : E \to \mathbb{R}_\infty$ for each n. Prove that:

(i) $p = \inf_n f_n$ is measurable.

(ii) $k = \sup_n f_n$ is measurable.

(b) Let E be a measurable subset of $\mathbb{R}$ with $\mu(E) < \infty$, and let f and $\{f_n\}_{n=1}^{\infty}$ be measurable and finite almost everywhere on E. Prove that for each pair of positive real numbers δ and ϵ, there exists a measurable set $A \subseteq E$ and an integer n_0 such that $\mu(E - A) < \epsilon$ and $|f(x) - f_n(x)| < \delta$ for all $x \in A$ and $n \geq n_0$.

(c) State and prove Egorov's theorem.

4. (a) Let

$$f_n(x) = \begin{cases} -n^2, & x \in (0, \frac{1}{n}) \\ 0, & \text{otherwise.} \end{cases}$$

(i) Show that

$$\lim_{n\to\infty} \int_{\mathbb{R}} f_n(x)dx \neq \int_{\mathbb{R}} \lim_{n\to\infty} f_n(x)dx.$$

(ii) Explain why this does not contradict Fatou's lemma.

(b) State and prove the Dominated Convergence theorem.

(c) For $n \geq 1$, let

$$f_n(x) = \begin{cases} 2n^2, & \frac{1}{2n} \leq x \leq \frac{1}{n} \\ 0, & x \in (0, \frac{1}{2n}) \cup (\frac{1}{n}, 1). \end{cases}$$

(i) Verify that the Dominated Convergence theorem does not hold for the sequence $\{f_n\}$.

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

Department of Mathematics & Statistics

MID TERM FINAL EXAMINATIONS

MAT5311—LEBESGUE MEASURE AND LEBESGUE INTEGRATION

March, 14, 2014

Time allowed : THREE(3) HOURS

Instructions : There are six(6) questions. Answer **ANY FIVE (5)** questions. All questions carry equal marks. Show all your working to earn full marks.

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1. (a)(i) Define an algebra on a non empty set X.

 (ii) Define a σ-algebra on a non empty set X.

 (iii) Let $\mathcal{A} = \{A \subseteq \mathbb{R} : \text{ Either } A \text{ or } A^c \text{ is countable}\}$. Prove that $\mathcal{A}$ is a σ-algebra.

 (b) Let X be a set. Prove that the intersection of an arbitrary non empty collection of σ-algebras on X is a σ-algebra on X.

 (c)(i) Let $\mathcal{A}$ be a σ-algebra of subsets of X and let $Y \subseteq X$. Show that

$$\mathcal{A} \cap Y = \{E \cap Y : \ E \in \mathcal{A}\}$$

 is a σ-algebra of subsets of Y.

 (ii) Let X be a set, and let $\mathcal{A}$ be an algebra on X. Prove that $\mathcal{A}$ is a σ-algebra if $\mathcal{A}$ is closed under the formation of unions of increasing sequences of sets.

2. (a)(i) Define the outer measure μ^* of a subset E of $\mathbb{R}$.

 (ii) Define the Lebesgue measure μ of a subset E of $\mathbb{R}$ in relation to the outer measure.

 (iii) If A is such that $\mu^*(A) < \infty$ and there is a measurable subset $B \subseteq A$ with $\mu(B) = \mu^*(A)$, show that A is measurable.

(b)(i) Let $f: E \to \mathbb{R}_\infty$ be a function, where E is a measurable subset of $\mathbb{R}$. When is f said to be a measurable function?

(ii) If f and g are measurable functions on E and their sums and squares are also measurable, prove that fg is also measurable.

(iii) Show that if f is measurable, then the set $\{x : a < f(x) < b\}$ is measurable for each pair of real numbers a and b.

(c) Prove that μ^* is countably additive on the collection of all measurable subsets of $\mathbb{R}$.

3. (a) Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of measurable functions with $f_n : E \to \mathbb{R}_\infty$ for each n. Prove that:

(i) $p = \inf_n f_n$ is measurable.

(ii) $k = \sup_n f_n$ is measurable.

(b) Let $\mathcal{A}$ be a σ-algebra and $f : A \to \mathbb{R}$ where $A \in \mathcal{A}$.
Define $\mathcal{F} = \{B \in \mathbb{R} : f^{-1}(B) \in \mathcal{A}\}$. Prove that $\mathcal{F}$ is a σ-algebra.

(c) State and prove Fatou's lemma..

4. (a) Let

$$f_n(x) = \begin{cases} -n^2, & x \in (0, \frac{1}{n}) \\ 0, & \text{otherwise.} \end{cases}$$

(i) Show that

$$\lim_{n\to\infty} \int_{\mathbb{R}} f_n(x)dx \neq \int_{\mathbb{R}} \lim_{n\to\infty} f_n(x)dx.$$

(ii) Explain why this does not contradict Fatou's lemma.

(b) State and prove the Dominated Convergence theorem.

(c) For $n \geq 1$, let

$$f_n(x) = \begin{cases} 2n^2, & \frac{1}{2n} \leq x \leq \frac{1}{n} \\ 0, & x \in (0, \frac{1}{2n}) \cup (\frac{1}{n}, 1). \end{cases}$$

(i) Verify that the Dominated Convergence theorem does not hold for the sequence $\{f_n\}$.

(ii) Explain why the Dominated Convergence theorem can not work.

5. (a) Let

$$f(x) = \frac{1}{\sqrt[3]{x}(1+\ln x)}.$$

Show that $f \in L^3[1, \infty)$.

(b) If $1 \leq p < q < \infty$ and $\mu(E) < \infty$, prove that $L^q(E) \subset L^p(E)$.

(c) If $\mu(E) < \infty$, f is measurable and $f \in L^3(E)$, prove that f is integrable by directly applying Holder's inequality.

6. (a) If $0 < p < q < r \leq \infty$, prove that $L^p \cap L^r \subset L^q$ and $\|f\|_q \leq \|f\|_p^{\lambda} \|f\|_r^{1-\lambda}$, where $\lambda \in (0,1)$ is defined by

$$\frac{1}{q} = \frac{\lambda}{p} + \frac{1-\lambda}{r}.$$

(b) State and prove Holder's inequality. You may use the fact that if $p > 0$ and a and b are any nonnegative reals, $ab \leq \frac{a^p}{p} + \frac{b^q}{q}$.

(c) Use Holder's inequality to show that if $\mu(E) = K$ for some constant K, then if $1 < p < \infty$ and $f \in L^p$, we have that $f \in L^1$ and $\|f\|_1 \leq \|f\|_p K^{\frac{1}{q}}$ where p and q are conjugate indices.

END.

(ii) Explain why the Dominated Convergence theorem can not work.

5. (a) Let

$$f(x) = \frac{1}{\sqrt[3]{x}(1 + \ln x)}.$$

Show that $f \in L^3[1, \infty)$.

(b) If $1 \leq p < q < \infty$ and $\mu(E) < \infty$, prove that $L^q(E) \subset L^p(E)$.

(c) If $\mu(E) < \infty$, f is measurable and $f \in L^3(E)$, prove that f is integrable by directly applying Holder's inequality.

6. (a) If $0 < p < q < r \leq \infty$, prove that $L^p \cap L^r \subset L^q$ and $\|f\|_q \leq \|f\|_p^{\lambda} \|f\|_r^{1-\lambda}$, where $\lambda \in (0, 1)$ is defined by

$$\frac{1}{q} = \frac{\lambda}{p} + \frac{1-\lambda}{r}.$$

(b) State and prove Holder's inequality. You may use the fact that if $p > 0$ and a and b are any nonnegative reals, $ab \leq \frac{a^p}{p} + \frac{b^q}{q}$.

(c) Use Holder's inequality to show that if $\mu(E) = K$ for some constant K, then if $1 < p < \infty$ and $f \in L^p$, we have that $f \in L^1$ and $\|f\|_1 \leq \|f\|_p K^{\frac{1}{q}}$ where p and q are conjugate indices.

END.

The University of Zambia
School of Natural Sciences
Department of Mathematics and Statistics
Supplementary Examination
MAT 5342 Operator Theory

Duration: Three (3) hours

Instructions:

(i) There are five questions in this paper

(ii) Answer any four

(iii) All questions carry equal marks

---

1. Let $T : V \to W$ be a linear operator between vector spaces. Prove the following:

   (a) i. The range $R(T)$ is a subspace of W.

   ii. If $\dim(V) < \infty$, say $\dim(V) = n$, then $\dim(R(T)) \leq n$.

   iii. The null space $N(T)$ is a subspace of V.

   (b) i. If a normed space X is finite dimensional, prove that every linear operator on X is bounded.

   ii. If $T : X \to Y$ is a linear operator between normed spaces, prove that T is bounded if and only if it is continuous.

   iii. Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be the linear operator defined by $(x_1, x_2, ..., x_n) \mapsto (-x_1, x_2, ..., x_n)$. Verify that $\dim(\mathbb{R}^n) = \dim(\mathcal{N}(T)) + \dim(R(T))$.

2. (a) Let $T : V \to W$ be a linear operator between vector spaces. Prove the following:

   i. T is injective if and only if $N(T) = \{0\}$.

   ii. If T^{-1} exists, then it is a linear operator.

   iii. If T is bijective and $S : W \to Z$ is another bijective linear operator, then $(ST)^{-1} : Z \to V$ exists and $(ST)^{-1} = T^{-1}S^{-1}$.

   (b) i. Suppose that X is an n dimensional vector space and $E = \{e_1, e_2, ..., e_n\}$ is a basis for X, prove that $\dim(X^*) = n$.

   ii. If X is a normed space and Y is a Banach space, prove that the vector space $B(X, Y)$ of all bounded linear operators from X into Y is a Banach space.

iii. Show that the dual space of ℓ^1 is ℓ^∞.

3. (a) i. Let $T : \mathbb{C}^2 \to \mathbb{C}^2$ be the linear operator defined by $(a, b) \mapsto (a - bi, a + 2bi)$. Find T^*.

   ii. Suppose that X and Y are normed spaces and $T : X \to Y$ is a bounded linear operator. Prove that $N(T)$ is a closed subspace of X.

   iii. Show that a bounded linear operator $T : H \to H$ on a complex Hilbert space H is normal if and only if $||T^*x|| = ||Tx||$ for all $x \in H$. Hence show that for a normal linear operator, $||T^2|| = ||T||^2$.

   (b) i. Prove that every bounded linear functional f on a Hilbert space H can be represented as $f(x) = \langle x, z \rangle$, for any $x \in H$ and where z depends on f.

   ii. Prove that if H is a complex Hilbert space, then a bounded linear operator T on H is unitary if and only if it is isometric and surjective.

   iii. Show that for any bounded linear operator T on a Hilbert space, the operators $T_1 = \frac{1}{2}(T + T^*)$ and $T_2 = \frac{1}{2i}(T - T^*)$ are self-adjoint.

4. (a) i. Show that the linear operator $T : \ell^2 \to \ell^2$ defined by $(x_1, x_2, ...) \mapsto (x_1, 0, x_3, x_4, ...)$ is bounded, self-adjoint and positive. Find the square root of T.

   ii. Let $T : H \to H$ be a bounded positive self-adjoint linear operator on a complex Hilbert space. Using the positive square root of T, show that for all $x, y \in H$, $|\langle Tx, y \rangle| \leq \langle Tx, x \rangle^{\frac{1}{2}} \langle Ty, y \rangle^{\frac{1}{2}}$.

   iii. If P_1 and P_2 are projections on a Hilbert space H, show that $P = P_1 - P_2$ is a projection on H if and only if $P_2 \leq P_1$.

   (b) i. Prove that a bounded linear operator P on a Hilbert space is a projection if and only if P is self-adjoint and $P^2 = P$.

   ii. Let X and Y be normed spaces and $T : X \to Y$ a linear operator. Prove that T is compact if and only if it maps every bounded sequence (x_n) in X onto a sequence (Tx_n) in Y which has a convergent subsequence.

   iii. Let H be a Hilbert space, P_1, P_2 projections on H, $Y_1 = P_1(H)$ and $Y_2 = P_2(H)$. Prove that $P = P_1P_2$ is a projection on H if and only if $P_1P_2 = P_2P_1$, and that P projects H onto $Y_1 \cap Y_2$.

5. (a) i. Show that the operator $T : \ell^2 \to \ell^2$ defined by $(x_1, x_2, ...) \mapsto (0, \frac{x_1}{1}, \frac{x_2}{2}, ...)$ has no eigenvalues. Hence $\sigma(T) = \sigma_r(T) = \{0\}$.

   ii. If (P_n) is a sequence of projections defined on a Hilbert space H and $P_n \to P$, show that P is a projection on H.

iii. Let $T : X \to X$ be a compact linear operator and $S : X \to X$ a bounded linear operator on a normed space X. Show that TS and ST are compact.

(b) Let X be a normed space and $T : X \to X$ a compact linear operator. Prove the following:

i. If $0 \neq \lambda \in \mathbb{C}$, then $\dim(\mathcal{N}(T - \lambda I)) < \infty$.

ii. If $0 \neq \lambda \in \mathbb{C}$, then $\dim(T_\lambda{}^n) < \infty$ and $\{0\} = \mathcal{N}(T_\lambda{}^0) \subset \mathcal{N}(T_\lambda) \subset \mathcal{N}(T_\lambda{}^2) \subset ...$, where $T_\lambda = T - \lambda I$.

iii. If $\dim(X) = \infty$, then $0 \in \sigma(T)$.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS

2013 ACADEMIC YEAR FINAL EXAMINATIONS

MAT5611 : STATISTICAL INFERENCE

TIME ALLOWED: Three (3) Hours

INSTRUCTIONS:
1. Answer any **Four** (4) Questions
2. Show All Essential Working

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1. (a) Define the following:
   (i) a sufficient statistic T.
   (ii) a complete statistic T.

   (b) Let $X_1, X_2, \dots, X_n$ be a random sample from a distribution with p.f. $f_\theta(x) = \theta^x (1-\theta)^{1-x},\ x = 0, 1$.
   (i) Find a sufficient statistic for θ.
   (ii) Show that the statistic in (i) is a minimal sufficient statistic.
   (iii) Find a UMVUE of $\tau(\theta) = \theta(1-\theta)$.

   (c) Let $X_1, X_2, \dots, X_n$ be a random sample from $f_\theta(x) = \frac{e^{-\theta}\theta^x}{x!}$, $x = 0, 1, 2, \dots$
   (i) Find $E[(-1)^{X_1}]$.
   (ii) Find $P(X_1 = x | T = t)$ where $T = \sum_{i=1}^{n} X_i$.
   (iii) Hence or otherwise find the UMVUE of $\tau(\theta) = e^{-2\theta}$.

2. (a) (i) State Basu's theorem.
   (ii) Prove Basu's theorem.

   (b) Let $X_1, X_2, \dots, X_n$ be a random sample from $f_\theta(x) = e^{-(x-\theta)}$, $x > \theta$.
   (i) Show that $f_\theta(x)$ is a location invariant family distribution.
   (ii) Show that $T = X_{(1)}$ is a complete sufficient statistic for θ.
   (iii) Show that $X_{(1)}$ and $X - X_{(1)}$ are independent.

3. (a) Define the following:
   (i) exponential family distribution.
   (ii) location equivariant estimator.

(b) Let $X_1, X_2, \dots, X_n$ be a random sample from

$$f_\theta(x) = \left(\frac{\lambda}{2\pi}\right)^{\frac{1}{2}} x^{-\frac{3}{2}} e^{-\frac{\lambda}{2\mu^2 x}(x-\mu)^2}, \quad x > 0,\ \lambda > 0,\ \mu > 0$$

where $\theta = (\lambda, \mu)$.
   (i) Show that $f_\theta(x)$ is a regular exponential family distribution.
   (ii) Find a complete sufficient statistic for $\theta = (\lambda, \mu)$.
   (iii) Find the maximum likelihood estimator for $\theta = (\lambda, \mu)$.

4. (a) (i) State the information inequality.
   (ii) Prove the information inequality.

(b) Let $X_1, X_2, \dots, X_n$ and $Y_1, Y_2, \dots, Y_m$ be independent random samples from independent normal distributions $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ respectively.
   (i) Show that the generalized likelihood ratio test for testing $H_0: \mu_1 = \mu_2, \sigma_1^2 = \sigma_2^2$ against $H_1: \mu_1 \neq \mu_2, \sigma_1^2 \neq \sigma_2^2$ is given by

$$\Lambda(x, y) = \frac{\left(\frac{1}{n+m}\left[\sum_{i=1}^{n}(x_i - \hat{\mu})^2 + \sum_{j=1}^{m}(y_j - \hat{\mu})^2\right]\right)^{\frac{n+m}{2}}}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^{\frac{n}{2}}\left(\frac{1}{m}\sum_{j=1}^{m}(y_j - \bar{y})^2\right)^{\frac{m}{2}}}$$

where $\hat{\mu} = \frac{n\bar{x} + m\bar{y}}{n+m}$.
   (ii) Find the asymptotic distribution of $\Lambda(x, y)$ under H_0.

5. (a) Define the following:
   (i) locally most powerful test.
   (ii) an unbiased estimating function.

(b) Let $X_1, X_2, \dots, X_n$ be a random sample from the distribution with p.d.f. $f_\theta(x) = \frac{e^{-\frac{x}{\theta}}}{\theta}, x > 0$.
   (i) Find a size α locally most powerful test for testing $H_0: \theta = \theta_0$ against $H_1: \theta > \theta_0$.
   (ii) Find the wald test for testing $H_0: \theta = \theta_0$ against $H_1: \theta \neq \theta_0$.
   (i) Find an approximate 95% confidence interval for θ.

(c) Let $X_1, X_2, \dots, X_n$ be a random sample from $f_\theta(x) = \frac{1}{\theta - 1}, \ 1 < x < \theta$.
   (i) Find the maximum likelihood estimator of θ.
   (ii) Show that the M.L. estimator is a consistent estimator of θ.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS

2013 ACADEMIC YEAR
END OF YEAR EXAMINATIONS

MAT5632 : DESIGN AND ANALYSIS OF EXPERIMENTS

TIME ALLOWED: Three (3) Hours

INSTRUCTIONS:
1. Answer any **Four** (4) Questions
2. Show All Essential Working

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1. (a) (i) State and define two principles of experimental design.
   (ii) State two reasons why each of the two principles stated in (i) is important.

   (b) A manufacturer of television sets was interested in the effect of five coating types on tube conductivity for colour picture tubes. The following conductivity data were collected:

						Total
Coating type	1	80	83	83	85	331
	2	75	75	79	79	308
	3	67	72	74	74	287
	4	62	62	67	69	260
	5	60	61	64	66	251
						1437

(Additionally $\sum_{i=1}^{5}\sum_{j=1}^{4} Y_{ij}^2 = 104471$)

   (i) Write down a model for the above study. Explain all the terms in your model and state all the assumptions.
   (ii) Show that for any contrast $C = \sum_{i=1}^{5} c_i \tau_i$ the estimate $\widehat{C} = \sum_{i=1}^{5} c_i \hat{\tau}_i$ is the same for the fixed effects models and the cell means model.
   (iii) Construct an ANOVA table for the study.
   (iv) Test for the effect of coating type on conductivity. Use $\alpha = 0.05$.
   (v) Find the sum of squares of the contrast that compares coating types 1 and 2 to the other three coating types.
   (vi) Use (v) to test for the difference between coating types 1 and 2 and the other three coating types.
   (vii) Construct a 95% confidence interval for a contrast that compares coating types 1, 2 and 3 to coating types 3 and 4.

2. (a) Given a one factor fixed effects model

$$Y_{ij} = \mu + \tau_i + \varepsilon_{ij} \quad , \quad i = 1, 2, ..., t \quad ; \quad j = 1, 2, ..., r$$

where $\varepsilon_{ij} \sim N(0, \sigma^2)$,

Prove that

(i) $E(MSTrt) = \frac{r}{t-1}\sum_{i=1}^{t}(\tau_i - \bar{\tau}.)^2 + \sigma^2$, where MSTrt is the mean treatment sum of squares.

(ii) $E(MSE) = \sigma^2$, where MSE is the mean square error.

(b) The effects of four factors on some response Y are studied. Each factor is varied at two levels in a 2^4 factorial experiment and the following data recorded:

Treatment Combination	Response (Y)
(1)	8
a	9
b	9
ab	11
c	10
ac	16
bc	13
abc	22
d	7
ad	7
bd	5
abd	10
cd	7
acd	14
bcd	10
abcd	13
Total	171

(Additionally $\sum_i \sum_j \sum_k \sum_l Y_{ijkl}^2 = 2093$)

(i) Estimate the effects AB, BC and ABC.

(ii) Estimates of all the other effects are:

A = 4.125	AC = 2.125	ABD = -0.375
B = 1.875	AD = -0.375	ACD = -0.875
C = 4.875	BD = -1.125	BCD = -0.625
D = -3.125	CD = -1.125	ABCD = -1.375

Plot all the effects on a normal probability paper. What effects appear significant?

(iii) Prepare an ANOVA table that only includes the effects identified as significant in (ii). Test for the significance of the effects and comment on your findings. Use $\alpha = 0.05$.

3. (a) An experiment was conducted to study the effect of three factors A, B and C, each at two levels, on the response Y. The following treatment layout was proposed as it was difficult to get homogeneous blocks of size 8:

Run	Block	A	B	C	Y
1	1	–	–	–	5
2	1	+	+	–	18
3	1	–	–	+	8
4	1	+	+	+	18
5	2	+	–	–	21
6	2	–	+	–	13
7	2	+	–	+	13
8	2	–	+	+	7
9	3	–	–	–	6
10	3	+	–	–	21
11	3	–	+	+	6
12	3	+	+	+	18
13	4	–	+	–	13
14	4	+	+	–	17
15	4	–	–	+	8
16	4	+	–	+	17
Total					209

(i) Identify the treatment effect(s) confounded with blocks in this design. Show your steps clearly.

(ii) Write down a layout of the blocks under each replication clearly showing what observations appear in each block and the effect(s) confounded.

(iii) Estimate the effects A, B, AB and BC.

(iv) Copy and complete the following ANOVA table.

Source	SS	df
Blocks		
Replications		
Blocks within replications		
A		
B		
Error		
Total	482.9375	

(b) Consider a 2^{5-2} factorial design with generators $I = ABC$ and $I = BDE$.
   (i) Write down a complete alias structure of the design.
   (ii) Find the resolution of the design.
   (iii) Obtain a quarter fraction of the design.

4. (a) Define the following
   (i) orthogonal contrasts.
   (ii) principal block.

(b) A study was conducted to determine the effect of inoculation on root yield of sugar beets planted at different in-row spacings. One of two equal-sized plots at each of the three locations used in the study was randomly assigned to receive an inoculation. Furthermore, each of the plots was subdivided into four equal subplots where sugar beets were planted at different in-row spacings between plants (4, 6, 12 and 18 cm). Assume that inoculation is a fixed factor, and that location and plant spacing are random factors. The following results were obtained:

		Plant spacing (cm)					
Location	Inoculation	4	6	12	18	Sub-total	Total
1	Yes	17	17	16	13	63	
	No	20	20	22	20	82	145
2	Yes	16	17	15	12	60	
	No	18	20	21	19	78	138
3	Yes	17	19	17	13	66	
	No	18	21	19	19	77	143
Total		106	114	110	96		426

(i) What design was used in the experiment? Explain.
(ii) Write down a model for the experiment. Explain all the terms in the model and state all the assumptions.
(iii) Find the expected mean squares of the factors in the model.
(iv) Copy and complete the following ANOVA table.

Source	SS	df	MS	F*
location				
inoculation				
location-inoculation	4.75			
spacing				
location-spacing	4.417			
inoculation-spacing	20			
Error	2.25			
Total	160.5			

(v) Test for the effects of location and spacing main effects. Use $\alpha = 0.05$.

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR FINAL EXAMINATIONS

MAT 5642: STATISTICAL METHODS IN EPIDEMIOLOGY

INSTRUCTIONS:
1. Answer any FIVE (5) questions.
2. Calculators are allowed.
3. You may use statistical tables or formulae provided if necessary.
4. Show all your work to earn full marks.
5. A table of formulae has be provided.

TIME: THREE (3) Hours

Q1 A simple blood test could offer many women an early warning of breast cancer even if they do not inherit genes linked to the disease. Scientists have identified a molecular "switch" in blood samples that increases a woman's chances of having breast cancer.

Lead researcher Professor Martin Widschwendter, from University College London, said: "We identified an epigenetic signature in women with a mutated BRCA1 gene that was linked to increased cancer risk and lower survival rates. "Surprisingly, we found the same signature in large cohorts of women without the BRCA1 mutation and it was able to predict breast cancer risk several years before diagnosis. "The data is encouraging since it shows the potential of a blood-based epigenetic test to identify breast cancer risk in women without known predisposing genetic mutations."

The scientists analysed blood sample DNA from 119 postmenopausal women who went on to develop breast cancer over a period of up to 12 years, and 122 who remained cancer-free. Their results appear in the online journal Genome Medicine.

(a) (i) List any four major applications of epidemiology.

(ii) Under what application does the researchers' work described in the article fall?

(b) (i) Discuss briefly, why epidemiologists view disease from both a biological and a sociological perspectives.

(ii) Under what perspective are the researchers in the article viewing disease? Support your argument based on the article.

(c) The last paragraph of the article gives us some figures regarding risk of developing breast cancer.

(i) Calculate an appropriate estimate of the risk of developing breast cancer, incorporating all the bits of information given in the paragraph.

(ii) Interpret the result in (i).

(iii) identify the following; the study design, exposure and criteria for selecting the subjects in this article.

Q2 The Bayes theorem allows one to change direction of prediction so that, for example, a probability of disease given exposure can be transformed into a probability of exposure given disease. Let events and quantities be defined as below:

D+ = an individual has disease and D- an individual has no disease.
E+ = an individual is exposed and E- an individual is free of exposure.

$$\psi = \frac{\Pr(E+|D+)/\Pr(E-|D+)}{\Pr(E+|D-)/\Pr(E-|D-)}$$

$$\mathrm{CIR} = \frac{\Pr(D+|E+)}{\Pr(D+|E-)}$$

(a) Show that $\mathrm{CIR} = \psi \times \left[\frac{\Pr(E+|D-)}{\Pr(E-|D-)}\right] \times \left[\frac{\Pr(E-|D+)\times\Pr(D+)+\Pr(E-|D-)\times\Pr(D-)}{\Pr(E+|D+)\times\Pr(D+)+\Pr(E+|D-)\times\Pr(D-)}\right]$

(b) Show that the expression in (a) is equivalent to:

$$\mathrm{CIR} = \psi \times \left[\frac{\Pr(E+|D-)}{\Pr(E-|D-)}\right] \times \left[\frac{1-\Pr(E+)}{\Pr(E+)}\right]$$

(c) Suppose that true population probabilities showing the distribution of disease and an associated exposure are as shown below.

	Disease status	
Exposure status	D+	D-
E+	0.23 = Pr(D+ and E+)	0.15 = Pr(D- and E+)
E-	0.17 = Pr(D+ and E-)	0.45 = Pr(D- and E-)

(i) Obtain a 2×2 table of the true case-control probabilities for the population.

(ii) Using (i) obtain the value of ψ.

(iii) Assuming Pr(E+) = 0.3 obtain the value of CIR.

Q3 (a) (i) List any four stages of research in which bias may occur.

(ii) Briefly describe, using a specific example, what is meant by selection bias.

(b) Suppose you have the following quantities defined as:

θ = the target parameter, the true effect measure in the population of interest

$\hat{\theta}$ = the estimate of θ computed from a study population

θ_0 = the parameter actually being estimated such that $E(\hat{\theta}) = \theta_0$

(i) Show that the mean square of $\hat{\theta}$, defined by $E(\hat{\theta}-\theta)^2$, may be expressed as:

$$MSE(\hat{\theta}) = E(\hat{\theta}-\theta)^2 = Var(\hat{\theta}) + [Bias(\hat{\theta})]^2,$$

where $Var(\hat{\theta}) = E(\hat{\theta}-\theta_0)^2$ and $Bias(\hat{\theta}) = \theta_0 - \theta$.

(ii) Suppose that θ represents the Cumulative Incidence Difference, i.e., θ = CID for the target population, $\theta_0 = CID_0$ for the study population and $\hat{\theta}$ = an estimate of θ_0. Assume that the value of θ in the target population is known but not θ_0 and data from the study population yields the table below.

Exposure Status	Disease Status D+	Disease Status D-	
E+	a	b	m_1
E-	c	d	m_2

Obtain an expression for the estimate of $MSE(\hat{\theta})$ assuming large samples.

(c) The association of gallstone disease with age is to be conducted in women aged 90 and above versus those aged 50 to 89. The cumulative incidence difference CID ($CI_{\geq 90} - CI_{50-89}$) is of interest. Unfortunately, women available are aged 80 and above and the results from the study are shown below. Assume large sample.

Age category	Gallstones Disease Yes	Gallstones Disease No
≥ 90	20	5
80 - 89	22	14

(i) State why $\widehat{CID} \pm Z_{1-\alpha/2}\sqrt{MSE(\hat{\theta})}$ is an appropriate $100(1-\alpha)\%$ confidence interval for $CI_{\geq 90} - CI_{50-89}$

(ii) Provide a 95% confidence interval for CID if bias is thought to be around 0.01

Q4 Meloni et al, studied bacterial infection in people with glucose-6-phosphate dehydrogenase (G6PD) deficiency on the island of Sardinia, where bacterial infections are endemic. From 1976 to 1988, there were 476 children with bacterial infections admitted to the only pediatric hospital in the district. The bacterial diseases that were examined are Mediterranean spotted fever (MSF), Typhoid (T) and Meningitis (M). The 2×2×2 classification of gender, G6PD and disease is shown below.

Gender	G6PD Deficiency	Disease MSF	Disease T	Disease M
Boys	Yes	10	5	2
	No	132	63	31
Girls	Yes	15	9	4
	No	101	74	30

Let D+ indicate the disease of interest and collapse the other two columns to form D-, e.g., for boys MSF; (E+, D+) = 10 and (E+, D-) = 5 + 2 = 7, where E+ = G6PD deficiency.

(a) (i) Obtain the cumulative incidence ratio (CIR) for each of the three diseases among Boys to 3 decimal places.

(ii) Obtain the cumulative incidence ratio for each of the three diseases among Girls to 3 decimal places.

(iii) Which disease has the greatest difference in CIR between boys and girls?

Let CIR_B be the cumulative incidence ratio among boys for the disease identified in (a) (iii). Similarly, let CIR_G be the cumulative incidence ratio among girls for the disease identified in (a) (iii).

(b) Obtain an estimate of $Var[Ln(CIR_B) - Ln(CIR_G)]$

(c) (i) Obtain a 95% confidence interval for the true $Ln(CIR_B) - Ln(CIR_G)$ assuming large samples for both groups.

(ii) Based on (ii) what is the conclusion on the hypotheses below?:

H_0: $CIR_B = CIR_G$ versus H_1: $CIR_B \neq CIR_G$ at a 5% significance level.

Q5 Suppose that in some community there are 2000 children, some are orphans (both parents deceased) and others are not. Some of the children are vulnerable in the sense that there is really no help available for daily sustenance because either they are orphans or their parents are incapable of providing it. A nongovernmental organization is contracted by the Government to conduct a baseline study to determine the true status of the children.

Define:

D+ = Child is vulnerable.

D- = Child is not vulnerable.

E+ = Child is an orphan.

E- = Child is not orphan.

Suppose that the true situation in the community is as follows:

	Vulnerability status		
Orphanage status	D+	D-	TOTAL
E+	60	740	800
E-	40	1160	1200
TOTAL	100	1900	2000

(a) Determine the relative risk of vulnerability between the two groups of children using odds ratio.

(b) Suppose that at the end of baseline study there is no misclassification of orphanage by the NGO but that there was misclassification of vulnerability with the following probabilities of misclassification:

$\phi_{D+|E+}$ = Pr(D+|D+E+) = 0.8 $\phi_{D+|E-}$ = Pr(D+|D+E-) = 0.8

$\psi_{D-|E+}$ = Pr(D-|D-E+) = 0.9 $\psi_{D-|E-}$ = Pr(D-|D-E-) = 0.9

(i) Construct an actual misclassified table of the data.`

(ii) Calculate the estimate of the relative risk similar to that in (a) using (i).

(iii) Determine the relative bias of the estimate in (ii).

(c) What are the practical implications of such a misclassification to the children involved.

Q6 (a) Below is an article that appeared on The Daily Telegraph website on 17/07/2014.

Asthma inhalers make children half a centimetre shorter

Children who use corticosteroids inhalers to control asthma are likely to be 0.5cm shorter than they should be, because the drug stunts growth, a major review has found

Children who use common asthma inhalers are likely to be shorter because the drugs stunt growth, scientists have found Photo: GETTY

(i) Describe an experiment you would conduct to either validate or invalidate the claim.

(ii) Suggest two reasons why such a finding is important.

(b) Define the following quantities in a community exposed to some factor and under threat of some acute disease:

$\widehat{CI}_1$ = the cumulative incidence rate in the exposed group

$\widehat{CI}_0$ = the cumulative incidence rate in the unexposed group

$\widehat{CI}$ = the cumulative incidence rate in the entire population

$\widehat{AR} = \left(\widehat{CI} - \widehat{CI}_0\right)/\widehat{CI}$, the attributable risk to the factor

$\widehat{CI} = \hat{p}\,\widehat{CI}_1 + \left(1-\hat{p}\right)\widehat{CI}_0$

$\hat{p}$ = the proportion of the total candidate population exposed to the risk factor.

(i) Show that $\widehat{AR} = \dfrac{\hat{p}(\widehat{CIR} - 1)}{\hat{p}(\widehat{CIR} - 1) + 1}$

(ii) What is the limit of $\widehat{AR}$ as $\widehat{CIR}$ increases without bounds.

(c) A retiree learning farming planted some tomato seedlings on two different types of soils, A and B. The seedlings sprouted but some died later. The farmer kept a record of days since the seedlings were planted until death, suppose the data are shown below:

	Type A soil	Type B soil	TOTAL
Death during season	25	36	61
Person-Time (days)	69	139	208

Test the hypothesis H_0: IDR = 1 versus H_A: IDR > 1 using the normal approximation to the binomial at 5% level of significance.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS & STATISTICS

2013 ACADEMIC YEAR
FINAL EXAMINATIONS

MAT 5662 – THEORY OF NON-PARAMETRIC STATISTICS

TIME ALLOWED: THREE (3) HOURS

INSTRUCTIONS: Attempt any four (4) questions. All questions carry equal marks.

Full credit will only be given when all the necessary work is shown.

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1. (a) Define the following:

   (i) Equal in distribution
   (ii) Signed rank Statistics
   (iii) Estimable parameter

   (b) Prove the following:

   (i) If W^* treats the n iid random variables $X_1, X_2, \ldots, X_n$ symmetrically with $E(W^*) = 0$ and V^* is a projection of W^* on $\mathcal{V}$, then $E(W^* - V^*)^2 \leq E(W^* - V)^2$ for any $V \in \mathcal{V}$.

   (ii) If Z is a continuous random variable with distribution that is symmetric about zero, then the random variables $|Z|$ and $\psi = \psi(Z)$ are stochastically independent, where

   $$\psi(x) = \begin{cases} 1 & if \quad x \geq 0 \\ 0 & if \quad x < 0 \end{cases}$$

   (iii) Convergence in quadratic mean implies convergence in probability.

   (c) (i) State Slutsky's theorem.

   (ii) Prove that if a sequence of random variables $\{V_n\}$ has asymptotic distribution with cdf $F(v)$ and $\{W_n\}$ is a sequence of random variables such that the sequence $\{W_n - V_n\}$ converges in probability to zero, then the limiting distribution of $\{W_n\}$ also has cdf $F(w)$.

2. (a) Define the following:

(i) Distribution-free Statistics
(ii) Non-parametric distribution-free statistics
(iii) U-statistic for Estimable parameter

(b) Prove the following

(i) Given two independent random samples $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ from $F(x)$ and $F(y-c)$ respectively, then the distribution of $W_y = \sum_{i=1}^{n} R_i$, the Wilcoxon rank statistic is symmetric about the value $\frac{n\,(n+m+1)}{2}$.

(ii) Given $Z_1, Z_2, \ldots, Z_n$ a random sample from a continuous distribution that is symmetric about zero, let Q be the number of positive $Z_{i's}$. When $Q = q$, let $S_1 < S_2 < \ldots S_q$ denote the ordered absolute ranks of those $Z_{i's}$ that are positive, then
$Prob\,\{Q = q, S_1 = s_1\,,\, S_2 = s_2, \ldots, S_q = s_q\} = \left(\frac{1}{2}\right)^n,$
for $q = 0, 1, 2, \ldots, n$.

(c) Construct the distribution of Wilcoxon signed rank statistics under H_0 for $n = 5$

3. (a) Define the following:

(i) Convergence in Probability
(ii) Convergence in Quadratic mean
(iii) Limiting distribution

(b) Prove the following where μ_r is the mean and $F_r(x)$ is the cdf of the r^{th} order statistics, $W = X_{(n)} - X_{(1)}$ and $P(x)$ is the cdf of the distribution of the original distribution, then

(i) $\mu_r = \int_0^\infty (1 - F_r\,(x) - F_r\,(-x))\, dx$

(ii) $E(W) = \int_{-\infty}^{\infty} \left(1 - \left(P_{(x)}\right)^n - \left(1 - P_{(x)}\right)^n\right) dx$

(c) Let $X_1, X_2, \ldots, X_n$ be a random sample from uniform on the interval $(-\theta, \theta)$, $\theta > 0$. Using the results in (b) above find

E(W)

4. (a) (i) State the one – sample U – statistics theorem

(ii) Prove the one – sample U – statistics theorem

(b) Given a random sample $X_1, X_2, X_3 \ldots, X_n$ from a continuous with pdf given by

$$P(x) = \beta\, x^{\beta-1}\, e^{-x^{\beta}},\ x > 0\,,\ \beta > 0$$

Find the following:

(i) pdf of $X_{(r)}$

(ii) pdf for the range $W = X_{(n)} - X_{(1)}$

(c) Given two independent random samples X_1, X_2, X_3 and Y_1, Y_2, Y_3, i.e. $m = 3$ and $n = 3$.

(i) Construct the distribution of Mann-Whitney/Wilcoxon test statistic under $H_0 : C = 0$

(ii) What would be the rejection region for testing $H_0 : C = 0$ vs $H_a : C > 0$ at 0.10 level of significance.

5. (a) (i) Define the projection of W onto some space $\mathcal{V}$

(ii) Derive the general expression for the variance of a U – statistics for an estimable parameter.

(b) Let $Z_1, Z_2, \ldots, Z_n$ be a random sample from a continuous distribution that is symmetric about zero. Let R^+ be the vector of absolute ranks of the Z_i's , $I = 1, 2, \ldots, n$ and $\psi_i = \psi(Z_i)$ where $\psi(x)$ is as given in 1 b (ii).

Prove the following

(i) The random variable $\psi_i , \psi_2 , \ldots, \psi_n , R^+$ are mutually independent

(ii) Each ψ_i is Bernoulli random variable with $P = \frac{1}{2}$.

(iii) R^+ is uniformly distributed over the set of all permutations of the set $\{1, 2, 3, \ldots, n\}$

(c) (i) Given a random sample $X_1, X_2, \ldots, X_n$ from a continuous distribution that is symmetric about θ. Prove that W^+ the Wilcoxon signed rank statistics, its distribution is symmetric about its mean when H_0: $\theta = \theta_0$ is true.

(ii) Let $U(X_1, X_2, \ldots, X_n)$ be a U – statistic with a symmetric kernel $h(X_1, X_2, \ldots, X_k)$. Prove that if $E(h^2(X_1, X_2, \ldots, X_k)) < \infty$, then $\lim_{n \to \infty} n\, var\{U(X_1, X_2, \ldots, X_n)\} = k^2\, \xi_1$.

End

The University of Zambia

Department of Mathematics & Statistics

2012 Academic Year Second Semester Final Examinations

MAT5662 - Theory of Non-Parametric Statistics

Time allowed : **Three (3) hours** Full marks : 100

Instructions:
- Attempt **any four (4)** questions. **All** questions carry **equal** marks.
- Full credit will only be given when all the necessary work is shown.
- Indicate your **computer number** on all answer booklets used.

This paper consists of 3 pages of questions.

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1. (a) Define the following:
   i. Distribution-free statistic.
   ii. Non-parametric distribution-free statistic.
   iii. Signed rank statistic.

   (b) Given a random sample $X_1, X_2, \ldots, X_n$ from a continuous distribution with pdf $p(x)$ and cdf $P(x)$, derive the following:
   i. pdf of $X_{(r)}$
   ii. joint pdf of $X_{(r)}$ and $X_{(s)}$ with $r < s$.
   iii. pdf for the range $W = X_{(n)} - X_{(1)}$.

   (c) Given a random sample $X_1, X_2, \ldots, X_n$ from

$$p(x) = \alpha x^{\alpha-1} e^{-x^{\alpha}}, \quad x > 0, \quad \alpha > 0.$$

   Using the results found in (b) above, find the following:
   i. pdf of $X_{(r)}$
   ii. joint pdf of $X_{(r)}$ and $X_{(s)}$ with $r < s$.
   iii. pdf for the range $W = X_{(n)} - X_{(1)}$.

2. (a) Let $X_1, X_2, \ldots, X_n$ be a random sample from an exponential distribution with mean 2. Find the following:

   i. mean of the r^{th} order statistic.

   ii. covariance of the r^{th} and s^{th} order statistic with $r < s$.

   iii. variance of the r^{th} order statistic.

   (b) Prove the following where μ_r is the mean of the r^{th} order statistic, $F_r(x)$ is the cdf of the r^{th} order statistic, $W = X_{(n)} - X_{(1)}$, and $P(x)$ is the cdf of the original distribution.

   i. $\mu_r = \int_{-\infty}^{\infty} (1 - F_r(x) - F_r(-x))\, dx$.

   ii. $E(W) = \int_{-\infty}^{\infty} (1 - (P(x))^n - (1 - P(x))^n)\, dx$.

   (c) i. Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous distribution whose pdf is symmetric about its mean μ. Show that the pdf for $X_{(r)}$ and $X_{(n-r-1)}$ are mirror images of each other, that is, $f_r(\mu + x) = f_{n-r-1}(\mu - x)$.

   ii. The midpoint range denoted by m for a random sample of size n is defined as $m = \frac{1}{2}(X_{(n)} - X_{(1)})$. If $X_1, X_2, \ldots, X_n$ is a random sample from a continuous distribution with cdf $P(x)$, show that the cdf of m is given by

$$F_m(y) = \int_{-\infty}^{y} (P(2y - x) - P(x))^{n-1}\, p(x)\, dx.$$

3. (a) Define the following:

   i. Estimate parameter.

   ii. U-statistic for estimate parameter.

   iii. Convergence in quadratic mean.

   (b) Prove the following:

   i. Let $W_y = \sum_{i=1}^{n} R_i$ be the wilcoxon rank statistic when $H_0 : c = 0$ is true given $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ are independent random samples from $F(x)$ and $F(y - c)$ respectively. The distribution of W_y is symmetric about $\mu = \frac{n(m+n+1)}{2}$.

   ii. Let $Z_1, Z_2, \ldots, Z_n$ be a random sample from a continuous distribution that is symmetric about 0. Let Q be the number of positive Z_i's, and for $Q = q$, let $s_1 < s_2 < \ldots < s_q$ denote the ordered absolute ranks of those z_i's that are positive, then

$$P\{Q = q,\ \ S_1 = s_1, S_2 = s_2, \ldots, S_q = s_q\} = \left(\frac{1}{2}\right)^n,$$

   for $q = 0, 1, 2, \ldots, n$.

(c) i. State slutsky's theorem.

ii. Prove that if a sequence of random variables $\{V_n\}$ has asymptotic distribution with cdf $F_{(v)}$ and $\{W_n\}$ is a sequence such that $\{W_n - V_n\}$ converges in probability to zero, then the limiting distribution of $\{W_n\}$ is also given by the cdf $F_{(w)}$.

4. (a) Define the following:

i. Convergence in probability.

ii. Limiting distribution.

iii. Projection of a U-statistic.

(b) i. Derive the general expression for the variance of a U-statistic.

ii. Prove that if W^* treats the n iid random variables $X_1, X_2, \ldots, X_n$ symmetrically, $E(W^*) = 0$ and V^* is a projection of W^* onto $\mathcal{V}$, then for any $V \in \mathcal{V}$,

$$E\{(W^* - V^*)^2\} \leq E\{(W^* - V)^2\}.$$

(c) Let $Z_1, Z_2, \ldots, Z_n$ denote a random sample from a continuous distribution that is symmetric about 0. Let R_i^+ denote the vector of absolute ranks of the Z_i's. Prove the following.

i. If $\Psi_i = \Psi(Z_i)$, for $i = 1, 2, \ldots, n$, then the n+1 random variables $\Psi_1, \Psi_2, \ldots, \Psi_n, R^+$ are mutually independent.

ii. Each Ψ_i is bernoulli random variable with $P = \frac{1}{2}$.

iii. R^+ is uniformly distributed over $\mathcal{B}$, the set of all permutations of integers $(1, 2, 3, \ldots, n)$.

5. (a) Let $\gamma = E(x_1^2 - X_1X_2)$, that is, $h(X_1, X_2) = X_1^2 - X_1X_2$ is a kernel for the variance γ.

i. Find the symmetric kernel.

ii. Find the U-statistic of γ.

iii. Find the Projection of $U - \gamma$, where U is the U-statistic in (ii) above.

(b) i. State the one-sample U-statistic theorem.

ii. Prove the one-sample U-statistic theorem.

(c) i. Prove that if W^+ is the wilcoxon signed rank statistic and $H_0 : \theta = \theta_0$ is true, then the distribution of W^* is symmetric about its mean $\mu = \frac{n(n+1)}{4}$, given a random sample $X_1, X_2, \ldots, X_n$ from a continuous distribution that is symmetric about θ.

ii. Construct the distribution of wilcoxon signed rank statistic under H_0 for $n = 5$.

END OF PAPER

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS & STATISTICS

2012 ACADEMIC YEAR
FIRST SEMESTER FINAL EXAMINATIONS

MAT 5911 – STOCHASTIC PROCESSES

TIME ALLOWED: THREE (3) HOURS

INSTRUCTIONS: There are five (5) questions in this question paper. Answer any four (4) questions. Normal distribution table will be provided.

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1. (a) Let X and Y be any two random variables.

   (i) Explain what do you understand by $E(X|Y)$.

   (ii) If X and Y are discrete random variables, show that
   $$\sum_y E(X|Y=y)\,P(Y=y) = E(X)$$

   (iii) Let X and Y be any two independent continuous random variables having densities f_X and f_Y respectively. Show that
   $$P(X<Y) = \int_{-\infty}^{\infty} F_X(y) f_Y(y)\,dy$$
   where $F_X(y) = P(X \le y)$

   (b) An Insurance company supposes that the number of accidents that each of its policy holders will have in a year is Poisson distributed with the mean of the Poisson depending on the policy holder. If the Poisson mean of a randomly chosen policy holder has a gamma distribution with density function
   $g(\lambda) = \lambda e^{-\lambda}$, $\lambda \ge 0$
   find the probability that a randomly chosen policy holder has exactly n accidents next year.

   (c) A group of children has four boys under 10, six boys over 10, six girls under 10 and x girls over 10. Find the value of x if age and gender are independent.

   (d) Let X be a discrete random variable taking values on the set of non negative integers. Show that $E(X) = \sum_{n=0}^{\infty} P(X>n)$.

2. (a) Define the following terms:

(i) A Stochastic process
(ii) State space of a Stochastic process
(iii) A Markov process
(iv) A Markov chain
(v) A homogeneous Markov chain with discrete time space

(b) Consider a homogeneous Markov chain $\{X_n, n = 0, 1, 2, \dots\}$. Let

$P_{ij}{}^{n} = P(X_{n+k} = j \mid X_K = i),\ n \geq 0;\ i,\ j \geq 0$

(i) Show that

$$P_{ij}{}^{n+m} = \sum_{K=0}^{\infty} P_{iK}{}^{n} P_{kj}{}^{m}$$

(ii) A market survey of 1500 consumers having a choice of three brands A, B, and C shows the following results:

$$N = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 250 & 175 & 75 \\ 100 & 450 & 250 \\ 50 & 100 & 50 \end{pmatrix} \end{array}$$

where (i, j) th element of N describes number of consumers changing from brand i to brand j in 1 year, i, j = A, B, C.
Assume transiting between brands in one year is modeled as a Markov chain, find the transition matrix of the chain and also the probability that a consumer sticks to brand A at the end of the second year given that he was using brand A at the beginning of the first year.

(c) Consider a homogeneous Markov chain $\{X_n, n = 0, 1, 2, \dots\}$.

(i) Define a recurrent and a transient state.

(ii) Show that if state i is recurrent then $\sum P_{ii}{}^{n} = \infty$

(iii) Prove that if state i is recurrent and state i communicate with state j, then state j is recurrent.

(d) Consider a Markov chain whose state space consists of the integers $i = 0, \pm 1, \pm 2, \dots$ and have transition probabilities given by

$$P_{i,\ i+1} = P_{i,\ i-1} = \frac{1}{2},\ i = 0, \pm 1, \pm 2, \dots$$

Prove that all the states of the Markov chain are recurrent.

$\left(\textit{Stirling's approximation}: n! \approx n^{n+\frac{1}{2}} e^{-n} \sqrt{2\pi} \right)$

3. (a) Define the following for a homogeneous Markov chain $\{X_n, n = 0, 1, 2, \ldots\}$

(i) State j is accessible from state i
(ii) An irreducible Markov chain
(iii) An absorbing state
(iv) A Positive recurrent state
(v) An aperiodic state
(vi) An ergodic state

(b) Consider a Markov chain with states space $\{1, 2, 3, 4\}$ and transition matrix

$$P = \begin{pmatrix} \cdot2 & \cdot8 & 0 & 0 \\ \cdot1 & \cdot9 & 0 & 0 \\ 0 & \cdot5 & 0 & \cdot5 \\ \cdot3 & 0 & \cdot7 & 0 \end{pmatrix}$$

(i) Show that state one is recurrent

(ii) Find the mean recurrence time of state 1

(iii) Classify the states of the chain

(iv) For transient states i and j, let s_{ij} denote the expected number of time periods that the Markov chain is in state j, given that it starts in state i. If $S = (s_{ij})$ and $S = (I - P_T)^{-1}$, where elements of P_T are transit probabilities from transient states into transient states, find s_{43}.

(c) A total of three white and three black balls are distributed among three urns, with each urn containing exactly three balls. At each stage, a ball is randomly selected from each urn and two selected balls are interchanged. Let X_n denote the number of black balls in urn 1 after the nth interchange.

(i) Give the transition probabilities of the Markov chain X_n, $n \geq 0$.
(ii) Find the limiting probabilities and interpret them.

(d) Consider a population in which each individual produces i new off-spring until at the end of his lifetime with probability P_i, independent of the other individuals. Let μ be the expected value of the number of off-springs of each individual.

Let X_n denote the number of individuals at the nth generation, who are the off-springs of the individuals at the $(n-1)$ th generation.

Assume $X_0 = 1$

Show that $E(X_n) = \mu^n$ for all $n > 0$

4. (a) (i) Define a Poisson process of rate λ, $\lambda > 0$

(ii) Events occur independently in a certain stochastic process. The chance of an event occurring in the time interval $(t, t+\Delta t)$ is independent of the behavior of the process prior to time t and equals $ae^{-t/b}\,\Delta t + (\Delta t)$. Show that the chance of no event occurring in the interval (U, V) is $\exp\left\{-ab\left(e^{-\frac{u}{b}} - e^{-\frac{v}{b}}\right)\right\}$

(b) Let $\{N(t), t \geq 0\}$ represents a Poisson process of rate λ. Let T_n denote the elapse time between $(n-1)$ st and the nth event, $n = 1, 2, \ldots$.

(i) Show that T_1 has exponential distribution with mean $\frac{1}{\lambda}$

(ii) Show that probability density function of S_n is $\frac{\lambda e^{-\lambda t}(\lambda t)^{n-1}}{(n-1)!}$, where $S_n = T_1 + T_2 + \ldots + T_n$

(c) Suppose immigrants to Zambia arrive at a Poisson rate of ten per month and each immigrant is of Asian descent with probability $\frac{1}{12}$. Show that the number of immigrants to Zambia of Asian descent is a Poisson process of rate $120 \times \frac{1}{12}$ per year.

(d) In a bacterial culture, each organism will generate a successor with probability $\lambda\Delta t$ in any small interval of length Δt and there are no deaths. Thus, if there are k members of the population at time t, the probability that there will be a successor in $(t, t+\Delta t)$ is $k\lambda\Delta t$. Given that the size of the population at time $t = 0$ is one, show that the expected size of the population at time t is given by $m(t) = e^{\lambda t}$

You may use the following:
If $M_X(t)$ is the m.g.f. of r.v.X then
$\frac{d}{dt} M_X(t) \mid_{t=0} = E(X)$

5. (a) Customers arrive at a two chair shoeshine stand at rate eight per hour. The average length of a shoeshine is 4 minutes. There is only one attendant, so that one chair is used as waiting position. Customers who find both chairs occupied go away.

(i) Assuming Poisson input and exponential service times, write and solve the statistical – equilibrium probability state equations.

(ii) Find the expected number of customers in the system

(b) Define the following terms:

(i) A Stochastic process $\{ X(t), t \geq 0 \}$ has independent increments
(ii) A Stochastic process $\{ X(t), t \geq 0 \}$ has stationary increments
(iii) A Brownian motion process

(c) In a standard Brownian motion process $\{ X(t) \}$, for any $s < t$, the conditional distribution of $X(s)$ given that $X(t) = B$ is normal with

$$E(X(s) \mid X(t) = B) = \frac{s}{t} B$$

$$V(X(s) \mid X(t) = B) = \frac{s}{t} (t - s)$$

Let $\{ Y(t), 0 \leq t \leq 1 \}$ be a Brownian motion with $V(Y(t)) = 16t$

Find the following:

(i) $P(Y(1) > 0 \mid Y(1/2) = 4)$
(ii) $P(Y(1/2) > 0 \mid Y(1) = 4)$

(d) The number of particles emitted from a radioactive source is a Poisson process with parameter λ. If n particles were observed in the interval $(0, t)$, find the probability that K of these particles were observed in the interval $(0, \tau), 0 < \tau < t$.

End of Examination

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS & STATISTICS

2013 ACADEMIC YEAR
MID – YEAR EXAMINATIONS

MAT 5911 – STOCHASTIC PROCESSES

TIME ALLOWED: THREE (3) HOURS

INSTRUCTIONS: There are five questions in this question paper. Each question has four parts. Answer any three parts from each question.

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1. (a) Random variables X and Y have joint pdf $f(x,y)$ given by
$f(x,y) = \frac{4(x+y)}{5x^3}$, $0 \le y \le 1$, $1 \le x < \infty$
Find the value of A such that
$$P(0 < Y < \tfrac{1}{2} \mid X > A) = \frac{5}{16}$$

   (b) Given that X and Y are any two discrete random variables, show that $E(X) = E(E(X \mid Y))$

   (c) Let X be exponential random variable with mean $\frac{1}{2}$. Find $E(X \mid X > 1)$

   (d) The number of customers entering a store on a given day is Poisson distributed with mean 10. The amount of money spent by a customer is uniformly distributed over [0,100]. Find the mean amount of money that the store takes in on a given day.

2. (a) Let $\{X_n, n = 0, 1, 2, \ldots\}$ be a Markov Chain with one step transition probabilities matrix P

      (i) State and prove Chapman – Kolmogorov equations

      (ii) Let $P^{(n)}$ be the n step transition probabilities matrix. Show that $P^{(n)} = P^n$

(b) Consider the Markov Chain $\{ X_n, n = 0, 1, 2, ... \}$, in which the transition probability from state i to state j in n steps is $P_{ij}^{(n)}$.
Let row vector $P_{(n)}$ denote the unconditional probabilities at the n^{th} transition i.e. $P(X_n = j)$ is the j^{th} element of $P_{(n)}$.
Let $P_{(0)} = (\alpha_0, \alpha_1, \alpha_2, ...)$ be the initial distribution. By conditioning on the distribution of X_0,

(i) Show that $P(X_n = j) = \sum_{i=0}^{\infty} P_{ij}^{(n)} \alpha_i$

(ii) $P_{(n)} = P_{(0)} P^n$

(c) Diabetic retinopathy (DR) is a common disease among people with insulin – dependent (type I) diabetes. Patients with early DR tend to have progressive retinal changes, which may cause blindness. The illness can be classified into four states depending on the progression of the DR, ranging from best (state 1) to worst (state 4): State 1 indicates no DR and State 4 indicates total blindness. Following 693 diabetic patients for DR diagnostic, the following one year transition matrix was constructed:

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} \cdot 65 & \cdot 27 & \cdot 08 & \cdot 00 \\ \cdot 16 & \cdot 59 & \cdot 24 & \cdot 01 \\ \cdot 00 & \cdot 13 & \cdot 74 & \cdot 13 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

Given that the initial probability distribution of patients is
$P_{(0)} = (\cdot 337 \quad \cdot 449 \quad \cdot 214 \quad 0)$
Find what percent of patients will become blind after two years?

(d) Let i and j be any two states of a Markov Chain. Explain the following terms:

(i) i and j communicate
(ii) What should be $\sum_{n=1}^{\infty} P_{ii}^{(n)}$, if i is recurrent.
(iii) Prove that if state i is recurrent and states i and j communicate then state j is recurrent.

3. (a) Consider a Markov Chain whose state space consists of integers $i = 0, \pm 1, \pm 2, ...$ and has transition probabilities given by $p_{i,\, i+1} = p_{i,\, i-1} = \frac{1}{2}$, *for every* i. Show that state 0 is recurrent.
(You may need to use the following approximation: $n! \approx n^{n+\frac{1}{2}} e^{-n} \sqrt{2\pi}$)

(b) Job occupations in a society are classified as upper, middle or lower class. In this society, the occupation of a child depends only on his or her parents' occupation and social mobility between classes is described by a Markov Chain with transition probabilities matrix

$$P = \begin{bmatrix} 0.45 & 0.48 & 0.07 \\ 0.05 & 0.7 & 0.25 \\ 0.01 & 0.5 & 0.49 \end{bmatrix}$$

with States E_0: upper class jobs; E_1: middle class jobs; E_2: lower class jobs

(i) State if this chain is irreducible
(ii) State if the states are ergodic
(iii) Determine what percent of the society will be in the upper class jobs in the long run.

(c) Consider a population in which each individual produces i new off-springs by the end of his lifetime with probability P_i, independent of the other individuals. Let μ be the mean number of off-springs from each individual. Let X_n be the number of individuals in the nth generation. Assume $X_0 = 1$.

(i) Show that $E(X_n) = \mu^n$
(ii) Given $\mu < 1$, show that the population will be extinct with probability 1.

(d) Consider a Markov Chain with transition probabilities matrix

$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{3}{4} & \frac{1}{4} \end{pmatrix}$$

and states E_1, E_2, E_3 and E_4

(i) Specify the classes of the Markov Chain
(ii) Find the expected number of time periods that the chain is in state 1 given that it started from state 2.

4. (a) (i) Define a Counting process
(ii) Define a Poisson process of rate λ, where $\lambda > 0$.
(iii) Show that in a Poisson process of rate λ, the probability of occurrence of one event in an interval of length t is $e^{-\lambda t}(\lambda t)$.

(b) In a Poisson process of rate λ, let W_n define the waiting time until the nth event. Find the probability density function of W_n.

(c) Customers arrive at a bank as a Poisson process with rate of 10 per hour. Suppose two customers arrived during the first hour. Find the probability that

(i) Both arrived during the first 20 minutes
(ii) At least one arrived during the first 20 minutes.

(d) Events occur independently in Stochastic process $\{ X(t), t \geq 0 \}$. The chance of an event occurring in the interval $(t, t + \Delta t)$ is independent of the behaviour of the process prior to time t and equals $2e^{-t} \Delta t + 0(\Delta t)$. Prove that chance of no event occurring in the interval (t_1, t_2) is $exp\,[-2(e^{-t_1} - e^{-t_2})]$.

5. (a) A gas station has one diesel fuel pump exclusively for trucks and has room for three trucks only (one at the pump and two lined up waiting for service). Trucks that arrive to find the line full leave for another gas station. Given that trucks arrive according to Poisson process with mean number of arrivals 5 per hour and service time of a truck is exponential with mean 6 minutes, find the probability distribution of the queue system under statistical distribution.

(b) A drug is administered in a bacteria culture which can only result in deaths and no births of organisms. Assume deaths occur independently and chance of a death in any small interval of length Δt is $\mu \Delta t + 0(\Delta t)$. Let $X(t)$ be the number of bacteria in the culture at time t. Assuming $X(0) = K$, show that $X(t)$ has a Binomial distribution with parameters K and $e^{-\mu t}$.

You may use the following:
The MGF of a Binomial random variable with parameters n and p is
$$(1 - p\,(1 - e^t))^n$$

(c) Consider a Stochastic process $\{X(t), t \geq 0\}$ whose state space is an interval of real line.

(i) State the conditions under which $\{ X(t) \}$ has independent increments
(ii) State the conditions under which $\{ X(t) \}$ has stationary increments
(iii) Given that $\{ X(t) \}$ has independent increment, for any two time points s and t, such that $s < t$, show that the joint density of $X(s)$ and $X(t)$ can be expressed as a product of densities at times s and $t - s$.

(d) (i) Define a Standard Brownian Motion
(ii) Let $\{ X(t) \}$ be a Brownian motion with $V(X(t)) = 9t$.
You are given that for $s < t$, $E(X(s) \mid X(t) = K) = \frac{s}{t} K$ and $V(X(s) \mid X(t) = K) = \frac{s}{t} (t - s)$

Find $P(X(2) > 0 \mid X(1) = 3)$
and $P(X(1) > 0 \mid X(2) = 3)$

END OF EXAMINATION. GOOD LUCK!

UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF PHYSICS
MID-YEAR UNIVERSITY EXAMINATIONS
2013/2014 ACADEMIC YEAR

B.Sc. PHYSICS
PHY5021
MATHEMATICAL METHODS FOR PHYSICS

DURATION: Three hours.

INSTRUCTIONS: Answer any four questions from the six given. *Each question carries 25 marks with the marks for parts of questions indicated.*

MAXIMUM MARKS: 100

DATE: Tuesday, 4th March 2014.

Formulae that may be needed:

1. Fourier series:

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right], \qquad n = 1, 2, 3, \ldots,$$

with

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx, \quad a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)\,dx, \quad b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)\,dx$$

2.

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$$

3. Laplace transform:

$$\mathcal{L}(f) = F(s) = \int_0^{\infty} e^{-st} f(t)\,dt$$

4. Laplace transform of a derivative of any order

$$\mathcal{L}(f^{(n)}) = s^n \mathcal{L}(f) - s^{n-1} f(0) - s^{n-2} f^{(1)}(0) - \ldots - f^{(n-1)}(0)$$

5. Shifting theorem along the s-axis for Laplace transforms

$$\mathcal{L}[e^{at}f(t)] = F(s-a)$$

6. Shifting theorem along the t-axis for Laplace transform (inverse form)

$$\mathcal{L}^{-1}[e^{-as}F(s)] = f(t-a)u(t-a),$$

where $u(t-a)$ is the Heavyside function.

Materials included

1. Tables of Laplace transforms.

QUESTION 1

Using the method of separation of variables, find the solution of the following equations satisfying the given initial condition:

(i)

$$3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0, \qquad u(x,0) = 4e^{-x},$$

(12 marks)

(ii)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \qquad u(x,0) = \sin\frac{\pi}{2}x.$$

(13 marks)

QUESTION 2

(i) Using the transformation

$$v = x + y, \qquad z = 3x + y,$$

solve the equation

$$u_{xx} - 4u_{xy} + 3u_{yy} = 0.$$

(13 marks)

(ii) D'Alembert's solution of the wave equation is

$$u(x,t) = \phi(x+ct) + \psi(x-ct).$$

Beginning with this solution, obtain the solution satisfying the initial conditions

$$u(x,0) = f(x), \qquad \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0.$$

(12 marks)

QUESTION 3

Derive the two-dimensional wave equation for a vibrating membrane. Include a diagram, and clearly state any assumptions used. (25 marks)

QUESTION 4

Consider a circular plate of unit radius, which has it's faces insulated, and which satisfies the following boundary conditions:

$$u(1,\theta) = c_1, \quad \text{for} \quad 0 < \theta < \pi \tag{1}$$

$$u(1,\theta) = c_2, \quad \text{for} \quad \pi < \theta < 2\pi. \tag{2}$$

The requirement that the temperature $u(r,\theta)$ at each point of the plate is finite leads to the condition:

$$u(r < 1, \theta) = M, \tag{3}$$

i.e., $u(r,\theta)$ is bounded. Find the steady state temperature $u(r,\theta)$ after equilibrium is reached. Hints: The solution of the separated ordinary differential equation involving r is

$$F(r) = Cr^k + \frac{D}{r^k}.$$

Apply condition (3) in step 2 of the product method. In step 3, to satisfy the boundary conditions (1) and (2), first form a series solution from $n = 0$ to $n = \infty$. Note that $u(r,\theta)$ must have period 2π. (25 marks)

QUESTION 5

Derive the the Laplacian ∇^2 in polar coordinates (r,θ). (25 marks)

QUESTION 6

Using Laplace transforms, taken with respect to t, solve the partial differential equation

$$\frac{\partial^2 y(x,t)}{\partial t^2} - c^2 \frac{\partial^2 y(x,t)}{\partial x^2} + A = 0, \qquad x \geq 0, \ t \geq 0,$$

with the initial condition

$$y(x,0) = 0,$$

and with the boundary conditions

$$y(0,t) = 0, \qquad \text{and} \qquad \lim_{x \to \infty} \frac{\partial y(x,t)}{\partial x} = 0,$$

where A is a constant. Check that the solution of the linear ordinary differential equation obtained by taking Laplace transforms is

$$\hat{y}(x,s) = c_1 e^{sx/c} + c_2 e^{-sx/c} - \frac{A}{s^3}.$$

(25 marks)

END

UNIVERSITY OF ZAMBIA
DEPARTMENT OF PHYSICS
M.Sc.
2013 SECOND SEMESTER UNIVERSITY EXAMINATIONS

PHY5022
MATHEMATICAL METHODS FOR PHYSICS

DURATION:	Three hours.
INSTRUCTIONS:	Answer any four questions from the six given. *Each question carries 25 marks with marks indicated in brackets.*
MAXIMUM MARKS:	100
DATE:	19th August 2013

Formulae that may be needed

In this question paper the following labels will be used: r=number of classes, d_i=dimensions of an irreducible representation i, N=number of elements in the group, h_k= number of elements in class c_k, and $c_{ij,s}$=class multiplication coefficients.

1.
$$[lm, n] = \frac{1}{2}\left(\frac{\partial g_{ln}}{\partial x^m} + \frac{\partial g_{mn}}{\partial x^l} - \frac{\partial g_{lm}}{\partial x^n}\right)$$
$$\left\{ \begin{matrix} s \\ lm \end{matrix} \right\} = g^{sn}[lm, n]$$

2.
$$\frac{\partial^2 x^r}{\partial \overline{x^j} \partial \overline{x^k}} = \overline{\left\{ \begin{matrix} n \\ jk \end{matrix} \right\}} \frac{\partial x^r}{\partial \overline{x^n}} - \frac{\partial x^i}{\partial \overline{x^j}} \frac{\partial x^l}{\partial \overline{x^k}} \left\{ \begin{matrix} r \\ il \end{matrix} \right\}$$

3. In spherical polar coordinates:
$$g^{\mu\mu} = \frac{1}{g^{\mu\mu}}$$

4. For orthogonal systems $g_{\mu\nu} = 0$ for $\mu \neq \nu$

5.
$$\phi(x) = f(x) - \lambda \sum_{n=1}^{\infty} \frac{\int_a^b \phi_n(t) f(t)\, dt}{\lambda - \lambda_n} . \phi_n(x)$$

6.
$$c_j . c_k = \sum_s c_{jk,s} c_s$$

7.
$$h_i h_j \chi_i^{(k)} \chi_j^{(k)} = d_k \sum_{s=1}^{r} c_{ij,s} h_s \chi_s^{(k)}$$

8.
$$\sum_{i=1}^{r} d_i^2 = N,$$

9.
$$\sum_{k=1}^{r} h_k \left(\chi_k^{(i)}\right)^* \chi_k^{(j)} = N\delta_{ij}$$

10.
$$c_i = \frac{1}{N} \sum_{p=1}^{r} h_p \left(\chi_p^{(i)}\right)^* \chi_p$$

11.
$$\frac{1}{N} \sum_{p=1}^{r} h_p \chi_p^* \chi_p \quad \begin{cases} = 1 & \text{IRREDUCIBLE} \\ \neq 1 & \text{REDUCIBLE,} \end{cases}$$

12.
$$du_i = \nabla u_i . d\vec{r}, \qquad i = 1, 2, 3, \ldots$$

QUESTION 1

Consider a general curvilinear coordinate system $u_1(x, y, z)$, $u_2(x, y, z)$, and $u_3(x, y, z)$.

(a) Show that $\frac{\partial \vec{r}}{\partial u_i}$ is a vector tangent to the coordinate curve u_i, where $\vec{r}$ is a position vector specifying a point on u_i and $i = 1, 2, 3$. Hence, define unit tangent vectors to the coordinate curves u_i. Define the scale factors. (6 marks)

(b) Define vectors orthogonal to the coordinate curves u_i, and from these vectors define unit vectors perpendicular to the coordinate curves u_i. (3 marks)

(c) By using directional derivatives prove that $\nabla f(x, y, z)$ is perpendicular to surfaces with $f(x, y, z) =$ constant.

(9 marks)

(d) Prove that the set of vectors $\frac{\partial \vec{r}}{\partial u_i}$ and the set of vectors ∇u_i are recipricol sets of vectors, i.e., show that

$$\frac{\partial \vec{r}}{\partial u_i} . \nabla u_j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}, \qquad i, j = 1, 2, 3.$$

(7 marks)

QUESTION 2

(a) Determine the metric tensor in cylindrical coordinates. (8 marks)

(b) If A_p is a tensor show that:

$$A_{p;q} = \frac{\partial A_p}{\partial x^q} - \left\{ \begin{matrix} s \\ pq \end{matrix} \right\} A_s, \quad \text{and}$$

is also a tensor. (8 marks)

(c) Find the physical components of the velocity with contravariant components $v^k = \frac{dx^k}{dt}$ in cylindrical coordinates. You are given that in cylindrical coordinates $g_{22} = \rho^2$, $g_{33} = 1$ and $g_{11} = 1$. (9 marks)

QUESTION 3

(a) Consider the one-dimensional inhomogeneous equation

$$Lu(x) - \lambda u(x) = f(x), \tag{1}$$

where L is a hermitian operator λ is a constant, and $u(x)$ satisfies homogeneous boundary conditions. Derive the formula for the solution $u(x)$ in terms of a Green's function expressed as eigenfunctions of L. (16 marks)

(b) The equation of a string of length l, vibrating with angular frequency ω is

$$\frac{d^2u(x)}{dx^2} + k^2 u(x) = 0, \tag{2}$$

satisfying the boundary conditions

$$u(0) = u(l) = 0.$$

The constant $k = \frac{\omega}{c}$, where c is the velocity of the wave ~~is~~ in the string. Determine the Green's function for the problem using the formula for the Green's function that results from the derivation of part (a). The eigenvalues and normalised eigenfunctions of eq. (2) are

$$\lambda_n = -\left(\frac{n\pi}{l}\right)^2, \quad u_n = \sqrt{\frac{2}{l}} \sin \frac{n\pi x}{l}, \quad n = 1, 2.3. \ldots,$$

respectively. (5 marks)

(c) Write down the differential equation of the Green's function of eq. (1). Give the physical interpretation of this equation. (4 marks)

QUESTION 4

(a) Use the algebraic method to show that the eigenvalues and un-normalised eigenfunctions of the integral equation

$$\phi(x) = \lambda_n \int_{-1}^{1} (t + x)\phi(t)\, dt$$

are given by

$$\lambda_1 = \frac{\sqrt{3}}{2}, \quad \lambda_2 = -\frac{\sqrt{3}}{2}, \quad \phi_1(x) = A\frac{\sqrt{3}}{2}\left(x\sqrt{3}+1\right), \quad \text{and} \quad \phi_2(x) = A\frac{\sqrt{3}}{2}\left(x\sqrt{3}-1\right),$$

respectively, with A a constant. (12 marks)

(b) Solve the integral equation

$$\phi(x) = x + \frac{1}{2}\int_{-1}^{1} (t + x)\phi(t)\, dt$$

using the Hilbert-Schmidt method and the results from part (a). You are given that the normalisation constants for the eigenfunctions are $N_1 = A\sqrt{3}$ and $N_2 = -A\sqrt{3}$. (13 marks)

QUESTION 5

(a) Give the defining properties of a group and an abelian group. (5 marks)

(b) Consider the following elements of permutation of six objects: $a = [146253]$, $b = [351624]$, $c = [425163]$, $d = [456123]$, $e = [243165]$ and $f = [654123]$. By writing these elements in cyclic notation, determine which elements are equivalent. (4 marks)

(c) The classes of the group of A_4 of even permutations are: $c_1 = I$, $c_2 = A,\ D,\ E,\ H$, $c_3 = B,\ C,\ F,\ G$, and $c_4 = J,\ K,\ L$,. The multiplication table A_4 is rearranged so that the identity element lies along the leading diagonal. The resulting table is:

I	A	B	C	D	E	F	G	H	J	K	L
B	I	A	H	L	J	D	E	K	G	C	F
A	B	I	K	F	G	L	J	C	E	H	D
D	G	K	I	C	B	J	L	F	H	E	A
C	L	E	D	I	K	H	A	J	F	B	G
F	J	H	A	K	I	E	D	L	C	G	B
E	C	L	J	G	F	I	K	B	A	D	H
H	F	J	L	B	C	K	I	G	D	A	E
G	K	D	E	J	L	A	H	I	B	F	C
J	H	F	G	E	D	B	C	A	I	L	K
K	D	G	F	A	H	C	B	E	L	I	J
L	E	C	B	H	A	G	F	D	K	J	I

(i) From the table determine the characters of the matrices in the regular representation. This should be done without determining the matrices themselves. (Hint: you only need to determine the diagonal elements of the matrices, which can be done directly from the above table.) (4 marks)

(ii) Use the characters from part one to determine whether the regular representation is reducible.

(5 marks)

(iii) From the table determine the matrices of each element of class c_3 in the regular representation. Find a matrix which commutes with each of the these matrices. From this matrix state whether or not the regular representation is reducible giving a reason or reasons for you answer. Does your answer agree with your answer to part (ii)? (7 marks)

QUESTION 6

Consider S_8, the symmetry group of a square. Its 8 elements are divided into 5 classes: $c_1 = E$, $c_2 = C_{2z}$, $c_3 = C_{4z}^+, C_{4z}^-$ and $c_4 = \sigma_x, \sigma_y$ and $c_5 = \sigma_1, \sigma_2$. Complete the following character table for S_8:

	$\chi^{(1)}$	$\chi^{(2)}$	$\chi^{(3)}$	$\chi^{(4)}$	$\chi^{(5)}$		
c_1							
c_2				1	-2		
c_3			-1	-1			
c_4			1	-1			
c_5		-1	-1	1	0		

The multiplication table for the group ~~is~~ S_8 is :

E	C_{2z}	C_{4z}^+	C_{4z}^-	σ_x	σ_y	σ_1	σ_2
C_{2z}	E	C_{4z}^-	C_{4z}^+	σ_y	σ_x	σ_2	σ_1
C_{4z}^+	C_{4z}^-	C_{2z}	E	σ_1	σ_2	σ_y	σ_x
C_{4z}^-	C_{4z}^+	E	C_{2z}	σ_2	σ_1	σ_x	σ_y
σ_x	σ_y	σ_2	σ_1	E	C_{2z}	C_{4z}^-	C_{4z}^+
σ_y	σ_x	σ_1	σ_2	C_{2z}	E	C_{4z}^+	C_{4z}^-
σ_1	σ_2	σ_x	σ_y	C_{4z}^+	C_{4z}^-	E	C_{2z}
σ_2	σ_1	σ_y	σ_x	C_{4z}^-	C_{4z}^+	C_{2z}	E

(25 marks)

END

The University of Zambia

School of Natural Sciences

Department of Physics

2013/2014 Academic Year

End of Year University Examination

Computational Physics and Modeling II-PHY 5032

Instructions **Total Marks 100**

- **Time Allocation:** Three (3) Hours.
- **All questions carry equal marks.**
- Marks for each question are shown in the square brackets [ ].
- Whenever necessary, use the information given in the **appendix**
- **Answer:**
  - i) Question one (1) and
  - ii) Any three (3) questions from 2, 3, 4, 5 and 6.

Q.1 **(a)** Find the Cholesky factorization of the following matrix

$$\mathbf{A} = \begin{bmatrix} 4 & 12 & 8 & 4 \\ 1 & 7 & 18 & 9 \\ 2 & 9 & 20 & 20 \\ 3 & 11 & 15 & 14 \end{bmatrix}$$

[13]

(b) Given the boundary value problem

$$y'' = \frac{2x}{x^2+1}y' - \frac{2}{x^2+1}y + x^2 + 1$$

with bounday conditions $y(0) = 2$, $y(1) = 5/3$. Convert this equation to a system of four first-order initial-value problems giving initial conditions to each of them.

[12]

Q.2 Given the following linear system

$$\begin{aligned} 2x_1 - 3x_2 + x_3 &= 6 \\ x_1 + x_2 - x_3 &= 4 \\ -x_1 + x_2 - 3x_3 &= 5 \end{aligned}$$

i) Without finding the determinate, find the inverse of the characteristic matrix for this system,

ii) Use the inverse obtained in (i) to solve the linear system.

[25]

Q.3 The function

$$f(x) = x^4 - x - 10$$

has a unique zero near 2. Given that

$$g(x) = \sqrt[4]{x+10}$$

is a fixed point function associated with $f(x)$, estimate the root of $f(x)$ after **two** iterations.

[25]

Q.4 Given the initial-value problem

$$y'' + \frac{-6x^2}{1+x^3}y' + \frac{-6x}{1+x^3}y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

on $[0, 1]$. Taking $h = 0.5$, approximate the solution to of the above ordinary differential equation using the Runge-Kutta method.

[25]

Q.5 Approximate the solution to the following initial-value problem

$$y' = f(x, y) = xe^{3x} - 2y, \quad 0 \leq x \leq 1, \quad y(0) = 0$$

at $x = 0.8$ with $h = 0.2$ using the Adams fourth-order predictor-corrector equations;

$$y_4 = y_3 + \frac{h}{25}\left[55f(x_3, y_3) - 59f(x_2, y_2) + 37f(x_1, y_1) - 9f(x_0, y_0)\right]$$

as the predictor equation and

$$y_4^* = y_3 + \frac{h}{25}\left[9f(x_4, y_4) + 19f(x_3, y_3) - 5f(x_2, y_2) + f(x_1, y_1)\right]$$

Hint: Use the Euler's method to determine (x_1, y_1), (x_2, y_2), and (x_3, y_3).

[25]

Q.6 The system

$$u_1' = u_2, \quad u_2' = -\frac{2}{x}u_2, \quad u_3' = u_4, \quad u_4' = -\frac{2}{x}u_4$$

with initial conditions

$$u_1(1) = 10, \quad u_2(1) = 0, \quad u_3(1) = 0, \quad u_4(1) = 1$$

on the interval $[1, 2]$, arises in the solution of the differential equation describing the electrostatic potential between two concentric spheres, one of radius 1 and the other of radius 2. Using the Runge-Kutta method with $n = 2$ (i.e. $h = 0.5$) calculate the values of each component of the solution as a function of x.

[25]

⋆⋆⋆⋆⋆⋆⋆ **End of Examination** ⋆⋆⋆⋆⋆⋆⋆

Appendix

Newton-Raphson:

Iteratively approximates the root of a non-linear function $f(x)$ starting with an initial guess x_i

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Euler's Method:

The most basic technique for finding the approximate solution of a first-order ODE $y' = f(x, y)$ with initial condition $y(x_0) = y_0$ on an interval $[x_0, x_n]$. The interval $[x_0, x_n]$ is divided into n equally spaced points (nodes) and the distance between each node $h = (x_n - x_0)/n$ is called the step-size. This method approximates solution values $y_1, y_2, \ldots, y_n$ at the node points $x_1, x_2, \ldots, x_3$.

$$y_{i+1} = y_i + hf(x_i, y_i)$$

Classic Runge-Kutta Method:

The most basic technique for finding the approximate solution of a first-order ODE $y' = f(x, y)$ with initial condition $y(x_0) = y_0$ on an interval $[x_0, x_n]$. The interval $[x_0, x_n]$ is divided into n equally spaced points (nodes) and the distance between each node $h = (x_n - x_0)/n$ is called the step-size. This method approximates solution values $y_1, y_2, \ldots, y_n$ at the node points $x_1, x_2, \ldots, x_3$.

$$y_{i+1} = y_i + (k_1 + 2k_2 + 2k_3 + k_4)/6$$

where

$$k_1 = hf(x_i, y_i), \qquad k_2 = hf(x_i + 0.5h, y_i + 0.5k_1),$$

$$k_3 = hf(x_i + 0.5h, y_i + 0.5k_2), \quad k_4 = hf(x_i + h, y_i + k_3)$$

THE UNIVERSITY OF ZAMBIA
DEPARTMENT OF PHYSICS
FIRST SEMESTER EXAMINATIONS 2012
PHY 5311 THEORETICAL PHYSICS I

TIME : THREE HOURS
ANSWER : ANY FOUR QUESTIONS
MAXIMUM MARKS : 100

Useful formulas:

$$\begin{vmatrix} H'_{11} - E^{(1)} & H'_{12} & \cdots & H'_{1\alpha} \\ H'_{21} & H'_{22} - E^{(1)} & \cdots & H'_{2\alpha} \\ \cdots & \cdots & \cdots & \cdots \\ H'_{\alpha 1} & H'_{\alpha 2} & \cdots & H'_{\alpha\alpha} - E^{(1)} \end{vmatrix} = 0$$

$$E_n^{(1)} = \langle \psi_n | H' | \psi_n \rangle$$

$$c_{ba}^{(1)} = (i\hbar)^{-1} \int_{t_0}^{t} H'_{ba}(t') \exp(i\omega_{ba} t') dt'$$

Eigenfunctions of a particle in a square well located between $x = 0$ and $x = L$:

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

Under a unitary transformation the operator A changes to A' according to:

$$A' = UAU^{\dagger}$$

---

1. (a) A particle moves in a two-dimensional box with walls placed at $x = 0$, $x = L$, $y = 0$ and $y = L$. Thus, the potential is zero inside the box and infinite outside.

(i) Show that the eigenfunctions

$$\psi_{n_x n_y}(x, y) = \frac{2}{L} \sin \frac{n_x \pi x}{L} \sin \frac{n_y \pi y}{L}, \quad n_x, n_y = 1, 2, 3, ... \ .$$

satisfy the time-independent Schroedinger equation for the system, provided that the energy values are

$$E_{n_x, n_y} = \frac{\pi^2 \hbar^2}{2mL^2} \left(n_x^2 + n_y^2 \right)$$

[4marks]

(ii) Show that the eigenfunctions obey the orthonormality relation

$$\int_{y=o}^{L} \int_{x=0}^{L} \psi^*_{n_x n_y}(x, y) \psi_{n'_x n'_y}(x, y) dx dy = \delta_{n_x n'_x} \delta_{n_y n'_y}$$

[5marks]

(b) The particle in (a) above is acted upon by the perturbation $H' = k$, where k is a constant. Use first-order perturbation theory to show that

(i) the perturbed energy of the ground state of the particle is

$$E = \frac{\hbar^2 \pi^2}{mL^2} + k$$

[4marks]

(ii) the perturbed energy of the first excited state is

$$E = \frac{5\hbar^2 \pi^2}{2mL^2} + k$$

[8marks].

(iii) Explain why your results are exact and why they must apply to all the states of the system. [2marks]

(iv) Hence, deduce the eigenfunctions of the perturbed system. [2marks]

(2) (a) A particle of charge q in a square well of width L positioned between $x = 0$ and $x = L$ is acted upon by the perturbation $-\varepsilon_0 q x e^{-\gamma t}$, where ε_0 is a constant. If the particle is in the ground state at $t = 0$, calculate the probability that it is in the first excited state as $t \to \infty$. [11marks]

You may need the integral

$$\int_0^L x \cos \frac{m\pi x}{L} dx = \frac{L^2}{m^2 \pi^2} [(-1)^m - 1], \text{ with } m \text{ an integer.}$$

(b) The Hamiltonian of the harmonic oscillator is

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

while the ladder operators for the problem are

$$a_\pm = \frac{1}{\sqrt{2}}[\frac{p}{(m\hbar\omega)^{1/2}} \pm i(\frac{m\omega}{\hbar})^{1/2}x]$$

(i) Working in momentum space, show that

$$[x, p] = i\hbar$$

[2marks]

(ii) Given the commutation relations $[H, a_\pm] = \pm\hbar\omega a_\pm$, show that a_+ is a raising operator while a_- is a lowering operator. [6marks]

(iii) Explain why if ϕ_0 is the ground-state eigenfunction, then

$$a_-\phi_0 = 0$$

[2marks]

(iv) Given that ϕ_0 has the form

$$\phi_0 = e^{-\alpha p^2}$$

determine the value α must have for ϕ_0 to be the correct ground-state eigenfunction. [4marks]

3. (a) (i) Explain the main advantage of the variational method over perturbation theory in solving quantum systems. [2marks]

(ii) The use of the variational method for estimating ground-state energies and eigenfunctions is based on the result

$$\langle H \rangle \geq E_0$$

where E_0 is the ground-state energy and $\langle H \rangle$ is the expectation value of the Hamiltonian for a selected trial function. Prove this result. [6marks]

(b) A particle of mass m moves in the one-dimensional square-well potential

$$\begin{aligned} V &= 0; \;\; 0 < x < L \\ V &= \infty; \; x < 0, x > L \end{aligned}$$

(i) Consider the variational function

$$\phi_\alpha(x) = x^\alpha(1-x)^\alpha$$

Write down an expression for the expectation value $\langle H \rangle_\alpha$ of the energy, but do not attempt to evaluate the integrals. [4marks]

(ii) The integrals may be evaluated to give

$$\langle H \rangle_\alpha = \frac{\hbar^2}{2mL^2}\frac{2\alpha(4\alpha+1)}{2\alpha-1}$$

where only positive values of α are allowed. Use the variational principle to obtain an estimate of the ground-state energy and compare it with the exact value

$$E = \frac{\pi^2\hbar^2}{2mL^2}$$

[10marks]

(iii) Compute the percentage error and comment on your result. [3marks]

4. (a) The commutation relations for the components of the operator for the orbital angular momentum are

$$[L_i, L_j] = i\hbar L_k, \quad \text{with } i, j, k \text{ taken in cyclical order}$$

(i) Use this information to show that the ladder operators $L_\pm = L_x \pm iL_y$ satisfy the commutation relations

$$[L_z, L_\pm] \overset{+}{=} \hbar L_\pm$$

[6marks]

(ii) Prove that L_+ is a raising operator, while L_- is a lowering operator. [6marks]

(b) (i) Given that

$$L_x = i\hbar(\sin\varphi\frac{\partial}{\partial\theta} + \cot\theta\cos\varphi\frac{\partial}{\partial\varphi}),$$

$$L_y = i\hbar(-\cos\varphi\frac{\partial}{\partial\theta} + \cot\theta\sin\varphi\frac{\partial}{\partial\varphi})$$

show that

$$L_+ = \hbar e^{i\varphi}\left(\frac{\partial}{\partial\theta} + i\cot\theta\frac{\partial}{\partial\varphi}\right)$$

and

$$L_- = \hbar e^{-i\varphi}\left(-\frac{\partial}{\partial\theta} + i\cot\theta\frac{\partial}{\partial\varphi}\right)$$

[6marks]

(ii) Given that

$$Y_{10}(\theta,\varphi) = \sqrt{\frac{3}{4\pi}}\cos\theta$$

obtain the other spherical harmonics for $l = 1$. They do not have to be normalised. [4marks]

(iii) Show that

$$L_+ Y_{11}(\theta,\varphi) = 0$$

and explain the meaning of this result. [3marks]

5. **(a)** Show how the differential eigenvalue equation $A\psi = \lambda\psi$ can be transformed into the matrix eigenvalue equation

$$[A]\,\boldsymbol{\psi} = \boldsymbol{\psi}$$

Give the expressions for the operator $[A]$ and the vector $\boldsymbol{\psi}$. [8marks]

(b) **(i)** State the defining property of unitary operators. [2marks]

(ii) Prove that the operator

$$[U] = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

is unitary. [4marks]

(iii) The z component of the spin operator for spin 1 is

$$[S_z] = \begin{pmatrix} \hbar & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\hbar \end{pmatrix}$$

so that the eigenvalues are $\hbar$, 0 and $-\hbar$ with the respective eigenvectors

$$\boldsymbol{\chi}_+ = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix},\quad \boldsymbol{\chi}_0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\ \textit{and}\ \boldsymbol{\chi}_- = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Show that under the unitary transformation mediated by $[U]$, the eigenvalues of $[S_z]$ are unaffected. [5marks]

(iv) Show that the eigenvectors transform correctly under $[U]$. [4marks]

(v) Comment on this result. [2marks]

6. The Pauli spin matrices are

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \text{and} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The Hamiltonian of an electron in a uniform magnetic field B is

$$H = \frac{\mu_B}{\sqrt{2}} B(\sigma_x + \sigma_y)$$

where μ_B is the Bohr magneton.

(i) Calculate the energy eigenvalues of the electron and show that the normalised eigenvectors are given by

$$\boldsymbol{\chi}_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{2}(1+i) \end{pmatrix} \quad \text{and} \quad \boldsymbol{\chi}_2 = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{2}(1+i) \end{pmatrix}$$

[15marks]

(ii) Verify that the eigenvectors are orthogonal. [2marks]

(iii) Verify that the eigenvectors satisfy $\sum_k |k\rangle \langle k| = I$. [4marks]

(iv) Obtain the expectation value of $S_z = \frac{\hbar}{2}\sigma_z$, the z component of the spin in the lower-energy state. [4marks]

******END OF EXAMINATION*****

THE UNIVERSITY OF ZAMBIA
DEPARTMENT OF PHYSICS
UNIVERSITY OF ZAMBIA
SECOND SEMESTER EXAMINATIONS 2013
PHY 5322 THEORETICAL PHYSICS II

TIME: THREE HOURS
ANSWER: ANY FOUR QUESTIONS
MAXIMUM MARKS: 100

In Lagrangian mechanics,

$$p_i = \frac{\partial L}{\partial \dot{q}_i}, \quad \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0$$

$$H = \sum \dot{q}_i \frac{\partial L}{\partial \dot{q}_i} - L, \quad L = T - V$$

The Dirac matrices are

$$\begin{aligned} \alpha_1 &= \begin{pmatrix} 0 & \sigma_x \\ \sigma_x & 0 \end{pmatrix}, \quad \alpha_2 = \begin{pmatrix} 0 & \sigma_y \\ \sigma_{xy} & 0 \end{pmatrix}, \quad \alpha_3 = \begin{pmatrix} 0 & \sigma_z \\ \sigma_z & 0 \end{pmatrix} \\ \alpha_0 &= \rho_3 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \text{ where } I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned}$$

the Pauli spin matrices are

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Note that the Pauli spin matrices obey the angular momentum algebra

$$[\sigma_i, \sigma_j] = i\hbar\sigma_k \text{ with } i, j, k \text{ in cyclical order}$$

---

1. **(a)** Explain the difference between a discrete and a continuous symmetry and give an example of each.. [4marks]

(b) The unitary operator for the infinitesimal continuous symmetry transformation δS is

$$U_{\delta S}(\varepsilon) = I + i\varepsilon F_S$$

where ε is an arbitrary infinitesimal quantity and F_S is a Hermitian operator.

(i) Show that if the Hamiltonian H is invariant under the symmetry transformation δS, then

$$[F_S, H] = 0$$

[3marks].

(ii) The time-evolution operator

$$U(t, t_0) = \exp\left(-\frac{i}{\hbar}H(t - t_0)\right)$$

is the unitary operator that mediates time translations. Given that the expectation value of the operator A is

$$\langle A \rangle = \left\langle \frac{\partial A}{\partial t} - \frac{i}{\hbar}[A, H]\right\rangle$$

show that the energy is conserved when H is not a function of the time.[5marks]

(c) **(i)** Show that under a symmetry transformation the transformed wave function ψ' of a physical system is related to the original wave function by

$$\psi'(\mathbf{r}) = \psi(\mathbf{r} - \delta\mathbf{r})$$

where $\delta\mathbf{r}$ is the change in the position vector under the symmetry transformation. [3marks]

(ii) The rotation matrix for a rotation of ϕ about the x axis is

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{pmatrix}$$

Use this information to show that the generator of infinitesimal rotations about the x axis is the operator

$$\widehat{L}_z = i\hbar\left(z\frac{\partial}{\partial y} - y\frac{\partial}{\partial z}\right)$$

for the x component of the angular momentum. [10marks]

2. (**a**) It is known that if the potential in the Lagrangian is

$$V = q\phi - q\mathbf{v} \cdot \mathbf{A}$$

then the Euler-Lagrande equations yield the Lorentz force

$$\mathbf{F} = q(-\nabla\phi + \mathbf{v} \times \mathbf{B})$$

which is the correct force acting on a particle subject to the magnetic field $\mathbf{B}$ and the electrostatic potential ϕ.

(**i**) Show that this Lagrangian gives the generalized momenta

$$P_i = p_i + qA_i \quad i = x, y, z$$

[4 marks]

(**ii**) Show that the Hamiltonian corresponding to this Lagrangian is

$$H = T + q\phi$$

where T is the kinetic energy. [6marks]

(**iii**) Use this information to explain how you would incorporate the electromagnetic field into the description of a physical system. [2marks]

(**b**) The transition rate for absorption of radiation for an atom in a radiation field of intensity $I(\omega_{ba})$ is given by

$$W_{ba} = \frac{\pi I(\omega_{ba})}{3c\hbar^2\epsilon_0} |\hat{\epsilon} \cdot \mathbf{D}_{ba}|^2$$

where

$$\mathbf{D}_{ba} = -e \int \psi_b^* \mathbf{r} \psi_a d\tau$$

is the dipole matrix element.

An isotropic three-dimensional harmonic oscillator of frequency ω in the ground state is in interaction with an electromagnetic wave which is polarized in the direction

$$\epsilon = \frac{1}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

The wave functions of the oscillator are given by

$$\psi_a = \psi_{n_{ax}n_{ay}n_{az}}(x, y, z) = X_{n_{ax}}(x)Y_{n_{ay}}(y)Z_{n_{az}}(z)$$

while the energy values are

$$E_{n_{ax}n_{ay}n_{az}} = \left(n_{ax} + n_{ay} + n_{az} + \frac{3}{2}\right)\hbar\omega$$

(**i**) Determine the possible final states of the oscillator. [6marks]

(**ii**) Determine the respective transition rates.. [7marks]

You may use the result

$$x_{nm} = \left\{ \begin{array}{c} 0, \quad m \neq n \pm 1 \\ \frac{1}{\alpha}\left(\frac{n+1}{2}\right)^{1/2}, \quad m = n+1 \\ \frac{1}{\alpha}\left(\frac{n}{2}\right)^{1/2}, \quad m = n-1 \end{array} \right\}$$

for the position matrix elements of the harmonic oscillator, where $\alpha = \sqrt{m\omega/\hbar}$.
[8marks]

3. (**a**) What is the dipole approximation in the treatment of the interaction of radiation with matter? [2marks]

(**b**) (**i**) Explain the meaning of the parity operation and justify why measurable quantities must be invariant under it. [4marks]

(**ii**) Show that in spherical polar coordinates, the parity operation is equivalent to the transformation

$$(r, \theta, \phi) \longrightarrow (r, \pi - \theta, \pi + \phi)$$

[3marks]

(**c**) Stimulated absorption and emission or as well as spontaneous emission of radiation all depend on the dipole matrix element

$$\mathbf{D}_{ba} = \langle \psi_b \left| \mathbf{D} \right| \psi_a \rangle = -e \int \psi_b^*(\mathbf{r}) \mathbf{r} \psi_a(\mathbf{r}) d\mathbf{r}$$

where $\mathbf{D} = -e\mathbf{r}$ is the electric dipole operator, $\psi_b(\mathbf{r})$ is the state of higher energy and $\psi_a(\mathbf{r})$ is the state of lower energy. For a hydrogen-like atom, the matrix element $\mathbf{D}_{ba}$ contains the angular factors

$$I_x(l'm', lm) = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} Y_{l'm'}^*(\theta, \phi) \sin\theta \cos\phi Y_{lm}(\theta, \phi) \sin\theta d\theta d\phi$$

$$I_y(l'm', lm) = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} Y_{l'm'}^*(\theta, \phi) \sin\theta \sin\phi Y_{lm}(\theta, \phi) \sin\theta d\theta d\phi$$

and

$$I_z(l'm', lm) = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} Y_{l'm'}^*(\theta, \phi) \cos\theta Y_{lm}(\theta, \phi) \sin\theta d\theta d\phi$$

where m and m' are magnetic quantum numbers, l and l' are angular momentum quantum numbers. Given that the spherical harmonics $Y_{lm}(\theta, \phi)$ transform as

$$Y_{lm}(\theta, \phi) \longrightarrow -(-1)^l Y_{lm}(\theta, \phi)$$

under the change $(\theta, \phi) \longrightarrow (\pi - \theta, \pi + \phi)$,

(**i**) Determine the selection rules arising from the parity operation.[8marks]

(**ii**) Determine the selection rules pertaining to the magnetic quantum numbers.[8marks]

4. Three identical non-interacting fermions are trapped in a one-dimensional box with walls at $x = 0$ and $x = L$.

(i) Show that the single-particle states satisfy the Schroedinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx_i^2}\psi_i(x_i) = E_i\psi_i(x_i), \quad i = 1, 2, 3$$

[5marks]

(ii) Show that the wave function

$$\psi_i^{(n_i)}(x_i) = \sqrt{\frac{2}{L}}\sin\left(\frac{n_i\pi x_i}{L}\right)$$

satisfies the Schroedinger equation for the single-particle states provided that the energy is given by

$$E_{n_i} = \frac{n_i^2\pi^2\hbar^2}{2mL^2}$$

[4 marks]

(iv) Determine the ground-state energy of the three-particle system. [3marks]

(v) Obtain the wave function of the three-particle system. [5marks]

(vi) Using any two particles, verify the Pauli exclusion principle for the system. [5marks]

(vii) Give the energy and the wave function of the ground state if the particles are bosons. [3marks]

5. The total angular momentum of two spin-1/2 particles is $\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$.. When a uniform magnetic field $\mathbf{B}$ is applied to the system, each individual particle has energy $-\boldsymbol{\mu}\cdot\mathbf{B}$, where μ is the magnetic moment for a spin-1/2 particle.

(i) Show that the possible values of the quantum number s for the combined system are $s = 0, 1$. [2marks]

(ii) Show that the states of the combined system are

$$\Phi^{00}_{\frac{1}{2}\frac{1}{2}} = \frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 + \begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2\right]$$

$$\Phi^{1,-1}_{\frac{1}{2}\frac{1}{2}} = \begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2, \quad \Phi^{11}_{\frac{1}{2}\frac{1}{2}} = \begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2$$

and

$$\Phi^{10}_{\frac{1}{2}\frac{1}{2}} = \frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 - \begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2\right]$$

Use the table of Clebsch-Gordan coefficients provided. [10marks]

(iii) Determine the energies of all the compound states. [2marks]

(iv) Determine the behaviour of these states under interchange of the particles and hence state are suitable for describing fermions and which for describing bosons. [6marks]

(vi) Given that

$$S_{1x} = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}_1, \quad S_{1y} = \frac{\hbar}{2}\begin{pmatrix}0 & -i\\i & 0\end{pmatrix}_1 \quad \text{and } S_{1x} = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}_1$$

confirm that the anti-symmetric state is an eigenfunction of S_1^2. [5marks]

6 **(a)** **(i)** Obtain the Klein-Gordon relativistic wave equation

$$(-\hbar^2 c^2 \nabla^2 + m_0^2 c^4)\Psi(\mathbf{r}, t) = -\hbar^2 \frac{\partial^2}{\partial t^2}\Psi(\mathbf{r}, t)$$

[4 marks]

(ii) Obtain the Dirac equation

$$[c(\boldsymbol{\alpha} \cdot \mathbf{p}) + \alpha_0 m_0 c^2]\Psi(\mathbf{r}, t) = i\hbar \frac{\partial \Psi(\mathbf{r}, t)}{\partial t}$$

[7marks]

(iii) The Klein-Gordon equation was proposed before the Dirac equation. Explain why it was considered unsatisfactory. [2marks]

(iv) Use any two of the Dirac matrices given on the front page to prove the commutation properties. [4marks]

(b) The Dirac Hamiltonian for a particle moving under the central potential $V(r)$ is

$$H = c\alpha_1 p_x + c\alpha_2 p_y + c\alpha_3 p_z + \rho_3 m_0 c^2 + V(r)$$

where $\mathbf{p} = (p_x, p_y, p_z)$ is the momentum and $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$, $\alpha_0 = \rho_3$ are the Dirac matrices.

(i) Show that

$$[H, L_z\} = -i\hbar c(\alpha_1 p_y - \alpha_2 p_x)$$

[4marks]

(ii) In fact, it can be shown that in addition

$$[H, L^2] \neq 0$$

However, if

$$\mathbf{J} = \mathbf{L} + \frac{\hbar}{2}\boldsymbol{\sigma}$$

where $\boldsymbol{\sigma}$ is the Pauli spin operator, it can be shown that

$$[H, J_x] = 0 \text{ and } [H, J^2] = 0$$

Explain the significance of all these results.. [4marks]

****

END OF EXAMINATION

Table 6.3 Clebsch–Gordan coefficients.

$\langle j_1 \frac{1}{2} m_1 m_2 \vert jm\rangle$		
j	$m_2 = \frac{1}{2}$	$m_2 = -\frac{1}{2}$
$j_1 + \frac{1}{2}$	$\left(\frac{j_1 + m + \frac{1}{2}}{2j_1 + 1}\right)^{1/2}$	$\left(\frac{j_1 - m + \frac{1}{2}}{2j_1 + 1}\right)^{1/2}$
$j_1 - \frac{1}{2}$	$-\left(\frac{j_1 - m + \frac{1}{2}}{2j_1 + 1}\right)^{1/2}$	$\left(\frac{j_1 + m + \frac{1}{2}}{2j_1 + 1}\right)^{1/2}$

$\langle j_1 1 m_1 m_2 \vert jm\rangle$			
j	$m_2 = 1$	$m_2 = 0$	$m_2 = -1$
$j_1 + 1$	$\left[\frac{(j_1 + m)(j_1 + m + 1)}{(2j_1 + 1)(2j_1 + 2)}\right]^{1/2}$	$\left[\frac{(j_1 - m + 1)(j_1 + m + 1)}{(2j_1 + 1)(j_1 + 1)}\right]^{1/2}$	$\left[\frac{(j_1 - m)(j_1 - m + 1)}{(2j_1 + 1)(2j_1 + 2)}\right]^{1/2}$
j_1	$-\left[\frac{(j_1 + m)(j_1 - m + 1)}{2j_1(j_1 + 1)}\right]^{1/2}$	$\left[\frac{m^2}{j_1(j_1 + 1)}\right]^{1/2}$	$\left[\frac{(j_1 - m)(j_1 + m + 1)}{2j_1(j_1 + 1)}\right]^{1/2}$
$j_1 - 1$	$\left[\frac{(j_1 - m)(j_1 - m + 1)}{2j_1(2j_1 + 1)}\right]^{1/2}$	$-\left[\frac{(j_1 - m)(j_1 + m)}{j_1(2j_1 + 1)}\right]^{1/2}$	$\left[\frac{(j_1 + m + 1)(j_1 + m)}{2j_1(2j_1 + 1)}\right]^{1/2}$

and the symmetry relations

$$\begin{aligned}
\langle j_1 j_2 m_1 m_2 | jm\rangle &= (-)^{j_1+j_2-j}\langle j_2 j_1 m_2 m_1 | jm\rangle \\
&= (-)^{j_1+j_2-j}\langle j_1 j_2 - m_1 - m_2 | j - m\rangle \\
&= \langle j_2 j_1 - m_2 - m_1 | j - m\rangle \\
&= (-)^{j_1-m_1}\left(\frac{2j+1}{2j_2+1}\right)^{1/2}\langle j_1 j m_1 - m | j_2 - m_2\rangle \\
&= (-)^{j_2+m_2}\left(\frac{2j+1}{2j_1+1}\right)^{1/2}\langle j j_2 - m m_2 | j_1 - m_1\rangle.
\end{aligned} \tag{6.292}$$

Details concerning the explicit computation of the Clebsch–Gordan coefficients, together with tables, can be found for example in Edmonds (1957) or Rose (1957). In Table 6.3 the Clebsch–Gordan coefficients $\langle j_1 j_2 m_1 m_2 | jm\rangle$ are given for the cases $j_2 = 1/2$ and $j_2 = 1$. By using the symmetry relations, all the coefficients with any one of j_1, j_2 or j equal to 1/2 or to 1 can be obtained.

Addition of the orbital angular momentum and spin of a particle

As a first example, let us consider a particle of spin s. Let $\mathbf{L}$ be its orbital angular momentum operator and $\mathbf{S}$ its spin angular momentum operator. The total angular momentum operator of the particle is therefore $\mathbf{J} = \mathbf{L} + \mathbf{S}$. We shall denote by m_l, m_s and m_j the quantum numbers corresponding to the operators L_z, S_z and J_z, respectively. In the position representation the simultaneous eigenfunctions of the operators $\mathbf{L}^2$ and L_z are the spherical harmonics $Y_{lm_l}(\theta, \phi)$ with $l = 0, 1, 2, \ldots$, and

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ANSWER: ANY FOUR QUESTIONS
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$$[L_i, L_j] = i\hbar L_k, \text{ with } i, j, k \text{ in cyclical order}$$

Recursion relation for Legendre polynomials .$(2l+1)(1-w^2)^{1/2}P_l^{|m|}(w) = P_{l+1}^{|m|}(w) - P_{l-1}^{|m|}(w)$ where $w = \cos\theta$.
Orthogonality relation for Legendre polynomials:

$$\int_{-1}^{+1} P_l^{|m|}(w)P_{l'}^{|m|}(w)dw = \frac{2}{2l+1}\frac{(l+|m|)!}{(l-|m|)!}\delta_{ll'}$$

$$\int x\cos x \, dx = \cos x + x\sin x + C$$

Question 1.(a) Describe all the ways in which matter and electromagnetic radiation can interact. [6marks]

(b) The expression for the Lorentz force acting on an electron in an atom is

$$\mathbf{F} = -e(\mathbf{E} + \mathbf{v}\times\mathbf{B})$$

Given that the amplitude of the electric field and that of the magnetic field in an electromagnetic wave are related by

$$B_0 \approx \frac{1}{c}E_0$$

explain why we can treat the interaction of radiation with matter in terms of the electric field only. You may assume that the velocity of an electron in a one-electron atom is

$$v = \frac{Ze^2}{4\pi\varepsilon_0 n\hbar}$$

where n is its principle quantum number. [4marks]

(c) A certain particle in a three-dimensional infinite potential well of dimension L is in interaction with an electromagnetic wave which is polarized in the x direction.

(i) Sketch the solution of the time-independent Schroedinger equation implicitly for this particle and thus show that its wave functions in the state a are given by

$$\Psi_a = \Psi_{n_{ax}n_{ay}n_{az}}(x, y, z) = \psi_{n_{ax}}(x)\psi_{n_{ay}}(y)\psi_{n_{az}}(z)$$

while its energy values are

$$E_{n_{ax}n_{ay}n_{az}} = E_{nax} + E_{nay} + E_{naz}$$

Here $\psi_{n_{ai}}(x_i)$ is the eigenfunction for a particle in a one-dimensional square well and is given by

$$\psi_{n_{ai}}(x_i) = \sqrt{\frac{2}{L}} \sin \frac{n_{ai}\pi x_i}{L}$$

[9 marks]

(ii) The transition rate for stimulated emission of radiation by the particle is

$$W_{ba} = \frac{\pi I(\omega_{ba})}{\hbar^2 c\varepsilon_0} |\hat{\boldsymbol{\epsilon}} \cdot \mathbf{D}_{ba}|^2$$

where $\hat{\boldsymbol{\epsilon}} = \hat{\mathbf{i}}$ is the polarization vector and

$$\mathbf{D}_{ba} = -e \int \psi_b^* \mathbf{r} \psi_a d\tau$$

is the dipole matrix element, If the particle is initially in the ground state ($n_x = n_y = n_z = 1$), deterrmine the probability that the particle remains in the ground as a result of the transition.. [6 marks]

Question 2. The transition probabilities for the various processes that occur when electromagnetic radiation and matter interact vanish if the dipole matrix element

$$\mathbf{D}_{ba} = \langle \psi_b | \mathbf{D} | \psi_a \rangle = -e \int \psi_b^*(\mathbf{r}) \mathbf{r} \psi_a(\mathbf{r}) d\mathbf{r}$$

is zero. Here $\mathbf{D} = -e\mathbf{r}$ is the electric dipole operator, $\psi_b(\mathbf{r})$ is the atomic state of higher energy and $\psi_a(\mathbf{r})$ is the atomic state of lower energy. The matrix element $\mathbf{D}_{ba}$ contains the factors

$$K_x(m, m') = \int_0^{2\pi} \exp\left[i\left(m - m'\right)\phi\right] \cos\phi d\phi$$

$$K_z(m, m') = \int_0^{2\pi} \exp\left[i\left(m - m'\right)\phi\right] d\phi$$

and

$$L^+(l', l, m) = \int_0^{\pi} P_{l'}^{m+1}(\cos\theta) \sin\theta P_l^m(\cos\theta) d\theta$$

where m and m' are magnetic quantum numbers, l and l' are angular momentum quantum numbers and $P_l^m(\cos\theta)$ are associated Legendre polynomials.

(i) Show that the factor $K_x(m, m')$ leads to the selection rule

$$\Delta m = \pm 1$$

for magnetic quantum numbers. [11marks]

(ii) Show that the factor $K_z(m, m')$ leads to the selection rule

$$\Delta m = 0$$

for magnetic quantum numbers. [5marks]

(iii) Show that the factor $L^+(l', l, m)$ leads to the selection rule

$$\Delta l = +1.$$

for angular momentum quantum numbers. [9marks]

Remember that

$$\lim \frac{\cos x}{x} \underset{x \to 0}{\longrightarrow} 0 \text{ and } \lim \frac{\sin x}{x} \underset{x \to 0}{\longrightarrow} 1$$

Question 3. (a) With the aid of the heuristic rules for converting dynamical quantities into operators, use the relativistic expression for the energy

$$E^2 = p^2c^2 + m^2c^4$$

to obtain the one-particle Klein-Gordon relativistic wave equation

$$\left(-\hbar^2c^2\nabla^2 + m^2c^4\right)\Psi(\mathbf{r},t) = -\hbar^2\frac{\partial^2}{\partial t^2}\Psi(\mathbf{r},t)$$

Note that $\mathbf{p}$ is the momentum and m the rest mass of the particle.[4 marks]

(ii) Show that the continuity equation corresponding to this equation is

$$\nabla\cdot\mathbf{j}+\frac{\partial\rho}{\partial t} = 0$$

and obtain the expressions for ρ and j. [12 marks]

(b) (i) Explain why Dirac sought to linearize the relativistic equation

$$E^2 = p^2c^2 + m^2c^4$$

for the energy. [3 marks]

(ii) Show that the linearization procedure of Dirac

$$E = c\sum_{\mu=0}^{3}\alpha_u p_\mu$$

is possible if the quantities α satisfy

$$\alpha_\mu\alpha_{\mu'} + \alpha_{\mu'}\alpha_\mu = 2\delta_{\mu\mu'} \text{ and } \alpha_\mu^2 = 1.$$

[8 marks]

Question 4. (a) The two independent spins $\mathbf{J}_1$ and $\mathbf{J}_2$ are added to give the total spin $\mathbf{J} = \mathbf{J}_1 + \mathbf{J}_2$.

(i) Show that $\mathbf{J}$ commutes with $\mathbf{J}_1$ and $\mathbf{J}_2$. [4marks]

(ii) Explain the implications of this. [2 marks]

(b) A certain quantum system is composed of two particle of spin $S = 1$ which are added to give $\mathbf{J} = \mathbf{S}_1 + \mathbf{S}_2$.

(i) Show that the possible values of the total angular momentum of the system are $J = 0, 1$, and 2. [2marks]

(ii) Give all the wave functions of the compound system without evaluating the Clebsch-Gordan coefficients except for the cases $J = 2$ and $M = \pm 2$. [8 marks]

(iii) Explain why the $J = 2$ and $M = \pm 2$ states are

$$\Phi_{11}^{J=2,M=2} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_2 \text{ and } \Phi_{11}^{J=2,M=-2} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_1 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_2$$

[5 marks]

(iii) Show that the expectation values of the operator

$$[J_z] = [S_{1z}] + [S_{2z}] = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}_1 + \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}_2$$

in the states $J = 2$ and $M = \pm 2$ are respectively $\langle J_z \rangle = \pm 2\hbar$. [4marks]

Question 5. The ladder operators for angular momentum L are defined by

$$L_{\pm} = L_x \pm i L_y$$

(i) Show that

$$[L_-, L_z] = \hbar L_-$$

and

$$[L_+, L_z] = -\hbar L_+$$

[4marks]

(ii) Show that L_+ and L_- are raising and lowering operators respectively. [4marks]

(iii) Given that

$$L_x = i\hbar\left(\sin\phi\frac{\partial}{\partial\theta} + \cot\theta\cos\phi\frac{\partial}{\partial\phi}\right)$$

and

$$L_y = i\hbar\left(-\cos\phi\frac{\partial}{\partial\theta} + \cot\theta\sin\phi\frac{\partial}{\partial\phi}\right)$$

show that

$$L_+ = \hbar e^{i\phi}\left(\frac{\partial}{\partial\theta} + i\cot\theta\frac{\partial}{\partial\phi}\right)$$

and

$$L_- = \hbar e^{-i\phi}\left(-\frac{\partial}{\partial\theta} + i\cot\theta\frac{\partial}{\partial\phi}\right)$$

[4marks]

(iv) Use

$$Y_{22}(\theta,\phi) = \sqrt{\frac{15}{32\pi}}e^{2i\phi}\sin^2\theta$$

to show that $Y_{21}(\theta,\phi)$. has the form

$$Y_{21}(\theta,\phi) = Be^{-i\phi}\sin\theta\cos\theta$$

[4marks]

(v) Normalize this function. [5marks]

(vi) Show that there is no state corresponding to $m = 3$. [4marks]

Question 6. The Hamiltonian of the harmonic oscillator is

$$H = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

and the ladder operators are defined as

$$a_\pm = \frac{1}{\sqrt{2}}[\frac{p}{(m\hbar\omega)^{1/2}} \pm i(\frac{m\omega}{\hbar})^{1/2}x]$$

(i) Show that

$$H = (a_+a_- + a_-a_+)\frac{\hbar\omega}{2}$$

[6marks]

(ii) Prove one of the commutation relations

$$[H, a_\pm] = \pm\hbar\omega a_\pm$$

[6marks]

(iii) Show that the ground state ψ_0 of the harmonic oscillator satisfies the equation

$$\frac{1}{\sqrt{2}}[-i\left(\frac{\hbar}{m\omega}\right)^{1/2}\frac{d}{dx} - i(\frac{m\omega}{\hbar})^{1/2}x]\psi_0 = 0$$

[6marks]

(iv) Prove that the solution of this equation is

$$\psi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\exp(-\frac{m\omega}{2\hbar}x^2)$$

[3marks]

(v) Show that the ground-state energy is $E_0 = \frac{1}{2}\hbar\omega$. [4marks]

END OF EXAMINATION

UNIVERSITY OF ZAMBIA
DEPARTMENT OF PHYSICS

2013/14 MID-TERM UNIVERSITY EXAMINATIONS

PHY 5811
SOLAR ENERGY AND APPLICATIONS

DURATION: Three (3) hours
INSTRUCTIONS: Answer **any four (4)** questions. The marks for each question are given in square brackets.

MAXIMUM MARKS: 100

DATE: Friday 07th March, 2014

Some useful identities and formulae are given below:

NOTE: Unless otherwise stated, all symbols have the usual meaning.

$$I_{SC} = 4.921 MJ/m^2.hr \qquad \delta = 23.45\sin\left[\frac{360}{365}(284+n)\right] \qquad \omega = 15\left[(ST-12)+\frac{MM}{60}\right]$$

$$E_o = 1+0.033\cos\left(\frac{360n}{365}\right) \qquad \frac{\overline{H}}{\overline{H}_o} = \overline{K}_T \qquad \omega_s = \cos^{-1}(-\tan\phi\tan\delta)$$

For $\omega_s \geq 81.4^o$ and $0.3 \leq \overline{K}_T \leq 0.8$

$$\frac{\overline{H}_d}{\overline{H}} = 1.311 - 3.022\overline{K}_T + 3.427\overline{K}_T^{\,2} - 1.821\overline{K}_T^{\,3}$$

For $\omega_s \leq 81.4^o$ and $0.3 \leq \overline{K}_T \leq 0.8$

$$\frac{\overline{H}_d}{\overline{H}} = 1.311 - 3.560\overline{K}_T + 4.189\overline{K}_T^{\,2} - 2.137\overline{K}_T^{\,3}$$

Liu and Jordan model is given as;

$$\overline{H}_t = \overline{H}\left(1-\frac{\overline{H}_d}{\overline{H}}\right)\overline{R}_b + \overline{H}_d\left(\frac{1+\cos\beta}{2}\right) + \overline{H}\rho\left(\frac{1-\cos\beta}{2}\right)$$

Correction coefficients in Ångström formula are;

$$a = -0.110 + 0.235\cos\phi + 0.323\left(\frac{\bar{S}_a}{\bar{S}_p}\right), \quad b = 1.449 - 0.553\cos\phi - 0.694\left(\frac{\bar{S}_a}{\bar{S}_p}\right)$$

$$U_b = \frac{K_{in}}{L_{in}}, \qquad \frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots\ldots\ldots\ldots,$$

$$U_s = \frac{2(l_1 + l_2)l_3 K_{in}}{l_1 l_2 L_s}, \qquad \sigma = 5.67x10^{-8} W/m^2K^4$$

$$R_b = \frac{\frac{\pi}{180}\omega_s' \sin\delta\sin(\phi-\beta) + \cos\delta\cos(\phi-\beta)\sin\omega_s'}{\frac{\pi}{180}\omega_s \sin\delta\sin\phi + \cos\delta\cos\phi\sin\omega_s}, \qquad (\tau\alpha)_{eff} = \frac{\tau\alpha}{1-(1-\alpha)\rho_d}$$

$$r_b = \frac{\sin\delta\sin(\phi-\beta) + \cos\delta\cos(\phi-\beta)\cos\omega}{\sin\delta\sin\phi + \cos\delta\cos\phi\cos\omega}, \quad \dot{q}_L = U_L A_p(T_p - T_a), \quad \tau_a = \exp\left(\frac{-NkL}{\cos\theta_2}\right)$$

$$\tau_r = \frac{1}{2}(\tau_{rs} + \tau_{rp}), \quad \tau_{rs} = \frac{1-\rho_s}{1+(2N-1)\rho_s}, \quad \tau_{rp} = \frac{1-\rho_p}{1+(2N-1)\rho_p}, \quad \dot{q}_{ab} = \dot{q}_L + \dot{q}_u$$

$$\eta_i = F_R\left[(\tau\alpha)_{eff} - U_L\frac{(T_i - T_a)}{I_t}\right], \quad \dot{q}_u = F' A_p\left[\dot{q}_{ab} - U_L(T_f - T_a)\right], \quad r_d = \frac{1+\cos\beta}{2}$$

$$r_r = \frac{\rho(1-\cos\beta)}{2}, \quad \frac{\bar{H}}{\bar{H}_o} = a + b\left(\frac{\bar{S}_a}{\bar{S}_p}\right), \quad \bar{I}_T = \bar{I}_t\left(1 - \frac{\bar{I}_d}{\bar{I}_t}\right)\bar{r}_b + \bar{I}_d\left(\frac{1+\cos\beta}{2}\right) + \bar{I}_t\rho\left(\frac{1-\cos\beta}{2}\right)$$

$$q_{rad} = h_r(T_p - T_c) \qquad h_r = \sigma\varepsilon_{eff}\frac{\left[(T_p+273)^4 - (T_c+273)^4\right]}{T_p - T_c} \qquad \varepsilon_{eff} = \left[\frac{1}{\varepsilon_p} + \frac{1}{\varepsilon_c} - 1\right]^{-1}$$

Q1. (a) Define the following terms with reference to the Sun-earth angle geometry.

(i) Solar altitude angle
(ii) Declination of the Sun angle
(iii) Solar azimuth angle
(iv) Zenith angle. [2+2+2+2]

(b) Calculate the
(i) zenith angle at 10:30 solar time,
(ii) solar azimuth angle at 10:30 solar time,
(iii) sunrise hour angle, and
(iv) day length,
on October 16 in Lusaka (15° S latitude). [4+6+4+3]

Q2. (a) The daily extraterrestrial solar radiation on a horizontal surface may be expressed as

$$H_o = \frac{24}{\pi} I_{SC} E_o \left[\frac{\pi}{180} \omega_s \sin\delta \sin\phi + \cos\delta \cos\phi \sin\omega_s \right].$$

(i) Estimate the daily extraterrestrial solar radiation on a horizontal surface located at the equator on June 11, 1996. [7]
(ii) Estimate the daily extraterrestrial solar radiation received on a horizontal surface at latitude 28° N, on May 15. [6]

(b) (i) Derive an expression for the hourly solar radiation on a horizontal surface and show that it reduces to

$$I_o = I_{sc} E_o (\sin\delta \sin\phi + \cos\delta \cos\phi \cos\omega_i). \qquad [10]$$

(ii) State any assumptions that you may have made. [2]

Q3. (a) (a) Estimate the daily global solar radiation on a horizontal surface at a location 28° N, on September 15, if the monthly average daily hours of "bright" sunshine observed is 11.5 hrs. The day's solar radiation on a horizontal surface outside the earth's atmosphere is 40.0 MJ/m^2day. [15]

(b) A collector is to be installed at latitude 43° N, inclined at an angle of 60° to the horizontal facing due south. The monthly average daily solar radiation on horizontal surface for January H = 3.85MJ/m^2.day. Use the Liu and Jordan Model (also known as the isotropic diffuse model) to estimate the monthly average global radiation incident on the collector for the month of January. The clearness index for the month of January is 0.44 and the ground albedo is 0.7. Comment on your answer. [10]

Q4. **(a)** Use the prediction equation to calculate the monthly average daily global solar radiation falling on a horizontal surface on a town located 21°N latitude during the month of January. The monthly average sunshine hours for January are 9.8hrs. [10]

(b) Calculate the monthly average hourly solar radiation falling on a tilted flat-plate collector facing due south ($\gamma = 0$) at a town 13° N latitude between 11:00 hrs and 12:00hrs on October 15, given the following parameters.

$\bar{I}_t = 2.41\,\text{MJ/m}^2\text{hr}$, $\bar{I}_d = 1.07\,\text{MJ/m}^2\text{hr}$, $\rho = 0.2$, $\beta = 15^o$. [15]

Q5. (a) (i) Calculate the threshold radiation flux for the following values of $(\tau_o\alpha_o) = 0.80,\ 0.60, 0.40,$ and 0.20. The solar collector has the following parameters; $T_p = 100°$ C, $T_a = 16°$ C and $U_L = 6$ W/m² °C. [8]

(ii) Comment on your results. [2]

(b) The collector system operates under the following parameters;

Instantaneous collector efficiency	= 44.7%
Effective transmittance-absorptance product	= 0.727
Overall heat loss coefficient	= 4.0W/m²°C
Fluid inlet temperature	= 60°C
Ambient temperature	= 25°C
Solar radiation falling on collector area	= 852.7W/m²

Calculate the collector heat removal factor for a flat-plate collector with a double glass cover and a selective absorber surface. [15]

End of Examination

UNIVERSITY OF ZAMBIA
DEPARTMENT OF PHYSICS

2012 SECOND SEMESTER UNIVERSITY EXAMINATIONS

PHY 5822
SOLAR ENERGY MATERIALS

DURATION: Three (3) hours

INSTRUCTIONS: Answer **any four (4)** questions. The marks for each question are given in square brackets.

MAXIMUM MARKS: 100

DATE: Monday 26th August 2013

Some useful identities and formulae are given below:

$k = 1.38x10^{-23} J/K$ $\quad \hbar = 1.055x10^{-34} J.s$ $\quad \eta = \frac{I_{sc}V_{oc}}{I_N A} FF$ $\quad \delta = \frac{2\pi nd}{\lambda}$

$n = N_c \exp\left[-\frac{E_c - E_F}{kT}\right]$ $\quad p = N_v \exp\left[-\frac{E_F - E_v}{kT}\right]$ $\quad n = n_i \exp\left[-\frac{E_i - E_F}{kT}\right]$ $\quad N = n + ik$

$E_F = E_c + kT\ln\left(\frac{N_D}{N_C}\right)$ $\quad I = I_o\left[e^{\frac{qV}{kT}} - 1\right]$ $\quad E_F = E_v - kT\ln\left(\frac{N_A}{N_V}\right)$ $\quad R = rr^*$

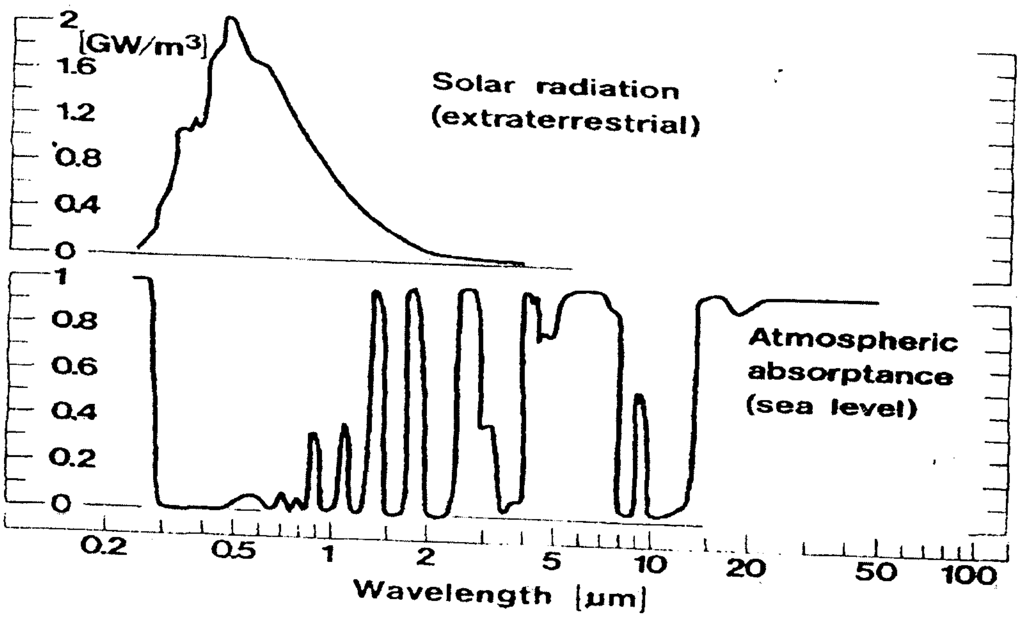
$\nabla^2 = i\frac{\partial^2}{\partial x^2} + j\frac{\partial^2}{\partial y^2} + k\frac{\partial^2}{\partial z^2}$ $\quad \alpha = \frac{4\pi k}{\lambda}$ $\quad q = \frac{\omega}{c}N$ $\quad T = \frac{T_1 T_2 \exp(-\alpha d)}{1 - R_1 R_2 \exp(-2\alpha d)}$

$R = R_1 + \frac{R_1(1 - R_1^2)\exp(-2\alpha d)}{1 - R_1^2 \exp(-2\alpha d)}$ $\quad R = \frac{(1-n)^2 + k^2}{(1+n)^2 + k^2}$ $\quad I = I_o e^{-\alpha x}$ $\quad \alpha = \frac{4\pi k}{\lambda}$ $\quad T = tt^*$

$T_\perp = \frac{N_2}{N_1}\left|\frac{2N_1}{N_2 + N_1}\right|^2$ $\quad R_\perp = \left|\frac{N_1 - N_2}{N_1 + N_2}\right|^2$ $\quad n_i^2 = np$ $\quad c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$ $\quad \varepsilon = \varepsilon_1 + i\varepsilon_2$

$R_p \approx \frac{\left[(n^2 + k^2)\cos^2\theta - 2n\cos\theta\right] + 1}{\left[(n^2 + k^2)\cos^2\theta + 2n\cos\theta\right] + 1}$ $\quad R_s \approx \frac{\left[(n^2 + k^2) - 2n\cos\theta + \cos^2\theta\right]}{\left[(n^2 + k^2) + 2n\cos\theta + \cos^2\theta\right]}$

Q1. (a) In the figure below, the top curve shows the solar radiation outside the atmosphere and the bottom curve shows a typical absorption spectrum vertically across the full atmospheric envelope at clear weather conditions. Briefly, explain the term *spectral selectivity* based on the figure below. [10]



(b) The optical constants of austenitic stainless steel are $n = 1.9$; $k = 3.5$ at 500nm and $n = 5.4$; $k = 7.5$ at 2000nm. Calculate the reflectance at these wavelengths at

(i) normal incidence ($\theta_i = 0^o$),

(ii) glancing incidence ($\theta_i = 85^o$). [15]

Q2. (a) Car dealers often use "tinted windows" as a sales argument. Tinting means that the glass has been made slightly absorbing.

(i) If glass is 3mm thick, what extinction coefficient gives a transmittance of about 25% instead of the clear glass value of 92%? Use $\lambda = 0.5\mu m$.

(ii) (ii) Calculate the reflectance of such a tinted window. Use the non-coherence case. [13]

(b) Write short notes on the following:

(i) Solar mirrors, Solar selective absorbers and Heat mirrors [2+2+2]

(ii) Thermal evaporation, Sputtering and Spray pyrolysis [2+2+2]

Q3. **(a)** The variation of band gap energy with temperature for silicon is given by the relation,

$E_g(T) = E_g(0) - \frac{aT^2}{T+b}$, where $a = 7\text{x}10^{-4}$ eV/K, $b = 1100$ K, and $E_g(0) = 1.16$ eV.

(i) Calculate the band gap energy for silicon at 40 °C.

(ii) Determine the temperature for zero band gap energy for silicon and explain for the choice of your answer. [13]

(b) A gallium arsenide solar cell with a band gap of 1.42 eV at a temperature of 300K has a short circuit current of 2.34 A under normal illumination. Calculate the corresponding open circuit voltage given that the dark saturation current is $I_o = 1.5x10^{11} \exp\left(-\frac{E_g}{kT}\right)$. [12]

Q4. **(a)** The reflectance of an anti-reflecting coating is generally given as $R = \left(\frac{n_3 - n_2^2}{n_3 + n_2^2}\right)^2$, where n_3 is the refractive index of the substrate, and n_2 is the refractive index of the thin film (coating). A thin semiconductor film has been deposited on quartz with refractive index 1.5. If the reflectance of the film is 0.2, calculate the

(i) thickness of the film

(ii) refractive index of the film

(iii) band gap energy when the cut-off wavelength is 0.43μm. [11]

(b) Calculate the shift in the Fermi-level in a silicon crystal doped with a group V impurity of concentration $1\text{x}10^{15}\text{cm}^{-3}$. The effective density of states in the conduction band is $2.82\text{x}10^{19}\text{cm}^{-3}$ and the band gap energy for silicon is 1.11eV at a temperature of 27°C. [8]

(c) (i) What is ellipsometry?

(ii) An ellipsometer has a light source, a polarizer, a compensator, an analyzer and a detector. Draw a labeled diagram of an ellipsometer. [6]

Q5. **(a)** The I-V characteristic of a PV cell under illumination may be expressed as

$$I = I_o\left[\exp\left(\frac{eV}{kT}\right) - 1\right] - I_L,$$

where the symbols have their usual meaning.

(i) Derive an expression for the voltage when the overall cell current is zero.

(ii) Hence, calculate the voltage for a silicon solar cell with the following parameters at 40°C; $I_L = 4.2A$, $I_o = 1.5x10^5 \exp(-E_g / kT)$ and $E_g(S_i) = 1.11eV$. [13]

(b) In a certain experiment, the total reflectance of silicon was found to be 30% at zero photon energy i.e $\omega = 0$. Use this information to calculate the static dielectric constant, $\varepsilon_1(0)$ of silicon. Hint: Determine the refractive index of silicon. [12]

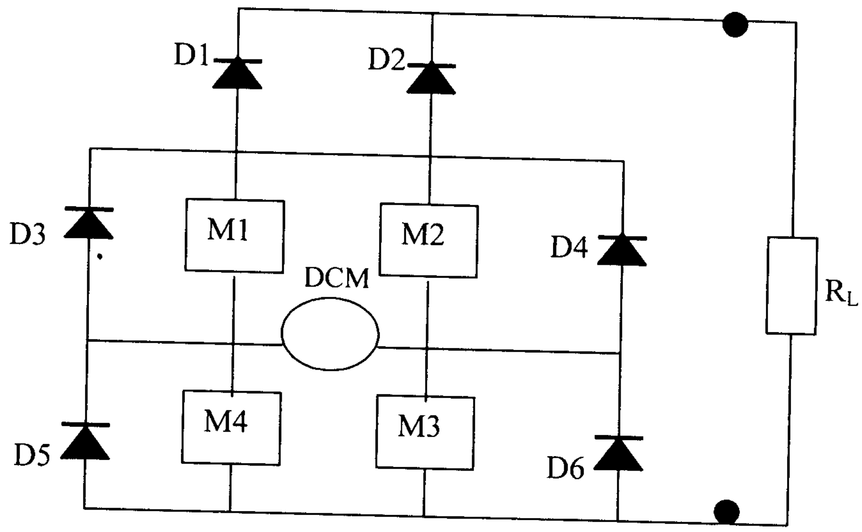
Q6. **(a)** The effect of the parasitic resistances (series resistance and shunt resistance) is to reduce the fill-factor of a solar cell module. Draw the corresponding I-V curves in each case to show the effects of these parasitic resistances on the fill-factor. [6]

(b) A sample of germanium is made of p-type material by adding $1.1x10^{20}$ of acceptor atoms. If the concentration of intrinsic carriers $n_i = 2.5x10^{19}\ m^{-3}$ at 300K and assuming that all the acceptor atoms are ionized at 300K, calculate the density of electrons and also compare it to the intrinsic charge carrier concentration. [8]

(c) In the figure below, the modules (M1 – M4) are connected in a bridge circuit where DCM is a DC motor and R_L is the load resistance.

(i) Identify the diodes (D1- D6) whether they are blocking diodes or by-pass diodes.

(ii) Explain the functions of each diode. [11]



--------------End of Examination--------------

UNIVERSITY OF ZAMBIA
DEPARTMENT OF PHYSICS

2013 SECOND HALF UNIVERSITY EXAMINATIONS

PHY 5822
SOLAR ENERGY MATERIALS

DURATION:	Three (3) hours
INSTRUCTIONS:	Answer **any four (4)** questions. The marks for each question are given in square brackets.
MAXIMUM MARKS:	100
DATE:	Thursday 17th July 2014

Some useful identities and formulae are given below:

$$k = 1.38x10^{-23} J/K \qquad \hbar = 1.055x10^{-34} J.s \qquad \eta = \frac{I_{sc}V_{oc}}{I_N A}FF \qquad \delta = \frac{2\pi nd}{\lambda}$$

$$n = N_c \exp\left[-\frac{E_c - E_F}{kT}\right] \qquad p = N_v \exp\left[-\frac{E_F - E_v}{kT}\right] \qquad n = n_i \exp\left[-\frac{E_i - E_F}{kT}\right] \qquad N = n + ik$$

$$E_F = E_c + kT\ln\left(\frac{N_D}{N_C}\right) \qquad I = I_o\left[e^{\frac{qV}{kT}} - 1\right] \qquad E_F = E_v - kT\ln\left(\frac{N_A}{N_V}\right) \qquad R = rr^*$$

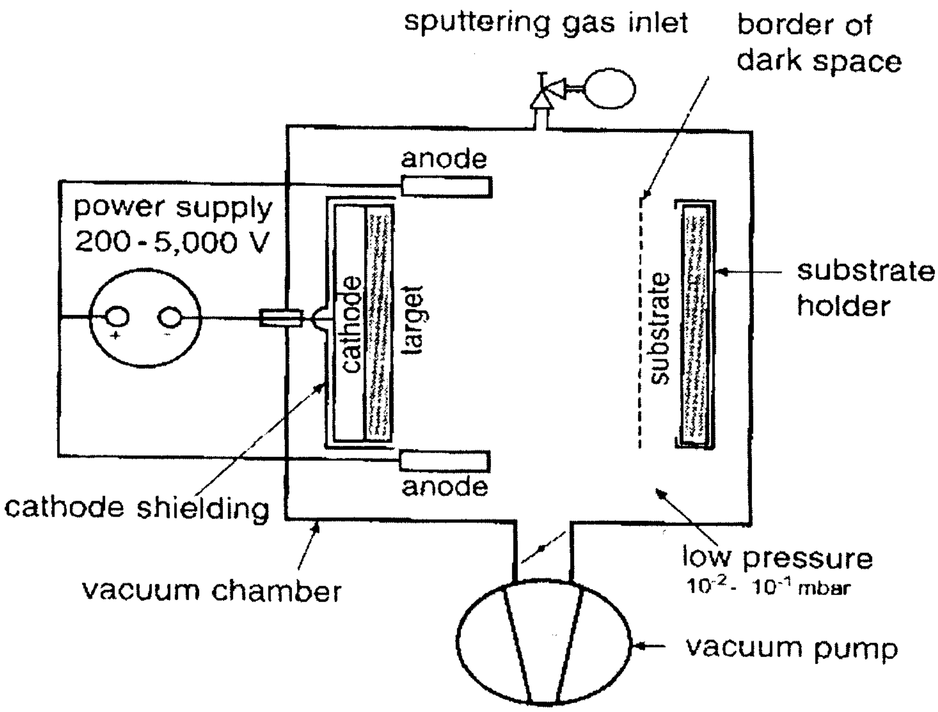
$$\nabla^2 = i\frac{\partial^2}{\partial x^2} + j\frac{\partial^2}{\partial y^2} + k\frac{\partial^2}{\partial z^2} \qquad \alpha = \frac{4\pi k}{\lambda} \qquad q = \frac{\omega}{c}N \qquad T = \frac{T_1T_2\exp(-\alpha d)}{1 - R_1R_2\exp(-2\alpha d)}$$

$$R = R_1 + \frac{R_1(1 - R_1^2)\exp(-2\alpha d)}{1 - R_1^2\exp(-2\alpha d)} \qquad R = \frac{(1-n)^2 + k^2}{(1+n)^2 + k^2} \qquad I = I_o e^{-\alpha x} \qquad \alpha = \frac{4\pi k}{\lambda} \qquad T = tt^*$$

$$T_\perp = \frac{N_2}{N_1}\left|\frac{2N_1}{N_2 + N_1}\right|^2 \qquad R_\perp = \left|\frac{N_1 - N_2}{N_1 + N_2}\right|^2 \qquad n_i^2 = np \qquad c = \frac{1}{\sqrt{\mu_o\varepsilon_o}} \qquad \varepsilon = \varepsilon_1 + i\varepsilon_2$$

$$R_p \approx \frac{\left[(n^2 + k^2)\cos^2\theta - 2n\cos\theta\right] + 1}{\left[(n^2 + k^2)\cos^2\theta + 2n\cos\theta\right] + 1} \qquad R_s \approx \frac{\left[(n^2 + k^2) - 2n\cos\theta + \cos^2\theta\right]}{\left[(n^2 + k^2) + 2n\cos\theta + \cos^2\theta\right]}$$

Q1. **(a)** The figure below shows one of the physical vapour deposition techniques used to deposit thin films called the sputtering technique. [5]



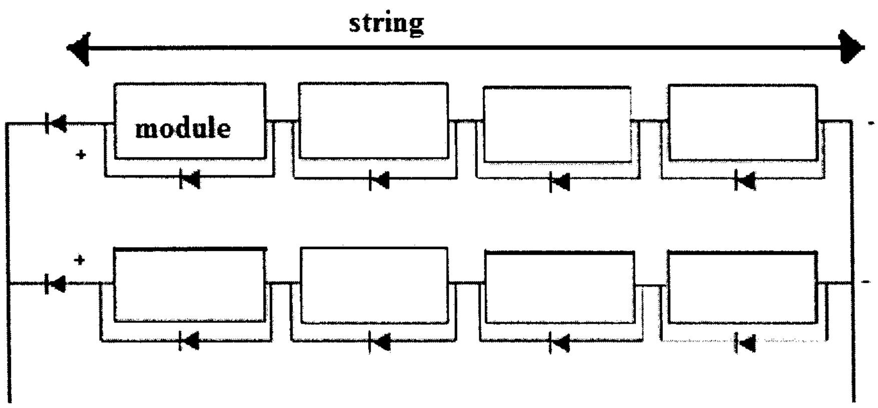
List any five (5) conditions favourable for the successful deposition of thin films on the substrate.

(b) The optical constants of austenitic stainless steel are $n = 1.9$; $k = 3.5$ at 500nm and $n = 5.4$; $k = 7.5$ at 2000nm. Calculate the reflectance at these wavelengths at

(i) normal incidence ($\theta_i = 0^o$),

(ii) glancing incidence ($\theta_i = 85^o$). [12]

(c) In the figure below, the modules are arranged in series making a module string. Identify the diodes in this figure and explain their functions. [8]



Q2. **(a)** The total amplitude reflectance coefficient for a single layer thin film may be expressed as

$$r = \frac{r_1 + r_2 e^{-2i\delta}}{1 + r_1 r_2 e^{-2i\delta}}$$

(i) Show that the total reflectance R for this thin film may be expressed as

$$R = \frac{r_1^2 + r_2^2 + 2r_1 r_2 Cos2\delta}{1 + r_1^2 r_2^2 + 2r_1 r_2 Cos2\delta} \qquad [6]$$

(ii) Similarly, show that the total transmittance T for this thin film may be expressed

$$T = \frac{1 + r_1^2 r_2^2 - (r_1^2 + r_2^2)}{1 + r_1^2 r_2^2 + 2r_1 r_2 Cos2\delta} \qquad [5]$$

(b) The table below shows the optical constants for gold as a function of photon energy. Calculate the experimental values of the real and imaginary parts of the complex dielectric function, and the angular frequency ω. [9]

$E = \hbar\omega(eV)$	0.89	1.02	1.14	1.26	1.51	1.64
n	0.43	0.35	0.27	0.22	0.16	0.14
k	9.52	8.15	7.15	5.66	5.08	5.54

(c) Write short notes on the following:

(i) Solar mirrors, Solar selective absorbers and Heat mirrors [1+1+1]

(ii) Thermal evaporation, and Spray pyrolysis [1+1]

Q3. (a) The variation of band gap energy with temperature for silicon is given by the relation,

$E_g(T) = E_g(0) - \dfrac{aT^2}{T+b}$, where $a = 7\text{x}10^{-4}$ eV/K, $b = 1100$ K, and $E_g(0) = 1.16$ eV.

(i) Calculate the band gap energy for silicon at 40°C.

(ii) Determine the temperature for zero band gap energy for silicon and explain for the choice of your answer. [13]

(b) A gallium arsenide solar cell with a band gap of 1.42 eV at a temperature of 300K has a short circuit current of 2.34A under normal illumination. Calculate the corresponding open circuit voltage given that the dark saturation current is $I_o = 1.5x10^{11} \exp\left(-\dfrac{E_g}{kT}\right)$ A. [12]

Q4. (a) The reflectance of an anti-reflecting coating is generally given as $R = \left(\dfrac{n_3 - n_2^{\,2}}{n_3 + n_2^{\,2}}\right)^2$, where n_3 is the refractive index of the substrate, and n_2 is the refractive index of the thin film (coating). A thin semiconductor film has been deposited on quartz (substrate) with refractive index 1.5. If the reflectance of the film is 0.2, calculate the

(i) thickness of the film

(ii) refractive index of the film

(iii) band gap energy when the cut-off wavelength is 0.43μm. [13]

(b) The spectral response of a solar cell is defined as the Amps generated per watt of incident light. Thus

Spectral response $= \frac{q\lambda}{hc}.QE$.

Where QE is the quantum efficiency of the solar cell, q is the electronic charge and h is the Planck's constant. Calculate the short-circuit current of a solar cell of area 4cm^2, illuminated by light of wavelength 800nm at an intensity of 200W/m^2. Take the quantum efficiency of the cell to be 0.80. [12]

Q5. **(a)** In a certain experiment, the total reflectance of silicon was found to be 30% at zero photon energy i.e $\omega = 0$. Use this information to calculate the static dielectric constant, $\varepsilon_1(0)$ of silicon. (Hint: Determine the refractive index of silicon). [12]

(b) A silicon ingot is doped with 10^{22} phosphorous atoms per cubic meter. Find the

(i) carrier concentrations (n and p) and

(ii) Fermi level measured from the conduction band and

(iii) Fermi level measured from the intrinsic Fermi level.

The values $n_i = 1.45x10^{16} m^{-3}$ and $N_c = 2.8x10^{25} m^{-3}$ for silicon are at room temperature (300K) and complete ionization is assumed. [13]

---------------End of Examination--------------

The University of Zambia

Department of Physics

University First Semester Examination

2012/13 Academic Year

PHY 5911

Computational Physics and Modelling I

Instructions **Max. Marks 100**

- ***Time allowed:*** *Three (3) Hours.*
- ***All questions carry equal marks.***
- *Marks for each question are shown in the square brackets [ ].*
- *Whenever necessary, use the information given in the* ***Appendix.***
- ***Answer any four (4) questions.***

Q.1 (a) In Cubic Spline approximation, we write the n cubic polynomial pieces as

$$S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3$$

on each subinterval $[x_i, x_{i+1}]$ for $i = 0, 1, \ldots, n-1$ where a_i, b_i, c_i and d_i represent $4n$ unknown coefficients, which when joined up as smoothly as possible result in the spline $S(x) = f(x) = y$. Defining $m_i = S_i''(x_i)$ and $h_i = x_{i+1} - x_i$ and using the fact that

1. $S_i''(x_{i+1}) = s_{i+1}''(x_{i+1})$
2. $S_i(x_i) = y_i$ and $S_i(x_{i+1}) = y_{i+1}$
3. $S_i'(x_{i+1}) = S_{i+1}'(x_{i+1})$,

show that

$$h_i m_i + 2(h_i + h_{i+1})m_{i+1} + h_{i+1}m_{i+2} = 6\left[\frac{y_{i+2} - y_{i+1}}{h_{i+1}} - \frac{y_{i+1} - y_i}{h_i}\right]$$

[10 Marks]

(b) Using the expression obtained in **a**, find the Natural spline for the following data points

i	0	1	2	3	4
x_i	-2	-1	0	1	2
y_i	4	-1	2	1	8

[15 Marks]

Q.2 (a) Using appropriate linear combinations of the Taylor expansions for $f(x+2h)$, $f(x+h)$, $f(x)$, $f(x-h)$ and $f(x-2h)$, show that

$$f^{(3)}(x) \approx \frac{1}{2h^3}\left[f(x+2h) - 2f(x+h) + 2f(x-h) - f(x-2h)\right]$$

and

$$f^{(4)}(x) \approx \frac{1}{h^4}\left[f(x+2h) - 4f(x+h) + 6f(x) - 4f(x-h) + f(x-2h)\right]$$

[15 Marks]

(b) Let $f(x) = x\ln x + x$, and $x = [0.9,\ 1.3,\ 2.1,\ 2.5,\ 3.3]$. Use the formula in **Q.2 (a)** for the approximation of the third derivative of the function $f(x)$ to approximate value of $-1/x^2$ at $x = 1.7$. Also compute the absolute error.

[10 Marks]

Q.3 (a) If A is the true (unknown) value of the quantity approximated by a low-order approximating formula $A(h)$ (for differentiation or integration) and that the error in the approximation can be expressed as

$$A - A(h) = a_2 h^2 + a_4 h^4 + \cdots$$

where h is the step-size and the coefficients of the error terms do not depend on it, derive the first-level Richardson extrapolation formula by separately using step-sizes h and $h/2$ to improve the approximation of A. Also, show that the error in the improved approximation can be expressed as

$$A - B(h) = b_4 h^4 + b_6 h^6 + \cdots$$

[10 Marks]

(b) Consider the data below;

x	1	2	3	4	5
$f(x)$	2	4	8	16	32

By taking $h = 2$, use the central-difference formula and first-level Richardson extrapolation to estimate $f'(3)$.

[15 Marks]

Q.4 (a) For a two variable function $u(x, y)$, a general point can be denoted as (x_i, y_j) and the value of the function at this point as $u_{i,j}$ and the step-size in the $x-$ and $y-$directions as $\Delta x = h$ and $\Delta y = k$ respectively. Show that the central finite-difference approximation formula for the first-order partial derivative with respect to x at the point (x_i, y_j) is

$$\frac{\partial}{\partial x} u(x_i, y_j) = \frac{u_{i+1,j} - u_{i-1,j}}{2h}$$

[10 Marks]

(b) Let the values of u at (x_i, y_j) be recorded in the matrix:

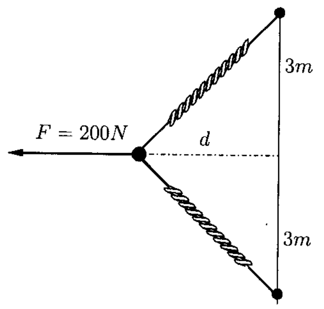
$$u_{i,j} = \begin{pmatrix} 5.1 & 6.5 & 7.5 & 8.1 & 8.4 \\ 5.5 & 6.8 & 7.8 & 8.3 & 8.9 \\ 5.5 & 6.9 & 9.0 & 8.4 & 9.1 \\ 5.4 & 9.6 & 9.1 & 8.6 & 9.4 \end{pmatrix}$$

Assume the indices begin at 1, i is the index for the rows and j index for the columns. Suppose that $h = 0.5$ and $k = 0.2$, use the central finite-difference formula to determine

$$\left.\frac{\partial u}{\partial y}\right|_{2,4}$$

[10 Marks]

Q.5 (a) To determine the displacement d of a spring of stiffness 400 N/m and unstretched length 6 m when a force of 200 N is applied, as illustrated in the figure below, two expressions are found for the tension T in each half of the spring.



First, T is half the horizontal component of the applied force; i.e.,

$$T = 100\sqrt{9 + d^2}/d$$

Second, T is the product of the sprint constant and the amount by which the spring is stretched; i.e.,

$$T = 400(\sqrt{9 + d^2} - 3)$$

Show that the equation involving only d is

$$4(\sqrt{9 + d^2} - 3)d - \sqrt{9 + d^2} = 0$$

[10 Marks]

(b) Using the Secant method, approximate the distance d in **Q.5 (a)** after three iterations given that d lies between 1.5 m and 2 m.

[15 Marks]

Q.6 (a) Determine the Lower (**L**) and Upper (**U**) and triagular matrices of **A**

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 0 \\ 2 & 6 & 2 & 2 & 1 & 0 \\ 3 & 14 & 12 & 3 & 4 & 1 \\ 1 & 2 & 1 & 2 & 6 & 12 \\ 0 & 2 & 2 & -1 & -3 & -6 \\ 2 & 4 & 3 & 3 & 2 & 2 \end{bmatrix}$$

[10 Marks]

(b) Given that $\mathbf{b} = [4 \quad 12 \quad 23 \quad -21 \quad 15 \quad 3]'$ use the *Doolittle* method to solve the system

$$\mathbf{Ax} = \mathbf{b}$$

[15 Marks]

⋆⋆⋆⋆⋆⋆⋆ **End of Examination** ⋆⋆⋆⋆⋆⋆⋆

Appendix

Taylor Expansion

For an arbitrary x_0 and $x = x_0 + h$, the Taylor expansion about x_0 can be written as

$$f(x \pm h) = \sum_{k=0}^{n} \frac{(\pm h)^k}{k!} f^{(k)}(x) + \frac{(\pm h)^{n+1}}{(n+1)!} f^{(n+1)}(\xi)$$

for $\xi \in [x_0, x]$.

Hermite Interpolation

A Hermite approximation polynomial of degree $2n+1$, at most, that matches both function values and some of the derivative values at some specified values of the independent variable is given by

$$P(x) = \sum_{i=0}^{n} [H_i(x)y_i + K_i(x)z_i]$$

where

$$\begin{aligned} H_i(x) &= [L_i(x)]^2(1 - 2L_i'(x_i)(x - x_i)) \\ K_i(x) &= [L_i(x)]^2(x - x_i) \\ L_i(x) &= \prod_{\substack{k=0 \\ k \neq i}}^{n} \left(\frac{x - x_k}{x_i - x_k} \right) \\ z_i &= y_i' \end{aligned}$$

Partial Derivatives

For a function of two variables, $u(x,y)$, its Taylor series for $u(x+\Delta x, y)$ about (x,y) is given by;

$$u(x + \Delta x, y) = u(x,y) + \Delta x \frac{\partial u}{\partial x}(x,y) + \frac{(\Delta x)^2}{2!}\frac{\partial^2 u}{\partial x^2}(x,y) + \frac{(\Delta x)^3}{3!}\frac{\partial^3 u}{\partial x^3}(x,y) + O[(\Delta x)^4]$$

and for $u(x - \Delta x, y)$ about (x,y) is

$$u(x - \Delta x, y) = u(x,y) - (\Delta x)\frac{\partial u}{\partial x}(x,y) + \frac{(\Delta x)^2}{2!}\frac{\partial^2 u}{\partial x^2}(x,y) - \frac{(\Delta x)^3}{3!}\frac{\partial^3 u}{\partial x^3} + O[(\Delta x)^4]$$

Cubic Spline

A function $f(x)$ can be approximated by n cubic polynomial pieces

$$S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3$$

The resulting spline $S(x)$ satisfies the following conditions

1. $S(x) = S_i(x)$ on each subinterval $[x_i, x_{i+1}]$ for $i = 0, 1, \dots n - 1$
2. $S_i(x_i) = y_i$ for $i = 0, 1, \dots n$
3. $S(x)$, $S'(x)$ and $S''(x)$ are continuous on $[x_0, x_n]$

Assuming that the second derivatives of $S(x)$ at the first (x_0) and last (x_n) knots are equal to zero, leads to a "Natural Spline". If it is assumed that the first derivatives at the first and last knots equal the first derivatives of the function $f(x)$, leads to a "Clamped Spline".

Secant Method

For a nonlinear function $f(x)$, the approximation to its zero,x_{k+1}, between x_{k-1} and x_k is given by

$$x_{k+1} = x_k - \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1}}f(x_k)$$

LU-factorization

The matrix $A = [a_{ij}]$ can be written in terms of its factor matrices L and U as $A = LU$. The entries of the matrices $L = [m_{ij}]$ (with main diagonal elements equal 1) and $U = [u_{ij}]$ are computed as follows

$$\begin{aligned}
u_{1j} &= a_{1j} \qquad \text{for } j = 1, \dots, n \\
m_{i1} &= \frac{a_{i1}}{u_{11}} \qquad \text{for } i = 2, \dots, n \\
u_{ij} &= a_{ij} - \sum_{s=1}^{i-1} m_{is}u_{sj} \qquad \text{for } j = i, \dots, n \qquad i \geqslant 2 \\
m_{ij} &= \frac{1}{ujj}\left(a_{ij} - \sum_{s=1}^{j-1} m_{is}u_{sj}\right) \qquad \text{for } i = j+1, \dots, n \qquad j \geqslant 2
\end{aligned}$$