DETERMINATION OF AN IMPROVED GEOID MODEL OVER ZAMBIA

BY

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A thesis submitted to the University of Zambia in partial fulfilment of the requirements of the Degree of MASTER OF ENGINEERING in Geodesy and Geo-informatics

THE UNIVERSITY OF ZAMBIA

LUSAKA

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DECLARATION

I hereby truthfully declare that the dissertation represents my own work, and that it has not previously been submitted for an award at any university or learning institution:

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APPROVAL

This thesis of Stanslous Saka has been approved as fulfilling the partial requirements for the award of Master of Engineering in Geodesy and Geo-informatics.

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ABSTRACT

In the determination of an improved geoid over Zambia, the Modified Stokes' formula employing the RCR technique was used. The formula combined EGM2008 data, terrestrial classical free-air gravity anomalies over Zambia, and the SRTM 3″ DEM over Zambia which represent the long, intermediate, and short-wavelength geoid respectively.

Preliminary accuracy evaluation of the proposed new geoid model termed ZG2016 at 4 GPS/Leveling stations reveal a rms error of 7.0cm compared to 69cm for the old model (ZG96: Nsombo, 1996). The higher accuracy of the new model is attributed to its use of a higher resolution GGM and DEM.

Suffice to mention that EGM2008-only gave a rms error of 12.2cm when evaluated at the 4 stations. This suggests that EGM2008, though a global model, may optionally be used as a geoid model over Zambia.

The evaluation of these two models using only 4 GPS/Leveling stations is not statistically acceptable. However, this is dictated by the fact that most of the established benchmarks in the country have been removed. More benchmarks will have to be found for better evaluation. If satisfactorily evaluated at more benchmarks, ZG2016 may play an important role in orthometric height determination by use of more efficient GNSS technology, thereby leading to minimisation of the use of the generally costly and labour intensive conventional leveling method in this regard. A good geoid model may also prove helpful to a number of earth-related sciences such as geology and geophysics.

Key words: Geoid, RCR technique, GPS/Leveling, GNSS technology, EGM2008

DEDICATION

This thesis is specially dedicated to my wife Lillian and to our beautiful daughter Lusungu.

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Suffice to mention that usually behind every successful work are people who rarely get the accolades they deserve for their valuable contributions. I gladly choose to show gratitude towards everyone who contributed to the success of my research.

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Above all, to God be the glory! Only through His good will can we achieve any small thing under the sun. A horse is made ready for the day of battle, but victory belongs to the Lord.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF APPENDICES

ACRONYMS

AC	Additive Correction
BGI	Bureau Gravimétrique International
CGIAR-CSI	Consultative Group of International Agricultural Research Consortium for Spatial Information
DEM	Digital Elevation Model
DTM	Digital Terrain Model
DWC	Downward Continuation
EGM	Earth Gravity Model
EGM96	Earth Gravity Model of 1996
EGM2008	Earth Gravity Model of 2008
FFT	Fast Fourier Transform
GGM	Global Geopotential Model
GNSS	Global Navigation Satellite Systems
GOCE	Gravity field and steady-state Ocean Circulation Explorer
GPS	Global Positioning System
GRACE	Gravity Recovery and Climate Experiment
GRS80	Geodetic Reference System of 1980
ICGEM	International Centre for Global Earth Models
KTH	Royal Institute of Technology
LSMS	Least Squares Modification of Stokes' formula
MATLAB	Matrix Laboratory

MOZGEO 2002	Mozambique Geoid Model of 2002
MSL	Mean Sea Level
NASA	National Aeronautics and Space Administration
NGA	National Geospatial-Intelligence Agency
RCR	Remove-Compute-Restore
RMS	Root Mean Square
RTM	Residual Terrain Model
SRTM	Shuttle Radar Topography Mission
THAI12G	Thailand Gravimetric Geoid Model of 2012
UGG2014	Uganda Gravimetric Geoid Model of 2014
WGS84	World Geodetic System of 1984

CHAPTER 1 INTRODUCTION

A vertical reference frame or datum forms the basis for all developmental projects in which heights are used. In particular any project involving the impounding, transport and distribution of water is critically dependent upon the appropriate vertical reference frame being used. Heights are generally considered to refer to mean sea level (MSL), and most vertical reference frames attempt to approximate mean sea level as the datum for heights. In principle, the geoid (a level surface which globally best fits MSL) is the ideal datum (Merry, 2003). The geoid is an equipotential surface of the earth's gravity field that best fits to global mean sea level in a least squares sense (Jekeli et al., 2009).

The geoid is used as a reference surface to which orthometric heights are referred, and the vertical datum is usually established using tide gauge measurements of mean sea-level in conjunction with geodetic leveling (Featherstone et al., 1998).

The orthometric height of a point refers to its vertical height above the geoid. According to Heiskanen and Moritz (1967), spirit leveling combined with gravity measurements furnishes potential differences which are basic to the theory of orthometric heights, i.e., orthometric heights are derived from potential differences. To accurately determine the orthometric heights of points, we need to carry out precise leveling, and this should be coupled with gravity measurements. However, this would prove to be highly tasking, strenuous, and costly especially for large scale projects.

Conversely, the orthometric heights can be determined from two data sets in the form of ellipsoidal heights and geoidal heights (geoidal undulation). The difference between the ellipsoidal height and the geoidal height at any given point gives the orthometric height of that point. The ellipsoidal heights can easily be obtained from Global Navigation Satellite Systems (GNSS) receivers whenever these are employed in point positioning. Thus the advent of GNSS technology provides a quicker means of obtaining orthometric heights by reduction of ellipsoidal heights. The fact that satellite surveying is quicker and hence cheaper in the long term makes this method advantageous to classical terrestrial surveying techniques.

The geoidal height required to reduce GNSS ellipsoidal heights can be determined gravimetrically. The geoid-WGS84-ellipsoid separation (geoidal height) can be computed from terrestrial gravity measurements in conjunction with a global geopotential model and a Digital Terrain Model (Featherstone et al., 1994).

1.1 Problem background and research justification

The geoid has been held by many as the fundamental reference surface of geodesy, and its precise determination has been and still is the centre of discussion for many geodesists. Traditionally, the geoid has served as the reference surface for orthometric heights and other vertical heights. The geoid represents in some way the Earth's physical surface and more importantly, it is closely associated with the Earth's gravity field (Nsombo, 1996).
Most surveying measurements are made in relation to the geoid because the equipment is aligned with the local gravity vector, usually through the use of a spirit bubble. As such geodesists have chosen an oblate ellipsoid of revolution, flattened at the poles, to approximate the geoid in order to simplify survey data reduction and mapping. On a global scale, the geoid departs from the WGS84 ellipsoid by approximately ±100m (Featherstone et al., 1998).

Due to the significance of the geoid as a reference surface for vertical heights, and the physical surface of the earth, and its close association to the earth's gravity field, it is so necessary that this surface be precisely determined over Zambia. According to Nsombo (1996), prior to the year 1996, there were not any systematic computation of the geoid in Zambia, save only erratic computations in limited areas. Due to the need of knowledge of the geoid over Zambia, a preliminary geoid over Zambia (ZG96) was gravimetrically determined in 1996 (Nsombo, 1996, 1998).

Then, the geoid model was computed from available information; terrestrial gravity, Doppler and GPS data, gravity anomalies from geopotential coefficients (EGM96), and topographic height information.

This research is aimed at computing a better geoid model taking advantage of improvements in data quality and quantity. The geoid still has to be determined using gravimetric means owing to the fact that this approach is cheaper and practically attainable compared to the method of geoid determination using leveling coupled with gravity measurements as the data sets required to compute an improved geoid using the gravimetric approach are readily available.

It is 20 years now since the preliminary geoid over Zambia was computed from data available then. This research is aimed at closing the gap in knowledge between what was known about the geoid then, and what can now be known. The undertaking of this research has a number of advantages as given below

1. The use of the higher resolution GGM produces a better geoid model. This time around the geoid was computed using the higher resolution EGM2008, as opposed to the EGM96. The EGM96 is a gravity model of the earth resulting from the spherical harmonic expansion of the geopotential up to degree and order 360, implying a spatial resolution of 30′ (30 angular minutes) whereas the EGM2008 is the expansion of the geopotential up to degree 2190, and order 2159 resulting in a spatial resolution of 5′ (5 arc minutes) . Over EGM96, EGM2008 represents improvement by a factor of six in resolution, and by factors of three to six in accuracy depending on gravitational quantity and geographic area, (Pavlis et al.,2012).The use of such a model may help provide gravity anomalies in all areas where there is scanty terrestrial gravity data.

2. Some corrections to terrestrial gravity anomalies are to be made with the use of a Digital Elevation Model (SRTM 3″/90m resolution) mostly to handle the effects of topographic masses above the geoid. This new model has a higher resolution than the previous models used back then. This helps improve the accuracy of the model.

3. To facilitate the practical utilisation of the determined geoid, a computer program/algorithm is to be written in one of the latest programming languages (e.g Visual Basic, MATLAB, etc.) and to eventually be made available to surveyors, engineers, and other users, so that they will be able to compute orthometric heights of points by inputting geocentric coordinates into the program. Before this research such a computer program was not available.

4. Nowadays most surveys (control, geodetic, and engineering) are established using satellite positioning systems such as GPS. The reference frame for GPS is the Geodetic Reference System 1980 (GRS80), where heights are referred to the GRS80 ellipsoid, and not to MSL. In order for Global Navigation Satellite Systems (GNSS) derived ellipsoidal heights to have any physical meaning in a surveying or engineering application, they must be transformed to orthometric heights.

   The Global Positioning System (GPS) provides surveyors with three-dimensional coordinates with respect to the geocentric World Geodetic System 1984 (WGS84). Before the GPS-derived positions can be used in the local reference frame however, two distinctly different coordinate transformations must be applied. The horizontal transformation from WGS84 latitude and longitude to the local horizontal datum is relatively straight-forward when using conformal or projective transformation models. In the future however, these horizontal coordinate transformations may become unnecessary as different countries move to the use of a geocentric datum for surveying and mapping (Featherstone et al., 1998).

   It is however, unlikely that ellipsoidal heights will ever be used for practical surveying, engineering, or geophysical applications as they have no physical meaning. For instance, when using ellipsoidal heights, there is the possibility that water will appear to flow up-hill because the physical force of gravity is not considered. Therefore, it will always be necessary to transform GPS-derived ellipsoidal heights to orthometric heights, using knowledge of the position of the geoid with respect to the WGS84 ellipsoid (Featherstone et al., 1998). Thus we see that in engineering survey applications such as in dam construction, the use of orthometric heights with a physical meaning, as opposed to the geometric ellipsoidal heights becomes necessary for correct design of dams.

The author has also observed that in Zambia static GNSS measurements are used in international boundary surveys because of the advantages offered by GPS over conventional methods in carrying out surveys especially for large scale projects. Use of GNSS is generally cheaper, quicker, and more convenient. However, the final adjusted coordinates of the boundary beacons, which have been built at an average spacing of 500m, are in WGS 84 coordinates with heights being ellipsoidal. The countries desire to have these heights referenced to mean sea level (orthometric heights). It would be very costly and difficult to determine the orthometric heights of these beacons on a very long and mountainous stretch using conventional leveling. It is for such reasons that a good geoid model becomes handy. It is ideal to have a model whose height accuracy is as close as possible to the vertical accuracy inherent in GNSS surveys.

The aforementioned advantages emphasize the need for re-computation of the geoid over Zambia.

1.2 Research objectives

1.2.1 Main objective

To gravimetrically determine an improved geoid over Zambia using the Earth Gravity Model of 2008 (EGM2008) and Digital Elevation Model (SRTM 3″/90m resolution)

1.2.2 Specific objectives

1. Assessment of the requirements and strategies required to come up with a better geoid model
2. Computation of the geoid model in (1) above.

3. Evaluation of the determined geoid model in terms of accuracy
4. Facilitation of the practical utilisation of the new model by way of an interpolation program

1.3 Research questions

1. What are the benefits of computing a gravimetric geoid model?
2. What data is currently available, suitable, and obtainable to facilitate the gravimetric geoid determination over Zambia?
3. What numerical /computational strategies will be employed in the gravimetric determination of the geoid?
4. How will the accuracy of the newly determined geoid over Zambia be assessed?
5. How will the new geoid model be practically utilised?

1.4 Significance of study

The new geoid model will ensure a quicker way of determining orthometric heights by utilising GPS/GNSS technology, but at the same time guaranteeing improved sub-metre accuracy (as seen in literature) in orthometric heights which is acceptable for many practical applications in surveying and mapping. The resulting geoid model from this study may also prove useful to a number of earth-related sciences, e.g. Geology, Geophysics, etc. This is because the position of the geoid below the topography reveals important truths about the structure and characteristics of subsurface materials. Information about the geoid provide manifestations about geologic conditions and geologic features. Variations in geoid pattern may result from different subsurface mass distributions. Finally, the current study may act as a platform for further study and model development

CHAPTER 2 LITERATURE REVIEW

The geoid is an equipotential surface of the earth's gravity field that best fits to global mean sea level in a least squares sense (Jekeli et al., 2009). The geometric quantities of the earth such as its size and shape can be determined from its physical quantities such as gravity.

In this chapter literature is reviewed with a view to gaining understanding of the subject of gravimetric geoid determination. The requirements and strategies for gravimetric geoid determination are explored. Finally, some case studies of recent national/regional geoid models are presented. The data and methodologies/strategies used, together with the achieved model accuracies are presented with a view to taking a leaf from such studies.

2.1 Geodetic boundary-value problems

According to Heiskanen and Moritz (1967), the potential V satisfies Poisson's equation:

$$\Delta V = -4\pi k\rho \tag{2.1}$$

where

$$\Delta V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \tag{2.2}$$

The symbol Δ, called the Laplacian operator, has the form

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Outside the attracting bodies, i.e. in empty space, the density ρ is zero and 2.2 becomes (Heiskanen and Moritz, 1967)

$$\Delta V = 0 \tag{2.3}$$

This is the famous Laplace's equation whose solutions are called harmonic functions. For this reason the potential of gravitation is a harmonic function outside the attracting masses, but not inside the masses where the density ρ is not zero. There it satisfies Poisson's equation.

Thus the gravitational potential is harmonic at all points where there are no attracting masses, and, consequently, so is the outer potential of the earth if we disregard the atmosphere and the centrifugal force. This is the reason for the basic importance of harmonic functions in physical geodesy (Heiskanen and Moritz, 1967).

2.1.1 Dirichlet's problem (1st Boundary-value problem of potential theory)

Stokes' theorem states that a function V harmonic outside a surface S is uniquely determined by its values on S. In general, however, there are infinitely many mass distributions which have the given harmonic function V as exterior potential.

According to Heiskanen and Moritz (1967) it is impossible to determine uniquely generating masses from the external potential. They state that the inverse problem of potential theory (determination of the masses from the potential) has no unique solution and that it occurs in geophysical prospecting by gravity measurements where invisible masses are inferred from disturbances of the gravity field. They also mention that the problem can be solved more completely, if additional information is furnished, for example, by geology or by seismic measurements.

On the other hand, they mention that the direct problem of potential theory i.e. the determination of the potential from the masses does have a unique solution.

Stokes' theorem states that there is only one harmonic function V that assumes given boundary values on a surface S, provided that such a harmonic function exists. The assertion that for arbitrarily prescribed boundary values there always exists a harmonic function V that assumes on S the given boundary values is called Dirichlet's principle (Heiskanen and Moritz, 1967).

The problem of computing the harmonic function (inside or outside S) from its boundary values on S is Dirichlet's problem, or the first boundary value problem of potential theory according to Heiskanen and Moritz (1967). Dirichlet's problem can be solved by Poisson's integral which can be an explicit solution of Dilichlet's problem for the exterior of the sphere, which has many applications in physical geodesy.

2.1.2 Nuemann's problem (2nd Boundary-value problem of potential theory)

In Nuemann's problem, the normal derivative $\partial V/\partial n$ is given on the surface S, instead of V itself. The normal derivative is the derivative along the outward-directed surface normal n to S. For a sphere the solution of this boundary value problem is easily expressed in terms of spherical harmonics.

2.1.3 Robin's problem (3rd Boundary-value problem of potential theory)

In the third boundary-value problem a linear combination of V and its normal derivative

$$hV + k\frac{\partial V}{\partial n}$$

is given on S. According to Heiskanen and Moritz (1967) and Sjöberg (1990), the third boundary-value problem is particularly relevant to physical geodesy, because the determination of the undulations of the geoid from gravity anomalies is just such a problem. This problem is called boundary-value problem of physical geodesy. Just as the first boundary-value problem can be solved directly by a surface integral (Poisson's integral), similar integral formulas also exist for the second and third problems. An integral formula that solves the boundary-value problem of physical geodesy is Stokes' integral.

The so-called the fundamental equation of physical geodesy which relates the measured gravity anomaly Δg to the unknown anomalous potential T is given by (Heiskanen and Moritz, 1967; and Sjöberg, 1990)

$$-\frac{\partial T}{\partial n}+\frac{1}{\gamma}\frac{\partial \gamma}{\partial n}T=\Delta g \tag{2.4}$$

where Δg is assumed to be known at every point of the geoid, then we see that a linear combination of T and $\partial T/\partial n$ is given upon that surface. Here $\partial \gamma/\partial n$ is the normal gradient of normal gravity. Thus we see that the determination of T (the small difference between the actual gravity potential, W, and the normal potential U) is a third boundary-value problem of potential theory. If it is solved for T, then the geoidal height, which according to Heiskanen and Moritz (1967) is the most important geometric quantity in physical geodesy, can be computed by Bruns' formula which is given by equation 2.7.

Heiskanen and Moritz (1967) further state that the basic problem of physical geodesy, which is the determination of the geoid from gravity measurements, is essentially a third boundary-value problem of potential theory.

2.2 Gravimetric determination of the geoid

According to Amos (2010) the most common method of establishing a vertical datum has been to determine Mean Sea Level (MSL) at a tide gauge and then transfer the level to benchmarks in the hinterland by precise leveling. He mentions that precise leveling is a labour intensive and expensive method of transferring heights that only provides heighted benchmarks along the leveling routes. What this means is that where there are no roads it is not possible to efficiently implement a national vertical datum based on precise leveling alone.

If the orthometric height of every point on the earth's surface was known, then the geoid would precisely be modeled. According to Amos and Featherstone (2003, 2009) a geoid model can be computed using the gravimetric method, provided spatially dense and accurate gravity and terrain data are available. The gravimetric method furnishes the geoidal height (N) from gravity and terrain data.

The primary practical application of the geoidal height N in land surveying and geodesy is to transform the geometric GNSS-derived ellipsoidal heights h to orthometric heights H which are related to the earth's gravity field and as such have a physical meaning. The geoid can be obtained globally when the ellipsoidal height of every point on the earth's surface is reduced by subtracting the geoidal undulation at that particular point (Refer to Figure 2.1). Thus the geoid is gravimetrically determined using the simple formula (Amos, 2010)

$$H = h - N \tag{2.5}$$

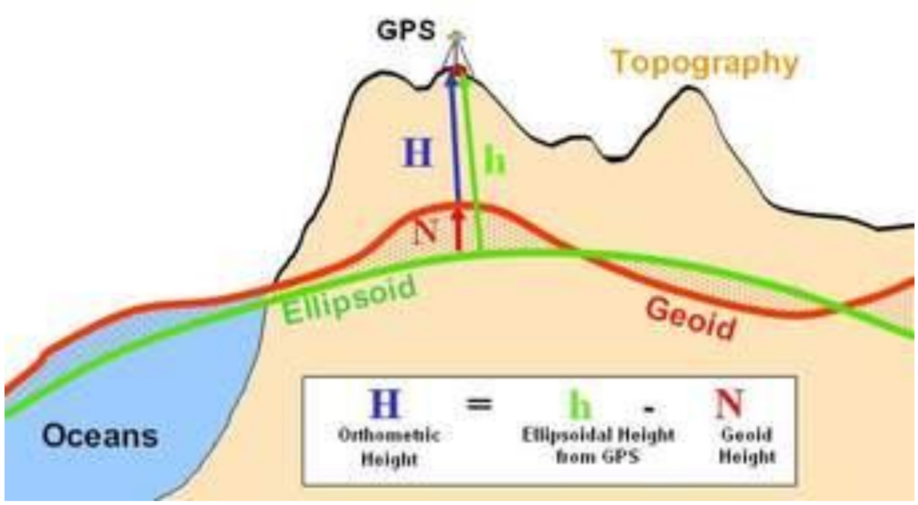

Figure 2.1: Orthometric, ellipsoidal, and geoidal height (Source: http://principles.ou.edu/earth_figure_gravity/geoid/index.html)

Amos and Featherstone (2003) emphasise that the use of a geoid model in conjunction with GPS provides a very attractive alternative to geodetic spirit leveling, especially over long distances and steep terrain.

As can be found in most physical geodesy literature, a level ellipsoid is chosen to represent the earth because the gravity field of an ellipsoid is of fundamental practical importance because it is easy to handle mathematically and the deviations of the actual gravity field from the ellipsoidal "normal" field are so small that they can be considered linear. On the contrary, the geoid, which is supposed to be the reference for both heights and position (x,y) has rather disagreeable mathematical properties. It is a smooth, but complicated surface with discontinuities of curvature. Thus, it is not suitable as a surface on which to perform mathematical computations directly, as on the ellipsoid.

The physical height H of a point above sea level (also called the orthometric height) is measured along the curved plumb line, starting from the geoid (Figure 2.1). On the other hand, the geometric (vertical) height of a point above the ellipsoid is called ellipsoidal height h, and it differs from the orthometric height H by the geoidal undulation N (Heiskanen and Moritz, 1967). Figure 2.1 illustrates the three heights.

The ellipsoidal height is the elevation above the ellipsoid and it is measured by global navigation satellite systems (GNSS). Borge (2013) states that a satellite needs a reference for its measurements and uses a reference ellipsoid to give coordinates and heights. He mentions that a reference ellipsoid is used as reference for satellites so that they can give coordinates and heights in this system. He also makes the point that the ellipsoid is used because it is a good approximation to the earth's shape and also a mathematical surface.

Once again, the important point coming out of literature is that if the geoidal height and ellipsoidal height is known at every point on the earth, then the geoid can be accurately determined using equation (2.5).

2.2.1 Stokes' integral

An integral formula that solves the boundary-value problem of physical geodesy (equation 2.4) is Stokes' integral. The disturbing potential T, can be computed from the famous Stokes' formula (Heiskanen and Moritz, 1967) as

$$T = \frac{R}{4\pi} \iint_{\sigma} \Delta g \, S(\psi) \, d\sigma \tag{2.6}$$

Then by Bruns' theorem,

$$N = \frac{T}{\gamma} \tag{2.7}$$

the geoidal height is finally obtained as (Heiskanen and Moritz, 1967):

$$N = \frac{R}{4\pi\gamma} \iint_{\sigma} \Delta g \, S(\psi) \, d\sigma \tag{2.8}$$

where

R is the mean radius of the earth (6371km)

γ is the normal gravity on the reference ellipsoid

Δg is the measured terrestrial gravity anomaly; which is the difference in magnitude between the actual gravity vector at a geoid point P, and the normal gravity vector at a corresponding point Q on the ellipsoid (P and Q lie on the same ellipsoidal normal through Q), i.e., $\Delta g = g_P - \gamma_Q$. Refer to Figure 2.2.

dσ is the element of area in which the gravity anomalies are defined; the integration being performed over a unit sphere σ

S(ψ) is the Stokes' function written in full as:

$$S(\psi) = \frac{1}{sin(\frac{\psi}{2})} - 6\,sin\left(\frac{\psi}{2}\right) + 1 - 5cos\psi$$
$$- 3cos\psi\, ln\left(sin\left(\frac{\psi}{2}\right) + sin^2\left(\frac{\psi}{2}\right)\right) \tag{2.9}$$

This formula (2.8) was published by George Gabriel Stokes in 1849, and is therefore, called Stokes' formula or Stokes' integral. It is by far the most important formula of physical geodesy because it makes it possible to determine the geoid from gravity data.

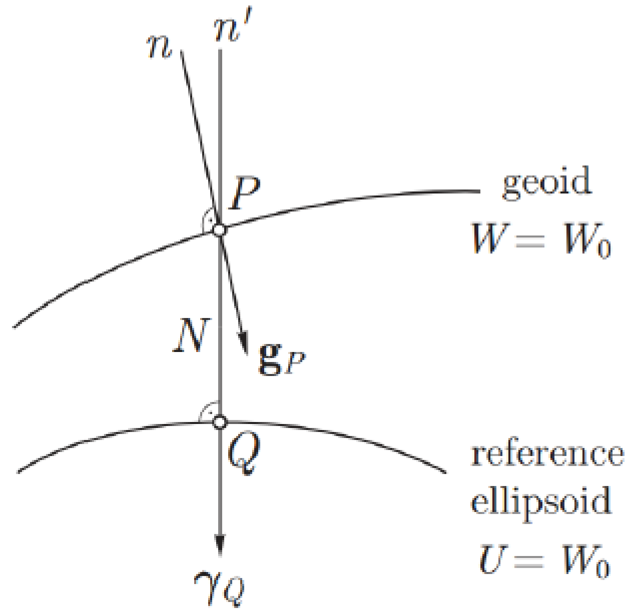

Figure 2.2 Geoidal undulation and gravity anomaly (Hofmann-Wellenhof & Moritz (2006) page 91)

According to Heiskanen and Moritz (1967) and Sjöberg (1990), Stokes' integral in its original form (equation 2.8) holds only for a reference ellipsoid that

1. Has the same potential $U_o = W_o$ as the geoid,
2. Encloses a mass that is numerically equal to the earth's mass, and
3. Has its centre at the centre of gravity of the earth

Finally, T is assumed to be harmonic outside the geoid. This means that the effect of the masses above the geoid must be removed by suitable gravity reductions. The reader is referred to Heiskanen and Moritz (1967), Sjöberg (1990), and Bajracharya (2003) for a detailed explanation on gravity reductions.

2.2.2 Practical evaluation of Stokes' integral

In the previous section we have learned that the classical solution to the geodetic boundary value problem can be given by Stokes' formula which is a surface integral. Since we do not know the exact analytical expression of the gravity anomaly Δg, Stokes' integral (2.8) cannot be evaluated analytically. Furthermore, what we are able to obtain from measurements is g $(or\ \Delta g)$ at a finite number of discrete points, other than the continuous Δg at every point on the surface of the earth. Therefore, according to Heiskanen and Moritz (1967) and Fan (1989) the above analytical integral has to be approximated by numerical integrations in practical applications. They present two methods in physical geodesy for numerical integrations, depending on how the surface element $\Delta\sigma$ is formed. The two are the template method and the grid method. The template method was mostly used for manual calculation in the earlier days when electronic computers were not available, while the grid method is particularly suitable for large computations by computers. For this reason only the grid method is described in this section.

Assume that we have divided the whole surface of the earth (or the unit sphere) by parallel circles and meridian circles into a set of grid blocks (Figure 2.3).

The size of each block is expressed by the latitude difference $\Delta\varphi$ and longitude difference $\Delta\lambda$. This means that there are $M_1=360°/\Delta\lambda°$ blocks on each parallel circle and $M_2=180°/\Delta\varphi°$ blocks on each meridian circle.

In practice, Δg is often available only inside a regional area σ_o with minimum/maximum latitude $\varphi_{min}/\varphi_{max}$ and minimum/maximum longitude $\lambda_{min}/\lambda_{max}$. In this case the original

Stokes' integral has to be truncated into integrations only over the area σ_o. Assume that σ_o is divided into blocks of size $\Delta\varphi$ by $\Delta\lambda$ and inside each block $\Delta\sigma_{ij}$ (Figure 2.3), the mean gravity anomaly $\overline{\Delta g}_{ij}$ is given. Now the number of blocks on each meridian circle and parallel circle is $M_1 = (\varphi_{max} - \varphi_{min})/\Delta\varphi$ and $M_2 = (\lambda_{max} - \lambda_{min})/\Delta\lambda$, respectively. The total number of blocks for the whole region σ_o is $M_1 * M_2$. In this case the geoidal undulation can be calculated from Δg inside σ_o as

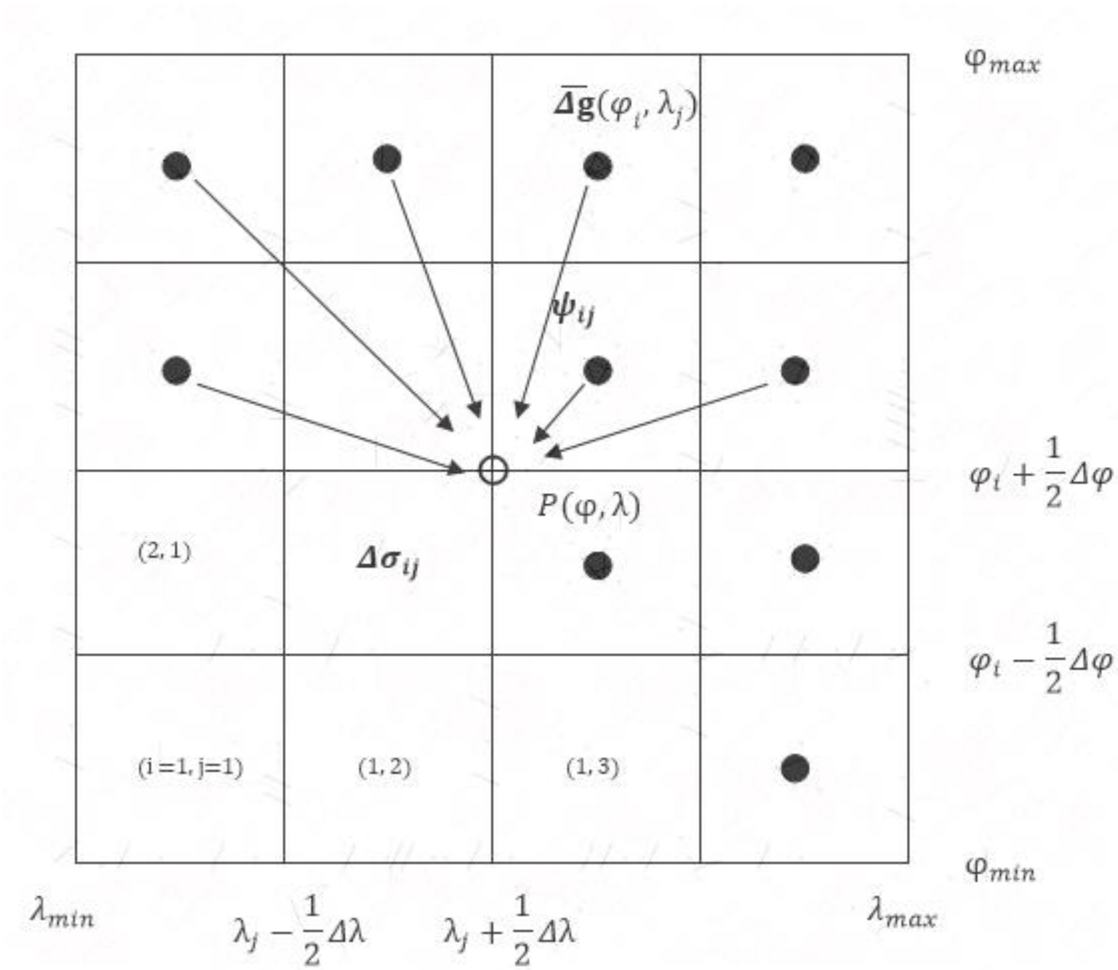

Figure 2.3: Grid method of numerical evaluation of Stokes' formula

$$\tilde{N}\,(\varphi,\lambda) = \frac{R}{4\pi\gamma}\iint\limits_{\sigma_o} \Delta g\; S(\psi)\; d\sigma = \sum_{i=1}^{M1}\sum_{j=1}^{M2}\frac{R}{4\pi\gamma}\iint\limits_{\Delta\sigma_{ij}} \Delta g\; S(\psi)\; d\sigma \qquad (2.10)$$

If we approximate Δg in $\Delta\sigma_{ij}$ by $\overline{\Delta g}_{ij}$ (i = 1,2,…, M_1; j = 1,2,…., M_2) and S(ψ) by $S(\psi_{ij})$ (where ψ_{ij} is the spherical distance from the computation point P (φ, λ) to the centre of block $\Delta\sigma_{ij}$), then equation (2.10) becomes

$$\tilde{N}\,(\varphi,\lambda) \cong \sum_{i=1}^{M1}\sum_{j=1}^{M2}\frac{R}{4\pi\gamma}\iint\limits_{\Delta\sigma_{ij}} \overline{\Delta g_{ij}}\; S(\psi_{ij})\; d\sigma$$

Simplifying this equation further gives

$$\tilde{N}\,(\varphi,\lambda) = \left(\frac{R}{4\pi\gamma}\right)\sum_{i=1}^{M1}\sum_{j=1}^{M2} \overline{\Delta g_{ij}}\; S(\psi_{ij})A_{ij} \qquad (2.10')$$

where A_{ij} denotes the area of block $\Delta\sigma_{ij}$ and is given by

$$A_{ij} = \iint\limits_{\Delta\sigma_{ij}} d\sigma = \int_{\varphi_i-\frac{1}{2}\Delta\varphi}^{\varphi_i+\frac{1}{2}\Delta\varphi}\int_{\lambda_j-\frac{1}{2}\Delta\lambda}^{\lambda_j+\frac{1}{2}\Delta\lambda} \cos\varphi\; d\varphi d\lambda$$

i.e.

$$A_{ij} = 2\Delta\lambda \sin\left(\frac{1}{2}\Delta\varphi\right) cos\varphi_i = A_i \qquad (2.11)$$

where φ_i and λ_j denote the latitude and longitude of the centre of the block $\Delta\sigma_{ij}$ and they can be calculated from the following formulae

$$\varphi_i = \varphi_{min} + \left(i - \frac{1}{2}\right)\Delta\varphi \qquad (2.12a)$$

and

$$\lambda_j = \lambda_{min} + \left(j - \frac{1}{2}\right)\Delta\lambda \qquad (2.12b)$$

Finally ψ_{ij} can be obtained from

$$cos\psi_{ij} = sin\varphi sin\varphi_i + cos\varphi cos\varphi_i \cos(\lambda - \lambda_j)$$

Stokes function can also be written in a much simpler form as

$$S(t) = 1 - 5t - 3\sqrt{2 - 2t} + \sqrt{\frac{2}{1 - t}} - 3t\ln\left(\sqrt{\frac{1 - t}{2}} + \frac{1 - t}{2}\right) \qquad (2.13)$$

where

$$t = cos\psi$$

2.3 Modification of Stokes' integral

The surface integral in Stokes' formula (2.8) has to be applied over the whole Earth. According to Nsombo (1996) and Abdalla (2009), the original Stokes' formula has a major drawback since it calls for a continuous coverage of gravity anomalies. However, practically the area is limited to a small spherical cap σ_o, around the computation point due to limited coverage of available gravity anomaly data. Hence, the surface integral cannot be extended all over the Earth. Accordingly the surface integral has to be truncated to gravity anomaly area σ_o , and then we get an estimator of the geoidal height:

$$\tilde{N}\,(\varphi, \lambda) = \frac{R}{4\pi\gamma} \iint_{\sigma_o} \Delta g\, S(\psi)\, d\sigma \qquad (2.14)$$

The difference between geoidal height in equation (2.8) and the new estimator in equation (2.14) δN is called the truncation error of Stokes' formula (Abdalla, 2009):

$$\widehat{N}(\varphi, \lambda) = \frac{R}{4\pi\gamma} \iint_{\sigma-\sigma_o} \Delta g\, S(\psi)\, d\sigma \qquad (2.15)$$

where $\sigma - \sigma_o$ is called the remote zone (the area outside the gravity area). As a result of this truncation, the computed geoid undulation suffers from a truncation error (Nsombo, 1996).

It is explained that the truncation error of the remote zone can be reduced when Stokes' formula combines the short-wavelength terrestrial gravity anomalies and long-wavelength (up to maximum degree M) Global Geopotential Model (GGM). When combining information from the GGM with Stokes' integration over local gravity data, regional geoid models may be estimated. Abdalla (2009) highlights two components that are considered in geoid modeling. These are the long-wavelength component provided by a GGM (using spherical harmonics) and the short-wavelength component from local gravity observations. He explains that by using local gravity data, Stokes' formula will be truncated to an inner zone, and that this results in truncation errors due to the lack of gravity data in remote zones.

Various other authors including Kuroishi (1995), Lemonie et al. (1996), and Smith and Milbert (1999) have discussed different approaches to the minimisation of the truncation error as well as the possible compensation for the lack of a global coverage of gravity data by employing, among other methods, a combination of terrestrial gravity with global geopotential models (GGM) through the modification of Stokes' formula.

In addition to these, Amos et al. (2003) mentions that a modern gravimetric geoid uses a combination of three primary input data sources (Amos et al., 2003):

(i) A global geopotential model which provides most of the long and intermediate wavelength geoid undulations

(ii) Terrestrial gravity data (from land or ship-based observations, or derived from satellite altimetry in open and maritime areas) in and surrounding the area of interest, which supply most of the intermediate wavelengths.

(iii) A high resolution digital elevation model (DEM), which supplies most of the short wavelengths, and is also required to satisfy theoretical demands of geoid computation from the geodetic boundary- value problem

2.3.1 The RCR technique

According to Sjöberg (2005) the Remove-Compute-Restore (RCR) technique is the most well-known method for regional gravimetric geoid determination today. Its basic theory is the first-order approximation of either Molodensky's method or the classical geoid modelling by Helmert's second method of condensing the topography onto the geoid. He explains that the method involves the removal of the less precise long-wavelength terrestrial gravity anomaly field from Stokes' integral by utilising a higher order reference field represented by a more precise Earth gravity model (EGM) and the restoration of the EGM as a low-degree geoid contribution in order to produce a high accuracy geoid model. He explains that further improvement is achieved also by removing and restoring a residual topographic effect, which favourably smoothes the gravity anomaly to be integrated in Stokes formula.

Sjöberg (2005) makes the important point that the use of terrestrial gravity data in a Stokes-type solution has been improved for regional geoid determination by using a higher-order reference field taken from an EGM. In these combined solutions, the EGM is primarily intended to represent the long-wavelength gravity field, while a Stokes-type integral with residual gravity computes the high frequency signal. The removal and restoration of the high-frequency topographic effects is done with the help of a digital terrain model (DTM).

In the RCR technique Stokes' integral does not operate on the full gravity anomaly, but only on a residual gravity anomaly reduced by the EGM and DTM (Sjöberg, 2005).

Bajracharya (2003) as well as Srinivas et al. (2012) indicated that a global geopotential model, local gravity information and digital terrain model represent the low, medium and high frequency part of the gravity signal, respectively. They explain that the residual gravity anomaly Δg_{res} is obtained after the removal of the long-wavelength gravity anomalies (using a global geopotential model) and the topographic effect from the terrestrial free-air gravity anomalies. First, the gravity anomalies are reduced in a remove step using a mass reduction scheme to formulate boundary values on the geoid, which can be expressed as (Bajracharya, 2003):

$$\Delta g_{res} = \Delta g_F - \Delta g_T - \Delta g_{GGM} \tag{2.16}$$

where Δg_F represents the free-air anomalies, Δg_T the direct topographical effect on gravity for a particular reduction method used, and Δg_{GGM} is the reference gravity anomaly from a geopotential model.

The direct topographical effect on gravity Δg_T for each mass reduction scheme can be expressed as (Bajracharya, 2003):

$$\Delta g_T = A_T - A_{(Inv,Cond,Comp,Ref)} \tag{2.17}$$

where A_T is the attraction of all topographic masses above the geoid and $A_{(Inv,Cond,Comp,Ref)}$ represents the attraction of either inverted topographical masses, or the condensed masses, or the compensated masses, or the reference topographic masses for the Rudzki, Helmert, Airy-Heiskanen or Pratt-Hayford, and Residual Terrain Model (RTM) reduction schemes, respectively.

The reference gravity anomaly Δg_{GGM} and geoidal undulation N_{GGM} at a computation point P (r, θ, λ) are expressed by (Heiskanen and Moritz, 1967; Pavlis et. al. (2012) and Srinivas et al., 2012):

$$\Delta g_{GGM}(r,\theta,\lambda) = \left(\frac{GM}{r^2}\right) \sum_{n=2}^{n_{max}} \left(\frac{a}{r}\right)^n (n-1) \sum_{m=0}^{n} P_{nm}(\cos\theta)[\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda] \tag{2.18}$$

$$N_{GGM}(r,\theta,\lambda) = \left(\frac{GM}{r\gamma}\right) \sum_{n=2}^{n_{max}} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} P_{nm}(\cos\theta)[\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda] \tag{2.19}$$

where GM is the geocentric gravitational constant referring to the total mass (earth's body plus atmosphere), (r, θ, λ) are the spherical polar coordinates of the computation point P (geocentric radius, co-latitude, and longitude, respectively), γ is the mean normal gravity, a is the semi-major axis of the ellipsoidal earth model, $\bar{C}_{nm}$ and $\bar{S}_{nm}$ are the fully normalized spherical harmonic coefficients of a global geopotential model (GGM), $\bar{P}_{nm}$ are the fully normalized Associated Legendre polynomials, n and m are the degree and order, respectively, of the expansion of a GGM.

In the compute step the Stokes' integral with residual gravity Δg_{res} computes the high frequency signal $N_{\Delta g}$ (residual geoid) using equation 2.14.

The total geoid obtained as the result of the restore step can be expressed as (Smith and Roman, 2001; Bajracharya, 2003; Dumrongchai et al., 2012; Srinivas et al., 2012):

$$N = N_{\Delta g} + N_{GGM} + N_{ind} \tag{2.20}$$

where $N_{\Delta g}$ represents residual geoid obtained by using Δg_{res} from equation (2.16) in Stokes' formula, N_{GGM} denotes the long wavelength part of the geoid obtained from a global geopotential model, and N_{ind} is the indirect effect on the geoid which depends on the mass reduction scheme used.

The removal or shifting of masses which underly the gravity reductions changes the gravity potential and hence the geoid. This change of the geoid is an indirect effect of the gravity reductions. The surface computed by Stokes's formula without considering the indirect effect on geoid is called the cogeoid, which is not the geoid. This surface is also called the regularized geoid since it is obtained by regularizing the external masses above the geoid surface as Stokes's approach requires (Heiskanen and Moritz, 1967). Figure 2.4 shows the relation between geoid and the co-geoid. The vertical distance between geoid and co-geoid caused by the change in potential due to the gravimetric reduction process is called the indirect effect on geoid. The indirect effect for Helmert's second method of condensation can be obtained in planar approximation as (Wichiencharoen, 1982; Smith and Milbert, 1999; Smith and Roman, 2001)

$$N_{ind} = -\frac{\pi G\rho}{\gamma} h_p^2 \tag{2.21}$$

where h_P is the elevation of point P, ρ is the density of the earth's crust, G is the Universal gravitational constant, and γ is the normal gravity.

Finally, making use of equation 2.14, we see that the total geoid for the RCR technique is given by

$$N(\varphi,\lambda) = \frac{R}{4\pi\gamma}\iint_{\sigma_o} \Delta g_{res}\, S(\psi)\, d\sigma + N_{GGM} + N_{ind} \tag{2.20'}$$

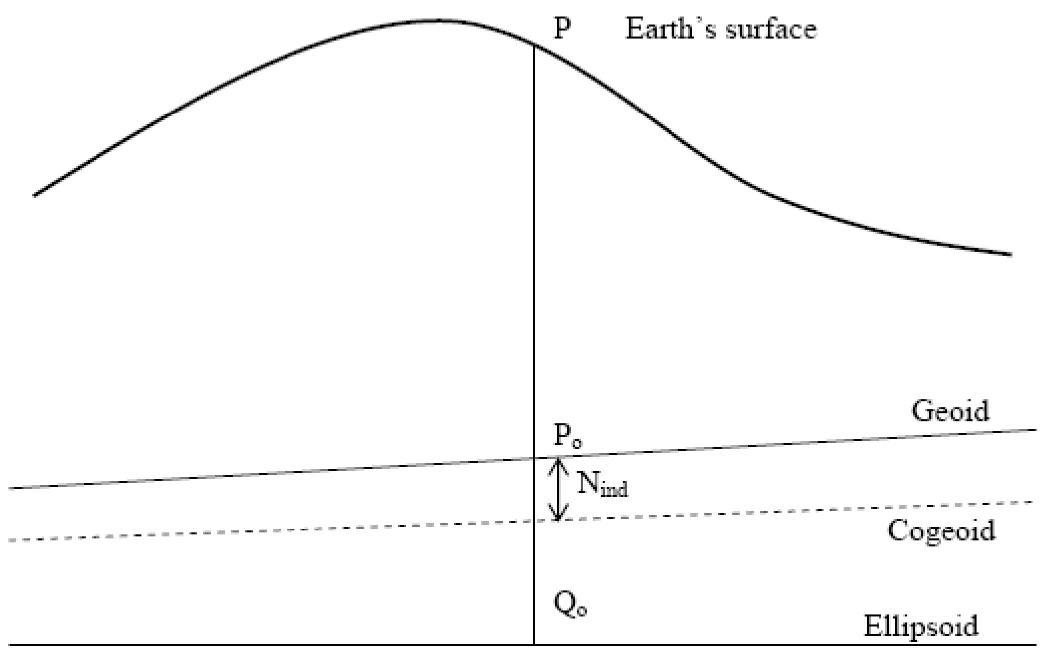

Figure 2.4: The indirect effect on the geoid (Bajracharya, 2003)

2.3.2 Terrestrial gravity measurements

According to Fan (2002) terrestrial gravity measurements have been the main information source for geoid determination since Stokes derived his well-known formula. It is the direct measurement of the magnitude of the gravity vector. However, due to different reasons not all parts of the world are covered by gravity surveys. This lack of continuous global coverage of terrestrial gravity measurements makes it impossible to apply Stokes' formula in its original form, hence its restriction to a spherical cap (with adequate gravity information) and modification to incorporate long-wavelength satellite-based gravity field measurements for the region outside the spherical cap.

2.3.3 Global Geopotential Models (GGMs)

GGMs are spherical harmonic models of the Earth's gravitational potential. This is according to Fan (2002) who alludes to the fact that Global Geopotential Models are primarily derived from satellite orbit tracking data and that high degree GGMs also use mean values of terrestrial gravity measurements as well as gravity anomalies over oceans derived from satellite altimetry data. He mentions that a GGM can best describe the global structure of the gravity field and thus the global shape and form of the geoid. However, it lacks details of the gravity field and the geoid due to high altitude of the satellites and sampling of point gravity data.

EGM2008 is a spherical harmonic model of the Earth's gravitational potential (a GGM), developed by a least squares combination of the ITG-GRACE03S gravitational model and its associated error covariance matrix, with the gravitational information obtained from a global set of area-mean free-air gravity anomalies defined on a 5 arc-minute equiangular grid. This grid was formed by merging terrestrial, altimetry-derived, and airborne gravity data. Over areas where only lower resolution gravity data were available, their spectral content was supplemented with gravitational information implied by the topography (Pavlis et al., 2012).

According to Pavlis et al. (2012) EGM2008 is complete to degree and order 2159, and contains additional coefficients up to degree 2190 and order 2159. Over areas covered with high quality gravity data, the discrepancies between EGM2008 geoid undulations and independent GPS/Leveling values are on the order of 5 to 10 cm. EGM2008 vertical deflections over USA and Australia are within 1.1 to 1.3 arc-seconds of independent astrogeodetic values. These results indicate that EGM2008 performs comparably with contemporary detailed regional geoid models. EGM2008 performs equally well with other GRACE-based gravitational models in orbit computations. Over EGM96, EGM2008 represents improvement by a factor of six in resolution, and by factors of three to six in accuracy, depending on gravitational quantity and geographic area. The reader is referred to Pavlis et al. (2008, 2012) for details on the EGM2008.

It is important to note here the achievable accuracy of EGM2008 reported by Pavlis et al. (2012). This is very important because it is clear that the use of this high resolution EGM2008 is likely to improve significantly the accuracy of regional gravimetric geoid models. This is so because the contribution of the GGM to the total geoidal undulation solution is greater in the Modified Stokes' formula compared to that of the terrestrial gravity field. The terrestrial gravity field which supplies most of the intermediate wavelength only contributes towards the residual cogeoid height $N_{\Delta g}$(see equation 2.20).

2.3.4 Digital Terrain Models (DTM)

DTMs are terrain heights in regular grid format. They provide detailed description of the surface of the earth on which gravity measurements are made and thus contribute to the determination of the high-frequency information of the gravity field and the geoid (Fan, 2002).

Stokes formula assumes that there are no masses outside the geoid and that the gravity anomalies are given on the geoid. These two assumptions are not true and gravity reduction is thus needed. As topographic masses near the point of geoid computation has direct and significant influences on the gravity and the geoid at the computation point, terrain data can be used to model the high frequency details of the gravity field and the geoid (Fan, 2002).

2.4 Determination of the geoid from ground level anomalies

It is by now clear that the reduction of gravity to sea level necessarily involves assumptions concerning the density of the masses above the geoid. This is so because it is impossible to determine the density of the masses at every point between the geoid and the ground. In practice an assumption that $\rho = 2.6g/cm^3$ is usually made.

To avoid this assumption, Molodensky in 1945 proposed a different approach. Figure 2.5 shows the geometrical principles of this method.

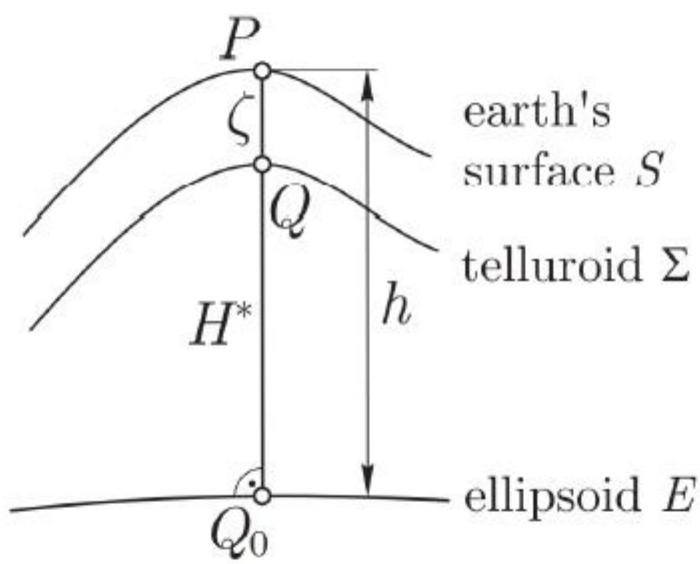

Figure 2.5: The height anomaly (Hofmann-Wellenhof & Moritz (2006) page 297)

The ground point P is projected onto the ellipsoid according to Helmert. However, the geometric height h is now determined by (Heiskanen and Moritz, 1967):

$$h = H^* + \zeta \tag{2.22}$$

where $H^*$is the normal height replacing the orthometric height H, and the height anomaly ζ replacing the geoidal undulation N. The vertical distance from the ellipsoid to the telluroid is the normal height H^*. The height anomaly is given by (Heiskanen and Moritz, 1967):

$$\zeta = h - H^* \qquad (2.22')$$

closely corresponding to the geoidal undulation N = h – H, which is the difference between the geometric and the orthometric height.

The gravity anomaly (According to Molodensky's theory) is now defined as (Heiskanen and Moritz, 1967; Abdalla, 2009):

$$\Delta g = g_P - \gamma_Q \tag{2.23}$$

It is the difference between the actual gravity as measured on the ground and the normal gravity on the telluroid.

Molodensky anomalies are referred to ground level, whereas the classical gravity anomalies are referred to sea level. A direct formula for computing γ at Q is (Heiskanen and Moritz, 1967; Abdalla, 2009):

$$\gamma = \gamma_o[1 - 2(1 + f + m - 2f sin^2\varphi)\frac{H^*}{a} + 3\left(\frac{H^*}{a}\right)^2] \qquad (2.24)$$

where γ_o is the corresponding value on the ellipsoid, a is the ellipsoid's semi-major axis, and m is as given in Heiskanen and Moritz (1967, page 75).

The height anomaly is given by the equation (Heiskanen and Moritz, 1967; Sjöberg, 2005):

$$\zeta = \zeta_0 + \zeta_1 = \frac{R}{4\pi\gamma_Q}\iint_\sigma \Delta g\ S(\psi)\ d\sigma + \frac{R}{4\pi\gamma_Q}\iint_\sigma G_1\ S(\psi)\ d\sigma \qquad (2.25)$$

The reader is referred to Heiskanen and Moritz (1967) as well as Sjöberg (2005) for a description of the terms in equation 2.25.

Finally, Heiskanen and Moritz (1967) and Sjöberg (2005) state that the geoidal undulation N, can be obtained from the height anomaly ζ as follows:

$$N = \zeta + H^* - H = \zeta + \frac{\bar{g} - \bar{\gamma}}{\bar{\gamma}}H \cong \zeta + \frac{\Delta g_B}{\bar{\gamma}}H \qquad (2.26)$$

where $\bar{g}$ is the mean gravity along the plumb line between geoid and ground, and $\bar{\gamma}$ is the mean normal gravity along the normal plumb line between ellipsoid and telluroid, and the term $\bar{g} - \bar{\gamma}$ is approximately equal to the Bouguer anomaly Δg_B.

2.5 Case studies

1. Mozambique geoid model (MOZGEO 2002)

To develop the MOZGEO 2002, various data sets related to geoid determination were collected. Terrestrial gravity measurements made in Mozambique during the period of Portugal rule and later by UK Military Service, as well as gravity measurements made in Malawi, South Africa, Tanzania, Zambia, and Zimbabwe were used. Digital terrain heights in Southern Africa from the 30″ by 30″ GLOBE digital terrain model were secured. A global geopotential model GPM98C, complete to degree and order 1800 was used in geoid determination by Modified Stokes' formula (Fan, 2002).

5′ by 5′ mean gravity anomalies were derived from the terrestrial gravity measurements. It turned out, however, that only about 12 percent of the 5′ by 5′ blocks were covered by gravity measurements in Southern Africa. Within Mozambique, the northern part of the country lacked gravity measurements except 59 UK gravity data points, while the southern part was relatively better surveyed. The remaining empty blocks were filled by GPM98C-derived gravity anomalies, leading to a complete set of 5′ by 5′ for the data area.

As a test of MOZGEO 2002, geoid heights were computed at 13 GPS/Leveling stations in the Mozambique GPS network. The gravimetrically computed geoid heights were compared to GPS/leveling derived geoid heights. For the total of 13 stations, an average difference of 16cm, with a standard deviation of 1.467 metres was found. At 5 GPS stations located in Southern Mozambique, the differences were all below one metre, while large differences existed in the poorly surveyed Northern Mozambique (Fan, 2002).

The Modified Stokes' formula after Smith and Milbert (1999) and Smith and Roman (2001) was employed as follows

$$N(\varphi,\lambda) = \frac{R}{4\pi\gamma}\iint\limits_{\sigma_o} (\Delta g - \Delta g_{GGM})\, S(\psi)\, d\sigma + N_{GGM} + N_{ind}$$

where

$$\Delta g = \Delta g_F + C_P + A$$

Here Δg is the reduced terrestrial free-air gravity anomaly, Δg_F is the terrestrial free-air gravity anomaly, C_P is the terrain correction, and A is the atmospheric correction. The terrain correction is given by

$$C_p = \frac{1}{2} G\rho R^2 \iint_\sigma \frac{(H - H_P)^2}{l_o^3}\, d\sigma$$

where

R = mean earth radius

ρ = mean crustal distance

l_o = $2Rsin(\frac{\psi}{2})$ = spherical distance between computation and running point

H_P = orthometric height of computation point

H = orthometric height of running point

G = Universal gravitational constant

The atmospheric correction is given by

$$A = 0.8658 - 9.727 * 10^{-5} * H_P + 3.482 * 10^{-9} * H_P^2$$

Δg_{GGM} is as given in equation 2.18

N_{GGM} is as given in equation 2.19

N_{ind} is as given in equation 2.21

2. The Uganda geoid model (UGG2014)

According to Sjöberg et al. (2015) the remove-compute-restore (RCR) is perhaps the most well-known approach to gravimetric geoid determination and has been applied in most parts of the world. They further mention that, as an alternative, the Least Squares Modification of Stokes formula (LSMS) with additive corrections (AC), commonly called the KTH method, has been gaining prominence since winning the geoid modeling competition at the International Hotine-Marussi Symposium in 2009. The method was developed at the Royal Institute of Technology (KTH) Division of Geodesy by Sjöberg. Sjöberg et al. (2015) reveal that, compared to other methods, this method is superior because it is the only method that minimizes the expected global mean square error of the estimated geoid height. They state that in contrast to most other methods of modifying Stokes' formula, which only strive at reducing the truncation error, the KTH method matches the errors of truncation, gravity anomaly and the Global Geopotential Model (GGM) in a least squares sense.

The computation of a gravimetric geoid model over Uganda (UGG2014) was done by Sjöberg et al. (2015) using the KTH Method who mentions that for many developing countries such as Uganda, precise gravimetric geoid determination is hindered by the low quantity and quality of the terrestrial gravity data. With only one gravity data point per 65 square kilometres, gravimetric geoid determination in Uganda appears an impossible task. However, recent advances in geoid modelling techniques coupled with the gravity-field anomalies from the Gravity Field and Steady-State Ocean Circulation Explorer (GOCE) satellite mission have opened new avenues for geoid determination especially for areas with sparse terrestrial gravity. UGG2014 was derived from sparse terrestrial gravity data from the International Gravimetric Bureau, the 3 arc second SRTM ver4.1 Digital Elevation Model from CGIAR-CSI and the GOCE-only global geopotential model GO_CONS_GCF_2_TIM_R5. To compensate for the missing gravity data in the target area, the surface gravity anomalies extracted from the World Gravity Map 2012 were employed. Using 10 Global Navigation Satellite System (GNSS)/levelling data points distributed over Uganda, the RMS fit of the gravimetric geoid model before and after a 4-

parameter fit is 11 cm and 7 cm respectively. These results show that UGG2014 agrees considerably better with GNSS/levelling than any other recent regional/global gravimetric geoid model. The results also emphasize the significant contribution of the GOCE satellite mission to the gravity field recovery, especially for areas with very limited terrestrial gravity data. With an RMS of 7 cm, UGG2014 is a significant step forward in the modelling of a "1-cm geoid" over Uganda despite the poor quality and quantity of the terrestrial gravity data used for its computation (Sjöberg et al., 2015).

The Least Squares Estimator of the KTH method is given by Sjöberg et al. (2015) as

$$\tilde{N}\,(\varphi,\lambda) = \frac{R}{4\pi\gamma}\iint_{\sigma_o} \Delta g\, S^L(\psi)\, d\sigma + c\sum_{n=0}^{M}(Q_n^L + s_n)\Delta g_n^{GGM} + \delta N_{comb}^T + \delta N_{dwc} + \delta N_{tot}^a + \delta N_{tot}^e$$

where σ_o is the spherical cap, R is the mean Earth radius, γ is mean normal gravity on the reference ellipsoid, $S^L(\psi)$ is the modified Stokes' function, c = R/2γ, s_n are the modification parameters, M is the maximum degree of the GGM, L is the maximum degree of modification, Q_n^L are the Molodensky truncation coefficients, Δg is the unreduced surface gravity anomaly, Δg_n^{GGM} is the Laplace surface harmonic of the gravity anomaly determined by the GGM of degree n. The estimator in the equation is the so-called combined estimator, which means that the truncated Stokes' formula is applied to the unreduced surface gravity anomaly after which the final geoid height is determined by adding a number of additive corrections, i.e. , δN_{comb}^T - the combined topographic correction, δN_{dwc} the downward continuation correction, δN_{tot}^a - the total atmospheric correction and δN_{tot}^e- the total ellipsoidal correction. For details regarding the additive corrections, the reader is referred to Sjöberg et al. (2015).

i. Global geopotential model used

For the computation of UGG2014, the GOCE-only model GO_CONS_GCF_2_TIM_R5 up to degree 280 was used, whose standard deviations of 37 cm and 29 cm before and after the 4-parameter fitting respectively are the lowest for the satellite-only GGMs. This was preferred in order to guard against correlations that may arise between the errors in the GGM and the terrestrial gravity anomalies in the case of the combined model. However, these results also highlight the contribution of the GOCE satellite mission to the gravity field recovery as the difference in standard deviations between the GOCE-only model to degree and order 280 and the combined model (EGM2008) complete to degree and order 2159 is approximately only 17cm.

ii. The DEM used

The 3 arc second SRTM ver4.1 Digital Elevation Model from CGIAR-CSI was used in geoid computation. Height information was used in the computation of the Bouguer correction, which was used in the conversion of the surface free-air anomalies to Bouguer anomalies which were then used in the gridding procedure. In addition, heights were required in the computation of the combined topographic correction and the downward continuation (DWC) effect, which are additive corrections to the approximate geoid height.

3. The Sudan gravimetric geoid model

Abdalla (2009) determined a gravimetric geoid model of Sudan using the method of the Royal Institute of Technology (KTH) developed by Professor L.E Sjöberg as in the case of the Uganda geoid model. He reports that the method is based on least-squares modification of Stokes' formula (LSMS). Herein the modified Stokes' function is applied instead of the original one, which has a very significant truncation bias unless a very large area of integration is used around the computation point.

In the KTH method, the surface gravity anomaly and GGM are used with Stokes' formula, providing an approximate geoid height. Previously, several corrections must be added to gravity to be consistent with Stokes' formula. In contrast, here all such corrections (Topographic, Downward Continuation, Ellipsoidal and Atmospheric effects) are added directly to the approximate geoid height. This yields the corrected geoid height, which may be tested against geometrical geoid height derived from the GPS/levelling data, so as to assess the precession of the gravimetric geoid model.

Finally, the assessment of the new gravimetric geoid was done in absolute sense by computing the global mean square error as an internal accuracy, while the external accuracy determined by comparing with GPS/levelling in term of relative sense. The standard deviation is served as an indication of accuracy in absolute sense, and in the study the standard deviation of the agreement between the new geoid and 19 GPS/levelling points after 7-parameter fitting was estimated to 0.29 m. The additive corrections had improved the standard deviation of the final gravimetric geoid height after using 7-Parameter fitting to become 0.29 m instead of 0.42m of the uncorrected geoid heights.

The computation formula applied here is the same as that applied in the case of Uganda above (KTH Method).

4. Gravimetric geoid of a part of South India

Srinivas et al. (2012) computed a geoid model for a part of South India using the RCR technique. Computed geoid undulations are compared with geoid obtained from global geopotential models such as EGM2008 and EIGENGRACE02S and measured GPS-levelling records at 67 locations. Statistical analysis of comparison suggested that the computed gravimetric geoid model had a good match with the geoid determined from GPS-levelling with rms of 0.1 m whereas EGM2008 has 0.46 m. The differences of GPS-levelling with EGM2008 at majority of stations fell in the range of ±0.5 m, which indicates that EGM2008 may be used for orthometric height determination with an accuracy of <0.5 m in the south Indian region and offers a reasonably good transformation platform from ellipsoid to local datum. However, Srinivas et al. (2012) insist that local determination of geoid is necessary for better accuracy of orthometric height from GPS. The gravimetric geoid calculated from the available gravity data showed considerable improvement to the global model and can be used to achieve orthometric height with an accuracy of 0.1 m.

They also mention that hybrid global geopotential models such as EGM2008 use long wavelength data from satellites and short wavelength data from available terrestrial gravity, and that these models provide fairly good information over a region of small geoidal anomalies, or when substantial data from that region are used in developing the model; which is not the case for South India (and here geoidal height decreases up to -106 m in the Indian ocean).

The geoidal undulations were computed as in equation (2.20′) and this is repeated here:

$$N(\varphi,\lambda) = \frac{R}{4\pi\gamma}\iint_{\sigma_o}(\Delta g_{res})\, S(\psi)\, d\sigma + N_{GGM} + N_{ind}$$

where Δg_{res} is as given in equation (2.16).

5. The Thailand geoid model (THAI12G)

According to Dumrongchai et al. (2012), the non-tidal EGM2008 global geopotential model from degree 2 to 2190, and 3949 terrestrial gravity measurements were used to contribute long and medium-scale information of geoid structure for THAI12G gravimetric model using the RCR technique.

They mention that since the gravimetric determination of the geoid requires adequate and accurate gravity data over the area of computation, significant errors may result from where there are data gaps in gravity measurements. Therefore, in the mountainous terrains devoid of gravity measurements, topography-implied gravity anomalies were simulated using high resolution residual terrain model (RTM) data from a three-arcsecond digital elevation model (DEM). The Shuttle Radar Topography Mission (SRTM) three-arc-second DEM contributed high frequency gravity field signals and was also used to generate terrain corrections for the computation of Faye anomalies (free air anomalies plus terrain corrections). EGM2008-only gravity anomalies were used to pad coastal and marine areas as well as neighbouring countries to reduce spurious features during gridding of the areas.

Fits of 200 GPS/leveling reference points to THAI12G showed a 60.6cm root mean square (rms).

It was also noted that THAI12G performs equivalently to EGM2008, i.e. there is no significant difference between the two models and that this may indicate that EGM2008 can be used alone over Thailand. They further claim that these numerical findings signify that the addition of Thailand local gravity data does not deteriorate the long-and-medium wavelength structures of EGM2008 in THAI12G. Finally, they conclude that the accuracy of THAI12G can be significantly improved if more terrestrial gravity data becomes available (Dumrongchai et al., 2012).

The computation formulae in this case is as given in equation (2.20′). The reduced free-air anomalies are Faye anomalies.

These appropriate case studies (research) together with the other literature herein reviewed provided the enlightenment required for geoid determination over Zambia. These case studies and literature were specifically referred to for the following reasons

i. They reveal important and practical principles in gravimetric geoid determination
ii. They reveal the recent and popular methods in geoid determination across the world
iii. They help give an insight into the levels of accuracy that may be expected in gravimetric geoid determination in different scenarios
iv. They reveal the not so obvious techniques required for geoid determination especially in areas like Zambia with scarce gravity data. For instance, they reveal

how to overcome such a challenge through void-filling and good interpolation techniques in order to come up with a gridded data set which is a prerequisite in geoid determination using Stokes' formula.

v. They highlight the strength or accuracy of the GGMs in regional gravity modeling. It is for this reason that even for areas like Zambia, with scanty gravity data, there is hope that the few terrestrial gravity measurements can help improve the already practically accurate EGM2008, albeit by a small margin.

The methodology and strategies for geoid determination over Zambia will now be presented.

CHAPTER 3 RESEARCH METHODOLOGY

In this chapter the research approach and design is discussed. Basically, the strategy, data and steps required to determine a better geoid model for Zambia are outlined.

3.1 Study area

The research aimed at coming up with a geoid model over Zambia, i.e. between 18°S $\leq \varphi \leq$ 8°S and 22°E $\leq \lambda \leq$ 34°E. Figure 3.1 shows the study area.

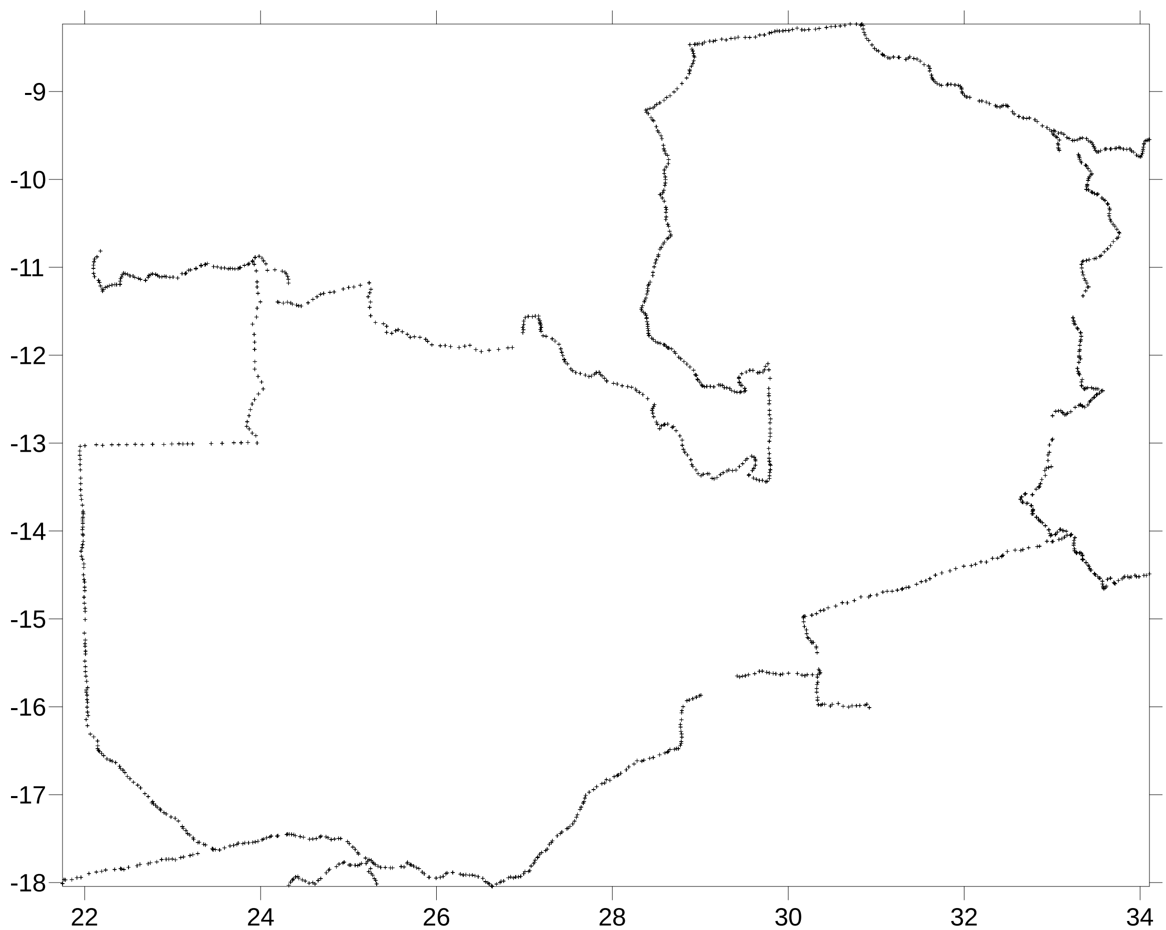

Figure 3.1: Study area (Zambia: 18°S $\leq \varphi \leq$ 8°S and 22°E $\leq \lambda \leq$ 34°E)

3.2 Research approach and design

As discovered in literature review, the Remove-compute-restore (RCR) is the most common technique for gravimeric geoid determination in the world at the moment. This method has served its purpose so well thus far.

According to Sjöberg et al. (2015), the Least Squares Modification of Stokes formula (LSMS) with additive corrections (AC), commonly called the KTH method is an alternative to the RCR technique. They mention that the KTH method is superior because it is the only method that minimizes the expected global mean square error of the estimated geoid height i.e. in contrast to most other methods of modifying Stokes' formula, which only strive at reducing the truncation error, the KTH method matches the errors of truncation, gravity anomaly and the Global Geopotential Model (GGM) in a least squares sense.

Sjöberg (2005) argue that the RCR technique must employ the modified Stokes' kernel technique and all the refined topographic, atmospheric, and other corrections to gravimetric geoid determination available today. They mention that if this is not taken care of, a precise EGM that in future will result from the satellite gravity and gradiometry missions, in combination with the RCR technique will most probably not achieve today's demands for a 1-cm geoid model.

For this study, however, the well-known RCR technique described in Section 2.3.1 and employed in geoid determination by the likes of Smith and Milbert (1999), Fan (2002), Srinivas et al. (2012), and Dumrongchai et al. (2012) was adopted by the author for its simplicity and convenience. The KTH method which is beginning to gain prominence may be theoretically superior to the RCR technique, but it will not be used for now as it requires much study. As has been noted in literature review, accuracies of geoid models in the range of 10cm are achievable in certain parts of the world using the RCR technique.

3.2.1 Computation formula

The geoid computation formula employed in this study is the Modified Stokes' formula based on the RCR technique as described in Section 2.3.1. The formula is given by equation 2.20′, and is repeated here.

$$N(\varphi,\lambda) = \frac{R}{4\pi\gamma}\iint_{\sigma_o} \Delta g_{res}\, S(\psi)\, d\sigma + N_{GGM} + N_{ind} \tag{2.20′}$$

All the terms are as described in Sections 2.2.1 and 2.3.1.

Suffice to mention that the geoidal undulations were also approximated using surface free-air anomalies by employing equations 2.25 and 2.26. However the small correction term containing G_1 (equation 2.25) which represents the effect of the topography (or the terrain correction) was neglected. The major reason for its exclusion is the complication associated with its computation. According to Heiskanen and Moritz (1967) and Dumrongchai et al. (2012) the terrain correction computation method requires the use of rectangular prisms or templates and fast fourier transforms (FFT), a method which at present is a challenge. Nevertheless, Heiskanen and Moritz (1967) argue that the terrain correction is a small correction, and that even for mountains 3000 metres in height, the terrain correction is only of the order of 50 mgals. Hence, for practical geoid determination, it may be neglected, but theoretically it is important.

3.2.2 Computation procedure

The surface integral in equation 2.20′ above was solved numerically using the technique described in Section 2.2.2. The integration was restricted to the study area i.e. between 18°S ≤ φ ≤ 8°S and 22°E ≤ λ ≤ 34°E.

Determination of the geoid requires that an appropriate grid of geoidal undulations over a study area be computed. The task was to compute this grid of geoidal undulations N at a set of pre-defined computation points forming a 5′ by 5′ grid over Zambia. With this grid of geoidal undulations, the geoid over Zambia could be well described, and interpolation at any other point would be possible. The geoid determination procedure was as follows:

1. Creation of a 5′ by 5′ grid over Zambia corresponding to the 5′ by 5′ resolution of the EGM2008 used in the geoid computation. The grid intersections are the computation points at which the geoidal undulations $N(\varphi, \lambda)$ are to be calculated. Between 18°S ≤ φ ≤ 8°S and 22°E ≤ λ ≤ 34°E we get a grid of 121 rows and 145 columns corresponding to 17,545 computation points.

2. Creation of a set of coordinates for each 5′ by 5′ block (grid centres) at which gravity anomalies are to be provided (both terrestrial free-air anomalies and free-air anomalies calculated from a global geopotential model (EGM2008). The block centres are calculated using equation 2.12. Between 18°S ≤ φ ≤ 8°S and 22°E ≤ λ ≤ 34°E we have a total of 17,280 block centres.

3. Computation of mean heights at each block centre using the SRTM 3″ DEM

4. Determination of mean terrestrial free-air gravity anomalies at the block centres in order to produce a gridded data set required for numerical evaluation of Stokes' integral.

5. Computation of residual gravity anomaly Δg_{res} at each block centre by using equation 2.16. However, in this study the direct topographical effect (or terrain correction) was not applied due to the reasons already alluded to in Section 3.2.1.

6. Application of Stokes' formula on the residual free-air gravity anomaly at each computation point to obtain the residual co-geoid height $N_{\Delta g}$

7. Finally, the total geoidal undulation at each computation point is obtained by adding the contribution of the GGM and the indirect effect (equation 2.20′ above)

All computations were done in MATLAB (Student version R2016a). All the inputs into the formula (2.20′) were matrices. The computations were performed using diverse matrix manipulations.

3.2.3 GPS observations at benchmarks

The research required static GPS observation of as many benchmarks as possible to facilitate evaluation of the determined geoid model through the criteria found in Section 3.5. These benchmarks which are currently so few in Zambia had to be found for this purpose. Benchmarks in Zambia were established along the main roads starting from the main reference benchmark that was located in Chirundu. Unfortunately, it is very hard to find benchmarks as most of them have been removed during road works or for some other reasons. This is a very sad state of affairs. The Zambia Survey Department some 5 years ago undertook an exercise to take stock of the remaining benchmarks along the main roads from Livingstone to Kapiri-Mposhi via Lusaka. Only a few benchmarks were found intact. Nevertheless four (4) benchmarks were found intact along the Lusaka-Mazabuka and Lusaka-Kabwe road. Table 3.1 below gives the details of the benchmarks utilised in model evaluation.

Table 3.1: Benchmarks used for model evaluation

BENCHMARK	WGS84 LONGITUDE	WGS84 LATITUDE	LOCATION
BM18M5	28.19315556°	-15.74516389°	Kafue
BM19M5	28.19537500°	-15.89355000°	Along Mazabuka road
BM10M15	28.21038611°	-15.05557778°	Along Kabwe road
BM10M20	28.22565278°	-15.12163611°	Along Kabwe road

Figure 3.2 below is a location map of the benchmarks BM18M5 and BM19M5 whereas Figure 3.3 is a map showing the location of benchmarks BM10M15 and BM10M20. Figure 3.4 is a location map showing all the 4 benchmarks together, but at a smaller scale.

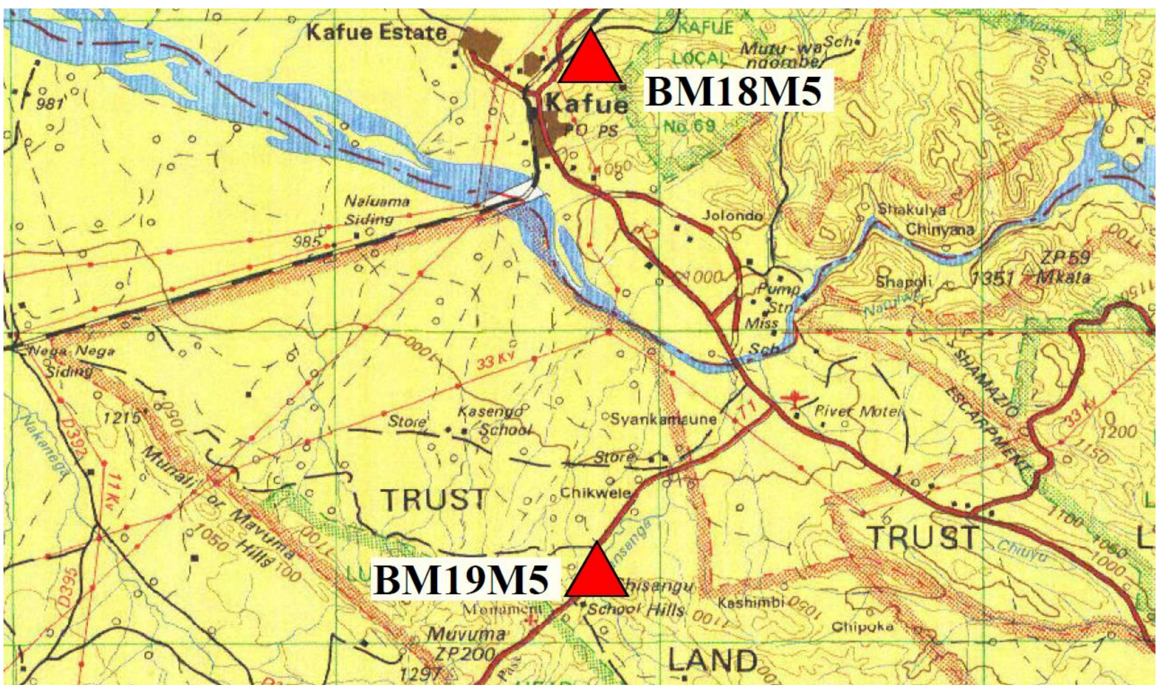

Figure 3.2: Location map of benchmarks BM18M5 and BM19M5

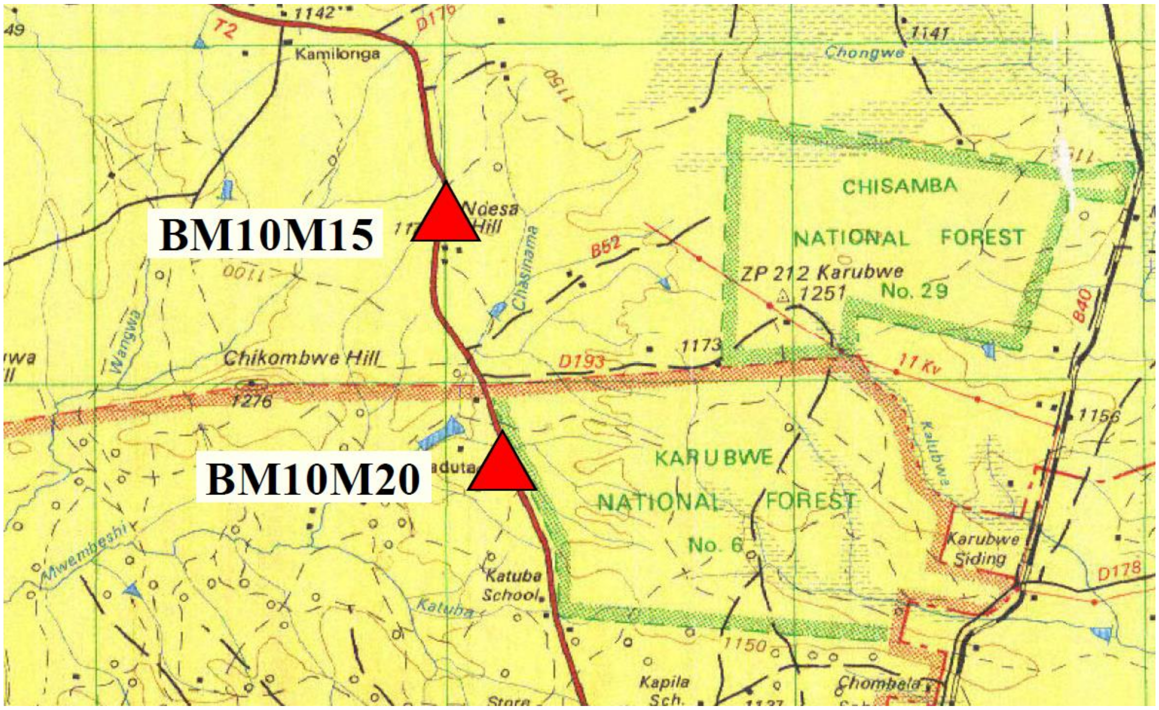

Figure 3.3: Location map of benchmarks BM10M15 and BM10M20

It is still expected that more benchmarks can be found in other parts of the country. However, due to time and financial constraints the search for benchmarks could only be limited to the area around Lusaka province.

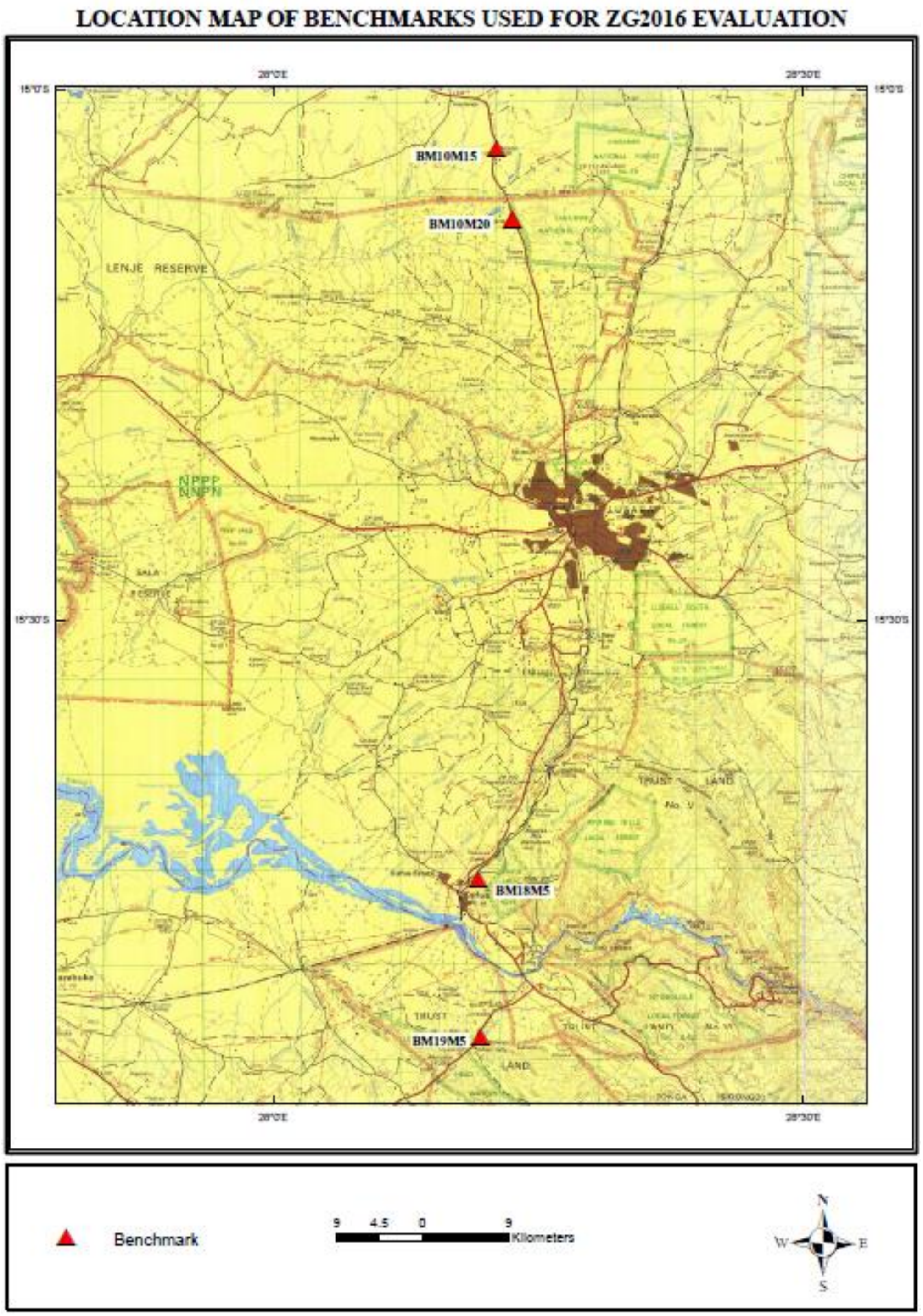

Figure 3.4: Location map of the 4 benchmarks used for Model evaluation

Figure 3.5 below displays a summary of the geoid determination procedure

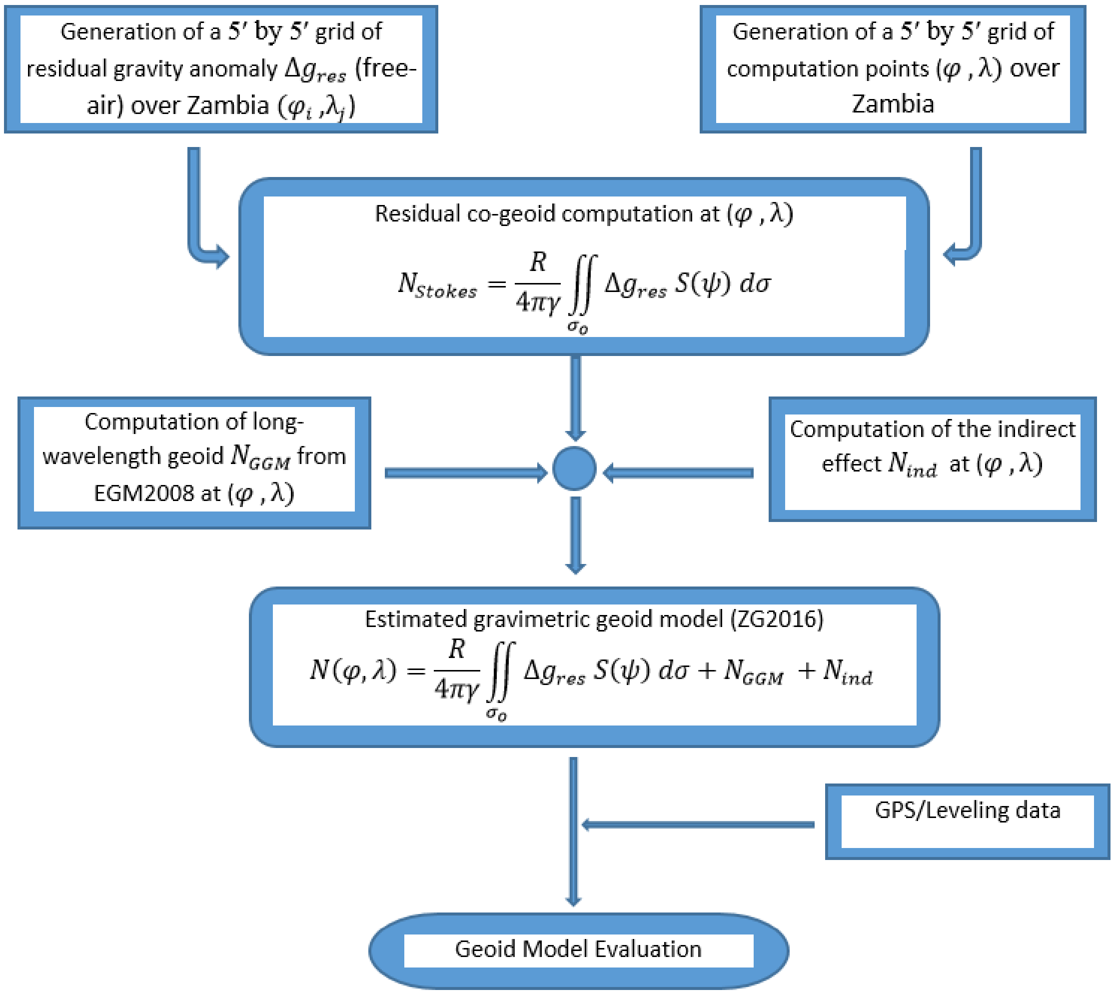

Figure 3.5: Geoid determination procedure

3.3 Data

As already highlighted three sets of data are required for regional gravimetric geoid determination: terrestrial gravity anomalies, global geopotential model coefficients, and elevation information supplied by a digital elevation model (DEM). The three data sets used in this study are briefly described in the following sections.

3.3.1 Terrestrial gravity anomalies over Zambia

The gravity measurements over Zambia are scanty with an average density of 1 gravity measurement per 97 square kilometres (Nsombo, 1998). This poses a challenge in gravimetric geoid determination. The author was able to secure a data set of 12560 point free-air anomalies from the research supervisor. This data set originated from the Zambia Geological Survey Department. Figure 3.6 shows the distribution of these gravity anomalies. Note the gaps in data over the study area. Table 3.2 gives the statistics of these terrestrial point free-air anomalies over Zambia

In this study, surface free-air anomalies over the study area were also acquired from the International Gravimetric Bureau (BGI) via their official website http://bgi.omp.obs-mip.fr/data-products/Gravity-Databases. The overall task of BGI is to collect, on a world-wide basis, all measurements and pertinent information about the earth's gravity field, to compile them and store them in a computerised data base in order to redistribute them on request to a large variety of users for scientific purposes. BGI has a global gravity database (which combines terrestrial gravity measurements and EGM2008 gravity data) from which grids of surface free-air anomalies can be extracted and downloaded. These were used to determine the geoidal undulations via the height anomaly. Figure 3.7 shows the distribution of these gravity anomalies.

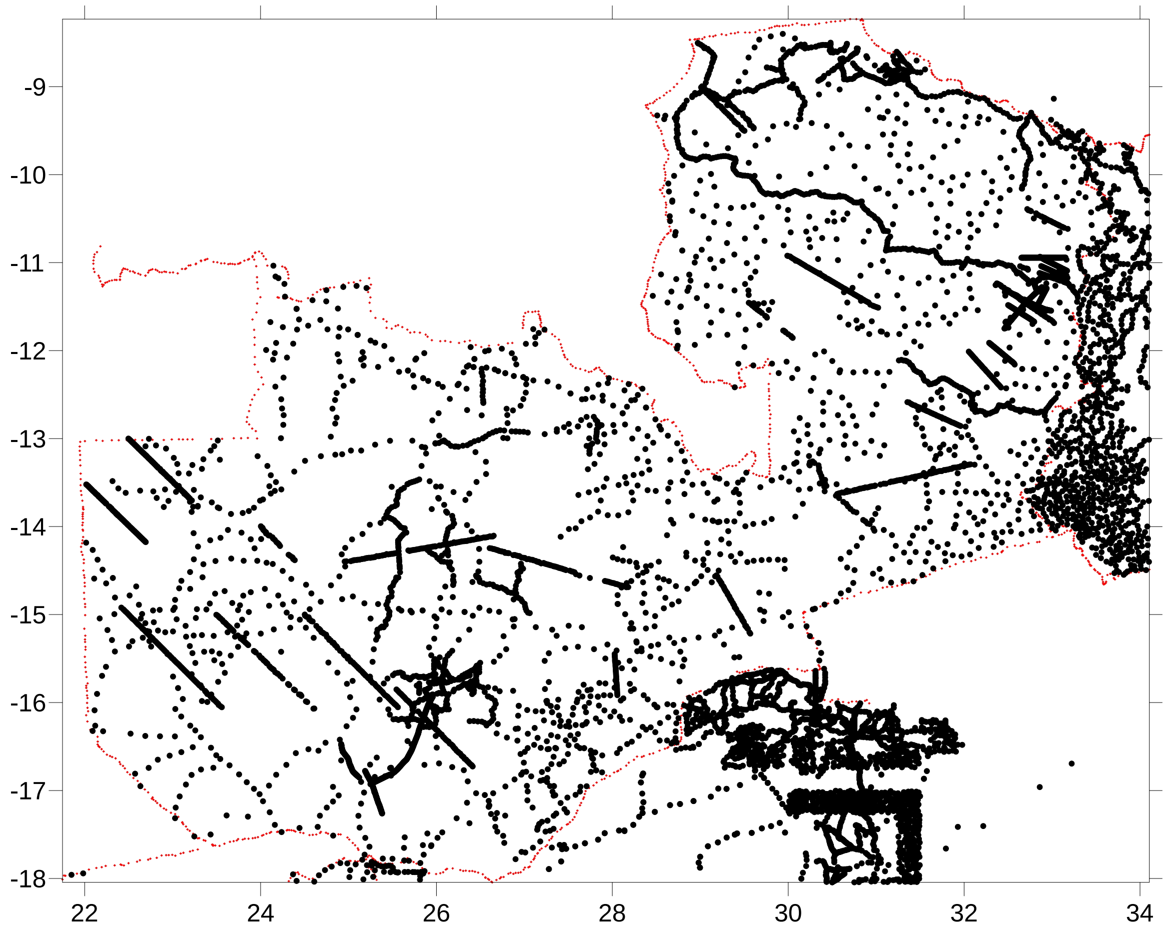

Figure 3.6: Terrestrial point free-air anomalies over Zambia (Source: Zambia Geological Survey)

Table 3.2: Statistics of point terrestrial free-air anomalies over Zambia. Unit is mGals. No. of points = 12560

Maximum	181.13
Minimum	-477.44
Mean	-1.77
Standard deviation	43.63

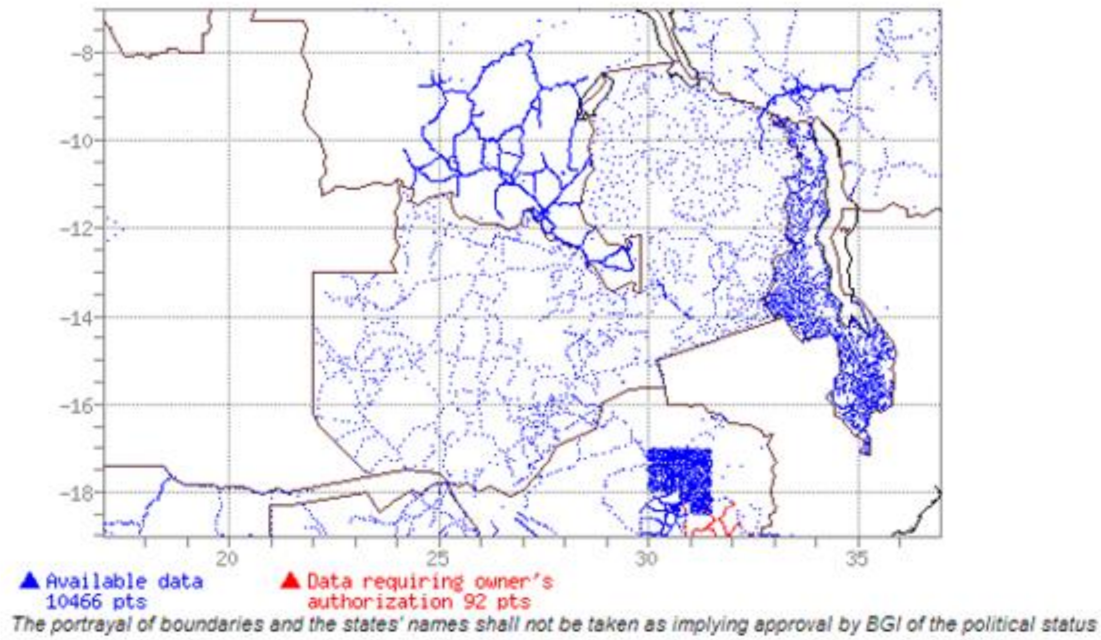

Figure 3.7: BGI land gravity data over Zambia (Source: http://bgi.omp.obs-mip.fr/data-products/Gravity-Databases)

3.3.2 Global geopotential model

EGM2008 was the global geopotential model selected to be used in this study owing to its higher resolution when compared with other existing models. EGM2008 is complete to degree and order 2159, and contains additional coefficients up to degree 2190 and order 2159. EGM2008 was used to supply the long-wavelength part of the gravity field.

3.3.3 Digital elevation model (DEM)

The Shuttle Radar Topography Mission (SRTM) 3″ (or 90m) DEM was used to represent the high-frequency part of the gravity signal. The data covering the study area was downloaded from http://dds.cr.usgs.gov/srtm/version2_1/SRTM3/Africa. SRTM boasts the most complete near-global high-resolution database of the Earth's topography. The

Shuttle Radar Topography Mission is an international project spearheaded by National Geospatial-Intelligence Agency (NGA) and National Aeronautics and Space Administration (NASA).

The data was downloaded in tiles (.hgt zip files) and was loaded into Global Mapper (Version 10.01) for visualization and extraction of XYZ grid i.e. latitude, longitude, and elevation text file. Figure 3.8 reveals the characteristics of the SRTM heights over Zambia whereas Table 3.3 shows the statistics of the SRTM heights over Zambia.

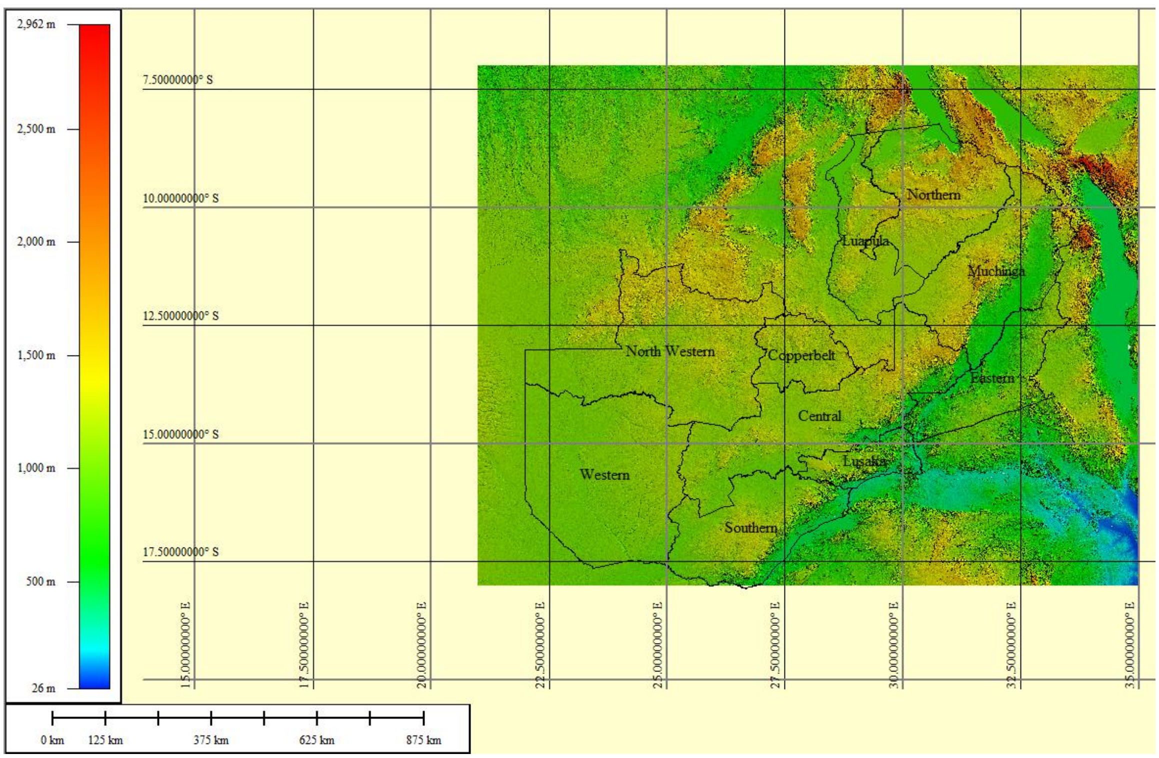

Figure 3.8: Relief over Zambia (SRTM 3″ DEM-generated)

Table 3.3: Statistics of SRTM heights over Zambia. Unit is metres.

Maximum	2741
Minimum	110
Mean	1077
Standard deviation	282

3.4 Data preparation

In order to use the formula for geoid computation, a gridded data set over the entire study area is required. The void areas or the gaps in terrestrial free-air gravity anomalies (See Figure 3.6) needed to be filled and a 5′ by 5′ grid of terrestrial gravity anomalies generated over the entire study area. Thus there was need to prepare the data sets going into the calculations. These data sets were prepared as follows:

1. Terrestrial gravity anomalies

A total of 12560 point free-air gravity anomalies over Zambia (Figure 3.6) were available. As can be clearly seen, the density and distribution of these anomalies is not good enough for geoid determination. Densification to a 5′ by 5′ grid corresponding to the resolution of EGM2008 was required. Void areas were filled with EGM2008-only gravity anomalies following the principle used by Fan (2002), Abdalla (2009), Dumrongchai et al. (2012), and Sjöberg et al. (2015). This is important in order to diminish interpolation errors in geoid determination. Infact, interpolation of gravity anomalies to void areas (based on terrestrial gravity anomalies alone) gave results that produced large errors in geoidal heights when the evaluation was done at the GPS/leveling points. The void or sparsely-filled areas are in 7 blocks (B1 to B7): see Figure 3.9 below.

The EGM2008-derived Bouguer anomalies were generated at a spacing of 5′ in these blocks using the online calculation service of the International Centre for Global Earth

Models (ICGEM) at http://icgem.gfz-potsdam.de/ICGEM/. The ICGEM is mainly a web based service and provides a web-interface to calculate gravity functionals (such as height anomaly, gravity anomalies, and geoidal undulations) from spherical harmonic models on freely selectable grids.

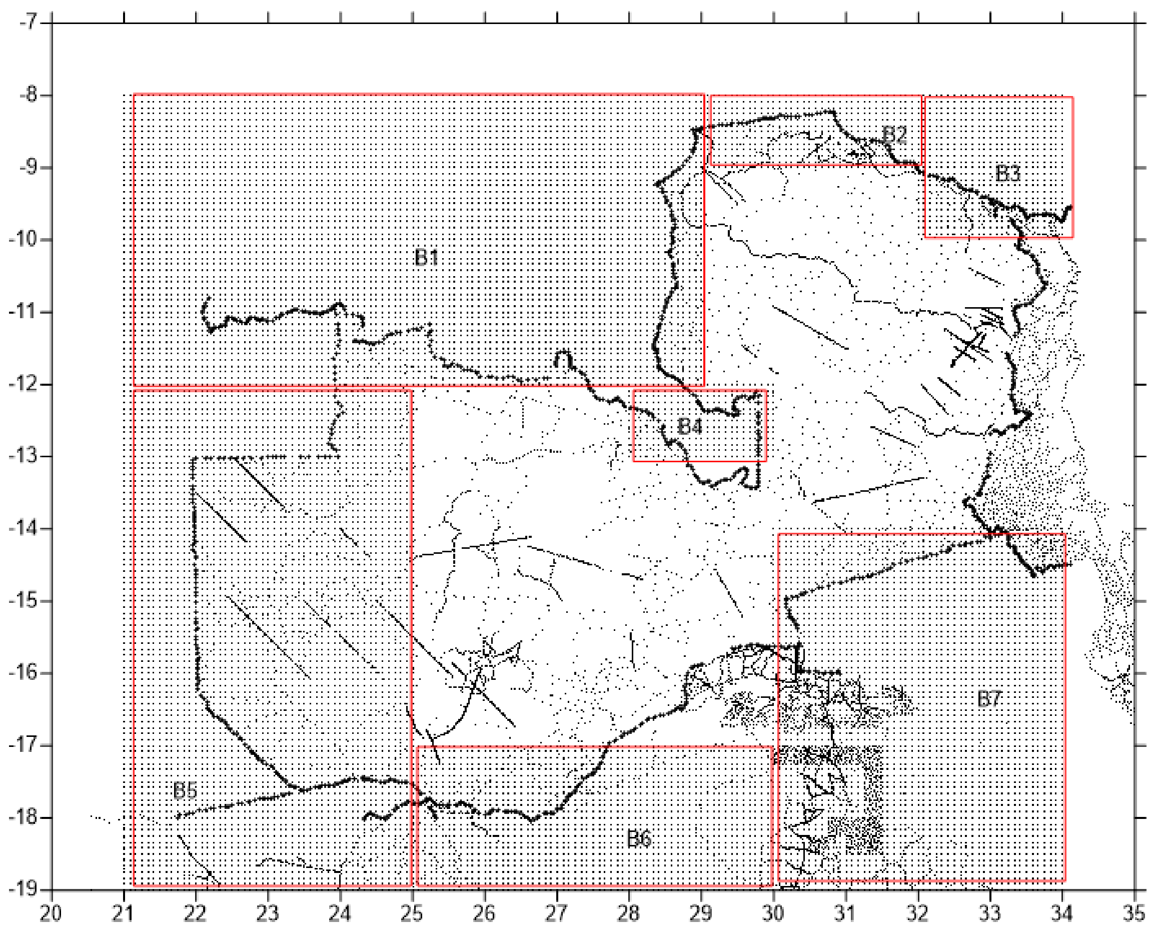

Figure 3.9: Fill-in gravity anomaly areas

The original 12560 point free-air anomalies were converted to Bouguer anomalies using the elevations of the gravity stations by application of the Bouguer plate reduction $(-2\pi G\rho H)$. Then the Bouguer anomalies in the 7 blocks were added to the 12560 terrestrial Bouguer gravity anomalies. The result was a data set of 27423 point Bouguer gravity anomalies which were used in interpolation to create a $2.5'$ by $2.5'$ gridded data set over the study area. The Bouguer anomalies have good interpolatory properties whereas free-air anomalies are extremely dependent on topography, so that their interpolation is very inaccurate (unless their correlation with height is removed). This procedure is followed in light of what is discussed in literature (Heiskanen and Moritz (1967), Smith and Milbert (1999), Featherstone and Kirby (2000), Smith and Roman (2001), Dumrongchai et al. (2012) and Sjöberg et al. (2015)).

The 2.5′ by 2.5′ grid of interpolated Bouguer anomalies was later converted to free-air anomalies by restoration of the Bouguer plate $(+2\pi G\rho H)$. The SRTM DEM was very useful in this regard in providing the elevations at the gravity anomaly points. From the 2.5′ by 2.5′ grid of free-air anomalies, a coarser grid of 5′ by 5′ mean free-air anomalies was generated. This resulted in a grid of 17280 mean free-air anomalies which were used for geoid computation.

The interpolation was performed in Surfer software (from Golden Software Inc., Colorado) using Kriging interpolation method. Kriging is one of the most flexible methods and is useful for gridding almost any type of data set. With most data sets, Kriging with a linear variogram is quite effective. In general this is the method that the manufacturer (Golden Software Inc.) most often recommends. Kriging is the default gridding method in Surfer because it generates the best overall interpretation of most data sets. Kriging attempts to express trends that are suggested in data, so that, for example, high points might be connected along a ridge, rather than isolated by bull's-eye type contours. For large data sets, however, it can be very slow.

Actually, this Kriging with linear variogram method has been used by Abdalla (2009), Ulotu (2009), and Sjöberg et al. (2015) for gravity anomaly interpolation in areas with sparse gravity data (South Sudan, Tanzania, and Uganda respectively).

2. Global geopotential model (EGM2008) data

The Modified Stokes formula requires as input long-wavelength gravity anomalies and geoidal undulations generated by means of GGM spherical harmonic coefficients. The ICGEM online calculation service was used to compute a 5′ by 5′ grid of both gravity anomalies and geoidal undulations. The long-wavelength gravity anomalies were computed at all block centres, whereas the long-wavelength geoidal undulations were computed at all computation points (grid intersections). These grids were loaded as matrices in the MATLAB program.

Suffice to mention that the long-wavelength gravity anomaly grids computed included both the classical free-air anomalies and the surface (Molodensky) free-air anomalies. This is so because an attempt was made to calculate the geoid using both the classical Stokes' approach (using classical free-air anomalies referred to the geoid) and the Molodensky approach (using surface free-air anomalies referred to the ground).

3.5 Geoid model evaluation

The accuracy of the gravimetric geoid model was tested at the four stations (benchmarks) with well-known orthometric heights H. To accomplish this, the ellipsoidal height h for each benchmark needs to be well-known. GPS static observations were made at the benchmarks and the results were post-processed using online processing services. The Online Positioning User Service (OPUS) provided by the National Geodetic Survey (United States), Australian Positioning Service (AUSPOS) provided by Geoscience Australia, and the Canadian Spatial Reference System Precise Point Positioning Service (CSRS-PPP) provided by Natural Resources Canada were used. AUSPOS and CSRS-PPP results agreed well (similar results) and provided more information regarding the measurement errors, so were adopted as the final post-processing coordinates for these stations. The processed ellipsoidal heights for these GPS/leveling stations were then used for geoid model evaluation. The obtained ellipsoidal height of each benchmark was reduced to orthometric height H_{Model} by subtracting the geoidal undulation N_{Model} provided by the geoid model i.e.

$$H_{Model} = h - N_{Model}$$

The computed orthometric height H_{Model} at each benchmark was then compared with the known orthometric height H, the error ε being given by

$$\varepsilon = H_{Model} - H \tag{3.1}$$

Finally, the root mean square error (RMS) was used to evaluate the absolute error of the model as follows:

$$rms = \sqrt{\left(\frac{1}{n}\sum_{i=1}^{n} {\varepsilon_i}^2\right)} \tag{3.2}$$

For comparison's sake the rms was computed for the model based on the classical Stokes' solution as well as that based on surface free-air anomalies. In fact, the rms obtained by EGM2008 as a model on its own was also determined to check if the regional models utilising terrestrial gravity anomalies performed better than the EGM2008 over Zambia.

The research findings and their discussion will now follow.

CHAPTER 4 FINDINGS AND DISCUSSION

In this chapter the findings of the research are presented and discussed.

4.1 Geoid model comparisons

In chapter 3 it was made clear that an attempt was made to compute the geoid over Zambia using the RCR technique, but in two different ways. Firstly, the geoid model was computed by the RCR technique using classical free-air anomalies in equation 2.20′. For convenience this model will be referred to as ZG2016. Secondly, a geoid model based on Molodensky's surface free-air anomalies was computed using equations 2.25 and 2.26. For convenience the model will be referred to as ZG2016M.

In addition to the two regional gravimetric geoid models, the EGM2008 only (global model) is analysed and compared with the two models in the accuracy evaluation based on equation 3.2. This is a deliberate move aimed at checking how the two regional models, utilising terrestrial gravity data, perform when compared with EGM2008 (which gives the intermediate to long-wavelength geoid over Zambia).

Tables 4.1 through to 4.3 below show the results of the fit of the three geoid models at the four GPS/Leveling points (benchmarks) already presented in Table 3.1 (Section 3.2.3). The evaluations are based on equations 3.1 and 3.2.

Table 4.1: Evaluation of geoid model ZG2016

Benchmark	Measured h (m)	Model N (m)	Model H (m)	Known H (m)	Error (m)
BM18M5	1021.008	1.305	1019.703	1019.841	-0.138
BM19M5	1031.503	1.524	1029.979	1029.975	0.004
BM10M15	1124.189	-0.208	1124.397	1124.411	-0.014
BM10M20	1101.612	-0.053	1101.665	1101.643	0.022
				RMS	**0.070**

Table 4.2: Evaluation of geoid model ZG2016M

Benchmark	Measured h (m)	Model N (m)	Model H (m)	Known H (m)	Error (m)
BM18M5	1021.008	0.986	1020.022	1019.841	0.181
BM19M5	1031.503	1.168	1030.335	1029.975	0.360
BM10M15	1124.189	-0.515	1124.704	1124.411	0.293
BM10M20	1101.612	-0.357	1101.969	1101.643	0.326
				RMS	**0.298**

Table 4.3: Evaluation of EGM2008 (only)

Benchmark	Measured h (m)	Model N (m)	Model H (m)	Known H (m)	Error (m)
BM18M5	1021.008	1.363	1019.645	1019.841	-0.196
BM19M5	1031.503	1.574	1029.929	1029.975	-0.046
BM10M15	1124.189	-0.105	1124.294	1124.411	-0.117
BM10M20	1101.612	0.040	1101.572	1101.643	-0.071
				RMS	**0.122**

To begin with, it can readily be noted that, of the three models analysed, ZG2016 model has the best accuracy as indicated by its lower rms error value (7.0cm). In these preliminary tests, it is quite amazing to see such high accuracies for EGM2008-only as geoid model over Zambia. It highlights the possibility of using this global model alone for geoid modeling over Zambia with errors of up to 12cm. The national/regional model combining an Earth Gravity Model (EGM2008) data and terrestrial gravity measurements betters EGM2008-only by only about 5cm. Things might get even better in future with the development of higher resolution GGMs. It is clear that the accuracy of regional gravimetric geoid models is highly dependent on the accuracy of GGMs used in their computation. This is expected because the contribution of the GGM to the total geoidal undulation solution is greater in the Modified Stokes' formula compared to that of the terrestrial gravity field. The terrestrial gravity field which supplies most of the short wavelength only contributes towards the residual cogeoid height $N_{\Delta g}$(see equation 2.20).

These preliminary findings offer support to the claim by Pavlis et al. (2012) that over areas covered with high quality gravity data, the discrepancies between EGM2008 geoid undulations and independent GPS/Leveling values are of the order of 5 to 10 cm. Pavlis et al. (2012) state that EGM2008 performs comparably with contemporary detailed regional geoid models. Furthermore, they indicate that over EGM96, EGM2008 represents improvement by a factor of six in resolution, and by factors of three to six in accuracy, depending on gravitational quantity and geographic area. This is a very important piece of information as in this study an attempt was made to determine a better geoid model compared to ZG96 which was based on EGM96. Nsombo (1996) found at best an rms of 0.69m when evaluating ZG96 at 17 GPS/Doppler points. The accuracy of these points may not have been so good, but still the preliminary results seem to conform to what Pavlis et al. (2012) suggest about the accuracy of EGM2008 in comparison with EGM96. Thus the improvement in the new models over ZG96 would mostly be attributed to the use of this higher resolution EGM2008, and higher resolution SRTM 3″ DEM.

It is so important to mention that further evaluation of the models is required using more benchmarks which should also have a good spatial distribution over the study area. This is important for the results to have statistical significance.

The errors in ZG2016 may be attributed to the fact that some corrections to the geoid solution were not considered. The terrain correction, even if it may be small, would have to be computed to meet theoretical demands. Even the modification of Stokes' kernel suggested by Sjöberg (2005) might further improve the results.

If after more evaluations of these models (in future), their accuracy does not significantly change, then the models would be good enough for orthometric height determination for a number of engineering, surveying, and geodetic applications.

Of the three models analysed, ZG2016M gives the lowest accuracy. The errors may be attributed to neglecting the term containing G_1 in equation 2.25 which represents the effect of the topography. This term was avoided due to computational challenges. However, Heiskanen and Moritz (1967) refer to it as a small correction, together with the geoid-quasigeoid separation represented by the second term in equation 2.26. In this study only the geoid-quasigeoid separation was computed even though it did not improve the result (rms = 29.8cm). The rms for the model obtained by neglecting the two corrections was 18.6cm which is better. This difference may originate from the fact that only one correction is applied omitting the other with which they may cancel out. This may be the case because according to Heiskanen and Moritz (1967) the terrain correction is always positive (page 131) whereas the geoid-quasigeoid separation is usually negative on the continents (page 328). Some of the errors in ZG2016M model may also have originated from the errors in surface free-air anomalies downloaded from the BGI website.

4.2 Description and discussion of the geoid over Zambia

In estimating and describing the geoid over Zambia, use is made of the ZG2016 model. Table 4.4 gives the statistics of the geoid over Zambia.

Table 4.4: Statistics of the geoid over Zambia (ZG2016). Unit is metres.

Maximum	12.959
Minimum	-16.943
Mean	-3.434
Standard deviation	7.110

The geoid over Zambia (based on the Intenational Gravity Standardisation Net of 1971(IGSN71) and GRS80) rises from north-east to south-west. This implies that the international reference ellipsoid (GRS80 in particular) is above the geoid in the northern and eastern parts of the country, but is below the geoid in the southern and western parts of the country, while it coincides with the geoid in areas between these two regions. This implies that in areas around Lusaka, for instance, the geoid and the reference ellipsoid coincide. This means that in such areas the geoidal undulations are zero or close to zero.

Such regions possess an advantage in that ellipsoidal heights from GNSS measurements may be treated as orthometric heights (assuming elevation datum bias is negligible) owing to the fact that the geoidal undulations required to reduce the ellipsoidal heights to orthometric heights are zero or close to zero in such a region.

Information about the geoid (such as the undulations of the geoid) provide manifestations about geologic conditions and geologic features. Variations in geoid pattern may result from different subsurface mass distributions. Thus a good geoid model will be of value to a number of earth-related sciences such as geology and geophysics.

Figure 4.1 is a contour plot of the geoidal undulations over Zambia (based on GRS80) with 1m contour interval while Figure 4.2 is a three-dimensional depiction of the geoidal undulations over the country.

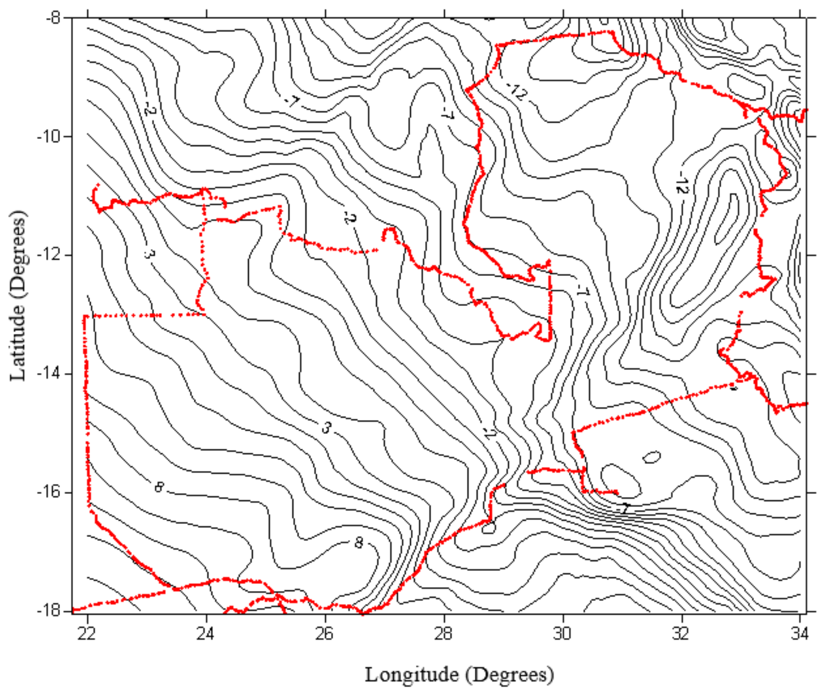

Figure 4.1: Geoidal undulations contour map over Zambia (Based on ZG2016)

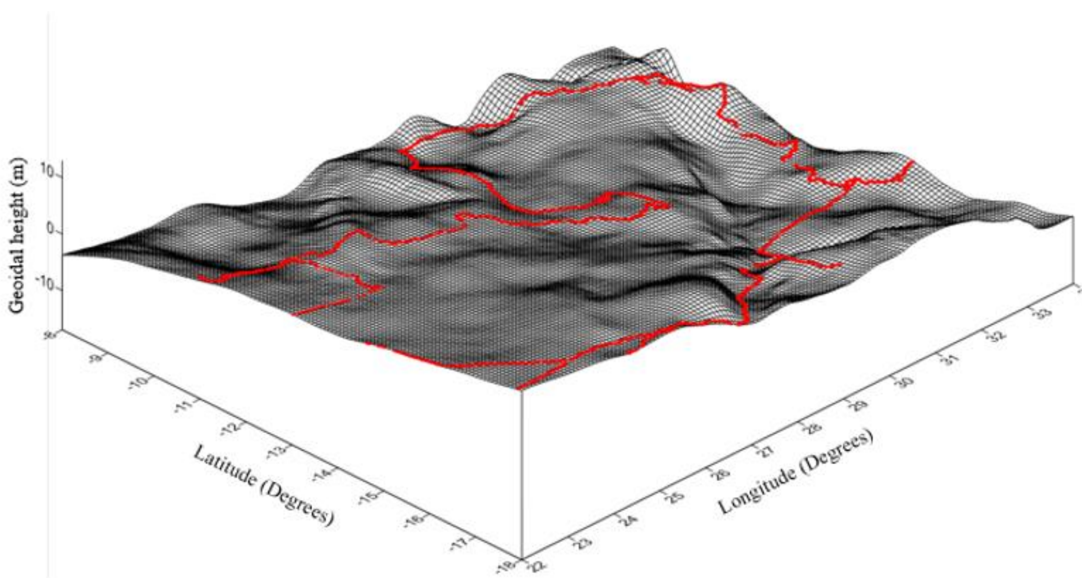

Figure 4.2: 3D depiction of geoidal undulations over Zambia (Based on ZG2016)

4.3 ZG2016 Interpolation Program

A program in MATLAB has been written to estimate the geoidal undulation at any point over Zambia given the point's latitude and longitude in decimal degrees. The program interpolates geoidal undulations from the computed grid of 17545 geoidal undulations over Zambia (ZG2016). The 17545 values forming a 5′ by 5′ grid of geoidal undulations over Zambia are used as sample points for the prediction of values at any point over Zambia. The cubic interpolation method is employed. The reader may refer to Appendix A for the interpolation program code, and to Appendix B for a sample of the results given by the interpolation program.

A text file for the grid of computed geoidal undulations over Zambia is also available in case someone wants to come up with an independent interpolator. Furthermore, a complete set of all MATLAB program codes and raw data pertaining to ZG2016 computations are available on data CDs prepared for safe keeping and archiving. The CDs also contain a user manual for the various computations relating to geoidal undulations computation over Zambia.

To summarise this study, the research conclusion and recommendations will now follow.

CHAPTER 5 CONCLUSION AND RECOMMENDATIONS

In this chapter the thesis conclusion and recommendations are presented

5.1 Conclusion

The geoid over Zambia has been estimated gravimetrically using the RCR technique, employing three sets of data, namely; terrestrial gravity data over Zambia, EGM2008 data, and the SRTM 3″ DEM data over Zambia.

A proposed geoid model termed ZG2016 is in place. This model is based on the use of classical free-air anomalies. Preliminary evaluation of this model at 4 GPS/Leveling stations reveals a rms error of 7.0cm. Thus ZG2016 can tentatively be said to be ten times more accurate than ZG96 (rms error of 69cm) owing to the higher accuracy EGM2008 and SRTM 3″ DEM used in its computation. EGM2008-only gave a rms of 12.2cm when evaluated at the same stations. Thus based on preliminary findings, ZG2016 model is an improvement on EGM2008-only resulting from the use of the terrestrial gravity anomalies over Zambia in conjunction with EGM2008. This indicates that terrestrial gravity data, even of low density, may offer a little improvement in geoid modeling when combined with a GGM in regional gravimetric geoid modeling. However, the confidence level will only go up if more evaluations are done at more GPS/Leveling stations country-wide. Evaluation at only 4 stations is not statistically acceptable, but this has been dictated by challenges already highlighted in Section 3.2.3. More evaluations can be done as more benchmarks are found. It is still expected that more benchmarks can be found in other parts of the country. However, due to time and financial constraints the search for benchmarks could only be limited to the area around Lusaka province.

The accuracy of EGM2008 obtained in this study (12.2cm) is close to the accuracy (5 – 10cm) claimed by the EGM2008 developers (Pavlis et al., 2012). They reported that this accuracy is achievable over areas covered with high quality gravity data. Therefore, based on literature and initial findings in this study, EGM2008 can also be used, as a second option, to estimate orthometric heights over Zambia to accuracies of about 12cm.

A better geoid for Zambia can be obtained with improvements in both the quality and quantity of terrestrial gravity data, as well as the use of a higher resolution GGM which is anticipated in future as satellite missions aimed at modeling the earth's gravity field intensify. Secondly, corrections such as the terrain correction which was omitted in this study owing to reasons provided in Section 3.2.1 will have to be added in future. In addition, the use of the modified Stokes' kernel suggested by Sjöberg (2005) might further improve the results.

Once ZG2016 model is satisfactorily evaluated, it may be used to determine orthometric heights at any point in Zambia in a more convenient way. The model may be used to reduce geometric GNSS ellipsoidal heights to the physically meaningful orthometric heights, which are so important in surveying, geodesy, and engineering. By so doing the usual dependence on spirit leveling which is a labour intensive and expensive method may be minimized or completely eliminated. By use of a geoid model which furnishes geoidal undulations, we can take advantage of the advent of efficient GNSS technology to determine orthometric heights by use of equation 2.5. The model may also prove helpful to a number of earth-related sciences such as geology and geophysics

Finally, ZG2016 model has been used to describe the geoid over Zambia. The maximum and minimum values of the geoidal undulations over Zambia are about 13m and -17m, respectively, implying a range of 30m. The mean geoid over Zambia (8°-18°S and 22°-34°E) is -3.434m with a standard deviation of 7.11m. The geoid over Zambia (based on

GRS80 and IGSN71) rises from north-east to south-west, and coincides with the GRS80 ellipsoid in areas between these two regions.

5.2 Recommendations

The following are the recommendations at the end of this study

1. There is need for continued evaluation of ZG2016 model, as well as EGM2008, whenever new benchmarks are found and observed with GNSS receivers. These benchmarks should preferably have a good spatial distribution over the study area.
2. The Zambia Survey Department is urged to take stock of all the remaining benchmarks country-wide, and ensure proper maintenance of these survey infrastructure which are also very important for academic exercises like this research.
3. Based on initial findings, ZG2016 may be used in the determination of orthometric heights of international boundary pillars whose heights are ellipsoidal (GPS-measured heights).
4. More terrestrial gravity surveys need to be undertaken in Zambia to aid the development of a high accuracy geoid model.
5. Further studies in geoid modeling are encouraged in order to develop the best possible geoid model. The author is convinced that there is still much to be learnt and improved.

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APPENDICES

Appendix A: ZG2016 Program Code

The following is the MATLAB code for the interpolation program used to compute geoidal undulation of points over Zambia (ZG2016_3options.m):

………………………………………………………………………………………………

```
function [ N ] = ZG2016( LONGITUDE,LATITUDE )
%This function program computes geoidal height at a point by prompting
for
%latitude and longitude input of a single point.

fprintf('This program computes geoidal heights in Zambia at points
whose latitude and longitude are given. Option 1: Geoidal height of a
single point, Option 2: Geoidal heights of points saved in an Excel
file (lat,lon), Option 3:Geoidal height at a user-defined grid of
points \n')

OPTION=input('Enter the option number (1,2 or 3):\n');

if OPTION==1   %Option 1 selected then proceed with commands coming
immediately below


[xq,yq]=meshgrid(22:0.0833333:34,-8:-0.08333:-18);

filename1='N_EGM2008_GRID.xlsx';
filename2='ZG2016 RESIDUAL GEOID_GRID';
filename3='IndirectEffect_GRID.xlsx';


Negm=xlsread(filename1);
NRes=xlsread(filename2);
IEOT=xlsread(filename3);
TotalGeoid=NRes+Negm+IEOT;
Z=(xq+yq)*0+TotalGeoid;


fprintf('This program computes geoidal height in Zambia at a point
whose longitude and latitude are given\n')
LONGITUDE=input('Enter the longitude of the point in decimal
degrees:\n');
LATITUDE=input('Enter the latitude of the point in decimal
degrees:\n');

zi=griddata(xq,yq,Z,LONGITUDE,LATITUDE,'cubic');   %Interpolation to a
single point
N=zi;
```

```
dlmwrite('myFile3.txt',[N],'delimiter',' ','precision',4)% writes
results to a text file called myFile3
elseif OPTION==2
%This function program computes geoidal height at a set of points by
%prompting for a file with longitude and latitude pairs of points

%ENTERING DATA FILE NAME
fprintf('This program computes geoidal heights at a set of points by
prompting for an Excel point coordinates file\n ')
filename0=input('Enter Excel data file name below. It must be between
quotes. The data file must be saved within the Matlab environment in
the Matlab folder on the PC. The data file must be a matrix of two
columns without headers. The first column is for latitude values given
as negatives south of the Equator, while the second column is for
longitude values:\n')


%THE CODE BELOW IS CONCERNED WITH PRINTING THE RESULTS TO A TEXT FILE
fprintf('Printing to a file\n');
fprintf('=================\n');
filename=input('This function computes geoidal heights at a given set
of points whose latitude and longitude pairs are given in an Excel file
and writes the results to a text file created by user on this PC.
Specify file path. Enter file to write to(between quotes):\n');
u=fopen(filename,'w');   %open output file


fprintf('========================================================\n')
fprintf(u,'\n========================================================\r');
fprintf('LATITUDE       LONGITUDE       Geoidal Height (m)\n');
fprintf(u,'\n LATITUDE      LONGITUDE    Geoidal Height (m) \r');
fprintf('========================================================\n');
fprintf(u,'\n========================================================\r');


[xq,yq]=meshgrid(22:0.0833333:34,-8:-0.08333:-18);

filename1='N_EGM2008_GRID.xlsx';
filename2='ZG2016 RESIDUAL GEOID_GRID';
filename3='IndirectEffect_GRID.xlsx';


Negm=xlsread(filename1);
NRes=xlsread(filename2);
IEOT=xlsread(filename3);
TotalGeoid=NRes+Negm+IEOT;
Z=(xq+yq)*0+TotalGeoid;


M=xlsread(filename0);

lat=M(:,1);
lon=M(:,2);
L=length(lat);

for n=[1:L]
```

```
    LATITUDE=lat(n,1);
    LONGITUDE=lon(n,1);


zi=griddata(xq,yq,Z,LONGITUDE,LATITUDE,'cubic');   %Interpolation to a
set of (lat,lon) points contained in specified Excel data file
N=zi;


dlmwrite('myFile3.txt',[LATITUDE, LONGITUDE,N],'delimiter','
','precision',7)% writes results to a text file called myFile3 within
MATLAB


fprintf('%+0.5f       %+6.5f     %+10.3f\n',LATITUDE,LONGITUDE,N);
fprintf(u,'\n%+0.5f       %+6.5f     %+10.3f\r',LATITUDE,LONGITUDE,N);

end

fprintf('================================================\n');
fprintf(u,'\n================================================\r');
fclose(u);                %close output file


elseif OPTION==3
%This function program computes geoid height at a grid of points by
%prompting for a user-generated grid  of computation points between -8
and
%-18 deg. South and 22 to 34 deg. East.


%GRID GENERATION
fprintf('This program computes geoidal heights at a grid of points by
prompting for a user-generated grid  of computation points between -8
and-18 deg. South and 22 to 34 deg. East\n')
phimin=input('Enter the minimum latitude for the grid in decimal
degrees. NOTE: Latitudes are negative south of the equator:\n');
phimax=input('Enter the maximum latitude for the grid in decimal
degrees:\n');
deltaphi=input('Enter the grid latitude separation in decimal
degrees:\n');
lambdamin=input('Enter the minimum longitude for the grid in decimal
degrees:\n');
lambdamax=input('Enter the maximum longitude for the grid in decimal
degrees:\n');
deltalambda=input('Enter the grid longitude separation in decimal
degrees:\n');

%THE CODE BELOW IS CONCERNED WITH PRINTING THE RESULTS TO A TEXT FILE
fprintf('Printing to a file\n');
fprintf('==================\n');
filename=input('This program computes geoidal heights over Zambia at
points given by a user-defined grid and writes the results to a text
file created by user on this PC. Specify file path. Enter file to write
to(between quotes):\n');
u=fopen(filename,'w');   %open output file
```

```
fprintf('=======================================================\n')
fprintf(u,'\n=====================================================\r');
fprintf('LATITUDE       LONGITUDE       Geoidal Height (m)\n');
fprintf(u,'\n LATITUDE      LONGITUDE    Geoidal Height (m) \r');
fprintf('=======================================================\n');
fprintf(u,'\n=====================================================\r');


[xq,yq]=meshgrid(22:0.0833333:34,-8:-0.08333:-18);

filename1='N_EGM2008_GRID.xlsx';
filename2='ZG2016 RESIDUAL GEOID_GRID';
filename3='IndirectEffect_GRID.xlsx';


Negm=xlsread(filename1);
NRes=xlsread(filename2);
IEOT=xlsread(filename3);
TotalGeoid=NRes+Negm+IEOT;
Z=(xq+yq)*0+TotalGeoid;


mlon=(lambdamax-lambdamin)/deltalambda;
nlat=(phimax-phimin)/deltaphi;
m=[1:mlon];
n=[1:nlat];


for phi=phimax-(n-0.5)*deltaphi; % Latitude of computation point
for lambda=lambdamin+(m-0.5)*deltalambda;% Longitude of computation
point

LATITUDE = phi;
LONGITUDE = lambda;

zi=griddata(xq,yq,Z,LONGITUDE,LATITUDE,'cubic');   %Interpolation to a
set of lat,lon points contained in user-specified grid
N=zi;


fprintf('%+0.5f       %+6.5f     %+10.3f\n',LATITUDE,LONGITUDE,N);
fprintf(u,'\n%+0.5f       %+6.5f     %+10.3f\r',LATITUDE,LONGITUDE,N);

end
end

fprintf('============================================\n');
fprintf(u,'\n==============================================\r');
fclose(u);                %close output file


       else
            error ('No option selected. Please select an option')

end

end
```

Appendix B: ZG2016 Interpolation Program Sample Result

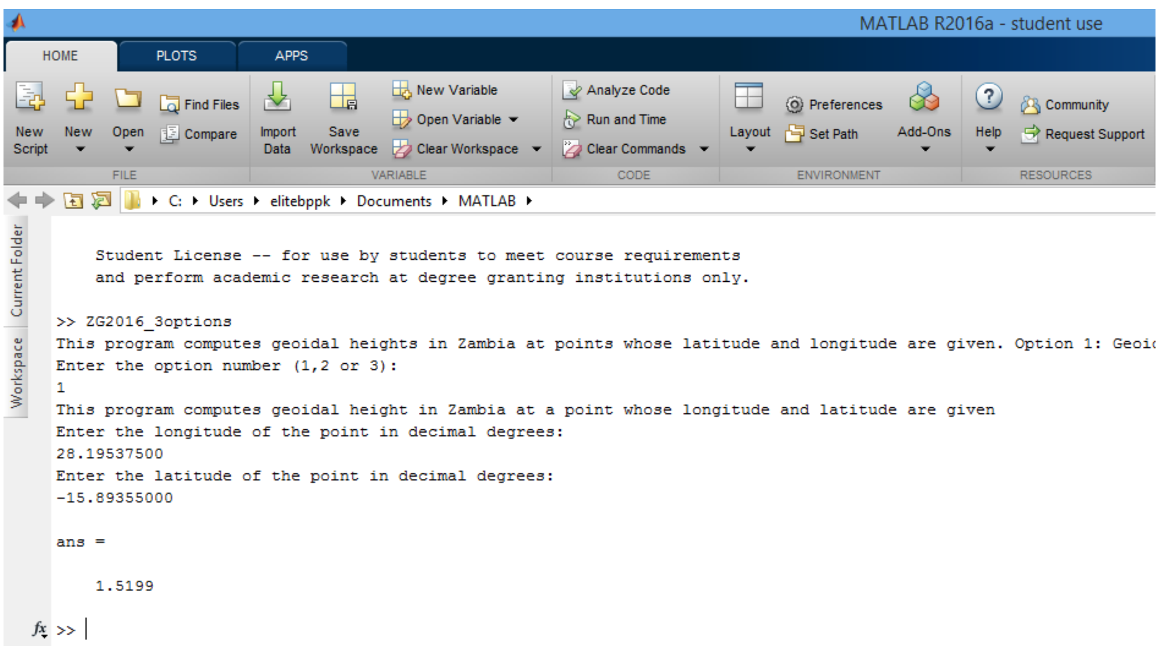
The MATLAB geoidal undulation interpolation program (Section 4.3) is termed ZG2016_3ptions.m. It provides three options for the calculation of geoidal undulations.

OPTION 1: For a single point input

OPTION 2: For an Excel file containing (lat,lon) coordinates of computation points

OPTION 3: For a grid of points input

The figure below shows a sample result for the Model value of geoidal undulation for BM19M5 (Using Option 1 above). The value of the geoidal undulation at BM19M5 is 1.5199m. This differs by 4mm from the value shown in Table 4.1 due to interpolation errors. Direct computation without interpolation gives a geoidal undulation value of 1.524m.



For options 2 and 3 of the program, the results are printed to a user-specified text file whose file path must be specified within quotes at the command prompt. For example as: 'C:\Users\elitebppk\Desktop\MATLAB_Results\geoidheights.txt'. The results are printed in the format (Latitude, Longitude, Geoidal Height) for each computation point.