FIRST YEAR STUDENTS' UNDERSTANDING OF SPECIFIC CONCEPTS IN SELECTED MATHEMATICS TOPICS : THE CASE OF THE UNIVERSITY OF ZAMBIA

By

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ABSTRACT

This study investigated the understanding which University of Zambia (UNZA) first year students of mathematics had of specific concepts in selected mathematics topics. Procedural and conceptual understanding underpinned the investigation. It was also the intention of the study to determine whether there exists any relationship between students' confidence levels and their procedural and conceptual understanding of particular concepts. Further investigations were done to determine if there is a relationship between students' confidence and their actual performance in procedural and conceptual mathematics problems. The study also sought to develop and design instruments for measuring understanding of concepts in mathematics. A quantitative approach was followed and specifically a case study design was employed. Three hundred and seventy eight (378) randomly sampled first year students of mathematics wrote a test which was based on binomial expansions, systems of equations, inequalities, set theory, binary operations, partial fractions, polynomials, functions, trigonometry, quadratics and complex numbers, as taught in first year at UNZA. Three of the five lectures of first year mathematics at UNZA were also asked to answer the questionnaire after studying the questions in the test for students in terms of how they rated procedural and conceptual understanding they might expect. They were also asked to give their opinion on how they rated procedural and conceptual levels of difficulty associated to the questions in the test. Internal consistency of input variables was measured using Cronbach alpha. The correlation matrix was used rather than the variance-covariance matrix because it conformed to the research design of the study. The scoring criteria for the test items was made to measure procedural and conceptual understanding before the test was administered. It was found that procedural internal consistency was 0.879 while conceptual internal consistency was 0.842 which suggested that the test as an instrument prepared for research was reliable because all its components after testing for internal consistency were in the required range of 0.65 to 0.95. The test data was analysed using standard indices while the data derived through questionnaires was analysed using multivariate techniques. The study revealed that procedural understanding was 9.78 while conceptual understanding was 22.6. These results shows that the smaller the result the more explained the result was. Standard indices indicated that, students at UNZA understands procedural concepts more as compared to conceptual ones. The results also showed that the procedural confidence of understanding possessed by students was 6.56 while conceptual confidence of understanding was 12.9. The results indicated that students were twice more confident to answer procedural concepts as compared to conceptual. The overall conclusion from this study is that, there was a significant relationship between students' confidence levels and their procedural and conceptual understanding. On this premise, the findings further indicate that there exists a relationship between students' confidence and their actual performance in procedural and conceptual mathematical problems. Furthermore, the study revealed that there was a positive correlation between students' understanding of mathematics concepts and their ability to execute procedures. Based on the findings, UNZA lecturers of mathematics should focus on teaching methods which would enhance students' conceptual understanding of concepts in mathematics. Further recommendations are that, the current study may stimulate further research of understanding of mathematics concepts at universities in Zambia and beyond.

Key words: Procedural understanding, Conceptual understanding, Confidence levels, Standard indices, Multi-variate techniques.

DEDICATION

To dad and mum, Mr and Mrs T.B. Mwape post-humously. How I wish you were there to see your seed excel and blossom into a doctor. Thank you for your teaching; not only to work with valour at school but also to show love and care to the underprivileged, unheeded and abandoned in society.

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LIST OF ABBREVIATIONS AND ACRONYMS

AMTE	Association of Mathematics Teacher Education
CBU	Copperbelt University
CCUI	Conceptual Confidence of Understanding Index
CIS	Critical Interpretive Synthesis
CJ	Comparative judgement
CLS	Conceptual Learning Strategies
CPD	Continuous Professional Development
CPI	Conceptual Performance Index
CRI	Confidence of Response Index
CU	Conceptual Understanding
CUI	Conceptual Understanding Index
ECZ	Examination Council of Zambia
EFA	Exploratory Factor Analysis
HOD	Head of Department
ID	Intellectual Disability
ICT	Information and Communication Technologies
ILI	Information Literacy Instruction
MEDP	Malaysian Educational Development Plan
MAT	Mathematics
MDS	Multi-Dimensional Scaling
MLS	Mathematics Learning Strategy
NCTM	National Council for Teaching Mathematics

NE	None Examinable
NMR	Nested Multivariation Reasoning
NS	Natural Sciences
PCA	Principal Component Analysis
PBL	Project-Based Learning
PC	Principal Components
PCUI	Procedural Confidence of Understanding Index
P-C	Procedural Conceptual
PF	Procedural Fluency
PISA	Programme for International Student Assessment
PLS	Procedural Learning Strategies
PPI	Procedural Performance Index
PUI	Procedural Understanding Index
RR	Representational Reasoning
SNS	School of Natural Sciences
SPSS	Statistical Package for Social Sciences
SR	Systematic Research
STEM	Science Technology Engineering and Mathematics
TPT	Typical Performance Test
UK	United Kingdom
UNZA	University of Zambia
US	United States

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CHAPTER ONE
INTRODUCTION

1.1 Introduction

This chapter is divided into five sections. The first section will look at the background to the study. The second section will look at the five strands of mathematical understanding. The third section will look at the University of Zambia first year students performance in mathematics (2014-2016). The fourth section will look at procedural and conceptual understanding of mathematics. The chapter will then state the problem which was investigated. Additionally, the purpose and the objectives of the study, and specific research questions. This will be followed by the significance of the sudy, operational definition of key terms, the scope of the study, and limitations and delimitations of the study. Furthermore, the chapter will highlight the layout of the thesis and provide a summary of the first chapter.

1.1.1 Background to the study

First year mathematics taught at the University of Zambia provides basics for subsequent admittance into several disciplines such as medicine, agriculture, engineering to mention but a few. Some of the students of mathematics ultimately become teachers of the subject in secondary schools and colleges. Despite the significance of the subject, little scholarly work has been conducted in Zambia to assess students' understanding of mathematics concepts. Arising from this premise, it has been observed that it is difficult to measure students understanding of concepts. The current study sought to develop and design valid and reliable instruments for measuring understanding of mathematics concepts . In this research, a case study approach was employed and specifically first year students were picked as a sample. This was so because it was difficult or even impossible to look at all the streams who learn mathematics at UNZA. First year students were picked because the mathematics course at this level cuts across all discipline. If you want to do medicine, engineering or agriculture you should pass first year mathematics. Above and beyond, most students come with good grades from ordinary level mathematics but still performs poorly. Additionally, looking at the records in the mathematics department it was revealed that students performance in mathematics at first year was poor as compared to any other year of study. The records showed that it was not all the concepts of mathematics at this level were students had difficulties but only on specific concepts. Hence, this prompted the researcher to only focus on first year students understanding of specific concepts in selected mathematics topics.

The field of general education struggles to measure conceptual understanding (Rittle-Johnson, Schneider & Star, 2015 and Wojcik, 2017). Additionally, the difficulty of measuring conceptual understanding presents a barrier to progress in the development and practice of high-quality mathematics education interventions (Jones, et al, 2015). Furthermore, despite a long history of research on the relationship between conceptual knowledge and procedural knowledge, the findings throughout the world have not been categorically conclusive, as divergent views about this relationship are highlighted (Khashan, 2014). In addition, it seems like the methodologies used before have not been scientifically significant. There is a broad recognition that procedural fluency and conceptual understanding are intertwined. However, the methods for measuring conceptual understanding remain elusive (Wojcik, 2017). Most of the studies worldwide show deep explainations of concepts, writing solutions and pointing out errors as the standard of measuring understanding of mathematics concepts which have been challenged by other scholars to possess some limitations. Consequently, this prompted the researcher to try out other methodologies which have not yet been explored in the measuring of mathematical understanding.

Currently, no standardized approaches for assessing conceptual and procedural understanding with proven validity, reliability, and objectivity have been developed (Schneider & Stern, 2010 and Rittle-Johnson & Schneider, 2015). Additionally, some scholars want more standardized testing of mathematical skills, while others want more authentic assessments that are based on mathematical standards (Ellis and Berry, 2005: 7). Furthermore, Crooks and Alibali (2014) postulate that lack of coherent and specific definitions of conceptual knowledge presents a challenge for researchers seeking to measure it. It was observed that, previously, researchers depended on deep explanations to measure procedural and conceptual understanding of mathematical concepts which were not viable means of carrying out vital experiments (Hosein et al., 2008). Crooks and Alibali (2014) stipulate that, however difficult knowing exactly how to measure conceptual understanding it has not stopped researchers from trying, leading to a wide range of tasks. By and large, there is an alternative assessment tool that could quantify both students' procedural and conceptual understanding of mathematics that may result in alternative instructional materials and methodologies (De Zeeuw et al, 2013). In contrast, an increasing emphasis on the K-12 curriculum of the United

States of America (USA) which necessitates the development of valid, reliable assessments of conceptual understanding that take into account the distinction between mathematical and statistical reasoning (Whitaker, Foti, Steven; and Jacobbe, Tim, 2015). The current study sought to investigate the understanding of specific concepts in selected mathematics topics at the University of Zambia (UNZA) by integrating standard indices and multivariate analysis in the analysis and interpretation of data.

It was observed from literature reviewed that there is little effort in the development and designing of instruments to measure procedural and conceptual understanding of mathematics concepts. Arising from the aforesaid, to achieve increased conceptual understanding in classrooms, there was need to have valid and reliable measures of conceptual understanding (Bisson et al., 2016). Star (2005) suggests that the ways that students come to know, use, and understand mathematical procedures have not been a prominent focus of mathematics education research for atleast 10 years now. Additionally, measuring knowledge of a given concept with acceptable validity and reliability is a major challenge for mathematics education researchers (Crooks and Alibali, 2014 and Bisson et al., 2016). Above and beyond, Audrey et al (2013) suggests that, mathematics assessment tools still focus solely on procedural side of understanding mathematics instead of equally important conceptual aspect of learning mathematics. This prompted the researcher to investigate on an alternative approach of assessing mathematical understanding of concepts by ensuring that the instruments used are valid and reliable.

Therefore, since mathematics is one of the most important subjects in universities, students must learn it with full understanding of its concepts. Mathematics is a progression of learning concepts that build upon each other (Edwards, 2015). It is offered as a prerequisite subject to most of the courses in Zambia and beyond. Consequently, the subject is at the centre of most of the disciplines offered at the University of Zambia. It is against this background that mathematics is a compulsory course at first year level in the fields of physical and biological sciences at UNZA. The subject affects different fields of study because there are calculations in most of the disciplines. Therefore, students should focus on understanding mathematical concepts.

Teaching mathematics for understanding may yield positive gains in student learning and retention of knowledge (Belter, 2009). However, learners must not only be determined to master basic skills of the subject but also persistently pursue means and ways of building new concepts on old ones. Therefore, each student must learn mathematics with understanding (Marchionda, 2006; Ghazali & Zakaria , 2011; Fatqurhohman, 2016 and Mutawah et al., 2019). However, we do not yet understand well the implications of students understanding of the relevance of mathematical concepts in the learning process. Ellis and Berry (2005) argue that students repetitively practice facts, skills, and procedures in an effort to memorise them and then tested to discern what has been learned. This should not be the case as learning means more than memorization and repetition of facts but should include the understanding of specific concepts in mathematics. The implication of rote learning is that it does not create a building block on which knowledge can be built, and does not provide a skill or knowledge that can be connected with other skills or knowledge (Long, 2005:63). Nevertheless, students often memorize procedures to solve mathematics problems without understanding how these procedures work (DeCaro, 2016). Aytekin and Sahiner (2020) argued that students who focus only on procedural knowledge try to memorize the rules of operations by practicing without focusing on understanding the conceptual basis. Subsequently, Edwards (2015) stipulated that, if students memorize the procedures, skills, or facts but they don't understand the reasoning involved, they will not know when or how to apply the knowledge unless the situation is identical to what they have memorized (p.54). Additionally, students mindlessly apply these procedures to solve new problem types even when these strategies are no longer efficient or correct (McNeil & Alibali, 2005).

İncikabi et al., (2015) suggested that procedural learners perceive mathematics as a set of rules and procedures that are independent from each other, and believe that success in mathematics usually comes from memorization of these rules and procedures.(p.2). It is noteworthy to understand that beliefs or values about learning can encourage students to apply or discourage students from applying, their mathematical knowledge (Zerpa, Kajander and Van Barneveld, 2009). Some students develop the perception that mathematics is just made of numerous rules, formulas and equations that they must memorize. The focus on memorization can impede the student's ability to think deeply about a subject. Knowing facts is not the same as understanding how they can be of use. In addition, Bossé and Bahr (2008) indicated that historically, traditional mathematics instruction has been characterized by an extreme commitment to the rote memorization of procedures with little concern for the associated concepts that underlie them (p.3). However, the aim of teaching mathematics at university is to develop deep understanding of the subject and to produce conceptually trained students who can creatively solve unforeseen problems (Khalid and Ekholm, 2015).

There is need for students to understand and conform to new strategies for them to solve new problems correctly. Weber and Lockwood (2014) suggested that, students' ways of thinking should complement a focus on students' understanding of specific mathematical content. Zuya et al (2017) suggested that,

> Competence in mathematics requires the knowledge of concepts and procedures. The possession of subject matter knowledge is necessary for a mathematics teacher to be content and effective in teaching. This is because subject matter knowledge is a combination of the knowledge of both concepts and procedures .(p.30).

To illustrate this, this study investigated procedural and conceptual understanding of mathematics concepts of first year students at UNZA was done. The study focused on students understanding of specific concepts in selected mathematics topics. It is against this background that it was sought to address students understanding of mathematical concepts at university level. This was because literature shows that little erudite work has been done on understanding of concepts by university students in Zambia. For decades, the major emphasis in university mathematics was on procedural knowledge. Rote learning was the norm, with little attention paid to understanding mathematical concepts. Rote learning is not the answer in mathematics especially when students do not understand the fundamentals (Mutawah et al., 2019). Bossé and Bahr (2008,20) stipulated that memory must also be seen as a necessary component of learning. Retention of concepts along with the procedures which apply to, and can be employed in expanding upon, those concepts is vital to learning. Memory and retention must not be seen as in opposition to learning; they must be recognized as a necessary and valuable component of learning.

The study concentrated on the understanding of specific concepts in selected mathematics topics by first year students at the University of Zambia. Martin (2011) in support of Mccloskey (2007) explains that, 'understanding spatial relations is crucial to reading maps, whole numbers and fractions are important for being able to manage money, and probabilities are required for proper financial planning'. Understanding is a complex, multidimensional integration of information into a learner's own conceptual framework (Banda, Mumba and Chabalengula, 2016). Students show mathematical understanding of specific concepts if they can be able to explain, interpret and apply the knowledge gained to a new situation correctly without difficulties. Presenting mathematical concepts from simple to complex using concrete models in early stages of university mathematics learning will help in the abstraction of the concept. Actively engaging students in the learning process will help them in making connections which help to achieve a greater understanding of mathematical concepts (Mutawah et al, 2019).

In many domains, students must learn both fundamental concepts and correct procedures for solving problems. It is in this regard that, prospective mathematics teachers must posses a good knowledge of mathematical concepts and procedures (Zuya, 2017). Furthermore, learners' conceptual understanding is influenced by their familiarity with concepts (Tanner & Allen, 2005 and Banda, Mumba and Chabalengula, 2016) and thus familiarity with concepts increases their understanding of concepts in learning mathematics. Further, Belter (2009) suggested that,

> Mathematics is a multidimensional discipline. It is about procedures, concepts, and number facts. It is about predicting, estimating, and verifying. It can be algorithmic, and yet it is also robust and permits flexibility and seeks efficiency. It is grounded in reality and yet it defines a reality of its own. Too often, school mathematics overlooks these intriguing dimensions and focuses on training students to become proficient only at following prescribed recipes in the context in which they were learned. (p.1).

Procedural and conceptual understanding of mathematics at universities across the globe and within Africa and the sub-region has been of special concern for mathematics lecturers. Some studies show that success in solving mathematical problems is supported by ideas that allow a deep understanding (Fatqurhohman, 2016). Engelbrecht, Hardings and Potgieter (2005:701) argue that 'Mathematics pedagogy based on Vygotskian theory approaches mathematics as a conceptual system rather than a collection of discrete procedures' which is not in agreement with the suggestion by the same authors that mathematics understanding is procedural (Engelbrecht et al., 2009). A significant indicator of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes (National Research Council, 2001:116). Rittle-Johnson & Schneider (2015) argues that considerable progress has been made in understanding the development of conceptual and procedural knowledge of mathematics. Conceptual understanding can be measured in various ways, mainly involving providing definitions, explanations and reasons. Procedural fluency can be measured by checking the accuracy or the procedure of solving problems (Star and Rittle-Johnson, 2008, Verschaffel et al., 2009 and Mutawah et al., 2019). An important next step is to develop a more comprehensive model of the relations between conceptual and procedural understanding of mathematical concepts. Noteworthy, competence in mathematics requires the knowledge of concepts and procedures (Zuya et al., 2017).

Nevertheless, when students systematically use incorrect rules or correct rules in an inappropriate domain, there are likely to be misconceptions. Therefore learning with understanding is essential to empower students to solve the problems they will inevitably face in the future without difficulties (Mutawah et al., 2019). It is evident that a survey of journals and publicly available databases indicates that the development of students' procedural knowledge has not been a recent focus of research in mathematics education (Star, 2005). Mathematics concepts are taught in lectures and the emphasis is on memorization of formulas and procedures to solve specific tasks. Nevertheless, it has been argued that, when teaching mathematics, lecturers should embed mathematics concepts in real life situation so that learners see mathematics with a human face and not an abstract one. Furthermore, the learning of mathematics at all levels, be it early childhood education, primary, secondary, college and or university; students study to pass examinations without minding whether they understand the concepts in mathematics. At each level mainly there are continuous professional development (CPD) held to improve results. Hagger (2014), in support of Wang (2013), stipulates that promoting better skills, learning, and attainment in mathematics is important given the prominent role of mathematical competency in Science, Technology, Engineering and Mathematics (STEM) subjects.

1.1.2 Structure of Chapter One

To understand why there was a need to conduct this study at the University of Zambia, it was necessary to know something about mathematics in general and then look at procedural and conceptual aspects of university mathematics. In view of this, Section 1.1.2.1 looks at the five strands of mathematical understanding which are: Conceptual Understanding, Procedural Fluency, Strategic Competence, Adaptive Reasoning and Productive Disposition. Then the second Section 1.1.2.2, explains, the University of Zambia First Year Students' Performance in Mathematics (2014-2016). Consequently, Section 1.1.2.3 illustrates the Rational for providing Procedural and Conceptual Understanding of Mathematis Concepts in Mathematics and then Section 1.1.2.4 addresses the **e**mpirical review of other studies and also Section 1.1.2.5 illustrates the Comparative Study of Procedural and Conceptual Understanding of mathematical concepts which made the study to be undertaken so that we are aware of the two strands of mathematical understanding. Finally, Section 1.1.2.6 addresses the underpinning perspective on procedural and conceptual understanding of mathematics.

1.1.2.1 Five Strands of Mathematical Understanding

The primary learning outcome of mathematics at university education ought to be mathematical proficiency. Likewise, the five strands of mathematical proficiency are intertwined and interconnected in that the development of one strand aids the development of others.

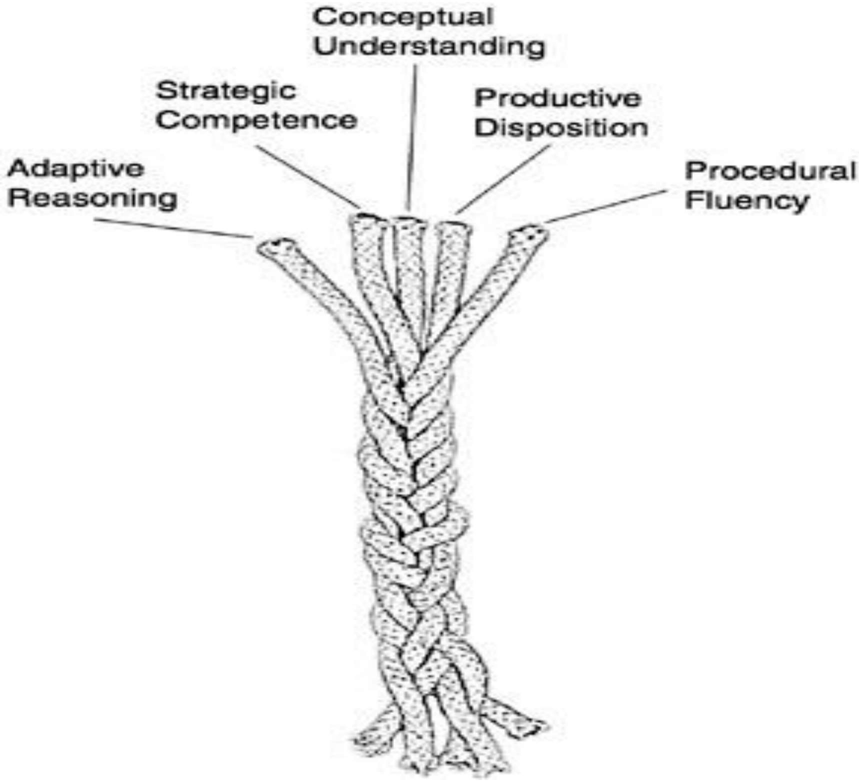


Figure 1.1 The Braided Conceptual Model of Mathematical Cognitive Processes. (Kilpatrick, Swafford & Findell, 2001:5).

Mathematical understanding have five components. These are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition (National Research Council, 2001:116) (figure 1.1). Additionally, Aytekin and Sahiner (2020) in agreement with Kilpatrick, Swafford, and Findell (2001) state that mathematical competence consists of five interrelated elements expressed as conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. The components of understanding mathematics are interrelated and intertwined. Unfortunately, these five aspects of understanding mathematics are not emphasised at most of the levels of learning. The National Research Council (2001) suggest that, ‘to be mathematically proficient, a student must have:

- conceptual understanding: comprehension of mathematical concepts, operations, and relations
- procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- strategic competence: ability to formulate, represent, and solve mathematical problems
- adaptive reasoning: capacity for logical thought, reflection, explanation, and justification
- productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy. (p.1).

It is believed that all the five components of mathematical understanding are interrelated in one way or another. In this regard, Bautista (2013) supports Samuelsson (2010) in stating that, 'students who are able to use their language efficiently to discuss mathematical problems seem to have a positive effect on students' conceptual understanding, strategic competence and adaptive reasoning'. If our learner should understand and do mathematics, they must be proficient in all areas but in our school systems, only procedural fluency is emphasized making learners incompetent. Furthermore, Bautista (2013) stipulates that:

> Students learn to integrate and form a functional grasp of mathematical ideas in conceptual understanding. This enables students to earn new learning schema by connecting those ideas to what they already know supportive to retention that prevents common errors. They then carry out their skills and meaningful procedures in a flexible, accurate, efficient, and appropriate manner (procedural fluency). Strategic competence includes their ability to formulate plans that enables them to use symbolic analysis in solving problems. Using the capacity for a logical thought, the student-learners learn how to reflect, explain and justify procedures/ plans /strategies in a diagnostic manner (adaptive reasoning). Corollary to the real life situations, the student-learners see the mental cognition as sensible, useful and worthwhile towards self-efficacy (p.3).

Graven (2012) argues that, 'productive disposition in mathematics understanding is neglected, no link is made between knowledge of learner dispositions to how teachers might access practice and learning because students mathematical learning disposition is generally ignored (p.52). In some studies, only the first four strands are considered. But, most often, only the first two strands of mathematical understanding are looked at in research. For example, studies done in Zambia and the sub-region have also only looked at the first two strands of mathematical understanding, even though all the five strands are crucial for students to understand and use in mathematics. It has been observed that, conceptual and procedural fluency are mainly applied in order to make students acquire mathematical ideas and relate to new situations of academic endeavours. However, this usually makes lecturers teach methods and solutions to problems to the learners without making them understand the concepts of mathematics content. Most often, many students are able to reproduce the content taught during an exercise or test and when they fail to remember, they get stuck because they do not combine productive thinking to reproductive thinking. In other ways, our learners never even think but simply remember solutions and methods. But students with conceptual understanding know more than isolated facts and methods, know why a mathematical idea is important and the kinds of contexts in which it is useful, are able to learn new ideas by connecting them to ideas they already know, and are able to remember or retain them (Kilpatrick, Swafford & Findell, 2001: 118). Even though all the five strands are important in the understanding of mathematics as Bautista (2013) supports Kilpatrick, Swafford and Findell (2001) in pointing it out that the five strands of mathematical proficiency present the interdependence of five components of learner's mathematics expertise in solving questions. The current study only picked procedural and conceptual components. Not only did procedural and conceptual understanding conform to lower order of learning which constitutes first year mathematics learning but also the work would have been too bulky if it were to look at all the five strands of mathematical proficiency. Discussing all the five strands was beyond the scope of this research. By and large, strategic competence, productive disposition and adaptive reasoning calls for higher order learning where learning have to be justified by showing sense in logical thought process. Additionally, the last three components of mathematical understanding calls for deep learning methods here learners are expected to be more practical in acquiring skills. Strategic competence is necessary to determine the direction of problem solving and to understand which stages to follow through in order to reach a solution (Egodawatte and Stoilescu, 2015). These approaches are mainly used in higher mathematical concepts here a small error may lead to poor output of big projects such as constructions of buildings and bridges. If there was an error in construction the buildings or bridges may collapse which can lead to lose of life.

To add on, at secondary school level most of the teachers of mathematics teach in an instrumental way that brings in memory at the expense of understanding which is in conflict with Gilmore and Papadatou-Pastou (2009) who suggested that mathematics concepts could develop through the encapsulation or reification of a process or procedure. By and large (Zuya, 2017) argues that, 'a good understanding of mathematical concepts and procedures gives the mathematics teacher competence in the mathematics classroom.' In this mind, we need to teach in a relational way to promote relational understanding. Additionally, it has been observed that the distinction between instrumental knowledge and relational knowledge is perhaps the most widely used among mathematics educators in recent years. Relational knowledge is defined as the ability to know ‘what to do and why’ while ‘instrumental knowledge’ is the ability to use ‘rules without understanding the reasons (Skemp,). Hagger et al., (2014) argues that, the mechanism by which autonomous motivation leads to adaptive educational outcomes is through greater interest, effort and application toward instruction, and, particularly, greater involvement in self-directed study outside of the class . To be able to relate similar concepts in mathematics and related subjects. Nevertheless, we have to discourage instrumental understanding where learners understand topics as isolated parts, thus lack roots which they easily forget. One who learnt construction in drawing class will still fail the same in mathematics because of lack of connectivity of concepts which is called conceptual understanding. To add on, it appears that students who knows procedures are good at mastering rules usually by memorising rules, algorithms and procedures while those students who are good at conceptual knowledge are termed as good problem solvers. Recently, the teaching process has shifted its focus towards a balance between procedural and conceptual understanding (Ghazali & Zakaria, 2011 and Engelbrecht, Bergsten & Kågesten, 2017).

However, we have to encourage relational understanding where learners combine knowledge of similar topics which is not common to exhibit by most of the students. Bautista (2013) supports Samuelsson (2010) by pointing it out that, students who are able to use their language efficiently to discuss mathematical problems seem to have a positive effect on students’ conceptual understanding, strategic competence and adaptive reasoning. The combined understanding cements ideas and create roots so that forgetting of concepts in mathematics learning becomes an issue of the past. However, Mills (2016) stated that , memorizing procedures without understanding is consistent with curricular approaches that focus on content rather than conceptual understanding. Sometimes, memorization of mathematics concepts helps students in the mastering of basic skills such as the multiplication table. Students who have memorized the multiplication table will not be bothered to use the calculator to solve simple mathematical manipulations.

It has been observed worldwide that, ‘even if more conceptually-oriented tasks are promoted in national tests, tasks produced by teachers for local assessment are mainly procedural in nature which shows that mathematics focus mostly on computational skills and less on deeper conceptual thinking’ (Boesen, 2006 and Engelbrecht, Harding, & Potgieter, 2005). Likewise, Engelbrecht, Bergsten and Kågesten (2012) suggest that, ‘‘the increasing availability and efficiency of computational tools, such as calculators and computers, seem to imply that at least part of what is commonly included in the notion of procedural knowledge can be achieved without deeper conceptual understanding''(p.3). Few studies have used fine-grained measures that can detect gradual changes in understanding. According to most of the literature revealed it showed that among the five strands of mathematical competence mostly procedural and conceptual understanding were highly used. Nevertheless, this prompted the researcher to only look at the two aspects of understanding mathematics. Furthermore, it would have been difficult or impossible to look at all the five strands of mathematical competence. Consequently, the researcher looked at conceptual and procedural understanding of first year mathematics students at UNZA.

1.1.2.2 First Year UNZA Students' Performance in Mathematics (2014-2016)

To start with, the study looked at the performance of first year students before an investigation into means and ways of measuring effectively the understanding of mathematics concepts (procedural or conceptual). It could have been impossible to look at all the streams of students doing mathematics at the University of Zambia. Addionally, first year students were chosen because of the huge numbers doing mathematics and that they come with good grades from ordinary level but perform poorly when it comes to university mathematics. I analysed results which I had obtained from the mathematics department at UNZA with the permission of the head of department after getting the letter of approval from UNZA ethical committee. These were some of the results I got after analysing them.

Table 1.1: University of Zambia Final Examination Assessment in MAT 1100 from 2014-2016

SUBJECT	ENTERED	SAT	ABSENT	GRADES								PASS % B-A^+	PASS % C-A^+
				A^+	A	B^+	B	C^+	C	D^+	D	Quality	Quantity
MAT 1100 2014-2015	1534	1383	151	5	20	61	116	185	526	1	449	14.6%	66%
MAT 1100 2015-2016	1411	1251	160	10	50	94	148	198	418	0	335	24.1%	73.4%

Table 1.1 above shows first year final results in mathematics (MAT1100) for 2014/ 2015 and 2015/2016 academic years. Table 1.1 shows that bare pass of students; passing from a grade of C up to A^+ for 2014-2015 academic year was 66%. Furthermore, the author generated the table and named quality pass as the marks that may enable a student enter the school of his/her choice which was 4.6%. Then the pass of C to $A^+$was defined by the author as quantity pass. Concurrently, for 2015-2016 academic year quantity pass was 73.4%. The same period for quality pass measuring from B to A^+ which may help a student enter a course of choice was only 24.1%. These results make most of the students end up in the programmes which they were not interested in at first. Additionally, it is observed that the grade of D for the two academic years had more students scoring it. This meant that there were more students to repeat the course in the next academic year.

Table 1.2: Mathematics MAT 1100 Final Results 2014-2015 Academic Year Before Moderation

Grades	**90-100** $\boldsymbol{A^+}$	**80-89** $\boldsymbol{A}$	**70-79** $\boldsymbol{B^+}$	**60-69** $\boldsymbol{B}$	**50-59** $\boldsymbol{C^+}$	**40-49** $\boldsymbol{C}$	**35-39** $\boldsymbol{D^+}$	**0-34** $\boldsymbol{D}$	**NE**
Male	**4**	**18**	**43**	**89**	**134**	**166**	**73**	**323**	**121**
Female	**1**	**2**	**18**	**27**	**51**	**78**	**37**	**240**	**123**
Total	**5**	**20**	**61**	**117**	**185**	**244**	**110**	**654**	**244**

As can be seen in Table 1.2, it states the grades of students by gender for the period 2014-2015 before moderation was done by the department. The total number of male and female students when adding up all the grades was 971 and 578. The number of male students was more than that of females by 393. Nevertheless, the difference for those who did not write the final examination was more on the female. As shown in Table 1.2, 123 females did not write the final examination as compared to 121 males. The percentage for females who did not write the examination (none examinable percentage) was twice as large as for the males. There were 21.3% of females who did not write the examination while we had 12.5% of males entered to write but did not for whatever reasons advanced. Despite the number of females being small, they had a bigger percentage of failures as compared to the males. After calculating it was indicating that 47.9% females failed and also 40.8% of males. For quality pass, meaning the pass grades for students to be put in their course of choice usually is to get a grade of B to A^+. As for the males the quality pass percentage was 15.9% and females was 8.3%. This means that altogether, only a small number of students may enter to do the course of choice. Only 154 males out of 971 and also 48 females out of 578 may enter to do the course of their choice. After looking at quantity pass it shows some rise in numbers of students passing. The quantity pass is measured from $C\ \ to\ A^+$. For males the quantity pass was 46.8% and females was 30.6%. This meant that if we look at numbers then we would have 454 males passing out of 971 and also 177 females passing out of 578. Still on both gender the percentage fail was more than the pass percentage. Nevertheless, since the percentage for failing was more than passing it meant that very few candidates understood the concepts taught.

Table 1.3: Mathematics MAT 1100 Final Results 2014-2015 Academic Year After Moderation

Grades	90-100 A^{+}	80-89 A	70-79 B^{+}	60-69 B	50-59 C^{+}	30-49 C	0-29 D	NE	TOTAL
Male	**4**	**18**	**44**	**90**	**134**	**352**	**208**	**121**	**971**
Female	**1**	**3**	**18**	**27**	**51**	**174**	**166**	**123**	**563**
Total	**5**	**21**	**62**	**117**	**185**	**526**	**374**	**244**	**1 534**

As can be seen in Table 1.3, after moderation of results a number of significant figures changed. The percentage fail for males dropped from 40.8% to 21.4%. Meaning the failure percentage almost dropped by half implying that moderation of results helped a good number of students to proceed to the next level of university education. Females fail percentage dropped from 47.9% to 29.5%. this great reduction in failure percentage also helped a good number of female students proceed to the next academic year at UNZA. This meant that despite a good number of students proceeding to the next stage the understanding of concepts in mathematics was still low. This is so, because of moderation of results which helped a good number of students advance to the next level. For instance, the quantity pass percentage ($C\ to\ A^{+}$) for males increased from 46.8% to 66.1%. The increase in pass percentage for males was about 20%. Females pass percentage also increased from 30.6% to 48.7% after moderation of results. The increase in pass percentage for female candidates after moderartion was about 18%. The female pass percentage increase after moderation was not very different from the male percentage pass increase after moderation. When we look at the quality and quantity pass percentage, we observe that the quality pass for males was 16.1% whilst the quantity was 66.1% after moderation of results. It means that out of 66.1% of all the males who passed about 50% end up doing courses which were not there first choice. However, it showed that only 16.1% did the program which they had hoped for. Females pass percentage difference between quantity and quality was 48.7% less 8.7% giving us the value of 40% exactly. It meant that only 8.7% of the females did the program of their choice and 40% ended up being put in the fields which they had not opted for. Table 1.3 shows that we had more than half the number of females failing mathematics since the pass percentage after moderation was 48.7% it meant that we had 51.3% of females failing to proceed to the next academic year. Furthermore, it was observed that the number of both males and females increased greatly after moderation of results. It was so because before moderation 166 males had a grade of C and 78 females had the same grade. Likewise, after the moderation we had 352 males scoring a grade of C whilst females scoring the same grade was 174. As from the illustration above, it clearly shows that most of the students fail before moderation and only pass after moderation. This indication confirms that very few students understands the concepts since a good number of students only pass the examinations after moderation of results by the department.

Table 1.4: Mathematics (MAT 1100) Final Results 2015-2016 Academic Year Before Moderation

Grades	**90-100** A^+	**80-89** A	**70-79** B^+	**60-69** B	**50-59** C^+	**40-49** C	**35-39** D^+	**0-34** D	**NE**
Male	**9**	**35**	**77**	**133**	**146**	**194**	**65**	**264**	**148**
Female	**0**	**15**	**17**	**35**	**51**	**81**	**39**	**72**	**48**
Total	**9**	**50**	**94**	**138**	**197**	**275**	**104**	**336**	**196**

As from Table 1.4 above, the figures revealed that the performance of first-year students in mathematics was not good. When the performance is bad it meant that even the understanding of mathematical concepts was low. We had 148 male students who entered to sit for mathematics but did not write the final examinations. On the other hand, we had 48 female candidates who had entered to sit for the final examinations but did not write the final examinations. As from the table the percentage fail of the male having a D grade was 28.8% while for female was 25.9%. This result shows that we had more males failing than the females. For the D^+grade we had 7.1% males and 14% females. To add on, we had a total of 35.9% of males failing and also 39.9% of females failing. These results revealed that we had 4% more females failing than males in 2015-2016 academic year before moderation of results. On quality pass, where we measure the pass percentage from B to A^+ grade, for male candidates, we had 27.5% while females we had 21.6%. This implies that we had more male candidates performing well hence the understanding of concepts by male was slightly more than that of female candidates. The quantity pass is defined (By the author) from C grade to A^+ was calculated and stated. As for male candidates, we had 64.4% while for females was 64.2%. This indicated that there were big difference on the quantity pass for the two gender.

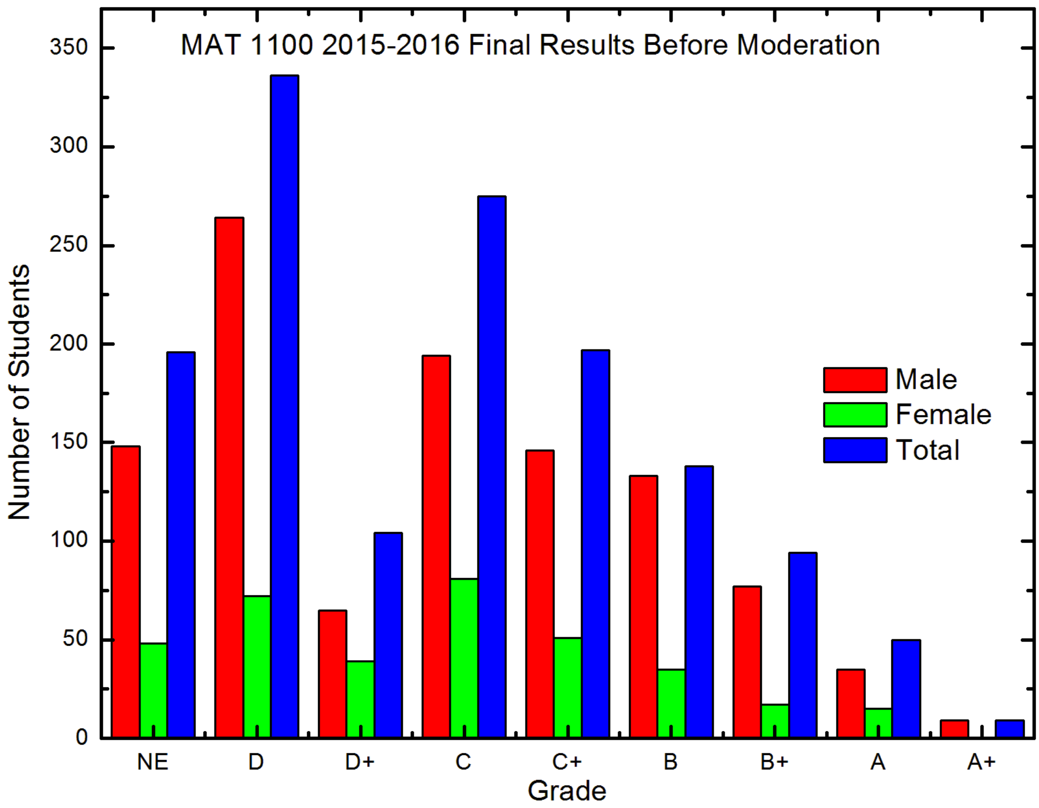


Figure 1.2 MAT 1100 2015-2016 Final Results Before Moderation

As from Figure 1.2 above, the highest numbers were for the students who had failed the final examinations. The category which followed was those students who had a bare pass, a C grade. Table 1.2 above shows the number of candidates passing with a C^+ is almost equal to the number of candidates who entered for the final examinations but did not write the examinations. Table 1.2 shows that A^+ had the smallest number of candidates, all of them males followed by A which had also had few candidates. Nevertheless, a good number of candidates had D meaning that there was a big failure rate amongst all the categories. This shows that low performance had a lot of candidates. Hence or otherwise, this meant that even understanding of mathematics concepts was low because of low performance. It was revealed by Engelbrecht et al (2005) that mathematics performance correlates with understanding. Good performance predicts good understanding and the opposite is correct.

Table 1.5: Mathematics MAT 1100 Final Results 2015-2016 Academic Year After Moderation

Grades	90-100 A^+	80-89 A	70-79 B^+	60-69 B	50-59 C^+	30-49 C	0-29 D	NE	TOTAL
Male	**9**	**35**	**77**	**133**	**146**	**397**	**121**	**148**	**1085**
Female	**0**	**15**	**17**	**35**	**51**	**99**	**61**	**48**	**326**
Total	**9**	**50**	**94**	**168**	**197**	**496**	**270**	**127**	**1 411**

Final results analysis after moderation of results for Mathematics (MAT 1100 2015-2016) academic year revealed that before moderation the pass percentage was 64.4% for quantity pass (i.e. grade of C to A^+) for male candidates and after moderation, 86.8% . This shows that there were an improvement of 22.4% in the pass percentage for males after moderation of results. On the other hand, there was 64.2% pass percentage for females obtaining the quantity pass. Furthermore, after moderation of results we had 78.1% of females getting quantity pass. This shows 13.9% increase in the pass percentage for females after moderation of results. The results revealed that there was 22.4% pass percentage increase for males and 13.9% pass percentage increase for females. This shows that moderation favoured more males than females. It may be concluded that, after moderation, the fail percentage dropped drastically for males from 35.9% to 13.2% while for females it dropped from 39.9% to 21.9%. It can be stated that moderation of results at UNZA helps a good number of students to proceed to the next academic year without students fully understanding concepts in the initial course because a good number pass the final examination through the moderation of results.

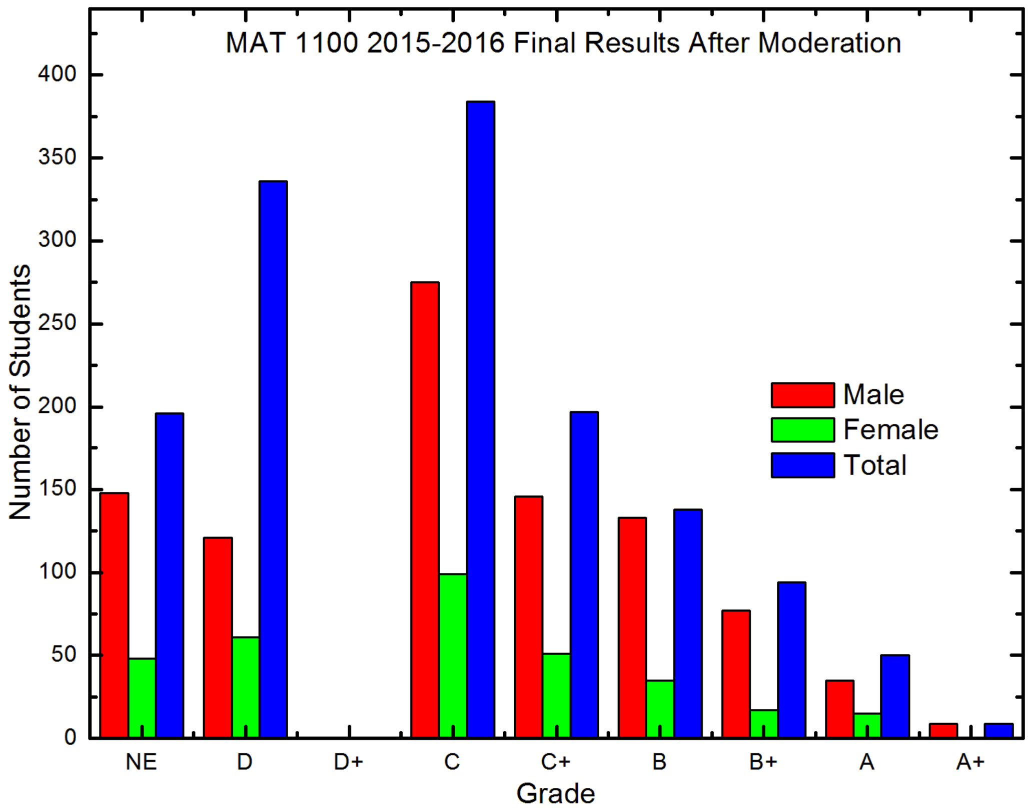


Figure 1.3 MAT 1100 2015-2016 Final Results After Moderation

Figure 1.3, shows results after moderation by the mathematics department indicating an increase in the pass percentage. The bars for D^+ were completely removed because all the students who had initially obtained a D^+ were upgraded to a C grade. This reveals that the grade of D^+ was now not allocated to any student because all the students with a grade of D^+ after moderation of results obtain a pass grade of C. Nevertheless, the pass grade of C^+ is equal to the number of students who entered to write mathematics but did not write the final examinations based on their own reasons (Not Examined). The number of students scoring an A^+ was 9, all of them male. The numbers of students passing mathematics in the final examination at UNZA incressed very much after moderation of results.

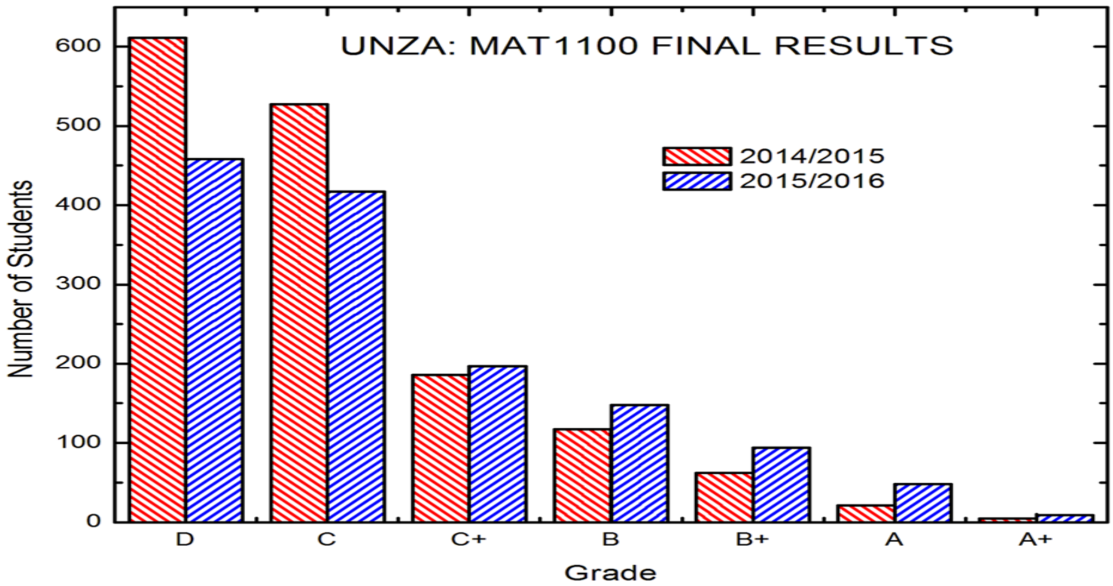


Figure 1.4 MAT1100 Final Assessment (Analysis of Results)

The results in Figure 1.4 show that good grades in mathematics to enable a candidate enter into a career of choice was small. The grades of B^+ to A^+ for the two consecutive years were less than two hundred. To add on, it was observed that about half of the number of students with a grade of C had failed initially but survived due to moderation of results. The results were moderated because very few candidates passed mathematics examination.

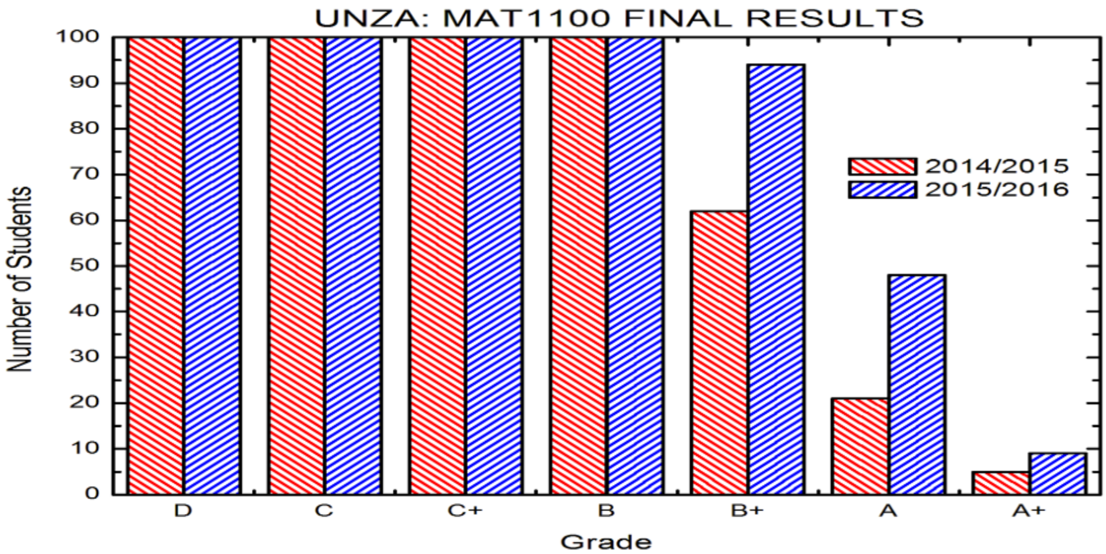


Figure 1.5 Analysis of Results

As from the Figure 1.5, it shows that for the consecutive years students with good grades were less than 100. Figure 1.5 tries to show the true values for the grades of B^+, A and A^+. This means that equally the number of candidates who understood the concepts in mathematics was very low. This is because we compare getting questions correctly and giving examples as components of understanding under the conceptualising of learning model. Ngesu and Simotwo (2018) argues that despite the subject (mathematics) being one of the longest taught from primary through secondary into some higher learning courses like engineering and medicine, it has registered unsatisfying results.

The discussion above showed that the performance and understanding of mathematics by first year students at UNZA was low. If university students do not perform well in mathematics it means teachers of mathematics to be produced will be less competent. Therefore, understanding of first year mathematics at UNZA is critical because if left unchecked, it may affect the nation as UNZA produces a big number of graduates in many fields where mathematics is of optimal importance. It has been observed by the current researcher that there has been few studies of the problem at hand by previous researchers because literature shows a bias towards studying of students performance only. Therefore, it is not surprising that many lecturers at tertiary level are finding it difficult to lecture, or teach mathematics to first year university students (Tulasman & Hulsman, 2013). Most of the methods for measuring mathematical understanding used previously proved to be unliable this made the researcher to use standard indices and multivariate analysis to measure procedural and conceptual understanding of mathematics by first year students at UNZA. Therefore, it was important to develop and design instruments which may be used to measure students understanding of specific concepts without difficulties. The ability to analyse student work and thought processes is important since students often find ways to solve problems that deviate from traditional methods (Marchionda, 2006). It is believed that conceptual understanding may help students to avoid errors of any magnitude in mathematics learning. Ideally, if a student is confident, he or she responds well to questions and, hence, excel in examinations. Not only did the current study seek to establish the correlation between understanding and confidence exihibited by first year students of mathematics but also performance and confidence they had in the subject. Hence, precautions may be taken so that students learn mathematics with understanding. Learning with understanding is essential to enable students to solve emergent problems throughout their lives (Mutawah et al, 2019).

1.1.2.3 Rationale for Providing Procedural and Conceptual Understanding of Mathematical Concepts

Mwape and Musonda (2014) in support of ECZ (2012) argued that, ‘‘One of the objectives of teaching the mathematics curriculum is to build up understanding and appreciation of mathematical concepts and computational skills in the learners in order for them to apply them in other subject areas and everyday life’’. Students should be able to understand procedural and conceptual aspects of mathematics. However, some schools only emphasize procedural understanding and do not directly focus on conceptual understanding as they do on passing examinations alone (Ghazali and Zakaria, 2011). Berger (2014) suggests that effective mathematics instruction needs to provide opportunities for students to build procedural fluency through conceptual understanding.

Furthermore, Hallett, Nunes, Bryant and Thorpe, Christina (2012:469) argues that “recent research on children’s conceptual and procedural knowledge has suggested that there are individual differences in the ways that children combine these types of knowledge across a number of mathematical topics. Cluster analysis has demonstrated that some children have more conceptual knowledge, and some children have more procedural knowledge, and some children have an equal level of both”. However, a teacher must be able to do more than use an algorithm. She must also understand the concept well enough to be able to look at mathematics as a dynamic discipline (Marchionda, 2006). By and large, teachers are doing their best with what they know. The criticism is that procedures-first, understanding-second approach which, by default, is designed to teach procedures’ not to generate conceptual understanding is not a reliable procedure as it needs a number of adjustments. As supported by Andrew (2016:5-6) argues that,

> A procedures-first-understanding-second approach is easier to use year after year. It requires little modification over time compared to alternative approaches.
>
> The approach is teacher-centric, and teacher-centricity is what the majority of teachers are most familiar with. Moving away from a teacher-centered approach is difficult for may teachers simply because it is unfamiliar and because it is rare, and there is no concrete roadmap provided.
>
> The approach can be reasonably effective at enabling students-those gifted with a good memory-to gain high results in short-term mathematics tests. And this appears to be the primary outcome sought by most parents and school administrators.
>
> Going forward, there is need to avoid students spending significant volumes of time not understanding what they are working on in class but rather understanding of concepts must be developed in lessions.

Despite the argument being insightful as stipulated in the discussion the current study sought to add on the use of cluster analysis in the data analysis to other multi variate models such as factor analysis, principal component analysis, discriminant analysis and multi dimensional analysis.

1.1.3 Review of Empirical Studies

Since the study of mathematics at university level is so crucial in the development of individuals and the nation, there is need to improve the understanding of mathematical concepts. Maglipong and Bongolto (2017) stipulate that, 'mathematical patterns help to improve students conceptual understanding but also suggest that conceptual understanding is the key component of mathematical expertise'. However, Alibali and Crooks (2014) postulate that the task used to measure conceptual knowledge does not always align with theoretical claims about mathematical understanding hence making it a challenge to understand the relationship between conceptual and procedural understanding. Noteworthy, Zuya et al., (2017) observed that the ability to commit into memory that two negative numbers can result into a positive number when multiplied or divided, is not the same as understanding the reason for the product or quotient to be a positive number. The ability to explain the reason for the answer to be positive is evidence of possessing conceptual knowledge. Despite the study being discerning on the understanding of mathematics however the current study sought to develop and adapt standard indices as new measuring equipment of mathematical understanding.

Not only do mathematical concepts become interesting and enjoyable if done through investigation of patterns that requires deep and critical conceptual understanding but also the use of effective visuals help learners with a variety of learning styles to develop a strong conceptual understanding of mathematical concepts (Maglipong and Bongolto, 2017; Galvez, 2009). However, it is argued that teaching mathematics should begin with definitions and theorems and then move on to work on computations in a procedural manner (Bergsten et al., 2015). Maglipong and Bongolto (2017) in agreement with Schulz (2011) postulates that, 'mathematical models and patterns help students to understand mathematical concepts. Maglipong and Bongolto (2017), argues that not only should teachers motivate students to love the subject but also must be updated with the recent trends and techniques in teaching mathematics in order to help improve students' conceptual understanding. Ghazali and Zakaria (2011) in agreement with Maglipong and Bongolto (2017) suggests that, if students lack understanding of concepts, they are unlikely to construct the desired algebraic ideas because of lack of conceptual knowledge obtained at secondary school level in algebra. While the study by Malipong, Bongolto and Zakaria were informative in that they encouraged deep appreciation of mathematics pattern in the understanding of mathematics. The current study not only advocated for the use of multivariate analysis but also focussed how to measure specific concepts in first year university mathematics.

Furthermore, Alibali and Crooks (2014) in agreement with the (National council of Teachers of Mathematics (2010), stipulates that, 'the general consensus, in research on mathematical thinking and in mathematics education, is that having conceptual knowledge confers benefits above and beyond having procedural skill'. This statement is considered invalid because, again, there is another scholar (Baroody et al., 2007) who suggests that it is difficult to distinguish conceptual from procedural knowledge at some points in development, because the two forms of knowledge are deeply intertwined. Eventhough, it is difficult for previous scholars to distinguish between procedural and conceptual understanding the present study tried to separated procedural and conceptual concepts by involving informed guidance of UNZA lecturers of mathematics to ascertain as what is procedural or oncptual.

Largely, Graven (2012) reveals that, knowledge of students involves knowing learners' levels of mathematical competence, and what they are able to do and not do. This may enhance students abilities of remediating and reinforcing understanding of concepts in mathematics learning (procedural or conceptual) understanding of underlying concepts in mathematics. Furthermore, this helps in making sure that the current study defined adequately the concepts what was to be procedural or conceptual.

1.1.4 A Comparative Study of Procedural and Conceptual Understanding of Mathematics.

It is important to know about procedural and conceptual understanding of first year students in universities and also about the University of Zambia. The Mathematics Association of America (2000) suggests that, "Educational accomplishments in mathematics often exert a strong influence on career accomplishments." University mathematics study must be built on and extended on prior experiences. Students entering university education must have a diverse preparation for higher mathematics but due to many factors including background, time since high school graduation, age, and work experience make them to lack behind in understanding of first year mathematics. As a result, mathematics departments in universities have difficulties in placing students in their first university mathematics courses by using only data such as high school rank- in-class, grade point average, or record of high school mathematics courses. To avoid such anomalies, Universities should be able to conduct admission tests in order to measure students' readiness for university mathematics education knowledge and skills. It has been argued by the Mathematics Association of America (2000) that good placement tests assess computational skills in unexpected contexts and a balance of procedural fluency and conceptual understanding. Changes in high school curriculum and variability of mathematical preparation of students makes them to lag behind as far as understanding of mathematical concepts are concerned. Unfortunately, most school mathematics curricula are overly concerned with developing procedural knowledge in the form of speed and accuracy in using computational algorithms rather than the development of higher order thought processes. Moreover, students are prone to using procedures rather than knowing how the procedures are achieved (Ghazali and Zakaria, 2011). Additionally, some theories from cognitive psychology and mathematics education suggest that children's understanding of mathematics concepts develops together with their knowledge of mathematical procedures (Gilmore and Papadatou-Pastou, 2009). However, the current study seeks to bring in a balance between procedural and conceptual understanding of concepts in mathematics since they are equally important in the learning of mathematics.

Additionally, Hannigan et al (2013) postulated that interest and attitudes can play important roles in developing openness to learning and create the potential for deep conceptual understanding. Furthermore, it was revealed that if teachers cannot identify when students have conceptual understanding of a mathematical area, and when not, than even the best learning activities prove quite ineffective. Identifying when a student does or does not have conceptual understanding can be quite difficult, especially compared to grading of a traditional test where we solely checks for a correct numerical solution (Korn et al, 2014: 16). The current study considered all the steps of the working in the given question to students in the awarding of marks. This helps a lot because the work of students is appreciated at whatever level they reach when solving mathematics.

1.1.5 Underpinning Perspective on Procedural and Conceptual Understanding of Mathematics

There is by far a number of significant reasons that underpins the understanding of mathematics concepts at first year level. No wonder it is a teachers job to be able to analyse and invent strategies to see where children's misconceptions are and also determine the validity of the strategy (Marchionda, 2006). Nonetheless, the knowledge of concepts and procedures is imperative for competence in mathematics (Zuya, 2017). It is against this background that, an important goal in mathematics is to flexibly use and apply multiple efficient procedures and to understand why these procedures work (DeCaro, 2016). Furthermore to teach for procedural understanding could mean to present pre-formulated definitions notations and procedures without first having provided contexts of meaning for the concepts and methods involved (Bergsten et al, 2016). Eventhougth (Yurniwat & Yarmi, 2020) argued that, a prospective teacher needs to have a depth understanding of procedural knowledge which is in conflict to (Manandhar, 2018) who stipulates that conceptual knowledge can be acquired by connecting whatever we have learnt before. Ideally, the fundamental strategy should be to incorporate both procedural and conceptual understanding of mathematics concepts in order to enhance mathematical understanding. Not only did Crooks and Alibali (2014) postulates that, lack of coherent and specific definitions of conceptual understanding presents a challenge for researchers seeking to measure it but also Englebrecht et al., (2005) did the same.

Likewise, Kilkenny (2017) suggested that mathematics educators should promote both skills and procedural facility as well as conceptual understanding to being essential in the teaching and learning of mathematics. On the other hand, Rittle-Johnson and Schneider (2015) argue that not only may conceptual knowledge help with the construction, selection, and appropriate execution of problem-solving procedures but also implementing procedures may help students develop and deepen understanding of concepts. It is against this background that Crooks and Alibali (2014) argues that,

> Despite a clear movement in both research and educational practice towards emphasizing conceptual knowledge in addition to procedural knowledge, there are several obstacles standing in the way of a comprehensive understanding of conceptual knowledge. One major hurdle for researchers is that there does not appear to be a clear consensus in the literature as to what exactly conceptual knowledge is and how best to measure it (p.345).

Additionally, Lee et al (2016) suggested that, understanding of mathematics concept should be a primary goal for all the mathematics teachers who are teaching the subject. This is so, because understanding mathematics concepts is viewed as being able to think and act flexibly with the concept and it is more than following steps in a mathematics procedure. Therefore, conceptual understanding in statistics (LOCUS) project aims to create valid and reliable assessment for conceptual understanding of statistics (Whitaker, Douglas, Foti, Steven; and

Jacobbe, Tim, 2015). Alternatively, the knowledge of concepts involves understanding of meaning and not just ability to recall definitions, rules or procedures (Zuya, 2017). Particularly, sophisticated mathematical thinking builds on personal interpretations of mathematical concepts (Chin and Pierce, 2019) as the ideal means of progress in mathematics understanding. Consequently, the current study sought to investigate first year students understanding of specific concepts in selected mathematics topics at UNZA. This was done by measuring procedural and conceptual concepts of mathematics.

1.2 Statement of the Problem

The University of Zambia has been training mathematics students for a long time. Notwithstanding, stakeholders usually focus on students final scores attained in tests and examinations. They also normally concentrate on generic aspects such as how many students passed or failed the assessments. This is done without in-depth consideration of the understanding students acquire of the mathematics concepts studied. It is a known fact that first year university mathematics plays a critical role to place students into respective programs of choice. For a student to be quoted in a program of choice he/she should pass mathematics with good grades. Hence, the understanding of first year mathematics means may lead to produce credible scientists, engineers, doctors, agriculturalists and mathematicians. Unfortunately, little scholarly work with empirical evidence has been conducted at UNZA to establish the understanding students acquire as they study mathematics concepts. A study done at UNZA by Malambo (2015) observed that preservice students demonstrated lack of in-depth understanding of school mathematics concepts despite having studied advanced mathematics courses in the School of Natural Sciences (SNS). Therefore, it appears that few research works have been conducted to ascertain the problem students have concerning the understanding of mathematics concepts studied.

If the situation remains unresolved, working stations may continue to receive incompetent teachers of mathematics skills. Producing teachers with little understanding of mathematics concepts means that these teachers may fail to teach learners in school. To further understand the problem, the current study investigated understanding of specific concepts in selected mathematics topics by first year UNZA students.

1.3 Purpose of the Study

The purpose of this study was to investigate the understanding of specific concepts in selected mathematics topics by first year students at the University of Zambia.

1.4 Objectives of the Study

The objectives of the study were:

- Develop and design valid and reliable instruments for measuring understanding of concepts in mathematics.
- Determine the kind of understanding possessed by first year UNZA mathematics students of procedural and conceptual concepts in mathematics.
- Examine the relationship between the confidence levels of students and their understanding of procedural and conceptual concepts in mathematics.
- Investigate the relationship between students' confidence and their actual performance in procedural and conceptual mathematics problems.

1.5 Research Questions

The study was guided by the following research questions.

- How to develop and design valid and reliable instruments for measuring understanding of concepts in mathematics?
- What kind of understanding do first year UNZA students have of procedural and conceptual concepts in mathematics?
- What is the relationship between students' confidence levels and their understanding of procedural and conceptual concepts in mathematics topics?
- What relationship is there between students' confidence and their actual performance in procedural and conceptual mathematics problems?

1.6. Significance of the Study

This study is expected to contribute to the body of knowledge which may help the University of Zambia lecturers align their teaching methods which would enhance students' procedural and conceptual understanding of concepts in mathematics. It has been observed that relatively there is little empirical research done related to the researcher's topic at different levels in Zambia. Therefore, this work sought to contribute to the existing literature on the understanding of first year university mathematics. Researchers afterwards may use standard indices to measure students procedural and conceptual understanding of concepts. Therefore, the results of the study may make an original contribution to research in the area of procedural and conceptual understanding of university mathematics. The study could also be useful to several stakeholders such as university lecturers of mathematics content and methodology and also curriculum planners and various examination bodies. Furthermore, the outcome of the study may inform policies of the university on how to prepare examinations on procedural and conceptual tasks in mathematics.

Specifically, the results of this study could:

- Show the extent of the relation between students' conceptual and procedural understanding in mathematics.
- Reveal the significance of the relationship between students' confidence levels when handling procedural and conceptual problems.
- May create awareness on the relationship between students' confidence and their actual performance in procedural and conceptual mathematical problems.
- May provide information that might be used by other researchers who may deal with the related problem in future.

1.7 Definitions of Terms

The following terms in the study where used to mean as explained:

A Conceptual Leap: If MDS works for physical distances, then it may also work for data that can be interpreted as 'conceptual distances' (Jacob, 2012).

Adaptive reasoning: The capacity for logical thought, reflection, explanation, and justification.

Cluster analysis: Kothari (1985) suggest that, 'Cluster analysis consists of methods of classifying variables into clusters. Technically, a cluster consists of variables that correlate highly with one another and have comparatively low correlations with variables in other clusters. The basic objective of cluster analysis is to determine how many mutually and exhaustive groups or clusters, based on the similarities of profiles among entities, really exist in the population and then to state the composition of such groups. '(p.337).

Communality ($\boldsymbol{h^2}$): Communality, symbolized as h^2, shows how much of each variable is accounted for by the underlying factor taken together. A high value of communality means that not much of the variable is left over after whatever the factors represent is taken into considerations.

Computational procedures in MDS: Given the number of dimensions, k, the aim of MDS is to find a configuration in k dimensions such that the stress criterion used is minimised.

Most ordinal MDS computer packages start with an initial configuration in k dimensions, and then iteratively improve the configuration by moving the points short distances in such a manner as to reduce the stress (Cox. T.F, and COX M.A.A, 2001).

Conceptual confidence performance: This is the ability to achieve solving a problem through conceptual understanding by being able to interpret and apply mathematical concepts in relation to the level of confidence to perform that task.

Conceptual confidence understanding: This is the ability to understand the concepts in the question and then confidently obtain the correct solutions.

Conceptual knowledge: This is basically the type of knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationship are as prominent as the discrete pieces of information. Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network. (Lefevre, 1986 and Rittle-Johnson and Schneider, 2015).

Conceptual performance: This is the performance attained by students when they solve problems by using concepts.

Conceptual understanding: This consists of relationship constructed internally and connected to already existing ideas. It involves the understanding of mathematical ideas and procedures and includes the knowledge of basic arithmetic facts. Students use conceptual understanding of mathematics when they identify and apply principles (Engelbrecht, Harding, Potgieter; 2005).

Confidence level: This is the desired degree of precision.

Correlation coefficient: Correlation coefficient between variables is applied as a measure in cluster analysis. But, it is not widely used. The correlation coefficient is a kind of similarity coefficient. So, grouping of objects is done based on the maximum value of such coefficient. (Panneerselvam, 2012:471).

Correlation matrix: It is the matrix of correlation coefficients of the original observations between different pairs of input variables.

Eigen value: This is the sum of squares of the factor loadings of the variables on a factor.

Factor: A factor is an underlying dimension that account for several observed variables. There can be one or more factors, depending upon the nature of the study and the number of variables involved in it (Kothari & Garg,1985).

Factor Analysis: Factor analysis is a technique used when there is a systematic interdependence among a set of observed variables and the researcher is interested in finding out something more fundamental or latent which creates a certain commonality among observed variables. Panneerselvam (2012) argues that:

> In many real-life applications, the number of independent variables used in predicting a response variable will be too many. The difficulties in having too many independent variables in such exercise are as follows:
>
> - Increased computational time to get solution
> - Increased time in data collection
> - Too much expenditure in data collection
> - Presence of redundant independent variables
> - Difficulty in making inferences.
>
> These can be avoided using factor analysis. Factor analysis aims at grouping the original input variables into factors which underlie the input variables. Each factor will account for one or more input variables. Theoretically, the total number of factors in the factor analysis is equal to the total number of input variables. But, after performing factor analysis, the total number of factors in the study can be reduced by dropping the insignificant factors based on certain criterion. (p.431).

Factor analysis seeks to resolve a large set of measured variables in terms of relatively few categories, known as factors (Kothari & Garg, 1985:349).

Factor loadings: Factor Loadings, $L_i(j)$: This is a matrix representing the correlation between different combinations of variables and factors. $L_i(j)$: is the factor loading of the variable j on the factor I, where $i = 1,2,3, \dots, n$ and $j = 1,2,3, \dots, n$. Factor Loadings are those values which explain how closely the variables are related to each one of the factors discovered. They are also known as factor-variable correlations.

Measures of similarity between variables: We may wish to revise the roles of variables and objects. Instead of clustering variables and objects, which is our main concern, we may sometimes cluster variables among and within the intended variables for measure in the current study. This duality arises with all analysis that start from a data matrix. If we wished to carry out an MDS analysis on variables, we would need measures of similarity between columns of the data matrix instead of between the rows (Borg, I. and Groenen, P.J.F, 2005).

Minimum eigenvalue criterion: If the eigenvalue (sum of squares of the factor loadings of all variables on a factor) of a factor is more than or equal to 1, then that factor is to be retained; otherwise, the factor is to be dropped (Panneerselvam,2012).

Multidimensional scaling: Multidimensional Scaling (MDS) allows a researcher to measure an item in more than one dimension at a time (Kothari,1985: 338). Nevertheless, in some other sense, multidimensional scaling is one of the several multivariate techniques that aim to reveal the structure of a data set by plotting points in one or two dimensions. To add on, Jacoby (2012) defines Multidimensional scaling as, 'a family of procedures for constructing a spatial model of objects, using information about the proximities between the objects.

Multidimensional scaling is one of the several multivariate techniques that aim to reveal the structure of a data set by plotting points in one or two dimensions. The input data for MDS is in the form of a distance matrix representing the distances between pairs of objects. (Borg, I. and Groenen, P. J.F, 2005). In addition, Kothari and Garg (1985) argues that:

> The significance of MDS lies in the fact that it enables the researcher to study 'The perceptual structure of a set of stimuli and the cognitive processes underlying the development of this structure…MDS provides a mechanism for determining the truly salient attributes without the judge to appear irrational.' With MDS, one can scale objects, individuals or both with a minimum of information. The MDS analysis will reveal the most salient attributes which happen to be the primary determinants for making a specific decision. (p.405).

However, MDS may be used to determine whether the distance matrix may be represented by a map or configuration in a small number of dimensions such that distances on the map reproduce, approximately, the original distance matrix $\{\delta_{ij}\}$. Furthermore, the input data for MDS should be in the form of a distance matrix representing the distances between pairs of objects.

Multidimensional Scaling (MDS) is one of the extensions of multivariate techniques for measuring human perceptions and preferences. Multidimensional scaling deals with the judgments of respondents about the degree of similarity of pairs of stimuli on similarity basis or distance basis (Panneerselvam, 2012).

Firstly, there is no ambiguity about what we mean by 'distance' between variables. Secondly, it is preferable to use multidimensional scaling in two dimensions. This is so because the points in two dimensions can be easily projected on the two-dimensional space. Finally, the input data for MDS is in the form of a distance matrix representing the distances between pairs of objects (variables).

Procedural confidence performance: This is the ability to achieve solving a problem through manipulation of mathematical skills, such as procedures, rules, formulae and symbols used in mathematics.

Procedural confidence understanding: This is the ability to understand the procedures in the question and then confidently obtain the correct solutions.

Procedural fluency: is defined as the skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.

Procedural knowledge: The knowledge obtained by solving problems using procedures.

Procedural performance: This is the performance attained by pupils when they solve problems by using procedures.

Procedural understanding: This is the level of understanding that consists of methods and strategies to solving the problems correctly by following the given procedures. Procedural understanding is the ability to solve problems by using a procedure that is efficient, accurate, true to add, subtract, multiply and divide (Askew, 2012, Kilpatrick et al., 2001 and Fatqurhohman, 2016).

Productive disposition: The inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy. (National Research Council, 2001:116).

Profile Dissimilarities: Jacob (2012) argues that, one important type of conceptual distance data:

> Each of k objects has scores on each v variables.
>
> Each object's vector of scores is called its profile.

For each pair of objects, take the sum of squared differences across the v variables and, optionally, take the square root of the sum.

For objects i and j, each of which have scores on variables $x_{1,}\ x_{2,\ldots.}x_v$ the profile dissimilarity is:

$$\delta_{ij} = \left[\sum_{i=1}^{v}\left(x_{ik} - x_{jk}\right)^2\right]^{0.5}$$

δ_{ij} is the profile dissimilarity between the two objects; it can be interpreted as the distance between them in $v -$ dimensional space (p.27).

Scree plot criterion: Scree plot criterion requires the researcher to plot the eigenvalues of the factor by taking the factor number on $X - axis$ and the eigenvalues on the $Y - axis$. Identify the factor number at which the slope of the line connecting the points changes from step to a gradual trailing off towards the right of the identified factor number. Such change in slope in the graph is known as the scree point from the origin are to be retained for future study and all the factors to the right of the scree point are to be dropped from the study (Panneerselvam,2012).

Strategic competence: This is the ability to formulate, represent, and solve mathematical problems. Bautista (2013) states that, Strategic competence includes the ability to formulate plans that enables learners to use symbolic analysis in solving problems.

1.7.1 Scope of the Study

The study focused on procedural and conceptual understanding of concepts in mathematics which made the researcher investigate the extent of success in confidence of solving either procedural or conceptual aspects of mathematics. Furthermore, the study focused on the understanding of specific concepts in selected mathematics topics at first year level at the University of Zambia in the School of Natural Sciences.

1.8 Limitations of the Study

The study had a number of limitations that needed to be taken into consideration. The study only focused on first year students studying mathematics at the School of Natural Sciences at the University of Zambia. Since only part of the students doing mathematics was picked this meant that the validity and reliability of the study was limited to only first year students. The implication is that the results though generalizable may not reflect the same results for second, third and fourth year students. This is because of the variations in methodologies used by lecturers at different levels of students in the university. Also, it is difficult to generalise the results obtained from a case study. Going forward a number of tests should be prepared for each year to have a consolidated view of students understanding of mathematics concepts. Subsequent studies may overcome these limitations stipulated above.

1.9 Delimitations of the Study

The study was limited to the amount of data to be covered and investigated. The study focussed on objectives which were quantitative in nature because they suited the scope of the study. Even though, a number of students take mathematics at UNZA it was observed that first year students had more difficulties than any other stream. Hence the study opted only to use results of first year students at UNZA. The study was conducted at UNZA with a sample of only randomly selected 378 first year students. Similarly, the number of students who were picked were among all those pursuing first year mathematics course at UNZA offered in the School of Natural Sciences. The study further looked at the first two components of mathematical understanding of braided cognitive process because there are of a lower order (procedural fluency and conceptual understanding). The study did not look at strategic competence, productive disposion and adaptive reasoning because there are of a higher order of mathematical understanding.

1.10 Layout of the Thesis

The thesis has seven chapters. To begin with, chapter one summarised the background to the research and stated the problem. It also outlined the research objectives and questions. Chapter two offers the philosophical orientation, research paradigm, theoretical and conceptual framework. Chapter three discusses literature as reviewed by the current researcher focussing on previous scholars but relating to the topic at hand. Chapter four presents the methodology. It shows the methods used to collect data with a bias to primary data of which the test was given using standard indices followed by multivariate analysis (i.e. factor analysis, multi-dimensional scaling and cluster analysis using both SPSS and XLSTAT with Excel). Chapter five shows research findings by presenting research results. Chapter six discusses research results and finally chapter seven provides conclusions and recommendations of the research.

1.11 Summary of chapter 1

Chapter one laid the foundation for the thesis. It introduced the background to the study, stated the research problem and research questions. Then the research was justified by giving a synopsis of UNZA performance in mathematics by first year students. The chapter adequately stated limitations and definitions of the study. Finally, an outline of the summary of the chapter was given.

CHAPTER TWO
THEORETICAL AND CONCEPTUAL FRAMEWORK

2.1 Introduction

This chapter contains philosophical orientation, philosophical worldviews, research paradigms and theoretical and conceptual frameworks of the study.

2.2 Philosophical Orientation

The researcher picked all the three important components of research paradigm as approaches to research which involved realist ontology, philosophical assumptions as well as distinct methods or procedures in conducting research. The research approach which was instituted composed of a plan to conduct research, it involved the philosophical orientation, research designs, and specific methods. The researcher used operationalizing the concept of learning model to develop his own conceptual framework that he used to explain the interaction of components in the study. However, the researcher picked positivism as a philosophical orientation in the research. Positivism was picked because it suited the study since it recognises results that can be scientifically verified as argued by Henslin (1998) that positivism philosophical theory stipulates laws and their operation which derive validity from the fact of having been enacted by authority or of deriving logically from the existing decisions, rather than from any moral considerations. Additionally, positivists believe that before the study, the researcher should be involved in planning of a study so that an appropriate scientific method is engaged. Besides, the researcher thought through the philosophical worldview assumptions that he brought to the study, the research design that related to this worldview, and the specific methods or procedures of research that translated the approach into practice. Moreover, Moore (1987) argues that, ‘positivists believe that sociology tries to be a science and use as many of the methods which the physical sciences uses as is possible by gathering statistics.(p.16). Furthermore, the same author postulates that, research is another way of trying to arrive at the truth. The person engages in research, to find out as many relevant facts and pieces of information about the area under investigation.

2.3 Philosophical Worldviews

Although philosophical ideas remain largely hidden in research (Slife & Williams, 1995), they still influence the practice of research and need to be identified. It was suggested that philosophical ideas of positivism in the study embraced explanation of the research for which the researcher chose quantitative approaches for the research. The philosophical worldview proposed in the study by definition acted as the basic ideas of that worldview. It was observed that the worldview in this study was based on scientific inclinations and experiences. Therefore, quantitative methods and approaches were used in this research. Therefore, quantitative research explains that the phenomenal by which numerical data is collected and analysed mathematically is based on specified statistical methods. While it is important to use the right data analysis tools, it is even more important to use the right research design and data collection instruments (Muijs, 2004).

2.4 Positivism

There is by far a number of variations in explaining positivism that may be equal to the number of authors who address the topic of philosophical research. Nevertheless, in its essence, positivism is based on the idea that science is the only way to learn about the truth. As a philosophy, positivism adheres to the view that only 'factual' knowledge gained through observation, including measurement, is trustworthy (Collins, 2010). Studies which engage positivism as a method have their data analysed scientifically and findings produce results which are observable and quantifiable. In addition, the philosophical implication of the study is based on the paradigm shift taken or the methodological orientation. In this case a positivist approach was used, not only did it involve observable and measurable variables but also quantitative approaches were used. In this study, the researcher was independent from the research because he was operating within clearly defined parameters which made the study to be objective. Given that, the researcher made sure that the research was focused on observed and measurable variables. Our study was operated on clearly defined parameters by having objectives being scientific and impartial in that it involved measuring of variables and concepts using evidence based tenets of research.

Ramanathan (2008) argues that, 'positivism depends on quantifiable observations that lead to statistical analyses. On the other hand, it has been noted that "as a philosophy, positivism is in accordance with the empiricist view that knowledge stems from human experience. It has an atomistic, ontological view of the world as comprising discrete, observable elements and events that interact in an observable, determined and regular manner".

However, in positivism studies the researcher is independent from the study and there are no provisions for human interests within the study. Crowther and Lancaster (2008) argue that as a general rule, positivist studies usually adopt deductive approach, whereas inductive research approach is usually associated with a phenomenology philosophy. Moreover, positivism relates to the viewpoint that the researcher needs to concentrate on facts, whereas phenomenology concentrates on the meaning and has provision for human interest. Therefore, developing an understanding of the concept for positivism is helpful before conducting concept analysis (Pawlikowski, 2018).

Wilson (2010) in agreement with Easterby-Smith, Thorpe, & Jackson, (2008) suggested that, "if you assume a positivist approach to your study, then it is your belief that you are independent of your research and your research can be purely objective. Independent means that you maintain minimal interaction with your research participants when carrying out your research." Crowther, & Lancaster, (2008) further postulates that, "research methods in other words are studies with positivist paradigm that are based purely on facts and consider the world to be external and objective. Wilson (2010) stipulates that:

> The five main principles of positivism research philosophy can be summarized as the following:
>
> 1. There are no differences in the logic of inquiry across sciences.
> 2. The research should aim to explain and predict.
> 3. Research should be empirically observable via human senses. Inductive reasoning should be used to develop statements (hypotheses) to be tested during the research process.
> 4. Science is not the same as the common sense. The common sense should not be allowed to bias the research findings.
> 5. Science must be value-free and it should be judged only by logic.

In this study, a systematic and scientific approach was made and followed at all costs. It was observed that logical means and ways were instituted in the collection of data for the project in oder to avoid bias. In line with the aforementioned, it was easy to explain and predict the entire process of developing measuring instruments of mathematics concepts. The already alluded to approach was followed because it made the study be guided by informed literature, science and empirical evidence. Arising from the aforesaid, then the developed instruments were used not only to measure mathematics understanding of concepts but also procedural and conceptual confidence of learners exhibited to solve tasks correctly.

2.5 Theoretical and Conceptual Framework

2.5.1 Introduction

This section of the study elaborates on the relationship that is there between the theoretical and conceptual framework of the study. This component of the study highlights three theoretical frameworks and three conceptual frameworks which conformed to the study. Firstly, Auguste Comte, Emile Durkheim and Logical positivism comprised of theoretical framework. Secondly, operationalizing the concept of learning model and knowledge translation framework composed of the two conceptual framework of the study. Thirdly, the author developed his own model for the research after analysing the test which he gave to the students. Furthermore, the development of the research model by the researcher was based on the research questions, research approach and also literature surveys.

2.5.2 Theoretical Framework

Positivists approach was picked as theoretical frameworks which suited the study because the researcher did not want to compromise the findings of the study by remaining objective and also avoid any interractions between and among the subjects under investigation. Therefore, in this study Auguste Comte's, Emile Durkheim and Logical Positivism theorems were used in the research. This was so that the shortfall of one theorem maybe uplifted by the strength of the other and vice-versa. Section 2.5.3 outlines Auguste Comte's theory.

2.5.3 Auguste Comte's Theory

Auguste Comte (1798-1857) was the founder of sociology, as he is widely identified. He analysed the society based on social order. Most importantly, he argued that, 'the scientific method should be applied to the study of society. Henslin (1998) points out that, 'the idea of applying scientific method to social world is known as positivism and was apparently first proposed by Auguste Comte (p.6). To add on, Comte argued that, the right way is to apply scientific method to social life and this is what is today called sociology. Nevertheless, Comte stipulated that we must observe and classify human activities in order to uncover society's fundamental laws. Auguste Comte first described the epistemological perspective of positivism in 'the Course in Positive Philosophy,' a series of manuscripts published between 1830 and 1842. These texts were followed by the 1844 work, A General View of Positivism (published in French 1848, English in 1865). Gilson and Irving (2012) in agreement with Giddens (1974) argues that, 'the first three volumes of the course dealt chiefly with the physical sciences already in existence (mathematics, astronomy, physics, chemistry, biology), whereas the latter two emphasized the inevitable coming of social science. Observing the circular dependence of theory and observation in science, and classifying the sciences in this way, Comte may be regarded as the first philosopher of science in the modern sense of the term. Pickering (1993) argues that, ‘the physical sciences had necessarily to arrive first, before humanity could adequately channel its efforts into the most challenging and complex "Queen Science" of human society itself’. His view of Positivism therefore set out to define the empirical goals of sociological method.

Schaefer (2012:9) argued that not only did Auguste Comte had credit of being one of the most influential of philosophers of the early 1800s but also was considered as a theoretical sciencentist of society and a systematic investigator of behaviour of his time and was influencial to the improvement of society. Giddens and Sutton (2013) suggest that, ‘Auguste Comte describes sociology as an emerging science that should adopt the successful and positivist methods of natural sciences (p.12). However, science involves the use of systematic methods of empirical investigation, the analysis of data, theoretical thinking and the logical assessment of arguments to develop a body of knowledge about a particular subject matter.

Gilson, Gregory and Irving Levinson (2012) suggest that, Comte would say: "from science comes prediction; from prediction comes action. “It is a philosophy of human intellectual development that culminated in science. The irony of this series of phases is that though Comte attempted to prove that human development has to go through these three stages, it seems that the positivist stage is far from becoming a realization. This is due to two truths: according to Schunk (2008) who further suggests that, the positivist phase requires having a complete understanding of the universe and world around us and requires that society should never know if it is in this positivist phase. To add on, Anthony Giddens (1974) argues that, since humanity constantly uses science to discover and research new things, humanity never progresses beyond the second metaphysical phase. Marsh (1996) points out that, ‘Auguste Comte helped to establish the idea of the study of society as a project and gave it the name while Emile Durkheim gave sociology its academic credibility and influence (p.47). The next section focuses on Auguste Comte’s theory in relation to the current study.

Auguste Comte’s Theory (Relation to the Current Study)

It was observed that Auguste Comte’s theory in relation to the current study signalled that science is the only valid knowledge supported by facts which are the only objects of knowledge. This is in line with Auguste Comte's theory who suggested that the concept of positivism is made up of experience, the system of facts, objective, human, and natural phenomena (Pawlikowski, Rico, Van Sell, 2018). Likewise, it has been revealed that Comte was a scientific thinker, systematically reviewing available data. This means that Auguste Comte was an empiricist or that he believed in actual observations of experiements in order to have empirical evidence (i.e. evidence supported by facts obtained from the empirical study of procedures of a study. In this regard, the researcher had to obtain data using empirical methodology by applying logical empiricists in the interpretation of results (Kitchener, 2004). The following section outlines the researchers personal perspective of Auguste Comte's theory towards the study at hand.

Auguste Comte's Theory (Personal Perspective towards the Study)

My personal perspective towards the study of Auguste Comte's theory is that positivism approach has of late been viewed as a momentous approach of conducting research based on objectivity, empiricism, and natural phenomena. Above and beyond, scientific methods in research are highly encouraged. This empowered me to only use quantitative methods in my research study. To add on, I believe that observation and reason are true components of Auguste Comte's theory. This is so because all explanation and prediction in this theory are based on lawful succession of empirical facts. This may be achieved only when the researcher uses quantitative approaches in data collected, interpretation and analysis of results. I observed that Auguste Comte was a scientific philosopher who believed in systematic reviewing of data which concided with the edifice of the current study. This prompted me to use only scientific methodologies because they matched the way I handled, organized and analysed data in my study. Section 2.5.4 outlines Emile Durkheim's theory.

2.5.4 Emile Durkheim's Theory

Henslin (1998) argues that, 'Durkheim's identification of the key role of social integration in social life remains central to sociology today (p.7). Giddens and Sutton (2013) suggest that, Durkheim study on social phenomenon helps the society to appreciate research when we consider people's behaviour and also going beyond individual interaction in order to conduct sound research (p.78). Furthermore, Durkheim identified social integration as the degree to which people are tied to their social group, as a key social factor. To add on, the primary professional goal of Emile Durkheim (1858-1917) was to get sociology recognized as a separate academic discipline. However, another goal for Durkheim was to show how social forces affect people’s behaviour. The modern academic discipline of sociology began with the work of Émile Durkheim (1858–1917). While Durkheim rejected much of the details of Comte's philosophy, he retained and refined its method, maintaining that the social sciences are a logical continuation of the natural ones into the realm of human activity, and insisting that they may retain the same objectivity, rationalism, and approach to causality. Thibodeaux (2016) in agreement with Schunk (2008) argues that, ‘Comte was the only major sociological thinker to postulate that the social realm may be subject to scientific analysis in exactly the same way as natural science, whereas Durkheim saw a far greater need for a distinctly sociological scientific methodology’. Ferrante (2011) argues that,’ Emile Durkheim suggested that the system of social ties acts as a cement binding people to each other and to the society (p.14). Schaefer (2012) argues that, ‘Emile Durkheim established an impressive academic reputation (p.9). Giddens and Sutton (2013) suggest that, ‘ Durkheim explains social facts as institutions and rules which constrain or channel human behaviour (p.12). The next section looks at Emile Durkheim’s theory in relation to the current study.

Emile Durkheim’s Theory (Relation to the Current Study)

The French sociologist Emile Durkheim firmly believed that knowledge acquired through the scientific investigation of society could be used to help men build a better tomorrow (Young, 1962). Emile Durkheim sought to develop a scientific study of any phenomena by observation. Likewise, the current study used scientific methodologies which are objective in order to avoid bias in the study. The study used scientific methods which demanded observable evidence which was gotten after using multivariate analysis methods and standard indices as clear and scientific objective methods of the study. Furthermore, Emile Durkheim conceived sociology to be not only a positive science and a method of investigation, but also to be the foundation of an integrated social philosophy which men could use to raise social standards (ibid, 1962). Section 2.5.4. b The following section outlines the researchers personal perspective of Emile Durkheim’s theory towards the study at hand.

Emile Durkheim’s Theory (Personal Perspective towards the Study)

My personal perspective towards the study in relation to Emile Durkheim’s theory of positivists nature is that research is a scientific way of organizing and interpretation of data. I believe that research should be done in a scientific way which agrees with the advocates of Emile Durkheims who suggested that research must be developed in a scientific way and findings of the study should be established through a well thought observation. I do believe that for results of the study to be declared authentic, the research should employ objective, scientific approaches and quantitative methods of data collection and analysis. All these scientific steps of conducting research should be clear and straight forward so that any qualified researcher may be able to follow the methods used without difficulties. Section 2.5.5 outlines Logical Positivism theory.

2.5.5 Logical Positivism

Flynn (2007) stipulated that, logical positivism (also known as logical empiricism or logical neopositivism) was a philosophical movement risen in Austria and Germany in 1920s, primarily concerned with the logical analysis of scientific knowledge, which affirmed that statements about metaphysics, religion and ethics are void of cognitive meaning and thus nothing but expression of feelings or desires; only statements about mathematics, logic and natural sciences have a definite meaning (p,1). Additionally, Giddens (1974) in agreement with Gilson, and Irving (2012) laments that:

> ‘Shortly after the end of the First World War, a group of mathematicians, scientists, and philosophers began meeting in Vienna to discuss the implications of recent developments in logic, including Wittgenstein's Tractatus. Under the leadership of Moritz Schlick, this informal gathering (the "Vienna Circle") campaigned for a systematic reduction of human knowledge to logical and scientific foundations. Because the resulting logical positivism (or "logical empiricism") allowed only for the use of logical tautologies and first-person observations from experience, it dismissed as nonsense the metaphysical and normative pretensions of the philosophical tradition. Although participants sometimes found it difficult to defend the strict principles on which their programme depended, this movement offered a powerful vision of the possibilities for modern knowledge.

Additionally, logical positivism was used because it has a very narrow perspective toward science, whereby it is thought as a science or set of laws, theories and principles. It is revealed that logical positivism has two components. The first one is the context of discovery. In this context it shows how a researcher is supposed to do a research following an appropriate approach in order to do a scientific inquiry be it personal, contextual or philophical paradigm in order to formulate new discovery. Subsequently, we have the context of justification where the researcher is capable of defending his discovery using scientific means or theories. In this regard, logical positivism may help researchers identify right methods of scientific inquiry that are capable of constituting authentic knowledge. On the basis of the arguments presented above, this may help researchers to know the difference between authentic and pseudo statements because authentic data is always verifiable. Not only is logical positivism objective but also has anti-metaphysics attitude which makes science be a priviledge of knowledge which made the researcher to opt to use it in the current study. The next section looks at Logical Positivism theory in relation to the current study.

Logical Positivism (Relation to the Current Study)

The reality on how to conduct a research study based on logical positivism is that the researcher must use objective research methods in order to uncover the truth. By developing reliable measurement instruments, we can objectively study the physical world (Muijs, 2004). According to Kitchener (2004),

> What makes a theory an adequate one, or one theory better than another? The obvious answer is empirical evidence. A theory has some positive epistemic status if available empirical evidence adequately supports it; one theory is better than another if the available evidence supports it more adequately than it supports the other theory.(p.45).

Notwithstanding, logical positivism means that the actual observations of an experiment must show empirical evidence only after using authentic methodologies. Likewise, this may help the researcher to analyse the test instrument before giving it to the participants in the study. It can be noted that, when using logical positivism the researcher should be detached from the research as much as possible and use methods that maximise objectivity and minimise the involvement of the researcher (Muijs, 2004). Quantitative research was picked in this study because it allows generalizability of results and also increased understanding of research results. Additional mechanism to solve issues or uncover new problems are thereby enhanced. The following section outlines the researchers personal perspective of Logical Positivism theory towards the study at hand.

Logical Positivism (Personal Perspective towards the Study)

The study used logical positivists because it accepts epistemological and semantic claims that normally hold radical metaphysical conclusions. Logical positivism cultivate in students the habit of critical thinking leading to a coherent vision of reality. Firstly, the study accepts empiricism, which is the claim that we can only come to know invulnerable truths through sense experience. In this case the study sought to verify the findings of the research by analysing data which can only be accepted to be true after empirical results. Secondly, logical positivism accepts verificationist theory of meaning, according to which the meaning of synthetic claim is exhausted by the sense experience we could expect to undergo if claim were true. Above and beyond, the study needed to be investigated by scientific experiments by using authentic, valid and reliable methodology in order to verify facts.

Logical positivism is a combination of the two approaches upheld by positivism and symbolic logic. Positivism is a particular school of knowledge which advocates that valid knowledge must be based on sense knowledge. Therefore, logical positivism emphasis is on the use of symbolic logic. It was this symbolic logic which helped the researcher to come up with the four standard indices in the study. It was easy to do that because symbolic logic helped the researcher to develop and formulate logical principles which were heavily used in the adjustment of standard indices borrowed from Pretoria University which are the brain child of Engelbrecht et al (2005). Symbolic logic was used because it has the advantage of clarity and exactness and anything in science should be translated in terms of symbolic logic. Hence or otherwise, if science is formulated in mathematical logic then science is clear, distinct and exact. It is the aforementioned tenets of logical positivism which made the author to develop the four standard indices which are used later in the study to measure procedural and conceptual (understanding and confidence).

Therefore, in this study four sets of formula were developed and designed then utilized in the analysing of data and also multi-variate analysing tools were used as a set of principle to measure procedural and conceptual (understanding and confidence). It was further revealed that logical positivism uses empirical research and logical analysis of results which have to be scientifically proven to be true. On the whole, logical positivism which is sometimes called empirical research since it uses conventional science to logically analyse the findings by using appropriate methodologies is commonly used because it involves the use of symbolic logic which helps to represent research findings adequately to respective audience. In summary, the research results should be supported by well interpreted results obtained after following scientific procedures.

It is evident that, the study used Auguste Comte's theory, Emile Durkheim's theory and Logical positivism as all the three theories are quantitative in nature. The three theories were similar in a number of aspects, such as advocating for the use of scientific method which should involve systematic methods of empirical investigation. In addition, the three theories encouraged objectivity through reviewing of available data. In this study, it was observed that the three theories stated above helped in the stages of instrument development and designing. This was so because the aforementioned theories were not only enevitable in the stages of instrument development but also were well-articulated for the foundations of instrument development. On the whole, the theories are positivists in nature and they advocate for a systematic and scientific approach of conducting a study which was instituted in this project.

Despite having a good number of similarities the three theories had few differencies. Auguste Comte's theory helped the study to come up with standard indices which were scientific in nature and latter applied to society. This was in line with Auguste Comte's theories which states that any scientific method instituted should be applied to the study of society. In this aspect, standard indices were applied to measure mathematical understanding and also confidence of understanding. In addition, Emile Durkheim's theory helped the study to make standard indices be accepted in society by using social integration. This was so because of the advocation of building a better tomorrow. Consequently, the new adapted standard indices may be used even in other STEM subjects to measure understanding of other scientific ocncepts. Last but by no means the least, Logical positivism was used because it helped to formulate standard indices because its prominence in the use of symbolic logic which made the contributed standard indices to the body of knowledge acceptable since there were clear and exact. Furthermore, Logical positivism helped greatly in the formulation of new discovery (standard indices) in that before the contributed new knowledge were made public there were justified using scientific means. By and large, Karl Popper (1962) was interrogated on the falsification of conjectures. The earlier standard indices of Engelbrecht et al., (2005) were falsified and then improved upon by the current researcher. This lead to the contribution of four standard indices as new knowledge to the body of knowledge.

2.5.6 Conceptual Framework of the Research

Conceptual frameworks were used to help the researcher focus on the variables in the study. Firstly, operationalising of concept of learning model was used. Secondly, knowledge translation framework was used. Lastly, the researcher developed his own conceptual model. Therefore, three conceptual frameworks were used in the study in order to achieve a detailed investigation in measuring of understanding of mathematics concepts. This was to be achieved by measuring of understanding of specific concepts of selected topics in mathematics at first year level at UNZA.

2.5.7 Operationalizing the Concept of Learning Model

The study was grounded by Operationalizing of the Concept of Learning Model. Operationalizing the Concept of Learning Model was used in the current study because it suited the research at hand. Above and beyond, it can be deduced from the conceptual framework that it made easier for the explanation of procedural and conceptual concepts. Againist this background, operationalizing the concept of learning model helped in the development of the test which was used as an instrument for research in order to develop and design valid instruments for measuring of mathematical concepts. In view of the aforesaid, Kent and Foster (2016) rigorously allowed learners to operationalize mathematical concepts in the learning process. Additionally, the model assisted the researcher to determine how to measure procedural and conceptual concepts in mathematics. Arising from the above, the instruments developed were also used to measure procedural and conceptual confidence levels of the students. On the whole, not only was understanding of mathematics concepts delineated with answering questions correctly but also giving appropriate examples as emphasised in the model exemplified below. The current study made sure that concepts were correctly operationalized in order to obtain valid measures of understanding (procedural or conceptual). It was the intention of the present study to make sure that the concepts measured were the intended ones as supported by (Sekaran, 2003:182) who contends that concepts are supposed to be described as observable characteristics in order to measure the required concepts appropriately. To understand fundamental properties of mathematics, statements from algebra, concepts in analysis or geometry, or mathematical text which may represent concepts and theorems which have been methodologically done correctly there is a definite need to operationalize the concepts for easier understanding of concepts by learners (Lupu, 2014).

In order to formulate the operationalization of the concept of learning model, it was the wish of the researcher to simplify major variables by operationalizing them so that it would be easier for learners to understand mathematics concepts. In this way, it is argued that lecturers should teach mathematics with an emphasis towards understanding of mathematics concepts by learners. This may only be achieved by operationalizing mathematics concepts so that they are related to real life situations. If this is done, it means understanding of procedural and conceptual concepts in mathematics may improve. Lectures' strategies and epistemological assumptions towards the teaching of mathematics concepts should change so that abstract concepts in mathematics are elaborated in a more clear and friendly manner. It is further argued that, there is need to operationalize the constructs of knowledge in the development of context when teaching and learning mathematics for easier understanding of mathematics concepts (Nezhnov, 2014).

Personal perspectives towards the theory of operationalizing the concept of learning model were in more ways than one. To start with, the study adopted Nowell et al (2017) who pointed out that conceptual framework should be adjusted in order to conform to the current study. In this way, the study was clear and explicit about how the research would answer research objectives. To this end, an audit trajectory was done by expressing the researchers own conceptual framework to show how understanding of knowledge translation may be achieved from research variables to research findings. Therefore, the conceptual framework underpinning the whole study was developed by structured test items which were organized as either procedural or conceptual. Nevertheless, the process of conducting research was composed of students who had to write the test. The students were ready to write the test at the end of the term because they were tested on all the topics covered in that particular term. The outcomes consisted of performance of the students in relation to confidence learners exhibited when solving questions correctly. Furthermore, the outcome of the test may inform policies of the university on how to prepare examinations on procedural and conceptual problems in university mathematics. Additionally, it was observed that, knowledge continues to develop to a more deep-level understanding of conceptual relations which underlying learned procedures and , finally interpretes data to the highest level of understanding that allows a person to see the boundaries of knowledge acquired (Nezhnor, 2014). Then, next section 2.5.8. outlines the operationalizing the concept of learning model.

2.5.8 Operationalizing the Concept of Learning Model

Operationalizing the concept of learning model gives the opportunity to researchers to design components of the study into elements which are well elaborated in order to conform to the day-to-day practices of interpreting studies in a more detailed manner (Basham et al., 2016).

OPERATIONALIZING THE CONCEPT OF LEARNING MODEL

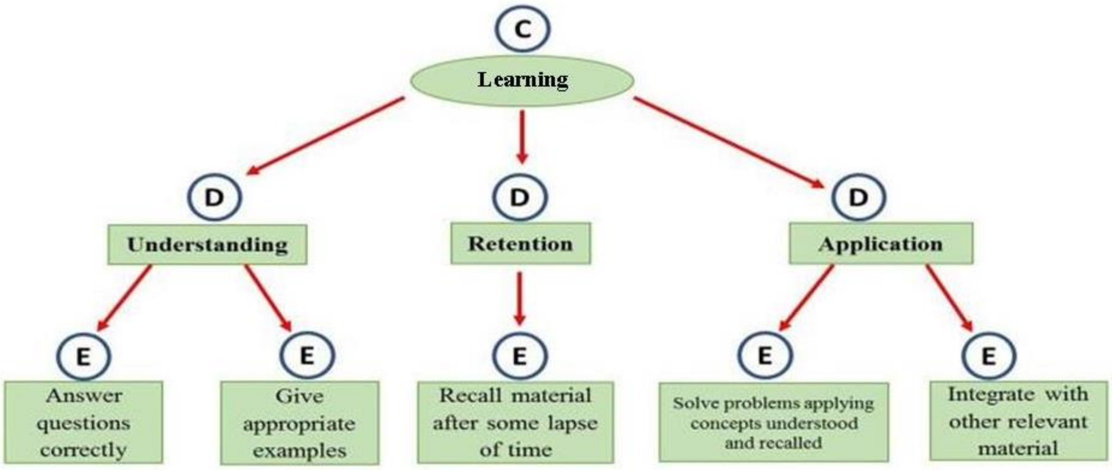


Figure 2.1 Operationalizing the Concept of Learning Model (Sekaran, 2003: 183) Research Methods for Business. A Skill Building Approach

In this model above (Figure 2.1) the researcher picked the component of understanding and married it with answering the questions correctly and also give appropriate examples in order to measure understanding of conceptual and procedural concepts. This model helped the researcher to measure understanding by formulating a test which was used to measure understanding by employing standard indices, factor analysis, multidimensional scaling and cluster analysis.

Learning

Learning may be defined as the rate of understanding of concepts after instructions given by the teacher. Furthermore, learning may also be defined as the ability of retention of information and also application of learnt concepts to answer real life situations. In summary, conceptual definition of learning successfully convey the message about different theoretical perspectives on learning but suffer from lack of operationalizing of concepts (Harel & Koichu, 2010).

Understanding

Understanding of concepts may be measured by giving questions to the learners after the learning session, if students answer the questions correctly then it means they have understood concepts learnt. Usually, students should give correct answers with the aid of examples.

Retention

Retention entails the ability of learners to remember learnt concepts after a long period of time since the learning occurred.

Application

Students should not only be able to apply the concepts understood and remember to solve real life situation problems but also to solve similar problems in different subjects. The five components of Operationalizing the Concept of Learning Model was married to the Braided Conceptual Model of Mathematical Cognitive Processes. As stipulated above in the diagram we wish to elaborate fully the conceptual framework as here under clarified.

Answer questions correctly

In this study answering questions correctly was correlated to procedural understanding of concepts. Therefore, answering questions correctly in the test or examination helps the lecturer to know if the students have understood the procedures of mathematical concepts taught.

Give appropriate examples

The study has shown that giving appropriate examples correlates to conceptual understanding of concepts. If learners are able to give examples after learning then it means they have understood concepts learnt.

Recall materials after some lapse of time

It can be deduced from the conceptual framework above that recall material after some lapse of time correlates to strategic competence. Lapse of time in this study meant over a period of a year or more but students were tested at the end of the test. Hence, only immediate concepts in that particular term were tested on to students. Even though the five components of Kilpatrick et al., (2001) are interrelated and intertwined basically the first two are of a low order as compared to the last three. In addition Malubila (2020) supports Christiansen (2019) that the five strands of understanding are interrelated and intertwined but they have been falsified by Kalungia (2020) who argues that the first two are of the lower order and the last last are of the higher order and needs deep understanding as compared two the first two. Kalungia (2020) used the last three components of Kilpatrick et al., (2001) to measure students understanding of pharmany concepts of third year students at UNZA and stipulated that they formulated deep understanding of concepts. This prompted the researcher not to use the last three components in the current study since they were considered to be of a higher order of mathematical proficiency even though there are interralted and intertwined. Instead, the current study considered only two lower levels of mathematical proficiency. Learning is achieved when the students are able to recall the concepts taught after some period of time. Consequently, for students to be good at school they are supposed to remember the concepts taught in different subjects and use the concepts appropriately in order to find solutions to new problems.

Solve problems by applying concepts understood and recalled

According to the conceptual framework above, solving problems by applying concepts understood and recalled required that students should not only be able to apply concepts learnt in order to solve current problems but also to solve problems in the subject area. This means that learners should be able to understand concepts fully so that they may be able to recall what they had learnt in order for them to apply them to other situations. Arising from the arguments presented above, it may be concluded that this aspect was not applied to the current study as it is considered to be a bit above the strands considered.

Integrate with other relevant material

By and large students are able to integrate concepts with other relevant materials if they are able to apply the concepts learnt in different situations correctly. This means that concepts learnt in mathematics should be applied to solve problems in related science subjects. Therefore, integrating with other relevant material was not picked in the current study because it was not part of the scope of the study.

2.5.9 Knowledge Translation (KT)

With reference to Figure 2.2 on the next page, it has been observed that researchers within and across research disciplines have settled on using knowledge translation framework. Likewise, this is because of its vast application by policy makers, planners and managers across disciplines. It is with respect to the aforementioned qualities of KT that the study adopted to use it in the interpretation of research results. It was observed that not only does KT involve creation of new knowledge and applications but it is also used to analyse specific content of variables under investigation. There is need to use a conceptual framework that stipulates conditions for achieveing critical goals such as provoking students' intellectual need to learn mathematics, helping them to construct mathematical ways of understanding and ways of thinking (Harel & Koichu, 2010). Likewise, the study opted to use KT because it helped in the generation of new instruments to use when measuring procedural and conceptual understanding of mathematics concepts and students confidence to either perform a procedural and/or conceptual mathematics problem. Knowledge translation (KT) is a complex and multidimensional concept that demands a comprehensive understanding of its mechanisms, methods, and measurements, as well as of its influencing factors at the individual and contextual level and interaction between both these levels (Sudsawad, 2007).

KNOWLEDGE TRANSLATION MODEL

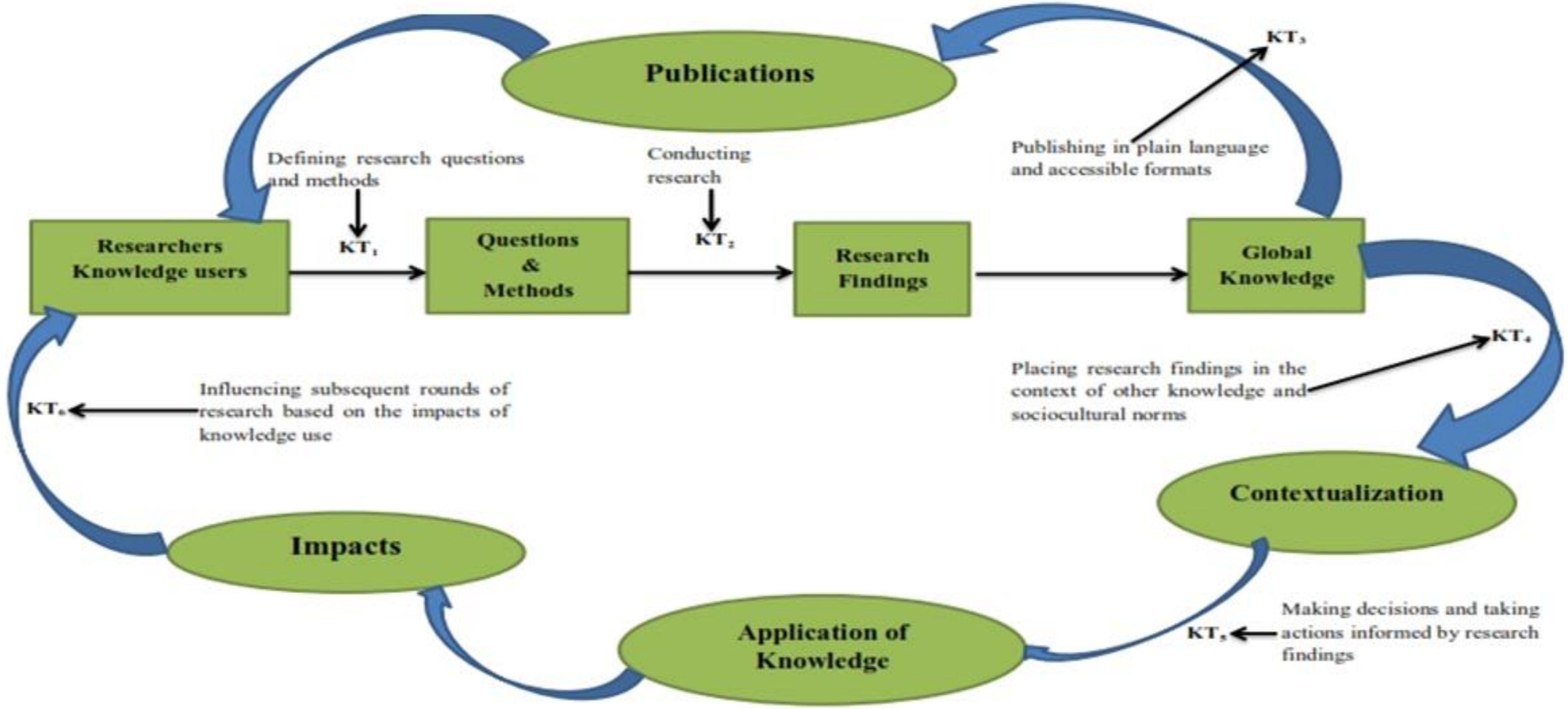


Figure 2.2: Source (Sudsawad, 2007)

2.5.10 Operationalizing the Concept of Learning Model vs Knowledge Translation

The present study which is an attempt to investigate first year students' understanding of specific concepts in selected mathematics topics at UNZA. Likewise, this was to be achieved by developing and designing valid and reliable instruments for measuring mathematics understanding (procedural and conceptual). Operationalizing the concept of learning model helped the researcher to measure specific concepts in mathematics. The study hopes to delve further into the development and designing of valid and reliable instruments for measuring mathematical understanding and confidence of learners (procedural or conceptual). It is important to mention that the test in the study should neither be too difficult or simple for the respondents but it should be a standard test by using the typical performance test (TPT). The test should be objective in that it should measure understanding of mathematics concepts (procedural or conceptual). This may be achieved when students are able to answer questions correctly and also give appropriate examples of mathematical concepts.

On the other hand, Knowledge Translation (KT) was used because of its three viable phases. These involve knowledge inquiry, knowledge synthesis and knowledge product. The current study sought to investigate the understanding of mathematics concepts by first year students at UNZA. Notwithstanding, KT was used since it makes knowledge to be conceptualized empirically. In this regard, four standard indices were generated which helped us intervene and innovate to aid the measuring of students understanding of specific concepts in mathematics. The new instruments for measuring procedural and conceptual understanding are sometimes called 'standard indices'. The high validity and reliability of standard indices may consequently result in the accurate recording of test performance of learners which in turn may help lecturers make sound judgements as to which concepts to shade more light on while teaching.

As alluded to above, if standard indices are properly used to measure procedural and conceptual understanding then the actual understanding of concepts will be captured. Therefore, students test performance should improve meaning that the learners may fully understand mathematical concepts learnt. Lastly but by no means the least, it means that the overall understanding of concepts when using operationalizing the concept of learning model may improve. If the test performance improves then also the overall understanding of concepts in the learning process may improve since they is a significant correlation between performance and understanding of concepts. This implies that procedural and conceptual understanding of specific concepts in selected mathematics tests may improve if new instruments (standard indices) are utilized fully. Furthermore, the understanding of procedural and conceptual concepts in mathematics shall be virtuous. However, both standard indices and multivariate analysis were used in the current study. This was so, because the short-fall of one method maybe uplifted by the strength of the other. On the whole, this meant that the adopted instruments in the study may help to have adequate feedback in form of marked scripts which may help learners improve their understanding of different concepts in specific concepts in mathematics topics at first year level at UNZA. The findings in the use of operationalizing of learning model and knowledge translation framework prompted the researcher to develop his own conceptual framework.

2.5.11 Justification for Developing and Designing Authors Conceptual Framework

As alluded to in the above arguments on the operationalizing of the concept of learning model, the author adopted a conceptual framework from Nowell et al., (2017) model to use in the measuring of procedural and conceptual understanding of mathematics concepts. Additionally, the developed instruments (standard indices) also measures procedural and conceptual confidence exihibited by learners in order for them to solve mathematical problems without difficulties. Nevertheless, the researcher developed his own conceptual framework as he combined the benefits obtained from the use of both operationalizing of the learning model and knowledge translation framework. Likewise, this was possible, because positives were picked from the two frameworks so that it maybe easier to show how specific concepts in mathematics maybe measured. The author's conceptual framework explains how the input variables, in this case, the specific concepts of which students had difficulties to understand were processed by integrating them into a standard test made with the help of expert judgement from UNZA lecturers and the outcome being the results obtained after a scientific process of scrutinising of the test by using standard indices and multivariate analysis.

2.6 Author's Conceptual Framework Model (2020)

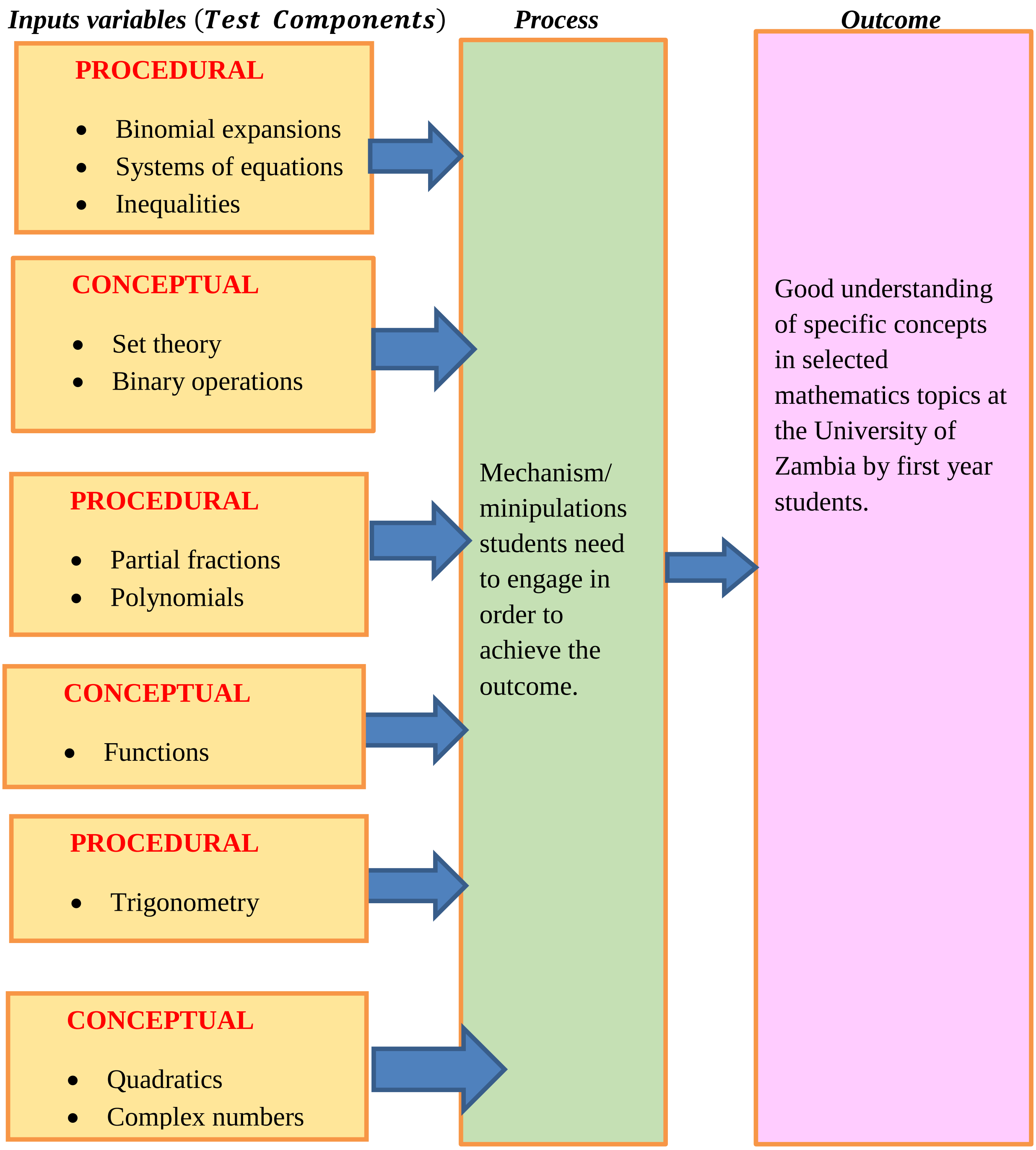


Figure 2.3 Conceptual Framework Model (Source: Mwape, 2021)

The conceptual framework above was derived from the conceptual frameworks used in the study. Likewise, two conceptual frameworks were utilized in the study. To start with, Operationalizing the Concept of Learning Model combining it with Knowledge Translation Framework cumulated into the researchers own conceptual framework model as clarified. The conceptual framework shows that there are a number of factors affecting first year students' procedural and conceptual understanding of mathematics at the University of

Zambia. These variables include: Binomial expansions, Systems of equations and Inequalities as procedural, Set theory and Binary operations as conceptual, Partial fractions and Polynomials as procedural. Functions as conceptual. Trigonometric identities as procedural and lastly but by no means the least Quadratics and Complex numbers as conceptual. The aforementioned were gotten after the researcher checked previous scripts of students results from 2010 up to 2016. It was found that the aforesaid were the specific concepts in selected mathematics topics at the University of Zambia where students had difficulties. Then a standard test was developed in which lecturers of first year students at UNZA had expert judgement by contributing to the inputs of the test which was used in the research and the other stake-holders were the supervisors of the study. However, the criteria used to classify topics as procedural or conceptual was assigning questions to the three lecturers who where teaching first year mathematics to go through the questions and then state which question was procedural or conceptual. Above and beyond, the two supervisors and I rigorously went through the same questions and stated which questions were procedural or conceptual. Then, the two results were scrutinised to come up with an authentic report of the concepts which where either procedural or conceptual. To sum up, the above illustration explains in full the context used to assign questions in the study as either procedural or conceptual.

Authors conceptual framework helped to transform knowledge into the context which focused on questions where students were able to solve mathematics problems with specific concepts considered to be difficult. Notwithstanding, the aforesaid, concepts maybe either procedural or conceptual. The formulated conceptual framework guided to simplify concepts into conceptualized form so that when preparing procedural and conceptual questions the test should be a standard one. To add on, the new conceptual framework aided to empirically investigate the understanding of specific concepts of first year students at UNZA. As from the aforesaid above, the author’s conceptual framework shows all the specific questions which were prepared in the test. The conceptual framework helps to see how knowledge is transformed from difficult specific concepts at first year level to impoved understanding of mathematics concepts at UNZA. This may be achieved by using standard indices to measure data obtained from the structured test and also using multivariate analysis (factor analysis, multi-dimensional scaling and cluster analysis) to measure data from questionnaires.

Binomial expansions, Systems of equations and Inequalities

Binomial expansions , Systems of equations and Inequalities were picked as specific concepts were students had difficulties on procedural skills at the University of Zambia. It was observed from previous examinations scripts that students failed basic mathematical manipulations. Nonetheless, in the current study the researcher constructed a structured test on the captioned components with expert judgement of UNZA first year lecturers of mathematics.

Set theory and Binary operations

Set theory and Binary operations were picked as specific concepts where students had difficulties at first year university level of mathematics. This was after the researcher had a thoroughly check of preceding scripts from 2010 to 2016 at UNZA. The difficulties students had on set theory was proving of De Morgan’s laws and on Binary operations students had difficulties to determine whether or not the binary operation was associative or not. Consequently, structured questions on set theory and binary operations were set with the help of supervisors and UNZA lecturers of mathematics at first year level. In order to make sure that conceptual understanding of concepts in selected mathematics topics improves. The current study measured understanding of concepts in mathematics. This was to be managed from the conceptual framework and then develop valid and reliable instruments of measuring understanding and confidence of conceptual concepts.

Partial fractions and Polynomials

Partial fractions and Polynomials were picked under procedural concepts were students had problems. It was established that students had difficulties on partial fractions involving one linear and non-factorizable factor. It is imperative to mention that the study set out to identify specific concepts in mathematics where first year students had problems. It was expected that results of this study may help to develop and design valid and reliable instruments of measuring understanding and confidence (procedural and conceptual).

Functions

The researcher identified functions and specifically definitions of domain and range as components where students had snags. It is evident from previous tests and examinations that first year students performed badly on the aforementioned concepts. Current evidence suggests that it is difficult to measure conceptual understanding (Wojcik, 2017) as supported by Hirschfeld-Cotton (2008) who argues that creating opportunities for students to examine, apply, prove and communicate mathematics will not only give meaning to the discipline, but also develop a deep understanding of mathematics concepts. In the current study, it was observed that students had a number of difficulties to find the domain and large of the function. Furthermore, few students were able to find the range especially using interval notation. Atleast, a small number of students tried to given the correct domain and range using set-builder notation.

Trigonometric Identities

Trigonometric identies were also identified as one of the specific procedural concepts where students had hitches. This study set out to investigate how to measure procedural concept by using valid and reliable means. It is against this background that the study sought to develop and design valid and reliable means of measuring understanding and confidence of procedural concepts. It was observed that a good number of students failed to solve trigonometric identities correctly.

Quadratics and Complex numbers

It was observed that finding the roots of Quadratics and solving operations on Complex numbers using a number of manipulation were also identified as specific concepts where students had worries because of poor performance which was recorded in the aforesaid period of 2010 to 2016. This meant that the researcher had a task of preparing structured questions on Quadratics and Complex numbers with the help of expert judgement from other stake holders. Additionally, it was the intention of the study to measure conceptual components without complications. This may help students to understand conceptual components of mathematics at first year level. Good understanding of concepts in selected mathematics topics at the University of Zambia by first year students may only be achieved if the instruments developed may be valid and reliable.

2.7 Chapter summary

The study used three sociological theorists; Auguste Comte, Emile Durkheim and logical positivism. The study incorporated all the three sociological theories because the shortfall of one theory was uplifted by the strength of the other. Auguste Comte emphasized the application of the research finding to the society after a rigorous scientific methodology of reviewing available data whilst Emile Durkheim supports social integration of distinct contribution to the body of knowledge. Not only does Logical positivism ropes symbolic logic which gave clarity and exactness to the standard indices adapted and developed in the study but also makes it easier to verify the final contribution to knowledge base in the field of mathematics education. The study used three conceptual frameworks. The first conceptual framework was the operationalizing the concept of learning model. This model was used because it helped to operationalize the concepts so that it is easier to measure mathematical understanding and confidence of solving concepts. The second conceptual framework was knowledge translation framework which helped greatly to transform concepts into understandable forms. This helped the researcher to develop his own conceptual framework by combining the two conceptual frameworks already elucidated. Measuring understanding of mathematics concepts at the University of Zambia (UNZA) may be attained by using valid and reliable instruments. This may be achieved by measuring both procedural and conceptual understanding of specific concepts in mathematics. The following chapter presents current scholarly research findings on procedural and conceptual understanding of mathematics. Additionally, the upcoming chapter explains existing knowledge gaps regarding measuring instruments of procedural and conceptual understanding in mathematics and how these gaps in literature may be filled in by the current study.

CHAPTER THREE
LITERATURE REVIEW

3.1 Introduction

This section of the chapter clarifies on what other researchers have done and concluded upon on similar topics. However, in this section the researcher writes about the current knowledge with information relevant to the study, more importantly related to the objectives and research questions of the study.

This study used Critical Interpretive Synthesis literature review based-methodology. It involved a critical synthesing of evidence and illustration of information related to the study. Critical Interpretive Synthesis (CIS) was used because it had the capacity of allowing a critical scrutiny of other literature relating to the current study by identifying limitations of previous research through a rigous process of comparing evidence and theory (Carey, 2012).

Instead of using the systematic and narrative literature review based-methodologies the current study opted for critical interpretive systhesis because it helped in the building of the new theory and consolidation of the process to prioritise things that can be measured rather than just identifying concepts that are considered significant. Importantly, a CIS helped the study to prioritise quantitative over qualitative methods of findings and instead looked at interrogating all sources of evidence whilst attempting to build and develop theory (Dixon-Woods et al., 2006). However a CIS was adopted over RS because of its immerse qualities of being more coherent, organized and rigor in the interrogation of evidence. The CIS was of help in my study since I was developing and adapting a new theory concerning measuring of understanding of mathematical concepts.

The current study used an hourglass approach of literature review. An hourglass approach involves a scientific writing with the introduction covering a broad overview of the topic. Then the focus of the study was shifted to specific ideas relating to the research questions. Then, how data was collected, analysed and consequently summarise of key findings of the research. The study interrogated literature at international level, then at sub-region before finally settling on the local stage. Not only did the current study scientifically interrogated previous literature methods but also results. This helped to put across new thoughts on how to improve instruments used to measure mathematical understanding. By and large, the new scientific knowledge contributed to the body of knowledge in the study was then published in peer-reviewed journals.

3.2 Conceptual and Procedural Understanding of Mathematics at International Level

In order to solve any problem correctly, students need both applications of understanding of conceptual and procedural knowledge (Cracolice et al., 2008). Surif et al (2012) in agreement with Cracolice at al (2008) argues that, most students are weak in conceptual knowledge and that they continue to rely on algorithm problem solving techniques. This shows that students are only able to memorize and remember the formula and the process involved without understanding the concepts. Memorizing facts and isolated procedures was assumed to be a deterrent to understanding. However, Mills (2016) in his analysis suggests that, some memorization was seen as integral to forming the basis of understanding (p.6). Furthermore, when the procedures learned are not suitable for solving the problem at hand, students are lost (Khalid and Ekholm, 2015). It is against this background that students need to understand clearly both conceptual and procedural aspects of mathematics concepts. Jóhannsdóttir Björg (2013) justifies that,

> Conceptual knowledge relates to something already known, while procedural knowledge is a chain of operations to be performed with or without understanding. Three correlated dimensions of mathematical knowledge; algorithmic, formal and intuitive knowledge. Algorithmic knowledge consists of rules, procedures and their theoretical justifications. Formal knowledge involves axioms, definitions, theorems and proofs and intuitive knowledge is the type of knowledge usually accepted as being obvious, like mental models used for representing mathematical concepts and operations. (p.22).

Rittle-Johnson Bethany et al (2001) in agreement with (Ghazali and Zakaria, 2011 and Marchionda, 2006) argues that, not only does conceptual and procedural knowledge influence each other but also conceptual mathematics understanding is knowledge that involves a thorough understanding of underlying concepts behind the algorithms performed in mathematics. The increase in one type of knowledge leads to the increase in the other type of knowledge. Khalid and Ekholm (2015) points it out that:

> For the conceptually oriented student, each new aspect of a mathematical concept-definition, procedure, graphical representation, examples-adds a new layer to a deeper understanding. This is not the case for the procedurally oriented student, who memorises procedures to pass the examinations. Each new aspect introduced in mathematics is thus perceived as a further burden on the memory.

In addition, Rittle-Johnson, Siegler and Alibali (2001) argues that, ‘initial conceptual knowledge predicts gains in procedural knowledge, and gains in procedural knowledge predict improvements in conceptual understanding (p.346). On the whole (Artigue, 2007) suggests that procedural knowledge may underpin conceptual knowledge in support of Adeleke (2007) in agreement with (NCTM, 2006) who states that, ‘it is the duty of the teacher to teach mathematics in a way to encourage the understanding of the required basic structure of mathematics. Bernstein (2000), argues that, ‘conceptual and procedural understanding in mathematics must be operationalized into criteria for what characterizes a solution to a task as mainly conceptually or procedurally oriented. One way of achieving this is through a careful and thoughtful selection of appropriate strategy that will help in promoting student’s ability to create meaning of mathematical concepts rather than passive reception of ideas. To add on, procedural knowledge may only lead to greater conceptual knowledge under certain circumstances, such as after extensive experience using the procedure, or when the relation between the procedure and the underlying concepts is relatively transparent.

The previous argument supports the current study in that the two did not encourage memorization of concepts as a model of learning. Instead there was an advocation of operationalizing of concepts for easier understanding of either procedural or conceptual concepts. The following section looks at conceptual and procedural understanding of mathematics in Sweden.

3.3 Conceptual and Procedural Understanding of Mathematics in Sweden

Khalid and Ekholm (2015) argues that, the delays in graduation for many students is mainly because of the unpreparedness for abstract thinking in mathematics among several other reasons and also the simplified procedural teaching approach of teaching from early education in Sweden. Many mathematics books from senior high school are designed in a way that stresses solving of mathematical problems without paying enough attention to conceptual understanding (Ibid, 2015). However, research suggests that if students can understand procedures the chances for them to understand concepts in mathematics is equally high. Bergsten et al (2013: 2) in agreement with (Alpers 2010 and Cardella 2008) pointed it out that,‘mathematical activities occur as the contextual embedding of mathematical models, as well as concepts and procedurals, that use objects drawn on understanding of mathematical notations and graphics. Engelbrecht, Bersgsten and Kagesten (2012) found that first year engineering students tended to proceduralise tasks having a conceptual focus. Commonalities of outcomes between the two countries such as a high confidence in their performance on procedural tasks than conceptual tasks and the view that both categories are relevant for their engineering students indicate that efforts need to be made to increase students' confidence in conceptual oriented tasks (Engelbrecht, Bergstern and Kágesten 2012).

Engelbrecht et al (2013) in agreement with (Brown, Seidelmann and Zimmermann 2006; Star 2005 and Khalid and Ekholm 2015) argues that, ' the teaching of undergraduate mathematics should focus on fostering conceptual understanding that might draw on a contextualized problem in order to uncover patterns and relationships for providing a contextual basis for new mathematical knowledge requiring students to connect to their prior knowledge. However, Engelbrecht et al (2012) suggests that, 'in both South Africa and Sweden, mathematics teaching at upper secondary level often has an emphasis on procedural skills rather than conceptual understanding'(p.2).

It was observed that there were a number of similarities of procedural and conceptual understanding of mathematics in Sweden and what the current study focused on. In Sweden the teaching of concepts in mathematics had been procedural from early education through to higher education. This makes learners to perform well in procedural oriented questions as compared to conceptual. The situation is similar to the current study which observed that the teaching and learning of mathematics had been procedural in Zambia from early education upto higher education. This has lead to students at UNZA performimg well in procedural questions as compared to conceptual. The current study recommends a balance of procedural and conceptual emphasis of concepts in the learning of mathematics at all the levels of education. The next section focuses on conceptual and procedural understanding of mathematics in South Africa.

3.4 Conceptual and Procedural Understanding of Mathematics in South Africa

Engelbrecht et al (2009) suggests that even though students at universities in South Africa improve performance in mathematics over a semester there is need to make the transition from secondary to university mathematics somewhat soother (p.3). Furthermore, Engelbrecht et al (2009) in agreement with Hourigan and O' Donoghue (2007) contends that, there is a big difference between the nature of first year students' mathematics experience at pre-university level and that which they experience at university in mathematics courses. It has been observed, the unpreparedness of students causes permanent damage to students' further mathematics careers at university. As a consequence, students beginning their university studies have less training in deeper conceptual thinking. University teachers often complain that first-year students have little understanding of basic concepts of pre-calculus and even the high achieving students are only better in a procedural way of thinking (Engelbrecht, Harding, & Potgieter, 2005).

Bersgsten et al (2015) points out that, conceptual understanding of mathematics may start by providing a contextual basis for the new mathematical knowledge requiring students to connect to their prior knowledge while for procedural knowledge lecturers should define notations, procedures without providing meaningful contexts to concepts and the methods to be used. However, Umalusi (2014) in agreement with Umalusi (2009) suggests that, the curriculum that tries to cover too much does so at the risk of losing depth of understanding. In mathematics, the number of sub-topics were increased to 15% of the total. This increase in breadth could lead to teachers either omitting certain sub-topics, or compromising on the depth at which the sub-topics are dealt with. Bergsten et al 2015 argues that:

> Procedural knowledge is defined by two parts, procedures for solving of mathematical tasks, on the other hand, and then the knowledge of the symbolic representations used in such procedures. To be competent in mathematics, then it involves not only knowledge of concepts and procedures but also of relations between these two types of knowledge. Research suggests that there is a complex interplay between conceptual and procedural knowledge; conceptual knowledge may influence or even become procedural with repeated exposure, while procedural knowledge may support the development of conceptual knowledge.

Umalusi (2014) stipulates that, in mathematics the breadth and depth were significantly more demanding hence poor performance in conceptual and procedural understanding (p.59). Umalusi (2018) argues that to deepen the understanding of procedural and conceptual performance as well as confidence in solving first year university mathematics correctly quantitative or scientific method should be used (p.77). Furthermore, Engelbrecht et al (2012) stipulates that, the increasing availability and efficiency of computational tools, such as calculators and computers, seem to imply that at least part of what is commonly included in the notion of procedural knowledge can be achieved without deeper conceptual understanding (p.3).

Conceptual and procedural understanding of mathematics in South Africa compared to the study at hand shows that students in both countries are not ready for university mathematics.

Not only did the study acknowledged that first year students had little understaniding of basic concepts at mathematics in South Africa but also in Zambia. The use of scientific calculators in the two countries has reduced the conceptual understanding of mathematics. This now means that mathematics is mostly learnt in a procedural way without a deeper conceptual understanding of mathematics concepts. Section 3.5 outlines conceptual and procedural understanding of mathematics in Zambia.

3.5 Conceptual and Procedural Understanding of Mathematics in Zambia

In Zambia, few scholarly work has been done on procedural and conceptual understanding of mathematics concepts. There is a huge knowledge gap in this area, hence the study was conducted to make a significant scientific contribution to the knowledge of the current literature and also to investigate on the modalities of improving the measuring of understanding of mathematics concepts. Pre-service student teachers had poor understanding of conceptual concepts as compared to procedural concepts at UNZA after studying advanced mathematics up to fourth year (Malambo, P. 2015; Malambo, P. et al. 2018; Malambo,P. et al. 2019 and Malambo, P.2020). Not ony did the current study compare performance and confidence of first year university mathematics at UNZA in Zambia but also focused more on understanding which learners had with respect to confidence they exhibited in solving mathematical tasks. Bergsten et al., (2013:2) compared performance and confidence between second and fourth year groups of students in their answers to a questionnaire comparing conceptual and procedurally focused mathematical problems. Furthermore, a closer look into senior high school books shows that, although the students have been exposed to most of what is taught in the first calculus course at university, they still show little understanding of the abstract conceptual mathematics and find it problematic (Khalid and Ekholm, 2015). This can also be shown in Zambia, despite the Ministry of General Education putting in place the revised curriculum of 2013 to include new major topics such as calculus, earth geometry, sequences and series and linear programming in mathematics syllabus students still find it difficult to do university mathematics with confidence. Section 3.6 outlines conceptual and procedural understanding of mathematics at elementary level.

3.6 Conceptual and Procedural Understanding of Mathematics at Elementary Level

There is by far a lot of much information concerning conceptual and procedural knowledge at elementary level. To achieve this, a number of scholars argue in order to ascertain the significance of conceptual and procedural knowledge at elementary level. When the students in elementary schools do not grasp a complete understanding of fractions, the misconceptions or misunderstandings of this crucial mathematical concept could transfer to their next level of education into middle school (Edwards, 2015). Martin (2011) points it out that, procedural skills and conceptual knowledge in kindergarten predict mathematical fluency, computation and applied reasoning performance in grade 1, which are direct antecedents of formal arithmetic. To add on, Adeleke (2007) argues that, ‘in mathematics the relationship between concepts and procedures has been examined but this has been limited to the lower and elementary mathematics (Gilmore and Bryant, 2006) suggest that there is a growing awareness that children may differ in their conceptual understanding and procedural fluency. This is seen in the performance of children in tests and final examinations where children are successful to carry out routine procedures to pass examinations, but fail to understand underlying concepts in mathematics. For instance, in number talks students are provided with opportunities to make connections between strategies and adjust their own thinking (Berger, 2016). This is usually done at an elementary level where the contextual understanding of mathematical concepts is limited to memorization of mathematical concepts. Concomitantly, students’ procedural fluency is influenced by their mathematical knowledge and abilities to perform without problems computation procedures that shows initial learning and understanding of mathematics concepts.

At this level misconceptions and misunderstanding of mathematics concepts are critical to the foundation of mathematics learning because once missed the understanding of basic concepts maybe distorted. At elementary level pupils pass examinations through memorization of concepts which continues up to higher education. Most of first year students at UNZA focus on memorization of key concepts instead of trying to understand the concepts. Nevertheless, the current study sought to bring a balance between procedural and conceptual understanding of mathematics concepts. The next section concentrates on conceptual and procedural understanding of mathematics at secondary level.

3.7 Conceptual and Procedural Understanding of Mathematics at Secondary Level

Adeleke (2007) points out that, most students have some mathematical knowledge but they have almost no understanding of the basic structure of mathematics, thereby making them resort to memorization of mathematical facts and concepts. It is inevitable to do mathematics with full understanding of concepts and not to solve by trial and error method. It is against this background that the teaching process has shifted its focus towards a balance between procedural and conceptual understanding (Ghazali and Zakaria, 2011). In support of the aforesaid, they define 'conceptual understanding of mathematics as the value of learning in terms of understanding and not merely in terms of reproducing the teacher's mathematics or algorithms'. In addition, (Yusof and Tall, 1996) support the accession by indicating that there is a growing awareness that many students are successfully learning how to carry out routine procedures to pass examinations, but there is a concern that the system may not be providing students with experiences to encourage them to be creative and reflective (p.3). Pupils at secondary school are often given mathematical problems that consist of difficult procedures and concepts which may involve deep calculations of algorithms of which they only manage to memorize in order to solve the questions correctly which is not supposed to be the case. Students should be able to understand mathematical concepts before they can engage themselves in solving the given task be it procedural or conceptual. Even though, memory may play a significant role in the understanding of mathematical concepts. It is what is remembered and how it is remembered that distinguishes those who understand from those who do not (Yusof and Tall, 1996).

However, Berger (2016) points it out that, number talks can be used in the mathematics classroom from kindergarten through twelfth grade to build a strong procedural fluency upon which conceptual understanding and mental calculation may act as a foundation of comprehending mathematical concepts and abstract entities of involving mental effort to construct relationships between the ideas. Furthermore, NCTM (2000) suggests that:

> Students demonstrate conceptual understanding in mathematics when they provide evidence that they can recognize, label, and generate examples of concepts; use and interrelate models, diagrams, manipulatives, and varied representations of concepts; identify and apply principle; know and apply facts and definitions; compare, contrast, and integrate related concepts and principles; recognize, interpret, and apply the signs, symbols, and terms used to represent concepts. Conceptual understanding reflects a student's ability to reason in setting involving the careful application of concept definitions, relations, or representations of either (p.2).

Bautista (2013) suggests that, students learn to integrate and form functional grasp of mathematical ideas in conceptual understanding (p.51). Effective mathematics instruction needs to provide opportunities for students to build procedural fluency through conceptual understanding (Berger, 2016). Understanding the procedural nature of solving mathematics has an important role in developing a conceptual understanding. This is because the component of procedural understanding exists on aspects of conceptual understanding. Thus, enables the acquiring of several aspects that support the process of transition from procedural understanding to a conceptual understanding (Fatqurhohman, 2016). Notwithstanding, Gilmore and Bryant (2006) appear to suggest that children differ in the relationship between their conceptual understanding and calculation skill. Berger (2016) demonstrates how to avoid mistakes when multiplying a trinomial by a binomial. It is important to let all strategies be heard before helping students to adjust to their thinking to correct mistakes. (See Figure: 3.1).

The above discussion points out that students at secondary level tend to memorise facts instead of understanding facts and procedures involved in performing given tasks in order to obtain correct solutions. It was revealed that at this stage mostly students do not understand basic structures of mathematics and resort to memorization of mathematics concepts. However, the current study advocates for full understanding of procedural and conceptual concepts.

Student number 1:

$$(x^2 - 3x + 5)(2x - 1)$$

	x^2	-3	5
$2x$	$2x^3$	$-6x^2$	$10x$
-1	$-x^2$	$3x$	-5

$$= 2x^3 - 7x^2 + 13x - 5$$

Student number 2: $(x^2 - 3x + 5)(2x - 1) = 2x^3 - x^2 + 10x - 5$

Student number 3: $(x^2 - 3x + 5)(2x - 1) = 2x^3 - x^2 - 6x^2 + 3x + 10x - 5$

$$= 2x^3 - 7x^2 + 13x - 5$$

Figure 3.1 Number Talks

Number Talk prompt for High School students and resulting strategies (Berger, 2016:6)

Berger (2016) argues that students should be availed with a number of strategies so that they may have a good number of choices for good procedure to solve the algebraic problem correctly. This is inevitable because it may make students improve competencies in procedural and conceptual understanding of mathematical problems. Providing students with opportunities to use their own reasoning strategies and methods for solving problems (NCTM, 2014:47). By providing students with the opportunity to engage in Number Talks, teachers facilitate discourse among students about their thinking, help students build connections among strategies, and present students with opportunities to put their knowledge into practice as they solve problems (Berger, 2016).

It was observed that other studies revealed engaging students with the opportunity to use number talks because they may help to improve the understanding of mathematics concepts. This was so because it is believed that it may improve the rigor in the thinking of students. Therefore, the current study suggests that students should be engaged in the solving of mathematics in routine programs such as mathematics tutorials and clinics. Section 3.8 outlines conceptual and procedural understanding of mathematics at tertiary level.

3.8 Conceptual and Procedural Understanding of Mathematics at Tertiary Level

Students find great difficulties in constructing their own mathematical understanding (Davis & Vinner, 1986; Martin & Wheeler, 1987; Sierpinska, 1988; Eisenberg, 1991; Williams, 1991) and have a narrow view of the mathematics that shapes their mathematical behavior (Schoenfeld, 1989; Vinner, 1994). Berger (2016) believe that part of the problem is that as students advance through coursework, opportunities to explore numbers in conceptually meaningful, rigorous ways diminish. Kilpatrick et al (2001:18) in support of Fatqurhohman (2016) suggests that:

> There are three aspects of conceptual understanding that comprehension of mathematical concepts, operations, and relations. Comprehension of mathematical concepts can be done by identifying and using appropriate concepts to solve problems. Relations can be done through a representation (picture or symbol numeric). Thus, the process of transition from procedural understanding to conceptual understanding of using the three aspects, which can identify problems in the use of an algorithm, the process of an algorithm, connecting some of concepts to transform into another shape through the representation of symbol of picture (p.183).

Furthermore, Fatqurhohman (2016) argues that, ‘procedural understanding has important aspects of the conceptual understanding of learning mathematics, as stated in the purpose of their study on the description on the process of transition from procedural understanding to conceptual understanding of solving mathematical problems at tertiary level. In addition, having students compare incorrect procedures to correct ones aids conceptual and procedural understanding and reduce misconceptions (Durkin & Rittle Johnson, 2012). On the whole, it is inevitable to compare procedures that may help students gain conceptual and procedural understanding, nevertheless, it is more substantial if learners have sufficient prior knowledge of concepts learnt. Generally, Khalid and Ekhilm (2015) argues that:

> Most students can follow procedures and algorithms, based on memory, to solve known problems. However, if translation between different mathematical expressions is required or if they are asked whether a given number is the correct answer to an equation, they hardly know what to answer. Engelbrecht et al 2005 conclude that such behavior indicates a poor understanding of the abstract concepts involved.

This makes students view mathematics as a subject of unrelated rules, algorithms and formulas. Consequently, students labelling mathematics to be a difficulty subject. Hence or otherwise a good number of students only pass mathematics by memorizing formulas, definitions, algorithms and rules immediately after writing examinations they forget most of the concepts.

It was revealed that at tertiary level students have the tendency of memeorizing facts to reproduce them in examinations instead of understanding concepts. This is against the current study which advocates for students to understand mathematics learning by constructing their own mathematical understanding by engaging in routine practice of solving mathematics questions rather than depending on memorization of facts. The following three sections looks at literatures, findings and conclusions from related studies to the study at hand.

3.9 Literatures, Findings and Conclusions from Related Studies (2005-2009)

A study carried out by Engelbrecht et al (2005) concluded that the emphasis when teaching mathematics should be based on understanding and interpretation of concepts. Furthermore, it was revealed that the findings from the study were not surprising, as the methodologies used by lecturers encourage conceptual understanding rather than drilling learners to memorize routine procedures used in the computations of mathematical problems. Ibid (2005) investigated procedural and conceptual skills of first year students and also attempted to measure their confidence in solving mathematical problems. The objective of the study was to determine whether there is any relation between students' conceptual and procedural performance of mathematics and whether there is any relation between their confidence levels when solving mathematical problems. To do this, a test was prepared which consisted of 10 multiple choice questions items of which five were considered to be chiefly procedural and five chiefly conceptual. A sample of 235 students was selected from a group of first year students in life sciences. Additionally, to achieve construct validity the test items were mixed (procedural and conceptual) and also the test was thoroughly scrutinized by lecturers in the mathematics department. This was done to allow expert judgement fulfilled in order to have unbiased view of percentage of procedural and conceptual knowledge. The study revealed that correlations between students' confidence and their actual performance for both procedural and conceptual mathematical items was about (0.4) which was significant for the sample size. Furthermore, it was also revealed that the result seems to indicate that students were more realistic about their ability to solve procedural aspects of items, which, may be attributed to the style of teaching at secondary school level which is more procedural in nature. To add on, the study revealed that misconceptions were more likely to appear in conceptual thinking than procedural. Above and beyond, the study seems to indicate that in calculus for life sciences, the performance was better in conceptual as opposed to procedural. Students were more confident to answer conceptual problems than procedural ones. Consequently, the general notion that 'doing' is easier than 'thinking' was disputed in this study. Hence or otherwise, the outcome of the study may be different if it is done to different streams or at a different university because of different methodogies applied in different universities. Students were not taught how to answer university entrant examinations which usually consists of procedural knowledge, but were provided with enough scope for them (students) to learn mathematics through exploration of mathematical concepts. Nevertheless, a suggestion was made to change the approach when teaching from teacher-centered to student-centered approach.

Literature revealed that the relationship between classroom emphasis on conceptual understanding and procedural knowledge of linear algebra concepts and students achievement in algebraic reasoning was correlated (Joffrion, 2005). To ensure reliability of coding results, graduate students of mathematics education were selected to be part of the study. Furthermore, it was observed that teachers use both conceptual and procedural methods of construction when teaching students to solve algebraic problems. It was further revealed that successful balancing of conceptual and procedural emphasis of classroom instructions may enable students develop both conceptual and procedural understanding of algebraic concepts. The study revealed that there is supposed to be a smooth transition to teaching students from arithmetic to algebraic concepts in order to avoid misconceptions encountered especially when solving algebraic equations and inequations.

Long (2005) looked at mathematics concepts in teaching procedural and conceptual knowledge of number bases. This study was underpinned by the debate of procedural and conceptual knowledge student have in manipulations of number bases. Certainly, (ibid, 2005), revealed that for students to understand the relationship between place value on either procedural or conceptual algorithms. Students should be fully cognoscente of the link between procedural and conceptual understanding of mathematics concepts. In this regard, the conceptual link to procedural one remain elusive. Furthermore, Kilpatrick et al (2001) tries to circumvent the distorting among the two strands (procedural fluency and or conceptual understanding).

On the other hand, the review of literature by Star (2005) postulates that reconceptualization of procedural and conceptual knowledge is noteworthy. However, it has been observed that little empirical evidence from previous studies shows that the data of procedural knowledge in mathematics only explains to satisfy a few certain aspects of mathematical understanding. It is against this background that it is gratifying to note that there has been lack of research on the development of procedural knowledge in mathematical education. By and large, current research in mathematics education has not been concentrating on procedural understanding of mathematical concepts. In the 1980's studies in mathematics education focused on how to highlight mathematical errors on procedural questions. Previously, the methods used to measure procedural understanding as well as conceptual understanding of mathematical concepts has not be valid and reliable, this prompted the research to look at other alternative methods. Research in mathematical education avoided detailed and careful studies that might invigorate development of valid and reliable means of measuring procedural skills and conceptual understanding of specific concepts in mathematics. In this regard, the implications of reconceptualizing procedural knowledge meant that the methods and procedures for assessing students procedural knowledge may drastically improve and make the understanding of specific concepts in mathematics easy. The current study under review focused on measuring mathematical understanding by pointing out the errors committed on procedural questions which is different from my research which looked at both procedural and conceptual understanding by using standard indices to measure mathematical understanding. Consequently, my study also measured procedural and conceptual confidence using standard indices and the results where authenticated by using multi-variate analysis.

Literature reviewed by Marchionda (2006) sought to look at preservice teachers procedural and conceptual understanding of fractions and their effects of inquiry-based learning on understanding. It was revealed that if a preservice teacher is unable to explain why a procedure works then she will probably teach students to memorise the procedure which will only perpetuate the problem of placing emphasis on only rules and procedures. It was observed that students could not explain why the algorithm works and therefore did not conceptually understand the mathematics behind the algorithm. Not only did preservice teachers have low level of confidence in mathematics but also their abilities to learn and teach mathematics were compromised. It was revealed that it is important to use active-learning, inquiry-based instead of the teacher telling students what to do and think, the teacher engages the students by questioning, investigating, discussing, and reflecting on the topics of interest. Data collected took place during two semesters. In this regard observations data were collected via video tapes in order to determine students beliefs and attitudes about mathematics. In this study pretest, quiz, unit test, posttest, and final examinations were all scored using their corresponding rubrics. Furthermore, it was revealed that by the end of the semester, most of the participants had already demonstrated that they possessed adequate procedural knowledge. Marchionda (2006) focused on preservice teachers procedural and conceptual understanding of fraction while my study looked at first year students understanding of specific concepts in selected mathematics topics. The previous study under review pointed out that students mainly used procedures without understanding why they got correct solutions that's why they memorized the working to solve questions correctly. Above and beyond, pre-service teachers had low confindence in mathematics as shown from the data collected over two semesters. In my study, data was collected within a semester and no videos were recorded but only a standard test was constructed and given to a sample of the entire first year taking mathematics.

A review of literature revealed that findings were consistent with other previous works on the use of procedural and conceptual learning strategy in mathematics. However, Adeleke (2007) stipulates that, most of the students who have some mathematical knowledge do not have basic structures of mathematical concepts and may resort to memorization of concepts. In this study, a modified non-equivalent pretest post test control group design was used. Furthermore, the results revealed that, pretest performance of students achievement was obtained using analysis of variance (ANOVA). Eventhough, it is argued that teachers have a duty to teach mathematics in a way to encourage understanding of specific concepts in mathematics. From this, students will be able to understand basic structures of mathematics. In this regard, it was observed that science teachers were used in the experiment because of their knowledge in mathematics concepts. Hence or otherwise, it was further revealed that conceptual learning strategies (CLS) are by far good learning and teaching strategies as compared to procedural learning strategies (PLS). This is so because CLS enables students to solve both routine and non-routine mathematical problems in order to solve for some specific concepts in mathematics. Epistemological variance in the current study under review was that it used (CPL) and (PLS) which enabled students to solve correctly both routine and non-routine mathematical problems in order to solve specific concepts in mathematics. Whilst my study used a standard test for students to solve then after use standard indices to measure mathematical understanding.

Reviewing of literature on the balance between procedural knowledge and conceptual understanding in mathematics teacher education by Bossé and Bahr (2008) stated that the association of mathematics teacher education (AMTE) looked at the initial survey and subsequent conference discussions. In this facet responses of the prepared questions were allowed for to determine the relationship between procedural and conceptual understanding of mathematics concepts. This was so because the sample which was picked was small which consisted of AMTE members representing the entire population of the study. In addition, the survey was blended with multiple choice questions and also open ended questions for the study. However, some responses required candidates to give short answers. It was observed that procedural knowledge gave accuracy of algorithmic efficacies. Particularly, the purpose for conceptual understanding and procedural knowledge not only complemented each other but also showed the contextual underpinning of the survey. Nonetheless, there was a number of notable inconsistencies in opinions and practice concerning the balance between procedural fluency and conceptual understanding of mathematics concepts. Furthermore, it was revealed that the balance between procedural knowledge and conceptual understanding may help mathematics educationalist know how to implement 21st century skills and strategies to foster sustainable pedagogical methodologies in the learning of new concepts in mathematics. The study under review picked a small sample which was not proportion to the entire population which was in conflict with my sample which was proportional to the study. My sample was 378 whilst the population was 1500 meaning that the sample covered 25.3%

of the entire population which was in line with the standard procedure to pick a sample in the range of 15 to 30 percent of the entire population.

Dunham (2008) looked at the impact of procedural and conceptual understanding towards knowledge development as there is little empirical studies comparing the impact of procedurally based to conceptually based instruction. Very few studies have compared the use of traditional methods to post modernity methodologies. In the current study, the researcher sought to look at the connections between conceptual knowledge and procedural knowledge. This study established the underlying meaning of mathematical symbols. It was revealed that students who did well in conceptual component equally did well in procedures and vice versa. By design, the study did not directly test a causal link between procedural and conceptual knowledge, as the result were not statistically significant because there was few scholarly evidence of the claims to support the findings. Nevertheless, it was revealed that the understanding and use of conceptual explanation methods were directly linked to high levels of performance. The study under review explained the meaning of mathematical symbols which correlates well with my study which used standard indices which are highly symbolized. However, standard indices are explained for users to pick them and use them accordingly.

Literature reviewed by Hirschfeld-Cotton (2008) argues that, one of the biggest obstacles for mathematics instruction is teachers use of an inappropriate approach to the nature of mathematics and learning. Therefore, mathematics communication of conceptual understanding and students attitudes towards classwork should improve. In general, few teachers expect and require mathematics to be meaningful, this is so because students usually learn and believe what they are taught. Additionally, students attitudes are shaped by their teachers. However, a shift must occur in mathematics instruction from rote memorizing and performing algorithms, to critical thinking and conceptual understanding. It was revealed that oral and written work in mathematics learning should promote deeper conceptual understanding. The study incorporated a number of methods to examine research questions. Consequently, it was revealed that it was inevitable to engage discussion that might allow deeper conceptual understanding of concepts by students. Finally, it was revealed that oral and written communications are intertwined and benefit one another because if well instituted students will be able to enjoy learning mathematics on a daily basis and students will develop interest in the subject. Therefore, these two methodologies should be developed because they allow a clearer understanding of mathematics topics. Methodological variances were observed in the study under review as one of the major contributing factors leading to inappropriate approaches which contributed to rote learning. However, my study advocates for the 21st century methodologies which engages more learners in the learning process.

A review of pedagogical literature by Hosein et al (2008) was inconsistent with previous findings from other studies which exposed a number of gaps between procedural and conceptual understanding without using established softwares. The current study Ibid (2008) suggested that mathematical thinking of undergraduate students should therefore, use three soft-wares in order to enhance mathematical thinking. Againist this premise students conceptual understanding of mathematical concepts may improve. In this regard, it was observed that there are limited studies which determines whether students learning are better promoted by either glass-box or black-box softwares. In this research, three softwares were used; black-box, glass-box and open-box software. All the softwares were used to help promote learning without using a teacher as a facilitator of the learning process. The study helped to promote conceptual understanding of mathematical concepts by graduate students. As a rule, procedural knowledge of learners might improve (formal and algorithms); discrete knowledge or procedural knowledge. A sample of 36 undergardute students of mathematics were picked for the study. Linear programming was picked as a topic of discussion were three tasks were picked to give students to solve using softwares. Undergarduate from (U.K, Trinidad and Tobago were observed using video tele-conferencing). Problems were rotated to all the undergraduate students in order to avoid carry-over effects. Furthermore, questionnaires were given to students to indicate their opinions, views, ideas and emotions from mathematics students. However, it was observed that previously researchers depended on deep explanations to measure procedural and conceptual understanding of mathematical concepts which were not viable means of carrying out vital experiments. Going forward students used softwares to confirm conjectures of determining which strand is more than the other (conceptual or procedural). From this study, it was revealed that students with low confidence in mathematics had also lower marks. Furthermore, it was revealed that students who were not certain of their working in mathematics resorted to real-life explanations and performed well in tests. The pedagogical variance of the current study was that it used XL-STRATA with Excel to analyse data while in the literature reviewed they used black-box, glass-box and open-box softwares .To add on, my study gave out the questionnaire to find out the confidence of answering questions correctly be it procedural or conceptual while in the reviewed study a questionnaire was used to solicit for views, opinions,ideas and feelings.

Furthermore, this study used a sample of 36 undergraduate students and tested them on linear programming whereas my study used a sample of 378 first year students in the school of natural sciences taking mathematics out of the population of 1500. Finally, the reviewed used used deep expalnations to measure procedural and conceptual and while the study being undertaken used standard indices.

Wachira and Onchwari (2008) stipulated that mathematics preservice teachers' beliefs and conceptions of appropriate technology use helped them to make students understand mathematics concepts. On the other hand, it was observed that many preservice teachers had little or no exposure to the use of technology in the teaching and learning of mathematics. This time period of technology most students work have been lessened especially concerning calculations because of the high usage of scientific calculators. Students can learn mathematics more deeply with appropriate and responsible use of education technology in mathematics classroom. Appropriate technology use will in the long run support the growth of conceptual development and therefore enhance the assimilation and understanding of concepts. The effective use of technology in the learning process will proceduce growth of students use of software, graphing calculators and other data analysis softwares amicably in the understanding of mathematics concepts. Nonetheless, many mathematics teachers had little exposure of the use of technology in the learning of mathematics. However, in this study 20 preservice middle mathematics methods undergraduate students were picked of which 15 were female and 5 males. The findings revealed that it is important to make strides and be consistent in the use of technology in the teaching and learning of mathematics in order to proceduce desired results. The study at hand and my study did not differ in the advocation of technological use. The study under review emphasized effective use of technology in the learning of mathematics which conforms with the current study which suggests the use of video tele-conferencing, teams, google-meet, zoom and other platforms to be engaged in the teaching and learning of mathematics.

Literature reviewed by Belter (2009) enthralled on the impact of teaching algebra with a focus on procedural understanding. In this regard, the researcher sought to develop a procedural understanding framework which helps to illustrate students understanding of algebraic concepts with easy. Nonetheless, at the end of course work students were given specific questions with instructions well tabulated for assessment purposes. It was revealed that, teaching for understanding in mathematics classroom may be enhanced by integrating mathematics procedures in the learning process of algebraic concepts. In this study, a sample of 13 teachers was picked to participate in a summer institute conference engrossed on the methodologies on how to improve the teaching of mathematics for understanding. In this regard, a framework for understanding was developed which was utilized to analyze data by creating specific rubrics to be used for marking of the test. However, from a gigantic number of tests in mathematics a subset of 830 tests were picked with a view of creating rubrics for each specific test. The researcher used a t-test which was compared to test scores from test 1 to test 3 for skill and understanding. It was revealed that, there was no significant difference in gain scores. Finally, it was recommended that more effective, appropriate and reliable approaches should be put in place since certain algebraic procedures were not understood by most of the learners. In the literature reviewed there was a development of a procedural understanding framework which illustrated students understanding of algebraic concepts which was in conformity with the study being undertaken were the authors conceptual frame was formulated under interrogating the operationalizing the concept of learning model and the knowledge translation framework.

Engelbrecht, Bergsten and Kågesten (2009) argues that undergraduate students' preference for procedural to conceptual solutions of mathematical problems are not debated accordingly. Hence, the current study sought to evaluate routine and procedural components of the two further apart countries (South Africa and Sweden). In this regard, the researchers investigated at Pretoria University of South Africa and Linköping University of Sweden. It was revealed that students in the two universities had adequate experience in mathematics with a gigantic focus on symbol manipulation skills, and had less training in deeper conceptual thinking. Most undergraduate students in South Africa and Sweden only have a bit of competences in procedural skills whilst they struggle to master conceptual components of mathematical reasoning. Furthermore, it was the intention of the researchers to develop a valid and reliable instrument to use for measuring of procedural and conceptual knowledge of second year students of engineering departments of (Pretoria and Linköping Universities). It is believed that the instrument once developed may improve the measuring of conceptual and procedural knowledge of mathematics. It was revealed that students in the two countries had difficulties in the assimilation of conceptual components of literal equations. On the other hand it was discovered that students had challenges of connecting conceptual and procedural concepts in mathematics learning of engineering students. Engelbrecht et al (2009) postulates that, students should be taught first procedures before they are introduced to concepts. Additionally, it was revealed that one aspect of learning mathematics influences the other (procedural and conceptual). Finally, it was revealed that there is need to develop a valid and reliable instrument of measuring conceptual and procedural knowledge of mathematics so that it may not only cover the two countries (South Africa and Sweden) but also to other countries. In this regard, a test instrument was developed which was further scrutinized by colleagues of the researchers which had open-ended questions to be analysed using qualitative analysis and also the number of students to be interpreted quantitatively. The literature reviewed shows that South Africa and Sweden had a rigor in symbol manipulations of mathematical skills as compared to conceptual learning of concepts. However, this coincides with the current study which showed that at UNZA students had more procedural understanding as opposed to conceptual after developing and adapting standard indices as new instruments to be used in the measuring of mathematical understanding.

A study was conducted by Gilmore and Papadatou-Pastou (2009) to examine patterns of individual differences in conceptual understanding and arithmetic procedural skills. This was done using the approach of meta-analysis . Children were put in various sub-clusters using cluster analysis. It was observed that children in this research were given a number of tasks in mathematics on addition and subtraction. However, it was revealed that, not all the children who were able to solve addition problems correctly got subtraction questions. Furthermore, to determine whether these differences were reliable and reflective. An alternative path of development of data set was examined in 14 diffferent studies of children’s understanding of inversion. Finally, evidence from the study revealed that reliable patterns of individual differences in conceptual understanding and arithmetical skills were attained. The reviewed study was related in methodology to the current study in that both studies used cluster analysis to related various groups of subjects in order to measure mathematical understanding.

A study conducted by Zerpa, Kajander and Christina Van Barneveld (2009) on factors that impact preservice teachers’ growth in conceptual mathematical knowledge after acquiring a reform based mathematics methods course. It was revealed that preservice initial levels of conceptual and procedural mathematical knowledge and values did not affect the delivery of the lesson by the teacher. In other ways the competences of the teacher in lesson delivery and methodologies and teaching strategies is not affected by his or her initial training. It was observed that teachers of mathematics need to have a deep conceptual understanding of the mathematics that they are teaching and not to teach in the traditional way which cultivates skills and neglects conceptual understanding of the underlying domain. In this study one hundred and eleven (111) participants were sought from grades four to ten preservice teachers. The instrument used was Perceptions of Mathematics (POM) instrument which was established to measure preservice teachers' conceptual and procedural mathematical values. Furthermore, the results indicated that teachers beliefs about mathematics may influence students' perceptions of mathematical concepts either positively or in a negative way. Additionally, teachers' beliefs may encourage or discourage students performance and or understanding of mathematical concepts. Nonetheless, regression model was used to bring to light preservice teachers change in conceptual mathematical knowledge after doing a mathematics methods course. However, a Pretest-Posttest design was used in the research mainly because the mathematics methods course was compulsory. At primary and a high school it was revealed that preservice teachers engaged (111) students from grades four to ten on their procedural and conceptual competencies. By and large, the study under review stressed that teachers beliefs about mathematics may affect students either in a positive or negative form. This was also observed by the current study that students need to be given hope at UNZA that they can make it especially if they had to be attending tutorials and mathematics clinics regularly.

3.9.1 Lessons learnt from literature review related to my study during the era (2005-2009)

The review of literature during the era (2005-2009) revealed that there were a critical knowledge gaps in the methods and approaches used to conduct research involving the understanding of mathematics concepts. It was observed that there was a high discrepancy between what we know and what we need to know concerning students conceptual and procedural confidence. There was a definite need to improve in the measuring of mathematical understanding as most of the research revealed that thaere is a gigantic gap of having good and reliable instruments of measuring mathematical understanding. Notwithstanding, it was revealed that most of the students who had mathematical knowledge did not have basic structures of mathematical concepts, hence they depended on memorization to master what they knew. Furthermore, it was observed that during the era captioned students who did well in conceptual mathematics also did well in procedural. Teachers were using inappropriate approaches in the teaching and learning of mathematics. It was observed that very few teachers were using 21st century methods (video teleconferencing, power point, zoom facilities and other latest methodologies) in the teaching process. During this era (2005-2009) research revealed that teachers were not exposed to use new technologies in the teaching of mathematics. Likewise, research showed that researchers were still using deep explanations to measure procedural and conceptual understanding. Relating the lessons learnt during the captioned era to my study it was observed that the current study being undertaken fought for new valid and reliable instruments to be used in the measuring of mathematical understanding and confidence which students have towards the solving of either procedural or conceptual oriented questions. However, there was no standardized forms of measuring mathematical understanding during this time. Hence, my study may contribute greatly to the measuring of mathematical concepts and also other STEM subjects.

3.10 Literatures, Findings and Conclusions from Related Studies (2010-2015)

In spite of the substantial amount of critical work that has been produced on procedural and conceptual understanding of mathematics. There is still a lot of debate as to which is used more in the learning of mathematics. However, Hallett, Nunes and Bryant (2010) argue that in research there are a number of conflicting findings as to which learners favour more either procedural or conceptual and or both. Furthermore, Ibid (2010) suggested that there are individual differences in the extent to which children understand procedural and conceptual knowledge on fractions. Nevertheless, it was revealed that even though it is possible to focus on both (procedural or conceptual understanding) some learners were more competent in one aspect or another. In this study procedural knowledge was defined as the insight in learning which follows certain sequences of anticipated actions in order to produce desired results. Even though procedural and conceptual understanding are two distinct strands of mathematical proficiency there is no restriction for learners to possess one of the two or even both. This study focused on fourth and sixth graders who were selected as the sample as requested to complete three different questions that reflect conceptual understanding. Additionally, fifth graders also were asked to complete five different questions measuring conceptual understanding. It was observed that all the questions which were asked sought to give the answer to the understanding of equivalence and ordering of fractions at the already specified stage of learning. To measure procedural knowledge, researchers used the performance which pupils obtained on addition and multiplication of fractions. Henceforth, the results from this study suggested that children in all the grades understood conceptual items more than procedural ones. In conclusion, it was revealed that pupils learn by understanding, and procedural knowledge is only a tool used to enhance conceptual understanding of concepts on fractions. Other research previously have used (concept-first and or procedural-first fashion) as argued by (Rittle-Johnson et al., 2001). Therefore, in this study, unlike Rittle-Johnson and others Hallet, Nunes and Bryant (2010) did not assume that one aspect of learning directs to the other but used cluster analysis which verified the results by putting the data in specific subgroups before analysis was done. In this way, it was easier to test the hypothesis of the study. The study further revealed that procedures for each individual student should complete an assessment of knowledge on fractions and verbal instructions which included pictures on the laptop. However, it was further observed that the items were coded with respect to the reflectiveness of each item as being either of conceptual understanding or showing procedural knowledge. The study under review used performance of pupils to measure procedural knowledge while the current study developed and adapted standard indices and used them to measure procedural and conceptual understanding.

Forrester and Chinnappan (2010) observed the predominance of procedural knowledge in the solving of fractions were highly utitized at all levels of learning. However, it was observed that procedues involves understanding of rules and routines of mathematics while conceptual knowledge involves understanding of mathematical relationships. Nevertheless, the purpose of the study was to examine quality of pre service teachers representations of fraction concepts. Furthermore, the study sought to find out the relative use of procedural and conceptual knowledge when preservice teachers represent fraction problems that involve subtraction. Additionally, the study sought to determine the relative use of procedural and conceptual knowledge of preservice teachers representation of fraction problems that involve multiplication. In this regard, 186 students were picked as a sample of which 22 were males and 164 were females respectively. A final assessment task for first year mathematics content and pedagogy unit was used in the study. It was revealed that students were able to show all steps including any visual representation by using quantitative data analysis with the aid of SPSS. The current study recommends the use of softwares in the analysis and interpretation of data. In this regard, the study being undertaken used XL STRATA with excel and also original software in the analysing of data which supports the reviewed article which also advocates for software usage.

The review of literature by Zakaria et al (2010) on the conceptual knowledge and mathematics achievement of matriculation students indicated that there should be a balance between conceptual knowledge and skill development. Furthermore, it was revealed that students possess mastery of mathematics procedures and concepts at the matriculation level. It was observed that the understanding of certain concepts and procedures in the process of carrying out specific actions and operations maybe improved after the study. In this regard, it was argued that the learning of mathematics should be coupled with active involvement of learners. Therefore, lecturers should not sacrifice procedural skills because teaching is based on concepts which have a better effect of improving the understanding of students. However, the study consisted of a sample of 250 students, of which 92 were male and 158 were female obtained by using random cluster sampling. Additionally, 130 accounting students and then both physical sciences and life sciences had 60 students. A standard, validated and reliable instrument of conceptual knowledge test of sequences and series were given to learners. Cronbach's alpha reliability procedure of 0.85 was obtained which showed that the instrument was reliable. There was a positive correlation between conceptual knowledge and mathematics achievement in the courses of study. The study revealed that there was a significant difference between conceptual knowledge of students majority in accounting and that of sciences. The results further revealed a significant relationship between conceptual knowledge and mathematics achievement in both courses of the study. Based on this study, conceptual knowledge of accounting students is lower when compared to science students. To add on, it was observed that accounts students should be given more attention during their matriculation level. In relation to the study being undertaken there were a number of similarities not only in the picking of the sample but also the use of Cronbach's alpha reliability of procedural which was at 0.710 and for conceptual was at 0.651 all of them were in the range standard range of reliaablity of 0.65 to 0.95 as stipulated by Zakaria and others.

A review of literature shows that most of the learners do not have mental computation and conceptual understanding of mathematics concepts. Even though, Ghazali and Zakaria (2011) argues that, the goal in mathematics teaching has shifted towards an emphasis on both procedural and conceptual understanding. The study showed the importance of gaining procedural and conceptual understanding of mathematics concepts. A survey method was used which consisted of 132 respondents from lower secondary schools in Malaysia who were selected using cluster analysis. The researcher gave a test in algebra which had 14 conceptual and procedural items of which 8 items were procedurally oriented and also 6 measuring conceptual understanding. Furthermore, expert judgement was sought by giving the test instrument to lecturers of mathematics for validation. The algebra test was to measure students procedural and conceptual understanding. Internal consistency with Cronbach Alpha was used and it was 0.804. This was so, because the reliability coefficient for procedural items was 0.73, and for conceptual items 0.67. Therefore, the test instrument was reliable.

The relationship between procedural and conceptual understanding was determined using pearson's correlation. The study revealed that students had a higher procedural understanding of concepts than conceptual understanding. Furthermore, the results showed a significant relationship between mathematics procedural understanding and conceptual understanding. The study under review used pearson's correlation which is not closer to reality while my study used multi-variate anlaysis and specifically factor analysis, discrimanat analysis, cluster analysis and multi-dimensional analysis which gives values closer to reality as stipulated by Kothari (1985).

Akgün et al (2012) looked at the transfer of mathematical knowledge by making learners obtain necessary skills and competences of the subject which are related to real life situational examples. In this regard, researchers sought not only to look at means and ways of enhancing the teaching of problem solving skills but also the ways of thinking mathematically. This maybe achieved by addressing the understanding of mathematical language and symbolism of leaners so that students may improve their procedural knowledge. Descriptive research design was used in this study to help with the analysis of data. As a rule, it was revealed that the transfer of knowledge about series to problem solving was done. Additionally, the ratio of students current answers was on the high side to procedural oriented tasks. Furthermore, it was revealed that students failed to solve real life problems as it was difficult for them to draw the relationship with other concepts and misconceptions were highly observed in the study. The study under review coincides with my study in the use of mathematical language and symbolism. This helped greatly in the construction of standard indices which were later used to measure mathematical understanding of either procedural or conceptual concepts.

Appleton (2012) sought to develop a sequence of instruction tasks based on foundational concepts of fraction and decimals. The study further made sure that there is a balance between conceptual understanding and procedural knowledge of typical fractions and decimals. It was observed that students understanding of operating on fractions consisited of rote procedures without connections to other mathematical concepts. It was revealed that students' conceptual understanding of fractions and decimals can begin from some ideas of whole number knowledge in order to increase deep understanding of mathematics by students. The study under review focused on fractions to determine procedural understanding which was different from my study which looked at the understanding of specific concepts in selected mathematics at UNZA be it procedural or conceptual.

A study by Engelbrecht et al (2012) focused on conceptual and procedural approaches of mathematics in engineering curriculum with respect to students conceptions and performance. This study reports on the projects done by two higher institutions of two separate countries. That was University of Pretoria of South Africa and Linköping University of Sweden showing that students benefited more on conceptual oriented questions than procedurally ones. This was achieved by comparing students performance to their confidence of answering correctly either procedural or conceptual questions. Computational tools like calculators have reduced the significance of measuring procedural knowledge as this may be done by using calculational gadgets. Nevertheless, students wth competencies in conceptual approaches scored more than in both conceptual and procedural tasks. Confidence of response on mathematical tasks may affect the performance of learners. Concomitantly, to ensure construct validity the test was thoroughly and independently scrutinized by other colleagues to get unbiased views of the levels of procedural or conceptual understanding (Engelbrecht, Bergsten and Kâgesten 2012). A multiple choice test was given to second year students to investigate their preference of mathematics tasks either procedural or conceptual. Pearson correlation coefficient between performance and students conceptual and procedural confidence were highlighted. It was observed that Swedish students performed better than South African students in both procedural and conceptual components of mathematics even though the margin was small. The methodological variance was that the study under review used used multiple choice test both in South Africa and Sweden while my study used structured questions which gives students more opportunities to explain their solutions with the aid of examples. Additionally, the reviewed study used pearson correlation coefficient to measure conceptual and procedural confidence while my study used multi-variate analysis to measure cprocedural and conceptual confidence. Therefore, the study being undertaken shows that factor analysis, cluster analysis and multi-deminsioninal scaling explained results closer to reality.

The purpose of this study was to investigate the effect of cooperative learning with metacognitive scaffolding on mathematics conceptual understanding and procedural fluency (Jbeili, 2012). In this regard it meant that learning mathematics with understanding should involve more than competency of basic skills. Nonetheless, students should interact with each other in order to receive feedback and be informed on the current mathematical understandings of general concepts. By and large, cooperative learning may improve not only students cognitive performance but also students' mathematical understanding and also procedural fluency with the help of good learning environments. To add on, all the materials and instruments used in this study were translated in Arabic. The sample of the study was randomly picked and consisted of 240 male students who were later assigned to three groups. In order to study students mathematical conceptual understanding and procedural fluency textbooks were utilized. In order to test for validity and reliability of the instruments used in the research. Two experienced mathematics teachers, two education mathematics supervisors, and two mathematics education university lecturers reviewed the test. Each looked at the question and assessed which of the mathematical proficiency strand (CU or PF), represented the question. Cronbach's Alpha reliability coefficient was tested and it was 0.88. The findings of the research stipulated that cooperative learning process should be scaffolded appropriately, and modelled through metacognitive learning which uses instructional design, curriculum design, computer based design or web-based design. Additionally, a close examination of results revealed that cooperative learning alone is an insufficient form of scaffolding unless metacognitive scaffolding is integrated adequately in order to develop mathematics conceptual understanding and procedural fluency. The methodological difference was that the reviewed study only picked 240 male students and assigned them to three groups while my study picked a sample of 378 of which 240 were male and 138 were female. Even though the number of males were equal the current study was dfferent in that it als had females. The current study used a standard test as a tool to measure mathematical understanding while the study under review used textbooks in the measuring of procedural and conceptual understanding.

In this study, (Hallett et al, 2012) investigated the possibility of individual differences in conceptual and procedural knowledge in mathematics. The study examined the impact of individual differences by using percentiles or standard deviations as a unit of measurement. Nonetheless, cluster analysis was used in the study to determine the overall differences in some profiles and identify whether children can be classified into groups based on differences of understanding of either procedural or conceptual mathematics concepts. The study attempted to identify profiles in the combination of conceptual and procedural knowledge of multiplication and also to explain discrepancies between the conceptual and procedural understanding. In this regard, it was necessary to create profiles of individual as alluded by Hallett at al (2010) who performed cluster analysis with the help of residualized scales. The sample in this research consisted of 123 Grade 6 students from four different primary schools and 118 Grade 8 students from two different secondary schools. Furthermore, written measure was composed of 26 items and was presented to students written task. It was observed that, to test reliability of the instrument cronbach's alpha of 0.677 was recorded as internal consistency of the scales which was within the recommended range of acceptance. Henceforth, it was significant to reiterate that most of the children in the study had higher scores in procedural than conceptual ones. It was further revealed that, cluster analysis demonstrates that individual differences existed at both Grade 6 and 8 samples. The result of this study supports and confirms the idea that individual differences in conceptual and procedural knowledge of fractions exists. Nevertheless, the findings from this study hinged on the use of adequate and repeated assessments, which led to the identification of a profile that includes students whose procedural knowledge was stronger than conceptual knowledge. It was observed that, if some teachers focus more on conceptual knowledge and others focus more on procedural knowledge, this would lead to individual differences in the combination of conceptual and procedural knowledge. From the discussion, we may conclude that, there is an observed individual difference in conceptual and procedural knowledge of fractions. At primary and secondary school students were eagaged in tasks which helped the researcher to use cluster analysis to observe individual differences of understanding of either procedural or conceptual concepts. This is in agreement with the suggestion by the current study being undertaken which suggests that cluster analysis helped to measure procedural and conceptual confidence.

A review of literature by Graven (2012) postulated that there is need in mathematics education to determine the impact of learners mathematical dispositions of young learners by accessing and assessing their understanding of concepts. On the other hand, this was done in order to know students mathematical learning disposition as this component was initially neglected. This was so, because the study revealed that there was more to be investigated to determine the relationship of mathematical dispositions in conformity to other strands of mathematical proficiency. In this regard, the researcher developed an instrument to measure mathematics competence and confidence of students in order to enhance deep conceptual understanding of mathematical concepts. This is possible, because for a long time mathematical dispositions ignored mathematics assessments. Nonetheless, this study sought to look at mathematics disposition by seeing oneself as an effective learner and doer of mathematics in order to integrate this strand to combine well with the other four strands of mathematical proficiency. This study after review supports the current study, in that it also fought to develop and adapt standard indices to use in the measuring of procedural and conceptual understanding of concepts which was the key significant scientific contribution to the body of knowledge.

A review of pedagogical literature on procedural and conceptual knowledge of students understanding of mathematics functions was highlighted. In this regard, Lauritzen (2012) suggested that, conceptual and procedural knowledge of mathematical functions maybe either procedurally or conceptually constructed through procedures. In this study, it was observed that development of mathematical understanding maybe enhanced by involving mathematical concepts of functions to be presented in various forms (graphical, conceptual and or procedurally). This was done by exploring the use of functions in economic learning. Firstly, this was done by comparing procedural and conceptual knowledge of functions. The researcher sought to find out how procedural and conceptual knowledge of functions may be measured and also how they relate to each other. Furthermore, the researcher revealed that, procedural and conceptual knowledge of functions are related to one another. It was further revealed that the ability to apply functions heavily depend on procedural and conceptual knowledge of functions. On the other hand the study sought to determine how procedural and conceptual knowledge of functions may be measured. To add on, the study looked at the ability to apply functions depending on procedural and conceptual knowledge of functions by investigating intermediate effects of two explanatory variables. However, the study also looked at how students perform procedural tasks and to what extent they reveal procedural knowledge. This is so because of the relationship between intermediate effects and also explanatory variables among and between variables in the functions that are mostly disregarded. Finally, it was observed that, the study involved a number of models (measurement model, latent variable model and the complete model). The study was conducted in Norway at the Norwegian Business School and the study sample was made of 283 students who were sampled randomly. The study under review was similar to the current study in that both used simple random sampling to get the sample. Additionally, my study also focused on developing standard indices to be used in the measuring of procedural and conceptual understanding of mathematical concepts. This is similar to the reviewed study which developed a model for measuring procedural and conceptual knowledge.

Correspondingly, Tmninic and Abrahamson (2012) postulated the significance of the body of knowledge in terms of rethinking of mathematical concepts as signified embodied procedures. Noteworthy, little scholary agreement over the relations of procedural and conceptual understanding of mathematical concepts has been discovered by researchers.

Notwithstanding the fact that (procedures and concepts) in mathematics at university first year level are not well defined. Consequently, the discipline of mathematics should involve theoretical and epistemological enhancement in the teaching and learning of concepts. Preferably, there should not be a distinction between procedural performance and conceptual understanding of mathematics concepts. In this respect, it shows that it is substantial to promote both (procedural or conceptual understanding of mathematical concepts). The epistemological difference is that my study tried to explain more on the theoretical and conceptual models concerning mathematical understanding of procedural and conceptual concepts. Additionally, the current study tried to explain the difference between procedural nad conceptual understanding and also the confidence of understanding of either (procedural or conceptual).

It was synonymous to look at the development of measuring instruments in mathematics education. In this regard, a look at Audrey et al (2013) suggested that there is need to see how scholars may develop and design instruments to measure conceptual understanding of mathematics. This was so because he had earlier indicated that mostly mathematics assessment tools often focus solely on procedural side of understanding mathematics instead of equally substantial aspect of conceptual understanding of mathematics. It was also observed that previous studies looked at traditional way of using instrument mainly of multi-choice assessment as supported by Engelbrecht et al (2005) who also looked at undergraduate students understanding of procedural and conceptual performance and confidence in calculus. Audrey et al (2013) sought to develop an alternative assessment tool that could quantify both students and procedural understanding of mathematics. In response to this, it resulted in putting forward instructional materials and methodologies that may suit the approach at hand. This study transformed NetLogo a computer deriven soft-ware for it to be re-designed as a tool to assess students conceptual understanding of 'rate of change' a component in calculus. To add on, other researchers may redesign a NetLogo simulation to assess students conceptual understanding of mathematics. It can thus be deduced from the findings that, this project can potentially create a tool that will assess a skill that previous tools could not assess in mathematics. In conclusion, this study sought to put across less inextricable assessment tools of measuring conceptual understanding in mathematics. The study under review only focus on assessment tools that measure procedural understanding while my study focused on developing and adapting tools that measure both procedural and conceptual understanding of mathematics concepts. To add on, the study under review used multiple choice questions in the instrument to measure mathematical understanding while my study used structured questions. Finally, the other discrepancy was that my study used standard indices to measure procedural and conceptual understanding of mathematical concepts while the other study used NetLogo a computer driven software to assess students conceptual understanding.

Related literature shows that students fluency in problem solving must complement their mathematical-scientific explanations in their decision making process. Furthermore, Baustista (2013) reported that there is a possibility for students to solve mathematical questions with understanding and communicate ideas in a productive way. However, five strands of mathematical proficiency by Kilpatrick, Swafford and Findell (2001) was used in order to show that there was a clear grasp and understanding of mathematical concepts when using worthwhile strategies and evocative procedures and vindicated reasoning. Furthermore, Baustista (2013) designed a study which analyzed procedural fluency and written-mathematical explanation to construct meaningful responses of students. In this regard, written-mathematical explanations were sought by using a descriptive-correlation-comparative research design in oder to analyse data. In this respect, written work was transcribed following a rubric which was developed by the researcher. In this way, students may learn to integrate and form a purposeful grasp of mathematical ideas in conceptual understanding. A descriptive-correlation-comparative research design was used to measure academic performance of students in english and mathematics and it was found to correlate. However, a high performance was recorded in physics as the subject is taught using mathematical components of graphical, trigonometry and algebra. Reliability of the achievement test was 0.87. Additionally, it was revealed that pearson-r-correlation of 0.01 significant level was used. This revealed that the test items were consistent in this research. Additionaly, it was also revealed that students performed well in their written-mathematical explanations. This indicated that scholars can translate problems by drawing. It was revealed that students' procedural fluency is controlled by their mathematical ability in algebra and trigonometry. On the other hand, it was further revealed that students' procedural fuency is influenced by their mathematical knowledge and abilities. If this is done accordingly, then students procedural and conceptual understanding of mathematics concepts will be improved. The study under review correlated to my study in the use of Kilpatrick and others braided conceptual model of cognitive development. My study focused only on procedural and conceptual understanding of mathematical concepts while the study under review focused on all the five levels.

Getenet and Beswick (2013) looked at how to develop instruments to measure mathematics teacher educators' knowledge . In this study the focus was on how to develop the measuring instrument using a number of stages. Researchers had to compare the new instrument to the already validated existing instrument. This enabled them to create a draft instrument for their study. In order to ensure content validity of the study the researchers not only did they involve expert review of the questions but also did a pilot testing of the instrument using a small portion of the sample. This paper described the procedures used to develop an instrument to measure knowledge of technology which is integrated in teaching of mathematics. The methodological similarity between the study studies were that my study also fought to develop and adapt measuring instruments of mathematical understanding while the study under review also developed instruments to measure mathematical knowledge of teachers.

In a study where Hannigan et al (2013) reviewed literature by investigating prospective secondary mathematics teachers conceptual knowledge and attitudes of statistics. It was revealed that there was lack of awareness of the difference between statistical and mathematical thinking in students. It was also observed that there was no standard assessment instrument to use which may focus on conceptual understanding rather than procedural. Furthermore, prospective mathematics teachers encountered a number of errors because of lack of exposure to analyse and explore statistical data. It was revealed that prospective teachers may need strong subject matter knowledge so that they are equipped with adequate knowledge to enable them to go and teach mathematics without difficulties. Therefore, a survey of attitudes towards statistics (STATS) and Cronbach's alpha coefficient was used to measure the reliability of each component on the STATS-36. The study sought to correlate the results of CAOS test to AATS on prospective secondary mathematics teachers. The research revealed that positive attitudes towards statistics did not strongly correlate with conceptual understanding. It was observed that the study under review had no standard assessment instrument to measure conceptual understanding even though it had for procedural. This prompted the current study to develop and adapt valid and reliable instruments for measuring mathematical understanding.

Nik Azis and Tapsir (2013) did a study on the analysis of instruments used for measuring values of mathematics education. However, previous literature argues that mathematics is value free. The gap in literature prompted the current study to be conducted. The paper proposed a model of an instrument to measure values which may suit Malaysian Educational

Development Plan (MEDP) because previous instruments designed suited Western Mathematics Education. With the aforesaid points the study sought to develop a holistic instrument suitable for the education system of Malaysia. The study was grounded on expectancy-value theory of achievement motivation. This theory promotes individual choices, persistence and performance. To develop the instrument a test was constructed with 20 constructivist problems and 14 positivist. The study used Cronbach Alpha to measure internal consistency coefficient and it was found to be 0.74 and 0.64 for constructivist and positivist respectively. Furthermore, the instrument developed sought to establish psychometric properties which may reduce the need for redundant research design so that it may be applied across different subjects. The study revealed that, it is time educationalist must focus on developing instruments which are able to measure values in mathematics education in order to assist students and teachers to gain personal and social identities that affect choices people make in mathematics and mathematics education. My study was similar to the study under review in that both of them advocated for developing of instruments to measure mathematical understanding. Even though the other study mainly focused on developing an instrument to be used to measure mathematical values in education.

Roschelle (2013) pointed out that students have the desire to develop abilities of carrying out routine procedures and algorithms in more satisfying manner. This is done by developing a deep understanding of the meaning and application of mathematical concepts. For understanding to take place, students should be accustomed to ideas which are connected to authentic or recognized ideas of mathematical reasoning. This means that students must see how concepts learnt earlier connects to more advanced mathematics concepts. In conclusion, the study revealed that technology can help immensely to build conceptual understanding of mathematics concepts. The study being undertaken agrees with the study under review in that it recommended students to find time to attend routine programs such us mathematics clinics and tutorials as suggested in the reviewed study were routine procedures are encouraged.

A study carried out by Star and Stylandes (2013) concluded that it is important to understand what is meant by conceptual or procedural understanding of mathematics concepts. On the other hand, other researchers in mathematics education have formed different conceptual and procedural frameworks. Nonetheless, the purpose of the study was to explore procedural and conceptual knowledge in the context of instructional situation. The framework which help learners achieve their goal of learning mathematics concepts was coined by Star and Stylandes (2013). It was observed that teachers should have a common knowledge before policy implementation of what conceptual knowledge should be in mathematics education. Consequently, lack of mutual thoughtfulness of procedural and conceptual understanding of mathematical concepts may lead to learners failing to have a deep level understanding of mathematical concepts.Therefore, assessing procedural and conceptual knowledge in a mathematics task should include concept maps in order to show how concepts are connected to one another. Therefore, if students continue to focus on knowledge quality this may improve learners understanding of fundamental concepts in either procedural or conceptual knowledge. On the other hand, the researcher in this study sought to explore the distinction between procedural and conceptual knowledge. Arguably, as described by psychologists conceptual and procedural knowledge are difficult to assess. Hence the current study sought to use other viable method with huge validity and reliability to assess students procedural and conceptual understanding of mathematical concepts.

Literature reviewed by Tularam and Hulsman (2013) looked at a study of first year tertiary students mathematical knowledge of conceptual and procedural knowledge, logical thinking and creativity. It was observed that students were no longer higher achievers but more students obtained an average level of high school mathematics were entering university with an aim of achieving higher education qualifications. Most of the learners are relaxed because they see mathematics to be too abstract or not of immediate use in everyday life situations. However, the main aim of the study was to examine environmental science students' learning focus in terms of conceptual, procedural and logical aspects of first year university mathematics. Therefore, a critical examination of students written work was conducted to help expose the nature of their knowledge base in mathematics soon after they arrive to higher education from high school. Nevertheless, a sample of 133 students from 155 was picked. It was revealed that first year environmental science degree students were requested to prepare a focus sheet for their learning of preparation of a sheet for applied mathematics course. This was done by looking at the details of topics taught regarding students competences of procedural competence, conceptual competences, logical competences and creativity based framework (linear, Quadratic and limits). There was a small variance in the picking of topics as compared to the current study where binomial expansions and systems of equations, partial fractions and polynomials and trigonometric identies were picked as procedural and also set theory (DeMorgan's Laws) and function and functions and complex numbers were picked as conceptual. My study looked at the broader perspective of topics as far measuring of procedural and conceptual understanding of concepts are concerned.

A study done by Chinnappan and Forrester (2014) of which literature was reviewed in order to ascertain pre-service teachers procedural and conceptual knowledge of fractions. To do the task the researchers defined procedural knowledge as the understanding which involves rules and routines of mathematics while conceptual knowledge as that which enables learners to understand relationships of mathematical nature. Furthermore, it was revealed that prospective and practicing teachers need to have a sound understanding of both knowledge categories that involves fractions. The conceptual framework used in the study composed of Representational reasoning model (RR) of mathematical understanding. In this case the decision to use the (RR) model was based on the desire to better understand, capture, and support pre-service teachers abilities to construct a balance between conceptual and procedural knowledge. Hands-on activities were encouraged so that the findings of the study are easily contextualized. Quantitative data analysis was used to analyse data statistically. The methodological similarity was that the study under review used quantitative approach which also the current study used in the analysis and interpretation of data. The current study recommended quantitative methodologies based on positivists philosophers which advocates for scientific approaches in data analysis.

Crooks and Alibali (2014) performed a study which looked at different ways of defining and measuring conceptual knowledge in mathematics. In this research, the researchers sought to identify and establish means and ways of measuring conceptual knowledge. In this study, researchers sought to establish a tool which should be used to measure conceptual knowledge with high validity and reliability. This was done, in order to help researchers use a standard formular when measuring conceptual knowledge. However, reviewers explored the possible differences which may occur within and across domains as far as defining and measuring of conceptual knowledge is concerned. It is believed that, defining and measuring conceptual knowledge consistently fosters a deeper and more integrated understanding of mathematical concepts. Students tasks were to measure knowledge underlying procedural and conceptual understanding. This was done by observing learners and see to it that they are able to solve problems correctly. However, this method of measuring procedural and conceptual understanding lacks validity and reliability. Hence, the researcher; sought to develop standard indices as an instrument to use in the measuring of procedural and conceptual understanding of mathematical concepts. Not only are variations in measurement techniques inevitable due to different specific concepts being used by different age groups but also lack of precision of distinguishing conceptual from procedural understanding. It was observed that the study under review tried to measure conceptual knowledge with high validity and reliability while the current study fought to develop and adapt standard indices to be used in the measuring of procedural and conceptual understanding of mathematical concepts.

It has been claimed that the five strands of mathematical proficiency of Kilpatrick et al's of (2001) has been at the centre to give direction of mathematical understanding. Nevertheless, Graven and Stott (2014) contended that it was momentous to develop a procedural fluency of spectrum that can help to determine procedural understanding of mathematical concepts. In this regard, the researchers revealed a range of methods that leads to accurate answers in the responses to questions (procedural fluency and conceptual understanding). Therefore, the developed spectrum may be used to separate the two strands such that one shows flexibility and efficiency of conceptual understanding to be high which is interwined with procedural fluency. In the end the distinction between the two strands is murky. However, oral interviews instruments were used in this study which helped in the coding of responses (correct or incorrect) from participates. However, not only did the method fail to reveal the flexibility and fluency of the instrument but also failed to capture the aspect of learning on the part of the subjects (students). Consequently, this led to a redesign of the instruments and coding systems that may enable tracking of learner movements in terms of procedural fluency. For instance, if the learner answered the question correctly, regardless of the method used then the researchers were tempted to say that they had achieved procedural fluency. In this regard, it meant that we may conclude that at the end of the spectrum the researchers attained procedural fluency that was more elaborate with full flexibility. Finally, the writing adopted one of Kilpatrick et al's (2001) five strands of mathematical proficiency into a spectrum (procedural fluency). Not only did this enable researchers to adopt the methods but also helped them explain the progress over time of students procedural fluency. The epistemic variance was that the study under review used oral instruments to determine procedural understanding of concepts while the current study used structured based questions coupled with a questionnaire to assess students procedural and conceptual (understanding and confidence).

Khashan (2014) investigate conceptual and procedural knowledge of rational numbers for Riyadh elementary school teachers. In this regard, it was pointed out that, teachers should acquire a deep conceptual knowledge base in order for them to teach effectively. Likewise, it has been noticed that, there are several misconceptions in the teaching of mathematics because of the gap in teachers methodology pedagogy acquired from instituitions for them to teach at that level. Nonetheless, it was revealed that most of the teachers do not go far in the mastering of mathematics conceptual and procedural knowledge hence it is difficult for them to teach effectively when they are lacking teaching competencies. In this study, a sample of 27 novice and 30 experienced teachers were picked for a study. To achieve the objectives of the study, a test was prepared with 15 questions on conceptual and the other 15 on procedural. To test the validity of the test expert judgement was sought from specialists in mathematics, curriculum planners and teachers of methodology. In this regard, the questions were tested and those with an average estimate of 70% were deleted and replaced or proposed for amendments in order for them to conform to the required standard. Internal consistency was measured and it was 0.86 for both procedural and conceptual components of the test prepared. Furthermore, it was revealed that the procedural knowledge exhibited by primary school teachers of rational numbers in Riyadh elementary schools was more than the conceptual knowledge. The methodological difference was that the study under review observed teachers at elementary schools while the current study focused on first year students understanding of specific concepts at UNZA. At the the two separate instances it was revealed that students had more procedural than conceptual understanding. This, may be attributed to the conventional methods used and techniques used which support the use of procedural oriented methodologies.

To get more insight into the conceptual understanding of mathematics via a hands-on approach at Liberty University, Korn at al (2014) used the three Rs (reading, writing and arithemetic) in the learning process. It was revealed that using a hands-on approach presents an effective way of teaching conceptual understanding of mathematics. Notwithstanding, educators should have knowledge and skills to teach conceptual understanding of mathematics. Therefore there must be a common understanding of what teaching mathematics entails for educators to be able to teach conceptual understanding of mathematics through hands-on learning. Researchers wish to involve the theory of variability and how well it can be implemented for a specific theory in order to enhance conceptual understanding of mathematics. Furthermore, it was revealed that in employing a hands-on approach to teach conceptual understanding, there is need to use manipulative tools. The study concluded that the goal towards using a hands-on approach in teaching conceptual understanding in mathematics cannot remain in the realm of academia but must be applied to the average classroom. The study under review encouraged hands on approach to teaching mathematics while the current recommends a balance of procedural and conceptual learning of concepts.

The study done by Long and Dunne (2014) revealed that the teaching of primary school mathematics should include planning of assessments.This was aimed at developing from initial intuition to more explicit and general conceptions of learning mathematics concepts by either abstraction or induction. Finally, it was also revealed that, a conceptual field approach was required in order to deepen insights into the underpinning of mathematical concepts (procedural or conceptual). It was established that there is no common perception of mathematics knowledge at primary level concerning procedural and conceptual mathematical understanding. The study under review conforms to my study in that both of them suggests developing a conceptual understanding of mathematics concepts.

NCTM (2014) focused on procedural fluency in mathematics. This was done by looking at how procedural fluency extends students' conceptual fluency and applies this competence into all the five strands of mathematics proficiency. It was observed that there is need to analyse students' procedures often since it was revealed that this appears as insights towards misunderstanding in planning instructions. If done properly, this may help teachers adjust in planning of instructions and steps. However, the discrepancy was in the epistemology in that the study under review focused on all the five strands of mathematics fluency while my study only focused on procedural and conceptual understanding of mathematics concepts. Consequently, this may help teachers justify choices of procedures to take that might strengthen mathematical understanding of concepts.

Literature revealed that, up to now, there have been no direct ways of assessing different levels of mastery within the outlined theoretical framework (Nezhnov, 2014). Additionally, there is no current available quantitative instruments that measure theoretical understanding of learners' mathematical knowledge. The study further revealed that, the extent of understanding mathematics is relatively narrow which is composed of knowledge of specific concepts (algorithms) in mathematics. Finally, the study revealed that, going forward not only will students understand how to solve mathematics problems but also know how to solve algorithms either in a standard or novel way. The epistemological variance was that the study under review stated that there was no quantitative instrument to use for measuring theoretical understanding of mathematical concepts. However, the current study developed and adapted valid and reliable instruments to use for measuring procedural and conceptual understanding of mathematics concepts.

Sarwali and Shahrill (2014) postulated that understanding students mathematical errors and misconceptions may be looked at different pedagogical instructions. It was revealed that systematic errors may be a consequence of students misconceptions. This means that students may fail to make connections with what they already know. In this regard, it was revealed that students errors are unique because they reflect their understanding of concepts in the learning of mathematics.The sample was in three phases, denoted as X, Y and Z. Data was obtained through a written test consisted of 13 questions. Furthermore, the topics covered were algebra, statistics and geometry. The results revealed that confidence does not affect performance. This was so because there was no clear-cut division between high and low confidence levels after the study was done. The discrepancy to the study under review was that it focused on students mathematical errors and misconceptions which do not reflect the true picture of mathematical understanding of concepts. Nevertheless, the current study focused on developing valid and reliable instruments (standard indices) to be used for measuring mathematical understanding (procedural or conceptual) which differed with the study under review.

The duality between ways of thinking and ways of understanding was put under introspection by Weber and Lockwood (2014). In this regard the two scholars contend that, the duality ways of thinking and understanding should combine well with the learning trajectories in mathematics education for the smooth teaching and learning of mathematical concepts. Therefore, there should be always a reflexive relationship and interaction between the two already mentioned components of mathematical knowledge. It was observed that the ways of thinking and ways of understanding helps to uncover new insights for researchers about students conceptual and procedural understanding of mathematics concepts. Furthermore, in the same vain, this may help lecturers to balance the explanation of learning trajectories to students in order for them to appreciate various categories of conceptual levels of concepts in mathematics learning. Preferably, ways of understanding corresponds to the subject matter knowledge. This may be observed in the ways of thinking exihibited by students in the sense that conceptual tools will definitely correspond to the ways of understanding as a concept of duality principle. In this regard feedback of students after writing a test, assignment and examinations should be encouraged because it impacts positively on the understanding of mathematical concepts. Understanding the current knowledge of students involves generating a model of students ways of thinking and students ways of understanding. This is seen in reviewed literature which is grounded in the interaction with and among the students. In this scenario, it implies that many existing learning trajectories of discrete levels of conceptual change should suggest a complemently explicit attention of how learning occurs between levels. In the same vain, it is an obligation for students to practice reasoning in order to internalize desirable ways of understanding and ways of thinking in mathematics learning. Consequently, if students are to develop ways of understanding and ways of thinking that stands a test of time then learners must engross problem solving skills that may help to develop easily and also do so frequently over a stipulated time. The study under review encouraged to internalize ways of understanding and ways of thinking in the learning and teaching of mathematics. Above and beyond, the current study also supports the promotion of understanding of mathematical concepts and also having confidence or exhibiting adequate zeal to solve the questions correctly be it procedural or conceptual.

The findings of the current study may be able to add on the existing literature that technological advancement improves the understanding of mathematical concepts. In this study, Aydos (2015), sought to look at the impact of teaching mathematics with GeoGebra on the conceptual understanding of limits and continuity the case of gifted and talented students in Turkey. The study sought to determine the impact of teaching limits and continuity topics in GeoGebra mathematical soft-ware. This was done to improve conceptual understanding and attitudes of students towards the learning of mathematical concepts through technology. In this study, the researcher constructed an open-ended type of test questions to measure conceptual understanding. Besides, data was analysed using an independent sample t-test on gain scores for control and experimental groups. Notably, there was a massive gain in conceptual understanding in a variety of mathematical topics. Teachers should learn to use mathematical programs in their teaching and learning process. Nevertheless, the ultimate goal of the project was to increase the use of information and communication technologies (ICT) in the classroom. The first objective of the study was to use smart boards, projectors, internet access, copiers and printers. This could be achieved by having in-serving training of teachers in the use of modern equipments in the teaching and learning of mathematics. The study revealed that there was massive need to investigate into the impact of introducing GeoGebra on Turkish curriculum of learning mathematics. This meant to use computer program in the teaching of mathematics particularly calculus. Additionally, after the study it was revealed that the use of GeoGebra as a free-available and open source soft ware increased. The software is constantly being improved upon by computer experts so that the teaching of calculus improves. In this regard, a pre-post test design was employed in the study to determine the impact of GeoGebra software on conceptual understanding of limits and continuity under calculus. It was further revealed that students in the experimental group out performed their peers in the control group. It was recommended that subject-specific technology use should be encouraged in all schools across the country. Finally, it was concluded that, in order to ensure GeoGebra and other similar computer programs are adequately used professional development should be encouaraged so that a number of opportunities are created for practicing mathematics teachers to up grade in mathematics competences of using soft-ware without challenges. The study under review was similar to the current study in that both of them recommends learning of mathematical concepts through technology. The methodological difference which was observed between the two studies was that the study under review used an open ended test to measure conceptual understanding while my study consisted of a structured questions test in order to measure procedural and conceptual understanding of mathematics concepts. Additionally, the study under review used an independent sample t-test while my study used multi-variate analysis tools in the interpretation of data.

Reviewing literature on the development of fraction concepts and procedures in U.S and Chinese children by Bailey et al (2015) observed that Chinese middle school children have greater knowledge of fraction concepts than some grade peers of U.S. It was further revealed that Chinese children were superior in both conceptual and procedural knowledge of fraction at grade 6 level and that the difference in procedural knowledge was large. Additionally, it was determined that Chinese and U.S participants encountered fraction concepts and procedures at similar times during their schooling. Hence it was not difficult to compare the difference and similarities in the aspect mention between the two countries. Even though the study under review focused on conceptual and procedural knowledge at grade 6 level it coincided with my study in that both of them recommended a balanced in the learning of procedural and conceptual concepts of mathematics.

Literature was reviewed to assess the individual differences in students' knowing and learning about fractions. This was done through an in-depth qualitative study done by Bempeni and Vamvakoussi (2015), however ninth graders were chosen to assess their competence of conceptual and procedural knowledge of fractions. However, participants were from Athens which consisted of three girls and four boys. The instrument developed was a thirty questions fractions tasks which was in groups of four categories. Data was analysed according to phases (i.e from category A upto D). For instance in the first category each student was asked to solve fraction tasks and students were encouraged to think aloud and no duration was assigned to the task. This gave students ample time to assess the strategies which suited the work. Finally, it was revealed that three different students profiles were utilized. Of the three students it was observed that one student was exceptionally good in all the categories. The study under review opted for a qualitative study and picked grade 9 while my study was purely quantitative and consisted of first year students at UNZA. To add on, the study under review concentrated on conceptual and procedural knowledge of fraction at grade 9 while the current study focused on specific concepts of selected mathematics topics at first year university level.

Maglipong and Bongolto (2015) argues that, ‘mathematical patterns of procedural and conceptual knowledge can be linked to computer assisted learning’. Furthermore, computers can be used to change mathematics teaching by decreasing the time needed for procedural skills and increasing the time for conceptual understanding. Additionally, a significant correlation between procedural knowledge scores and conceptual knowledge scores on the post-test, which was not present on the pre-test was observed. The study under review recommended computer assisted learning which is in conformity with my study which also encouraged computer usage in the learning of mathematics concepts.

A study was done by Bergsten et al (2015) on conceptual and procedural understanding of mathematics as viewed by qualified engineers from two countries. This study was conducted in South Afrca and Sweden by interviewing second year students on the impact of procedural and conceptual mathematics. It was observed that teaching of mathematics should focuss on conceptual understanding in order to provide a contextual basis for students to connect to their prior understanding of mathematics concepts. In this regard, it was observed that students tend to proceduralize questions designed to have a conceptual focus. To add on, the study explored views and ideas concerning procedural and conceptual mathematics for engineering students. The study revealed that the problem was with mathematics courses for engineering students which were procedural in nature and failed to correspond well with the application part of engineering work which is conceptual in nature. Students in the study under review proceduralized questions which were designed to have a conceptual focus this correlated to the current study which showed that first students at UNZA performed well in procedural questions as compared to conceptual. This was so because most of the students were only focused to do procedural work as compared to conceptual.

Christina, Maglipong and Bongolto (2015) state that mathematical patterns impact positively on students' performance as far as the manipulations of college algebra. In this regard, it was noted that the development of pattern based thinking and the use of patterns to analyze and solve problems help hugely as powerful tools for doing and understanding mathematics. In the same mode, if students lack understanding of concepts they are unlikely to construct desired ideas. The current study revealed that the major problem was lack of connecton of concepts from secondary to higher educaton level. Consequently students' with deep approaches of learning generally demonstrate a more sophisticated understanding of learning opportunities in mathematics. Furthermore mathematical patterns help students immeasurably to observe and connect number, pictures and figures without problems. Additionally the use of mathematical patterns helps learners to develop critical thinking skills for them to analyze practical problems without difficulties. Mathematical pattern improve students conceptual understanding of mathematical concepts. Notwithstanding, mathematical patterns are enjoyable and interesting. The study under review supports my study which encourages the implementation of developing patterns in the teaching and learning of mathematics concepts.

A review of related literature was established by comparing mathematical patterns to students performance of college algebra (Christina, Maglipong and Bongolto, 2015). On the other hand, researchers sought to develop a pattern of based thinking that uses patterns to analyse and solve problems in an extremely powerful way in order to improve mathematical understanding of concepts. In this regard, if students lack conceptual understanding of mathematics, then they are unlikely to construct worthwhile algebraic ideas. It was revealed that the use of patterns in the teaching and learning of mathematics correlates highly to visual approaches of teaching mathematical materials at various levels of education. Furthermore, the study revealed that problems by students connect to their lack of learning of concepts at secondary level. Notably, students with an adequate knowledge of approaches generally demonstrate to be more sophisticated understanding of learning opportunities of mathematical concepts. Mainly, learning mathematical patterns are done through investigations and students observe and connect numbers, pictures and figures correctly after serious reasoning. The study revealed that the use of mathematical patterns helps to develop students critical thinking and analysis skills of practical problems in mathematics. Ideally, mathematical patterns helps to improve students conceptual understanding. Particularly, conceptual understanding is the ultimate key component of mathematical understanding of specific concepts. Furthermore, the study revealed that, mathematical patterns are enjoyable and interesting, which allows learners to develop their critical thinking in order to enhance mathematical understanding of concepts. The study under review supports the use of patterns and visual approaches in the teaching of mathematics concepts. This is in line with the recommendation made by the current study that the use of pictures, videos in the teaching and learning of mathematics concepts may improve drastically the performance of students in conceptual understanding.

A study by Edwards (2015) contributed immmersely to the body of mathematics education literature on procedural and conceptual understanding of mathematical concepts. Edwards suggests that teachers of elementary education should use conceptual and procedural methods for teaching of fractions. The study postulates that fractions and the four basic mathematics operations are the key for mathematics understanding of specific concepts. This means that the teaching and learning of fractions should be improved by enhancing strategies of teaching mathematics. Generally, the elementary mathematics teachers approaches should be improved. Tests should be standardized from kindergarten through primary and secondary education upto tertially level. Elementary mathematics teachers should be able to prepare standard tests not only in mathematics but also in reading, science and writing. The researcher recommended microteaching coupled with simulations in the teaching process of a conducive environment which supports smooth transition of concepts. This can only be achieved where mathematics teachers have good qualifications, adequate prescribed and recommended textbooks of mathematics for both lecturers and students. In this aspects the textbooks should be of the same level to that of students. The concepts of mathematics should not be too abstract but of readability level that is not confusing to either the lecturer or the student. In this regard, the teachers of mathematics should not only be able to learn how to conceptualize the teaching of fractions but also increase their self-efficacy of teaching fractions. Mathematics teachers should be aware of mathematics anxiety affects both parties (teachers and pupils). On the other hand, the study was preceded by the pre-test and post-test which helped heavily to improve the teaching of fractions in order to develop an in-deepth understanding of concepts. Continuous professional development (CPD) was discovered to be among the major factors that helps greatly in the understanding of fractional concepts of a mathematics nature. In the long run students will be able to read properly for them to solve fractional concepts without difficulties. The study revealed that the major misconception is that school mathematics for elementary education is taken lightly as if it is very easy to teach, when it is not the case that all elementary teachers of mathematics should be able to have a full understanding of mathematics concepts before they can go to teach. The study under review agrees to the current study in methodology used. A pre-test and post test was used in the teaching of fractions at elementary level which corresponds to my study which gave a pilot test to 30 first year students from the same cohort of 2016/2017 academic year. Finally, adjustments and corrections were made to the test then it was administered to 378 first year mathematics students at UNZA in the School of Natural Sciencs (SNS).

Reviewing literature by two scholars Egodawatte and Stoilescu (2015) recommended that instructional and routine problems should be solved by using techniques which were learned in the classroom. Then, non-routine problems are typically solved by using special techniques such as drawing a diagram, constructing a pattern, or solving a similar problem. It was further observed that procedural knowledge nurtures skills needed for solving problems. Grade 11 university college students were used of which 30 randomly sampled from a urban secondary school in Ontario. Based on the findings, it was revealed that the test showed students who had low attainment in conceptual understanding equally did poorly in procedural. The study under scrutiny supports the current study in that both of them encourages consistency in routine programmes. My study recommends that students should be finding time to attend mathematics clinics and tutorials so that they maybe helped in certain components where they may be lacking behind.

Notably, İncikabi et al (2015) looked at Seventh Grade Students' Performances on Conceptual, Procedural and Graphical Problems Regarding Circles. However, a case study design was used in this study. Additionally, a pilot study of 24 students was taken to validate the study. Likewise, an achievement test which consisted of the concept of the circle was scruitinized in the study. The study was conducted in Middle School that had an average equilavent to that of National Assessments. The test had to measure, conceptual, graphical or procedural concepts of the circle. Data was analyazed by external experts and researchers. Furthermore, the findings revealed that students preferences of question types corresponded with their preference according to their stated areas of interests. The study under inspection corresponded to my study in methodology in that it picked a pilot study of 24 students with 16 males and 8 females while the current study selected 30 students composed of 20 males and 10 females respectively. The study under analysis specifically interrogated performance of students on conceptual, procedural and graphical problems while my study focused on procedural and conceptual understanding of first year university students.

Khalid and Ekholm (2015) did a study on contextualizing calculus with everyday examples to enhance conceptual learning. A diagnostic test and the final examination in calculus was given to engineering students in Sweden followed by a questionnaire to solicit students opinions. The examination was an intervention group and recorded better results than the diagnostic test results. This helped the researchers to know that contextualizing teaching of mathematics through concrete examples helps students to focus attention on developing conceptual understanding of concepts. Furthermore, the results from the final examination showed that students afterwards were able to become more motivated in doing mathematics hence further developed a better grasp of abstract thinking in mathematics. Besides this helped students to start solving mathematics problems by using everyday examples to abstract mathematics problems consequently eliminating fear from students and replacing it with confidence. The study revealed that the use of everyday examples made it easier for many students to understand abstract concepts. Final results showed that using everyday examples in mathematics teaching not only helps students develop conceptual understanding of mathematics concepts but also attain in future comptences in solving procedural problems without difficulties. The methodological similarity between the study under investigation and my study is that both of them blended the test prepared as a tool for data collection with the questionnaire to solicit views and opions which students had towards the solving of mathematical questions.

Reviewing literature concerning the validity and reliability of mathematics measuring instruments. I looked at Rabab'h and Veloo (2015) who argue that, the validity and reliability of mathematics learning strategy instruments among Middle School Students in Jordan should be improved. It is evident that, Mathematics Learning Strategy (MLS) should be the main focus behind the requirement of valid and reliable measure to help students learn mathematics. Not only does MLS play a pivotal role in academic but also helps learners to have good career choices. In this study, the sample size consisted of 360 schools from 8th grade to 12th grade of which 178 (49%) male and 182 (51%) female. A Likert scale of 5-point measurement was used which had a number of categories as 1 (Strongly disagree), 2 (Disagree), 3(Moderately agree), 4 (Agree) and 5 (Strongly agree). This was to allow participants get information from a wide range of views from the students. In this study, information was gotten from mathematics attitude, mathematics motivation, mathematics self-regulation, mathematics self-efficacy and mathematics anxiety. In this study, data on the research protocols were coded and scored according to instruments which were provided by the author. For instance, mathematics attitude instrument consisted of 24 items while the validity of the items of the instrument after validation dropped from 83 to 65 items. In conclusion, we may state that, the findings provided evidence of a good psychometric property for the Arabic version of MLS instrument. The study under evaluation stressed the need for validity and reliability of mathematics measuring instruments which coincided with my study which made sure that there was validity and reliability of mathematics constructs before the research instrument tool was used.

Despite an extensive history of the relatonship between conceptual and procedural knowledge of mathematics, it appears that the findings from research are not conclusive hence Rittle-Johnson and Schneider (2015) tried to show the development of conceptual and procedural knowledge of mathematics over a range of methods. In this regard, the way to measure conceptual and procedural knowledge reveals that the two knowledge types are different. However it was observed that the measuring of conceptual knowledge may involve multi-dimensional constructs. Furthermore, to understand the difference between the two knowledge types there is need to study them separately. To add on, it was revealed that the strength of the relationship between procedural and conceptual understanding of mathematical components varies across studies and over time, but it is clear that the two types of knowledge are often related. Even though the two knowledge may differ as studies have shown that it is diffcult to separate them. This is because the gain in one type of knowledge results in the gain in the other type. The study under review postulates that the methods of measuring mathematical understanding has been different from one scholar to the other. Nevertheless, the study under enquiry corresponds to the current study in that it contends that conceptual understanding maybe measured using multi-dimensional scaling of constructs between and among variables while my study states that we may use a number of multi-variate analysis tools. Howver, in the current only factor analysis, multi-dimensional scaling and cluster analysis was used in order to find out the constructs concerning the level of confidence exhibited by students to solve questions correctly.

Additionally, Rittle-Johnson, Schneinder and Star (2015) sought to look at the bidirectional relations between procedural and conceptual knowledge of mathematics. In this aspect, it was revealed that the learning of mathematics concepts is not a one-way street but it may be either way. Even though most of the studies show the movement from conceptual to procedural.

However, it was further revealed that basic understanding of mathematics should proceed from symbolic representation and skills promotion to the development of both procedural and conceptual understanding of specific concepts in mathematics. In this regard, the goal of the current paper was to review empirical evidence of the relation between conceptual and procedural knowledge. The researcher tried to review empirical evidence on the relations between conceptual and procedural knowledge over a pro-longed period of time. It was observed that previously research designs were not appropriate to the study and that it was difficult to measure conceptual knowledge because the instruments used were not valid and reliable. Hence evidence for bidirectional relations between procedural and conceptual knowledge comes from longitudinal studies on the predictive relation between the two types of knowledge over time. It was revealed that conceptual to procedural was the most common or the best identified in a number of studies. The study under enquiry focused on the bidirectional relations between procedural and conceptual knowledge of a mathematics test while the current study sought to bring a between between procedural and conceptual understanding of mathematics concepts at first level.

Whitaker et al (2015) piloted the assessment that consisted of four stages. In this study the total piloted item consisted of 48, 55 and 53 multiple-choice items at the beginning, intermediate and advanced levels. However, the piloted sample consisted of 2075 students at the beginning and or intermediate level while 1249 during the intermediate and advanced levels. The criteria used to select the sample was in three aspects; performimg students, gender and non responses. Furthermore, permission was obtained from parents and only those students whose parents signed the guidelines forms participated in the study. This was done in order not to violate ethical issues. Furthermore, selected students were given a $5 as an incentive for participating in the study. It was revealed that the types of items that appeared in the LOCUS assessment were intended to measure conceptual understanding of statistical concepts. In addition, the LOCUS assessment recognized data that was analysed which needed more procedural knowledge of graphical and numerical summaries. Noteworthy, the study interpreted and generalized results beyond the sample. The study was proved to be valid and reliable. The reliability test after measuring the stratified alpha was between 0.70 and 0.72 for each proportion. Furthermore, the LOCUS assessment, whose aim was to measure conceptual rather than procedural understanding of statistics, was appropriately designed by researchers. In this regard, results of the LOCUS pilot assessments provided a snapshot of what students with prior statistics of 6-12 grades exihibited more procedural than conceptual understanding of statistical concepts prior to the full implementation of the entire research program. Also, the levels of conceptual understanding in statistics (LOCUS) project aimed to create valid and reliable assessment tools for conceptual understanding of statistical concepts. The study under scrutiny had three piloted study from the populations. The first pilot study had 48 out of 2075 while the second and third pilot study had 55 and 53 from 1249 respectively. It was observed that only students who had parents signing the guidelines forms of participation took part in the study. To add on, my study had a pilot study of 30 students of which 20 were males and 10 were female from a population of 1500.

Literature review in the past has debated on the procedural and conceptual understanding of mathematics but has left a knowledge gap on the understanding of specific concepts in mathematics at university level . Notably, Wriston (2015) sought to look at the conceptual understanding and application of students knowledge in the assimilation of content. However, teachers used individualized teaching methods and strategies in order to build new concepts for learners to develop a smooth understanding of other unknown concepts. On the other hand, it was observed that students who did not understand basic mathematical components find difficulties to comprehend higher concepts in mathematics learning. In this study, to validate the clarity of all questions on both assessments, sample assessments were given to nine pre-service teachers. The test was only given to mathematics teachers and 10 professors in the mathematics department to give expert judgement of the research instrument prepared. Additionally, it was revealed that many students failed to master the understanding of fractions. Finally, pre-service teachers took the assessments further by trying to reveal the clarity of each and every question prepared. It was observed that a good foundation of kindergarten through to the eighth grade may help students to master fractions, ratio and proportions concepts without difficulties. The study under analysis gave the questions prepared to professors to seek for expert judgement of the formaulated questions which agrees with the current study which gave test questions to supervisors for crtical analysis before including them in the test questions.

3.10.1 Lessons learnt from literature review related to my study during the era (2010-2015)

Although there has been plethora of research on procedural and conceptual mathematics, little research has been done on measuring of procedural and conceptual understanding of mathematics. It was revealed that, previous studies had no restrictions on learners procedural or conceptual understanding and also lacked knowledge of measuring procedural and conceptual understanding. As from the literature reviewed during this period captioned it was observed that unreliable measuring tools were used to measure mathematical understanding. Hence, a shift in methodology was sought to move from having a good number of measuring tools which only led to have misconceptions and misunderstanding of mathematics concepts (procedural or conceptual). Likewise, a number of studies used Cronbach alpha to determine the reliability of instruments used in their studies. To add on, it was observed that a good number of studies used computer soft-ware to assess mathematical knowledge such as NetLogo, GeoGebra and SPSS in order to measure mathematics understanding. Pearson regression analysis and cluster analysis were used to analyse data quantitatively. It was further revealed that, the use of scientific calculators has significantly reduced procedural competences in the learning of mathematics especially at secondary school level. Some studies advocated that the learning of mathematics should be a balance between conceptual knowledge and skill development. This may be achieved by active involvement of the learners in the learning process. However, to a lesser extent, the difficult in assimilation of mathematics concepts by previous studies revealed that there was a definite need to steer the emphasis on mathematical language and symbolism so students may align themselves to the recognized mathematics register. To sum up, it was argued that the teaching and learning should include real life situation examples so that research findings are easily related to current situations concerning mathematical understanding. However, there were a number of ways of which the current study learnt from previous scholars. Firstly, the gap in methodology was observed of which my study focused on developing and adapting new measuring instruments used to measure procedural and conceptual understanding of mathematics concepts. Secondary, the study undertaken corresponds to a number of scholars who advocates for involvement of computer platforms in the measuring of mathematical understanding. In this facet, my study employed XL STRATA with Excel to use to analyse data using multivariate analysis tools and also original software 'O' to analysis descriptive graphs.

3.11 Literatures, Findings and Conclusions from Related Studies (2016-2020)

A study done by Aydin and Özgeldi (2016) sought to look at elementary mathematics teachers' contextual, conceptual, and procedural knowledge. However, in this study researchers revealed that students mathematics performance in international large-scale assessments such as the Programme for International Student Assessment (PISA) of which the task was to investigate difficulties encountered by preservice elementary mathematics teachers to acquire competences in mathematics when using different types of mathematical knowledge. Ideally, students contextual knowledge can be elicited by presenting problems in a story context or by using appropriate variable, symbols, diagrams for presenting everyday life situational examples. The study used a sample of 52 preservice teachers of which 38 were females and 14 males. Calculus and linear Algebra were picked as topics under investigation. It was revealed that prospective teachers had difficulty in understanding the context of the problem. The main problem was of interpreting geometrical model of real life situation. Furthermore, it was revealed that procedural knowledge received more attention than conceptual and contextual knowledge. Hence or otherwise, going forward it was believed that conceptual and contextual approaches should receive more attention since the study had received little attention. The study under observation emphasized the need to know how to measure variables and also how to interprate diagrams correctly. The study under review related to my study being undertaken in more ways than one, ideally I wish to discuss only one area. The use of symbols was encouraged which made the current study to develop and adapt standard indices which were completely made of symbols. This was so because of understanding and appreciation of mathematical rigor and brevity as debated by logical positivism theories.

In a study conducted by Bergsten et al (2016) on conceptual and procedural approaches to mathematics in the engineering curriculum with an emphasis on comparing views of Junior and Senior Engineering Students in South Africa and Sweden showed similar results. This was done by using mathematical models in the teaching of procedures and concepts to engineering students in the already captioned countries. It was observed that, usually mathematics is studied as procedural fluency as compared to its conceptual orientation. This leads to engineers having a computational mind towards engineering work instead of having an understanding focus. It was revealed that most of the students had higher confidence in their performance on procedural tasks as compared to conceptual ones on either Junior or Senior level in both countries. The correlations in both countries at Junior level showed that students had more confidence in procedural tasks as compared to conceptual ones. Above and beyond, senior students in both countries were more confident in conceptual questions. The major similarity between the study under review and my study was that both recorded students being more confident to handle procedural questions as comopared to conceptual.

In a study done by Bisson et al (2016) they looked at how to measure conceptual understanding using comparative judgement as it improves students'understanding of core concepts in mathematics. The aforementioned was to be achieved by ensuring increased conceptual understanding in classrooms with an emphasis on ensuring valid and reliable measures of conceptual understanding are upheld. On the other hand, another approach to measure conceptual understanding was to use one-to-one clinical interviews and developing a scoring rubric in order to rate the quality of participant understanding of concepts. The emipirical data from this study suggested, CJ used no detailed assessement criteria or scoring rubric than the final rank order instead was grounded by the collective expertise of the judges. In this study a number of experiments were done to students in Applied Statistics in England. For instance, 20 participants were picked to do a 13 item multi-choice instrument. Another instrument consisted of 20 open-ended questions. However, the author used this instrument because it is underpinned by decades of rigorous research in developing childrens understanding. Hence the instrument is considered to be valid and reliable. Eventhough, it is believed that CJ is far more better than other instruments when it comes to measure conceptual understanding. The study under enquiry used a rubric to measure the scores awarded by expert judges which relates highly to my study which used also a rubric to award marks to specific questions of the test given and than related it to the descriptive graphs generated using original software.

A study was conducted by DeCaro (2016) on how to induce mental set constructs procedural flexibility and conceptual understanding in mathematics. Findings revealed that problem solving may help or hinder both flexibility in applying the mathematical strategy in using deep conceptual thinking when solving mathematical problems. Nonetheless, the current study sought to examine consequences that may occur when applying mental set on both procedural and conceptual concepts. In this regard, it was observed that the test to learners was able to test their problem-solvng flexibility as compared to conceptual understanding exhibited by students. However, findings of this study suggested that potential factors for reducing students flexibility and understanding constraints in mathematics may be uplifted by involvng students in the solving of mathematical problems to enhance understanding of procedural and conceptual mathematical problems. The current study relates to the study under scrutiny in that both of them suggests that problem solving should play a pivotal role in the learning of procedural and conceptual constructs in mathematics.

Fatqurhohman (2016) looked at the process of transition from procedural understanding to conceptual understanding of mathematical concepts. However, in this study three aspects of problems were used. Firstly, identification of the problem followed by the use of algorithms and finally processing algorithms. It was revealed that procedural understanding of learning mathematics has a purpose to fulfil learning by describing the transition from procedural understanding to conceptual understanding of solving mathematical tasks. To add on, the study was made up of the sample of 20 students of fifth grade. The instrument of the study was a test composed of structured questions. Additionally, the researcher prepared also structured interviews to solicit opinions on the understanding of transition process of procedural to conceptual understanding of mathematical concepts. However, all the 20 participants volunteered to participate in the study. The researcher gave adequate instructions and rules to students before writing of the given test. Furthermore, data was obtained from students who had met the basic standards of transition of procedural to conceptual understanding of mathematical concepts. Data obtained was analysed either quantitatively or qualitatively. In this regard, qualitative data was used to describe the process of transition based on the specific aspect of algorithm. Based on the findings it was revealed that students had more understanding of procedural concepts than conceptual ones. Finally, it was also further revealed that students with a conceptual understanding had the capacity to see the relationship between concepts and procedures that can provide arguments to explain facts appropriately. The study under examination used quantitative and qualitative methods while my study only used quantitative methods because they conformed to the scope of the current study. Even though there was a small variance in methodologies utilized both of the studies used structured questions in the test prepared. To add on, in the study under review participants volunteered to be part of the study while my study used simple random sampling to pick contestants. This meant that my study gave each and every member of the population an equal chance of being part of the study.

Keene and Fortune (2016) looked at the framework for integrating conceptual and procedural understanding of the first two years of undergraduate mathematics at North Carolina State University. In this study, researchers sought to investigate the common instructional approach of the first two years of undergraduate mathematics by critically looking at the teaching and learning of calculus and especially where procedures were involved to find the solutions of the problems. This was to be done by integrating the teaching of procedures and concepts and also to encourage the learning of concepts before procedures are taught. The study examined the components of relational understanding by looking at the outcome of carrying out procedures before the study of concepts. In this way, it was significant to identify when it was appropriate to use a specific procedure especially when it was needed to give a solution. In this way, students were able to understand the reasons why procedures work and the steps for using procedures appropriately in the solving of mathematical problems. The study revealed that students were able to symbolically or graphically verify the correctness of a procedure without repeating the steps used to arrive at the correct solution. The study under examination integrated the conceptual framework in order to understand more on procedural and conceptual aspects of mathematics. The study being undertaken used three conceptual framework to break down concepts affecting procedural and conceptual understanding for it to produce acceptable results of mathematical understanding.

Historically, gigantic amount of literature has contributed immensely to the understanding of procedural and conceptual concepts of mathematics. In this regard, it has been revealed that, there are a number of theories which enhance the understanding of communication especially in the teaching and learning of mathematical concepts. However, previous scholars argue that there is definite need to add rigour and details to the level of social ontology and hence or otherwise allow for more sophistication of operationalising of concepts. It is only when concepts are adequately operationalized that learners may be able to identify each and every specific concept of mathematics without problems. This may help in solving mathematics problems correctly (procedural or conceptual tasks). Concomitantly, Kent and Foster (2016) concluded that theories of communication affect greatly the understanding capacity of students in learning mathematical concepts. The study under scrutiny argued that there is need to add rigor in the understanding of procedural and conceptual ocncepts of mathematics. This was supported by the study being undertaken which allowed rigor by operationalizing mathematical concepts in the learning process.

Literature reviewed by Kusuma and Masduki (2016) used descriptive qualitative study to analyse data which was collected by applying observation, test and interview methods. Typically, this was done to find out how students solve logarithms in order to determine the conceptual and procedural understanding. Based on the test results it was shown that out of 24 students, 7 students were in a high category, 10 of the medium category and 7 in a lower category. It was revealed that if students got the solutions correctly and also explained well the procedures of how to get the answers then those students were put in the high category. Furthermore, if students got the questions correctly but unfortunately failed to explain how the solutions where obtained then those students were in the medium category. Finally, if the students failed the questions and did not know how to explain the workings then those students were in the low category. It was revealed that most of the students were in the medium category where students knew how to solve logarithms correctly but failed to explain the rules of logarithms for addition and subtraction. The study under review observed students by giving them a test and then interviews to give opinions and views on how they were supposed to answer the questions correctly. However, the current study prepared a structured test then at the end of the test a questionnaire was put for students to indicate their confidence which they may exhibit to answer the questions correctly.

Mills (2016) postulate that conceptual understanding plays a critical role in the understanding of concepts in mathematics. Nevertheless, the term conceptual understanding was used in the study to appreciate how to determine educators who may help students attain understanding in a concept based curriculum. Notwithstanding, the intention of concept analysis which is used to uncover what it means to have conceptual understanding but also to reveal understanding in a progressive way. It may only be achieved, when memorization of facts is avoided as opposed to conceptual understanding. In this regard, the study further revealed that concept analysis be used to organize around the dimensions of conceptual understanding. Most of the time, the process of conceptual understanding was attained with the help of factual or procedural knowledge. It is evident from the literature of mathematics education and psychology that the viable process of how to attain conceptual understanding begin with salient dimensions identified. The study under investigation supports the current study in that both of them advocates for conceptual understanding of concepts as opposed to memorization of facts. Furthermore, my study suggests to put up a balance between procedural and conceptual understanding of mathematics concepts.

Rittle-Johnson et al (2016) evaluated the effect of instructions on concepts and procedures within the same lesson. This was done by experimenting the effects of evaluated instruction type since the author believed that mathematical understanding maybe equivalent to conceptual knowledge. In this study, the sample was second grade children from 13 classrooms in three public schools (i.e. middle-to-upper-middle class). The assessment used was adapted from a previously validated assessment instrument which had procedural and conceptual knowledge concepts. In this regard, a brief and improved version of the assessment was created for use at pretest and midtest sessions. A problem solving workbook was developed which had 17 problems to be solved by 52 small groups of 3-5 children each.

Consequently, results showed that there was no interaction between instruction type and instruction order, which was contrary to the expectations. A multivariate ANCOVA on the four primary outcomes was instituted. Furthermore, to explore conditions was observed that there was need to perform manipulations which impacted performance on secondary analysis of interventions of measures. This study highlights the need for future research to evaluate the equivalent amount of time required on conceptual instruction for effective learning of either procedural or conceptual concepts in mathematics lesson. The study under review used multivariate ANCOVA while the study being undertaken used multivariate analysis tools specifically factor analysis, multidimensional scaling and cluster analysis. This shows that both studies used multivariate analysis tools because of their capacity to analyse massive data and organize them into manageable and sizeable data forms. To add on they have the ability to show relationships between and among variable and also they have the power to drop variables which are not well expalained according to the topic under discussion.

Sinay and Nahornick (2016) sought to add literature by reviewing the teaching and learning of mathematics. It was revealed that not only are mathematical skills and confidence essential for students in the learning of mathematics but also a supportive and engaging classroom environment where learners are free to ask. Furthermore, the teaching of mathematics should include class dialogue (Math Talk) i.e. a way to have structured mathematical discussions that construct knowledge and meanings. It is hoped that instructional practices will be incorporated into classroom to improve students mathematical abilities and attitudes. To crown it all, it was concluded that the teaching and learning of mathematics should include the aspect of teaching for conceptual understanding by always having high expectations of students achievements in tests as a core of good educators. The study under scrutiny supports the current study in that both of them supports mathematical skills as essential in the learning of mathematics. By and large, the two studies strongly suggest that students should have confidence to answer questions correctly be it procedural or conceptual.

In an attempt to review literature by looking at the study done by Bouhjar et al (2017) examining students' procedural and conceptual understanding of eigenvectors and eigenvalues in the context of inquiry-oriented instruction at Florida State University. It was revealed that inquiry-oriented instructional materials and strategies help students develop more robust conceptual ways of reasoning about linear algebra. This was done by comparing two groups; where one had to use inquiry based and then the other one used other methods. It was further revealed that the learning of Linear Algebra of students usually struggle to master procedural and conceptual understanding of mathematical concepts. In this regard, mostly students struggle to coordinate algebraic with geometric interpretations. Nonetheless, the study focused on the in-depth analysis of students' reasoning of assessment questions related to eigenvectors and eigenvalues. It was further revealed that students find it challenging to make sense of the symbolic shift (for instance, $Ax = \lambda x$) to geometric. The study under enquiry focused on in-depth analysis of students reasoning of assessment of questions related to eigenvalues and eigenvectors which corresponds to the study being undertaken which looked at the factors associated with the confidence of students to answer correctly either procedural or conceptual concepts by analysing eigenvalues and eigenvectors in the extraction matrix and correlation matrix .

The study done by Danquah (2017) focused on factors that contribute to the underperformance of mathematics among middle school students. This was done by using the academic approach to find solutions that are used to improve instructions in the context of urban schools. In this study 12 middle school mathematics educators were chosen and then subjected to oral interviews which lasted for 15-20 minutes. Furthermore, it was observed that there was lack of well-equipped and quality teachers hence depriving urban city students of good achievement. In this regard, the author focused on finding solutions to low mathematics achievement from the perspective of what educators may do academically in order to boast students mathematics performance. It was revealed that teachers can use various forms of technology for instructional purposes. The teaching of mathematics may be enhanced by completing and submitting assignments online and creating communication tools. Online and face-to-face modes should be available to users throughout in the teaching of mathematics. Furthermore, it was observed that multimedia technological components should be upheld as means of instructions. Additionally, to develop a teaching plan, requires focus and rigor. Not only should teachers engage students in Project- Based Learning (PBL) instruction method when coaching mathematics but also professional development. The study under review supports my study in recommending technology use in the teaching and learning of mathematics concepts (procedural or conceptual).

A study of (25) qualified engineers from South Africa and Sweden done by Engelbrecht et al (2017) on conceptual and procedural knowledge which engineers may acquire when learnng mathematics. It was observed that the engineerng curriculum for mathematics which should be studied must underscore computational mathematics which is procedural in nature. Not only was the study based on 25 professonal engineers but also university lecturers of engineering subjects. Nevertheless the study revealed that some of the engineering students were not sure on how mathematics may assist them in their career. In this regard, the teaching of mathematics should focus on fostering conceptual understanding that includes contextualised problem solving by using patterns and mathematical relationships in the learning process. Procedural questions were more prominent as compared to conceptual. In the same way students were more confident to answer procedural questons than conceptual ones. The sample size was made up of 9 qualified engineers of which 3 where from South Africa and 6 from Sweden. Moreover, the study had 5 qualified engineers in managerial positions that is to say 2 from South Africa and also 3 from Sweden. To add on, the study had 11 university lecturers in applied engineering of which 6 where from South Africa and 5 from Sweden. Thermatic content analysis was used to analyse data qualitatively. It was revealed that there was lack of application to real life situation of the mathematical content learnt. Furthermore, most of the interviewees strongly recommended that procedural problems should be explained in details on how they may be applied in industries. Finally it was observed that conceptual problems are essential to solve more engneering problems than procedural ones. The methodological variance of the study under scrutiny and my study was that the study under review used thermatic content analysis to analys qualitative data while my study used quantitative methods to analyse data (standard indices and multivariate analysis tools).

Gilmore et al (2017) identified that good conceptual understanding is important for success in mathematics. The sample size consisted of 75 children of which 41 were male from two suburban primary schools. It was observed that children who spoke English language fluently had more advantages than those who could not speak. In this study, children conceptual understanding was measured by giving them two tasks of counting and solving arithmetic. Additionally, children also completed separate verbal and visual spatial working memory tasks. The relationship between procedural skill, conceptual understanding and working memory and mathematics achievement was investigated using linear regression model. Likewise, this meant that the relationship between children's working memory, conceptual and procedural scores in mathematics achievement test was measured. Furthermore, the three skills on children's mathematics achievement test was examined using subgroups of children within the whole sample. A hierarchical cluster analysis (using Ward's method) on children's procedural skill, conceptual understanding and working memory was employed. The study revealed that good conceptual understanding may help children overcome working memory difficulties by allowing them identify and use strategies which have lower working memory demands. The study under review used linear regression analysis to anlyse data quantitatively while my study used multivariate analysis (factor analysis, cluster analysis and multidimensional scaling). The study being undertaken opted for multivariate analysis because they analyse data closer to reality than linear regression analysis.

A review of literature by Khoule et al (2017) focused on the impact of conceptual and procedural knowledge on students mathematics anxiety. In this study, the research sought to look at remedial mathematics students who once failed mathematics examinations. However, 164 countries were involved of which 128 different native languages were used in the study. The study was on elementary algebra course which constituted of 105 remedial mathematics students from four elementary algebra sections of the mathematics syllabus. Nonetheless, a random sample of '2' elementary algebra sections were placed to stand for a conceptual treatment. Participants in all groups were not repeating elementary algebra course. To do the study, the research used an equal number of lesson plans for both procedural and conceptual (seven). However, the objective of the lesson plan was to help students gain a deeper understanding of concepts. For instance, to understand the concept of gradient. A slope of a curve is stated using a conceptual approach. This may develop clear understanding of concepts by learners. Furthermore, the teacher is supposed to make additional illustrations on how to find the gradient of the curve. The independent t-test was used and it indicated that there was no statistical evidence that the variance between the anxiety-difference of the two groups fluctuated. Finally, the study revealed that teachers must make efforts to balance their use of conceptual and procedural approaches in the teaching and learning of mathematics concepts. The study under enquiry used the independent t-test to analyse data while my study used multivariate analysis.

A review of literature done by the two scholars Mainini and Banes (2017) argued that differentiating instructions are normally used to increase conceptual understanding and engagement of university mathematics as a routine matter. It has been observed that often times teachers struggle to use differential instructions in order to meet the needs of students in their class. It was revealed that it maybe inevitable that a single classroom may be used to represent a wide range of students abilities, learning styles, strengths and needs of students. However, most of the teachers of mathematics used small groups of students to teach mathematical concepts to the learners. In this study, a sample of 28 students was picked. English language was used by learners to discuss the work which was given to be done under a specified period of time. Researchers had to put students in flexible groups using systematic and inquiry based methods. This was done to engage students in mathematics activities. Finally, it was revealed that the flexible small-groups instruction could be used to enhance both conceptual understanding and behavioural engagement of students to do mathematics activities without difficulties. The study under review used inquiry based methods to analyse data by systematically putting subjects in flexible groups while the current study used quantitative methods in form of multivariate analysis and standard indices.

It was revealed that exposing conceptual and procedural difficulties encountered by students through their experiences that assist students to master factorisation is vital. The researcher used a case study as a lens of enquiry selected 30 level 4 students, of which 5 were selected for face-to-face interviews in order to probe their conceptual and procedural difficulties based on their responses. Not only was data collected but also interpreted using authentic measurement. Notwithstanding, Naicker (2017) stipulates that, students of level 4 should understand procedures and flexibly use factorisation to solve other mathematics problems. Interviews were conducted to gain an understanding of the mathematical processes experienced by students. The study revealed that misconceptions existed in integrating mathematics concepts that is drilling mathematics procedures without attempting to provide full level of conceptual understanding. The researcher respected and protected issues of confidentiality and privancy of personal rights. The study under examination revealed that students lacks conceptual understanding which is in conformity to the current study which shows that students at UNZA had low conceptual understanding of concepts.

Robertson (2017) looked at developing valid and reliable survey scale. To do this appropriately he argued that scale development should consist of thirteen steps. Firstly, he pointed it out that researchers should decide what they want to measure. Secondly, there is need to develop the theoretical foundations. Thirdly, to generate items pool of tasks to be tested. Fourthly, to write down consciously items under investigation. Then to decide on the number of items to test the subjects. Consequently, establish content and face validity of the items selected for the study. In line with this measure, not only should the researcher decide on a scaling approach to take but also the number of response options. Then a pre-test should be undertaken before a pilot scale is developed. Arising from all the steps alluded to, data should be collected. After data had been gathered systematically, the researcher is supposed to reduce the number of items in the study. Likewise, the researcher is supposed to establish dimensionality and validity of the study. To sum up the entire scale development process establishing reliability of the scale. In order to bridge the gap in literature Ibid (2017) pointed it out that lack of a suitable measure or lack of resources in survey prompts research teams to develop suitable measurement frameworks and subsequent measuring instruments. It was argued that, efforts should be made to ensure that each item reflects the underlying constructs that every researcher is measuring. It is believed that, this may be achieved by putting in place a continually theoretical underpinning of the framework. It was observed that, a good sampling plan is inevitable in order to conduct necessary statistical analysis. In conclusion, as from the empirical data collected by the current researcher; it is stated that a good research begins with good instrument. This can be achieved if only a systematic and scientific process is followed.

Additionally, a review of literature by Saiman, Puji Wahyuningsih, and Hamdani (2017) suggest that, conceptual or procedural mathematics for engineering students at University of Samudra entails the mixing of contextual, mathematical models and understanding of mathematical notations and graphics. However, all this could be achieved when mathematics teachers improve upon their pedagogical skills in the teaching of procedural and conceptual components of mathematics. As a rule, engineering students were interviewed on the relevance of procedural and conceptual aspect of mathematics education. In response, it was revealed that mathematics education is viewed as the pillar of students to depend on when performing interpretation of mathematical concepts and procedures. However, in this study the researcher picked 25 engineering students all from three engineering departments. On the other hand, the three departments of engineering where students were picked from were mechanical, civil and industrial engineering. To add on, the researchers used a combination of convenient and snowball sampling in the collection of data while my study used simple random sampling. Finally, it was revealed that in all the three engineering departments students felt that conceptual mathematics is more important than procedural mathematics for engineering students. The study being undertaken suggested that both procedural and conceptual understanding are significant in the learning of mathematical concepts. Therefore, there is need to have a balance between procedural and conceptual concepts in the learning of mathematics.

Reilly (2017) looked at how to address misconceptions of mathematical understanding. This was done by focusing on mathematics proficiency when addressing the low level mathematical components of understanding which are procedural and conceptual understanding. The study sought to examine the consistency of equivalent attribution of two learning constructs. The two learning constructs of procedural fluency and conceptual understanding of mathematics concepts were heavily utilized in this study. On the other hand, it was observed that a mixed research approach was followed (Quantitative and Qualitative). In this regard, a social, cultural and historical constructivism approach was followed. However, cognition and constructivism were used by following a conceptual framework. To add on, a number of epistomologists were used in the study which included Jean Piaget, Lev Vygotsky and John Dewey. Nonetheless, the purpose of the study was to identify valuation teachers had towards procedural fluency and conceptual understanding of mathematics concepts. This was achieved by involving secondary teachers in the valuation of competences in either procedural or conceptual understanding of mathematical concepts. In this study purposive sampling was used and it was revealed that only honest and accurate reflections provided by participants during data collections helped the researcher to obtain viable results. It was further revealed that an amalgamation of constructionist theory and situated cognition in mathematics classroom was achieved only by humanizing mathematics teaching and learning process. The results of this study were dependent on the honest and accurate reflections provided by participants during data collections. The study under review used purposive sampling to get the data while my study used simple random sampling. Furthermore, the study under scrutiny was constructivist in nature while my study was positivist. To add on the, the epistemological variance between the study under examination and the study being undertaken was that my study used Auguste Comtes' theory, Emile Durkheim theorys' and Logical positivism while the other one used Jean Piagets' theory, Lev Vygotskys' theory and John Dewey's theory.

Romano (2017) spent 20 years teaching mathematics to students of mechanical engineering and he had to find out that mostly students focus on acquiring procedural skills of solving mathematical tasks correctly than understanding the mathematical concepts which made then to get the questions correctly. Furthermore, he reviewed that middle and high school mainly have incompetent teachers who fail to develop in the learners a spirit of mathematical thinking. If only students may develop a spirit of inquiry in their learning of mathematics concepts will they acquire the abilities of understanding mathematics fully. The study under enquiry focused on middle and high school teachers competencies in mathematics while the current study looked at first year students understanding of specific concepts in selected mathematics concepts at UNZA.

Wojcik (2017) postulates that, academic expectations of students with learning disabilities in the areas of reading and mathematics are not good. This prompted the researcher to look at the Intellectual Disability (ID) of students with learning disabilities in mathematics. To add on, the researcher was sceptical of students' basic academic abilities and also standardized students assessment process. This study was done in order to minimize the use of negative feedback towards learners. However, the purpose of the study was to show that individuals with ID can learn algebra. Additionally, new cohorts of students with ID found how to demonstrate the ability of completing algebra skills (technologies). In this respect, usually graphical calculators help a lot to do the intended mathematical manipulations. The researcher used a single-case experimental multiple-baseline across participants design to monitor the performance of six students with ID. Subsequently, a conceptual framework for algebra instruction was developed. In this regard, algebra was viewed as a tool to unlock the conceptual understanding of mathematics. The study included learning programs from Kindergarten to Grade 8. Furthermore, there are a number of empirical research publications related to algebra achievements for students with ID. Nonetheless, a multi-stage systematic search for documentary literature on algebra interventions for students with ID were employed. It was revealed that both students with conceptual understanding and procedural fluency can improve their ID. The study was piloted in the spring of 2014 at Central Virginia High School. The study explored abilities of participants of completing algebra skills. Evidence-based interventions were explored in the study. It was revealed that 3 out of 6 participants improved their algebraic skills. Finally, at the end of the study, it was further revealed that the researcher understood the importance of academic activities for individuals with ID adequately. The study under review related to the current study in that both of them focused on the need to improve algebraic skills in order to do well in procedural and conceptual concepts of mathematics.

The knowledge of concepts and procedures is imperative for competence in mathematics (Zuya, 2017). It is against this background that prospective mathematics teachers must possess good knowledge of mathematical concepts and procedures. Normally, a good understanding of mathematical concepts and procedures gives mathematics teachers confidence in the mathematics classroom. Additionally, the knowledge of concepts which involves understanding of meanings and not only recalling of rules, definitions and procedures should be upheld. Therefore, in this study, the researcher sought to find out the knowledge content of prospective mathematics teachers' of procedural and conceptual knowledge in mathematics teaching and learning. However, this was done with special emphasis on algebra. Nevertheless, the hypothesis was tested at 95% confidence level and 54 undergraduate students were randomly picked as a sample. Expert judgement was attained by involving lecturers of mathematics in the construction of a 20-item test instrument. However, the test consisted of 10 procedural and also 10 conceptual questions with the reliability of apha 0.87. The test instrument which the researcher prepared was composed of structured questions only. Nonetheless, the finding revealed that prospective mathematics teachers performed below average in conceptual tasks and above average in procedural tasks. The study under review constructed a 20-item test instrument which had 10 procedural and also 10 conceptual structured questions while my study had 6 items of which 3 were procedural and also 3 conceptual structured questions. There was a variance in the degree of precision in that the study under review had used a 95% confidence level while the current study used 99% confidence level because I wanted to be closer to reality in obtaining my sample size. The methodological similarity in the two studies were that both involved expert judgement in the construction of test instruments.

Literature of concepts and procedures concerning the knowledge of pre-service teachers in geometry was highly debated by Zuya et al (2017). In this regard, conceptual understanding of students was defined by means of justifying if a statement in mathematics is true or not. However, expertise in mathematics was sought in the construction of the test which was used as an instrument for the experiment. Generally, knowing mathematics has to do with the understanding of concepts and procedures. Additionally, competence in mathematics knowledge involves students understanding of concepts and procedures. In this study, it was revealed that the study of geometry may encourage students understanding and appreciating of the environment we live in. Consequently, this may lead to the understanding of interrelations among geometric shapes. However, the study revealed that pre-service mathematics teachers need the knowledge of geometry for them to appreciate the teaching and learning of mathematics. Ideally, the study sought to investigate conceptual and procedural knowledge of pre-service mathematics teachers in geometry. This was done by determining the extent to which pre-service mathematics teachers possess conceptual knowledge in geometry. To add on, not only did the extent to which pre-service mathematics teachers possess procedural knowledge in geometry but also to determine how the two components are related to their conceptual and procedural knowledge in geometry. However, a survey research design was implemented. Nevertheless, independent variables picked in the study were conceptual and procedural knowledge using quantitative methods. Researchers picked 28 undergraduate students enrolled to train as secondary school mathematics teachers. The test instrument prepared comprised of open-ended tasks. Cronbach alpha was use to measure the reliability of the test and its conceptual value was 0.76 and procedurally 0.81. However, results of pre-service mathematics teachers scores were determined by using pearson product moment correlation which gave ($r = 0.640$, $n = 28$). Additionally, it was revealed that pre-service mathematics teachers performed above average in conceptual knowledge test. Students had a better understanding of conceptual than procedural concepts which was in agreement with Engelbrecht et al (2005) whose results of undergraduate students of health sciences in calculus of 10 multiple choice tasks of which 5 were conceptual and 5 were procedural showed a high understanding of conceptual concepts. It was further revealed that competence in mathematics knowledge in geometry requires students to have competence of both conceptual and procedural aspects of the specific component of geometry. The study under examination focused on pre-service mathematics teachers knowledge of geometry while my study looked at the understanding of specific concepts in first year university mathematics. To add on, the study under review had the test instrument with open-ended tasks while my study was composed of structured questions.

Seeing dwindling competence of pre-service mathematics teachers in sets and functions prompted Aksu, Konyalioğlu and Kul (2018) to look at conceptual knowledge of Binary operation. This study argued that the purpose of learning mathematics should be of a balance towards learning achieving procedural and conceptual competencies in equal amounts. Additionally, it was pointed out that pre-service mathematics teachers should be familiar with basic concepts of sets, functions and numerical operations. Ibid (2018) sought to determine if pre-service mathematics teachers had knowledge on binary operations by evaluating correct or incorrect answers mathematically as they were coded. In response to the research undertaken, it was revealed that the analysis of the answers for the questions posed indicated that the concepts were not internalised and conceptual learning did not take place. Furthermore, it was noted that the methodology implored was not objective but subjective. The study used descriptive frequencies which do not give full information when analysing of results. In this regard, it was further revealed that pre-service mathematics teachers depended on conceptual explanations that uses memorization of facts rather than understanding. The study under scrutiny used methodology which was subjective while the current study used objective methodologies. To add on, the study under review focused labelled binary operations as conceptual concepts which was in conformity to my study which demonstrated that the mentioned was as it where.

Cartwright (2018) used a qualitative approach to look at the deep interpretations of results by examining the data collected in the study. Above and beyond, questionnaires and semi-structured interviews were employed. This was inevitable in order to explore teachers conceptions of mathematical fuency. It was observed that interviews recorded not only captured rich details of experiences but also enabled the researcher to marry information obtained to the written conversations earlier conducted prior to the interview. Thermatic analysis was done after the audio were recorded and later transcribed. This strategy helped to determine efficient and flexible pedagogical approaches teachers used in the teaching of mathematics concepts. It was revealed that teachers were able to teach from known to unknown. Furthermore, the study revealed that teachers conceptions of mathematics fluency relates to procedures were mathematics fluency was categorised as strategical competence, conceptual understanding and adaptive reasoning. Mathematics fluency from the teachers perspective meant that the teacher should be able to apply and demonstrate properly mathematics concepts to learners for smooth understanding of the materials. The study under enquiry used a qualitative approach while the current study used a quantitative. The methodological variance was that the study under scrutiny used questionnaires and semi-structured interviews in order to get data while my study used a test and a questionnaire at the end. This was so to determine procedural and conceptual understanding and confidence exhibited by students in order for them to get the question correctly.

Literature reviewed by Kadijevich (2018) points out the teaching of mathematics in relation to procedural and conceptual knowledge of the subject matter. It was revealed that one of the important challenge is concerned with developing and relating conceptual and procedural mathematical knowledge. However, the study sought to find means and ways of promoting relations between procedural and conceptual knowledge. To achieve this, the researcher made an assumption that one type of knowledge is based upon the other. Additionally, it was found out that the relation between the two types of knowledge may also be affected by the leaner's P-C profile. The study under inspection was similar to my study in that the two studies advocated for a balance of knowledge for procedural and conceptual understanding of mathematics concepts.

Ngesu and Simotwo (2018) postulates that, ‘there are a number of factors that influence pupils performance in mathematics at primary level of Turkana township in Kenya’. Nonetheless, the study adopted a sample size of 266 respondents consisting of headteachers, teachers and learners inclusive. Stratified and purposive sampling were used to come up with the sample. Likewise, questionnaire was administered to get the opinions concerning the thinking of stake-holders on the performance in mathematics by learners. Descriptive survey design were used to interprete the data. It was revealed that there was no significant relationship between teachers qualifications and pupils performance. Parents not assisting learners in home work contribute in some way to the lower performance in mathematics concepts. Furthermore, it was revealed that, teaching resources plays a critical role in enhancing the performance of learners in mathematics. Not only did the researcher point out that learners attitude affects their performance in mathematics but also lack of assistance by parents to do home work. In this regard, it was recommended that school libraries should stock prescribed books covering the syllabus and also parents should supplement by buying books for children. The methodological variance between the two studies were that the study under enquiry picked stratified and purposive sampling to get the sample while my study used simple random sampling.

Novaliyosi (2018) looked at the development of instruments to measure students mathematical logical thinking ability in Kapita Selekta. Not only was it synonymous for this research to develop a test instrument but also to test the ability of the instrument before using it. Based on the assumptions by the researcher it was observed that several topics were looked at in order to ascertain opportunities, combinations, logic and mathematical measurements of constructs. In line with this measure, the results achieved during this study were in conformity with the validation of the instruments by experts and expert panelists.The panel of experts consisted of 12 professors and 8 teachers. It was observed that among a number of constructs the study revealed that there were a number of correlations between and among the number of constructs like probability, mathematical logic and combinatorics. In conclusion it maybe suggested that the instrument developed had 5 packages of logical thinking ability. The study under analysis developed a test instrument then tested it to ascertain its abilities before starting to use it. This approach corresponds to my study which also developed a test instrument and tested it using the pilot study and also using the same cohort to tset the instrument.

Recent literature shows that undergraduate students of mathematics have difficulties of understanding mathematics concepts. It is evident that, Chin and Pierce (2019) looked at undergraduate mathematics instructors who often report that students make careless errors on key mathematical ideas and strategies. Nonetheless, in this study university students' conceptions of mathematical symbols and expressions were highly explored. To do this, there was need to determine the complexity in comprehending of mathematical symbols that leads students to avoid taking mathematics courses because of the unsubstantiated belief that mathematics is difficult. To add on, the research sought to identify students problematic conceptions related to mathematical symbols in their undergraduate mathematics subjects. On the whole, the significance of mathematical domains in the teaching and learning of university mathematics were highly explored. The findings of the study indicated that there is a pattern of students' errors in making sense of mathematical symbols and this pattern is based on individual conceptions that are generally developed from prior learning of mathematical concepts and symbols. The results of the study were based on the responses collected from 125 teacher educators and experienced teachers. Not only are students supposed to follow the approved mathematical register by mathematics community but should be able to understand the materiality, syntax and meaning of the symbols without problems. The study had a number of theoretical framework. This was so that one framework may complement the other one. Students errors were explored in the understanding of mathematical symbols such as $f^{-1}(x)\ as\ \frac{1}{f(x)}$ which is not true. Also, $\sin^{-1}(x)\ as\ \frac{1}{\sin(x)}$ which is also not true. This was so because of students memorization of the exponent rule of $x^{-1} = \frac{1}{x}$ which they end up misusing even in aspects where it is not applicable. Furthermore, the study identified a number of students problematic conceptions related to symbol use at undergraduate level in mathematics learning. The study under review observed that students made a number of mistakes on the use of symbols or mathematical resigister. This was also observed by the current study being undertaken were a number of students made errors on how to represent certain key mathematical ideas and strategies which resulted in students performing below average in tests.

The study sought to explore cognitive processes in developing understanding of chain rule and implicit differentiation and related rates. Jeppson (2019) looked at how well it may be done to improve explicitly the use of (Nested Multivariation Reasoning) NMR in the understanding of multiplicative nature of chain rule. The study revealed that students developed meanings for chain rule and implicit differentiation through meaningful contexts. This was achieved through the first draft of the study which consisted of the pilot study of learning activities and also questions which were intended to help student development of multiplicative nature of the chain rule. The study under investigation corresponded with the current study in that both of them focused on how to improve the understanding of mathematics concepts at university level.

Malambo et al., (2019) investigated mathematics student teachers understanding of key school concepts after completing advanced mathematics courses at UNZA. This qualitative case study consisted of 22 final year mathematics student teachers who wrote an open-ended structured test to assess competences on functions and trigonometry. Expert judgement was sought to ascertain the suitability of the test. However, the study accessed student teachers on how they can explain and justify reasoning on unpacking of concepts. This study provides evidence that studying advanced mathematics does not make a student teacher capable of teaching ordinary mathematics without difficult. Furthermore, the study revealed that final year students may have misconceptions regarding key aspects of functions and non-functions and inverse functions. Finally, the study revealed that mathematics student teachers may possess procedural knowledge while lacking conceptual knowledge of mathematics which is required for them to teach effectively. The study under examination correlated to my study in that it looked at 22 final year students competencies in ordinary mathematics while my study focused on 378 first year students at the same university. The study under scrutiny generated an open-ended structured test on functions and trigonometry while the current study focused on six different specific concepts in a variety of mathematics topics. The methodological similarity was that both of the studies involved expert judgement of UNZA lecturers of first year mathematics in the construction of the test. However, the methodological variance was that the study under inspection used a qualitative case study while my study used a quantitative case study.

Recently, Mutawah et al (2019) reviewed literature to try to find ways and means of improving conceptual understanding and problem-solving skills in order to reach the desirable stage of mathematical proficiency. This was done by looking at the impact of conceptual understanding, procedural knowledge and problem-solving skills in mathematics of high school graduates. As a rule, the study consisted of 350 students of which 50 were males and 300 females. Furthermore, a quantitative approach was used which comprised of five domain questions in mathematics (numbers and operations, algebra, geometry, measurements, data analysis and probability). The test paper had 60 questions of which 20 were conceptual, 20 procedural and also 20 problem-solving oriented. It was revealed that most of the students performed well in procedural knowledge and poor in both conceptual and problem-solving questions. Furthermore, it was revealed that the teaching system in Bahrain emphasized more on how to do and what to do instead of why and when to do. These approaches cumulated in learners achieving more on procedural component of mathematics than conceptual and problem-solving. Based on the results of the study it may be stated that there was a positive correlation of improved achievement in conceptual items leads to improved attainment in procedural and problem solving using pearsons correlation. The study under review was insightful since it focused on how to develop mathematical proficiency (five strands of mathematical understanding) in the learners of high school graduates it was different from my study which only considered two components of mathematical proficiency at the university (procedural and conceptual).

VanScoy (2019) explores the point-of-need Information Literacy Instruction (ILI) by adopting ACRL Framework through the of conceptual and procedural knowledge. It was observed that ACRL Framework for Information Literacy for Higher Education (2015) provides as a standard to the point-of-need for ILI. The study further sought to show how conceptual and procedural knowledge manifest in point-of-need ILI. This was done by gathering 1260 transcripts from text-based and online reference transactions done by Librarians. Webpage descriptive coding was done by the author and four graduate students on some selected school libraries, academic and public libraries. Each transcript was read and categorized by one team member which was reviewed by another team after analyzing the transcript thermatically. This was done after initial coding and using the transcript in the study. The differences were discussed by different team members until the agreement was reached. Of the 164 transcripts in the study, where ILI occurred, it was found that 26 contained conceptual instruction, 100 had procedural instruction and 38 required both. The study revealed that the point-of-need of ILI tends not to be understood as a form of instruction with its own particular needs. Therefore, future research should examine users' challenges and how service provides to ILI. The epistemological variance between the study under inspection and my study was that this study looked at procedural and conceptual knowledge by using descriptive coding on school mathematics. This meant that the study under review used thermatics content analysis which is qualitative in nature while my study looked at procedural and conceptual understanding of specific concepts in selected mathematics concepts. Above and beyond current study undertaken used standard indices and multivariate analysis tools to analyse the quantitative data.

Aytekin and Sahiner (2020) used a three-tiered teaching experiment as a method to ascertain preservice teachers competence towards the teaching of division of fractions. The experiment was meant to improve the planning of activities developed by preservice teachers of mathematics. Reflective thinking was to be developed by preservice teachers when collaborating themselves to the Central Anatolia Region University which was picked as a sample. Consequently, the main implementation of the study was framed according to the data obtained from a pilot study. This qualitative study was done with the use of interviews and video recording of lessons done by preservice teachers of mathematics. Furthermore, not only was one-to-one interviews employed but also group interviews. Each preservice mathematics teacher prepared a lesson plan which was to be discussed in a group for improvement. In this three-tiered experiment, preservice teachers of mathematics examined the teaching of procedures and conceptual concepts of division of fractions. The study revealed that some students had high level of procedural knowledge and a low level of conceptual knowledge. It was recommended that going forward procedural and conceptual knowledge should be developed simultaneously. The study under investigation used both one-to-one interviews and group interviews to examine procedural and conceptual competencies of learning mathematics while my study focused on constructing of a test in order to use it in the measuring of procedural and conceptual understanding of mathematics concepts.

Literature reviewed by (Malambo, 2020) contends that the understanding of university mathematics by student teachers on the tangent function is not upto date. Likewise, this attest to the uniqueness of the tangent function which has a period of 180° as compared to the sine and cosine function whose period is 360°. The argument is that pre-service mathematics teachers who have studied advanced mathematics in a university may not necessarily have an in-depth understanding of mathematics concepts taught at school level. In this qualitative case study where 22 pre-service teachers were picked at UNZA after studying advanced mathematics. It was revealed that student teachers demonstrated a discrepancy in the understanding of mathematics concepts (tangent function). Notwithstanding, pre-service teachers failed to explain conceptual difference of the tangent function from an algebraic representation to a graphical form. It was revealed that a good number of students failed to plot the tangent function meaning that they failed to interpret symbolic tangent function into a graphical representation. When student teachers were asked to justify what they had drawn they failed to comprehensibly defend what they had drawn respect to the symbolic form of the tangent function. Based on the analysis of the collected data, it was concluded that pre-service teachers lacked conceptual understanding of concepts even though they had a bit of procedural understanding since there were able to justify symbolic form of the tangent function but failed to elucidate graphical form explicitly. Even though the study under review focused on first year students at UNZA just like my study there were different in methodologies used and the scope of the study. The study under equiry used qualitative analysis tools while my study used quantitative.

3.11.1 Lessons learnt from literature review related to my study during the era (2016-2020)

Prospective teachers of mathematics had contextual difficulties of understanding mathematics, this meant that they were supposed to move from computational focus to an understanding perspective approach. Likewise, it was observed that students lacked rigor in conceptual understanding of mathematics concepts. This prompted researchers to use differentiating instructions in order to enhance increased conceptual understanding of mathematics concepts. There is need to upscale the knowhow of mathematical symbolism at undergraduate level because a good numbers of students were making mistakes as they have memorised rules instead of understanding them. When we look at the exponential rule of $x^{-1} = \frac{1}{x}$ it is correct but most students interprate it wrongly even where it is not applied. For instance, the exponential rule does not apply for $f^{-1}(x)$ as $\frac{1}{f(x)}$ which is not true and $\sin^{-1}(x)$ as $\frac{1}{\sin(x)}$ which is also not true. This is because of the memorization of the exponential rule by students and they fail to notice that it does not make sense in other situations. However, it was observed that students had more confidence handling procedural tasks as compared to conceptual. Cluster analysis was used using (Wards methods) to analyse conceptual understanding of mathematics concepts and it was revealed that students had more misconceptions in conceptual tasks as compared to procedural. It was revealed without hesitation that a lot of elusiveness still existed in the methodologies used to measure mathematical understanding. Hence the need of developing new instruments to measure mathematics understanding (procedural and conceptual).

3.12 Conceptual and Procedural Understanding of Mathematics in the 21st Century

Adeleke (2007) argues that, 'in the twenty-first century, developing nations have come to realize the increasing role of science and technology with strong mathematical content in sustainable national development'. One way by which this can be realized is to teach mathematical problem solving skills at the core of science and technological education early enough in the education of the students. Bautista (2013) argues that, 'students' procedural fluency in problem solving must complement their mathematical-scientific explanations in their decision making. Even though memory plays an important role in the understanding of mathematics concepts as supported by Yusof and Tall (1996) who suggested that, it is what is remembered and how it is remembered that distinguishes those who understand from those who do not. Likewise, Bautista (2013) supports Rusell (2000) by suggesting that, 'complementary forces between conceptual understanding and procedural fluency will predict the success in solving computational problems correctly.

Standard indices, factor analysis, multidimensional scaling and cluster analysis were used in answering research objectives in this study. Unlike previously Pearson-r correlation at 0.01 level of significance was used which depicted that there was a high significant relationship between procedural fluency and conceptual understanding of 0.992, and the p-value of less than 0.001 as pointed out by Samuelsson (2010) and Bautista (2013). In the current study, multivariate analysis was used which has a tendency to condense massive data and show the in-depth correlation between and among the variables. Additionally, it was observed that previously there was a lot of elusiveness in the methodology used in order to measure procedural and conceptual understanding. Fatqurhohman (2016) prepared a problems test (multiple questions) which were given to all participants (20 students) who voluntarily agreed to participate in solving a given problem. But, Kothari and Garg (1985) argued that, 'convenience sampling at times may give very biased results and it may work in qualitative research where the desire happens to develop hypothesis rather than to generalize to larger populations. It was observed that Fatqurhohman (2016) used both quantitative and qualitative approach which did not align with picking of the sample.

3.13 Chapter summary

In summary, after the review of literature on understanding of concepts in mathematics. It was observed that a good number of studies used elusive methodologies to measure mathematical understanding of procedural and conceptual concepts. For instance, some studies used writing to learn mathematics and also answering questions correctly as methods to measure mathematics understanding. These methods proved not to be valid and reliable because they were subjective in nature and compromised the results obtained of the studies. The review of literature indicated that few scholars used new methodologies such us computer software to generate and analyse mathematical understanding of concepts. Unfortunately as from literature review, it was revealed that some researchers were still using unreliable methodologies which are unscientific in nature to be used to measure mathematical understanding such as looking only at the generic performance of students in mathematics as a measure of mathematics understanding. Additionally, some of the scholars made their students to use writing to learn mathematics as a way of measuring students understanding which were not valid and reliable. Generally, others used the correct answers given by learners as a method to determine students understanding of concepts. In addition, some were concerned with the errors committed by learners as a method of measuring students understanding of concepts. Even though most of the studies reviewed used unreliable procedures and methods which were not valid they informed by study on the number of key issues. The literature revealed that most of the students understood more procedural concepts as compared to conceptual. Above and beyond, literature informed my study on the way students viewed mathematics as thought-provoking which led to a number of them lose focus because of fear. After reviewing literature it was revealed that most of the studies concluded that students performance on procedural aspects was good as compared to conceptual. My study wish to conclude that there is a definite need to have a balance between procedural and conceptual understanding of mathematics concepts in the learning processs be it at primary, secondary or higher learning institutions. The next chapter looks at the methodology used in the current study.

CHAPTER FOUR
METHODOLOGY

4.1 Introduction

This chapter presents the research methodology used in carrying out this study. The chapter outlines the research paradigm, research approach and research design of the study. Then the area of study, target population, sample and sampling procedures are deliberated. Finally the chapter discusses the research instruments, reliability of instruments, the process of developing and designing instruments for measuring mathematics understanding, data analysis, ethical considerations then the summary concludes the chapter.

4.2 Research Paradigm

A research paradigm is an approach of conducting research which is based on verified procedures (Mouton, 2001). There are four major research strategies or paradigms which are mostly used in research and these are; inductive, deductive, abductive and retroductive. These four strategies of generating new scientific knowledge provide different ways of answering research questions by specifying the research procedures from the beginning to the end of the study. In this study, inductive strategy was used which starts with data collection, followed by data analysis and then development of generalizations with further testing of results. Furthermore, procedures were systematic and objective in order to produce results which maybe generalized as true results of the study and accepted by the general population as authentic evidence of results of the current study. By and large, inductive paradigm is committed to the method of objective observation of research which involves capturing of facts as objective as possible.

It is believed that from research paradigms we have three branches of research philosophies which are outlined as; ontology, epistemology and methodology. Moon and Blackman (2014) suggested that,'understanding philosophy is important because social science research can only be meaningfully interpreted when there is clarity about the decisions that were taken and affect the research outcomes'. The ontological assumptions used in the study were in conformity to (Ahmed, 2008) who suggested that as a researcher it is weighty to know the nature of reality in order to find out what can now be known as true knowledge to be contributed to the body of study. In this study all the three branches of philosophy were utilised. The study was science based and it used philosophies that provides the general principles of theoretical thinking, a method of cognition, perspective and self-awareness, all of which are used to obtain knowledge of reality and to design, conduct, analyse and interpret research and its outcomes (Moon and Blackman, 2014).

It is widely believed that ontological assumption of the study comes first. However, the current study analysed all the five types of ontology (Naïve realism, structural realism, critical realism, bounded relativism and relativism) and opted for naïve realism which is sometimes called realist ontology. However, Naïve realism (realist ontology) was used by (Ahmed, 2008) which corresponds to the current study which opted to use a number of research methods in the investigations and interpretations of results. Hence, the study sought to investigate on understanding of mathematics concepts by first year UNZA students. This helped in the study to develop and design standard indices to use in the measuring of procedural and conceptual concepts of mathematics. To add on, realist ontology (naïve realism) suited the objectives of the study which strived to make sure that reality was understood concerning the understanding of mathematics concepts by first year students at UNZA. It was revealed that is was possible to interpret results by incorporating appropriate methods in the measuring of mathematical understanding. Hence, in this regard naïve realism helped to come up with apposite methods in the study. Subsequently, the hallmark of the study was to use a case study as a methodology which focused on the understanding of first year UNZA students on procedural and concepts of mathematics.

It has been revealed that there are three main categories of epistemology (objectivism, constructionism and subjectivism). Epistemological stance used in the study was objectivism because it corresponded to the objectives of the study which sought to interpret the meaning that exists within the objects under investigation. In this paradigm we use science to come up with the truth. The current study sought to show how it conducted research using scientific means and how it come up with final results of the study which had to be adequate and legitimate to be appreciated by scholars in the same field. Epistemlogical helped the researcher to come up with three theories which were used in the study. The utopian theories which covered the study were positivist in nature because they conformed to the scope of the study. The theories which underpinned the study were Auguste Comte's theory, Emile Durkheim's theory and Logical positivism and have been elaborated in chapter two and theoretical frameworks. Consequently, the study used positivists methodologies which advocate for empiricism to authenticate the collected data. Therefore, the paradigm upon which this study was based on was positivism. For this worldview, knowledge is based on natural phenomena and also shows properties and relationships.

Subsequently, the studies theoretical perspective or methological way was positivist in nature which allowed the researcher to use multiple scientific methods in the analysis and interpretation of results. Therefore, information derived from sensory experience, interpreted through reason and logic, forms the exclusive source of all certain knowledge (Amory, 1999). Positivism holds that valid knowledge or truth is found only in a scientific way. Giddens (1974) argues that, 'Verified data received from the senses are known as empirical evidence; thus positivism is based on empiricism. The methodology in the study was in conformity to (Ahmed, 2008) who advocates for positivism as the theoretical perspective which underpins the current study. Positivism asserts that all authentic knowledge allows verification and assumes that only credible and valid knowledge is scientific. The study used observable means as ways of obtaining data and analysing it in order to obtain valid facts and rejects speculations. Hence, positivism was used since it is linked to empirical science. The study opted for scientific method because of the scholars in this orientation believes observations and practical work with data could be verified by authentic methods. Since, all the methods are imperfect by nature, the current study sought to develop and design valid methods to use in the measuring of procedural and conceptual understanding of mathematics concepts. This meant that the current study was to develop and adapt reliable measuring instruments (standard indices) to be used to measure either procedural or conceptual concepts in mathematics.

Positivism adheres to the view that only factual knowledge gained through observation, including measurement is trustworthy. The researcher in the current study solely used data which was collected from 378 subjects and interpretated in an objective way. This meant that bias was avoided because of the procedures made the researcher to remain objective throughout data interpretation. Additionally, the process was not subjective to allow the researcher to know the subjects under discussion. The observations were quantified which led to statistical analysis using descriptive and multivariate analysis (factor analysis, multidimensional scaling and cluster analysis). The study opted for the positivist approach because it suited the study as it advocates for objective measuring of constructs using scientific and systematic ways of quantification of data in order to enhance precision in the description of parameters.

The study focused on developing and adapting instruments to use for measuring constructs from the test. This was achived by measuring procedural and conceptual understanding of mathematics concepts at first year level at UNZA with the help of standard indices as well as multivariate analysis (factor analysis, multi dimensional scaling and cluster analysis). This idea was supported by Fatqurhohman (2016) who stated that analysis of quantitative data obtained in his study showed the percentage of each aspect of the transition of concepts from either procedural or conceptual or vice versa was valid.

4.3 Research Approach and Design

A case study was used because it uses several methods to obtain data so that a wide variety of information is sought to come up with new authentic solution to the current problem under investigation. The research followed a case study design which was composed of first year students at UNZA in the School of Natural Sciencies. Specifically, this involved understanding of students conceptual and procedural concepts of first year students at UNZA. Quantitative research is a means for testing objective theories by examining the relationship among variables. These variables, in turn, can be measured, typically with instruments, so that numbered data can be analysed using statistical procedures (Creswell, 2009). Cauvery et al, (2008) argued that, ‘the first essential quality of a successful research worker is that he must possess a scientific frame of mind, and have the determination and ability to get the naked facts and not to be influenced by one’s own wishes’.

The research approach in this study was composed of a plan of procedures for conducting the research that spanned the steps from broad assumptions to detailed methods of data collection, analysis, and interpretation. For instance, Aliyu et al (2014) suggested that,

> The positivist paradigm emphasizes that genuine, real and factual happenings could be studied and observed scientifically and empirically and could as well be elucidated by way of lucid and rational investigation and analysis. The decisive factor for assessing and appraising the soundness and validity of a systematic scientific and logical theory is whether a researcher’s facts view point (i.e., theory-based on guesses and hunches) are reliable consistent and dependable by means of the knowledge researchers are capable to achieve by means of their senses.(p.83).

This decision involved the philosophical characteristics of the study such as; procedures of inquiry (called research designs); and specific research methods of data collection, analysis, and interpretation. Furthermore, the selection of a research approach was based on the nature of the research problem or issues being addressed, research objectives and research questions of the study the researchers' personal experiences, and the audiences for the study. Thus, in this study, the research approach, research designs, and research methods were the three main terms that represented how the research was conducted from inception up to the conclusion of the study.

However, a test was conducted were it was administered to the selected sample of 378 students out of the 1 500 registered first year. Each test paper had a questionnaire for the student to indicate how confident they were to answer the question correctly be it procedural or conceptual. The test was prepared in such a way that the questions were alternating procedural followed by conceptual upto the end. The test was marked using two marking rubrics; one was prepared by the researcher then the other was done by lecturers of first year mathematics at UNZA. This allowed the researcher to compare and construct the two marking scheme in order to come up with one marking scheme which was well consolidated by the researcher, supervisors and also the three lecturers out of the five who where teaching first year students mathematics in the School of Natural Sciences.

The results obtained from the test were analysed using standard indices. This helped the researcher to determine procedural understanding and also conceptual understanding. May reference be made to the next chapter to appreciate how standard indices were utilized to measure procedural and conceptual understanding of mathematics concepts. To add on, the results from the questionnaire were analysed using factor analysis, multidimensional scaling and cluster analysis. The researcher picked three multivariate analysis tools so that the shortfall of one component maybe uplifted by the strength of the other. However, the software XL STRATA with EXCEL helped in the analysis of data using mulitivariate analysis. It was observed that when using factor analysis out of the six factors only two factors were highly explained which were both procedural in nature. These factors composed of question one and three and they measured up to 43.6% of the total. The rest of the factors were dropped off because they did not show high correlation. However, it was observed that the same result was obtained after using multidimensional scaling and cluster anlysis. Procedural and conceptual understanding and confidence of understanding were measured using standard indices, factor analysis, multidimensional scaling and cluster analysis.

Furthermore, in this study, among other significant findings there were also testing of objectives of theories by examining the relationship among and between variables. These variables, in turn, were measured, typically on instruments, so that numbered data were analysed using statistical procedures. Kothari (1985) stipulates that ‘the formidable problem that follows the task of defining the research problem is the preparation of the design of the research project, popularly known as the ‘‘research design’’(p.31). Additionally, Msabila and Nalaila (2013) defines a research design as a plan on how a study will be conducted or a comprehensive outline on how an investigation will take place (p.27). Not only was a case study design selected but also was explained why it was opted for a study to be purely quantitative type. However, at the beginning of the research preliminary considerations of the three choices of research design were made. It was only that a case study design suited the research and combined well with the research questions and objectives.

Taylor (2006) in agreement with Kothari (1985) further asserts that the design which minimises bias and maximises the reliability of the data collected and analysed is a good design. Bias may enter the research design process as a result of the researcher’s preconceptions or subject behaviours. Needless to state that, the researcher’s preconceptions may include beliefs, stereotypes, prejudices and cultural aspects. Since either the researcher or the subjects may behave intentionally or unintentionally in such a matter that may distort the study findings.

4.4 Area of Study

The study was conducted at the University of Zambia in Lusaka Province of Zambia. The University of Zambia was selected since it is the biggest institution of learning in the country with the largest number of students in mathematics. It also had the longest history of offering mathematics courses. In addition, UNZA was highly accessible from various parts of the country since it is centrally located.

4.5 Target Population

The word population is used to denote the aggregate from which the sample is chosen (Cochran, 2015:5). Target population only involved first year students at the University of Zambia who were taking mathematics in the School of Natural Sciences (SNS). At the time of research, the total number of registered first year students who took mathematics was 1 500.

4.6 Sample

There are many formulae used to calculate sample size for any study. This study, the formular given below was used to determine the sample size because it suited the procedures for obtaining the sample in this particular study.

$$n = \left(\frac{Z\sigma}{d}\right)^2$$

Where n = Sample size

z = Value at a specified level of confidence or desired degree of precision.

s or σ = Standard deviation of the population

d = Difference between population mean and sample mean

The steps in computing the sample from the above formulae were as follows:

i. Select the desired degree of precision. i.e. specified level of confidence and designate it as small z (at 1% level of significance or 99% confidence level, the value is 2.58 and at 5% level of significance or 95% confidence level 1.96). In the current study the desired degree of precision picked was at 1% significant level because the researcher wanted to be closer to reality and it suited the scope of my study.
ii. Multiply the z selected in step 1 by the standard deviation of the universe which may be assumed.
iii. Divide the product or the preceding step by the standard error of mean or difference between population and sample mean, squared the resultant quotient. The result is the size of the sample required.

In this study, to determine the sample size, the researcher worked out first the standard deviation from the sample of 378 and obtained the value of $\sigma = 22.7$, from the population of 1 500 students the researcher worked work the mean of students to have the value of; mean = 24, additionally, from the sample of 378 the researcher calculated the sample mean = 21 and decided that the results should be closer to reality and opted for confidence level of 99% as the desired degree of precision which was z = 2.576.

Solution

$$\boldsymbol{n} = \left(\frac{\boldsymbol{z\sigma}}{\boldsymbol{d}}\right)^{\mathbf{2}}$$

$\sigma = 22.7, d = 24 - 21, z = 2.576$ (at 99% confidence level of significance, the value of z is 2.576).

Substituting the values yields:

$$n = \left(\frac{2.576 \times 22.7}{3}\right)^2$$

$$n = \left(\frac{58.4752}{3}\right)^2$$

$$n = (19.4917)^2$$

$$n \approx 379.927$$

$$\therefore n = 380$$

Hence, the sample size of the study was 380 students. The number was generated by the calculations above. The lottery method was used to randomly select the elements of the sample, i.e. the students to measure.

At the time of research, UNZA mathematics department had five lecturers teaching mathematics at first year level. The intention was to include all the five lecturers in the study undertaken. This was for lecturers to ascertain the level of difficulties associated to either procedural or conceptual questions. However, it was observed that at the time of research only three lecturers were available as the other two were engaged with other duties.

Table 4. 1 : Lottery of 380 subjects picked from 1 500 of the entire population

1	114	241	343	451	595	718	833	983	1096	1219	1357	1482
4	123	242	344	458	596	721	834	985	1099	1220	1359	1483
17	127	243	346	464	608	724	844	987	1101	1222	1360	1484
19	152	250	351	474	612	726	845	989	1103	1224	1365	1488
20	156	257	355	477	614	727	852	990	1104	1236	1369	1491
21	157	258	356	479	617	730	854	993	1108	1240	1373	1492
24	164	262	359	482	618	735	856	1000	1111	1242	1375	1496
30	166	264	360	484	623	747	864	1006	1123	1245	1376	1497
32	171	267	361	485	626	749	880	1012	1125	1248	1381	
33	174	270	362	492	628	752	886	1014	1130	1252	1386	
36	179	271	365	497	630	753	888	1020	1133	1258	1388	
48	181	273	367	515	635	759	894	1022	1134	1262	1389	
49	184	277	373	516	638	762	896	1023	1142	1268	1395	
50	189	278	381	520	642	766	897	1027	1149	1274	1396	
51	190	279	390	525	646	772	902	1036	1151	1279	1401	
52	203	282	397	530	654	777	908	1038	1155	1282	1402	
53	207	288	399	537	657	785	917	1041	1159	1283	1404	

58	209	290	409	539	664	786	927	1049	1160	1290	1405	
63	210	292	415	543	669	787	929	1050	1177	1292	1407	
66	213	296	417	544	672	789	930	1054	1178	1297	1410	
69	215	297	421	547	677	792	934	1055	1179	1298	1429	
74	216	299	423	553	681	794	935	1057	1181	1299	1431	
76	220	304	425	556	682	797	937	1061	1184	1300	1434	
79	221	306	426	557	696	800	941	1068	1189	1311	1437	
84	225	308	429	560	701	801	949	1070	1191	1321	1443	
87	226	309	431	565	705	807	953	1074	1195	1327	1444	
98	229	313	432	567	706	810	957	1082	1207	1328	1446	
105	231	316	433	569	708	816	960	1085	1210	1343	1453	
106	232	318	444	576	711	821	961	1087	1214	1346	1455	
107	234	319	447	580	713	827	967	1090	1216	1350	1460	
109	235	333	448	581	716	828	978	1093	1218	1355	1470	

4.7 Study Sample and Sampling Procedure

The researchers' target population was drawn from all the first year students at the University of Zambia doing mathematics. Simple random sampling was employed so that each subject had equal chance of being selected as supported by Khoule et al (2017) who also used random sample in his study where he compared conceptual and procedural teaching. However, a random sample can be selected by using a table of random numbers. Nevertheless, the advantage of simple random sampling is that it is simple and easy to apply when small populations are involved. However, because every person or item in a population has to be listed before the corresponding random numbers can be read, this method is very cumbersome to use for large populations and cannot be used if no list of the population of items is available. It can also be very time consuming to try and locate every person included in the sample. There is also a possibility that some of the persons in the sample cannot be contacted at all. Therefore, in this study, the researcher sampled 380 subjects to participate in the study using a lottery. The researcher, commanded on internet to generate a lottery from I to 1 500 picking only 380 subjects. Mostly, when picking the sample from the population the researcher should pick either 15 to 30 percent of the population. In the current study the sample picked was 25.3% of the total population. This meant that the sample was highly representative of the entire population. In some other instances the sample is supposed to represent about a quarter of the whole population. Nevertheless, after considering all aspects the researcher picked 380 subjects which was closer to a quarter of the entire population meaning that the sample was presentative of the entire population of the study.

4.8 Justification for using Sampling in the Study

Sampling methods were used in the study because they suited the research. To add on, sampling procedure was probabilistic which helps to generalise the results. Furthermore, sampling was done to make the results of the research have a worthwhile meaning as allulled to by Gupta (2010). Sampling is cheaper in data collection since only a small fraction of the population is selected and also the data collected is analysed quickly than in a census. The current study showed that since the population was definite and data were available then scientific methods were to be used in the study. Furthermore, in sampling, following-up, non-response are much easier to follow than in a census.

4.9 Validity of Instruments

To enhance content and face validity, the administered test was preceded by document analysis, piloting, and expert judgement by UNZA lecturers of mathematics. This was supported by Zakaria et al (2010) who argued that, instruments were validated by two expert teachers and one lecturer from the department of mathematics in his study where he looked at the conceptual knowledge and mathematics achievement of students in tests. Additionally, Engelbrecht (2015) stipulated that, for construct validity the test was thoroughly and independently scrutinized by other colleagues to get unbiased view of the levels of procedural or conceptual knowledge needed to complete each item successfully. Bisson et al (2013) argues that, comparative judgemnt performs valid and reliable forms in a variety of contexts. To add on, Audrey et al (2013) suggested, a full assessment of the model validility which included an assessment of what is neglected in the model as well as the accuracy of the representation as supported by (Theofanids & Fountouki, 2019) who picked 30 variable in the study and the scores measured actually what it was supposed to measure meaning it was valid. A measuring framework combines theory and data to describe a condition necessary to achieve an objective (Robertson, 2017). This prompted the researcher to compare constraints properly before the instruments were used for the study. Noteworthy, the constraints which were invalid were discarded from the study instrument. Hence, for the study to be considered valid, it must measure what it intends to measure (Mouton, 2001).

4.10 Reliability of Instruments

The current study used Cronbach's Alpha reliability test and it was supported by Zakaria et al (2010) who also made the instruments reliable by conducting Cronbach's Alpha reliability procedure. In this study a test which was constructed using all the procedures for test construction was based on binomial expansions, sets, partial fractions and polynomials, functions, trigonometry and complex numbers. The test consists of 6 items, 3 items measuring procedural understanding and also 3 items measuring conceptual understanding. Of the six items listed; binomial expansions, partial fractions and polynomials and trigonometric identities were defined to be procedural in nature while sets, functions and complex numbers were defined to be conceptual. The study was supported by Egodawatte and Stoilescu (2015) who argued that, the first phase of the study involved the standardization of the test instrument. Nevertheless, the test was given to UNZA mathematics lecturers for validation. For the instrument to be reliable an alpha value must be in the range of 0.65 to 0.95 (Ghazali, Nor Hasnida Che and Zakaria Effandi, 2011). The reliability coefficient for the second trial was 0.88 (Egodawatte and Stoilescu, 2015). In response to the above, Mohajan (2017) argued that reliability and validity are the two most important and fundamental features in the evaluation of measurement instrument for a good research. Hence, the aforesaid, ushered in opportunities to increase confidence levels exihibited by students to solve specific mathematical concepts at first year university level. Hence, if given measures are implemented by another researcher, then the study may replicate similar results meaning that the study is accurately reliable.

4.10.1 Internal Consistency

The efficacy of measuring internal consistency largely depends on correlation of input variabbles. In line with the aforesaid statement, (Nik Azis and Tapsir, 2013) argued that factor analysis and Cronbach alpha were in their study to identify items with high structure correlation. Aydos (2015) used Cronbach's alpha in order to determine internal consistency analysis methods which was in conformity with the required range of 0.65 to 0.95 the results were in agreement with (Theofanidis & Fountouki, 2019) who stipulated that, for a measurement instrument to be reliable, the reliability coefficient (Cronbach alpha) must be above 0.70.

4.10.2 Cronbach Alpha

Where N equals the number of items $\sum \sigma^2 \, yi$ equals the sum of item variance and $\sigma^2 x$ equals the variance of the total composite. Therefore, Cronbach Alpha is given by:

{ Cronbach alpha: $\alpha \ = \frac{N}{N-1}\left(1 \ - \frac{\sum \sigma^2 yi}{\sigma^2 x}\right)$ }.

In this study, the correlation matrix was used rather than the variance-covariance matrix hence the alpha reduced to the following formula.

alpha $= Np/\,[1 \ + p(N \ - 1)]$

where N equals the number of items and p equals the mean inter-item correlation. The scoring criteria for the test items was made to measure procedural and conceptual understanding before the test was administered. This was given to UNZA mathematics lecturers for their expert judgement of the instrument. The assessment item had a scale to determine procedural and conceptual understanding by solving the questions prepared for testing first year students. Previously, Sarwali and Shahrill (2014) also used Cronbach alpha in the construction of the test and it gave the alpha value of 0.9 suggesting that the test items had relatively high internal consistency.The working below implied that Cronbach's alpha was driven by assessing the understanding of the knowledge expected to come out from the responses of students.

Table 4.2: Mean values of Procedural and Conceptual Understanding of first year students at UNZA in mathematics

Tests	**Binomial expansions**					**Partial fractions & Polynomials**				
Group A Procedural understanding	0.2	0.8	0.4	0.8	1.0	0.6	0.4	0.8	0.8	1.0
Group B Procedural understanding	0.6	0.6	0.8	0.6	0.8	1.0	0.8	0.6	1.0	0.8
Tests	**Procedural understanding Trigonometry**					**Conceptual understanding Sets: Set Theory**				
Group A	0.6	1.0	0.6	0.8	0.8	0.6	1.0	0.4	0.8	0.8
Group B	0.2	0.8	0.6	1.0	0.4	0.4	0.6	0.4	1.0	0.8
Tests	**Functions: Domain & Range**					**Linear & Quadratic Functions Complex numbers**				
Group A Conceptual understanding	0.6	1.0	0.2	1.0	0.2	0.4	0.4	0.6	1.0	0.8
Group B Conceptual understanding	0.4	0.6	0.8	1.0	0.4	1.0	0.6	0.8	1.0	0.4

Internal consistency with Cronbach alpha

alpha $= Np/\,[1 + p(N - 1)]$

The number of test items was 3. The average inter-correlation of the items for **procedural understanding** as from test given to the first **group A**:

$$= \frac{0.2 + 0.8 + 0.4 + 0.8 + 1.0 + 0.6 + 04 + 0.8 + 0.8 + 1.0 + 0.6 + 1.0 + 0.6 + 0.8 + 0.8}{15} = \frac{10.6}{15} = 0.707$$

But we know that; alpha $= Np/ [1 + p(N - 1)]$

Hence, replacing the values in the formula we obtain.

alpha = $3(0.707)/[1 + (0.707)(3 - 1)]$

$$= \frac{2.121}{2.414}$$

$= 0.879$ therefore, procedural questions prepared were reliable.

alpha $= Np/ [1 + p(N - 1)]$

The number of test items was 3. The average inter-correlation of the items for **procedural understanding as from the test given to group B:**

$$= \frac{0.6 + 0.8 + 0.8 + 0.6 + 0.8 + 1.0 + 0.8 + 0.6 + 1.0 + 0.8 + 0.2 + 0.8 + 0.6 + 1.0 + 0.4}{15} = \frac{10.8}{15} = 0.72$$

But we know that; alpha $= Np/ [1 + p(N - 1)]$

Hence, replacing the values in the formula we obtain.

alpha = $3(0.72)/[1 + (0.72)(3 - 1)]$

$$= \frac{2.16}{2.44}$$

$= 0.885$ therefore, procedural questions prepared were reliable.

alpha $= Np/ [1 + p(N - 1)]$

The number of test items was 3. The average inter-correlation of the items for **conceptual understanding as from the test given to group A:**

$$= \frac{0.6 + 1.0 + 0.4 + 0.8 + 0.8 + 0.6 + 1.0 + 0.2 + 1.0 + 0.2 + 0.4 + 0.4 + 0.6 + 1.0 + 0.8}{15} = \frac{9.6}{15} = 0.64$$

But we know that; alpha $= Np/ [1 + p(N - 1)]$

Hence, replacing the values in the formula we obtain.

alpha = $3(0.64)/[1 + (0.64)(3 - 1)]$

$= \frac{1.92}{2.28}$

$= 0.842$ therefore, conceptual questions prepared were reliable.

alpha $= Np/ [1 + p(N - 1)]$

The number of test items was 3. The average inter-correlation of the items for **conceptual understanding as from the test given to group B:**

$$= \frac{0.4 + 0.6 + 0.4 + 1.0 + 0.8 + 0.4 + 0.6 + 0.8 + 1.0 + 0.4 + 1.0 + 0.6 + 0.8 + 1.0 + 0.4}{15} = \frac{10.2}{15} = 0.68$$

But we know that; alpha $= Np/ [1 + p(N - 1)]$

Hence, replacing the values in the formula we obtain.

alpha = $3(0.68)/[1 + (0.68)(3 - 1)]$

$= \frac{2.04}{2.36}$

$= 0.864$ therefore, conceptual questions prepared were reliable.

As from the above working we may suggest that the test as an instrument prepared for research was reliable because all its components after testing for internal consistency were in the required range of 0.65 - 0.95.

Table 4.3: Mean values of Procedural and Conceptual understanding as determined by UNZA first year students of mathematics using SPSS.

Model Summary					
Dimension	Cronbach's Alpha	Variance Accounted For			
		Total (Eigenvalue)	Inertia	% of Variance	
1	.710	2.449	.408	40.825	
2	.651	2.188	.365	36.459	
Total		4.637	.773		
Mean	.682[a]	2.318	.386	38.642	
a. Mean Cronbach's Alpha is based on the mean Eigenvalue.					

As from the table, we see that the procedural level of understanding attached to questions 1,3 and 5 was 0.710 which falls within the recommended range of reliability. Furthermore, it shows that the internal consistency for conceptual level of understanding was 0.651 which also falls within the standard range of reliability. However, the table reveals that the two internal consistency values from students are correct because they fall within the given internal consistency range of 0.65 to 0.95.

4.11 Developing and designing Instruments for Measuring Understanding of Concepts in Mathematics.

Figure 4.1 shows how the study developed and designed instruments to measure mathematics understanding of students.

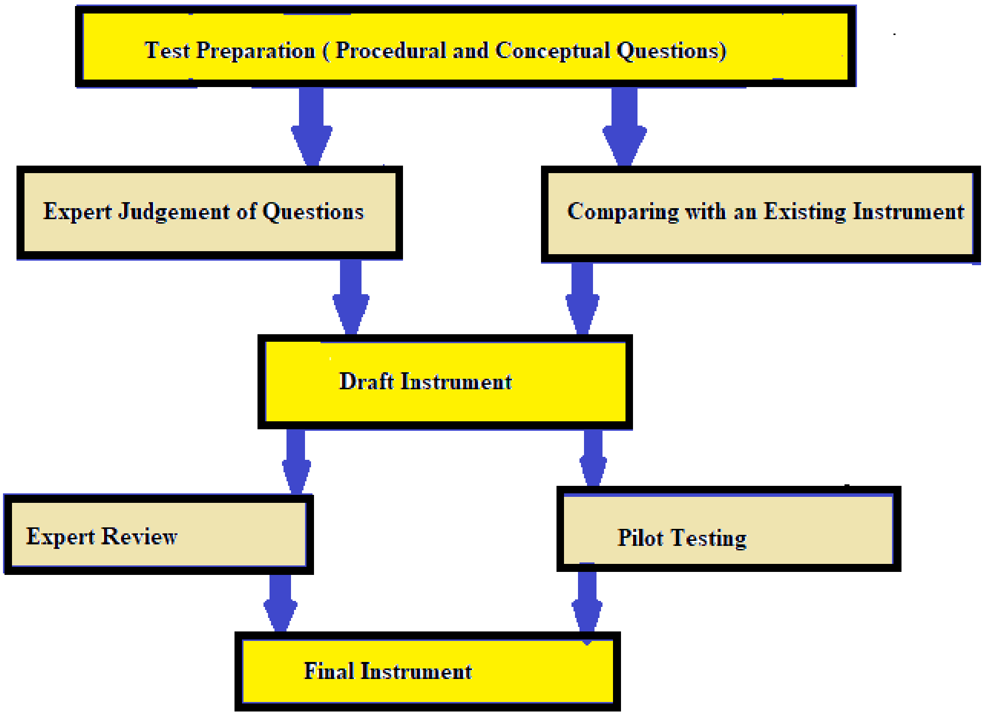


Figure 4.1 Process of developing and designing instruments to measure mathematics understanding of students (procedural and conceptual)

Test Preparation (Procedural and Conceptual Questions)

In the preparation of the test the researcher followed standard procedures in test construction. However, the test prepared adequately covered the specific concepts in selected mathematics topics at first year level at UNZA. This was achieved since expert judgement was involved by allowing lecturers at UNZA handling first year students to participate in the construction of the test. The current test instrument was validated as supported by Sarwali and Shahrill (2014) who suggested that the written test was previously validated because it was used as an instrument for a previous mini research conducted by another author. Henceforth, to ensure construct validity, the test was thoroughly and independently scrutinized by other colleagues to get unbiased view of the levels of procedural or conceptual knowledge needed to complete each item successfully according to the working definitions of the constructs provided (Engelbrecht et al., 2005). On the whole, Audrey et al (2013) suggested that standard, paper and pencil tests do not accurately measure 21st century skills. Hence, the current study sought to improve on test preparation by addressing all the shortcomings so that the test may start measuring intended test constructs such us originality and conceptual concepts of mathematics. Therefore, in this paper, the test preparation was also supported by Hallett, Nunes and Bryant (2010) who suggested that, in order to assign the items in this measure to either a conceptual or a procedural scale, we coded each of the items as reflective of either more conceptual understanding or more procedural knowledge. Arising from the above, the current study with the guidance of the school syllabus which incorporated all the themes and or skills as prescribed in the curriculum of mathematics at first year level at UNZA. Makoya (2018) suggests that:

> Each question paper is critically judged against the following criteria: content coverage; text selection, type and quality of questions; cognitive skills; language and bias; predictability; development of marking guidelines; conformity of the marking guideline with the question paper; accuracy and reliability of marking guideline; and general impression and remarks. (p.8).

The current study undertaken by Audrey De Zeeuw et al., (2013) suggested that test the prepared should be original, creative and structured in order to enable students give detailed solutions which is in conflict with (Engelbrecht et al, 2005) who used multiple-choice test to measure procedural and conceptual performance and confidence of learners. As from the above, the present research focussed on using structured questions to measure procedural and conceptual understanding and confidence of learners in solving first year university mathematics. In addition, the study employed item analysis as supported by (Nik Azis and Tapsir, 2013) who also used item analysis in order to verify the instruments predictive validity. To add on, Audrey et al, (2013) suggested that while assessing conceptual understanding in mathematics, it was inevitable to look at the consequence of misalignment between the call for reforms and current assessment, teachers using weak but rapid instructional methods to pass limited scope of assessment instead of students preparing for 21st century citizenship. In line with the aforesaid, the study made sure that the sole purpose of the research was to measure procedural and conceptual understanding of mathematics concepts. This maybe achieved by using scientific approaches in the entire process of data management upto data analysis. Therefore, quantitative analysis was used to measure the test items difficult using both validity and reliability. High reliability and validity can be built into a test in advance through item analysis (Anastasi & Urbina 2017:160). Comparative judgement was used as it is supported by Bisson et al., (2016) who argues that CJ approach allows test questions for any topic to be developed rapidly. Engelbrecht, Harding and Phiri (2010) stipulated that; to ensure that students are mathematical proficient.

- Enhance students' perceptions of the beauty, vitality, and power of the mathematical sciences.
- Enhance students' understanding of mathematics as a creative endeavour.
- Increase students' confidence ability to communicate quantitative ideas orally and in writing (and since a precursor to communication is understanding, improve students' ability to interpret information, organize material, and reflect on results).
- Encourage students to continue taking courses in the mathematical sciences (Saxe & Braddy, 2015, pp. 12-13).

The teaching and learning of mathematics should not focus on finishing the syllabus but on the understanding of mathematics concepts at that specified level of education. Nevertheless, in the currenct study the researcher prepared a structured test in order to measure mathematics understanding of first year students at UNZA. The researcher gave the prepared test to the selected sample which he marked and these were in total of 378 scripts being a team leader data for additional mathematics at (ordinary level) 'O' levels and also a part-time lecturer at Rusangu University it was as if he was doing one of his routine tasks. The University of Zambia mathematics department made the marking key which was used together with the one for the researcher. To sum up, it is believed that the hallmark of an effective test with a good rubric is to give reliable and valid results after conducting a mal-practice free examination at universities. Furthermore, interest and attitudes can play important roles in developing openness to learning and create the potential for deep conceptual understanding (Hannigan et al, 2013).

To add on, Nezhnov et al., (2014) argued that, it is critical to provide educators with measurement tools that will allow them to better understand the level of their students mathematical learning. In this study a questionnaire was drafted which was used and it was validated by UNZA lecturers of mathematics and mathematics education for educational measurement for openness, fluency, appropriateness of language structure, expression and relevancy to confirm content validity (Nik Azis and Tapsir, 2013). Furthermore, it was observed by Bisson et al., (2016) point it out that comparative judgement does require a detailed rubric to represent conceptual understanding of a topic. This prompted the researcher of the current study to make a detailed rubric for procedural and conceptual questions developed for the study (see Appendix A). Figure 4.1 explains the process of instrument development and makes sure that the newly designed instrument were valid and reliable. The instrument development ought to be systematic yet creative requiring continual refinement and revision (Getenet and Besick, 2013). In order to come up with test questions, UNZA lecturers of mathematics were consulted for expert input. Audrey et al (2013) states that, if the goal of assessment is to measure particular expectations, and the the current call for students is to exhibit higher-order thinking skills, then the assessment themselves need to change. In response to this, the study involved specialised knowledge of mathematics concepts of the stake-holders who helped to make a valid and reliable test. It was observed that instructional repertoire were clarified by providing guidance on determing actually what students should understand (Smith et al, 2018). In line with this argument it can be stated that other researchers may pick the instruments developed and add on something new because the study on instrument development is continuous.

Test preparation of procedural and conceptual questions was done in an open manner by involving experts. The first year mathematics syllabus was analysed; it has a number of topics with sub-topics and a number components. Generally, there were a lot of topics which gave trouble to first year students at UNZA. To know the topics where students had difficulties in assimilation of mathematical concepts, the researcher had to get informed consent from the Head of Department mathematics at the School of Natural Sciences. The researcher was allowed to have access to previous final examination scripts of first year students at UNZA from 2010 to 2016**.** It was from these scripts that the researcher had to find out what topics students had difficulties with. Procedural and conceptual understanding underpinned the investigation. It was also the intention of the study to determine whether there existed any relationship between students' confidence levels and their procedural and conceptual understanding of particular concepts. In this regard, a quantitative approach was followed and specifically a case study design was employed.

Firstly, the current study sought to look at the questions in an interchanging manner (procedural followed by conceptual question). In this regard, question one was procedural in nature and consisted of binomial expansions, systems of equations and inequalities as procedural-oriented questions. Nevertheless, the specific concept where students had problems was on binomial expansions specifically finding the n^{th} term of the binomial coefficient. On the other hand, the researcher determined that students had difficulties of solving correctly systems of equations specifically on the concept of one linear and one quadratic. The first question ended with inequalities as procedural questions. It was observed that, the specific concept on inequalities where students had difficulties on inequalities involving quotients.

Secondly, the question which followed was conceptual oriented and it was on set theory and real number system. The specific concept where students had difficulties was on De Morgan's laws and binary operations on real numbers.

Thirdly, the question prepared was procedural in nature and consisted of partial fractions and polynomial functions. This was arrived at after the researcher had already found out that students had difficulties of understanding concepts. The specific concepts where students had difficluties were on partial fractions with one linear and non-repeating quadratic factor, on polynomials it was observed that students had difficulties of finding unknown factors. This is when computing factors of the polynomial using factor theorem either using synthetic division or long division.

Fourthly, the conceptual question organized was on functions. In the current study, it was observed that most of the learners had problems of determining the specific concepts of domain and range of functions correctly.

Fifthly, the procedural question prepared was on trigonometry. It was discovered that students had difficulties of solving correctly on specific concept of trigonometric identies.

The last question consisted of conceptual questions. After, a thorough scrutinizing of the previous scripts of 2010-2016 final examinations of mathematics past papers. It was further noted that students had challenges in understanding concepts on quadratic functions and complex numbers. It was revealed that the specific concept of which students had difficulties on were of finding the roots of quadratic functions. Also, students had problems on complex numbers. It was observed that, the specific concept of solving correctly on arithmetic operations on complex numbers was a challenge to most of the students. My personal expertise coupled with the guidance of the supervisors and expert judegement of UNZA lecturers of first year mathematics helped me to determine whether the questions for the test where either procedural or conceptual. Hence or otherwise, it was possible to come up with credible components of procedural and conceptual aspects of questions.

Hence, the current study used standard indices as well as multi-variate analysis techniques to determine the understanding of specific concepts in selected mathematics topics of first year students at UNZA. Against this background, it is imperative for students to have confidence when solving mathematics. The study used empirical observation in order to determine the understanding possessed by UNZA first year students in mathematics (procedural or conceptual). This was supported by Nik Azis and Tapsir (2013) who took an initial step in developing an instrument to measure values in mathematics which were positivist in nature such as objectivity, control, mystery, accuracy and clarity. Credence is given to Audrey et al (2013) who proposed that research should focus on the development of a tool that will be used to assess calculus students ability to apply their conceptual understanding of mathematics concept to the scientific phenomena through modelling. The present study attempts to investigate on the understanding possessed by first year students in the learning of mathematics. This was to be achieved by measuring students understanding and confidence to solve mathematical tasks correctly. Engelbrecht et al (2015) argues that,

> In our attempt to investigate the importance of the distinction between conceptual or procedural thinking in the undergraduate mathematics curriculum of engineering students, we attempted to develop an instrument consisting of items expected to be solved by a mainly conceptual or procedural approach by the target group that can be used with sample groups of students. The construction of the items, and the decision about items are to be classified as mainly conceptual or as mainly procedural for the students, were made by mathematicians and mathematics educators.(p.932).

Engelbrecht et al (2005) investigated undergraduates students' performance and confidence in procedural and conceptual mathematics where a test was constructed which consisted of ten multiple choice items of which five were considered to be procedural and five conceptual. It has been observed that multiple choice questions usually measures lower level constructs as Audrey et al (2013) argues that higher-order thinking skills, related to sophisticated cognition are difficult to measure with the use of multiple-choice tests. In line with the previous measures the current study developed three procedural and three conceptual questions which were structured in nature in order to measure original skills and constructs of concepts. The credibility of analysing data was enhanced by using XL STATA with Excel in order to come up with multivariate analysis by analyzing data from questionnaires.

Additionally, data was analysed from the test using standard indices which were developed and adjusted to suit the current study. Methodologies which were objective in nature were used which were in agreement with (Muijs, 2004) who used methods that are detached him from the research as far as possible in order to maximise objectivity and minimise researcher involvement in the study.

Nonetheless, Nik Azis and Tapsir (2013) had a strong determination in developing a holistic instrument suitable for the education system of Malaysain Education. Additinally, Creswell (2002) argues that:

> Most students develop instruments with little feedback from advisors or consultants about how to design the instrument. Instead of developing your own instrument, I would encourage you to locate or modify an existing instrument. (p.168).

It has been noted that beginning researchers develop their own instruments rather than taking time to locate an existing instrument suitable for their study

Questionnaire

A questionnaire was administered to elicit the participating students' experiences as they solved the test items. The questionnaire sought to measure students confidence which is a component of values in mathematics education. Questionnaire with too many items can create problems with respondent fatigue or response biases. Shorter questionnaires are cheaper to administer and thus increase survey sustainability (Robertson, 2017). When formulating the questionnaire, it was inevitable to use a Likert scale as it is seen as the most suitable method in assessing latent construct like values (Nik Azis and Tapsir, 2013). In this study a 5-point Likert scale was used, 1 (Total guess), 2 (Not certain), 3 (Certain), 4 (Fairly certain) and 5 (Completely certain). This procedure is in agreement with Rabab'h and Veloo (2015) who also used a Likert scale instrument of 5-point which was from 1 (Strongly disagree), 2 (Disagree), 3 (Moderately agree), 4 (agree) and 5 (Strongly agree). In line with the debate above, the Likert scale of 5-point scale was used to find out the level of confidence of students of answering questions correctly on procedural and conceptual concepts. According to (Muijs, 2004) the confidence of the students to answer questions correctly was boasted because of the well prepared questionnaire which was used in data collection. Above and beyond the same scale was used to find out the level of difficult attached to answer questions correctly and this was asked to UNZA lecturers of first year mathematics. The study set out to investigate the relationship between students understanding of mathematical concepts (procedural or conceptual) and their confidence in doing the work. This reasonates with the procedures adopted by Forrester and Chinnappan (2010) who compared students workings in mathematics and compared procedural to conceptual graphs usings quantitative data analysis with the aid of SPSS version 18. Exploratory factor analysis was employed to identify both the sub-factors and verification of the structural design of the questionnaire (Nik Azis and Tapsir, 2013). May reference be made to the questionnaire on Appendix A and B.

Expert Judgement of Questions

The items were revised based on the inputs provided by the experts where some items needed to be rewritten but no items were deleted (Nik Azis and Tapsir, 2013). Additionally, (Yosi,2018) supports the ascension that it was the aim of the research to produce an instruments for tests that will be able to measure test constraints in mathematics. The questions used were prepared by the researcher which he gave to the supervisors to have expert inputs. There after, aIso gave the same questions to UNZA lecturers of first year mathematics to have expert judgement. In response to this, (Cotton and Convert, 2007) supports (Getenet and Beswick, 2013) by stating that the instrument designed to measure initial knowledge of mathematics that links each item conducted to ensure full understanding of concepts. As in any scientific endeavour, we need to follow a systematic process when building a measuring instrument (Robertson, 2017). Arising from the above, the capstone of the study was to make sure that the questions selected for the study were either procedural or conceptual. The current study suggested that the picking of a question in the study to be either procedural or conceptual should always involve expert judgement of concerned lecturers. All the stake-holders involved in this current study were well informed.

Comparing with an Existing Instruments

The current study developed and designed instruments to measure procedural and conceptual understanding of mathematics concepts. This idea was borrowed from Engelbrecht et al (2005) who used standard indices to measure procedural and conceptual performance of undergraduate students of life sciences at Pretoria university in calculus. This idea relates to Getenet and Beswick (2013) who also constructed an instrument to measure mathematical constraints. Based on the analysis of the two instruments the study come up with its own instrument as debated in the next chapter. This was easy because the existing instruments were revised and adjustments made to conform to the present study. In doing so, some items were revised, included or even discarded based on the new conceptualisation of the instrument. In order to conceptualise the instrument the operationalisating of the concept of learning model was used. The concepts to be measured were identified and defined appropriately in order to have a sound judgement of the concepts measured as procedural or ocneptual. The idea was to develop an instrument that can measure student mathematical logical thinking abilities on various topics of interest (Yosi, 2018).

Draft Instrument

The draft instrument was synonymous to the development of a valid and reliable means of measuring procedural and conceptual understanding of mathematics concepts. However, Robertson (2017) suggesedt that, it is necessary to develop new instruments that are valid and reliable measures of phenomena of interest. This enabled the study to compare the draft instrument with an existing instrument in order to adjust the instrument so that it can measure the constructs (concepts) under the current study. This helped to construct the draft instrument for measuring the already mentioned constructs without difficulties. Comparing the draft instruments to the ones which were developed and designed earlier by Engelbrecht et al (2005) and also Novaliyosi (2018) whose instruments developed can be used in the process of lecture so that the research could be carried out in other research developments, it was observed that there were similiraties.This made the current study to develop and adapt the previous instruments and then use them in the study at hand. Even though, there are a number of challenges in the development and designing of instruments and most of the researchers are at loggerheads on the same aspect of instrument development. The study of instrument development and designing has not halted as a number of scholars are still testing various instruments for possible development. In response to this current outcry the study thought of developing and designing valid instrumnets which maybe used to measure procedural and conceptual understanding of concepts. In view of the aforesaid, the study investigated the confidence levels exhibited by students in the solving of procedural and conceptual oriented tasks.

Expert Review

The review of instruments was aimed to pinpoint means and ways on how to redefine constructs and also to operationalise concepts to be measured. Based on the results of expert and expert panelists validations indicated that the instrument designed was easy to read and well understood by the students hence the improvements made to the instrument proved vital

(Yosi, 2018). Professor Engelbrecht Johann of Pretoria University reviewed the instruments at the 12th Southern Africa Association of Education Assessment (SAAEA) 2018 Conference in Pretoria, South Africa. Expert review of standard indices was done and authority was given to make adjustments to the instrument to conform to the research objectives. In line with this measure outlined above, Getenet and Beswick (2013) involved two experts in mathematics education to review the questionnaire and also made several changes to the questions. Expert review of instruments ushered in opportunities to adjust in the framing of questions from multi-choice to open-ended and structured. Credence to the need alluded to above expert recommendations enable the researcher to only use quantitative methods because they conformed to the current study.

Pilot Testing

The instrument was reviewed by content experts (UNZA) mathematics lecturers, then a pilot test with a small group of the same cohort of first year students was done. The pilot-test assisted in addressing problems that might occur during administration (Getenet and Beswick, 2013). The pilot study consisted of 30 students of which 20 were males and 10 females. The pilot testing revealed that some test items were vague and below standard. Hence, some items were replaced and corrections were made even to the part of the questionnaire where the question was not clear. For example, the question on binary operations on real numbers a conceptual oriented question corrections were made to it. To add on, the question on trigonometrical identities a procedural question where were also some amendents effected. In this regard, a standard pre-test of the instrument on a small sample of the population should be undertaken in order to make the collected data authentic. The corrected test instrument was later given to the selected sample of which results were analysed using standard indices. Hence, the new standard indices were structured in a way that there were different from the old standard indices as they showed components which helped to measure mathematics understanding. These improvements on standard indices enabled the current researcher to make significant improvements of the measuring instruments. However, the data from the test was then analysed using developed and designed stardard indices which conformed to the study. The study made sure that the instrument developed were supported by evidence-based approached which were used before. In this regard, the current study improved upon old instruments and adjusted them to suit the current project. On the whole, from the discussions we may wish to state that the new instruments may add value to mathematical measuring instruments of procedural and conceptual concepts.

Final Instrument

The study developed and designed valid and reliable instruments used to measure conceptual and procedural understanding and confidence of students exhibited when solving mathematics concepts. Audrey et al (2013) argued that, the final step of analysis is validation, which is the process of assessing the adquancy of a model to represent a particular system and situation. Credence to mention that, a perfect designed and excuted measuring instrument in mathematical understanding may proceduce unbiased data only if the underlying measurement are not scientifically sound. Failure to design measuring instruments scientifically will result in poor-quality data and ultimately poor decision-making. It may be concluded that this paper described the procedure used to develop instruments to mesasure mathematical understanding. Furthermore, the aforementioned process gives detailed information on how would be researchers interested in the same field use the same instrument . Based on this study, the new developed instrument maybe used in other subject areas and maybe improved upon by other interested researchers. For instance, reference maybe made to the final instrument in Appendix A.

4.12 Analysis of Data

The research was purely quantitative because of the nature of research approach, research objectives, research questions and the literature reviewed. The data from the test was analysed using standard indices. Standard indices were used because they proved to be valid and reliable consistently. Furthermore, the test and the questionnaire were attached to each other because after answering the test then the students were supposed to answer the questionnaire based on the same questions of the test on how confident there were to answer questions. The data obtained from the questionnaire was quantitative in nature and it was subjected to statistical analysis using XL STATA with excel. This soft ware helped to generate multivariate components of factor analysis and multi-dimensional scaling to know the levels of confidence attached by learners in answering questions correctly on procedural and conceptual components. Furthermore, also SPSS was used to generate statistical analysis of cluster analysis in relation to performance of students. Multivariate analysis were opted for because of the tendency to determine or analyse interrelations among correlated variables. Additionally, multivariate analysis are concerned with the relationship between sets of dependent variables. Multivariate analysis of late are highly utilised because a series of univariate statistics carried out separately for each variable may sometimes lead to incorrect interpretations of results (Kothari, 1985). Univariate statistics ignores the correlations or inter-dependence among variables which may results in mis-representation or mis-information of facts. Hence, multi-variate analysis has emerged as a powerful method to analyse data in terms of many variables. In this particular research, factor analysis, multi-dimensional scaling and cluster analysis were used to interpret the questionnaire quantitatively. Three multi-variate methods were chosen so that the short fall of one method maybe uplifted by the strength of the others.

Standard Indices

The researcher developed and designed standard indices to measure understanding of concepts and confidence using the information from the test. The change made in the instrument developed (standard indices) was the denominator which was changed and also the units. Initial indices were measuring performance and confidence but for the current study indices measured understanding of mathematics concepts.

Multivariate Analysis

More recently, quantitative strategies have involved complex experiments with many variables and treatments such as factorial designs. In particular factor analysis was used as one of the methods for this study to interpret the questionnaire quantitatively. Kothari (1985) in agreement with the aforementioned statement about factor analysis supports Fontana (1985) who postulates that:

> Recent research using factor analysis rather suggests that such models may approximate closer to reality than Spearman's 'g' and 's' model and the other so called 'hierarchical' model based to a greater or lesser degree upon it. Briefly, factor analysis is a technique that examines a range of scores to establish whether groups of them correlate more strongly amongst themselves than they do with the rest. (p.112).

Furthermore, the information from the questionnaire was used to analyse data using multivariate techniques. This was done so that the short fall of one method was to be uplifted by the strength of the other one and vice versa. Additinally, Kothari and Garg (1985), argue that:

> Of late, multivariate techniques have emerged as a powerful tool to analyse data represented in terms of many variables. The main reason being that a series of univariate analysis carried out separately for each variable may, at times, lead to

> incorrect interpretation of results. This is so because univariate analysis does not consider the correlation or inter-dependence among variables. As a result, during the last fifty years, a number of statisticians have contributed to the development of several multivariate techniques. (p.399).

Multivariate analysis tools were opted for because of their capacity to analyse gigantic data and put them into sizeable manageable data form. Above and beyond, they have the tendency to scrutinize the relationships between and among independent variables.

Factor Analysis

Rabab'h and Veloo (2015) used factor analysis in the study which helped them to determine the questionnaires construct validity and also to determine whether or not a multiple dimension or a single dimension underlies the items in the question. In line with the aforesaid, the proponents of factor analysis believe that it helps to deal with data sets where there are large numbers of observed variables that are thought to reflect a smaller number of underlying/latent variables. It is one of the most commonly used inter-dependency techniques and is used when the relevant set of variables shows a systematic inter-dependence and the objective is to find out the latent factors that create a commonality. Factor analysis was used in the current study because of its tendency to show all the hinden relationship between and among variables. By and large, factor analysis has also the tendency to drop factors which do not conform to the study at hand.

Multi-dimensional scaling (MDS)

Multidimensional scaling is a means of visualizing the level of similarity of individual cases of a data set. It refers to a set of related ordination techniques used in information visualization, in particular to display the information contained in a distance matrix. In addition, (Jacoby, 2012) stipulates that, multidimensional scaling is a family of procedures for constructing a spatial model of objects, using information about the proximities between the objects. Kothari and Garg (1985) in agreement with Jacoby (2012) points out that MDS has many applications in social research such as reducing dimensionality, modelling perceptions of survey respondents or experimental subjects, flexible with respect to input data, useful measurement tool and graphical output. Furthermore, MDS form a non-linear dimensionality reduction as Kothari (1985) argues that:

> Multidimensional scaling (MDS) allows a researcher to measure an item in more than one dimension at a time. The basic assumption is that people perceive a set of objects

> as being more or less similar to one another on a number of dimensions (usually uncorrelated with one another) instead of only one. (p.338).

Kothari and Garg (1985) argues that, MDS is used when all the variables (whether metric or non-metric) in a study are to be analysed simultaneously and all such variables happen to be independent (p.85). MDS was used in the study in order to triangulate the study; this was done, so that the shortfall of using factor analysis was to be uplifted by the other multivariate analysis tool.

Cluster Analysis

In this sampling plan, the total population was divided into groups (known as clusters) and a simple random sample of the groups was selected. On the other hand, (Hallet et al, 2010) used cluster analysing in the analysing of data by performing two residualized scales one conceptual and one procedural (p.400). The study used the already aforesaid method because it helped to analyse procedural and conceptual understanding of mathematics concepts in various tutorial groups at UNZA . Ideally, this multi-variate component conformed to the analysing of data at hand hence it was used in the study. In this regard, Kothari and Garg (2017) argued that, cluster analysis divide large group of objects or observations, like customers or products, into smaller groups such that the observations within each group are similar or close (or homogeneous) and the observations in different groups are dissimilar or far away. These smaller groups are called clusters. Thus, resulting clusters exhibit high internal (within cluster) homogeneity and high external (between clusters) heterogeneity (p.387). Furthermore, in cluster analysis observations are based on particular set of variables which in turn affect the clusters obtained using cluster analysis. Conspicuously, in the current study since it involved a fixed sample, it meant that the expected random error was supposed to be very small of which most of the variation within the population was to be present internally in the groups and not between the groups.

4.13 Ethical Consideration

The researcher ensured that all the participants were treated with respect. Henceforth, consent was sought from participants before participating in the study. Furthermore, the researcher assured the subjects that the results from the study were to be confidential and only to be used for academic purpose.

Taylor et al (2011) in agreement with Creswell (2002) suggests that:

> Data collection should be ethical and it should respect individuals and sites. Obtaining permission before starting to collect data is not only a part of the informed consent process but is also an ethical practice. Protecting the anonymity of individuals by assigning numbers to returned instruments and keeping the identity of individuals confidential offers privacy to participants. During data collection, you must view the data as confidential and not share it with other participant or individuals outside of the project. You need to respect the wishes of individuals who choose not to participate in your study. Even when they consent to participate, people may back out or not show up for an observation or interview. Attempts to reschedule may be futile and you may need to select another person for data collection rather than force an individual to participate. (Taylor et al, 2011:52 & Creswell, 2002: 169-170).

The researcher ensured that all the participants were treated with respect. Henceforth, consent was obtained from participants before they participated in the study. Furthermore, the researcher assured the subjects that the results from the study were to be confidential and only to be used for academic purposes. To add on, participants had the right to understand what the researcher was doing and the researcher had to share the findings with the subjects and got their reactions of the results of the research. However, the researcher ensured that the interests and integrity of respondents was upheld. In conclusion, the researcher obtained a clearance certificate from the University of Zambia Ethics Committee.

4.14 Chapter summary

Chapter 4 discussed the methodology that underpin the current study. The study used ontological, epistemological and methodological assumptions as they all suited the case study design which was used. Probability sampling was employed and specifically simple random sampling was used in the selection of the sample of first year students studying mathematics at the School of Natural Sciences (SNS) at the University of Zambia (UNZA) which was 380 participants. Since the study was objective, therefore positivists approaches were utilized. This meant that all the methods of data collection and interpretations were scientific in nature. To add on, issues relating to developing and designing instruments to measure mathematical understanding and confidence of learners were highlighted. Issues relating to test preparation and administration were also highlighted. Consequently, a test was prepared with the help of expert judgement by UNZA lecturers of mathematics. Cronbach alpha was used to determine the internal reliability of the instrument formulated. Ethical issues were adequately addressed.

CHAPTER FIVE
PRESENTATION OF FINDINGS

5.1 Introduction

This chapter presents the results of the study and the responses from UNZA lecturers of mathematics at the School of Natural Sciences. The chapter outlines mathematics results obtained by first year students admitted at UNZA from 2014 to 2016. The present study was an attempt to investigate procedural and conceptual understanding and confidence of solving mathematics exhibited by first year students at UNZA. The chapter also presents new instruments designed and developed for measuring understanding of mathematics concepts.

5.1.1 MAT 1100 SCHEME 2016/17

The researcher analyzed MAT1100 SCHEME 2016/17 syllabus at UNZA. It was found that the syllabus had many topics to be covered for a year. The already mentioned cohort was picked because it was within the time-frame when the researcher was doing the study. It would have be difficult or impossible to look at the entire topics in the syllabus. Hence the researcher picked only the part of the syllabus for term one and opted to look at specific concepts in the selected mathematic topics. In this study questions 1, 3 and 5 were considered to be procedural and questions 2,4 and 6 were considered to be conceptual. This was so after a comprehensive analysis of the constructs of each question by the researcher, supervisors and mathematics lecturers of first year students at UNZA. This was to avoid bias in the selection of the questions to be picked and to put them in the category of either procedural or conceptual. Therefore, each question picked was exceedingly scrutinized by all the three stake-holders so as to be sure and clear that the the questions were put in the right category before the test was given to students. Further adjustments were also made after the pilot study was made as earlier pointed out in the previous chapter.

Three topics were picked for procedural and the same number for conceptual aspects**.** **Question one was procedural in nature. 1.** (a) Binomial expansions in the scheme constituted of Pascal’s triangle; factorials; Binomial coefficients; Binomial formula for positive integral exponents; Binomial formula for rational exponents. But in the test which first year students wrote, the researcher was testing on the Binomial coefficients. A good number of learners had difficulties in finding the n^{th} term of the Binomial expansion; hence the researcher decided to give them the same question on binomial to see if they might be an improvement in the understanding of the n^{th} term of the binomial expansion. To add on, the current study sought of determining how to measure understanding of specific concept on binomial theorem.

(b) **The scheme has Equations in the following categories;** Quadratic; polynomials; involving radicals, quotients and absolute value; system of equations in two and three unknown variables and equations of which we have one linear and one quadratic. The researcher picked the problem for one linear and one quadratic equations because it was found that students had problems in obtaining the correct solution to these types of questions.

(c) **The third question of number one was on Inequalities which consists of linear**, quadratic; polynomials; involving quotients, radicals and absolute value. It was discovered that a good number of learners had difficulties with inequalities involving quotients. Hence or otherwise the researcher gave the specific problem on inequalities involving quotients. To add on, the second question was conceptual in nature which consisted only of two questions.

$2.(a)$ **Set theory in the scheme consists of definitions; subsets; set operations;** De Morgan's laws. With reference to the previous final examinations it was discovered that students had difficulties in answering questions on De Morgan's law correctly. This made the researcher opt for testing the specific concept on set theory on De Morgan's law.

(b) **Real numbers from the scheme was gotten from; Sets of numbers:** Natural numbers; integers; rational numbers; real numbers; complex numbers; arithmetic operations on complex numbers; surds. However, it was observed that students had doubts in answering questions on real numbers hence the researcher gave a question to measure the specific concept of binary operations on real numbers.

Question three was procedural in nature and it consisted of partial fractions and polynomial functions.

$3.(a)$ **As from the scheme (partial fraction constituted of)** Denominator with: linear factors none of which is repeating; linear factors of which some are repeating; quadratic factors none of which is repeating. It was observed that most of the students in previous tests and examinations had problems with partial fraction with one linear and none repeating quadratic. Hence, the researcher gave a question on partial fraction with the denominator having one linear and none repeating quadratic factor. It was observed that students still have snags in solving correctly these types of questions.

(b) **On polynomial functions we have:** Polynomials; addition of; multiplication of; division of; remainder theorem; factor theorem; factorization of; graphs of; looking back at previous performance it was observed that students had doubts on the remainder theorem. Therefore, the researcher prepared on finding the unknown variable when the function is divided by one factor the remainder is twice when divided by a different factor. This question was more of problem solving. It was seen that a number of students had doubts to get the correct solution.

The fourth question was conceptual in nature.

4.**The scheme on functions is as follows:** Binary operations; relations; functions; domain and range; many-to-one function; one-to-one functions; inverse functions; composite functions; even and odd functions. It was observed that students had difficulties in finding the domain and range of functions. Hence the researcher asked only on how to find the domain and range of four different types of functions on functions. Nevertheless, students had anomalies on how to determine the domain and the range of functions appropriately.

The fifth question was procedural in nature.

5.**Transcendental functions (Trigonometric functions)** as from the scheme, this consisted of: Trigonometric ratios; ratios of angles; degrees and radian measures; identities; trigonometric functions; domain; graphs; identities; trigonometric equations. By looking at the dead scripts of first year students it was discovered that a large number of candidates had difficulties solving trigonometric identities. Therefore, the researcher prepared the two questions on trigonometric identities.

Question six was conceptually oriented

6.**Questions six $(\boldsymbol{a})$ and $(\boldsymbol{b})$ consisted of Quadratic roots and were obtained from the scheme:** Completing the square method; maximum and minimum; values of quadratic functions (i.e. roots of quadratic functions); graphs of quadratic functions and applications. It was observed that students had problems finding the roots of quadratic functions.

(c) **Sets of numbers:** This portion consisted of; Natural numbers; integers; rational numbers; real numbers; complex numbers; arithmetic operations on complex numbers; surds. It was observed that students had difficulties on arithmetic operations on complex numbers. Hence the question picked was on arithmetic operations on complex numbers. Even after the test was given it was observed that students still had difficulties on the concepts of complex numbers.

Table 5.1: Overall Data Collection

Appendices	Expected Collection	Actual Collected
Appendix A	380	378
Appendix B	5	3
Total	385	381

As can be seen above, it shows that the actual collected instruments were less to the expected data only by 1%. This good record was because of good leadership exhibited by the head of department for mathematics at SNS. The head of department was eager to know on the findings of the study. Hence, he helped by facilitating the research and being accommodative. Two lecturers for first year mathematics were busy hence the researcher only managed to capture three. For the students, out of the 380 randomly sampled participantes only two did not manage to write the test. Students were not forced to write the test because they had the rights to withdraw from participating in the study at any moment. Consequently, the sample size reduced to 378 first year students to participate in the study.

5.2 Research Originality

The current study was undertaken to bridge the gap in literature concerning the instruments used before to measure procedural and conceptual understanding of concepts. A campaign was ushered to create opportunities of developing and designing instruments to measure mathematics constructs. Previous studies show that it was difficult to measure mathematics understanding and confidence (procedural or conceptual). In view of a number of literature reviewed, it was observed that previously most of the researchers used unscientific means of measuring mathematical understanding. These included writing to learn mathematics, defining concepts and also answering questions correctly meant that students had understood the concepts. In some cases other researchers measured mathematical understanding by looking at the number of mistakes committed in answering tasks. Some of the researchers looked at misconceptions and misunderstanding of concepts in mathematics learning. Most of the aforementioned strategies of measuring mathematics understanding of concepts were not properly done. This was so because most of the methods used previously were not scientific, hence lacked validity and reliability in their usage.

The gap in literature motivated the current study to bring in novelty and create new literature on procedural and conceptual understanding of mathematics concepts. This chapter explains and clarifies the value of my study towards literature. Researchers in the same field may use my study to find out on procedural and conceptual understanding of mathematics concepts. Academic scholars may use my standard indices to measure understanding of any of the STEM courses (Science, Technology, Engineering and Mathematics).

This prompted the current study to look at alternative ways of measuring mathematics understanding by developing and designing instruments of measuring procedural and conceptual understanding of concepts. There was need to develop and design or improve upon existing instruments in order to change the status quo. It was inevitable to develop and design instruments then use them to measure mathematics understanding of students. This thesis makes an important contribution to the field's understanding of students' procedural and conceptual understanding of mathematical concepts. The originality of the study is contextual in that no such study has been conducted at UNZA to measure first year students' procedural and conceptual understanding of selected mathematics concepts.

This chapter demonstates my original thinking and significant scientific contribution towards the problem under investigation. The research now verifies something which we did not know but because of this study we now know. This is my original contribution to the body of knowledge which is the pinnacle of my academic study as here-under explained. To start with, the originality is seen as a factor of novel and creativity which the current study has done by developing and adapting standard indices in the measuring of mathematical understanding. In order to make my study contribute to the knowledge base in mathematics education, I had to refute earlier works which were measuring mathematical knowledge and mathematical performance. My study managed to collect and analyse data and made amendments to earlier formulars and made new ones which were validated and proved to be reliable. We may use these new standard indices contributed to measure procedural and conceptual understanding of specific concepts in mathematics. This chapter clarifies my research originality and the final documentation of the contribution to knowledge base in procedural and conceptual understanding. This was possible because enough evidence was collected and verified by using standard procedures to prove that the new standard indices were viable and workable and that my research questions were adequately answered. In this regard, I picked standard indices which were developed in South Africa in 2005 by Englebrecht Johann, Harding Ansie and Portiegtier Martigitier then modified the instruments to suit the study at hand. Therefore in other words research is an academic conjecture which may be refuted by other scholars and then improve upon. Therefore, going forward other academician may pick my study and refute it with evidence then improve it further. This demonstrates that my study is not the final documentation of knowledge in this area but it is a stepping stone for further research. The modified instruments used in this study are here under explained.

5.3 Measurement of understanding of concepts in mathematics.

Research question one sought to develop and design instruments for measuring understanding of concepts in mathematics. In view of the aforementioned question one sought to develop and design instruments for measuring understanding of concepts in mathematics. In the current study the researcher picked part of first year students as a sample at UNZA. To answer the first research question the researcher developed and designed valid and reliable instruments for measuring mathematics understanding of concepts as here-under explained (standard indices).

Standard Indices

Procedural Understanding Index (PUI)

$$\boldsymbol{PUI} = \frac{\sum \boldsymbol{\alpha}^{*} \boldsymbol{U_p} * \boldsymbol{d_p}}{\mathbf{1000} \sum \boldsymbol{U_p} * \boldsymbol{d_p}}$$

To find the Procedural understanding index. The researcher used the captioned index. The researcher prepared a questionnaire for lecturers to find out the procedural understanding knowledge that they attached to the questions and also the level of difficult for each question then, the researcher had to find the average as Average for Procedural understanding.

<u>For the numerator</u>

$\alpha = 1$, if the question is answered correctly

$\alpha = 0$, if the question is answered wrongly

$U_p{}^*$ Is the procedural understanding knowledge lecturers have by looking at the questions prepared for testing students.

d_p Is the procedural level of difficult lecturers associate to the questions prepared.

<u>For the denominator</u>

$U_p *$ Is the procedural understanding knowledge, we calculate, the marks obtained by students answering the questions.

d_p Is the procedural level of difficult students have and it is stated in the questionnaire at the end of the question paper.

$$\boldsymbol{PUI} = \frac{\sum \boldsymbol{\alpha^* U_p * d_p}}{\mathbf{1000} \sum \boldsymbol{U_p * d_p}}$$

Conceptual Understanding Index (CUI)

$$\boldsymbol{CUI} = \frac{\sum \boldsymbol{\alpha^* U_c * d_c}}{\mathbf{1000} \sum \boldsymbol{U_c * d_c}}$$

To find the Conceptual understanding index. The researcher used the captioned index. The researcher prepared a questionnaire for lecturers to find out the conceptual understanding knowledge that they attached to the questions and also the level of difficult for each.

<u>For the numerator</u>

$\alpha = 1$, if the question is answered correctly

$\alpha = 0$, if the question is answered wrongly

$U_c{}^*$ Is the conceptual understanding knowledge lecturers have by looking at the questions prepared for testing students.

d_c Is the conceptual level of difficult lecturers associate to the questions prepared.

<u>For the denominator</u>

$U_c *$ Is the conceptual knowledge, we calculate, the marks obtained by students answering the questions.

d_c Is the conceptual level of difficult students have by stating in the questionnaire at the end of the question paper.

$$\boldsymbol{CUI} = \frac{\sum \boldsymbol{\alpha}^* \boldsymbol{U_c} * \boldsymbol{d_c}}{\mathbf{1000} \sum \boldsymbol{U_c} * \boldsymbol{d_c}}$$

Procedural Confidence of Understanding Index (PCUI)

$$\boldsymbol{PCUI} = \frac{\sum \boldsymbol{\beta}^* \boldsymbol{U_p} * \boldsymbol{d_p}}{\mathbf{1000} \sum \mathbf{3}^* \boldsymbol{U_p} * \boldsymbol{d_p}}$$

To find the Procedural confidence of understanding index. The researcher used the captioned index. The researcher prepared a questionnaire for lecturers to find out the procedural knowledge that they can attach to the questions and also the level of difficult for each question.

<u>For the numerator</u>

(Where the value of β being 4, 3, 2,1 or 0, indicating complete certainty, fairly certainty, certain, less certain and uncertain or a total guess respectively).

$U_p{}^*$ Is the procedural understanding knowledge lecturers have by looking at the questions prepared for testing students.

d_p Is the procedural level of difficult lecturers associate to the questions prepared.

<u>For the denominator</u>

Where 3 is the constant.

$U_p *$ Is the procedural understanding knowledge, we calculate, the marks obtained by students answering the questions in the test.

d_p Is the procedural level of difficult students have by stating in the questionnaire at the end of the question paper.

$$\boldsymbol{PCUI} = \frac{\sum \boldsymbol{\beta}^* \boldsymbol{U_p} * \boldsymbol{d_p}}{\mathbf{1000} \sum \mathbf{3}^* \boldsymbol{U_p} * \boldsymbol{d_p}}$$

Conceptual Confidence of Understanding Index (CCUI)

$$\boldsymbol{CCUI} = \frac{\sum \boldsymbol{\beta}^* \boldsymbol{U_c} * \boldsymbol{d_c}}{\mathbf{1000} \sum \mathbf{3}^* \boldsymbol{U_c} * \boldsymbol{d_c}}$$

To find the Conceptual confidence of understanding index. The researcher used the captioned index. The researcher prepared a questionnaire for lecturers to find out the conceptual understanding knowledge that they attached to the questions and also the level of difficult for each question then.

For the numerator

(Where the value of β being $4,\ 3,\ 2{,}1\ or\ 0,$ indicating complete certainty, fairly certainty, certain, less certain and uncertain or a total guess respectively).

$U_c{}^*$ Is the conceptual understanding knowledge lecturers have by looking at the questions prepared for testing students.

U_c Is the conceptual level of difficult lecturers associate to the questions prepared.

For the denominator

$U_c *$ Is the conceptual understanding knowledge, we calculate, the marks obtained by students answering the questions in the test.

d_c Is the conceptual level of difficult students have by stating in the questionnaire at the end of the question paper.

$$\boldsymbol{CCUI = \frac{\sum \beta^* U_c * d_c}{1000 \sum 3^* U_c * d_c}}$$

All the four standard indices were credible, as they have been validated and imbedded within a measurement theory (Robertson, 2017). Likewise, standard indices developed by Engelbrecht et al (2005) are discussed in chapter six. The standard indices which Engelbrecht and others did helped a lot in that, not only did they acted as a stepping stone but also as a guide. I had to refute them and develop my own simply because they measured mathematical knowledge and confidence while by study measured mathematical understanding and confidence.

5.4 : Understanding of concepts in mathematics by first year UNZA students.

To answer the second research question, the researcher used the data obtained after students wrote a test and answered a questionnaire at the end of the question paper. The kind of understanding which first year UNZA students have of concepts in mathematics are procedural and conceptual understanding. Henceforth, in this study a test was used to compare procedural and conceptual understanding of first year students at UNZA in mathematics.

5.4.1: Procedural Understanding Index *(PUI)*

$$\boldsymbol{PUI} = \frac{\sum \alpha^{*} U_{p} * d_{p}}{1000 \sum U_{p} * d_{p}}$$

To find the procedural understanding index. The captioned index was used. The number of students picked was 378. The questionnaire for lecturers was prepared and given to them to find out the procedural understanding that they attached to the questions and also the level of difficult for each question. There were only three lecturers by then handling first year classes at UNZA. Therefore, I had to find the procedural understanding and difficulty as observed by lecturers. There were three questions considered to be procedural (question 1,3 and 5).

Average Procedural Understanding as observed by lecturers

$$= \frac{0.2 + 0.4 + 0.6}{3} + \frac{0.8 + 0.8 + 0.6}{3} + \frac{1.0 + 0.2 + 0.6}{3}$$

$$= \frac{1.2}{3} + \frac{2.2}{3} + \frac{1.8}{3}$$

$$= 0.4 + 0.7 + 0.6$$

$$= 1.7$$

Average for Procedural level of difficulty as observed by lecturers

$$= \frac{0.6 + 0.4 + 0.6}{3} + \frac{0.8 + 0.6 + 0.8}{3} + \frac{0.6 + 0.2 + 0.4}{3}$$

$$= \frac{1.6}{3} + \frac{2.2}{3} + + \frac{1.8}{3}$$

$$= 0.5 + 0.7 + 0.4$$

$$= 1.6$$

Replacing the values above in the index and then computing for all the 378 students we obtain:

$$\boldsymbol{PUI} = \frac{\sum \alpha^{*} U_p * d_p}{1000 \sum U_p * d_p}$$

$$\text{PUI} = \frac{(1)(1.7)(1.6)}{\frac{4}{14}\times(1)} + \frac{1(1.7)\,(1.6)}{\frac{5}{8}\times(1)} + \frac{0(1.7)\,(1.6)}{\frac{0}{10}\times(0.4)} + \cdots + \frac{1(1.7)(1.6)}{\frac{14}{14}\times 0.6} + \frac{1(1.7)(1.6)}{\frac{8}{8}\times 0.6} + \frac{1(1.7)(1.6)}{\frac{10}{10}\times 0.6}$$

$$= \frac{2.72}{0.28571429} + \cdots + \frac{2.72}{0.625} + \frac{0}{0} \ldots \frac{2.72}{0.6} + \frac{2.72}{0.6}\ \frac{2.72}{0.6}$$

$$= 9.51999986 + \cdots + 4.352 \ldots .4.53 + 4.53 + 4.53$$

$= 13.8719999 + \cdots + 13.6$, then we divide the solution by 1000 we obtain.

$$= 0.0138719999 + \cdots + 0.0136$$

$$\approx 9.7813326612$$

$= 9.78$ to 3 significant figures.

Procedural understanding index was calculated in such a way that each student had to answer question 1, 3 and 5 which were procedurally oriented. After the test was analysed it was observed that out of the 378 randomly selected students only 30 students got a zero. I had to find the percentage of students who got zeros against the total number of students . However, it was found that 7.94% got a zero mark which had no significant interference from the answer obtained. Therefore, students who got the zero mark were considered to be part of the error of measurements. This was so because the true measurement is defined as the measurement taken plus or minus the error encountered. This clarifies why the zero marks were tolerated. This was so because it did not have a significant impact to the total figure obtained.

Answering the same second research question, we see that there are two forms of understanding of mathematics by first year UNZA students. The other form of understanding is conceptual understanding. Hence or otherwise, we use the conceptual understanding index for all the 378 first year students sampled at UNZA.

5.4.2: Conceptual Understanding Index *(CUI)*

$$\boldsymbol{CUI} = \frac{\sum \boldsymbol{\alpha^{*} U_{c} * d_{c}}}{\mathbf{1000} \sum \boldsymbol{U_{c} * d_{c}}}$$

To find the conceptual understanding index. The captioned index was used. The number of students picked was 378. The questionnaire for lecturers was prepared and given to them to find out the conceptual understanding that they attached to the questions and also the level of difficult for each question. There were only three lecturers by then handling first year classes at UNZA. Therefore, I had to find the conceptual understanding and difficult as observed by lecturers.

Average Conceptual Understanding as Observed by Lecturers

$$= \frac{0.2 + 0.6 + 0.6}{3} + \frac{0.6 + 0.8 + 0.8}{3} + \frac{0.6 + 1.0 + 0.6}{3}$$

$$= \frac{1.4}{3} + \frac{2.2}{3} + \frac{2.2}{3}$$

$$= 0.467 \quad + +0.73 \ + 0.73$$

$$= 1.927$$

$$= 1.9$$

Average Conceptual level of Difficulty as Observed by lecturers

$$= \frac{0.4 + 0.8 + 0.4}{3} + \frac{0.8 + 0.8 + 0.6}{3} + \frac{0.8 + 0.6 + 0.6}{3}$$

$$= \frac{1.6}{3} + \frac{2.2}{3} + \frac{2.2}{3}$$

$$= 0.53 \ + 0.73 \ + 0.667$$

$$= 1.9267$$

$$= 1.9$$

Replacing the values above in the index and then computing for all the 378 students we obtain:

$$\text{CUI} = \frac{1(1.9)(1.9)}{\frac{1}{10} \times 1} + \frac{1(1.9)(1.9)}{\frac{3}{10} \times 0.4} + \frac{1(1.9)(1.9)}{\frac{5}{18} \times 1} + \cdots + \frac{1(1.9)(1.9)}{\frac{9}{10} \times 0.8} + \frac{1(1.9)(1.9)}{\frac{8}{10} \times 0.6} + \frac{1(1.9)(1.9)}{\frac{18}{18} \times 0.6}$$

$$= \frac{3.61}{0.1} + \frac{3.61}{0.12} + \frac{3.61}{0.278} + \ldots + \frac{3.61}{0.72} + \frac{3.61}{0.48} + \frac{3.61}{0.6}$$

$$= 36.1 + 30.1 + 12.986 + \cdots + 5.0139 + 7.52 + 6.017$$

$= 79.19 + \cdots + 18.6$,then we divide the solution by 1000 we obtain.

$$= 0.07919 + \cdots + 0.0186$$

$$= 22.60562581$$

Conceptual understanding index was calculated in such a way that each student had to answer question 2, 4 and 6 which were conceptually oriented. After the test was analysed it was observed that out of the 378 randomly selected students only 20 students got a zero. I had to find the percentage of students who got zeros against the total number of students . However, it was found that 5.29% got a zero mark which had no significant intrusion from the answer obtained. Students who got the zero mark were considered to be part of the error of measurements. This clarifies why the zero marks were tolerated. This was so because it did not have a significant influence to the total figure obtained.

As from the above working, it shows that the smaller the value the more understanding on the part of the learners. Hence, it shows that first year students at UNZA had more understanding of procedural as compared to conceptual. The results above show that:

$$\boldsymbol{PUI}\ \frac{\sum \boldsymbol{\alpha}^* \boldsymbol{U_p} * \boldsymbol{d_p}}{\mathbf{1000} \sum \boldsymbol{U_p} * \boldsymbol{d_p}} = 9.78$$

$$\boldsymbol{CUI} = \frac{\sum \boldsymbol{\alpha}^* \boldsymbol{U_c} * \boldsymbol{d_c}}{\mathbf{1000} \sum \boldsymbol{U_c} * \boldsymbol{d_c}} = 22.6$$

Procedural understanding was twice more than the conceptual understanding as shown from the calculation as from the formula for the research question two.

5.5 Relationship between students confidence levels and their understanding of procedural and conceptual concepts in mathematics topics.

To answer the third research question, the researcher used data obtained after students wrote a test and answered a questionnaire at the end of the question paper. The relationship that exists between students confidence levels and their understanding of concepts in mathematics topics are: procedural confidence of understanding and conceptual confidence of understanding. To measure the two aspects, standard indices were used and also multivariate analysis tools.

5.5.1: Procedural Confidence of Understanding Index *(PCUI)*

$$PCUI = \frac{\sum \beta^{*} U_p * d_p}{1000 \sum 3^{*} U_p * d_p}$$

To find the procedural confidence of understanding. I used the captioned index. The number of students picked was 378. The questionnaire for lecturers was prepared and given to them to find out the procedural understanding that they attached to the questions and also the level of difficult for each question. As from the formula above, (where the value of β being 4,3,2,1 or 0, indicating completely certain, fairly certain, certain, less certain, not certain or a total guess respectively). The calculations show the first and the last term where the terms in between are also included.

$$\boldsymbol{PCUI} = \frac{4(1.7)(1.6)}{3\left(\frac{4}{14}\right)(1)} + \frac{3(1.7)(1.6)}{3\left(\frac{5}{8}\right)(0.8)} + \frac{1(1.7)(1.6)}{3(0)(0.4)} + \ldots + \frac{2(1.7)(1.6)}{3\left(\frac{14}{14}\right)(0.6)} + \frac{2(1.7)(1.6)}{3\left(\frac{8}{8}\right)(0.6)} + \frac{2(1.7)(1.6)}{3\left(\frac{10}{10}\right)(0.6)}$$

$$= \frac{10.88}{0.857} + \frac{2.72}{0.5} + \propto + \cdots + \frac{5.44}{1.8} + \frac{5.44}{1.8} + \frac{5.44}{1.8}$$

$$= \mathbf{12.695} + \mathbf{5.44} + \cdots + \mathbf{3.02} + \mathbf{3.02} + \mathbf{3.02}$$

$$= \mathbf{18.135} + \cdots + \mathbf{9.06}$$

$$\approx \mathbf{6562.134}$$

Dividing the final solution by 1000 yields $\boldsymbol{PCUI} = \mathbf{6.562134}$

Hence or otherwise $\boldsymbol{PCUI} = 6.56$ to 3s.f.

Procedural confidence of understanding index was calculated in such a way that each student had to state how confident they were to answer question 1, 3 and 5 which were procedurally oriented. After the information was analysed from the test it was revealed that 11.38% of the students got a zero or undefined. This meant that out of the total of 378 students 43 had either a zero or undefined. Therefore, students who got the zero or undefined were considered to be part of the error of measurements. This was so because the value obtained was insignificant and can not affect or change the result already obtained.

5.5.2: Conceptual confidence of understanding index (CCUI)

$$\boldsymbol{CCUI} = \frac{\sum \beta^{*} U_c * d_c}{1000 \sum 3^{*} U_c * d_c}$$

To find the conceptual confidence of understanding. I used the captioned index. The number of students picked was 378. The questionnaire for lecturers was prepared and given to them to find out the conceptual understanding that they attached to the questions and also the level of difficult for each question. As from the formula above, (where the value of β being 4,3,2,1 or 0, indicating completely certain, fairly certain, certain, less certain, not certain or a total guess respectively). The calculations show the first and the last term where the terms in between are also included.

$$\boldsymbol{CCUI} = \frac{\mathbf{4(1.9)(1.9)}}{\mathbf{3}\left(\frac{\mathbf{1}}{\mathbf{10}}\right)\mathbf{(1)}} + \frac{\mathbf{3(1.9)(1.9)}}{\mathbf{3}\left(\frac{\mathbf{3}}{\mathbf{10}}\right)\mathbf{(0.6)}} + \frac{\mathbf{4(1.9)(1.9)}}{\mathbf{3}\left(\frac{\mathbf{5}}{\mathbf{18}}\right)\mathbf{(1.0)}} + \ldots + \frac{3(1.9)(1.9)}{3(0.9)(0.8)} + \frac{2(1.9)(1.9)}{3(0.8)(0.6)} + \frac{2(1.9)(1.9)}{3\left(\frac{18}{18}\right)(0.6)}$$

$$= \frac{14.44}{0.3} + \frac{3.61}{0.18} + \frac{14.44}{0.83} + \cdots + \frac{3.61}{0.72} + \frac{7.22}{1.44} + \frac{7.22}{1.8}$$

$$= 48.13 + 20.06 + 17.398 + \cdots + 5.01 + + 5.0138 + 4.01$$

$$= 85.59 + \cdots + 14.03$$

$$\approx 12\,903.70949$$

Dividing the final solution by 1000 yields $\boldsymbol{CCUI} = \mathbf{12.90370949}$

Hence or otherwise $\boldsymbol{CCUI} = \mathbf{12.9}$ to 3 s.f.

Conceptual confidence of understanding index was calculated in such a way that each student had to state how confident they were to answer question 2, 4 and 6 which were conceptually oriented. As from the data obtained from the test it was revealed that 9.79% of the students got a zero or undefined. This meant that out of the total of 378 students 37 had either a zero or undefined. For a sample to be significant it must cover about 15 to 30 percent of the total population. The study shows that only 9.79% students were affected which was below the values to make a significant impact to the overall value obtained. Hence, the figure was considered to be part of the error of measurements. This was so because the value obtained was insignificant and can not affect or change the result already obtained.

As from the working above it shows that the smaller the result the more explained the result is. This meant that students were more confident to answer procedural questions as compared to conceptual. Hence, the two indices above may be summarized as follows:

$$\boldsymbol{PCUI} = \frac{\sum \boldsymbol{\beta}^{*} \boldsymbol{U_p} * \boldsymbol{d_p}}{\mathbf{1000} \sum \mathbf{3}^{*} \boldsymbol{U_p} * \boldsymbol{d_p}} = \mathbf{6.56}$$

$$\boldsymbol{CCUI} = \frac{\sum \boldsymbol{\beta}^{*} \boldsymbol{U_c} * \boldsymbol{d_c}}{\mathbf{1000} \sum \mathbf{3}^{*} \boldsymbol{U_c} * \boldsymbol{d_c}} = \mathbf{12.9}$$

Procedural confidence of understanding from the indices above for research question three shows that students were more confident to handle procedural questions than conceptual questions. The results show that students were twice more confident to answer procedural as compared to conceptual questions.

Table 5.2: The Correlation Matrix

	Level of confidence to answer the question by students on set theory	Level of confidence to answer the questions by students on partial fractions and polynomial functions	Level of confidence to answer the questions by students on functions the domain and range	Level of confidence to answer the questions by students on transcendental functions trigonometric functions	Level of confidence to answer the questions by students on linear, quadratic functions and complex numbers	Level of confidence to answer the questions by students on binomial expansions and systems of equations
Level of confidence to answer the question by students on set theory	1.000	.335	.319	.286	.268	.269
Level of confidence to answer the questions by students on partial fractions and polynomial functions	.335	1.000	.391	.331	.346	.434
Level of confidence to answer the questions by students on functions the domain and range	.319	.391	1.000	.274	.323	.268
Level of confidence to answer the questions by students on transcendental functions trigonometric functions	.286	.331	.274	1.000	.273	.320
Level of confidence to answer the questions by students on linear, quadratic functions and complex numbers	.268	.346	.323	.273	1.000	.385
Level of confidence to answer the questions by students on binomial expansions and systems of equations	.269	.434	.268	.320	.385	1.000

Column sums: 2.477 2.837 2.575 2.484 2.595 2.676

Sum of the column sums(T) $= \mathbf{15.644}$ $\therefore$ $\sqrt{T} = \mathbf{3.955}$

First centroid factor $= \frac{2.477}{3.955}, \quad \frac{2.837}{3.955}, \quad \frac{2.575}{3.955}, \quad \frac{2.484}{3.955}, \quad \frac{2.595}{3.955}, \quad \frac{2.676}{3.955}$

$$= \mathbf{0.626}, \ \mathbf{0.717}, \ \mathbf{0.651}, \mathbf{0.628}, \mathbf{0.656}, \mathbf{0.677}$$

The second centroid factor B shall be found in the similar manner. Then

$$(First\ Centroid\ Fctor)^2 \ + \ (Second\ Centroid\ Factor\ )^2 \ = \ Communality\ (x^2)$$

The variance of one variable is accounted for by the centroid factor A and B and the remaining percent is a portion due to errors of measurement involved in assessing variables. As from the correlation matrix it shows that the Level of confidence to answer the questions by students on Binomial expansions and systems of equations correlated highly to the Level of confidence to answer the questions by students on partial fractions and polynomials all the two questions (i.e. questions 1 and 3 in the test paper given to students were predominantly procedural in nature. This shows that students were more confident to answer procedural questions as compared to conceptual.

Table 5.3: Factor Loadings Concerning First Centroid Factor A

Variables	*Factor loadings concerning First Centroid factor A*
1	0.626
2	0.717
3	0.651
4	0.628
5	0.656
6	0.677

To obtain the second centroid factor B, we first of all develop the first matrix of factor cross product, Q_1 :

Table 5.4: First Matrix of Factor Cross Product

First Matrix of Factor Cross Product (Q_1). First centroid Factor A

	0.626	0.717	0.651	0.628	0.656	0.677
0.626	0.392	0.449	0.408	0.393	0.411	0.424
0.717	0.449	0.514	0.467	0.450	0.470	0.485
0.651	0.408	0.467	0.424	0.409	0.427	0.441
0.628	0.393	0.450	0.409	0.394	0.412	0.425
0.656	0.411	0.470	0.427	0.412	0.430	0.444
0.677	0.424	0.485	0.441	0.425	0.444	0.458

Now we obtain first matrix of residual coefficient (R_1) by subtracting Q_1 from R as shown below:

Table 5.5: First Matrix of Residual Coefficient

First Matrix of Residual Coefficient (R_1) *Variables*

Variables

	1	2	3	4	5	6
1	0.608	-0.114	-0.089	-0.107	-0.143	-0.155
2	-0.114	0.486	-0.076	-0.119	-0.124	-0.051
3	-0.089	-0.076	0.576	-0.135	-0.104	-0.173
4	-0.107	-0.119	-0.135	0.606	-0.139	-0.105
5	-0.143	-0.124	-0.104	-0.139	0.570	-0.059
6	-0.155	-0.051	-0.173	-0.105	-0.059	0.542

Reflected Matrix of Residual Coefficients (R_1') and Extracted of 2^{nd} Centroid Factor (B)

Table 5.6: Matrix of Factor Loadings

Variables

	1	2	3	4	5	6
1	0.608	0.114	0.089	0.107	0.143	0.155
2	0.114	0.486	0.076	0.119	0.124	0.051
3	0.089	0.076	0.576	0.135	0.104	0.173
4	0.107	0.119	0.135	0.606	0.139	0.105
5	0.143	0.124	0.104	0.139	0.570	0.059
6	0.155	0.051	0.173	0.105	0.059	0.542

Variables

Column sums: 1.216, 0.97, 1.153, 1.211, 1.139, 1.085

Sum of column sums (T)= 6.774 $\therefore$ $\sqrt{T} = 2.603$

Second Centroid Factor B: $\frac{1.216}{2.603}, \frac{0.97}{2.603}, \frac{1.153}{2.603}, \frac{1.211}{2.603}, \frac{1.139}{2.603}, \frac{1.085}{2.603}$

$$= 0.467,\ 0.373,\ 0.443,\ 0.465,\ 0.438,\ 0.417$$

Now the matrix of factor loadings can be written as under:

Table 5.7: Factor Loadings

Variables	*Factor loadings*	
	Centroid Factor A	*Centroid Factor B*
1	*0.626*	*0.467*
2	*0.717*	*0.373*
3	*0.651*	*0.443*
4	*0.628*	*0.465*
5	*0.656*	*0.438*
6	*0.677*	*0.417*

Table 5.8: Factor Loadings and Communality

Working out the communality and eigen values from the final results as shown below:

Variables	*Factor loadings*		*Communality*(h^2)
	Centroid factor A	*Centroid factor B*	
1	*0.626*	*0.467*	$(0.626)^2 + (0.467)^2$ =0.610
2	*0.717*	*0.373*	$(0.717)^2 + (0.373)^2 = 0.653$
3	*0.651*	*0.443*	$(0.651)^2 + (0.443)^2 = 0.620$
4	*0.628*	*0.465*	$(0.628)^2 + (0.465)^2 = 0.611$
5	*0.656*	*0.438*	$(0.656)^2 + (0.438)^2 = 0.622$
6	*0.677*	*0.417*	$(0.677)^2 + (0.417)^2 = 0.632$
Eigen value (Variance accounted for i.e., common variance)	3.955	2.603	6.558
Proportion of total variance	.46 (46%)	.35 (35%)	.81 (81%)
Proportion of common variance	.60 (60%)	.40 (40%)	1.00 (100%)

Each communality in the above table represents the proportion of variance in the corresponding (row) variable and is accounted for by the two factors (A and B). The last row shows that of the common variance approximately 60% is accounted for by factor A and the other 40% by factor B. After using factor analysis, I had to use another method by employing Principal component analysis to check if the solution shall be the same.

Table 5.9: Normalizing Factor (A)

	1	2	3	4	5	6
1	1.000	.335	.319	.286	.268	.269
2	.335	1.000	.391	.331	.346	.434
3	.319	.391	1.000	.274	.323	.268
4	.286	.331	.274	1.000	.273	.320
5	.268	.346	.323	.273	1.000	.385
6	.269	.434	.268	.320	.385	1.000

Column sums: U_{a1} 2.477 2.837 2.575 2.484 2.595 2.676

Normalizing factor= $\sqrt{(2.477)^2 + (2.837)^2 + (2.575)^2 + (2.484)^2 + (2.595)^2 + (2.676)^2}$

$$= \sqrt{40.87998}$$

$$= 6.394$$

Normalizing U_{a1} we obtain V_{a1} i.e., ${}^{U_a}/_{Normalizing\ factor^*}$

$$= \frac{2.477}{6.394}, \quad \frac{2.837}{6.394}, \quad \frac{2.575}{6.394}, \quad \frac{2.484}{6.394}, \quad \frac{2.595}{6.394}, \quad \frac{2.676}{6.394}$$

$$= 0.387, \ 0.444, \ 0.403, \ 0.389, \ 0.406, \ 0.419$$

Table 5.10: Matrix of Factor Cross Product

Then we obtain U_{a2} by accumulatively multiplying V_{a1} row by row into R and the result comes as under:

First Matrix of Factor Cross Product (Q_1)

	1	2	3	4	5	6
1	0.387	0.444	0.403	0.389	0.406	0.419
2	0.444	0.197	0.179	0.173	0.180	0.186
3	0.403	0.179	0.162	0.157	0.180	0.169
4	0.389	0.173	0.157	0.151	0.158	0.163
5	0.406	0.180	0.164	0.158	0.165	0.170
6	0.419	0.186	0.169	0.163	0.170	0.176

First Centroid Factor A

Table 5.11: First Matrix of Residual Coefficients

	1	2	3	4	5	6
1	0.613	*-0.109*	*-0.084*	*-0.103*	*-0.138*	*-0.150*
2	*-0.109*	*0.803*	*0.212*	*0.158*	*0.166*	*0.248*
3	*-0.084*	*0.212*	*0.838*	*0.117*	*0.159*	*0.099*
4	*-0.103*	*0.158*	*0.117*	*0.849*	*0.115*	*0.157*
5	*-0.138*	*0.166*	*0.159*	*0.115*	*0.835*	*0.215*
6	*-0.150*	*0.248*	*0.099*	*0.157*	*0.215*	*0.824*

Table 5.12: Normalizing Factor (B)

	1	2	3	4	5	6
1	0.613	*0.109*	*0.084*	*0.103*	*0.138*	*0.150*
2	*0.109*	*0.803*	*0.212*	*0.158*	*0.166*	*0.248*
3	*0.084*	*0.212*	*0.838*	*0.117*	*0.159*	*0.099*
4	*0.103*	*0.158*	*0.117*	*0.849*	*0.115*	*0.157*
5	*0.138*	*0.166*	*0.159*	*0.115*	*0.835*	*0.215*
6	*0.150*	*0.248*	*0.099*	*0.157*	*0.215*	*0.824*

Column sums U_{a2} = *1.197* *1.696* *1.509* *1.499* *1.628* *1.693*

Normalizing factor= $\sqrt{(1.197)^2 + (1.696)^2 + (1.509)^2 + (1.499)^2 + (1.628)^2 + (1.693)^2}$

$$= \sqrt{14.35}$$

$$= 3.788$$

Normalizing U_{a1} we obtain V_{a1} i.e., ${}^{U_a}/_{Normalizing\ factor^*}$

$$= \frac{1.197}{3.788}, \quad \frac{1.696}{3.788}, \quad \frac{1.509}{3.788}, \quad \frac{1.499}{3.788}, \quad \frac{1.628}{3.788}, \quad \frac{1.693}{3.788}$$

$$= 0.316, \ 0.448, \ 0.398, \ 0.396, \ 0.430, \ 0.447$$

Since U_{a2} : [1.197, 1.696, 1.509, 1.499, 1.628, 1.693] and that

V_{a2} : [0.316, 0.448, 0.398, 0.396, 0.430, 0.447]

Comparing V_{a1} *and* V_{a2}, we find the two vectors are almost equal and this shows convergence has occurred. Hence V_{a1} is taken as the characteristic vector, V_a. Finally, we compute the loadings on the first principal component by multiplying V_a by the square root of the number that we obtain for normalizing U_{a2}. The result is as explained in the next diagram.

Table 5.13: Finding the Principal Component I

Variables	(Characteristic Vector V_a)	×	Normalizing factor= $\sqrt{normalizing\ factor\ of\ U_{a2}}$	=	Principal Component I
1	0.387	×	3.788	=	1.466
2	0.444	×	3.788	=	1.682
3	0.403	×	3.788	=	1.527
4	0.389	×	3.788	=	1.474
5	0.406	×	3.788	=	1.538
6	0.419	×	3.788	=	1.587

Table 5.14: Finding the Principal Component II

Variables	(Characteristic Vector $V_{a)}$	× Normalizing factor= $\sqrt{normalizing\ factor\ of\ U_{a2}}$		= Principal Component II
1	0.316	×	6.394	= 2.021
2	0.448	×	6.394	= 2.865
3	0.398	×	6.394	= 2.545
4	0.396	×	6.394	= 2.532
5	0.430	×	6.394	= 2.750
6	0.447	×	6.394	= 2.858

Table 5.15: Principal Components Versus Communality

Variables	Principal Components		Communality
	I	II	
1	1.466	2.021	$(1.466)^2 + (2.021)^2$ = 6.2335
2	1.682	2.865	$(1.682)^2 + (2.865)^2$ = 11.037
3	1.527	2.545	$(1.527)^2 + (2.545)^2$ = 8.8087
4	1.474	2.535	$(1.474)^2 + (2.535)^2$ = 8.5963
5	1.538	2.750	$(1.538)^2 + (2.750)^2$ = 9.9279
6	1.587	2.852	$(1.587)^2 + (2.852)^2$ = 10.686
Eigen value i.e. common variance	9.274	15.574	24.848
Proportion of total variance	0.373 (37.3%)	0.627 (62.7%)	(100%)

Table 5.16: Communalities

Level of Confidence to answer questions correctly	Initial	Extraction
Level of confidence to answer questions on binomial expansion and systems of equations correctly	1.000	0.450
Level of confidence to answer questions on set theory correctly	1.000	0.335
Level of confidence to answer questions on partial fractions and remainder theory correctly	1.000	0.514
Level of confidence to answer questions on functions correctly	1.000	0.377
Level of confidence to answer questions on trigonometric identies correctly	1.000	0.326
Level of confidence to answer questions on quadratic functions and complex numbers correctly	1.000	0.395

Extraction Method: Principal Component Analysis

It was observed that students were more confident to answer the first, third and fifth questions which are procedural in nature as compared to answering of question two, four and six which are conceptual in nature. However, even when we sum the three procedural questions together and also the three conceptual ones separately. It was observed that, procedural questions had more amount as compared to the conceptual ones as captioned in the table of communalities.

Procedural confidence: $0.450 + 0.514 + 0.326 = 1.26$

Conceptual confidence: $0.335 + 0.377 + 0.395 = 1.107$

We see that students where more confident to answer procedural questions as compared to conceptual by 0.183. It follows that, students were more confident in procedural questions as compared to conceptual no wonder they did well in procedural ones.

Table 5.17: Extraction Method: Principal Component Analysis

Components	Initial Eigen values			Extraction Sums of Squared Loadings		
	Total	**% of Variance**	**Cumulative %**	**Total**	**% of Variance**	**Cumulative %**
1	2.616	43.596	43.596	2.616	43.596	43.596
2	0.793	13.212	56.808			
3	0.744	12.403	69.211			
4	0.676	11.270	80.481			
5	0.652	10.864	91.345			
6	0.519	8.655	100.000			

The Total Variance Explained is shown by Eigen-values and Factor Loadings which correlates to the cumulative variable. However, in Table 5.17 only one factor was extracted by applying principal component analysis from the six initial variables explains up to 43.6% of all the cases.

However, the Total Variance Explained maybe calculated using a formula as below:

$\frac{\lambda_1}{\lambda_1 + \lambda_2 + \cdots} \times 100\%$ Percentage explained by λ_1

Using Principal Component Analysis which is concerned with the explanation of the variance-covariance structure of a set of variables through a few linear combinations of the variables. However, the general objectives of PCA are:

- Data reduction
- Identification of group of inter-correlated variables
- To rewrite the data set into an alternative form

Other Principal component are required to reproduce the total system variability. Often much of this variability may be accounted for by a small number k of the principal components. PCA reviews relationships that would not necessarily result into the anticipated results.

Let $A = \sum_{ij}^{n} L^2{}_{ij} = R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$

From the correlation matrix we can calculate the Eigen values and the eigen vectors.

For instance, λ_1 will be associated with the vector $v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$

The elements v_1 are the elements of the first Eigen vector. The Eigen vector is in component form. Algebraically, Principal Components are particularly linear combinations of ρ random variables $x_1.x_2.x_3 \ldots.x_p$. The Principal Components depended on Covariance matrix $\sum$ or correlation matrix ρ of $x_1.x_2.x_3 \ldots.x_p$. Therefore development does not require a multi-variate normal assumption.

Consider the linear combination

$$\gamma_1 = a_1 X = a_{11}X_1 + a_{12}X_{2\ldots} + a_{1p}X_p$$

$$\gamma_1 = a_2 X = a_{21}X_1 + a_{22}X_{2\ldots} + a_{2p}X_p$$

$$\gamma_p = a_p X = a_{p1}X_1 + a_{p2}X_{2\ldots} + a_{pp}X_p$$

The Principal Component are those correlated linear combination of $y_1 y_2 y_3 \ldots y_p$.

Total Variance $= \sigma_{11} + \sigma_{22} + \ldots\sigma_{pp}$

$$\lambda = \lambda_1 + \lambda_2 + \ldots\lambda_p$$

Proportional of Total Variance due to the K^{th} Principal Component

$$\frac{\lambda_1}{\lambda_1 + \lambda_2 + \cdots\lambda_p}, K = 1,2,3, \ldots$$

For instance, calculating the population principal components. Suppose we are given the covariant matrix.

$$\sum = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

It could be verified that the Eigen value and associated Eigen value vectors are (i.e picking from the above, gives:

$\lambda_1 = 8.592, \quad \rho = [0.141, -0.454, 0 \ ]$

$\lambda_2 = 9.730, \quad \rho = [0,0,1 \ ]$

$\lambda_3 = 7.968, \quad \rho = [0,0,0\ ]$

Therefore, the principal components become

$\gamma_1 = e_1\chi \ = \ 0.141\chi_1 \quad - 0.452\chi_2 \quad + 0\chi_3 \quad = \ 0.141\chi_1 \quad - 0.452\chi_2$

$\gamma_2 = e_2\,\chi \ = \ 0\chi_1 \quad + 0\chi_2 \quad + 1\chi_3 \quad = \chi_3$

$\gamma_3 \ = e_3\,\chi \ = \ 0\chi_1 \quad + 0\chi_2 \quad + 0\chi_3 \quad = 0$

Percentage explained for λ_1

$\frac{8.592}{17.698} \times 100\% = 48.547858508645\% = 48.5478585$ to 7d.p $= 48.548$ to 3d.p.

In this case the component γ_1 could replace the original six variables with very little loss of information.

As from **Table 5.17** Principal Component Analysis has reduced 6 factors to only 1 factor which significantly explain the major correlations. As from **Table 5.17** the one extracted Eigen value explains up to 43.596% which is only 4.952 different from the calculated value which is 48.548%. It shows that the difference between the table value and the calculated value is minimum which maybe due to extraneous factors affecting each and every factor.

Model Fit, Graphical Assessment

Eigenvalues are related to the variance in the double-centered distances that is “explained” by each eigenvector. The “Scree plot” below shows the eigenvalues plotted in the order that they were factored from the dissimilarities matrix. Useful visual representation of model fit Logic:

The “important” dimensions in a metric MDS solution should account for a large part of the variance in the dissimilarities data. Dimensions associated primarily with error should account for very little variance.

Scree Plot for Metric MDS Solution

Graph of eigenvalues versus order of extraction in metric MDS of distances between each and every eigenvalue.

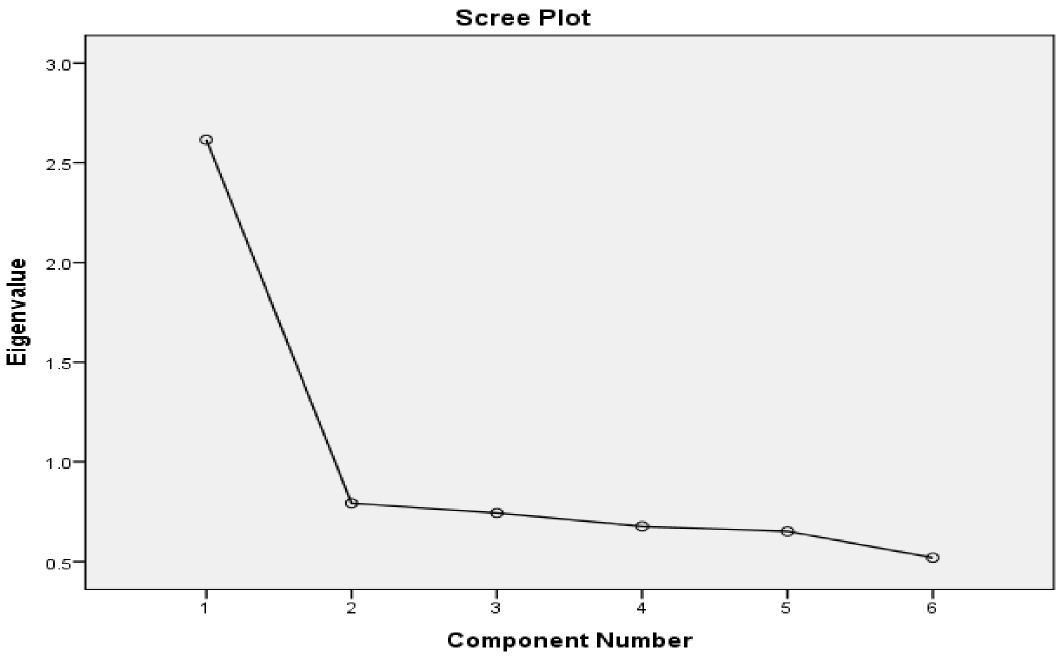


Figure 5.1 Scree Plot Result

Goodness of Fit Measure for Metric MDS

Eigenvalues measure variance associated with each dimension of the MDS solution Sum of first m eigenvalues relative to sum of all q eigenvalues (usually q = k):

Fit = $\frac{\sum_{i=1}^{m}\varphi^2}{\sum_{i=1}^{q}{\varphi^2}_i}$ Here, first eigenvalue is 2.616 and the sum of the eigenvalues is 6.

Fit = $\frac{2.616}{6} = 0.436$

= 43.6%.

5.6: Relationship between students confidence and their actual performance in procedural and conceptual mathematical problems.

To answer the fourth research question, firstly the actual performance of students in procedural and conceptual mathematics problems was addressed. May reference be made to Table 5.18.

Table 5.18: Procedural Understanding Versus Conceptual Understanding

Procedural Understanding Questions	Conceptual Understanding Questions
(1) $\frac{1504}{12096} \times 100\% = 12.4\%$ (3) $\frac{1188}{12096} \times 100\% = 9.8\%$ (5) $\frac{1504}{12096} \times 100\% = 12.4\%$	(2) $\frac{1264}{13764} \times 100\% = 9.2\%$ (4) $\frac{1267}{13764} \times 100\% = 9.2\%$ (6) $\frac{1743}{13764} \times 100\% = 12.7\%$
Total Correct Percentage of Procedural Understanding $= 12.4\% + 9.8\% + 12.4\% = 34.6\%$	Total Correct Percentage of Conceptual Understanding $= 9.2\% + 9.2\% + 12.7\% = 31.1\%$
Procedural Understanding Questions	**Conceptual Understanding Questions**
(1) $\frac{1504}{5292} \times 100\% = 28.4\%$ (3) $\frac{1188}{3024} \times 100\% = 39.3\%$ (5) $\frac{1504}{3780} \times 100\% = 39.8\%$	(2) $\frac{1264}{3780} \times 100\% = 33.4\%$ (4) $\frac{1267}{3780} \times 100\% = 33.5\%$ (6) $\frac{1743}{6204} \times 100\% = 28.1\%$
Total Correct Percentage of Procedural Understanding $= \frac{28.4\% + 39.3\% + 39.8\%}{3} = \frac{107.5}{3} = 35.8\%$	Total Correct Percentage of Conceptual Understanding $\frac{33.4\% + 33.5\% + 28.1\%}{3} = \frac{95}{3} = 31.7\%$

<u>Procedural Understanding Questions</u>

Total Marks for Question one (1) = 5 292

Total Marks for Question three (3) = 3 024

Total Marks for Question five (5) = 3 780

Total Marks = 12 096

Percentage Marks Scored for Question one (1): $\frac{1\,504}{12\,096} \times 100\% = 12.4\%$

Percentage Marks Scored for Question three (3): $\frac{1\,188}{12\,096} \times 100\% = 9.8\%$

Percentage Marks Scored for Question five (5): $\frac{1\,504}{12\,096} \times 100\% = 12.4\%$

Total Correct Percentage $= 12.4\% + 9.8\% + 13.9\% = 34.6\%$

Hence or otherwise procedural performance in percentage was 34.6% and the procedural fail percentage was 65.4%.

Conceptual Understanding Questions

Total Marks for Question two (2) $= 3\ 780$

Total Marks for Question four (4) $= 3780$

Total Marks for Question six (6) $= 6\ 204$

Total Marks $= 13\ 764$

Percentage Marks Scored for Question two (2): $\frac{1\ 264}{13\ 764} \times 100\% = 0.0918337692 \quad \times 100\%$

$= 9.2\%$ to 2 s.f

Percentage Marks Scored for Question four (4): $\frac{1\ \ 267}{13\ 764} \times 100\% = 0.0920517291 \quad \times 100\%$

$= 9.2\%$ to 2 s.f

Percentage Marks Scored for Question six (6): $\frac{1\ 743}{13\ 764} \times 100\% = 12.7\%$

Total Correct Percentage $= 9.2\% + 9.2\% + 12.7\% = 31.1\%$

Hence or otherwise conceptual pass performance in percentage was 31.1% and the conceptual fail percentage was 68.9%. In the aforesaid study, it was found that students' confidence on conceptual items was at (average 65.5%) significantly with a p-value (p=0.006) which was higher than their confidence levels for procedural items which was at (average 62.5%). As from the above working it shows that first year students at UNZA performed well in procedural questions as compared to conceptual ones. However, the table below outlines the performance of students from the test which they were given.

Table 5.19: Marks Obtained by First Year Students at UNZA in Procedural and Conceptual Oriented Questions

Marks Obtained	0-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
Procedural Understanding	61	68	34	51	55	24	24	19	17	25
Conceptual Understanding	69	72	72	40	45	21	26	21	9	3

It shows that we had 25 students having A^{+} in procedural questions as compared to conceptual were we only had 3 students. Furthermore, for A we had 17 students in procedural questions and in conceptual we had only 9 students. To add on, between 0-10 marks in procedural questions we had 61 students whilst the same range we had 69 students in conceptual oriented type of questions. To sum up, between $11 - 20$ we had 68 students for procedural questions and 72 for conceptual. Additionally, between 21-30 marks it was observed that for procedural performance we had 34 while conceptual performance we had 72 students. The table above revealed that students performed well in procedural questions as compared to conceptual oriented questions.

Table 5.19 shows that; we had 164 students passing out of 378 in procedural questions or scoring above 40% as compared to 125 students passing out of 378 in conceptual questions. The results showed that students performed well in procedural questions at first year level in mathematics at the University of Zambia.

Table 5.20: Mathematics MAT 1100 Test Results 2016-2017 Academic Year for the Research done at Natural Sciences Mathematics Department

Grades	90-100 A^{+}	80-89 A	70-79 B^{+}	60-69 B	50-59 C^{+}	40-49 C	35-39 D^{+}	0-34 D	NE	TOTAL
Male	8	10	19	22	13	33	2	133	1	241
Female	1	4	8	5	11	5	2	102	1	139
Total	9	14	27	27	24	38	4	235	2	380

The results as recorded by the researcher from the test given to subjects for the project. The results shown in table 5.20 were not moderated. This shows the actual performance of first year students in mathematics. Notwithstanding the performance of students in mathematics correlates greatly with the understanding of concepts. Students usually solves problems in mathematics correctly when they have atleast understood the underlying concepts and procedures in the questions. As from the results captioned it reveals that more male students did well as compared to the female. In this regard, we observe that we had 8 male students scoring a grade of A^+ as compared to only one female candidate scoring the same grade. The quantity pass percentage (C $to\ A^+$) as defined by the researcher; for males it was 43.6% as compared to the female which was 24.5%. Additionally, the quality pass percentage (B $to\ A^+$) as defined by the researcher for both male and female was found to be 13.2% of the total which was very low. The quality pass percentage for males was 24.5% as compared to females which was 12.9%. As from the results, we may state that the pass percentage for males and females is below average and also the quality pass to enable a student enter a course of his/her choice is very low 24.5% for males and 12.9% for females. The results further reveals that we had a good number of students failing with very low marks since the number of D^+ was very small for both the male and female students.

Furthermore, the results shows that in all the grades the number of males passing was more than the number of females. The participants had a right to withdraw from the research hence I did not force them to write hence the number reduced by two. The sample initially was 380 but due to the two none examinable students the sample reduced to 378 participates. Since the pass rate was small as shown above it means students failed because they did not understand mathematics concepts adequately. This is so because the same research on confidence levels of students to answer the questions most of them indicated that they were confident to answer the questions correctly without any difficulties.

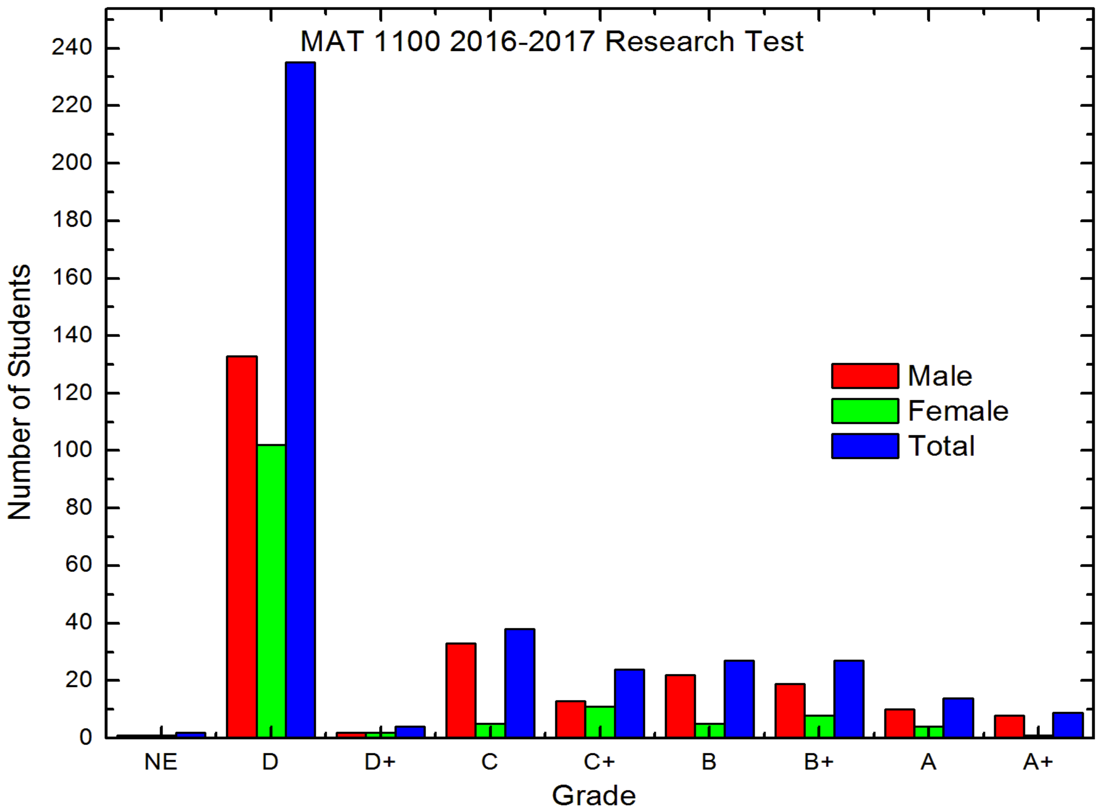


Figure 5.2

As from Figure 5.2, it reveals that more students had a D grade in the test which the researcher had given to the students. Many students had very low marks that is why we had very few candidates in D^+grade. The fail percentage for both male and females was adding up to 62.9%. Since the sample picked was 25.3% of the actual population then it means that it was highly representative of the population. Henceforth, we may be allowed to general our results as the true reflection of the total population. This reveals that most of the students passes final examination in first-year mathematics after moderation of results. The moderation of results were not done in this study hence the results remained unchanged showing a very big failure rate. In general, there is always moderation of results which helps a good number of students to make it to the next level of university education.

Table 5.21: Mathematics MAT 1100 Test Results 2016-2017 Academic Year for Research done at UNZA Students obtaining less than 10%

Marks	9	8	7	6	5	4	3	2	1	0
Males	2	0	4	3	0	1	4	0	2	1
Female	9	0	7	1	0	2	5	0	5	0
Total	11	0	11	4	0	3	9	0	7	1

Despite the fact that UNZA at SNS number of female taking mathematics is small, after writing the test of first year students it was revealed that there were more females under performaing than males. As from the Table 5.21 calculating the percentage for females scoring less than 10% plus those who were absent was at 21.1% whilst for males it was 7.5%. Hence, we may wish to conclude that performance in test directly corresponds to the understanding of mathematics concepts. Therefore, the understanding of concepts in mathematics for females is very low as compared to the male students.

5.6.1 Findings from the students (Answers to the given test)

The findings from the students has been analysed step by step as demonstrated in this chapter of the current study. This is to give the descriptive outline of the test result as obtained in actual form. The following component demonstrates how test results were discussed in my study.

5.6.1.1 Introduction

The researcher wish to illustrate some examples of student solutions obtained in the given test. Nonetheless, the study was quantitative in nature. Hence or otherwise, we wish to quantify the number of students who used the method correctly in the study. However, we counted the number of candidates who either answered part of the question correctly or got the entire question correctly. Then we had to find the percentage of each component of the question from the given test. In this quantitative analysis of the solution. I used the test rubric in the allocation of marks to each specific concept.

1. (a) Find the 5th term in the expansion of $\left(x^3 - \frac{1}{2x}\right)^6$ [4]

In this task of the experiment, with the help of expert judgement by UNZA mathematics lecturers the first question was picked to stand for procedural type of question. The researcher measured specifically procedural understanding of the n^{th} term of the Binomial Expansion because it was the specific concept were students had difficulties. However, 188 out of 378 got the question correctly and it was solved by one of the students as shown in the figure below:

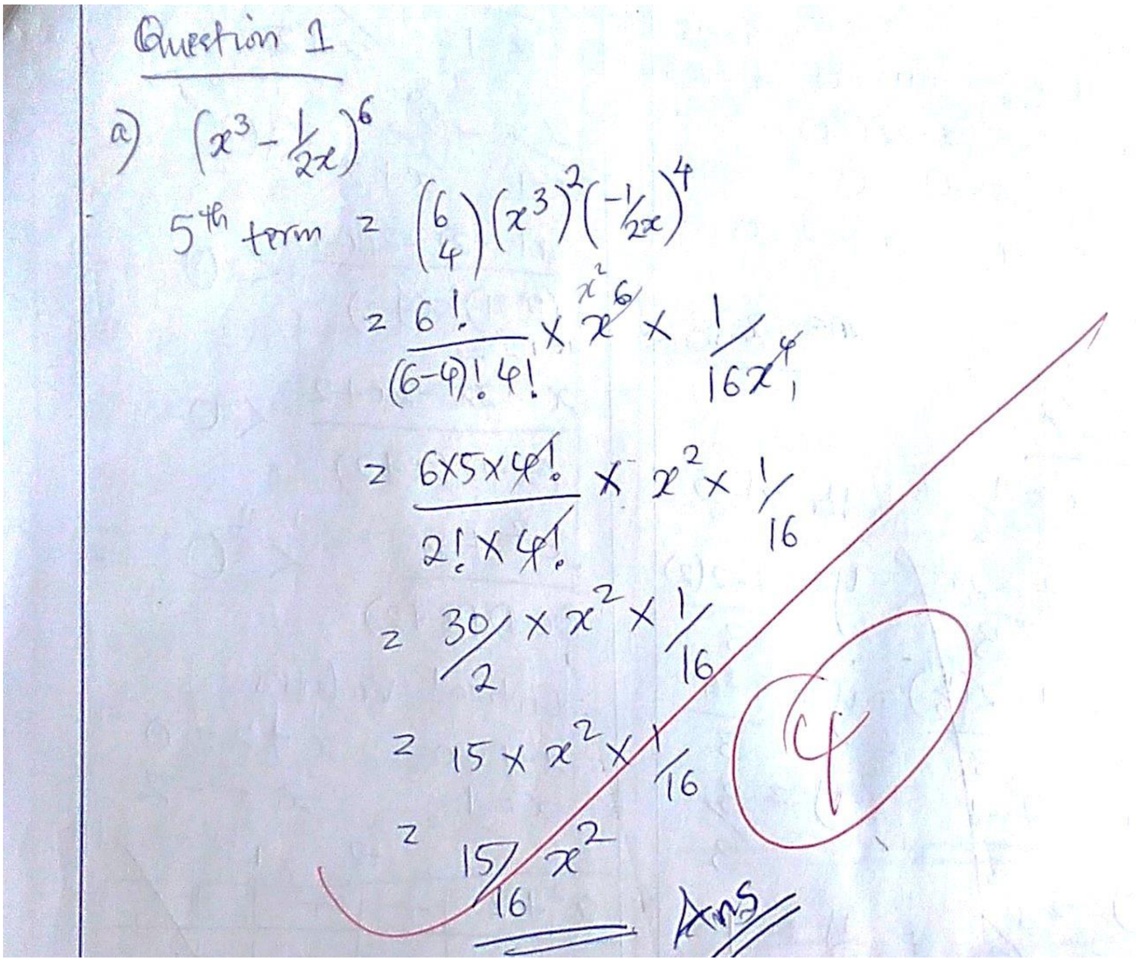


Figure 5.3 Student 1 Work Sample of Marked Question 1.a.

It was observed that a good number of candidates got the question correctly. However, about 49.7% of the candidates who wrote the test got the question correctly. This number included those who got from 1 out of 4 upto those who got 4 out of 4. The results revealed that only 132 out of 188 students got the whole question correctly. The results further revealed that 132 out of a total of 378 got all the marks correctly. This meant that students about 34.9% of students who wrote the test got question (1.a) correctly. Hence, the difference of 14.8% was observed of the students who got everything from those who got either everything or part of the question correctly. The results showed that students had challenges of finding the specific concept of the n^{th} term of the Binomial Expansion after a test was given to first year students at UNZA. In this task, which we had considered to be a procedural task, we were expecting students to realize that; firstly there were supposed to master the formula for finding the nth term of the Binomial Expansion. The study revealed that few students were able to master the formula of finding the nth term of the Binomial Expansion. Students had a bit of skills in finding the nth term of the Binomial Expansion because a good number of those who had attempted to answer the question got above average from the mentioned question. Subsequently, we may wish to state that atleast students understood a bit of specific concepts of finding the the nth term of the Binomial Expansion.

(b) Solve the simultaneous equations

$$2x^2 + 3y^2 + x = 13, \quad 2x + 3y - 7 = 0 \qquad [5]$$

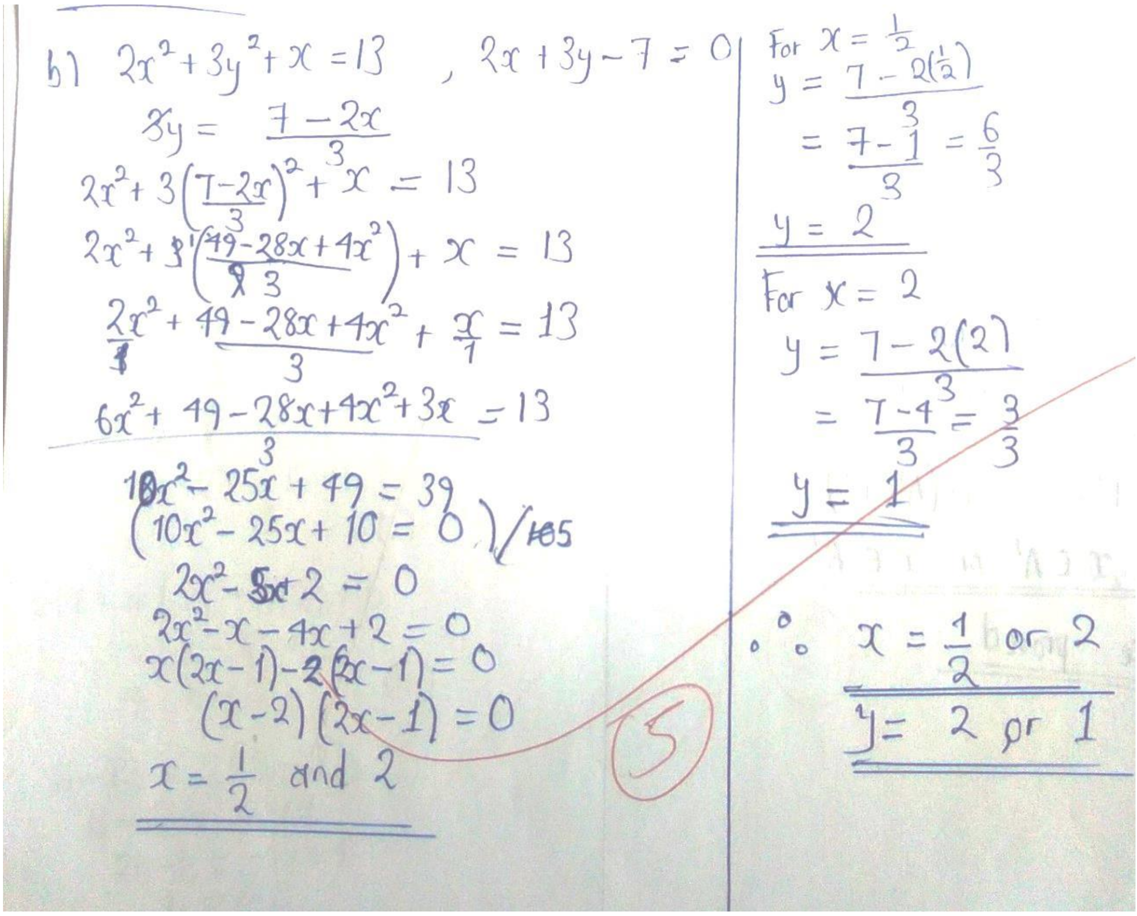


Figure 5.4 Student 2 Work Sample of Marked Question 1.b.

Although, the question was a procedurally task. It was observed that most of the students had difficulties of mastering procedural skills of solving a simultaneous equation in one linear and one quadratic. On the other hand, it was revealed that only 118 students out of 378 got either part the question or the entire question correctly. This meant that 31.2% of the students got something out of the question on simultaneous equation of one linear and one quadratic.

Furthermore, it was revealed that 104 out of 118 students got the entire question correctly. This meant that 27.5% of the students who wrote the test got everything correctly on question (1.b). The difference of students percentage of those who got everthing correctly and those who got part or everything correctly was 3.7% . This shows that atleast students had understood a bit of concepts on how to solve the simultaneous equation of one linear and one quadratic correctly.

(c) Solve the inequality $\frac{x}{x-1} < \frac{2}{x+2}$ [5]

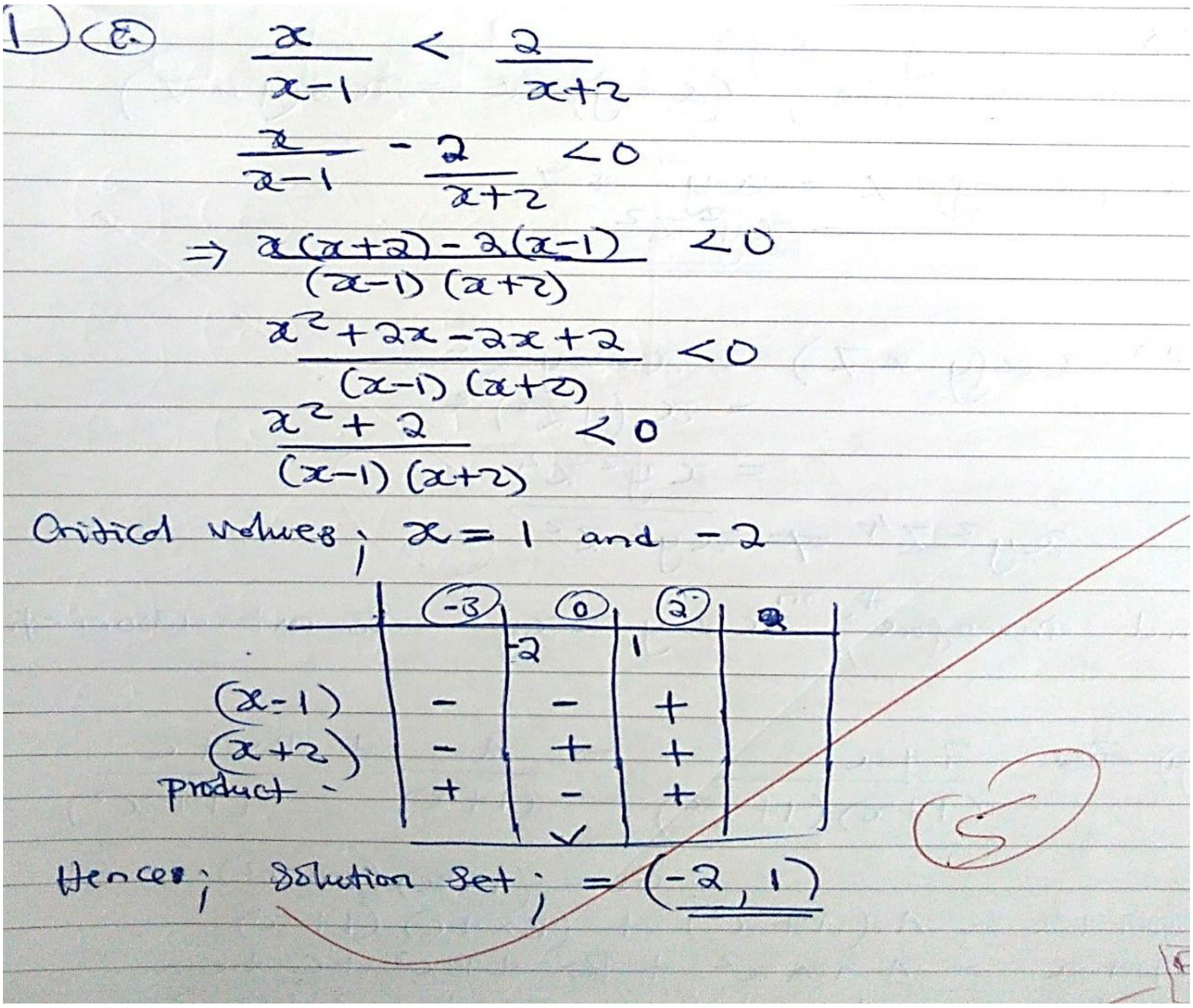


Figure 5.5 Student 3 Work Sample of Marked Question 1.c.

The solution above was gotten from student working who happened to have solved the question correctly. It was observed that 169 students got part of the question correctly while 119 of the students got everything correctly. In the same vain, it was observed that 44.7% of the students got either part of the question or the whole question correctly. Besides, the study revealed that 31.5% of the students got the entire question correctly on inequalities involving fractions. However, as from the statistics above, the study gave a difference of 13.2% between those who got everything correctly from those students who got part of the question correctly combining with those who got everything correctly. Nonetheless, it was revealed that atleast students had some competences of understanding specific concepts of solving the procedural question on inequalities involving fractions.

2. Prove that

$(a)\ (A \cap B)' = A' \cup B'$ [5]

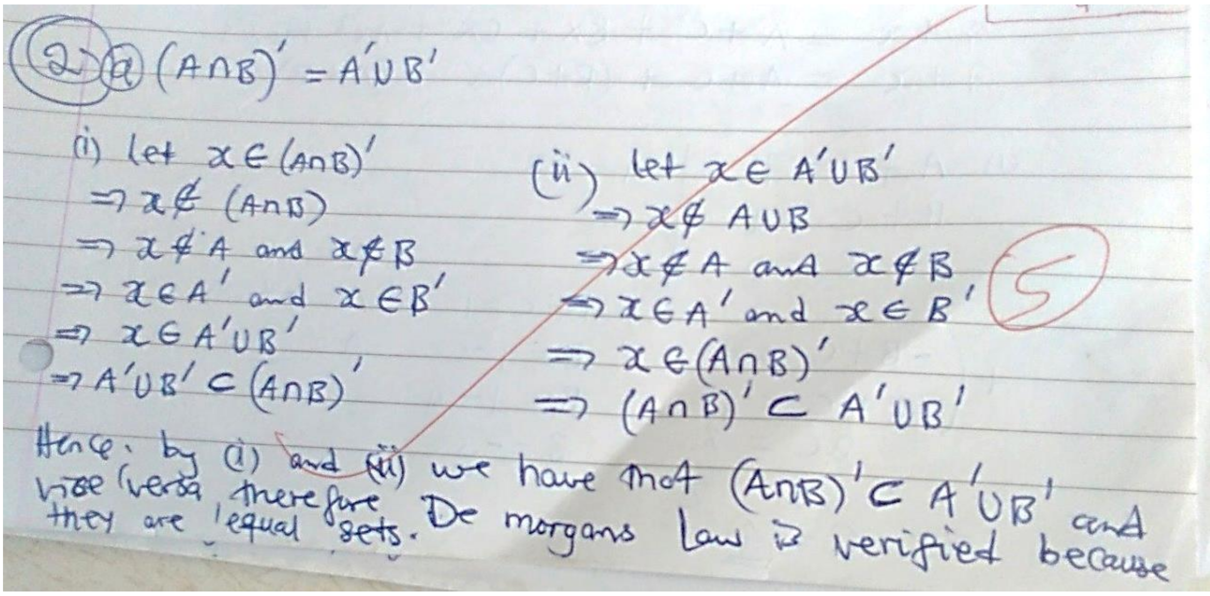


Figure 5.6 Student 4 Work Sample of Marked Question 2.a.

This task was designed to test the conceptual understanding of mathematics concepts, specifically De Morgans' laws on set theory. It was revealed that more than half of the number of students were solving only the first part of the question. A small number of students was able to solve the second part (the converse) of the problem. In this reverence, 67.5% of the candiadates who wrote the test got part of the question correctly. Normally, it was revealed that only 8.2% of the total (378 or 100%) of the entire students populance got everything correctly. However, this meant that 255 students out of 378 got part of the task correctly. On the other hand, 31 students out of 378 got everything correctly. This reveals that students had difficulties of understanding conceptual concepts of De Morgans laws on Set Theory. Futhermore, the results shows that the number or percentage for those students who got everything correctly to those who only got part of the task correctly was about $\frac{1}{8}$ of the total. Furthermore, the difference in percentage for the students who got part of the question correctly and also who got the entire question correctly was 59.3% meaning that there was a big difference in the marks for students getting everything and part of the question. This shows that students had challenges of understanding mathematical concepts of set theory. This meant that students had a lot of difficulties in solving the entire question on De Morgans laws correctly. However, it was further revealed that students had challenges of understanding conceptual components of mathematics.

(b) Determine whether or not the binary operation defined by

$x * y = xy^2$ on $\mathbb{R}$ is associative. [5]

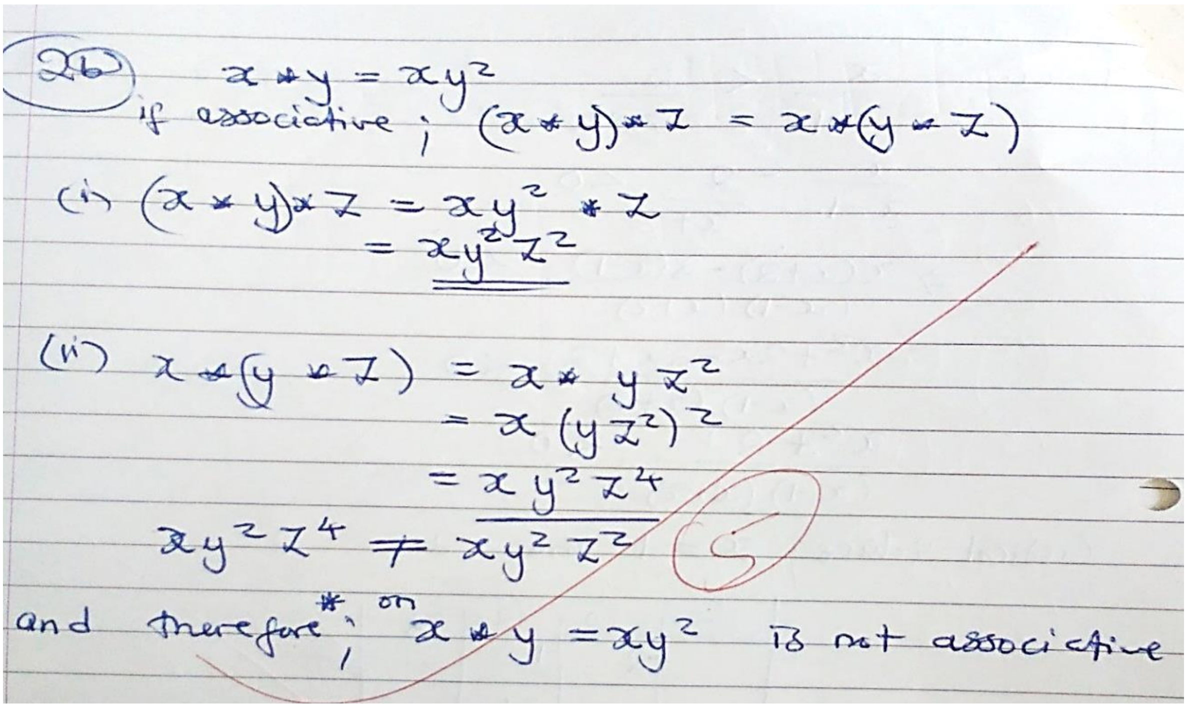
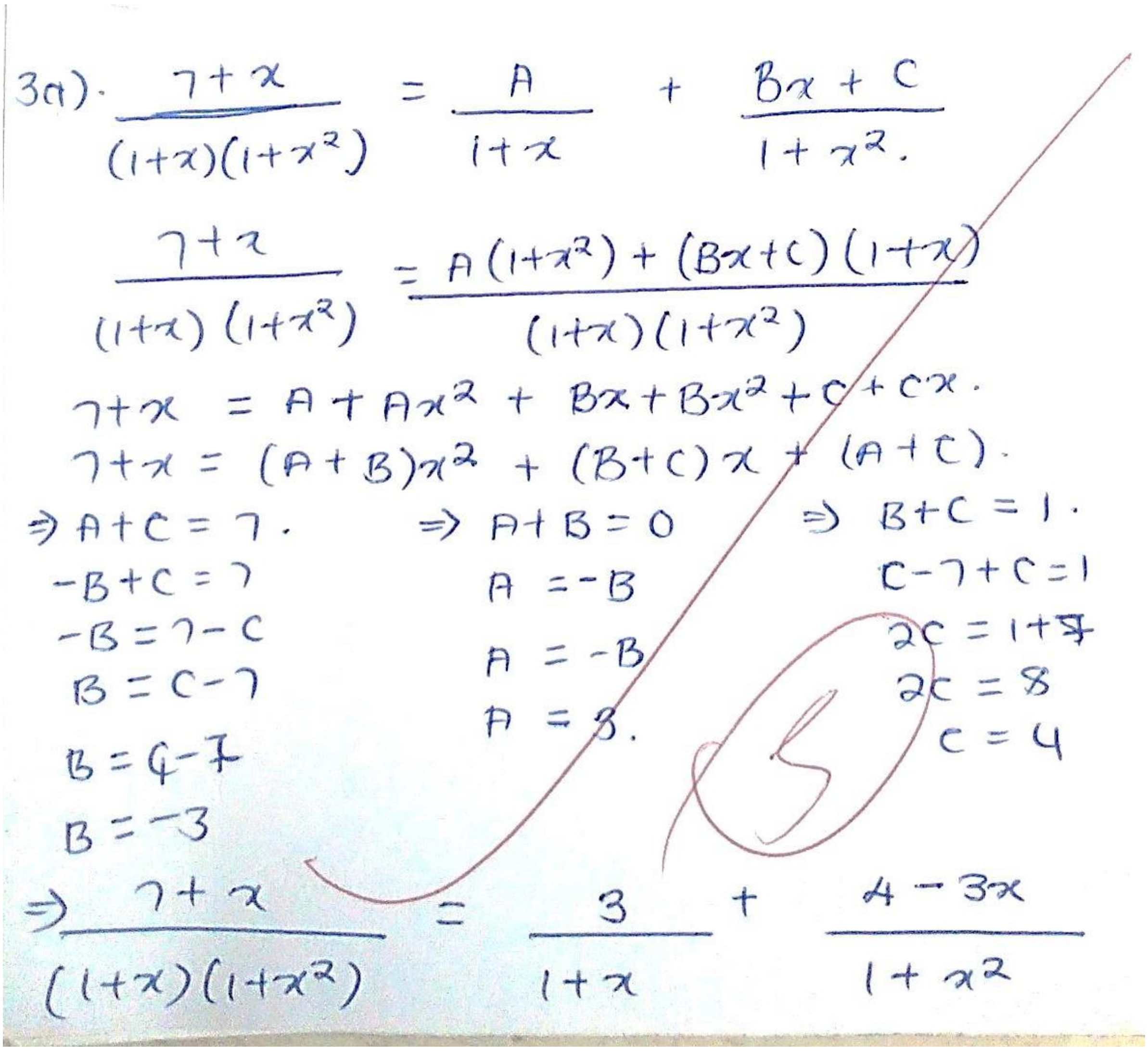


Figure 5.7 Student 5 Work Sample of Marked Question 2.b.

The solution above was gotten from one of the students who had solved the question correctly on conceptual component of the question. It was discovered that 216 students got part of the task correctly while 104 got the entire question correctly. This meant that about half of the number of the students who got part of the question correctly got everything correctly. As from the findings it was revealed that 57.1% of the students got part of the question correctly. Conversely, the findings showed that 27.5% of the students got everything correctly. It was revealed that the difference in percentage for students who partially got some marks and whose who got everything was 29.6%. meaning the difference was big. This showed that students did not understand the conceptual component of the question elaborated above. The results showed that students had challenges of understanding concepts of mathematics.

3 (a) Resolve $\frac{7+x}{(1+x)(1+x^2)}$ into partial fractions [5]



3a). $\frac{7+x}{(1+x)(1+x^2)} = \frac{A}{1+x} + \frac{Bx+C}{1+x^2}$

$\frac{7+x}{(1+x)(1+x^2)} = \frac{A(1+x^2)+(Bx+C)(1+x)}{(1+x)(1+x^2)}$

$7+x = A + Ax^2 + Bx + Bx^2 + C + Cx$

$7+x = (A+B)x^2 + (B+C)x + (A+C)$

$\Rightarrow A+C=7$ $\Rightarrow A+B=0$ $\Rightarrow B+C=1$

$-B+C=7$ $A=-B$ $C-7+C=1$

$-B=7-C$ $A=-B$ $2C=1+7$

$B=C-7$ $A=3$ $2C=8$

$B=4-7$ $C=4$

$B=-3$

$\Rightarrow \frac{7+x}{(1+x)(1+x^2)} = \frac{3}{1+x} + \frac{4-3x}{1+x^2}$

5

Figure 5.8 Student 6 Work Sample of Marked Question 3.a.

Question three (a) is a procedural component question gotten from the students script who got the entire question correctly. It was observed that 238 students got part of the question correctly while 162 got the entire question correctly. In this regard, it was observed that 63% of the students got part of the question correctly. On the other hand, 42.9% of the candidates got the entire question correctly. The difference in percentage of students who got everything and those who partially got something out of the question was 20.1%. This shows that the understanding of concepts of partial fractions (procedural understanding) of mathematical concept was good.

(b) The remainder when $x^3 - 5x + b$ is divided by $x + 3$ is twice the remainder when it is divided by $x - 2$. Find the value of b . [3]

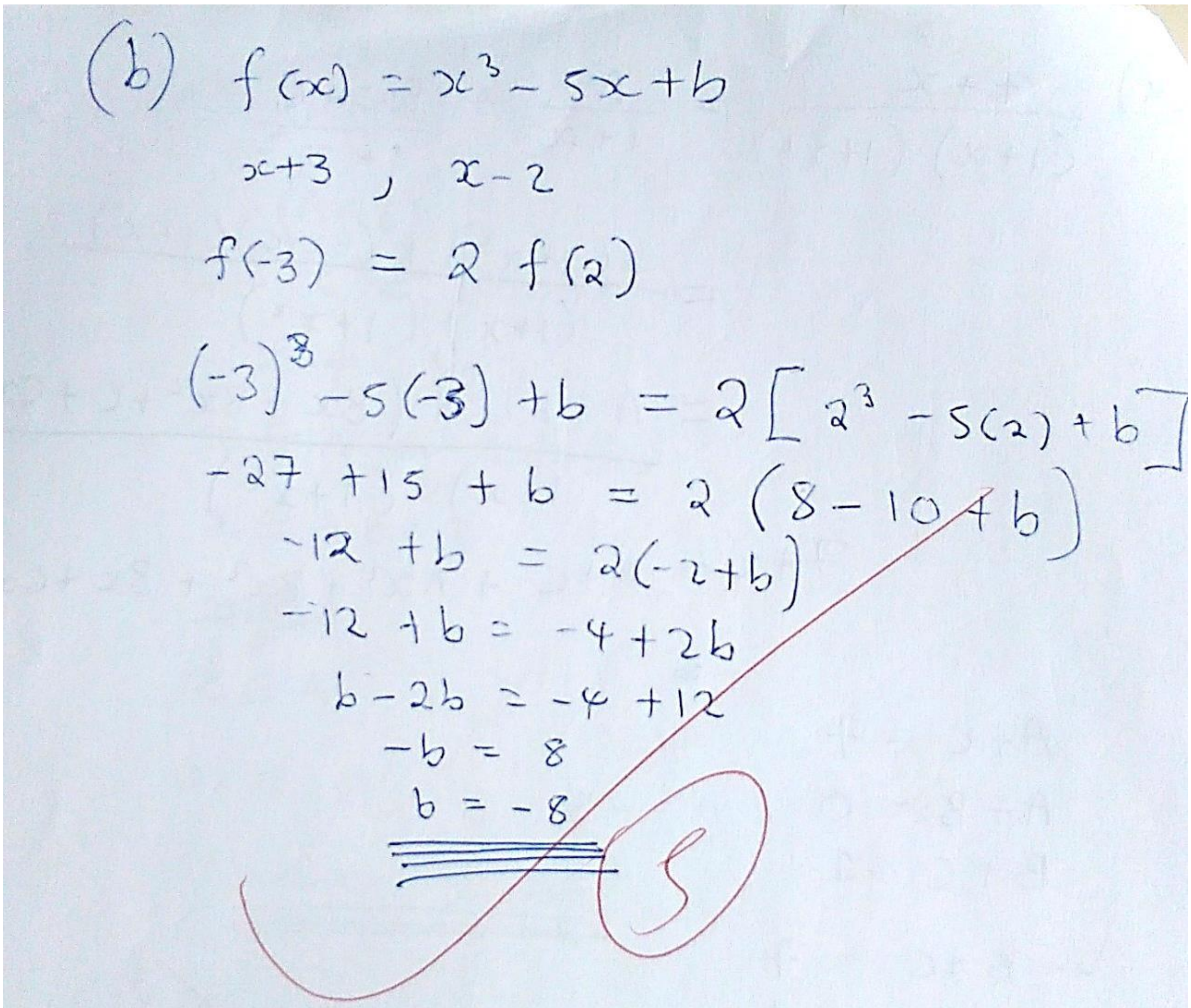


Figure 5.9 Student 7 Work Sample of Marked Question 3.b.

It was revealed that 103 of the students who had attempted the question got everything correctly. This cumulated to 27.3% as pass percentaged of the students on the procedural component of factor theory . This showed that the students had procedural understanding of the concept on factor theory. There was no difference at all in pass percentage. This was so because all the students who had attempted the question got the entire question correctly.

4. Find the domain and range of each of the following functions.

$(i)\ f(x) = \sqrt{3x - 4}$ [2]

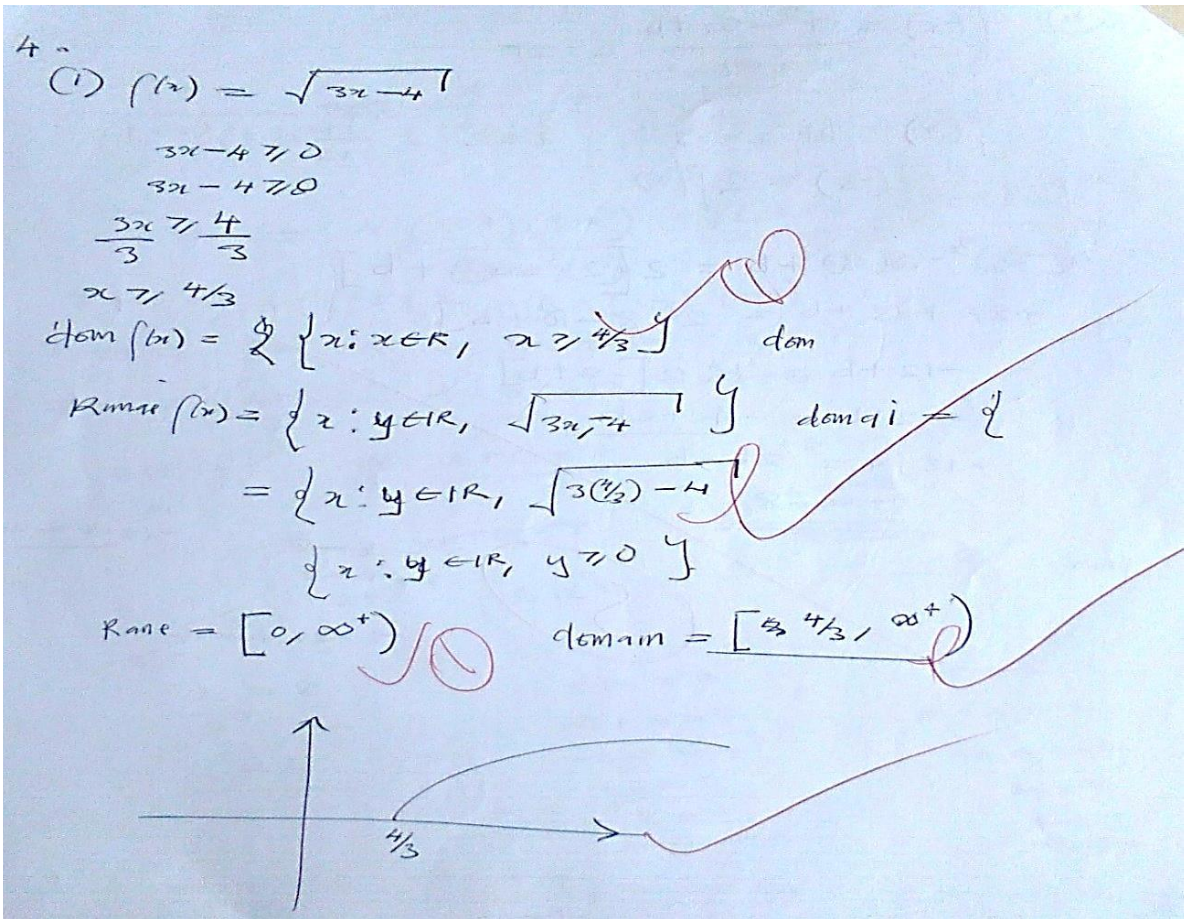


Figure 5.10 Student 8 Work Sample of Marked Question 4.i.

The task captioned above shows the working of one of the students who had gotten everything correctly on the conceptual question. The task sought to determine the mathematical understanding students had of concept under functions (Domain and Range). Most of the time, it was observed that students had a lot of difficulties in finding the range of the function. It was revealed that a good number of students only managed to determine the domain of the function and failed to find the range of the function. In this task, 234 students only managed to get part of the question correctly while 57 managed to get the whole question correctly. Nonetheless, it was observed that, 61.9% of the candidates only managed to get part of the question correctly while 15.1% managed to get the entire question correctly. It was further revealed that the difference between the candidates with partially correct and those who got everything correct was 46.8% . This showed a good number of students had difficulties in solving the whole questions correctly. However, the study revealed that students had low conceptual understanding of concepts in finding the domain and range of functions.

(ii) $f(x) = x^2 - 2$ [2]

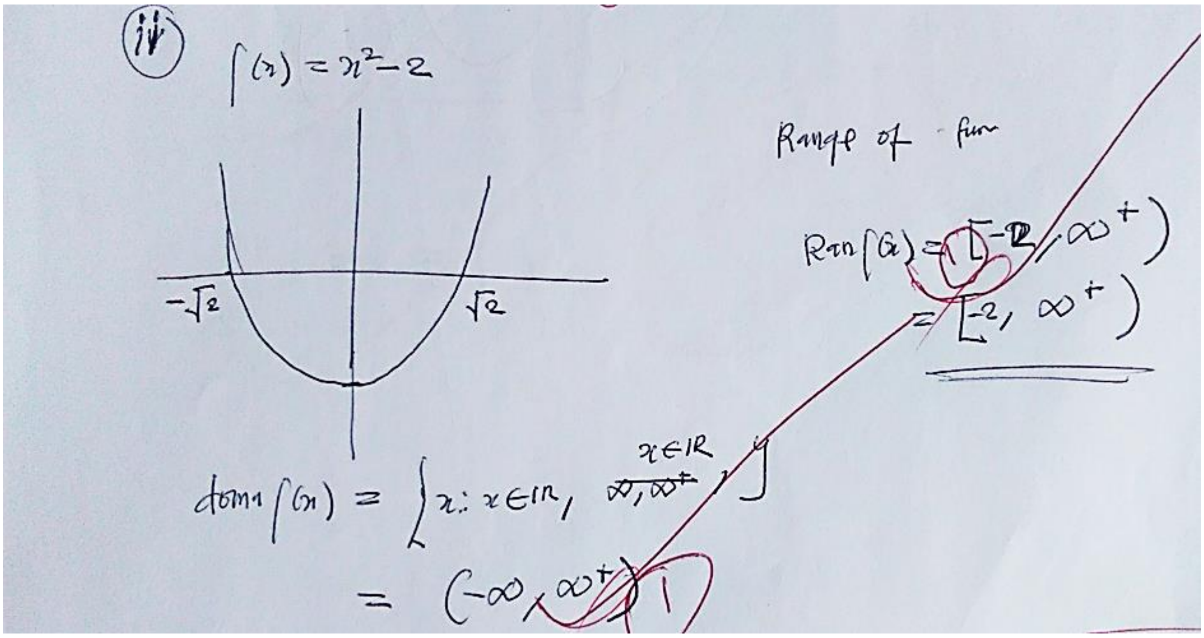


Figure 5.11 Student 9 Work Sample of Marked Question 4.ii.

The study revealed that students had a lot of difficulties in the understanding of conceptual components of mathematics under functions. Most of the students tried a bit to find the domain and failed to determine the range of the functions. Then again, a good number of students failed even to draw the graph of the function in order for them to find the domain and range of the functions easily. The study showed that 243 students got part of the question correctly while 24 candidates got everything correctly. This showed that students did not understand the conceptual concepts of functions. To add on, it was further revealed that 64.3% of the students partially got the question correctly while 6.4% of the learners got everything correctly. The difference of students percentage of having everything correctly from those who partially obtained some questions correctly was 57.9%. This shows that students had a gigantic problem of understanding conceptual concepts under functions. It was observed that the component were a good number of candidates got the task wrong was on finding the range of the function. A big number of students had difficulties in sketching the domain and the range of the function. From this study, it was revealed that only a small number of students were able to draw correctly the sketch graph of the range of functions.

(iii) $f(x) = -\sqrt{x}$ [3]

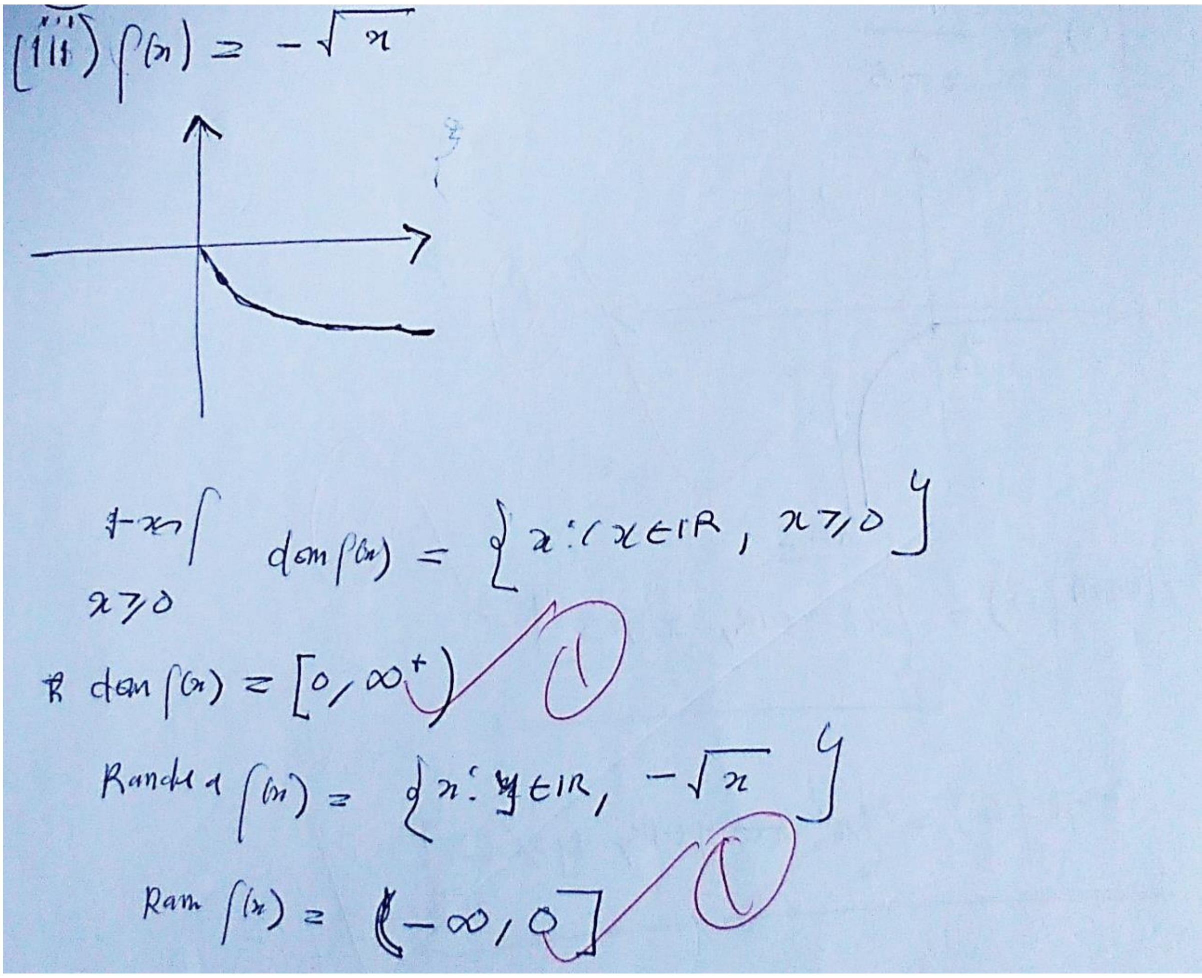


Figure 5.12 Student 10 Work Sample of Marked Question 4.iii.

It was observed that a small number of students got the entire question correctly. This was so, because only 8 out of 378 got the entire question correctly. To add on, we had 198 students getting part of the question correctly. However, the results showed that students had challenges of understanding conceptual components of functions.

Furthermore, it was revealed that 52.4% of the students got part of the question correctly while only 2.1% of the learners got everything correctly. The percentage difference of students who got everything correctly and those who got part of the question correctly was 50.3%. This was a very big difference to show that a large number of learners had difficulties in the understanding of conceptual components of mathematics (functions-domain and range). A good number of students failed to draw the graphical representation of the domain and range of functions. It was further, observed that students had difficulties especially determining the range of the function either using set builder notation and or interval notation.

$(iv)\quad f(x) = \frac{1}{x-3}$ [3]

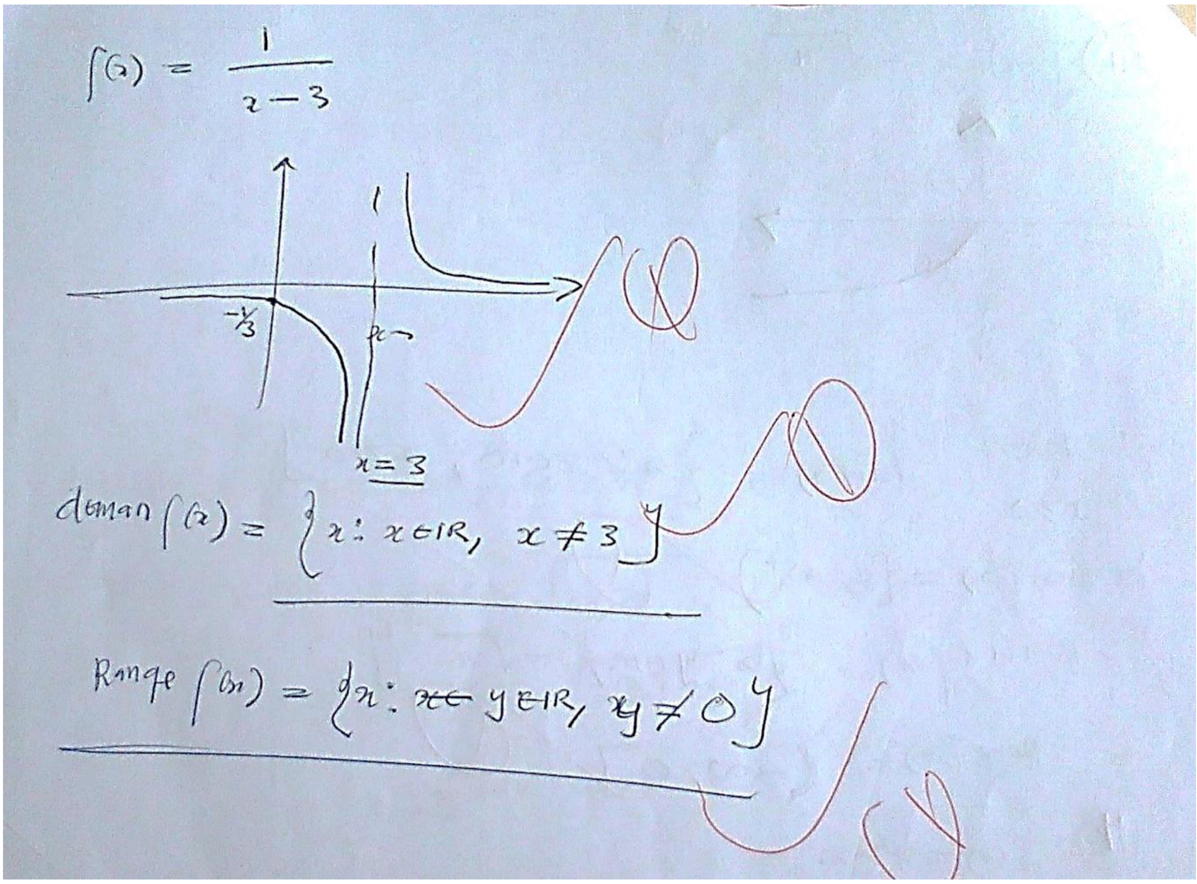


Figure 5.13 Student 11 Work Sample of Marked Question 4.iv.

The study revealed that 223 students atleast got something correctly . Furthermore only 8 students got everything correctly meaning that a big number of 215 did not get everything correctly. The study revealed that very few students were able to find the solution using the diagram. However, 59% of the students got part of the question correctly and 2.1% got everything correctly. The results showed that there was a difference of 56.9% between the percentage of students who got part of the question correctly and those who got the entire task correctly. These results revealed that students had a lot of challenges of finding the domain and range of functions. We may wish to state that students did not understand fully the concepts of functions. The conceptual component of the function was not adequately addressed by the learners. Out of all the questions on functions it was observed that the fourth question was the one where many students did not fully understood the conceptual component of the manipulations which were involved in solving the task correctly.

5. Prove that

(a) $\frac{1}{\tan^2\theta+1} + \frac{1}{\cot^2\theta+1} \equiv 1$ [5]

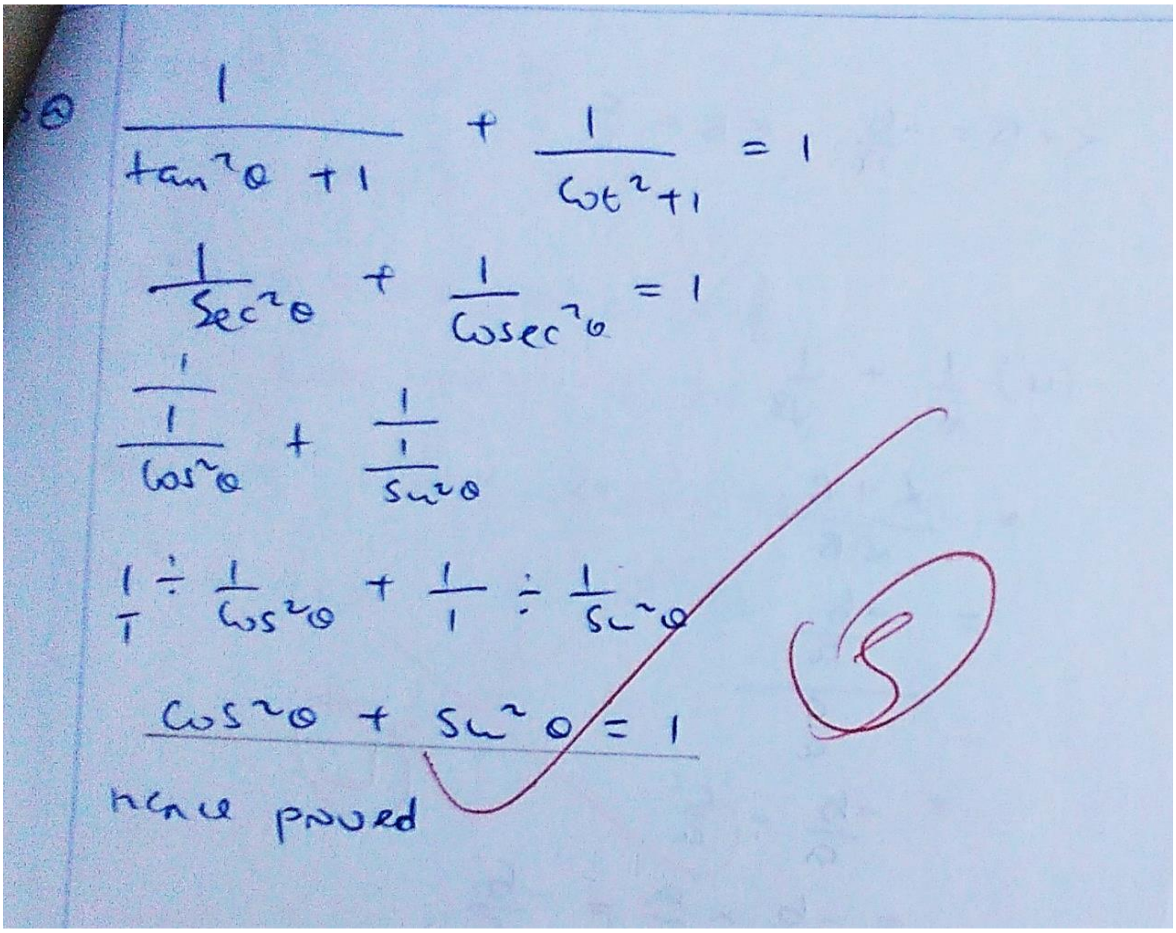


Figure 5.14 Student 12 Work Sample of Marked Question 5.a.

The solution was obtained from one of the students who had solved the question correctly. However, it was revealed that 254 students out of 378 got part or everything correctly of the entire question. Furthermore, the study showed that 250 students got everything correctly. It was revealed that the difference between students who got everything and part and or everything correctly on this part of the task was only four students. Nevertheless, about 67.2% of the students either got part or everything correctly while 66.1% of the students got all the entire question correct. This meant that the difference in percentage in terms of performance was 1.1%. However, the results shows that students performed well on this specific concept of solving the trigonometric identity. We may wish to state that, students understanding of the concept of this task was overwhelming.

(b) Prove the identity $\sqrt{\left(\frac{1-\sin x}{1+\sin x}\right)} \equiv \sec x - \tan x$ [5]

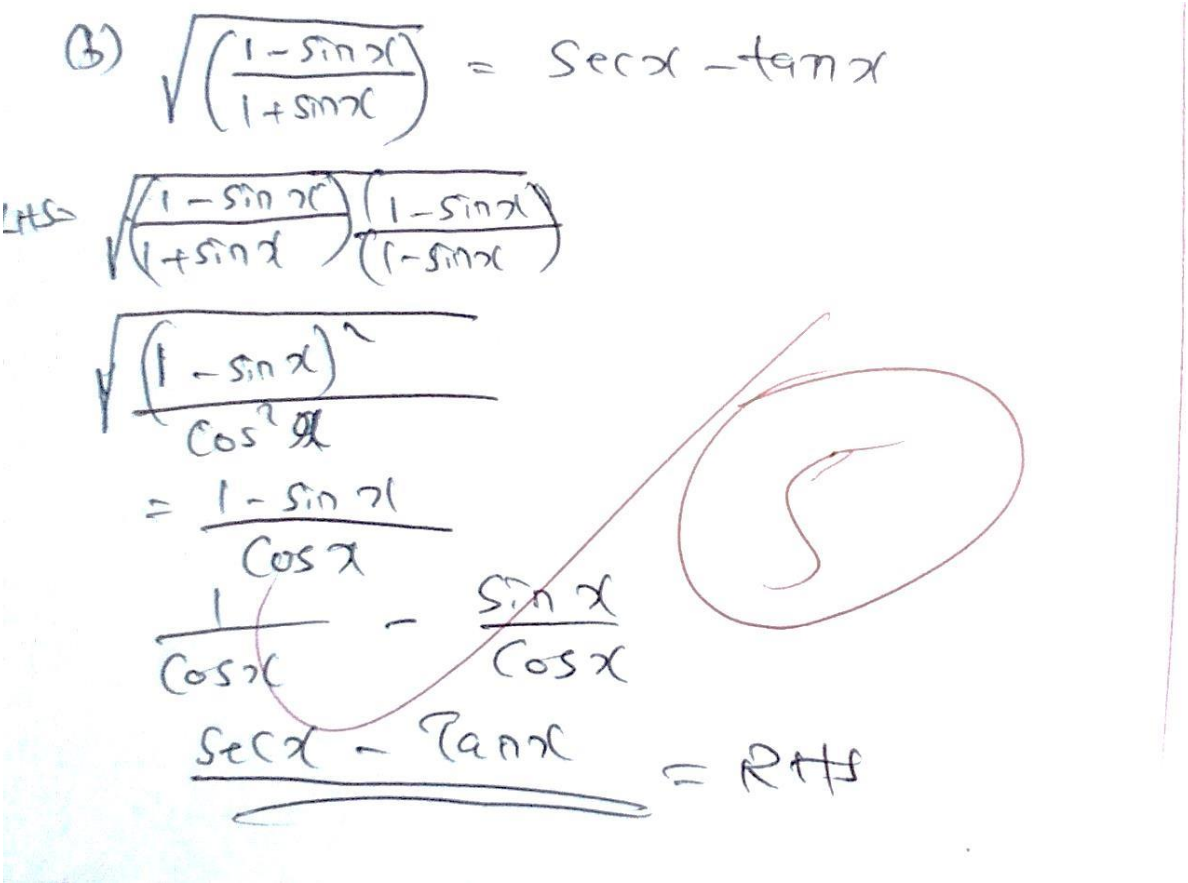


Figure 5.15 Student 13 Work Sample of Marked Question 5.b.

It was observed that 112 students either got part or every question correctly and 104 students got the entire question correctly. This meant that about 29.6% of the students got either part or the entire question correct while on the other side 27.5% got the entire question correctly. Likewise, looking at the difference in performance concerning percentage it was revealed that there was a small difference of only 2.1%. This showed that the understanding of the concept by students was a bit okey for this particular concept of trigonometric identity. Eventhough, the performance or the understanding of the concept in question 5.b was low as compared to the concept in 5.a. Correspondingly, it was revealed that the difference in understanding between concept in (5.a) to that of (5.b) was twice in magnitude as compared to the other one.

6. (a) If α and β are the roots of the equation $ax^2 + bx + c = 0$.

Obtain in terms of a, b and c the expression

(i) $\alpha + \beta$ [2]

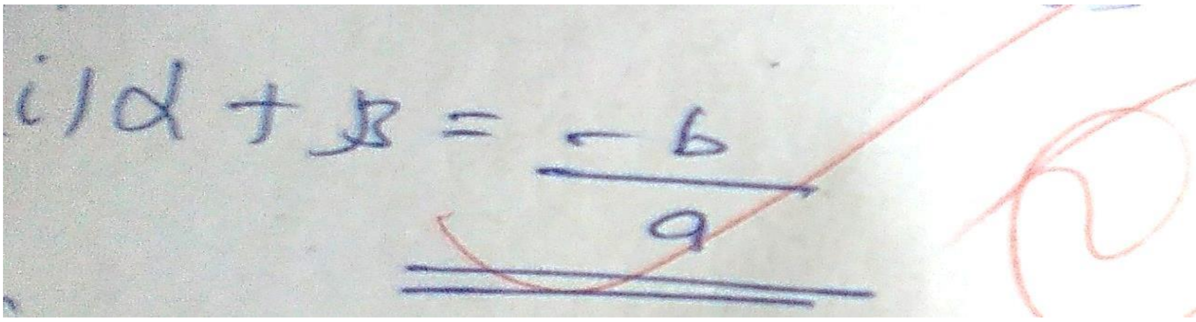


Figure 5.16 Student 14 Work Sample of Marked Question 6.a.i.

Even if, the question was conceptual in nature; repeated solving of the above task tends to be solved as if it is a procedural question. Nevertheless, it was revealed that 228 students got everything correctly while 232 students got something out of the entire marks. The difference in percentage for the students getting something and everything correctly was 1.1%. Students performance was encouranging. This clearly indicated that the conceptual understanding of the concept above was good.

$(ii)\ \frac{1}{\alpha}\ +\frac{1}{\beta}$ [2]

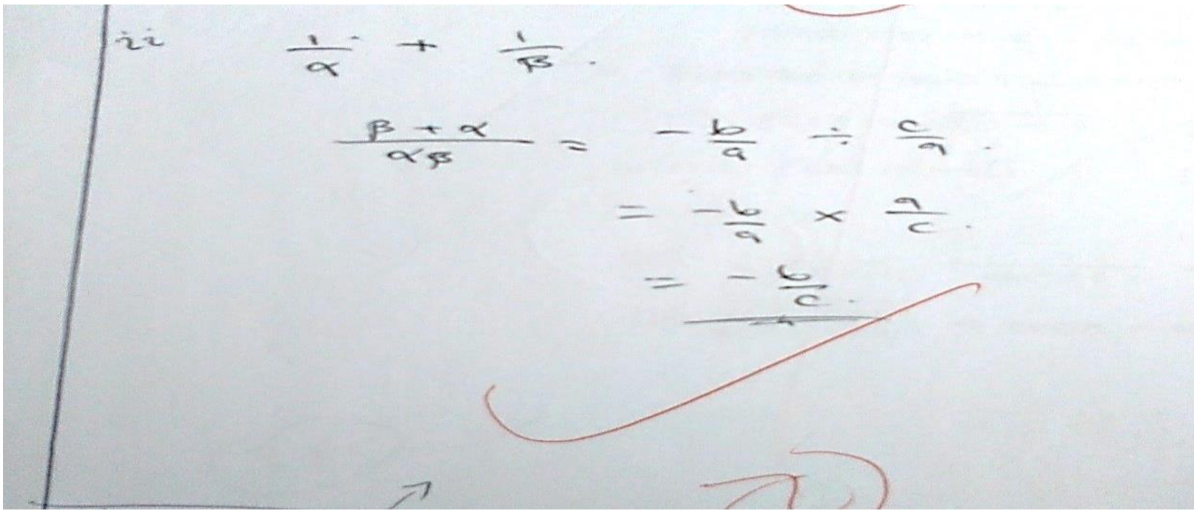


Figure 5.17 Student 15 Work Sample of Marked Question 6.a.ii.

The task was conceptual in nature and the performance of the students was good. It was revealed that 195 students out of 378 got the entire question correctly. However, 211 students out of 378 got something correctly. The performance of students getting everything correctly was 51.6% and for those getting atleast something was 55.8%. The difference of those who got everything correctly and those who got something was 4.2%. Nonetheless, the results above shows that the performance of conceptual concept on this task was good.

$(iii)\ \alpha^2 + \beta^2$ [2]

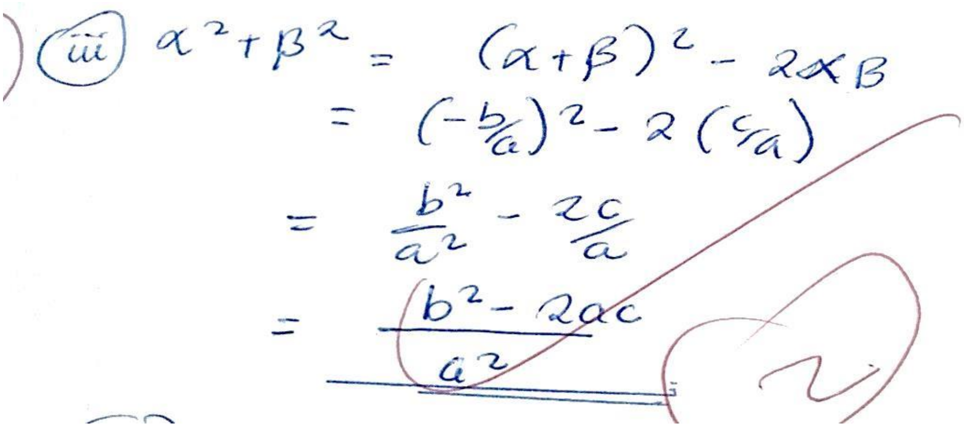


Figure 5.18 Student 16 Work Sample of Marked Question 6.a.iii.

It was revealed that only 120 students got the entire question correctly on this component of the question. Furthermore, 122 students got atleast something correctly. The difference in percentage of the students who got everything correctly and whose who atleast got something was 0.5%. However, the percentage pass on the captioned task was 31.8% (students recorded everything correctly). Notably, those students who got atleast something was 32.3%. Furthermore, the illustration above shows that students performance of the captioned task was below average. Therefore, it clearly indicates that the understanding of the concept in this instance which is conceptual was low.

$(iv)\quad \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ [2]

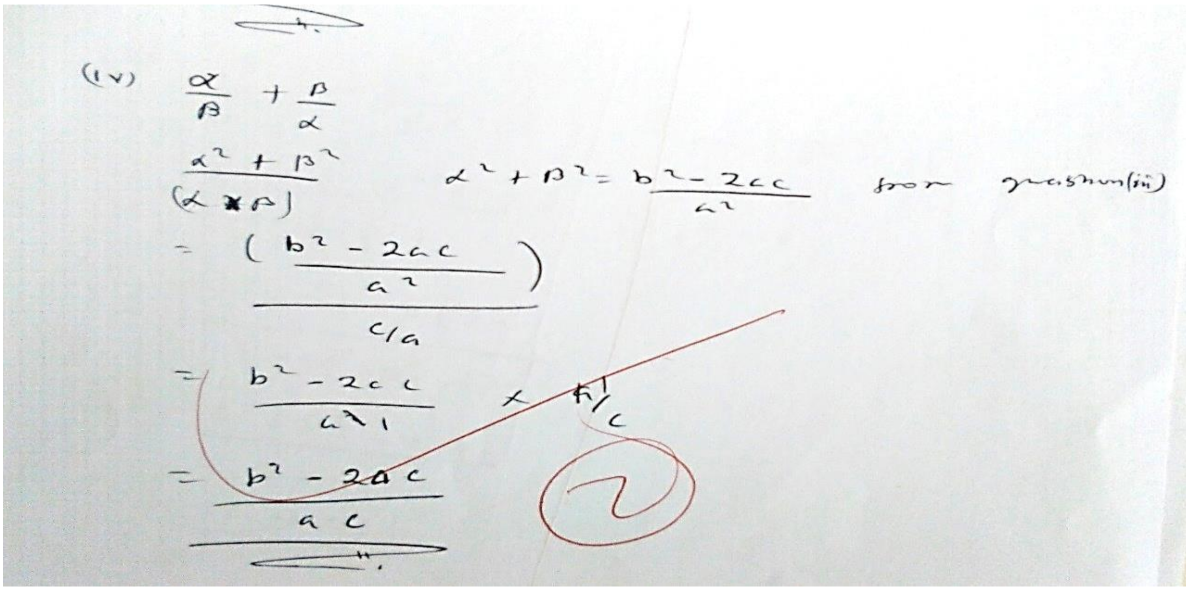


Figure 5.19 Student 17 Work Sample of Marked Question 6.a.iv.

The captioned solution was achieved by 83 students out of the total of 378 first year students doing mathematics in the school of natural sciences at the University of Zambia. By and large, 84 students got something out of the the total of 378. Even though the difference in percentage was very low of 0.2% it revealed that the performance of the captioned concept was very low. Ideally, we may wish to conclude that the understanding of the conceptual concept of the captioned task was low. Meaning that students had a very low understanding capacity of conceptual concept of the already captioned question.

(b) Find identical roots of the equation $(x + k)^2 = 2 - 3k$ [5]

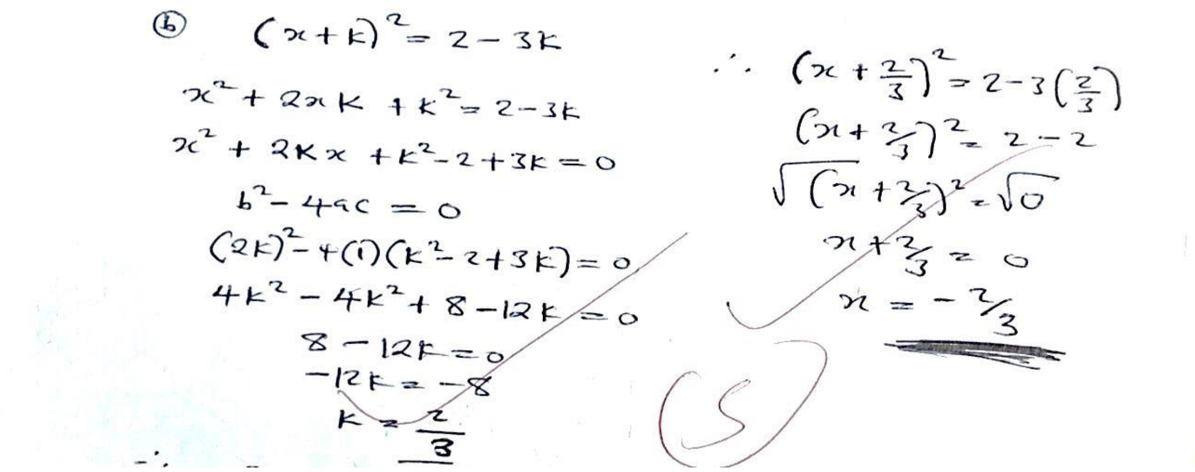


Figure 5.20 Student 18 Work Sample of Marked Question 6.b.

It was observed that only 36 out of the total of 378 students got the whole task correctly. Additionally, it was observed that 76 students got atleast something out of the entire marks allocated to the question. On the other hand, this shows that 9.5% of the students got the entire question correctly whilst 20.1% got atleast something out of the whole given task. The study reveals that about twice the number of students who got everything correctly matched with those who got part of the question correctly. Gargantuan effort is supposed to be put in the teaching and learning of the concept stipulated above. This is so because only a small percentage of the learners were able to get the entire task correctly. To add on, this shows that students performance is very low on the captioned conceptual concept. Therefore, we may wish to conclude that since the performance was very low hence or otherwise the understanding of the conceptual concept of the captioned task was very low. As a rule, students should be able to put in much more attention on the assimilation of the conceptual task captioned above.

(c) Express in the form $a + ib \in \mathbb{R}$:

$$\frac{i^3}{2+i} - \frac{i^4}{2-i}$$ [5]

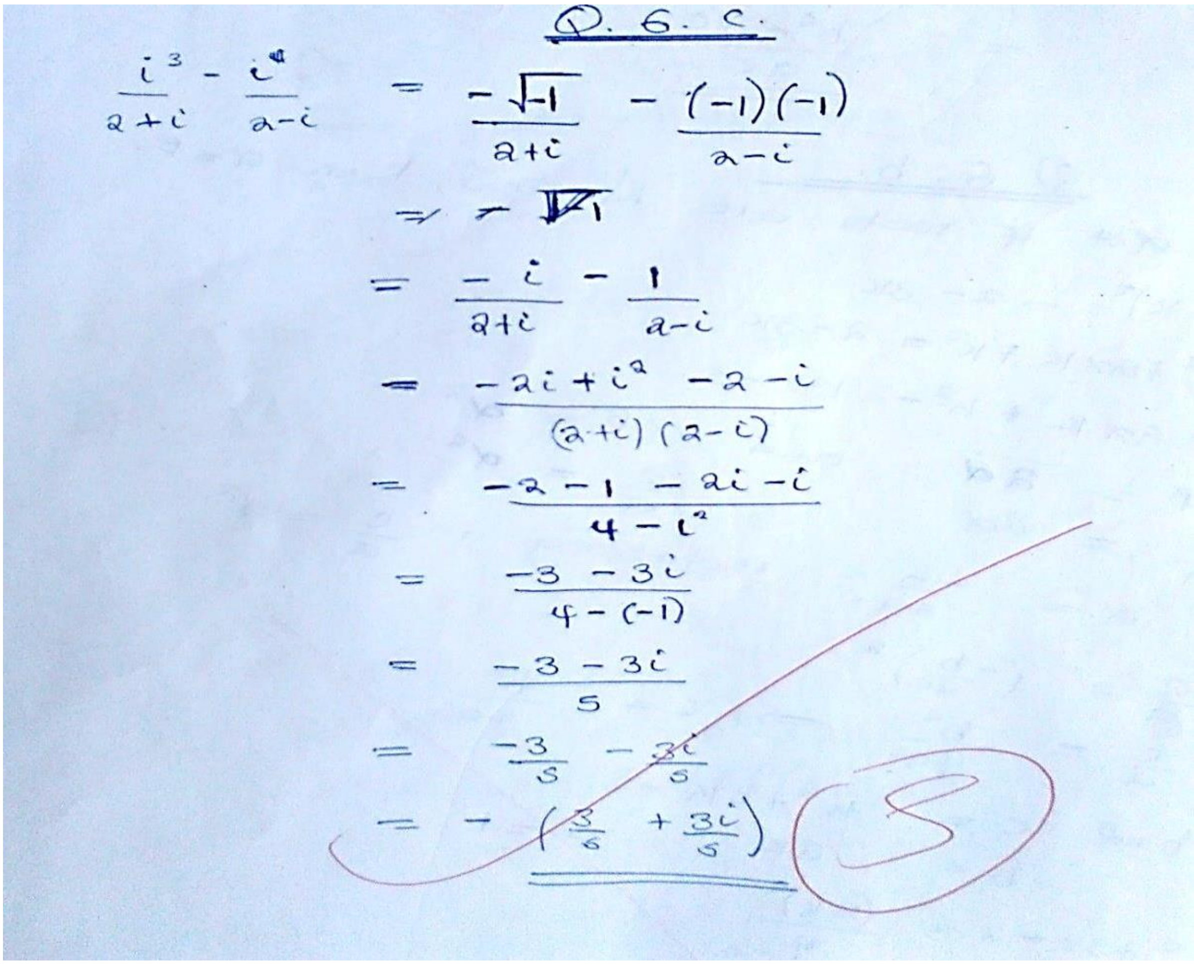


Figure 5.21 Student 19 Work Sample of Marked Question 6.c.

The captioned working was obtained from one of the students who had answered the conceptual task correctly. Ordinarily, the study revealed that 104 students out of the total of 378 got the full task correctly. Furthermore, 112 students in any case got something out of the conceptual task given from the same sum of 378 learners. This meant that 27.5% of the students got everything correctly whislt 29.6% got something out of the total marks. The difference between students who got everything correctly and whose who partly got something was very minimum (i.e it was only 2.1% difference). This shows that students performance of the captioned conceptual task was low. Hence, we may wish to state that since the performance was low it follows that the understanding of the specific concept of arithmetic operations on complex numbers was low.

5.6.2 Findings from the students test and questionnaire

The following analysis show the findings and discussions from the test and the questionnaire.

5.6.2.1 Introduction

There are many factors included for presentation of research results from the students questionnaire. However, the results obtained from the test and questionnaire are presented as follows.

Table 5.22: Statistical Table (A)

Question Number	**Questions prepared from the test**	Mean	Std. Deviation	Analysis N
2	Level of confidence to answer the questions by students on set theory	3.4735	1.22986	378
3	Level of confidence to answer the questions by students on partial fractions and polynomial functions	3.6429	1.20871	378
4	Level of confidence to answer the questions by students on functions the domain and range	3.4101	1.30845	378
5	Level of confidence to answer the questions by students on transcendental functions trigonometric functions	3.4841	1.24078	378
6	Level of confidence to answer the questions by students on linear, quadratic functions and complex numbers	3.3228	1.30559	378
1	Level of confidence to answer the questions by students on binomial expansions and systems of equations	3.4471	1.33056	378

However, it has been revealed from the discussion of the test which was given to first year students; which shows that questions 1,3 and 5 are predominantly procedural while 2,4 and 6 are conceptual in nature. As from the table above it shows that students were more confident in answering questions 1,3 and 5 as compared to 2,4 and 6.

Table 5.23: Statistical Table (B)

Procedural Confidence		Conceptual Confidence	
Question number	Mean	Question number	Mean
Question 1	3.4471	Question 2	3.4735
Question 3	3.6429	Question 4	3.4101
Question 5	3.4841	Question 6	3.3228
Total	10.5741	Total	10.2064

The table presents the level of confidence exhibited by first year students to answer the questions in the given test correctly. As shown in the table, the students were more confident answering procedural questions correctly as compared to conceptual problems. We can see that, the total for procedural confidence was 10.5741 and conceptual confidence was 10.2064. This meant that students were more comfortable answering procedural questions as compared to conceptual ones. It could be further argued that, students might have problems to understand conceptual oriented problems.

However, comparing the information from the three tables we may conclude that first year students at the University of Zambia were more confident to answer procedural questions as compared to conceptual. The difficulty in conceptual questions hampered them to get questions correctly and the confidence in procedural helped them to perform well in procedural ones. Notwithstanding the fact that students performed well in the questions where they were confident and also performed badly where the confidence was low. The confidence in conceptual questions was low and also the performance was equally low.

A Conceptual Leap

The researcher, used physical distances as input data. If MDS works for physical distances, then it may also work for data that can be interpreted as "conceptual distances." Many types of data can be interpreted as conceptual distances.

Usually correspond to ideas like "proximity" and similarity (or dissimilarity)

Table 5.24 Two Dimensional Representation Space

Results for a 2-dimensional representation space:			
Configuration:			
		Dim1	Dim2
	0.335	0.012	0.055
	0.319	-0.301	0.369
	0.286	0.437	0.233
	0.268	-0.347	-0.287
	0.269	0.200	-0.370

Profile Dissimilarities Matrix

Profile dissimilarities matrix, $\Delta*$:

Torgeron's Procedure for Metric MDS, III

The factoring process is carried out by performing an eigen decomposition on $\Delta*$

$\Delta* = \nu\Lambda^2\nu'$

$\boldsymbol{\nu}$ is the k ×q matrix of eigenvectors, Λ^2 is the q×q diagonal matrix of eigenvalues, and q is the rank of $\Delta*$ (usually equal to k).

We create X from the first m eigenvectors (Vm) and the first m eigenvalues (Λ^2m):

X = VmΛm

X contains point coordinates such that the interpoint distances have a least-squares fit to the entries in Δ. The table below gives the above illustration.

Table 5.25: Distances measured in the representation space

:

	0.335	0.319	0.286	0.268	0.269
0.335	0	0.443	0.461	0.496	0.465
0.319	0.443	0	0.751	0.657	0.893
0.286	0.461	0.751	0	0.941	0.648
0.268	0.496	0.657	0.941	0	0.554
0.269	0.465	0.893	0.648	0.554	0

After using MDS the current study revealed that the result obtained was a distance matrix because it satisfied the three properties of a distance matrix. However, MDS is used to show the dissimilarity between examples of pairs. As from Table 5.25 it shows that the result represents an MDS for a $N \times N$ matrix where each entry has a non-negative entry. Furthermore, the results showed the 'identity' property; i.e. the distance as from Table 5.25 shows zeros in the leading diagonal meaning bisecting the table into two equal componeats where the two parts are identical. Additionally, the distance in the table from which ever diagonal we pick we symmetrical meaning that the MDS has given out a proper distance matrix as a result. Lastly but by no means the least Table 5.25 satifies the inequality property.

Table 5.26: Comparative Table

Pair	Dissimilarity	Disparity	Distance	Rank (Dissimilarity)	Rank (Disparity)	Rank (Distance
0.335 - 0.269	0.566	0.566	0.465	1	1	3
0.335 - 0.319	0.609	0.609	0.443	2	2	1
0.268 - 0.269	0.615	0.615	0.554	3	3	5
0.335 - 0.268	0.654	0.654	0.496	4	4	4
0.335 - 0.286	0.669	0.669	0.461	5	5	2
0.319 - 0.268	0.677	0.677	0.657	6	6	7
0.286 - 0.269	0.680	0.680	0.648	7	7	6
0.319 - 0.286	0.726	0.726	0.751	8	8	8
0.286 - 0.268	0.727	0.727	0.941	9	9	10
0.319 - 0.269	0.732	0.732	0.893	10	10	9

Assessing Errors in Metric MDS Solution

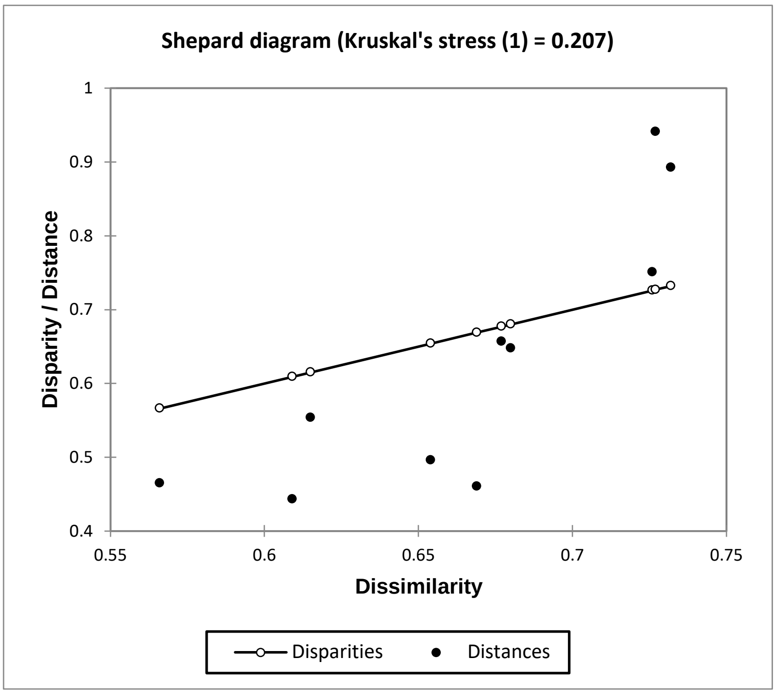


Figure 5.22: Shephard Diagram for Metric MDS of Eigen-values

The goodness of fit for data reduction techniques such as MDS assessed with shephard diagram as elaborated above. Notwithstanding distances in MDS solution do not appear to be a linear function of the dissimilarities. Instead, distances seem to be monotonically related to dissimilarities. It is rather assumed that distances should be linear functions of dissimilarities, after performing MDS and assuming that d_{ij} are a monotonic function of $\delta_{ij'}$ s: $d_{ij} = f^m(\delta_{ij}) + e_{ij}$

In this expression, if f^m means monotonic function and e_{ij} is an error term. Then, we need to show using **Torferson's Procedure for Metric** MDS, which states that:

If $k \times k$ dissimilarities matrix, Δ

We assume the dissimilarities correspond to distances in $m - dimensional$ space

(except for random error).

Then $k \times k$ matrix of distances between points is $\boldsymbol{D}$.

The $k \times k$ matrix, $\boldsymbol{E}$, contains random errors.

But we know the Hypothesis

$$\Delta = D + E$$

Hence or otherwise, we may find the coordinate matrix, $\boldsymbol{X}$, such that entries in $\boldsymbol{E}$ are close to zero. Hence the result is a straight line since we shall remain with a straight line:

$d_{ij} = f^m(\delta_{ij})$ which is a linear function graph as shown in figure above.

5.6.2.2 Marks obtained from the test given per question and confidence of response

The researcher had to compare the actual performance of students on each and every question to the level of confidence attached to the questions. To find out more on the confidence exihibited by students in solving mathematics problems in the test, descriptive statistics were generated as here-under discussed. The questions were divided into two categories procedural and conceptual components. This shows that question 1,3 and 5 were procedurally oriented while question 2,4 and 6 were conceptually oriented.

Percentage marks of students on binomial expansions and systems of equations

(Question one (1): Procedural oriented question)

NB:a) Specific concepts of Binomial coefficients, i.e finding the n^{th} term of the Binomial coefficients.b) One linear and one quadratic. c) Inequalities involving quotients.

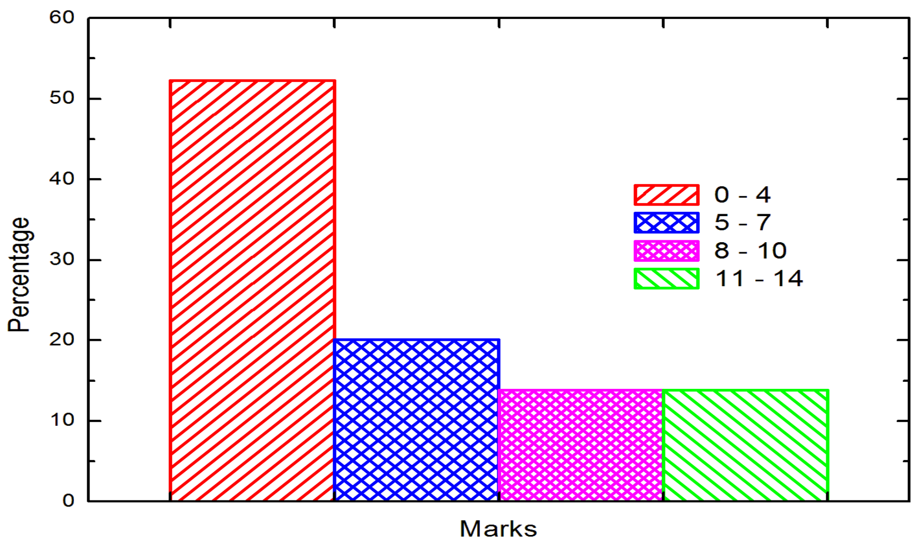


Figure 5.23 a Marks obtained of Specific concepts of:

a) finding the n^{th} term of the Binomial coefficients.

b) One linear and one quadratic.

c) Inequalities involving quotients.

Question one had 14 marks. As shown from the table above most of the students had between 0-4 marks included because this yielded a bigger percentage of 52.3%. Furthermore, as from the figure above it shows that about 50% mark was for students obtaining 5 upto 14 marks. However, it was revealed that the performance of students was slightly above average. However, between 5 to 10 out of 14 this was giving us 27.6%. Additionally, as from the table above students who scored between 11-14 inclusive (75% and above) was 13.8% of the total who wrote the test. This Figure 5.23a above clearly shows that atleast moderate results or average performance was recorded on binomial expansions and systems of equations. Furthermore, we may wish to acknowledge that students understood the procedural concepts in the first tasks given to students.

Level of confidence to answer the questions by students on binomial expansions and systems of equations

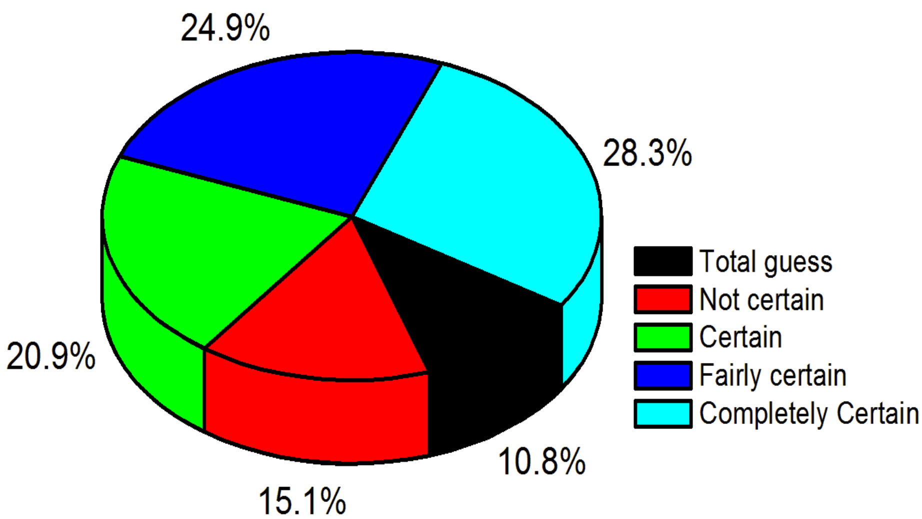


Figure 5.23 b Level of confidence to answer questions by students of Specific concepts of :

a) finding the n^{th} term of the Binomial coefficients.

b) One linear and one quadratic.

c) Inequalities involving quotients.

As from Figure 5.23b above, it shows that students who lacked confidence were very few in numbers. The total percentage for total guess and not certain was 25.9% .The percentage increased with the level of certainity. As from the figure above, it showed that certain had 20.9%, fairly certain 24.9% and completely certain was 28.3%. Unfortunately, on the performance we had 52.3% obtaining between 0-4, 20.1% of the students obtaining between 5-7 inclusive and also students getting between 8-10 and the range of 11-14 respectively added up to the percentage of 13.8%. In this case, the study revealed that there were more misconceptions of concepts by students of mathematics on binomial expansions and systems of equations. This was because there were more wrong solutions recorded and high confidence as indicated in the diagram above. Furthermore, we may wish to state that, some of the students did not have misconceptions but were over confident and made a lot of mistakes in the working mainly due to numerical errors.

Percentage marks of students on set theory

(Question two (2): Conceptual oriented question)

NB:a) Specific concepts on set theory, i.e De Morgan's laws. b) Real numbers (binary operations on real numbers).

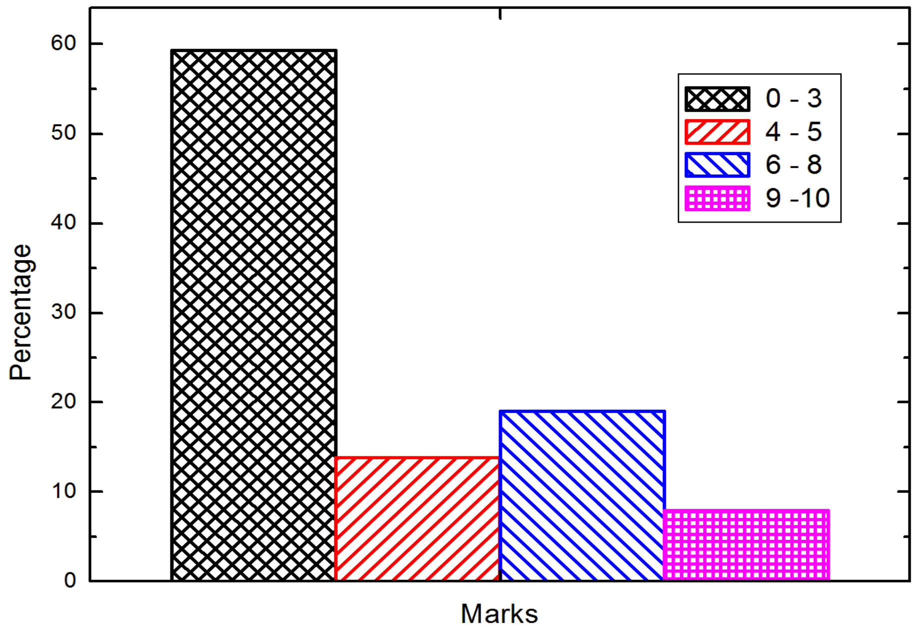


Figure 5.24 a Marks obtained of Specific concepts of:

a) Set theory, i.e De Morgan's laws.

b) Real numbers (binary operations on real numbers).

As from the table above, it shows that question two had 10 marks. The portion of 0-3 marks had 59.3%, 4-5 marks had 13.8% of the students, 6-8 marks recorded 19% and last but by no means the last 9-10 marks had only 7.9%. This shows that most of the students did not perform well in the test which was given by the researcher. As well as very few students obtained very good grades as shown above, only 7.9% of the entire students did very well on set theory. This may help us to state that, students had a lot of misconceptions on set theory because only a small percentage did well. This clearly shows lack of understanding of mathematical concepts.

Level of confidence to answer the questions on set theory and number systems correctly

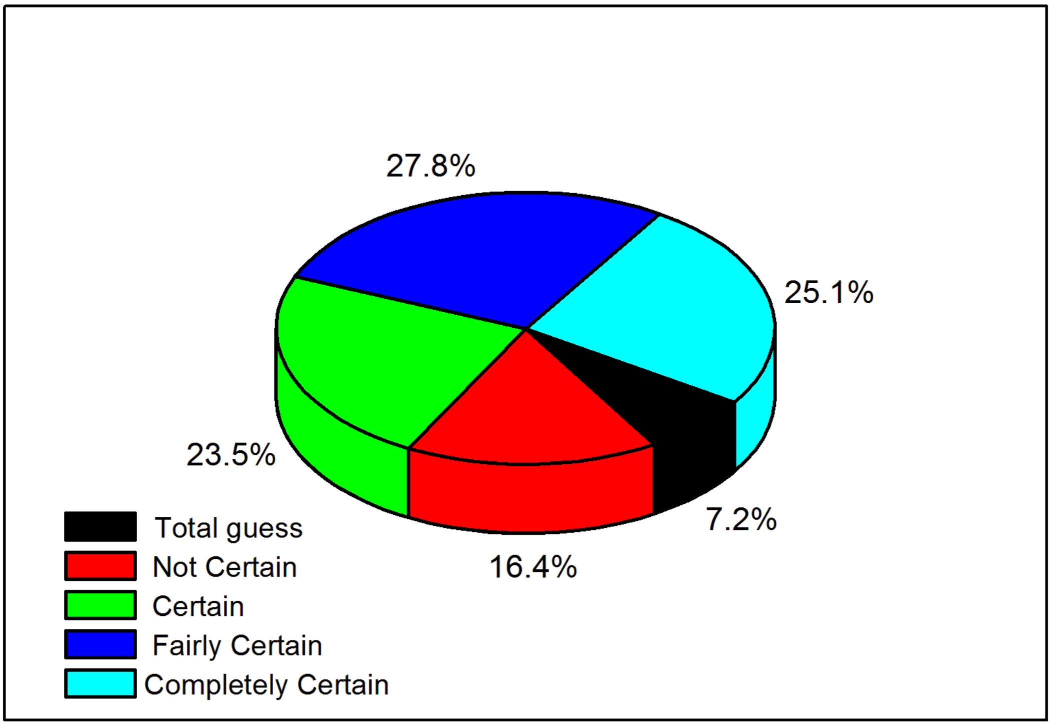


Figure 5.24 b Level of confidence to answer questions by students of Specific concepts of :

a) Set theory, i.e De Morgan's laws.

b) Real numbers (binary operations on real numbers).

The diagram above shows that students were very confident to answer the questions correctly. This was because only a very small percentage of students of 23.6% indicated that there were not certain or guessed the answers. Nevertheless, as earlier indicated students failed the question on set theory despite indicating that they can perform correct calculations.

Besides, 76.4% of the entire students were confident to answer the question on set theory correctly but after the test only 7.9% of the students did well which was almost equivalent to those very few who wished to perform but only a guess of 7.2%. In this respect, most of the learners where confident to answer the question on set theory correctly but they got the question wrongly. This means that, the students had high confidence but low knowledge or the level of understanding of the concepts on set theory was not there (wrong answer and high confidence indicates misplaced confidence of the subject matter or misplaced confidence to understand the concepts in mathematics by the learners).

Percentage marks of students on partial fractions and polynomial functions

NB:a) Specific concepts on partial fractions, i.e partial fraction with one linear and none repeating quadratic factor. b) Polynomials; finding the unknown factors.

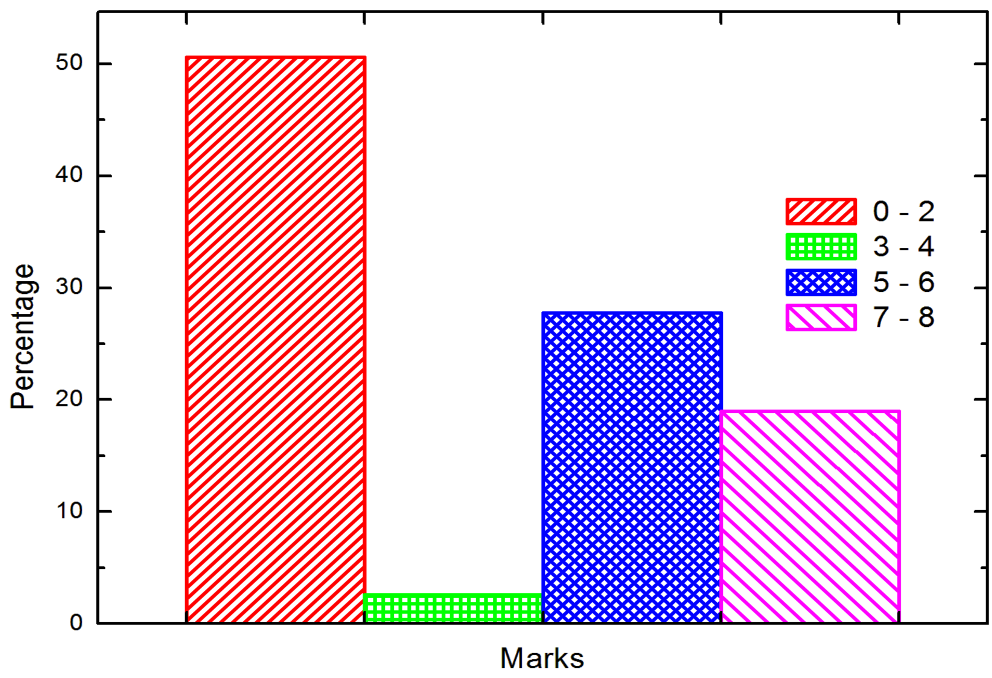


Figure 5.25 a Marks obtained of Specific concepts of:

a) Partial fraction with one linear and none repeating quadratic factor.

b) Polynomials; finding the unknown factors.

As from the Figure 5.25a above, it shows that 50.6% of the students had gotten 0-2 marks inclusive which was a bad performance. Hence or otherwise, more than half of the students failed the test. On the other side, when we add the percentage for 5-6 and 7-8 marks we obtain 36.8% as the pass percentage. Question three shows the highest pass percentage of the students. We may wish to conclude that students had a bit of procedural understanding when solving questions on partial fractions and polynonials. Conversely, maybe the methodologies used when teaching the already mentioned two components of procedural questions might have been more student centered such that the procedures involved when solving the questions may reduce or be with no difficulties. Atleast, students performance was overwhelming since a good number of learners about 177 out of 378 did well in the test. This meant that, 201 student failed the question on partial fractions and polynomials. From the above illustration ,it was revealed that atleast the confidence levels marched with the performance. This means that students atleast understood the concepts on partial fractions and polynomials because the performance tried to be the same as the amount confidence they exhibited in order to solve the question. Eventhough, the percentage for confidence was slightly more than performance. With reference to Figure 5.25a it shows that students did well in procedural questions which reckons to the confidence students had to solve the tasks correctly. Good performance of students indicates good understanding of concepts. reckons

Level of confidence to answer the questions by students on partial fractions and polynomial functions

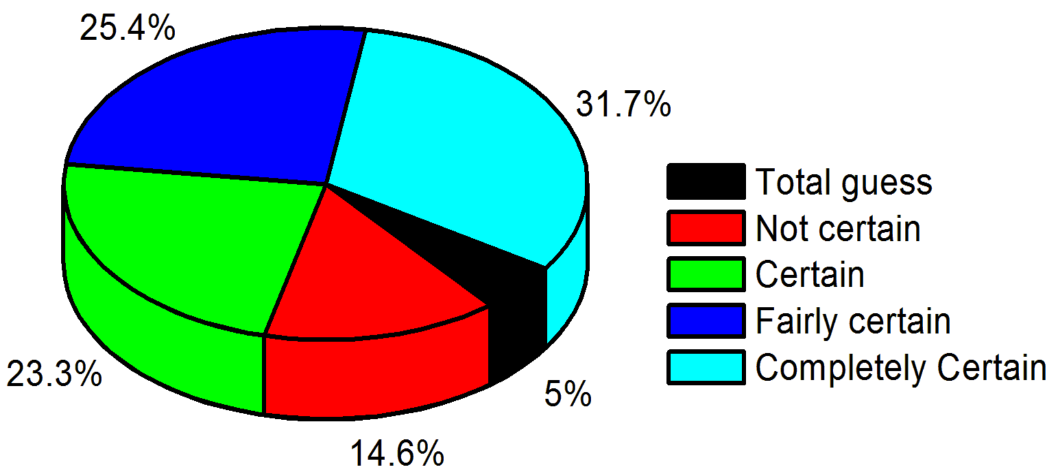


Figure 5.25 b Level of confidence to answer questions by students of Specific concepts of :

a) Partial fraction with one linear and none repeating quadratic factor.

b) Polynomials; finding the unknown factors.

As from Figure 5.25b, it was revealed that the level of confidence to answer correctly questions on partial fractions and polynomials was 80.4%. Nonetheless, the performance shown above gave a percentage of about 45% passing. This shows that students were confident to solve the questions correctly but the confidence was twice more than the actual performance of the learners. This reveals that students were either over confident or had misplaced confidence. Many times, over confidence in learners may lead to students making careless mistakes or errors when solving questions. Moreover, when the performance is low and the confidence is high we may state that the students could have a number of misconceptions of concepts. The results above shows that atleast on question three the level of confidence was close in magnitude to the performance of the students even though confidence levels out-numbered performance. We may wish to conclude from the discussion that procedural performance of learners on partial fractions and polynomials was a bit better as compared to other components of concepts underpinning the study.

NB: Specific concepts on set, i.e determining the domain and range of functions.

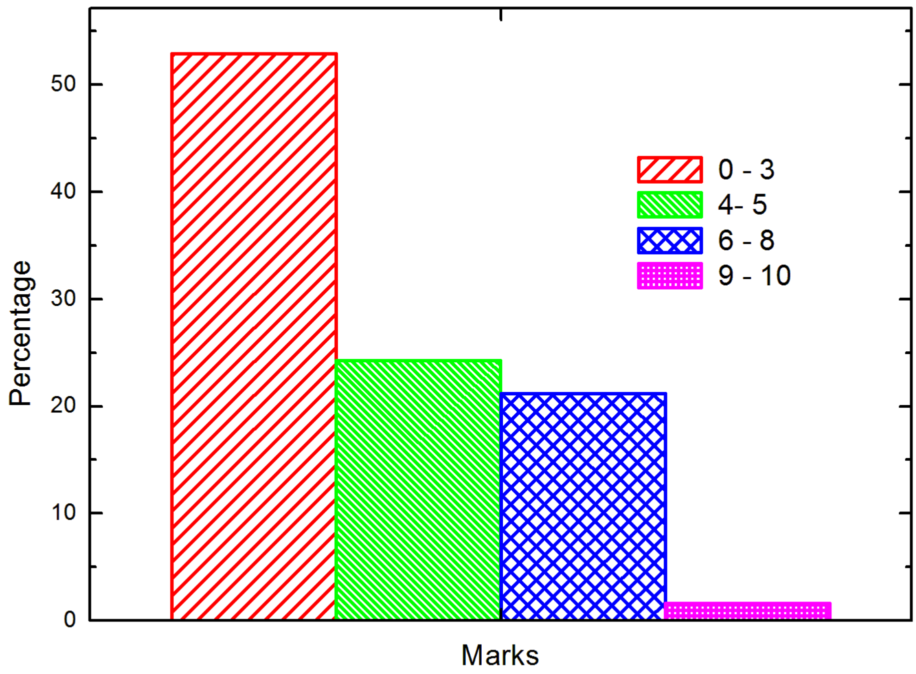


Figure 5.25 c Percentage marks of students on functions

As from Figure 2.25c, it shows that the pass percentage of students on the concept of finding the domain and range of the function was 25%. It clearly reveals that students had difficulties in finding the domain and range of functions. Furthermore, it was revealed that, about 53% of the students who wrote the test got between 0-3 marks out of 10. Low scores translated into low understanding of concepts on functions. To add on, about 2% of the learners scored between 9-10 inclusive. Meaning that very few students were able to have very good marks on functions.

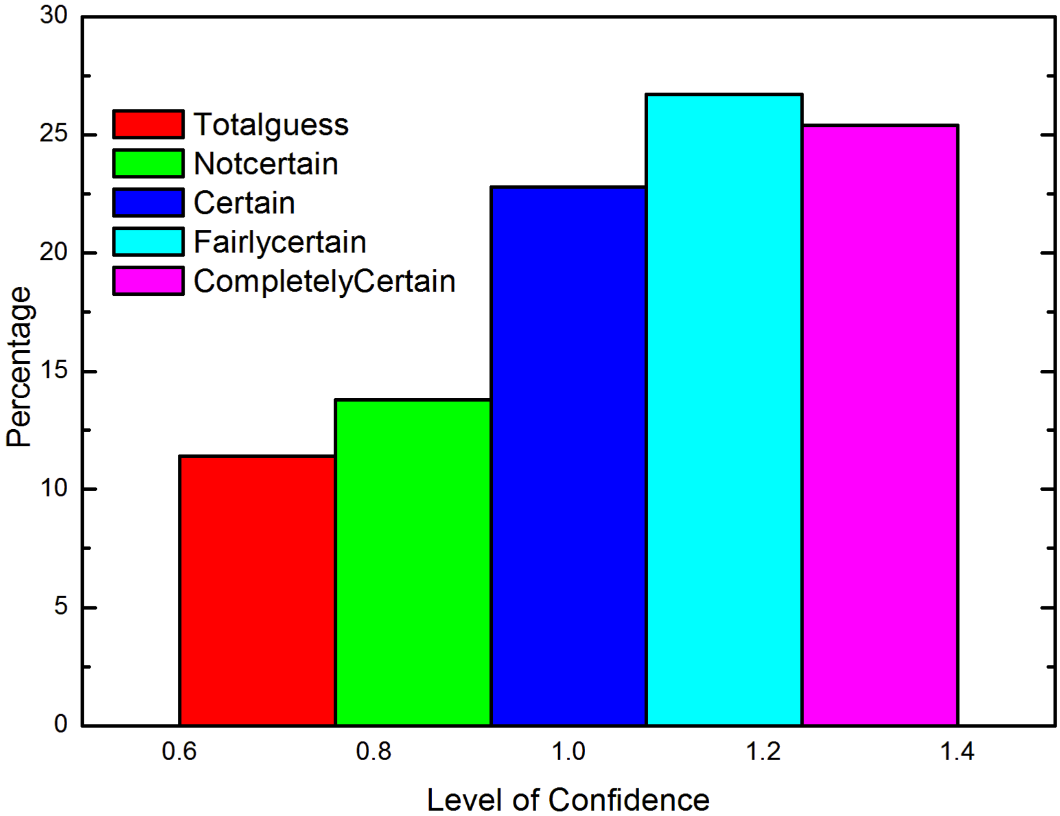


Figure 5.25 d Level of confidence to answer the questions by students on functions

Figure 5.24d, reveals that about 26% of the students were not certain to get the questions correctly on the concept of finding the domain and range of the given functions. This shows that students who were not confident to answer the questions correctly were equal to the students who had scored between 6-10 marks correctly. This results shows that a good number of students were confident to answer questions correctly when actually they were unable to. This shows that either students were over confident or had misconceptions on how to find the domain and range of functions.

NB: Specific concepts on trigonometric identities.

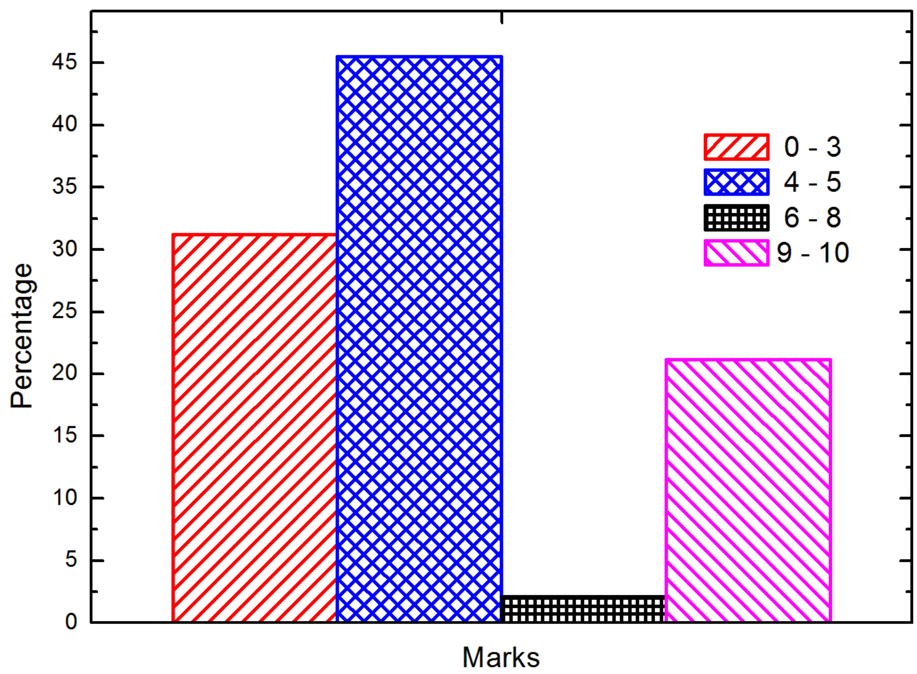


Figure 5.25 e Percentage marks of students on trigonometry

Trigonometric identities were procedural in nature by definition as from the researcher. However, the table reveals that 20% of the students scored 9-10 marks in the test which was a good result. Students had atleast good understanding of procedural concepts on trigonometric identities because of the good percentage recorded of students scoring higher marks. Nonetheless, it shows that about 77% of the students scored below average in the test given specifically on trigonometric identities. Figure 5.25e above shows that a good number of students got below average marks. Although, the performance was generally low the percentage pass of students with high marks was good. This means that the understanding of concepts by students on trigonometric identities was good.

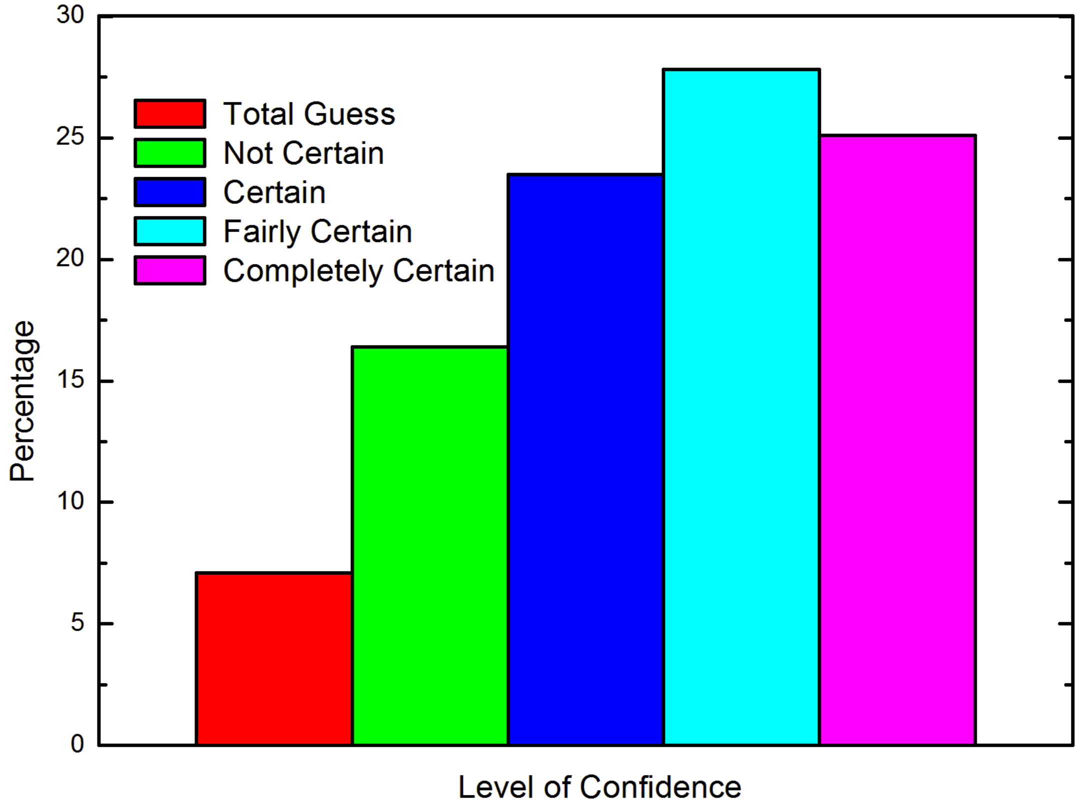


Figure 5.25 f Level of confidence to answer the questions by students on trigonometric identities

As from Figure 5.25f, it was revealed that the percentage for uncertainty (total guess and not certain) was 23%. This entails that a big number of students were confident to answer the questions correctly. This was not what happened because as from Figure 5.25f above it shows that the performance was less while the confidence in students was high. In this regard, we may state that the confidence was misplaced or the confidence in students was too high such that in the process students made mistakes or errors due to over confidence in working. Notwithstanding, students should be able to learn trigonometric concepts with full understanding because trigonometry is not a topic but a branch of mathematics. Therefore understanding of trigonometric identies is must to students at this level of education.

NB: Specific concepts on a & b) finding the roots of quadratic functions.

c) Arithmetic operations on complex numbers.

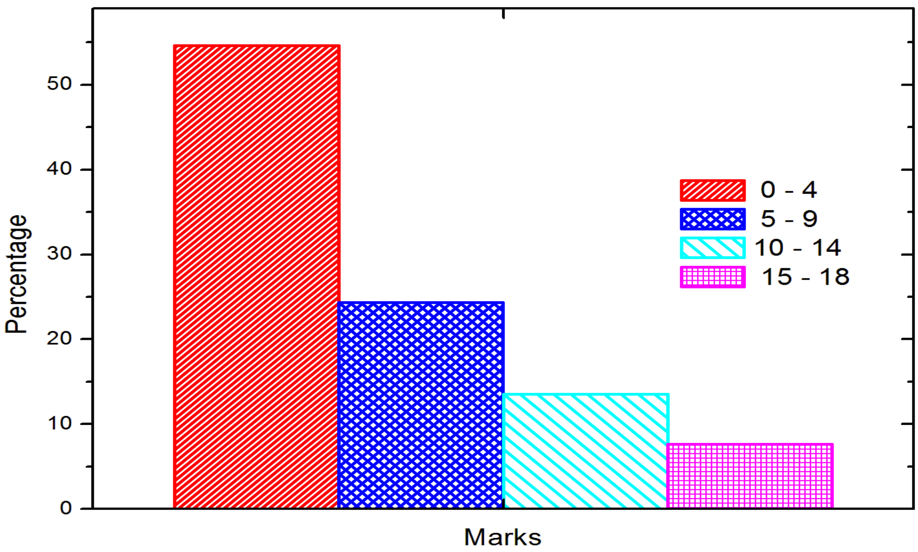


Figure 5.25 g Percentage marks of students on quadratic functions and complex numbers

The performance on complex numbers as observed from the figure was low. This reveals that learners who had scored 10 marks and above were only 21% of the total. Nevertheless, we had 79% of the students failing to make it on the concepts of complex numbers and the roots of quadratic equations. Furthermore, it is observed that the mark of 0-4 which is the smallest mark range had the highest percentage of learners (55%). In this regard, we may wish to state that a good number of students had less marks. Also, the percentage marks reduces as we increase the mark range.

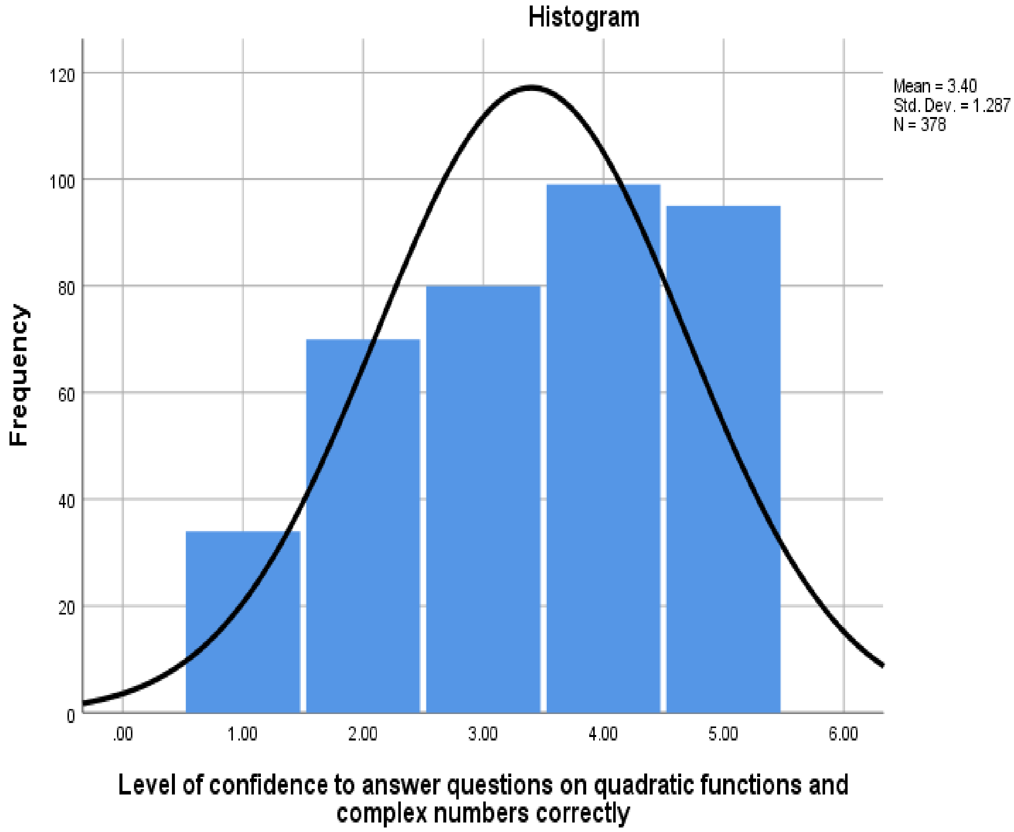


Figure 5.25 h Level of confidence to answer the questions by students on quadratic functions and complex numbers.

The histogram shows that students were confident to answer questions correctly on quadratic functions and complex numbers. Likewise, the performance was low while the confidence was very high. This means that, despite recording poor percentage marks in the test students were confident to answer the questions correctly. In this regard, lecturers may look at possible ways to correct students misconceptions so that sooner than later misconceptions are corrected. Ultimately, students performance and confidence may correspond in measure.

Table 5.27 Summary of Findings

Research Objectives	Significant Original Contribution to Knowledge of Mathematics Education
To develop and design valid and reliable instruments for measuring understanding of concepts in mathematics	Justified true knowledge contributed to the body of knowledge **(Four Standard Indices).** These standard indices consisted of two procedural and the other two conceptual.
To determine the kind of understanding possessed by first year UNZA mathematics students of procedural or conceptual concepts in mathematics.	The study revealed that the kind of understanding possessed by first year UNZA students in mathematics are procedural and conceptual. It was further revealed that procedural understanding was twice more than conceptual understanding. To add on, it was revealed that students who were confident in procedural tasks did well in conceptual components.
To examine the relationship between the confidence levels of students and their understanding of procedural and conceptual concepts in mathematics.	The study revealed that there was a positive relationship between students confidence and their understanding of procedural and conceptual concepts of mathematics at UNZA among first year students. Based on the research findings, the study further revealed that procedural confidence of understanding was twice more than conceptual.
To investigate the relationship between students' confidence and their actual performance in procedural and conceptual mathematics problems.	Based on the results of the study, it was revealed that students performed well in procedural tasks where they were more confident. To add on, the study revealed that either students at UNZA had high confidence but low knowledge or the level of understanding of concepts was little.

5.7 Chapter summary

The chapter presented research results of the study related to first year students at the University of Zambia (UNZA) as indicated by the objectives in chapter one. The chapter started by developing and designing measuring instruments (Standard Indices) used for measuring mathematics understanding. Issues relating to procedural and conceptual understanding of mathematical concepts were determined. To add on, the chapter also used specific indices to measure procedural and conceptual confidence of understanding exhibited by students to solve mathematics tasks correctly. Relationship between procedural and conceptual understanding was determined. Multivariate analysis (i.e. factor analysis, multi-dimensional scaling and cluster analysis) were used to determine the correlation between and among specific concepts in mathematics at first year level. Finally, the chapter presented research results concerning the relationship between students confidence and their actual performance in procedural and conceptual mathematics problems. The next chapter discusses research results of the study.

CHAPTER SIX
DISCUSSION OF RESULTS

6.1 Introduction

This chapter is divided into six sections. The first section gives the overview of the discussion of results based on the objectives of the study. The following section discusses in-depth the adoption, development and designing of new measuring instruments (standard indices) with a focus of comparing them to the initial measuring instruments. Likewise, there is a discussion of results on the measuring of mathematics understanding using standard indices and multi-variate analysis tools. Above and beyond, the chapter discusses the results obtained concerning the relationship between students confidence levels and their mathematics understanding. Furthermore, the chapter discusses the relationship between students confidence and their actual performance in procedural and conceptual mathematics problems. Finally the chapter concludes with the summary of the chapter.

6.2 Overview of the chapter

The researcher used standard indices to interpret results on procedural and conceptual understanding and confidence of answering questions in the test. Furthermore, he opted to use multivariate analysis because the research composed of many variables which were supposed to be correlated with each other and further observation revealed that other probabilistic model could not have appropriately provided the intended results. Kothari (1985) argues that, 'multivariate techniques are largely empirical and deal with the reality; they possess the ability to analyze complex data (p.316). Accordingly, in most of the applied and behavioural researchers, we generally resort to multivariate techniques for realistic results. There is by far a number of components of multivariate analysis but in this study only factor analysis, multi-dimensional scaling and cluster analysis suited the study.

On the whole, the researcher's views were supported by Zuya (2017) who argued that expert judgement attained by involving lecturers of mathematics in the construction of a 20-item test instruments. Additionally, Lautitzen (2012) suggested that the tasks developed were to find out how students perform on procedural tasks and to what extent they may reveal procedural understanding. Furthermore, it was evident that Zuya et al (2017) suggested expertise in mathematics to be sought in the construction of the test which was used as an instrument for the experiment. In order to ascertain as to which component of the test needed to test students on. The current study was supported by Zakaria et al (2010) who picked 15 items in the subjective test, which included subtopics of arithmetic series, geometric series, binomial expansion, and mathematical induction which were all considered to be procedural in nature.

6.3 Discussion of results on the measurement of understanding of concepts in mathematics.

Measurements are largely standardised but can be customised for local contexts (Robertson, 2017). In view of the aforesaid, the current study was supported by Engelbrecht et al (2009) who believes that instruments once developed may improve the measuring of conceptual and procedural knowledge of mathematics. It is usually good practice to eventually have atleast three to five items for each construct being measured (Robertson, 2017). However, Rittle-Johnson, Schneider and Star (2015) argued that previous research design were not appropriate because they produced instruments which were not valid and reliable hence it was difficult to measure conceptual knowledge. The researcher used standard indices adopted from (Engelbrecht et al, 2005) which were adjusted and developed to conform to the current study. However, standard indices developed by Engelbrecht et al (2005) are here-under explained.

Procedural performance index (PPI)

$$\boldsymbol{PUI} = \frac{\sum \boldsymbol{\alpha^{*} k_p * d_p}}{\sum \boldsymbol{k_p * d_p}}$$

To find the Procedural performance index. Researchers used the captioned index. Researchers prepared a questionnaire for lecturers to find out the procedural knowledge that they can attach to the questions and also the level of difficult for each question then, the researchers had to find the average as Average for Procedural performance and also average for procedural difficult.

<u>For the numerator</u>

$\alpha = 1$, if the question is answered correctly

$\alpha = 0$, if the question is answered wrongly

${k_p}^{*}$ Is the procedural knowledge lecturers have by looking at the questions prepared for testing students.

d_p Is the procedural level of difficult lecturers associate to the questions prepared.

For the denominator

$k_p *$ Is the procedural knowledge, we calculate, the marks obtained by students answering the questions.

d_p Is the procedural level of difficult students have and it is stated in the questionnaire at the end of the question paper.

$$\boldsymbol{PUI} = \frac{\sum \boldsymbol{\alpha}^* \boldsymbol{k_p} * \boldsymbol{d_p}}{\sum \boldsymbol{k_p} * \boldsymbol{d_p}}$$

Conceptual performance index (CPI)

$$\boldsymbol{CPI} = \frac{\sum \boldsymbol{\alpha}^* \boldsymbol{k_c} * \boldsymbol{d_p}}{\sum \boldsymbol{k_c} * \boldsymbol{d_c}}$$

To find the Conceptual performance index. Researchers used the captioned index. Researchers prepared a questionnaire for lecturers to find out the conceptual knowledge that they can attach to the questions and also the level of difficult for each question then, the researchers had to find the average as Average for Conceptual performance and also average for conceptual difficult.

For the numerator

$\alpha = 1$, if the question is answered correctly

$\alpha = 0$, if the question is answered wrongly

${c_p}^*$ Is the conceptual knowledge lecturers have by looking at the questions prepared for testing students.

c_p Is the conceptual level of difficult lecturers associate to the questions prepared.

For the denominator

$c_p *$ Is the conceptual knowledge, we calculate, the marks obtained by students answering the questions.

c_p Is the conceptual level of difficult students have and it is stated in the questionnaire at the end of the question paper.

$$\boldsymbol{CPI = \frac{\sum \alpha^* k_c * d_p}{\sum k_c * d_c}}$$

Procedural confidence index (PCI)

$$\boldsymbol{PCI = \frac{\sum B^* k_p * d_p}{\sum 3^* k_p * d_p}}$$

To find the Procedural confidence index. Researchers used the captioned index. Researchers prepared a questionnaire for lecturers to find out the procedural knowledge that they can attach to the questions and also the level of difficult for each question then, the researchers had to find the average as Average for Procedural performance and also average for procedural difficult.

<u>For the numerator</u>

(Where the value of β being 3,2,1 or 0, indicating completely certainty, moderate certainty or a total guess respectively).

${k_p}^*$ Is the procedural knowledge lecturers have by looking at the questions prepared for testing students.

d_p Is the procedural level of difficult lecturers associate to the questions prepared.

<u>For the denominator</u>

$k_p *$ Is the procedural knowledge, we calculate, the marks obtained by students answering the questions.

d_p Is the procedural level of difficult students have and it is stated in the questionnaire at the end of the question paper.

$$\boldsymbol{PCI = \frac{\sum B^* k_p * d_p}{\sum 3^* k_p * d_p}}$$

Conceptual confidence index (CCI)

$$\boldsymbol{CCI = \frac{\sum B^* k_c * d_c}{\sum 3^* k_c * d_c}}$$

To find the Conceptual performance index. Researchers used the captioned index. Researchers prepared a questionnaire for lecturers to find out the conceptual knowledge that they can attach to the questions and also the level of difficult for each question then, the researchers had to find the average as Average for Conceptual performance and also average for conceptual difficult.

<u>For the numerator</u>

(Where the value of β being 3,2,1 or 0, indicating completely certainty, moderate certainty or a total guess respectively).

${c_p}^*$ Is the conceptual knowledge lecturers have by looking at the questions prepared for testing students.

c_p Is the conceptual level of difficult lecturers associate to the questions prepared.

<u>For the denominator</u>

$c_p *$ Is the conceptual knowledge, we calculate, the marks obtained by students answering the questions.

c_p Is the conceptual level of difficult students have and it is stated in the questionnaire at the end of the question paper.

$$\boldsymbol{CCI = \frac{\sum B^* k_c * d_c}{\sum 3^* k_c * d_c}}$$

The standard indices developed by Engelbrecht et al (2005) differs from the one I developed in a number of ways. Firstly, the initial standard indices were measuring generic aspects of mathematics performance of students while the new ones measured mathematics understanding which is of a higher order. Likewise, Engelbrecht et al (2005) standard indices measured procedural and conceptual performance while the current ones measured procedural and conceptual understanding of mathematics concepts. The constructs in the initial standard indices focused on measuring students knowledge while the newly developed ones focused on students understanding of mathematics. Since the current study was concerned with the understanding of mathematics concepts. This left the researcher with no option but to adapt, then adjust and develop new standard indices to measure the construct of understanding of specific concepts in mathematics.

Empirical data was used from literature reviewed to know the specific constructs the current study intended to measure. In line with the current study which aimed at investigating the understanding of first year students of mathematics concepts (procedural or conceptual). It was believed that the investigation into the understanding of mathematics concepts by first year students at UNZA maybe achieved by developing and designing measuring instruments of mathematics concepts. To develop deep abstract conceptual thinking in mathematics, students have to experience contextualized mathematics by relating it to their own experience and connecting it to the concrete instances from which it has been abstracted (Khalid and Ekholm, 2015). Ghazali and Zakaria (2011) argues that, students are prone to use procedures rather than knowing how the procedures are achieved and also they focus more on calculation procedures not conceptual (p.2).

Additionally, some variations in measurement techniques are inevitable due to the different specific concepts being measured and the different participant populations and age groups that are typical in each domain (Crooks and Alibali, 2014: 366). A good number of literature reviewed showed that most of the students are good at procedures as compared to concepts. On the other hand, Zuya et al (2017) observed that the performance of preservice teachers on conceptual items of open-ended question on geometry was more than procedural tasks which is in agreement with Engelbrecht et al (2005) who had to find out how undergraduate students of life sciences at Pretoria University performed well on conceptual tasks than procedural ones of the multiple choice question on Calculus. However, in the current study the researcher developed a structured test which he used to measure procedural and conceptual understanding and also procedural and conceptual confidence of understanding. Nevertheless, many studies on conceptual and procedural understanding of mathematics at universities agrees with the current study that mainly students understand more on procedural concepts than conceptual as supported by (Sarwali & Shahrill, 2014 and Mutawah et al, 2019).

6.4 Discussion of results on the understanding of concepts in mathematics by first year UNZA students.

As from the current study, results revealed that students did well in procedural components as opposed to conceptual. However, if both teaching and examination of university mathematics is geared towards procedural knowledge, then the students cannot be blamed for not developing the conceptual knowledge hoped for (Bergqvist, 2007). The results revealed that the procedures used by students while solving mathematical problems shows various levels of students' conceptual understanding (Ghazali and Zakaria, 2011: 690). The attainment of conceptual understanding was aided by the inclusion of meaningful learning activities. In other ways, Crooks and Alibali (2014) supports the idea by stating that, there has been a shift in the way procedural understanding is measured and that it has become relatively standardized: participants solve a set of problems, and a score is calculated based on correct answers they obtain or based on the specific procedures they use to arrive at those answers (p.345). In this regard students were not able to understand how to use mathematical symbols appropriately in solving problems. Above and beyond, the higher the level of procedural understanding, the higher the level of students' conceptual understanding, and vice versa (Ghazali and Zakaria, 2011). Much of the literature describing how students achieved conceptual understanding included a description of meaningful learning activities that took place in a social and active learning environment (Mills, 2016). The procedures used by the students while solving mathematical problems at various levels of students' conceptual understanding (Ghazali and Zakaria, 2011).

It was observed with dismay that students were failing to understand the basic rules of logic, including the role of axioms and to formulate mathematical statement from problems. It was further revealed that most of the students were failing to recognise and appreciate the connections between theory and applications. This lead to the poor understanding of conceptual concepts in mathematics. Research revealed that students' level of procedural understanding is high whereas the level of conceptual understanding is low and hence they suggested that a reformation in teaching is needed to boost conceptual understanding among students to minimize the use of algorithms and memorization (Mutawah et al, 2019). The progressive dynamics specific to the basic understanding of conceptual mathematics has received relatively little research attention hence low understanding recorded for conceptual. Therefore, there is a call for conceptual understanding of mathematics ; it is no longer sufficient for students merely to memorize rules and algorithms (Marchionda, 2006).

6.5 Discussion of results on the relationship between students confidence levels and their understanding of procedural and conceptual concepts in mathematics topics.

The researcher looked at a number of literature which related to the components of the topic at hand. It was observed that most of the literature reviewed emphasised on performance and confidence exhibited by students towards understanding of mathematics concepts. On the other hand, some literature reviewed focussed on how questionnaires were developed whilst others highlighted the development and designing of measuring instruments in mathematics. On the whole, some research focussed on developing tests to measure procedural and conceptual understanding in mathematics. This was done by identifying constructs of interest through literature review and also findings related to the current study. Furthermore, undergraduate mathematics instructors often report that students make careless errors or have not previously learned key mathematical ideas and startegies (Chin and Pierce, 2019). Nonetheless, the current study revealed that there was a strong relationship between students confidence levels and their understanding of procedural and conceptual concepts in mathematics. This meant that confident students outperformed those who were not in solving mathematics tasks. The current study further revealed that those who were confident in conceptual tasks did well also in procedural. On the other hand, those who were confident in procedural tasks did not perform well in conceptual. In this study just like the study done by Engelbrecht et al (2005) argued that,

> The Confidence of Response Index (CRI) has its origin in the social sciences, where it is used particularly in surveys and where a respondent is requested to provide the degree of certainity he has in his own ability to select and utilise well-established knowledge, concepts and laws to arrive at an answer. In an academic examination environment a student is asked to provide an indication of confidence of response along with each answer set. This indication is usually based on some scale (0-5), where 0 implies a total guess and 5 implies complete confidence. Irrespective of whether the answer is correct or not, a low confidence indicates a quess which, in turn, implies a lack of understanding. However, if the confidence is high and the answer is wrong it points to a misplaced confidence on the subject matter, either misjudging his own ability or a sign of existence of misconceptions. The use of confidence of response, in conjunction with the correctness or not of a response can be used to distinguish between lack of understanding (wrong answer and low confidence) and a misconception (wrong answer and high confidence). This may not always be the case; students could just be over confident or in procedural problems students with high confidence may make numerical errors.

Nevertheless, in this study we looked first at the question which was procedural in nature. How was the performance of the students before we had to analyse the confidence of response exihibited by the learners in conjunction with the questionnaire which was given to the students after writing the test in order to elicit options. In the same line, Novaliyosi (2018) suggested that, to think logically give full confidence to say something because everything that is said to be based on a logical reason as well as the support of strong evidence. However, Engelbrecht et al (2005) argued that the panel of experts were asked to indicate (in their opinion), the level of difficulty on procedural (dp) and conceptual level (dc) of each item in the test. Hence, to bridge the gap in measurement the current study used factor analysis, multi-dimensional scaling and cluster analysis when comparing constructs while Engelbrecht et al (2005) used the scatter plot for data analysis to compare constructs and pearson correlation coefficient and statistical t-test. The study used multivariate analysis because of they measure closer to reality as supported by Fontana (1985) who postulates that recent research using factor analysis rather suggests that such models may approximate closer to reality than spearman's 'g' and 's' model. The current study looks at trigonometry specifically at trigonometric identities in agreement with (Malambo et al, 2018) who postulated that, trigonometry occupies a central position in mathematics curriculum designed for secondary school learners and it is taught through to university level.

Learners performed poorly on the Annual National Assessment diagnostic tests, and show deteriorating performance as they move into successively higher grades. This suggests that learners are entering each successive grade with accumulated deficits in knowledge and skills from the previous grade. Essentially, Umalusi (2015) pointed it out that, many South African learners have not been able to keep up with the curriculum as they move through each grade (p.45). The previous argument is supported by Mutawah et al (2019) who laments that, even after graduating from high school, it is apparent that students do not possess an appropriate level of conceptual understanding. Notwithstanding, this makes it difficult to pass or understand first year mathematics at UNZA. It was revealed that most of the students at UNZA learn mathematics in order to complete the study material than understanding of the materials learned, students mainly prefer memorizing of formulars towards writing quizzes, tests and or examinations. It was further revealed that, this makes students forget concepts and formulars when they are writing tests which leads to misconceptions and misunderstanding of concepts. Additionally, not providing concrete examples to contexualize concepts denies students the opportunity of making connections between the abstract mathematical concepts and their own experience (Chin and Pierce, 2015).

The current study revealed that students possessed more of procedural understanding as compared to conceptual which is in agreement with (Malambo et al, 2019) whose paper provided evidence to show that final year student mathematics teachers at UNZA possessed more procedural knowledge while lacking conceptual knowledge of the mathematics they are required to teach. Therefore, to pass university mathematics students need to develop both conceptual and procedural understanding (Mary and Heather, 2006 & Khoule et al, 2017 and Rittle- Johnson, Schneider and Star, 2015). Eventhough, Aytekin and Sahiner (2020) observed that some students may have a high level of procedural knowledge and have a low level of conceptual knowledge, nevertheless the study was concluded by indicating that procedural and conceptual knowledge develops simultaneously.

The study revealed that there was a significant relationship between procedural and conceptual understanding. Students who understood procedural questions equally had no difficulties in understanding conceptual ones as supported by Dunham (2008) who argues that empirical evidence attests to the existence of the relationship between conceptual and procedural knowledge. It is time that educationists accentuate the efforts on developing instrument which are able to measure values in mathematics education to assist students and teachers to gain their personal and social identities affecting choices they made concerning mathematics and mathematics education (Nik Azis and Tapsir, 2013). There should be a reformation in teaching to boost conceptual understanding among students in order to minimize memorization of facts (Ghazali and Zakaria, 2011). Nonetheless, the current study agreed with the study which was done in South Africa by Dhlamini and Luneta (2016) whose findings revealed that most of the leaners after writing a test in mathematics exhibits more proficiency in procedural applications of tasks than conceptual understanding. It was revealed that lecturers should be encouraging and giving hope to under-achievers in mathematics at UNZA as supported by (Zakaria, 2010) who argued that individual attention should be given to weak and low-performing students in mathematics at matriculation level.

6.6 Discussion of results on the relationship between students confidence and their actual performance in procedural and conceptual mathematics problems.

The section of the chapter outlines the relationship between students confidence and their actual performance in procedural and conceptual mathematics problems at first year level at UNZA. Despite my study being more insightful in the methodology engaged in the research otherwise it was the same with the research done by Engelbrecht et al (2009) and Ghazali and Zakaria (2011) which revealed that there is a significant relationship between students confidence and understanding of procedural and conceptual concepts in mathematics. This is because procedural understanding of skills helps learners to understand the mathematical concepts required to perform mathematical manipulations. Students often can do the procedure when they know what is needed (Keene and Fortune, 2016). Lauritzen (2012) suggests that a set of tasks that are meant to reflect a student's procedural knowledge of functions, should ideally measure procedural knowledge of functions (p.35). However, Lauritzen (2012) suggested that,

> Some students have problems to reflect on an answer and detect inconsistencies in the result. Maybe they are not able to detect that the properties of the answer are

inconsistent with the given problem. It seems that some have not developed control mechanism to control that the solution should meet certain properties. (p.5).

The researchers results were similar to Ghazali and Zakaria (2011) whose results revealed that, students in procedural and conceptual understanding of the test in algebra showed a high level of procedural understanding but a low level of conceptual understanding (p.689). The current study shows that students performed well in procedural concepts and not conceptual which is not in agreement with Engelbrecht et al (2005) in the study of undergraduate procedural and conceptual performance and confidence in calculus conducted to life science students at Pretoria University.

Table 6.1: Discriminant Analysis

Discrimination Measures			
	Dimension		
	1	2	Mean
Level of confidence to answer questions on binomial expansion and systems of equations correctly	.439	.395	.417
Level of confidence to answer questions on set theory correctly	.338	.356	.347
Level of confidence to answer questions on partial fractions and remainder theory correctly	.544	.499	.521
Level of confidence to answer questions on functions correctly	.372	.312	.342
Level of confidence to answer questions on trigonometric identities correctly	.341	.247	.294
Level of confidence to answer questions on quadratic functions and complex numbers correctly	.414	.379	.397
Active Total	2.449	2.188	2.318
% of Variance	40.825	36.459	38.642

Table 6.8 shows results of the variables underpinning the study which postulates that procedural aspects were more visible as compared to conceptual components. In this research, it was revealed that discriminant analysis was used in order to discriminat between the two components of procedural and conceptual understanding. Nonetheless, discriminant analysis is used to discriminant between two or more mutually exlusive and exhaustive

groups on the basis of some explanatory variables Kothari and Garg (1985:373). Table 6.8 was discriminanted into three groups after discriminant analysis. Using Table 6.8 it was observed that group 1 collected 1.324 for procedural as compared to 1.125 in the same group for conceptual. To add on, for group two we had 1.141 for procedural while 1.047 was for conceptual. Furthermore, the group mean recorded 1.23 for procedural and a group mean of 1.086 for conceptual. As from the debate above, we may wish to state that using discriminant analysis, it was observed that procedural understanding was more than conceptual understanding. This means that students at the University of Zambia were more confident to answer questions correctly on procedural aspects as compared to questions of conceptual nature. In the same vain, we may postulate that, following the measurements in a horizontal plane. The sum of the first, third and fifth row (i.e. $1.251 + 1.564 + 0.882 = 3.697$). The first, third and fifth rows are all procedural in nature and are more explained than the conceptual number. On the other hand, the sum of the second, fourth and sixth rows are all conceptual in nature. The sum of the second, fourth and sixth (i.e. $1.041 + 1.026 + 1.19 = 3.257$).

From the demonstrations above, it clearly reveals that students at the University of Zambia were more confident to answer procedural questions as compared to conceptual ones. From the data above, we may wish to state that, students were more confident to answer question three more than the first question . This means that, students were confident to answer on partial fractions specifically on selected concepts of (partial fractions: denominator with: linear factors none of which is repeating and also polynomial functions specifically the remainder theorem. This is followed by question one consisting of binomial expansions (factorials; binomial coefficients) and also on equations with the specific concepts of procedurally oriented question systems of equations in two unknowns. On question number five it was a bit astonishing because, despite low confidence to answer questions on trigonometric identities it was revealed that a good number of students atleast got the question correctly. This aspect showed that learners lacked confidence on how to solve trigonometric identies when actually they could solve without difficulties. Furthermore, it was also found that there is a positive relationship between conceptual understanding and mathematics achievement (Ghazali and Zakaria, 2011). Even though, in some university mathematics courses emphasis has shifted to a more theoretical approach for which the new intake of students is clearly not ready (Engelbrecht, Harding and Phiri, 2010:3).

Table 6.2: Cluster Analysis

Final Cluster Centers		
	Cluster	
	1	2
Level of confidence to answer questions on binomial expansion and systems of equations correctly	2.71	4.23
Level of confidence to answer questions on set theory correctly	2.97	4.03
Level of confidence to answer questions on partial fractions and remainder theory correctly	3.03	4.31
Level of confidence to answer questions on functions correctly	2.82	4.06
Level of confidence to answer questions on trigonometric identities correctly	2.95	4.07
Level of confidence to answer questions on quadratic functions and complex numbers correctly	2.67	4.03

Cluster analysis is a technique which groups persons/ objects/ occasions into unknown number of groups such that the members of each group are having similar characteristic/ attributes (Panneerselvam, 2012:467) and also (Kothari and Garg, 1985: 387) argues that, the purpose of cluster analysis is to divide large groups of observations, like customers or products, into smaller groups such that the observations within each group are similar or close (or homogeneous) and far away. From the Table 6.9, it shows that the first cluster had procedural understanding of 8.69 while in the same cluster conceptual understanding was 8.46. This shows that students at the University of Zambia were more confident to answer questions of procedural nature more than conceptual ones by 0.23. In the second cluster for procedural confidence of understanding was 12.61 while conceptual confidence was 12.12. in this regard, This shows that, in the second cluster the difference of conceptual understanding from procedural understanding was 0.49. As Table 6.9 indicates above, the difference in confidence in students in the first and second clusters. The big difference was observed in the second cluster between procedural and conceptual understanding which was twice more than that in cluster one. When we had the procedural confidence from questions; one, three and five compared to question ; two, four and six (i.e. procedural questions $6.94 + 7.34 + 7.02 = 21.3$ and also for conceptual questions $7 + 6.88 + 6.7 = 20.58$). The information above revealed that students at the University of Zambia were more confident to answer procedural questions as compared to conceptual ones. The study revealed that there is a positive correlation between student's understanding of mathematical concepts and their ability to excute procedures (Long,2005:62). Furthermore, it was revealed that the more confident learners were the more well they performed either procedurally or conceptually. From this result, we may wish to state that, since students were more confident to answer procedural questions as compared to conceptual. We may conclude that students understanding of concepts was more in procedural questions than conceptual and they were more confident in answering procedural tasks than conceptual questions.

Arising from the aforementioned, the study revealed that most of the students had procedural understanding and not conceptual understanding. This is supported by Bouhjar et al (2017) who stipulates that students performed better on procedural item than on the conceptual item in a matriculation level of mathematics. Research reveals that most students display high levels of procedural understanding but a low level of conceptual understanding (Ghazali and Zakaria, 2011). Students learn a procedure without understanding the will power of requiring extensive training so that steps of procedure can be performed easily and correctly (Fatqurhohman, 2016). Furthermore, it was revealed that both performance and confidence of procedurally oriented questions were good as compared to conceptual tasks. Bempeni and Vamvakoussi (2015) stipulated that in literature there is an overarching distinction between the deep approach to learning, associated with the individual's intention to understand; and the surface approach, associated with the individual's intention to reproduce. Being able to explain why something is or why something works is a crucial part of teaching (Hope, 2006). It was revealed that students should be given additional practice questions in form of homework and assignments so that they improve their mathematical skills.

The current evidence suggests that students at UNZA performed well in procedural questions as compared to conceptual as supported by Engelbrecht et al (2017) who argued that procedural questions were more prominent as compared to conceptual. The study was also supported by Fateel et al (2019) whose study revealed that, students performed well in procedural knowledge and poor in conceptual understanding. According to the findings of the current study, students with low confidence performed low in the test as supported by Hosein et al (2008) whose study revealed that students with low confidence in mathematics had lower marks. Belter (2009) argues that students had difficulties particularly in the understanding of conceptual concepts because after giving a test the questions produced very poor results.

6.7 Chapter summary

This chapter provides an amalgamation of the discussions of the results of the study. Issues relating to instruments used for measuring mathematics understanding were addressed. In line with this, there was a discussion of using standard indices and also multi-variate analysis tools in the measuring of mathematics understanding of first year students at UNZA. This chapter also showed discussions concerning the relationship between students confidence and their mathematics understanding (procedural or conceptual). Thereafter, a synthesis of results on the relationship between students confidence and performance was highlighted. However, it was revealed that the researcher adopted standard indices from (Engelbrecht et al, 2005) which were adjusted and developed to conform to the current study. To add on, the study at hand revealed that students were more confident to answer procedural tasks as compared to conceptual. The study being undertaken revealed that there was a strong relationship between students confidence levels and their understanding of procedural and conceptual concepts in mathematics. The study reveals that students at UNZA performed well in procedural questions as opposed to conceptual. The next chapter focuses on the conclusion, recommendation and any further research to be done similar to the current study.

CHAPTER SEVEN
CONCLUSIONS AND RECOMMENDATIONS

7.1 Introduction

This chapter is organized as follows; Introduction, Conclusions, Conclusions derived from all the research questions, Significant Scientific Contribution to the Body of Knowledge, Recommendations and Chapter summary.

7.2 Conclusions

It was revealed that students at UNZA had more procedural understanding of first year mathematics concepts than conceptual. It was observed that most of the students had high confidence but low understanding of concepts either procedural or conceptual. Despite getting solutions correctly some students indicated that they were not confident to get the solution correctly in the questionnaire. This meant that students got correct solutions by trial and error methods. Furthermore, students who had good grades in conceptual problems had equally correct procedures for procedural questions while learners who were good at procedures often forgot correct ways and degenerated to old incorrect procedures.

Individuals who become used to solving problems in a certain way may overlook important features which may limit both procedural and conceptual understanding of mathematical concepts. In the current study, the researcher used standard indices which showed that the procedural understanding of mathematics concepts was more than conceptual understanding. Factor analysis, multi-dimensional scaling and cluster analysis exposed similar results which revealed that students who did well in conceptual problems also performed well in procedural questions. Unfortunately, students who did well in procedural tasks did not perform well in conceptual tasks. This meant that students conceptual understanding of concepts was low. Conversely, not only did the study revealed students reproducing mathematical routines without a thoughtful developed conceptual understanding of specific concepts but also had challenges of mastering concepts because of a good number of misunderstandings and misconceptions recorded. Noteworthy, the study further revealed that the majority of the learners only managed to memorize formulars and rules without grasping the real application of mathematical facts. Hence, the study showed that students had high confidence but failed to solve the tasks correctly. Concomitantly, the confidence and performance in procedural tasks was a bit better than conceptual ones.

7.3 Conclusions derived from all the research questions

This part of the chapter outlines the conclusions obtained from each research question. However, the study being undertaken clearly justifies each and every research questions findings as here-under elaborated.

7.3.1 Measuring of understanding of mathematics concepts.

In this regard, I developed and designed four measuring instruments in order to answer research question one. Based on the steps under methodology, the measuring instruments were in the form such that the first two measured procedural and conceptual understanding while the last two measured procedural and conceptual confidence exihibited by learners. I made sure that the developed measuring instruments for mathematical understanding were scientifically valid and reliable as argued in the methodology opted for the study. This was possible because the developed instruments atleast measured two of the mathematical proficiency. The current study tried to provide new instruments to measure mathematical understanding despite a lot of challenges I managed to develop four mathematical measuring instruments (standard indices).

7.3.2 Understanding of mathematics concepts by first year UNZA students

The study revealed that the kind of understanding possessed by first year UNZA students in mathematics are procedural and conceptual. Based on the findings from the study (Stanadard Indices 1 & 2). It was revealed that procedural understanding was twice more than conceptual understanding (i.e. the smaller the value the more understanding on the part of the students). In conclusion, the findings from standard indices 1 & 2 were compared to the findings from multivariate analysis (factor analysis, multi-dimensional scaling and cluster analysis) and the findings where similar.

7.3.3 Confidence levels of students in solving mathematics tasks

The study revealed that there was a positive relationship between students confidence and their understanding of procedural and conceptual concepts of mathematics at UNZA among first year students. Based on the research findings, the study further revealed that procedural confidence of understanding was twice more than conceptual confidence of understanding. This meant that students were more confident to answer procedural questions as compared to conceptual. It was further revealed that students did well in questions 1, 3 and 5 which were procedural in nature and also had high level of confidence to answer the same questions which were procedural . This clearly shows that there was a significant relationship between students confidence levels and their procedural and conceptual understanding. To add on, it was observed that there was a positive correlation between students understanding of mathematics concepts and their ability to excute procedures. Likewise, students did well on binomial expansions and systems of equations where they had high confidence meaning also that they possessed high understanding of procedural concepts. Nonetheless, the study also revealed that the level of confidence by students to answer questions on partial fraction and polynomial functions was high. Moreso, the understanding using standard indices showed that students understood questions 1, 3 and 5 which gave same results that students were equally confident when using principal component analysis and multi-dimensional scaling and cluster analysis.

7.3.4 Confidence versas performance

Based on the results from the study it was revealed that students confidence correlated to actual performance of the test. Most of the students were confident to answer correctly procedural oriented questions. After the study it was observed that students performed well in procedural questions where they were more confident. Furthermore, students who performed well in conceptual tasks also did well in procedural. Unfortunately, students who did well in procedural tasks did not perform well in conceptual. However, the study revealed that either students had high confidence but low knowledge or the level of understanding of the concept was not there. Students at UNZA had more procedural understanding of concepts than conceptual.

7.4 Significant Scientific Contribution to the Body of Knowledge (Implications of the Study)

This study has demonstrated and clarified the significant scientific contribution to knowledge base of understanding of procedural and conceptual mathematics concepts. Unique contribution of this study towards the knowledge base in mathematics education was the four standard indices which have been clarifield through a logical thought processes in chapter five. However, (Gelling et al, 2014) point it out that the concept of originality is associated with something novel or unique. In this research, the topic itself was the first contribution to the Body of Knowledge since it has never been done at the University of Zambia. The study being undertaken developed and adapted new measuring instruments for procedural and conceptual understanding of mathematics concepts. To add on the study also developed standard indices to measure procedural and conceptual confidence exhibited by students for them to perform workings correctly. This piece of writing sought to demonstrate how new standard indices will be used in my discipline and other related subjects. This meant that my findings may be used to solve trending issues in mathematical measuring of concepts. This is so because previously scholars used approaches like writing to learn mathematics, mistakes and errors committed to measure mathematical understanding . These formulars were not accurate and reliable. Thus, the current study came to falsify and refute previous work and give alternative solution which were proved to be valid and reliable.

Furthermore, the test developed for the study may be adopted and used by other researchers because it conformed to good test specification. The items selected were short, without sacrificing clarity and also written at an appropriate level of the intended audience.

7.4.1 The Topic Itself

The study revealed that understanding of specific concepts in selected mathematics topics at UNZA by first year students underpinned the research. Not only was mathematical understanding of concepts being addressed but also students confidence in solving given tasks correctly. Notwithstanding the fact that mathematical understanding was the keystone of the study. However, it was shown that few empirical studies were done as pointed out in literature concerning the measuring of mathematics understanding using scientific procedures. The results of most of the studies showed unscientific methods of measuring students understanding of mathematics concepts such as pointing out errors in tests, asking questions on key concepts and writing to learn mathematics. All these methodologies proved not to be reliable. The topic was explicit and it has never being done at UNZA. Not only may it contribute to the understanding of procedural and conceptual aspects of first year mathematics but also to the confidence exhibited by students at that level. By and large, the study might help lecturers to measure mathematical understanding by using the newly developed and adapted standard indices.

7.4.2 Theoretical and Conceptual Framework

The theoretical and conceptual framework chosen suited the current study. The theoretical framework for the study helped to use positivists approaches which helped in the collection and analysis of data. Notwithstanding, the aspects of conceptual frameworks in that I used two conceptual frameworks opted which helped to develop my own conceptual framework.

Consequently, I used my conceptual framework in the construction of the test questions as procedural then followed by conceptual from question one upto six in an alternating way. My conceptual framework also helped in the analysis and interpretation of data. The conceptual framework helped in making a synthesis of things that have not been put together before. It was observed that the theoretical and conceptual framework helped me to establish what is unique and determine how this can be clearly explicated.

7.4.3 Reflections on Methodology

This study focussed on the understanding which University of Zambia (UNZA) first year students of mathematics had of specific concepts in selected mathematics topics: It was revealed that students who were confident in procedural tasks did well in conceptual component. Above and beyond, it was also observed that students understanding of procedural concepts was more than conceptual ones. Based on the current study, it was revealed that students had confidence to do well in procedural components (i.e. finding the n^{th} term of the binomial expansions, solving simultaneous equations of one linear and one quadratic, inequalities involving quotients all of question one and also partial fractions involving one linear and none repeating quadratic and the remainder and factor theorem all of question three lastly but by no means the least trigonometric identities for question five). In all the already mentioned specific concepts of procedurally oriented questions students understanding, confidence and performance was average. Additionally, as from the study it was further revealed that students confidence to do well in conceptual components was low (i.e. solving tasks on set theory using De Morgans laws, the task on real number involving binary operations on real numbers all of question two. Finding the domain and range of functions for question four and finally question six consisted of quadratic roots of functions and also arithmetic operations on complex numbers). It was revealed that students confidence, understanding and performance of conceptual concepts in mathematics was below average.

The methods used in the current study were unique. I had to combine standard indices and multivariate analysis in the analysis of data. Hence my research originality demonstrates the interpretation of research results in the light of the objectives and then widens to fit the current study into the theory upon which it was introduced. The methodology that was employed for this study enabled UNZA's first year students understanding of specific concepts (procedural or conceptual) to be upheld and achieved. Simple random sampling techniques were employed. The mathematics test prepared were structured test accompanied by a questionnaire to elicit participating views on the level of confidence to answer the questions. This was significant because of justification of students reasoning.

7.4.4 Research Findings

The practical implication of the study is that the new developed instruments (standard indices) may be used by other mathematics education experts. The study clarifies the significant scientific contribution to the body of knowledge in that my study has added four standard indices to be used in mathematics education to measure mathematical understanding. However, since research is an academic guess as supported by Karl Popper (1962) who argues that research is an intelligent academic opion that has undergone systematic scientific observation and has contributed to knowledge base in the particular field. Hence or otherwise the results of this study is the final embodiment of knowledge which has being demonstrated. Notwithstanding, the study shows that standard indices and multi-variate analysis (factor analysis, multi-dimensional scaling and cluster analysis) may be collaborated to find the understanding of the subject matter be it any of the subjects (i.e. science, technology,engineering and mathematics).

7.4.5 Reseach Contributions

Justified true knowledge contributed to the body of knowledge **(Four Standard Indices).** These standard indices consisted of two procedural and the other two conceptual. Demonstrating what is new requires indication of what was previously known. However, the study at hand has demonstrated what was there before establishing the new thing. Something which we did not know but because of this study we now know. This makes my study fit or appropriate to be a doctorate thesis because it has made significant scientific contribution to knowledge in mathematics education.

Therefore, the current study developed four measuring instruments (standard indices) used to measure mathematics understanding (procedural or conceptual) and also confidence exhibited by students to solve the mathematical tasks correctly. Standard indices may be used in any STEM subjects. Finally, this paper proposes the use of new measuring instruments (standard indices) for measuring of procedural and conceptual understanding as well as confidence of solving either of the two constructs. The current study proposes for a paradigm shift in measuring mathematics understanding from qualitative to quantitative ways because the adapted quantitative measurements are scientifically validated and proved to be reliable using scientific observations.

7.5 Recommendations

The current evidence suggests that mathematics lecturers at UNZA may adopt new measuring instruments of mathematics concepts developed and designed in the study (standard indices) so that they may use them. This may enhance measuring students mathematics understanding of concepts and confidence exhibited to handle the given tasks.

- Based on the findings, UNZA lecturers of mathematics should focus on teaching methods which would enhance students' conceptual understanding of concepts in mathematics.
- The current study may stimulate further research of understanding of mathematics concepts in universities in Zambia and beyond.
- Effective mathematics teaching should focus on the development of both conceptual understanding and procedural fluency.

To use mathematics effectively, students must be able to do much more than carry out mathematical procedures. They must know which procedure is appropriate and most productive in a given situation, what a procedure accomplishes, and what kind of results to expect. On the whole, the researcher strongly suggests that students should be given a test before starting first year learning to ascertain if they are mathematically competent. The students who perform well should be streamed in mathematics and sciences while those who may fail mathematics despite a good grade on the certificate should be allocated in non-sciences streams.

7.6 Chapter summary

The current study investigated first year students' understanding of specific concepts in selected mathematics topic at UNZA. The study revealed how to develop and design measuring instruments (standard indices) which were used in the measuring of procedural and conceptual understanding of mathematics concepts. Furthermore, the developed instruments were able to measure procedural and conceptual confidence exhibited by learners to solve tasks correctly. The newly developed measuring instruments were authenticated by collaborative efforts of using multivariate analysis tools (factor analysis, multi-dimensional scaling and cluster analysis) which gave similar results. In line with the aforesaid, we may wish to conclude that the study answered adequately all the four research questions. Not only did the study contributed to filling the knowledge gap in literature on the topic but also contributed immensely to the field of mathematics education. Furthermore, a contribution in the processs of developing and designing measuring instruments was underscored . Finally, the study contributed new knowledge to the development and designing of measuring instruments (standard indices) of mathematical understanding to the branch of mathematics and mathematics education.

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APPENDICES

Appendix A: Research instrument for Students (Test)

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS
MAT 1100 TEST   DURATION: 2 HOURS

Instructions: There are **(6)** six questions in this paper. Answer **all** questions.

At the end of the paper there is a questionnaire please fill it in as it is also part of the exercise. Please do not indicate your name:

Computer number:…………… Tutorial group:…………………………….

1. (a) Find the 5th term in the expansion of $\left(x^3 - \frac{1}{2x}\right)^6$ [4]

   (b) Solve the simultaneous equations

   $2x^2 + 3y^2 + x = 13, \quad 2x + 3y - 7 = 0$ [5]

   (c) Solve the inequality $\frac{x}{x-1} < \frac{2}{x+2}$ [5]

2. Prove that

   (a) $(A \cap B)' = A' \cup B'$ [5]

   (b) Determine whether or not the binary operation defined by

   $x * y = xy^2$ on $\mathbb{R}$ is associative. [5]

3 (a) Resolve $\frac{7+x}{(1+x)(1+x^2)}$ into partial fractions [5]

   (b) The remainder when $x^3 - 5x + b$ is divided by $x + 3$ is twice the remainder when it is divided by $x - 2$. Find the value of b . [3]

4. Find the domain and range of each of the following functions.

   (i) $f(x) = \sqrt{3x - 4}$ [2] (ii) $f(x) = x^2 - 2$ [2]

   (iii) $f(x) = -\sqrt{x}$ [3] (iv) $f(x) = \frac{1}{x-3}$ [3]

5. Prove that

   (a) $\frac{1}{\tan^2\theta + 1} + \frac{1}{\cot^2\theta + 1} \equiv 1$ [5]

(b) Prove the identity $\sqrt{\left(\frac{1-\sin x}{1+\sin x}\right)} \equiv \sec x - \tan x$ [5]

6. (a) If α and β are the roots of the equation $ax^2 + bx + c = 0$.

Obtain in terms of a, b and c the expression

(i) $\alpha + \beta$ [2] (ii) $\frac{1}{\alpha} + \frac{1}{\beta}$ [2]

(iii) $\alpha^2 + \beta^2$ [2] (iv) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ [2]

(b) Find identical roots of the equation $(x + k)^2 = 2 - 3k$ [5]

(c) Express in the form $a + ib \in \mathbb{R}$:

$\frac{i^3}{2+i} - \frac{i^4}{2-i}$ [5]

QUESTIONNAIRE: In your considered opinion. How do you rate your level of confidence to answer the following questions correctly?

Circle one response for each item using the scale as follows:

1. Total guess 2. Not certain 3. Certain 4. Fairly certain 5. Completely certain

1. **Binomial Expansion**: factorials; Binomial coefficients. **Equations:** Systems of equations in two unknowns.	1	2	3	4	5
2. **Sets: Set Theory:** De Morgan's laws; Set operations **Number Systems:** Binary operation on numbers	1	2	3	4	5
3. **Partial fractions: Denominator with:** linear factors none of which is repeating. **Functions: Polynomial Functions:** remainder theorem	1	2	3	4	5
4 **Functions:** domain and range.	1	2	3	4	5
5 **Transcendental Functions:** Trigonometric Functions: Identities.	1	2	3	4	5
6. **Functions: Linear and Quadratic Functions, complex** **numbers:** applications.	1	2	3	4	5

THANK YOU FOR YOUR COOPERATION

Appendix B: Research instrument for Lecturers

THE UNIVERSITY OF ZAMBIA

DIRECTORATE OF RESEARCH AND GRADUATE STUDIES

DEPARTMENT OF MATHEMATICS AND SCIENCE EDUCATION

LECTURERS' QUESTIONNAIRE

I am a bonafide student of UNZA carrying out a research on " First year students' understanding of specific concepts in selected mathematics topics : the case of the University of Zambia." You have been selected as one of the respondents. You are requested to answer the questionnaire after studying the test of first year students at the University of Zambia as honesty as you can. The information is only for academic purposes and will be kept confidential. I would be very grateful if you could spare some of your valuable time to complete the questionnaire.

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

MAT 1100 TEST DURATION: 2 HOURS

1. (a) Find the 5th term in the expansion of $\left(x^3 - \frac{1}{2x}\right)^6$ [4]

(b) Solve the simultaneous equations

$$2x^2 + 3y^2 + x = 13, \quad 2x + 3y - 7 = 0$$ [5]

(c) Solve the inequality $\frac{x}{x-1} < \frac{2}{x+2}$ [5]

2. Prove that

(a) $(A \cap B)' = A' \cup B'$ [5]

(b) Determine whether or not the binary operation defined by

$x * y = xy^2$ on $\mathbb{R}$ is associative. [5]

3 (a) Resolve $\frac{7+x}{(1+x)(1+x^2)}$ into partial fractions [5]

(b) The remainder when $x^3 - 5x + b$ is divided by $x + 3$ is twice the remainder when it is divided by $x - 2$. Find the value of b . [3]

4. Find the domain and range of each of the following functions.

(i) $f(x) = \sqrt{3x - 4}$ [2] (ii) $f(x) = x^2 - 2$ [2]

(iii) $f(x) = -\sqrt{x}$ [3] (iv) $f(x) = \frac{1}{x-3}$ [3]

5. Prove that

(a) $\frac{1}{\tan^2\theta + 1} + \frac{1}{\cot^2\theta + 1} \equiv 1$ [5]

(b) Prove the identity $\sqrt{\left(\frac{1-\sin x}{1+\sin x}\right)} \equiv \sec x - \tan x$ [5]

6. (a) If α and β are the roots of the equation $ax^2 + bx + c = 0$.

Obtain in terms of a, b and c the expression

(i) $\alpha + \beta$ [2] (ii) $\frac{1}{\alpha} + \frac{1}{\beta}$ [2]

$(iii)\ \alpha^2 + \beta^2$ [2] $(iv)\ \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ [2]

(b) Find identical roots of the equation $(x + k)^2 = 2 - 3k$ [5]

(c) Express in the form $a + ib \in \mathbb{R}$:

$$\frac{i^3}{2 + i} - \frac{i^4}{2 - i}$$ [5]

QUESTIONNAIRE: As a lecturer of mathematics in your considered opinion,

How do you rate procedural understanding you may expect by looking at the questions prepared for testing students?

Circle one response for each item using the scale as follows:

1 Total guess 2. Not certain 3. Certain 4. Fairly certain 5. Completely certain

1. Binomial Expansion: factorials; Binomial coefficients. **Equations:** Systems of equations in two unknowns. **Inequations:** Quadratic equations	1	2	3	4	5
3. Partial fractions: Denominator with: linear factors none of which is repeating. **Functions: Polynomial Functions:** remainder theorem	1	2	3	4	5
5.Transcendental Functions: Trigonometric Functions: Identities.	1	2	3	4	5

QUESTIONNAIRE: As a lecturer of mathematics in your considered opinion,

How do you rate conceptual understanding you may expect by looking at the questions prepared for testing students?

Circle one response for each item using the scale as follows:

1. Total guess 2. Not certain 3. Certain 4. Fairly certain 5. Completely certain

2. Sets: Set Theory: De Morgan's; Set operations. **Binary Operation:** Binary operations on Real Numbers.	1	2	3	4	5
4. Functions: domain and range	1	2	3	4	5
6. Functions: Linear and Quadratic Functions, Complex Numbers: Applications.	1	2	3	4	5

QUESTIONNAIRE: As a lecturer of mathematics in your considered opinion,

How do you rate procedural level of difficult you may associate to the questions?

Circle one response for each item using the scale as follows:

1 Total guess 2. Not certain 3. Certain 4. Fairly certain 5. Completely certain

1. Binomial Expansion: factorials; Binomial coefficients. **Equations:** Systems of equations in two unknowns. **Inequations:** Quadratic equations	1	2	3	4	5
3. Partial fractions: Denominator with: linear factors none of which is repeating. **Functions: Polynomial Functions:** remainder theorem	1	2	3	4	5
5.Transcendental Functions: Trigonometric Functions: Identities.	1	2	3	4	5

QUESTIONNAIRE: As a lecturer of mathematics in your considered opinion,

How do you rate conceptual level of difficult you may associate to the questions prepared?

Circle one response for each item using the scale as follows:

1. Total guess 2. Not certain 3. Certain 4. Fairly certain 5. Completely certain

2. Sets: Set Theory: De Morgan's; Set operations. **Binary Operation:** Binary operations on Real Numbers.	1	2	3	4	5
4. Functions: domain and range	1	2	3	4	5
6. Functions: Linear and Quadratic Functions, Complex Numbers: Applications.	1	2	3	4	5

THANK YOU FOR YOUR COOPERATION

Appendix C: Syllabus for Mathematics (MAT 1100)

Appendix C

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

MAT 1100 SCHEME 2016/17

TERM1

Week beginning	TOPIC	SUB TOPICS	No. of lectures
09/01/17	**1.1 Sets**	**1.1.1 Sets Theory:** Definitions; subsets; set operations; De Morgan's laws **1.1.2 Sets of Numbers:** Natural numbers; integers; rational numbers; irrational numbers; real numbers; Complex numbers; arithmetic operations on complex numbers; surds.	6 lectures
23/01/17	**1.2 Functions**	**1.2.1 Functions:** Binary operations; relations; functions; domain and range; many-to-one function; One-to-one functions; inverse functions; compound functions; even and odd functions **1.2.2 Linear and Quadratic Functions:** Completing the square; maximum and minimum Values of quadratic functions; graphs of quadratic functions; applications. **1.2.3 Polynomial Functions:** Polynomials; additions of; multiplication of; division of; remainder theorem; Factor theorem; factorization of; graphs of. **1.2.4 Rational Functions:** Domain of; range of; graphs of. **1.2.5 Modulus Functions:** Domain of; range of; graphs of. **1.2.6 Radical Functions:** Domain of; range of; graphs of.	9 lectures
06/02/17	**1.3 Equations**	**1.3.1 Equations:** Quadratic; polynomials; involving radicals, quotients and absolute value; systems of Equations; one linear and one quadratic	9 lectures
20/02/17	**1.4 Inequalities**	**1.4.1 Inequalities:** Linear; quadratic; polynomials; involving quotients, radicals and absolute value	4 lectures
27/02/17	**1.5 Partial Fractions**	**1.5.1 Denominator with:** linear factors none of which is repeating; linear factors of which some are Repeating; quadratic factors none of which is repeating.	3 lectures
06/03/17	**1.6 Binomial Expansion**	1.6.1 Binomial Expansion: Pascal's triangle; factorials; Binomial coefficients; Binomial formula for positive integral exponents; Binomial formula for rational exponents.	4 lectures
13/03/17	**1.7 Transcendental Functions**	1.7.1 Trigonometric Functions: Trigonometric ratios; ratios of angles; degrees and radian measures; Trigonometric functions; domain; graphs; identities; trigonometric equations.	11 lectures
		End of term 1 test	46 lectures

TERM 2 **MAT 1100 FOUNDATION MATHEMATICS SCHEME 2016/17**

Week beginning	**TOPIC**	**SUB TOPICS**	**No. of lectures**
17/04/17	**2.1 Transcendental Functions**	**2.1.1 Trigonometric Functions:** (Continued) **2.1.2 Inverse Trigonometric Functions:** Domain and range; graphs. **2.1.3 Exponential and logarithmic Functions:** Domain and range of; graphs of; their properties Inverses; equations of.	8 lectures
01/05/17	**2.2 Differential Calculus**	**2.2.1 Limits:** Limits of a function; continuity of a function. **2.2.2 The Derivative:** Differentiation of function from first principle; differentiation by formula; Derivative of; sum; product rule; quotient rule; chain rule; implicit differentiation. 2.2.3 Derivatives of Trigonometric functions. 2.2.4 Derivatives of Exponential function and logarithmic functions. 2.2.5 Derivatives of Inverse trigonometric functions. 2.2.6 Gradient functions; tangents and normal lines to a curve; increasing and decreasing functions; Stationary points (critical points); point of inflexion; relative maximum and minimum; related rates; curve sketching and asymptotes of rational functions.	20 lectures
05/06/17	**2.3 Mathematical Induction**	**2.3.1 The principle of Mathematical Induction**	4 lectures
		End of Term 2 Test	32 lectures

TERM 3 **MAT 1100 FOUNDATION MATHEMATICS SCHEME 2016/17**

Week beginning	TOPIC	SUB TOPICS	No. of lectures
19/06/17	**3.1 Coordinate Geometry 1**	**3.1.1 Coordinate Geometry:** Distance between two points; division of a straight line into a ratio; Equation of a straight line; parallel and perpendicular lines; distance between a point and a Line; equation of a circle; tangent and normal lines to a circle.	4 lectures
26/06/17	**3.2 Integral Calculus**	**3.2.1 Indefinite Integrals:** Integration as the reverse of differentiation; integration of trig functions; exponential functions; hyperbolic functions. **3.2.2 Methods of Integration:** Substitution, integration by parts, change of variable, partial fractions. **3.2.3 Definite Integrals:** Applications to areas.	16 lectures
24/07/17	**2.3 Vectors and Matrices**	**3.3.1 Matrices:** Sum; product; transpose; determinants; factorization of determinants; inverse matrix, solving system of equations by (i) matrices (ii) Cramer's rule. **3.3.2 Applications:** Solutions of system of linear equations by inverse matrix method; Cramer's rule. **3.3.3 Vectors:** Definition; vector addition; vectors in 3-dimension; dot product; vector (cross) product.	12 lectures
14/08/17	**3.4 Further Complex Numbers**	**3.4.1 Complex Numbers in Polar form:** Modulus and argument; De-Moivre's theorem; roots of a complex number	4 lectures
		End of Term 2 Test	36 lectures

Appendix D: Marking Rubric

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

MARKING RUBRIC OF 1100 TEST

- Understanding the flow of concepts in mathematics
- Understanding supporting concepts to the topic
- Understanding of the skills and procedures

0. Answer contains many errors and faults. Virtually no real understanding or appreciation of the concepts. Chaotic presentation of stages, complete wrong working. Misconceptions.
1. Below average, shows some basic understanding of the concepts but limited knowledge. Insufficient attention to organisation and presentation of work.
2. Clear presentation of concepts . Requires some level of supervision (more practice). Shows little ability to try to solve some problems unaided.
3. Good understanding of concepts in mathematics with minor slips. Generally good understaning of facts.
4. Clear indication of very good understanding. Good presentation of work.
5. Broad understanding of the concept in mathematics. Exemplory presentation. Discuss concepts coupled with insight and originality.

Appendix E: Miscellaneous

THE UNIVERSITY OF ZAMBIA

SCHOOL OF EDUCATION

Telephone: 291381
Telegram: UNZA, LUSAKA
Telex: UNZALU ZA 44370

PO Box 32379
Lusaka, Zambia
Fax: +260-1-292702

Date 05/10/2016

TO WHOM IT MAY CONCERN

Dear Sir/Madam

RE: FIELD WORK FOR ~~MASTERS~~/ PhD STUDENTS

The bearer of this letter Mr/~~Ms~~ MWAPE, JOHN Computer number.................................... is a duly registered student at the University of Zambia, School of Education.

He/She is taking a ~~Masters~~/PhD programme in Education. The programme has a fieldwork component which he/she has to complete.

We shall greatly appreciate if the necessary assistance is rendered to him/her/.

Yours faithfully

Emmy Mbozi (Dr)
ASSISTANT DEAN POSTGRADUATE STUDIES- SCHOOL OF EDUCATION

cc: Dean-Education
Director-DRGS

THE UNIVERSITY OF ZAMBIA

DIRECTORATE OF RESEARCH AND GRADUATE STUDIES

Great East Road | P.O. Box 32379 | Lusaka 10101 | **Tel:** +260-211-290 258/291 777
Fax: +260-1-290 258/253 952 | **Email:** director@drgs.unza.zm | **Website:** www.unza.zm

HUMANITIES AND SOCIAL SCIENCES RESEARCH ETHICS COMMITTEE

Approval of Study

19th January, 2021

REF. No. HSSREC: 2018-SEP-022

The Principal Investigator

Dear John Mwape

RE: "FIRST YEAR STUDENTS' UNDERSTANDING OF SPECIFIC CONCEPTS IN SELECTED MATHEMATICS TOPICS: THE CASE OF THE UNIVERSITY OF ZAMBIA."

Reference is made to your submission. The University Of Zambia Humanities And Social Sciences Research Ethics Committee IRB resolved to approve this study and your participation as Principal Investigator for a period of one year.

Review Type	**Expedited/Ordinary Review**	Approval No. **2018-SEPT-022**
Approval and Expiry Date	Approval Date: 20th OCTOBER, 2018	Expiry Date: 19TH OCTOBER, 2019
Protocol Version and Date	Version-Nil	-
Information Sheet, Consent Forms and Dates	• English.	To be provided
Consent form ID and Date	Version	To be provided
Recruitment Materials	Nil	Nil

1

There are specific conditions that will apply to this approval. As Principal Investigator it is your responsibility to ensure that the contents of this letter are adhered to. If these are not adhered to, the approval may be suspended. Should the study be suspended, study sponsors and other regulatory authorities will be informed.

Conditions of Approval

- Provide information sheets and consent letters as these were not attached. The information sheets should have had the essential features included. Please use the WHO templates which you could download at www.who.int/rpc/research_ethics/informed_consent/en/). REC would appreciate if the PI could customise the WHO templates and include the domains of what the submitted protocol is positing on tools and the sampling units (people who have been or shall be participating in this study.
- No participant may be involved in any study procedure prior to the study approval or after the expiration date.
- All unanticipated or Serious Adverse Events (SAEs) must be reported to the IRB within 5 days.
- All protocol modifications must be IRB approved by an application for an amendment prior to implementation unless they are intended to reduce risk (but must still be reported for approval). Modifications will include any change of investigator/s or site address or methodology and methods. Many modifications entail minimal risk adjustments to a protocol and/or consent form and can be made on an Expedited basis (via the IRB Chair). Some examples are: format changes, correcting spelling errors, adding key personnel, minor changes to questionnaires, recruiting and changes, and so forth. Other, more substantive changes, especially those that may alter the risk-benefit ratio, may require Full Board review and approval. In all cases, except where noted above regarding subject safety, any changes to any protocol document or procedure must first be approved by the IRB before they can be implemented.
- All protocol deviations must be reported to the IRB within 5 working days.
- All recruitment materials must be approved by the IRB prior to being used.
- Principal investigators are responsible for initiating Continuing Review proceedings. Documents must be received by the IRB at least 30 days before the expiry date. This is for the purpose of facilitating the review process. Any documents received less than 30 days before expiry will be labelled "late submissions" and will incur a penalty.
- Every 6 (six) months a progress report form supplied by The University of Zambia Humanities And Social Sciences Research Ethics Committee IRB must be filled in and submitted to us. There is a penalty of K500.00 for failure to submit the report.
- The University Of Zambia Humanities And Social Sciences Research Ethics Committee IRB does not "stamp" approval letters, consent forms or study documents unless requested for in writing. This is because the approval letter clearly indicates the documents approved by the IRB as well as other elements and conditions of approval.

Should you have any questions regarding anything indicated in this letter, please do not hesitate to get in touch with us at the above indicated address.

On behalf of The University of Zambia Humanities and Social Sciences Research Ethics Committee IRB, we would like to wish you all the success as you carry out your study.

Yours faithfully,

Dr. Jason Mwanza
BA, MSoc, Sc., PhD
CHAIRPERSON
The University Of Zambia Humanities and Social Sciences Research Ethics Committee IRB

C

Director, Directorate of Research and Graduate Studies
Assistant Director – Research, Directorate of Research and Graduate Studies
Vice Chairperson, Humanities and Social Sciences Research Ethics Committee
Assistant Registrar- Research, Directorate of Research and Graduate Studies
Senior Administrative Officer – Research Affiliation, Directorate of Research and Graduate Studies