THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

SECOND SEMESTER EXAMINATIONS

2011/2012 ACADEMIC YEARS

1. C482 –Inorganic Industrial Chemistry II
2. CS3251 –Electronics for Computing I
3. CST2032 - Fundamentals of Computer Architecture

4. CST 2041 – Introduction to operating systems

4. CST 3022 –
5. CST 3062 –Advanced Databases and Information systems
6. CST 3142 –Software Engineering II
7. CST 4012 –Advanced Operating Systems and Distributed Systems
8. CST 4122-Compilers
9. CST 4132-Computer Graphics
10. EM 312 –Engineering Mathematics IV
11. GEO111 –Introduction to Human Geography I
12. GEO 112 –Introduction to Human Geography II
13. GEO 175 –Introduction to Mapping Techniques in Geography
14. GEO 211 –The Geography of Africa
15. GEO 277 –Quantitative Techniques in Geography II
16. GEO 482 –Environment and Development II
17. GEO 492 –Natural Resources Economics
18. GEO 911 –Population Geography
19. GEO 912 –Geography of Refugees and Migration
20. GEO 951 –Climatology

21. GEO 955 –Geomorphology

22. GEO 962 –Biogeography

23. GEO 975 –Cartography

24. GEO 995 –Environmental and Natural Resources Management I

25. M III –Mathematical Methods I

26. M 112 -Mathematical Methods II –A

27. M114 -Mathematical Methods II –B

28. M 161 – Introduction to Mathematics, Probability and Statistics I

29. M16 - Introduction to Mathematics, Probability and Statistics II

30. M211 –Mathematical Methods III

31. M212 –Mathematical Methods IV

32. M221 –Linear Algebra I

33. M261 –Introduction to Statistics

34. M292 –Introduction to Probability

35. M325 –Group and Ring Theory

36. M412 –Functions of a Complex Variable II

37. M422 –Module and Field Theory

38.431 –Real Analysis V

39. M912 –Mathematical Methods VI

40. M962 –Time Series Analysis
41. P192 –Introductory Physics –II (Option A)
42. P198 –Introductory physics –II (Option B)
43. P212 –Magnetism in Matter and Atomic Physics
44. P252 –Classical Mechanics II
45. P302 – Computational Physics I
46. P332 –Statistical and Thermal Physics
47. P411 –Nuclear Experimental Techniques
48. P412 –Nuclear Physics
49. P441 –Analogue Electronics II
50. P442 –Digital Electronics II
51. P485 –The Physics of Renewable Energy & Environment

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2011 ACADEMIC YEAR, SECOND SEMESTER
FINAL EXAMINATION
May 23, 2012

C482: INORGANIC INDUSTRIAL CHEMISTRY II

TIME: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR QUESTIONS

Question 1.
Describe the properties, reactions of manufacture and use of:
(a) Compound fertilizers (give examples of synergism and antagonism of the mixed fertilizers).
(b) Ammonium sulphate from gypsum.
(c) Single and triple superphosphate.

Question 2.
(a) In the production of Nitric acid, Ammonia is mostly used. Outline the physicochemical foundation manufacturing dilute Nitric acid.
(b) Describe the technological process production of dilute nitric acid (write down the temperatures, pressures and concentrations of the main components in the liquid and gaseous phases).
(c). State the production process of concentrated Nitric acid.

Question 3.
(a) Write down the reactions production of Phosphorus and phosphorus pentoxide.
(b) Briefly explain how ortho-phosphoric acid is produced by wet-method.
(c) Describe the production process of Hydrochloric acid.

Question 4.

In the production of Sulphuric acid Iron pyrite and Sulphur are usually used.
(a) What are the advantages and disadvantages associated with the use of these raw materials?
(b) Describe the SO_2 oxidation to SO_3 process (indicate kindling and other temperatures if Vanadium catalyst is used).
(c) State the properties of 98.3% sulphuric acid and why this acid is used for absorption of SO_3 containing gas?

Question 5.
Write down the reactions and outline the major steps involved in the production of:
(a) Ammonia.
(b) Ammonium nitrate.
(c) Urea.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

COMPUTER STUDIES DEPARTMENT

UNIVERSITY EXAMINATIONS

CS3251: ELECTRONICS FOR COMPUTING I

SEMESTER 1 EXAM **5th DECEMBER 2011**

TIME: THREE HOURS

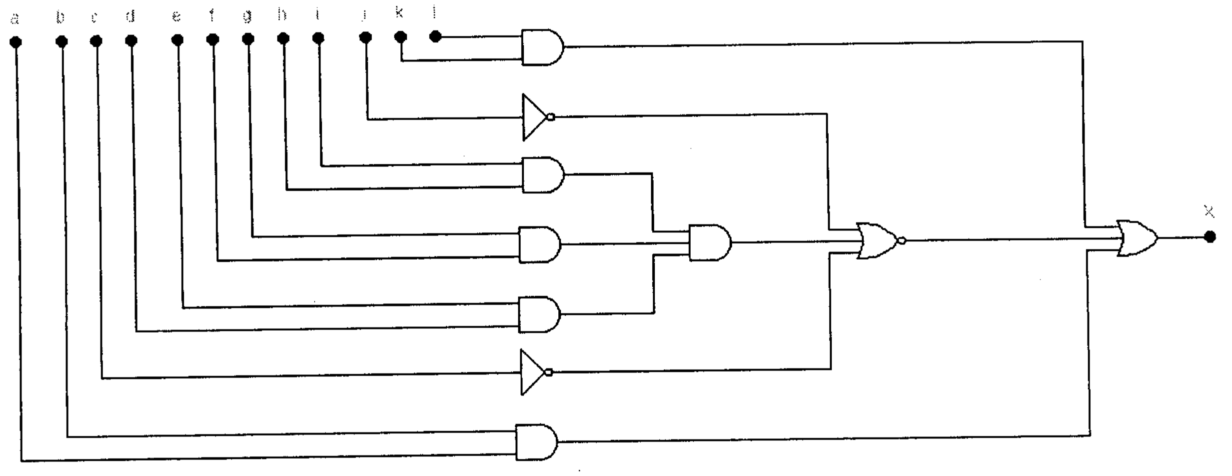
ANSWER: ALL QUESTIONS

QUESTION 1
Using the kanaugh maps and your output F in question1, redesign the circuit diagram (removing exclusive elements / excluding the use of exclusive or/nor) to produce the same signal F.

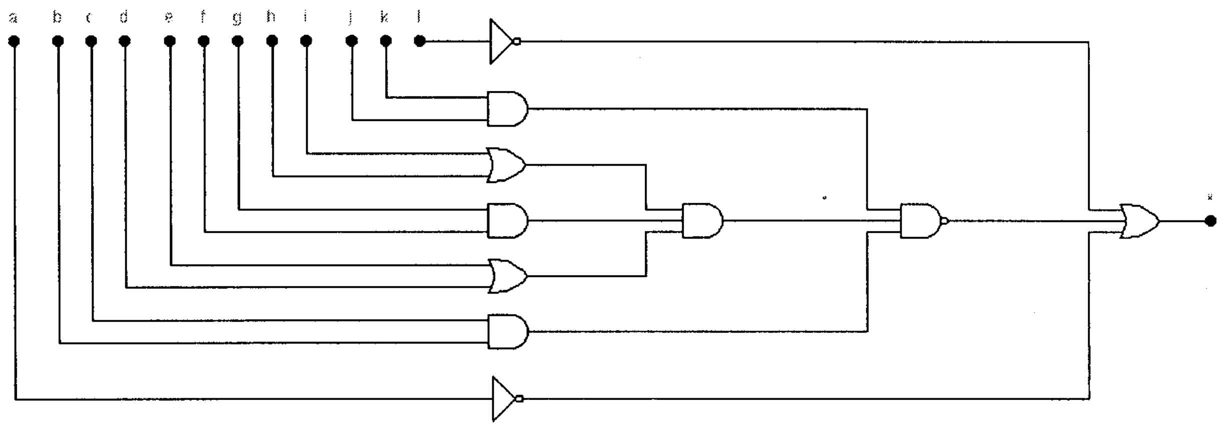
QUESTION 2
If in question3 e=g, d=h, c=I, b=j, a=k and m=l reduce the Boolean expressions accordingly to come up with a simplified expression.

QUESTION 3
Write the Boolean expression for two digital circuit diagrams below. (Choose any two diagrams)

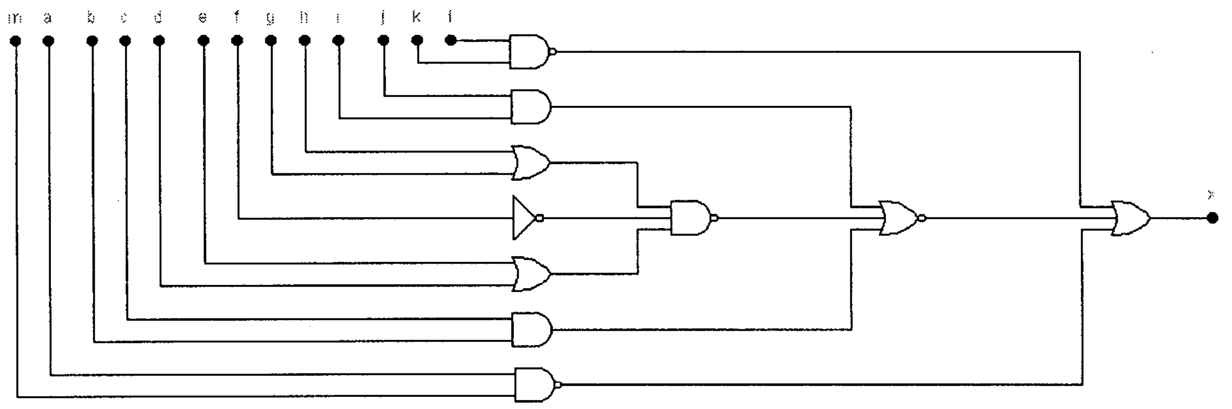
1.



2.

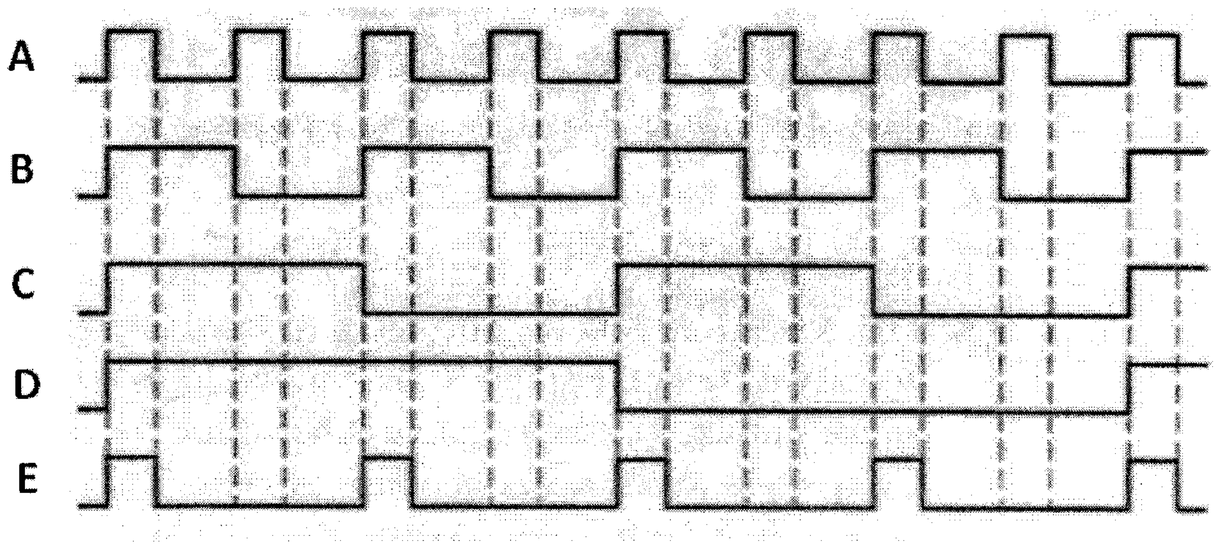


3.

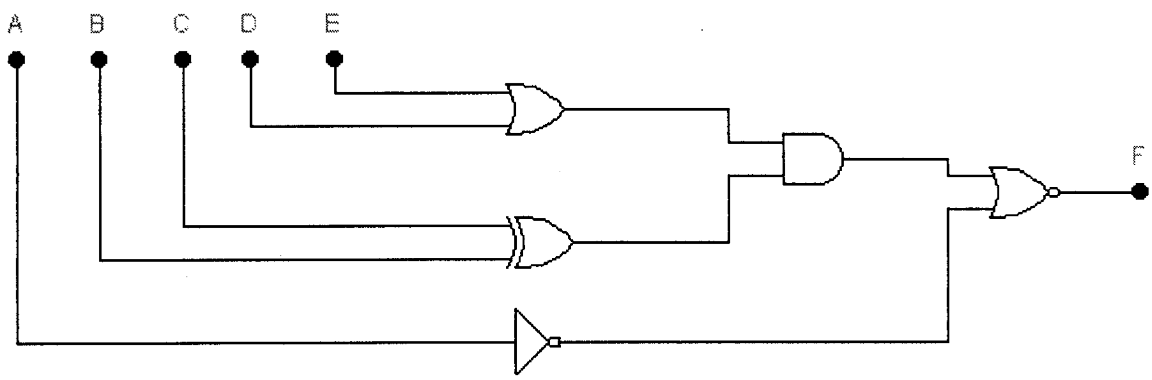


QUESTION 4

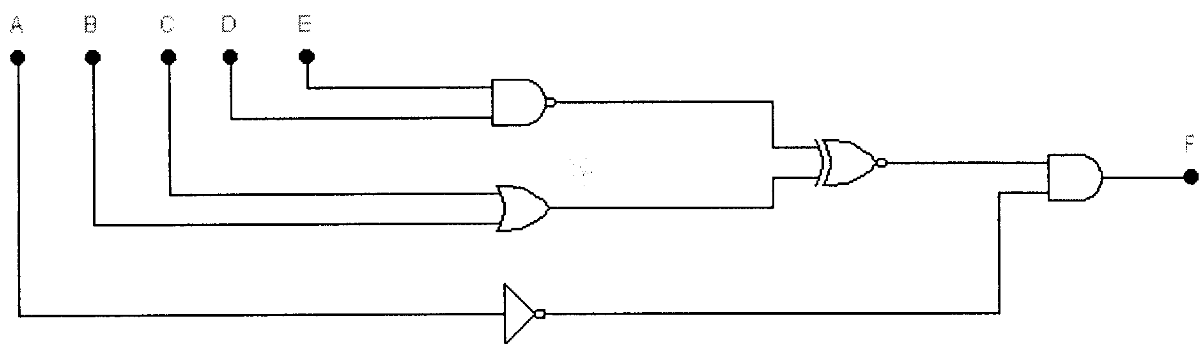
Using the input signal (A, B, C, D and E) below, determine the output signal F for two (2) of the digital circuit diagrams 1, 2 and 3. (Choose any two diagrams)



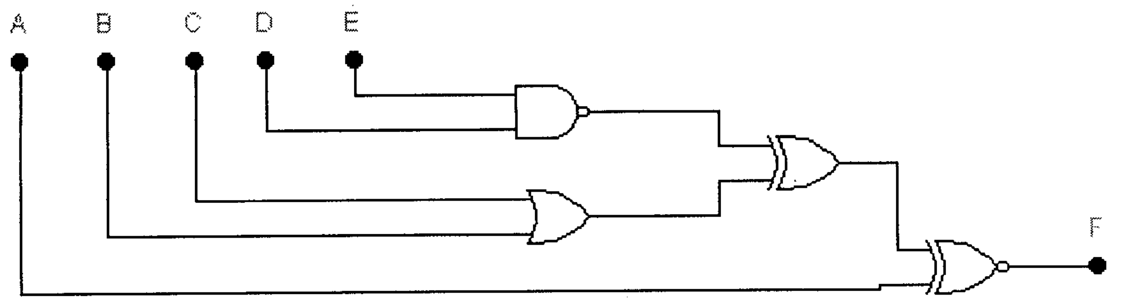
1



2

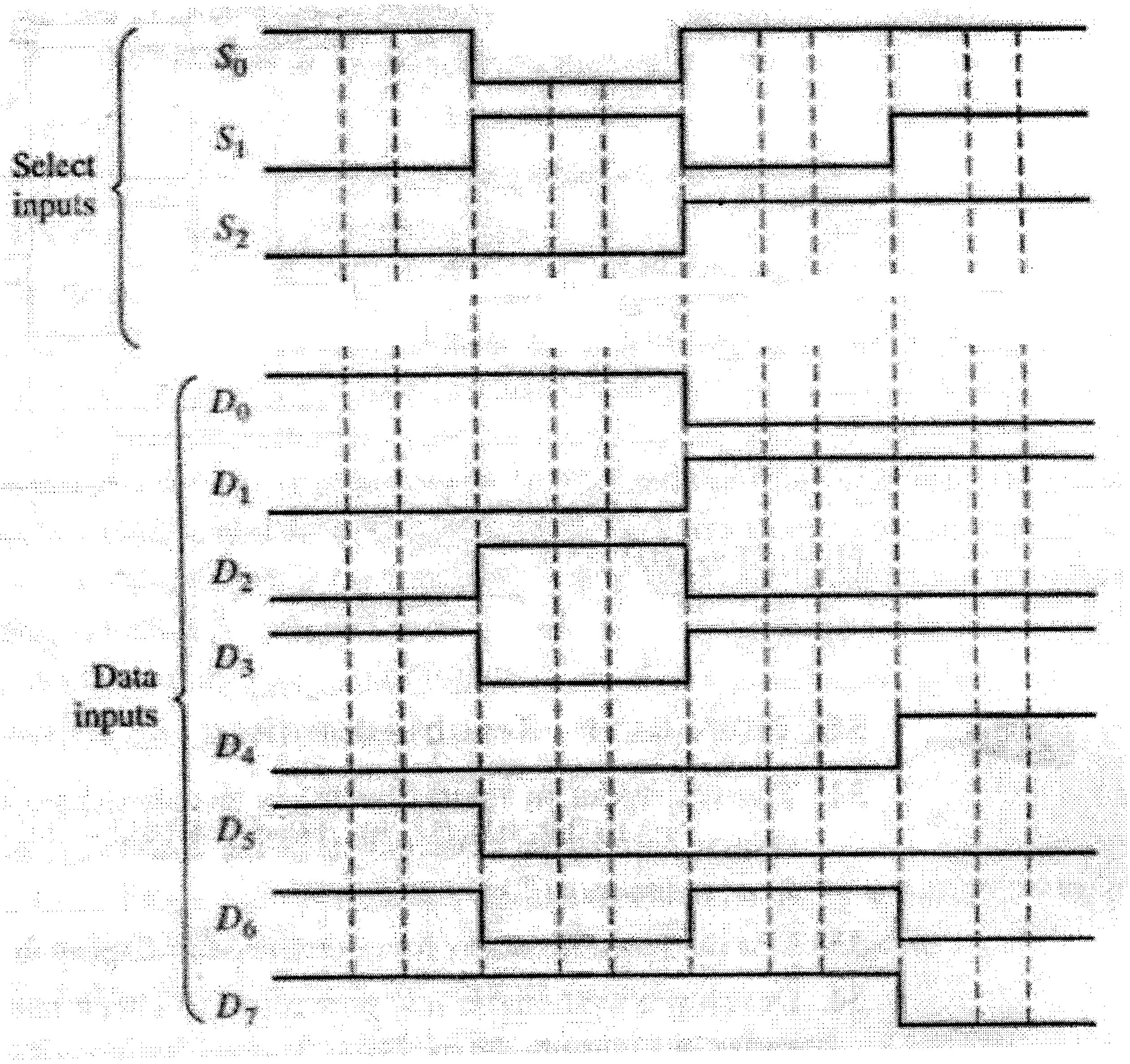


3

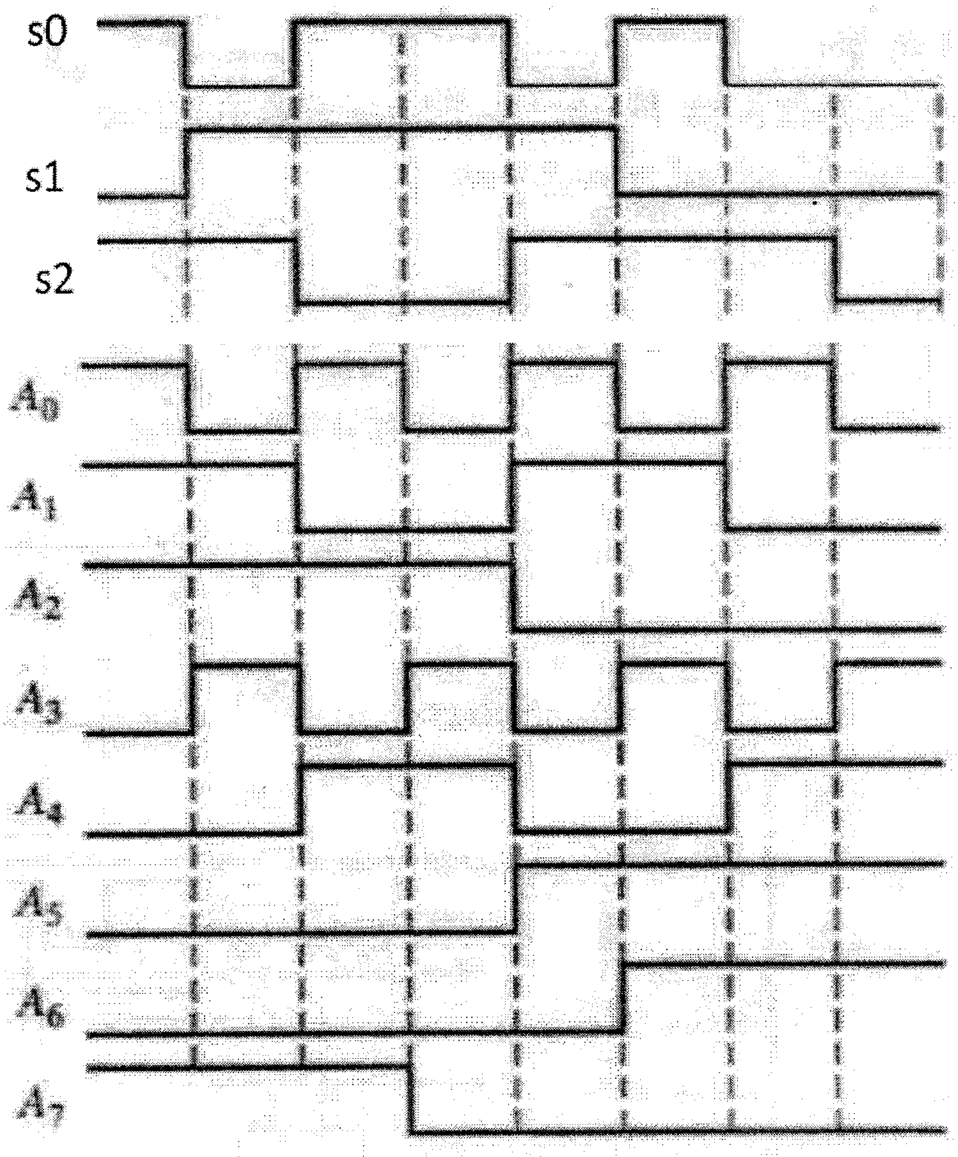


QUESTION 5
Design a multiplexor and determine the output using one of the input pulse lines below. (Choose one)

1.



2.



END OF EXAM

CST 2032 Exam
Thursday 7th June 2012

Answer any five questions. All questions carry equal marks.
Write clearly and explain steps precisely.

Lecturer: Dr. John Regan

1.

a) An IEEE floating point number is made up of 3 parts, the sign bit, the significand and the exponent. Explain each of these terms. For a single precision 32 bit value how many bits are assigned to the sign, exp and fractional part.

b) Given the decimal number 15213.0 write down how the number is stored in a computer as a floating point number on a 32 bit system.

c) Explain the difference between **normalised** and **denormalised** numbers.

d) Consider the following 8 bit floating point binary numbers. Compute the floating point additions – round if required. Assume an 8-bit OS with 4 bits for the exponent.

i. $00100011_2 + 01001101_2$

ii. $01010101_2 + 01101010_2$

2.

a) Explain the difference between the **Little Endian** system and the **Big Endian** system. Give an example of a system that uses Little Endian and one that uses Big Endian.

b) Convert 15123 which is a decimal number to hexamdecimal.

c) Given the following three unsigned integers compute their product:

i. 0011

ii. 1010

iii. 1100

In Order to compute the product of the three integers you should convert to decimal to verify your answer.

d) Given the following signed integer compute its value in decimal

i. 1100010010010011

Next compute its value when you right shift it by 2 places (i.e. >> 2). Explain how you achieve this and check against the expected decimal value. Explain the discrepancy and describe what needs to be done so that the result correctly rounds towards zero.

3.

a) Explain what a **process** is.
b) **Context switching** can happen for several reasons, give two reasons.
c) Consider the following simple program

```
int main()
 {
     int pid = 0;
     pid = fork();
     if(pid = = 0)
     {
            printf("My pid is %d\n", pid);
     }

     printf("My pid is %d\n", pid);
 }
```

Assume the process identity of the parent is 23 and the process identity of the child is 24. What is the output of the above program? Explain clearly your reasoning.

d) Explain clearly the terms **reaping** and **zombie processes**.
e) Explain how **zombie processes** can be avoided.

4.

a) What is a **signal**?
b) Under what circumstances will the kernel send a signal to a process – name at least two.
c) A process can react in three ways to a signal – what are they?
d) Consider the following C code.

   i. What does the function call `signal(SIGINT, handler);` do? Who calls it the parent or the child?
   ii. What does the `wait(&child_status)` function do? Who calls it the child or the parent?
   iii. Given the pid of child 1 is 1333 and the pid of child 2 is 1334 what is the output of the program below?

```c
#include <stdio.h>
#include <signal.h>
#include <stdlib.h>

#define N 2

void handler(int sig) {
    printf("Process %d received signal %d - exiting\n", getpid(),
sig);
    exit(0);
}

int main() {
    pid_t pid[N];
    int i, child_status;
    signal(SIGINT, handler);

    for (i = 0; i < N; i++)
      {
        if ((pid[i] = fork()) == 0)
          {
               while(1); /* child infinite loop */
          }
      }
    for (i = 0; i < N; i++)
      {
        kill(pid[i], SIGINT);
      }

    for (i = 0; i < N; i++)
      {
        pid_t wpid = wait(&child_status);
        if (WIFEXITED(child_status))
            printf("Child %d terminated with exit status %d\n",
                   wpid, WEXITSTATUS(child_status));
        else
            printf("Child %d terminated abnormally\n", wpid);
    }
}
```

5.

a) Explain what the function of the %esp and %ebp in the Intel IA32 architecture is.

b) Suppose that %edx = 0xF000 and %ecx = 0x0100 fill in the following table:

Expression	Computation	Address
0x8(%edx)		
(%edx, %ecx)		
(%edx, %ecx, 4)		
0x80(, %edx, 2)		

a) Given the following Assembly code:
   1. Copy the assembly code into your answer sheet and comment each line of the assembly explaining clearly what each line does.
   2. Fill in the missing spaces in the given C code which corresponds to the assembly code.

```
pushl %ebp
movl  %esp,%ebp

movl  12(%ebp),%eax
xorl  8(%ebp),%eax
sarl  $17,%eax
andl  $8185,%eax

popl  %ebp
ret
```

```
int logical(int x, int y)
{
  int t1 = __________;
  int t2 = __________;
  int mask = __________;
  int rval = __________;
  return __________;
}
```

6.

a) Given the following values calculate the capacity of a disk:
   i. 512 bytes/sector
   ii. 300 sectors/track (on average)
   iii. 20,000 tracks/surface
   iv. 2 surfaces/platter
   v. 5 platters/disk
b) Draw a block diagram showing how the I/O bus connects to the I/O bridge, the CPU and to Main Memory. Include any other elements you thing are important in the diagram.
c) Explain each of the following terms:
   i. Seek Time
   ii. Rotational Latency
   iii. Transfer Time
d) The access time for retrieving a piece of data is defined as $T_{access} = T_{seek} + T_{rotational} + T_{transfer}$ given the following information calculate the access time.
   i. Rotational Rate = 7200 RPM
   ii. Average Seek Time = 9 ms
   iii. Average number of sectors per track = 400

7.

a) Explain briefly what the purpose of a cache is.
b) Given the following 16 bit addresses determine the hit ratio for a 2 way cache, assume 16 byte cache lines and a cache capacity of 128 bytes
   i. 0x0000
   ii. 0x0004
   iii. 0x2204
   iv. 0x0002
   v. 0xde0b
c) Explain the difference between a "Write Through" cache and a "Write Back" cache.
d) The Cache is only part of the memory hierarchy, another equally important part if the Translation Lookaside Buffer. Explain it's function with respect to virtual memory management. Include such terms as address translation and locality of reference.

8.

a) Write down and explain the 4 layers of the TCP/IP model giving examples of protocols that operate at each layer. You should give at least two examples of a protocol per layer.
b) Explain clearly the difference between TCP and UDP giving use cases for both protocols.
c) Explain the purpose of the Domain Name System and how it operates.
d) Explain what the purpose of the socket interface is and list 3 well known socket interfaces.

CST 2041 Exam 2011

09.00 - 12.00 December 2, 2011

GLT
Time: 3 hours
Answer any five questions.
All questions carry equal marks.
Write clearly and explain steps precisely.
Lecturer: Dr. John Regan

Q1

a. What is the difference, in general, between a thread and a process. The Linux Operating System doesn't distinguish between a thread and a process - explain clearly why not.

b. Explain the difference between user mode and kernel mode.

c. What is the purpose of system calls. Give an example of five different system calls. Explain the function of the call and the operating system in which it is defined.

d. If only kernel processes can access hardware, including the CPU, then how do user processes execute? You should include information about how a program's address space looks in virtual memory and about how the scheduler operates.

Q2

a. Explain the difference between the buddy scheme and the slab allocation scheme in Linux.

b. What part of the OS uses these schemes and why?

c. In a typical Linux environment user processes are given 3GB of virtual address space in a 32 bit system. What is the maximum size of each process's page table assuming a 4 KB page sizes and assuming a single level page table?

d. Explain the term ZONE_HIGHMEM in relation to Linux Memory Management. In terms of the size of RAM when does it become important and why?

Q3

a. Explain the logic behind an **inverted** page table.

b. Consider a computer system with a 32 bit virtual address space and 4 KB page sizes. The hardware on this system can support up to at most 512 MB of RAM. How many entries are there in each of the following?

   1. A conventional single level page table
   2. An inverted page table

c. Assuming a 16 bit virtual address space and a 1 KB page size what are the page numbers and offsets for the following address references (given in decimal). Give your answers in decimal format.

   1. 100
   2. 798
   3. 4587
   4. 2695
   5. 24

d. Suppose a desktop computer running a 32 bit operating system is fitted with 8 GB of physical memory. What problems will this cause for the OS and why? What would be a potential solution?

Q4

a. What is a *LAN* and what is a *WAN*? Briefly explain each term.

b. What is meant by contention on a network link? There are two main techniques for avoiding data corruption on a link, CSMA/CD and Token Passing. Describe each technique.

c. The Internet Model consists of four layers. Name and describe each layer. Include a diagram in your answer.

d. Every networked device on a system has a unique byte number called the medium access control (MAC) address assigned to it. What is the purpose of this address? How is an ARP packet related to a MAC address?

Q5

a. What is a PCB? Include a diagram in your answer including at least five entries in the PCB.

b. What is the purpose of the CPU scheduler?

c. What is meant by a context switch? Give four examples under which a context could happen. What is the typical time taken for a context switch on modern operating systems?

d. What is the difference between a preemptive and non-preemptive kernel? Under what circumstances may preemption be disabled?

Q6

a. What is meant by Symmetric Multi-Processing?

b. What is meant by Processor Affinity? What is the difference between soft affinity and hard affinity?

c. Consider the following set of processes, with the length of the CPU burst given in milliseconds.

Process	Burst Time	Priority
P_1	10.0	3
P_2	1.0	1
P_3	2.0	3
P_4	1.0	4
P_5	5.0	2

The processes are assumed to have arrived in order P_1, P_2, P_3, P_4, P_5 all at time 0. The operating system supports three scheduling algorithms: FCFS, SJF and RR (time quantum = 1 ms)

1. What is the turnaround time of each process for each of the scheduling algorithms supported by the OS?
2. What is the waiting time of each process for each of these scheduling algorithms?
3. Which of the algorithms results in the minimum average waiting time (over all processes)?

Q7

a. What is meant by a single level directory structure - use a diagram to support your answer. What is its primary disadvantage?

b. Most modern operating systems use a tree structured file system. What is the advantage of a tree structures file system. Draw a simple diagram outlining the structure of a tree structured file system.

c. What is meant by **journalling** is terms of file systems. Give an example of a modern file system that includes journalling and one that does not. Under what circumstances may you want to disable journalling?

d. File Control Blocks are part of the metadata of the file system. What are they used for? How many FCBs will you have per file system? Include a diagram of a typical FCB structure with your answer.

Q8

a. In terms of frame allocation what is meant by a local allocation policy and global allocation policy? Which policy is favoured by modern operating systems?

b. In terms of virtual memory usage what is meant by thrashing?

c. An operating system will usually attempt to allocate a process the correct number of pages to service its needs. What is this model called and how is this number calculated?

d. Assume our operating system has demand paging invoked. The page table is held in registers. It takes 5 milliseconds to service a page fault if an empty frame is available or if the replaced frame is not modified and 15 milliseconds if the page to be replaced is modified. Memory access time is 100 nanoseconds. Assume that the page to be replaced is modified 85 percent of the time. What is the maximum acceptable page fault rate for an effective access time of 150 nanoseconds? Express your answer as a percentage - i.e. the maximum percentage of page faults allowed in this case.

CST 3022 Exam
Tuesday 29th May 2012

Answer any **five** questions. All questions carry equal marks.
Write clearly and explain steps precisely.
Lecturer: Dr. John Regan

1.

a) What is the purpose of a program stack? Give an example of three elements that might be stored in a program's stack.

b) Programming Languages may also make use of the heap. Using the Java programming language as an example what type of variables does Java use the heap for?

c) Explain clearly what is meant by the ***current referencing environment.***

d) Consider the following Java (pseudocode) program, at the point marked CRE write down all the variables (and functions etc) that make up the current referencing environment: (you should of course assume static scoping)

```
private static int var = 0;
int compute_square(int x)

{
      int result = 0;
      result = x * x;
      return result;
}

int do_something(double kappa)
{
      int z =10, res = 0;
      res = compute_square(z);              //CRE
      System.out.println("z = " + z);
      return res;
}
```

2.

a) Explain the term **binding lifetime** of an object[1].

b) Explain the terms **static allocation, stack based allocation** and **heap based allocation** giving examples of each.

c) Explain the term **garbage collection** and how it relates to programming languages. Give an example of a language that uses garbage collection and one that doesn't.

d) What is meant by the **scope** of an object? Explain the difference between **static scoping** and **dynamic scoping.**

e) Consider the following pseudocode:

1Recall that an object in this sense refers to anything that might have a name: variables, constants, types and so on and is not related to the concept of an object in an object orientated language.

```
x : integer;                              --global

procedure set_x(n : integer)
      {
            x := n;
      }
procedure print_x()
      {
            write_integer(x);
      }
procedure first()
      {
            set_x(12);
            print_x();
      }
procedure second()
      {
            x : integer;
            set_x(19);
            print_x();
      }
set_x(7);
first();
print_x();
second();
print_x();
```

What does the program print if the language is **statically scoped**? What does it print with **dynamic scoping**? Explain briefly the reason for the value printed in each case.

3.

a) In programming languages what is meant by an **l-value** and what is meant by an **r-value**. In the following expression identify the l-value and the r-value

```
result = compute_square(x);
```

b) In terms of **side effects** explain why the following expression may be considered dangerous:

```
A[get_index(i)] = A[get_index(i)] + 256;
```

Outline at least two ways in which the above expression may be made safe against the threat posed by side effects.

c) What is meant when a language is said to be **statically typed**? Give two examples of a statically typed language and two examples of a dynamically typed language.

d) In terms of data types explain each of the following terms:
1. Type Equivalence
2. Type Compatibility
3. Type Inference

Give an example of when you might encounter each scenario in your favourite programming language.

4.

a) Explain what is meant by the terms ***actual*** parameters and ***formal*** parameters.
b) Explain what the function of the ***stack pointer*** and ***frame pointer*** is.
c) Explain the role of **Dope Vectors** in arrays. Describe what is meant by a **conformant array**.
d) Assume that a subroutine takes as its only argument the size of an array e.g.

```
void foo(int size) {
   double M[size][size];
   ...
   return;
}
```

Using the above subroutine as a template sketch out the stack frame for the above subroutine.

5.

a) A purely functional language is one that has no notion of side effects.
   1. What is a side effect?
   2. Scheme is a functional language but it does allow side effects. Give an example of a side effect in Scheme.
b) Scheme (and other functional languages) also have both **First Class Values** and **Higher Order functions.** Define what is meant by both.
c) Write a scheme function called **sum***. **sum** should sum all of the elements of a list. If an element in the list is not a number then the function should print that not a number was found and output the sum up until that point, otherwise it should return the sum of all of the elements of the list. Include a description of the logic of your code as well.

6.

a) **Locking** is often required in concurrent programming. Why is it required?
b) What is the difference between **"SpinLocks"** and **"Mutexes"**?
c) Explain clearly the difference between the concurrent programming paradigms of **Functional Decomposition** and **Domain Decompositions.**
d) The heat equation is given by the well known formula:

$$U_{x,y} = U_{x,y} + C_x * (U_{x+1,y} + U_{x-1,y} - 2*U_{x,y}) + C_y * (U_{x,y+1} + U_{x,y-1} - 2*U_{x,y})$$

where $U_{x,y}$ is the heat in a given cell. X and Y are the indices of the cell in a grid and C_x and C_y are constants corresponding to the heat transfer in the X and Y planes respectively. Write a pseudocode program that correctly uses N threads to update the heat in each cell over T timesteps.

* The program should be as syntactically correct as possible. However, no marks will be deducted for small syntax errors.

7.

a) BASH is a scripting language that is **weakly typed.** What is meant by **weak typing**?
b) BASH scripts are not compiled like C or Java programs are. How then do they run? How is the script converted into machine code?
c) Write a BASH script* that outputs the top 5 **directories** in your home directory and below in terms of size. The output should be to the screen and of the form:
Directory #: Size: Directory Name:

Note: Don't worry about the formatting too much, I'm more interested in the logic and use of commands.

8.

a) Explain clearly each of the following terms:
   1. Call-by-Value
   2. Call-by-Reference
   3. Call-by-Value/Result
   4. Call-by-Sharing
b) Consider the following pseudocode:

```
X : Integer;

void compute_double(Y : Integer)
{
      Y = Y * Y;
      print(X);
      return;
}

X = 12;
compute_double(X);
print(X);
```

Considering the above parameter passing models; call-by-reference, call-by-value, call-by-value/result and call-by-sharing methods. Write down the output that would be expected from all of those models. Assume that print(A) prints the value A to the screen.

c) Given that the C language allows for the possibility of passing values by reference and passing values by value, give three examples of cases where it would be preferential to pass a value by reference rather than by value.

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* The program should be as syntactically correct as possible. However, no marks will be deducted for small syntax errors.

University of Zambia

School of Natural Sciences

Department of Computer Studies

CST3062 Final Exam

Advanced Databases and Information Systems

This exam has seven questions answer any five questions.

Question 1 (XML technologies)

(a) Define XML and give two advantages of using it. **[4 points]**

(b) When is a document said to be a well formed XML document and valid XML document? **[2 points]**

(c) What is the importance of schemas and dtds in XML technologies? **[2 points]**

(d) Given that a computer always has a make(brand name), it can also optionaly have a floppy disk drive, has multiple hard drives, and either a dvd or cd writer, one monitor, a keyboard, a mouse and a year of manfucture;

I. write a dtd defining this data structure **[4 points]**

II. write a sample XML document implementing this dtd given that the dtd will be on another machine on the network. Populate this XML database wih sample data aswell. **[2 points]**

III. write an XQuery for finding all computers that have a dvd writer and where manufactured later than 2006. **[2 points]**

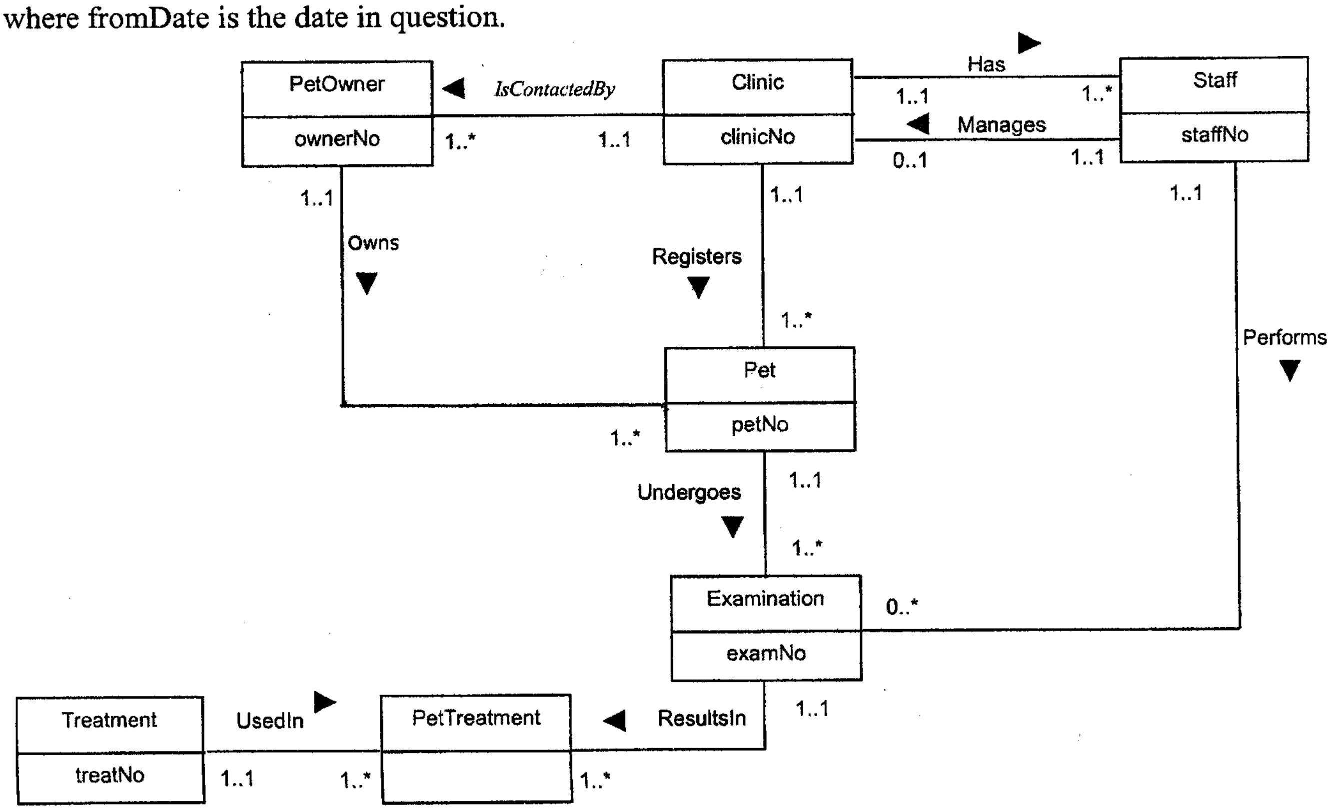
IV. what technology would you use to transform these results into html? Do the the transoformation. **[4 points]**

Question 2(Programming Relational Databases)

To calculate the period in months from a given date use the function

period_diff((date_format(now(),'%Y%m')), (date_format(fromDate,'%Y%m')))

where fromDate is the date in question.



(a) Define Assertions, Triggers and Procedures. **[3 points]**

(b) List two advantages and two diadvantages of using

I. Assertions. **[2 points]**

II. Triggers. **[2 points]**

III. Procedures. **[2 points]**

(c) Using the given entity relationship diagram and MySQL as the DBMS
   I. Write the SQL that cab be used to create the database. **[4 points]**
   II. Write a procedure that can be used to retrieve records for members of staff that have reached the age of 55. **[3 points]**
   III. Employees are always supposed to be older than 18. Write a trigger that checks that before an employee can ge registered we check if they are of age. The staff relation has an extra attribute called dateOfBirth. **[4 points]**

Question 3(Object Oriented Databases)

(a) Define an Object-Oriented Database Management System (OODBMS). **[2 points]**
(b) Give three most important advantages of a DDBMS over a relational DBMS. **[3 points]**
(c) Describe how relationships can be modeled in an OODBMS. **[3 points]**
(d) You are a newly employed database administrator and have been requested to implement a simple object oriented database application using db40. The following are the requirements; The databse is supposed to store information about;
Employees(fname, lastname, salary, dateOfBirth),
Spouse(fname, lastname, dateOfBirth), (Spouse to Employee)
Dependents(fname,lastname,dateOfBirth). (Dependents of employee)

   I. Using db4O as the OODBMS write the required code to create the database. **[4 points]**
   II. Write a sample query to
      1. insert a record for an employee married and with two children. **[4 points]**
      2. to increase the salary of the employee in (a) by 10%. **[2 points]**
      3. to delete the record of the employee in (a). **[2 points]**

Question 4(Distributed Database Systems)

(a) Discuss two advantages and disadvantages of a DDBMS. **[4 points]**
(b) Explain with illustrations how the four transaction types supported by IBMs Distributed Relational Architecture (DRDA) work. **[4 points]**
(c) In distributed database systems what do the following terms mean
   I. fragmentation. **[1 point]**
   II. Allocation. **[1 point]**
   III. Replication. **[1 point]**
(d) Consider the following simplified relational schema for InstantBuy:

OrderDetail(orderNo, itemType)	10,000 records stored in London
Client(clientNo, cCity)	1,000 records stored in Glasgow
ClientOrder(clientNo, orderNo)	100,000 records stored in London

For simplicity, assume that each tuple in each relation is 10 characters long, there are 100 clients who have ordered item 'TV3190', there are 10 clients in Edinburgh and computation time is negligible compared to communication time. The communication system has a data transmission rate of 10,000 characters per second and a 1-second access delay to send a message from one site to another.

   I. Write an SQL query that will list the clients in Edinburgh who have ordered items of type 'TV319'. **[3 points]**
   II. Give three possible strategies for retrieving data for this query. **[3 points]**

III. Calculate the amount of time it would take to run the query under each of the strategies stating any assumptions you make to support your calculations. **[3 points]**

Question 5 (Current and Imaging Trends)

(a) Give a definition of a data warehouse. Discuss the benefits of implementing a data warehouse. **[4 points]**

(b) Why would we want to dynamically generate web pages from data held in the operational database? List five general requirements for web-database integration. **[4 points]**

(c) Give a definition of an ORDBMS. What are the advantages and disadvantages of an ORDBMS. **[4 points]**

(d) Define replication servers. Describe the expected functionality of a replication server. **[4 points]**

(e) Give five reasons why the weaknesses of the relational data model and relational DBMSs may make them unsuitable for advanced database applications. **[4 points]**

Question 6 (Multi topic)

(a) Define horizaontal, vertical and mixed fragmentation. Illusrtate this using relational algebra. **[6 points]**

(a) A DDBMS must ensure that no two sites create a database object with the same name. One solution to this problem is to create a central name server. What are the disadvantages with this approach? Propose an alternative approach that overcomes these disadvantages. **[4 points]**

(b) Create an XML dtd and XML sample document for the following schema. Use your own sample data(minimum two records for each relation). **[6 points]**
Branch(branchNo, street, city, postcode)
Staff(staffNo, fName, lName. Position, sex, DOB, salary, branchNo)

(c) Create a style sheet to display this information in a browser **[4 points]**

Question 7 (Multi topic)

The following tables form part of a database held in a relational DBMS:

Hotel	(<u>hotelNo</u>, hotelName, city)
Room	(<u>roomNo</u>, <u>hotelNo</u>, type, price)
Booking	(<u>hotelNo</u>, <u>guestNo</u>, <u>dateFrom</u>, dateTo, roomNo)
Guest	(<u>guestNo</u>, guestName, guestAddress)

where Hotel contains hotel details and hotelNo is the primary key;
Room contains room details for each hotel and (roomNo, hotelNo) forms the primary key;
Booking contains details of the bookings and (hotelNo, guestNo, dateFrom) forms the primary key;
and Guest contains guest details and guestNo is the primary key.

(a) What four strategies can be used to place data with regards to distributed systems **[4 points]**

(b) Produce an object-oriented schema from the relational schema putting in mind methods that can be useful in this schema. **[4 points]**

(c) Write a dtd for the schema. **[4 points]**

(d) Write an SQL implementation of the schema. **[4 points]**

(e) Write a trigger that checks wether the user has already booked a room or not before they make a booking. **[4 points]**

University of Zambia

School of Natural Science

Department of Computer Studies

CST3142 Software Engineering II

29^{th} May 2012

9.00 Hrs to 12.00 Hrs

Instructions:

1. *This examination has two sections*
2. *Answer any five questions in Section A. Each question carries 5 marks*
3. *Answer any five questions in Section B. Each question carries 15 marks*
4. *Present your answer neat and understandable*

Section – A **(5 x 5 = 25 Marks)**

Answer any five questions. Each question carries 5 Marks

1. What is a project, and what are its main attributes? How is a project different from what most people do in their day-to-day jobs?

2. What is the role of project manager? What are suggested skills for all project managers and for information technology project manager?

3. Write short notes on weighted scoring model and balanced score card?

4. What is project scope management? What are the processes involved in project scope management?

5. Consider the following table, Network Diagram Data for a Small Project. All duration estimates or estimated times are in days; and the network proceeds from Node 1 to Node 9.

Table: Network Diagram for a small project

Activity	Initial Node	Final Node	Estimated Duration
A	1	2	2
B	2	3	2
C	2	4	3
D	2	5	4
E	3	6	2
F	4	6	3
G	5	7	6
H	6	8	2
I	6	7	5
J	7	8	1
K	8	9	2

i. Draw an AOA network diagram representing the project. Put the node in numbers in circles and draw arrows from node to node, labelling each arrow with the activity letter and estimated time.
ii. Identify all of the paths on the network diagram and note how long they are.
iii. What is the critical path for this project and how long is it?
iv. What is the shortest possible time it will take to complete this project

6. Give examples of when you would prepare rough order magnitude, budgetary, and definitive cost estimates for an information technology project.

7. Explain about cost of quality? What are the five major cost categories related to quality?

Section – B **(5 x 15 = 75 Marks)**

Answer any five questions. Each question carries 15 Marks

1. a. Explain project life cycle and product life cycle? How does a project life cycle differ from product life cycle?

   b. Explain the four frames of organizations. How can they help project managers understand the organizational context for their projects?

2. a. What is project charter? What is the basic information in the project charter?

   b. What is a project management plan? Explain in detail about the common elements of a project management plan?

3. Explain in detail about Work Breakdown Structure (WBS), WBS dictionary and Scope baseline? Describe different ways to develop a WBS? Give some advice for creating a WBS and WBS dictionary.

4. Explain the following schedule development tools and concepts: Gantt Charts, critical path method, PERT, and critical chain scheduling?

5. a. Explain about earned value management and its terms

   b. Given the following information for a one-year project, answer the following questions.

Planned Value (PV)	= $35,000
Earned Value (EV)	= $30,000
Actual Cost (AC)	= $40,000
Budget at completion (BAC)	= $150,000

   i. What is the cost variance, schedule variance, cost performance index (CPI), and schedule performance index (SPI) for the project
   ii. How is the project doing? Is it ahead of schedule or behind of schedule? Is it under budget or over budget?
   iii. Use the CPI to calculate the estimate at completion (EAC) for this project, Is the project performing better or worse than planned?
   iv. Use the SPI to estimate how long it will take to finish this project.
   v. Sketch the earned value chart based for this project.

6. Explain about the quality control and the main categories of outputs for quality control? Explain in detail about the four tools and techniques for quality control?

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF COMPUTER STUDIES

CST4012 Advanced Operating Systems and Distributed Systems

Final Exam

Tuesday, June 5TH, 2012

Instructions:

Answer any five questions.

Duration: 3 hours.

Make sure that you write your name and computer number on all your answer sheets.

- GOOD LUCK -

Q1

a. Classify multiprocessors using Flynn's Taxonomy.**[4 points]**

b. Given a process P, migrating from machine A to machine B explain the five transfer strategies that can be used to transfer its address space.**[5 points]**

c. If one is to design a multiprocessor scheduler, what are the important issues that one must be put into consideration? **[2 points]**

d. One of the important metrics for characterizing multiprocessors is the frequency of synchronization

   I. Why is the frequency of synchronization an important metric **[2 points]**

   II. What are the different levels of frequency synchronization and how do they relate to distributed systems and their support.**[5 points]**

e. A multi-computer with 256 CPUs is organised as a 16x16 grid. What is the worst case delay (in hops) that a message might have to take? **[2 points]**

Q2

a) Define a petrinet. **[2 points]**

b) An important aspect of petrinets is that of transition:

   I. What is transition enabling and firing? **[2 points]**

   II. Illustrate transition enabling and firing using petrinets. **[4 points]**

c) What is a finite state machine **[2 points]**

d) Given two cups; a blue cup holds **5mls** of liquid and a red cup holds **10mls** of liquid. You are required to use these cups to feel either a **15mls** or **20mls** dish. Given any process of feeling any dish you can only transfer a maximum of **20mls**.

   I. Draw a finite state machine illustrating how these cups can be used to feel either dish. **[6 points]**

   II. Identify the decision points in the petrinet in Q2.d.(i). **[4 points]**

Q3

a) Among the many proposals for multiprocessor thread scheduling and processor assignment protocols, four general approaches stand out and these are; Load sharing, Gang scheduling, Dedicated Processor Assignment and Dynamic scheduling. Without going into detail explain how each one these works. **[4 points]**

b) Explain how the three variants of load sharing protocol for thread scheduling and processor assignment work. **[3 points]**

c) What is the advantage of using a multi-core processor over multiple processors? **[3 points]**

d) Suppose we have four applications, two with six threads, one with two threads and another with three threads in an environment with six processors. Using gang scheduling with uniform allocation time how much processor time is wasted? **[4 points]**

e) What is dynamic scheduling? Outline the protocol used in dynamic processor allocation using dynamic scheduling. **[6 points]**

Q4

a) Process migration is one of the fundamental concepts of distributed systems. Define process migration. **[2 points]**

b) Processes can be transferred using either preemptive of non-preemptive transfers. Define and differentiate the two. **[4 points]**

c) How does centralized deadlock detection work? Illustrate using an appropriate diagram. **[5 points]**

d) What are the three conditions under which process migration can be started? **[3 points]**

e) Consider the behavior of two machines in a distributed system. Both have clocks that are supposed to tick 1000 times per millisecond. One of them actually does, but the other ticks only 990 times per millisecond. If UTC updates come in once a minute, what is the maximum clock skew that will occur? **[6 points]**

Q5

a) Name two advantages and two disadvantages of distributed systems over centralized ones. **[4 points]**

b) One of the methods that can be used to send messages between processes in a distributed system is remote procedure calls. What are remote procedure calls? **[2 points]**

c) Explain how the ring election algorithm works. **[4 points]**

d) Briefly explain how a

   I. centralized algorithm **[2 points]**

   II. distributed algorithm (citing an example) **[2 points]**

   can be used to achieve mutual exclusion in a distributed system.

e) A distributed system may have multiple, independent critical regions. Imagine that process 0 wants to enter critical region A and process 1 wants to enter critical region B. Can Ricart and Agrawala's algorithm lead to deadlocks? Explain your answer. **[6 points]**

Q6

a) What is a transaction? **[2 points]**

b) What are the five transaction primitives? **[5 points]**

c) One of the challenges in implementing nested transaction is that of trying to maintain the ACIDity of the transaction.

   I. Define ACIDity **[2 points]**

   II. Using an appropriate diagram explain how private space can be used to implement nested transactions with high ACIDity. **[6 point]**

d) Given that

```
X = 0; Y = 0;
BEGIN_TRANSATION
X = X + 1;
Y = Y + 2;
X = Y * Y;
END_TRANSACTION
```

   I. Explain how writelogs are used to recover state from a failed transaction. **[2 points]**

   II. Show a write log for the transaction above. **[3 points]**

Q7

a) Define a distributed system. **[2 points]**

b) What are the six desirable properties of a distributed system? **[6 points]**

c) How does Christians clock synchronization algorithm work? How does it compare to the Berkeley and Averaging algorithms. **[4 points]**

d) What is the difference between a Multiprocessor and a Multicomputer? **[4 points]**

e) Illustrate and explain the architecture of multi-core processors. **[4 points]**

THE UNIVERSITY OF ZAMBIA
DEPARTMENT OF COMPUTER STUDIES

CST4122 - COMPILERS

UNIVERSITY EXAMINATION

Thursday, May 24, 2012; 09:00Hrs

INSTRUCTIONS : There are SIX(6) questions in this examination and you are required to answer ANY FOUR of them. Good luck!

DURATION : 3 Hours

1.
   a. Describe the phases of the compilation process from the point of view of the analysis and synthesis of a program. Outline the functions of each phase stating the form of its input and output. [you can use a labelled diagram]
   b. Discuss the role that the symbol table plays in the compilation process.
2.
   a. Write a program in the P language that reads two numbers A and B and swaps the values contained in them (i.e. A get what is in B and B gets what is in A)
   b. Hand translate this code into the M code and optimise if possible.
3.
   a. Consider the following regular expression
   b. 1(0|1)*1.
   c. What type of strings does the RE generate
   d. Construct an NFA for your grammar above.
   e. Convert NFA into a DFA.

4. For each of the following grammars
   a. State why it is not LL parsable
   b. convert it into an LL parsable grammar.

   i. $A \rightarrow A\ \alpha$
      $A \rightarrow \beta$
      $A \rightarrow \gamma$

   ii. A -> B C D E F
       A -> B C D G

   iii. A -> B D
        B -> A C | E

5. Consider the grammar given the below

```
E -> E + T | E
T -> T * F | F
F -> id | num
```

Where id and num are terminals representing identifiers and numbers respectively

a. Draw the syntax tree for a + b - c
b. Discuss the precedence and associativity of the operators *, +.
c. By using brackets, show how the following expressions are evaluated
   i. a + b * c + d * e
   ii. a + b + c + d + e

6.

a. Describe the difference between top-bottom (LL) and bottom-up (LR) parsing
b. There are the two modes of implementing an LL parser i.e. using a recursive descent parser and using a stack. Describe these two methods.

c. Given the following grammar

```
E -> id ET
ET -> + E | - E | ε
```

   i. Show how the stack implementation will parse the following expression a + b - c. [you need a parse table]

********************END OF EXAMINATION********************

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF COMPUTER STUDIES

COURSE NAME: COMPUTER GRAPHICS

COURSE CODE: CST 4132

UNIVERSITY EXAMINATIONS

Time : 3 hours

You MUST answer ALL questions from Section A which are worth 24 marks

AND ALL questions from Section B which are worth 76 marks.

CALCULATORS AND ELECTRONIC DEVICES ARE NOT PERMITTED

Section A

1) What is the motivation of computer graphics? (2 marks)

2) Define the following terms and give examples: graphical modeling and rendering. (4 marks)

3) The display processor will provide assistance for a number of Operations in the raster display architectures. List the operations. (5 marks)

4) The most important task for the video controller is the constant refresh of the display. List the types of refresh. (2 marks)

5) what does the function glutInitDisplayMode() do? (2 marks)

6) Callbacks are used for two purposes. Name the purposes. (2 marks)

7) List 5 common call backs you know. (5 marks)

8) Given the colouring function as void glColor3d(GLdouble red, GLdouble green, GLdouble blue), can you rewrite the function in two other ways? (2 marks)

Section B

9) Discuss algorithms of drawing the circle. (11 marks)

10) Computer graphics today has many applications. List and explain more than 12 applications. (6 marks)

11) Discuss Affine Transformations. (13 marks)

12) Illustrate the graphical system architecture. (4 marks)

13) Discuss polygon filling. (8 marks)

14) Write a complete program of the rotating teapot using Opengl. (8 marks)

15) Discuss line drawing algorithms. (8 marks)

16) Briefly explain the following display technologies:
Electroluminescent (EL), Electrophoretic, Direct-view storage tube (DVST) and GE light-valve projection system. (5 marks)

17) Discuss clipping. (13 marks)

The University of Zambia

School of Natural Sciences

Department of Mathematics & Statistics

Second Semester Examinations - -May 2012

EM312 - Engineering Mathematics IV

Time allowed : Three (3) hours   Full marks : 100

Instructions: • This paper consists of **two** sections, **Section A** and **Section B**. Attempt **any four** questions from **Section A** and **any one** from **Section B**. All questions carry equal marks.

- **Full credit** will only be given when **necessary work** is shown.
- Indicate your **computer number** on all answer booklets.

This paper consists of 4 pages of questions.

Section A

1. a) Given the equation $x^2 - y^2 + z^2 - 4x - 2y - 2z + 4 = 0$.

   (i) Identify the surface described by the above equation.

   (ii) Sketch the surface.

   b) Evaluate the integral

$$\int_0^4 \int_{\sqrt{y}}^2 y \cos x^5 \, dx dy \ .$$

   c) (i) Find a parametric representation of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the cone $z = \sqrt{x^2 + y^2}$.

   (ii) Find the equation of the tangent plane to the parametric surface $x = u + v$, $y = 3u^2$, $z = u - v$ at the point $(2, 3, 0)$.

Please Turn Over/...

2. a) (i) Find all the points at which the direction of fastest change of the function $f(x,y) = x^2 + y^2 - 2x - 4y$ is $\mathbf{i} + \mathbf{j}$.

(ii) Find the directions in which the directional derivative of $f(x,y) = ye^{-xy}$ at the point $(0,2)$ has value 1.

b) Let $\mathbf{F}(x,y,z) = \langle e^{x^2} - 2xz, 2x, y^2 \rangle$ and let S be the portion of the paraboloid $2z = 17 - x^2 - y^2$ lying above the hemisphere $z = \sqrt{25 - x^2 - y^2}$.

(i) Obtain parametric equations for the curve of intersection C of the paraboloid and the hemisphere.

(ii) Calculate curl $\mathbf{F}$.

c) Use spherical coordinates to find the mass of the solid lying below the hemisphere in b) and above the plane $z = 4$, if the density function is given by $\rho(x,y,z) = z$.

3. a) (i) Sketch the region R bounded by the planes $x + 2y + z = 2$, $x = 0$, $x = 2y$, and $z = 0$.

(ii) Evaluate

$$\iiint_R dV$$

b) (i) State the Fundamental Theorem of line integrals.

(ii) Show that the vector field $F(x,y,z) = yz\,\mathbf{i} + xz\,\mathbf{j} + (xy + 2z)\,\mathbf{k}$ is conservative and find a function f such that $F = \nabla f$.

(iii) Use the results in (ii) to evaluate $\int_C F \cdot d\bar{r}$, where C is the line segment from $(1,0,-2)$ to $(4,6,3)$.

c) Find a formula for

$$\iint_S F \cdot \hat{n}\, dS \,,$$

for the case where S is given by $x = h(y,z)$ and $\hat{n}$ is the unit normal that points forward.

4. a) (i) State Green's theorem.

(ii) A particle starts at the point $(-2,0)$, moves along the x-axis to $(2,0)$ and then along the semicircle $y = \sqrt{4 - x^2}$ to the starting point. Find the work done on this particle by the force field

$$F(x,y) = \langle x, x^3 + 3xy^2 \rangle \,.$$

Please Turn Over/...

b) Let E be the solid bounded by the cylinder $x^2 + z^2 = 9$ and the planes $y = 0$, $x = 3y$ and $z = 0$ in the first octant.

(i) Sketch E.

(ii) Evaluate the integral

$$\iiint_E dV .$$

c) Evaluate the integral

$$\iint_R e^{(x+y)/(x-y)}\, dA$$

where R is the trapezoidal region with vertices $(1,0)$, $(2,0)$, $(0,-2)$, and $(0,-1)$.

5. a) (i) State Gauss (Divergence) Theorem.

(ii) Sketch the region E bounded by the parabolic cylinder $z = 1 - x^2$ and the planes $z = 0$, $y = 0$ and $y + z = 2$.

(iii) Evaluate

$$\iint_E F \cdot dS ,$$

where S is the surface of E and $F(x,y,z) = xy\,\mathbf{i} + (y^2 + e^{xz^2})\,\mathbf{j} + \sin xy\,\mathbf{k}$.

b) Given

$$F(x,y) = \frac{x}{\sqrt{x^2+y^2}}\,\mathbf{i} + \frac{y}{\sqrt{x^2+y^2}}\,\mathbf{j}$$

(i) Sketch F.

(ii) Evaluate $\int_C F \cdot d\bar{r}$, where C is the parabola $y = 1 + x^2$ from $(-1,2)$ to $(1,2)$.

c) A wire takes the shape of the semicircle $x^2 + y^2 = 1$, $y \geq 0$. Find the mass of the wire if the linear distance at any point is proportional to its distance from the line $y = 1$.

Section B

6. a) Events A and B are such that $P(A) = \frac{2}{3}$, $P(A|B) = \frac{2}{3}$, $P(B) = \frac{1}{4}$.

(i) Find $P(A \cap B)$ and $P(B|A)$.

(ii) Are A and B independent?

Please Turn Over/...

b) Three members of a private country club have been nominated for the office of President. The probability that Mr Phiri will be elected is 0.3, the probability that Mr Mwale will be elected is 0.5 and the probability that Ms Sihwaya will be elected is 0.2. Should Mr Mwale or Ms Sihwaya be elected, the corresponding probabilities for an increase in membership fees are 0.1 and 0.4; respectively. Should Mr Phiri be elected, the corresponding probability for an increase in fees is 0.3.

(i) What is the probability that there will be an increase in membership fees?

(ii) If someone is considering joining the club but delays, only to find out that the fees have been increased, what is the probability that Mr Mwale was elected President of the club?

c) Let X be a random variable with density function

$$f(x) = \begin{cases} ax^2 & -1 < x < 2\ , \\ 0 & \text{otherwise}\ . \end{cases}$$

(i) Find the value of a.

(ii) Find $P\left(-\frac{1}{3} \leq X \leq 1\right)$.

(iii) Find $P\left(X \leq 0 | -\frac{1}{3} \leq X \leq 1\right)$

7. a) In a sample space, events A and B are independent, events B and C are mutually exclusive, and A and C are independent. If $P(A \cup B \cup C) = 0.9$, $P(B) = 0.5$ and $P(C) = 0.3$, find $P(A)$.

b) The probability that a regularly scheduled flight departs on time is $P(D) = 0.83$, the probability that it arrives on time is $P(A) = 0.82$. The probability that it departs and arrives on time is $P(D \cap A) = 0.78$. Find the probability that a plane

(i) arrives on time given that it departed on time.

(ii) departed on time given that it has arrived on time.

c) Let X be the life in hours of a light bulb with density function $f(x) = 0.001e^{-0.001x}$, $x \geq 0$.

(i) Show that $f(x)$ is indeed a density function for the variable X.

(ii) Find $P(X \leq 1000)$.

(iii) Find $P(X \geq 100 | X \leq 1000)$.

END!

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2011 ACADEMIC YEAR FIRST SEMESTER
FINAL EXAMINATIONS

GEO 111: INTRODUCTION TO HUMAN GEOGRAPHY I

TIME: THREE HOURS

INSTRUCTIONS: ANSWER QUESTION 1(40%) AND ANY OTHER THREE QUESTIONS (20% EACH)

---

1. The data given in Table 1 shows the distribution of hospital beds in various districts in a given Province. Study the table and answer the questions that follow.

Table 1: Number of hospital beds in Districts

District	Population	Hospital beds
A	315,370	11,300
B	335,710	22,700
C	211,000	22,000
D	711,820	28,000
E	904,057	30,000
F	224,550	15,000

Source: Hypothetical

a) Calculate the ratio of advantage of each of the six districts
b) Draw the Lorenz curve for the distribution of hospital beds in the six districts and show the inequality gap and line of perfect equality.
c) Explain the meaning of the curve you have drawn.

2. "Environmental determinism is regarded by many as overly simplistic because it neglects the cultural factors that effect human behaviour" (Stoddard et al., 1986: 99). Discuss.

3. Write short explanatory notes on **ALL** of the following
a) Functional complexity
b) The relationship between society and space
c) The Rank Size Rule
d) Consumer behaviour

4. Examine the main approaches used in the study of Human Geography from the beginning of the 19^{th} Century to date.

5. Discuss the ways in which the urban structures of African cities depart from the models of urban structure suggested by Burgess (1926) and Hoyt (1939).

6. Evaluate the factors of industrial location showing their relevance to the location of industries in Zambia.

END OF EXMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2011 ACADEMIC YEAR SECOND SEMISTER FINAL EXAMINATIONS

GEO 112: INTRODUCTION TO HUMAN GEOGRAPHY II

TIME: **THREE HOURS**

INSTRUCTIONS : **Answer any four questions.**
Candidates are advised to make use of illustrations and examples wherever appropriate.

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1. Explain the causes of industrialization in England and comment on the lessons that Africa can learn from this process as well as from 'Asian Tigers', in order to facilitate rapid socio- economic development.

2. Define culture, and describe its functions according to Ali Mazrui (2002).

3. 'Land tenure is not static but dynamic'. Comment on this statement with respect to the evolution of land tenure systems in Africa.

4. Describe Rostow's stages of economic growth model and show whether it is applicable to the situation in Africa or not.

5. "A resource is a means to attain given ends" (Zimmermann, 1964: 8). Discuss.

6. Describe the Malthusian theory on population, and comment on whether the African continent is over or under populated.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2011 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS
GEO 175: INTRODUCTION TO MAPPING TECHNIQUES IN GEOGRAPHY

PAPER II: THEORY

CLASSIFICATION OF NUMERICAL DATA, CONSTRUCTION OF TABLES, STATISTICAL MAPS AND DIAGRAMS

TIME: Three Hours

INSTRUCTIONS: Answer any four questions.
The use of a Philip's University Atlas and a certified calculator is allowed. Candidates are encouraged to make use of illustrations wherever appropriate.

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1. Write short explanatory notes on **all** of the following:

    (a) Desire-line maps [5 marks]

    (b) Ratio scale of measurement [5 marks]

    (c) Two entry tables [5 marks]

    (d) Divergence line graphs [5 marks]

    (e) Statistical diagrams [5 marks]

2. Examine the data given in Table 1 showing the length of time spent studying some Geography practical exercises by a group of randomly selected students over a period of one week.

Table 1: Hours spent studying Geography Practical Exercises

2.6	2.7	3.4	3.5	2.8	2.9	4.0	2.4	4.0	3.3
2.5	2.2	2.1	2.0	3.1	3.0	3.7	3.8	2.2	2.3
2.9	1.5	1.2	1.6	1.0	3.6	2.9	1.7	3.9	2.8
2.5	4.0	3.3	3.1	3.4	2.6	4.0	2.0	1.2	1.5
1.2	2.5	2.9	1.6	1.0	3.6	2.9	1.7	3.9	2.8

Source: Field Survey

Table 1 comprises fifty observations recorded as correct to the nearest tenth of a unit ranging in value from 1.0 to 4.1 hours. If it is decided to use eight classes of width 0.4 hours and to begin the first class at 0.95 hours:-

(a) Find the class boundaries of the eight classes. [4 marks]

(b) What are the class limits of the eight classes? [5 marks]

(c) Calculate the class marks of the eight classes. [5 marks]

(d) Construct a frequency distribution table for the data in Table 1. [8 marks]

(e) Calculate the relative frequency of the fifth class. [2 marks]

(f) How should the first class interval be written so that it becomes an open class? [1 mark]

3. Study the data given in Table 2 and then answer the questions that follow.

Table 2: Zambia's Population Distribution by Province in 2010

Province	Size (Km^2)	Population Size
Central	94, 394	1, 267, 803
Copperbelt	31, 328	1, 958, 623
Eastern	69, 106	1, 707, 731
Luapula	50, 567	958, 976
Lusaka	21, 896	2, 198, 996
Northern	147, 826	1, 759, 600
North-western	125, 826	706, 462
Southern	85, 283	1, 606, 793
Western	126, 386	881, 524
Total	752, 612	13, 046, 508

Source: Adapted from GRZ (2011) *Zambian Census of Population and Housing*, Lusaka, CSO. P2

(a) Use the most appropriate statistical mapping technique to show the data in Table 2 on the outline map of Zambia (Figure 1) provided. [15 marks]

(b) What are the merits and demerits of the technique that you have used? [5 marks]

(c) If you were asked to draw a distribution of population map, which information provided in Table 2 would you discard and why? [5 marks]

4. You have been appointed personal assistant to the financial manager of a book printing company. The company has planned to present information to the company employees in an attempt to encourage greater interest in the company's business operations. You have been requested to contribute to this by providing a suitable data presentation. The relevant data are as follows:

Table 3a: Cash Flow of a Book Printing Company 2008 - 2011

Description of Item	Value for the different time periods (K'million)			
	2008	2009	2010	2011
Sales	60	58	45	40
Wages	25	22	19	13
Production costs	8	6	5	4
Material costs	18	16	13	10
Taxation	0	3	1	2
Other costs	8	7	5	5
Profit	1	4	2	6
Price index	1700	1500	1150	1000

Source: Hypothetical

There are three products by the company and their share of sales is given as follows:

Table 3b: Share of sales of the three products of the company

Product	2008 (%)	2009 (%)	2010 (%)	2011 (%)
A	35	40	45	35
B	10	10	08	12
C	55	50	47	53
Total	100	100	100	100

Source: Hypothetical

(a) Prepare a visual display to show sales by product. [15 marks]

(a) Comment on the major highlights of your diagram [5 marks]

(b) What are the merits and demerits of the technique that you have used in (a)? [5 marks]

5. Study Figure 2, an outline map of an oceanic island with spot heights indicated and then answer the questions that follow.

(a) On an oceanic island map (Figure 2) provided, interpolate contours at a100 metre contour interval. [15 marks]

(b) What are the advantages of the mapping technique that you have used in (a) over the dot map? [4 marks]

(c) Describe any two methods that can be used in selecting a contour interval if it is not given. [6 marks]

6. The climatic data in Table 3 was recorded at an unidentified school weather station. Study the data and then answer the questions that follow.

Table 4: Climatic data from unidentified school weather station

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Temp. (° C)	26	26	27	29	31	33	35	34	33	32	30	27
Rainfall (mm)	38	15	15	20	00	00	00	00	00	08	18	36

Source: Hypothetical

(a) Use the most suitable statistical diagram to show the data in Table 4. [15 marks]

(b) Using your diagram, comment on the relationship between temperature and rainfall at the unidentified school. [5 marks]

(c) What are the merits of using the technique that you have employed in (a) presenting climatic data? [5 marks]

END OF EXAMINATION

COMPUTER No. ....

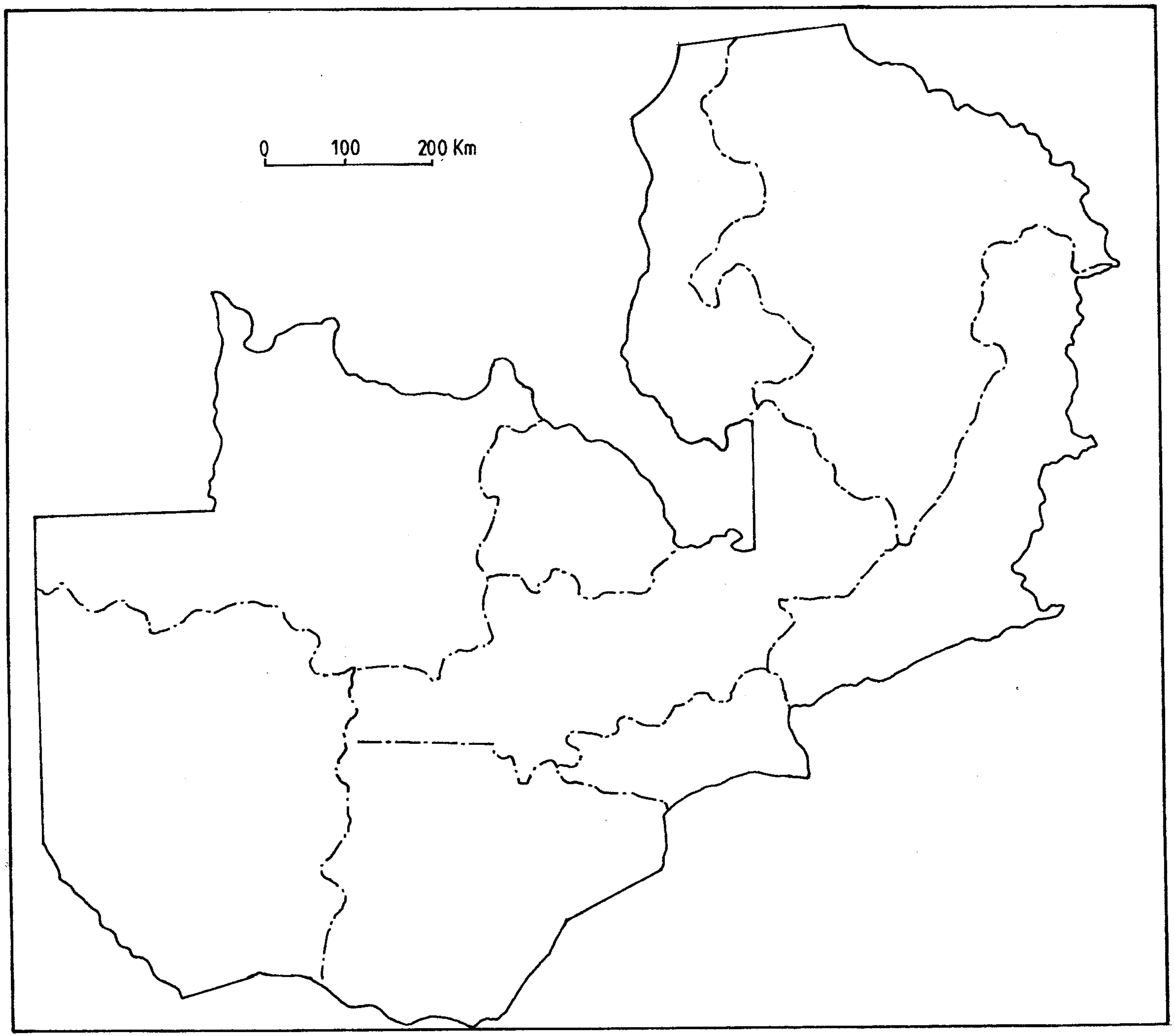

Figure 1

COMPUTER No. ....

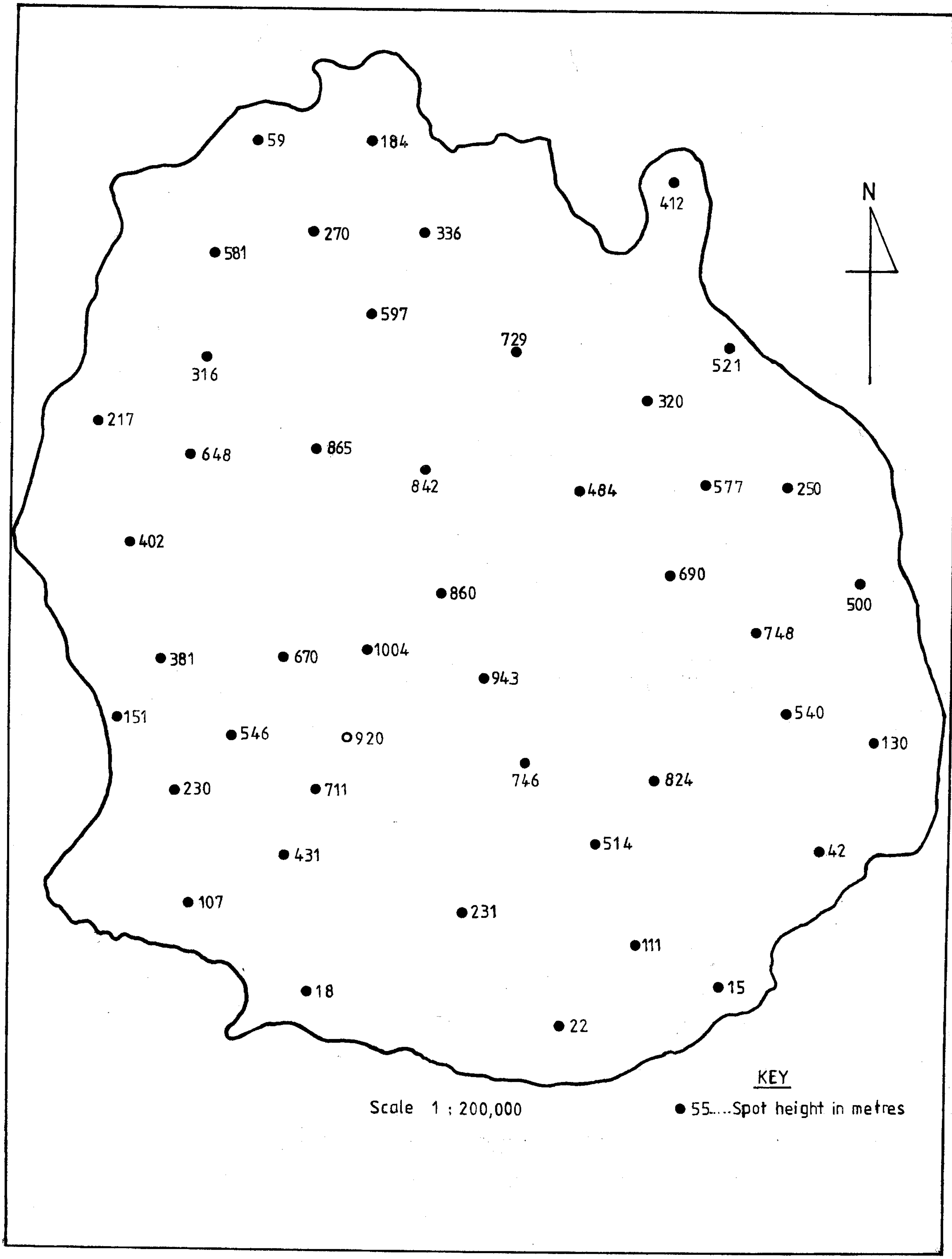

Figure 2

THE UNIVERSITY OF ZAMBIA
INSTITUTE OF DISTANCE EDUCATION

2011 ACADEMIC YEAR DISTANCE EDUCATION FINAL EXAMINATIONS

GEO 211 : THE GEOGRAPHY OF AFRICA

TIME: **THREE HOURS**

INSTRUCTIONS : **Answer any four questions.**
Candidates are advised to make use of illustrations and examples wherever appropriate.

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1. Write short explanatory notes on **ALL** of the following aspects of Africa:

   (a) Fold mountain ranges
   (b) Coral reef
   (c) Volcanic activity
   (d) Drainage
   (e) Ocean currents.

2. 'Water is the most important landscape forming agent in deserts'. Discuss this statement with reference to Africa.

3. Explain the causes of the socio-economic crises in Africa according to Griffiths and Binns (1988), and suggest possible solutions.

4. Discuss the challenges that African people face in promoting national unity in view of racial and linguistic diversity.

5. Explain the shifts in development strategies that Tanzania has experienced since independence.

6. In what ways is the human population in Africa both a potential resource and a challenge for sustainable socio-economic development?

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
INSTITUTE OF DISTANCE EDUCATION

2011 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS
GEO 272: QUANTITATIVE TECHNIQUES IN GEOGRAPHY II

TIME: **Three hours**

INSTRUCTIONS: **Answer any FOUR questions**
All questions carry equal marks
Use of certified calculator is allowed

Q1. Write short explanatory notes on **ALL** of the following:

a) Class limits and class boundaries
b) Type I and type II errors in hypotheses testing
c) Limitations of statistics
d) Scales of measuring geographical data
e) Normal and skewed distributions

Q2. Is your chance of getting a cold influenced by the number of social contacts you have? A group of 266 healthy men and women were grouped according to their number of social contacts (such as parent, friend, church member, neighbour). The data are provided in Table 1.

Table 1: Colds and number of social contacts

Condition	**Number of social contacts**		
	Three	**Four**	**Five or more**
Cold	49	43	34
No Cold	31	47	62

Source: Hypothetical

Do the data provide sufficient evidence to indicate that susceptibility to colds is affected by the number of social contacts you have? Test at the 5% significance level.

Q3. Kitwe and other copperbelt towns have seen an increase in hazy skies due to air pollution emanating from the mining activities in recent years. In Mbaula Compound, however, air pollution from mining activities is so serious that it has resulted in an increase in the reported cases of chest-related diseases at Mbaula Clinic in 2011. This has increased the medical expenses of the local residents not to mention the attendant health implications. The data are provided in Table 2.

Table 2: Sulphur dioxide emissions and reported cases of chest-related diseases at Mbaula Clinic in 2011

Month	Sulphur dioxide emissions (grams)	Reported cases of chest-related diseases
1	14.2	215
2	16.4	325
3	11.9	185
4	15.2	332
5	18.5	406
6	22.1	522
7	19.4	412
8	25.1	614
9	23.4	544
10	18.1	421
11	22.6	445
12	17.2	408

Source: Hypothetical

Assuming the samples are random and normally distributed, what conclusion would you draw at 5% significance level?

Q4. A professor of geography noticed that the marks in his course were normally distributed. He handed back his graded mid-semester marked scripts in class by calling out the name of each student and personally handing the marked script to each one of them. At the end of the process he noted that there were several marked scripts that remained with him, the result of students missing that particular class.

He formed the theory that the absence was caused by poor performance by those students on the test. He recorded the marks (out of 50) for the scripts that remained with him in column A and the marks of the scripts that were returned to students in column B. The data are presented in Table 3.

Table 3: Test results of the mid semester test in Geo 272

A	B
12.6	20.3
7.6	30.6
10.5	10.8
5.9	25.3
11.8	45.0
15.6	33.4
8.9	24.4
16.3	16.3
17.8	36.2
18.6	
20.5	

Source: Hypothetical

Do the data support the professor's theory? Allow for 99% accuracy in your analysis.

Q5. Youth unemployment is one of the greatest challenges that Zambia is facing at the moment. To improve their chances of getting any job, the youth must first obtain a tertiary qualification. However, most of the tertiary institutions require that prospective candidates must have five credits or better that must include English and Mathematics to be admitted. Table 4 shows the marks obtained in English and Mathematics by a group of 10 high school graduates who were seeking entry into tertiary institutions in 2011.

Table 4: Marks obtained in English and Mathematics by 10 students

Student No.	English	Mathematics
1	56	66
2	75	70
3	45	40
4	71	60
5	61	65
6	64	56
7	58	59
8	80	77
9	76	67
10	61	63

Source: Hypothetical

Assuming the data are not normally distributed, why do tertiary institutions emphasize on admitting only students who have obtained credits or better in English and Mathematics among other subjects? Test at 1% significance level.

Q6. Car dealers in Zambia use the car odometer reading to determine the value of used cars that their customers trade in. A car dealer recorded the price and the number of kilometers on the odometer. The data are presented in Table 5.

Table 5: Car odometer reading and value of second hand cars

Odometer reading (000 km)	Selling price (million kwacha)
37	54
44	51
45	50
30	58
32	57
34	53
40	55
32	53
24	58
48	54
39	50
37	52
40	54

Source: Hypothetical

(a) Plot the data.
(b) Conduct a regression analysis so as to come up with a regression equation.
(c) Draw a line of best fit in your scatter diagram.
(d) Define your regression equation.
(e) How much would a car with an odometer reading of 72000 km cost?

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
2011 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS

GEO 482: ENVIRONMENT AND DEVELOPMENT II

TIME: Three Hours
INSTRUCTIONS: Answer any four questions. All questions carry equal marks.

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1. 'The major cause of Wetland loss and degradation are due to both human actions and natural factors'. Discuss.

2. Explain the social and ecological challenges of water pollution in the Kafue River that is emanating from point and non point sources of human activities.

3. The environmental pollution effects of the Bhopal pollution disaster of 1984 in India are evident enough to indicate the negative consequences. Outline the major environmental pollution effects of this disaster.

4. The management of waste materials in Zambia has been difficult and challenging. This difficulty has physical and visible manifestations. Explain with a known case example.

5. Discuss how poor communities living adjacent to a large national park could both hinder and enhance wildlife conservation.

6. Outline and briefly explain the following:

   (a) The three (3) policy measures commonly used for regulating fish catch in state/ publicly managed fisheries and

   (b) The main pillars essential for a successful fisheries co-management program.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2011 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS

GEO 492: NATURAL RESOURCES ECONOMICS

Time: Three hours

Instructions: Answer any four questions.
All questions carry equal marks. You are encouraged to use illustrations wherever appropriate.

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1. Describe the effects of extraction costs on Hotellings model of resource depletion.

2. Demonstrate how the supply of a non renewable but recyclable resource can be increased over time.

3. Demonstrate the effects of using private and social discount rates in public projects.

4. Using an illustration, explain at what point deforestation becomes an economic problem.

5. 'The Maximum Economic Yield (MEY) is a social as well as a private optimum'. Discuss.

6. Evaluate the use of Hedonic Pricing Method in the valuation of natural resources.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2011 ACADEMIC YEAR FIRST SEMESTER FINAL EXAMINATIONS

GEO 911: POPULATION GEOGRAPHY

Time: Three hours

Instructions: Answer Qu estion 1 and any other three. Question 1 carries 40 Marks of this paper while the rest carry 20 marks each. Use of a certified calculator is allowed.

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1. Study Table 1 and answer the questions that follow:

Table 1: Ages, number of women, births of female babies to women in age group and age specific rates

	Ages of women	Number of women	Births of female babies to women in age group
Country A	15 - 19	8,990,000	488,941
	20 - 24	9,574,000	967,472
	25 - 29	10,928,000	1,139,256
	30 - 34	10,924,000	1,803,547
	35 - 39	9,600,000	369,518
	40 - 44	8,155,990	40,776
Country B	15 - 19	6,890,000	200,451
	20 - 24	9,574,000	552,236
	25 - 29	10,928,000	648,628
	30 - 34	10,924,000	721,273
	35 - 39	9,600,000	334,759
	40 - 44	6,155,990	20,383
Country C	15 - 19	7,900,572	1,466,823
	20 - 24	8,678,000	3,202,416
	25 - 29	10,900,645	3,717,768
	30 - 34	10,880,600	2,410,641
	35 - 39	8,999,875	808,554
	40 - 44	8,155,990	122,328

Source: Population Reference Bureau (PRB, 2011).

1. Assuming the method for computing Total Fertility Rate (TFR) is the same as the one for calculating ~~Growth~~ Gross Reproduction Rate (GRR), where only female babies are used as the numerator:
   - i) Calculate the Gross Reproduction Rates for countries A, B and C. **[12 Marks]**
   - ii) In case there is a tendency of obtaining high specific birth rates for cohorts 25 to 29 in each country as compared to cohorts 15 to 19 and 40 to 44 explain the reasons for this scenario. **[10 Marks]**

iii) Explain the differences or similarities (If any) among the three countries regarding GRRs.**[08 Marks]**

iv) What, in your view, would be the reasons for either the differences or the similarities referred to in 1(iii)? **[10 Marks]**

2. Ascertain why Clarke (1965), argues that Trewartha is a highly respected scholar in Geography.**[20 Marks]**

3. 'High population growth rates and high absolute population numbers are correlated with crop land, forests and energy.' Discuss. **[20 Marks]**

4. Based on what has happened to some western countries like Germany, Romania and Bulgaria argue why you would or not recommend the pronatalist policy to the Zambian government? **[20 Marks]**

5. Discuss the applicability of any five schema of Huffman and Huffmans' (1977) non-economic values of children with respect to Zambia. **[20 Marks]**

6. Male circumcision is now being advocated as one of HIV prevention methods in Zambia. Given Zambia's cultural context, to what extent is this likely to succeed? **[20 Marks]**

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2011 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS

GEO 912: GEOGRAPHY OF REFUGEES AND MIGRATION

TIME : Three hours

INSTRUCTIONS : Answer any FOUR questions. ALL questions carry equal marks. Use of a certified calculator is allowed.

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1. Discuss the effects of the rural and urban control, and adjustment sub-systems in relation to normal migration.

2. 'The refugees who flee from Rwanda can not only be blamed on the internal political, ethnic and socio-economic situations but on external interference as well.' Elucidate.

3. With reference to Petersen (1970) typology of migration assess the effect of the interaction of the physical environment and man.

4. Examine what is meant by `Burden sharing' in protecting and assuring the reasonable livelihoods of refugees within the asylum countries.

5. The United States of America and the Australian refugee policies leave much to be desired. Discuss.

6. Examine all four reservations made by Zambia to the United Nations (UN) (1951 convention and 1967 protocol relating to refugees.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2011 ACADEMIC YEAR FIRST SEMESTER FINAL EXAMINATIONS

GEO 951: CLIMATOLOGY

TIME: **Three hours**

INSTRUCTIONS: Answer any **FOUR** questions.

All questions carry equal marks. Candidates are advised to make use of illustrations and examples wherever appropriate.

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1. Write short explanatory notes on ALL of the following:

   a) Methods of assessing climate change and variability
   b) Conservation of absolute angular momentum
   c) Climate sensitivity
   d) Advantages and disadvantages of CFCs
   e) VOC abatement

2. Compare and contrast between wave cyclone and tropical cyclone.

3. How does the Three Cell Model explain the observed general circulation of the atmosphere?

4. 'One of the most important functions of the atmosphere is to provide the surface with protection from solar radiation'. Discuss.

5. Explain how the study of climatology enhances the understanding of the linkage between the environment and public health.

6. Identify five natural and five anthropogenic sources of air pollution; and discuss the control methods advocated for the reduction of particulate and nitrogen dioxide pollutants.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2011 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS

GEO 955: GEOMORPHOLOGY

TIME: Three hours

INSTRUCTIONS: Answer any FOUR questions.

All questions carry equal marks. Candidates are advised to make use of illustrations and examples wherever appropriate.

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1. Write short explanatory notes on ALL of the following:

    a) Contact and cataclastic metamorphism
    e) Buried and exhumed palaeosol
    c) Types of channel resistance
    d) Froude Number and Etchplain
    d) Probable maximum flood

2. Discuss the characteristics of seismic waves and the factors affecting the extent of damage caused by earthquakes.

3. With the use of diagrams outline the typical bed forms in the lower and upper flow regimes for alluvial sand bed channels.

4. State five dating methods and discuss the importance of sediment studies with reference to climate change in central Africa in the last thirty millennia.

5. Describe and explain the geomorphological legend of Zambia.

6. Define a *dambo* and discuss typical characteristics of *dambos* and how their existence is threatened by human activities in tropical Africa.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2011 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS

GEO 962: BIOGEOGRAPHY

Time: Three hours

Instructions: Answer any four questions. All questions carry equal marks.

The Philips University Atlas and a certified electronic calculator are allowed.

Candidates are encouraged to make use of illustrations wherever appropriate.

1. Write short explanatory notes on the ALL of the following:
    a) Spermatophyta
    b) Trophic levels
    c) Patterns of migration
    d) Types of fires in tropical savanna
    e) Betasian and mullerian mimicry

2. 'Continental drift facilitated the development of separate, distinctive fauna and flora on each continent in the world'. Discuss.

3. a) Describe the Theory of Island Biogeography.
   b) How has the Theory of Island Biogeography influenced the criteria used in the selection and creation of protected areas?

4. Elucidate on the factors which control the distribution and abundance of plants and animals in the world.

5. Explain the role of evolution in the process of speciation of organisms.

6. In what ways and to what extent do the terrestrial biomes influence the conservation of biodiversity and agricultural development in the world?

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2011 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS

GEO 975: CARTOGRAPHY

TIME: Three hours.

INSTRUCTIONS: Answer any four questions.

All questions carry equal marks.

---

1. Write short explanatory notes on ALL of the following:
   a) Sub-groups of a point symbol
   b) Subtractive colour process
   c) Semantic accuracy
   d) Map background hues
   e) Map projection recognition based on aspect

2. Explain the basic principles of the Universal Transverse Mercator (UTM) grid system used for position determination on the 1:50,000 Zambian topographic map sheets.

3. Explain how you would achieve clarity on a monochrome map showing various features of the University of Zambia campus.

4. 'Map production follows a logical sequence that culminates into a printed map'. Elucidate using a theme of your choice.

5. Discuss the necessity of generalization in mapping.

6. Outline and illustrate three graphic elements commonly used in making distinction among nominal point, line and areal geographic features.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2011 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATION

GEO 995: ENVIRONMENTAL AND NATURAL RESOURCES MANAGEMENT I

TIME: Three hours

INSTRUCTIONS: Answer any **four** Questions. All questions carry equal marks. Use of illustrations were appropriate is highly encouraged

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1. Explain what is meant by the terms nutrient cycling and energy flow and discuss their importance in an ecosystem.

2. Write short explanatory notes on ALL of the following

    a. Species habitat and ecological niche
    b. Renewable and non-renewable resources
    c. Characteristics of forest biomes
    d. The ecosphere and biosphere
    e. Fortress conservation

3. Discuss the main characteristics of the 'payments for ecosystem services' conservation paradigm and the challenges that arise in translating it into operational practice.

4. Outline and discuss the three levels of biodiversity and their implications for natural resources conservation.

5. Using mineral and energy resources for illustration, discuss the Resource Curse Hypothesis.

6. Use either Conservation Agriculture **or** Agro forestry to examine the opportunities and challenges of the practice of sustainable agriculture among smallholder farming households.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

INSTITUTE OF DISTANCE EDUCATION

Department of Mathematics and Statistics

2011/2012 Academic Year

Semester I

M111 Mathematical Methods I

FINAL EXAMINATION

Time Allowed: Three (3) Hours **May, 2012**

Instructions:

1. You must write your **Computer Number**, on each answer booklet you have used.
2. There are Seven (7) questions in this paper, Attempt **Any Five (5)** questions. All questions carry equal marks
3. Calculators are **Not** allowed.

1. (a) (i) Express $0.21\overline{555}$ in the form $\frac{a}{b}$ where a and b are integers.

(ii) Express $\frac{3}{2-3i}$ in the form $a+ib$ where a and b are rational numbers.

(iii) Express $\frac{2+2\sqrt{5}}{3+\sqrt{5}}$ in the form $a+b\sqrt{5}$ where a and b are real numbers.

(b) Prove each of the following identities:

(i) $\sin^2 x = \frac{1-\cos 2x}{2}$

(ii) $\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$

(c) Let α and β be roots of the equation $9x^2 - 6x + c = 0$.

(i) Find in terms of c an equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

(ii) Find the range of values of c if the roots α and β are complex roots.

2. (a) Define an operation $(*)$ on the set of real numbers by $a * b = a^b$.

(i) Is $(*)$ a binary operation on the set of real numbers? Give reason for your answer.

(ii) Is the operation commutative?

(iii) Evaluate $(2*-1)*5$

(b) Let $f(x) = \frac{cx^2+3}{x-2}$ where c is a constant. Find the value of c if $f'(3) = -9$.

(c) The polynomial $f(x) = ax^3 + 5x^2 - bx - 6$ leaves a remainder of -12 when divided by $x-1$. If also $x+2$ is a factor of $f(x)$,

(i) find the values of a and b

(ii) factorize the polynomial, $f(x)$, completely.

(iii) solve the inequality $f(x) \leq 0$

3. (a) Given that X and Y are sets, simplify as far as possible

(i) $(X \cup Y) \cap (X \cup Y')$

(ii) $[(X \cap Y')' \cup X']'$

(b) Let $z_1 = 1-2i$ and $z_2 = 2+3i$ be two complex numbers.

(i) Express $z_2 - \frac{1}{z_1}$ in the form $a + ib$ where a and b are real numbers.

(ii) Given that $2z_1 = (x - iy)z_2$, find the values of x and y

(c) Let $f(x) = -1 + \sqrt{2x+4}$ be a function.

(i) Find the **domain** and the **range** of $f(x)$.

(ii) Sketch the graph of $f(x)$.

(iii) Solve the inequality $f(x) \leq 3$.

4. (a) Given that $f(x) = \frac{2}{2-x}$ and that $g(x) = \frac{3+x}{x+1}$,

(i) find the **domain** and the **range** of $f(x)$.

(ii) find $(gof)^{-1}(x)$

(iii) solve the equation $(gof)(x) = \frac{3}{2}$.

(b) Find the solution to the equation $\sin 2x - \sqrt{3}\cos x = 0$ for values of x in the interval $[0, 2\pi)$.

(c) Given that $A = \{x \in \mathbf{R} \mid x \geq -1\}$, $B = \{x \in \mathbf{R} \mid -7 < x \leq 8\}$ and $C = \{x \in \mathbf{R} \mid x \leq -12 \text{ or } 3 < x < 20\}$,

(i) find B'

(ii) find also $B - A$ and express it on a number line.

(iii) express the set C using interval notation and express it on a number line.

5. (a) Find $\frac{dy}{dx}$

(i) $y = (2x^3 - 1)^7$

(ii) $y = x\sin(x^2)$

(iii) $y = e^{-x^2}$

(b) Find all values of x such that $-\frac{1}{2} < \frac{x}{x-1} \leq \frac{1}{3}$.

(c) Let $f(x) = 2 - x - 3x^2$ be a quadratic function.

(i) By completing the square express $f(x)$ in the form $f(x) = -3(x+p)^2 + q$ where p and q are constants.

(ii) Sketch the graph of $f(x)$.

(iii) Find the roots of the equation $f(x) = -2$.

6. (a) Differentiate each of the following functions from the **First Principle**

(i) $f(x) = \sqrt{x}$

(ii) $g(x) = x^2 + 2x$

(b) Express the polynomial $P(x) = x^3 + 2x^2 - 1$ in the form $P(x) = (2x+1)Q(x) + R$ where $Q(x)$ is the quotient and R is the remainder when $P(x)$ is divided by $2x+1$.

(c) Let $f(x) = 1 - 2\sin\frac{x}{2}$ be a function.

(i) Determine the amplitude and the period of the function $f(x)$.

(ii) Sketch the graph of $f(x)$ for values of x in the interval $-2\pi \le x \le 3\pi$.

(iii) Find all solutions to the equation $f(x) = 2$ in the interval $0 \le x \le 3\pi$.

7. (a) Find the value of each of the following limits:

(i) $\lim_{x \to 0} \frac{3x}{x^3 - 5x}$

(ii) $\lim_{x \to 4} \frac{4 - x}{\sqrt{x} - 2}$

(iii) $\lim_{x \to -\infty} \frac{x + 5x^2}{3x^2 + 4x}$.

(b) Given that A is an obtuse angle and B is an acute angle such that $\sin A = \frac{3}{5}$ and $\cos B = \frac{5}{13}$, find the value of $\tan(A+B)$.

(c) Let $f(x) = 1 + |2x - 3|$.

(i) Is the function $f(x)$ even, odd or neither?

(ii) State the range of $f(x)$.

(iii) Sketch the graph of $f(x)$.

THE UNIVERSITY OF ZAMBIA

School of Natural Sciences

Department of Mathematics and Statistics

2011/2012 Academic Year

Semester II

M112Mathematical Methods II – A

FINAL EXAMINATION

Time Allowed: Three (3) Hours **May, 2012**

Instructions:

1. You must write your **Computer Number**, on each answer booklet you have used.
2. Indicate your Tutorial Group **TG**, on your main answer booklet.
3. There are Seven (7) questions in this paper, Attempt **Any Five (5)** questions. All questions carry equal marks
4. Calculators are **Not** allowed.

1. (a) Given the matrices $A=\begin{pmatrix}2 & -1\\ 1 & 3\end{pmatrix}$, $B=\begin{pmatrix}7 & 4\\ 2 & -3\\ 5 & 1\end{pmatrix}$ and $C=\begin{pmatrix}1 & 1 & 1\\ -6 & 2 & 5\end{pmatrix}$, find the following matrices:

(i) $B^T + C$

(ii) $5A - CB$

(iii) $(CB)A$

(b) A curve is given by the equation $x^2 - y^2 + 10x - 2y = 0$

(i) Find the equation of the tangent to the curve at the origin.

(ii) Find the equation of the normal to the curve at the origin.

(iii) A straight line L: $y = mx$ makes an angle of $\frac{3\pi}{4}$ with the normal to the curve in (ii). Find the equation of the line L.

(c) Sketch the graph of the function $f(x) = 1 + \log_2(x+2)$.

2. (a) (i) Prove the identity $\cosh 3x = 4\cosh^3 x - 3\cosh x$

(ii) Solve the equation $\sinh^2 x - 5\cosh x + 5 = 0$ for real values of x.

(iii) Solve the equation $(\log_{10} x)^2 - \log_{10} x = 2$

(b) (i) Find the modulus and the argument of the complex number

$$w = \frac{\left[\sqrt{3}(\cos\theta + i\sin\theta)\right]^4}{\cos 2\theta - i\sin 2\theta}.$$

(ii) Find the cube roots of the complex number $z = -2 + 2i$

(c) Find $\frac{dy}{dx}$ given that $y = \ln^2(a^x)$

3. (a) Prove using Mathematical Induction that:

(i) if $A=\begin{pmatrix}2 & a\\ 0 & 1\end{pmatrix}$, then $A^n = \begin{pmatrix}2^n & (2^n - 1)a\\ 0 & 1\end{pmatrix}$ for every positive integer n

(ii) $n^3 + 5n + 6$ is divisible by 3 for every positive integer n.

(b) (i) A farmer is to fence a rectangular plot which is bounded by a river on one side. There is no fence required on the side where there is a river. The cost of fencing the front side which is opposite the river is K15,000 per meter while the cost of fencing along the sides is K10,000 per meter. Find the dimensions of the largest plot which can be fenced for K3,000,000.

(ii) Find the integral $\int \frac{\cos\left(\frac{1}{x^2}\right)}{x^3}\,dx$

(c) If the remainder when $f(x) = 3x^3 + 4x^2 - ax + 1$ is divided by $3x - 2$ is $\frac{7}{3}$

(i) find the value of a.

(ii) Express $f(x)$ in the form $f(x) = (3x-2)Q(x) + R$ where Q(x) is the quotient and R is the remainder when $f(x)$ is divided by $3x - 2$.

4. (a) Find the following integrals:

(i) $\int x^2\sqrt{1+2x^3}\,dx$

(ii) $\int -xe^{-x}\,dx$

(iii) $\int \frac{2}{9+x^2}\,dx$

(b) Given that A is a point $(-1, 2)$, B is the point $(2, 3)$ and C is the point $(3, 5)$. P is a point on BC and divides the line segment BC in the ratio $4:3$. Q is a point on the line segment AB such that $AQ = \frac{3}{5}AB$.

(i) Find the coordinates of P

(ii) Find the coordinates of Q

(iii) Find the equation of a line which passes through the point $(4, -1)$ and which parallel to the line segment AB.

(c) If n is a positive integer, show that $\left(\sqrt{3}-i\right)^n = 2^n\left(\cos\frac{n\pi}{6} - i\sin\frac{n\pi}{6}\right)$.

5. (a) (i) Find the distance from the point $(-2, 5)$ to the line $x+3y=9$.

(ii) Find the center and the radius of the circle C_1 whose equation is $2x^2+2y^2-11x+6y-8=0$.

(iii) If a circle C_2 with the same center as C_1 in (ii) is tangent to the $x-$axis, find the equation of the circle C_2.

(b) (i) Evaluate the integral $\int_0^{\frac{\pi}{3}}(x-\sin x)dx$

(ii) Calculate the area bounded by the curve $f(x)=x^2+2x$ and the $x-$axis between the lines $x=-3$ and $x=0$.

(iii) Find $\frac{dy}{dx}$ given that $\log\left(\frac{y}{x}\right) - xy = 1$.

(c) Let $f(x) = 1-2^{1-x}$ be a function.

(i) State the domain and the range of $f(x)$.

(ii) Find also $f'(0)$.

6. (a) (i) Find the third term and the constant term in the expansion of $\left(x-\frac{2}{x}\right)^{12}$.

(ii) Expand $(1-2x)^{-\frac{1}{2}}$ in ascending powers of x up to and including the term in x^3. State the range of values of x for which your expansion is valid.

(iii) Hence use your expansion in (ii) to find the approximate value of $\frac{1}{\sqrt{0.8}}$ to 3 decimal places.

(b) Given the function $f(x)=-\frac{2}{3}x^3-\frac{1}{2}x^2+x$;

(i) Determine intervals where the function is decreasing.

(ii) Use second derivative test to classify the extreme points of the function.

(iii) Sketch the graph of the function.

(c) A certain town had a population of 9, 000 when the 2000 census was taken and a population of 12, 500 when the 2010 census was taken. Assuming an exponential law of growth, $P=P_0e^{kt}$, where P is the population at time t years and P_0 is the initial population, estimate the population of the town at the 2020 census.

7. (a) (i) Find the value(s) of x for which the matrix $A=\begin{pmatrix} 2-x & 1 & 1 \\ 2 & 3-x & 4 \\ -1 & -1 & -2-x \end{pmatrix}$ is a singular matrix.

(ii) Find the inverse of the matrix $\begin{pmatrix} 1 & -1 & 2 \\ 1 & 2 & 3 \\ 3 & 0 & 1 \end{pmatrix}$.

(iii) Use Cramer's Rule to solve the system of equations:

$$x-y+2z=4$$
$$x+2y+3z=2.$$
$$3x+z=4$$

(b) Let $h(x)=\frac{2-3x}{x+2}$ be a rational function.

(i) Determine the vertical and horizontal asymptotes of the curve of the function.

(ii) Hence sketch the graph of $h(x)$.

(c) Express $\frac{5x+2}{(2x-1)(x+1)}$ into partial fractions and hence find the integral

$$\int\frac{5x+2}{(2x-1)(x+1)}dx$$

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
2012 ACADEMIC YEAR
SECOND SEMESTER EXAMINATIONS
M114: MATHEMATICAL METHODS II (B)

TIME ALLOWED: THREE (3) HOURS

INSTRUCTIONS:

(a) Write your computer number on each answer booklet used

(b) There are **SIX** questions in this paper. Answer any **FIVE(5)** questions only. All questions carry equal marks.

(c) No calculators to be used.

---

1. (a) (i) Expand $\sqrt{4-x}$, in ascending powers of x, up to and including the term in x^3.

   (ii) Find values of x for which the matrix

$$\begin{pmatrix} 2-x & 1 & 1 \\ 2 & 3-x & 4 \\ -1 & -1 & -2-x \end{pmatrix} \text{ is singular}$$

   (b) (i) Sketch the curve $y = xe^{-\frac{x}{2}}$

   (ii) Determine the coordinates of the turning point

   (ii) The region R is bounded by the curve, the x-axis and the line x= 2, find the area of R.

   (c) Solve the equation $z^3 + 1 = 0$, giving your answer in the form a + ib, where a and b are real, show the solutions on an Argand diagram.

2. (a) The Tetrahedron ABCD has vertices A(0,1,0), B(1,1,2) ,C(-2,1,3) and D(2,0,1). Find the volume of the Tetrahedron..

   (b) (i) Find $\int \frac{5x+2}{(2x-1)(x+1)}dx$

   (ii) By the method of mathematical induction, prove that $x^{2n} - y^{2n}$ is divisible by x-y.

   (c) Given the equation of an ellipse $\frac{(x+1)^2}{4} + \frac{(y+2)^2}{9} = 1$. Find

   (i) the eccentricity

   (ii) the coordinates of the foci and vertices

   (iii) the equations of the directrices

   Hence, sketch the graph of the ellipse.

1/6

3 (a) Find $\frac{dy}{dx}$ of the following:

(i) $y = \ln(1 + \tan^2 x)$ (ii) $x^2 + y^3 = 12$

(b) (i) By using the substitution $x = \sin\vartheta$, Show that

$$\int_0^{\frac{1}{2}} \frac{x^4}{\sqrt{1-x^2}}dx = \frac{(\pi - 7\sqrt{3})}{64}$$

(ii) Using the definition of sinhx and coshx . Show that

$ar\cosh x = \ln[x + \sqrt{x^2 - 1}\,]$, $x \geq 1$

(c) (i) A curve has equation $yx^2 - 3x = \ln y$. Find the equation of normal to the curve at the point (3,1).

(ii) Express $\cos^4\vartheta$ in terms of $\cos 4\vartheta$ and $\cos 2\vartheta$.

4. (a) The angle between the vector $a = i + j$ and $b = 2i + j + \lambda k$ is $\frac{\pi}{4}$.

(i) Find the possible values of λ.

(ii) Find a vector which is perpendicular to both a and b for each value of λ, found in part (i) above.

(b) Given that a circle passes through the points (2,0) , (8,0) and (10,4)

Find the equation of the circle

(c) (i) Find $\int \frac{\cos(\frac{1}{x^2})}{x^3}dx$

(ii) Find the fourth roots of 1 + i

1 3 3 1
1 4 6 4 1

$\frac{10-2}{4} = \frac{8}{4} = 2.$

$\cos^4\theta$ $\cos 4\theta$ & $\cos 2\theta$

$(\cos\theta + i\sin\theta)^4 = \cos 4\theta$

$(z + \frac{1}{z})^4$ $z + z^{-1}$

$z^4 + 4z^3(\frac{1}{z}$ $z + \frac{1}{z}$

$z + \frac{1}{z}$

$2\cos$

$a = i + j + 0k$

$b = 2i + j + 2k$

$a \times b \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 2 & 1 & -2 \end{vmatrix} = -2i + 2j + (1-2)k$

$-2i + 2j + -k$

$-2i + 2j \cdot \lambda$

$= 2i - 2j + (1-2)k$

$= 2i - 2j - k$

5 (a) (i) Find the inverse of the matrix A,

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & 0 \\ 1 & -1 & -2 \end{pmatrix}$$

(ii) Solve the system of equations given below;

$$x + 2z = 1$$
$$3x + y = -2$$
$$x - y - 2z = 1$$

(b) For the hyperbola with equation $4y^2 - x^2 = 12$

(i) Find the eccentricity and centre

(ii) Find the foci and vertices of the hyperbola

(iii) Find the equation of the asymptotes

(iv) Sketch the graph of the hyperbola

(c) Solve the equation 4 tanhx – sech x = 1

6. (a) Sketch the curve with equation $y = -\frac{2}{3}x^3 - \frac{1}{2}x^2 + x$, showing

(i) points where it meets the x-axis

(ii) The coordinates of the turning points

(iii The ranges of x, where the function is increasing and decreasing

(b) (i) By the method of mathematical induction, prove that the following statement is valid for all positive integral values of n

$$1.2 + 3.4 + 5.6 + \ldots + (2n-1)(2n) = \frac{n(n+1)(4n-1)}{3}$$

(ii) Evaluate $\sqrt[3]{1.01}$

(iii) The radius of a spherical balloon is increasing at the rate of 0.2m/s. Find the rate of increase of the surface area of the balloon at the instant when the radius is 1.6m?

(c) Given the curve $y = \frac{x^2 + 9}{x^2 - 1}$

(i) Determine the critical and horizontal asymptotes

(ii) Determine the turning points, if there are any.

(iii) Sketch the curve

(iv) Determine the intervals when the curve is concave up and down

END of EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS
SEMESETER I EXAMINATIONS – 2011

M161 – INTRODUCTION TO MATHEMATICS, PROBABILITY AND STATISTICS 1

INSTRUCTIONS:
1. Answer any **five (5)** questions.
2. All questions carry equal marks.
3. Show all the necessary work to earn full marks.
4. Write down the questions attempted on the front page of the main booklet.
5. Use of calculators is **NOT** allowed.

TIME ALLOWED: Three **(3)** hours.

---

1. **[a]** [i] Express 1. 02999999........as a rational number to its lowest term.

[ii] Let the universal set $E = \Re$, be the set of real numbers such that $A=(3,6]$, $B=[5,7)$ and $C=[2,4]$
Find $A \cap (B' \cap C')$ and display it on the number line.

[b] If A, B, C, D are subsets of the universal set E, such that
$E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{x \in E : x \text{ is odd}\}$, $B = \{x \in E : x \text{ is divisible by } 4\}$,
$C = \{x \in E : \sqrt{x} \text{ is a whole number}\}$, $D = \{x \in E : x+6 \in E\}$,

Find each of the following:
[i] $(A \cap B) \cup (C \cap D)$
[ii] $A \cap (D - C)$

[c] Let $A = \{r, s, u, v\}$ and define the operation $*$ as given in the table below:

*	r	s	u	v
r	r	s	u	v
s	s	r	v	u
u	u	v	r	r
v	v	u	s	s

[i] Evaluate $s * (r * v)$
[ii] Is $*$ associative? Justify your answer.
[iii] Find if any, the identity on set A with respect to $*$.

2. [a] Given that the roots of the given equation $x^2 + 4x + 2 = 0$ are α and β, form an equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$

[b] Solve for $x \in \Re$:

[i] $|x-3| = 2|x-2|$

[ii] $\sqrt{2x-1} = 1 + \sqrt{x+3}$

[c] Find the set of values of x for which:

$$\frac{x(x+2)}{x-3} < x+1$$

3. [a] Given $f(x) = \dfrac{2x-2}{x-2}$ and $g(x) = x^2$,

[i] Show whether f is an even or odd function or neither.

[ii] Show that $f^{-1}(x) = f(x)$

[iii] Find $(f \circ g)(x)$

[iv] Hence solve the equation $f[g(x)] = 3$

[b] When the cubic polynomial $x^3 + ax^3 - 3x + 4$ is divided by $x-3$, the remainder obtained is twice the remainder when the polynomial is divided by $x-2$. Calculate the value of a.

[c] [i] Let X and Y be two sets.

Show that $(X' \cup Y)' \cap (Y \cup X)' = \phi$

[ii] Find a, b and c if: $a + b\sqrt{c} = \dfrac{1}{2+\sqrt{3}} + \dfrac{3}{2-\sqrt{3}}$

4. [a] Solve the inequality $\left|\dfrac{1}{1-3x}\right| = 2$

[b] The quadratic polynomial $p(x) = ax^2 + bx + c$ leaves a remainder of 3 on division by $x-1$, a remainder of 12 on division by $x-2$ and no remainder on division by $x+2$.

Find $p(x)$ and solve $p(x) = 0$

[c] When a certain car factory produces x cars per day its profit $\$x$ is given by

$$p(x) = 5x^2 - 100x.$$

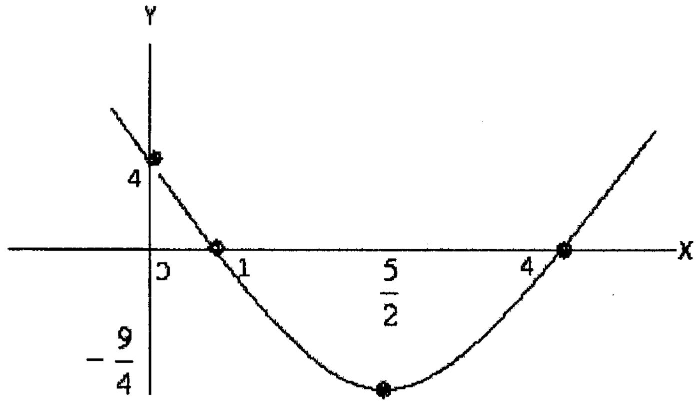
How many cars per day must the factory produce:

[i] to make a profit.

[ii] to make a profit of $ 24, 000 per day.

[iii] to make the worst possible loss.

5. **[a]** The sketch of the curve $f(x) = ax^2 + bx + c$ is given below:



Using the curve above,

[i] Write down the zeros of $f(x)$

[ii] Calculate the value of a, b and c.

[iii] Copy and sketch in the same diagram the graph of $|f(x)|$.

[b] [i] Given that $\sqrt{3} \approx 1.732$, find, without using a calculator, to 3 decimal places, the value of $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

[ii] Find the integer k given that $k = (\sqrt{x+1} + \sqrt{x})(\sqrt{x+1} - \sqrt{x})$

[c] Solve for x and y:

$$\frac{x}{1-i} + \frac{y}{1+3i} = \frac{2}{1+8i}$$

6 **[a]** [i] Solve the simultaneous equation

$xy = 1$

$x - y = 12$

[ii] Expand and simplify: $\left(x - \frac{1}{x}\right)^4$

[b] Find the constants A, B and C in the identity

$$\frac{x^2}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

[c] Solve equation:

[i] $2^{2x+3} - 33(2^x) + 4 = 0$

[ii] $\log_2 x + 4\log_x 2 = 1$

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS
SEMESETER I EXAMINATIONS – 2011

M161 – INTRODUCTION TO MATHEMATICS, PROBABILITY AND STATISTICS 1

INSTRUCTIONS:
1. Answer any **five (5)** questions.
2. All questions carry equal marks.
3. Show all the necessary work to earn full marks.
4. Write down the questions attempted on the front page of the main booklet.
5. Use of calculators is **NOT** allowed.

TIME ALLOWED: Three **(3)** hours.

---

1. **[a]** [i] Express 1. 02999999........as a rational number to its lowest term.

[ii] Let the universal set $E = \Re$, be the set of real numbers such that $A = (3,6]$, $B = [5,7)$ and $C = [2,4]$
Find $A \cap (B' \cap C')$ and display it on the number line.

[b] If A, B, C, D are subsets of the universal set E, such that
$E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{x \in E : x \text{ is odd}\}$, $B = \{x \in E : x \text{ is divisible by } 4\}$,
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Find each of the following:
[i] $(A \cap B) \cup (C \cap D)$
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*	r	s	u	v
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s	s	r	v	u
u	u	v	r	r
v	v	u	s	s

[i] Evaluate s * (r * v)
[ii] Is * associative? Justify your answer.
[iii] Find if any, the identity on set A with respect to * .

2. **[a]** Given that the roots of the given equation $x^2 + 4x + 2 = 0$ are α and β, form an equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$

[b] Solve for $x \in \Re$:

[i] $|x-3| = 2|x-2|$

[ii] $\sqrt{2x-1} = 1 + \sqrt{x+3}$

[c] Find the set of values of x for which:

$$\frac{x(x+2)}{x-3} < x+1$$

3. **[a]** Given $f(x) = \dfrac{2x-2}{x-2}$ and $g(x) = x^2$,

[i] Show whether f is an even or odd function or neither.

[ii] Show that $f^{-1}(x) = f(x)$

[iii] Find $(f \circ g)(x)$

[iv] Hence solve the equation $f[g(x)] = 3$

[b] When the cubic polynomial $x^3 + ax^3 - 3x + 4$ is divided by $x-3$, the remainder obtained is twice the remainder when the polynomial is divided by $x-2$. Calculate the value of a.

[c] [i] Let X and Y be two sets.
Show that $(X' \cup Y)' \cap (Y \cup X)' = \phi$

[ii] Find a, b and c if: $a + b\sqrt{c} = \dfrac{1}{2+\sqrt{3}} + \dfrac{3}{2-\sqrt{3}}$

4. **[a]** Solve the inequality $\left|\dfrac{1}{|1-3x|}\right| = 2$

[b] The quadratic polynomial $p(x) = ax^2 + bx + c$ leaves a remainder of 3 on division by $x-1$, a remainder of 12 on division by $x-2$ and no remainder on division by $x+2$.
Find $p(x)$ and solve $p(x) = 0$

[c] When a certain car factory produces x cars per day its profit $\$x$ is given by

$$p(x) = 5x^2 - 100x.$$

How many cars per day must the factory produce:

[i] to make a profit.

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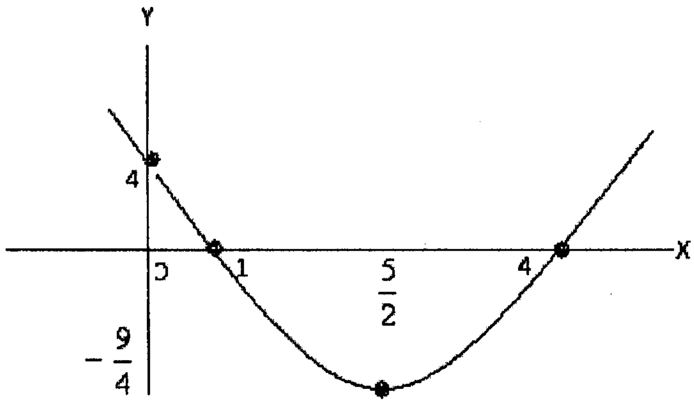
5. **[a]** The sketch of the curve $f(x) = ax^2 + bx + c$ is given below:



Using the curve above,

[i] Write down the zeros of $f(x)$

[ii] Calculate the value of a, b and c.

[iii] Copy and sketch in the same diagram the graph of $|f(x)|$.

[b] [i] Given that $\sqrt{3} \approx 1.732$, find, without using a calculator, to 3 decimal places, the value of $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

[ii] Find the integer k given that $k = (\sqrt{x+1} + \sqrt{x})(\sqrt{x+1} - \sqrt{x})$

[c] Solve for x and y:

$$\frac{x}{1-i} + \frac{y}{1+3i} = \frac{2}{1+8i}$$

6 **[a]** [i] Solve the simultaneous equation

$$xy = 1$$
$$x - y = 12$$

[ii] Expand and simplify: $\left(x - \dfrac{1}{x}\right)^4$

[b] Find the constants A, B and C in the identity

$$\frac{x^2}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

[c] Solve equation:

[i] $2^{2x+3} - 33(2^x) + 4 = 0$

[ii] $\log_2 x + 4\log_x 2 = 1$

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

UNIVERSITY SEMESETER II EXAMINATIONS

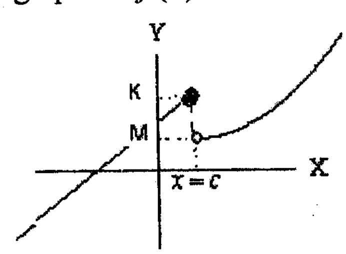
30th MAY 2012

M 162 – INTRODUCTION TO MATHEMATICS, PROBABLITY AND STATISTICS II

INSTRUCTIONS:
1. Answer **ANY FIVE (5)** Questions.
2. Show all the necessary work to earn full marks.
3. Write down the questions attempted in the first column on the front page of the main booklet.
4. Use of a calculator is **NOT** allowed.

TIME ALLOWED: Three (3) hours.

Q1 **[a]** Given the graph of $f(x)$ as shown:



Find the following where possible:

[i] $f(c)$

[ii] $\lim_{x \to c^-} f(x)$

[iii] $\lim_{x \to c} f(x)$

[b] If $f(x) = x^2 + 5x - 1$

[i] Find $f(x+h)$ and $f(x+h) - f(x)$ in simplest form.

[ii] Hence use [i] to find $f'(x)$

[iii] Find the equation of the tangent and normal lines to $f(x)$ at P (1, -1).

[c] [i] Find $\frac{dy}{dx}$ given that $y = x^3 \ln x + \cos x$

[ii] Given that $\int_0^2 x(x+2)(x-1)\,dx = k$, where k is a constant, find k.

Q2 **[a]** Evaluate the following limits:

[i] $\lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$

[ii] $\lim_{x \to 9} \frac{x^2 - 7x - 18}{\sqrt{x} - 3}$

[iii] $\lim_{x \to \infty} \frac{7 + 5x + 3x^2}{8 + 4x^2}$

[b] Find the value of $\frac{dy}{dx}$ given that

[i] $x^2 - 3xy + 2y^2 - 2x = 4$ at the point $(1, -1)$

[ii] $y = e^{x^2 - x}$ at $x = 1$

[iii] $x = t^3$ and $y = t^2$ at $t = 2$ using chain rule.

[c] Find the $\int x \ln x \, dx$

Q3 **[a]** [i] Evaluate: $\int_1^2 \frac{x^4 - 1}{x^3} \, dx$

[ii] Find A and B for which $\frac{x+2}{x^2 - 4x - 5} = \frac{A}{x-5} + \frac{B}{x+1}$

Hence find $\int \frac{x+2}{x^2 - 4x - 5} \, dx$

[b] [i] If $x^2 + 3y^2 + 2y = 0$, and $\frac{dx}{dt} = 2$ when $x = 3$ and $y = -1$, find $\frac{dy}{dt}$.

[ii] Find y' given that $y = x^2 \sin x$

[c] Given the equation of the curve $f(x) = x^3 + 6x^2 + 9x$,

[i] Find the x and y intercepts.

[ii] Find the stationary points.

[iii] Test the nature of the stationary points.

[iv] Find the point(s) of inflection.

[v] Sketch the graph of the curve labeling all the necessary features.

Q4 **[a]** [i] Find $\int \left(9x^2+6\right)\sqrt{x^3+2x-1}\;dx$

[ii] For the function $y = 2x^3 - 4x$,

Find the value of $\dfrac{y'}{\sqrt{1+y''}}$ when $x = 2$.

[b] The dimensions of a rectangular cardboard ABCD are 8 cm and 3cm. A square of side $x\ cm$ is removed from each of the corners and the remainder is folded to form an open box of depth $x\ cm$ and volume $v\ cm^3$.

[i] Show that $V = 4x^3 - 22x^2 + 24x\ cm^3$

[ii] Find the value of x for which $\dfrac{dV}{dx} = 0$

[iii] Find the value of x for which the volume of the box is maximum. Calculate the maximum volume.

Q5 **[a]** For the data set,

18.3	9.6	18	23.4	18
9.7	24.6	29.4	17.1	14.2
14.1	21.8	14.3	21.7	13.1
25.4	14.4	18.8	16	15.8
15.1	26.4	14.4	14.4	9.7

Starting with the interval 9.5 – 13.5, 13.5 – 17.5 etc,
[i] Set up a frequency distribution.
[ii] Construct a histogram and a frequency polygon on the same diagram.

[b] Given the following data set:

CLASS INTERVAL	FREQUENCY
15 - 19	2
20 - 24	6
25 - 29	7
30 - 34	5

Determine the approximate:
[i] Modal class
[ii] Median class
[ii] Sample mean for the data

Q6 [a] The following table shows the probabilities of two events A and B. Copy and complete the table given that

$P(A) = 0.47$, $P(B) = 0.40$ and $P(A \cap B) = 0.25$

	B	B^C	
A	0.25	-----	0.47
A^C	-----	-----	----
	0.40	-----	

Hence, find:

[i] The probability that either A occurs or B occurs.

[ii] The probability that one of these events occurs and the other does not occur.

[b] Suppose that $P(A) = 0.68$, $P(B) = 0.55$ and $P(A \cap B) = 0.32$, Find:

[i] $P(A \cup B)$

[ii] $P(B / A)$

[iii] $P(B / A^C)$

[c] Two production lines contribute to the total amount of a company's product.

Line 1 (L_1) provides 30% of the total and 15% of its products are defective (D).

Line 2 (L_2) provides 70% of the total and 5% of its products are defective.

[i] What percentage of the items in the total collection of products is defective?

[ii] Suppose an item is randomly selected from the total collection of products and is found to be defective.
What is the probability that it came from Line 1 (L_1)

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
DEPARTMENT OF MATHEMATICS AND STATISTICS

SEMESETER I EXAMINATIONS - 2012
DISTANCE EDUCATION
M211 – MATHEMATICAL METHODS III

INSTRUCTIONS:
1. Answer any **four (4)** questions.
2. All questions carry equal marks.
3. Show all the necessary work to earn full marks.
4. Write down the questions attempted on the front page of the main booklet.
5. Use of calculators is allowed.

TIME ALLOWED: Three (3) hours.

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1. [a] Discuss and sketch the graph of the equation

$$9x^2 - 4y^2 - 54x - 16y + 29 = 0$$

[b] Given the function $41x^2 - 24xy + 34y^2 - 25 = 0$

[i] Identify the conic given by this equation.

[ii] Use a suitable rotation of axes to find an equation for the graph in an $X'Y'$ plane.

[iii] Sketch the graph.

2. [a] Given the equation of the conic $\dfrac{10}{3+2\cos\theta}$,

[i] Find the eccentricity and classify the conic.

[ii] Sketch the graph showing all its characteristics.

[b] Find the volume of the solid obtained by revolving the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ about the x – axis.}$$

[c] A ball is dropped from a height of 6 meters and begins bouncing. The height of each bounce is three- fourths the height of the previous bounce. Find the total distance travelled by the ball.

3 [a] If $f(x) = \frac{1}{4}x^2 + 1$, show that f satisfies the hypotheses of the mean value theorem on the interval [-1,4] and find the number c in (-1,4) that satisfies the conclusion of the theorem.

[b] Let $f(x) = 2x^3 + 7x^2 + x - 6$

[i] Find a formula for the $4th$ Taylor polynomial of f in powers of $(x-2)$.

[ii] Hence calculate $p_4(1)$.

[c] [i] Find $\int \frac{1}{\sqrt{1+x} + \sqrt{x}}\, dx$

[ii] Find $\int_0^\infty \operatorname{sec}h\, x\, dx$

4. [a] Show that the reduction formula for $\int \tan^n x\, dx$ is $\frac{\tan^n x}{n-1} - \int \tan^{n-2} x\, dx$

Hence find the $\int \tan^3 x\, dx$

[b] [i] Find $\lim_{x \to +\infty} \frac{x^2+1}{x\sqrt{3x^2+1}}$

[ii] Evaluate $\int \frac{1}{x\sqrt{x^4-9}}\, dx$

[c] Sketch the graph of $y = 1 - x^2$ and find the curvature at the point $P(2,-3)$.

5 [a] [i] Find the arc length of the curve $f(x) = \frac{x^3}{6} + \frac{1}{2x}$ from $x = \frac{1}{2}$ to $x = 2$

[ii] Show that the curvature of the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the curve

$x^3 + y^3 = 3xy$ is $-\frac{8\sqrt{2}}{3a}$

[b] Evaluate $\int \tan x \sin^2 x \cos^5 x\, dx$

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
DEPARTMENT OF MATHEMATICS AND STATISTICS

SEMESETER II EXAMINATIONS - 2012
DISTANCE EDUCATION
M212 – MATHEMATICAL METHODS IV

INSTRUCTIONS:
1. Answer any **four (4)** questions.
2. All questions carry equal marks.
3. Show all the necessary work to earn full marks.
4. Write down the questions attempted on the front page of the main booklet.
5. Use of calculators is allowed.

TIME ALLOWED: Three (3) hours.

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1 [a] [i] Given the vectors $\underset{-}{a} = (a_1, a_2, a_3)$, $\underset{-}{b} = (b_1, b_2, b_3)$, $\underset{-}{c} = (c_1, c_2, c_3)$

Show that $\underset{-}{a} \bullet (\underset{-}{b} + \underset{-}{c}) = \underset{-}{a} \bullet \underset{-}{b} + \underset{-}{a} \bullet \underset{-}{c}$.

[ii] For $\underset{-}{a} = (1,1,1)$, $\underset{-}{b} = (2,1,0)$, $\underset{-}{c} = (0,0,1)$,

Find $\underset{-}{a} \bullet (\underset{-}{b} \times \underset{-}{c})$. Hence find the volume of the box with $\underset{-}{a}$, $\underset{-}{b}$ and c as adjacent sides.

[b] [i] Find the distance from the point $D(-1, 1, 2)$ to the plane through the points $3x - 2y + 2z = 1$

[ii] Find the curvature for the curve $\bar{r}(t) = \frac{1}{3}t^3\, i + \frac{\sqrt{2}}{2}t^2\, j + t\, k$ at $t = 1$

2. [a] Suppose the dimensions (in meters) of a rectangular box change from 9, 6, and 4 to 9.02, 5.97 and 4.01, respectively.

[i] Use differentials to approximate the change in volume.

[ii]Find the exact change in volume.

[b] [i] Let $z = f(u - v, v - u)$, show that $\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v} = 0$

[ii] If the line l has parametric equations $x = 5 - 3t$, $y = -2 + t$, $z = 1 + 9t$,
Find parametric equations for the line through P (5 , 4,-3) that is parallel to l.

3. [a] Solve the equation:

[i] $\cos x\frac{dy}{dx}+y\sin x=\sec^2 x$

[ii] $x\frac{dy}{dx}+y=y^2\ln x$

[b] [i] If $u=\ln\frac{x^2+y^2}{x+y}$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=1$

[ii] Verify Euler's theorem for $z=x^n\ln\left(\frac{y}{x}\right)$

4 [a] If $f(x,y)=\frac{1}{3}x^3+\frac{4}{3}y^3-x^2-3x-4y-3$,

find the local extrema and saddle points of f.

[b] Solve the following differentials

[i] $\frac{dy}{dx}+\frac{3}{x}y=\frac{4}{x^2}+10x$

[ii] $y''+10y'+29y=0$

5 [a] [i] Find $\frac{\partial z}{\partial x}$ and $\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$ given that $z=e^y\sin xy$

[ii] Let l be the line of intersection of the two planes

$2x-y+4z-4=0$ and $x+3y-2z-1=0$.

Find parametric equations for the line l.

[b] [i] Find the distance between the point $Q\,(1,5,-4)$ and the plane given by $3x-y+2z=6$.

[ii] Find the limit if it exists:

$$\lim_{(x,y)\to(0,0)}\frac{3x^3-2x^2+3y^2x-2y^3}{x^2+y^2}$$

[iii] Find the second partial derivatives of $f(x,y)=3xy^2-2y+5x^2y^2$ and determine the value of $f_{xy}(-1,2)$.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS

2011 ACADEMIC YEAR
FIRST SEMISTER FINAL EXAMINATIONS

M221: LINEAR ALGEBRA I

TIME ALLOWED: THREE HOURS.
INSTRUCTIONS: (i) ATTEMPT ANY 5 QUESTIONS
(ii) SHOW ALL WORKING

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1. (a)What is meant by the following terms

(i) Subspace of a vector space V.

(ii) Linear combination of elements of $S=\{v_1,v_2,\ldots,v_n\}$.

(b).Show that if $S=\{v_1,v_2,\ldots,v_n\}$ is a linearly dependent set of vectors, then at least one of the vectors is a linear combination of the vectors preceding it.

(c) If S and T are subspaces of $V_4(\mathbb{R})$ defined by $S=\{(\alpha,\beta,\gamma,\delta)|\alpha+\beta+\gamma=0\}$ and $T=\{(\alpha,\beta,\gamma,\delta)|\gamma=-\delta\}$,find the $\mathbb{R}$ -bases for $S\cap T$ and $S+T$.

(d)Find the (i) null space

(ii)Nullity and rank,

of the system of linear equations where $\lambda=1$, $\begin{cases}2x+2y+z=5\\3x-y+\lambda z=2\\x+5y-2z=5\end{cases}$.

2. (a) Define the following

(i) Basis of a vector space V.

(ii) Linear transformation.

(b).Find the R-basis for the solution space of the system of linear equations

$$\begin{cases}x-2y+3z+w=0\\2x+y-z-w=0\end{cases}$$

(c) Given two subspaces W_1 and W_2 of a vector space V. Prove that the following are also subspaces

(i) $W_1\cap W_2$.

(ii) W_1+W_2.

(d)Show that $T:V_4(\mathbb{R})\to V_4(\mathbb{R})$ defined by

$T(\alpha,\beta,\gamma,\delta)=(\alpha-\beta,\alpha+2\beta,\delta,2\alpha+\beta+\delta)$ is a linear transformation.

3. (a) What is meant by a

(i) Normal matrix.

(ii) Homogenous system of linear equation.

(b)Evaluate the matrix equation below for X,

$$\begin{pmatrix} 2 & -3 & 3 \\ 2 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix} X = \begin{pmatrix} 2 & -3 \\ 2 & 5 \\ 6 & 6 \end{pmatrix}$$

(c) Find the invertible matrices P and Q such that PAQ is in normal form given

$$A = \begin{pmatrix} 2 & -1 & 7 \\ 1 & 1 & 2 \end{pmatrix}$$

(d).Show that if A and B are invertible $n \times n$ matrices, then AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$.

4. (a) Define the following

(i) Row operations on a matrix (ii) Elementary matrix

(b)Prove that for any mxn matrices A and B, $(AB)' = B'A'$.

(ii)Hence, given matrices $A = \begin{pmatrix} 2 & 1-i \\ 6 & 2-i \\ -2 & i+1 \end{pmatrix}$ and $B = \begin{pmatrix} 2+i & 2+3i \\ i & 6 \end{pmatrix}$, show that $(AB)' = B'A'$

(c)For an invertible matrix $A = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & 1 \\ 0 & 3 & -2 \end{pmatrix}$, find the elementary matrices for which $A = E_1 \cdot E_2 \cdot E_3 \cdot E_4 \cdot I_3$

5. (a)Define the following

(i)Singular matrix (ii) Column equivalent matrices

(b) If A and B are $n \times n$ matrices, prove that $\det(AB) = (\det A)(\det B)$.

(c) Given the system of linear equations

$$\begin{cases} 2x + 2y + z = 5 \\ 3x - y + \lambda z = 2 \\ x + 5y - 2z = \mu \end{cases}$$

Find the conditions for which λ and μ must satisfy for the system of equations to have

(i)no solutions?

(ii) a unique solution?

(iii) Infinite solutions?.

Find all the solutions (in terms of λ and μ for infinite solutions) whenever possible.

d).Given that $D_n = 2D_{n-1} - D_{n-2}$ where $D_n = \begin{vmatrix} 2 & 1 & 0 & 0 \cdots 0 \\ 1 & 2 & 1 & 0 \cdots 0 \\ 0 & 1 & 2 & 1 \cdots 0 \\ \cdots & \cdots & \cdots & \cdots\cdots \\ 0 & 0 & 0 & 0 \; 1 \; 2 \end{vmatrix}$ and $D_n \in \det(M_n(K))$.

Evaluate D_5.

6. (a) Find the values of λ for which the system below will be consistent and for those values, find the complete solution.

$$\begin{cases} x+5y+3=0 \\ 5x+y-\lambda=0 \\ x+2y+\lambda=0 \end{cases}$$

(b)What is meant by row equivalent matrices?Hence show that matrices given below are row equivalent,

$$\begin{pmatrix} 1 & 0 & -1 & 1 \\ 2 & 1 & 0 & 1 \\ -2 & 2 & 0 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 0 & -1 & 1 \\ -1 & 1 & 0 & 2 \\ 0 & 3 & 0 & 5 \end{pmatrix}.$$

(c)Prove that a matrix A is non-singular if and only if its determinant is not equal to zero.

(d)Prove that every mxn matrix is row equivalent to an mxn reduced echelon matrix.

END OF EXAM

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2011 ACADEMIC YEAR
FIRST SEMESTER EXAMINATIONS

M261 INTRODUCTION TO STATISTICS

Time Allowed: Three (3) Hours

Instructions:
1. Answer any **Five (5)** Questions
2. Show All Essential Working
3. Statistical Tables will be provided
4. Calculators are Allowed

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1. (a) Define the following:
   (i) a parameter
   (ii) descriptive statistics

   (b) The number of hours that 50 employees of a certain organization spend in the office in a week is given in the following grouped cumulative distribution table:

Hours	Cumulative frequency
15 – 17	2
18 – 20	6
21 – 23	13
24 – 26	22
27 – 29	32
30 – 32	44
33 – 35	50

   (i) Construct a grouped frequency distribution table.
   (ii) Construct a histogram on the absolute frequency scale.
   (iii) Construct a frequency polygon on the relative frequency scale.
   (iv) Describe the distribution of the number of hours spent in the office.

   (c) The time taken to run a road race is normally distributed with a mean of 180 minutes and a standard deviation of 14 minutes.
   (i) Find the probability that a randomly selected runner completes the race between 170 and 215 minutes.
   (ii) If 11.9% of the runners complete the race in less than k minutes, find the value of k.

2. (a) Define the following:
   (i) a random sample
   (ii) a stratified sample

(b) The number of kilometres 30 taxi drivers cover on a given day are:

11	8	26	31	62	19	7	3	14	75
33	30	42	15	18	23	29	13	16	6
50	9	10	17	12	20	22	24	32	25

   (i) Prepare an ordered stem and leaf plot for the data.
   (ii) Construct a boxplot for the data and identify the outliers, if any.

(c) Delivery times for all food orders at a fast food restaurant during lunch hour are normally distributed with a mean of 8.4 minutes and a standard deviation of 1.8 minutes. Find the probability that the mean delivery time for a random sample of 16 such orders at this restaurant is
   (i) between 8 and 9 minutes.
   (ii) within 1 minute of the population mean.

3. (a) (i) Define and state two causes of response bias.
   (ii) Define a probability sample.

(b) A politician claims that the percentage of female voters exceeds that of male voters by more than 5%. A random sample of 500 female registered voters showed that 300 of them voted in the last presidential election. Another independent random sample of 400 male voters showed that 208 of them voted in the same election.
   (i) Construct a 98% confidence interval for the difference between the true proportions of female and male voters in the last election.
   (ii) Test the politician's claim at the 5% level of significance.

(c) A medical expert claims that the average age of first-time mothers in Zambia is 25 years or less. The standard deviation of the age of first-time mothers is known to be 4.8 years. Ministry of health officials take a random sample of 57 first-time mothers and decide the claim should be rejected if the mean age is greater than 27 years.
   (i) Explain how one can commit a type I error in this context.
   (ii) Find the probability of a type I error if the claim is true.
   (iii) Find the probability of a type II error if the true mean age of first-time mothers is 28 years.

4. (a) Define the following:
   (i) a statistical hypothesis
   (ii) a type II error

(b) A consumer agency wanted to find out if the mean time taken by each of three brands of medicine to provide relief from a headache is the same. The following table gives the time (in minutes) taken by each patient to get relief from a headache for patients randomly assigned to the three drugs.

Drug

1	2	3
25	15	44
38	21	39
42	19	54
65	25	58
47		73
52		

   (i) What design was used in the experiment?
   (ii) Write down a model for the above design. Explain all the terms in your model. State all the assumptions.
   (iii) Is the mean relief time the same for each of the three drugs. Use the $\alpha = 0.05$ level of significance.

(c) A sample of 13 cans of brand A diet soda gave a mean number of 23 calories with a standard deviation of 3 calories. Another independent sample of 11 cans of brand B diet soda had a mean of 25 calories with a standard deviation of 5 calories. Assume the amounts of calories of diet soda are normally distributed.
   (i) Is there a difference in the mean number of calories for the two brands. Carry out a test using a 5% level of significance assuming the variances of the two brands are equal.
   (ii) Construct a 90% confidence interval for the ratio of variances of the two brands.

5. (a) (i) State two properties of a normal distribution curve.
(ii) Define a sampling distribution.

(b) The following data give the experience (in years) and annual salaries (in millions of Kwacha) of 9 randomly selected employees of a certain organization.

Experience (x)	14	3	5	6	4	9	18	5	16
Annual salary (y)	62	29	37	43	35	60	67	32	60

(You may use the following summary statistics:

$\sum_{i=1}^{6} x_i = 80$, $\sum_{i=1}^{6} x_i^2 = 968$, $\sum_{i=1}^{6} y_i = 425$, $\sum_{i=1}^{6} y_i^2 = 21841$, $\sum_{i=1}^{6} x_i y_i = 4404$)

(i) Draw a scatter diagram. Comment on the pattern.
(ii) Estimate the simple linear regression equation.
(iii) Explain the meaning of the estimated parameters.
(iv) Copy and complete the following ANOVA table.

Source of variation	Sum of squares (SS)	Degrees of freedom (df)	Mean square (MS)	F*
Regression Error			 35.000	
Total				

(v) Is there a significant linear relationship between experience and annual salary? Use a 5% level of significance.
(vi) Compute a 95% confidence interval for the slope parameter.

6. (a) (i) Define the coefficient of determination.
(ii) State two properties of the correlation coefficient.

(b) Recent recession and bad economic conditions forced many people to hold more than one job to make ends meet. A sample of 500 persons who held more than one job produced the following two way table.

	Marital status		
Gender	Single	Married	Other
Male	65	230	25
Female	40	120	20

Test at the 10% level of significance whether gender and marital status are related for all people who hold more than one job.

(c) A computer company claims that the mean time to learn how to use its new software is not more than 2 hours. A random sample of 12 persons took the following times (in hours) to learn how to use the software:

1.75	2.25	2.40	1.90	1.50	2.75
2.15	2.25	1.80	2.20	3.25	2.60

(i) Test at the 1% level of significance whether the company's claim is true.

(ii) Construct a 98% confidence interval for the population standard deviation of times taken to learn the software. Is it significantly different from 0.5 hour? Explain.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2011 ACADEMIC YEAR
SECOND SEMESTER EXAMINATIONS

M292: INTRODUCTION TO PROBABILITY

Time Allowed: Three (3) Hours

Instructions:
1. Answer any **Five (5)** Questions
2. Show All Essential Working
3. Calculators are Allowed

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1. (a) State and prove Boole's Inequality.

(b) Suppose that 30% of bottles produced in a certain plant are defective. If a bottle is defective, the probability is 0.9 that an inspector will notice it and remove it from the filing line. If a bottle is not defective, the probability is 0.2 that the inspector will think it is defective and remove it from the line.

(i) If a bottle is removed from the line, what is the probability that it is defective?

(ii) If a customer buys a bottle that has not been removed from the line, what is the probability that it is defective?

(iii) If a customer buys one bottle at a time until two defective bottles are found, find the probability that he needs to buy 10 bottles.

(c) Let X and Y be continuous random variables with joint probability density function

$$f(x,y) = \frac{x}{5} + ky \quad , \quad 0 < x < 1 \quad , \quad 1 < y < 5$$

(i) Show that $k = \frac{1}{20}$.

(ii) Find $f_Y(y)$.

(iii) Find $E(X \mid y)$.

2. (a) (i) Define independence of two events A and B.

(ii) If $M_X(t)$ is the moment generating function of a random variable X, prove that the moment generating function of a random variable $Y = aX + b$ is given by $M_Y(t) = e^{tb} M_X(at)$ where a and b are constants.

(b) The integers 1, 2, 3, … , 9 are arranged in a row, resulting in a 9 – digit number . Find the probability that the

(i) resulting number is even.

(ii) resulting number is divisible by 5.

(iii) digits 4 and 6 are next to each other.

(c) Suppose that X and Y are discrete random variables with joint probability function

$$f(x, y) = c(x + y) \quad , \quad x = 0, 1, 2 \quad ; \quad y = 0, 1, 2$$

(i) Show that $c = \frac{1}{18}$.

(ii) Find the marginal probability functions $f_X(x)$ and $f_Y(y)$.

(iii) Find $\rho(X, Y)$.

3. (a) We are interested in the sequence of male and female births in families that consist of three children.

(i) List all the elements of the sample space.

(ii) Find the probability that the first child and the third child are male.

(iii) Given that a family has at least two female children, find the probability that the firstborn is male.

(b) A continuous random variable X has probability function

$$f(x) = \begin{cases} kx & , \quad 0 < x < 1 \\ \frac{k}{x^4} & , \quad x > 1 \\ 0 & , \quad otherwise \end{cases}$$

(i) Show that $k = \frac{6}{5}$.

(ii) Find E(X).

(iii) Find Var(X).

(iv) Find the cumulative distribution function of X.

(c) The joint moment generating function of two random variables X and Y is given by

$$M_{X,Y}(t_1, t_2) = e^{3 - 2e^{t_1} - e^{t_2}}$$

(i) Find the marginal moment generating functions of X and Y.

(ii) Are X and Y independent? Explain.

4. (a) Given that events A and B are such that $P(A) = 0.3$, $P(B) = 0.7$ and $P(A \cup B) = 0.9$, find

(i) $P(A \cap B)$

(ii) $P(A \cup B')$

(iii) $P(B \mid A')$

(b) Let X be a gamma random variable with probability density function

$$f(x) = \frac{\lambda}{\Gamma(r)} (\lambda x)^{r-1} e^{-\lambda x} \quad , \quad x > 0 \quad ; \quad \lambda > 0 \quad , \quad r > 0$$

(i) Show that the moment generating function of X is given by

$$M_X(t) = \left(\frac{\lambda}{\lambda - t} \right)^r \quad , \quad t < \lambda$$

(ii) Use the moment generating function in (i) to find E(X) and Var(X).

(c) A biased coin that shows heads with probability 0.15 is tossed. Find the probability that

(i) 5 heads are obtained in 15 tosses.

(ii) the first head is obtained on the 8^{th} toss.

(iii) at least 25 tosses are required to obtain the first head.

5. (a) Suppose X and Y are independent random variables with E(X) = 2, E(Y) = 3, Var(X) = 4 and Var(Y) = 16. Find

(i) E(2Y – X)

(ii) Var(5X – Y)

(b) A soldier who hits a target with probability $\frac{2}{3}$ fires shots until he hits the target or 5 shots have been fired. Let X denote the number of shots fired. Find the

(i) probability distribution of X.

(ii) mean of X.

(c) The length of phone calls at a public telephone booth follows an exponential distribution with a mean of 10 minutes. If someone arrives immediately ahead of you at a public telephone booth, find the probability that you will wait

(i) more than 10 minutes.

(ii) between 10 and 20 minutes.

(iii) an additional 15 minutes or more if the person is still on the phone 5 after you arrive minutes.

6. (a) (i) Define the joint moment generating function of random variables $X_1, X_2, \ldots, X_n$.

(ii) For jointly distributed random variables X and Y, prove that $Var(X) = E[Var(X \mid Y)] + Var[E(X \mid Y)]$

(b) Molecules of a rare gas occur at an average rate of 3 per cubic metre of air and follow a Poisson distribution.

(i) Find the probability that a cubic metre of air contains none of the molecules.

(ii) Find the probability that 3 cubic metre of air contain exactly 4 of the molecules.

(iii) The probability that at least one molecule is found in 0.99. Find how much air is taken as sample.

(c) The joint probability density function of random variables X and Y is given by

$$f(x, y) = e^{-(x+y)} \quad , \quad x > 0 \quad , \quad y > 0$$

(i) Find the marginal probability density functions $f_X(x)$ and $f_Y(y)$.

(ii) Are X and Y independent? Why or why not?

(iii) Find $P(X > 2)$.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

Department of Mathematics and Statistics

2011/2012 Academic Year

Semester II

M325 Group and Ring Theory

FINAL EXAMINATION

Time Allowed: Three (3) Hours **May, 2012.**

Instructions:

1. You must write your **Computer Number**, on each answer booklet you have used.
2. There are Seven (7) questions in this paper, Attempt **Any Five (5)** questions. All questions carry equal marks
3. Should you have any problem or if you need more answer booklet(s), put up your hand, an invigilator will come to attend to you.

1. (a) Let α and β be two integers.

(i) Define $\gcd(\alpha, \beta)$, the greatest common divisor of α and β.

(ii) Show that if c is a common divisor of α and β then c divides $\gcd(\alpha, \beta)$.

(iii) If $\alpha = 1365$ and $\beta = 1200$, find $\gcd(\alpha, \beta)$ and express it in the form $\alpha(s) + \beta(t)$ where s and t are integers.

(b) Let X be a set and $x, y \in X$

(i) Define an equivalence relation $x \equiv y$ on X.

(ii) Let G be a group acting on the set X as follows: x and y are related, $x \equiv y$ if there exists an element $g \in G$ such that $y = gx$. Show that this relation is an equivalence relation on X.

(c) Given that x is an integer such that $1 \le x \le 12$, find the solution set of x such that $x^2 \equiv 1 \bmod 15$.

2. (a) Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Define a function $\Phi : A \to A$ by $\Phi(n)$ = the remainder when $7n$ is divided by 9 for $n \in A$.

(i) Show that Φ is a permutation on A.

(ii) How many permutations in S_8 have the same cycle structure as Φ?

(iii) Determine the **order** and the **parity** of Φ.

(b) Let $G = S_3$ be a group of all permutations on three symbols. If $H = \langle(1 \;\; 2)\rangle$ and $K = \langle(1 \;\; 3)\rangle$ are subgroups of G;

(i) Find HK

(ii) Determine with reason if HK is a subgroup of G

(iii) Find all the distinct cosets of K in G.

(c) Prove that if H is a subgroup of index 2 in a group G, then $g^2 \in H$ for every $g \in G$.

3. (a) Let G be a group and $x \in G$.

(i) Define the center of G

(ii) Define the centralizer of $x \in G$.

(iii) Prove that the centralizer of $x \in G$, $C_G(x)$, is a subgroup of G.

(b) (i) Define a normal subgroup of a group G

(ii) Let H and K be any groups and let $G = H \times K = \{(h, k) : h \in H, k \in K\}$ be a group with the operation defined by: $(h_1, k_1)(h_2, k_2) = (h_1h_2, k_1k_2)$ for (h_1, k_1) and (h_2, k_2) in G. If $\overline{H} = \{(h, 1) : h \in H\}$, show that $\overline{H}$ is a normal subgroup of G.

(c) Let $H = \langle(1 \;\; 2 \;\; 3)\rangle$ be a subgroup of S_3.

(i) List all the elements of H

(ii) Show that H is a normal subgroup of S_3

4. (a) Let $(G, *)$ and $(H, \circ)$ be two groups with the given operations $*$ and $\circ$ respectively.

(i) Define a homomorphism $\varphi: G \to H$

(ii) Define the kernel and the image of φ.

(iii) If G is the multiplicative group of all non zero complex numbers, define a mapping φ by $\varphi(a+ib) = a^2 + b^2$. Show that φ is a homomorphism and find its kernel.

(b) (i) State Lagrange Theorem.

(ii) Let G be a finite group with subgroups H and K. If further, H is a subgroup of K, show that $[G:H] = [G:K][K:H]$.

(c) Let G be a group of order 4 such that $x^2 \neq 1$ for each non identity element $x \in G$. Show that G is a cyclic group.

5. (a) (i) Define a ring homomorphism

(ii) Define an **ideal** of a commutative ring $\Re$.

(iii) Let A and $\Re$ be two commutative rings, and let $f: A \to \Re$ be a ring homomorphism. Prove that the kernel of f is a proper ideal in A.

(b) (i) State the First Isomorphism Theorem.

(ii) Let $S^1 = \{z \in \mathbf{C} : |z| = 1\}$ with the operation on S^1 being multiplication of complex numbers. Let **R** be the additive group of real numbers. Define $f: \mathbf{R} \to S^1$ by $f(x) = e^{2\pi ix}$, $x \in \mathbf{R}$. Show that $\mathbf{R}/\mathbf{Z}$ is isomorphic to S^1, that is $\mathbf{R}/\mathbf{Z} \cong S^1$ where **Z** is the set of integers.

(Note $e^{ix} = \cos x + i \sin x$)

(c) Let G be a group of order 63. Show that G contains a normal subgroup and determine its order.

6. (a) (i) Define a subring of a commutative ring $\Re$.

(ii) Let $Z[i] = \{z \in \mathbf{C} : z = a + ib, a, b \in \mathbf{Z}\}$. Show that $Z[i]$ is a subring of **C**, the ring of complex numbers.

(b) (i) Define an integral domain.

(ii) Prove that if D is an integral domain, and $a, b \in D$ are non zero elements such that $a \mid b$ and $b \mid a$, then there is a unit $u \in D$ such that $b = ua$.

(iii) Show that Z_{21} is not an integral domain.

(c) Show that Z_5 is a field.

7. (a) (i) State Eisenstein Criterion for irreducibility.

(ii) State Gauss Theorem for irreducibility.

(iii) Determine whether $h(x) = x^4 + 15x^3 + 7$ is irreducible in $Q[x]$.

(b) Let $f(x) = x^4 + x^2 + 1$.

(i) Find the roots of $f(x)$ in Z_3.

(ii) Hence obtain the factorization into irreducibles of $f(x) = x^4 + x^2 + 1$ in $Z_3[x]$.

(c) Consider the polynomial $\Phi_5(x) = \dfrac{x^5 - 1}{x - 1} = x^4 + x^3 + x^2 + x + 1$. Show that $f(x) = \Phi_5(x+1)$ is irreducible in $Q[x]$.

End of Exam.

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

Department of Mathematics & Statistics

SECOND SEMESTER FINAL EXAMINATIONS

May, 2012

M412—FUNCTIONS OF A COMPLEX VARIABLE II

Time allowed : THREE(3) HOURS

Instructions : There are six(6) questions. Answer **ANY FIVE (5)** questions. All questions carry equal marks. Show all your working to earn full marks.

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1. (a) State and prove Liouville's theorem.

   (b) Find the Laurent series expansion of

   $$f(z) = \frac{1}{z^2(1-z)}$$

   in the annular domain $1 < |z| < \infty$.

   (c) (i) State the Mean Value theorem.

   (ii) Hence, considering the function $f(z) = \sin z$ on the unit circle, show that

   $$\int_0^{2\pi} \cos(\cos\theta)\sinh(\sin\theta)\mathrm{d}\theta = 0.$$

2. (a) State the Minimum Modulus Principle.

(b) Find the minimum value of $|f(z)| = |z^2 + 2|$ over the closed region $|z| \leq 1$.

(c) In each case below write the principal part of the function at its isolated singularity. Then determine if that singularity is a pole, an essential singularity, or a removable singularity.

(i) $f(z) = \frac{1}{z}e^z$. (ii) $f(z) = \frac{\cos z}{z}$.

(iii)

$$f(z) = \frac{1}{1 + z + z^2 + z^3 + z^4 + z^5}.$$

3. (a) Without using the Residue theorem, by integrating $f(z) = \frac{ze^{iz}}{z^2+a^2}$ around a suitable contour, prove that

$$\int_0^\infty \frac{x \sin x}{x^2 + a^2} dx = \frac{\pi}{2e^a}, \quad \text{where} \quad a > 0.$$

(b) Let C denote the circle $|z| = 2$ described in the positive sense. Determine the value of $\Delta_c \arg f(z)$ for the function

$$f(z) = \frac{z - (\frac{1}{2} + \frac{i\sqrt{3}}{2})}{z^3(z - 2\pi)^2}.$$

(c) Find the maximum value of

$$|f(z)| = |z^2 + 3z - 1|$$

on the unit disk $|z| < 1$.

4. (a) State and prove the Cauchy Inequality theorem.

(b) Verify the Cauchy Inequality theorem for the function $f(z) = \frac{1}{2z+1}$ if z lies on the circle $|z - 3| = 2$.

(c) Prove that

$$\int_{-\infty}^{\infty} \frac{x}{x^3 - 8} dx = \frac{\pi\sqrt{3}}{6}.$$

5. (a) (i) State Rouche's theorem.

(ii) Using Rouche's theorem, show that the roots of the equation $z^4 + 6z + 1 = 0$ lie within the circle $|z| < 2$ but one root lies inside the circle $|z| < \frac{3}{2}$.

(b) Evaluate

$$\int_C \frac{f'(z)}{f(z)} dz$$

if C is the circle $|z| = 3\pi$ for

$$f(z) = \frac{\sqrt{2}\sin z - 1}{(z-1)^2(z+5)}.$$

(c) (i) Find the Laurent series expansion of $f(z) = \frac{e^{-z}}{(z-2)^4}$ in the domain $0 < |z-2| < R$, for arbitrarily large R.

(ii) Hence find the residue of $f(z)$ at its singularity.

6. (a) State the residue theorem.

(b) Using the residue theorem, evaluate

$$\int_C \frac{1}{z(2z-5)(z-4)} dz,$$

where $C = \{z : \ |z+2| + |z-2| = 6\}$, positively oriented.

(c) Locate and identify the singularities of:

$$(i) \quad f(z) = \frac{1+z}{\cos z - 2} \qquad (ii) \quad f(z) = \frac{1}{1-e^{-z}}.$$

END.

The University of Zambia

School of Natural Sciences

Department of Mathematics & Statistics

2011 Academic Year Second Semester Final Examinations

M422 - Module and Field Theory

Time allowed : **Three (3) hours** Full marks : 100

Instructions:
- There are **six (6)** questions in this paper.
- Attempt **any five (5)** questions. **All** questions carry **equal** marks.
- Indicate your **computer number** on all answer booklets.

This paper consists of 3 pages of questions.

SECTION A - MODULE THEORY

1. **a)** Let R be a ring with 1; define the following:

   i) a left R-**module**;

   ii) a **torsion submodule** of an R-module M.

   b) i) Let $I \subset R$ be an ideal and $J \subset I$ be any ideal. Define $(r+J)m = rm$; for all $r \in I$ and $m \in M$. Show that M is a left I/J-module.

   ii) Let A and B be submodules of an R-module M. Show that their sum, defined by

   $$A + B = \{a + b | a \in A; b \in B\},$$

   is a submodule of M.

   c) Let R be an integral domain and let M be an R-module. Prove that

   i) the set of torsion elements M_r of M is a submodule of M;

   ii) M/M_r is torsion free.

2. **a)** What is the meaning of each of the following:

   i) M is a **free** R-module;

   ii) x is a **primitive element** of an R-module M?

   b) i) Show that $\mathbb{Q}$ is a torsion free $\mathbb{Z}$-module that is not free.

   ii) Let R be an integral domain and let M be a free R-module. Show that M is torsion free.

   c) i) Let R be a PID and let M be a free R-module with basis $S = \{x_j\}_{j\in J}$. If $x = \sum_{j\in J} a_j x_j \in M$, show that x is primitive if and only if $gcd(\{a_j\}_{j\in J}) = 1$.

   ii) Let M be a finite free R-module over a PID R. Show that every basis of M has $\mu(M)$ elements; where $\mu(M)$ is the rank of M.

3. **a)** Give the meaning of the following:

   i) an R-module **homomorphism** ;

   ii) M is a **direct sum** of its R-submodules $M_1, M_2, \ldots, M_n$.

   b) i) Let $f_i : M_i \to N;\ 1 \le i \le n$ be R-module homomorphisms and let $f : M_1 \oplus \cdots \oplus M_n \to N$ be defined by $f(x_1, \ldots, x_n) = \sum_{i=1}^{n} f_i(x_i)$. Show that f is an R-module homomorphism.

   ii) Let M be a direct sum of $M_1, M_2, \ldots, M_n$. Show that $M \cong M_1 \oplus M_2 \oplus \cdots \oplus M_n$.

   c) Let $f : M \to N$ and $g : N \to M$ be R-module homomorphisms such that $fg(y) = y$ for all $y \in N$. Prove that $M \cong Ker(f) \oplus Im(g)$.

SECTION B - FIELD THEORY

4. **a)** Define the following:

   i) an **irreducible polynomial** $f(x)$ over a field $\mathbb{E}$;

   ii) an **algebraic element** over $\mathbb{E}$.

   b) i) By considering the factorisation $x^4 + 2 = (x^2 + ax + b)(x^2 + cx + d) \in \mathbb{Z}_5[x]$; where $\mathbb{Z}_5 = \{0, 1, 2, 3, 4\}$ or otherwise, show that $x^4 + 2$ is irreducible over $\mathbb{Z}_5$.

   ii) Let $f(x) \in \mathbb{E}[x]$ with root α. Show that α is a multiple root if and only if $f'(x) = 0$.

   c) i) Show that $\alpha = \sqrt{1 + \sqrt[3]{2}}$ is algebraic over $\mathbb{Q}$.

   ii) Hence, find the basis for the extension $\mathbb{Q}(\sqrt{1 + \sqrt[3]{2}}) : \mathbb{Q}$.

5. **a)** What is meant by each of the following:

   i) a **Galois group** of the field extension $\mathbb{F} : \mathbb{E}$;

   ii) a **splitting field** of a polynomial $f(x)$ over the field $\mathbb{E}$?

   b) Find the Galois groups of the following:

   i) an extension $\mathbb{Q}(\sqrt{2}, \sqrt{5}) : \mathbb{Q}$;

   ii) a polynomial $f(x) = x^4 + x^2 - 6$ over $\mathbb{Z}_7$.

   c) Construct splitting fields for

   i) $f(x) = (x^2 - 3)(x^3 + 1)$ over $\mathbb{Q}$;

   ii) $f(x) = x^4 + 3x^2 - 18$ over $\mathbb{Z}_7$.

6. **a)** Give a meaning to each of the following:

   i) a **radical field extension** $\mathbb{F} : \mathbb{E}$;

   ii) a polynomial f is **solvable by radicals**.

   b) i) Show that the Galois group of a cyclotomic field of order n over $\mathbb{E}$ with $Char\mathbb{E} = 0$ is abelian.

   ii) Let $\mathbb{F} : \mathbb{E}$ be an extension and $f(x) \in \mathbb{E}[x]$. Prove that the Galois group of $f(x)$ over $\mathbb{F}$ is isomorphic to a subgroup of the Galois group of $f(x)$ over $\mathbb{E}$.

   c) Show that the following are solvable by radicals:

   i) $f(x) = x^7 - 5 \in \mathbb{Q}[x]$;

   ii) a quadratic polynomial $p(x) \in \mathbb{Q}[x]$.

END OF THE EXAMINATION!

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

Department of Mathematics & Statistics

FIRST SEMESTER FINAL EXAMINATIONS

2nd December, 2011.

M431—Real Analysis V

Time allowed : THREE(3) HOURS

Instructions : There are seven(7) questions. Answer **ANY FIVE (5)** questions. All questions carry equal marks. Show all your working to earn full marks.

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1. (a) Define a separable space.

   (b) Prove that the set of real numbers, $\mathbb{R}$, is a separable space.

2. (a) Let X be a metric space and $A \subset X$.

   (i) Define the interior of A.

   (ii) Define the closure of A.

   (iii) When is A said to be nowhere dense?

   (b) (i) Give an example of a nowhere dense set.

   (ii) Show that the set in 2.(b)(i) is nowhere dense.

3. (a) Define the following:

   (i) l^p and $||x||_p$, for $1 \leq p < \infty$ and $x \in l^p$.

   (ii) l^∞ and $||x||_\infty$, for $x \in l^\infty$.

   (b) If $1 \leq p < q < \infty$ show that

   (i) $l^p \subset l^q$.

(ii) $l^p \subset l^\infty$.

(c) If $x = \{a_1, a_2, a_3, \ldots, a_N, 0, 0, \ldots\}$, show that

$$\|x\|_\infty = \lim_{p\to\infty} \|x\|_p.$$

4. (a) Let $1 \le p < \infty$.

(i) If $a, b \in \mathbb{C}$, prove that $|a+b|^p \le 2^{p-1}(|a|^p + |b|^p)$.

(ii) If $B = \{x \in l^p : \|x\|_1 \le 1\}$, show that $\sup\{\|x-y\|_p : x, y \in B\} = 2$.

(b) Give an example of a sequence in l^3 that is not in l^2, showing it is in l^3 but not in l^2.

5. (a) Let (X, d_x) and (Y, d_y) be metric spaces and $f : X \to Y$ be a function between them. Define the following:

(i) f is Lipschitz continuous.

(ii) f is a contraction.

(iii) f is uniformly continuous on a compact subset A of X.

(b) (i) Let $X = Y = [1, \infty)$ and let X and Y both have the metric induced by the Euclidean metric on $\mathbb{R}$. For each fixed $k > \frac{1}{2}$, define the function $f_k : X \to Y$ by $f_k(x) = k(\frac{1}{x} + x)$.

Prove that f_k is uniformly continuous on X.

(ii) Let $X = Y = (0, \infty)$, $d_x = d_y$ both be metrics induced by the usual metric on $\mathbb{R}$, and let $f(x) = \frac{1}{x}$ for $x \in X$.

Prove that f is not uniformly continuous on X.

(c) Let z be a fixed point in the metric space (X, d). Define the function $f : X \to \mathbb{R}$ by $f(x) = d(x, z)$. Show that f is uniformly continuous.

6. (a) Let (X, d_x) and (Y, d_y) be metric spaces. Define the following:

(i) A subset B of X is sequentially compact.

(ii) A subset B of X is totally bounded.

(b) (i) Show that every totally bounded set is bounded.

(ii) Show that the set $B = \{\{x_n\} \in l^2 : \|\{x_n\}\|_2 \leq 1\}$ is bounded but not totally bounded.

(c) State, with reasons, whether the set

$$A = \{(x, y) \in \mathbb{R}^2 : \frac{x^2}{4} + \frac{y^2}{9} = 4\}$$

is compact or not.

7. (a) Let (X, d_x) and (Y, d_y) be metric spaces and $f : X \to Y$ be a bijection. Define the following:

(i) f is a homeomorphism.

(ii) f is an isometry.

Let X and Y be metric spaces and $f : X \to Y$ be a continuous function.

(b) If A is a compact subset of X, show that $f(A)$ is a compact subset of Y.

(c) If X is a compact metric space, Y any metric space and $f : X \to Y$ a continuous bijection, prove that f is a homeomorphism.

END.

The University of Zambia

School of Natural Sciences

Department of Mathematics & Statistics

2011 Academic Year Second Semester Final Examinations

M912 - Mathematical Methods VI

Time allowed : **Three (3) hours** Full marks : 100

Instructions:
- There are **SIX (6)** questions in this paper.
- Attempt **any FIVE (5)** questions. **All** questions carry **equal** marks.
- Indicate your **computer number** on all answer booklets.
- **Calculators** are **not** allowed

This paper consists of 3 pages of questions.

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1. **(a)** Find $\int_T (x^3y + \cos x)dA$, where T is the triangle consisting of all points (x, y) such that $0 \leq x \leq \pi/2$, $0 \leq y \leq x$. Sketch the region T.

   (b) Find the volume V of the solid in the first octant under the graph of $z = 3 - \sqrt{x^2 + y^2}$ and inside the right cylinder having equation $(x-1)^2 + y^2 = 1$. Sketch the solid.

   (c) **i.** Evaluate $\int_0^1 \int_{3y}^3 e^{x^2} dxdy$.

   ii. without proof, state the Mean Value Theorem for Double Integral.

2. **(a)** **i.** Evaluate the double integral $\int_D (a^2 - y^2)^{\frac{1}{2}} dA$, where D is the set of points (x, y) such that $0 \leq x \leq a$ and $0 \leq y \leq (a^2 - x^2)^{\frac{1}{2}}$

   ii. Find the volume of one of the wedges cut from the cylinder $4x^2 + y^2 = a^2$ by the planes $z = 0$ and $z = my$.

   (b) State and prove Fubini's Theorem of Double Integrals.(Prove one way)

   (c) **i.** Let R be the rectangle $[-2, 1] \times [0, 1]$ and let $f(x, y) = y(x^3 - 12x)$. Evaluate $\int_R f(x, y)dA$.

ii. Evaluate

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{dydx}{\sqrt{1-x^2-y^2}}.$$

3. (a) Given

$$I = \int_0^1 \int_0^{(1-x^2)} \int_0^{(1-x)} dydzdx.$$

Sketch the volume represented by the integral I and evaluate I using the order $dzdxdy$.

(b) Given $f(x) = k,\ (0 < x < 3)$

i. Sketch the graph of f and the corresponding periodic odd extension, where k is a negative constant. Hence find the Fourier sine series of f in the given interval.

ii. Given that $\sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$, use the series in (i) to find the value of

$$1 - \frac{1}{5} + \frac{1}{7} - \frac{1}{11} + \frac{1}{13} - \frac{1}{17} + . - \ldots$$

(c) Given

$$\mathbf{F}(x, y, z) = y^2\mathbf{i} + (2xy + e^{3z})\mathbf{j} + 3ye^{3z}\mathbf{k}.$$

i. Find a function f such that $\nabla f = \mathbf{F}$

ii. Evaluate the line integral $\int_C \mathbf{F}.d\mathbf{r}$ where C is the straight line from $(1, 1, 0)$ to $(2, 3, 1)$.

4. (a) Let $T(u, v) = [x(u, v), y(u, v)]$ be the mapping defined by $T(u, v) = [4u, 2u + 3v]$. Let D^* be the rectangle $[0, 1] \times [1, 2]$.

i. Find $D = T(D^*)$.

ii. Evaluate $\int\int_D (x-y)dxdy$ by making a change of variables to evaluate the integral over D^*.

(b) Use the definition of Laplace transform to find the laplace transform of $f(t) = \cosh 2t$. State the range of values of s for which the transform exists.

(c) Given the solid Q which is the first octant portion of the cylinder $y^2 + z^2 = 9$ between $x = 0$ and $x = 4$.

i. Sketch Q.

ii. Given that S is the surface of Q, write S in parametric form and use your result to evaluate $\int\int_S (x + z)dS$.

5. (a) Given $f(x) = \begin{cases} -x, & \text{if } -\pi < x < 0 \\ x, & \text{if } 0 < x < \pi \end{cases}$
assumed to have period 2π.

i. Sketch the graph and obtain the Fourier series of f.

ii. Use the result in (i) to find the value of

$$1+\frac{1}{9}+\frac{1}{25}+\frac{1}{49}+\frac{1}{81}+\dots$$

(b) Use Green's theorem to evaluate $\int_C (y+x)dx+(2x-y)dy$ where C is the boundary of the region lying between the graphs of $y=x$ and $y=x^2-x$.

(c) Use the first shifting theorem $\left(i.e,\ \mathrm{L}\{e^{at}f(t)\} = F(s-a), \text{where } F(s) = \mathrm{L}\{f(t)\}\right)$ to show that

$$\mathrm{L}(\cosh t \sin t) = \frac{s^2+2}{s^4+4}$$

6. **(a)** Given the function $f(x)=e^{-|x|}$, $-1<x<1$.

i. Sketch the graph of f.

ii. Find the fourier integral of $f(x)$.

(b) Let Q be the solid region bounded by the coordinate planes and the plane $2x+2y+z=6$ and let $\mathbf{F}(x,y,z)=x\mathbf{i}+y^2\mathbf{j}+z\mathbf{k}$.

i. Sketch the region Q.

ii. Evaluate $\int\int_{\mathbf{S}} \mathbf{F}\cdot\hat{\mathbf{n}}\,d\mathbf{S}$, where $\mathbf{S}$ is the surface of Q.

(c) Given $\mathbf{F}(x,y,z)=(y^3+1)\mathbf{i}+(3xy^2+1)\mathbf{j}$ and C is the semi-circular path from $(0,0)$ to $(3,0)$. Show that $I=\int_C \mathbf{F}\cdot d\mathbf{r}$ is independent of path. Hence evaluate I.

THE END

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS

2011/2012 ACADEMIC YEAR
SECOND SEMESTER FINAL EXAMINATIONS

M962: TIME SERIES ANALYSIS

TIME ALLOWED : Three (3) Hours

INSTRUCTIONS : There are six questions in this paper. Answer any five questions. All questions carry equal marks.

Every where in this question paper, a_t, t=0, 1, 2 ,….. represents a zero mean, normal random process with variance σ_a^2.

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1. (a) The following data represents number of cars imported from Japan in Zambia during the last six months.

Month	Cars
November	200
December	135
January	195
February	197
March	310
April	175

(i) Construct exponential smoothing forecast using $\alpha = 0.5$ and 0.9 and $\hat{X}_1 = X_1$.

(ii) Compute SSE for each set of forecast and state which value of α is optimal.

(iii) Make a forecast for May.

(b) The annual profit of Konkola Copper Mines in the last five years is given in the following table:

Year	Profit (Billion Kwacha)
2007	20
2008	42.5
2009	95.1
2010	207.8
2011	414

(i) Suggest which of the two trend equations will be appropriate

$$\text{Profit} = a + b\,(\text{year})$$
or
$$\text{Profit} = ab^{\text{year}}$$

(ii) Determine the trend equation you have suggested in part (i)

(Restrict calculations to 1dp).

(iii) Project profit for 2012.

(c) The quarterly sales (Billion Kwacha) at Manda Hill Shoprite since the first quarter of 2010 is given below.

Year	Q1	Q2	Q3	Q4
2010	78	63.2	80	95
2011	102.7	97.1	85.2	105.8
2012	99.2			

Compute the quarterly centered moving averages.

2. (a) Define the following:

(i) A stochastic process
(ii) A time series
(iii) A covariance stationary stochastic process

(b) Find the mean, variance and covariance functions of the following processes:

(i) $Y_t = a_t a_{t-1}$
(ii) $Y_t = a_t + a_{t-1}$

(c) Determine which of the following process are covariance stationary:

(i) $X_t = \sum_{i=1}^{t} a_i$

(ii) $X_t = A + Bt + a_{t-1}$, A and B are real constants

(iii) $X_t = (-1)^t a_t$

3. (a) (i) The first ten autocorrelations of a time series of 50 values were computed as follows:

Time lag :	1	2	3	4	5	6	7	8	9	10
Auto correlation:	-0.15	-0.01	-.04	.275	.025	-.18	-.03	-.04	-.09	.14

Test at 5% level of significance if the time series can be modeled as a random process.

(ii) The first seven autocorrelations and partial autocorrelations of a time series of 364 values are given in the following table:

Lag K	1	2	3	4	5	6	7
$\hat{\rho}_K$	.73	.49	.30	.20	.02	−.01	−.03
$\hat{\varphi}_{kk}$	·73	−·09	−·04	·04	−·03	−·12	·07

Identify the significant autocorrelations and partial autocorrelations and fit an appropriate ARMA model.

(iii) The value of statistic $Q = n\sum_{j=1}^{10} r_j^2$ for a time series of length 200 is found to be 26.55. Test at 0.05 level of significance, $H_o : \rho_1 = \rho_2 = \ldots = \rho_{10} = 0$ against H_a : at least one autocorrelation is not zero.

(b) Let $\{Z_t\}$ be a Stochastic process described by

(i) $\dot{Z}_t = 0.75\ \dot{Z}_{t-1} - 0.5\,\dot{Z}_{t-2} + a_t$. Show that $\{Z_t\}$ is stationary.

(ii) Find the first autocorrelation of $\{Z_t\}$.

(c) The first ten partial autocorrelations of a time series of 250 values are given below.

K	1	2	3	4	5	6	7	8	9	10
$\hat{\phi}_{KK}$	.67	−.45	−.04	−.08	.05	−.01	.03	−.01	−.04	−.1

Discuss with appropriate tests of hypothesis if these sample PACF indicate an AR(2) process.

4. (a) Show that for the stationary process

$Z_t - \phi Z_{t-1} = a_t - .4\, a_{t-1}$

(i) $\gamma_0 = \frac{(1.16-0.8\,\phi)\sigma_a^2}{1-\phi^2}$

(ii) $\gamma_1 = \frac{(-.4\,\phi^2+1.16\phi-.4)\sigma_a^2}{1-\phi^2}$

(b) Assume the process in part (a) was fitted into a time series for which $\widehat{\gamma_0} = 10$ and $\hat{\rho}_1 = 0.5$. Find initial estimates for ϕ and σ_a^2.

(c) Show that the process $\{X_t\}$ defined by $X_t = a_t + \frac{1}{2}(a_{t-1} + a_{t-2} + \ldots)$ is non stationary. Identify the process represented by the series of the first differences of $\{X_t\}$. Comment on the invertibility for the process of the first differences.

5. (a) Given the process $Z_t = \mu + a_t + \lambda a_{t-1}$

(i) Find the values of λ for which the process is invertible.

(ii) Find the autocorrelation function of the process.

(iii) If ρ_k is the k th autocorrelation coefficient of the process, show that

$$|\rho_k| < \cdot 5$$

(b) Find an invertible process which has the following autocorrelation function:
$\rho_o = 1, \rho_1 = -\frac{1}{2.9}$ and $\rho_K = 0 \; for \; K \geq 2$

(c) Consider the process $(1 - B)\, Z_t = (1 - \cdot 4B)\, a_t$

(i) Find the π weights for the following AR representation of the process.

$Z_t = (\pi_1 Z_{t-1} + \pi_2 Z_{t-2} + \ldots) + a_t$

(ii) Show that $\sum_{j=1}^{\infty} \pi_j = 1$

(iii) Suppose we have observations $Z_{80} = 62.6$, $Z_{79} = 70.4$, $Z_{78} = 64.7$, $Z_{77} = 58.9$ and $Z_{76} = 60.4$. Forecast Z_{81} and Z_{82} as a weighted average of previous observations.

6. (a) Consider a process $\{Z_t\}$ defined by $Z_t = a_t + \psi_1 a_{t-1} + \psi_2 a_{t-2} + \ldots$.
Let l step ahead forecast of Z_{n+l} made at time point $t = n$ be

$\widehat{Z_n(l)}$ where $\widehat{Z_n(l)} = \psi_l\, a_n + \psi_{l+1}\, a_{n-1} + \psi_{l+2}\, a_{n-2} + \cdots$.

(i) Show that $Z_{n+l} - \widehat{Z_n(l)} = a_{n+l} + \psi_1\, a_{n+l-1} + \ldots + \psi_{l-1}\, a_{n+1}$

(ii) Show that $\widehat{Z_n(l)}$ is an unbiased estimator of Z_{n+l}

(iii) Show that $V(\widehat{Z_n(l)}) = \sigma_a^2 \sum_{j-o}^{l-1} \psi_j^2$

(b) The stationary process

$Y_t - 4.77 = 1.54\,(Y_{t-1} - 4.77) - 0.67\,(Y_{t-2} - 4.77) + a_t$

was fitted to the annual time series of Zambia bituminous coal production in million tons from 1980 to 2010. The last four values in the time series are given below:

Year :	2010	2009	2008	2007
Y_t (million tons) :	5.3	5.53	5.77	5.8

(i) Forecast coal production for years 2011, 2012 and 2013.

(ii) Find 95% forecast limits for the forecast in (i).

(iii) Update forecasts given that coal production in the year 2011 is 4.97 million tons.

(You may use the following updated forecast equation:
$\widehat{Z_{n+1}(l)} = Z_n\,\widehat{(l+1)} + \psi_l\,(Z_{n+1} - Z_n\,\widehat{(1)})$

(c) The following data represents first ten values of a stationary time series:

5, 1, 4, −4, 3, −3, 7, 0, −1, 8

Find the first three sample autocorrelations.

End of Examination

TABLE A2 Chi-Squared Distribution[a]

	$p = 0.750$	0.900	0.950	0.975	0.990	0.995	0.999
$k = 1$	1.323	2.706	3.841	5.024	6.635	7.879	10.83
2	2.773	4.605	5.991	7.378	9.210	10.60	13.82
3	4.108	6.251	7.815	9.348	11.34	12.84	16.27
4	5.385	7.779	9.488	11.14	13.28	14.86	18.47
5	6.626	9.236	11.07	12.83	15.09	16.75	20.51
6	7.841	10.64	12.59	14.45	16.81	18.55	22.46
7	9.037	12.02	14.07	16.01	18.48	20.28	24.32
8	10.22	13.36	15.51	17.53	20.09	21.96	26.13
9	11.39	14.68	16.92	19.02	21.67	23.59	27.88
10	12.55	15.99	18.31	20.48	23.21	25.19	29.59
11	13.70	17.28	19.68	21.92	24.73	26.76	31.26
12	14.85	18.55	21.03	23.34	26.22	28.30	32.91
13	15.98	19.81	22.36	24.74	27.69	29.82	34.53
14	17.12	21.06	23.68	26.12	29.14	31.32	36.12
15	18.25	22.31	25.00	27.49	30.58	32.80	37.70
16	19.37	23.54	26.30	28.85	32.00	34.27	39.25
17	20.49	24.77	27.59	30.19	33.41	35.72	40.79
18	21.60	25.99	28.87	31.53	34.81	37.16	42.31
19	22.72	27.20	30.14	32.85	36.19	38.58	43.82
20	23.83	28.41	31.41	34.17	37.57	40.00	45.32
21	24.93	29.62	32.67	35.48	38.93	41.40	46.80
22	26.04	30.81	33.92	36.78	40.29	42.80	48.27
23	27.14	32.01	35.17	38.08	41.64	44.18	49.73
24	28.24	33.20	36.42	39.37	42.98	45.56	51.18
25	29.34	34.38	37.65	40.65	44.31	46.93	52.62
26	30.43	35.56	38.89	41.92	45.64	48.29	54.05
27	31.53	36.74	40.11	43.19	46.96	49.64	55.48
28	32.62	37.92	41.34	44.46	48.28	50.99	56.89
29	33.71	39.09	42.56	45.72	49.59	52.34	58.30
30	34.80	40.26	43.77	46.98	50.89	53.67	59.70
40	45.62	51.81	55.76	59.34	63.69	66.77	73.40
50	56.33	63.17	67.50	71.42	76.15	79.49	86.66
60	66.98	74.40	79.08	83.30	88.38	91.95	99.61
70	77.58	85.53	90.53	95.02	100.4	104.2	112.3
80	88.13	96.58	101.9	106.6	112.3	116.3	124.8
90	98.65	107.6	113.1	118.1	124.1	128.3	137.2
100	109.1	118.5	124.3	129.6	135.8	140.2	149.4
z_p	0.675	1.282	1.645	1.960	2.326	2.576	3.090

For $k > 100$ use the approximation $w_p = (\frac{1}{2})(z_p + \sqrt{2k-1})^2$, or the more accurate $w_p = k\left(1 - \frac{2}{9k} + z_p\sqrt{\frac{2}{9k}}\right)^3$, where z_p is the value from the standardized normal distribution shown in the bottom of the table.

SOURCE: Abridged from Table 8, Vol. I of Pearson and Hartley (1976), with permission from the *Biometrika* Trustees.

[a] The entries in this table are quantiles w_p of a chi-squared random variable W with k degrees of freedom, selected so $P(W \le w_p) = p$ and $P(W > w_p) = 1 - p$.

TABLE III

The Normal Distribution

$$\Pr(X \le x) = N(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-w^2/2}\, dw$$

$$[N(-x) = 1 - N(x)]$$

x	N(x)	x	N(x)	x	N(x)
0.00	0.500	1.10	0.864	2.05	0.980
0.05	0.520	1.15	0.875	2.10	0.982
0.10	0.540	1.20	0.885	2.15	0.984
0.15	0.560	1.25	0.894	2.20	0.986
0.20	0.579	1.282	0.900	2.25	0.988
0.25	0.599	1.30	0.903	2.30	0.989
0.30	0.618	1.35	0.911	2.326	0.990
0.35	0.637	1.40	0.919	2.35	0.991
0.40	0.655	1.45	0.926	2.40	0.992
0.45	0.674	1.50	0.933	2.45	0.993
0.50	0.691	1.55	0.939	2.50	0.994
0.55	0.709	1.60	0.945	2.55	0.995
0.60	0.726	1.645	0.950	2.576	0.995
0.65	0.742	1.65	0.951	2.60	0.995
0.70	0.758	1.70	0.955	2.65	0.996
0.75	0.773	1.75	0.960	2.70	0.997
0.80	0.788	1.80	0.964	2.75	0.997
0.85	0.802	1.85	0.968	2.80	0.997
0.90	0.816	1.90	0.971	2.85	0.998
0.95	0.829	1.95	0.974	2.90	0.998
1.00	0.841	1.960	0.975	2.95	0.998
1.05	0.853	2.00	0.977	3.00	0.999

The University of Zambia
Physics Department
University Examinations 2011/12
P-192: Introductory Physics- II
(Option A)

All questions carry equal marks. The marks are shown in brackets. Question 1 is compulsory. Attempt four more questions. Clearly indicate on the answer script left column on the cover page the questions you have answered.

Time : Three hours. **Maximum marks = 100.**

Do not forget to write your computer number clearly on the answer book as well as on the answer sheet for Question 1. Tie them together.

==

Wherever necessary use :

$g = 9.8 m/s^2$

1 metric ton = 1000 kg

$P_A = 1.01 \times 10^5\ N/m^2$

1 cal. = 4.18 J

$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$

$c = 3 \times 10^8$ m/s

$h = 6.63 \times 10^{-34}$ J-s

1 pascal =1 N/m^2

1eV = 1.6×10^{-19} J

$e = 1.6 \times 10^{-19}$ C

$m_e = 9.11 \times 10^{-31}$ kg

$k = 9 \times 10^9\ Nm^2/C^2$

$\mu_0 = 4\pi \times 10^{-7}\ N/A^2$

$\rho_{water} = 1000\ kg/m^3$

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

Question 1 : For <u>each</u> correct answer, 2 marks will be given. For <u>each</u> wrong answer, 0.67 will be deducted. For no answer, zero mark will be given. The minimum total mark for Question 1 is zero.]

(A) Theoretically the Carnot efficiency is 100 %:

a) When the low temperature reservoir is at 0° C.
b) When the low temperature reservoir is at 0 K.
c) When the low temperature and the high temperature reservoirs are the same.
d) When a Carnot engine is used.

(B) When two waves combine in such a way as to produce beats, the difference between the combining frequencies is equal to:

a) One fourth the number of beats per second.
b) One half the number of beats per second.
c) The number of beats per second.
d) Twice the number of beats per second.

(C) It is a common characteristic of all types of wave motion that:

a) Energy is transferred without the transport of particles.
b) Particles move up and down.
c) Particles move back and forth.
d) A material medium transmits the disturbance.

(D) The period of a simple pendulum is independent of:

a) Length.
b) Acceleration due to gravity
c) Mass.
d) Total energy.

(E) The heat accepted and rejected in a Carnot engine operating between two heat reservoirs defines:

a) The inefficiency of the engine.
b) The ratio of absolute temperature of reservoirs.
c) The ideal gas scale of temperature.
d) The numerical value of the temperature at the triple point of water.

(F) The electric field intensity from a point charge varies:

a) Directly as the distance from the charge.
b) Directly as the square of the distance from the charge.
c) Inversely as the distance from the charge.
d) Inversely as the square of the distance from the charge.

(G) To a stationary observer the frequency of a sound source when the source is moving towards the observer appears to be:

a) Lower than the actual frequency.
b) Same as the frequency of the actual source.
c) Higher than the actual frequency.
d) It is difficult to tell from the given information.

(H) The first law of thermodynamics is concerned with the conservation of:

a) Number of moles.
b) Energy.
c) Number of molecules
d) Temperature.

(I) When two progressive waves of the same frequency and same amplitude traveling in opposite directions in a straight line superimpose, they give rise to:

a) Beats.
b) Destructive interference.
c) Stationary (standing) waves.
d) Harmonics.

(J) A soldier sets his watch on hearing a distant siren which sounds on the hour. His watch will record:

a) Less than actual time.
b) More than actual time.
c) Perfectly correct time.
d) Nothing can be said.

Attempt any four questions from the following:

Q 2 (a) A person blows a whistle with a note of 800 Hz as the person moves from a stationary listener towards a wall with a speed of 1.5 ms^{-1}. How many beats per second are heard by the listener? (Take velocity of sound as 340 ms^{-1}) [10]

(b) A Carnot engine whose heat source is at 130 °C takes 5000 J of heat each cycle and exhausts 3500 J to the cold reservoir. Find the temperature of the cold reservoir. [7]

(c) Why are soldiers told to break step (move at different individual paces) when crossing a bridge? [3]

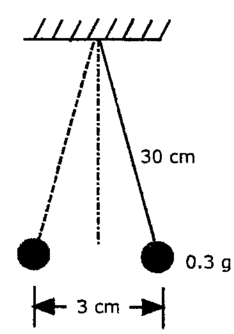
4 **Q.3 (a)** How much energy does a freezer have to remove from 2 litres of water at 20 °C to make ice at -10 °C? [8]

(b) The cone of a loudspeaker vibrates in SHM at a frequency of 262 Hz. The amplitude at the centre of the cone is $A = 1.5 \times 10^{-4}$m, and at t = 0, x = A.

i) What equation describes the motion of the centre of the cone?
ii) What is the velocity as a function of time?
iii) What is the position of the cone at $t = 1.0 \times 10^{-3}$s. [9]

(c) Define the specific heat capacity of a substance. [3]

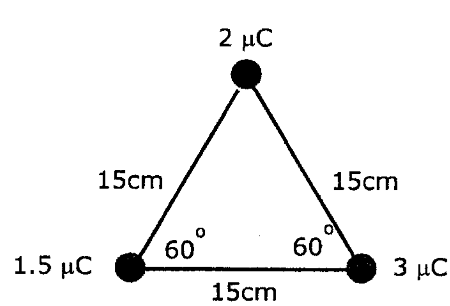
Q.4(a) Two small metallic spheres, each of mass 0.30 g are suspended as pendulums by light strings from a common point as shown. The spheres are given the same electric charge, and it is found that the two come to equilibrium when they are 3 cm apart. If the strings are 30 cm long, what is the charge on each sphere? [10]



(b) The sound level measured 50 m from a jumbo jet is 120 dB. What is the the sound level at 250 m? (ignore reflections from the ground) [10]

Q.5 (a) A cylinder of 20 litres capacity contains oxygen at a gauge pressure of 15 atmospheres and temperature 27 °C. When some amount of oxygen is withdrawn from it, its gauge pressure drops to 10 atm and the temperature to 20 °C. Find the amount of oxygen taken out from the cylinder. [10]

(b) The charges shown in the diagram below are stationary. Find the force on the 2 μC charge due to the 1.5 μC and 3 μC charges. [10]

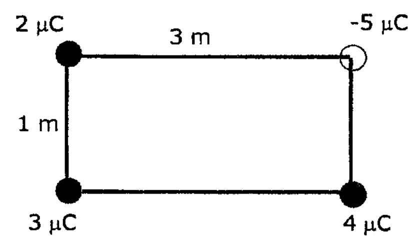


Q.6 (a) A barrel of diameter 134.122 cm at 20 °C is to be enclosed by an iron band. The circular band has an inside diameter of 134.110 cm at 20 °C. It is 7.4 cm wide and 0.65 cm thick. (α_{steel} =12 x 10^6/°C, Young's modulus 100 x 10^9 Nm^2)

i) To what temperature must the band be heated so that it will fit over the barrel?

ii) What will be the tension in the band when it cools to 20 °C? [10]

(b) Charges are arranged at corners of a rectangle as shown. Find the magnitude of the electrostatic force on the 3 μC charge. [10]



Q.7 (a) A balloon has 0.820 litre of dry air in it when sunk into a lake. Assume the pressure of dry air is 95% of the external pressure at all times, what is the volume of dry air in the balloon at a depth of 10 m? Assume atmospheric pressure at the surface is 1.013 x 10^5 Pa. [7]

(b) A 215 g sample of a substance is heated to 330 °C and then plunged into a 105 g aluminium calorimeter cup containing 165 g of water and a 17 g glass thermometer at 12.5 °C. The final temperature is 35.0 °C. What is the specific heat capacity of the substance? Assume no water boils away. (c_{water} = 4184 J/kg.°C, c_{Al} = 900 J/kg.°C, c_{glass} = 840 J/kg.°C) [11]

(c) If a glass thermometer had the same coefficient of volume expansion as the mercury it holds, the thermometer would not be very useful. Explain. [2]

Q.8 (a) A wire fixed at both ends has a fundamental frequency of 120 Hz. Its mass is 40 g and it has a linear density of 0.45 g/cm. Calculate its tension. [8]

(b) Water is being boiled in a pan on a solid cooker plate. The pan has a bottom area of 250 cm^2 and a thickness of 2.5 mm. If 1.5 g of steam is produced each minute, calculate the difference in the temperature between the inner and outer surfaces of the pan. (k_{pan} = 45 J/s.m.°C and H_v = 2256 J/g) [6]

(c) A simple pendulum of length 95 cm makes 100 oscillations in 200 s. Find the value of the acceleration due to gravity at the place. [6]

END OF P 192 EXAMINATION

Some useful equations

Thermal properties of matter:

$Q/t = e\sigma AT^4$ $Q/\Delta t = (kA\Delta T)/\Delta L$ $\Delta Q = mc\Delta T = nC\Delta T$ $\Delta L = \alpha L\Delta T$

$\Delta V = \gamma V\Delta T$ $\Delta W = P\Delta V$ $\Delta W = nRT.\ln(V_f/V_i)$ $C_v = C_p - R$

$PV = nRT$ $P_1V_1^\gamma = P_2V_2^\gamma$

Thermodynamics:

$Q = \Delta U + W$ Carnot engine, $e = 1 - T_2/T_1 = \frac{\text{work done}}{\text{input heat at high temperature}}$

$S = k\ln\Omega$ $\Delta S = Q/T$ Efficiency $= W/Q_h$ $COP_{fridge} = Q_c/W_{in}$

$COP_{heat\ pump} = Q_h/W_{in}$ $COP_{max\text{-}fridge} = T_c/(T_h - T_c)$ $COP_{max\ h.\ pump} : T_h/(T_h - T_c)$

Waves and vibrations:

$F = -kx$ $\omega = 2\pi f$ $f = (1/2\pi)\sqrt{(g/L}$ $a_c = \omega^2 x_0$

$P.E. = (1/2)kx^2$ $1/2)kx^2 + (1/2)mv^2 = (1/2)kx_0^2$ $\omega = \sqrt{(k/m}$

$f = (1/2\pi)\sqrt{(k/m)}$ $v = f\lambda$ $f_n = v/\lambda_n = n(v/2L)$ $L = n(\lambda_n/2)$

$v = \sqrt{T}/\mu$

Sound waves:

$v = \sqrt{(Y/\rho)}$ $v = \sqrt{(B/\rho)}$ $I_o = 10^{-12}\ W/m^2$ $I(dB) = 10\log(I/I_o)$

$I(r) = P/4\pi r^2$ $f' = f\left(\frac{v + v_l}{v - v_s}\right)$ (moving source and moving listener)

Electric forces and fields, electric potential:

$\mathbf{F} = q\mathbf{E}$ $\mathbf{E} = kQ/r^2$ $\mathbf{F} = (k\, q_1\, q_2)/r^2$ $V_{AB} = Ed$

$V = kq/r$ $\Delta PE = qEd$ $C = (\varepsilon_0 A)/d$ $W = qV_{AB}$

$T = 2\pi\sqrt{m/k}$

The University of Zambia
Physics Department
Second Semester University Examinations, 2011/12 Academic Year
P-198 : Introductory Physics- II (Option B)

All questions carry equal marks. The marks are shown in brackets. Question 1 is compulsory. Attempt four more questions. Clearly indicate on the answer script cover page which questions you have attempted.

Time: Three hours. **Maximum marks = 100**

Do not forget to write your computer number clearly on the answer book as well as on the answer sheet for Question 1. Tie them together !!

==

Wherever necessary use :

$g = 9.8 m/s^2$

1 metric ton = 1000 kg

$P_A = 1.01 \times 10^5$ N/m^2

$$\frac{\Delta Q}{\Delta t} = \frac{kA(T_2 - T_1)}{\Delta x}$$

1 cal. = 4.18 J ;

Isothermal process, $\Delta W = nRT \ln\left(\frac{V_2}{V_1}\right)$

Iso-volumetric process $\Delta U = 3/2nR\Delta T$

Specific latent heat of vaporization of water = 2257kJ/kg

$\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/Nm^2 ; $\mu_0 = 4\pi \times 10^{-7}$ N/A^2

$c = 3 \times 10^8$ m/s ; $\rho_{water} = 1000$ kg/m^3 $G = 6.67 \times 10^{-11}$ Nm^2/kg^2.

$h = 6.63 \times 10^{-34}$ J-s

$R = \rho \frac{L}{A}$

1 Pascal =1 N/m^2

1eV = 1.6×10^{-19} J

$e = 1.6 \times 10^{-19}$ C

$m_e = 9.11 \times 10^{-31}$ kg

$k = 9 \times 10^9$ Nm^2/C^2

$Q/t = e\sigma AT^4$ $Q/\Delta t = (kA\Delta T)/\Delta L$ $\Delta Q = mc\Delta T = nC\Delta T$ $\Delta L = \alpha L\Delta T$

$\Delta V = \gamma V\Delta T$ $\Delta W = P\Delta V$ $\Delta W = nRT.\ln(V_f/V_i)$ $C_v = C_p - R$

$PV = nRT$ $P_1V_1^\gamma = P_2V_2^\gamma$

Thermodynamics:

$Q = \Delta U + W$ Carnot engine, $e = 1 - T_2/T_1 = \frac{\text{work done}}{\text{input heat at high temperature}}$

$S = k\ln\Omega$ $\Delta S = Q/T$ Efficiency $= W/Q_h$ $COP_{fridge} = Q_c/W_{in}$

$COP_{heat\ pump} = Q_h/W_{in}$ $COP_{max\text{-}fridge} = T_c/(T_h - T_c)$ $COP_{max\ h.\ pump} : T_h/(T_h - T_c)$

Waves and vibrations:

$F = -kx$ $\omega = 2\pi f$ $f = (1/2\pi)\sqrt{(g/L}$ $a_c = \omega^2 x_0$

$P.E. = (1/2)kx^2$ $1/2)kx^2 + (1/2)mv^2 = (1/2)kx_0^2$ $\omega = \sqrt{(k/m}$

$f = (1/2\pi)\sqrt{(k/m)}$ $v = f\lambda$ $f_n = v/\lambda_n = n(v/2L)$ $L = n(\lambda_n/2)$

Sound waves:

$v = \sqrt{(Y/\rho)}$ $v = \sqrt{(B/\rho)}$ $I_o = 10^{-12}\ W/m^2$ $I(dB) = 10\log(I/I_o)$

$I(r) = P/4\pi r^2$ $f' = f\left(\frac{v + v_l}{v - v_s}\right)$ (moving source and moving listener)

Electric forces and fields, electric potential:

$\mathbf{F} = q\mathbf{E}$ $\mathbf{E} = kQ/r^2$ $\mathbf{F} = (k\ q_1\ q_2)/r^2$ $V_{AB} = Ed$

$V = kq/r$ $\Delta PE = qEd$ $C = (\varepsilon_o A)/d$ $W = qV_{AB}$

$Energy = \frac{\frac{1}{2}q^2}{C} = \frac{1}{2}qV = \frac{1}{2}CV^2$ $C_{par} = C_1 + C_2 + .. + C_n$ $C_{ser} = 1/C_1 + 1/C_2 + ...$

$C = Q/V$

Direct current circuits:

$V = IR$ $R = \rho\frac{L}{A}$ $\Delta R = R_o\alpha\Delta T$ $P = IV = I^2R = V^{@}/R$

$V_{terminal} = \varepsilon - Ir$ $R_{eq} = R_1 + R_2 + ...$series $1/R_{eq} = 1/R_1 + 1/R_2 + ...$parallel

SECTION A

Question 1 : Sample answers : F(a), G(d).... etc. **DO NOT <u>guess</u>** the answer. For <u>each</u> correct answer, 2 marks. For <u>each</u> wrong answer, 0.67 will be deducted. No answer, zero mark. The minimum total mark for Question 1 is zero. [$10 \times 2 = 20$]

(A) The electric field intensity of a point charge varies:

a) Directly as the distance from the charge.
b) Directly as the square of the distance from the charge.
c) Inversely as the distance from the charge
d) Inversely as the square of the distance from the charge

(B) Which of the following represents the force acting on simple harmonic oscillator?

(a) $F = -0.5x^2$
(b) $F = -2.3y$
(c) $F = 8.6x$
(d) $F = -0.4\sin\theta$

(C) In thermodynamics the isothermal and cyclic processes are related in the sense that:

(a) $\Delta U = 8.31$
(b) $\Delta Q = 0$
(c) $\Delta W = \Delta U$
(d) $\Delta U = 0$

(D) In wave motion, the phase difference $\Delta\phi$ and path difference Δd are related by the formula

(a) $\Delta\phi = \dfrac{2\pi\Delta d}{\lambda}$
(b) $\Delta\phi = \Delta d$
(c) $\sin\Delta\phi = \lambda\Delta d$
(d) $\dfrac{\Delta d}{\Delta\phi} = \lambda$

(E) In standing waves, a segment is the distance between

(a) A node and an anti-node
(b) A node and a crest
(c) Two adjacent nodes
(d) An anti-node and a trough

(F) If a material has a negative volume coefficient of expansion, it means that

(a) Its volume increases with increase in temperature.
(b) Its volume decreases with increase in temperature
(c) Its volume becomes negative.
(d) Its volume decreases with decrease in temperature.

(G) The rate of cooling of a body depends mainly on:
(a) The material of the body.
(b) The size of the body.
(c) The color of the body.
(d) The difference in temperature between the body and its surroundings

(H) In blood transfusion, it is dangerous to lower the bag containing the blood to the patient's bed. This is because

(a) Blood from the patient's vessels will flow back into the bag
(b) Blood will stop flowing into the patient's body but there will be no back flow
(c) The beating of the heart will decrease
(d) The veins will dilate.

(I) In the treatment of a gas using the kinetic theory, collisions between molecules are considered elastic. This means that the

(a) Molecular kinetic energy changes
(b) Temperature of the gas falls
(c) Molecular kinetic energy remains constant
(d) Volume of the container for the gas remains constant.

(J) One similarity between electric and magnetic field lines is that

(a) Magnetic field lines start from the north magnetic poles and end in south magnetic poles
(b) The density of either lines is proportional to the strength of the field in question
(c) Electric field lines are continuous in space including the charges while magnetic field lines emanate from north magnetic pole and end in south magnetic poles.
(d) The direction of magnetic field lines can cross while electric field lines cannot.

SECTION B

ATTEMPT ANY FOUR QUESTIONS FROM BELOW:

Q.2 (a) A gas sample expands isothermally from a state A to a state B, in the process absorbing 35 J of heat. It is then compressed isobarically to a state C, for which its volume is the same as that of state A. During the compression process, 22J of work is done on the gas. The gas is then heated at constant volume until it returns to state A.

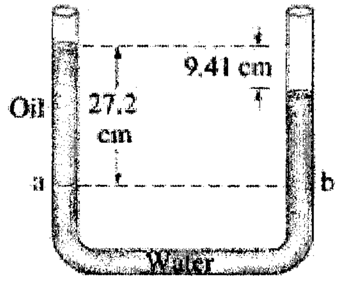
(i) Draw a *PV* diagram showing this process.
(ii) How much work is done by or on the gas as it traces this whole cyclic process?
(iii) How much heat is transferred to or from the gas as it goes from state B to state C and on to state A? [9]

(b) A 200-g mass is attached to a spring of force constant $k = 5.6\,\text{Nm}^{-1}$, and set into oscillation with amplitude $x_o = 25\,\text{cm}$. Determine

(i) the frequency in Hz
(ii) the period in seconds
(iv) the maximum velocity $v_{\max}$
(v) the maximum restoring force $F_{\max}$. [6]

(c) A string is clamped at both ends and tensioned until its fundamental vibration frequency is 85 Hz. If the string is then held rigidly at its mid-point, what is the lowest frequency at which it will vibrate? [5]

Q3 (a) Water and then oil (which don't mix) are poured into a U tube that is open at both ends. They come to equilibrium as shown. Find the density of the oil in kgm^{-3}? [5]



(b) A steel wire 0.85mm in diameter propagates transverse waves at 270ms^{-1}. Given that the density of steel is 7.9gcm^{-3}, find the tension T in the wire. [6]

(c) (i) If an inflated balloon is released on the Earth's surface, it rises until it comes to rest at a certain height. Explain the reason for this. [2]

(ii) In a weight lifting competition, it is found that the most massive concrete block the most powerful competitor can lift has a mass of 250kg. Suppose the competition is held under water, what is the most massive concrete block the same competitor can lift? [6]

Q4 (a) A tuning fork vibrating at 512Hz falls from rest and accelerates at $9.80 ms^{-2}$. How
How far below the dropping point is the fork when waves of frequency 485Hz reach the release point? Take speed of sound = $345 ms^{-1}$. [7]

(b) A pencil is weighted at one end so that it floats vertically in water with length L submerged. The pencil is then pushed vertically downward (without being totally submerged) and released. Show that it undergoes simple harmonic motion with period $T = 2\pi\sqrt{\frac{L}{g}}$. [8]

(c) A steel ball bearing 5.00 mm in radius is encased in a Pyrex glass cube 1.0 cm on a side. At 330K, the ball bearing fits tightly in its cavity. At what temperature will it have a clearance of 1.0μ m all round? [5]
Take $\alpha_{steel} = 11\times10^{-6}$ °C^{-1}, $\alpha_{pyrex} = 3.2\times10^{-6}$ °C^{-1}

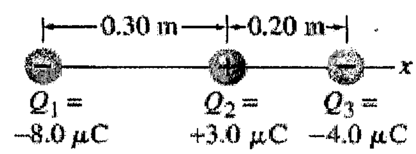
Q5 (a) A jet of steam at 100°C is directed at a 40 kg block of aluminium which is at a temperature of 50 °C. Assuming that all the steam hits the block and condenses, and that the resulting liquid water drops off before it can transfer more heat to the aluminium, how much steam must reach the aluminium before its temperature reaches 100°C?
Take specific heat capacity of aluminium $c_{Al} = 900$ J kg^{-1} °C^{-1} 4]

(b) A reversible engine contains 0.20 mol of an ideal monatomic gas, initially at 600 K and confined to a volume of 2.0 liters. The gas is taken through a cycle consisting of the following steps:

(i) Isothermal expansion to a volume of 4.0 liters
(ii) Iso-volumetric cooling to 300K
(iii) Isothermal compression to 2.0 liters
(iv) Iso-volumetric heating to 600K

Determine the **heat added** to the gas during the cycle and the **work done** by the gas as well as the **efficiency** of the engine. [12]

(c) Three charges Q_1, Q_2 and Q_3 are arranged as shown in the figure below. Calculate the net electric force on Q_3 due to the other two charges.

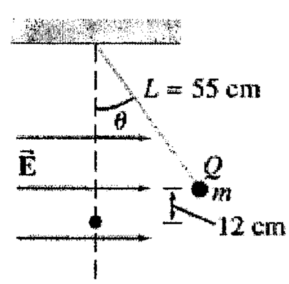


[4]

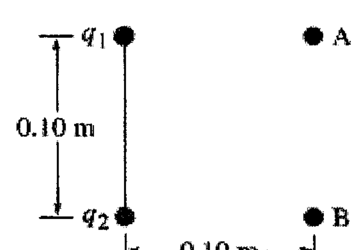
Q6 (a) A tuning fork produces a steady 400-Hz tone. When this tuning fork is struck and held near a vibrating guitar string, twenty beats are counted in five seconds. What are the possible frequencies produced by the guitar string? [4]

(b) A point charge of mass 1.0 g located at the end of an insulating string 55 cm long is observed to be in equilibrium with a uniform electric field of $2000 NC^{-1}$ when the pendulum hangs as shown. The charge is suspended 12 cm above the lowest point and the field points as shown. Find

(i) the tension in the string
(ii) the magnitude and sign of the point charge [9]



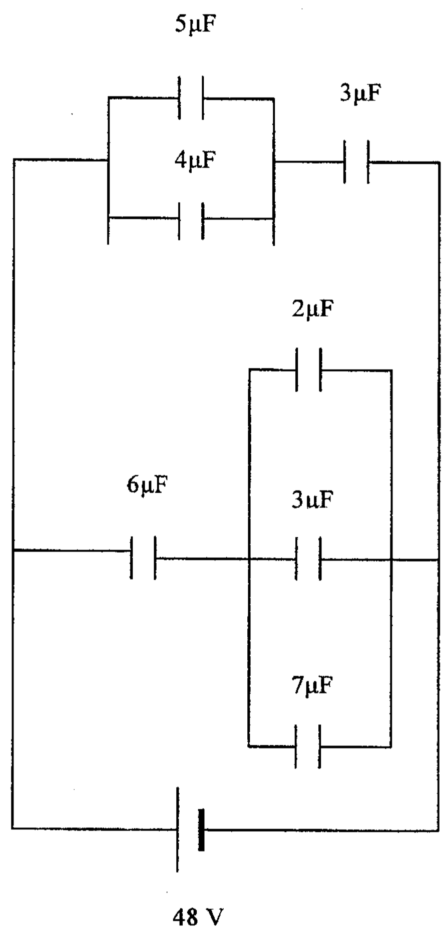
(c) Two charges are placed as shown in the figure below with $q_1 = 1.5\mu$ C and $q_2 = -3.3\mu$ C. Find the electric potential difference between points A and B, i.e. $V_A - V_B$ [7]



7 (a) How must the diameters of copper, d_{cu}, and aluminium, d_{al}, be related if they have to have the same resistance per unit length? [7]

Take $\rho_{Al} = 2.8 \times 10^{-8}\,\Omega\,\text{m}$ and $\rho_{Cu} = 1.7 \times 10^{-8}\,\Omega\,\text{m}$.

(b) Find the equivalent capacitance of the group of capacitors shown in the figure below [8]



(c) In the network of capacitors above, find the voltage across the 6μF capacitor and the charge stored in it. [5]

8 (a) Water flowing through a garden hose pipe of diameter 2.74cm fills a 25 - liter bucket in 1.5 minutes. What is the speed of the water leaving the bucket in ms^{-1}? [5]

(b) A liquid with a coefficient of volume expansion γ just fills a spherical flask of volume V_o at temperature T_i. The flask is made of a material that has a coefficient of linear expansion α. The liquid is free to expand into a capillary tube of radius r at the top of the flask.

(i) Show that if the temperature increases to a final value T_f, the liquid rises in the capillary by the amount

$$\Delta h = \frac{V_o}{\pi r^2}(\gamma - 3\alpha)(T_f - T_i) \qquad [4]$$

(ii) For a typical system such as a mercury in glass thermometer, why is it a good approximation to neglect the expansion of the flask? [2]

Data : Coefficient of volume expansion of mercury = 1.82×10^{-4} °C^{-1}
Coefficient of volume expansion of glass = 9×10^{-6} °C^{-1}

(c) A thin spherical shell of radius r_o possess a total net charge Q that is uniformly distributed on it. Use Gauss's law to derive

(i) the relation for the electric field for points outside the shell [2]
(ii) the relation for the electric field for points inside the shell [2]
(iii) a well labeled graphical plot for the fields cited in parts (i) and (ii) [3]

END OF EXAMINATION

The University of Zambia
School of Natural Sciences
Department of Physics
University Examinations 2011/12
Second Semester
P-212: Magnetism in Matter and Atomic Physics

Attempt <u>any five</u> questions only. All questions carry equal marks. The marks are shown in brackets. Write clearly your computer number on the answer book.

Time: Three hours. **Maximum marks = 100.**

Wherever necessary use:

$g = 9.8 m/s^2$	$N_{Av.} = 6.023 \times 10^{23}$ per mole
$e = -1.6 \times 10^{-19} C$	$1 eV = 1.6 \times 10^{-19} J$
$\mu_0 = 4\pi \times 10^{-7}$ Wb/A-m	$\varepsilon_0 = 8.85 \times 10^{-12}$ $C^2/N.m^2$
$m_p = 1.007825$ amu	$m_n = 1.008665$ amu
1 amu = 1.66×10^{-27} kg = 931.5 MeV	Stefan's law $E = \sigma T^4$
$m_e = 9.11 \times 10^{-31}$ kg = 0.511 MeV	$c = 3 \times 10^8$ m/s
Wien's constant $b = 2.9 \times 10^{-3}$ m.K	Stefan's constant $\sigma = 5.67 \times 10^{-8}$ $Wm^{-2}K^{-4}$
Boltzmann constant $k = 1.38 \times 10^{-23}$ $J.K^{-1}$	$1 \overset{0}{A} = 10^{-10}$ m. 1 nm = 10^{-9} m.
Luminosity of the Sun = 3.86×10^{26} watts.	$N = N_o e^{-\lambda t}$
$h = 6.63 \times 10^{-34}$ J.s = 4.136×10^{-15} eV.s	$E = mc^2$
Rydberg constant $R = 1.0974 \times 10^7$ m^{-1}	1 Ci = 3.7×10^{10} d/s
Rydberg equation $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$	$\psi_\lambda d\lambda = \frac{8\pi ch\lambda^{-5}}{e^{ch/kT} - 1} d\lambda$
Photoelectric equation $\frac{1}{2}mv^2 = h\nu - \varphi$	$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta)$
$I = I_o e^{-\mu x}$; $\lambda_m = \frac{2.90 \times 10^{-3} \text{m.K}}{T}$	$E_n = -\frac{13.6}{n^2}$ eV; $E = hf = \frac{hc}{\lambda}$
$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_0 c^2}[1 - \cos\theta]}$	$(1s)^2$ $(2s)^2$ $(2p)^6$ $(3s)^2(3p)^6$ $(4s)^2$ $(3d)^{10}$ $(4p)^6$ $(5s)^2$ ....

Q1(a). Show that the magnetic field intensity **B**, the magnetization **M** and the magnetic field strength **H** are related by the formula **B** = μ_0(**H**+**M**) [9]

(b) Show that the magnetic permeability μ and the susceptibility χ_m are related by $\mu = \mu_0(1+\chi_m)$, where μ_0 is the permeability of free space. [4]

(c) A toroidal winding carrying a current of 5 amps is wound with 60 turns/m of wire. The core is iron having a permeability of 5000 μ_0.
Find **H** and **B** inside the iron core. [4]

(d) Differentiate between paramagnetic and diamagnetic materials. [3]

Q2. The surface temperature of the Sun is about 5800 K.

(a) Assuming that it behaves like an ideal black-body, at what wavelength does the peak of the solar spectrum occur? [3]

(b) In which part of the spectrum is this wavelength situated? [2]

(c) What is the total radiated power per unit area of surface? [3]

(d) The wavelength of maximum radiation from the sun is 0.55 microns. Use this information to find the temperature of a star which has its maximum radiation intensity at 0.43 microns. [4]

(e) Give the quantity determined by each set of the following possible quantum numbers:

(i) 1, 2, 3, ... n
(ii) 0, 1, 2, 3, ... (n-1)
(iii) $-l$,.....0,.....$+l$
(iv) −½ , +½ [8]

Q3.(a) What is a radiation detector? Name three types of nuclear radiation detectors. Explain in some detail the workings of a scintillation detector; you can draw diagrams where necessary. [8]

(b) Which of the following electron configurations are possible, and which are not: [6]

(i) $1s^2, 2s^2, 2p^6, 3s^3$ (ii) $1s^2, 2s^2, 2p^6, 3s^2, 3p^5, 4s^2$ (iii) $1s^2, 2s^2, 2p^6, 2d^1$?

(c) In a particular Compton scattering experiment it is found that the incident wavelength λ_1 is shifted by 1.5% when the scattering angle $\theta = 120^0$.

(i) What is the value of λ_1?

(ii) What will be the wavelength λ_2 of the shifted photon when the scattering angle is $\theta = 75^0$?

Given $m = 9.1\times10^{-31}\,\text{kg},\ c = 3\times10^{8}\,\text{m/s},\ h = 6.63\times10^{-34}\,\text{J.s}$ [6]

Q4.(a) A beam of electrons bombards a sample of hydrogen. Through what potential difference must the electrons be accelerated if the first line of the Balmer series is to be emitted?

(b) Electrons in a television tube are accelerated through a potential difference of 25,000 volts. [4]

- (i) Calculate the speed of the electrons [2]
- (ii) Calculate the de Broglie wavelength of the electrons, [2]
- (iii) The speed of the electrons is known with an accuracy of 0.005%. With what accuracy can its position be known? [6]

Classical electron radius = 2.81794 x 10^{-15} m.

(c) Bohr's postulate states that in the one-electron atom, angular momentum is quantized.

$L = mvr = \dfrac{nh}{2\pi}$, where n = 1, 2, 3, ..., and m is the electron mass.

Show from this that the circumference of the electron orbits is an integral number of de Broglie wavelengths. [6]

Q5. (a) An X-ray beam of intensity I_0 is incident on a substance of thickness x. Derive an expression for the intensity I of the transmitted beam. [8]

(b) When ${}^{235}_{92}U$ undergoes fission, about 0.1% of the original mass is released as energy.

- (i) How much energy is released when 1 kg of ${}^{235}_{92}U$ undergoes fission? [2]
- (ii) How much ${}^{235}_{92}U$ must undergo fission per day in a nuclear reactor that provides energy to a 100-MW electric power plant that is 80% efficient in converting heat to electricity? [5]

(iii) When coal is burned, about 32.6 MJ/kg of heat is liberated. How many kilograms of coal, in metric tons, would be consumed per day by a conventional coal-fired 100-MW electric power plant of 50% efficiency? [5]

Q6. (a) A 50 gram sample of carbon is taken from the pelvis bone of a skeleton and is found to have a ^{14}C decay rate of 200 decays/minute. Given that carbon from a living organism has a decay rate of 15 decays per minute per gram of the substance, and that ^{14}C has a half-life of 5730 years, find the age of the skeleton. [6]

(b) Differentiate between negative and positive beta decay taking into account the following:

(i) The emitted particle's charge and mass; [3]
(ii) The nuclear charge and mass of the resulting daughter. [3]

(c) (i) State the incoming and outgoing particles a and b in the conversion of aluminium into phosphorous as represented by the reaction:

$$^{27}_{13}Al + a \rightarrow\ ^{30}_{15}P + b$$ [2]

(ii) Calculate the activity of 1 milligram sample of $^{90}_{38}Sr$ whose half-life is 28 years. [6]

Q7.(a) Differentiate between natural radioactivity and artificial radioactivity. [3]

(b) Write short notes on the three processes through which X-rays are mainly absorbed by matter. [6]

(c) The activity of a certain radioactive nuclide decreases by 15% of its original value in 10 days. Find its disintegration constant λ and its half-life $T_{1/2}$. [5]

(d) Study the part of the uranium decay series given below, and answer the questions that follow: [6]

$$^{238}_{92}U \rightarrow\ ^{234}_{90}Th \rightarrow\ ^{234}_{91}Pa \rightarrow\ ^{234}_{92}U \rightarrow\ ^{230}_{90}Th \rightarrow\ ^{226}_{88}Ra \rightarrow \ldots$$

(i) Name the particle emitted in each decay,
(ii) List the pairs of isotopes occurring in this part of the series
(iii) If the stable end product of the complete uranium series is ^{206}Pb, how many alpha particles are emitted between $^{226}_{88}Ra$ and the end of the series?

==End of P-212 Examination==

The University of Zambia

School of Natural Sciences

Department of Physics

Second Semester University Examinations - 2011/2012

Classical Mechanics II – P252

Duration: 3 Hrs

Date: 8th June 2012

Full Marks: 100

Time: 09:00 – 12:00 Hrs

Instructions

- **Use only your COMPUTER NUMBER on your answer sheets and NOT your name.**
- **This paper contains Seven (7) questions. Each question carries 20 marks.**
- **Attempt any Five (5) out of the Seven (7) questions given in this examination paper.**
- **This paper has a total of 100 marks. All questions carry equal marks.**
- **Show all your work clearly. Omission of essential work will result in a loss of marks.**
- **Marks allocated for each question are indicated in brackets [ ].**

You may need the following formulae:

$$A_n = \frac{2}{L}\int_0^L u_0(x)\sin\left(\frac{n\pi x}{L}\right)dx,\ B_n = \frac{2}{n\pi v}\int_0^L \dot{u}_0(x)\sin\left(\frac{n\pi x}{L}\right)dx,\ \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k} - \frac{\partial L}{\partial q_k} = 0,$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k} - \frac{\partial L}{\partial q_k} = Q_k,\ T = mc^2 - m_0c^2,\ t = \gamma t_o,\ \omega_n = \frac{n\pi v}{L},\ \dot{q}_k = \frac{\partial H}{\partial p_k},\ \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}},\ \frac{\partial^2 u(x,t)}{\partial t^2} = \frac{T}{\mu}\frac{\partial^2 u(x,t)}{\partial x^2},\ v = \frac{\omega}{k},\ L\gamma = L_0,\ -\dot{p}_k = \frac{\partial H}{\partial q_k}.$$

1. **(a)** The relativistic expression for the kinetic energy of a moving particle is given by

$$T = mc^2 - m_0 c^2$$

Show that for $v \langle\langle c$, this expression reduces to the Newtonian one. **[5]**

(b) The damped harmonic motion equation $\ddot{x} + 2\gamma\dot{x} + \omega^2 x = 0$ has the solution $x = e^{\lambda t}$ with $\lambda = -\gamma \pm \left(\gamma^2 - \omega^2\right)^{1/2}$. Briefly discuss the three cases that result as $\gamma^2 - \omega^2$ takes different values. Draw a diagram of x versus t for each case. **[6]**

(c) The equation of a transverse wave travelling along a very long string is given by

$$y = 6\sin(0.02\pi x + 4\pi t)$$

where y is in cm and t in seconds. Find the:

i) Amplitude **[1]**

ii) Wavelength **[2]**

iii) Frequency **[1]**

iv) Speed **[1]**

v) Direction of propagation of the wave **[1]**

vi) Maximum transverse speed of a particle in the string. **[3]**

2. **(a)** The square of the 4-vector momentum is invariant under the Lorentz transformations, i.e., $\vec{p}^2 = \sum_{\mu=1}^{4} p_\mu{}^2 = -m_0^2 c^2$, since the world velocity is a constant. Given that $p_\mu = \left(\vec{p}, i\, E/c\right)$, show that the total energy of a moving particle is given by $E^2 = c^2 p^2 + m_0{}^2 c^4$. **[4]**

(b) The motion of a forced harmonic oscillator is governed by the equation

$$m\ddot{x} + b\dot{x} + kx = F_0 \cos(\omega t + \theta_0)$$

The general solution of the above equation is

$$x = x(t) = x_i(t) + x_h(t)$$

where $x_i(t)$ is the particular solution for the transient motion and $x_h(t)$ is the homogeneous solution for the steady state motion.

i. Let $x_i(t) = A\cos(\omega t - \phi)$ and show that the forced harmonic oscillator equation of motion gives

$$\left(k - m\omega^2\right)\cos\phi + b\omega\sin\phi = F/A$$

and

$$\left(k - m\omega^2\right)\sin\phi - b\omega\cos\phi = 0\,.$$

[5]

ii. Hence, show that $\tan\phi = \dfrac{b\omega}{k - m\omega^2}$. **[1]**

iii. Recalling that $\dfrac{k}{m} = \omega_0{}^2$ and $\gamma = \dfrac{b}{2m}$, find expressions for $\sin\phi$ and $\cos\phi$. **[4]**

iv. Use the expressions obtained in parts (i) and (ii) to show that the amplitude of oscillation A is

$$A = \frac{F_0/m}{\sqrt{\left(\omega_0{}^2 - \omega^2\right)^2 + 4\gamma^2\omega^2}}$$

[5]

v. The full general solution is

$$x = x(t) = \frac{F_0/m}{\sqrt{\left(\omega_0{}^2 - \omega^2\right)^2 + 4\gamma^2\omega^2}}\cos(\omega t - \phi) + A_h e^{-\gamma t}\cos(\omega_1 t + \phi_h)$$

Explain what happens to the two terms on the right hand side after the system has been in oscillatory motion for a long time. **[1]**

3. **(a)** The differential equation governing the motion of a particle undergoing simple harmonic motion is

$$\ddot{x} + \omega^2 x = 0$$

Show that the solution of this equation is

$$x = a\cos\omega t + b\sin\omega t$$

[6]

(b) A string of length L and linear mass density μ under tension T is initially in the equilibrium position but has a velocity given by

$$v = ax\,,\ 0\langle x\langle \frac{L}{2}$$

$$v = a(x-L),\ \frac{L}{2}\langle x\langle L$$

i.) Show that $A_n = 0$ for all n. **[3]**

ii.) For the subsequent motion of the string

$$B_n = 0 \text{ if } n \text{ is odd, and}$$

$$B_n = \frac{-3aL^2}{n^2\pi^2 v}\cos(\frac{n\pi}{2}) \text{ if } n \text{ is even.}$$

The general solution for the motion of the string is given by

$$u(x,t) = \sum_{n=1}^{\infty}(A_n \sin\frac{n\pi x}{L}\cos\frac{n\pi vt}{L} + B_n \sin\frac{n\pi x}{L}\sin\frac{n\pi vt}{L})$$

Write the explicit solution $u(x,t)$ for the motion of the string for the first three harmonics. **[3]**

iii.) Plot the modes of vibration for the first three harmonics. **[3]**

iv.) How do all the modes of vibration for the motion manifest themselves on the string at an instant of time? **[1]**

(c) A 2 m long string of mass 50 g vibrates in the form of segments. If the wave has a wavelength of 0.5 m and a frequency of 60 Hz, find the tension in the string. **[4]**

4. (a) An electron (rest energy 0.51 MeV) is accelerated from rest through a voltage drop of 0.13 MeV, and then travels at constant velocity.

i) How long does it take for the electron (after it has reached its final velocity) to travel between two points 8.4 m apart? **[10]**

ii) What is the distance between the points as measured in the rest frame of the electron? **[2]**

(b) Prove that

$$y(x,t) = A\sin(kx)\cos(\omega t)$$

is a solution of the one dimensional wave equation for a stretched string where k is the wave number and ω is the angular frequency. **[8]**

5. **(a)** What are the most critical differences between the Lorentz transformations and the Galilean transformations? **[3]**

(b) A pendulum comprises a mass m attached to a spring of natural length l. It is pulled at an angle θ from the vertical.

i. Draw a diagram depicting the above described set up. **[1]**

ii. Show that the Lagrangian is

$$L(x,\theta,\dot{x},\dot{\theta}) = \frac{1}{2}m\left[\dot{x}^2 + (l+x)^2\dot{\theta}^2\right] - \frac{1}{2}kx^2 + mg(l+x)\cos\theta$$

iii. Prove that the equations of motion are

$$\ddot{x} - (l+x)\dot{\theta}^2 + \omega^2 x - g\cos\theta = 0$$

and

$+2\dot{x}\dot{\theta}+$

$$(l+x)\ddot{\theta} + 2\dot{x}\theta + g\sin\theta = 0$$

[16]

6. **(a)** (i) Explain two advantages of the Lagrangian formulation of mechanics over the Newtonian formulation. **[2]**

(ii) Explain the major advantage and the major disadvantage of the Hamiltonian formulation of mechanics over the Lagrangian formulation. **[2]**

(iii) Explain the importance of cyclic or ignorable coordinates in analytical mechanics. **[2]**

(b) A particle of mass m is attracted toward a given point by a force of magnitude

$$\left|\vec{F}(r)\right| = \frac{b}{r^2}$$

where b is a positive constant. Using Cartesian coordinates, show that the Hamiltonian is

$$H(x,y,p_x,p_y) = \frac{1}{2}m\left(\frac{p_x^{\ 2}}{m^2} + \frac{p_y^{\ 2}}{m^2}\right) - \frac{b}{\sqrt{x^2+y^2}},$$

and that the equations of motion are

$$m\ddot{x} + \frac{bx}{\left(x^2 + y^2\right)^{3/2}} = 0$$

and

$$m\ddot{y} + \frac{by}{\left(x^2 + y^2\right)^{3/2}} = 0$$

[14]

7. **(a)** A harmonic oscillator (HO) of mass m under damping force $-b\dot{x}$ is displaced a distance $+A$. Then it is released with zero initial velocity. Show that the equation representing the weakly-damped or under-damped motion is

$$x = \frac{A\omega}{\omega'} e^{-bt/2} \sin\left[\omega' t + \tan^{-1}\left(\frac{2\omega'}{b}\right)\right].$$

You may assume the solution for a weakly-damped harmonic motion to be $x = x_0 e^{-bt/2} \sin(\omega' t + \delta)$. **[14]**

(b) Frame S' has a speed $v = 0.6c$ relative to frame S. Clocks are adjusted so that $t = t' = 0$ at $x = x' = 0$.

i) An event occurs in S at $t = 2\times10^{-7}$ s at a point $x = 50$ m. At what time does the event occur in S'? **[3]**

ii) If a second event occurs at $x = 10$ m and $t = 3\times10^{-7}$ s in S, what is the time interval between the events as measured in S'? **[3]**

<u>END OF EXAMINATION</u>

The University of Zambia

Department of Physics

Computational Physics I

(P302)

University Second Semester

Examination-2011/2012

Instructions **Max. Marks 100**

- **Time allowed:** Three (3) Hours.
- **All questions carry equal marks.**
- Marks for each question are shown in the square brackets [ ].
- Whenever necessary, use the information given in the **appendix**
- **Answer:**

  i) Question one (1).

  ii) Any three (3) questions from 2, 3, 4, 5 and 6.

Q.1 **(a)** The following variable names are encountered in declaration statements of a C program;

i) `keyword`	iv) `gravity acc`	vii) `_char`
ii) `Matrix.A`	v) `d*t`	viii) `void`
iii) `statement`	vi) `row3`	ix) `6thname`

Determine which of the above are valid variable names. If invalid, explain why.

[9]

(b) Translate the following expressions to C expressions

i) $v = u \ln \left(\dfrac{m+u}{m-qt} \right)$ ii) $u = \dfrac{x^a + y^b}{z^{1/2}}$ iii) $y = a + \dfrac{b^{1/2}}{e^{-x}}$

[6]

(c) Convert the decimal number, 4.4433594×10^{-2}, into its 32-bit short, real-biased, normalised floating-point, binary equivalent. The bit distribution for the 32-bit single precision floating-point representation is shown below.

1 bit	8 bits	23 bits
Sign	Exponent	Fraction

[10]

Q.2 **(a)** Consider the well-known problem of rearranging (i.e sorting) a list of n integer quantities into a sequence of increasing values. Write a C program that will;

i) prompt the user to enter the value for n,

ii) read in the list of n interger values,

iii) sort the list in an ascending order,

iv) print the sorted list to the monitor.

[15]

(b) What will be printed by the following C program fragment?

```
int i = 0, x = 0;
while(i<20)
   {
     if(i%5==0 )
       {
         x += i;
         printf("%d " , x);
       }
   ++i;
   }
printf("\nx=%d\n",x);
```

[10]

Q.3 Suppose three jumpers are connected by bungee cords and are held in place vertically so that each cord is fully extended but unstretched, as shown in Figure 4.1a. Define three distances x_1, x_2 and x_3 as measured downward from each of their unstretched positions. After they are released, gravity takes hold and the jumpers will eventually come to the equilibrium position shown in Figure 4.1b. Assume that each cord behaves as a linear spring and follows Hooke's law, free-body diagrams can be developed for each jumper as depicted in Figure 4.1c.

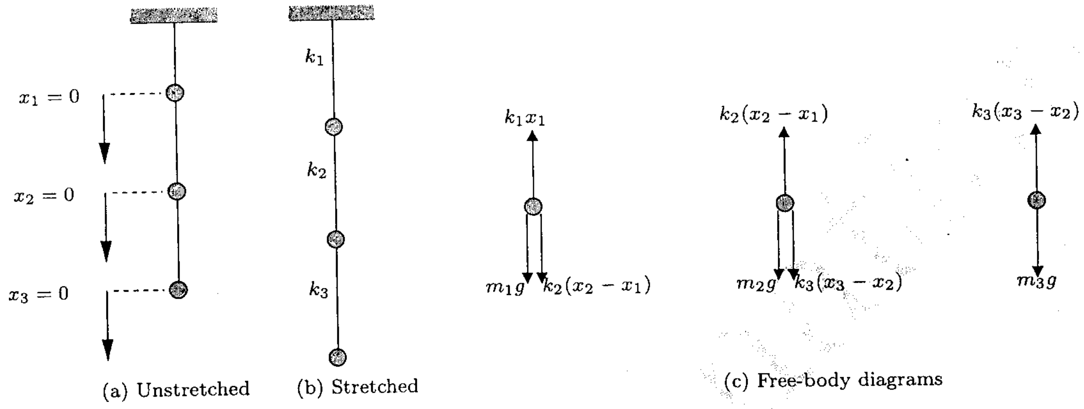


Figure 4.1

Taking m_i = the mass of jumper i (kg), k_j = the spring constant of cord j (N/m), x_i = the displacement of jumper i measured downward from the equilibrium position (m), and g = gravitational accerelation (9.81 m/s^2):

i) using Newton's law, write down a steady-state force balance equation for each jumper,

ii) by collecting terms, write down the linear system of equations in which x_1, x_2, and x_3 are the unknowns,

iii) given the parameters of the problem as

Jumper	Mass (kg)	Spring Constant (N/m)	Unstretched Cord Length (m)
Top (1)	60	50	20
Middle (2)	70	100	20
Bottom (3)	80	50	20

show that the matrix-vector form of this problem is

$$\begin{bmatrix} 150 & -100 & 0 \\ -100 & 150 & -50 \\ 0 & -50 & 50 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 588.6 \\ 686.7 \\ 784.8 \end{bmatrix}$$

iv) use the LU-factorization method to determine the values of the displacements x_1, x_2 and x_3.

[25]

Q.4 (a) The upward velocity of a rocket can be computed by the following formula:

$$v = u \ln\left(\frac{m_0}{m_0 - qt}\right) - gt$$

where v = upward velocity, u = the velocity at which fuel is expelled relative to the rocket, m_0 = the initial mass of the rocket at time $t = 0$, q = the fuel consumption rate, and g = the downward acceleration of gravity (assumed constant = 9.81 m/s^2). Take u = 2000 m/s, $m_0 = 150,000$ kg, and $q = 2700$ kg/s.
Write a C program to compute the time t at which $v = 750$ m/s within a relative error of 1%. What is the value of t after two iterations? (**Hint:** t is greater than 10s).

[15]

(b) The total mass of a variable density rod is given by

$$m = \int_0^L \rho(x) A_c(x) dx$$

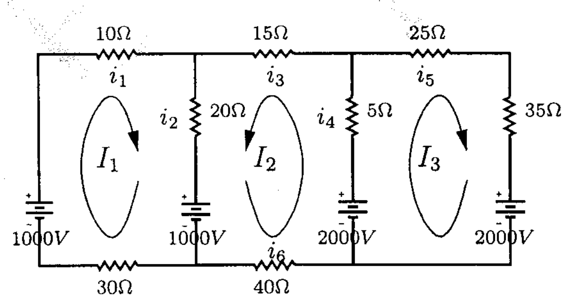
where m = mass, $\rho(x)$ = density, $A_c(x)$ = cross-sectional area, x = distance along the rod and L = the total length of the rod. The following data has been measured for a 10-m length rod.

x, m	0	2	3	4	6	8	10
ρ, g/cm^3	4.00	3.95	3.89	3.80	3.60	3.41	3.30
A$_c$, cm^2	100	103	106	110	120	133	150

Use the Trapeziod rule to determine the mass in grams.

[10]

Q.5 (a) Consider the problem of finding the currents in different parts of an electrical circuit, with resistors as shown in the diagram below;



Using Kirchoff's voltage law, write down the linear system of equations involving loop-currents I_1, I_2 and I_3 for the above circuit. Determine the values of the three loop-currents using the Gaussian elimination method. Also, determine the six branch currents i_1, i_2, i_3, i_4, i_5 and i_6.

[15]

(b) In an experiment to determine the relationship between the velocity v of a falling object and the drag force F, an individual is suspended in a wind turnel and the force measured for various levels of wind velocity. The data collected is listed in the following table.

v,m/s	10	20	30	40	50	60	70	80
F,N	25	70	380	550	610	1220	830	1450

Fit a straight line, $F = av + b$, to the above data. Wrtie a C program that determines the values of the coefficients, a and b.

[10]

Q.6 **(a)** An electrical circuit consists of a capacitor, $C = 1.1F$ in series with a resistor $R_0 = 2.1\Omega$. A voltage $E(t) = 110\sin(t)$ is applied at time $t = 0$. When the resistor heats up, the resistance becomes a function of current i. The differential equation for the current i is given by

$$\left(1 + \frac{2k}{R_0}i\right)\frac{di}{dt} + \frac{1}{R_0C}i = \frac{dE}{dt}$$

and the current-dependent resistance is

$$R(i) = R_0 + ki$$

where $k = 0.9$ and current $i = 0$ at $t = 0$. Taking $dt = 2$ sec and $0 \leq t \leq 60$ write a C program that will print the current, i and resistance R at time t to the standard output device. What are the values of the current i and resistance R at $t = 2$ and $t = 4$ sec?

[15]

(b) A car laps a race track in 84 seconds. The speed of the car at each 6-second interval is determined using a radar gun and it is given from the beginning of the lap, in feet/second, by the entries in the following table.

Time	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84
Speed	124	134	148	156	147	133	121	109	99	85	78	89	104	116	123

Using the composite Simpson rule, determine the length of the track.

[10]

⋆⋆⋆⋆⋆⋆⋆ **End of Examination** ⋆⋆⋆⋆⋆⋆⋆

Appendix

Commonly Used Library Funcitons:

Funtion	Type	Purpose
`abs(i)`	`int`	`Returns the absolute value of i`
`cos(d)`	`double`	`Returns the cosine of d`
`exp(d)`	`double`	`Raises` e `to the power d` ($e = 2.7182818\ldots$)
`fabs(d)`	`double`	`Returns the absolute value of d`
`log(d)`	`double`	`Returns the natural logarithm of d`
`log10(d)`	`double`	`Returns the logarithm (base 10) of d`
`pow(d1,d2)`	`double`	`Returns d1 raised to the d2 power`
`sin(d)`	`double`	`Returns sine of d`
`sqrt(d)`	`double`	`Returns the square root of d`
`tan(d)`	`double`	`Returns the tangent of d`

Newton-Raphson:

Given an equation $f(x) = 0$, the root can be obtained iteratively using the Newton-Raphson method:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Trapezoidal Method:

The integral I of a function $f(x)$,

$$I = \int_a^b f(x)dx$$

can be approximated numerically using the Trapezoidal rule:

$$I = \frac{h}{2}\left[f(a) + f(b) + 2\sum_{i=1}^{N-1} f(x_i)\right]$$

where N is the number of strips (Trapezoids) in the interval $[a, b]$, $h = (b-a)/N$ is the step-size. And $x_i = a + ih$.

Simpson's Method:

The integral I of a function $f(x)$,

$$I = \int_a^b f(x)dx$$

can be approximated numerically using Simpson's method using:

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a) + 2\sum_{i=2j}^{N-2} f(x_i) + 4\sum_{i=2j-1}^{N-1} f(x_i) + f(b)\right]$$

where N is an even number of strips in the interval $[a, b]$, $h = (b - a)/N$ is the step-size.

Euler's Method:

The Euler's method for finding the approximate solution of a first-order ODE, $y' = f(x, y)$, with initial condition$f(x_0) = y_0$, on an interval $[x_0, x_n]$ with step size $h = (x_n - x_0)/n$, is given by

$$y_{i+1} = y_i + hf(x_i, y_i)$$

where $h = (x_n - x_0)/n$ and n is the number of division between x_0 and x_n.

Least squares fit to a straight line:

$$y = ax + b$$

$$a = \frac{ns_{xy} - s_x s_y}{ns_{xx} - s_x s_x}$$

$$b = \frac{s_{xx}s_y - s_{xy}s_x}{ns_{xx} - s_x s_x}$$

where $s_x = \sum_{i=1}^{n} x_i,\quad s_{xx} = \sum_{i=1}^{n} x_i^2,\quad s_y = \sum_{i=1}^{n} y_i,\quad s_{xy} = \sum_{i=1}^{n} x_i y_i$

THE UNIVERSITY OF ZAMBIS
PHYSICS DEPARTMENT
UNIVERSITY EXAMINATIONS 2011/2012

P332
STATISTICAL AND THERMAL PHYSICS

DURATION: THREE (3) HOURS
TOTAL MARKS: 100

Instructions

- Answer any four (4) questions.
- Show all your working clearly. Omission of essential work will result in loss of marks.
- All questions carry equal marks. Marks allocated for each question are indicated in square brackets [ ].

You may need the following

$$\Delta S = \Delta Q/T$$

$$C_V = \left(\frac{\partial \overline{E}}{\partial T}\right)_V$$

$$\overline{E} = -\frac{\partial \ln Z}{\partial \beta}$$

$$\beta = \frac{\partial \ln \Omega}{\partial E}$$

$$\overline{p} = \frac{1}{\beta}\frac{\partial \ln \Omega}{\partial V}$$

$$\overline{X} = \frac{1}{\beta}\frac{\partial \ln Z}{\partial x}$$

$$\Omega(E) \propto V^N \chi(E)$$

1 (a) A box is separated by a partition which divides its volume by a ratio 3:1. The larger portion of the box contains 1000 molecules of H gas; the smaller, 100 molecules of He gas. A small hole is punctured in the partition and one waits until equilibrium is attained.

(i) Find the mean number of molecules of each type on either side of the partition [2 marks]

(ii) What is the probability of finding 1000 molecules of H in the larger portion and 100 molecules of He in the smaller portion? [4 marks]

(b) A system consists of N_1 molecules of type 1 and N_2 molecules of type 2 confined within a box of volume V. The molecules are supposed to interact very weakly so that they constitute an ideal gas mixture.

(i) Show that the total number of states $\Omega(E)$ in the range between E and $E + \delta E$ as a function of the volume V of this system is given by

$$\Omega(E_1, E_2) = V^{N_1+N_2}\chi(E_1)\chi(E_2)$$

You may treat the problem classically. [3 marks]

(ii) Use this result to find the equation of state of this system, i.e., to find its mean pressure $\bar{p}$ as a function of V and T. [6 marks]

(c) If a system has the partition function $Z = Z(x, \beta)$, prove the following:

(i) $S = k(\text{In}Z + \beta\overline{E})$ [5 marks]

(ii) $F = \overline{E} - TS = -kT\text{In}Z$ [5 marks]

2 (a) (i) State the difference between a purely thermal and a purely mechanical interaction, giving an example of each. [4 marks]

(ii) State the fundamental statistical postulate and explain its importance in statistical mechanics. [3 marks]

(iii) Explain the importance of quasi-static processes in statistical mechanics. [2 marks]

(b) Two systems A and A' insulated from their surrounding are in weak thermal interaction. Show that they achieve equilibrium when their temperature parameters are equal. [5 marks]

(c) Consider an ensemble of classical one – dimensional harmonic oscillators. Let the displacement x of an oscillator as a function of time t be given by $x = A\cos(wt + \varphi)$. Assume that the phase angle φ is equally likely to assume any value in the range $0 < \varphi < 2\pi$. The probability $w(\varphi)\,d\varphi$ that φ lies in the range φ to $\varphi + d\varphi$ is simply $w(\varphi)\,d\varphi = (2\pi)^{-1}\,d\varphi$. For any fixed time t, find the probability $P(x)\,dx$ that x lies between x and $x + dx$ by summing $w(\varphi)\,d\varphi$ over all angles φ for which x lies in this range and show that $P(x)dx$ can be expressed in terms of A and x as

$$P(x)dx = \frac{dx}{\pi\sqrt{A^2 - x^2}}$$

[11 marks]

3 (a) i) A particle is in equilibrium with a heat reservoir at absolute temperature T. Prove that the probability that it is in the state of energy E_r is

$$P_r = \frac{e^{-\beta E_r}}{\sum_r e^{-\beta E_r}} = \frac{e^{-\beta E_r}}{Z}$$

[10 marks]

ii) Show that the mean energy of such a particle is

$$\overline{E} = -\frac{\partial \ln Z}{\partial \beta}$$

[3 marks]

(b) Explain the difference between macrostate and a microstate and give an example of each [4 marks]

(c) A system consists of 2 spin ½ particles in weak interaction. A weak magnetic field H is applied to the system. Each particle can align parallel or anti-parallel with the field, with the energy of the particle being $-\mu H$ and μH respectively. This system is in contact with a heat reservoir at absolute temperature T

i) Obtain the partition function for this system [2 marks]

ii) Show that the probability that the system has zero energy is

$$P_0 = \frac{2}{e^{2\beta\mu H} + e^{-2\beta\mu H} + 2}$$

[2 marks]

iii) Show that the mean energy is

$$\overline{E} = -2\mu H \tanh(\beta\mu H)$$

[4 marks]

4 (a) Suppose that a system A is placed in thermal contact with a heat reservoir A' which is at an absolute temperature T' and that it absorbs an amount of heat Q in this process. Show that the entropy increase ΔS of A in this process satisfies the inequality $\Delta S \geq \Delta Q/T'$, where the equals sign is only valid if the initial temperature of A differs infinitesimally from the temperature T' of A'. [6 marks]

(b) Using the first law of thermodynamics

$$TdS = dE + pdV\ ,$$

show that

$$dH = TdS + Vdp$$

where $H = E + pV$ is the enthalpy. [2 marks]

(c) (i) Sketch and describe the main parts of a heat engine and show that its efficiency is given by

$$\eta \leq \frac{T_1 - T_2}{T_1}$$

[10 marks]

(ii) Hence find the efficiency of a Carnot engine operating between the temperatures 0°C and 100°C [3 marks]

5 Consider an isolated system consisting of a large number N of very weakly interacting localized particles of spin ½. Each particle has a magnetic moment μ, which can point either parallel or antiparallel to an applied field H. The energy E of the system is then $E = -(n_1 - n_2)\mu H$, where n_1 is the number of spins aligned parallel to H and n_2 the number of spins aligned antiparallel to H. Consider the energy range between E and $E + \delta E$ where δE is very small compared to E but is microscopically large so that $\delta E \gg \mu H$

(a) Show that the total number of states $\Omega(E)$ lying in this energy range is

$$\Omega(E) = \frac{N!}{\left(\frac{N}{2} - \frac{E}{2\mu H}\right)!\left(\frac{N}{2} + \frac{E}{2\mu H}\right)!}\frac{\delta E}{2\mu H}$$

[6 marks]

(b) Using Stirling's relation, for $N >> 1$, $\ln a! = N\ln N - N$, show that

$$In\Omega(E) = NInN - \left(\frac{N}{2} + \frac{E}{2\mu H}\right)\left(In\frac{N}{2} + In\left[1 + \frac{E}{\mu HN}\right]\right) - \left(\frac{N}{2} - \frac{E}{2\mu H}\right)\left(In\frac{N}{2} + In\left[1 - \frac{E}{\mu HN}\right]\right) + In\frac{\delta E}{2\mu H}$$

[6 marks]

(c) Using the results for part (b) or otherwise obtain the total energy of the system in terms of the absolute temperature T and show that it is given by

$$E = -N\mu H \tanh\left(\frac{\mu H}{kT}\right)$$

You may need the definition $\beta = \frac{\partial In\Omega}{\partial E}$ [8 marks]

(d) Under what conditions is T negative? [2 marks]

(e) The total magnetic M of this system is related to its energy E. Use the result of part (a) to show that M as a function of H and the absolute temperature T.

$$M = N\mu \tanh(\frac{\mu H}{kT})$$

[3 marks]

6 (a) Explain what is meant by an external parameter of a system, a mean generalized force and an equation of state. [6 marks]

(b) The entropy of a given system is given by

$$S = A(NVE)^{1/3}$$

where A is a constant, N is the number of particles, V is the volume and the E is the internal energy.

i) Show that the number of states accessible to the system in the energy range between E and $E+\delta E$ is $\Omega(E) = e^{\frac{A}{k}(NVE)^{1/3}}$

[2 marks]

ii) Obtain E as a function of S and V and show that it is given by

$$E = \frac{S^3}{A^3NV}$$

[1 marks]

iii) Use the first law of thermodynamics

$$TdS = dE + pdV$$

to show that [3 marks]

$$T = \left(\frac{\partial E}{\partial S}\right)_V \text{ and } -p = \left(\frac{\partial E}{\partial V}\right)_S$$

iv) Show that the molar specific heat capacity at constant volume is

$$c_V = \frac{1}{2N_A}\sqrt{\frac{A^3NVT}{3}}$$

where N_A is the Avogadro's number. [8 marks]

v) Show that the equation of state of the system is [3 marks]

$$p^2V = \frac{A^3NT^3}{27}$$

*********END OF EXAMINATIONS*********

The University of Zambia
School of Natural Sciences
Department of Physics
2011 Academic Year First Semester
Final Examinations
P-411: Nuclear Experimental Techniques

Attempt any four questions. All questions carry equal marks. The marks are shown in brackets.

Time: Three hours. **Maximum marks = 100.**

Write clearly your computer number on the answer book.

===

Wherever necessary use:

g = $9.8 m/s^2$

charge of an electron = 1.6×10^{-19}C

1 barn = 10^{-24} cm^2

mass of an electron = 9.1×10^{-31} kg

1 eV = 1.6×10^{-19}J

1 a.m.u. = 931.5 MeV = 1.66×10^{-27} kg

$N_{Av.}$ = 6.02×10^{23} per mole

1 curie = 3.7×10^{10} disintegrations/s

Planck's constant h = 6.63×10^{-34} J.s

$$\mu_a = \mu / N = \frac{\mu A}{\rho N_A}$$

Some equations you may find useful:

$$h\upsilon' = \frac{h\upsilon}{1+\frac{h\upsilon}{m_0 c^2}(1-\cos\theta)} \qquad \Omega = \frac{\pi a^2}{d^2} \qquad N = N_0 e^{-\Sigma_t x} \qquad m = \frac{n}{1+n\tau}$$

$$\mu_m = \frac{\mu}{\rho} = \frac{\sigma N}{\rho} = \frac{N_A \sigma}{atomic\,mass} \qquad n - m = nm\tau \qquad E_r = \frac{V_0}{r\ln(b/a)}$$

Q 1. (a) Derive from first principles the radioactive decay law and obtain the expressions relating the decay constant λ, and the half-life $T_{1/2}$ for a given radionuclide. [5]

(b) Explain the term *secular equilibrium*. Under what conditions does it exist? [5]

(c) One gram of natural potassium emits 29 beta particles per second due to the decay of ^{40}K (abundance ratio of 0.012 atom percent). Gamma rays are also emitted, and the ratio of gamma: beta = 0.12. The gamma rays follow electron capture in ^{40}K, and one photon is emitted for each orbital capture.

What is the half-life of ^{40}K? Given, $N_{Av.} = 6.02 \times 10^{23}$ per mole. [7]

(d) Draw a block diagram of the basic components of a scintillation spectrometer. Explain briefly the function of each component. [8]

Q2. (a) The nuclide $^{222}_{86}Rn$ emits three groups of alpha particles, with kinetic energies of 5.847, 5.779, and 5.613 MeV respectively. Associated with the alpha particles are gamma rays of energies 0.0687, 0.169, and 0.238 MeV.

Construct a decay scheme based on this data. [6]

(b) Mention two types of neutron sources and briefly describe how neutrons are produced in each case. [6]

(c) Neutrons do not interact with matter in the same way as charged particles do; explain why this is so, and describe briefly the type of interactions that neutrons may undergo. [6]

(d) Describe and contrast the current and pulse modes of detector operation stating the circumstances when they are used. [7]

Q3(a) (i) Describe the principal types of interaction through which swift charged particles such as electrons lose their kinetic energy when they traverse through matter. [4]

(ii) Distinguish between ionizing and radiative processes in the collision of fast electrons with matter. [4]

(iii) Indicate how the importance of each process depends on the electron energy and the Z of the absorber. [8]

(b) A ^{137}Cs source emitting 0.662MeV rays is placed in front of a 5cm x 5cm *NaI(Tl)* detector. ^{137}Cs emits a 0.662MeV gamma ray in 92% of its decays. The source-to-detector distance is 25cm. The activity of the source is 1 micro-curie = 3.7×10^4 d/s.

(i) If the number of counts in the *photopeak* for a 2-minute counting period is 13,000, calculate the *intrinsic photopeak efficiency* of the detector. [5]

(ii) Calculate also the *absolute efficiency* of the detector if the number of counts *outside* the photopeak for the same counting period is 20,000. [4]

Q4.(a) A portion of a pulse height spectrum taken with a Multi Channel Analyzer using a High Purity Germanium (HPGe) detector is given below:

Channel	Counts	Channel	counts	Channel	counts
27	162	34	511	41	430
28	199	35	506	42	418
29	251	36	550	43	366
30	301	37	599	44	329
31	347	38	590	45	295
32	432	39	510	46	232
33	434	40	491	47	159

(i) Plot the data and find the number of counts under the peak by direct summation assuming that the data consists of a constant background plus a Gaussian-shaped peak.
(ii) Estimate the constant background level.
(iii) If the energy calibration is 0.1513 keV/channel, what is the energy of the peak and its FWHM in keV? [12]

[Write your computer number on the graph paper and attach it firmly with the answer book]

(b) Gamma rays of energy 3.0 MeV are incident on an intermediate-sized *NaI(Tl)* detector.

(i)Draw the idealized response function; explain its various features. [3]

(ii) Describe how the various regions of the spectrum are related to the three well-known interaction processes of gamma rays with matter. [4]

(iii) For each process, sketch the variation of the interaction cross section with the gamma ray energy E_γ and the atomic number Z of the material. [6]

Q5.(a) Discuss three reasons explaining why the response function of a detector and its associated electronics to a mono-energetic source of radiation is NOT a mathematical delta function. [12]

(b) A 10-minute count rate of a source *plus* background gives a total of 846 counts. Background alone counted for 10 minutes gives a total of 73 counts.

What is the net counting rate due to the source alone, and what is its associated standard deviation?

[6]

(c) Counters *A* and *B* are *non-paralyzable* with dead time of 30 and 100 μs respectively.

At what true count rate will dead time losses in counter *B* be twice as great as those of counter *A*? [7]

Q6. (a) Write short notes on *full-energy peak efficiency*, *relative efficiency*, and *intrinsic efficiency* of a radiation detector. [9]

(b) Define

(i) *energy resolution* of a detector, and
(ii) *dead time* of a counting system. [6]

(c). If the resolution of a detector for 662keV gamma rays is 7%, what would be the resolution for 1280keV gamma rays? [7]

(d) When a fast neutron is captured by a heavy nuclide, the following reactions are possible: (n,α), (n,γ) and (n,p).

If the probability of the reaction is determined by the height of the potential barrier, write these equations according to their decreasing order of probability of occurrence. [3]

==End of P-411 2011 Exam==

The University of Zambia
School of Natural Sciences
Physics Department
University Examinations 2011/12
Second Semester
P-412: Nuclear Physics

Attempt <u>any four</u> questions. All questions carry equal marks. The marks are shown in brackets. Clearly indicate on the answer script cover page which questions you have attempted.

Time: Three hours. **Maximum marks = 100.**

Do not forget to write your computer number clearly on the answer book.

==

Wherever necessary use:

ε_0 = 8.85 × 10^{-12} C^2/Nm^2	$m_{hydrogen\ atom}$ = 1.007825 a.m.u.
m_n = 1.008665 a.m.u. = 939.551 MeV	m_{alpha} = 4.002603 a.m.u.
1 a.m.u. = 931.5 MeV = 1.6604 × 10^{-27} kg	m_p = 1.67 × 10^{-27} kg = 938.28 MeV
c = 3 × 10^8 m/s	m_e = 9.11 × 10^{-31} kg = 0.511 MeV
h = 6.63 × 10^{-34} J-s	e = 1.6 × 10^{-19} C
$\hbar$ = 6.58 × 10^{-22} MeV-s = 1.05 × 10^{-34} J-s	1eV = 1.6 × 10^{-19} J
1 fermi = 10^{-15} m	1 barn = 10^{-28} m^2
Avogadro's constant = 6 x 10^{23} per mole	Velocity of light = 3 x 10^8 m.sec^{-1}.
$\frac{e^2}{4\pi\varepsilon_0}$ = 1.44 MeV-fermi	$\hbar c$ = 197.33 MeV-fermi $m = \frac{m_0 c^2}{c^2} \equiv \frac{MeV}{c^2}$

$(1s_{1/2})^2$, $(1p_{3/2})^4$,$(1p_{1/2})^2$,$(1d_{5/2})^6$, $(2s_{1/2})^2$, $(1d_{3/2})^4$,$(1f_{7/2})^8$, $(2p_{3/2})^4$, $(1f_{5/2})^6$,$(2p_{1/2})^2$, $(1g_{9/2})^{10}$, [50]. $E = \frac{\hbar^2}{2\Im}\left[J(J+1) - BJ^2(J+1)^2\right]$. $\Delta E_C = \frac{3}{5}\frac{e^2}{R}\left[Z^2 - (Z+1)^2\right]$

Q1.(a) Define nuclear radius. Define skin thickness. Where do you expect skin thickness to be comparable to the nuclear radius? [9]

A form of charge distribution in the nucleus which is widely used is given by

$$\rho(r) = \frac{\rho_1}{1 + \exp[\frac{r - R_e}{Z_1}]}$$

where R_e and Z_1 are adjustable parameters.

(i) Explain how R_e can be determined from the knowledge of $\rho(r)$. [3]

(ii) Explain the physical significance of the identity $R = r_0\, A^{1/3}$. [3]

(b) Describe the principal types of evidence obtained from nucleon-nucleon interactions which suggest that nuclear forces are: [10]

i) repulsive at a very short distance

ii) of short range and attractive

iii) spin-dependent

iv) non-central and

v) charge-independent.

Q2.(a) Discuss the origin of each term in the semi-empirical mass formula: [7]

$$B(Z, A) = B_v + B_s + B_e + B_a + B_p$$
$$= a_v A - a_s A^{2/3} - a_e Z^2 A^{-1/3} - a_a \left(\frac{A}{2} - Z\right)^2 A^{-1} + a_p \delta A^{-1/2}$$

(b) The tabulated masses of ${}^{9}_{3}Li$, ${}^{9}_{4}Be$, and ${}^{9}_{5}B$ are 9.0268, 9.0122 and 9.0133 a.m.u. respectively.

Calculate in MeV the values of the Coulomb and asymmetry coefficients in the semi-empirical mass formula: [18]

$$M(A,Z) = ZM_H + NM_N - \alpha A + \beta A^{2/3} + \frac{\delta (A - 2Z)^2}{A} + \frac{\gamma Z^2}{A^{1/3}} + \lambda (A,Z)$$

Given M_H = 1.00783 amu, M_N = 1.00867 amu and 1 amu = 931.5 MeV.

Q3.(a) (i) Describe briefly the basic assumptions concerning the features of the forces involved in the single-particle shell model of the nucleus. [4]

(ii) Write down the rules for determination of the angular momenta and parities of nuclear ground states as obtained from the shell model. [8]

(iii) Does the shell model give correct values for the angular momenta of excited states, and for the magnetic moments? [4]

(b). Predict the spin and parity of the first excited state of the nuclei ${}^{31}_{14}Si$, ${}^{41}_{19}K$, and ${}^{49}_{21}Sc$.

Comment on the fact that that the observed values are

$$\frac{1}{2}^{+}, \frac{1}{2}^{+}, \text{ and } \frac{3}{2}^{+}.$$ [3+3+3]

Q4.(a) Describe briefly the rotational and vibrational motions in nuclei, indicating which types of nuclei exhibit rotational and which types exhibit vibrational spectra as their low-lying states. [7]

(b) The energy separation between the lowest member and any other higher-lying member of a rotational band in the spheroidal even-even nuclei is usually assumed to be of the form:

$$\Delta E = \frac{\hbar^2}{2\Im}[J(J+1) - BJ^2(J+1)^2]$$

where $\Im$ is the moment of inertia and B is a constant; the two parameters are obtained from a comparison ("fitting") of this equation with the observed excitation energies.

Explain the origin of the second term in the parenthesis on the right hand side of this equation. [4]

(c). In an experimentally determined ground state rotational band of, ${}^{166}_{70}Yb$, the first excited state with $J^{\pi} = 2^{+}$ has an excitation energy of 0.102 MeV and a higher-lying state with $J^{\pi} = 12^{+}$ is found to have an excitation energy of 2.17 MeV.

Obtain the angular momenta, parity, and expected excitation energies of the <u>second</u> and <u>third</u> excited states of this rotational band. [7+7]

Q5.(a) (i) Explain in short the terms *allowed* and *forbidden* beta transitions and *degree of forbiddenness.* [6]

(ii) Distinguish between the *Fermi* and the *Gamow-Teller* selection rules in beta decay of nuclei. [7]

On the basis of these selection rules, deduce:

(i) the degree of forbiddenness, and

(ii) the type (Fermi, G-T, or mixed) of the following J^{π} beta transitions:

$0^{+} \rightarrow 1^{+}$ $\quad \frac{5}{2}^{+} \rightarrow \frac{7}{2}^{-}$ $\quad \frac{1}{2}^{+} \rightarrow \frac{1}{2}^{+}$ $\quad 0^{+} \rightarrow 0^{+}$ [8]

(b) What do you understand by *super-allowed* transitions? Under what circumstances is such a transition most probable? [4]

Q6.(a) A fictitious nuclide has a $J = ½^{+}$ ground state and approximately evenly spaced excited states with $J = 9/2^{+}, 3/2^{-}, ½^{+}, 5/2^{+}, 7/2^{-}$ in order of ascending energy.

Draw an energy level diagram for this nuclide and indicate on this diagram the most likely gamma transition (down) from each level (five lines in all). Write beside the lines their multipolarity (E_1, M_2 etc). [10]

(b) In a certain nucleus the ground and first excited states have respectively the following values of

$$J^{\pi}: \frac{1}{2}^{+}, \frac{11}{2}^{-}, \frac{5}{2}^{+} \text{ and } \frac{7}{2}^{+}.$$

Identify the multipolarities of the following gamma-transitions:

$\frac{7}{2}^{+} \rightarrow \frac{5}{2}^{+}$, $\quad \frac{7}{2}^{+} \rightarrow \frac{11}{2}^{-}$, $\quad \frac{5}{2}^{+} \rightarrow \frac{1}{2}^{+}$, $\quad \frac{11}{2}^{-} \rightarrow \frac{1}{2}^{+}$ [8]

(c) Explain why the $0^{+} \rightarrow 0^{+}$ transition will not allow any gamma radiation to be emitted. Name and give some details of any possible mechanism by which the $0^{+} \rightarrow 0^{+}$ transition can take place. [4+3]

==End of P-412 Exam==

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF PHYSICS

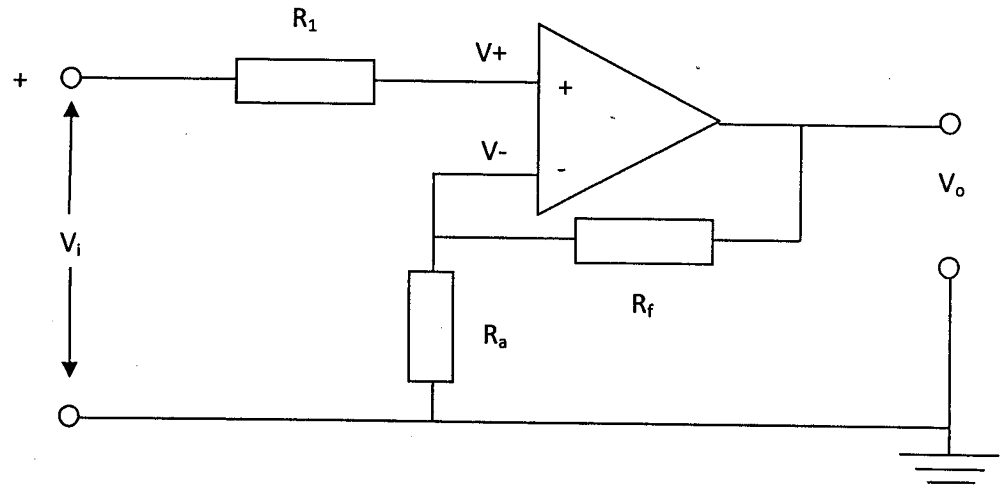
2011 ACADEMIC YEAR FIRST SEMESTER FINAL EXAMINATION

P441: ANALOGUE ELECTRONICS II

TIME: THREE HOURS MAXIMUM MARKS – 100

Attempt any four questions.
All questions carry equal marks.
The marks are shown in brackets.

Q1. (a) Find an expression for the output voltage of the following non-inverting operational amplifier circuit. **[4]**



$V_{out} =$

(b) An inverting amplifier with $R_1 = 1\ k\Omega$ and $R_f = 100\ k\Omega$ is nulled when the low dc supply voltage is 20 volts. Because of poor regulation, low dc voltage varies from 18 volts to 22 volts. Determine

(i) the change in the output offset voltage caused by the change in supply voltage **[4]**

(ii) the output voltage if $V_{in} = 10$ mV dc **[2]**

The operational amplifier is LM307 with $\frac{\Delta V_{io}}{\Delta V} = 15.85\mu V/V$ and $V_{cc} = \pm 10V$.

(c) Design a wide band pass filter with lower cut off frequency of 200Hz and upper cut off frequency of 1 kHz. The total pass band gain of the filter is 4. **[15]**

Q2. (a) Draw the circuit diagram of a peaking amplifier. What determines the peak frequency, fp, in the peaking amplifier? **[6]**

(b) The LM307 operational amplifier is used as an inverting amplifier with the following specifications.

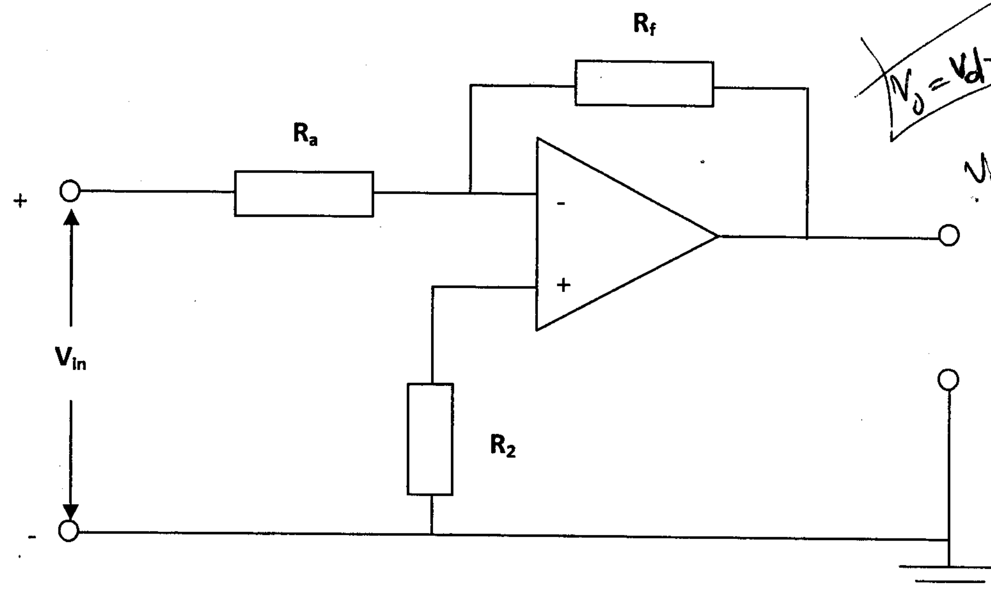
$\frac{\Delta V_{io}}{\Delta T} = 30\ \mu V/^{o}C$, $\frac{\Delta I_{io}}{\Delta T} = 300\ pA/^{o}C$, $V_s = \pm 15\ V$, $R_1 = 1\ k\Omega$, $R_f = 100\ k\Omega$, $R_L = 10\ k\Omega$

Assume that the amplifier is nulled at 25°C. Calculate the value of the error voltage at 35°C if (a) $V_{in} = 1$mV dc (b) $V_{in} = 10$ mV dc. **[9]**

$\frac{R_f}{R_1}(0.0003\ volts) \times$

$\frac{R_f}{R_1} = 4$

(c) Find an expression for the input resistance of an operational amplifier circuit with feedback shown below.



$V_o = V_d \times V_{in}$

$V_o =$

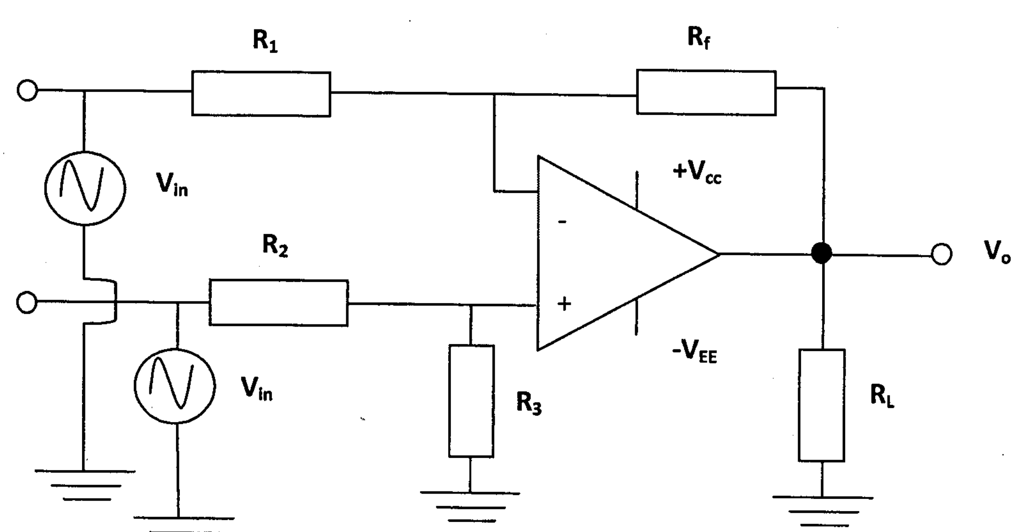
[10]

Q3. (a) In the circuit given below, $R_1 = R_2 = 1\ k\Omega$, $R_f = R_3 = 10\ k\Omega$, $V_d = 5$ mV sine wave at 1 kHz and $V_{in} = 2$ mV at 50 Hz. Calculate

(i) the output voltage at 1 kHz [3]

(ii) the amplitude of the induced 60 Hz noise at the output. [3]

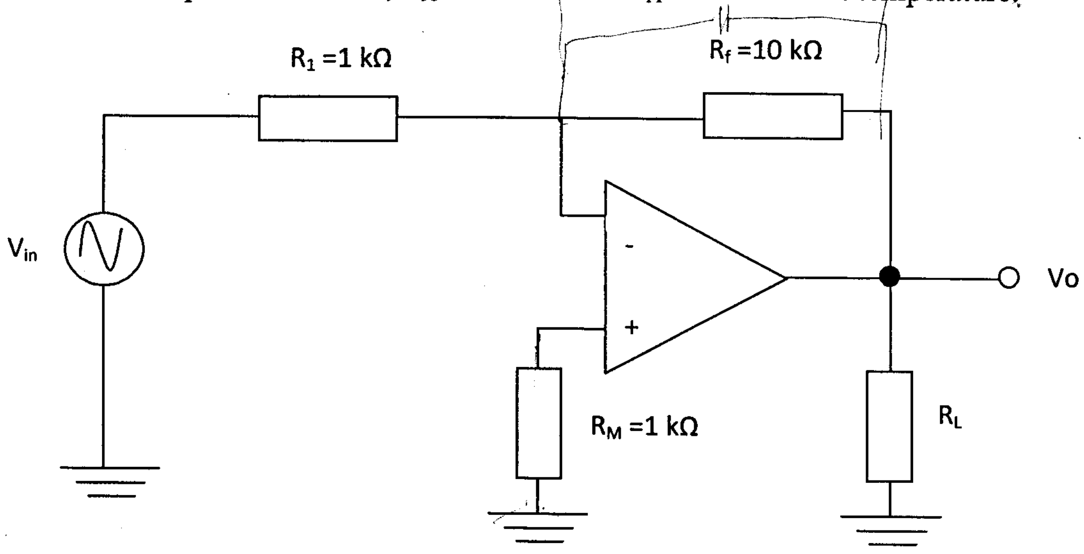
The operational amplifier is a μA741 with common mode rejection ratio, CMRR = 90 dB.



$$\frac{V_-}{R_1} - \frac{V_{in}}{R_1} + \frac{V_-}{R_f} + \frac{V_o}{R_f} = 0$$

(b) Compute the maximum possible total output offset voltages in the operational amplifier circuits given below. The operational amplifier is MC1536 with the following specifications. **[6]**

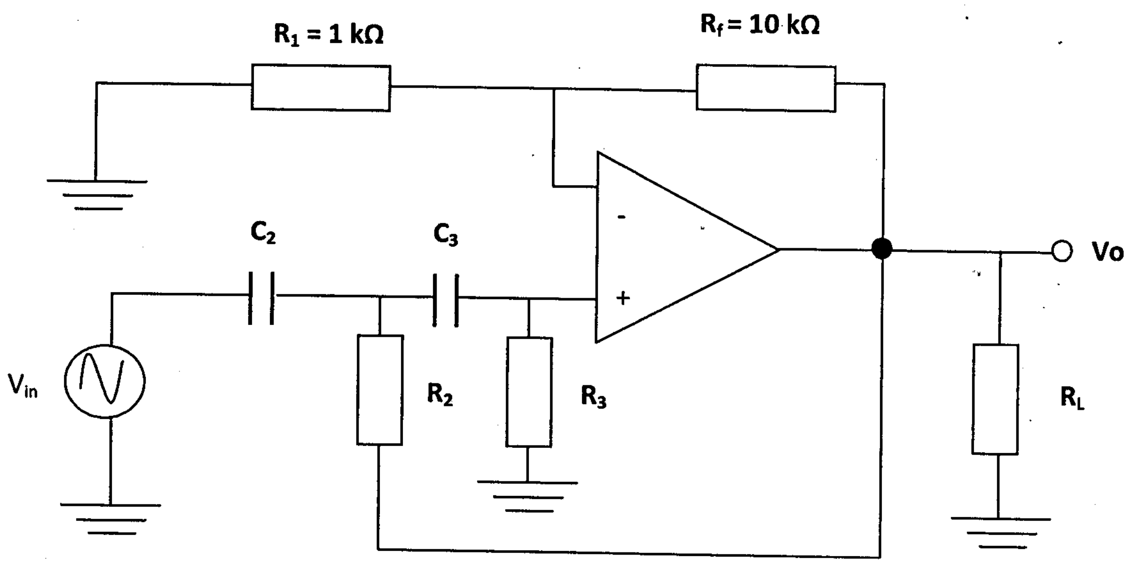
$V_{io\,max} = 7.5$ mV where $V_{io\,max}$ is the maximum input offset voltage, $I_{io\,max} = 50$ nA max where $I_{io\,max}$ is the maximum input offset current, $I_{B\,max} = 250$ nA max where $I_{B\,max}$ is the maximum input bias current, $T_A = 25°C$ where T_A is the ambient temperature.



(c) (i) Find an expression for the gain of the second order high pass Butterworth filter shown below.

(ii) Determine the magnitudes of the gain (dB) for the following frequencies of the second order high pass Butterworth filter for 1 Hz, 10 Hz, 100 Hz, 200 Hz, 500 Hz, 1000 Hz, 2000 Hz, 3000Hz.

(iii) Draw the frequency response plot of the second order high pass Butterworth filter given that R= 3.3 kΩ and C = 0.047μF. **[13]**



Q4. (a) In an experiment to investigate the discharge of a capacitor through a resistor R, the switch was first closed and capacitor allowed to charge fully to an e.m.f of a battery of V volts. The switch was then opened at a time (t = 0), and the capacitor was allowed to discharge through the resistor. Find an expression for the frequency of oscillation of the circuit. **[10]**

(b) Draw a circuit diagram for a differential amplifier and its small signal model equivalent circuit. **[9]**

(c) Write short notes on

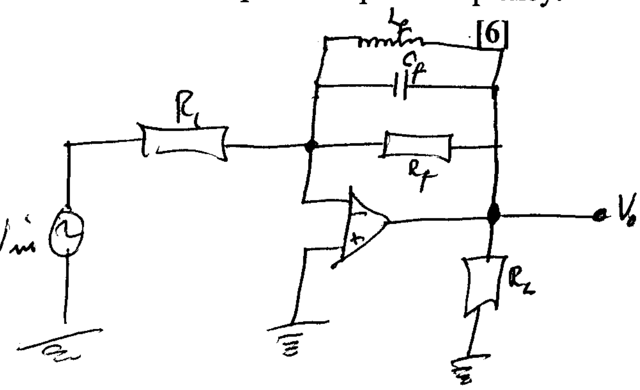
(i) Thermal voltage drift

(ii) Error voltage **[6]**

(iii) Input offset current

Q5. (a) (i) Define an oscillator circuit and give applications for oscillators. **[2]**

(ii) The peaking operational amplifier has the following values $R_1 = 1\ k\Omega$, $L = 100\ \mu H$, with internal resistance of 3Ω, $C = 0.01\ \mu F$, $R_f = 6.8\ k\Omega$ and $R_L = 10\ k\Omega$. Determine the peak frequency and gain of the operational amplifier at peak frequency. **[6]**

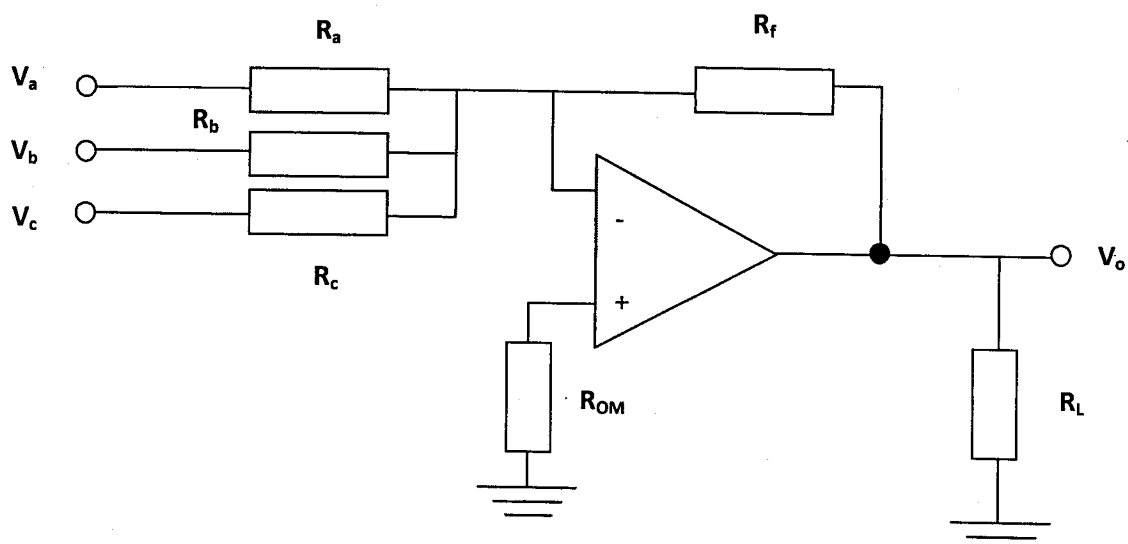


(b) In the figure shown below,

$V_a = 100$ mV $V_b = 200$ mV $V_c = 300$mV

$R_a = 3$ kΩ $R_b = 2.2$ kΩ $R_c = 1$ KΩ

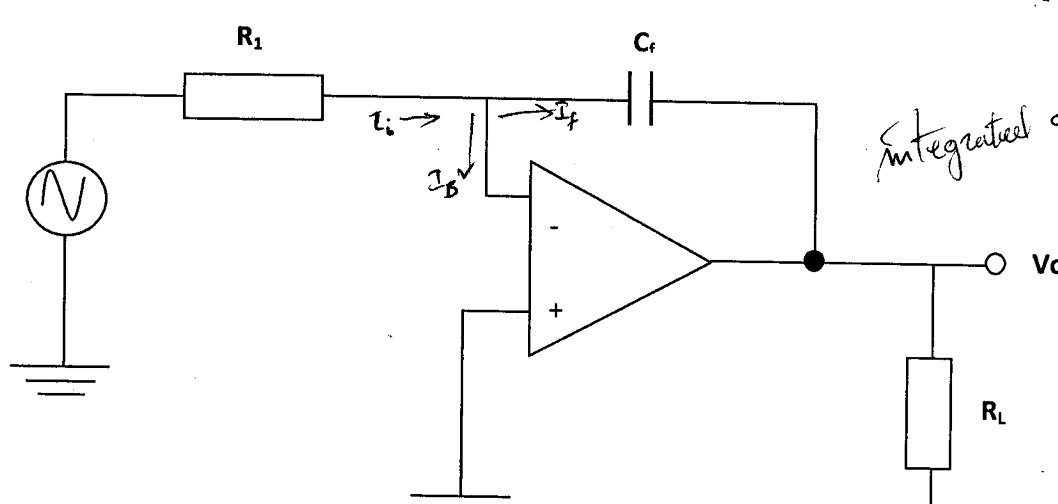
$R_f = 4.7$ kΩ $R_{OM} = 470$ Ω $R_L = 10$ KΩ



(i) Determine the output voltage. [5]

(ii) Identify the circuit from its operation. [1]

(c) Identify the circuit and derive an expression for the output voltage of the following operational amplifier circuit. [11]



Integrated operation amplifier.

therefore

$V_o = RC_f \frac{d}{dt} V$

Q6. **(a)** What is Schmitt trigger? Draw the circuit diagram and explain its operation. **[7]**

(b) For a Schmitt trigger circuit, $R_1 = 100\Omega$, $R_2 = 56\ k\Omega$, $V_{in} = 500$ mV peak-to-peak sine wave with maximum output voltage swing = ±14V.

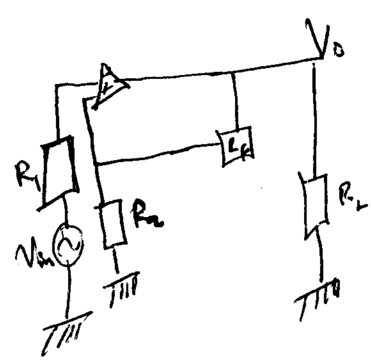
(i) Determine the threshold voltages V_{ut} and V_{Lt} **[7]**

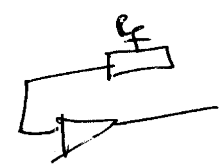
(ii) What is the value of the hysteresis voltage? **[6]**

(c) (i) What is a comparator? **[1]**

(ii) Draw the circuit of a non-inverting comparator and briefly explain its characteristics **[2]**

(iii) Draw the input and output waveforms when the reference voltage is $V_{ref} = 1$ Volt **[2]**





→ A circuit whose

A Compurentor is an inverted operation amp whose Rf has being Replaced by Cf, it

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF PHYSICS

2011/ 2012 ACADEMIC YEAR SECOND SEMESTER FINAL EXAM

P442: DIGITAL ELECTRONICS II

TIME: THREE HOURS
MAXIMUM MARKS – 100

Attempt ANY FOUR questions
All questions carry equal marks.
The marks are shown in brackets.

8085 / 8080A Instruction summary by Functional Groups

DATA TRANSFER (COPY)

Hex	Mnemonic	Hex	Mnemonic	Hex	Mnemonic	Hex	Mnemonic
40	MOV B,B	58	MOV E,B	70	MOV M,B	1A	LDAX D
41	MOV B,C	59	MOV E,C	71	MOV M,C	2A	LHLD
42	MOV B,D	5A	MOV E,D	72	MOV M,D	3A	LDA
43	MOV B,E	5B	MOV E,E	73	MOV M,E	02	STAX B
44	MOV B,H	5C	MOV E,H	74	MOV M,H	12	STAX D
45	MOV B,L	5D	MOV E,L	75	MOV M,L	22	SHLD
46	MOV B,M	5E	MOV E,M	77	MOV M,A	32	STA
47	MOV B,A	5F	MOV E,A	78	MOV A,B	01	LXI B
48	MOV C,B	60	MOV H,B	79	MOV A,C	11	LXI D
49	MOV C,C	61	MOV H,C	7A	MOV A,D	21	LXI H
4A	MOV C,D	62	MOV H,D	7B	MOV A,E	31	LXI SP
4B	MOV C,E	63	MOV H,E	7C	MOV A,H	F9	SPHL
4C	MOV C,H	64	MOV H,H	7D	MOV A,L	E3	XTHL
4D	MOV C,L	65	MOV H,L	7E	MOV A,M	EB	XCHG
4E	MOV C,M	66	MOV H,M	7F	MOV A,A	D3	OUT
4F	MOV C,A	67	MOV H,A	06	MVI B	DB	IN
50	MOV D,B	68	MOV L,B	0E	MVI C	C5	PUSH B
51	MOV D,C	69	MOV L,C	16	MVI D	D5	PUSH D
52	MOV D,D	6A	MOV L,D	1E	MVI E	E5	PUSH H
53	MOV D,E	6B	MOV L,E	26	MVI H	F5	PUSH PSW
54	MOV D,H	6C	MOV L,H	2E	MVI L	C1	POP B
55	MOV D,L	6D	MOV L,L	36	MVI M	D1	POP D
56	MOV D,M	6E	MOV L,M	3E	MVI A	E1	POP H
57	MOV D,A	6F	MOV L,A	0A	LDAX B	F1	POP PSW

ARITHMETIC

Hex	Mnemonic	Hex	Mnemonic	Hex	Mnemonic	Hex	Mnemonic
80	ADD B	CE	ACI	D6	SUI	23	INX H
81	ADD C	90	SUB B	DE	SBI	33	INX SP
82	ADD D	91	SUB C	09	DAD B	05	DCR B
83	ADD E	92	SUB D	19	DAD D	0D	DCRC
84	ADD H	93	SUB E	29	DAD H	15	DCR D
85	ADD L	94	SUB H	39	DAD SP	1D	DCR E
86	ADD M	95	SUB L	27	DAA	25	DCR H
87	ADD A	96	SUB M	04	INR B	2D	DCR L
88	ADC B	97	SUB A	0C	INR C	35	DCR M
89	ADC C	98	SBB B	14	INR D	3D	DCR A
8A	ADC D	99	SBB C	1C	INR E	0B	DCX B
8B	ADC E	9A	SBB D	24	INR H	1B	DCX D
8C	ADC H	9B	SBB E	2C	INR L	2B	DCX H
8D	ADC L	9C	SBB H	34	INR M	3B	DCX SP
8E	ADC M	9D	SBB L	3C	INR A		
8F	ADC A	9E	SBB M	03	INX B		
C6	ADI	9F	SBB A	13	INX D		

LOGICAL

Hex	Mnemonic	Hex	Mnemonic	Hex	Mnemonic	Hex	Mnemonic
37	STC	A9	XRA C	B3	ORA E	BD	CMP L
A0	ANA B	AA	XRA D	B4	ORA H	BE	CMP M
A1	ANA C	AB	XRA E	B5	ORA L	BF	CMP A
A2	ANA D	AC	XRA H	B6	ORA M	FE	CPI
A3	ANA E	AD	XRA L	B7	ORA A	07	RLC
A4	ANA H	AE	XRA M	F6	ORI	0F	RRC
A5	ANA L	AF	XRA A	B8	CMP B	17	RAL
A6	ANA M	EE	XRI	B9	CMP C	1F	RAR
A7	ANA A	B0	ORA B	BA	CMP D	2F	CMA
E6	ANI	B1	ORA C	BB	CMP E	3F	CMC
A8	XRA B	B2	ORA D	BC	CMP H		

BRANCHING

Hex	Mnemonic	Hex	Mnemonic	Hex	Mnemonic
C3	JMP	D7	RST 2	EC	CPE
C2	JNZ	DF	RST 3	F4	CP
CA	JZ	E7	RST 4	FC	CM
D2	JNC	EF	RST 5	C9	RET
DA	JC	F7	RST 6	C0	RNZ
E2	JPO	FF	RST 7	C8	RZ
EA	JPE	CD	CALL	D0	RNC
F2	JP	C4	CNZ	D8	RC
FA	JM	CC	CZ	E0	RPO
E9	PCHL	D4	CNC	E8	RPE
C7	RST 0	DC	CC	F0	RP
CF	RST 1	E4	CPO	F8	RM

CONTROL

Hex	Mnemonic
00	NOP
76	HLT
F3	DI
FB	EI
20	RIM
30	SIM

Q1. (a) If the BRA instruction is located at 40_{16} and the relative address is 10_{16}, then from which location will the next instruction be fetched? **[5]**

(b) (i) Define a microprocessor **[2]**

(ii) Draw a circuit block diagram of a microprocessor unit and in it show and name all the major features such as registers and the arithmetic logic unit. **[3]**

(iii) What are the functions of an accumulator in a microprocessor? **[2]**

(c) What are the functions of the following registers in a microprocessor?

(i) Data Register **[2]**

(ii) Address Register and **[2]**

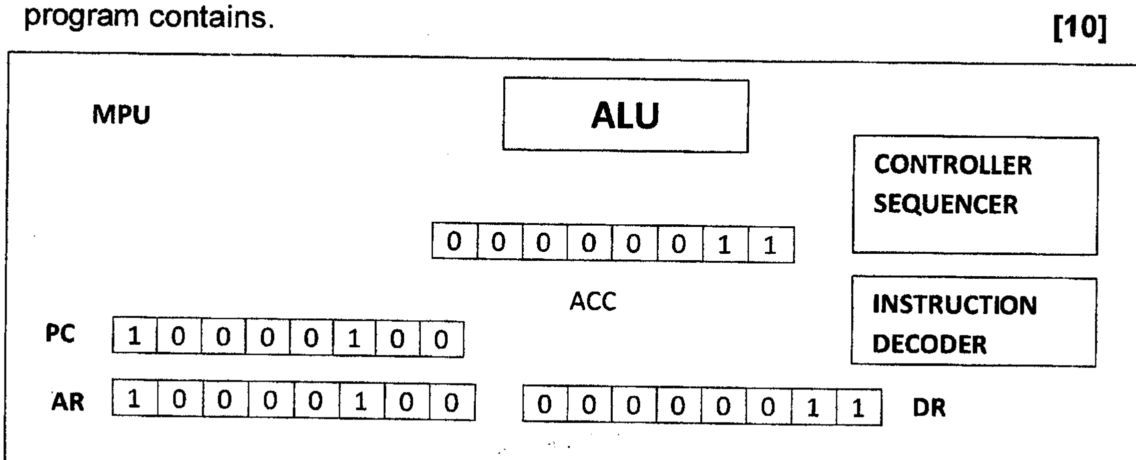
(iii) Program Counter **[2]**

(iv) What contents do accumulator, data register, program counter and address register of a microprocessor hold if the immediate addressing mode load accumulator, LDA, instruction is at location 83_{16} and the operand 03_{16} is at 84_{16}? **[7]**

Q2. (a) Since the program (a set of instructions) resides in memory, the counter must have a standard, fairly simple means of fetching the instructions, one at a time, from the memory and then executing them by the same microprocessor. What are the same series of operations that the fetch phase consists of in a microprocessor? **[5]**

(b) What is the relative address if we want to jump to location $A9_{16}$ and the BRA instruction is located at $B7_{16}$? **[10]**

(c) Assume that the microprocessor just completed the execution of the load accumulator, LDA, instruction. The state of the microprocessor after the load accumulator, LDA 03 is shown in the figure below. Find an error which the program contains. **[10]**



Address (Hex)	Memory Contents	MNEM
83	1000 0110	LDA
84	0000 0011	3
85	0010 0000	BRA
86	0000 0010	2
87	0011 1110	LILT
88	1000 1011	ADD
89	0000 0100	4

Q3.(a)To keep track of the internal conditions related to conditional branching instructions, most microprocessors have a group of single bit registers called condition code register flags. Explain in detail the functions of the following condition code registers in microprocessors.

(i) condition code zero (Z) register **[2]**

(ii) condition code negative (N) register **[2]**

(iii) condition code carry (C) register **[2]**

(iv) condition code overflow (V) register **[2]**

(b) Write and assemble a program to multiply 7 by 3 in which 7 must be added 3 times using the following instruction subset **[12]**

INSTRUCTION	MNEMONIC	OPCODE	BRANCH IF
Branching if carry clears	BCC	24	C=0
Branching if carry set	BCS	25	C=1
Branching if not equal to zero	BNE	26	Z=0
Branching if equal zero	BEQ	27	Z=1
Branching if plus	BPL	2A	N=0
Branching if minus	BMI	2B	N=1
Branching if overflow clear	BVC	28	V=0
Branching if overflow set	BVS	29	V=1

(c) Where does the microprocessor branch if the instruction BRA FD is located at F0? **[5]**

Q4. (a) Define memory map. Illustrate the memory map of 2K×8 (2048×8) memory chip with figure. [12]

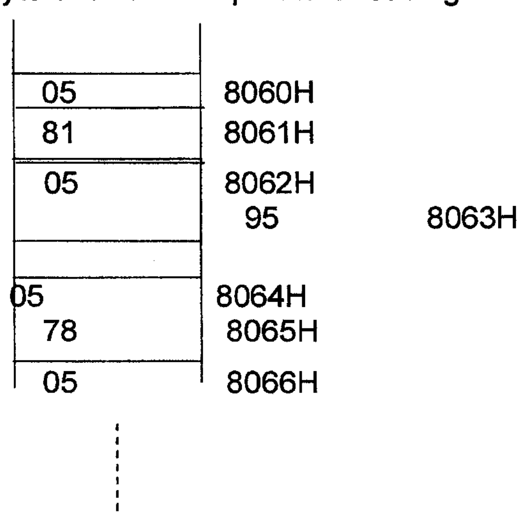
(b) Explain with figure how the memory map can be changed by modifying the hardware of the chip select line. [7]

(c) Write six differences between static RAM and dynamic RAM? [6]

Q5. A system is designed to monitor the temperature of a furnace. Temperature readings are recorded in sixteen bits and stored in memory locations starting at 8060H.

Temperature Readings (H): **05**81, **05**95, **05**78, **05**7A, **05**98

The higher order byte is stored first (eg. at memory location 8060H) followed by the lower order byte (eg. at memory location 8061H) and so on. However, the higher order byte of all the temperature readings is constant.



Draw the **flowchart** and write the **program** to transfer low order temperature readings to consecutive memory locations starting at 8080H and discard the higher order bytes. [25]

Q6. (a) Write short notes on

(i) Cache memory **(ii)** EEPROM **(iii)** Programmable Logic Devices [9]

(b) Show how the Filed Programmable Logic Circuit (FPLA) circuit can be programmed to implement the following functions. [8]

$$G_3 = A$$
$$G_2 = A\,\overline{B} + \overline{A}\,B$$
$$G_1 = B\,\overline{C} + \overline{B}\,C$$

(c) Write a program to swap the contents of two memory locations. The memory locations are 8050H with data 9EH and 8051H with data 72H. [8]

END OF P442 EXAMINATION

The University of Zambia

Department of Physics

Second Semester University Examinations - 2011

The Physics of Renewable Energy & Environment – P485

Duration: Three (3) Hours **Date: May 2012**

Instructions

- **This paper contains six (6) questions and has a total of 100 marks.**
- **Attempt any four (4) questions of your choice. Each question carries 25 marks.**
- **Show all your work clearly. Omission of essential work will result in loss of marks.**
- **Marks allocated for each question are indicated in square brackets [ ].**

Table 1: Some Physical Constants

Radius of the Sun, $R_\odot \simeq 6.96 \times 10^8$ m	Mass of the Sun, $M_\odot \simeq 1.99 \times 10^{30}$ Kg
Radius of the Earth, $R_\oplus \simeq 6.38 \times 10^6$ m	Mass of the Earth, $M_\oplus \simeq 5.97 \times 10^{24}$ Kg
1 Astronomical Unit, 1 AU $\simeq$ 149.6×10^9 m	Earth's Solar constant, $S \simeq 1352$ W.m^{-2}
Eccentricity of Earth, $\varepsilon = 0.0167$	Boltzmann constant, $k \simeq 1.38 \times 10^{-23}$ J.K^{-1}
Universal gas constant, $R = 8317$ J/Kg mole-K	Stefan-Boltzmann constant, $\sigma \simeq 5.67 \times 10^{-8}$ W.m^{-2}K^{-4}
Mean molecular mass of air, $\bar{M} \simeq 29.0$ amu	One atomic mass unit, 1 amu $\simeq 1.66 \times 10^{-27}$ Kg,
Speed of light (vacuum), $c = 2.9979 \times 10^8$ m.s^{-1}	Planck's constant, $h \simeq 6.63 \times 10^{-34}$ J-s
Specific heat of water, $c_w = 4.186$ KJ.Kg^{-1}.K^{-1}	Electron charge, $e \simeq 1.60 \times 10^{-19}$ Coulombs

›

Table 2: Values of $f(x) = 1/\sigma\left(\int_0^x \left(a/\left(x^5\left(e^{(b/x)}-1\right)\right)\right)dx\right)$ **for different** x**.**

x (μm-K)	$f(x)$	x (μm-K)	$f(x)$	x (μm-K)	$f(x)$
1100	0.001	4600	0.580	8100	0.860
1200	0.002	4700	0.594	6200	0.864
1300	0.004	4800	0.608	8300	0.868
1400	0.008	4900	0.521	8400	0.871
1500	0.013	5000	0.634	8500	0.875
1600	0.020	5100	0.646	8600	0.878
1700	0.029	5200	0.658	8700	0.881
1800	0.040	5300	0.669	8800	0.884
1900	0.052	5400	0.680	8900	0.887
2000	0.067	5500	0.691	9000	0.890
2100	0.083	5600	0.701	9100	0.893
2200	0.101	5700	0.711	0200	0.895
2300	0.120	5800	0.720	9300	0.898
2400	0.140	5900	0.730	9400	0.901
2500	0.161	6000	0.738	9500	0.903
2600	0.183	6100	0.746	9600	0.905
2700	0.205	6200	0.754	9700	0.908
2800	0.228	6300	0.762	9800	0.910
2900	0.251	6400	0.770	9900	0.912
3000	0.273	6500	0.776	10000	0.914
3100	0.296	6600	0.783	11000	0.934
3200	0.318	6700	0.790	12000	0.945
3300	0.340	6800	0.796	13000	0.955
3400	0.362	6900	0.802	14000	0.963
3500	0.383	7000	0.808	15000	0.969
3600	0.404	7100	0.814	16000	0.974
3700	0.424	7200	0.819	17000	0.978
3800	0.443	7300	0.824	18000	0.981
3900	0.462	7400	0.830	19000	0.983
4000	0.483	7500	0.834	20000	0.986
4100	0.499	7600	0.840	30000	0.995
4200	0.516	7700	0.844	40000	0.998
4300	0.533	7800	0.848	50000	0.999
4400	0.549	7900	0.852		
4500	0.564	8000	0.856		

Table 3: Formulae That May Be Useful

$\delta = 23.45\sin\left[\frac{360}{365}(d_n + 284)\right]$	$\dot{Q} = \frac{\left(T_1 - T_{(n+1)}\right)}{\sum_1^n \frac{L}{kA}}$
$n_f = \sqrt{n_g}$	$dq = du + Pdv$
$J = \varepsilon\sigma T^4$	$R = Nk$
$\text{cr}_{\max} = \left[\frac{2\sin\alpha}{0.0046}\right]^2$	$P = F^{dir} \times \pi R^2 \sin^2\alpha$
$F_\lambda^{(dir)} = \mu S_\lambda \exp\left(-\frac{\tau_\lambda}{\mu}\right)$ where $\mu = \cos\theta$	$\nabla\cdot(\boldsymbol{\kappa}\nabla T) + q_g = \rho c\frac{\partial T}{\partial t}$
$\frac{\dot{Q}}{l} = 2\pi\kappa\frac{(T_1 - T_2)}{\log_e(r_2/r_1)}$; $T_1 > T_2$ and $r_2 > r_1$	$T_{f,e} = T_B - \left[T_B - T_{f,i}\right]\exp\left(-\frac{U_L L}{\dot{m}c}\right)$
$s_f = \frac{m\lambda_0}{4n_f}$	$\text{cr} = \frac{A}{A'}$
$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$	$m = \frac{(f/p)}{(1 - f/p)}$
$r_\parallel = \left[\frac{n_r^2\cos\theta_i - n_i\sqrt{n_r^2 - n_i^2\sin^2\theta_i}}{n_r^2\cos\theta_i + n_i\sqrt{n_r^2 - n_i^2\sin^2\theta_i}}\right]^2$	$r_\perp = \left[\frac{n_i\cos\theta_i - \sqrt{n_r^2 - n_i^2\sin^2\theta_i}}{n_i\cos\theta_i + \sqrt{n_r^2 - n_i^2\sin^2\theta_i}}\right]^2$
$R = r\left[1 + \frac{\alpha^2(1-r)^2}{1 - \alpha^2 r^2}\right]$	$T = \frac{\alpha(1-r)^2}{1 - \alpha^2 r^2}$
$J_0 = DT^3\exp\left(-\frac{\varepsilon_g}{kT}\right)$	$\tilde{T}_m = \frac{\tilde{T}_0 - \tilde{T}_i}{\log_e\left(\tilde{T}_0/\tilde{T}_i\right)} = \frac{\tilde{T}_i - \tilde{T}_0}{\log_e\left(\tilde{T}_i/\tilde{T}_0\right)}$
$\left(\dot{Q}_{12}\right) = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right) + \left(\frac{1-\varepsilon_2}{\varepsilon_2}\right)\frac{A_1}{A_2} + \left(\frac{A_1 + A_2 - 2A_1F_{1-2}}{A_2 - A_1F_{1-2}}\right)}$	$\alpha = \exp\left(-\frac{n\kappa s}{\sqrt{n^2 - \sin^2\theta_i}}\right)$

The Quadratic Lagrange Interpolation Formula

Given a given a set of consecutive data $\left[x_{i-1}, f\left(x_{i-1}\right)\right], \left[x_i, f\left(x_i\right)\right]$ and $\left[x_{i+1}, f\left(x_{i+1}\right)\right]$, the Quadratic Lagrange polynomial can be used to estimate the value of $f(x)$ at any given x for $x_{i-1} \le x \le x_{i+1}$ as

$$\boxed{f(x) = L_{i-1}(x) f\left(x_{i-1}\right) + L_i(x) f\left(x_i\right) + L_{i+1}(x) f\left(x_{i+1}\right)},$$

where

$$L_{i-1}(x) = \frac{\left(x - x_i\right)\left(x - x_{i+1}\right)}{\left(x_{i-1} - x_i\right)\left(x_{i-1} - x_{i+1}\right)},$$

$$L_i(x) = \frac{\left(x - x_{i-1}\right)\left(x - x_{i+1}\right)}{\left(x_i - x_{i-1}\right)\left(x_i - x_{i+1}\right)} \quad \text{and}$$

$$L_{i+1}(x) = \frac{\left(x - x_{i-1}\right)\left(x - x_i\right)}{\left(x_{i+1} - x_{i-1}\right)\left(x_{i+1} - x_i\right)}.$$

1. (a) Assume the Sun to be a blackbody at 5777 K.

(i) Find the wavelength at which the maximum monochromatic emissive power occurs. **[2]**

(ii) Use the data given in table 1, and the Quadratic Lagrange Interpolation formula (refer to the given formulas above), to show that the energy of solar radiation with wavelengths less that $0.38\mu\text{m}$ is 10.0%. **[4]**

(iii) Use the data given in table 1, and the Quadratic Lagrange Interpolation formula, to show that the energy of solar radiation with wavelengths less that $0.78\mu\text{m}$ is 54.1%. **[4]**

(iv) Hence, or otherwise, find the energy fraction from the Sun that is in the visible part of the electromagnetic spectrum (i.e. 0.38 to $0.78\mu m$). **[1]**

(v) What is meant by "Intensity of radiation" ? **[3]**

(vi) State the Lambert's cosine law. **[3]**

(b) The density profile of an adiabatic expanding gas may be expressed as

$$\rho = \frac{\rho_o T_o}{T_o - Lz}\left[\frac{T_o - Lz}{T_o}\right]^{\frac{\gamma}{\gamma-1}},$$

where L is the adiabatic lapse rate, T_o is the absolute temperature of gas at sea level, ρ_o is the density of the gas at sea level and $\gamma = 1.4$ (for diatomic gas) is the ratio of specific heat capacity at constant pressure c_p to the specific heat capacity at constant volume c_v.

(i) Show that the corresponding pressure profile as a function of height is given by

$$P = P_0\left[1 - \frac{Lz}{T_0}\right]^{\frac{\gamma}{\gamma-1}}.$$ **[4]**

(ii) Use the given information to show that the adiabatic lapse rate for Lusaka with a vertical height of 1150m above sea level at an atmospheric pressure of 659mmHg is 9.477K/km. **[4]**

2 (a) The total optical thickness of a gray atmosphere and the single scattering albedo are 0.25 and 0.65 respectively. The ground reflectivity is about 0.25.

(i) Find the direct flux on a horizontal surface for the zenith angle of 45°. **[6]**

(ii) Estimate the total flux intercepted on a horizontal surface if the diffuse components of the radiation are assumed to be

$F^{\downarrow(diff)} = 123.0\text{W/m}^2$ and

$F^{\uparrow(diff)} = 46.0\ \text{W/m}^2$. **[6]**

(b) A standard cast iron pipe (inner diameter $= 40.0$ mm and outer diameter $= 45.0$ mm and thermal conductivity $\kappa = 20\ \text{W/m}^\circ\text{C}$) is insulated with magnesium insulation ($\kappa = 0.02\ \text{W/m}^\circ\text{C}$). Temperature at the interface between the pipe and insulation is 300°C. The allowable heat loss per unit length of pipe is 650 W/m and for safety, the temperature of the outside surface of insulation must not exceed 80°C.

(i) Write down the general formula for the heat lost per unit time per unit length of a cylindrical tube under steady state conditions, defining all variables in the equation. **[2]**

(ii) Using the expression in (i) show, with clear explanations that, the energy balance equation for this system can be expressed as

$$\frac{650}{2\pi} = \times 20.0 \times \frac{(T_1 - 300)}{\log_e(d_2/d_1)} = 0.02 \times \frac{(300-80)}{\log_e(2r_3/d_2)},$$

where T_1 = the temperature of the inside surface of the pipe, d_1, d_2 = diameters of inner and outer cylinders respectively, while r_3 = radial distance from centre of the pipe to the outermost surface of the insulator. **[6]**

(iii) Find the minimum thickness of insulation required. **[3]**

(iv) Calculate the inside surface temperature of the pipe. **[2]**

3. (a) The total thermal intensity from the entire atmosphere arriving on the horizontal ground at an angle of incidence θ_z is obtained by evaluating the following integral.

$$I^{thermal} = \frac{1}{\pi}\int_0^{\lambda_\tau} B_\lambda(T)\exp\left[-\frac{(\tau_\lambda - t_\lambda)}{\mu}\right]\frac{dt_\lambda}{\mu}.$$

If the atmosphere was isothermal and the thermal thickness is a constant and equal to τ over the spectral range $2\mu m \leq \lambda \leq 20\mu m$;

(i) Show that the above integral reduces to

$$I^{thermal} = \frac{B_\lambda(T_0)}{\pi}\left[1 - \exp(-\tau/\mu)\right].$$ **[9]**

(ii) The expression in (i) above may be integrated over the entire spectrum to obtain the sky temperature as

$$T_{sky} = \left[1 - 2E_3(\tau)\right]^{1/4} T_0,$$

where $E_3(\tau)$ = the Gold function of the third order. Calculate the air temperature if the sky temperature is 280K and $E_3(2) = 0.03$. **[4]**

(b) Water enters a 20.0 m long convoluted pipe at a temperature 27 °C and at a flow rate of 0.005 kg/s. If the walls of the pipe are maintained at $70^{\circ}C$ by a heat bath and the average heat transfer coefficient per unit length of pipe is $6.0\ W/m^{\circ}C$,

(i) Find the temperature of the water when it exists. **[5]**

(ii) Determine the heat extraction rate. **[3]**

(iii) Sketch the graph of water temperature rise against length of pipe for the above process. **[2]**

(iv) Explain why the rate of heat extraction would decrease if the mass flow rate of water were increased? **[2]**

4. (a) The reflection coefficient for the glass-air interface at normal incidence is given by

$$r = \left(\frac{n_g - 1}{n_g + 1}\right)^2.$$

(i) Show that the refractive index of a thin film n_f deposited on a glass substrate of refractive index n_g should be related as

$n_f = \sqrt{n_g}$. **[4]**

(ii) Hence, find the minimum thickness of this thin film deposited in order to make the glass non-reflective at normal incidence for light of wavelength $0.6\mu m$. Use $n_g = 1.5$. **[3]**

(b) The maximum concentration ratio for a spherical collector of radius 55cm and of rim angle of 35° is obtained to be 12,663. Find the

(i) aperture area of the spherical collector. **[3]**

(ii) minimum image area. **[3]**

(iii) average power falling on a small absorber at the focal point if the direct solar flux is $850 W/m^2$. **[3]**

(c) (i) What is meant by the term green-house effect? [2]

(ii) An ideal selective absorber coating has a sharp cutoff at $\lambda_c = 5\mu m$. Find the steady state temperature of this surface coating when the incident solar flux is 1000W/m^2. Neglect back losses and convection losses. [7]

5 (a) (i) Define bulk transitivity. [3]

(iii) A solar glazing 1.2 cm (0.2cm) thick has a refractive index of 1.53 and bulk extinction coefficient of 0.2cm^{-1} (0.08cm^{-1}). Find the angle of incidence if 96% of the solar flux passing through the upper surface reaches the lower surface after a single transit. [9]

(b) A wall of a furnace is made up of inside layer of silica brick 120.0 mm thick covered with a layer of magnesite brick 240.0 mm thick. The temperature at the inside surface of silica brick wall and outside surface of magnesite brick wall are 500°C and 80°C respectively. The contact thermal resistance between the two walls at the interface is 0.0035°C/W per unit area of wall. If thermal conductivities of silica and magnesite bricks are 1.7 W/m°C and 5.8 W/m°C, calculate

(i) the rate of heat loss per unit area of walls, [6]

(ii) the temperature drop at the interface. [7]

6. (a) (i) What is meant by the term "solar constant" ? [2]

(ii) What is meant by the term "air mass zero" ? [2]

(iii) State Kirchhoff's law of thermal radiation. [2]

$\sin\theta_i = \sqrt{n^2 - \left[\frac{-mcs}{\ln x}\right]^2}$

$\ln T = \ln 70 + 24000 \ln 43 \times 2.7$

$\ln T = 4.248 + 243725.76$

$T = e$

(b) The intensity of radiation from a surface of area element dA_1 to another of area element dA_2 is given by

$$I = \frac{dQ_{1-2}}{dA_1 \cos\theta \times \frac{dA_2}{r^2}},$$

where dQ_{1-2} = the rate of radiation heat transfer from dA_1 to dA_2 separated by distance r. Prove that if the intensity I is constant, then

$E = \pi I$,

where E = the total emissive power of a diffuse surface and I = radiant intensity emanating from the surface. **[9]**

(c) The flow rates of hot and cold water streams running through parallel flow heat exchanger are 0.2 Kg/s and 0.5 Kg/s respectively. The inlet temperatures on the hot and cold sides are 75°C and 20°C respectively. The exit temperature of hot water is 45°C. If the individual heat transfer coefficients for the heat exchanger on both sides are 650 $W/m^2.°C$, calculate the

(i) logarithmic mean temperature difference **[5]**

(ii) effective area of the heat exchanger. **[5]**

End of P485 Examination