LEARNERS' PROBLEM-SOLVING PROCESSES IN LINEAR PROGRAMMING AT SELECTED SCHOOLS OF LUSAKA DISTRICT OF ZAMBIA

BY

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DECLARATION

I, **Gabriel Sandambo,** hereby declare that the work contained in this dissertation has been composed and written by me and that this work is as the result of my own individual effort. I further sincerely declare that this research has not been previously published for any academic award at any other higher education institution, and that all the sources that I have used or quoted have been indicated and acknowledged accordingly by means of complete references.

Signature (candidate)……………………………… Date……………………....2024

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APPROVAL

This Dissertation of **Gabriel Sandambo** has been approved as fulfilling the requirements for the award of the Degree of Master of Education in Mathematics Education (M.Ed.- Mathematics Education) by the University of Zambia.

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ABSTRACT

This study investigated problem-solving processes of Grade 12 learners in linear programming. A total of 24 Grade 12 learners and 2 teachers of Lusaka district of Zambia were purposively sampled for the study from two selected secondary schools. The study employed the qualitative research approach which followed a descriptive case study design. Data was collected using lesson observations, semi-structured interviews and focus group discussions. Thematic analysis was used to analyse data. Polya's four stages of problem-solving: understand the problem, devise the plan(s), execute the plan(s) and looking back, and also the cognitive process dimension of the revised Bloom's taxonomy guided the analysis of the data. The study was guided by the following research questions: How do Grade 12 learners engage in solving problems in linear programming? What factors affect Grade 12 learners' problem-solving processes in linear programming problems? How can Grade 12 learners' problem-solving processes in linear programming be enhanced?

The findings showed that learners in their quest to understand linear programming questions would read the question(s) at least once, though it is not a guarantee for correct solution, they would identify key statements and list phrases with their associated mathematical symbols, and the cognitive demands at this stage were retrieving and recognising factual knowledge. At the devise-a-plan stage, learners were able to transform the conditional statements into inequalities, devise tables of values showing possible pair of coordinates and showed rough-workings. Although, some learners experienced difficulties in comprehending the language of linear programming which led to deriving incorrect inequalities. At this stage, cognitive demands included interpreting and translating the linear programming problem(s) to construct their own meaning and organise facts.

At the execute the plan(s) stage, learners could graph the inequalities, identify the feasible region with its boundary points, derive the profit function, test the boundary points into the profit function to determine the pairs of coordinates (x, y) that would yield the max/minimum profit and calculate the maximum/minimum profit. However, some learners experienced difficulties in arriving at correct solutions owing to lack of procedural and conceptual knowledge on how to graph the inequalities and calculating the maximum profit. This stage involved different cognitive levels from *Applying* through to *creating* and learners would engage in different cognitive processes from applying inequalities to deriving the feasible region, to evaluating the maximum/minimum profit. The findings further showed that after solving problems in linear programming, learners made little or no effort to engage in a critical examination of the solution arrived at, so as to ascertain whether it is correct and whether the plan can be used to solve another problem(s). At this stage, learners were also operating at the evaluating cognitive level and could engage in the cognitive process of verification and reflection on the problem-solving processes applied for mistake identification and knowledge consolidation.

The study showed that learners' problem-solving processes were constrained by the misunderstanding of the mathematical language of linear programming, poor foundation in pre-requisite knowledge of linear programming, inconsistency of shading the unwanted region at junior and senior secondary levels and lack of monitoring skills for checking the steps of their solving process affected the learners' problem-solving processes as they were unable to identify and correct errors and mistakes. To enhance or improve learners' problem-solving processes, it was suggested that there is need for teachers to be ensuring that learners have the prerequisite knowledge before introducing Linear Programming and there is need for consistency of shading either the unwanted or wanted region at both junior and secondary level to prevent learner misconception. It was further suggested that there is need to promote learner group-working as an approach to teaching linear programming. More learner interaction with authentic and situational problems is another measure that may enhance problem solving process in linear programming.

Key terms: Linear programming, problem-solving processes, cognitive demands.

DEDICATION

To

My Father and Mother

Richard Sandambo and **Alice Kamwaya**

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ABBREVIATIONS AND ACRONYMS

AIEMS Action to Improve English, Mathematics and Science

ECZ Examinations Council of Zambia

MoGE Ministry of General Education

MESVTEE Ministry of Science, Vocational Training and Early Education

NAS National Assessment Survey

NCTM National Council of Teacher of Mathematics

ZAMSTEP Zambia Mathematics and Science Teachers Education Project

CHAPTER ONE

INTRODUCTION

1.1 OVERVIEW

The first chapter presents the background of the study, statement of the problem, purpose of the study, study objectives, research questions, significance of the study, scope of the study (delimitations and limitations) theoretical and conceptual frameworks as well as the operational definition of terms.

1.2 BACKGROUND

Mathematics plays a critical role in personal, national and global development. It is essential in sciences, technology, business and many areas of ICT. Mathematics forms the basis of most scientific and industrial research. For instance, the comprehension of many complex systems and structures in the modern era is only possible through the use of Mathematics. Moreover, the development of every country depends upon the development of science and technology, and the development of science and technology depends upon the subject of Mathematics (Mehmood, 2014). In terms of personal development, Mathematics is a discipline that aids in the development of mental processes that improve, among others, logical and critical-thinking, accuracy and problem-solving skills needed in the making of decisions (Cockroft, 1982).

In this era of Internet of Things and Artificial Intelligence, apparently where robots are taking over the place of humans in industry, people who lack exceptional problem-solving skills may in all likelihood find it difficult to fit into this competitive environment, especially when it comes to job acquisition (Awuah, 2018). Incontestably, mathematics is the king, queen and servant or rather the backbone of all sciences (Cockcroft, 1982) and problem-solving is among the critical and essential skills of the 21st century (Mehmood, 2014). The ultimate reason of mathematics is to solve problems, Schoenfeld (1987) asserted that learners do not only learn mathematics while solving problems but also simultaneously develop problem-solving skills and strategies. Proficiency in problem-solving contributes to self-actualization and leads to greater opportunities for employment as well as contributing to economic growth (Awuah, 2018; Hanushek, Wößmann, Jamison & Jamison, 2008).

The problem-solving process requires individuals to use previously acquired knowledge, skills and understanding to satisfy the demands of an unfamiliar situation (Krulik & Rudnick, 1980). This implies that for one to be a good problem solver, one ought to have acquired certain skills that could engineer the easy solving of the problem from previous experience. The implication is that problems have some degree of difficulty that requires special skills to tackle. The key to becoming a good-problem solver lies in the cognitive domain, since the process is a cognitive one (Awuah, 2018). Schoenfeld (1992, p.3) opined that "the main goal of mathematics instruction should be to train learners to become competent problem-solvers." Problem-solving is one of the most essential cognitive skills needed in many professions as well as in everyday life (Cockroft, 1982).

The National Council of Teachers of Mathematics (NCTM, 1980) noted that, mathematics contains tools that help one in solving problems. In the early 1980s the NCTM proposed that problem-solving should be the priority in school mathematics (NCTM, 1980) and because of its importance, it has been included in most mathematics curricula of which the Zambian mathematics curriculum is no exception (MEVSTEE, 2013). It is not surprising that many researchers have found a strong relationship between mathematics and problem-solving (Che, Wiegert & Threlkeld, 2012; Sangcap, 2010). From this premise, it is a fact beyond dispute that a strong foundation in mathematical concepts prepares young people in the acquisition of essential problem-solving skills.

This notwithstanding, evidence abounds that the learning of mathematics in general and some topics (e.g., Linear programming) in particular seem to pose a challenge to most learners in school, locally and globally. In Zambian Grade 12 national examination results, very low percentage passes are recorded mostly in mathematics vis-à-vis other subjects (Examinations Council Zambia Reports, 2015, 2016, 2017 & 2018). For instance, in 2015, the percentage of failure was 50.6 percent of all the candidates who sat for Grade 12 GCE Examinations (ECZ, 2015), giving a pass percentage of 49.4 percent. According to ECZ (2017, p.27), "mathematics still poses a challenge to candidates even under the revised curriculum, it had the highest proportion of failure rate at 51.33 percent in 2016". The 2013 revised curriculum was implemented in Zambia starting from 2014, it adopted the Outcomes-Based Education which seeks to link education to real life experiences as it gives learners problem-solving skills to criticize, analyse and practically apply knowledge (MoGE, 2013). However, even with the revised curriculum, many learners continued failing the subject as revealed in the chief examiners' reports (ECZ, 2017). The Figure below shows performance in Mathematics at final examinations from 2015 to 2018.

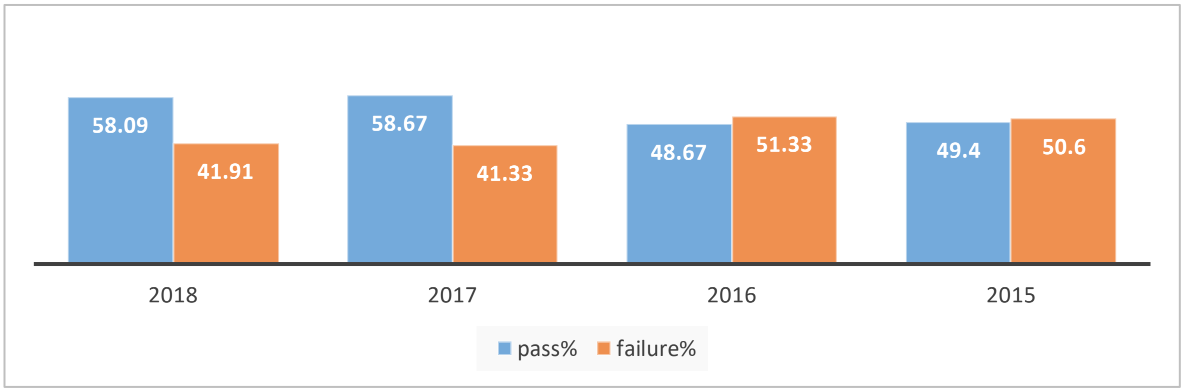


Figure 1.1: Annual Performance in Mathematics from 2015-2018.

In Zambia, underachievement in mathematics at grade 12 level has mostly been attributed to learners' lack of mastery of skills and poor essential workings. According to the ECZ Examination Performance Review Report, in 2015 low performance in Mathematics was as a result of candidates' poor presentation of workings and it was recommended that essential workings that lead to the final answer should be critically analysed by teachers when marking class work in order for them to provide appropriate feedback to the learners (ECZ, 2015).

In 2016, it was revealed that: "Some candidates omitted necessary steps in their solutions (poor working essentials) and it was recommended that teachers should be emphasizing the importance of showing essential working" (ECZ, 2016, p.28). Poor performance in 2017 was attributed to candidates' lack of mastery skills and poor essential working and it was recommended that teachers should ensure that showing essential working during instruction in the classroom is emphasized if candidates have to earn all the marks in any assessment as well as examinations (ECZ, 2017). Particularly, with regard to the topic Linear Programming in mathematics paper two, question 11, the report gives the following statement: *"…Learners had the challenges: to use the equality sign instead of inequality, failure to draw the inequalities correctly, failure to write correct objective function required to give maximum extreme values and use of wrong maximum values for maize and groundnuts arising from previous…"* (ECZ chief examiner's report, 2017, p.61).

From the reports of 2014, 2015, 2016, 2017 and 2018, the Examination Council of Zambia gives the following conclusive report: *"The sample scripts showed that some candidates have challenges that are related to* ***mastery of concepts and the skills in the syllabus rather than making errors in their calculation. Linear programming, Trigonometry, Earth Geometry and Calculus have been reported to be most challenging topics to the candidates even in the previous years.*** *Therefore, teachers of mathematics should always include these topics in their in-house Continuous Professional Development (CPD) and cluster meetings to address the challenges faced by candidates…"* (ECZ,2014, 2015, 2016, 2017 & 2018).

In all the reports, lack of Mastery Skills and poor working essentials has been cited as the common factor and a major deficiency in learners' solving of mathematical problems and according Polya (1988), this is the result of inadequate problem-solving processes which involve having challenges in: understanding the problem; devising a plan(s) to solve the problem, executing the plan(s) to solve the problem and looking back to ensure the solution realised is acceptable or otherwise (Polya, 1988).

In the effort to improve learning quality and achievement in mathematics, the government through the Ministry of Education and cooperating partners introduced a number of interventions (programmes), among others, such as Zambia Mathematics and Science Teachers Education Project (ZAMSTEP) in 1980 which aimed at improving the instructive skills and knowledge of teachers with diplomas, Strengthening of Mathematics, Science and Technology Education (SMASTE) (Carmody 2004, Mubanga 2020) and School Based Assessment (SBA) aimed at enhancing teaching and learning in the classroom through improved teacher capacity to identify what learners know, understand and can do (MESVTEE, 2019). However, assessment reports have consistently reported poor quality of mathematics education. Learner's lack of mastery skills and poor essential workings mostly contribute to low learner-achievement in mathematics in general and particularly in linear programming.

The purpose of education is preparing learners for life after school. It is therefore imperative that the teaching and learning of mathematics activities should be tailored in such a way that they should be realistic in nature. They should be providing the transformation of word problems of everyday life into mathematical problems whose intellectual power is motivation for further learning resulting into individual development and intellectual improvement (MoGE, 2013).

However, Boaler (1998, p.41) in her project report, stated that: "…*There is a growing concern among mathematics educators that many students are able to learn mathematics for 11 years or more but are then completely unable to use this mathematics in situations outside the classroom context. In various research projects adults and students have been presented with tasks in which they are required to make use of mathematics they have learned in school. These projects have shown that in real-world mathematical situations, adults and students do not use school-learned mathematical methods or procedures…"* (Boaler, 1998 p.41).

In Boaler's quotation, it is explicit that there is a breach between the use of real-life mathematics situations problems and the school-learnt mathematics which lead mostly to acquisition of inert concepts rather than the development of useful and robust knowledge. This is supported by Fruedenthal (1973) who posited that when children learn mathematics in an isolated fashion divorced from their real experiences, they will quickly forget it and will not be able to apply it. It is common for students to acquire algorithms, routines and de-contextualized definitions that they cannot use and therefore lie inert. Students can often manipulate algorithms, routines, and definitions they have acquired with apparent competence and yet cannot reveal to their teachers or themselves, that they would have no idea what to do if they came upon a real situation that would require them to apply the mathematical concepts they learnt in school. Brown, Collins and Duguid (1989), in proposing their model of situated cognition, argued that meaningful learning of mathematics will only take place if it is embedded in the social and physical context within which it will be used. This type of education provides an authentic context that reflects the way the knowledge will be used in real-life; it preserves the full context of the situation without fragmentation and decomposition, and it invites exploration and allows for the natural complexity of the real world (Brown, Collins & Duguid, 1989).

Linear programming has all the aforementioned ingredients in the teaching and learning of mathematics whose usefulness is feasible even in life after school. This establishes proper connections between mathematics as a discipline and the application of mathematics in the real-world contexts, and this should be highly promoted so as to bridge the gap between learning mathematics and use (MoGE, 2013). Linear programming is a mathematical technique for finding optimal solutions to problems that can be expressed using linear equations and inequalities. Real-world problems can be represented accurately by the mathematical equations of linear programming: the method can help to find the best solution to the problems and it can provide reasonably realistic representations of many real-world problems especially if a little creativity is applied in the mathematical formulation of the problem (Nkambule, 2009). Linear programming has the power in bringing real life problems of economics, business and decision-making skills into the classroom environment and in providing skills for determining best outcomes in a given mathematical model involving some linear relationship. It is against its powerfulness in bringing real-life problems to formal learning of mathematics that it was included even before in the school mathematics curriculum.

It is on this premise that this study explored learners' problem-solving processes in linear programming as a step towards solving the problem of learners' underachievement, with the hope of improving learners' problem-solving in the topic of linear programming and in mathematics in general. The findings of this study could be informative to various stakeholders about the weaknesses of learners in solving linear programming problems and help to improve their performance.

1.3 STATEMENT OF THE PROBLEM

The importance of mathematics cannot be overemphasised as it requires problem-solving which is among the essential skills of the 21st century. Linear Programming at Grade 12 level, is one of the topics that establishes the connection between mathematics as a discipline and the application of mathematics in the real-world contexts, through problem-solving (MoGE, 2013). However, the chief examiners' reports (ECZ 2014-2018) have consistently stated that most learners underachieve at Grade 12 final examination in mathematics in general and linear programming in particular and this is mostly attributed to learners' lack of mastery skills and poor essential workings (ECZ 2014, 2015, 2016 & 2017), and these are as a result of learners' inadequate problem-solving processes of: understanding the problem; devising plan(s) for solving the problem; executing the plan(s) to solve the problem; and looking back to ensure the solution realised is acceptable or otherwise (Mehmood, 2014; Polya, 1957). This notwithstanding, there is a lot of attention now given to competences and 21st century skills (Awuah, 2018). Problem solving is critical both to the attainment of competences in mathematics and the attainment of 21st century skills (Mehmood, 2014). As some mathematics education scholars have said, problem solving is at the heart of mathematics and a strong foundation in mathematical concepts prepares young people in the acquisition of essential problem-solving skills (Schoenfeld,1987). On this premise, this study aspired to explore and focus on understanding learners' problem-solving processes in Linear programming at grade 12 level. It also sought to establish constraints that affect learners' problem-solving processes and determine measures that would enhance learners' problem-solving in Linear programming.

1.4 PURPOSE OF THE STUDY

The purpose of this descriptive case-study was to explore Grade 12 learners' problem-solving processes involving linear programming.

1.5 STUDY OBJECTIVES

1.5.1 GENERAL OBJECTIVE

The general objective was to explore learners' problem-solving processes in linear programming

1.5.2 SPECIFIC OBJECTIVES

The specific objectives of the study were as follows:

i. To establish grade 12 learners' problem-solving processes in linear programming.
ii. To investigate factors that affect grade 12 learners' problem-solving processes in linear programming.
iii. To determine strategies that would help enhance learners' problem-solving processes in linear programming.

1.6 RESEARCH QUESTIONS

The study was guided by the following research questions:

i. How do Grade 12 learners solve problems in linear programming?
ii. What factors affect grade 12 learners' problem-solving in linear programming?
iii. How can learners' problem-solving processes in linear programming be enhanced?

1.7 SIGNIFICANCE OF THE STUDY

The findings of the study could form a basis for understanding learners' problem-solving processes in linear programming, factors that affect their problem solving as well as strategies that may improve problem solving in linear programming. This study could also be informative to various stakeholders such as policy makers and curriculum developers to see the need of incorporating and emphasizing problem-solving processes into the curriculum when making education policies concerning teaching and learning of mathematics in general and linear programming in particular. The study may also bring out relevant information to education providers for them to see the need of training teachers in problem-solving teaching approaches in order to promote exceptional higher thinking skills and problem-solving skills in learners.

1.8 SCOPE OF THE STUDY

1.8.1 Delimitation of the Study

The study was restricted to two senior secondary schools of Lusaka District of Zambia. The two schools were used in order to gain an in-depth understanding of learners' problem-solving processes in Linear Programming. In addition, it was restricted to the 12^{th} Graders only. According to MESVTEE (2013), Linear Programming is taken at Grade 12 level.

1.8.2 Limitations of the Study

Due to the COVID-19 pandemic, the data collection timeline was affected by the indefinite closures imposed on learning institutions which aimed at curbing the spread of the virus. Linear Programming being the second (2nd) topic in the O-level Mathematics Syllabus, was likely and is usually taught in the first term in Lusaka district of Zambia (MESVTEE, 2013). However, due to the indefinite closures imposed in the year 2020 and 2021, the topic was rescheduled and taught in the second term. The study did not look at how COVID-19 could have impacted on learners' problem-solving process in linear programming, as doing so could have overwhelmed and over-broadened the scope of the study. My visit may have affected participants by being anxious and have mixed feelings about my intentions of my study. However, they were assured that the research was part of academic progression and in partial fulfilment for the certificate and not for monitoring. These were the main weaknesses outside of the control of the researcher (Simon 2011).

1.9 THEORETICAL FRAMEWORK

This study was informed by the Gestalt (psychology of learning) theory of problem solving as expounded by Polya (1959). This theory holds that problem-solving occurs within a flash of insight which comprises; understanding the problem, devising a plan, solving the problem, and after getting a solution, looking back to see whether the answer arrived at is reasonable and acceptable (Polya, 1959; Wertheimer, 1959). This theory provides a phenomenological description of problem-solving as a problematic situation in which the problem solvers desire a solution to a problem but has no immediate solution to the problem (Mayer, 1995; Wertheimer, 2006).

Gestalt identified two types of thinking that take place during problem-solving namely; productive thinking and reproductive thinking. Productive thinking involves deep understanding of the phenomenal structure of a problem in order to create the solution. In light of productive thinking, Gestalt theory of problem solving holds that problem solving emphasizes the connection between reasoning and various cognitive abilities such as intelligence, intellect, attention and working memory (Wertheimer, 1959). From the cognitive point of view, Wertheimer (1995) submitted that working memory encompasses meta-level processes such as planning, monitoring and evaluating during problem solving. On the other hand, the Gestalt theory of problem-solving holds that reproductive thinking involves the mechanical application of previously learned concepts and experiences in order to solve new problem situations. Reproductive thinking leads to good performance on retention problems but poor on transfer problems. Wertheimer (1959) argued that in dealing with problem solving, the most essential aspect is what prior knowledge problem-solvers have and how efficiently they access it. He contended that successful problem-solving involves linking new knowledge to what is already known. These links can take different forms, such as adding to, modifying, or organising knowledge or skills (Wertheimer, 1995).

Due to failure of the Gestalt theory to explain on the indicators that can be used to divulge if learners have: understood Linear programming problems, devised plans, executed plans and evaluated the plans, Polya’s (1959) indicators of problem solving, helped to bridge this gap. This was found to be most expedient to the study because it clearly spells out how problems in mathematics are solved: getting a problem, understanding the problem, devising plans, solve the problem, and after getting a solution, looking back to establish whether the answer arrived at is reasonable and acceptable or otherwise. Hence, this theory informed and guided the study, and helped to explore Grade 12 learners' problem-solving processes in linear programming.

1.10 CONCEPTUAL FRAMEWORK

The conceptual framework for this study, is premised on the George Polya's model of problem solving and the cognitive dimension of the Revised Bloom's taxonomy, a classification system used to define and distinguish different levels of human cognition, that is, thinking, learning and understanding" (Bloom, 1994). The structure of the conceptual framework is in such a way that it presents an integrated way of looking at Learners' problem-solving processes. I believe the structure best explained the natural progression of the phenomenon under the study. It gives the main understanding that learners' problem-solving processes can be understood in terms of the four stages of problem solving and in terms of cognitive demands (from the cognitive point of view). It is only based on the incorporation of related literature on problem-solving processes involved in solving mathematical problems in general and Linear Programming, in particular.

The step in problem-solving process is to *Understand the problem*. This is where; the learner tries to understand the problem by engaging with the problem or task and want to actually solve it. (Polya, 1957). Some possible questions the learners could ask oneself include: Do I know what is the unknown and the data are? Do I know what is the condition or conditions involved in the problem? Can I draw a picture to help me understand the problem? Can I separate and write down the various parts of the condition? This stage involves reading, re-reading, identifying key words and drawing graphs. The main cognitive demand involved at this stage is *retrieving* and *recognizing* factual information as such it has been assumed to be related to the Remembering cognitive level. When the learner has understood the linear programming problem s/he moves into the second step of problem-solving which is *Devising a plan* step. At this point, the learner tries to find connections between the data and the unknown and should eventually obtain a plan of the solution (Polya, 1957). This is where the learner needs to create representations of their models or a plan based on devising formulas, simplifying rough workings. The main cognitive demands involved at this stage is *interpreting* and *translating* material from one form to another and as such it has been assumed to be related to the Understanding cognitive level.

The third step in the problem-solving process is to *execute the plan(s).* This is where the learner then carries out the plan developed in the previous step. The learners need to solve the linear programming problem by monitoring, applying appropriate formulas, while checking each step of the solution plan. The main cognitive demands involved at this stage include: *applying, analysing, evaluating* and *creating*. Therefore, this stage was assumed to be related the Applying, Analysing, Evaluating and Creating cognitive levels. After executing or carrying out the plan, the learner moves into the fourth stage which is *looking back*. At this point the learner examines the solution obtained. Possible questions are the following (Polya, 1957): Can I check the result, or argument? Can I derive the result differently? Can I use the result, strategy or method for another problem? This stage involves re-examining the solutions arrived at, and as such it is also assumed to be related to the evaluation cognitive demands. This is the interface between Polya's stages and the bloom's cognitive categories in the context of this study.

However, in relation to the interface between Polya's stages and the revised bloom's categories of the cognitive dimension, it must be put into perspective that the taxonomy is not a measure of difficulty, but an indication of the type of cognitive process required to answer questions correctly (Anderson, 2001). By implication, although the contextual interface looks linear, the cognitive categories can be subsumed into Polya's four stages. For instance, *creating* can still occur at the *devising-plan* stage, while it also possible to exhibit cognitive demands such as *remembering* and *understanding* at the *execute-plans* stage during the process of problem-solving.

The left sub-framework is such that it allows for "backtracking" between each step. It stresses the dynamic and cyclic nature of the problem-solving activity. The framework indicates that problem solving starts at *understanding the problem* proceeding clockwise. The arrows entail that when the problem solver fails to devise a plan s/he can still go back to the first stage which is understanding the problem and vice versa.

Due to the fact that Polya's model does not take into consideration the cognitive demands for solving the question but looks at the stages of solving the problem irrespective of the cognitive demand involved, as argued by Carson (2007), the study was partially informed by the Revised Bloom's Taxonomy a classification system that is used to define and distinguish different levels of human cognition, that is, thinking, learning and understanding (Bloom, 1994). Cognitive demands refer to the different levels of knowledge and cognitive abilities or function needed to answer question correctly (Cambridge Assessment, 2010; Johnstone & El-Banna, 1986)

Remembering cognitive demand deals with recall and recognising of terms, ideas, procedures, theories, formulas and so on. The Understanding cognitive demand examines learners on the ability to grasp the meaning of previously learned material. This may be shown by translating material from one form to another, interpreting material or summarizing or predicting consequences or effects. The Applying cognitive demand requires the ability to apply the material studied in new and real-life situations. It may include how to use rules, methods, concepts, principles, laws, theories, and equations. The Analysing cognitive demand requires the ability to disassemble material into its constituent parts so that its organizational structure may be understood. This exercise may include the identification of the different constituent parts, the examination of the relationships between the various parts and the understanding of the organizational principles involved. The Evaluating cognitive demand involves checking and critiquing. This may involve testing points and deriving abstract relations (Anderson, 2001). The Creating cognitive demand requires the ability to integrate parts to form a new whole. This may involve the production of a unique communication or a plan of operations.

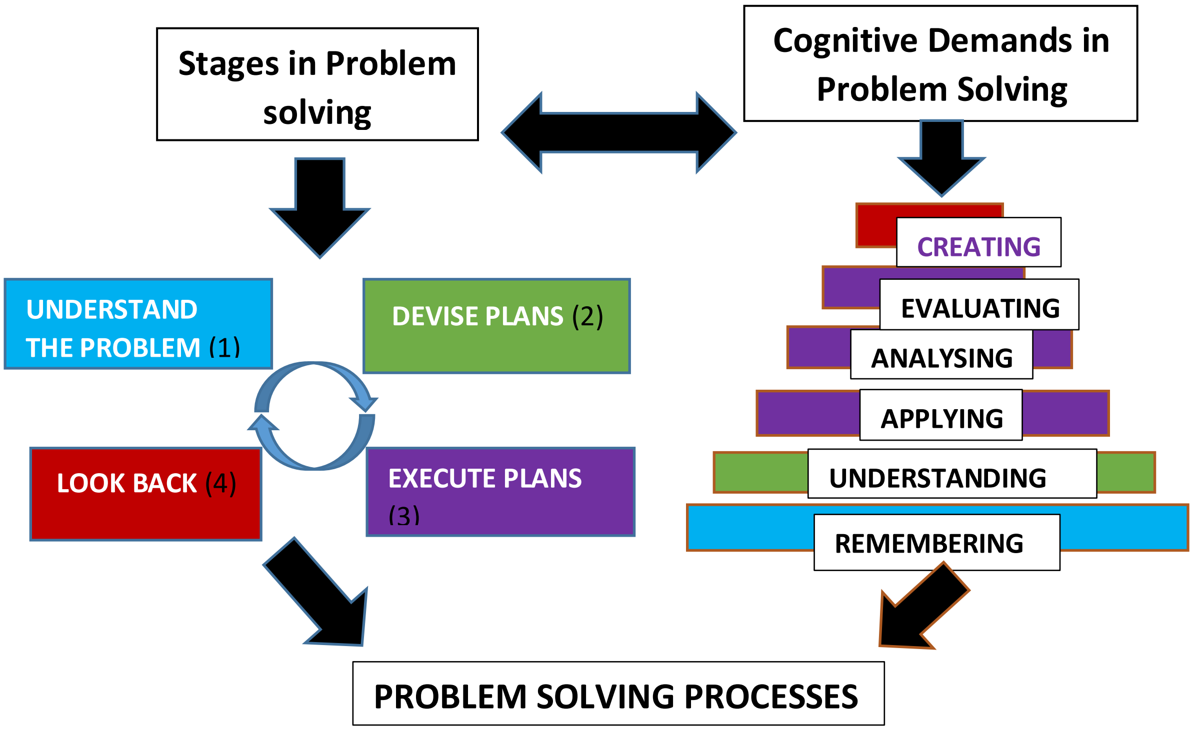


Figure 1.2: Conceptual Framework for the Study

1.11 Operational Definitions

A definition is a concise statement of the basic properties of an object or concept which unambiguously identify that object or concept (Hilton, 1986, p.48). In this study concepts were defined operationally; according to the contexts in which they were used.

Problem: Refers to a linear programming task that has no immediate solution or known solution strategy.

Linear Programming: A mathematical technique for finding optimal solutions to problems that can be expressed using linear equations and inequalities.

Problem-solving: Refers to engaging in a linear programming task for which the solution is not known in advance but may be realized through: understanding the task or problem, devising a plan(s) to solve the problem, executing the plan(s) and looking back.

Problem-solving processes: A step-by-step procedure by which the learner applies previously acquire knowledge and skills by identifying the linear programming problem, understand the problem, devise plans, carrying out the plans and looking back or evaluating the solution to check if the solution arrived at is correct or not.

Problem-solving strategies: Deliberate method(s) by which a linear programming problem can be solved.

Problem-solving skills: Learners' capabilities and abilities to solve problems from intellectual domains such as mathematics.

Cognitive demands: These are different levels of knowledge and cognitive abilities or functions needed to answer a question(s) correctly. These are related to remembering, understanding, applying, analysing, evaluating and creating.

1.12 Organization of the study

Chapter 1 introduced the study by giving the background of the study. The motivation of the study being: poor learner performance in mathematics in general, and Linear Programming in particular and why Linear programming; and the research gap. The chapter also outlined some key items such as the statement of the problem, purpose of the study, study objectives and their research questions, the significance of the study, scope (delimitation and limitation), the theoretical and conceptual framework and lastly the operational definitions of terms.

Chapter 2 outlines a review of related literature to the problem under exploration. Literature is presented under the following sub-headings: Problem-solving; Factors affecting learners problem-solving; George Polya's Problem-solving model; Bloom's Taxonomy; studies on Linear Programming and related topics, and gaps identified. The chapter concludes with the summary in tabular form showing the themes, key issues and their relation to the current study.

Chapter 3 provides the methodology which includes, the research paradigm, research design and approach, study area, study population, study sample, sampling techniques, data collection methods and instruments, data collection procedures and time line, data analysis, credibility and trustworthiness of the research, and ethical considerations. Lesson observation schedule was accompanied by video recordings. Semi-structured interviews, document analysis, and FGDs were used to supplement the lesson observations.

Chapter 4 presents the analysis of qualitative findings. For both vertical (within case) and horizontal (across case) analysis, the study used Polya's (1957) stages of problem-solving namely: understanding the problem, devising plans, executing plans and looking back, and the Bloom's (1956) taxonomy with its cognitive demand levels: remembering, understanding, applying, analysing, evaluating and creating. Data analysed was from Lesson Observations, Focus Group Discussions and Interviews. However, this document only reflects the horizontal analysis. The chapter ends with a summary of the findings.

Chapter 5 provides the discussion of the findings presented in the preceding chapter (Chapter 4) in line with the research objectives. The findings are further discussed in view of the literature reviewed and the theoretical foundations that informed the study.

Chapter 6 gives the conclusion and recommendations based on the findings. The conclusion summarises the study while recommendations provide suggestions to inform policy, practice and further research in mathematics education.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This section reviews the literature related to the subject of the study, namely: problem-solving; factors affecting problem-solving, Polya's problem-solving model, Bloom's Taxonomy and related studies in linear programming.

2.2 Problem-Solving

Problem-solving in mathematics has been perceived as an essential aspect of the teaching and learning of mathematics. It is one of the most essential cognitive skills needed in many professions as well as in everyday life (Jonassen, 2000). This is because everyone encounters problems in their daily activities. Some of these problems may require simple solutions. Others may require a series of steps before one arrives at the desired solution (Awuah, 2018). The act of solving problems and getting problems right, requires some level of skills. In this modern era, Individuals who have these skills have greater opportunities in their everyday life and profession. For example, people's capacity to solve complex problems enables them to easily adapt to changes in the environment and to learn from their mistakes (Awuah, 2018).

Proficiency in problem-solving contributes to self-actualization and leads to greater opportunities for employment as well as contributing to economic growth (Awuah, 2018; Hanushek, Wößmann, Jamison & Jamison, 2008). Due to its importance, in some countries such as Zambia and South Africa, there have been calls for the teaching of problem-solving, as well as the teaching of mathematics, through problem-solving to be included in the mathematics curriculum (Awuah, 2018). It is not surprising that the field has seen tremendous interest by researchers in mathematics education. As a result, the past decades have witnessed much research done on problem-solving in different disciplines. Studies on problem-solving have focused on different themes (Anderson, 2001; Jonassen, 2000; Mayer & Wittrock, 2006; Newell, Shaw & Simon, 1958). Among these themes is the emergence of a number of problem-solving models such as Polya's (1957) problem-solving models, problem-solving assessment tools, problem-solving as a teaching method and the identification of students' problem-solving skills. Researchers in the course of their studies have given different definitions of problem-solving. A few are captured here, Hepper and Krouskopf (1987) defined problem-solving as cognitive and effective behavioural processes for the purpose of adapting to internal or external demands or requests. Bingham (1988) defined problem-solving as a process that requires a series of efforts oriented towards eliminating the difficulties encountered in order to achieve a certain objective. Krulik and Rudnick (1980, p. 3) defined a problem as "a situation, quantitative or otherwise, that confronts an individual or group of individuals, that requires resolution, and for which the individual sees no apparent or obvious means or path to obtaining solution". Krulik and Rudnick (1980) asserted that the problem-solving process required individuals to use previously acquired knowledge, skills and understanding to satisfy the demands of an unfamiliar situation. This implies that for one to be a good problem solver, one ought to have acquired certain skills that could engineer the easy solving of the problem from previous experience. The implication is that problems have some degree of difficulty that requires special skills to tackle. The key to becoming a good-problem solver lies in the cognitive domain, since the process is a cognitive one. (Awuah, 2018). Of all the various definitions of problem-solving, Mayer and Wittrock (2006, p.287) definition of problem solving, "a cognitive process directed at overcoming obstacles" is the most widely accepted by problem-solving advocates. According to Meyer and Wittrock (2006, p.287) problem-solving is a means of "*transforming a given situation into a desired situation when no obvious method of solution is available*." The various definitions presented all have something in common, namely overcoming an obstacle to reach the desired solution.

These obstacles serve as barriers standing in between the problem and the solution (Funke, 2010). A popular mathematician, namely Halmos (1980, p.519), argued that "the mathematician's main reason for existence is to solve problems". Simply put, the act of mathematics is mainly about problem-solving. This notion is supported by Schoenfeld (1987) who asserted that learners not only learn mathematics while solving problems but also simultaneously develop problem-solving skills and strategies. The existence of a problem lies in the fact that learners are faced with a question that they do not recognize and where the mere application of a model is not appropriate (Awuah, 2018). However, this is seen as relative because to some learners what is needed to solve a problem may be the mere application of an algorithm (step-by-step procedure of solving a task). To others in the same grade, it will be an involving task to accomplish. A problem may be referred to as routine or non-routine based on the solver's familiarity with that particular problem. To a novice, a particular problem may be a mountain to climb but to an experienced person, that same problem may have an obvious solution. This argument supports the claim that to become a good problem solver one needs to be exposed to more questions to gain the necessary experience, as argued by Polya (1957), problem-solving is not inborn, but a skill that can be improved over time through one's exposure to questions.

These are the skills that one gains when one studies a subject like mathematics. As noted by the National Council of Teachers of Mathematics (NCTM, 1980), mathematics contains tools that help one in solving problems. In the early 1980s the NCTM proposed that problem-solving should be the priority in school mathematics (NCTM, 1980) because of its importance. It has been included in most mathematics curricula of which the Zambian mathematics curriculum is no exception (MEVSTEE, 2013).

Based on the importance of problem-solving, Schoenfeld (1992, p.3) opined that "the main goal of mathematics instruction should be to train learners to become competent problem-solvers." Therefore, it can be seen from the above literature that learners do not only learn mathematics while solving problems but also simultaneously develop problem-solving skills and strategies. Problem-solving is one of the most essential cognitive skills needed in many professions as well as in everyday life. It is on this motivation that the study focused to explore learners' problem-solving processes in linear programming, the factors that affect their problem solving and the strategies that could enhance their problem-solving processes in linear programming.

2.2.1 Concept of Mathematical Problem

Answer (2012), defined a Mathematical problem as a situation that involves some known and unknown variables in it and it requires some calculations to reach a solution. Polya (1973), in his famous book 'How to solve it': *"A New Aspect of Mathematical Method"*, described two types of problems: a '*problem to find'* and a '*Problem to prove'.* The main focus of the '*problem to find'* is to find the unknown certain value, object or the unknown of the problem. The unknown is also called "quaesutum" or the 'thing sought", or the 'variable required'. The problem may be theoretical, practical, concrete, abstract, or some puzzles. To solve the problem, one has to find all sorts of unknowns, by deriving, translating, constructing and inferring all kinds of variables (Polya, 1973), based on these specifications, a question on Linear Programming qualifies to be classified under the ‘problem-to-find’ mathematical problems. Other few examples of the ‘*problem to find*’ include the following: Finding the unknown profit percentage in a word problem about cost price and sale price, finding the 2nd angle given that one angle is 300 in supplementary angle and finding the Lowest Common Multiple of two numbers. According to Polya (1973) quoted by Mehmood (2014) a problem is composed of three components, “data, objectives and processes”. Data corresponds to information provided as a part of the problem. Objectives define the finalization required for solving the problem. Finally, processes are potential activities regarding accomplished objectives of the solution.”

The other type is the ‘*Problem to prove’*. The main focus of this type is to show categorically that a given claim is true or not true. The ‘*problem to prove’* has to answer the stated question either true or false. For showing the worth of the claim either true or false it has to provide 91 evidence which is a proof. The ‘problem to prove’ has two basic components: Hypothesis and Conclusion (which is to be proved) (Polya, 1973). For example, proving that the sum of the angles of triangle is 180 degrees and that diameter of circle is twice the radius of a circle. Also proving the theorem of Pythagoras and De-Morgan’s laws, are among the examples of *‘problem to prove’.*

A good Mathematical problem is the one that enhances the student’s intellectual abilities and problem-solving skills, and also which captures the student’s interest and curiosity (Betne, 2010; NCTM, 2010). Basic criteria for a Mathematical problem through which problem-solving skills can be developed was described in the light of the finding of the studies by Betne (2010); and D'Agostino (2011) as cited by Yuan (2013) as explained below:

1. The problem must involve calculations and Mathematical procedures.
2. The problem must contain high-level thinking and problem-solving skills.
3. The problem must help the leaner in clarity of concepts.
4. The problem should guide the teacher in the assessment of learners’ progress and difficulties.
5. The problem must have many solutions and develop the decision power of learners in selecting the right one.
6. The problem must stimulate interest and motivation in leaners learners.
7. The problem should develop higher abilities of cognition like application, analysis, evaluation and creativity.
8. The problem should be a source of learning of different Mathematical concepts.
9. The problem should create the use of skill(S) in the daily lives of learners.

10. The problem should be able to enhance the opportunity of using and practicing the skill in Mathematics.

To turn disadvantage into an opportunity, learners need to activate certain cognitive capabilities and psychological supports such as determination, courage and analytical thinking (Mehmood, 2014). In order to solve problems students, may need the ability through which Mathematical problems can be addressed, which is known as problem solving ability (Betne, 2010).

2.3 Factors that affect learners' problem-solving processes

2.3.1 The language of mathematics

Learning mathematics is like learning a new language as it involves its own unique and unambiguous statements or conditions and associated symbols. According to Pimm (1991), learning mathematics is actually acquiring control over the mathematics register. Halliday (1975) defined a mathematical register as a set of meanings that are appropriate to a particular mental function of language, together with the words and structures which expresses these meanings. The mathematical register includes words, phrases, symbols, abbreviations and ways of speaking, reading and writing that are specific to mathematics. Pimm (1991) argued that acquiring fluency in the mathematics register requires that learners learn to work within the mathematics register and understand its particularity. Learning mathematics in school comprises both informal and formal components. Informal language is the kind of language learners use in their everyday experiences to express their understanding of mathematical concepts. Formal mathematical language refers to the accepted use of terminology developed in schools.

Scholars recognizes language as a tool for teaching and learning. Mayer (1995) argues that humans use language to get things done and to engage in their interests. Using language to get things done depends on how well one is able to communicate his ideas to other people and how those people understand his ideas. In the same vein teachers negotiate meanings with learners in a language that they both parties understand. Learners use language to communicate with their peers and the teachers. Orton (1992), argued that language is both an individual and a social mode of thinking. Thinking is a process that assists learning and thus language structures that are not fully formed may set hurdles in learning. Learners have to understand the mathematics concepts as well as communicate their understanding of these concepts verbally and in writing. The teacher presents concepts written or verbally using mathematics language and everyday language. Students have to be proficient in both languages although competency in everyday language does not imply competency in mathematics language (Newman, 1983). Therefore, the understanding of the mathematical language of linear programming is key in building the conceptual understanding of linear programming.

2.3.2 Errors and Mistakes

Learners make a lot of mistakes, especially algebraic word problems. Studies have shown that there is a relationship between learners' poor performance in mathematics and the errors they make when solving algebra (Mukuka & Shumba, 2016; Norton, 2009). Fuchs, Fuchs, and Compton (2012) were of the notion that learners' poor performance could result mainly due to errors with mathematical concepts. Particularly, these errors occur when learners have to solve algebraic word problems. The aspect of creating equations to represent the relationship between the quantities also remains a challenge for most learners (Kieren, 2007). Drijvers, Goddijn, and Kindt, (2011) argued that learners find it difficult to transfer from arithmetic to algebra and this could be the reason for the errors they make in solving algebra. Nalube (2014) observed that conjoining error was common among 22 learners as they often replace a variable by any number for them to add and subtract algebraic expressions easily. Maat, Ibrahim and Zakaria (2010) also revealed that errors in solving equations such as algebraic word problems are due to lack of understanding the syntax and structure of algebra. Lack of conceptual, procedural and strategic knowledge by learners was one of the challenges faced in answering algebraic linear equations (Koji, Mulenga & Mukuka, 2016).

Lambdin (2009) stated that learners who have learnt mathematics without understanding are often successful only in solving problems similar to those they have already seen, and are unable to see how mathematical ideas are related or useful. These learners often see the subject itself in a negative light, viewing it as arbitrary and mysterious a subject which only geniuses can master. Some researchers used 'Newman's Error Analysis Model' (NEA Model) in learners' error analysis and established that in word problems learners made errors on the stages of reading and comprehension as the two were related to language (Mukunthan, 2013; Salihu, 2017). Scholars also revealed that the reasons for learners' challenges and the errors they make in Mathematics are linked more to their poor language proficiency than to their lack of computation ability (Boonen, Koning & Jolles, 2016; Ellerton, 1989; Oviedo, 2005; & Wright, 2014; Ural 2016;). This means learners' computation skills challenges are due to their inability to identify and understand key words in the problem statement. Montague (2005) indicated that key words play an important role in helping problem solvers to explain the meaning of word problems in their own way. Emphasising or highlighting key information is designed to help learners restate the mathematical problems in their own words, thereby strengthening their understanding of word problems (Cunningham & MacGregor, 2014; Polya, 1957). From the above studies, it can be noted that learners make a lot of errors in word problems due to a number of factors such as language problems, reading abilities, conjoining and failure to understand key issues in the problem. These errors could hinder their understanding of further mathematics which needs them to have knowledge of algebra, as it has been noted that algebra is used in many topics in mathematics, directly or indirectly, hence its importance cannot be overemphasised 23 (Musonda, 2017). It is vital that teachers probe learners' abilities at every stage of solving algebraic word problems in order to lessen the errors. It is imperative that error analysis is done to enable effective learning (Mubanga, 2019).

2.4 George Polya's Problem Solving Model

Polya was known as the great Mathematician due to his work on problem solving. He was Hungarian and taught at the Swiss Federal Institute of technology Zurich. He also worked at Stanford University. He was respectfully considered an authority in Mathematics' pedagogy. He was born in Budapest, Hungary on 13th December, 1887. George Polya is known as the father of the modern focus on problem solving in Mathematics education. He was a renowned Mathematician and had written many books on the subject of Mathematics but "*How to solve it*" was his famous book which he wrote in 1945 (Mehmood, 2014). In that book, Polya suggests a four-step model of solving a problem. According to Schoenfeld, 1992; Weber, 2008, this model gained a lot of currency as it was a method of teaching used to develop skills of problem-solving in mathematics. This great Mathematician died on 7th September, 1985 in California, USA (Yaun, 2013). According to Polya (1976), Problem solving is a method to develop the ability of Mathematical problem solving. It enables the students to become independent discoverers and by this method they can solve daily life Mathematical problems (Mehmood, 2014).

Polya's model is the oldest and most important problem-solving model (Bester, 2014). The model consists of four stages which work systematically to reach the solution of a mathematical problem.

According to Polya (1976), the first stage in problem-solving is to *understand the problem* as one needs to understand the problem before one can continue. Understanding the problem has to do with understanding the language of the problem statement, knowing what has been asked to be found or shown, being able to restate the problem in one's own words, being able to come up with a picture or diagram to represent what is being asked and also to acknowledge whether there is enough data to propose a solution.

The second stage of the model is devising a plan. According to Polya (1976), there may be many strategies by which a problem can be solved. This skill in choosing the appropriate strategy is best learned by solving many problems. Some of the known strategies include looking for the pattern, drawing a picture, using a formula to estimate and check and many more. The third principle of Polya's model is carrying out the plan. This step generally involves care and patience, particularly when one has the skill. It must be noted that if the plan selected is not working out one has to realise this and change it for another. The last stage of Polya's model is looking back. This involves a critical examination of the solution obtained to ascertain if the result is correct or whether the plan can be used to solve another problem. These four stages represent Polya's model. Researchers have found that learners execute the last stage of Polya's model with less care. They argue that many learners believe they are done with their mission of solving a problem and, as a result, they do not pay attention to the last stage ("look back") of the model (Lee, 2015).

According to Schoenfeld (1994), by teaching learners through this problem-solving model, learners can develop a deep understanding, analysing, synthesizing, evaluating and creating (Schoenfeld, 1994). Many research studies have proved the effectiveness of problem-solving teaching method on different faculties of mind (Mehmood, 2014; Polya, 1973; Schoenfeld, 2010;). The following are the few faculties of mind on which this method works very effectively: Cognition, Meta-cognition, Knowledge, Critical thinking, Creative thinking, Synthesis, and Analysis. The taxonomy of educational objectives is a framework through which we can get the results of instruction from the students. Hence to confirm the effectiveness of the Polya's problem solving method, it is necessary that it may be checked against some standard taxonomy (Aravena & Caamaño, 2007; Betne, 2010; Ellis, 2011; Mehmood, 2014). This is one of the weaknesses of Polya's model in assessing problem solving skills, it does not take into consideration the cognitive demands for solving the question but looks at the stages of solving the problem irrespective of the cognitive demand, as argued by Carson (2007) and Awuah (2018).

The Polya's Problem-Solving Model was critical to the focus of the study as it provided the four stages which are: *Understand the problem, Devise plan(s), Execute plan(s), Look Back.* These four stages with their specific descriptions and indicators partly informed the study, helped to conceptualise the phenomenon of problem-solving processes in terms of stages (See Section 1.9), and served as a lens through which learners' problem-solving processes were ascertained and explored.

2.5 Cognitive Domain of Bloom's Taxonomy

Bloom's taxonomy is a classification system used to define and distinguish different levels of human cognition that is, thinking, learning and understanding (Bloom, 1994). The taxonomy was originally published in 1956 by a team of cognitive psychologists chaired by Benjamin Bloom and consists of three main domains, namely: the cognitive domain, the affective domain and the psychomotor domain. The taxonomy is not a measure of difficulty, but an indication of the type of cognitive process required to answer questions correctly. This is an indication that attaining a given level of learning requires mastery of the previous level. The primary aim of the taxonomy was to design a logical framework for teaching and learning goals that would help develop new knowledge skills and understanding. Bloom classified the cognitive skill levels as "knowledge, comprehension, application, analysis, synthesis and evaluation" (Bloom, 1956). The taxonomy has been described as hierarchical in nature (Forehand, 2012) with the first three "knowledge, comprehension and application" representing the lower levels of cognition and "analysis, synthesis and evaluation considered as higher-order skills" (Bloom, 1956).

Knowledge cognitive demand deals with recall or recognition of terms, ideas, procedures, theories, formulas and so on. For example: remembering of procedures and concepts for drawing graphs of linear equations and inequations in one or two variables (preliminary knowledge for linear programming, learnt in junior secondary level). Comprehension cognitive demand involve the ability to grasp or construct meaning of previously learned material. Examples of verbs that relate to this function are: express, identify, describe, illustrate, interpret, draw, represent etc. This may be shown by translating material from one form to another, interpreting material (explaining or summarising) or predicting consequences or effects. For example, shading the wanted and unwanted regions.

Application cognitive demand requires the ability to apply the material studied in new and real-life or concrete situations. It may include how to use rules, methods, concepts, principles, laws, theories, and equations. For example, describing the wanted or unwanted region. Keywords that might be applicable in test construction items here are complete, construct, demonstrate, discover, solve and show.

Analysis cognitive demand requires the ability to disassemble material into its constituent parts so that its organizational structure may be understood. This exercise may include the identification of the different constituent parts, the examination of the relationships between the various parts and the understanding of the organisational principles involved. For example: Finding the values in the feasible region. The keywords normally used in test construction include “analysis”, “break down”, “compare”, “contrast”, “outline” and “distinguish”.

Synthesis cognitive demand requires the ability to integrate parts to form a new whole. This may involve the production of a unique communication (thesis or presentation) or a plan of operations (research proposal). For example: using the search line to determine the maximum and minimum values. During test construction keywords used in framing questions include categorizing, combine, compile, compose, create, modify and derive. Evaluation cognitive demand requires learners to make a judgment on ideas. The keywords used in framing questions include compare, conclude, defend, explain, and support. For example, applying the knowledge of linear programming in real life.

The original taxonomy is still in action and used by curriculum developers, researchers, teachers and evaluation experts. The old/original version of taxonomy has some criticisms, among others: overlapping of the cognitive levels, learning does not follow a sequential progression patron, and the Taxonomy does not support the modern learning theories such as constructivism, Metacognition and Self-structured learning (Suurkamm &Vezian, 2010).

2.6 The Revised Bloom's Taxonomy (Cognitive process dimension)

Bloom's taxonomy has stood the test of time because it has been used by educational role-players for many years and in the process has become the standard for developing frameworks for learning, teaching and assessment goals (Awuah, 2018). However, due to its long history and extensive use and due to the advancement in the field of Psychology and Education, it has been condensed, expanded and reinterpreted in a variety of ways to meet the diversified needs of the 21st century (Forehand, 2012). One of such revisions that stands out is the revised Bloom's taxonomy. Lorin Anderson, a former student of Bloom, gave an update of the original Bloom's taxonomy in 2001 with a supporting team of cognitive psychologists, curriculum theorists, instructional researchers and testing and assessment specialists when they published the book entitled: *The taxonomy for teaching, learning and assessment.*

There were three changes made to the original Bloom's Taxonomy. Firstly, the revised version of Bloom's Taxonomy changed noun to verb. Secondly, the process of synthesis was replaced with creativity and it is kept at the highest level in the Revised Bloom's Taxonomy because creativity is the result of combination of many parts of knowledge (Mehmood, 2014). Thirdly, as regards the structure, the original Bloom's Taxonomy had one dimension which was moving in a hierarchy from simple to complex, while the Revised Bloom's Taxonomy is of two dimensions. One dimension represents the '*knowledge dimension*' while the other is the '*cognitive process dimension'*. Hence the Revised Bloom's Taxonomy divided the noun and verb into two separate dimensions. The knowledge dimension is divided into four parts which is 'noun aspect' and the learning outcome on the student's part is divided into six parts which is verb aspect (Forehand, 2005; Mehmood, 2014).

There were three categories of knowledge in old Bloom's Taxonomy: Factual, Conceptual and Procedural. The Revised Bloom's Taxonomy includes a new addition of category: 'Metacognitive knowledge'. It is the way how knowledge can be acquired by someone, where students can adjust their ways of learning and can come to know about their strength and weaknesses. The Revised Bloom's Taxonomy provided an authentic and reliable tool to the curriculum developer, educational planner, teachers for instructional delivery and evaluators for assessment and evaluation (Forehand, 2012; Mehmood, 2014).

The Revised Bloom's Taxonomy has provided a large range of teaching activities for teaching and assessment and redefine the teaching learning process (Mehmood, 2014). This coincides with Pickard's (2007) assertion that the Revised Bloom's Taxonomy enables educators to identify which knowledge they expect students to use and to determine which cognitive process dimension is used.

Keeping in view the limitations of the old taxonomy, the revision published by Lorin Anderson and his fellows in 2001, by the name "*A Taxonomy for Learning, Teaching, and Assessing: A Revision of Bloom's Taxonomy of Educational Objectives"*. It also gained a lot of currency and acceptance as the old one. The revised Bloom's Taxonomy is the extension of the original old framework and not a new taxonomy (Munzenmaier & Rubin, 2013).

On the whole, the revised Bloom's Taxonomy enhanced the new understanding of learning and new methods of teaching. Among the new innovations it comes with include: addition of new category that is 'creating', conversion of nouns into verbs and one dimension to two dimensions. Further it represents students thinking processes not the behaviours, and the cognitive process and knowledge dimension are separate.

The taxonomy has several uses in the educational fraternity; among them is its usage in finding the development of teaching and learning outcomes. The ministry of Education recognizes and prescribes in the *Teachers' Curriculum Implementation Guide* (MEVTEE, 2013) on how teachers can employ the Revised Blooms Taxonomy in classifying their objectives and learners' class activities.

Therefore, since Polya's model does not take into consideration the cognitive demands for solving the problem but looks at the stages of solving the problem irrespective of the cognitive demands and skills, as argued by Carson (2007), Bloom's taxonomy helped to explore further learners' problem-solving processes in Linear Programming in terms of cognitive demands. For the fact that Problem-solving is a cognitive process (Krulik & Rudnick, 1980; Mayer & Wittrock, 2006), the study was partly informed by Revised Bloom's taxonomy of the Cognitive process dimension.

2.7 Studies in linear programming and related topics

Nkambule (2009), did a study on the teaching and learning of linear programming in a grade 11 multilingual mathematics class of English language learners: exploring the deliberate use of learners' home language. Data was collected through lesson observations for five consecutive days, reflective interview with teacher and clinical interview with two learners. This was a qualitative case study focusing on the teacher and his grade 11multilingual class in a township school in the East Rand. Analysis of data revealed that the teacher used learners' home language to probe learners' understanding of specific terms frequently used in linear programming concepts, for example terms such as 'at least' and 'utmost'. Learners' responses suggest that they drew on their home language for the meaning of these words. The finding of Nkambule's (2009) study were important to the focus of the study as they illuminated the importance of using home language in the teaching and learning of linear programming. The focus of this study was on the use of home language in the teaching and learning of linear programming, hence there was need to explore learners problem-solving processes in linear programming.

Haghverdi, Semnani and Seifi (2012), conducted a study aimed at establishing learners' difficulties in solving mathematical word problems from the viewpoint of teachers. The sample comprised 52 mathematics teachers who were randomly selected. The results showed that the learners' difficulties were due to their disabilities in representing and understanding of word problems, making a plan and defining the related vocabularies. The findings also revealed that, the causes of learner difficulties were text difficulties, unfamiliar contexts in problems and using inappropriate strategies. Moreover, the results from content analysis demonstrated that almost half of teachers' responses (51%) indicated that solving mathematical word problems was difficult for learners because they struggle with representation and understanding the problem. Two other difficulties cited in teachers' responses involved learners' inability to make a plan to solve the problem (31%) and a lack of vocabulary knowledge (10%). The study by Haghverdi et al, was important to the focus of the study as it provided a glimpse into the difficulties learners encounter in solving mathematical word problems and also helped to further validate findings of this study.

Mehmood (2014), conducted a true experimental study whose purpose was to find out the effect of George Polya's problem solving method of teaching on revised Bloom's Taxonomy of Educational Objectives, in the subject of mathematics at elementary level. True experimental pretest-post-test (double control group) design was used. An experiment was performed in a school situated in urban area of Islamabad. Multistage sampling technique used for the selection of school. The experiment involved a sample size of 132 Grade 8 learners. A Pre-test which consisted of 60 items of six cognitive processes was developed. Validity of the test was checked by the experts and reliability was checked through Alpha reliability analysis which was$\alpha = .89$. It was concluded on the bases of findings that Problem Solving Method showed significant improvement at all levels of knowledge dimension and hence all null hypotheses were not accepted. It was also concluded that Problem Solving Method works better than Conventional Method for teaching of Mathematics. In the light of these conclusions, 11 Problem Solving Method was recommended for teaching of Mathematics at Elementary level and also suggested to add it in teachers training programmes as well. The study by Mehmood (2014) was important to the focus of this study as it provided great insights into the significance of the Polya's model of problem-solving and the revised blooms taxonomy which mainly the informed and helped to conceptualise the phenomenon of problem-solving processes in linear programming.

Awuah (2018) investigated problem-solving skills of Grade 12 learners in probability. A total of 490 Grade 12 learners from seven schools South-Africa, categorised under four quintiles (socioeconomic factors) were purposefully selected for the study. The mixed method research methodology was employed in the study. Bloom's taxonomy and the aspects of probability enshrined in the Mathematics Curriculum and Assessment Policy Statement (CAPS) document of 2011 were used as a framework of analysis. A cognitive test developed by the researcher was used as an instrument to collect data from learners. The instrument used for data collection passed the test of validity and reliability. Results of the study also showed that with the exception of Bloom's taxonomy synthesis level, learners in Quintile 4 (fee-paying schools) had statistically significant (P-value < 0.05) higher achievement scores than learners in Quintiles 1 to 3, (i.e. non-fee-paying schools) at the levels of knowledge, comprehension, application, analysis and evaluation of Bloom's taxonomy.

The study by Awuah (2018), was important go the focus of the this as provided vital information on how to assess learners' problem-solving skills, via a cognitive test, in terms of cognitive demands using the six (6) cognitive levels of cognitive domain of the old bloom's taxonomy. This inspired this to partly explore problem-solving processes in terms of cognitive demands using a tasking whose question items were categorised according to the revised bloom's taxonomy. The study also provided great insights into the phenomenon of problem-solving.

Kaabo (2019), did a study in Zambia entitled teachers' strategies in the teaching of linear programming. This study interrogated teachers' strategies in the teaching of linear: the constraints and the suggested measures to overcome the constraints in the teaching and learning of linear programming. The descriptive research design was used to collect qualitative data from three teachers and 15 pupils, and was analysed using thematic analysis. The finding of the study revealed that to a large extent, teachers used linear equations when introducing linear programming. This strategy was used in order to bring the interaction inequality signs symbols that give the basis for formation of inequations from situational statements, plotting and shading to unveil the feasible region. The constraints included: the instruction for shading in grade nine and 12 were not consistent and the learners were not exposed to variety of situational problems under which the terms and their associated symbols would be used.

The findings of Kaabo's (2019) study were important to the focus of the study as they provided a glimpse into the teachers' strategies in the teaching and learning of linear programming and inspired this study to take a difference focus of exploring learners' problem-solving processes in linear programming.

2.8 Summary of the chapter

This chapter presented a number of issues on the: Problem-solving; Polya's problem-solving model; the Bloom's Taxonomy; the revised bloom's taxonomy, and gaps identified. All the studies highlighted in this chapter were significant to the focus of this study as they provided insights into the themes and key issues related to this study, they helped identify the research gap and helped to further substantiate the findings of this study. None of the related studies focused on learner' problem-solving processes in linear programming. Therefore, on that premise, this study explored learners' problem-solving processes in linear programming at grade twelve level in order to understand: how learners engage to solve problems in linear programming; the factors that affect learners' problem-solving processes; and the measures that may help enhance learners' problem-solving processes.

Table 2.1: Summary of themes and key issues relating to the study

Theme	Key Issue	Relating to the study
Problem-solving	▪ Essential cognitive skill in real life. ▪ A teaching and learning tool for promoting higher order thinking skills. ▪ An assessment tools. ▪ The concept of mathematical problem	▪ My study has been informed on the importance of problem-solving in real life. ▪ Understanding the importance of problem-solving in life led to my focus on linear programming and identification of the gap in the context of my study.
Factors that affect learners' Problem-solving processes	▪ Mathematical language ▪ Error and mistakes	▪ Informed the study on how mathematical language, mistake and errors affect learners' problem-solving processes.
Polya's Problem-Solving model.	▪ The four stages of problem-solving: *Understand the problem, Devise plan(s), Execute plan(s) & look back.*	▪ Helped conceptualise problem-solving processes in terms stages. Formed part of the conceptual framework
Bloom's Taxonomy	Historical perspective of the cognitive domain.	▪ Provided the historical perspective of the cognitive domain.
Revised Bloom's Taxonomy (Cognitive process dimension)	▪ The cognitive process dimension. ▪ The six cognitive levels: *remembering, understanding, applying, analysing, evaluating & creating.*	▪ Helped conceptualise problem-solving processes in terms of cognitive demands. ▪ Formed part of the conceptual framework.
Studies on learners' problem-solving processes in linear programming, related topics, and gaps identified	▪ Teaching and learning of linear programming in a grade 11 multilingual mathematics class English language learners. ▪ Learners' difficulties in solving mathematical word problems. ▪ Effect of Polya's problem solving method of teaching on revised Bloom's Taxonomy of Educational Objectives. ▪ Teachers' strategies in the teaching of linear programming.	▪ My study was informed on the focus of many studies reviewed. They provided great insight into related issues of the phenomenon under this study and inspired this study to explore learners' problem-solving processes in linear programming.

CHAPTER THREE

METHODOLOGY

3.1 Introduction

This chapter discusses the methodology: principles, procedures, and practices which governed the research study. Highlighted under this chapter is the research paradigm, research design and approach, study population, sampling techniques and study sample, data collection methods, data collection instruments, data analysis, rigor and ethical considerations that were employed in the study.

3.2 Research paradigm

The study employed a constructivist or interpretive paradigm (Mason, 2002). Constructivists view knowledge as socially constructed through language and interaction, and reality as connected and known through society's cultural and ideological categories. The Constructivist paradigm emphasizes the importance of examining the world from participants' point of view as they believe that both knowledge and reality are constructed and reproduced through communication, interaction, and practice (Creswell, 2014). This paradigm was found most appropriate as, I, the researcher was interested in exploring learners' problem-solving processes naturally in the classrooms where teaching and learning is done through communication, interaction and practice.

3.3 Research Design

Research designs are plans and the procedures for research that span the decisions from broad assumptions to detailed methods of data collection and analysis (Creswell 2009). Considering the qualitative nature of the research questions, the descriptive case study research design was used because it provides a systematic way to collect and analyse information and report results. Berg (2007) describes a descriptive case study as a method of collecting information by interviewing or observing a sample of individuals. It can also be used to collect information about people's attitudes, tone of voice, gestures, opinions or habits on any educational or social issue (Berg, 2007). The descriptive case study research design is appropriate when a researcher's goal is to gain insights, discovery and interpretations of a particular phenomenon (Merriam 1998). Therefore, the descriptive case design was found suitable for the study as it helped to explore learners' problem-solving processes as they occur in the natural setting. It also provided the opportunity to probe deeply, analyse thoroughly and get an in-depth and detailed understanding of the phenomena under review (Creswell, 2014). This was done in order to allow the teachers and learners to explain their thoughts, opinions, ideas or beliefs in detail.

3.4 Study Site

The study was conducted at two secondary schools in Lusaka district of Zambia. There are many motivating factors that influence the researchers' choice of the study site, such as; the nature and incidence of the problem, research time frame, and data accessibility, interest and instructions, resource availability, performance in particular field, goals and objectives of the study (Msabila & Nalaila, 2013). The researcher picked two secondary schools of Lusaka District on basis that one has been consistently among the top 10 highly ranked public secondary schools in Lusaka district and the other one is averagely ranked, this helped to understand how learners from different performance categories, engage in solving problems in linear programming.

3.5 Study Population

The individuals of interest included all grade twelve learners from secondary schools in Lusaka district of Lusaka Province of Zambia. The choice of the learners was based on the reason that they are at the level of learning where the topic linear programming is taught (MoGE, 2013).

3.6 Study Sample

This study employed a sample size comprising twenty-four (24) learners (12 from each school) and two Grade 12 teachers of mathematics from the selected schools, making a total sample of twenty-six in Lusaka District of Zambia. This is was reasonable sample size for a qualitative study because it provided the data that was needed as it fulfilled the requirements of efficiency, representativeness, reliability and flexibility (Merriam, 1998) as it also included Grade teachers (who teach linear programming), who are an important source of information as a way of understanding learners' problem-solving processes. In fact, a small sample for lesson observations, FDGs and interviews ensure a high level of reliability and helps to concentrate resources on obtaining reliable information (Henning, 2004).

3.7 Sampling Techniques

This study employed purposive sampling technique. Purposive sampling is appropriate to select unique cases that are especially informative as it uses the judgement of an expert in selecting cases with specific purpose in mind (Kothari, 2004; Neumann, 2014). Purposeful sampling is powerful in selecting information rich in cases for in-depth analysis (Merriam, 1998). Therefore, in order to ensure that grade twelve learners take part in this study, purposive sampling technique was employed (Maxwell, 2005). Twenty-four (24) learners were purposively selected with the help of the teachers. This involved three (3) categories: eight (8) high ability solvers, eight (8) middle ability solvers and eight (8) low ability solvers in linear programming. The teachers relied on their learners' continuous assessment records in Mathematics to classify learners according to the three categories. A variety of abilities of learners were revealed and helped to establish how learners of different abilities solve problems in linear programming.

3.8 Data Collection Methods

Data was collected using lesson observations, semi-structured interviews, and Focus Group Discussions. In a qualitative study, only qualitative data collection methods are used (Merriam, 1998). The following subsection provides a detailed description of the data collection methods and procedures.

3.8.1 Lesson Observations

A total of 24 lesson-periods were observed (12 periods for school **U** & 12 periods for school **V**). Lesson observation protocols and a video camera were used to collect data as the teachers were teaching and as the learners were learning in the classroom setting. The rationale for lesson observations was to see what and how grade twelve learners engage in problem solving processes: understanding the problem, devising plans, executing plans and looking back to see if the solution obtained is reasonable and acceptable, and the factors that affect learners' problem-solving processes in Linear Programming (See Appendix C). Creswell (2014), asserted that, direct observation is useful because some behaviour involves habitual routines of which people may not be aware of. In lesson observation, the information is sought by way of investigator's own direct observation without asking from the respondent (Kothari, 2004)

Patton (2002) as well observed that, observation is a data collection procedure whereby researchers try to understand what is happening in a given setting by paying attention, video recording, watching and listening carefully. In view of this, video recording was used to capture lesson observations (including vignettes: learners' workings from exercise books) in totality. According to Merriam (1998), one of the opportunities of using a video camera to record field observations is that it allows the researcher not only to capture the physical environment but also to revisit the images later and recall the experiences during analysis.

3.8.2 Semi-Structured interviews

Semi-structured interviews were conducted with two teachers after lesson observations. The rationale of the interviews with both the teachers was to have an in-depth understanding, opinions, and views pertaining to learners' problem-solving processes: understanding the problem, devising plans, executing plans and looking back to see whether the solution observed is reasonable, and regarding the factors that affect learners' problem solving and the strategies that may help enhance learners' problem-solving processes in Linear Programming (Kvale 1996) (See Appendix D for details). The interviews were audio-recorded to capture data in its totality.

3.8.3 Focus Group Discussions

In this study, Focus Group Discussion guides and the linear programming group task whose question items were aligned with the cognitive levels of the cognitive process dimension of the Revised Bloom's Taxonomy, (See Appendix E & F) were used to collect data on how grade twelve learners solve Linear Programming problems in terms of what cognitive demands they engage with to solve problems and in terms of whether learners engage in the understanding the problem, devising plans, executing plans and looking back to see whether the answer obtained is reasonable. This also helped to establish the challenges they encounter as they engage in solving problems in Linear Programming and, on the strategies, they would suggest to improve learners' problem-solving in Linear Programming (See appendix F). Focus Group Discussions were used in order to establish whether what was observed during lesson observations and semi-structured interviews with teachers was consistent with what the learners revealed. Cohen, Manion and Morrison (2007) described triangulation as the use of two or more methods of data collection in the study of some aspect of human behaviour and advised that a focus group usually comprises three to eight individuals who share certain characteristics which are relevant for the study. Focus Group Discussions were held after lesson observations since by that time, learners had gained enough knowledge in linear programming. This involved a total of 8 groups (4 groups from school **V** & 4 groups for school **U**), each group comprised three learners. In order to enhance trustworthiness of the qualitative data and findings, the proceedings of FDGs were audio-recorded and this helped to capture data in its totality.

3.8.3.1 The Group Task on Linear Programming

The group task on linear programming formed part of the Focus Group Discussions, it was the task which learners solved in groups of three. The task was adopted from the ECZ grade 12 ‘O’ Level past examination papers. The questions of the task aligned to the mathematics curriculum, they were already designed in the manner (standard) of Grade 12 examination questions and reflected the curriculum content. According to Ogbonnaya (2011), for a valid and reliable inference to be drawn from a study that is based on learners’ achievement, the study must make use of the assessment that is aligned with the curriculum standards expected to be learnt, this means that there must be coherence between the task and the examination guidelines of the curriculum or assessment guidelines. The task had no time constraint in order to eliminate unnecessary stress on learners and the possibility of errors being made because of time pressure. The task was a standard question adopted from the examination council of Zambia, as such it was already tested for validity and reliability.

However, to ensure that the of task’s question items were aligned with the cognitive levels of the revised Bloom’s Taxonomy (See Appendix D), the researcher categorised the question items of the linear programming task according to the six cognitive levels with guidance and with respect to the descriptions and indicators of the cognitive levels (See Table 3.1). This was necessary and essential so that the question items of the task would assess learners on the following cognitive demands: Remembering, Understanding, Applying, Analysing, Evaluating and Creating. For example, on the Remembering cognitive demand, learners were asked a question that involved recall and recognition of factual knowledge. For example, mathematical meanings of ‘*Not more than’*, ‘*At least’*, mathematical language and symbols. On Understanding cognitive demand, learners were asked a question that involved the cognitive process of interpreting and translating the situational conditions into inequalities. On Application cognitive demand, learners were asked a question that involved the application of factual, conceptual and procedural knowledge that is, drawing the lines of inequalities. For questions involving the analysing cognitive demand, learners were asked a question on describing and interpreting the wanted or unwanted region and the feasible region. On the Creating cognitive demand, learners were asked a question on determining the maximum and minimum value (See Table 3.1 and Appendix F).

Two experts were engaged to review the task's alignment with the cognitive levels. They moderated the questions to confirm their alignment with the six cognitive levels of the revised Bloom's Taxonomy. They judged the level of alignment of each question item against the cognitive levels by using a three-point rating scale (1= *not aligned;* 2= *fairly aligned*; 3=*much aligned*). All questions were retained following their judgement. The judgment of the two experts confirmed the task's proper alignment with the cognitive levels of the revised Bloom's taxonomy.

Table 3.1: Distribution of questions baarsed on Bloom's taxonomy and aspects of Linear Programming.

Question Number	Bloom's Cognitive Level	Aspects of Linear Programming in the Tasks
Question 1: Let x represent the no. of dresses and y the no. of suits. Write the inequality which represents the following conditions:		
1.1.1: The no. of dresses should not exceed 50. **1.1.2:** The no. of dresses should not be less than the no. of suits **1.1.3:** The cost of making a dress is k140 and of a suit k210. The total cost should be at least k10,500.	**Remembering** **Remembering** **Remembering**	- Recalling and recognising Mathematical statements with their associated symbols in linear programming.
1.1.1: The no. of dresses should not exceed 50. **1.1.2:** The no. of dresses should not be less than the no. of suits **1.1.3:** The cost of making a dress is k140 and of a suit k210. The total cost should be at least k10,500.	**Understanding** **Understanding** **Understanding**	▪ Interpreting and translating word statements into algebraic form (inequalities).
1.2: Using a scale of 2cm to represent 10 units on both axes, draw x and y for $0 \leq x \leq 60$ and $0 \leq y \leq 80$. Shade the unwanted region to indicate where the region (x,y) must lie.	**Applying**	▪ Applying graphical skills to draw lines of inequalities and shading the wanted region.
1.2: Using a scale of 2cm to represent 10 units on both axes, draw x and y for $0 \leq x \leq 60$ and $0 \leq y \leq 80$. Shade the unwanted region to indicate where the region (x,y) must lie.	**Analysing**	▪ Analysing to establish the feasible region with its boundary points.
1.3: The profit on a dress is k160 and on suit is k270. Find the no. of dresses and suits for maximum profit.	**Evaluating**	▪ Deriving the profit function to find the number of products for maximum profit.
1.4: Calculate the maximum profit.	**Creating**	▪ Calculating maximum or minimum profit.

3.9 Data collection procedure and time line

The study utilised the qualitative method to collect the required data from: lesson observations were conducted first. According to Kothari (2014), under observation method, the information is sought by way of investigator’s own direct observation without asking from the respondent. Then after, focus group discussions were conducted, since learners would have gained knowledge in Linear Programming from during their class lessons, this helped to see how they solved the task classified according to the cognitive process of the Revised Bloom’s Taxonomy. Then, lastly, data was collected from Semi-structured interviews as a follow up to the series of lessons observed and focus group discussions, respectively, this also served the purpose of triangulation of data, to see if what was observed during lessons was consistent with data from focus group discussions. The data was collected in the second term of the 2021 school calendar, from September to October. The period for data collection was six weeks (3 weeks for each school).

3.10 Data analysis

Collected data was thematically analysed using the *Six–step Guide to good thematic analysis* Framework by Braun and Clarke (2006), for the fact that this study involved purely qualitative data which required an analysis of themes in relation with problem-solving processes. Thematic analysis is the process of identifying patterns or themes within qualitative data.

For each research question, data recorded from the lesson observations, interviews and Focus Group Discussions was analysed through the six phases of Braun and Clarke (2006) Framework. The first phase was Familiarisation which involved transcribing data by reading and re-reading; noting down initial codes. The second phase was generating initial codes which involved coding interesting features in the data in a systematic fashion across the data set, collating data relevant to each code.

The third phase was searching for themes which involved collating or collecting codes into potential themes, gathering all data relevant to each theme. The fourth phase was reviewing themes this involved checking if the themes work in relation to the coded extracts and the entire data set; generating a thematic map.

The fifth phase was defining and naming themes, this involved ongoing analysis to refine the specifics for each theme; generating of clear names for each theme. The sixth phase was producing the report, this involved analysing and discussing the selected extracts, then relate them back to the research questions. Table 3.2 shows (from phase two) how data was analysed for research question 1 and shows the finalised themes for research question two.

Table 3.2.: Example of how data was analysed thematically using Braun & Clarke 2006 Framework

Themes under Research Question 1 (One): How do grade 12 learners solve problems in Linear Programming? (Objective: To establish Problem-solving processes in LP)	**Source of Data**
GENERATING INITIAL CODES & SEARCHING FOR THEMES (**Phases 2 & 3 of Braun & Clarke 2006 Framework**)	FDG, Lesson observation & Interviews
- Reading the Linear Programming questions	FDG, Lesson observation & Interviews
- Identifying and under lining key words.	FDG & Lesson observation
- Use of grade 9 linear Inequation and coordinate geometry (Preliminary Knowledge).	FDG, Lesson observation & Interviews
- Mathematical Language: use of mathematical symbols with their associated terms.	FDG, Lesson observation & Interviews
- Interpretation and Translation situational problem statements into inequations or inequalities.	FDG & Lesson observation
- Graphing the inequalities, shading the unwanted region to show the solution set.	Lesson observation & Interviews
- Use of trial-and-error method and boundary points (Vertices of the feasible region) when finding the maximum or the minimum cost/profit, Determining the maximum or minimum cost/profit	FDG, Lesson observation
- Interpreting Linear Programming questions.	FDG, Lesson observation & Interviews
- Checking through to if the answers realized are correct or wrong	

➢ <u>REVISED THEMES (MAJOR & SUB THEMES) GUAGED ACCORDING THE POLYA'S FOUR STAGES OF PROBLEM-SOLVING AND THE SIX LEVELS OF THE COGNITIVE DEMANDS.</u> UNDERSTANDING THE PROBLEM (**Phases 4,5&6 of Braun & Clarke 2006 Framework**) - Reading and re-reading the linear programming text or question. - Identifying and underlining keys words - Listing mathematical statements with their associated symbols. - Showing rough workings - *REMEMBERING- Recall and Recognition of Information retrieval of Mathematical language in linear programming.* DEVISING A PLAN - Interpreting, translating and writing down inequalities. - Scaling the graph paper. - Drawing tables values for the inequalities - Showing rough working skills. - *UNDERSTANDING- Grasping of meaning, interpretation, Translation and formation of inequalities.* CARRYING OUT THE PLAN - Checking each step when graphing the inequalities, shading the unwanted regions. - Identifying the feasible region and using its boundary points to determine maximum/minimum profits. - *APPLYING- Problem solving, usage of information in a new way.* - *ANALYSING- Analysing, separating and graphing inequalities and shading the unwanted region* - *EVALUATING- Using the information give to derive an equation* - *CREATING-*Calculating maximum or minimum profit. LOOKING BACK - Checking or evaluating to see if the answers or graphs, the wanted and unwanted are correct. - Interpreting the Linear programming answers. - *EVALUATING- Using the information give to derive an equation*	FDG, Lesson observation & Interviews FDG & Lesson observation. FDG & Lesson observation FDG & Lesson observation FDG & Lesson observation FDG & Lesson observation FDG & Lesson observation FDG & Lesson observation FDG & Lesson observation FDG & Lesson observation FDG & Lesson observation FDG & Lesson observation FDG & Lesson observation FDG & Lesson observation FDG & Lesson observation FDG & Lesson observation FDG & Lesson observation FDG & Lesson observation FDG & Lesson observation
Finalised Themes under research question 2: What factors affect grade 12 solving processes in Linear programming? (Phase 6)	
1. Mathematical Language **2.** Translating conditional statements into Inequations/ Inequations **3.** Graphing Inequalities **4.** Shading the unwanted region **5.** Calculating the maximum/minimum profit.	FDG, Lesson observation & Interviews FDG, Lesson observation & Interviews FDG, Lesson observation & Interviews FDG, Lesson observation & Interviews FDG, Lesson observation & Interviews

3.11 Trustworthiness of the Study

To ensure trustworthiness in the study, the participants' own words and vignettes (pictures) of pupils' exercises showing their problem-solving processes have been used in the presentation of findings. Themes that emerged after data analysis were subjected to expert view to see whether they were in line with recordings, and recognisable (Adler, 1996; Merriam, 1998;). My supervisor and colleagues cross examined them to ensure that they aligned with the study, and if they did not make sense alternative themes were suggested. According to Shenton (2004), credibility and trustworthiness can be attained by adopting research methods which are well established in qualitative approach such as observation, focus groups and individual interviews. In this view, I used these methods in data collection. Moreover, the use of lesson observations, semi-structured interviews, Focus Group Discussion and document analysis helped me to triangulate data thereby reducing the effect of investigator bias (Maxwell, 1992).

3.12 Ethical Considerations

Ethical considerations are an important aspect of any study in that they establish trust between the participants and the researcher. Before embarking on this study, the researcher sought ethical clearance from the Humanities and Social Sciences Research Ethics Committee of the University of Zambia and observed originality, quality and ownership of data as well as honesty (Creswell, 2014). After ethical clearance was granted by the committee to go out for research, permission was sought from Ministry of Education local authorities to gain entry into schools. For the fact that human beings are the objects of study in social sciences who need to know their privacy and their rights protected (Henning 2004, p.73), the participants were assured of their protection, anonymity and dignity. The researcher, also explained the purpose of the study to the participants in advance and explained that their participation in the study is on voluntary basis, and as such they can withdraw from participating at any stage of the study if they feel they can no longer continue participating. No names of participants and schools have been mentioned in this study, instead pseudonyms have been used.

3.13 Summary of the chapter

This chapter described the methodology that was used to carry out this study. The study used a constructivist research paradigm, and a descriptive case study design following the qualitative approach. Purposive sampling, homogenous in nature was used to select a total of 26 participants (i.e., 2 teachers and 24 learners) who participated in the study. Data was collected from lesson observation, focus group discussions for the learners and semi-structured interviews with teachers, respectively. Data analysis was done thematically using *the six-step guide to thematic analysis* by Braun and Clarke (2006).

Credibility and trustworthiness of the study was ensured by my supervisor who cross-examined the data and ensured that themes generated aligned with the study. Trustworthiness was also ensured by using 26 participants own words and writings and by triangulation of data from the lesson observations, focus group discussions and interviews. To adhere to ethical considerations, permission was sought from the various authorities and from the participants. The next chapter, presented the research findings according to the research questions of the study.

CHAPTER FOUR

PRESENTATION OF FINDINGS

4.1 Overview

The previous chapter described the research methodology that was used in the study to come up with the results which have been presented in this chapter. The themes that are presented in this chapter emerged from the data collected from lesson observations, semi-structured interviews and focus group discussions Data analysis was conducted, of the learners' solutions (workings), and a report on how learners performed in solving each category of questions has been presented based on the highlighted aspects taught in Linear Programming, Polya's problem solving stages and the cognitive demands. This study was specifically guided by the following research questions:

i. How do Grade 12 learners solve problems in linear programming?
ii. What factors affect grade 12 learners' problem-solving processes in linear programming?
iii. How can learners' problem-solving processes in linear programming be enhanced?

The themes that emerged from lesson observations, interviews as well as FGDs were categorized according to the three research questions depending on suitability. The following codes have been used for identification of the participants; Teacher V (Teacher at School V), Teacher U (teacher at School U); learners (V1, V2, V3, V4, V5, V6, V7, V9, V10, V11 and V12 at school V) and learners (U1, U2, U3, U4, U5, U6, U7, U8, U9, U10, U11, and U 12 at School U)

4.2 Research Question 1: How do Grade 12 learners solve problems in Linear Programming? (Gauged by the Polya's four stages to problem-solving and the cognitive demands from the Revised Bloom's taxonomy).

4.2.1 Understanding Linear Programming questions

4.2.1.1 Reading and rereading the Linear Programming text or question

The results from the research indicated that learners do read the Linear Programming question before solving it. From the lesson observations and during the Focus Group Discussion, all the twenty-four (24) learners could read the question before solving so as to try to understand it. The following are excerpts for four (4) learners during the Focus Group Discussions:

Interviewer: What did you do before solving the question? (Probe: why? How many times?)

*"…First, I read through the question many times to understand what they need…" (***V4***).*

*"…I, read, I went through the questions…at least twice, for better understanding…" (***U2***).*

*"…I always make sure I read question first, so I read through the questions...to know what the question is asking…Maybe three or five times…" (***U5***).*

*"…I first read through the questions to understand. I read many times until I understand…" (***V8***).*

(Verbatim excerpts, Focus group discussion)

Learners' responses showed and indicated that they read through the questions at least once before solving it, in order to gain understanding. It was also observed and ascertained during lessons and the focus group discussion that even though all learners could read and re-read the questions before solving, some leaners were unable to understand the questions and therefore, could not solve the question correctly.

4.2.1.2 Identifying, underlining and Listing keys words or statement

The results from the task under Lesson Observations and Focus Group discussions showed that the majority of the learners would underline keywords such as '*Not exceed'*, '*Not be less than'*, '*At least'*, '*x to represent the number of dresses'* and '*y to represent the number of suits*.' Learners also showed competence in identifying and listing keywords or statements with their associated symbols as noted in the following excerpts (Figure 4.1). Although, only 14 out of 24 learners, representing 58%, could correctly identify the keywords with their associated mathematical symbols and use them appropriately.

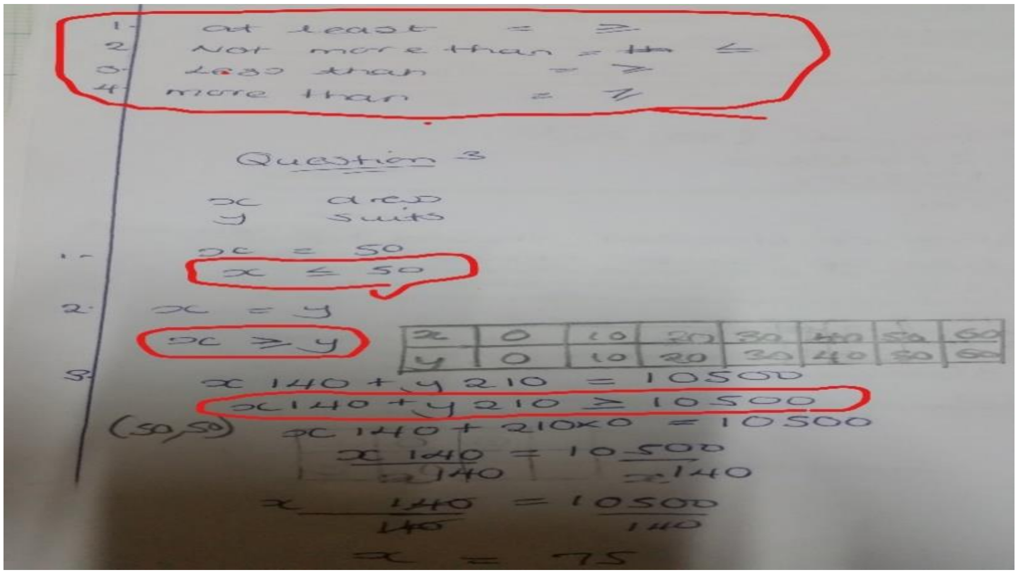


Figure 4.1: Learner V12 solving process.

4.2.2 Cognitive demands at '*Understanding the problem'* Stage.

4.2.2.1 Recognizing and retrieving factual knowledge.

The cognitive demands involved in problem-solving at the '*understanding the problem*' stage, were established on item **1.1** which was at the Remembering cognitive level (Refer to Appendix H), which required learners to write the inequalities which represented the three conditions of the question **1.1** and the results of the study revealed that most learners, in their quest to understand the question, were able to engage in the cognitive process of recognizing and retrieving relevant factual information needed in formulating the inequalities and solving the linear programming questions, although a few experienced challenges that led to incorrect solutions. Others were able to recognise the mathematical statements in the question (**1.1**) and recall their associated mathematical symbols as evidenced in the following vignette:

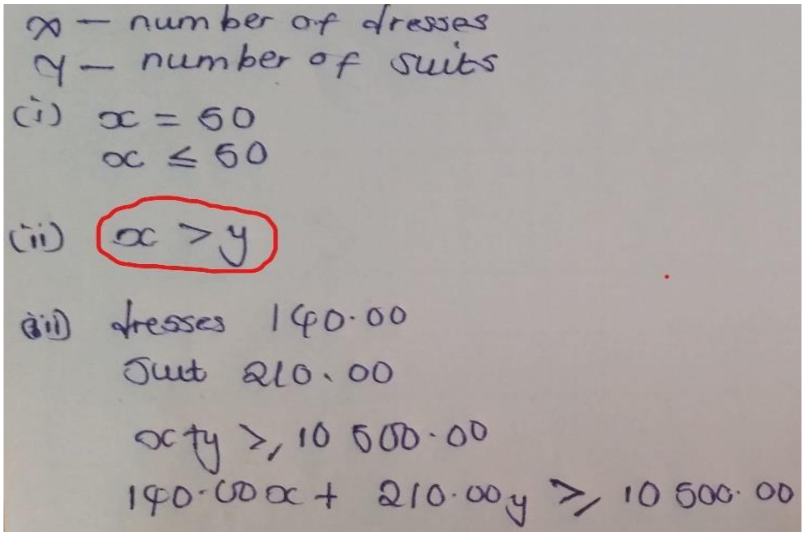


Figure 4.2: Learners U1, U2 & U3 solving process.

For the first condition, most learners (18 out of 24) were able to recognize the meaning of "*should not exceed 50"* with its associated mathematical symbol (≤) and were also able to recognize the meaning of "*At least"* for the second condition, with its appropriate mathematical symbol (≥). However, the majority of the learners (18 out of 24) experienced a challenge in recognizing the meaning of "*should not be less than"* (which implied, '*greater or equal to'*) with its appropriate mathematical symbol **(≥),** as shown in Figure **4.2**. When the learners (**U1, U2 & U3**) were probed, they had the following to say:

"…sir we are very sure it meant that...hmm since it says dresses should not be less than the number of suits…it means the number of dresses should be greater than the number of suits…so that's how we used the 'greater than' sign…" (Excerpt from interviews during Focus Group Discussion).

This substantiates the fact that some learners had challenges pertaining to retrieving and recognizing the meanings of the mathematical language with their mathematical symbols in linear programming. Most learners, however, did not have difficulty in remembering and recognizing mathematical statements with their associated symbols.

4.2.3 Devising plan(s) to solve a Linear Programming problem(s)

4.2.3.1 Writing down inequalities.

The results of the study indicated that learners had difficulties in forming Inequalities especially those that involved two variables. For instance, learners in 5 out of 8 groups could not correctly write down the inequality for the statements: "*The cost of making a dress is K140 and that of a suit is K210. The total cost should be at least K10500." and "The number of dresses should not be less than the number of suits"*, as shown in following vignette:

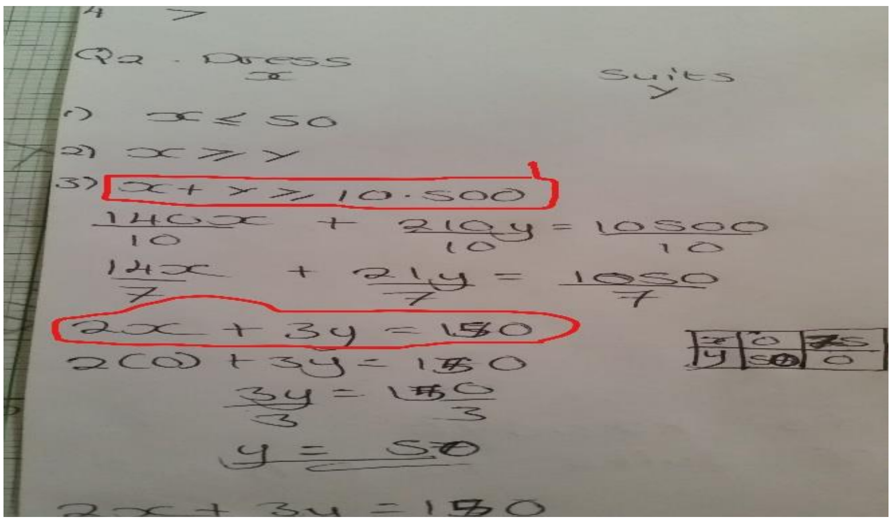


Figure 4.3: Learners V7, V8&V9 solving process.

As shown in Figure 4.3, some learners misrepresented the meaning of the statement "*not less than"* and *"at least"* instead of the *greater or equal symbol (≥),* they used the symbol for strictly '*greater than'* and the '*equal sign'*, as also shown in Figure **4.2**. Some learners also misunderstand the statements of the problem and that led them to writing incorrect inequalities. As shown in Figure **4.3**, some learners would first write a wrong inequality ($x + y \geq 10500$), then proceed to write the actual inequality.

4.2.3.2 Drawing tables of values for the inequalities and showing rough working skills

The results from the Focus Group Discussions and Lesson observations showed that learners (21 out of 24) representing 88%, in planning to solve the actual problems, drew tables of values and showed some rough-working skills as evidenced in the following vignettes.

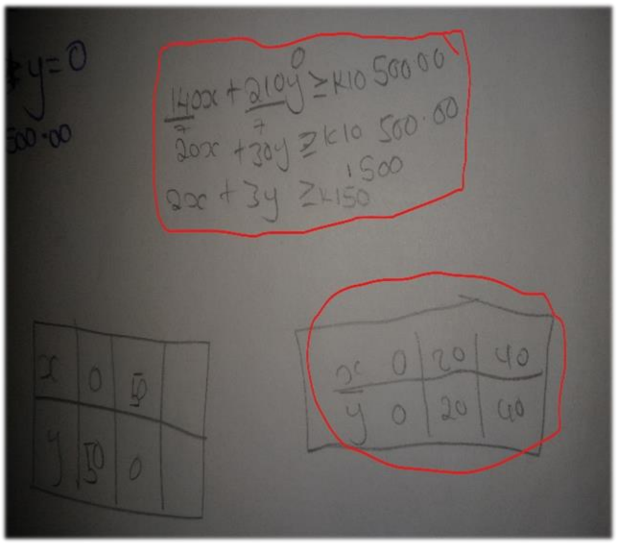


Figure 4.4: Learners U19, U20&U21 solving process.

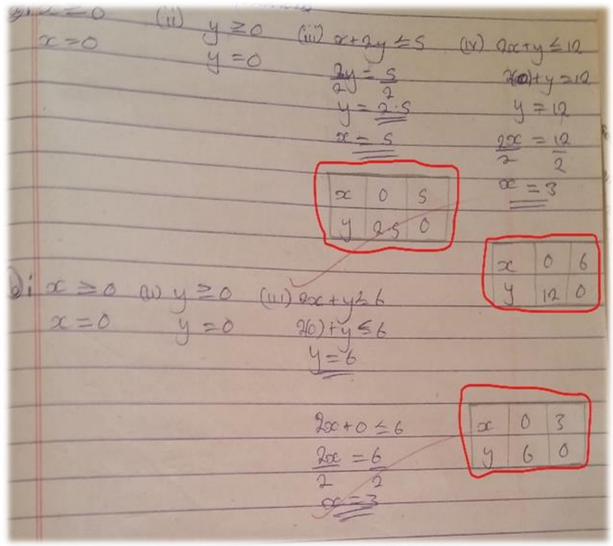


Figure 4.5: Learner V15.

The following verbatim from the focus group discussions align with what was observed during the lesson observations:

"…I always write some rough work on a separate piece of paper before writing the final equations in my exercise book…" **(**Learner **U8).**

"…first, I solve on a piece of paper, to make sure what the equation or what I am solving is correct…" ***(***Learner ***V5).***

Learners could draw the table of values in preparation to draw the graphs of inequalities, the majority (20 out of 24) could only find two pairs of coordinates correctly, the *x*-intercept (when *y*=0) and the *y*-intercepts (when *x*=0) (Figure 4.5), as emphasized by the teacher during the lesson observations and they could work out roughly on a piece of answer sheet as indicated in Figure 4.4, where they roughly formed a two-variable inequality, simplified it and roughly wrote its table of values.

4.2.4 Cognitive demands at *'Devising plan'* stage.

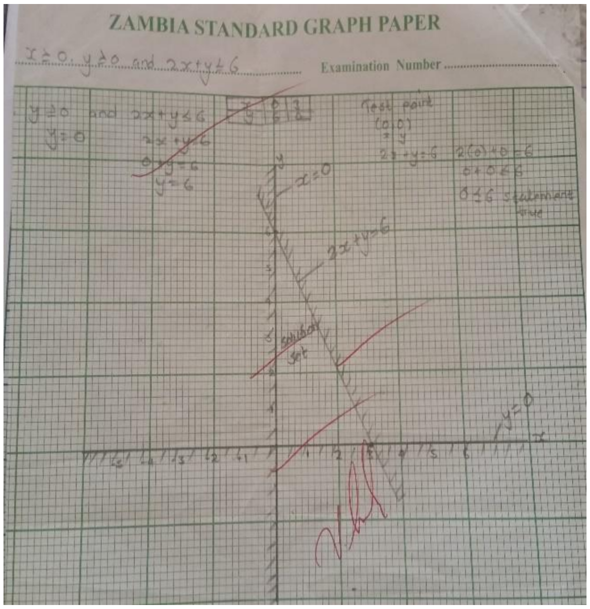
4.2.4.1 Interpretation and translation

The cognitive demands in problem-solving at '*Devising plans'* stage were also established on question **1.1** (See Appendix H), which was at both Remembering and Understanding cognitive levels. The findings showed that, in the quest to devise plans so as to solve the linear programming task, learners would engage in the process of interpretation and translation of the conditional statements to form mathematical inequalities. It was found that all the learners were able to recognize that the variable x represented the number of dresses and y, the number of suits. However, as evidenced in figure **4.2**, learners **U10, U11** and **U12** did not understand the meaning of "*should not be less than*", hence failed to correctly interpret and translate the statement and wrote an incorrect inequality as the majority (18 out of 24 learners). Learners **V7**, **V8** and **V9** were among the minority (6 out of 24) who correctly managed to interpret and translate the statements into inequalities for the second conditions. In writing down the inequality for the third condition, the results showed that some learners (8 out of 24) had a challenge in interpreting and translating the statement into the inequality, as evidenced in both Figure 4.3 and 4.2, some learners would first, write an incorrect inequality ($x+ y \geq 10500$), then proceed to write the actual inequality.

4.2.4 Carrying out the plan(s) to solve linear programming problems

4.2.4.1 Graphing Linear Inequations/Inequalities.

Results from the focus group discussions and the lesson observations showed that (15 out of 24) learners representing 67% showed competence in sketching the graphs of inequalities correctly: first drawing the associated equation of the inequality and then identifying and shading the unwanted regions. The following vignettes are the typical examples



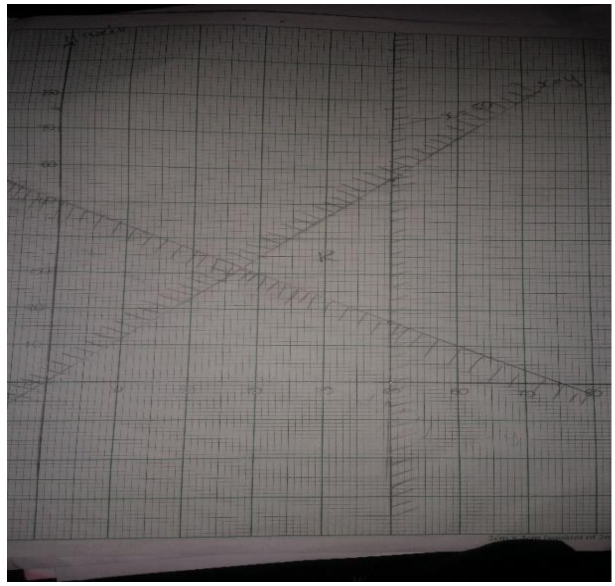
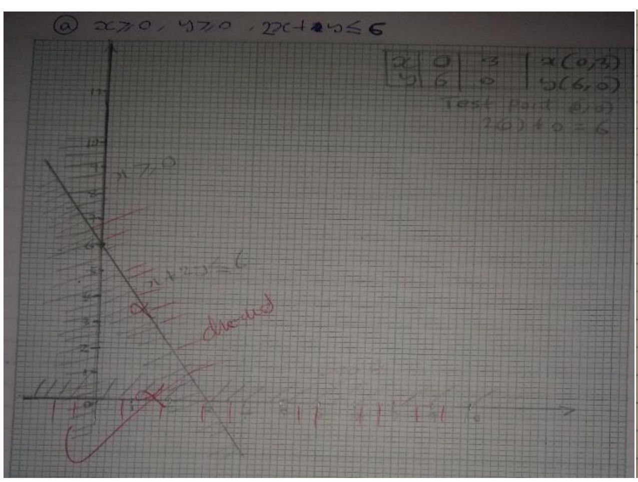


Figure 4.6: Learner V11 solving process.

Figure 4.7: Learners V4, V5& V6 solving process.

As shown from the above vignettes, learners were able to draw the graphs of the inequalities correctly: first by drawing the solid line because of the '*greater or equal to'* or the '*less or equal to'*, then proceed to identify the wanted and unwanted region by the testing two points from each side of the line, then shade the unwanted region.

On the other hand, the results indicated that learners experienced difficulties in drawing the graphs of the Linear programming inequalities, as evidenced from the following vignettes from the lesson observations:



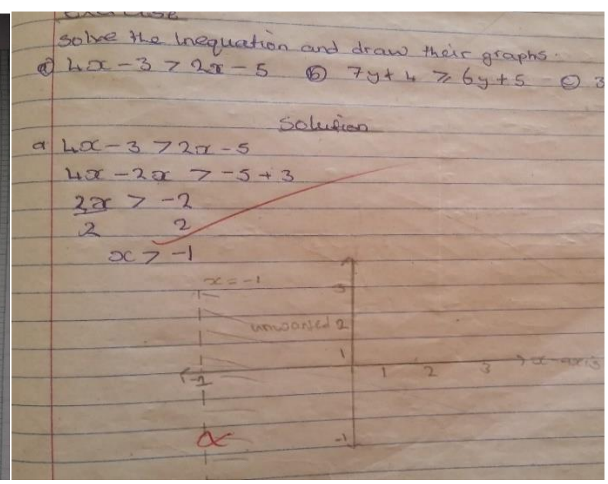


Figure 4.8: Learner V3 solving process.

Figure 4.9: Learner U8 solving process.

In Figure 4.8, learner (**V3**) was able to draw the associated equations of the inequalities but revealed a misunderstanding of shading the wanted region instead of the unwanted region. As preliminary work (grade 9 Inequations/Inequalities) to linear programming, learner **U8** was able to solve and draw the line of the inequality but had experienced a difficulty in determining the region or the side of the line to shade as the wanted region. The results revealed that (9 out of 24) leaners, representing 37.8%, experienced difficulties in determining the region of the line of the inequality to shade as the unwanted region. Despite the two teachers emphasising that:

*"…in linear programming, when drawing the inequalities we shade the unwanted region, **not** the wanted region as you were doing it in grade 9, …. this was just to remind us…"*
(Teacher **V:** Excerpt from lesson observation).

Learners still revealed the misconception of shading the wanted region instead of the unwanted region.

4.2.4.2 Identifying the feasible region and finding the maximum/ minimum profits

Results of the lesson observations revealed learners (11 out of 24) experienced difficulties in identifying the feasible region or the region for solution set, mainly due to incompetence in drawing the graphs of the inequalities correctly, as seen in Figure 4.8 and the vignettes below:

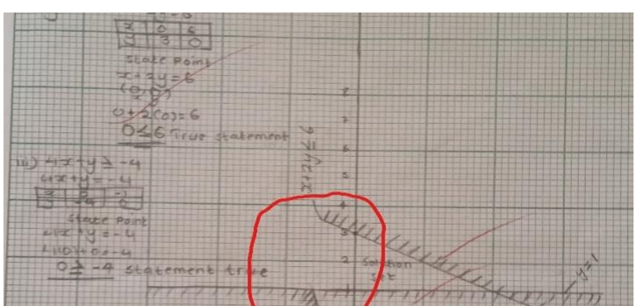


Figure 4.10: Learner U11 solving process.

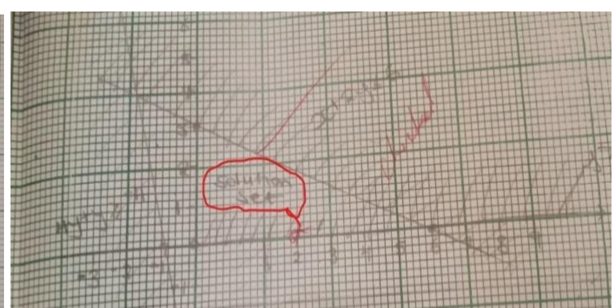


Figure 4.11: V6 Solving process.

Learner **V3** was unable to label the feasible region and when probed, he had the following to say:

"…the teacher said that the feasible region is the part that remains not shaded…this is sometimes confusing because all the parts sometimes become shaded…."

(Learner **V3:** Excerpt from lesson observation)

Learner **V6** and **U11** were able to identify the region of the feasible region despite not correctly drawing some of the lines of the inequalities.

The results from the focus group discussions also indicated that learners (12 out of 24, representing 50%), were able to correctly determine the maximum profit. However, half of the learners experienced difficulties in finding the maximum profit. For instance, the question: "*The profit on a dress is K160 and on a suit is K170. Find the number of dresses and suits the tailor must make for maximum profit and calculate the maximum profit?"* learners (21 out of 24) were able to derive the profit function which is *P(x) = 160x + 170y* but were unable to find the correct maximum profit due to incompetence in identifying and testing the point boundary points of the feasible region into profit function and others were testing the boundary points using an incorrect equation. The following vignettes are the typical examples:

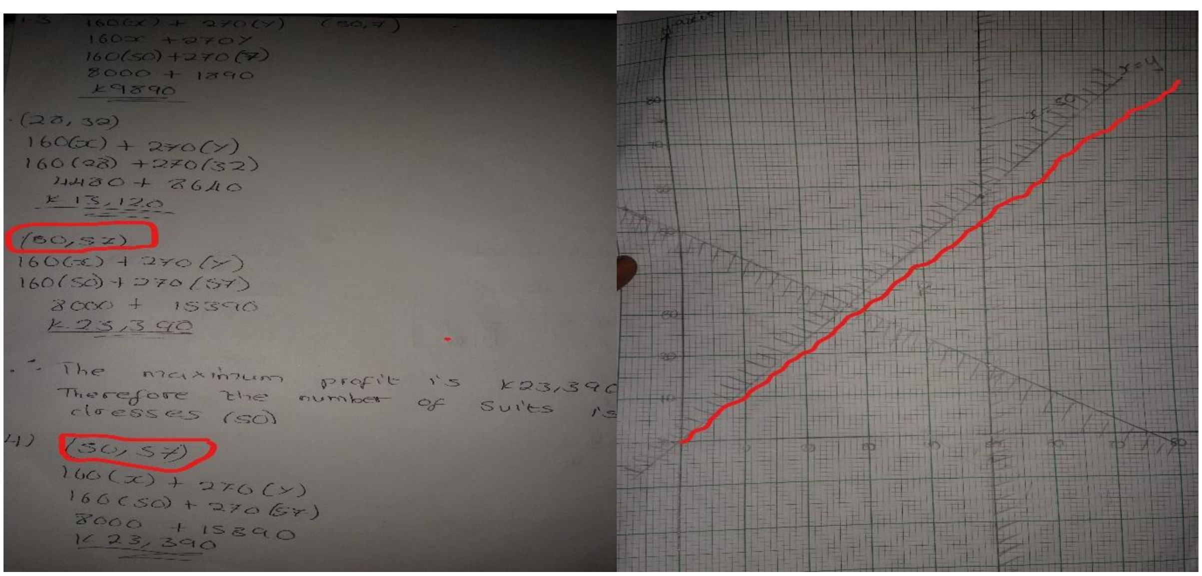


Figure 4.12: learner U6, U7 & U8 solving processes.

The learners, despite working collaboratively, could not find the correct maximum profit due to the incorrect drawing of the inequality $x \geq y$, which led to an incorrect pair of coordinates (50, 57), instead of the correct coordinates, (50, 50).

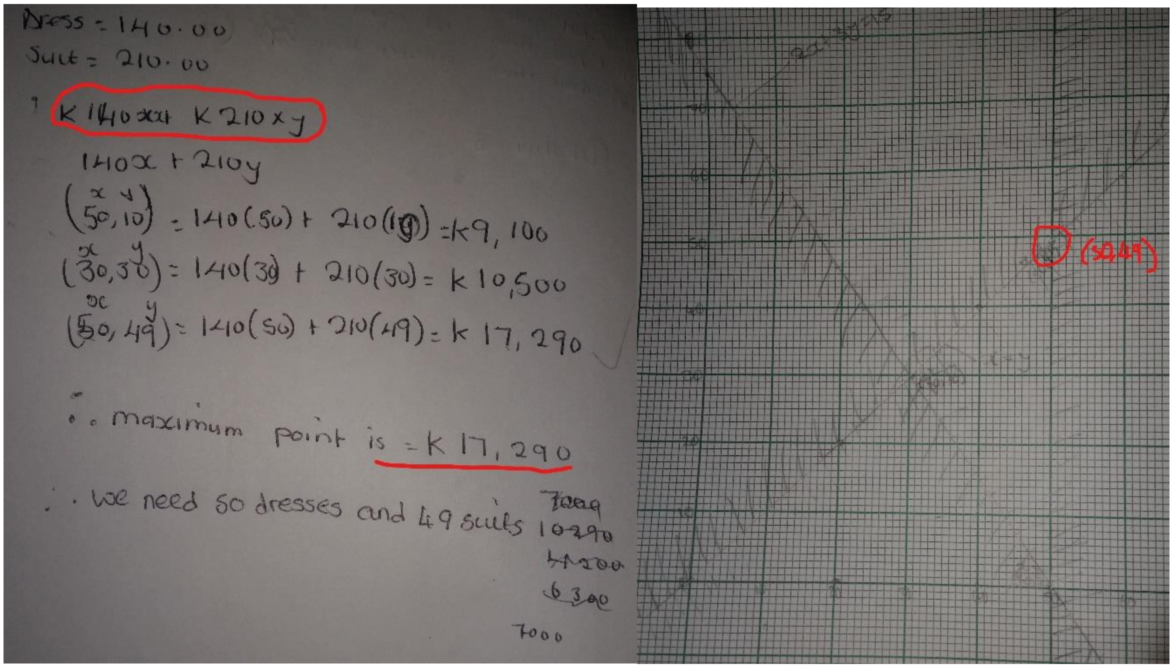


Figure 4.13: Learners V4, V5 & V6 solving process.

Learners could not arrive at the correct maximum profit, because instead of the profit function, they used one of the inequalities to test the boundary points for the maximum profit and when probed, they revealed a misconception on how to calculate the maximum profit as evidenced in the following verbatim:

"*…sir we used this inequality* (*140x + 210y*) *because to find the maximum profit…uhm you test the boundary points using* ***any*** *of the three inequalities…the biggest value you get, is the maximum profit…"*

(Learner **V5:** excerpt from the focus group discussion).

4.2.4.3 Checking every step in the solving process

The results from the focus group discussions showed that the majority of the learners (18 of out 24 learners) were not checking to see whether the steps of their workings were correct or not. This led to incorrect formulation of inequalities, incorrect graphing and eventually incorrect solution. The following verbatim provides evidence of the aforementioned finding:

"…. sometimes learner do not check through their work, as they solve, they just submit for marking. Others make simple mistakes, which can easily be identified and be corrected…" (Teacher U, Interview, September 2021).

"...how I check whether the answer is correct or wrong uhm…I go through my work, but sometimes even if you check, it is very hard to find the mistake, even if you check…unless the teacher marks our exercise books…" (Learner U4, FDG, September 2021).

According to the verbatim, learners did very little to check that each step of their solving processes were correct or not. Others cited time as the reason they don't check while others revealed lack of monitoring skills to check that their step was correct and whether it could be proven.

4.2.5 Cognitive demands at 'Carrying out the plan(s)' stage.

4.2.5.1 Problem-solving/Application: Graphing inequalities and shading the wanted region.

The cognitive demands involved at the '*carrying out the plan(s)'* stage, were established during focus group discussion and on question **1.2** (see Appendix H, FDG), which was at the Applying cognitive level. This required the application of the inequalities in graphing and shading the unwanted region. The results revealed that learners were able to engage in problem-solving process of applying acquired knowledge, facts, techniques and rules in graphing and shading the unwanted region, although some learners' workings were not mathematically correct. As evidenced in the vignettes below (Figure 4.14), learners (9 out 24) could not correctly draw graphs of the three inequalities: $x \geq 50$, $x \geq y$ and $2x + 3y \geq 10500$.

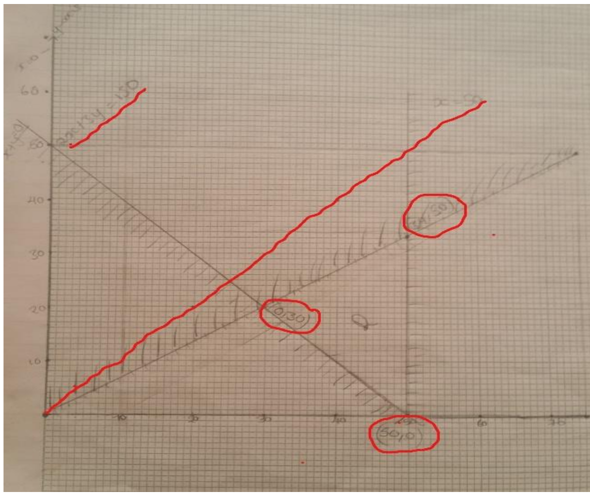


Figure 4.14: Learner V4, V5 & V6 solving process.

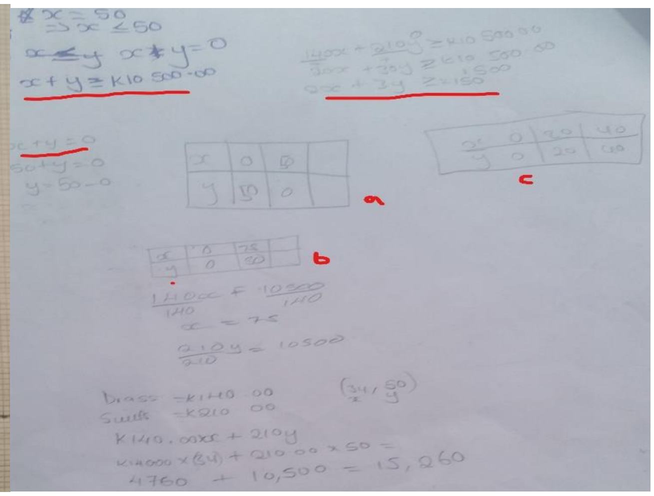
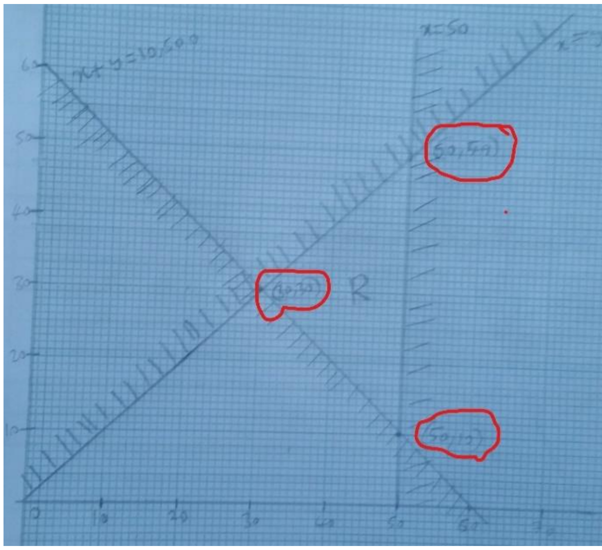


Figure 4.15: Learner V4, V5 & V6

In drawing the graph for the inequality $x \geq y$, learners had come up with a correct table of values (labelled c in Figure 4.17), but had used the table of values labelled b (Figure 4.15), which led to drawing an incorrect line with the correct shading of the unwanted region. In drawing the graph for the inequality $2x + 3y = 10500$, the results revealed that learners had experienced difficulties in drawing graphs for inequalities of two variables than with one variable. Learners would instead use the incorrect inequality $x + y \geq 10500$ and then write $x + y = 0$, then let $x = 50$, to get $50 + y = 0$, then get $y = 50$. Through this incorrect process learners came up with the table of values labelled ‘a’, with coordinates (0, 50) and (50, 0) which they used to draw an incorrect graph. The results also showed that learners (15 out of 24) did not have challenges in shading the unwanted region regardless of whether the graph arrived at was correct or incorrect. They were able to pick points on the sides of the graph and test them in the inequalities and shade the side whose point does not satisfy the inequality.

4.2.5.2 Analysing to establish the feasible region with its boundary points.

The finding showed that learners in carrying out the plans while operating at Analysing cognitive level could also engage in the cognitive process of analysis to establish the feasible region with its boundary points. This was established during lesson observation and on question **1.2** (refer to Appendix **H,**) of the task under the focus group discussion. The results indicated that learners (16 out of 24), despite working collaboratively, experienced challenges and could not arrive at the correct feasible region with correct boundary points.



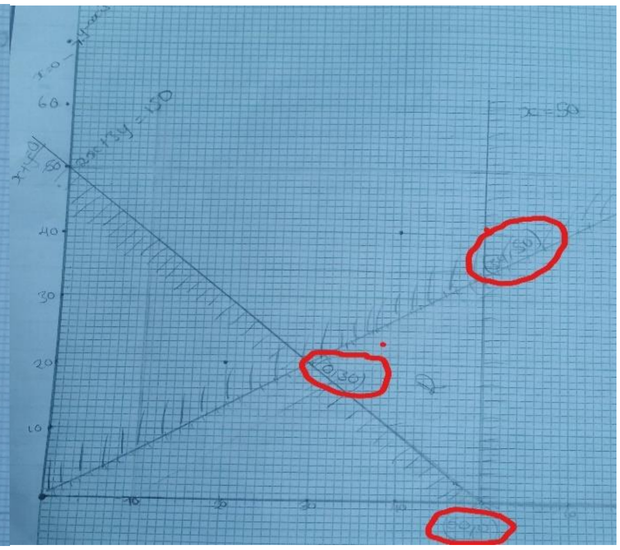
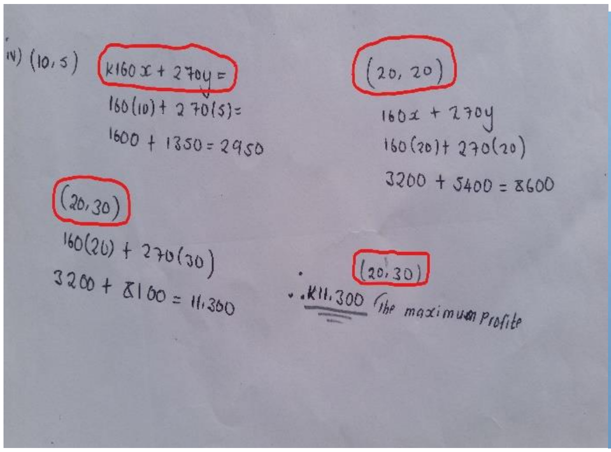


Figure 4.16: Learner U6, U7 &U8 solving process. ***Figure 4.17: Learner U2, U3 & U4 solving process.***

As seen in the above vignettes, some learners understood what the feasible region is, as the region that remains unshaded and it is where the solution for the three inequalities lies. However, some learners could not come up with the feasible region with correct boundary points owing to incompetence in drawing the graphs of the inequalities. For instance, in Figure 4.16, due to an incorrect graph for the inequality $2x + 3x \geq 150$, learners came up with inaccurate boundary points (*50,49), (30,30)* and (*50,10*), which are at total variance with the correct boundary points: (*50,50*), (*30,30)* and (*50,16*). In Figure 4.17, learners had difficulties in graphing the inequalities which lead to inaccurate boundary points (*34, 50*), (*10, 30*) and (*50, 10*), this also revealed lack of conceptual understating of coordinate geometry, learners had miss-represented the coordinates, for instance, instead of (***x, y***): (*50, 34*), (*30, 10)* and (*0, 10),* they would write **(*y, x*)** (*34, 50*), (*10, 30)*, (*50, 10*).

4.2.5.3 Derivation of abstract relations

Derivation of abstract relation was the third cognitive demand that emerged at the *carrying out plans* stage. This was established during lesson observation and on question **1.3** which was at the Evaluating cognitive level of the task under the focus group discussion (refer to appendix H). This involved deriving the profit function and establishing the correct boundary points. The results of the study revealed showed that learners (15 out of 24) were unable to derive the profit function and boundary points of the feasible region that could give the maximum profit, the following vignettes are the typical examples.



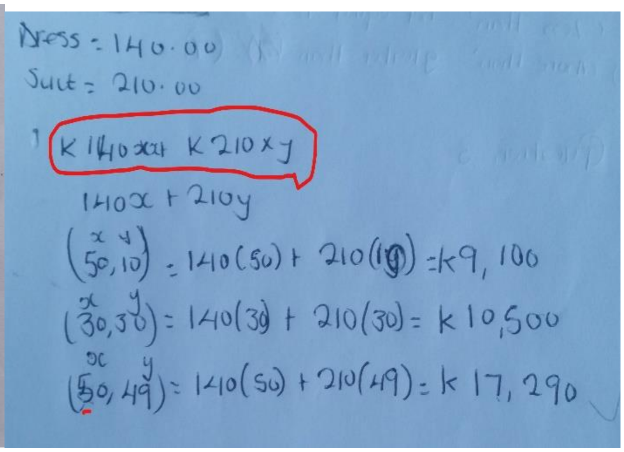


Figure 4.18: Learners U1, U2 & U3 solving process. *Figure 4.19: U10, U11 & U12 solving process.*

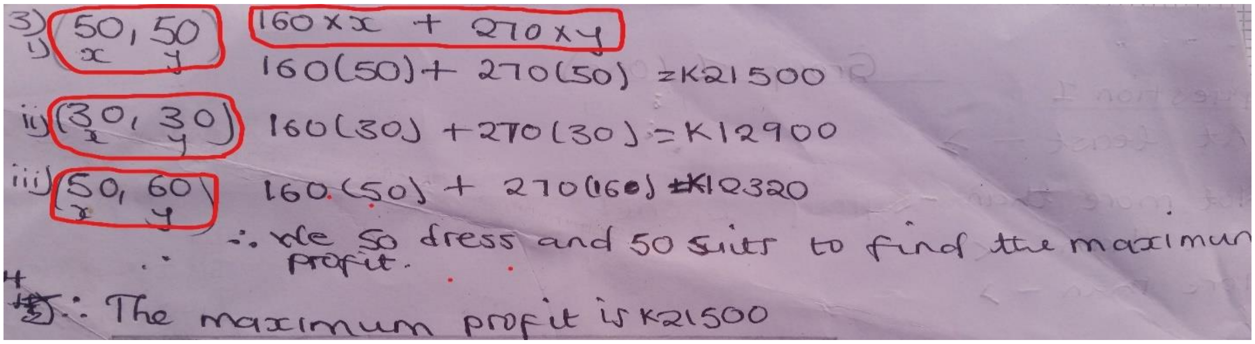


Figure 4.20: Learner V1, V2 &V3 solving process.

In Figure 4.18, learners were able to derive the correct profit function but could not establish the correct boundary points due to incorrect drawings for the inequality.

On the other hand, 9 out of 24 learners were able to derive the profit function correctly and managed to establish the boundary points that would be tested to yield the maximum profit as evidenced in the Figure 4.20.

4.2.5.4 Evaluating the maximum/minimum profit

The process of creating or generating the maximum/minimum profit emerged as another cognitive demand at '*carrying out plans'* stage. This was ascertained at question **1.3** and **1.4** under focus group discussion (See Appendix H). The questions were at evaluation cognitive level and required learners to deductively make judgements about the boundary points that would give the maximum profit and further determine the profit. The results of the study showed that learners (9 out of 24) could evaluate the maximum profit by testing boundary points of the feasible region. For instance, in Figure 4.20, learners revealed a misconception and lack of conceptual understanding for calculating the maximum profit, they had quite alright established the correct boundary points but had used an incorrect equation: 140x +210y as the profit function which led to an incorrect value for the profit function. Upon probing, learners had the following to say:

"...when finding the maximum profit.... uhm…first you get boundary points of the feasible region, test them in ***any*** *of the three inequalities…so that is how used this inequality (140x + 210y) …"* (Learners: **V1, V2& V3**).

Most learners were able to derive the profit function (*PR = 160x +270y*) but could not evaluate the correct maximum profit due to incorrect boundary points of the feasible region, which were as a result of learners' incompetence in drawing the graphs of the inequalities.

4.2.6 Looking back to check solving processes applied and solution arrived at after solving linear programming

4.2.6.1 Re-examining and reconsidering the solution arrived at

Results from the lesson observations also indicated that learners (20 out of 24) experienced difficulties in reconsidering and re-examining the results of their workings to check that they are correct or not. As evidenced in Figures: 4.9, 4.10, 4.11, 4.12, 4.13 and 4.14, learners did very little and experienced difficulties evaluating or looking back to ensure the answers or solutions arrived at are correct or not. For instance, in the aforementioned figures, results showed that learners could not examine whether the graphs of the inequalities, the feasible region and the minimum profit arrived at, were correct or not. The following verbatim which coincided with the views of other participants (such as learner **V3, U11** and **V7**), substantiates the above finding:

"...how I check whether the answer is correct or wrong uhm…I go through my work, but uhm sometimes even if I check, it IS very hard to find the mistake, even if you check…unless the teacher marks our exercise books…" (Learner U4, FDG, September 2021).

As evidenced in the above verbatim, it was established that learners experienced difficulties in looking back to evaluate and ensure that the solutions arrived at are correct or not.

4.2.7 Cognitive demands at *'Looking Back'* stage

4.2.7.2 Verification and reflecting on applied problem-solving processes

It was found that, at the *'looking back stage'* learners were operating at the evaluating cognitive level and would engage in the cognitive processes of reflection. This involved a critical reconsideration and re-examination of the solving processes used and the solution obtained so as to ascertain if the result is correct or whether the problem-solving processes plan can be used to solve another problem (Polya, 1976). However, as evidenced in Section 4.2.6.1, the majority of the learners did very little and experienced difficulties in verifying and reflecting on the processes used and the solution arrived at, and therefore could not identify errors and mistakes some of which could have been corrected during the problem-solving process.

4.2.12 Summary of research findings for research question one (1).

Table 4.1: Research question 1 on Grade 12 learners' problem-solving processes.

Solving Processes **(Stages & *cognitive demands*)**	**Specification of the solving processes**
Understanding the linear programming problem(s) 1. Learners do read linear programming problems. 2. Learners do re-read the linear programming. 3. Learners identified and underlined key statements. 4. Learners listed the statements with their associated symbols. ▪ *Remembering* *Learners could recognize and retrieve mathematical statements with their associated symbols, although others experienced difficulties.*	1. Reading 2. Re-reading 3. Identifying 4. Listing Phrases ▪ *Recognizing & Retrieving*
Devising a plan(s) to solve problems in linear programming 1. Learners interpreted and translated the conditional statements into algebraic form (inequalities), although others could not interpret and translate correctly. 2. Learners could draw table of values despite experiencing difficulties with inequalities with two variables. 3. Learners showed rough working skills. ▪ ***Understanding*** ▪ *Learners translated the mathematical statements into inequations, although other could not arrive correct solutions.*	1. Interpreting and translating. 2. Devising table of values. 3. Roughly working. ▪ *Interpreting & Translating*
Carrying out a plan(s) to solve linear programming problems 1. Learners drew the graphs of the inequalities, but others had challenges especially with the inequalities with two variables.	1. Graphing the Inequalities. 2. Identifying the feasible region.

2. Learners could construct the feasible region. Others had difficulties in constructing the correct feasible region due to inadequate skills in drawing graphs. 3. learners opted to use the method of testing the boundary points to find the maximum profit. Although, the majority could not arrive at the correct min/maximum profit. ▪ ***Applying*** ▪ *Learners experienced difficulties in applying the inequalities to graph and in shading the unwanted regions.* ▪ ***Analysing*** ▪ *Learners experienced difficulties in analysing and constructing the feasible region.* ▪ ***Evaluating*** ▪ *9 out of 24 learners were able to derive the profit function correctly and managed to establish the boundary.* ▪ ***Creating*** ▪ *Learners had difficulties in testing the boundary points and evaluating the maximum*	3. Calculating the Max/Minimum profit. ▪ *Application* ▪ *Analysis* ▪ *Deriving/testing* ▪ *Producing/ generating*
Looking back to check solving processes applied and solution arrived at after solving linear programming 1. Very few learners could reconsider and evaluate their solution arrived at. The majority did very little to check whether the answers were correct or not. ▪ *Reflection and verification* ▪ *Reflecting and reconsidering of processes and solutions was a challenge to the majority learners*	1. Reconsidering and Re-examining ▪ *Verifying and reflection on the processes.*

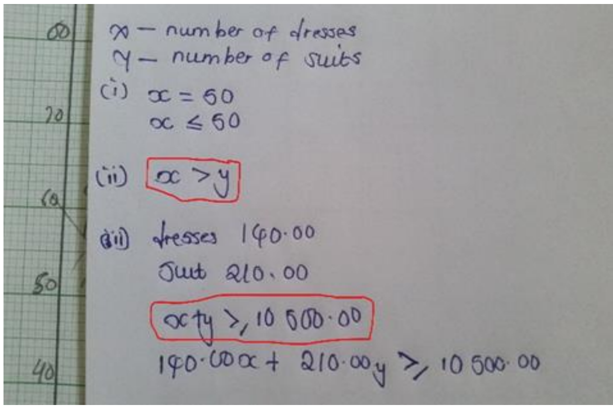
4.3 Research Question two (2): What factors affect grade 12 learners' problem-solving processes in linear programming?

Research question one focused on establishing and exploring factors that affect learners' problem-solving processes in linear programming. The following emerged as themes:

4.3.1 Insufficient teaching and exposure to mathematical language

One of the factors that affected learners' problem-solving processes in linear programming is insufficient teaching and exposure to mathematical language. Findings from lesson observations and Focus Group Discussion showed that teachers spend little time to explain and expose leaners to the mathematical language used in problems in linear programming. This is supported by the following verbatim: "*…our teacher did not take time to explain all the words we use in linear programming…Uhm some of the words are confusing…it can be nice if the teacher can take much to explain all the words…"* Learner **V8,** During Focus Group Discussion.

In the effort to understand the linear programming question, learners could read through the questions at least once. However, the findings showed that learners had difficulties understanding the conditional statements, which lead to misinterpretation and translation conditional statements into incorrect inequalities. Some learners (12 out of 24) were familiar with the common mathematical statements with their associated symbols such as '*greater than or equal to*' (≥), '*less than or equal to'* (≤), *'less than'* (<) and '*greater than'* (>). However, in instances where there was a slight twist of language, for instance, using '*should not exceed'* to mean '*less or equal to'* (≤), and using '*should not be less than'* or '*at least'* which imply '*greater or equal to' (≥),* learners (15 out of 24) had challenges in understanding the correct meanings implied, with their correct associated mathematical symbols. The following vignettes are the typical examples, which are excerpts from learners' workings on task under the focus group discussions.



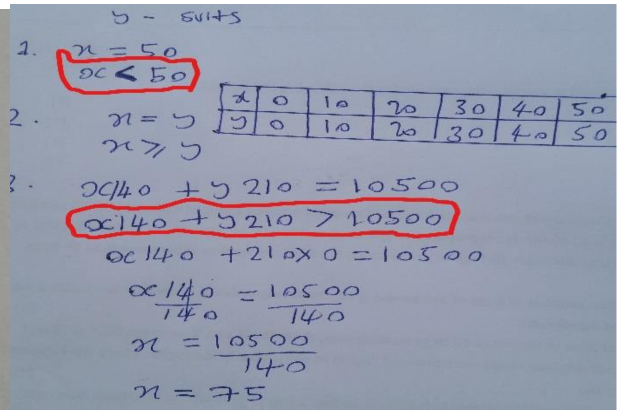


Figure 4.2: Learners U1, U2 & U3 solving process. *Figure 4.21: Learner V4, V5 & V6 solving process.*

As seen in Figure 4.2, learners could not correctly interpret and translate the statement '*should not be less than'*, they would write an incorrect inequality *(x >y)* while in figure 4.21, learners did not understand the meaning of '*at least'* which implied '*greater or equal to' (≥)*, and this led to translation into an incorrect inequality: $x140 + y210 > 10500$. This finding is further substantiated by an excerpt from the interview with Teacher **V**, which was a response from the question: "From your experience of teaching mathematics, what difficulties or challenges do learners encounter in solving linear programming question?

"*…among the challenges is misunderstanding of the mathematical statements...learners misinterpret the mathematical statements…which I think is due to language barrier…the majority of the learners are more familiar with the local language than English. For example, in local language when you hear someone use 'at least', it will mean the least value, that is a direct translation, yet the correct meaning is the 'given value or more'…*" (Excerpt from Teacher **V's** interview).

In general, failure to understand the language of linear programming stands out as one of the constraints in learners' problem-solving processes; of understanding the problem, planning to solve the problem; executing the plan and looking back to verify correct solutions. This therefore led to misinterpretation and translation of situational conditions into incorrect Inequalities.

4.3.2 Weak background in preliminary topics to Linear Programming

The results from the lesson observations, focus group discussions and semi-structured interviews as well revealed that some learners (9 out of 24) had a weak background and poor foundation in the preliminary topics (grade 9 and 10 topics): Coordinate Geometry and Graphs of Inequalities. These topics form the basis for the learning of linear programming. Due to this constraint, learners experienced difficulties in graphing the inequalities and in presenting the coordinates of the points correctly. The following vignette is a typical example:

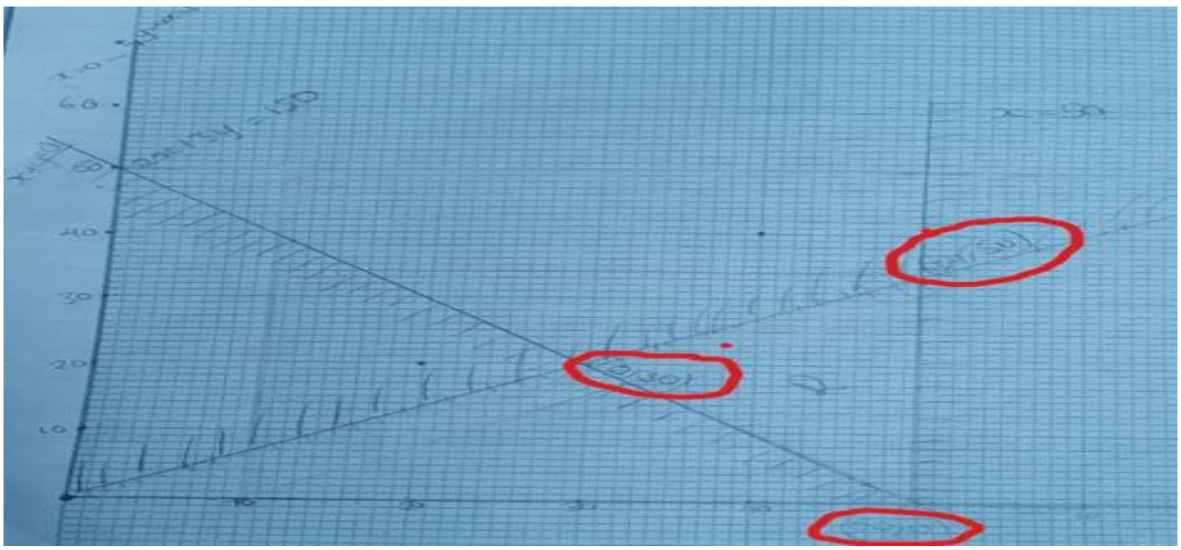


Figure 4.22: Learner U9 solving process (Lesson Observation)

As seen in the above Figure 4.21, due to poor foundation in Coordinate geometry, learners had difficulties in graphing the inequalities which lead to inaccurate boundary points (34, 50), (10, 30) and (50, 10), this further revealed learners' lack of conceptual understating of coordinate geometry. For instance, learners had mis-represented the coordinates, instead of (***x, y***): (50, 34), (30, 10) and (0, 10), soon learners would write **(*y, x*)** (34, 50), (10, 30), (50, 10). The following excerpt from semi-structured interviews with teacher **U,** as well, coincides with the aforementioned finding. *"...the other challenge I encounter is leaners fail to construct correct lines for the inequalities.... we teach leaners to come up with equations of straight lines when we teach* ***them coordinate geometry and also when they are learning Inequalities in grades 8 and 9****, but you find that when it comes to linear programming, they* ***forget to relate*** *the same concepts that they had learnt in those previous topics to relate them to the inequalities in linear programming..."*

(Teacher **U,** September, 2021).

It is explicit from the above excerpt that due to weak background and poor foundation in preliminary topics to linear programming, learners could forget and failed to relate the concepts in Coordinate Geometry and Inequalities, to linear programming and this leads to drawing incorrect graphs and misrepresentation of the coordinates.

4.3.3 Drawing the inequality graphs and shading the unwanted region.

Another constraining factor that affected learners' problem-solving processes in linear programming is shading the unwanted region to imply that the solution of the inequality(s) lies in the unshaded region. The results of the study showed that some learners (10 out of 24) would still shade the wanted region as was done at junior secondary level, instead of the shading the unwanted region in linear programming, and this would lead to coming up with an incorrect feasible region.

As seen in Figure 4.8 (Section 4.2.4.1), learner (**V3**) was able to draw the associated equations of the inequalities but revealed a misunderstanding of shading the wanted region instead of the unwanted region (For the inequalities $x + 2y \leq 6$ and $y \geq 0$), while learner **U8** (Figure 4.9, Section 4.2.4.1) was able to solve and draw the line of the inequality but had experienced a difficulty in determining the region or the side of the line to shade as the wanted region. The results revealed that (9 out of 24) leaners, representing 37.8%, experienced difficulties in determining the region of the line of the inequality to shade as the unwanted region. Despite the two teachers emphasizing that:

"…*in linear programming, when drawing the inequalities we shade the unwanted region, not the wanted region as you were doing it in grade 9, …. this was just to remind us…"* (Teacher **V:** excerpt from lesson observations).

Shading the wanted region in the preliminary topics to linear programming proved to be one of the constraining factors that affect learners' problem-solving processes at grade 12 level.

4.3.4 Calculating the maximum/minimum profit.

Another constraint that affects learners' problem solving to finding the maximum or minimum profit is identifying correct boundary points and testing them using the profit function. Learners revealed a misconception and lack of conceptual understanding for calculating the maximum profit, they had quite alright established the correct boundary points but had used an incorrect equation: 140x +210y as the profit function which led to an incorrect value for the profit function. (See Figure 4.20, Section 4.2.5.3). Upon probing, learners had the following to say:

"…when finding the maximum profit.... uhm… first you get boundary points of the feasible region, test them in ***any*** *of the three inequalities…so that is how we used this inequality (140x + 210y) …"* (Learners: V1, V2 & V3, September 2021, FDG).

Most learners (15 out of 24) were able to derive the profit function (PR = 160x +270y) but could not evaluate the correct maximum profit due to incorrect boundary points of the feasible region, which were as a result of learners' incompetence in drawing the graphs of the inequalities.

4.3.5 Lack of monitoring each step to the solution.

Out of 24, 20 learners experienced difficulties in checking and monitoring their workings to ensure they are on the right track. As evidenced in Figures: 4.9, 4.10, 4.11, 4.12, 4.13 and 4.14, learners did very little and experienced difficulties in checking and evaluating or looking back to ensure the answers or solutions arrived at are correct or not. For instance, in the aforementioned Figures, results showed that learners could not check the steps of their workings: in interpreting and translation word statements into inequalities, checking whether drawn graphs of the inequalities were correct by testing two points on the line then substituting them into the inequality and ascertain whether it satisfies the inequality or not, identifying the feasible region and in finding the maximum profit and this incompetence led to incorrect answers or solutions.

4.3.7 Summary of research findings for research question two (2).

Table 4.2: Research question two (2) on factors that affect Grade 12 learners' problem solving in Linear Programming.

Factor.	Specification of the factor.
▪ **Insufficient teaching and exposure to mathematical language** ▪ Learners misinterpreted and mis-translated word conditions into inequalities. Only 9 out of 24 could write correct inequalities.	▪ Misunderstanding and misinterpretation of conditional statements.
▪ **Weak background in preliminary topics.** ▪ Some learners had limited knowledge in coordinate geometry and graphs of inequalities: misrepresented pairs of coordinates and had challenges graphing inequalities.	▪ Poor foundation in grades 8 & 9 topics: Coordinate Geometry & Inequations and Inequalities.
▪ **Drawing inequality graphs and shading the unwanted region.** ▪ 9 out of 24 leaners, representing 37.8%, experienced difficulties in determining the region of the line of the inequality to shade as the unwanted region.	▪ Shading the wanted region in grade 9 Vs shading the unwanted region in grade 12.
▪ **Calculating the maximum/minimum profit.** ▪ Learners revealed a misconception and lack of conceptual understanding for calculating the maximum profit	▪ Identifying the correct boundary points and testing them using the profit function
▪ **Lack of monitoring and evaluating** ▪ Learners (20 out of 24) did very little and had difficulties in checking and monitoring to ensure the solutions arrived at are correct or not.	▪ Monitoring each step to ensure the right track to the solution.

4.4 Research question three (3): How can learners' problem-solving processes in linear programming be enhanced?

Research question three (3) focused on establishing and determining strategies that may help to enhance learner's problem-solving processes in linear programming. This was achieved through the interviews with the learners under the Focus Group Discussion and the semi-structured interviews with the teachers. Some of the strategies or measures that emerged from study are not specific to issues in linear programming, but may be applied in mathematics in general. The following are the themes that emerged after thematic analysis (Braun & Clarke, 2006).

4.4.1 Sufficient teaching and exposure to the mathematical language of teaching

Findings showed that in order to enhance learners' problem-solving processes in linear programming, teachers need to be spending much time to explain and expose learners to the mathematical language used in linear programming. This was seen in the following excerpt from the semi-structured interviews with teachers.

"*…among the challenges is misunderstanding of the mathematical statements...learners misinterpret the mathematical statements…which I think is due to language barrier…the majority of the learners are more familiar with the local language than English. For example, in local language when you hear someone use 'at least', it will mean the least value, that is a direct translation, yet the correct meaning is the 'given value or more…* ***I think to help learners, we teachers, need to be ensuring that our learners understand all terminology used in linear programming, by spending more to explain...uhm, in fact the whole language of linear programming.***..". (Excerpt from Teacher **V's** interview).

This finding was indirectly supported by the following complaint which coincided with the views of other learners: "*…our teacher did not take time to explain all the words we use in linear programming…Uhm some of the words are confusing…it can be nice if the teacher can take much to explain all the words…"* Learner **V8,** During Focus Group Discussion.

Ensuring that learners are well-conversant with the mathematical language used in linear programming is essential for the enhancement of their problem-solving processes.

4.4.2 Evaluation of learners' prior knowledge of concepts in Coordinate Geometry and Graphs of Inequalities prior to introducing linear programming

The findings of the study showed that ensuring that learners have the requisite knowledge of Coordinate geometry and Inequalities and Inequations: solving and graphing of Inequations and Inequalities is essential for the enhancement of learners' problem-solving in linear programming. This was seen in the following excerpt from the interview with the learners during the focus group discussion.

"*...for me, the biggest challenge is that I do not know how to plot the inequalities, our teacher did not spend enough time to teach us when introducing this topic (Linear Programming) …it can help some of us if our teacher can spend more time on how to plot the inequalities, especially those with two variables…" (*Learner **V8,** September, 2021)

The above concern coincided with the views held by a number of participants (such as Learners **V3, V6 & V9** among others) and their teachers. The following is an excerpt from the interview with teacher **U**, it was a response to the question: "*…from your experience, what are some of the challenges learners encounter in linear programming and what do you do to help learners to overcome the challenges…"* (Principal Investigator).

"…one of the most common challenges learners face is failure to plot correct lines for the inequalities…we teach learners how to come with straight lines in teaching them coordinate Geometry and also when they are learning Inequalities in grades 8 and 9…but when it comes to Linear Programming, learners forget and fail to relate the concepts they learnt previously…so now, before introducing Linear Programming I first spend some time to teach the preliminary topics, which are Inequations and Inequalities: how to solve and plot lines (graphs) of inequalities, I do this to remind the leaners of the knowledge needed in linear programming…" (Interview excerpt: Teacher **U**, September, 2021).

The findings revealed that ensuring that learners have the prior or pre-requisite knowledge of Coordinate Geometry and Inequalities and Inequations is one of the measures that can help and enhance leaners' problem-solving processes in linear programming.

4.4.3 Restricting the shading either to unwanted or wanted region in Linear Programming and the preliminary topics

The results of the study revealed that among the challenges learners in linear programming faced was shading the wanted region instead of the unwanted region. This misconception is attributed to the fact that at junior secondary level, under the topic: Graphs of Inequalities and Inequations, learners shade the wanted region to indicate where the solution of the inequality lies while at grade twelve level it is otherwise, they shade the unwanted to indicate that the solution of the inequality will lie in the unshaded region. The findings of the study indicated that consistency or restricting the shading of either the wanted region or unwanted region at both junior secondary level and senior level may help to enhance learners' problem solving in linear programming in the sense it would prevent learners from experiencing the misconception of shading the wanted region instead of the unwanted region at grade twelve level. Evidence of the above finding abounds in the following excerpts.

"…no matter, how many times you emphasize, some learners usually have the challenge of shading the unwanted region at senior secondary level, they shade the wanted region instead of the unwanted… this is because of shading the wanted region at junior secondary level, it can really help the learners if they can be doing the same thing both at junior and senior level…. if it is shading the wanted region, it should just be like that throughout…" (Interview excerpt: Teacher **V,** September, 2021).

4.4.4 More interaction with authentic situational problems

The results of the study indicated that due to lack of enough interaction with Linear programming problems embedded in real and authentic contexts, learners had challenges understanding the mathematical language of linear programming, as such, more learner interactions with situational problem may enhance their problem solving. From observation, learners were conversant with the common basic statements such as: '*greater than', 'less than', 'greater or equal to'* and *'lesser or equal to'*. However, in instances where there was a twist of language, for instance using *'should not exceed'* to mean *'less or equal to'* (≤), and using *'should not be less than'* or *'at least'* which imply *'greater or equal to'* (≥), learners would reveal misunderstandings which lead to misinterpretation and formation of incorrect inequalities and eventually incorrect graphs. The following interview excerpts resonates with the above observations:

"*…when you have taught and everything has been said the next thing is to give them* ***more*** *questions to* ***practice*** *so that they can apply what we have taught them...through practice learners get to understand more especially the mathematical statements and drawing the lines for the inequalities…"* (Interview excerpt: Teacher **V**, October, 2021).

"…so, we give them ***more*** *homework, sometimes even questions to research on in case they can discover some easier and simpler ways apart from what we have taught them, with more work learners are able to practice what they learnt…"* (Interview excerpt: Teacher **U**, September, 2021).

It is explicit that more learner interaction with situational problem may help and enhance learners' problem solving in linear programming.

4.4.5 Use of group-work approach to teach linear programming

Another measure, one of the teachers suggested, that can enhance and improve learners' problem-solving processes in linear programming is promoting group work, where learners are put in groups to work on an activity or problem in linear programming. This finding was substantiated by the following interview excerpt:

"…the challenges I face, is to draw the lines of the inequalities, especially with two variables…our teacher did not explain very much, but ***I have understood some things from my friends as we are solving the work you have given us, it can be nice if our teacher can also be putting us like this, in groups****…"* (Excerpt from FGD: Learner **V7,** September 2021).

"…the other way that we can help learners to improve uhm is to be giving them more work or questions for them to be working in groups, this can give them chance to discuss with one another, they will be sharing ideas and learn from one another…you know what, sometimes a learners will understand more when a fellow learner explains a concept to him or her, than when a teacher explains…" (Interview Excerpt: Teacher **U**, September 2021).

The above verbatim substantiates the finding that promoting group work may enhance and help improve learners' problem-solving processes in linear programming.

4.4.6 Summary of research findings for research question three (3).

Table 4.3: Research question three (3): How can learners' problem-solving processes in linear programming be enhanced?

Measure	Specification of the Measure	Finding
Sufficient teaching and exposure to the mathematical language of teaching	▪ Ensuring that learners are proficient with the mathematical language used in linear programming.	▪ More time for teaching and exposing learners to the mathematical language used in linear programming.
Evaluation of learners' prior knowledge of concepts in Coordinate Geometry and Graphs of Inequalities prior to introducing linear programming	▪ Ensuring that leaners have the pre-requisite knowledge of Coordinate Geometry and graphs of Inequalities before introducing the topic linear programming.	▪ Revision of Coordinate Geometry and Inequalities and In-equations before introduction to Linear Programming.
Restricting the shading either to unwanted or wanted region in Linear Programming and the preliminary topics	▪ Shading either the wanted region or unwanted region at both junior and senior secondary levels, to prevent the misconception at grade 12 level.	▪ Consistency in shading the wanted throughout, both at junior secondary level and senior level
More interaction with authentic situational problems.	▪ More learner interaction with situational problems of linear programming.	▪ More learner interaction with Linear programming problems via homework, research question and revision.
Use of group-work approach to teach L/P	▪ Learner-centred approach	▪ Promoting group work among learners.

4.5 Summary of the Chapter

This chapter presented finding from the participants on the processes of learners' problem solving in linear programming as presented in Table 7 below:

Table 4.4: Summary of the Chapter four

Objective 1: To establish learners' problem-solving processes in Linear programming	
Process	**Description**
Stages & *Cognitive levels*	
Understanding the problem ▪ *Remembering*	▪ Reading and Re-reading, ▪ Identifying key statements ▪ Listing Phrases with symbols ▪ *Retrieving and recognizing information.*
Devising plan to solve the problem ▪ *Understanding*	▪ Interpreting and translating. ▪ Devising table of values. ▪ Roughly working. ▪ *Interpreting and translating*.
Executing plans ▪ *Applying* ▪ *Analysing* ▪ *Evaluating* ▪ *Creating*	▪ Graphing the Inequalities. ▪ Identifying the feasible region. ▪ Calculating the Max/Minimum profit. ▪ *Applying* ▪ *Analysing* ▪ *Deriving, critiquing.* ▪ *generating and producing*
Looking back ▪ Evaluation	▪ Reconsidering and Re-examination ▪ *Reflection and Verification*
Objective 2: To explore factors that affect grade 12 learners' problem-solving processes in Linear Programming problems. **Factors**	
Insufficient teaching and exposure to the mathematical language of linear programming	▪ Understanding the language and symbols of Linear Programming.
Weak Background in preliminary topics to Linear Programming.	▪ Poor foundation in prerequisite knowledge of Linear Programming.
Drawing of Inequality graphs and shading the unwanted region	▪ Inconsistency of shading the unwanted region at junior and senior secondary levels.
Calculating the maximum/minimum profit	▪ Use of inappropriate strategy
Lack of monitoring and evaluating steps	▪ Lack of monitoring skills for checking the steps in solving the problems.

Objective 3: To determine strategies that would help improve learners' problem-solving processes in linear Programming. **Strategies**	
Sufficient teaching and exposure to the mathematical language of linear programming	▪ Ensuring learners are proficient with mathematical language of linear programming.
Evaluation of learners' prior knowledge of concepts in Coordinate Geometry and Graphs of Inequalities prior to introducing linear programming.	▪ Ensuring that leaners have the prior or pre-requisite knowledge of Linear programming.
Restricting the shading either to unwanted or wanted region in Linear Programming and the preliminary topics.	▪ Shading the wanted region throughout, to prevent the misconception at grade 12 level.
More learner interaction with authentic situational problems.	▪ More learner interaction with situational problems of linear programming.
Group-work approach to Linear programming.	▪ Promoting learners' group working.

The chapter presented research findings according to the research questions of study: what are learners' problem-solving processes in Linear Programming; what factors affect learners' problem-solving processes in Linear Programming and; what strategies would help enhance learners' problem-solving processes in linear programming. Table 4.4 summarises the chapter. The next chapter discusses the research findings in relation to the theoretical framework and literature review.

CHAPTER FIVE

DISCUSSION OF FINDINGS

5.1 Introduction

This chapter presents a discussion on the findings and the implications of these findings. As presented in the previous chapter, the discussion will be done in subsections. Section 5.2 discusses how learners engage in problem solving in linear programming. Sections 5.3 provides discussions on the factors that affect learners' problem processes in Linear programming and Section 5.4 discusses the strategies that can enhance learners' problem-solving skills in linear programming. Finally, Section 5.5 summarizes the chapter.

5.2 How do Grade 12 learners solve problems in Linear Programming? (With respect to Polya's stages of problem solving and the cognitive demands cognitive process dimension of the Revised Bloom's taxonomy)

5.2.1 Processes of problem-solving

This subsection provides a discussion of the stages in problem-solving as gauged by George Polya's model of problem solving and the cognitive demands of problem-solving.

5.2.1.2 Understanding the linear programming problem(s)

The discussion under this sub-section is based on the findings with respect to Polya's (1957) first stage of understanding the problem with its respective indicators: reading and re-reading, identifying key statements and listing phrases with their associated symbols.

The study showed that learners in their quest to solve the linear programming problems, first read the question at least once, with a view to gaining an understanding of what the question demands are. (Section 4.2.1.1). This finding was in line with the Gestalt psychology of learning (theory) as expounded by Werthimer (1959) who contended that reading a mathematical problem before solving it is an essential part of the problem-solving process as it helps the solver to understand the problem at hand. Through reading the question at least once, some learners were able to establish what the question required of them (the unknown), the data and conditions.

Under the Polya's stage of understanding the problem, the study also revealed that learners in the quest to understand the question, could also identify and underline keys statements, and list them with their associated mathematical symbols. (Section 4.2.1.2) According to Polya (1959), after one has read through the problem, one should be able to underline key words as this helps learners to restate the problem in their own interpretation thereby advancing their understanding of the problem. Learners did all these in the quest to understand the problem.

Polya (1957) argued that it is not wisdom to answer the question which you have not read and you do not understand, he further argued that, for understanding purposes, it is important to read a question before solving it. Needless to say, when a learner reads once through a question, s/he will understand the question, but s/he will understand even more when the question is read through at least once. On the other hand, the findings showed that reading the question at least once, does not guarantee correct solutions, during the lesson observation and focus group discussion, all the learners were able to read through the question at least once but only a handful of the learners managed to arrive at the correct solutions (Section 4.2.1.1). It is therefore, explicit that reading through the question at least once before solving is an integral part of the problem-solving process as it may help learners to: understand what the question requires of them (the unknown) and to identify and underline the given data and conditions or keywords with their associated mathematical symbols (what is known).

5.2.2 Cognitive demands at *Understanding the problem* stage.

5.2.2.1 Retrieving and recognizing.

Findings showed that learners in their effort to understand linear programming problems, would engage in the cognitive process of retrieving and recognizing or exhibit memory of previously learnt material. For instance, learners would retrieve and recognise conditional statements (factual knowledge) in order to understand what the question's demands are. On the other hand, the findings also showed that some learners had challenges in retrieving and recognizing the mathematical language of linear programming: the terms, meanings and their associated mathematical symbols needed in the formulation of inequalities and solving of linear programming questions. It can therefore be argued that having the ability to retrieve and recognize factual and conceptual knowledge is an essential aspect of problem-solving if learners are to achieve well in the solving of linear programming problems. Moreover, according to Mayer (1982), retrieving and recognizing terms, ideas and procedures is very essential in the process of problem-solving as it influences the problem solver's understanding of the problem as well as the choice of strategies that will be called upon in trying to solve the problem.

5.2.3 Devising plans to solve linear programming problems

This subsection provides a discussion of the findings according to Polya's second stage of problem-solving with respect to following indicators (sub-themes): Interpreting and Translating, devising table of values and roughly working.

The findings of the study showed that, learners in the quest to find the connection between the given data and the unknown (the key information and what is required of them in a linear programming problem), and eventually devise a plan for the solution, learners had difficulties in forming correct Inequalities, especially, those that involved two variables (Section 4.2.2.1). For instance, learners could not correctly write down the inequality for the statements: "*The cost of making a dress is K140 and that of a suit is K210. The total cost should be at least K10500.*" and "*The number of dresses should not be less than the number suits*". Some learners could not fully understand the meaning of the statement "*should not be less than*" and "*at least*" instead of the greater or equal sign *(≥),* they used the symbol for strictly '*greater than'* and the '*equal sign'*, as evidenced in Figure 4.2 of section 4.2.2.1., this led to mis-interpreting and formation of incorrect inequalities.

According to Hiebert and Lefevre (1986), learners' difficulties in interpreting and constructing relationships between pieces of information of a linear programming(s) can be attributed to their lack of conceptual and procedural knowledge i.e. knowing the formal language of linear programming, or the symbol representation system of linear programming, knowing the rules for completing and procedures, and knowing strategies for solving problems. (Hiebert & Lefevre 1986). These findings resonate with the findings of Haghverdi et al. (2012) which showed that the learners' difficulties in solving mathematical problems were due to their disabilities in representing and understanding of word problems, making a plan and defining the related vocabularies. The findings also revealed that, the causes of learner difficulties were text difficulties, unfamiliar contexts in problems and using inappropriate strategies (Haghverdi et al.,2012).

It can therefore, be argued that when learners are deeply rooted in conceptual and procedural knowledge of linear programming, they are well able to correctly interpret, translate and form correct inequalities, eventually devising a plan to solve the mathematical problem. This aligns with Polya's (1957) assertion that being able to devise a mathematical formula for use, is an essential skill, which can also guide learners on how the problem will be solved and how the solution will be arrived at.

The findings of the study also revealed that, before solving the linear programming question learners did draw tables of values and did rough workings of solving and graphing on a non-answer sheet (Section 4.2.2.2). This was all part of their quest to devise plans to solve the linear programming problems. In problem-solving, tables of values and rough workings play an importance role as they give the problem-solver clear direction of how the inequalities will be drawn and how the linear programming problem will be tackled. Moreover, according to Schoenfeld (1972), rough workings indicate the evidence of learners' thinking processes that was involved in the problem-solving process.

5.2.4 Cognitive demands at devising plans to solve the problem Stage

5.2.4.1 Interpreting and Translating

The findings of the study revealed that learners in their effort to devise a plan(s) to solve the problem, were operating at the Understanding cognitive level and were able to engage in the cognitive process of interpreting and translating of instructional and conditional massages of the linear programming problems in order to construct their own meaning, organize facts and state the main ideas about the problem. Learners were able to interpret, translate and organize the conditions of the problem into Inequalities and further drew tables of values. (Section 4.2.5).

This finding is in consonance with Nalube's (2014) argument that if learners are unable to interpret, translate and simplify expressions and solve problems: constructing equations from given problems and interpreting from context into algebraic language, it is a clear indication of lack of conceptual and procedural knowledge. Therefore, it can be argued that having the ability to grasp the meaning of previous learnt material (conceptual and procedural knowledge) is an essential skill of problem-solving as it can help learners in interpreting and translating material from one form into another.

5.2.5 Carrying out the plan(s) to solve Linear Programming problems

The subsection provides a discussion of the findings of the study as gauged by Polya's (1957) third stage of problem-solving with respect to the following indicators: graphing the inequalities, identifying the feasible region and calculating the Max/Minimum profit. This is the stage where learners are expected to actually solve the problem by carrying out the plan while checking or monitoring each step.

The findings of the study revealed that (15 out of 24) learners representing 67% showed competence in sketching the graphs of inequalities correctly: first drawing the associated equation of the inequality and then identifying and shading the unwanted regions to arrive at the correct graph of the inequality (Section 4.2.3.1). For instance, in Figure 4.6 and 4.7, learners were able to draw the graphs of the inequalities correctly: first by drawing the solid line because of the *'greater or equal to'* or the '*less or equal to'*, then proceed to identify the wanted and unwanted region by testing two points from each side of the line, then shade the unwanted region.

On the other hand, the results indicated that learners experienced difficulties in drawing the graphs of the Linear programming inequalities. For instance, in Figure 4.2g, learner **-V3** was able to draw the associated equations of the inequalities but revealed a misunderstanding of shading the wanted region to indicate where the solution lies, instead of the unwanted region, while learner-**U8** was able to solve and draw the line of the inequality but had experienced a difficulty in determining the region or the side of the line to shade as the wanted region. The transition from shading the wanted region at junior secondary level to shading the unwanted region in linear programming posed a challenge to learners, as they could not be able to arrive at correct feasible region.

The results further revealed that (9 out of 24) leaners, representing 37.8%, experienced difficulties in determining the region of the line of the inequality to shade as the unwanted region, despite the two teachers emphasizing the fact that in linear programming, when drawing the line of inequalities we shade the unwanted region, not the wanted region as was the case at Junior secondary level (grade 9). In general, learners experienced difficulties in graphing linear inequations which led to incorrect solutions. In problem solving, graphing is considered an aid, which can help learners to see, interpret and solve linear programming problems with some level of easy (Newman, 1983).

The findings also showed that some learners experienced difficulties in identifying the feasible region or the region for solution set, this can mainly be attributed to learners' inadequate skills of drawing the graphs of the inequalities correctly (Section 4.2.3.2). Therefore, it can be argued that having the procedural and conceptual understanding of graphing linear equations in linear programming is essential part of problem-solving if learners are to arrive at correct solutions (Anderson, 2001). Needless to say, the feasible region can only be clearly identified if and only if the lines of the inequalities are correctly drawn and the unwanted regions shaded. Alternatively, the feasible region comes out clearly as the region where all the inequality shadings converge.

The results also indicated that learners (12 out of 24, representing 50%), experienced difficulties in finding the maximum profit. (Section 4.2.3.2). For instance, the question: *"The profit on a dress is K160 and on a suit is K170. Find the number of dresses and suits the tailor must make for maximum profit and calculate the maximum profit?*" learners (21 out of 24) were able to derive the profit function which is P(x) = 160x + 170y but were unable to find the correct maximum profit due to incompetence in identifying and testing the point boundary points of the feasible region into profit function and others were testing the boundary points using an incorrect equation. (Figure 4.14). In another instance, some learners despite working collaboratively, could not find the correct maximum profit due to the incorrect drawing of the inequality $x \geq y$, which lead to an incorrect coordinate (50, 57), instead of the correct coordinate, (50, 50). Learners could not arrive at the correct maximum profit, because instead of the profit function they used one of the inequalities to test the boundary points for the maximum profit and when probed, this revealed a misconception on how to calculate the maximum profit.

In the actual solving of the linear programming problems, the study further showed that learners did very little in monitoring each and every step in their problem-solving process (Section 4.2.3.3), this affected their solving processes and lead to incorrect derivation of inequalities, graphs of inequalities and the maximum/minimum profit. The results from the focus group discussions revealed that the majority of the learners (18 of out 24 learners) made no or little efforts in checking to ensure that the steps of their workings were correct or not. Others cited the time factor as the reason they don't check each step while others revealed lack of monitoring skills to check that the step is correct and whether it can be proved. This finding aligns with Lee's (2015) assertion that many learners believe they are done with their mission of solving a problem and, as a result, they do not pay attention to the last stage ("look back") of the Polya's model (Lee, 2015). It can therefore be argued that, monitoring and checking each step in the solving process is essential as it helps learners to identify their errors or mistakes for correction during their solving process, and helps them to see clearly that they are in right and correct direction of the correct solution, (Polya, 1987).

5.2.6 Cognitive demands at *Carrying out the plan* stage.

The findings of the study revealed that at the *Carrying out the plan* stage, learners would operate at different levels of cognitive process dimension of the Revised Bloom's taxonomy from *applying* through to *creating* cognitive levels.

5.2.6.1 Problem-solving: Graphing inequalities and shading the unwanted region.

The cognitive demands, involved at the '*carrying out the plan(s)'* stage, were established during focus group discussion and on question **1.2** (see Appendix H, Focus Group Discussion) which was at the Applying cognitive level. This required the application of the inequalities in graphing and shading the unwanted region. The results showed that learners were able to engage in problem-solving process of applying acquired knowledge, facts, techniques and rules of linear programming (Previously learnt material) in graphing and shading the unwanted region, although some learners' workings were not mathematically correct. For instance, in drawing the graph for the inequality $x \geq y$, learners had come up with a correct table of values (labelled c in Figure 4.17), but had used the table of values labelled b (Figure 4.15), which lead to drawing an incorrect line with the correct shading of the unwanted region.

In drawing the graph for the inequality $2x + 3y = 10500$, the results revealed that learners had experienced difficulties in drawing graphs for inequalities of two variables than with one variable. Learners would instead use the incorrect inequality $x + y \geq 10500$ and then write $x + y = 0$, then let $x = 50$, to get $50 + y = 0$, then get $y = 50$. Through this incorrect process learners came up with the table of values labelled 'a', with coordinates $(0, 50)$ and $(50, 0)$ which they used to draw an incorrect graph. The results also revealed that learners (15 out of 24) did not have challenges in shading the unwanted region regardless of the whether the graph arrived at is correct or incorrect. They were able to pick points on the sides of the graph and test them in the inequalities and shade the side whose point does not satisfy the inequality. According to Lambdin (2009) and Anderson

(2001), the cognitive problem-solving process of applying acquired knowledge (Previously learnt material), facts, techniques and rules, is an essential cognitive demand in problem-solving especially if learners are to accurately draw the graphs of inequalities and in shading the unwanted region.

5.2.6.2 Analysing to establish the feasible region with its boundary points.

The findings showed that learners in carrying out the plans while operating at analysis cognitive level could also engage in the cognitive process of analysing to establish the feasible region with its boundary points. This was established during lesson observation and on question **1.2** (refer to Appendix **H,**) of the task under the focus group discussion. The results showed that learners (16 out of 24), while working collaboratively, experienced challenges and could not arrive at the correct feasible region with correct boundary points. Results showed that learners understood what the feasible region is, as the region that remains unshaded and it is where the solution for the three inequalities lies. However, some learners could not come up with the feasible region with correct boundary points owing to inadequate skills in drawing the graphs on the inequalities. For instance, in figure 4.16, due to an incorrect graph for the inequality $2x + 3x \geq 150$, learners came up with inaccurate boundary points *(50,49)*, *(30,30)* and *(50,10)*, which are at total variance with the correct boundary points: ***(50,50), (30,30)*** and ***(50,16).*** In figure 4.17, learners had difficulties in graphing the inequalities which lead to inaccurate boundary points *(34, 50)*, *(10, 30)* and *(50, 10)*, this also revealed lack of conceptual understating of coordinate geometry, learners had mis-represented the coordinates, for instance, instead of ***(x, y)****: (50, 34)*, *(30, 10)* and *(0, 10)*, they would write ***(y, x)*** *(34, 50), (10, 30), (50, 10).*

The Analysing cognitive demand is an integral part of problem-solving if learners are to answer questions correctly (Lambdin, 2009). Anderson (2001) asserted that, this requires the ability to be able to disassemble material into its constituent parts so that its organizational structure may be understood by the learner. This exercise may include the identification of the different constituent parts of the feasible region, the examination of the relationships between the inequalities, establishing the boundary points or the coordinates of the points at which the graphs of the inequality intersect and the understanding of the feasible region as the region where the solution of the inequalities lies.

5.2.6.3 Derivation of abstract relations.

Derivation of abstract relation was the third cognitive demand that emerged at the *carrying out plans* stage. This was established during lesson observation and on question **1.3** which was at the Evaluating cognitive level of the task under the focus group discussion (refer to Appendix H). This involved deriving the profit function and establishing the correct boundary points of the feasible region. The results of the study revealed that learners (15 out of 24), experienced difficulties in deriving the profit function and boundary points of the feasible region that could give the maximum profit. Learners were able to derive the correct profit function but could not establish the correct boundary points due to incorrect drawings for the inequality. On the other hand, 9 out of 24 learners were able to derive the profit function correctly and managed to establish the boundary points that would be tested into the profit function to yield the maximum profit.

According to Koji, Mulenga and Mukuka (2016), lack of conceptual, procedural and strategic knowledge by learners is usually one of the challenges faced in answering algebraic linear equations. Lambdin (2009) adds that learners who have learnt mathematics without understanding are often successful only in solving problems similar to those they have already seen, and are unable to see how mathematical ideas are related or useful. It can therefore be argued that having the conceptual, procedural and strategic knowledge of deriving abstract relations, particularly the profit function and also establishing the boundary points of the feasible region that can yield the maximum or minimum profit, is essential in the problem-solving process if learners are to achieve well in solving linear programming problems.

5.2.6.4 Evaluating the maximum/minimum profit.

Another cognitive demand that emerged at '*carrying out plans'* stage was the process of creating or generating the maximum/ minimum profit. This was ascertained at question **1.3** and **1.4** under focus group discussion (See Appendix H)**.** The question was at evaluation cognitive level and required learners to deductively make judgements about the boundary points that would give the maximum profit and further determine the profit. The results of the study revealed that learners (15 out of 24) had experienced difficulties in evaluating the maximum profit by testing boundary points of the feasible region. For instance, in Figure 4.20, learners revealed a misconception and lack of conceptual understanding for calculating the maximum profit, they had quite alright established the correct boundary points but had used an incorrect equation: *140x +210y* as the profit function which led to an incorrect value for the profit function. Most learners were able to derive the profit function *(PR = 160x +270y)* but could not evaluate the correct maximum profit due to incorrect boundary points of the feasible region, which were as a result of learners' lack of adequate skill in drawing the graphs of the inequalities. According to Anderson (2001), the evaluating cognitive demand is critical in problem-solving process of linear programming question if learners are to arrive correct solutions. Anderson and Krathwohl (2012) asserted that this skill requires learners to critically check, critique, test and make judgement about data or information, this makes it a higher thinking skill.

5.2.7 Looking back to check solving processes applied and solution arrived at after solving linear programming

The last stage of Polya's (1987) involves a critical examination of the solution arrived at so as to ascertain whether the result is correct or whether the plan can be used to solve another problem. Needless to say, at this stage learners would operate at the evaluation cognitive level. The results from the lesson observations also indicated that learners (20 out of 24) experienced difficulties in evaluating and examining the solutions of their workings to ensure they were correct or not. (Section 4.2.4.3). As evidenced in Figures: 4.9, 4.10, 4.11, 4.12, 4.13 and 4.14, learners did very little and experienced difficulties evaluating or looking back to ensure the answers or solutions arrived at are correct or not, this can be attributed to learners' lack of evaluative skills to ensure that the solution is correct, if anything finding means to prove the answers arrived at. This finding coincides with the finding of Lee (2015) who found that learners execute the last stage of Polya's model with less care, he argued that many learners tend to believe they are done with their mission of solving a problem and, as a result, they do not pay attention to the last stage ("look back") of the model (Lee, 2015).

It can therefore, be argued that having evaluative skills for ensuring that the solution arrived at is correct and having skills for proving the answers can really help learners in their problem-solving process. Polya (1976), adds that this is the step where students have to confirm their solution by applying it in a new situation. In this step, students seek new arguments and try to recheck their answers by comparing the known with unknown. Looking back means reconsidering, re-examining the results and the process of solution for the consolidation of their knowledge and to develop the ability to solve such examples independently in their daily life (Sheikh, 2014; Polya, 1976)

5.2.8 Cognitive demands at *looking back* stage

5.2.8.1 Verification and reflecting on applied problem-solving processes

The findings showed that learners at the *'looking back'* stage were operating at the evaluation cognitive level and would engage in the cognitive processes of verification and reflection on the problem-solving processes applied. This is where learners have to engage in a critical reconsideration and close re-examination of the solving processes they had used and the solution obtained so as to determine whether they are correct or whether the problem-solving processes and plan can be used to solve another problem (Polya, 1976). This is an important aspect or skill in the problem-solving process, as it can help learners identify errors and avoid mistakes, and promotes metacognition. Schoenfeld (1985) defined metacognition as the ability to reflect on one's own knowledge. This is a critical factor in learning and there is need to develop learners' metacognitive strategies and one effective technique is that adopted by Schoenfeld (1985) in teaching mathematical problem solving: he encouraged students to ask themselves questions continually about what they were doing, what they were trying to achieve and what they would achieve and do next. It can help learners to consolidate their knowledge and develop the ability to solve problems independently in linear programming and in their daily life (Sheikh, 2014).

5.3 Research Question two (2): What factors affect grade 12 learners' problem-solving processes in linear programming?

Research question two focused on establishing and exploring factors that affect learners' problem solving in linear programming. The following emerged as themes:

5.3.1 Insufficient teaching and exposure to mathematical language of linear programming

Insufficient teaching and exposure to mathematical language of linear programming emerged as one of the factors or constraints that affect learners' problem-solving in linear programming. was Finding showed that learners in their effort to understand the linear programming question, had difficulties understanding the conditional statements, which led to misinterpretation and translation of the mathematical statements into incorrect inequalities. A few learners were familiar with the common mathematical statements with their associated symbols such as '*greater or equal to*' (≥), '*less or equal to'* (≤), *'less than'* (<) and '*greater than'* (>). However, in instances where there was a slight twist of language, for instance, using '*should not exceed'* to mean '*less or equal to'* (≤), and using '*should not be less than'* or '*at least'* which imply '*greater or equal to' (≥),* leaners experienced difficulties in understanding the correct meanings implied, with their correct associated mathematical symbols. As evidenced in Figure 4.2b (Section 4.2.2.1) learners misinterpreted the statement '*should not be less than'* and wrote an incorrect inequality *(X >Y)* while in figure 4.3a, some learners did not understand the meaning of '*at least'* which implied '*greater or equal to' (≥),* and this led to translation into an incorrect inequality; *140X +210Y > 10500.* This finding is further substantiated during the interview with teacher **V**, who stated that among the challenges that learners encounter in linear programming is misunderstanding of the mathematical statements, learners misinterpret the mathematical statement due to language barrier, the majority of the learners are more familiar with the local language than English. For example, in local (or ordinary) language when you hear someone use 'at least', it will mean the least value, that is a direct translation, yet the correct meaning is the 'given value or more'…" (Teacher **V's,** September, 2021).

As aforementioned above, it was found that there is an indirect parallelism between everyday speech or usual language and mathematical language, some learners were more familiar with the former as such they could understand the mathematical language from the perspective of the usual language of communication and this led them to misinterpretation of the conditions of the linear programming problems. This finding is consistent with finding of Nkambule (2009), who found that learners, in understanding the language of linear programming drew on their home language for the meaning of mathematical statements. of Haghverdi et al. (2012) observed that learners' difficulties in solving mathematical problems were due to their disabilities in representing and understanding of word problems, making a plan and defining the related vocabularies. Their study further showed that findings, the causes of learner difficulties were text difficulties, unfamiliar contexts in problems and using inappropriate strategies.

In the Zambian context, the significant of the curriculum in building mathematical language which enables learners to construct and communicate knowledge of mathematics has not received due attention even as the pedagogical challenge faced by learners in learning mathematics. (Nkambule, 2009). In the context of this study, to master mathematical language implies being able to understand and to interpret oral expressions of mathematics, being able to decode written and graphical representations of mathematical ideas, to express quantitative ideas and statements orally and in writing in varied contexts. Therefore, learning mathematical language is not only valuable in itself but also useful in understanding other fields of knowledge especially science. (Abdul & Sarabi, 2015). Allen (1988), contended that mathematics is a language and proficiency in this language can be acquired only by long and carefully supervised experience in using it in real situations. In this view, from a learner perspective, mathematical language is not much easy to learn and demands continuous effort from the learner as much as to learn a second language.

In general, failure to understand the language of linear programming: text difficulties, unfamiliar contexts of the problem(s) and using inappropriate strategies, stood out as one of the constraints in learners' problem-solving processes, which led to misinterpretation and translation of situational conditions into incorrect Inequalities.

5.3.2 Weak background in preliminary topics (Pre-requisite Knowledge) to Linear Programming

The results from the lesson observations, focus group discussions and semi-structured interviews as well revealed that some learners (9 out of 24) had a weak background and poor foundation in the preliminary topics (grade 9 and 10 topics): Coordinate Geometry and Graphs of Inequalities. These topics form the basis for the learning of linear programming. Due to this constraint, learners had experienced difficulties in graphing the inequalities and in presenting the coordinates of the points correctly. As was evidence 4.21 (Section 4.3.2), due to poor foundation in Coordinate geometry, learners had difficulties in graphing the inequalities which lead to inaccurate boundary points *(34, 50), (10, 30)* and *(50, 10),* this further revealed learners lack of conceptual understating of coordinate geometry, some learners had miss-represented the coordinates, for instance, instead of ***(x, y)****: (50, 34), (30, 10)* and *(0, 10),* some learners would write ***(y, x)*** *(34, 50), (10, 30), (50, 10).*

It was explicit as observed during lessons that due to weak background and poor foundation in preliminary topics to linear programming, learners forget and fail to relate previous learnt concepts of Coordinate geometry and Inequalities, to linear programming and this led to drawing incorrect graphs and misrepresentation of the coordinates. Prior knowledge or pre-requisite knowledge has long been considered the most important factor that significantly influence learning and learner achievement. (Thompson & Zamboanga, 2003). Mayer (1982), Schoenfeld (1982), and Silver (1982) contended that prior knowledge is a key element in the problem-solving process. Prior knowledge influences the problem solver's understanding of the problem as well as the choice of strategies that will be called upon in trying to solve the problem. In fact, prior knowledge and prior experiences is all that a solver has to draw on when first attacking a problem.

Therefore, a strong background and foundation in pre-requisite topics to linear programming plays a critical role in learners' problem solving, as it can provide learners with quality of prior knowledge which positively influence both knowledge acquisition and the capacity to apply higher-order cognitive problem-solving skills. (Dresel M, et al, 1988). Generally, it helps learners to make connections with new information and helps them understand concepts, and when teachers make connections between the lesson and learners' background knowledge, they validate learners' previous knowledge and experiences which facilitates greater interest in the lesson. (Diaz-Rico, 2008; Echevarria & Graves, 2011).

5.3.3 Shading the unwanted region

Another constraining factor that affects learners' problem-solving processes in linear programming is the shading of the unwanted region to imply that the solution of the inequality(s) lies in the unshaded region. At junior secondary level (Grade 8&9), under the topic of Inequalities and Graphs, learners shade the wanted region as where the solution of the inequality lies while at senior secondary level learners shade the unwanted region to indicate that the solution of the inequality lies in the region that remains unshaded. This inconsistency is another factor that affect learners' problem solving in linear programming. It was found that some learners (10 out of 24) exhibited a misconception in linear programming, they forgot and failed to shade the unwanted region, instead they shaded the wanted region which led them to coming up with an incorrect feasible region.

As evidenced in Figure 4.8 (Section 4.2.4.1), learner (**V3**) was able to draw the associated equations of the inequalities but revealed a misunderstanding of shading the wanted region instead of the unwanted region (For the inequalities $x + 2y \leq 6$ and $y \geq 0$), while learner **U8** (Figure 4.2i, section 4.2.4.1) was able to solve and draw the line of the inequality but had experienced a difficulty in determining the region or the side of the line to shade as the wanted region. The results revealed that (9 out of 24) learners experienced difficulties in determining the region of the line of the inequality to shade as the unwanted region. Shading the wanted region in the preliminary topics to linear programming proved to be one of the constraining factors that affect learners' problem-solving processes at grade 12 level.

5.3.4 Calculating the maximum/minimum profit

Another constraint that affects learners' problem solving to finding the maximum or minimum profit is identifying correct boundary points and testing them using the profit function. Learners revealed a misconception and lack of conceptual understanding for calculating the maximum profit, some learners had quite alright established the correct boundary points but had used an incorrect equation: $140x + 210y$ as the profit function which led to an incorrect value for the profit function. (See figure 4.2t, section 4.2.5.3). Most learners (15 out of 24) were able to derive the profit function ($PR = 160x + 270y$) but could not evaluate the correct maximum profit due to incorrect boundary points of the feasible region, which were as a result of learners' incompetence in drawing the graphs of the inequalities.

5.3.5 Lack of monitoring skills for correct solution.

The findings showed that 20 out of 24 learners did very little in checking and monitoring their workings to ensure they were on the right track. As evidenced in Figures: 4.8, 4.9, 4.10, 4.11 and 4.12, learners did very little and experienced difficulties in checking and evaluating or looking back to ensure the answers or solutions arrived at are correct or not. For instance, in the aforementioned Figures, results showed that learners could not check the steps of their workings: in interpreting and translation word statements into inequalities, drawing graphs of the inequalities, identifying the feasible region and in finding the maximum profit and this led to incorrect answers or solutions.

This is the step where students have to confirm their solution by applying it in a new situation. Polya (1976) asserted that, in this step students seek new arguments and try to recheck their findings by changing the known with unknown. By looking back means reconsidering, re-examining the results and the process of solution for the consolidation of their knowledge and develop the ability to solve such examples independently in their daily life (Polya 1976, p.15). Monitoring each step of the problem-solving process is essential and plays a critical role in ensuring that learners are on the right track and in establishing whether the answers arrived at are mathematically correct or not.

5.4 Research question three (3): How can learners' problem-solving processes in linear programming be enhanced?

This section discusses the findings of research question three of the study. Although, some of the measures that emerged from the study are not specific to linear programming, but they can be applied in mathematics in general.

5.4.1 Sufficient teaching and exposure to the mathematical language of linear programming

One of the measures that emerged that can enhance learners' problem-solving processes was ensuring that learners are sufficiently taught and exposed to the mathematical language of linear programming. Learning mathematics is like learning a new language as it involves its own unique and unambiguous statements or conditions and associated symbols (Pimm, 1991). Scholars recognizes mathematical language as a tool for teaching and learning. According to Mayer (1995), teachers negotiate meanings with learners in a language that they both parties understand. Orton (1992), argued that language is both an individual and a social mode of thinking. Learners have to understand the mathematics concepts as well as communicate their understanding of these concepts verbally and in writing. The teacher presents concepts written or verbally using mathematics language and everyday language. Students have to be proficient in both languages although competency in everyday language does not imply competency in mathematics language (Newman, 1983). Therefore, ensuring that learners sufficiently taught and exposed to the mathematical language of linear programming is key in building the conceptual understanding of linear programming.

5.4.2 Evaluation of learners' prior knowledge of concepts in Coordinate Geometry and Graphs of Inequalities prior to introducing linear programming

The study established that prior knowledge of Coordinate geometry, Inequalities and Inequations: solving and graphing of Inequations and Inequalities is essential for the enhancement of learners' problem-solving in linear programming. The study revealed that learners had challenges in grasping the concepts of linear programming, due to lack of adequate pre-requite knowledge upon which linear programming builds. It was suggested that teachers should ensure that learners have the pre-requisite knowledge by evaluating learners' prior knowledge and spending enough time to teach the preliminary topics to linear programming and these include: Coordinate geometry and Graphs of Inequations: how to solve and plot lines (graphs) of inequalities. Prior knowledge has long been considered the most important factor that significantly influence learning and learner achievement. (Thompson & Zamboanga, 2003). Mayer (1982), Schoenfeld (1982), and Silver (1982) contended that prior knowledge is a key element in the problem-solving process as it influences the problem solver's understanding of the problem as well as the choice of strategies that will be called upon in trying to solve the problem. Generally, when teachers make connections between the lesson and learners' background knowledge, they validate learners' previous knowledge and experiences which facilitates greater interest in the lesson and pre-requisite knowledge helps learners to make connections with new information and helps them understand concepts, and (Diaz-Rico, 2008; Echevarria & Graves, 2011).

5.4.3 Consistency of shading the unwanted region in Linear Programming and the preliminary topics.

The results of the study revealed that among the challenges learners in linear programming faced was shading the wanted region instead of the unwanted region. This misconception was attributed to the fact that at junior secondary level, under the topic: Graphs of Inequalities and Inequation, learners would shade the wanted region to indicate where the solution of the inequality lies while at grade twelve level it was otherwise, they shaded the unwanted to indicate that the solution of the inequality will lie in the unshaded region. The findings of the study indicated that consistency in shading the wanted or unwanted region throughout, both at junior secondary level and senior level may help to enhance learners' problem solving in linear programming in the sense that it would prevent learners from experiencing the misconception of shading the wanted region instead of the unwanted region at grade twelve level.

This finding resonates with finding by Kaabo (2019) that the instruction on the region to be shaded for grade 9 and also the common schemes of work prepared by the provincial team demanded that the shading should be for the wanted region which provides the solution set to the inequation/inequalities in question, but this contradicts the instruction for grade 12 linear inequations in linear programming, which demands shading the unwanted region. It can be therefore be argued that proper and successful communication of mathematical ideas between levels (grade 9 & 12) of mathematical knowledge is key if learners are to develop comprehensive mathematical understanding (Thurston, 1995)

5.4.4 More interaction with authentic and real situational problems.

The study revealed that more learner interactions with more situational problems may enhance learners problem-solving. According to Brown, Collins and Duguid (1928), meaningful learning of mathematics will only take place if it is embedded in the social and physical context within which it will be used. This type of education provides an authentic context that reflects the way the knowledge will be used in real-life; it preserves the full context of the situation without fragmentation and decomposition, and it invites exploration and allows for the natural complexity of the real world (Brown, Collins & Duguid, 1989). Fruedenthal (1987), adds that when children learn mathematics in an isolated fashion divorced from their real experiences, they will quickly forget it and will not be able to apply it. It can therefore be argued that more learner interaction with the authentic and real situations could enhance learners' problem-solving processes, as learners will be able to appreciate how linear programming is used and applied in real-life and this will give them motivation for deep learning.

5.4.5 Giving homework

It was also suggested that giving learners more situational problems for practice through assignments and homework would give an opportunity to reflect, practice, discover new strategies and consolidate their understanding of mathematical concepts of linear programming learnt in class. Moreover, giving homework aligns with the Ministry of Education (1996) National Policy on Education: Educating our Future which stipulates that each school will be required to have a clear schedule of performance-monitoring activity that checks pupils' progress and that prominent among these will be homework given to pupils on a regular basis.

According to the National Assessment Survey (MoE-NAS, 2016), homework is defined as the time learners spend outside the classroom in assigned learning activities. It serves to develop regular study skills and the ability to complete assignments independently, and contributes toward building responsibility, self-discipline, and lifelong learning habits. Homework also serves to encourage learners on how to use time wisely, which is very cardinal for one to be successful. Furthermore, it is positively linked to learner academic achievement and it is an inexpensive method of improving learner academic performance (MoE-NAS, 2016).

5.4.6 Group-work approach to teaching linear programming

Another measure, one of the teachers and learners suggested, that can enhance and improve learners' problem solving in linear programming is promoting group work, where learners are put in groups to work on an activity or problem in linear programming. Learners despite having challenges in drawing the lines of the inequality with two variables, appreciated group working during the focus group discussion as they were able to understand certain concepts from their fellow learners. This was further substantiated by teachers who indicated that group work for learners is important as it gives learners an opportunity to discuss, share and learn from their follow learners, sometimes learners may understand more when their fellow learners explain a concept to them than during class lessons. This aligns with the social-cultural theory in which Vygotsky considers collaboration or group work as a critical aspect. A learner is able to perform a certain task alone, while in collaboration, is able to perform a greater number of tasks. Vygotsky recommends the use of assisted instruction and problem solving should be under the guidance of a competent adult or capable peer (Vygotsky, 1978). Therefore, it can be argued that promoting learner collaboration or learner group working, where learners are strategically placed in groups to solve linear programming problems would enhance their problem-solving skills.

5.5 Summary of the Chapter

This chapter discussed the findings of the study on learners' problem-solving processes in linear programming with regard to how learners engage in problem-solving of linear programming questions, the factors that affect learners problem-solving processes and the measures that may enhance learners problem-solving processes and eventually improve learner achievement particularly in linear programming and mathematics in general.

CHAPTER SIX

CONCLUSION AND RECOMMENDATIONS

6.1 Overview

This chapter provides the conclusion to the study and highlights the major contribution the study has made to research (Section 6.3). Thereafter, recommendations (Section 6.4) have been made that could improve learner achievement particularly in linear programming and in mathematics in general as reported by Examinations Council of Zambia 2014, 2015, 2016, 2017 and 2018. The chapter ends by suggesting further research in mathematics education.

6.2 Conclusion

The study explored learners' problem-solving processes by investigating: how learners engage in problem-solving of linear programming problems; the factors that affect their problem-solving processes and contribute to underachievement in mathematics and the possible measures that would enhance problem-solving processes in linear programming.

The results showed that learners in their quest to understand linear programming questions would read the question at least once though it is not a guarantee for correct solution, they would identify key statements and list phrases with their associated mathematics. Learners would do all this with a view to gaining an understanding of what the question demands are. The results showed that learners in their quest to understand the problem could operate at remembering cognitive level and would engage in the cognitive processes of retrieving and recognising factual information. In quest to devise a plan(s) to solve the problem, the results showed that learners were able to transform the conditional statements into inequalities, they were able to devise tables of values showing possible pair of coordinates and showed rough workings. Although, some learners experienced difficulties in comprehending the language of linear programming which led to deriving incorrect inequalities. It was found that learners in their effort to devise a plan(s) were operating at understanding cognitive level and were engaging in the cognitive process of interpreting and translating the linear programming problem(s) to construct their own meaning and organise facts.

The results also showed that learners in their effort to execute the plan(s) to solve the linear programming problem(s) were able to graph the inequalities, identify the feasible region with its boundary points, derive the profit function, test the boundary points into the profit function to determine the pair of coordinates (x, y) that would yield the max/minimum profit and calculate the maximum/minimum profit. However, some learners experienced difficulties in arriving at correct solutions owing to lack of procedural and conceptual knowledge on how to graph the inequalities and calculating the maximum profit. It was further found that learners in the effort to execute plan(s) to solve the linear programming question(s), were operating at different cognitive levels from *applying* through to *creating* and would engage in different cognitive processes from applying inequalities to derive the feasible region, to evaluating the maximum/minimum profit.

The findings further showed that after solving linear problems learners made little or no effort to engage in a critical examination of the solution arrived at so as to ascertain whether it is correct and whether the plan can be used to solve another problem(s). At this stage, learners were also operating at the *evaluating* cognitive level and could engage in the cognitive process of verification and reflection on the problem-solving processes applied for mistake identification and knowledge consolidation.

The study showed that learners' problem-solving processes was constrained by a number of factors, some of which were not specific to linear programming but can be applied in mathematics in general. Insufficient teaching and exposure to mathematical language was among the factors that affected learners' problem-solving processes in linear programming, learners could understand from the perspective of their local or home language and this led to incorrect answers. Poor foundation in pre-requite knowledge upon which linear programming builds was another factor. Inconsistency of shading the unwanted region at junior and senior secondary levels was another factor that led learners' misconception of Linear Programming concepts. The study further found that lack of monitoring to ensure that the steps of their solving process affected the learners' problem-solving processes as they were unable to identify and correct errors and mistakes.

To enhance or improve learners' problem-solving processes, it was suggested that there is need for teachers to be evaluating learners' prior knowledge of concepts in Coordinate Geometry and Graphs of Inequalities prior to introducing linear programming and ensure that learners have the requisite knowledge and there is need for consistency of shading either the unwanted or wanted region at both junior and senior secondary school levels to prevent learner misconception. It was further suggested that there is need to promote learner group working by teachers using the group-work approach to teaching linear programming. More learner interaction with authentic and situational problems is another measure that may enhance problem solving processes in linear programming. These are the measures that emerged from the study for the enhancement of learners' problem-solving processes, although some are not specific to linear programming, they can be applied to mathematics in general.

6.3 Contribution to the field

The study has academically contributed to the body knowledge of mathematics education with regard to learners' problem-solving processes, factors that affect learners' problem-solving process and the measures that may enhance learners' problem-solving processes in linear programming. The Gastalt psychology (theory) of learning according to Polya's (1957) four stages of problem-solving: understanding the problem, devising a plan(s) for solving the problem, executing the plan(s) and looking back to re-examine the solutions arrived at, and also the cognitive process dimension of the revised Bloom's taxonomy (Krathwol, 2001) with its six cognitive levels helped me to emphasise appropriate analysis of data on how learners engage in problem-solving of linear programming questions. This unique integration or combination could be a new construct or approach to exploring problem-solving processes in terms of Polya's four stages and the cognitive demands. This helped to explore learners' problem-solving processes in linear programming in terms of the four stages of problem solving and the cognitive demands needed to solve a problem.

6.4 Recommendations.

6.4.1 Policy

1. There is need for the Ministry of Education through CDC to incorporate and emphasise the Polya's Problem-solving stages and the revised Bloom's Cognitive levels of solving mathematical problems in the curriculum for promoting learners' higher order thinking skills and problem-solving skills.

6.4.2 Practice

2. School management should intensify Continuing Professional Development (CPD) so that teachers of mathematics can share good classroom practices regarding the teaching of mathematics and how to devise high-level cognitive demanding tasks especially for School Based Assessment, as such promote learners' problem-solving skills and greater learners' learning.
3. There is need for learners to be identifying key mathematical statements with their mathematical symbols in order to understand the linear programming problem.
4. There is need for learners to be interpreting and translating conditional statements of linear programming into algebraic form.
5. Teachers should be ensuring that learners are well-vested in the skills of plotting inequality graphs before introducing the concept of linear programming.
6. Teachers should be emphasising the mathematical language of linear programming during lessons.
7. Learners should be at the centre of their own learning of Linear programming as this could help the teacher divulge what learners can do unassisted, what they can only do with assistance and the kind of intervention needed through mediation.
8. Teachers should be using group-work approach to teaching linear programming as this approach not only increases interaction between learners and between teacher and learner but it also enhances the opportunity for relevant mathematical dialogue between the learners.
9. There is need for teachers to use teaching approaches that promote learners' monitoring and verification skills in ensuring that their problem-solving processes and final solutions are correct.
10. Learners should critically be re-examining and reconsidering the applied problem-solving processes and the answers arrived at as this may foster learners' problem-solving processes in linear programming.

6.4.3Further Research.

1. There is need to investigate learners' errors and misconceptions in linear programming.
2. There is need to examine the relationship between learners' attitude and problem-solving processes in linear programming.

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APPENDICES

Appendix A: Letter of Permission for School Managers

C/o Depart. Of Mathematics
And Science Education
The University of Zambia
P.O Box 32379
Great East Campus, Lusaka.
Date: …………………………2021.

The Head teacher,

…………………Secondary School,
P.O. Box……………………………,
Lusaka.
Dear Sir/ Madam,

Re: <u>Request for Permission to conduct research at your school</u>

The above captioned matter refers.

I am a student at the University of Zambia (UNZA) under the Master of Education programme in Mathematics Education and in partial fulfilment of the requirements for the Master's degree, I am expected to conduct research and compile a report thereafter. The title of this research is *Learners' Problem-solving Processes in Linear Programming at selected Secondary Schools in Lusaka Zambia*. This will involve class lesson observations, group discussions outside the classroom and interviews with both the learners and their teachers. Therefore, I humbly request for your permission to conduct my research at your school.

In accordance and compliance with the ethical requirements laid down by the University of Zambia, be rest assured that the identity of the school, the teachers and pupils who will participate, will be highly protected.

Thank you for the consideration.

Yours Faithfully,

SANDAMBO GABRIEL

+260 969805743, gabrielmsandambo@gmail.com

Appendix B: Extract showing the details of the current topic of Linear Programming in Secondary School

Topic	Subtopic	Specific outcome	Knowledge	Skills	Values
12.2 Linear – Programming	12.2.1 Linear programming	12.2.1.2 Draw graphs of linear equations and inequations in one and two variables (as a recap) 12.2.1.3 Shade the wanted and unwanted regions 12.2.1.3 Describe the wanted or unwanted regions. 12.2.1.3Determine maximum and minimum values 12.2.1.4 Use the search line to determine the maximum and minimum values 12.2.1.5 Apply knowledge of linear programming in real life	-Drawing graphs of linear equations and inequations in one and two variables (as a recap) -Shading the wanted and unwanted regions - Describing the wanted or unwanted region - Finding Values in the feasible region -Using the Search line to determine the maximum and minimum values - Applying knowledge of linear programming in real life	-Interpretation of the wanted or unwanted regions. -Shading of the unwanted region. - Determination of maximum and minimum values. - Application of linear programming in real life situation.	-Logical thinking in finding the wanted region. - Planning when using graph paper.

Source: MESVTEE (2013) - "O" Level Mathematics Syllabus Grades 10 to 12.

Appendix C: Class Observation Guide/Schedule for learners

School code: __Time: ___Boys ____ Girls: ___Observation sheet: ___S/Topic: _____Date: ________

LESSON ELEMENTS TO CHECK/OBSERVE			
Item #	Elements to check (learner activity)	Observation(s)	Comment
a.	**Understanding the problem**		
1.	Reading a Linear Programming text/ problem		
2.	Re-reading a Linear Programming text/problem		
3.	Underlining or highlighting key terms etc. on the problem/question		
4.	Showing understanding of the meaning key words such as ‘at least’, ‘wanted region’ etc.		
b.	**Devising a plan**		
1.	Listing phrases/words and the associated mathematical symbols from the Linear Programming		
2.	Translating and writing down the inequalities before solving the Linear Programming problem		
3.	Showing rough working skills before solving the Linear programming problems		
4.	Drawing tables of values for inequations.		
	Carrying out the plan		
1.	Checking and monitoring each step when solving Linear Programming problems		
2.	Applying, sketching and graphing of the inequalities		
3.	Identifying and using the vertices of the feasible region or both the boundary and the points of the feasible region		
4.	Determining maximum and minimum profits		
	Looking back		
1.	Checking or evaluating to see if the answers or graphs are correct		
2.	Checking and evaluating to see if the answers or the wanted or unwanted regions are correct		
3.	Interpreting the Linear Programming answers		

Appendix D: Semi-Structured interviews for Teachers

Interviewer: ________________Interviewee: _________________ sex (   ) school Code: _____________

Date: ________________Start time: ____________

Understanding Linear Programming problems

1. **What do learners do before solving linear the programming question?**
   (Probe)
2. **How many times do learners read the question before solving it?**
   (**Indicators:** Read, reread, identifying key words, showing rough workings etc.)

Devising Linear Programming plans

1. **How do learner plan to solve the linear programming question? (Probe)**
2. **What do learners need to know and do in order to solve the linear programming question? (Probe)**
   (Indicators: listing phrases/words and associated mathematical symbols, translating and write down inequalities, scaling the graph paper, showing rough working skills etc.)

Carrying out a plan/Implementing a solution plan

1. **How do learners solve linear programming questions? (probe)**
2. **How do learners find the maximum or minimum profits?**
   (Probe)

(Indicators: Monitoring each step, use of appropriate inequalities when shading the wanted region, use the vertices of the feasible region or both the boundary and the points of the feasible region to find the maximum or minimum costs/profits, independently or collaboratively).

Reflecting/Evaluating/ Re-examining

What do learners do to ensure the answer they have found is correct or wrong? (probe)

(Indicators: checking if the answer is correct or incorrect after solving, interpret the answer of the Linear Programming problem etc.

Challenges to solving Linear Programming

1. **What are some of the challenges do leaners encounter when solving Linear Programming problems?**

Strategies to improve problem solving skills in Linear Programming

1. **What can you do to help learners improve on solving linear programming questions?**
   (**Probe**)

Thank you for your time and cooperation

Appendix E: FOCUS GROUP DISCUSION FOR LEARNERS

Understanding Linear Programming problems

1. **What did you do before solving Linear Programming questions?**
   (Probe)
2. **How many times did you read the question before solving?** Why?

(**Indicators:** reading, re-reading, identifying the key words/information such as 'wanted or unwanted region', 'at least', 'more than' and 'not more than' etc.

Devising Linear Programming plans

1. **What did you need to know in order to solve the Linear Programming Questions? (Probe)**
2. **How did you prepare and plan in orders to solve the linear programming questions? (Probe).**

(**Indicators:** listing phrases/words and the associated mathematical, forming, translating and writing down the inequalities before solving, showing rough workings, scaling the graph paper etc.)

3. **Do you use the vertices of the feasible region or both the boundary and the points of the feasible region to find the maximum or minimum costs/profits? Why?**

Carrying out the plan/Implementing a solution

1. **How did you solve the questions? (Probe)**
2. **How did you find the maximum? (Probe)**

(**Indicators:** monitoring each step when solving, using appropriate inequalities, independently or collaboratively).

Reflection/Evaluation/Re-examining

1. **How did you know the answer you got is correct or wrong? (Probe).**

(**Indicators:** checking if the answer is correct or wrong, interpreting the answer of the Linear Programming problem?

Challenges in solving Linear Programming problems

1. **What challenges or difficulties did you come across when solving the linear programming question? (Probe)**

Strategies to improve problem-solving skills in Linear Programming

1. **What do you think should be done so that you do not have challenges and difficulties? (Probe).**

Does the teacher ask questions, explain or make that encourage students to be reflective about problem solving in Linear Programming? If yes, how?

End of Interview

Thank you very much for your participation!

Appendix F: The Group Task on Linear programming

Read the following instructions carefully before answering the questions
1. You are to be in groups of three (3)
2. Answer ALL sub-questions.
3. Clearly show all calculations, diagrams graphs et cetera that you have used in determining the answers
4. Do not use a calculator unless
6. An answer sheets have been attached to the question paper.
7. Number the answer correctly according to the numbering system used in this question paper
8. Present your work neatly.

SEX	Learner Group ID
………………..	……………………

Question 1

A tailor at a certain market intends to make dresses and suits for sale.

1.1. Let X represent the number of dresses and Y the number of suits. Write the inequalities which represents each of the conditions below.

1.1.1. The number of dresses should not exceed 50.

1.1.2. The number of dresses should not be less than the number of suits.

1.1.3. The cost of making a dress is K140.00and that of a suit is K210.00. The total cost should be at least K10 500.00.

1.2. Using a scale of 2cm to represent 10 units on both axes, draw X and Y axes for $0 \leq X \leq 60$ and $0 \leq Y \leq 80$. Shade the unwanted region to indicate clearly the region where (X, Y) must lie.

1.3. The profit on a dress is K160.00 and on a suit it is K270.00. Find the number of dresses and suits the tailor must make for maximum profit.

1.4. Calculate the maximum profit.

End of Task!

Appendix G: **Clearance letter for data collection from UNZA HSSREC.**

THE UNIVERSITY OF ZAMBIA

DIRECTORATE OF RESEARCH AND GRADUATE STUDIES

Great East Road Campus | P.O. Box 32379 | Lusaka 10101 | **Tel:** +260-290 258/291 777
Fax: (+260) 211 290 258/253 952 | **Email:** director.drgs@unza.zm | **Website:** www.unza.zm

APPROVAL OF STUDY

27th July, 2021

REF NO. **HSSREC: 2021 – MAY - 030**

Mr. Gabriel Sandambo
C/o University of Zambia
School of Education
LUSAKA

Dear Mr. Sandambo,

RE: "LEARNERS' PROBLEM SOLVING PROCESSES IN LINEAR PROGRAMMING AT SELECTED SECONDARY SCHOOLS OF LUSAKA DISTRICT OF ZAMBIA"

Reference is made to your submission of the protocol captioned above. HSSREC resolved to approve this study and your participation as Principal Investigator for a period of one year.

REVIEW TYPE	ORDINARY REVIEW	APPROVAL NO. HSSREC: 2021 MAY 030
Approval and Expiry Date	Approval Date: 27th July, 2021	Expiry Date: 26th July, 2022
Protocol Version and Date	Version - Nil.	26th July, 2022
Information Sheet, Consent Forms and Dates	• English.	To be provided
Consent form ID and Date	Version - Nil	To be provided
Recruitment Materials	- Nil	Nil
Other Study Documents	Interview Guide Focus group Discussion Document Analysis.	
Number of Participants Approved for Study		

Specific conditions will apply to this approval. As Principal Investigator it is your responsibility to ensure that the contents of this letter are adhered to. If these are not adhered to, the approval may be

Towards Improving Service and Excellence in High Education Beyond Fifty Years suspended. Should the study be suspended, study sponsors and other regulatory authorities will be informed.

Conditions of Approval

- No participant may be involved in any study procedure prior to the study approval or after the expiration date.
- All unanticipated or Serious Adverse Events (SAEs) must be reported to HSSREC within 5 days.
- All protocol modifications must be approved by HSSREC prior to implementation unless they are intended to reduce risk (but must still be reported for approval). Modifications will include any change of investigator/s or site address.
- All protocol deviations must be reported to HSSREC within 5 working days.
- All recruitment materials must be approved by HSSREC prior to being used.
- Principal investigators are responsible for initiating Continuing Review proceedings. HSSREC will only approve a study for a period of 12 months.
- It is the responsibility of the PI to renew his/her ethics approval through a renewal application to HSSREC.
- Where the PI desires to extend the study after expiry of the study period, documents for study extension must be received by HSSREC at least 30 days before the expiry date. This is for the purpose of facilitating the review process. Documents received within 30 days after expiry will be labelled "late submissions" and will incur a penalty fee of K500.00. No study shall be renewed whose documents are submitted for renewal 30 days after expiry of the certificate.
- Every 6 (six) months a progress report form supplied by The University of Zambia Humanities and Social Sciences Research Ethics Committee as an IRB must be filled in and submitted to us. There is a penalty of K500.00 for failure to submit the report.
- When closing a project, the PI is responsible for notifying, in writing or using the Research Ethics and Management Online (REMO),both HSSREC and the National Health Research Authority (NHRA) when ethics certification is no longer required for a project.
- In order to close an approved study, a Closing Report must be submitted in writing or through the REMO system. A Closing Report should be filed when data collection has ended and the study team will no longer be using human participants or animals or secondary data or have any direct or indirect contact with the research participants or animals for the study.
- Filing a closing report (rather than just letting your approval lapse) is important as it assists HSSREC in efficiently tracking and reporting on projects. Note that some funding agencies and sponsors require a notice of closure from the IRB which had approved the study and can only be generated after the Closing Report has been filed.
- A reprint of this letter shall be done at a fee.
- All protocol modifications must be approved by HSSREC by way of an application for an amendment prior to implementation unless they are intended to reduce risk (but must still be reported for approval). Modifications will include any change of investigator/s or site address

or methodology and methods. Many modifications entail minimal risk adjustments to a protocol and/or consent form and can be made on an Expedited basis (via the IRB Chair). Some examples are: format changes, correcting spelling errors, adding key personnel, minor changes to questionnaires, recruiting and changes, and so forth. Other, more substantive changes, especially those that may alter the risk-benefit ratio, may require Full Board review. In all cases, except where noted above regarding subject safety, any changes to any protocol document or procedure must first be approved by HSSREC before they can be implemented.

Should you have any questions regarding anything indicated in this letter, please do not hesitate to get in touch with us at the above indicated address.

On behalf of HSSREC, we would like to wish you all the success as you carry out your study.

Yours faithfully,

Dr. J. L. I. Ziwa
ACTING CHAIRPERSON
THE UNIVERSITY OF ZAMBIA HUMANITIES AND SOCIAL SCIENCES RESEARCH ETHICS COMMITTEE - IRB

cc: Director, Directorate of Research and Graduate Studies
Assistant Registrar (Research), Directorate of Research and Graduate Studies
Acting Senior Administrative Officer, Directorate of Research and Graduate Studies

Appendix H: Permission letter for data collection from school U.

C/o Depart. Of Mathematics
And Science Education
The University of Zambia
P.O Box 32379
Great East Campus, Lusaka.
Date: 8th June, 2021.

The Head teacher,

Mu[redacted]ry School,
P.O[redacted]55........,
Lusaka.
Dear Sir/ Madam,

Re: <u>Request for Permission to conduct research at your school</u>

The above captioned matter refers.

I am a student at the University of Zambia (UNZA) under the Master of Education in Mathematics Education and partial fulfilment of the requirements for the Master's degree, I am expected to conduct research and compile a report thereafter. The title of this research is *Learners' Problem Solving Processes in Linear Programing at selected Secondary Schools in Lusaka Zambia.* This will involve class lesson observations, group discussions outside the classroom and interviews with both the learners and their teachers. Therefore, I humbly request for your permission to conduct my research at your school.

In accordance and compliance with the ethical requirements laid down by the University of Zambia, be rest assured that the identity of the school, the teachers and pupils who will participate, will be highly protected.

Thank you for the consideration.

Yours Faithfully,

SANDAMBO GABRIEL

+260 969805743, gabrielmsandambo@gmail.com

Approved.

REPUBLIC OF ZAMBIA
MINISTRY OF GENERAL EDUCATION
08 JUN 2021
DEPUTY HEADTEACHER'S OFFICE
MU[redacted]Y SCHOOL
[redacted]AKA.

D/HEAD

F.Y.A HOD MATHEMATICS

Appendix I: Permission letter to collect data from school V

C/o Depart. Of Mathematics
And Science Education
The University of Zambia
P.O Box 32379
Great East Campus, Lusaka.
Date: 8th June, 2021.

REPUBLIC OF ZAMBIA
MINISTRY OF GENERAL EDUCATION
01 SEP 2021
DEPUTY HEAD TEACHER'S OFFICE
[redacted] SECONDARY SCHOOL
LUSAKA.

The Head teacher,

Mu[redacted]
P.O[redacted]
Lusaka.
Dear Sir/ Madam,

Re: Request for Permission to conduct research at your school

The above captioned matter refers.

I am a student at the University of Zambia (UNZA) under the Master of Education in Mathematics Education and partial fulfilment of the requirements for the Master's degree, I am expected to conduct research and compile a report thereafter. The title of this research is *Learners' Problem Solving Processes in Linear Programing at selected Secondary Schools in Lusaka Zambia.* This will involve class lesson observations, group discussions outside the classroom and interviews with both the learners and their teachers. Therefore, I humbly request for your permission to conduct my research at your school.

In accordance and compliance with the ethical requirements laid down by the University of Zambia, be rest assured that the identity of the school, the teachers and pupils who will participate, will be highly protected.

Thank you for the consideration.

Yours Faithfully,

SANDAMBO GABRIEL

+260 969805743, gabrielmsandambo@gmail.com

REPUBLIC OF ZAMBIA
MINISTRY OF GENERAL EDUCATION
H.O.D MATHEMATICS
31 AUG 2021
[redacted]

Approved