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Establishment of a Unified Country-Wide Plane Coordinate System for Zambia

By

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A thesis submitted in partial fulfilment of the requirement for the Degree of Master of Engineering in Geoinformatics and Geodesy

DECLARATION

I hereby truthfully declare that I am the sole author of this report and that all content is my original work that has not been presented before for an award at any university or learning institution:

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ABSTRACT

Planet Earth is approximately spherical in shape, and is three dimensional. To map the Earth on a flat piece of paper, in two dimensions, a map projection must be carried out. A map projection is a mathematical technique of how to represent the Earth's curved surface on a flat surface. In Zambia, the map projection used for national mapping is the Universal Transverse Mercator (UTM) in 6 degree zones. Globally UTM zones run from zone 1 to zone 60 with Zambia falling onto zones 34, 35 and 36, and with central meridians at 21^0 E, 27^0 E and 33^0 E, respectively. The central meridians and the equator form three separate plane coordinate systems, with origins at the intersection of the equator and the particular central meridian.

Map projections come with distortions since there is "stretching" or "shrinking" of the curved surface of the reference ellipsoid or spheroid. In order to compute distortions, a scale factor is introduced to determine scale errors from the central meridian. For UTM projection the scale factor at the central meridian is 0.9996. To avoid negative coordinates for the southern hemisphere and the western part of the central meridian, a false easting and northing of 500,000m and 10,000,000m are introduced, respectively.

The problem with the UTM projection system is that data from different zones cannot easily be combined to create integrated, seamless maps of geographic features across zone boundaries. Therefore, in this study, the UTM projection and grid system was modified to cover the whole country Zambia in a single zone. To achieve this, a computer program was written to determine the scale factor at central meridian suitable for country-wide mapping. The central meridian was set to 28^0 E, and the scale factor at the central meridian was reduced to 0.9984 to minimise the mean scale error of mapping. A False-Easting of 800,000m was applied to eliminate negative coordinates, while a False-Northing of 10,000,000m was maintained.

The new plane coordinate system is intended to be used for country-wide, seamless landcover/use mapping projects such as Task 151 of the Southern African Science Service Centre for Climate Change and Adaptive Land Management (SASSCAL) project.

Key words: map projection, scale factor, scale error, seamless mapping, coordinate system

LIST OF ACRONYMS

BMBF	Germany Federal Ministry of Education and Research
CLC	CORINE Land Cover
CORINE	Coordination of Information on Environment
EEA	European Environmental Agency
EPSG	European Petroleum Survey Group
ETM	Enhanced Thematic Mapper
GIS	Geographic Information System
GPS	Global Positioning System
GRS	Geodetic Reference System
GRZ	Government of the Republic of Zambia
GSD	Geological Survey Department
IDE	Integrated Development Environment
ITRF	International Terrestrial Reference Frame
MMMD	Ministry of Mines and Mineral Development
MMU	Minimum Mapping Unit
NATO	North Atlantic Treaty Organization
NRSC	National Remote Sensing Centre
SASSCAL	Southern African Science Service Centre for Climate Change and Adaptive Land-use Management
SEED	Support to Economic Expansion and Diversification (SEED) Programme
TM	Transverse Mercator
TP	Primary Traverse station
TS	Secondary Traverse station
TT	Tertiary Traverse station
TRF	Terrestrial Reference Frame
UPS	Universal Polar Stereographic
USGS	United States Geological Survey
UTM	Universal Transverse Mercator
WGS84	World Geodetic System 1984
WTM	Wisconsin Transverse Mercator
ZSD	Zambia Survey Department
ZTM	Zambian Transverse Mercator
ZP	Zambia Primary station
ZS	Zambia Secondary station
ZT	Zambia Tertiary station

CHAPTER 1 INTRODUCTION

Maps are one of the world's oldest documents. The surface of the earth had for a long time been wrongly perceived as a flat plane, with old maps of the world depicting the world with edges. In the process of making maps the ellipsoidal or spherical surfaces are used to represent the surface of the Earth. These curved reference surfaces are then transformed to a flat plane of the map by means of a map projection. A **map projection** is a mathematical technique of how to represent the Earth's curved surface on a flat map. Since a map is a small-scale representation of the Earth's surface it is necessary to apply some kind of scale reduction. Thus, each point on the reference surface of the Earth with geographic coordinates may be transformed to a set of two-dimensional (2D) Cartesian coordinates or map coordinates representing positions on the map plane. Hundreds of map projections are developed in order to accurately represent a particular map or to best suit a particular type of map. Thus a map coordinate system can be created by choosing a projection and then tailoring its parameters to fit any region on the Earth. Standard coordinate systems have been developed to simplify the process of choosing a system. The most important standard map coordinate system used is the Universal Transverse Mercator (UTM) (Knippers, 2009).

Generally, modern mapping methods which involve working with Geographic Information System (GIS), and Global Positioning System (GPS) require the utilization of a country-wide reference coordinate system. The starting point for any project, application, service or product which is reliant on some form of geo-referencing must be a uniform and reliable co-ordinate reference system. A uniform and consistent coordinate system covering the whole country provides a fundamental reference system for all geo-spatial information, planning and development projects across a wide spectrum of disciplines (Wonnacott, 2005).

Therefore, a study was carried out to establish a unified country-wide plane coordinate system for Zambia, through modification of UTM system parameters. The resulting coordinate system was termed Zambian Transverse Mercator (ZTM) since its application is tailored for Zambia only. The land cover/land use mapping of the

Task 151 of the Southern African Science Service Centre for Climate Change and Adaptive Land Management (SASSCAL) project is earmarked to be mapped in the ZTM projection and grid system.

1.1 Problem Background and Justification

Zambia, like many other African countries, has an adopted projected coordinate reference system based on Clarke 1880 reference ellipsoid, Arc 1950 datum. The country is essentially divided into three (3) UTM zones in the southern hemisphere, namely: zone 34, zone 35, and zone 36. All geo-referencing involving small scale mapping is based on this particular plane coordinate system. The UTM system works well for large to medium scale mapping, since the area involved mostly falls within a single zone boundary. However, the division into multiple zones has its unique challenges when mapping geographic features that span across zone boundaries. This study looked at the current challenges in mapping, analysed what and how other countries achieve seamless mapping, and finally devised methodologies for establishing a country-wide plane coordinate system for seamless mapping in Zambia. The following are the main problems or challenges with the current plane coordinate system in Zambia:

- There is no defined unified plane coordinate system for country-wide mapping. The current UTM plane coordinate system based on the standard 6^{o} zones means that each zone uses a different central meridian. Hence a separate coordinate system.
- The above entails that data from different zones cannot easily be combined to create seamless (or wall-to-wall) maps/datasets.
- The common practice, in GIS, of using UTM system with longitude 27^{o} east as central meridian for country-wide mapping is not supported by any scientific study with regards to the inherent errors and possible corrections.
- The alternative practice of country-wide mapping using geographic coordinates has limitations in planimetric quantifications of lengths and areas of mapped features because longitudes and latitudes are angular quantities.

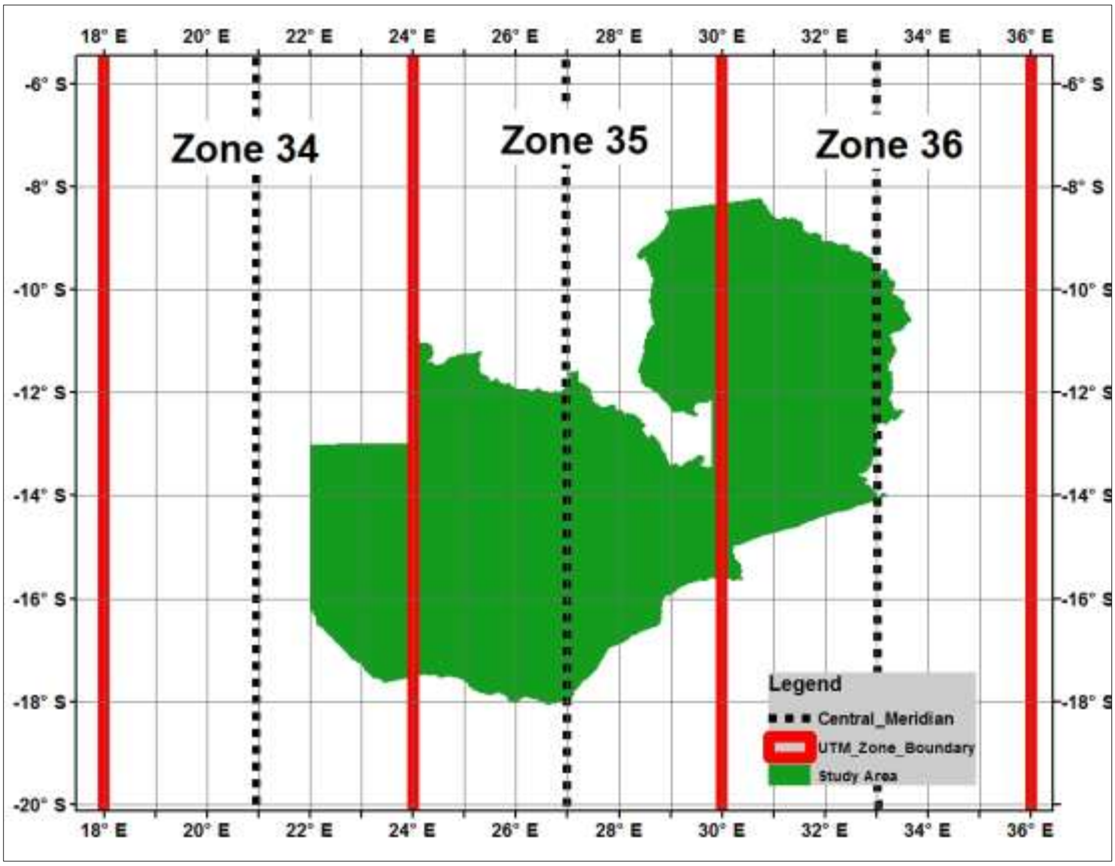


Figure 1.1: UTM zones and central meridians (dotted lines) across Zambia

1.2 Research Objectives

Main objective:

To establish a unified country-wide plane coordinate system for Zambia

Sub-objectives:

1. To modify the UTM projection parameters for Zambia
2. To determine scale distortions in the new projection system

1.3 Research Questions

The following research questions were answered in this study:

1. What UTM parameters need to be modified, and how will modification be carried out to achieve a unified country-wide mapping system?
2. How will the scale factors and distortions of the proposed plane coordinate system be maintained within acceptable limits?

1.4 Significance of the Study

The establishment of a unified country-wide plane coordinate system for Zambia is of great importance as the system is primarily intended to be used for country-wide land cover/ land use seamless mapping projects such as Task 151 of the Southern African Science Service Centre for Climate Change and Adaptive Land Management (SASSCAL) project.

SASSCAL is a joint initiative of Germany and five Southern African countries (Zambia, South Africa, Namibia, Botswana and Angola), responding to the challenges of global climate change. The mission of the project is to conduct problem-oriented research in the area of adaptation to climate change and sustainable land management, in order to provide evidence-based advice for all decision makers and stakeholders to improve livelihoods of people in the region and to create an African knowledge-based society (Matthias Muck, 2013).

Funded by the Germany Federal Ministry of Education and Research (BMBF), SASSCAL is scientifically coordinated by the University of Hamburg and is implemented in a collaborative effort by Southern African and Germany partners. The project has five thematic areas, namely: Climate, Water, Agriculture, Forestry, Biodiversity and Capacity Development. Each thematic area has a number of Tasks. Task 151 (SASSCAL) is under Agriculture and involves the development and improvement of integrated national and regional seamless land use assessment through mapping. Hence,the need for a country-wide plane coordinate system for Zambia.

Other possible uses of the proposed country-wide plane coordinate system are: country-wide seamless mapping in the National Land Audit programme of the Ministry of Lands and Natural Resources; seamless mapping for environmental monitoring, forestry, and any other generalised thematic mapping of various geographic phenomena that are of a country-wide nature.

CHAPTER 2 LITERATURE REVIEW

2.1 Introduction

Positions of any geographic features on the Earth surface can be specified using coordinates. A coordinate system is a set of mathematical rules used to tie coordinates and position. This system can be further developed by realizing the coordinate system through a datum, thus creating a coordinate based reference system, often referred to as a Terrestrial Reference Frame (TRF) (Sahl, 2014). Coordinates can be divided into three categories with respect to their nature: geodetic, geocentric, and projected coordinate system.

2.2 Geodetic Coordinate System

Geodetic coordinates or geographic coordinates consists of latitude (φ), longitude (λ), and ellipsoidal height (h), Figure 2.1. Latitude and longitude are angles which represent a point on the surface of an ellipsoid. The angles are defined through the use of meridians and parallels. Meridians are lines of constant longitude in the north-south direction, and parallels are lines of constant latitude in the east-west direction. One meridian, called the prime meridian, is assigned the value of zero degrees longitude.

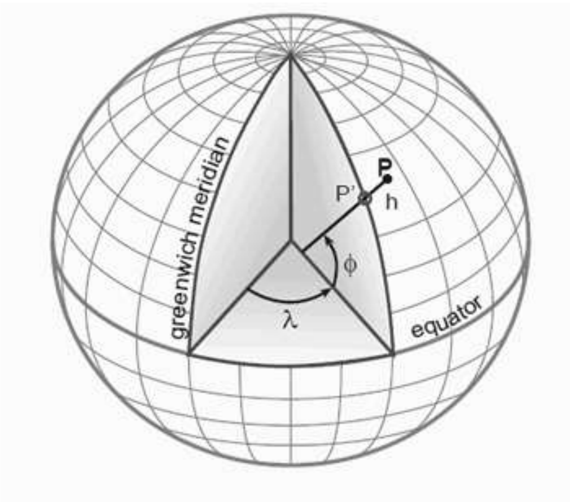


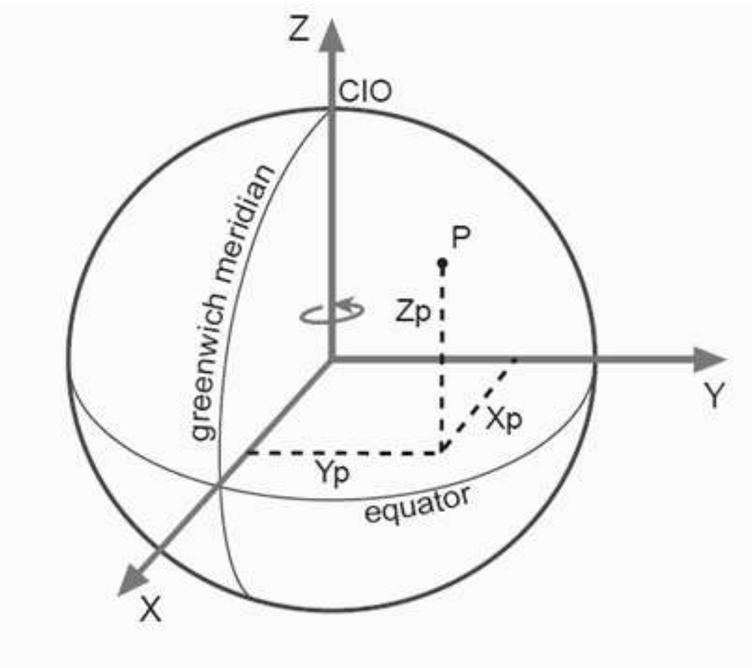
Figure 2.1: Geographic coordinate system (Knippers, 2009)

The ellipsoid is divided into two hemi-ellipsoids west and east of the prime meridian, where the longitude angles range from 0 to 180 degrees in either west or east respectively. The zero degrees reference for latitude is assigned to the equator of the ellipsoid, and the latitude angles range from 0 to 90 degrees in either north or south. The ellipsoidal height is the distance between the described point and the ellipsoid surface along a straight line perpendicular to the ellipsoid surface. Also in common use is the 2D geographic coordinate system where only latitude and longitude (φ**,** λ) are used to specify position.

2.3 Geocentric Coordinates (X, Y, Z)

Geocentric coordinates, also known as 3D Cartesian coordinates, uses three perpendicular axes, X, Y, and Z to describe a position in three dimensions, Figure 2.2. This coordinate system has the origin at the center of the Earth. The X-axis is defined as positive on that side of the geocenter which passes through the prime meridian, the Y-axis is defined as positive on that side of the geocenter which passes through 90 degrees east, and Z-axis is defined as positive on that side of the geocenter which passes through the North Pole. Similar to geodetic coordinates, a set of Earth-centered Cartesian coordinates are bound to the reference ellipsoid and would have to be transformed if used for a different ellipsoid.

Figure 2.2: Geocentric coordinate system (Mehlbreuer, 2009)



2.4 Projected Coordinate System

Projected coordinates are also known as plane coordinates, 2D Cartesian coordinates (X, Y), or grid based Eastings and Northings. These coordinates are intended to be used with a map, which is a two-dimensional plane surface projection of an area. The map coordinates of a point are computed from ellipsoidal latitude and longitude by means of a map projection. The coordinates are based on a simple two-dimensional Cartesian system which uses two axes known as easting and northing, where the distance from the axes are given in metres and referred to as Northings and Eastings. To better understand projected coordinate system a detailed review of map projections would be required. The following section on map projections is largely based on educational notes by Knippers (2009) with contributions from other authors. During literature review it was discovered that the author, R. Knippers, had a well-researched compilation on map projections.

2.5 Map Projections

As earlier defined in chapter 1, a map projection is a mathematical technique of how to represent the Earth's curved surface on a flat map. To represent parts of the surface of the Earth on a flat paper map or on a computer screen, the curved horizontal reference surface must be mapped onto a 2D mapping plane.

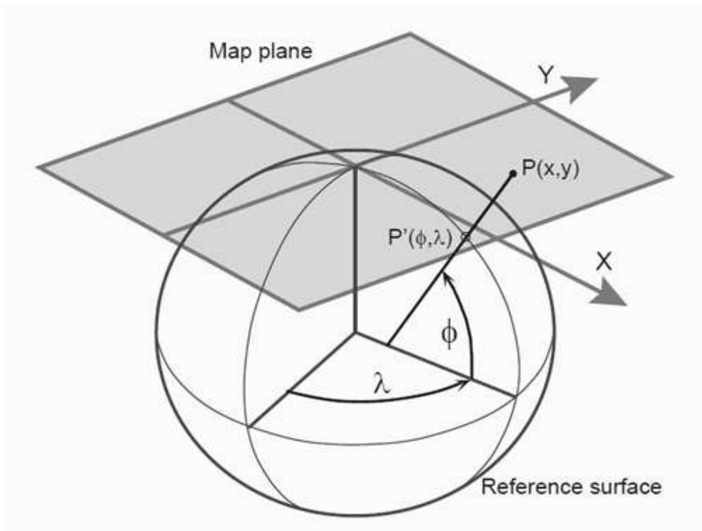


Figure 2.3: A map projection where geographic coordinates (φ, λ) of the reference surface are projected onto the 2D mapping plane with 2D Cartesian coordinates (*x*, *y*) (Knippers, 2009)

The reference surface for large-scale mapping is usually an oblate ellipsoid, and for small-scale mapping, a sphere. Mapping onto a 2D mapping plane means transforming each point on the reference surface with geographic coordinates (φ,λ) to a set of Cartesian coordinates (*x*, *y*) representing positions on the map plane (Figure 2.3).

The actual mapping cannot usually be visualized as a true geometric projection, directly onto the mapping plane as illustrated in the figure above. This is mostly achieved through mapping equations. A forward mapping equation transforms the geographic coordinates (φ, λ) of a point on the curved reference surface to a set of plane Cartesian coordinates (x, y), representing the position of the same point on the map plane:

$$(x, y) = f\,(\varphi, \lambda) \qquad (2\text{-}1)$$

The corresponding inverse mapping equation mathematically transforms the plane Cartesian coordinates (x, y) of a point on the map plane to a set of geographic coordinates (φ, λ) on the curved reference surface:

$$(\varphi, \lambda) = f\,(x, y) \qquad (2\text{-}2)$$

2.5.1 Classification of Map Projections

Map projections can be described in terms of their: *class* (cylindrical, conical or azimuthal); point of *secancy* (tangent or secant); *aspect* (normal, transverse or oblique); and distortion *property* (equivalent, equidistant or conformal). The three classes of map projections are *cylindrical, conical and azimuthal*. If the Earth's reference surface is projected on a map wrapped around the globe as a cylinder a cylindrical map projection is produced. Projected on a map formed into a cone gives a conical map projection. When projected directly onto the mapping plane it produces an azimuthal (*zenithal* or planar) map projection. Figure 2.4 shows the surfaces involved in these three classes of projections.

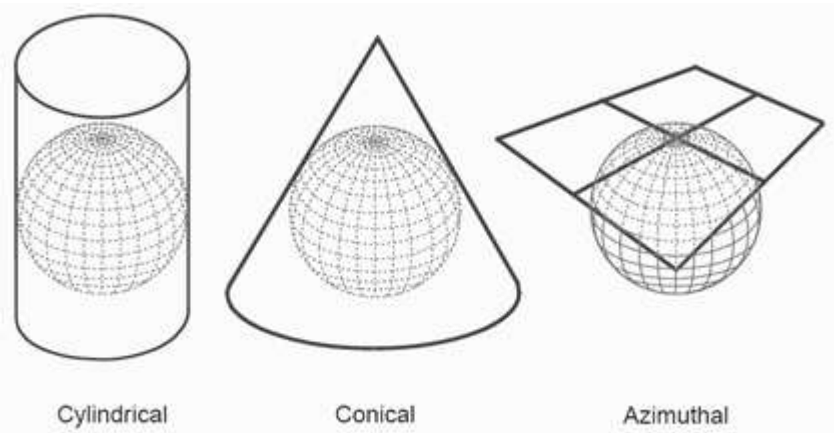


Figure 2.4: The three classes of map projections: cylindrical, conical and azimuthal (Knippers, 2009)

The planar, conical, and cylindrical surfaces in figure 2.4 above are all tangent surfaces; they touch the horizontal reference surface in one point (plane) or along a closed line (cone and cylinder) only. Another class of projections is obtained if the surfaces are chosen to be *secant* (to intersect) with the horizontal reference surface. In this way, the reference surface is intersected along one closed line (plane) or two closed lines (cone and cylinder). Secant map surfaces are used to reduce or average scale errors because the line(s) of intersection are not distorted on the map.

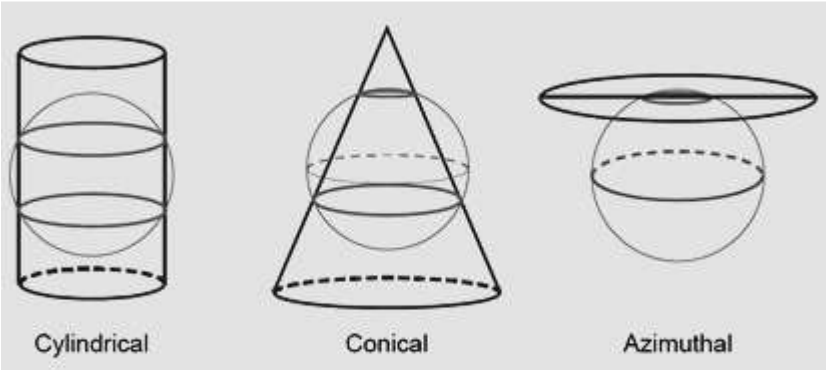


Figure 2.5: Three secant projection classes (Knippers, 2009)

Projections can also be described in terms of the direction of the projection plane's orientation (whether cylinder, plane or cone) with respect to the globe. This is called the aspect of a map projection. The three possible aspects are *normal*, *transverse* and *oblique*. In a normal projection, the main orientation of the projection surface is parallel to the Earth's axis. A transverse projection has its main orientation perpendicular to the Earth's axis. Oblique projections are all other, non-parallel and non-perpendicular, cases.

The manner in which the Earth's reference surface is projected onto the plane, cone or cylinder determines which kind of distortion properties the map will have compared to the original curved reference surface. The distortion properties of maps are typically classified according to what is not distorted on the map:

- In a ***conformal*** map projection the angles between lines in the map are identical to the angles between the original lines on the curved reference surface. This means that angles (with short sides) and shapes (of small areas) are shown correctly on the map.

- In an ***equal-area*** (equivalent) map projection the areas in the map are identical to the areas on the curved reference surface (taking into account the map scale), which means that areas are represented correctly on the map.
- In an ***equidistant*** map projection the length of particular lines in the map are the same as the length of the original lines on the curved reference surface (taking into account the map scale).

A particular map projection can have any one of these three properties. No map projection can be both conformal and equal-area. A projection can only be equidistant (true to scale) at certain places or in certain directions.

2.5.2 Scale Distortions on a Map

A map projection without distortions would correctly represent shapes, angles, areas, distances and directions, everywhere on the map. Unfortunately, any map projection is associated with scale distortions. There is simply no way to flatten out a piece of ellipsoidal or spherical surface without stretching some parts of the surface more than others (figure2.6). The type and amount of distortions a map will have, depends largely on the size of the area being mapped and the type of map projection that has been selected for mapping.

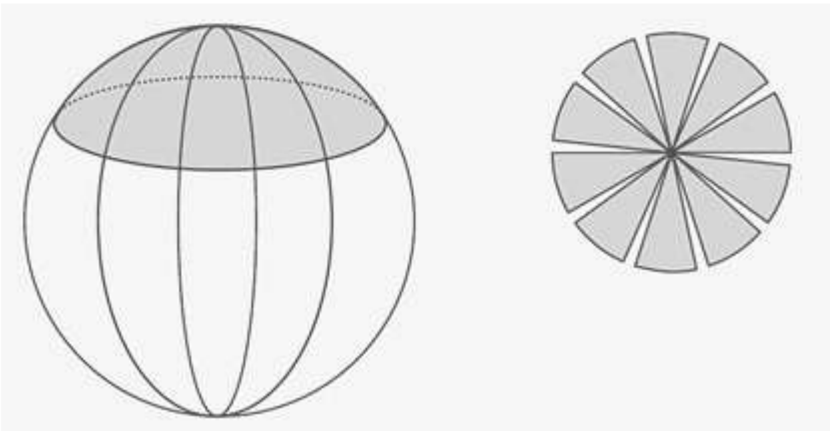


Figure 2.6: Scale distortions after flattening a piece of the ellipsoidal or spherical reference surface (Knippers, 2009)

Since there is no map projection that maintains correct scale all over the map, it may be important to know the extent to which the scale varies on a map. On a world map, the scale variations are evident where landmasses are wrongly sized or out of shape and the meridians and parallels do not intersect at right angles or are not spaced uniformly. These maps may have a *scale reduction diagram* to indicate the map scale at different locations, helping the map-reader to become aware of the distortions. On maps at larger scales, maps of countries or even city maps, the distortions are not evident to the eye. However, the map user could be aware of the distortions once distances, areas or angles are computed on the basis of measurements taken from these maps.

Scale distortions can be measured and shown on a map by *ellipses of distortion*. The ellipse of distortion, also known as *Tissot's Indicatrix*, shows the shape of an infinite small circle with a fixed scale on the Earth as it appears when plotted on the map. Every circle is plotted as circle or an ellipse or, in extreme cases, as a straight line. The size and shape of the ellipse shows how much the scale is changed and in what direction. The indicatrices on the map in figure 2.7 have varying degrees of flattening, but the areas of the indicatrices everywhere on the map are the same, which means that areas are represented correctly on the map. The distortion property of the map projection is therefore equal-area (or equivalent).

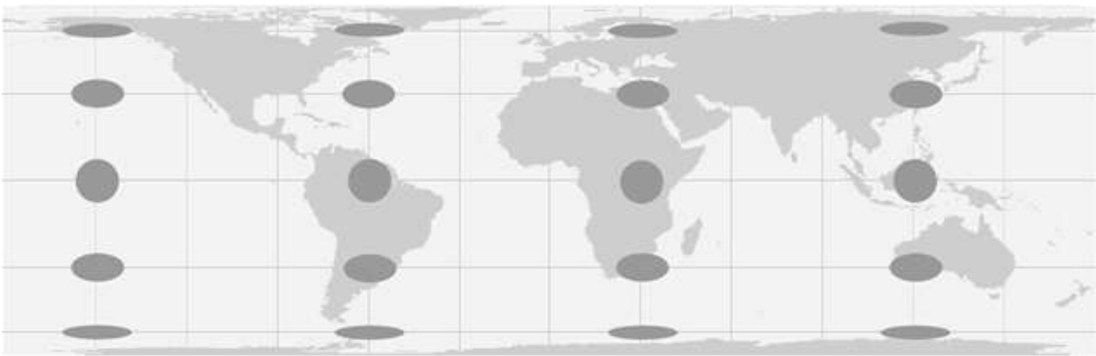


Figure 2.7: The ellipses of distortion plotted on the cylindrical equal-area projection (Knippers, 2009)

When the indicatrices are circles everywhere on the map, the angles and consequently shapes of small areas are shown correctly on the map. The distortion property of the map projection is therefore conformal (e.g. the Mercator projection).

Scale distortions on a map can also be shown by means of a *scale factor* (ratio of the scale at a given point to the true scale). Scale distortions exist at locations where the scale factor is smaller or larger than 1, e.g. a scale factor at a given point on the map is equal to 0.99960 signifies that 1000 metres on the reference surface of the Earth will actually measure 999.6 metres on the map. This is a contraction of 40 centimetres per kilometre.

According to Knippers (2009), the *nominal map scale* (given map scale) divided by the scale factor will give the actual scale. For example, a scale factor of 0.99960 at a given point on a map with a nominal scale of 1:10,000 will give a scale of 1:10,004 (10,000 divided by 0.99960) at the given point. This is a smaller scale than the nominal map scale. Scale distortions for both, tangent and secant map surfaces, are illustrated in figure 2.8. Distortions increase as the distance from the central point (tangent plane) or closed line(s) of intersection increases.

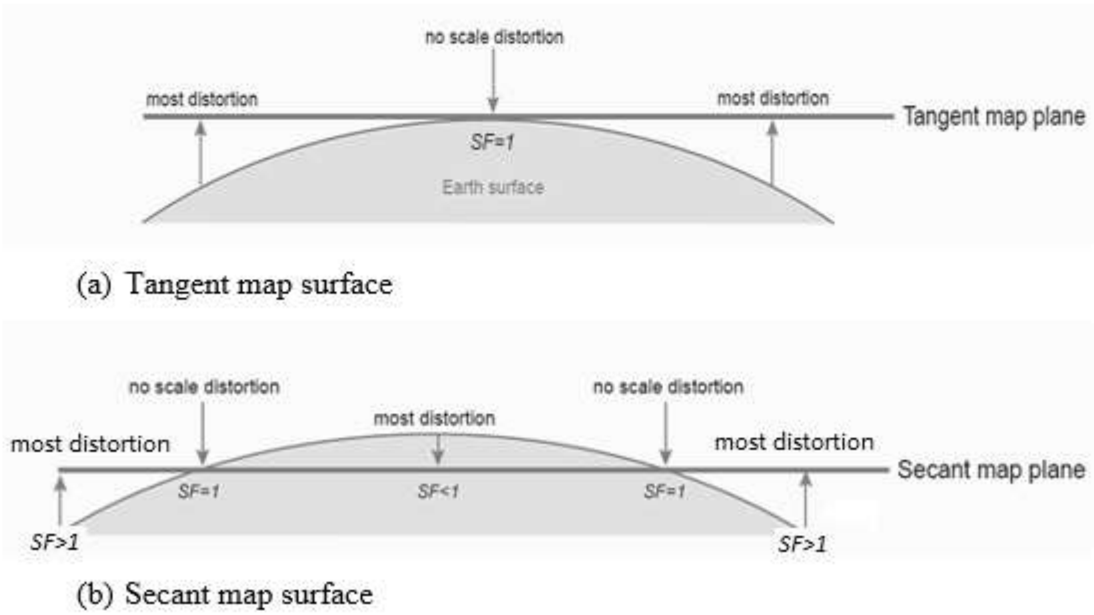


Figure 2.8: Scale distortions on a tangent map surface (a). The central point is not distorted on the map. Scale distortions on a secant map surface (b). Line(s) of intersection are not distorted on the map (Knippers, 2009)

On a secant map projection, the application of a scale factor of less than 1.0000 to the central point or the central meridian has the effect of making the projection secant, and the overall distortions are less than on projections that uses a tangent map surface. Most countries have derived their map coordinate system from a projection with a secant map surface for this reason (Knippers, 2009).

2.5.3 Choosing a Map Projection

Every map must begin with the choice of a map projection and its parameters. The cartographer's task is to ensure that the right type of projection is used for any particular map. A well-chosen map projection takes care that scale distortions remain within certain limits and that map properties match to the purpose of the map. Generally, normal cylindrical projections are typically used to map the world in its entirety (in particular areas near the equator are shown well). Conical projections are often used to map the different continents (the mid-latitudes regions are shown well), while the polar azimuthal projections may be used to map the polar areas. Transverse and oblique aspects of many projections can be used for most parts of the world, though they are usually more difficult to construct (Knippers, 2009).

In theory, the selection of a map projection for a particular area can be made on the basis of the ***shape*** of the area, the ***location*** (and orientation) of the area, and the ***purpose*** of the map. Ideally, the general shape of the mapping area should match with the distortion pattern of a specific projection. If an area is approximately circular it is possible to create a map that minimizes distortion for that area on the basis of an azimuthal projection. The cylindrical projection is best for a rectangular area and a conic projection for a triangular area. The choice of the aspect of a map projection depends largely on the location and orientation of the geographic area to be mapped. Optimal is when the projection plane is located along the main axis of the area to be mapped.

Once the class and aspect of the map projection have been selected, the distortion property of the map projection has to be chosen. The most appropriate type of distortion property for a map depends largely on the purpose for which it will be used. Map projections with a ***conformal distortion property*** represent angles and local shapes correctly, but as the region becomes larger, they show considerable area distortions. An example is the Mercator projection. Maps used for measurement of angles (e.g. aeronautical charts, topographic maps) often make use of a conformal map projection. Map projections with an ***equal-area distortion property*** on the other hand, represent areas correctly, but as the region becomes larger, it shows considerable distortions of angles and consequently shapes. Maps which are to be used for measuring areas (e.g. distribution maps) often make use of an equal-area map projection.

The ***equidistant distortion property*** is achievable only to a limited degree. That is, true distances can be shown only from one or two points to any other point on the map or in certain directions. If a map is true to scale along the meridians then the map is equidistant along the meridians. If a map is true to scale along all parallels then the map is equidistant along the parallels (i.e. no distortion in East-West direction). Maps which require correct distances measured from the centre of the map to any point (e.g. air-route, radio or seismic maps) or maps which require reasonable area and angle distortions (several thematic maps) often make use of an equidistant map projection.

In summary, the ***ideal map projection*** for any country would either be an azimuthal, cylindrical, or conic projection, depending on the shape of the area, with a secant projection plane located along the main axis of the country or the area of interest. The selected distortion property depends largely on the purpose of the map. For topographic and large-scale maps, conformality and equidistance are important properties. The equidistant property, possible only in a limited sense, however, can be improved by using secant projection planes. The *Universal Transverse Mercator* projection is a conformal cylindrical projection using a secant cylinder to meet conformality and reasonable equidistance.

2.5.4 Map Projections in Common Use

A variety of map projections have been developed, each with its own specific qualities. Only a limited number of map projections are frequently used. Here are some well-known projections described and illustrated. They are grouped into cylindrical, conical and azimuthal projections.

2.5.4.1 Cylindrical Projections

According to Knippers (2009), one of the best known cylindrical projection is *Mercator's* cylindrical projection. The transverse case and occasionally the oblique case of the Mercator projection are used in several countries for topographic mapping purposes. The *Transverse Mercator* and *Universal Transverse Mercator* projections are the best known examples. Two other well-known normal cylindrical projections are the *equidistant cylindrical* (or *Plate Carrée)* projection and *Lambert's cylindrical equal-area* projection. Normal cylindrical projections are typically used to map the world in its entirety (in particular, areas near the equator are shown well).

a) Mercator Projection

The *Mercator* projection is a normal cylindrical projection. The property of the projection is conformal. Parallels and meridians are straight lines intersecting at right angles, a requirement for conformality. Meridians are equally spaced. The parallel spacing increases with distance from the equator (Figure 2.9). The projection was originally designed to display accurate compass bearings for sea travel. Any straight line drawn on this projection represents a constant compass bearing or a true direction line (loxodrome or rhumb line). The oblique Mercator projection is sometimes used to align the cylindrical projection plane with a region that is oblique and follows neither a north-south nor an east-west axis. For example, this projection is used for mapping the Malaysian peninsula and the Alaska State (Knippers, 2009).

Figure 2.9: Mercator projection, Loxodromes in black are straight lines. Great circles in blue are curved (Knippers, 2009)

b) Transverse Mercator Projection

The *Transverse Mercator* projection is a transverse cylindrical conformal projection. The projection is also known as the *Gauss-Krüger* or Gauss conformal. Angles and shapes (of small areas) are shown correctly, as a result of conformality. Figure 2.10, shows a part of the world mapped on the Transverse Mercator projection.

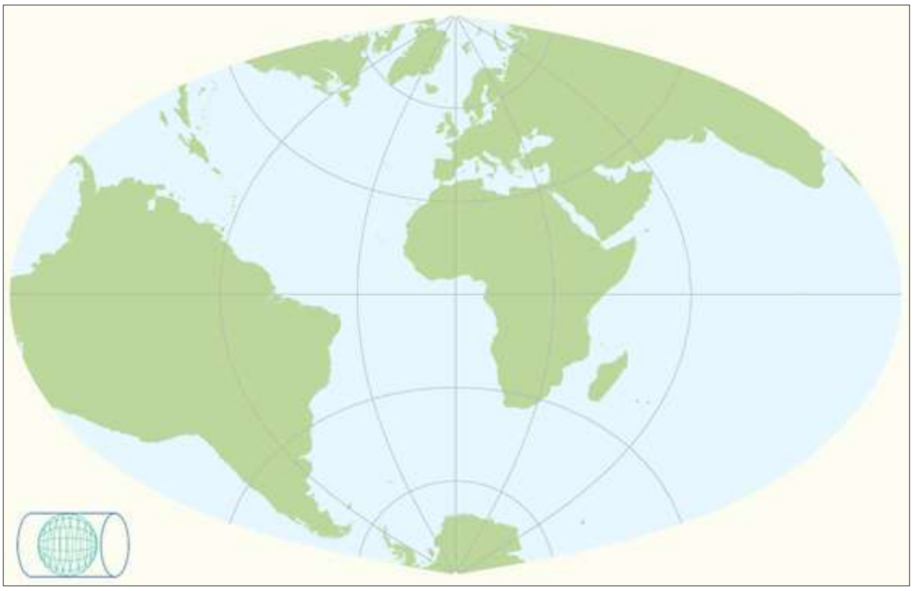


Figure 2.10: A part of the world mapped on a transverse cylinder in the Transverse Mercator projection (Knippers, 2009).

Various versions of the Transverse Mercator (TM) projection are used in many countries as the local map coordinate system on which the topographic mapping is based. Ghana uses TM projection with the central meridian located at 1°W of Greenwich. The projection is also used for aeronautical charts and recommended to the European Commission for conformal pan-European mapping at scales larger than 1:500,000 (Knippers, 2009).

c) Universal Transverse Mercator (UTM) Projection

The *Universal Transverse Mercator* (UTM) projection uses a transverse cylinder, secant to the reference surface (figure 2.11). The UTM divides the world into 60 narrow longitudinal zones of 6 degrees, numbered from 1 to 60. The narrow zones of 6 degrees (and the secant map surface) make the distortions so small that they can be ignored when constructing a map for a scale of 1:10,000 or smaller (Clynch, 2003).

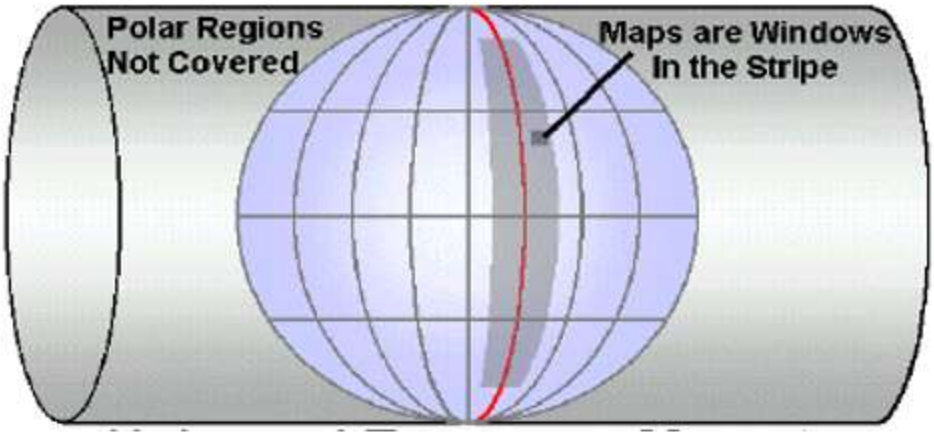


Figure 2.11: The projection plane of the UTM projection is a secant cylinder in a transverse position (Clynch, 2003).

The UTM projection is designed to cover the world, excluding the Arctic and Antarctic regions. The areas not included in the UTM system, regions north of 84°N and south of 80°S, are mapped with the *Universal Polar Stereographic* (UPS) projection. If a map series covers more than one UTM zone it is inconvenient to have the Eastings changing suddenly at a zone junction. For this reason a 40 kilometer overlap into an adjacent zone is allowed (figure 2.12). Mapping beyond this area will result in distortions at the edges of a UTM zone which may not be acceptable for the larger map scales. (A detailed description of UTM projection is provided in section 2.6.1).

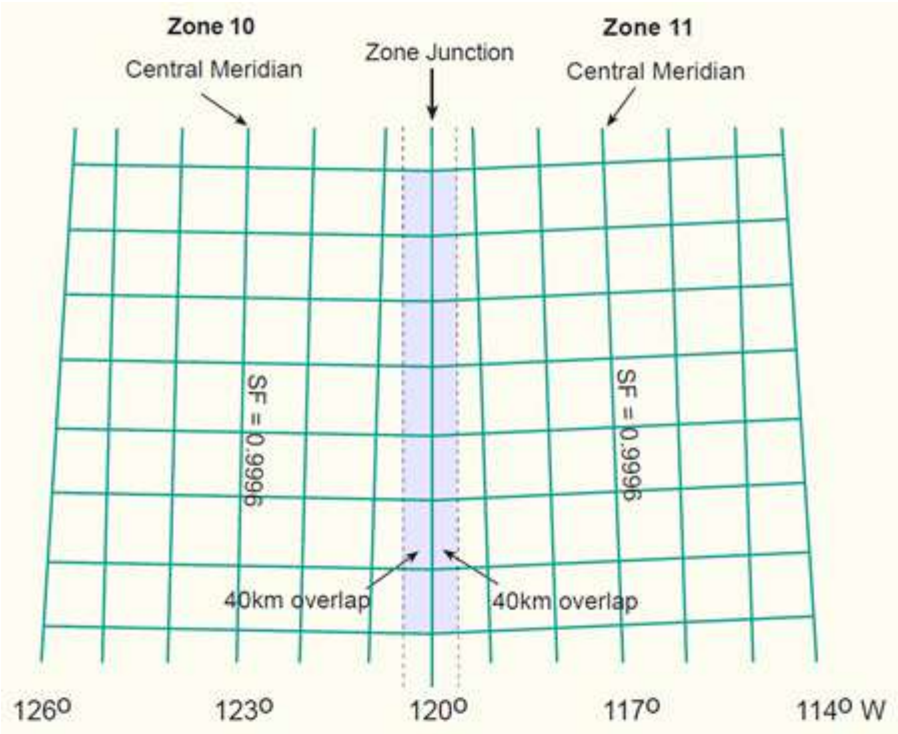


Figure 2.12: Two adjacent UTM-zones of 6 degrees longitude with a 40km overlap into the adjacent zone (Knippers, 2009)

d) Equidistant cylindrical Projection

The *equidistant cylindrical* projection, also called *simple cylindrical*, or *Plate Carrée,* has a true scale along all meridians (i.e. no distortion in north-south direction). The projection is also known as the *latitude/longitude* projection because the latitude and longitude are directly mapped into y and x respectively. Meridians are spaced at the same distances as the parallels, forming a grid of equal rectangles. Both shape and area are reasonably well preserved with the exception of polar regions. It is used for simple portrayals of the world or regions with minimal geographic data such as index maps. *Google Earth* uses the equidistant cylindrical (or simple cylindrical) projection for the display of its imagery base. The transverse version of this projection is known as the Cassini projection (Knippers, 2009).

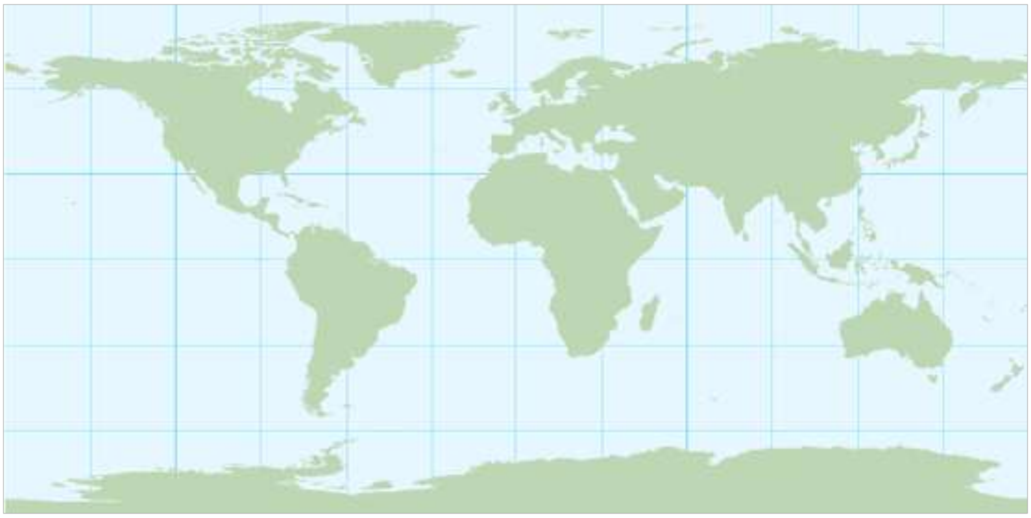


Figure 2.13: The equidistant cylindrical projection also called Plate Carrée projection (Knippers, 2009)

e) *Lambert's Cylindrical Equal-Area Projection*

The *Lambert's cylindrical equal-area* projection represents areas correctly, but it does have rather noticeable shape distortions towards the poles. Meridians are equally spaced and 0.32 times the length of the equator (Knippers, 2009). Parallels are unequally spaced and farthest apart near the equator. As a result of the distortions it is of little use for world maps.

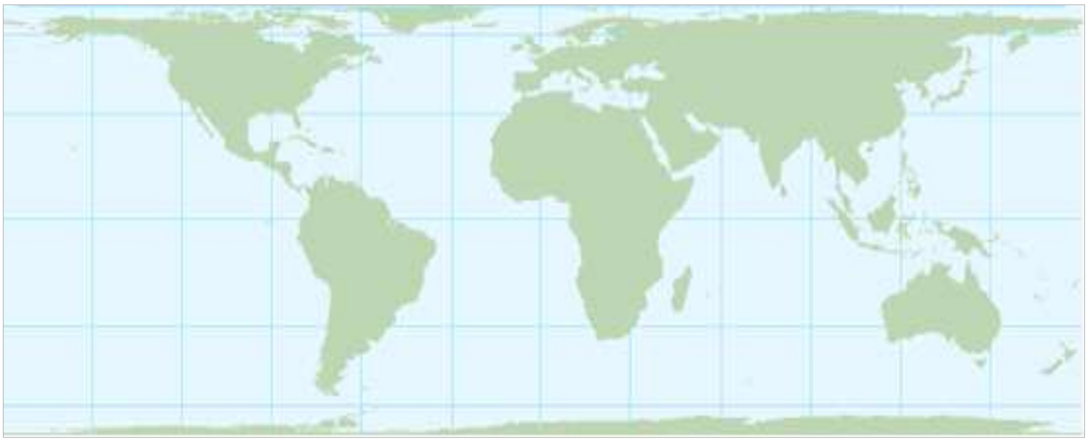


Figure 2.14: Lambert's cylindrical equal-area projection. The shape distortions are significant towards polar regions (Knippers, 2009)

2.5.4.2 Conic Projections

Four well-known normal conical projections are the *Lambert conformal conic* projection, the *simple conic* projection, the *Albers equal-area* projection and the *Polyconic* projection. They give useful maps of mid-latitudes for countries which have no great extent in latitude.

a) Lambert Conformal Conic Projection

The *Lambert conformal conic* projection is conformal. The parallels and meridians intersect at right angles (as in any conformal projection). Areas are, of course, inaccurate in conformal projections. Like with other conformal projections, Lambert's conical is also widely used for topographic maps. It is adapted in France and recommended to the European Commission for conformal pan-European mapping at scales smaller or equal to 1:500,000 (Mizake, 2008).

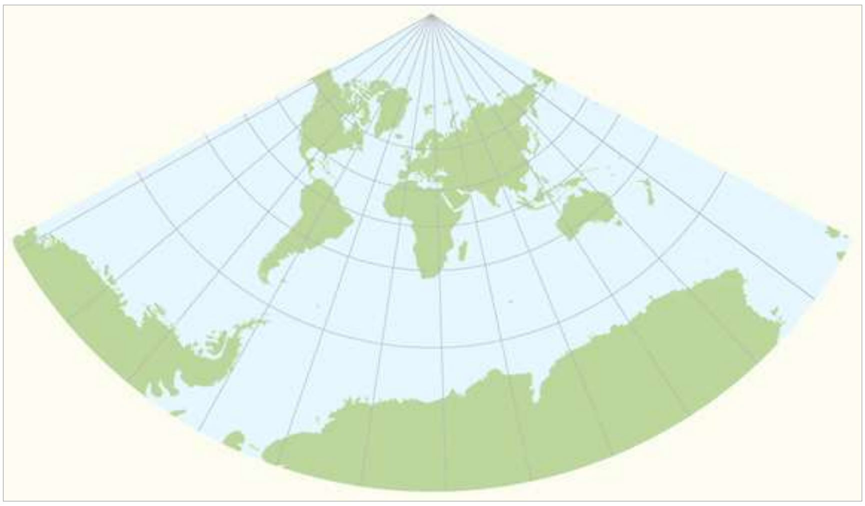


Figure 2.15: Lambert Conformal Conic projection (Knippers, 2009).

b) Simple Conic Projection

The *simple conic* projection is a normal conical projection with one standard parallel. All circular parallels are spaced evenly along the meridians, which creates a true scale along all meridians (i.e. no distortion in north-south direction). The map is therefore equidistant along the meridians. Both shape and area are reasonably well preserved. Whereas small countries are possibly shown on this projection, larger areas, such as Russia or Europe are better portrayed on the conic projection with two standard parallels. Others are *Albers equal-area*, and *Lambert's equal-area conic* projection with one standard parallel (Mizake, 2008).

2.5.4.3 Azimuthal Projections

Azimuthal (or zenithal or planar) projections are made upon a plane tangent (or secant) to the reference surface. All azimuthal projections possess the property of maintaining correct azimuths, or true directions from the centre of the map. In the polar cases, all the meridians radiate out from the pole at their correct angular distance apart. A subdivision may be made into perspective and non-perspective azimuthal projections. In the *perspective* projections, the actual mapping can be visualized as a true geometric projection, directly onto the mapping plane. For the *gnomonic* projection, the perspective point (like a source of light rays), is the centre of the Earth. For the *stereographic* this point is the opposite pole to the point of tangency, and for the *orthographic* the perspective point is an infinite point in space on the opposite side of the Earth. Two well-known *non-perspective* azimuthal projections are the *azimuthal equidistant* projection (also called Postel projection) and the *Lambert azimuthal equal-area* projection.

a) Stereographic Projection

The azimuthal *stereographic* projection is a conformal projection. Parallels and meridians intersect at right angles. In the polar aspect the meridians are equally spaced straight lines, the parallels are unequally spaced circles centered at the pole. Spacing gradually increases away from the pole. The scale is constant along any circle having its centre at the projection centre, but increases moderately with distance from the centre. The ellipses of distortion remain circles (indicating conformality). Areas increase with distance from the projection center. The polar stereographic projection is used in combination with the UTM coordinate system as *Universal Polar Stereographic* (UPS) for mapping regions north of 84°N and south of 80°S. Recommended for conformal mapping of regions approximately circular in shape; the Netherlands uses a modified version of the stereographic projection (Dutch double stereographic) known as RijksDriehoekstelsel (RD) (Stefanovic, 1996).

Figure 2.16: Polar azimuthal stereographic projection is a planar projection with a conformal property (Knippers, 2009).

b) Orthographic Projection

The *orthographic* projection is a perspective projection that views the globe from an infinite distance. *Google Earth* shows the Earth as it looks from an elevated platform such as an airplane or orbiting satellite. The projection used to achieve this effect is called the general perspective. This is similar to the orthographic projection, except that the point of perspective is a finite (near Earth) distance rather than an infinite (deep space) distance.

2.6 The Universal Transverse Mercator Projection and Grid System

In this section, the Universal Transverse Mercator projection and grid system will be studied in detail since the UTM modification is the basis of this study. Thereafter, a number of countries will be examined as to how they achieved the modification of the UTM coordinate system to produce various forms of the UTM plane coordinate system for either country-wide mapping or regional mapping. Emphasis would be placed on country-wide mapping in accordance with the aim of the study.

2.6.1 Universal Transverse Mercator

The Universal Transverse Mercator (UTM) projection and grid were adopted by the U.S. Army in 1947 for designating rectangular coordinates on large-scale military maps of the entire world. It was recommended for topographic mapping by the United Nations Cartography Committee in 1952. The UTM is the ellipsoidal Transverse Mercator to which specific parameters, such as central meridians, have been applied. The Earth, between latitudes 84° N and 80° S, is divided into 60 zones each generally 6° wide in longitude. Bounding meridians are evenly divisible by 6°, and zones are numbered from 1 to 60 proceeding east from the 180th meridian from Greenwich with minor exceptions. There are letter designations from south to north (figure 2.17). Thus, Lusaka city is in grid zone 35L, a designation covering a quadrangle from longitude 24° to 30° E and from latitude 8° to 16° S. Each of these quadrangles is further subdivided into grid squares 100,000 meters on a side with double-letter designations, including partial squares at the grid boundaries. From latitude 84° N and 80° S to the respective poles, the Universal Polar Stereographic (UPS) projection is used instead (Snyder, 1984).

Each geographic location in the UTM projection is given *x* and *y* coordinates, in meters, according to the Transverse Mercator projection, using the meridian halfway between the two bounding meridians as the central meridian, and reducing its scale to 0.9996 of true scale (a 1:2,500 reduction). The reduction was chosen to minimize scale variation in a given zone; the variation reaches 1 part in 1,000 from true scale at the Equator. The lines of true scale are approximately parallel to and approximately 180 km east and west of the central meridian. Between them, the scale is too small; beyond them, it is too great. In the Northern Hemisphere, the Equator at the central meridian is considered the origin, with an *x* coordinate of 500,000 m and a *y* of 0. For the Southern Hemisphere, the same point is the origin, but, while x remains 500,000m, y is 10,000,000 m. In each case, numbers increase toward the east and north (Snyder, 1984).

Negative coordinates are thus avoided. The ellipsoidal Earth is used throughout the UTM projection system, but the reference ellipsoid changes with the particular region of the Earth. For Zambia, the Clarke 1880 reference ellipsoid is used for UTM projection (Geomatic Solutions, 2011).

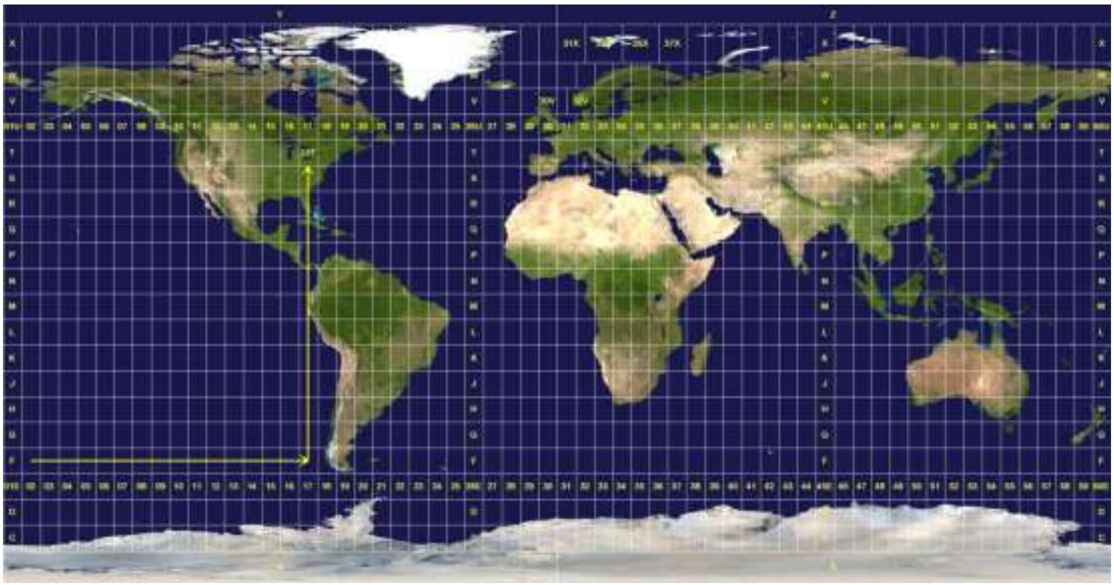


Figure 2.17: The Universal Transverse Mercator (UTM) grid zone designations for the world (Legallandconverter, 2016).

2.6.1.1 Formulas for the Sphere

A partially geometric construction of the Transverse Mercator for the sphere involves constructing a regular Mercator projection and using a transforming map to convert meridians and parallels on one sphere to equivalent meridians and parallels on a sphere rotated to place the equator of one along the chosen central meridian of the other. Such a transforming map may be the equatorial aspect of the Stereographic or other azimuthal projection, drawn twice to the same scale on transparencies. The transparencies may then be superimposed at 90° angles and the points compared. In an age of computers, it is much more satisfactory to use mathematical formulas. The following are rectangular coordinates for the Transverse Mercator applied to the sphere (Snyder, 1984):

$$x = \tfrac{1}{2} R k_0 \ln[(1+B)]/(1-B) \quad (2\text{-}3)$$

or

$$x = Rk_0 \arctan hB \quad (2\text{-}4)$$

$$y = Rk_0 \{\arctan[\tan = \phi / \cos(\lambda - \lambda_0)] - \phi_0\} \quad (2\text{-}5)$$

$$k = k_0 /(1 - B^2)^{1/2} \quad (2\text{-}6)$$

Where

$$B = \cos\phi \sin(\lambda - \lambda_0) \quad (2\text{-}7)$$

and k is the scale factor at any point (Φ, λ), while k_0 is the scale factor along the central meridian λ_0. The origin of the coordinates is at (Φ_0, λ_0). The Y axis lies along the central meridian λ_0, y is the northing and increases northerly, and the X axis is perpendicular through Φ_0 at λ_0, x is the easting and increases easterly. R is the radius of the sphere. The inverse formulas for (Φ_0, λ_0) in terms of (x, y) (Snyder, 1984):

$$\phi = \arcsin[\sin D / \cosh(x / Rk_0)] \quad (2\text{-}8)$$

$$\lambda = \lambda_0 + \arctan[\sinh(x / Rk_0) \cos D] \quad (2\text{-}9)$$

Where

$$D = y/(Rk_0) + \phi_0, \quad (2\text{-}10)$$

2.6.1.2 Formulas for the Ellipsoid

For the ellipsoidal form, the most practical form of the equations is a set of series approximations which converge rapidly to the correct centimetre or less at full scale in a zone extending 3° to 4° of longitude from the central meridian. Beyond this, the forward series as given here is accurate to about a centimetre at 7° of longitude, but the inverse series does not have sufficient terms for this accuracy. The forward series may be used with meter accuracy to 10° of longitude. Coordinate axes are the same as they are for the spherical formulas above. The formulas below are only slightly modified from those presented in standard references to provide mm accuracy at full scale (Snyder, 1987).

$$x = k_0 N\left[A + (1 - T + C)A^3/6 + (5 - 18T + T^2 + 72C - 58e'^2)A^5/120\right] \quad (2\text{-}11)$$

$$y = k_0 \left\{ M - M_0 + N \tan\phi \begin{bmatrix} A^2/2 + \left(5 - T + 9C + 4C^2\right)A^4/24 \\ + \left(61 - 58T + T^2 + 600C - 330e'^2\right)A^6/720 \end{bmatrix} \right\} \quad (2\text{-}12)$$

$$k = k_0 \begin{bmatrix} 1 + (1 + C)A^2/2 + \left(5 - 4T + 42C + 13C^2 - 28e'^2\right)A^4/24 \\ + \left(61 - 148T + 16T^2\right)A^6/720 \end{bmatrix} \quad (2\text{-}13)$$

Where k_0 =scale factor on the central meridian (this is 0.9996 for the UTM projection). *a* is the semimajor axis of the reference ellipsoid; *e* is the eccentricity of the ellipsoid defined as:

$$e'^2 = e^2 / \left(1 - e^2\right) \quad (2\text{-}14)$$

$$N = a / \left(1 - e^2 \sin^2 \phi\right)^{1/2} \quad (2\text{-}15)$$

$$T = \tan^2 \phi \quad (2\text{-}16)$$

$$C = e^2 \cos^2 \phi \quad (2\text{-}17)$$

$A = \left(\lambda - \lambda_0\right) COS\phi$, with λ and λ_0 in radians (2-18)

$$M = a \begin{bmatrix} \left(1 - e^2/4 - 3e^4/64 - 5e^6/256 - \ldots\right)\phi \\ - \left(3e^2/8 + 3e^4/32 + 45e^6/1024 + \ldots\right)\sin 2\phi \\ + \left(15e^4/256 + 45e^6/1024 + \ldots\right)\sin 4\phi - \left(35e^6/3072 + \ldots\right)\sin 6\phi + \ldots \end{bmatrix} \quad (2\text{-}19)$$

with *Φ* in radians. *M* is the true distance along the central meridian from the Equator to ***Φ*.** *N* is the radius of curvature of ellipsoid in the prime vertical. For computational efficiency the terms *A*, *C*, and *T* are used the series expansion.

To obtain UTM coordinates, the appropriate "false easting" is added to x and "false northing" added to y after calculation using (2-11) and (2-12). For the inverse formulas (Snyder, 1987):

$$\phi = \phi_1 - (N_1 \tan\phi_1 / R_1)\begin{bmatrix} D^2/2 - \left(5 + 3T_1 + 10C_1 - 4C_1^2 - 9e'^2\right)D^4/24 \\ + \left(61 + 90T_1 + 298C_1 + 45T_1^2 - 252e'^2 - 3C_1^2\right)D^6/720 \end{bmatrix} \tag{2-20}$$

$$\lambda = \lambda_0 + \begin{bmatrix} D - (1 + 2T_1 + C_1)D^3/6 + (5 - 2C_1 + 28T_1 - 3C_1^2 \\ + 8e'^2 + 24T_1^2)D^5/120 \end{bmatrix} / \cos\phi_1 \tag{2-21}$$

Where Φ_1 is the "foot point latitude" or the latitude at the central meridian which has the same y coordinate as that of the point (Φ, λ).

$$e'^2 = e^2 / \left(1 - e^2\right) \tag{2-14}$$

The polynomial coefficients and terms of the series expansion are defined as:

$$T_1 = \tan^2\phi_1 \tag{2-22}$$

$$C_1 = e^2 \cos^2\phi_1 \tag{2-23}$$

$$D = x/(N_1 k_0) \tag{2-24}$$

The radius of curvature in the meridian R_1 and the radius of curvature in the prime vertical N_1 are defined as:

$$R_1 = a\left(1 - e^2\right)/\left(1 - e^2\sin^2\phi_1\right)^{3/2} \tag{2-25}$$

$$N_1 = a/\left(1 - e^2\sin^2\phi_1\right)^{1/2} \tag{2-26}$$

To convert from tabular rectangular coordinates to Φ and λ, it is necessary to subtract any "false easting" from x and "false northing" from y before inserting x and y into the inverse formulas. To convert coordinates measured on an existing map, the correct central meridian must be used for the Y axis on the Transverse Mercator, but the X axis may cross it perpendicularly at any latitude chosen by the user.

2.6.2 Modified forms of Transverse Mercator

Most of the developed nations have over the past decades been developing country-wide plane coordinate reference systems, mostly by modification of the UTM projection parameters, namely: zone width, false easting and northing, scale factor and position of central meridian. Currently, these nations are reaping the benefits of having unified country-wide plane coordinate systems. Among the benefits are:

- All surveys correlate to a single reference framework. This means that all surveys, old and new, can be combined seamlessly to a consistent and contiguous mapping project.
- Data sharing among various users is simplified if everyone is working on the same reference system.
- Large projects can be surveyed in parallel as independent sections. As the work progresses, all sections can then be connected and the accuracy of the entire project would be maintained throughout.
- The control network and all details are positioned in a more homogeneous and accurate reference frame.

Among the countries that have been considered are Finland, Norway, Svalbard, New Zealand, Canada and Wisconsin State of the United States of America. Each of these countries went through a unique stage by stage process of establishing a country-wide or state plane coordinate reference system. Nonetheless, the general process involved: choice of reference ellipsoid and appropriate map projection, establishment and densification of the geodetic network in some cases like Finland, modification of standard projection parameters, testing and parameter adjustments to reduce map projection oriented distortions in distances, shape and areas.

There are a number of versions of the Transverse Mercator projection in existence, and plane coordinate grid systems used for surveying and mapping to suit specific accuracy requirements, as instituted by each country's mapping agency.

- **Canada**: In Canada, for provincial control surveys zones of 3-degrees in width are used instead of the standard 6-degree zone width of the UTM. The scale factor of the central meridian is set to 0.9999. This combination of narrower zone width and increased scale factor ensures higher accuracy of survey measurements. The resulting system is known as MTM, for “Modified Transverse Mercator”. Similarly, **South Africa** uses narrow zone widths of 2 degrees, central meridian scale factor = 1.000 X coordinate for “Southings”, Y coordinate for “Westings”. Zambian also uses 2 degrees narrow zone width, the *Longitude Zero* (L0), coordinate system for large scale cadastral surveying. An advantage of narrow zones is that ground-to-grid corrections are very small, and can often be omitted (Stuifbergen, 2009).

- **New Zealand:** The UTM in standard 6-degree zones, with a false Northing of 10 000 000 metres, for positive Northings in the southern hemisphere is used. However, the central meridian is shifted to 173 degrees east, in order to cover the whole country in a single zone.
- **Finland** uses a dual system, a ”Basic Grid” in 3-degree zones to benefit surveyors, and a ”Uniform grid” in a single 13 degrees wide zone to cover the entire country, for mapping purposes.
- In **Norway**, UTM zone 32V, in latitude band V (from 56 degrees N to 64 degrees N, central meridian at 9 degrees E) the zone is extended by 3 degrees farther west to longitude 3 degrees east, so that the south-western part of the country is covered within a single zone. The neighbouring zone 31V to the westward is accordingly trimmed by 3 degrees so that its coverage then is all over-water only. In the **Svalbard** (Spitsbergen) region, latitude band X (72 degrees N to 84 degrees N), the UTM zones are narrow due to meridian convergence towards the North Pole. In this case, only the odd-numbered UTM zones 31X, 33X, 35X and 37X are implemented. These zones are widened to cover the omitted zones 32X, 34X and 36X, thereby resulting in 12 degrees zone widths (Stuifbergen, 2009).
- In the **United States of America** a number of states use Transverse Mercator projection for state plane grids, in diverse zone-widths to provide their ”State Plane Coordinate System”, (DMA, 1990). The U.S. Army Map Service uses transverse Mercator for world-wide systematic coverage in UTM, (DMA, 1989).

The Transverse Mercator system is used officially in the United Kingdom, Ireland, Sweden, Norway, Finland, Germany, Poland, Russia, China, Bulgaria, former Yugoslavia, Portugal, Egypt, former British African colonies, Southern Africa, Australia and New Zealand, (Stuifbergen, 2009). All of these TM grids are modified forms of the

Gauss-Kruger projection, which could be regarded as the basic source algorithm from which to derive any one of the particular survey grids based on a Transverse Mercator projection.

2.6.2.1 Wisconsin Transverse Mercator

In the Universal Transverse Mercator coordinate system, Wisconsin state lies about equally in UTM zones 15 and 16 (figure 2.18). Federal and state agencies sometimes extend a zone so that surveying and mapping project works remain in one system. In these cases, the zone may be referred to as Zone 15 Extended, or Zone 15E.

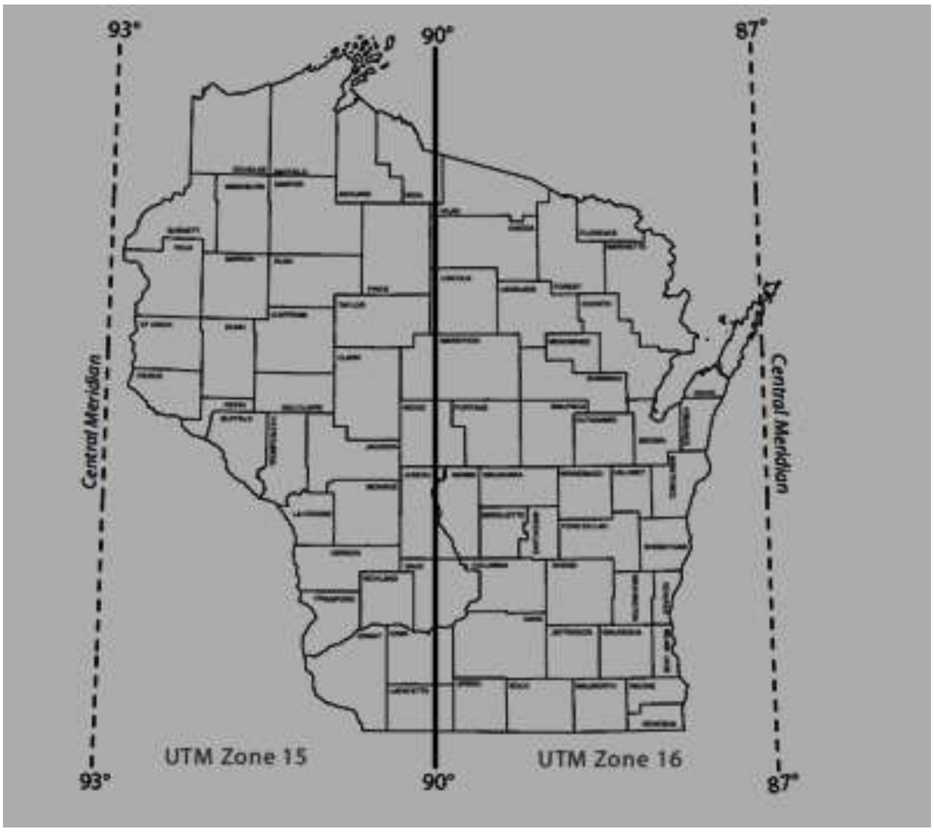


Figure 2.18: UTM zones 15 and 16 showing the difference in orientation based on each central meridians (Danielsen and Koch 2012).

Therefore, in the mid-1980s the Wisconsin Department of Natural Resources developed the Wisconsin Transverse Mercator (WTM) coordinate system to avoid having the state divided into two UTM zones. This system centres a UTM-like zone on the 90th meridian (west) thereby covering the state with one zone. The distortion in the projection increases from the centre toward the eastern and western extremes of the state. A false easting of 500,000 meters and a false northing of -4,500,000 meters are used to produce more convenient coordinate values for Wisconsin. WTM is an example of a coordinate system designed and created to satisfy a particular regional need and to avoid problems caused by use of two UTM zones (Danielsen and Koch, 2012).

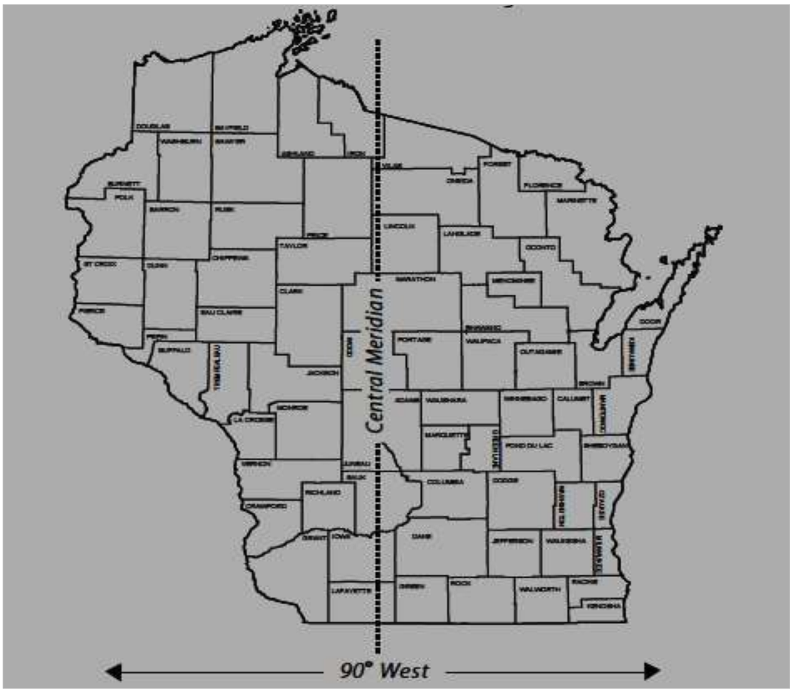


Figure 2.19: Wisconsin Transverse Mercator coordinate system (Danielsen and Koch 2012).

WTM coordinate system parameters

Projection:	Transverse Mercator	Scale factor:	0.9996
Central meridian:	90 degrees W	Latitude of Origin:	0 degrees
False Easting:	500,000 m	False Northing:	-4,500,000m
Unit:	metre		

2.6.2.2 The Finnish Coordinate Reference System

In Finland UTM projection is used with some modifications to fit the whole country in a single and wider projection zone, instead of the standard 6° zone. The coordinate system with wider projection zone is called the ETRS-TM35FIN for "European Terrestrial Reference System- Transverse Mercator 35 for Finland". ETRS-TM35FIN coordinates are based on the UTM projection and the GRS80 reference ellipsoid, (Ollikainen, 2004). As a deviation from the standard UTM, a wider projection zone of 13 degrees is used instead of the standard 6° wide zone. The central meridian is 27 degrees east while scale factor at the central meridian is maintained at 0.9996. The easting of the central meridian is set to 500,000 m as well as 8,500,000 m in special cases where the eastings would otherwise become negative.

Other modifications applied to standard UTM system are in the Basic Coordinate Frame. In this frame, Finland is divided into six projection zones, each 3° wide. The corresponding central meridians are 18°E (zone 0), 21°E (zone 1), 24°E (zone 2), 27°E (zone 3), 30°E (zone 4) and 33°E (zone 5). The grid of the basic coordinate frame is directed along the central meridian. The origin is the intersection of the equator and the central meridian. To avoid negative values for the x-coordinates the easting of the central meridian is set to 500,000 m. Furthermore, the y-coordinates of the different zones are distinguished by the ordinal number of the particular zone inserted before the actual coordinate value. Hence the false eastings of the central meridians are 18°E = 500,000 m, 21°E = 1,500,000 m, 24°E = 2,500,000 m, 27°E = 3,500,000 m, 30°E = 4,500,000 m and 33°E = 5,500,000 m. The basic coordinate frame is based on the International 1924 ellipsoid parameters, and scale factor at each central meridian is 1.0000 (Ollikainen, 2004).

The descriptions of various forms of the Transverse Mercator projection and grid system shows that the UTM system can be modified to suit the mapping purpose of a particular country or region. Narrower zones have higher accuracy and hence suitable for surveying and mapping at large scale, while wider zones have lower accuracy and are thus suitable for country-wide mapping at small scale.

2.7 Seamless Land Cover Mapping

Seamless land cover mapping is land cover mapping of a large piece of land in a continuous and integrated manner. A small piece of land such as a small town can be covered by one satellite image and hence mapped from boundary to boundary in one plane coordinate system with ease, seamlessly. However, a large piece of land covering, say the whole country, would require a suitable, tailored plane coordinate reference system and mosaicking of remotely sensed data to achieve seamless land-cover mapping.

Remotely sensed data come with features such as spatial resolution, which can help in determining the smallest possible feature that can be mapped. The knowledge of that smallest possible feature can in turn be used to customize the plane coordinate reference system in as far as scale errors are concerned. Therefore, land-cover maps derived from remotely sensed data are often presented using a Minimum Mapping Unit (MMU). MMU is defined as the smallest size areal entity to be mapped as a discrete entity. For example, a classification derived from Landsat Enhanced Thematic Mapper (ETM) data will typically have a nominal spatial resolution of 30 m (pixel size). A possible MMU for a classification created from ETM data may be 8100 m^2 or 90x90 m (3x3 pixels) (Knight and Lunetta, 2003).

Ideally, the smallest possible feature that could be mapped would be equal to one pixel. For a 30 meter image source this would be 30 meter by 30 meter area. For a one meter image this would be a 1 square meter area. But, it is generally agreed that the smallest observable feature that can reliably be identified would need to be four contiguous pixels in size i.e. 60 by 60 meters for a 30 meter image. This is because a 30x30m feature may not fall entirely within one given pixel but may instead be split among as many as 4 pixels, hence making up only a minority of any one of those pixels and not being the dominant feature reflected in any. The following section gives an account of how a group of European countries developed an integrated, seamless land-cover/ land-use database for various environmental and geographical uses within European Union member states.

2.7.1 The CORINE Land Cover

CORINE Land Cover (CLC) is a geographic land cover/land use database encompassing most of the countries of Europe. In the year 1985, the CORINE programme was initiated in the European Union. CORINE means 'Coordination of Information on the Environment' and it was a prototype project working on many different environmental issues. The CORINE databases and several of its programmes were organised by the European Environmental Agency (EEA). One of the programmes is an inventory of land cover in 44 classes organised hierarchically in three levels, and presented as a cartographic product, at a scale of 1:100 000. The first level (5 classes) corresponds to the main categories of the land cover/land use (artificial areas, agricultural land, forests and semi-natural areas, wetlands, water surfaces). The second level (15 classes) covers physical and physiognomic entities at a higher level of detail (urban zones, forests, lakes, etc.), finally level 3 is composed of all 44 classes (Corine Landcover, 2000).

CLC mapping was carried out on the basis of visual interpretation of satellite images (SPOT, LANDSAT TM and MSS). The following description of the Corine Landcover project is based on the Corine Landcover (2000) report on Methodology and Nomenclature. Ancillary data such as aerial photographs, topographic or vegetation maps, statistics and local knowledge, were used to refine interpretation. One of the major tasks undertaken in the framework of the CORINE programme was the establishment of a computerised inventory on the land cover. Data on land cover is necessary for the environment policy as well as for other policies such as regional development and agriculture. At the same time it provides one of the basic inputs for the production of more complex information on other themes such as soil erosion, pollutant emission into the air by the vegetation, etc. Therefore, the objectives of the land cover project were: to provide those responsible for and interested in the European policy on the environment with quantitative data on land cover which is consistent and comparable across Europe.

About 2.3 million square kilometres of area was covered in 12 countries at a working scale of 1:100, 000, and minimum mapping unit of 25 hectares. The process of land-cover mapping involved choice of the mapping scale, definition of the unit area and the minimum mapping unit.

2.7.1.1 Choice of the mapping scale

The working scale of 1:100 000 was chosen for the following reasons:

- Land cover data provided on smaller scales (1:250 000, 1:500 000) were not detailed enough to be useful to the Commission. At such scales, the size of the smallest unit mapped is very large. The chosen scale was well suited to serve as a basis for specific studies at larger scale within a country, such as preliminary investigations for civil development projects or environmental protection.
- It was also compatible with projects which use a smaller scale (e.g. 1: 1 000 000), since space-borne remote sensing allows generalisation at that basic cartographic scale.
- The scale was consistent with budgetary constraints and time limits for carrying out such a programme in the 12 Community countries; maps could be fairly easily updated on a regular basis; and this was a common mapping scale for most Community countries.

2.7.1.2 Definition of the unit area and size of the smallest unit mapped

The unit size area has the following main characteristics:

- It corresponds either to an area whose cover may be considered homogeneous (grass, water, forest, etc.) or to a combination of elementary areas (homogeneous as defined above) which, in its variations, represents characteristic land cover structures (covering large surfaces which can be considered to constitute a single type of land cover.
- Given the scale, the unit must represent a significant area of land, it should be clearly distinguishable from surrounding units, and its structure in terms of land cover should be stable enough to serve as a unit for the collection of more precise information.

Given these circumstances a spatial/statistical unit must be conceived, for all land cover mapping activities that meets the following two requirements: (a) its content must provide the thematic data required by the users, and (b) it must provide an acceptable representation of reality.

2.7.1.3 Size of the minimum mapping unit (MMU)

The surface area of the smallest unit mapped in the project was set at 25 hectares. Ideally, establishment of the minimum surface area to be mapped must comply with three basic requirements:

- Legibility of printed map, or in the case of the land cover project, easy digitization from the image interpretation manuscripts;
- The MMU must provide a representation of the essential features of the terrain in a way that serve the thematic objectives of the project;
- It must represent a trade-off between project operating costs and provision of land cover information requirements within overall project budgetary constraints.

On a scale of 1: 100 000, 25 hectares (500m x 500m) is represented by a 5 x 5 mm square or a circle with a 2.8 mm radius.

For map products that are not based solely on pixels, the Minimum Mapping Unit should be clearly defined and included as part of the metadata. For pixel based products, one can determine the appropriate resolution necessary for their applications based upon the smallest feature to be resolved. The pixel size must be half the smallest dimension of the feature in question. For instance if a house (of size ~13m x 8 m), is to be reliably identified on a map then the smallest dimension of that house is 8 m, so the pixel size must be no larger than 4 x 4 m (Herold, 2011).

2.7.1.4 Coordinate Reference System for CORINE Project

The CORINE project used a single coordinate reference system for all Europe called Lambert Azimuthal Equal Area for Europe also known as European Terrestrial Reference System 89 (ETRS89, European Petroleum Survey Group (EPSG) code:3035). This coordinate reference system is based on Lambert Azimuthal Equal Area projection and is used for statistical mapping at all scales and other purposes where true area representation is required (Corine Landcover, 2000).

2.8 Computer Programming for Map Projections

The complex mathematics inherent in map projections can best be handled using computer programs and algorithms. A computer programming language is a set of keywords and syntax rules that are used to tell the computer what the user want it to do. A list of instructions written in a programming language is called a program and is most often created using text editors or the text editor component of an Integrated Development Environment (IDE). Ultimately, these instructions must be translated from the programming language into the native language of the computer (assembly instructions). The translation is performed by specialised programs known as compilers. In order to use a programming language a compiler for a particular language must be installed on the computer.

The choice of programming language is driven in part by the nature of the project. C++ can be described as a middle-level, computer-programming language. It is highly portable in that there are compilers for almost all types of computers. C++ is also efficient and popular, allowing the development of powerful and efficient solutions to complex problems. A better way of managing large projects is to use Visual Studio. Once Visual Studio has been started, it is used to write, compile and run programs. One major advantage of Visual Studio is that it automatically includes a number of libraries that are used to help your program interact with the Windows operating system. A library is a collection of functions that have already been written and compiled (Hazela, n.d.).

2.8.1 Microsoft Visual C++

For simple C++ programs, the project should be a Visual C++ Console Application, also called a Win32 Console Application, located in the Win32 submenu of the Visual C++ project templates.

Microsoft Visual Studio is an integrated development environment from Microsoft Developer Network (2015). It can be used to develop console and graphical user interface application along with Windows Form applications, web sites, web applications, and web services in both native code together with managed code for all platforms supported by Microsoft Windows, Windows Phone, Windows CE, .NET Framework, .NET Compact Framework and Silverlight (Microsoft Developer Network, 2015).

2.8.1.1 Win 32 Console Application in C++ Visual studios

A console application accepts input and sends output to the console, which is also known as the command prompt. Console apps can be created to do basic work or to perform very sophisticated tasks. A console app can also be used as a proof-of-concept demonstration of functionality that one can later want to incorporate into a Windows app. Console apps can communicate with other desktop apps by means of pipes or other Remote Procedure Call (RPC) mechanisms (Microsoft Developer Network, 2015).

CHAPTER 3 RESEARCH METHODOLOGY

The study was undertaken to establish a plane coordinate system which would enable seamless mapping from one end of the country (Zambia) to the other, east to west and north to south, in a single grid reference system. In establishing a unified country-wide plane coordinate system, the current plane coordinate system for mapping at national level in Zambia, the standard UTM coordinate system was modified. To modify UTM system to the proposed ZTM system, the UTM parameters were modified. The parameters modified included the zone width, position of the central meridian, scale factor at the central meridian, and the false easting value. This chapter includes a review of the research method and design, a discussion of data collected, and problem analysis.

3.1 Study Area

The research was restricted to the following rectangular geographic space depicted in Figure 3.1, i.e. top left corner (8^0S, 22^0 E) and bottom right corner (18^0S, 34^0 E). The study area was the whole country Zambia covering 10 degrees of latitude by 12 degrees of longitude, approximately 752, 614 square kilometres.

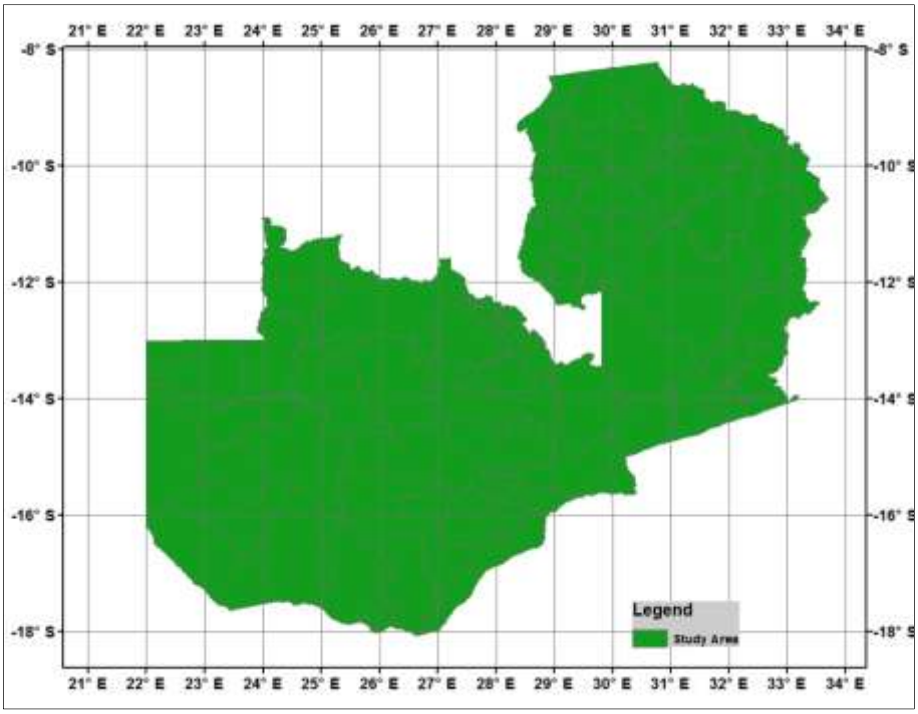


Figure 3.1: Study area covered the whole country Zambia

3.2 Research Approach and Design

The study was designed in such a way that the preliminary process involved reviewing relevant literature. Different map projections were studied, and eventually UTM projection was chosen to be modified. The choice of a map projection to modify was influenced by taking advantage of what is already in use in Zambia, and on the basis of extensive literature review on UTM. The ZTM coordinate system design and analysis was carried out with the help of C++ programming language in Microsoft Visual C++ (2010 Express version) integrated development environment, and Microsoft Excel 2013. Two separate computer programs were written for ZTM coordinate system design and analysis. The first program was written to help define the scale factor at the central meridian (28^{o} E) in the proposed 12^{o} wide zone Transverse Mercator projection, thereby defining the Zambian Transverse Mercator (ZTM) coordinate system. The second program converted geodetic coordinates to ZTM coordinates, and from ZTM coordinates back to geodetic coordinate system.

Preliminary results included modified UTM projection parameters, central meridian scale factor value, and predetermined values for zone width and false easting. An iterative approach to determining scale factor at central meridian was employed in such a manner as to assess the suitable parameters for reduced scale errors and distortions. After defining the range of acceptable scale errors based on standard UTM distortion pattern, the ZTM plane coordinate system was thus established. The new parameters were tested in ArcMap (ArcGIS Desktop 10.2) mapping and visualisation software. Then sample thematic mapping of features that are country-wide in extent, including land cover mapping of SASSCAL Task 151 was carried out in ArcMap. The ZTM projection file was then saved. Logical steps that were followed during the study are as shown in the process flow chart below (figure 3.2).

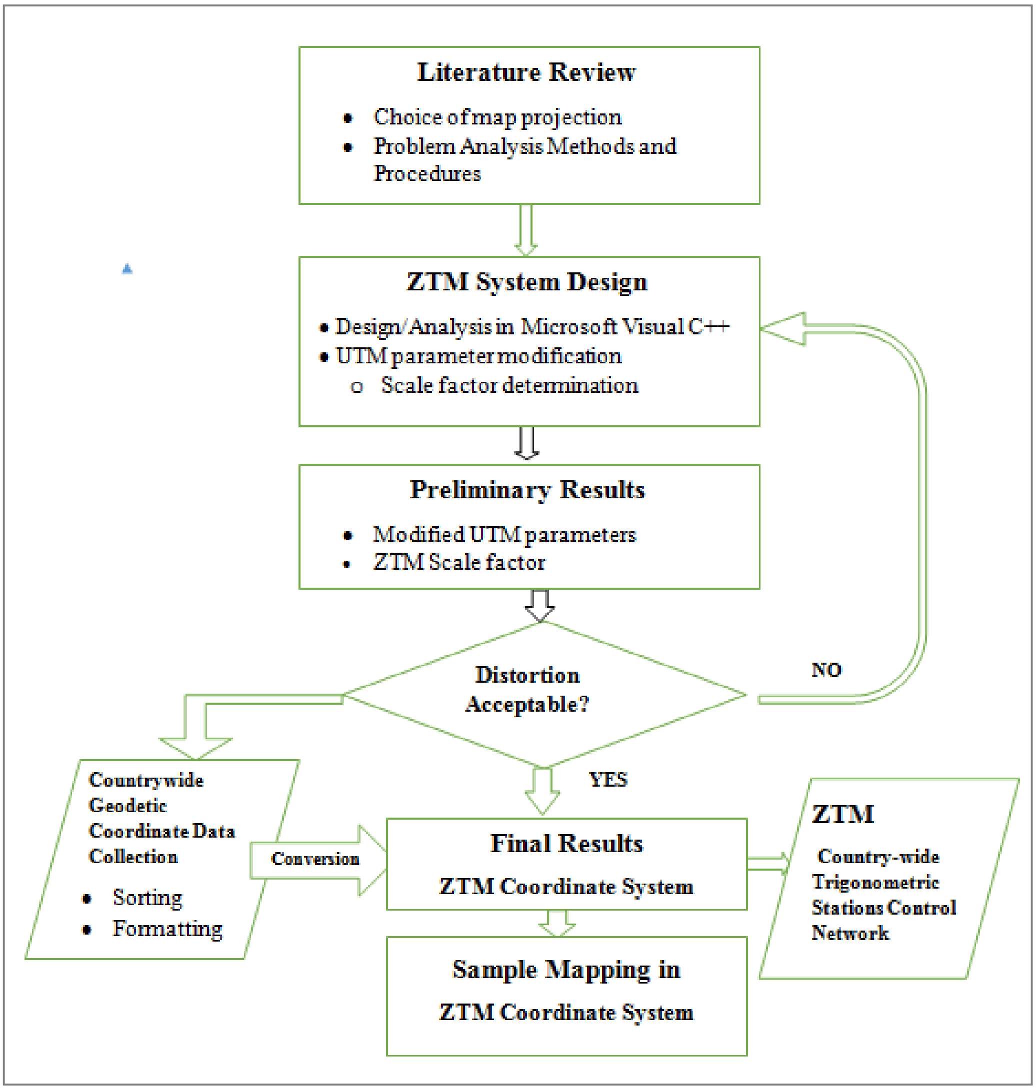


Figure 3.2: ZTM coordinate system establishment process flow chart

From the chart (figure 3.2), the Zambian country-wide geodetic control network's coordinate data was collected, sorted and formatted, and used as input in a C++ program for conversion from geodetic coordinate system to ZTM coordinate system. The computational problems of establishing a new coordinate system were particularly carried out in C++ programming language because C++ has predefined mathematical functions in its library making it suitable for analysing map projection problems which are mathematical by nature. Other programming software products such as Mapping Toolbox (MathWorks, 2015) which provides algorithms, functions, and an app for analysing geographic data and creating map displays in MATLAB were considered. However, learning C++ was easier due to prior knowledge of the language, availability of online tutorial resources and free copies (2010 Express version) of software from Microsoft Development Network (2015).

3.3 Geodetic Coordinate Data Collection

This section describes the data collected and data collection procedure used in the study. The classical geodetic coordinates of the trigonometric stations were collected from the University of Zambia's School of Engineering, Department of Geomatic Engineering in Excel file format.

3.3.1 The Zambian Classical Geodetic Network

The Zambian geodetic framework comprises triangulation stations of primary (ZP), secondary (ZS), tertiary (ZT), and quaternary (ZQ) order. The triangulation net was initially along the arc of the 30th meridian and formed part of the arc measurement for the definition of the Arc 1950 and Arc 1960 datums. Later, surveys in Zambia were tied to this 30th arc measurements using triangulation and traversing techniques. The traverse stations were often designated: TP, TS or TT for primary, secondary and tertiary traverse stations, respectively.

The geodetic coordinates collected were the ones obtained from the 2006 joint survey of trigonometric stations conducted by the Ministry of Mines and Mineral Development (MMMD), and Zambia Survey Department. The ministry (MMMD) had been executing the Gemstone Sector Development (GSD) component of the Support to Economic Expansion and Diversification (SEED) Programme. This program was financed under a credit agreement signed between the Government of the Republic of Zambia (GRZ) and the World Bank.

Under the GSD component, there was a sub-component of establishing a modern Mining Cadastre System. The undertaking of the modern Mining Cadastre System entailed a revision of the national geodetic framework. Thus the Zambia Survey Department was requested to carry out a GPS campaign comprising 41 primary trigonometric stations. This work, jointly administered by the Mines Development Department and Zambia Survey Department, was finally executed between April and June 2006 (Nsombo, 2012).

3.3.2 Data description

Coordinate data was collected in Microsoft Excel file format. The data file comprised of five (5) different spread sheets each with a different coordinate system. Figure 3.3 shows Arc 1960 Geodetic coordinates based on Clarke 1880 modified ellipsoid, in Microsoft Excel sheet number 4.

ZAMBIAN CLASSICAL GEODETIC NETWORK

Datum ARC60 Clarke 1880 mod

GEODETIC COORDINATES

MINING CADASTRE CODE	DEPT.SURVEY CODE	LATITUDE ° ' "	LONGITUDE ° ' "	ORTHO-HEIGHT m
MC01	ZP27	17° 39' 44.81800" S	26° 43' 07.82900" E	1350.566
MC02	TP32	14° 08' 04.09000" S	23° 53' 13.14000" E	1096.360
MC03	ZP268	14° 44' 06.74400" S	26° 54' 04.40600" E	1512.260
MC04	ZP234	15° 43' 17.85000" S	26° 02' 52.35000" E	1152.450
MC05	ZP44	16° 31' 20.39100" S	27° 37' 48.18600" E	1343.617
MC06	ZP46	15° 53' 26.44500" S	27° 46' 42.88400" E	1155.708
MC07	ZP66	13° 52' 47.90400" S	28° 40' 55.74000" E	1429.558
MC08	ZP22	14° 27' 47.73500" S	30° 46' 52.69800" E	936.649
MC09	ZP2	15° 32' 13.12000" S	29° 06' 21.84443" E	1411.779
MC10	TS236	15° 37' 31.01000" S	30° 24' 25.95000" E	375.600
MC11	MP41	12° 12' 25.72800" S	33° 10' 04.74900" E	1208.100
MC12	TS208	12° 32' 36.15000" S	30° 59' 36.59000" E	1555.400
MC13	ZP76	13° 20' 32.49600" S	29° 36' 36.36800" E	1895.990
MC14	TP55	17° 48' 21.36000" S	25° 44' 01.41000" E	962.860
MC15	TT51	8° 39' 01.77080" S	29° 10' 21.99332" E	1164.992
MC16	ZP141	10° 12' 57.23000" S	31° 14' 17.39000" E	1457.920
MC17	ZS273	10° 07' 18.02100" S	33° 22' 16.18500" E	1833.670
MC18	ZS44	10° 59' 47.70400" S	33° 16' 50.68000" E	1574.720
MC19	ZP150	9° 55' 31.96200" S	30° 20' 13.42000" E	1659.440
MC20	ZP138E_WITMK	n/a	n/a	n/a
MC21	ZP134	9° 02' 43.55438" S	31° 30' 30.23499" E	2067.150
MC22	ZP338	9° 33' 11.73600" S	32° 40' 42.22800" E	1558.130
MC23	TS612	9° 37' 51.44660" S	28° 44' 46.30931" E	952.600
MC24	ZP23	14° 08' 12.78700" S	32° 32' 55.32200" E	1314.600
MC25	ZP90	13° 12' 23.36900" S	27° 50' 21.77600" E	1296.251
MC26	ZP164	10° 28' 17.00100" S	29° 09' 13.14700" E	1368.850

ITRF2000 GEODETIC | ITRF2000 UTM | ITRF2000 GEOCENTRIC | ARC60 GEODETIC | ARC60 UTM

Figure 3.3: Control point data collected, in various coordinate systems.

Table: 3.1 shows the state of coordinate data in terms of the number of control points, coordinate system and reference ellipsoid. The source data had additional information on the accuracy of the measurements of country-wide control points.

Table 3.1: Summary of status of collected coordinate data

Spread sheet	No. of Control points	Coordinate System	Remarks
1	41	Geodetic (φ, λ, h)	Based on ITRF 2000 epoch 2006, GRS 80 ellipsoid.
2	41	UTM 27	Based on conversion from geodetic coordinates in sheet 1
3	41	XYZ Cartesian	Based on conversion from geodetic coordinates in sheet 1
4	50	Geodetic (φ, λ, H)	Based on Arc 1960 datum and Clarke 1880 ellipsoid
5	50	UTM (21,27 & 33)	Based on conversion from geodetic coordinates of sheet 4

The coordinate data that was selected to be used in data analysis was the sheet 4 control points based on the arc 1960 datum and Clarke 1880 ellipsoid. These points were selected based on the above reference ellipsoid and geodetic datum.

3.4 ZTM System Design and Analysis

The process of designing the ZTM coordinate system involved modification of UTM coordinate system parameters, determination of the minimum mapping unit, and continuous analysis of subsequent ZTM designs. It was during the designing process that the Zambian Transverse Mercator projection and grid was established. The following subsections outline the entire design and analysis process.

3.4.1 Establishment of the Zambian Transverse Mercator projection

In determining a suitable scale factor for the ZTM projection at the central meridian and consequently at zone boundaries, the purpose and technical requirements of the SASSCAL Task 151 mapping project was considered in detail. The purpose of SASSCAL Task 151 is to develop and improve integrated national and regional seamless land use assessment. Technical requirements included defining the minimum mapping unit (MMU), working scale, and display format: printed paper size (size A0 as maximum) or dynamic display scale computer screen, and maximum display scale (1:1,500 000 for A0 paper size). The process of determining scale factor for ZTM projection involved two (2) steps. The first step involved determining the minimum mapping unit, working scale and associated properties for displaying a legible map product such as display formats, print paper size and maximum display scale, for the intended mapping project. The second step aimed at determining scale factor at central meridian based on size of minimum mapping unit.

3.4.1.1 Determining the Minimum Mapping Unit

Land-Cover maps derived from remotely sensed data are often presented using a minimum mapping unit (MMU). An MMU, as described in Chapter 2, is "the smallest size areal entity to be mapped as a discrete entity". The predominant satellite image data to be used in Task 151 is Landsat 8, Landsat ETM, and TM. These datasets were chosen because they are readily available from United States Geological Survey (USGS) and the National Remote Sensing Centre (NRSC), in Lusaka, Zambia. In addition, the spatial and temporal resolutions of Landsat datasets are fair enough for country-wide thematic mapping. Going by the method of determining a minimum mapping unit from literature review (see section 2.6.1) the MMU was determined as 300m x 300m (9 hectare).

Using the CORINE example, the MMU for Zambian country-wide mapping was determined by considering the proportion of Total Project Area of CORINE to Total Project Area of Zambia:

Table 3.2: Determining the minimum mapping unit

Item	**EU's CORINE project**	**Zambia (SASSCAL Task 151)**
Number of Countries	12	1
Total Project Area	2.3 million sq. km	752,614 sq. km
MMU	25 hectares	**8.1806** hectares

From Table 3.2, the MMU value of 8.1806 hectares was computed by areal proportionality of the two projects. However, by considering other factors such as legibility of maps to be printed, ease of digitisation and extraction of information, the MMU value was rounded up to the next whole number and fixed at **9 hectares** (300x300m or 10 x 10 pixels of Landsat imagery at 30m spatial resolution). The 9 hectare MMU is sufficient even on the basis of 3x3 pixel requirement for the basic mapping unit for Landsat images. On a scale of 1: 100 000, MMU of 25 hectares (CORINE project) is represented by a 5 x 5 mm square or a circle with a 2.8 mm radius.

Similarly, an MMU of 9 hectares for countrywide seamless landcover mapping would be represented by a 3 x 3mm square or a circle with a radius of 1.7mm on a 1:100 000 scale. ***Therefore, the minimum ground distance to be correctly represented on a map in the proposed ZTM coordinate system would be 300 metres.*** In addition, a smallest possible map display scale of 1: 1,500 000 for seamless mapping on maximum paper size of A0 (841 x 1149 mm) would be used for final mapping, since the whole country is optimally covered at this scale. However, the whole seamless landcover map of Zambia or its regions can be mapped at various user defined display scales, according to output paper size and level of map details required.

3.4.1.2 Determining Suitable Scale factor at central meridian

Bearing in mind the minimum mapping unit defined above, four approaches to the problem of determining the suitable scale factor were devised. The first approach involved determining the scale factor by assuming scale factor at zone boundaries for both UTM and proposed ZTM to be equal. In the second approach, scale factor was determined by analyzing which position of secant lines (standard lines) resulted in minimum scale errors. The third approach established a mathematical relationship between UTM scale factor and longitudes within a zone, and then applied the relationship in determining ZTM scale factor. The fourth approach was a hybrid of second and third approaches, with consideration of the minimum mapping unit.

A computer program to compute and determine scale factors for both UTM and ZTM system was written in C++ programming language. The program was written in such a way that it had three (3) *functions* or sub-programs each carrying out a specific task. The first sub-program computed scale factor of any point in standard UTM projection system given latitude and longitude of that point. The second and third sub-programs were together used to determine ZTM scale factor at the central meridian and subsequently at any other location within the 12-degrees wide zone boundaries. The program source code is in appendix A, this section will merely highlight the input/output command prompt or console which pops up after debugging the code.

The formula used for computing scale factor in a C++ program was adapted from equation (2-13):

$$k = k_0\left[1 + (1 + C)A^2 / 2\right] \qquad (3\text{-}1)$$

Where k is the scale factor at any point

k_0 is the scale factor at central meridian

$$C = e^2 \cos^2 \phi \tag{2-15}$$

$$A = (\lambda - \lambda_0) \cdot COS\phi \text{, with } \lambda \text{ and } \lambda_0 \text{ in radians} \tag{2-16}$$

a. *Determining scale factor by assuming scale factor at zone boundaries for both UTM and proposed ZTM to be equal*

Using the first sub-program of the computer program, the scale factor at zone boundary of the standard UTM projection was computed as **1.0009**. The C++ console window is as shown in figure 3.4 below. The program prompted the user to choose the sub-program from the menu. In this case sub-program 1 was selected by entering '1' from keyboard.

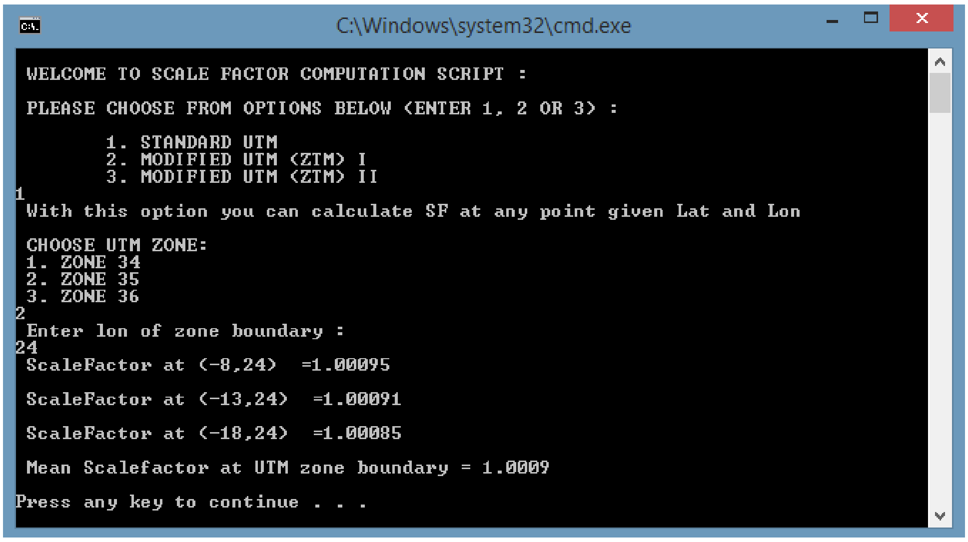


Figure 3.4: The C++ command prompt window for input/output showing scale factor computation results at UTM zone 35 boundary.

Again the user was asked to specify UTM zone. Choosing zone 35 by keyboard entry '2', the user was asked to enter longitude value of zone boundary (either 24oE or 30oE for UTM27). Entering 24oE, the average scale factor was calculated from three different latitude positions: 8oS at the extreme north-end, 13oS at the middle, and 18oS at extreme south-end of study area.

Scale factor was computed at three different latitudes because the value of scale factor depends on the latitude in UTM projection. The value of scale factor tends to reduce towards the poles due to curvature and reduced arc distance from central meridian. At latitude 8^{O}S scale factor is 1.00095 while at 18^{O}S scale factor is 1.00085 for the same longitude 24^{O}E. The mean value of scale factor (1.0009) at UTM zone boundary (24^{o}E) was used to calculate the scale factor at the central meridian for the ZTM projection. This was achieved by changing the subject of the formula in equation (3-1) to k_0:

$$k_0 = k / \left[1 + (1 + C) A^2 / 2\right] \tag{3-2}$$

where k=1.0009.

Sub-program 2 had modified UTM parameters hence the name "Modified UTM (ZTM) part I" in the output display. Central meridian was set to the mid longitude of study area, 28^{O}E; and the study area was covered in a single zone of 12^{O} width from 22^{O}E to 34^{O}E. Similar to the UTM sub-program, in "Modified UTM (ZTM) part I" the user was requested to enter the longitude of zone boundary (22^{O}E or 34^{O}E). Using formula (3-2) above, the scale factor of central meridian (28^{O}E) was computed on three latitude positions (8^{O}S, 13^{O}S and 18^{O}S). The mean scale factor was found to be **0.995708.** Figure 3.5 shows the C++ command prompt or console with the input/output steps described above.

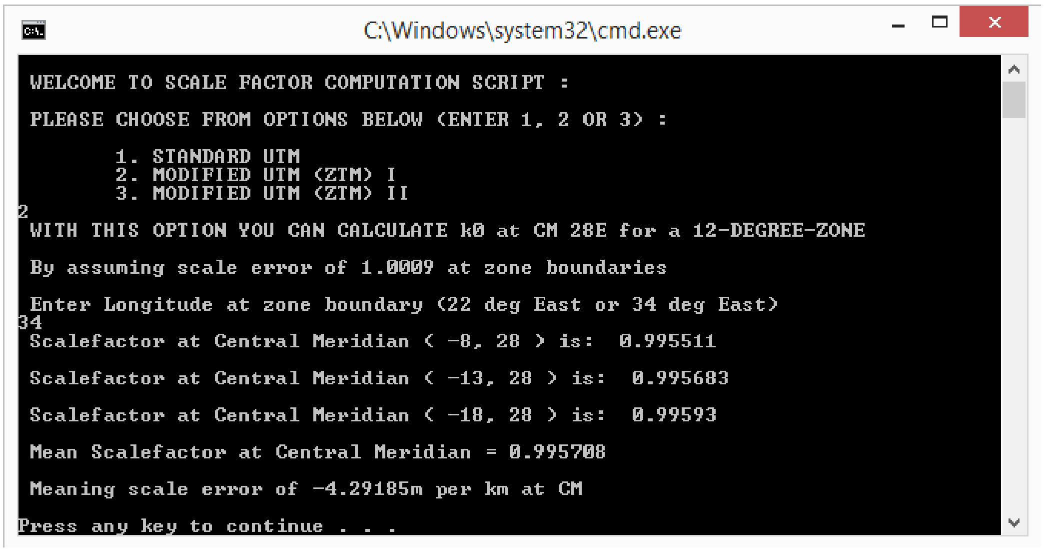


Figure 3.5: The C++ command prompt window for input/output showing scale factor computation results for central meridian.

b. ***Determining scale factor by analyzing positions of secant lines (standard lines) which resulted in minimum scale errors***

The aim of sub-program 3 was to determine the scale factor of central meridian 28^{O}E and compare the resulting value to the scale factor at the zone boundary 22^{O}E or 34^{O}E. Theoretically, the introduction of standard lines (lines of zero distortion, where k = 1) in the Transverse Mercator has the effect of balancing or smoothening the scale errors across the projected area. In the UTM system the standard lines are approximately 1.66^{o} east and west of the central meridian. This is a little over half way between central meridian and meridian of zone boundary.

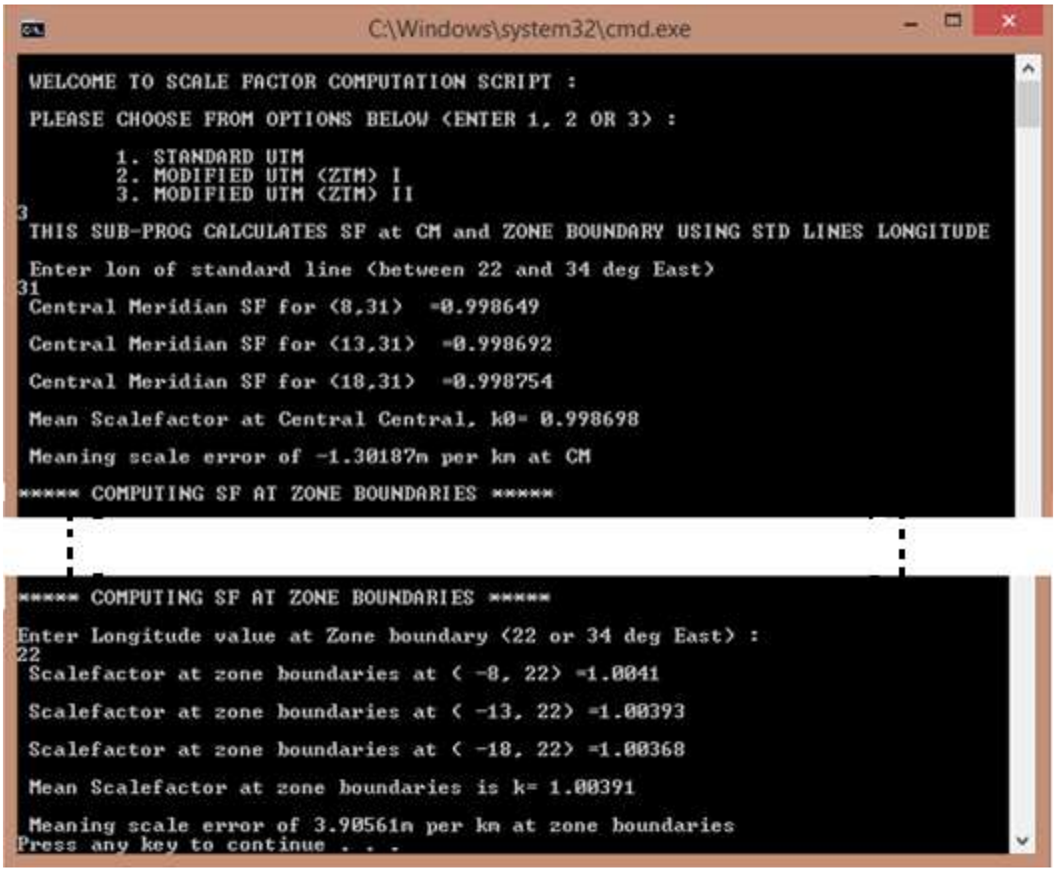


Figure 3.6: The C++ command prompt window for input/output showing scale factor computation results at central meridian using standard line 31^{o}E. The resulting scale factor is used in computing scale factor at ZTM zone boundary, 22^{o}E. Dotted lines indicates continuation of program.

Accordingly, the C++ program iteratively computed central meridian scale factor, $k_{0,}$ for twenty nine (29) proposed positions of standard lines from longitude 28^{O}E to 33^{O}E at an interval of 0.25^{O}. At 0.25^{O} interval a wide range of longitudes were assessed for possible positions of ZTM standard lines. In addition, from longitude 31^{o}E to 31.35^{o}E a finer interval of 0.1^{o} was used while a finest interval of 0.01^{o} was used between 31.30^{o}E and 31.35^{o}E. These additional intervals were introduced after noticing interesting relationships in scale errors between central meridian and zone boundary as analysed in the following illustrations.

By using equation (3-2) and setting k=1.00 for scale factor at standard line, the central meridian scale factor, k_0, was computed. The computed k_0 was then passed to second part of program and used to compute scale factor, $k_{boudary}$, using equation (3-3) as shown in figure 3.6 above.

Table 3.3: Comparison of scale factor at central meridian and corresponding scale factor at zone boundary for different positions of standard lines

No.	**Longitude of standard line**	**Scale factor**		**Scale Errors (metres)**		**ΔE (metres)**	**$E_0/E_{boundary}$ (%)**
		k_0	$k_{boundary}$	E_0	$E_{boundary}$		
1	28.00	1.00000	1.00521	0.000	5.200	5.20	0.00
2	28.25	0.99999	1.00521	0.000	5.200	5.20	0.00
3	28.50	0.99996	1.00518	-0.030	5.170	5.14	0.58
4	28.75	0.99992	1.00513	-0.080	5.130	5.05	1.56
5	29.00	0.99986	1.00507	-0.140	5.060	4.92	2.77
6	29.25	0.99977	1.00499	-0.220	4.990	4.77	4.41
7	29.50	0.99967	1.00489	-0.320	4.890	4.57	6.54
8	29.75	0.99956	1.00477	-0.440	4.760	4.32	9.24
9	30.00	0.99942	1.00463	-0.570	4.620	4.05	12.34
10	30.25	0.99927	1.00448	-0.730	4.480	3.75	16.29
11	30.50	0.99910	1.00431	-0.900	4.310	3.41	20.88
12	30.75	0.99891	1.00411	-1.090	4.110	3.02	26.52
13	31.00	0.99870	1.00391	-1.300	3.910	2.61	33.25
14	31.10	0.99861	1.00382	-1.380	3.810	2.43	36.22
15	31.20	0.99852	1.00373	-1.480	3.730	2.25	39.68
16	31.25	0.99847	1.00368	-1.520	3.670	2.15	41.42
17	31.30	0.99843	1.00363	-1.570	3.630	2.06	43.25
18	31.31	0.99842	1.00362	-1.580	3.620	2.04	43.65
19	31.32	0.99841	1.00361	-1.590	3.600	2.01	44.17
20	31.33	0.99840	1.00360	-1.600	3.600	2.00	44.44
21	31.34	0.99839	1.00359	-1.610	3.590	1.98	44.85
22	31.35	0.99838	1.00358	-1.620	3.570	1.95	45.38
23	31.50	0.99823	1.00343	-1.770	3.430	1.66	51.60
24	31.75	0.99797	1.00317	-2.030	3.160	1.13	64.24
25	32.00	0.99769	1.00289	-2.310	2.890	0.58	79.93
26	32.25	0.99739	1.00259	-2.600	2.590	-0.01	100.39
27	32.50	0.99708	1.00227	-2.920	2.260	-0.66	129.20
28	32.75	0.99674	1.00194	-3.250	1.940	-1.31	167.53
29	33.00	0.99639	1.00159	-3.60	1.59	-2.01	226.42

The values of scale factor were compiled (Table 3.3) and analyzed further. The highlighted values of scale factors and longitudes of standard lines in blue indicated possibility of suitable scale factor values and standard lines for adoption. This is because the differences (ΔE) in scale errors are moderate in this range (31.0^{O}E to 31.35^{O}E).

$$k_{boundary} = k_0 \left[1 + (1 + C) A^2 / 2\right] \quad (3\text{-}3)$$

Computation of scale errors at central meridian and zone boundary (Knippers, 1993):

$$\boldsymbol{E}_0 = 1000 \text{ x } \boldsymbol{k}_0 - 1000 \quad (3\text{-}4a)$$

$$\boldsymbol{E}_{\text{boundary}} = 1000 \text{ x } \boldsymbol{k}_{\text{boundary}} - 1000 \quad (3\text{-}4b)$$

And the difference in magnitude of scale errors:

$$\Delta\boldsymbol{E} = \boldsymbol{E}_{\text{boundary}} - |\boldsymbol{E}_0| \quad (3\text{-}5)$$

Where

$\boldsymbol{E}_0$ is the scale error at central meridian in metres

$\boldsymbol{E}_{\text{boundary}}$ is the scale error at zone boundary in metres

Scale error $\boldsymbol{E}_0$ along the central meridian can be explained as the amount of reduction in measured distance of 1000 metres (1km) when projected in ZTM. Similarly, ground distance of 1000 metres along ZTM zone boundary would increase by an amount $\boldsymbol{E}_{\text{boundary}}$ when projected in ZTM. For example, from Table 3.3 row number 13:

For scale factor, $\boldsymbol{k}_0$= 0.998698 a 1000m ground distance would measure 998.7m along central meridian on the map, giving a reduction of 1.30m (i.e. 1000 – 998.7). While along zone boundary the same ground distance of 1000m would measure 1003.91m on the map, resulting in an increase of 3.91m.

On further analysis, the percentage of scale error along UTM central meridian of the scale error along UTM zone boundary was found to be 44.44%. For example, zone 35 south (central meridian 27°E and zone boundary 30°E or 24°E):

$$\boldsymbol{E_0/E_{boundary}} = \frac{\textit{scale error at central meridian } (27^{o}E)}{\textit{scale error at zone boundary } (30^{o}E \text{ or } 24^{o}E)}$$

$$= (1-0.9996) / (1.0009-1)$$

$$= (0.0004/0.0009)\text{x}100\% \quad = 44.44\%$$

The last column of Table 3.3 presents equivalent percentages for the ZTM k_0-$k_{boundary}$ relationships. The values highlighted in green show a closer correlation with the standard UTM scenario. The position of standard lines in UTM is 1.66° from the central meridian on either side east and west. This scenario can be compared to one of the proposed ZTM standard lines at 31.33° and 24.67° E, Where there is 3.33° from the central meridian (28°E) on either sides east and west. Furthermore, the scale error percentage ($\boldsymbol{E_0/E_{boundary}}$) is 44.44% for scale factors (k_0 and $k_{boundary}$) determined by assuming ZTM standard line at 31.33° E and 24.67° E (same as 44.44% of UTM).

c. *Using a mathematical relationship between UTM scale factor and longitudes within a zone*

In this approach the aim was to determine a mathematical relationship between the scale factor at the central meridian and scale factor at any other meridian within the 6-degree zone of the standard UTM projection system. This relationship was proposed to be maintained for the 12-degree zone of ZTM system in spite of different values of scale factors.

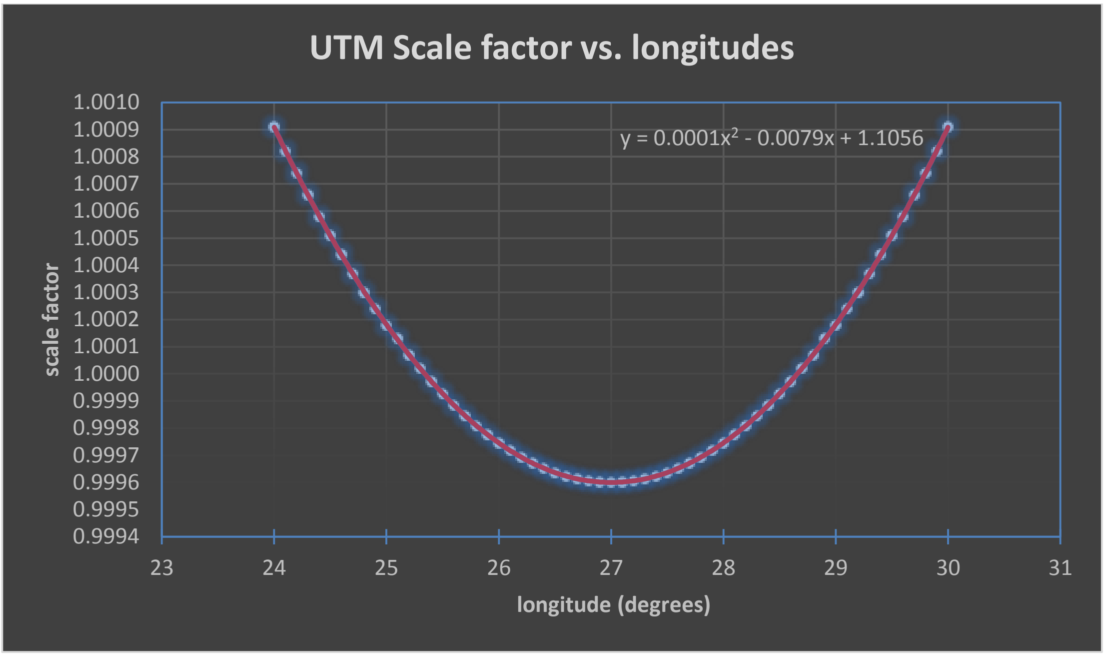


Figure 3.7: Relationship between central meridian scale factor and meridians of longitude for UTM zone 35S

Figure 3.7 shows a plot of scale factor values against meridians of longitude in UTM zone 35 south. The y-axis values for scale factor were computed from the first sub-program, and are plotted against corresponding longitude values in the x-axis. The latitude was kept constant at 13^{o} S, this value being the central latitude for study area and hence gave average results for scale factor. A total of sixty (60) scale factor values were computed at an interval of 0.1^{o} E of longitude, including at the central meridian 27^{o}E. At 0.1 degrees longitude interval, a good number (60) of points were considered resulting in a smooth curve.

The output data from C++ was copied, pasted and analyzed using Chart Tools in Microsoft Excel 2013. The chart tools in Excel 2013 have functionalities for chart elements such as Chart Title, Gridlines, Error Bars, Legends and Trendlines. With Trendline option one can extract the trend or tendency of plotted data by specifying the type of trend required from the list such as linear trend, power, exponential, polynomial, and logarithmic trend option. Each Trendline option selected results in a curve or line from which a corresponding mathematical relationship can be obtained. Guided by the shape of the curve, a second order polynomial trend option was specified and an equation of the curve was generated:

$$y = 0.0001x^2 - 0.0079x + 1.1056 \quad (3\text{-}3)$$

where, y =scale factor,

x =longitude in degrees,

the last term 1.1056 is the scale factor at longitude zero degrees (y-intercept).

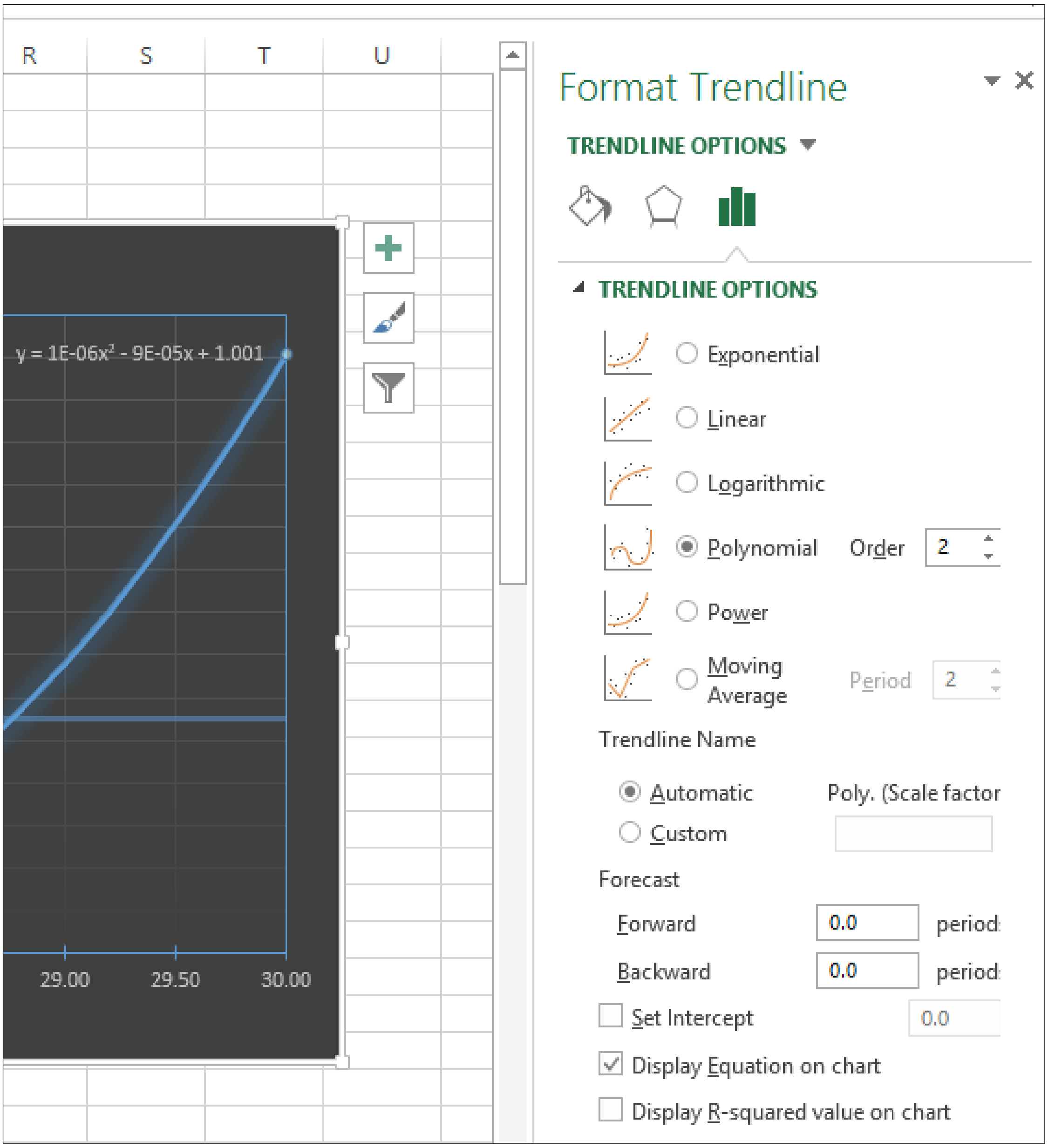


Figure 3.8: Data analysis in Microsoft Excel 2013

- **The Scale factor-Longitude relationship for proposed ZTM system**

  The procedure for plotting the graph was repeated for proposed ZTM system with central meridian at 28 degrees East, 12 degree-wide zone. The graphs obtained are as shown in figures 3.9 and 3.10 below.

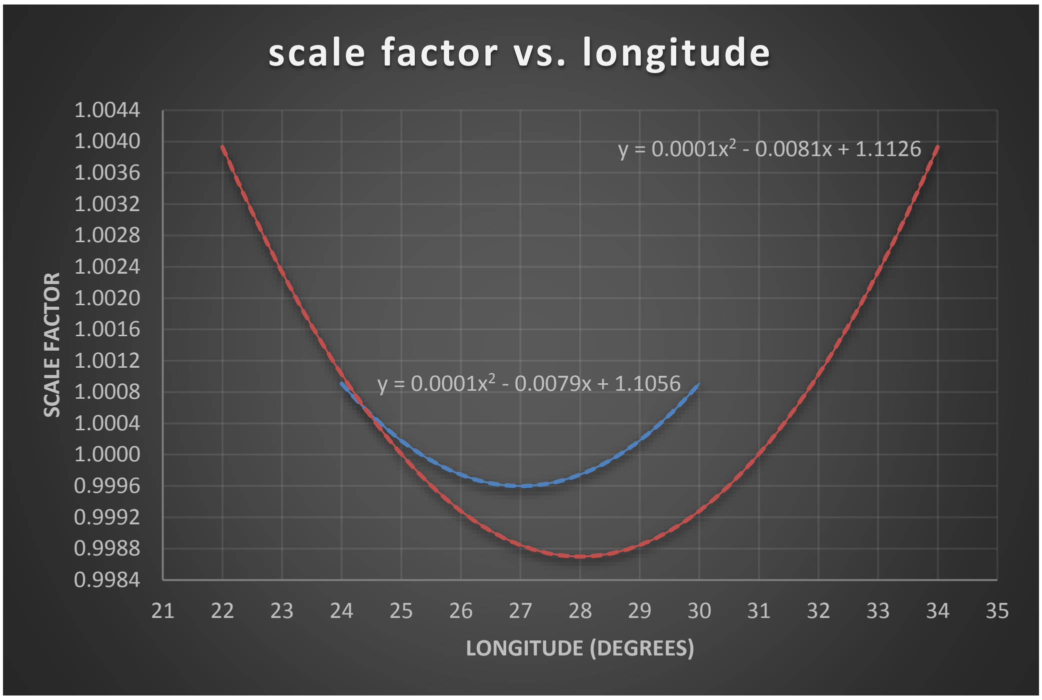


Figure 3.9: Graphical comparison of UTM27 curve in blue and proposed ZTM 28 (with k_0=0.9987) in red

The two curves intersect at (24.4^{O}E, 1.0006) within the range of longitude values 21^{o}E $\leq \lambda \leq$ 35^{o}E.

The resulting relationship between scale factor and longitudes have the quadratic equation of the form:

$$y = ax^2 + bx + c$$

The two equations are:

for UTM27: $y = 0.0001x^2 - 0.0079x + 1.1056$ (3-4)

for ZTM28: $y = 0.0001x^2 - 0.0081x + 1.1126$ (3-5)

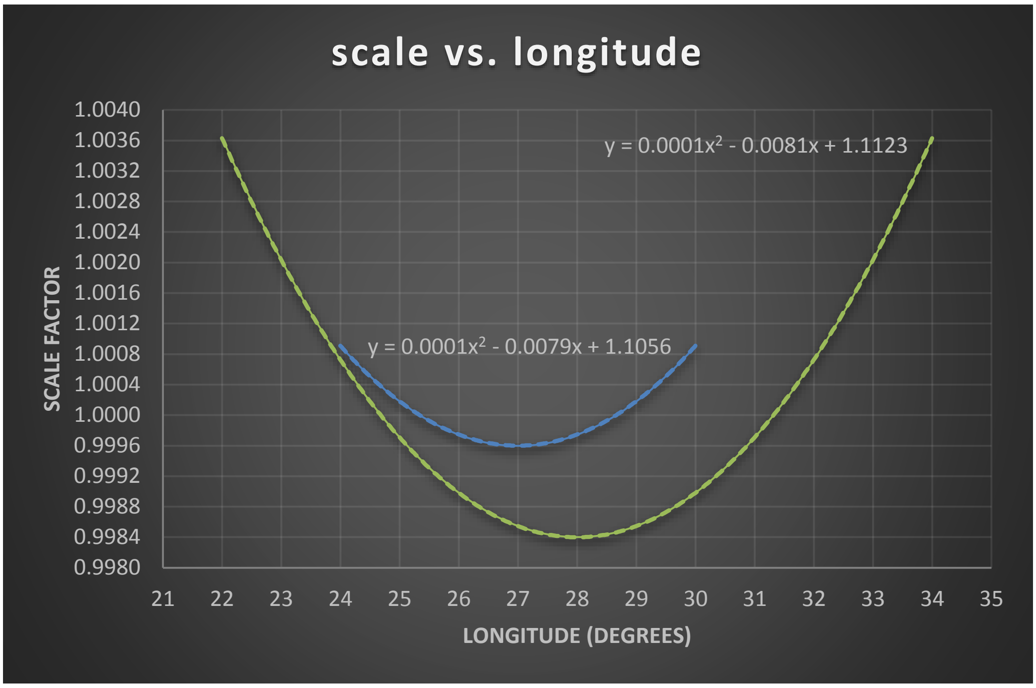


Figure 3.10: Graphical comparison of standard UTM27 in zone 35S in blue, proposed ZTM 28 with k_0=0.9984 in green.

CHAPTER 4 RESULTS AND DISCUSSION

The ZTM coordinate system design and analysis in the previous chapter presented various results depending on the hypothetical approach to determining the scale factor at the central meridian of the proposed ZTM projection for unified country-wide plane coordinate system. In this chapter the various findings from data analysis will be discussed with a view to coming up with a central meridian scale factor value upon which the proposed ZTM system will be based. The resulting parameters of ZTM coordinate system will then be tested by sample thematic mapping in ArcMap.

a) ***Determining scale factor by assuming scale factor at zone boundaries for both UTM and proposed ZTM to be equal***

The method of determining proposed ZTM central meridian scale factor (k_0) by letting the scale factor at the standard UTM 27 zone boundary (24^0E or 30^0E) to be the same as at the ZTM zone boundary (22^0E or 34^0E) yielded a scale factor value of **0.995708** as per procedure in chapter 3. Other standard UTM systems for Zambia such as UTM 21 and UTM 33 would also have produced the same results; UTM 27 was selected as a sample coordinate system from which to base comparisons. Using this value and at mid constant latitude of 13^0S, scale errors were computed, in a Microsoft visual studio C++ program, for all the meridians of longitude in the ZTM 12 degree-wide zone i.e. from longitude 22^0E to 34^0E. The scale errors were plotted against each longitude and analysed further in Microsoft Excel 2013 as shown in figure 4.1.

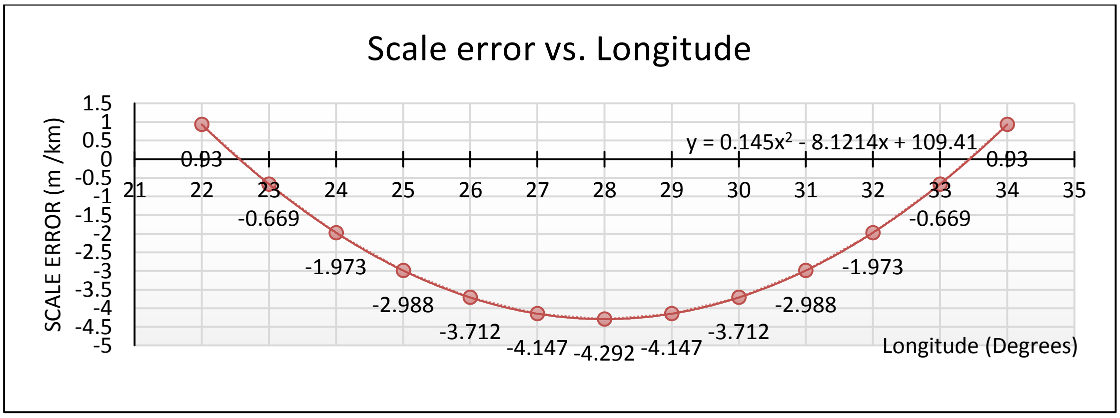


Figure 4.1: Comparison of scale errors at different longitudes of the ZTM projection system (central meridian scale factor of 0.995708)

The polynomial equation generated, $y = 0.145x^2 - 8.1214x + 109.41$, had roots at 22.553^0E and 33.456^0E. At these two positions the scale error was found to be zero, implying that these are the standard lines for the projection system if the scale factor at the proposed ZTM zone boundary is equivalent to the scale factor at UTM boundary. From the graph, the scale error at the central meridian was computed as -4.292 meters per kilometer. This value represents scale reduction at the central meridian. It means ground distance of 1000m would project to 995.708m on the ZTM map along the central meridian. The projected distance is 4.292 m less in every 1 kilometer of ground distance. This is the maximum scale reduction in this case.

From the central meridian, scale error improved gradually towards the standard lines until it reaches 0.00 (zero) at the standard lines. From standard lines, scale error increased from 0.00m/km to 0.93m/ km at the zone boundary.

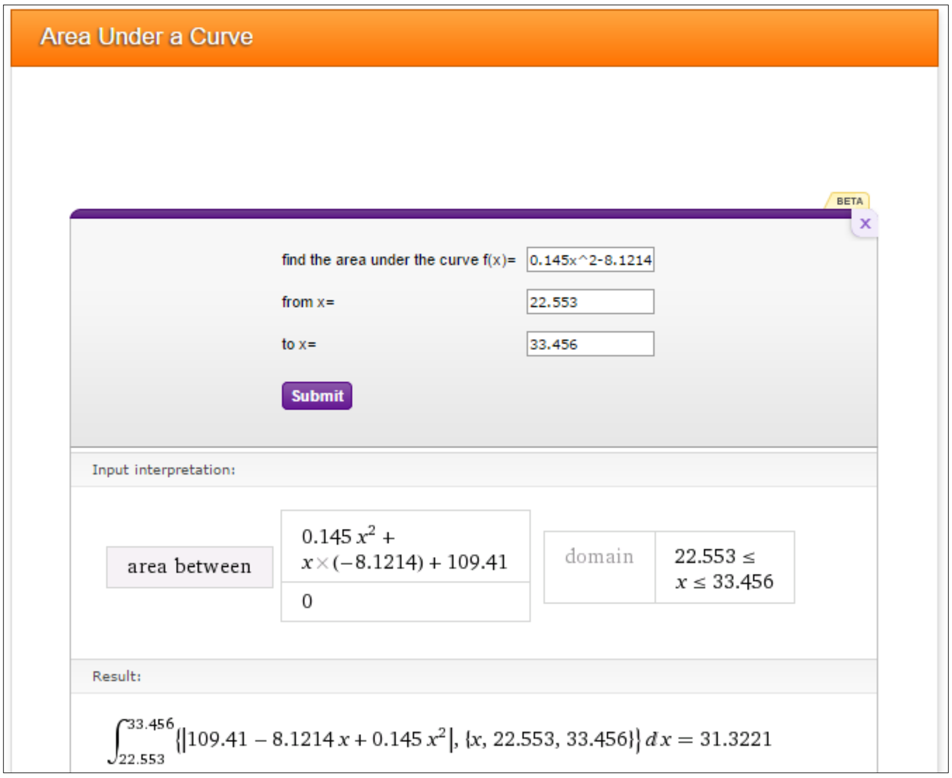


Figure 4.2: Calculation of area under the scale error curve (Wolframalpha, 2015)

On further analysis, the area under the curve of scale errors was calculated with the help of online "Area under the curve" calculator (Wolframalpha, 2015), based on calculus integrals. The procedure involved entering the function of the curve, the upper and lower limits as inputs. In this case the longitude of the standard lines and zone boundaries were the possible limits for area calculation. Accordingly, to calculate the area bounded by the curve and the x-axis, the upper limit was set to 33.456°E while the lower limit was set to 22.553°E, and the result was -31.322 units sq. The other part of the area under the curve was bounded by 33.456^{0}E and 34.000°E, and 22.553°E and 22.000°E, each resulted in +0.250 units sq. or +0.500 units sq. in total.

The areal unit is **m.km^{-1}. °E**, defined as total scale error in a projection of a group of longitudes delimited by the upper and lower limit of the scale error-curve. This is the sum of scale errors from boundary to boundary over the 12-degree zone width for a particular central meridian scale factor with specific positions of standard lines. The total area under the scale error curve for the first assumption was calculated as:

$$-31.322 + 0.500 = -30.822 \text{ m.km}^{-1}\ {}^{o}\text{E}.$$

The result -30.822 m.km^{-1} °E or simply -30.822 square units indicate the overall amount of scale reduction or scale distortion from boundary to boundary over the 12-degrees proposed ZTM zone width. Meaning, on average, scale distortion of -2.5685 meters per kilometer of length along a meridian of longitude would be expected i.e. -30.822 m.km^{-1} ° divided by 12 ° equals -2.5685 m.km^{-1}.

The results above are inconsistent with the basic requirement for a secant cylindrical transverse Mercator projection. Firstly, the scale error along the central meridian was larger than at the zone boundary (4.292 m/km compared to 0.93 m/km). Ideally, the scale error is supposed to be minimum between the standard lines and then to increase to maximum at the zone boundaries. Standard lines serve a purpose of averaging the scale error (or smoothening the scale variations) within a zone. However, the standard lines were too wide apart to average the scale errors in our first assumption. Therefore, using a projection with central meridian scale factor of 0.995708 would result in a map with highly reduced representation of ground distances in the middle. About 91% of the 12-degree zone width would have reduced scale due to position of secant lines which resulted from a fixed, assumed scale factor value of 1.0009 at the zone boundary. Seamless mapping with this projection would visibly shrink the distances in the middle of the map.

b) Determining scale factor by analyzing positions of secant lines (standard lines) which resulted in minimum scale errors

This method involved twenty-nine (29) trial positions of standard lines from which central meridian scale factor values were computed using equation (3-2) as outlined in the methodology. Ten (10) proposed positions were singled out and studied further because of the scale error averaging effect exhibited. Two of the ten proposed positions of standard lines are considered in the following discussion.

i. Standard lines at 25^0E and 31^0E

A pair of proposed standard lines, at longitudes 25^oE and 31^oE (row no. 13 in Table 3.3) resulted in the scale factor of **0.99870** at the central meridian of the proposed ZTM projection system. Standard lines at longitude 25^oE and 31^oE are exactly in the middle between central meridian at 28^oE and zone boundaries at 22^oE and 34^oE, respectively. It was therefore expected that this pair of standard lines would produce an even spread of scale errors across the zone width while keeping the central meridian scale error at 1.3 m/ km. Figure 4.3 shows the graph of scale errors against meridians of longitude for proposed ZTM projection with central meridian scale factor as **0.99870.**

The scale error at the central meridian was 1.30m/km, while at zone boundaries was 3.93 m/km as computed at constant mid latitude 13^oS. The results from calculations of area under the scale error-curve were as follows (Wolframalpha, 2015):

Area between standard lines 25^0E and 31^0E was found to be -5.013 units sq.
Area between 22^0E and 25^0E was +5.373
Area between 31^0E and 34^0E was +5.373
Total resultant Area under the curve was 5.373+5.373-5.013**= +5.733** units sq.

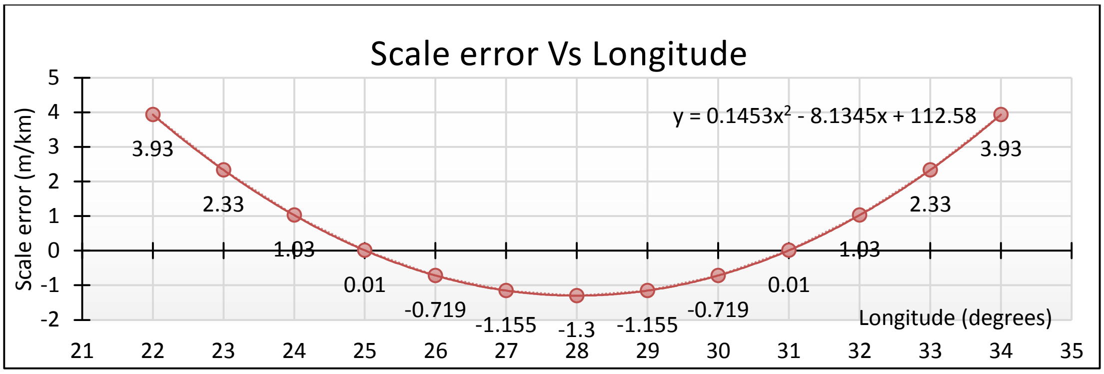


Figure 4.3: Comparison of scale errors at different longitudes of the ZTM projection system (central meridian scale factor of 0.9987).

The results from the *area under a curve* calculation show that there was an excess of +5.733 m/km scale errors across the 12-degree ZTM zone width. Scale reduction (negative scale errors) between central meridian and standard lines is extra compensated by increase in scale (positive scale errors) from standard lines to zone boundaries since there is an excess of +5.733 m/km of zone-wide scale error. Hence, the standard lines did not adequately smoothen the errors in this case as required. Meaning, on average, scale distortion of +0.4778 meters per kilometer of length along a meridian of longitude would be expected i.e. +5.733 $m.km^{-1\,o}$ divided by 12 ^{o} equals +0.4778 $m.km^{-1}$.

On further analysis, the ratio of scale error at central meridian to scale error at zone boundary was found to be **33.25%** compared to 44.44% of the standard UTM system, showing a deviation from standard UTM central meridian-zone boundary scale error relationship. The 44.44%, or ratio of scale error at the central meridian to scale error at zone boundary, is the standard scale distortion pattern that was implicitly agreed on by the NATO Allied Armies after the second World War when they designed the standard UTM projection and grid system with central meridian scale factor of 0.9996 and a zone width of 6 degrees (Knippers, 1993). In an attempt to modify the UTM system, it was hoped that the standard UTM distortion pattern be preserved even after altering other parameters.

ii. **Standard lines at 24.67ºE and 31.33ºE**

Similarly, Figure 4.4 shows the scale error-curve over the 12-degree zone width for central meridian scale factor of **0.99840.** This value was generated by assuming the standard lines to be at longitudes 24.67^{0}E and 31.33^{0}E. The resulting scale error at the central meridian was -1.6m/km while at the zone boundary it was found to be 3.63m/km. The mid latitude 13^{0}S was kept constant for all the computations of scale factors and errors at various meridians. The results of calculations of area under the scale error-curve were as follows (Wolframalpha, 2015):

Area between standard lines 24.67^{0}E and 31.33^{0}E was found to be -7.316 units sq.

Area between 22^{0}E and 24.67^{0}E was +4.267 units sq.

Area between 31.33^{0}E and 34^{0}E was +4.267 units sq.

Total resultant Area under the curve was:

+4.267+4.267-7.316=**+1.218** units sq.

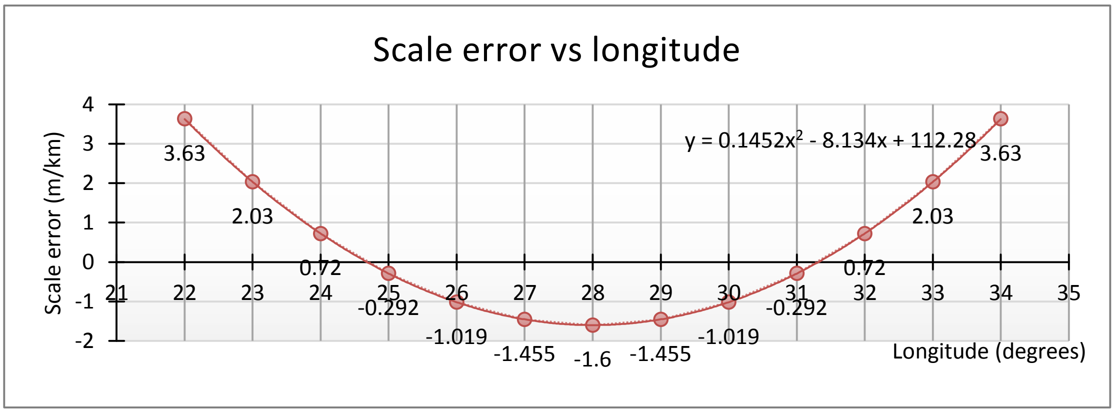


Figure 4.4: Comparison of scale errors at different longitudes of the ZTM projection system (central meridian scale factor of 0.9984).

The results from the *area under a curve* calculation show that there was an excess of +1.218 m/km scale errors across the 12-degree ZTM zone width. Scale reduction from central meridian to standard lines is fairly compensated by increase in scale from standard lines to zone boundaries with an excess of only +1.218m/km of scale error across the 12-degree zone width. Hence, the standard lines served their purpose by adequately smoothening the errors in this case. Meaning, on average, scale distortion of +0.1015 meters per kilometer of length along a meridian of longitude would be expected i.e. +1.218 $m.km^{-1}$ ^{o} divided by 12 ^{o} equals +0.1015 $m.km^{-1}$

On further analysis, the ratio of scale error at central meridian to scale error at zone boundary was found to be **44.44%** compared to 44.44% of the standard UTM system, showing a strong correlation with the standard UTM's central meridian-zone boundary scale error relationship. Therefore, setting the central meridian scale factor to 0.9984 and standard lines at 24.67^{o}E and 31.33^{o}E for a 12-degree proposed ZTM zone width produced the same scale distortion pattern as the standard UTM projection system. The standard UTM scale distortion pattern was preserved.

c) ***Using a mathematical relationship between UTM scale factor and longitudes within a zone***

The mathematical relationship between UTM scale factor and meridians of longitude within a zone was generated from graphical analysis of the scale error curve. The relationship obtained was a positive quadratic function with the parabolic graph of scale factor against longitudes opening upwards. The vertex of the parabola was the minimum value of scale factor at central meridian 0.9996. The function had no solutions as the graph did not cross the x-axis (Seward, 2010).

On plotting the graph of scale factor against meridians of longitude for the proposed ZTM system with central meridian scale factor of 0.9984, and from graphical analysis of polynomial equations the following features were deduced (*Please refer to figure 3.10).*

- The standard UTM graph had the form:

  $y=ax^2 + bx +c$

- The proposed ZTM graph had the form:

  $y=a(x-h)^2 + k$

where h is the horizontal shift and k is the vertical shift of the standard UTM graph.

This implies that the proposed ZTM graph, with central meridian scale factor of 0.9984, is a modified version of standard UTM graph, plotted on a wider range of x-values (12^{o}- wide zone instead of 6^{o}- wide zone) and with the vertical and horizontal shifts. From the graph (Figure 3.10) h is the horizontal shift of magnitude 1^{o}E (i.e. ZTM central meridian

28°E minus UTM central meridian 27°E). This had an effect of shifting the graph left-wards. *k* is the vertical shift of magnitude 0.0012. This is the difference in scale factor values between UTM and ZTM at the vertex (Seward, 2010):

$$0.9996-0.9984=0.0012$$

The value 0.0012 can be viewed in the following way: the scale error at UTM central meridian is 40 cm in every 1000m while at ZTM it is 160 cm in 1000m. 160 *minus* 40 *equals* 120 cm, which is 0.0012 *times* 1000m. Therefore, proposed ZTM scale errors are 120 cm larger than UTM scale errors at corresponding longitudes shifted 1° to the east.

4.1 Fixing the Central Meridian Scale Factor for Proposed ZTM System

The choice of UTM central meridian scale factor of 0.9996 was based on the accuracy of measurements required at the time of establishing the UTM projection and grid system. An acceptable accuracy requirement was decided upon and this was such that a measured distance of 50cm should have an error of 0.2mm. If 50cm is divided by 0.2mm the answer is 2500. Accuracy would be 2499 or 2501 units for a distance actually measuring 2500 units. 2499 divided by 2500 gives 0.9996 and this is how the scale factor was determined (Knippers, 1993).

In accordance with the results and discussion above, the working value for scale factor for the proposed ZTM coordinate reference system was fixed at **0.99840**. This means that at the central meridian ground distance of 1000 meters would measure 998.40 meters on the map in ZTM projection system. Giving an error of 1.6m in 1000m (1 km), and 3.6m in 1000m at the 12-degree zone boundaries. Thus the proposed minimum mapping length of 300 meters would be measured to 299.52 meters, resulting in an error of 48cm. On a working scale of 1:100, 000 a ground distance of 300m would be represented by 3mm, and the 48 cm error would be 0.0048mm, which is negligible for short distances.

The value 0.9984 was determined by the method of analyzing relationship between the longitude of standard lines and the resulting scale errors. The standard lines were located at longitude 24.67°E and 31.33°E. This value of scale factor showed a strong correlation with the standard UTM system characteristics. In addition, the graphical analysis method further supported the adoption of 0.9984 as central meridian for the proposed ZTM projection and grid system.

4.2 The Proposed Zambian Transverse Mercator Coordinate System

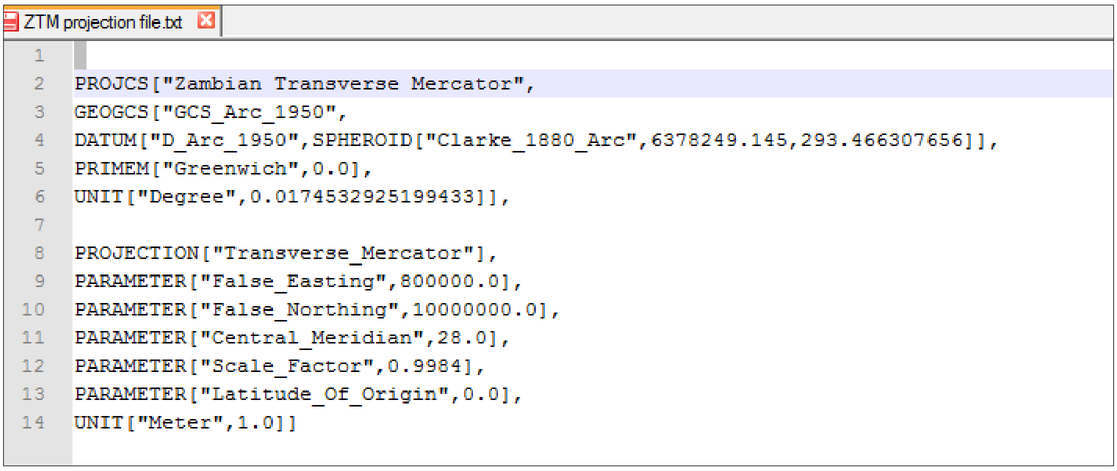
The Zambian unified country-wide plane coordinate system parameters

The proposed unified country-wide plane coordinate system parameters are:

Table 4.1: Proposed ZTM coordinate system parameters

Parameter	**Value/ Name**
Projection central meridian	28^0E
Scale factor at central meridian	0.9984
Zone width	12^0
False easting	800, 000m
False northing	10, 000, 000m
Reference Projection	Transverse Mercator
Reference ellipsoid	Clarke 1880 ellipsoid
Datum	Arc 1950

The parameters in the Table 4.1 above were used to carry out coordinate conversion and sample mapping in ArcMap 10.2. A false easting value of 800,000m was used in coordinate conversion from geodetic to ZTM and vice versa. An increase in false easting value from UTM's 500,000m to 800,000m sufficiently covered the whole study area, and avoided negative coordinates on the western side of the central meridian. The projection file that was obtained from ArcMap shape files, which had coordinate reference system edited to the proposed ZTM system, displayed the following projection information (Figure 4.5).



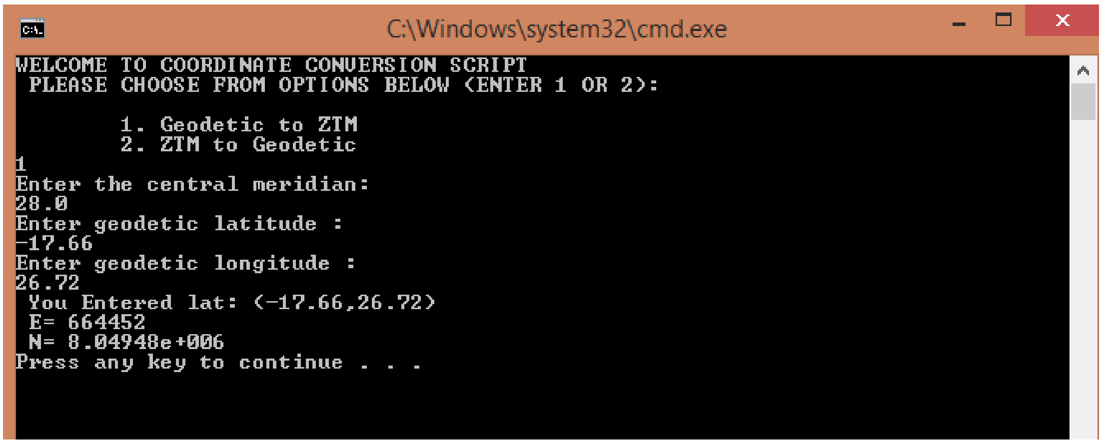
```
PROJCS["Zambian Transverse Mercator",
GEOGCS["GCS_Arc_1950",
DATUM["D_Arc_1950",SPHEROID["Clarke_1880_Arc",6378249.145,293.466307656]],
PRIMEM["Greenwich",0.0],
UNIT["Degree",0.0174532925199433]],

PROJECTION["Transverse_Mercator"],
PARAMETER["False_Easting",800000.0],
PARAMETER["False_Northing",10000000.0],
PARAMETER["Central_Meridian",28.0],
PARAMETER["Scale_Factor",0.9984],
PARAMETER["Latitude_Of_Origin",0.0],
UNIT["Meter",1.0]]
```

Figure 4.5: Projection file information from ArcMap as opened in text++ format

4.3 Coordinate Conversion

From the coordinate data collected, the geodetic coordinate data was converted to the proposed ZTM grid coordinate system using a C++ program developed in Microsoft Visual C++ (Express 2010) IDE. The C++ source code is as attached in Appendix A2.



```
WELCOME TO COORDINATE CONVERSION SCRIPT
 PLEASE CHOOSE FROM OPTIONS BELOW (ENTER 1 OR 2):

        1. Geodetic to ZTM
        2. ZTM to Geodetic
1
Enter the central meridian:
28.0
Enter geodetic latitude :
-17.66
Enter geodetic longitude :
26.72
 You Entered lat: (-17.66,26.72)
 E= 664452
 N= 8.04948e+006
Press any key to continue . . .
```

Figure 4.6: Sample C++ console window for coordinate conversion

The computer program was written in such an interactive way that it prompted the user to first select the type of conversion required i.e. either geodetic to ZTM or ZTM to geodetic coordinate system. Depending on the selection, the program further prompted the user to enter the input coordinates in one system, and then displayed the respective output coordinates in another coordinate system.

4.4 Sample Mapping in ArcMap

The following maps were compiled in ArcMap 10.2 based on the proposed ZTM coordinate system. The map compilation process was conducted in such a way that the features to be mapped were chosen on the basis of county-wide coverage. Therefore, features such as national parks and game management areas, national forests and forest reserves, and soil map of Zambia were sampled for mapping. Sample computations of areas and distances were also carried out in ZTM coordinate system. The computation results in ZTM coordinate system were then compared with corresponding results in *UTM coordinate system with central meridian 27^{0}E for country-wide mapping*. A check on relative accuracies of the two systems was conducted by comparing results from both systems with corresponding results from standard UTM zone by zone computation.

4.4.1 Sample map 1: Trigonometric Control Network

The map shows the 41 trigonometric control stations in proposed ZTM grid coordinate system. The points were converted from the Zambian classical geodetic network from SEED project as described in chapter 3.

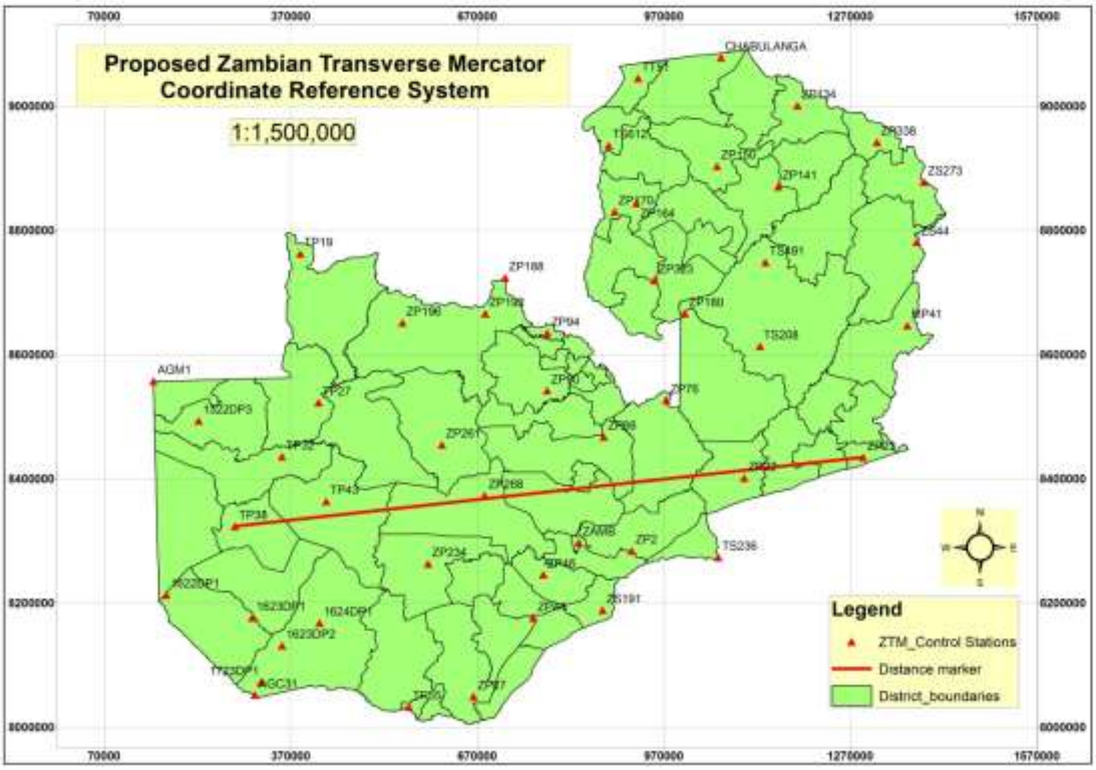


Figure 4.7: The proposed ZTM grid coordinate system with country-wide control stations

When carrying out sample mapping in ArcMap 10.2, the coordinate system properties of a new ArcMap data frame where customised to match the ZTM coordinate system parameters as in Figure 4.5. The sampled shapefiles for mapping in ZTM coordinate system had their projection subfiles edited in ArcCatalogue in order to conform to the ZTM coordinate system parameters. The edited shapefiles were then added to the ZTM-defined ArcMap data frame, and they displayed effectively in ArcMap geovisualisation environment. For a table of the new coordinates in ZTM coordinate system please refer to Appendix B.

In addition, sample distance measurements were carried out, for example, from control station TP38 in Mongu district to control station ZP23 in Chadiza district (Figure 4.7). The straight line distance spanning across UTM zones 34, 35 and 36 was measured as **1,016.473 km** in a single ZTM coordinate reference system. The standard UTM zone by zone distance computation resulted in **1016.339 km**. The ZTM result was **0.134 km** more than the more accurate UTM zone by zone computation result.

On the other hand, distance measurements for the same control stations (TP38 and ZP 23) were carried out in UTM coordinate system for country-wide mapping with central meridian 27^0E. The distance was measured as **1017.817 km**, which is **1.478 km** more than the more accurate UTM zone by zone computation results. These results show that having central meridian 27^0E for UTM countrywide mapping exaggerates distances because the standard UTM parameters are only meant for the 6^{-}degree zones width, beyond which, scale errors increase significantly. Alternatively, ZTM results were closer to the more accurate UTM zone by zone computation results because the ZTM system parameters are defined for a country-wide (Zambia) 12-degree zone width. The ZTM coordinate system parameters collectively minimises the scale distortions and errors inherent in seamless mapping across UTM zones.

Since ZTM coordinate system allows both mapping and computation of relatively accurate distances across zones, it can be useful in designing projects such as construction of an oil pipeline from Mongu through Lusaka to Chipata and ultimately up to Zambia-Malawi border area.

4.4.2 Sample map 2: Seamless Landcover mapping for Zambia

Figure 4.8 shows preliminary Landcover mapping results based on 1990 Landsat image classification using ENVI image processing software. The classes generally include vegetation, cropland, water bodies, wetlands, and built-up areas. It can be noted that the map legend only depicts district boundaries without the landcover classes. Full legend with land cover/use classes will be generated in the final field-validated map by the National Remote Sensing Centre. The map below only demonstrates the applicability of the ZTM system in seamless landcover mapping.

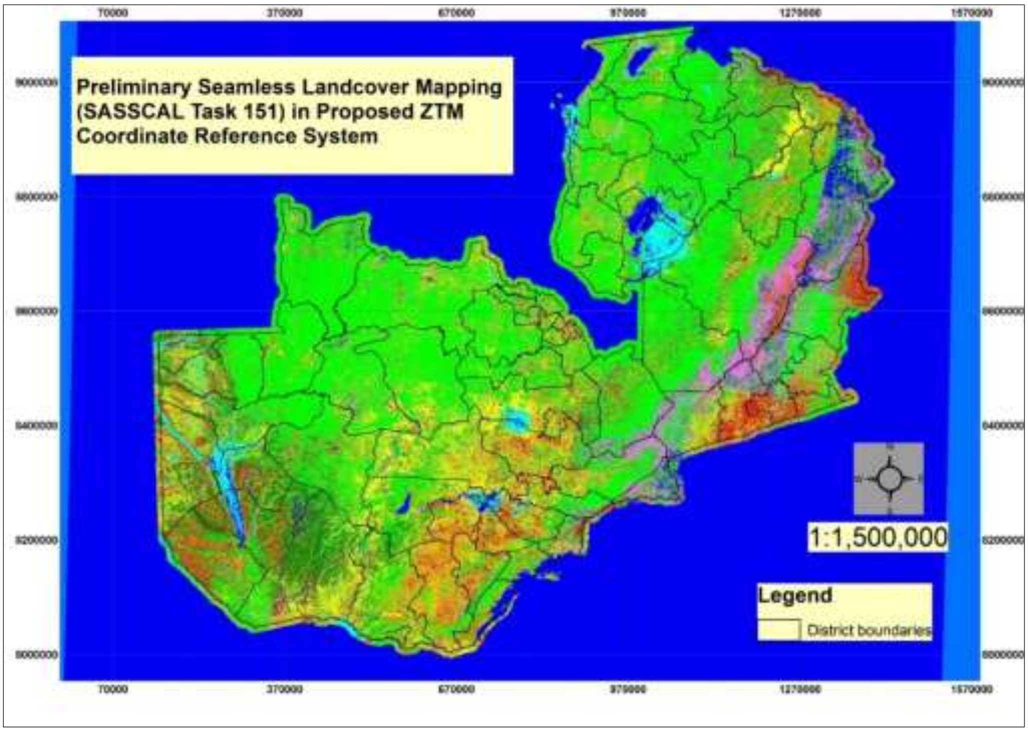


Figure 4.8: Preliminary Seamless Landcover mapping of SASSCAL Task 151 (*courtesy: NRSC 2015*)

The use of ZTM coordinate system for landcover mapping not only allow seamless mapping in plane coordinates, but also the quantification of landcover classes. For example, country-wide (across UTM zones) coverage area can easily be calculated for landcover classes such as all: water bodies, grasslands, croplands, wetlands, built-up areas and any other defined land cover class.

4.4.3 Sample map 3: National Parks and Forest Reserves

The projection subfiles of shapefiles for national parks, game management areas and forest reserves of Zambia were edited to match with ZTM coordinate system, and the following map was thus compiled in ZTM system. As can be seen in Figure 4.9, the mapped features cover all three UTM zones (34, 35 and 36).

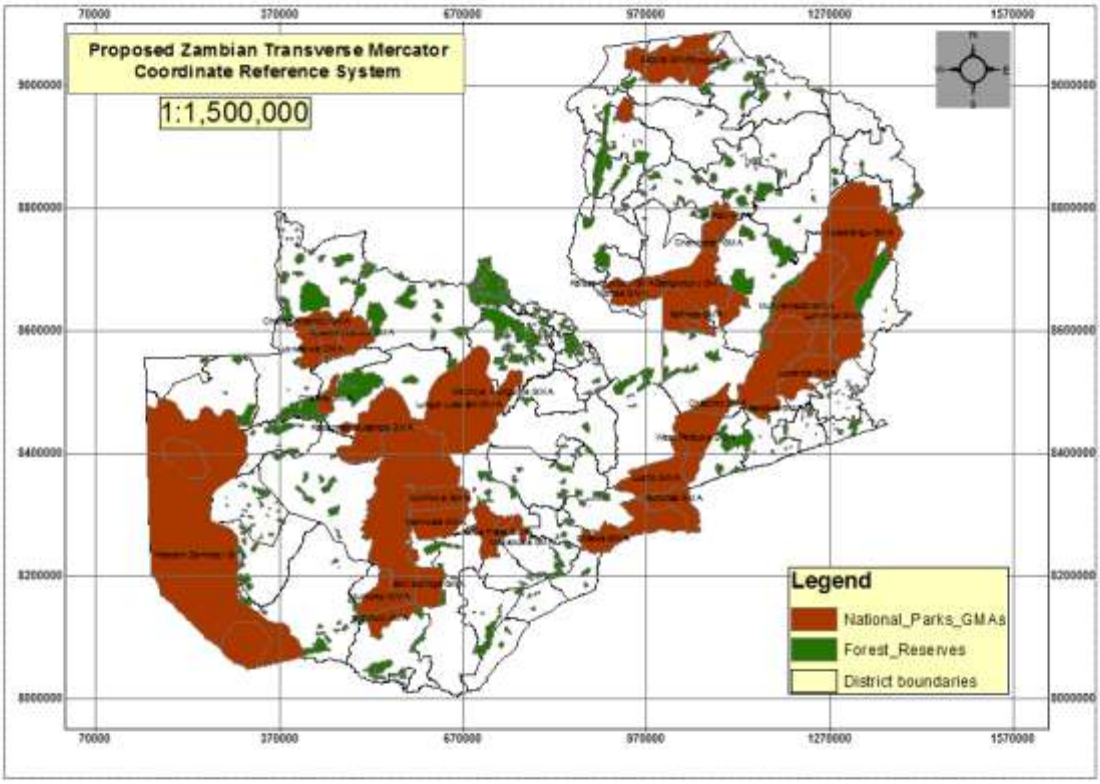


Figure 4.9: Zambian national parks and forest reserves as compiled in the proposed ZTM Coordinate system

Sample computations for total area covered by national parks and game management areas, and forest reserves were carried out in ZTM coordinate system and UTM for country-wide mapping with central meridian 27^0E, in ArcMap with shapefiles from NRSC (2015). Total area covered by all national parks and game management areas (GMAs) was computed as **226, 209.095 sq. km** in ZTM system, and **226,736.687 sq. km** in UTM for country-wide mapping with central meridian 27^0E. The more accurate, though cumbersome, computations were carried out using standard UTM coordinate system zone by zone (zone 34, zone 35 and zone 36), and the result was **226, 179.326 sq. km**.

The difference between ZTM and standard UTM zone by zone computation was **29.769** sq. km, representing an error of **0.013%** in area. On the other hand, the difference between UTM for country-wide mapping with central meridian 27^0E and standard UTM zone by zone computations was **557.361 sq. km**, representing an error of **0.246%** in area. The excesses in area in both ZTM and UTM for country-wide mapping with central meridian 27^0E systems can be attributed to increase in scale error as you move away from central meridians 28^oE and 27^oE, respectively. However, ZTM coordinate system proved to be more reliable for country-wide mapping as the error by area is relatively small.

4.4.4 Sample map 4: Zambian Soil Classification

The projection subfiles of shapefiles for soil classification of Zambia were edited to match with ZTM coordinate system, and the following map, depicting different types of soils in Zambia, was compiled in ZTM coordinate system. The map legend in Figure 4.10 only shows one class or type of soil (Gleysols) for convenience, since there are 90 other classes and subclasses which may render the legend illegible if shown.

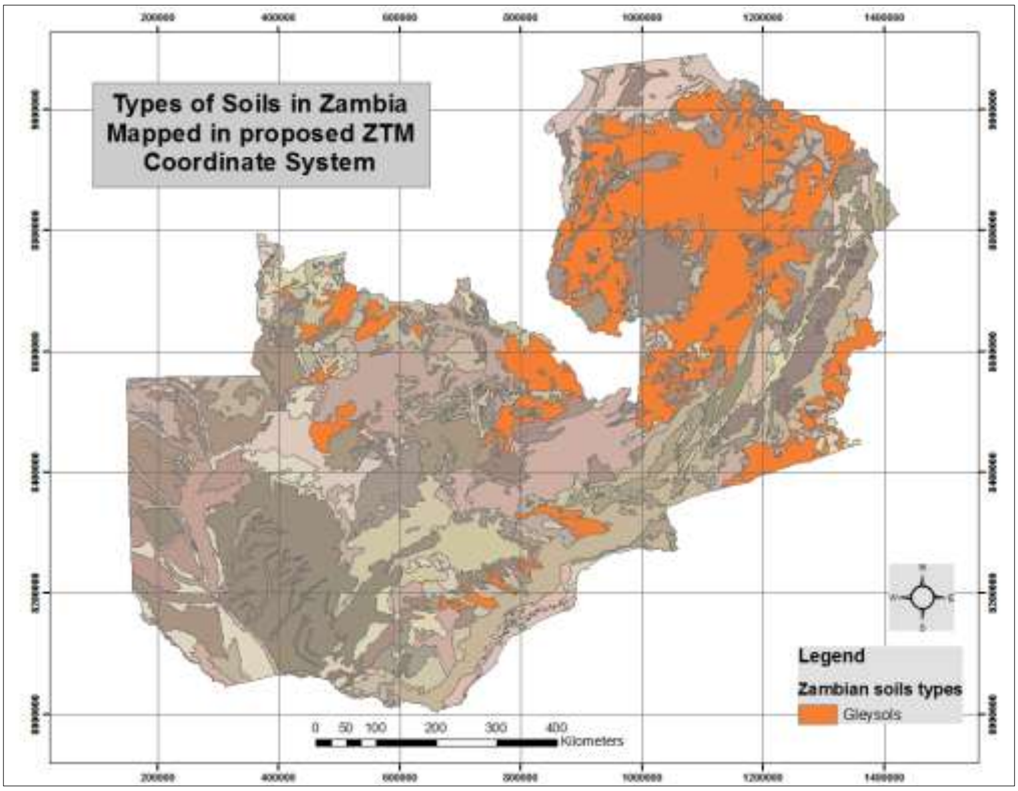


Figure 4.10: Soil types/classes (NRSC, 2015) of Zambia as compiled in ZTM coordinate system.

From area computations in ArcMap it was observed that the most prevalent soil type is Gleysols. Gleysols are a fine loamy to clayey soil, yellowish red to strong brown in colour, and of moderate drain for cultivation of field crops and trees. Area computation in UTM for country-wide mapping with central meridian 27^0E resulted in **152, 004.131 sq. km,** while in ZTM system it was **151, 400.552 sq. km**. Again computation results in UTM for country-wide mapping with central meridian 27^0E deviated even more from the **151, 380.873 sq. km** of the standard UTM zone by zone (zones 35 and 36) computation. The ZTM coordinate system parameters collectively minimises the scale distortions and scale errors inherent in seamless mapping across UTM zones, and hence showed less areal errors.

Although results from standard UTM zone by zone computations are most accurate, homogeneous features spanning across UTM zones have to be cut and separated into segments by zone. After which each segment is calculated within a particular zone and adding the results to come up with the sum. This process is cumbersome, especially if a lot of features are being mapped and analysed. Therefore, ZTM coordinate system eliminates these difficulties as both mapping and measurement of features can be done directly within moderate error; which error can be negligible at country-wide scale.

CHAPTER 5 CONCLUSION AND RECOMMENDATION

5.1 Conclusion

The establishment of a unified country-wide plane coordinate system for Zambia is of great importance as the current practice of using the standard Universal Transverse Mercator projection and grid system for country-wide mapping divides the country into three separate coordinate systems or zones. Data from different UTM zones cannot easily be integrated for seamless mapping. The alternative practice of country-wide mapping using geographic coordinates has limitations in planimetric quantifications of lengths and areas of mapped features because longitudes and latitudes are angular quantities.

In order to achieve seamless mapping the standard UTM parameters must be modified. The study devised methodologies for modifying the UTM parameters. The parameters modified were: position of central meridian, scale factor at central meridian, zone width, and false easting. Modifying these parameters resulted in the unified country-wide plane coordinate system for Zambia. The new system is named Zambian Transverse Mercator because of the scope of its application and that its projection is based on Transverse Mercator. The proposed ZTM projection and grid system has a central meridian at 28^0E with scale factor of 0.9984, a country-wide zone width of 12^0, a false easting of 800,000m; while the false northing of 10,000,000m and the Clarke 1880 reference ellipsoid were maintained.

The study involved a lot of computations using C++ computer programming language and most of the language techniques were learned during the study. This presented challenges in generating timely results, and in handling complex projection mathematics. The new plane coordinate system is intended to be used for country-wide, seamless landcover/landuse mapping projects such as Task 151 of the Southern African Science Service Centre for Climate Change and Adaptive Land Management (SASSCAL) project, among others.

5.2 Recommendations

The new country-wide plane coordinate system for Zambia is recommended for use in country-wide land audits, and seamless thematic mapping of features that span across the length and breadth of the country. Nevertheless, there is need for more research to be carried out in the area of reducing scale errors so that medium scale topographic mapping can also be performed in the proposed ZTM coordinate system.

With regards to coordinate system conversion, only conversion from geodetic to ZTM and vice versa was carried out in the study. Coordinate system conversion between ZTM and the standard UTM coordinate system will have to be devised as it is important. Future ZTM users should be able to convert UTM coordinates to ZTM coordinates easily.

There is need to study projection mathematics further so that a new projection tailored for the whole Southern Africa can be created other than the Transverse Mercator projection. In this regard, the possibility of using the WGS84 as reference ellipsoid for ZTM coordinate system can be considered in detail. Widening the scope of ZTM can enable an integrated land resource mapping system and a possible spatial database for the southern African region, just like the European countries' CORINE initiative.

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APPENDIX

APPENDIX A1: C++ source code for ZTM scale factor determination at central meridian

The following C++ source code was used to determine the proposed ZTM system scale factor at central meridian

```
// Scale Factor determination.cpp : Defines the entry point for the console
application.
//
#include "stdafx.h"
#include <iostream>
#include <iomanip>
#include <cmath>
#define PI 3.14159265

void Utm()
{
        using namespace std;
        double a,b,lat,lon,rad,k0,k,e2,ep,c,A;

                a=6378249.145;

                b=6356514.870;
                rad=PI/180;
                k0=0.9996; //SF at standard line

                e2=((a*a)-(b*b))/(a*a); //e squared
                ep=e2/(1-e2);  //e prime

                cout <<" With this option you can calculate SF at any point given Lat
and Lon"<<endl<<endl;
                cout <<" CHOOSE UTM ZONE:"<<endl;
                cout <<" 1. ZONE 34"<<endl<<" 2. ZONE 35"<<endl<<" 3. ZONE 36"<<endl;
                int choice;
                cin>>choice;

                if(choice==1)
                {
                cout <<" Enter Latitude value"<<endl;
                cin>>lat;
                cout <<" Enter Longitude value"<<endl;
                cin>>lon;
                    do
                    {
                       cout <<" WARNING! Point beyond zone limit: SF too
large!"<<endl;
                       }while(lon<18 && lon>24);

                c=ep*(cos(lat*rad)*cos(lat*rad));
                A=(lon-21.0)*rad*cos(lat*rad);
                k=k0*(1+(1+c)*A*A/2);
                cout <<" Scalefactor at ("<<lat<<"S,"<< lon<<"E) is:  "<<k<<endl;
                }
                else if(choice==2)
                {
                       /*
                       cout <<" Enter Latitude value"<<endl;
                       cin>>lat;
```

```
                        /*cout <<" Enter Longitude value"<<endl;
                        cin>>lon;
                        do
                     {
                        cout <<" WARNING! Point beyond zone limit: SF too
large!"<<endl;
                        }while(lon<24 && lon>30);
                        */

                        for (double lon=22.0;lon<=34.01; lon+=0.1)
                        {
                              lat=13.0;
                        c=ep*(cos(lat*rad)*cos(lat*rad));
                        A=(lon-28.0)*rad*cos(lat*rad);
                        k=0.9987*(1+(1+c)*A*A/2);
                       cout <<" Scalefactor at ("<< lon<<"E) is:  "<<k<<endl;
                        //cout <<" Scalefactor at ("<<lat<<"S,"<< lon<<"E) is:
"<<k<<endl;
                        }
                        /*

                        //This code added today 09092015 for figures in thesis
                                     double sum_k=0;
                                     cout <<" Enter lon of zone boundary :"<<endl;
                                     cin>>lon;
                                     for (int lat=-8;lat>=-18; lat-=5)
                                     {
                                     c=ep*(cos(lat*rad)*cos(lat*rad));
                                     A=(lon-27.0)*rad*cos(lat*rad);
                                     k=0.9996*(1+(1+c)*A*A/2);
                                     cout <<" ScaleFactor at ("<<lat <<","<<lon<<")
="<< k << endl<<endl;
                                     sum_k +=k;

                                      }
                                     cout <<" Mean Scalefactor at UTM zone boundary =
"<<  sum_k/3.0<< endl<<endl;

*/




               }
               else if(choice==3)
               {
                              cout <<" Enter Latitude value"<<endl;
                              cin>>lat;
                              cout <<" Enter Longitude value"<<endl;
                              cin>>lon;
                              do
                              {
                              cout <<" WARNING! Point beyond zone limit: SF too
large!"<<endl;
                              }while(lon<30 && lon>36);

                              c=ep*(cos(lat*rad)*cos(lat*rad));
                              A=(lon-33.0)*rad*cos(lat*rad);
                              k=k0*(1+(1+c)*A*A/2);
                              cout <<" Scalefactor at ("<<lat<<"S,"<< lon<<"E) is:
"<<k<<endl;
               }
               else
```

```
                      cout <<" Invalid entry!"<<endl;
}
//ZTM SCALE FACTOR DETERMINATION
void Ztm()
{
       using namespace std;
       double a,b,lat,lon,rad,k0,k,e2,ep,c,A;

               a=6378249.145;

               b=6356514.870;
               rad=PI/180;
               k0=0.99840; //SF at std UTM zone boundary

               e2=((a*a)-(b*b))/(a*a); //e squared
               ep=e2/(1-e2);  //e prime

               cout <<" WITH THIS OPTION YOU CAN CALCULATE k0 at CM 28E for a 12-
DEGREE-ZONE"<<endl<<endl;
               cout <<" By assuming scale error of "<<k<<" at zone boundaries"<<
endl<<endl;

/*{
               cout <<" Enter Latitude value (between 8 deg South & 18 deg
South)"<<endl;
               cin>>lat;
               cout <<" Enter Longitude at zone boundary (22 deg East or 34 deg
East)"<<endl;
               cin>>lon;
                   do
                   {
                      cout <<" WARNING! Point beyond zone limit: SF too
large!"<<endl;
                      }while(lon<22 && lon>34);

               c=ep*(cos(lat*rad)*cos(lat*rad));
               A=(lon-28.0)*rad*cos(lat*rad);
               k0=k/(1+(1+c)*A*A/2); // Doubling k at zone boundary
               cout <<" Scalefactor at Central Meridian (28 deg East) is:
"<<k0<<endl<<endl;
               cout <<" Meaning scale error of "<<(k0*1000)-1000<<"m per km "<<"at
CM"<< endl<<endl;
               cout <<" QUIET UNACCEPTABLE! at CM"<< endl;

               cout <<" And scale error of "<<(k*1000)-1000<<"m per km "<<"at zone
boundaries"<< endl;

               }*/
                                   //This code added today 09092015 for figures in
thesis
                                   //cout <<" Enter Longitude at zone boundary (22
deg East or 34 deg East)"<<endl;
                                   //cin>>lon;
                                   //double sum_k0=0;
                                   lat=13;

                                   for (int lon=22;lon<=34; lon+=1)
                                   {
                                   c=ep*(cos(lat*rad)*cos(lat*rad));
                                   A=(lon-28.0)*rad*cos(lat*rad);
                                   k=k0*(1+(1+c)*A*A/2);

                                   cout <<" k at "<<lon<<" E is:  "<<k<<endl;
```

```
                                    //sum_k0 +=k0;

                                     }
                                    //cout <<" Mean Scalefactor at Central Meridian =
"<<  sum_k0/3.0<< endl<<endl;
                                    //cout <<" Meaning scale error of
"<<((sum_k0/3.0)*1000)-1000<<"m per km "<<"at CM"<< endl<<endl;

}

void stdLineMethod()
{
       using namespace std;
       double a,b,lat,lon,rad,k0,k,e2,ep,c,A;

               a=6378249.145;

               b=6356514.870;
               rad=PI/180;
              // k=1.00; //SF at std UTM zone boundary

               e2=((a*a)-(b*b))/(a*a); //e squared
               ep=e2/(1-e2);  //e prime


               cout <<" THIS SUB-PROG CALCULATES SF at CM and ZONE BOUNDARY USING
STD LINES LONGITUDE"<<endl<<endl;

               cout <<" Mean k0 at Central Meridian     Mean k at Boundary"<<endl;
               cout <<" ---------------------------     ------------------"<<
endl<<endl;

               lon=31.30;
               while (lon<=31.40)
               {
                              double sum_k0=0;
                              for (int lat=-8;lat>=-18; lat-=5)
                              {
                                    c=ep*(cos(lat*rad)*cos(lat*rad));
                                    A=(lon-28.0)*rad*cos(lat*rad);
                                    k0=1/(1+(1+c)*A*A/2);
                                    //cout <<" CM SF for ("<<-lat <<","<<lon<<")
="<< k0 << endl<<endl;
                                    sum_k0 +=k0;
                              }

                             // cout <<"  k0= "<< sum_k0/3.0<< endl<<endl;



                                                                 double k1=sum_k0/3.0;
//cout <<" Meaning scale error of "<<(k0*1000)-1000<<"m per km "<<"at
CM"<<endl<<endl;

// Calculation of Scale factor at zone boundaries
//cout <<"***** COMPUTING SF AT ZONE BOUNDARIES *****"<<endl<<endl;
//cout <<"Enter Longitude value at Zone boundary (22 or 34 deg East) :"<<endl;
//cin>>lon;

double sum_k=0;
for (int lat=-8;lat>=-18; lat-=5)
                                                                  {
```

```
        c=ep*(cos(lat*rad)*cos(lat*rad));

        A=6*rad*cos(lat*rad);

        k=k1*(1+(1+c)*A*A/2);
//cout <<" Scalefactor at zone boundaries at ("<<lat<<", "<<34<<") ="<< k <<
endl<<endl;
        sum_k +=k;
        }
        cout <<"     "<< sum_k0/3.0<< "                    "<< sum_k/3.0 <<
        endl<<endl;
                                                                      //k=sum_k/3;
                                                                      //cout <<"
Meaning scale error of "<<(k*1000)-1000<<"m per km "<<"at zone boundaries"<< endl;

         lon +=0.01;

         }



}

int main()
{
        using namespace std;

                cout <<endl<<" WELCOME TO SCALE FACTOR COMPUTATION SCRIPT
:"<<endl<<endl;

                cout <<" PLEASE CHOOSE FROM OPTIONS BELOW (ENTER 1, 2 OR 3)
:"<<endl<<endl;
                cout <<"     1. STANDARD UTM"<<endl;
                cout <<"     2. MODIFIED UTM (ZTM) I"<<endl;
                cout <<"     3. MODIFIED UTM (ZTM) II"<<endl;

                int option;
                cin>>option;

                if (option==1)
                {
                      Utm();
                }

                else if (option==2)
                {
                      Ztm();
                }
                else if (option==3)
                {
                      stdLineMethod();
                }
                else

                cout<<" Invalid Entry!"<<endl;


    return 0;
}
```

APPENDIX A2: C++ source code for coordinate conversion between geodetic and proposed ZTM system

The following C++ source code was used to carry out coordinate conversion from geodetic to proposed ZTM and from proposed ZTM to Geodetic coordinate system

```
// MEngResearch.cpp : Defines the entry point for the console application.
//This Program converts between Geodetic Coordinates and Grid Coordinates

#include "stdafx.h"
#include <iostream>
#include <iomanip>
#include <cmath>
#define PI 3.14159265

void toZTM()
       {
       using namespace std;
       double J,n2,t2,lon,lat,cm,rad;

       //First we declare the constants
       double a,f,b,e2,eE2;
       //Formulae for the Spheroid
              a= 6378249.145;
              f=1/293.465;
              b= a*(1-f);
              e2=(a*a - b*b)/(a*a);
              eE2=(a*a-b*b)/(b*b);
              rad=PI/180;

       cout<<"Enter the central meridian:"<<endl;
       cin>>cm;
       cout<<"Enter geodetic latitude :"<<endl;
       cin>>lat;
       cout<<"Enter geodetic longitude :"<<endl;
       cin>>lon;
       J=((lon-cm)*rad)*cos(lat*rad);
       n2=e2*pow(cos(lat*rad),2.0)/(1-e2);
       t2=pow(tan(lat),2.0);
       //Definition of Meridian arc length M from equator to point
       double A,B,C,D,M,p,v,k,E,N;
       p=e2/8;
       k=0.99840;                        // scale factor on central meridian for
ZTM 28

       A=a*(1-2*p-3*p*p-10*p*p*p);
       B=a*(6*p+12*p*p+45*p*p*p)/2;
       C=a*(15*p*p+90*p*p*p)/4;
       D=a*(35*p*p*p)/6;
       M=A*(lat*rad)-B*sin(2*lat*rad)+C*sin(4*lat*rad)-D*sin(6*lat*rad);
       v=a/(sqrt(1-e2*pow(sin(lat*rad),2.0))); //Length of normal to spheroid
surface from a point to minor axis

       //Grid Coordinates E and N

       N=(M+v*tan(lat*rad)*(J*J/2+(pow(J,4.0)/24)*(5-
t2+9*n2+4*n2*n2)+(pow(J,6.0)/720)*(61-58*t2+t2*t2+270*n2-330*n2*t2)))*k;
       E=(v*(J+(J*J*J/6)*(1-t2+n2)+(pow(J,5.0)/120)*(5-18*t2+t2*t2+14*n2-
58*n2*t2)+(pow(J,7.0)/5040)*(61-479*t2+179*t2*t2-t2*t2*t2)))*k;

       cout<<" You Entered lat: ("<<lat<<","<<lon<<") "<<endl;
```

```
        //cout<<" You Entered lon: "<<lon<<endl;
        cout<<" E= "<<E+800000.00<<endl;                // false easting
        cout<<" N= "<<N+10000000.00<<endl;              // false northing
        }

void toGeodetic()
        {
                        using namespace std;
                                double J,n2,t2,lon,lat,cm,rad, N, E;

                                //First we declare the constants
                                double a,f,b,e2,eE2;
                                //Formulae for the Spheroid
                                        a= 6378249.145;
                                        f=1/293.465;
                                        b= a*(1-f);
                                        e2=(a*a - b*b)/(a*a);
                                        eE2=(a*a-b*b)/(b*b);
                                        rad=PI/180;

                                //J=((lon-cm)*rad)*cos(lat*rad);

                                //Definition of Meridian arc length M from equator to
point
                                double A,B,C,D,Mi,p,v,k,Ep,Np,phip, phi1,H;
                                p=e2/8;
                                cm=28.0;
                                k=0.99840;                          // scale factor on
central meridian for ZTM 28

                                A=a*(1-2*p-3*p*p-10*p*p*p);
                                B=a*(6*p+12*p*p+45*p*p*p)/2;
                                C=a*(15*p*p+90*p*p*p)/4;
                                D=a*(35*p*p*p)/6;
                                v=a/(sqrt(1-e2*pow(sin(lat*rad),2.0))); //Length of
normal to spheroid surface from a point to minor axis
                                Ep=800000.00;
                                Np=10000000.00;
                                //Geodetic Coordinates E and N
                                cout<<"Enter the Easting:"<<endl;

                                cin>>E;
                                cout<<"Enter Northing :"<<endl;
                                cin>>N;

                                Mi=(N-Np)*k;
                                phip=Mi/A;

                                //cout<<" Mi =:"<<Mi<<endl;
                                cout<<" phip =:"<<phip<<endl;

                                double Mp=(A*phip*rad-
B*sin(2*phip*rad)+C*sin(4*phip*rad)-D*sin(6*phip*rad)); //phi1 radian?
                                double dPhi=(Mi-Mp)/(A-2*B*cos(2.0*phip*rad));
                                        cout<<" dPhi =:"<<dPhi<<endl;
                                        while(abs (dPhi)<=pow(10.0,-9.0))
                                        {
                                                phip=phip+dPhi;
                                                Mp=(A*phip-
B*sin(2*phip)+C*sin(4*phip*rad)-D*sin(6*phip)); //phi1 radian?
                                                dPhi=(Mi-Mp)/(A-2*B*cos(2.0*phip*rad));
                                                cout<<" Mp =:"<<Mp<<endl;
                                                cout<<" Phi1 =:"<<phi1<<endl;
```

```
                                    }
                                    phi1=phip;
                                    H=(E-Ep)/(k*v);
                                    n2=e2*cos(phi1*rad)*cos(phi1*rad)/(1-e2);
                                    t2=pow(tan(phi1*rad),2);
                                    double C1, C2, C3, C11, C22,C33;
                                           C1=(1/cos(phi1*rad))*(H-
(H*H*H/6)*(1+2*t2+n2));

       C2=(pow(H,5.0)/120)*(5+28*t2+24*t2*t2+6*n2+8*n2*t2);

       C3=(pow(H,7.0)/5040)*(61+662*t2+1320*t2*t2+720*t2*t2*t2);

                                    lon=cm*rad+C1+C2-C3; //Geodetic longitude

                                           C11=(1+n2)*tan(phi1*rad)*(H*H/2-
(pow(H,4.0))/24);
                                           C22=(5+3*t2+n2-9*n2*t2-4*n2*n2);

       C33=(pow(H,6.0)/720)*(61+90*t2+45*t2*t2+46*n2-252*n2*t2-90*n2*t2*t2);
                                    lon=lon/rad;
                                    lat=phi1-C11*C22+C33;

                             cout<<" You Entered ZTM grid coordinates:
("<<E<<","<<N<<") "<<endl;

                             cout<<" Geodetic coordinates are:
("<<lat<<","<<lon<<")"<<endl;

       }

int main()
{
       using namespace std;

       cout<<"WELCOME TO COORDINATE CONVERSION SCRIPT"<<endl;
       cout <<" PLEASE CHOOSE FROM OPTIONS BELOW (ENTER 1 OR 2):"<<endl<<endl;
               cout <<"     1. Geodetic to ZTM"<<endl;
               cout <<"     2. ZTM to Geodetic"<<endl;

               int option;
               cin>>option;

               if (option==1)
               {
                     toZTM();
               }
               else if (option==2)
               {
                     toGeodetic();
               }
              else

               cout<<" Invalid Entry!"<<endl;

    return 0;
}
```

APPENDIX B: Country-wide Trigonometric Control Station Coordinates in ZTM Coordinate System

The table below present the country-wide Trigonometric station control coordinates in ZTM coordinate system. The notation for station point ID is as was presented in the source geodetic data, by the Geological/Zambia Survey Department during the SEED project.

Station ID	Easting	Northing
1322DP3	222130.661	8493271.486
1622DP1	170067.476	8213000.697
1623DP1	308160.123	8176137.060
1623DP2	356171.765	8131245.338
1624DP1	416292.052	8167772.366
1723DP1	322234.167	8072050.996
AGC31	312343.914	8051645.278
AGM1	149465.060	8557194.436
CHABULANGA	1062323.461	9078137.702
MP41	1362184.641	8646909.556
TP19	385503.621	8762489.405
TP27	414769.345	8522461.685
TP32	356277.241	8435522.581
TP38	280398.553	8323482.534
TP43	427022.956	8363549.342
TP55	560056.790	8032368.030
TS208	1124953.114	8613303.832
TS236	1057786.608	8273280.175
TS491	1133872.472	8748476.584
TS612	881775.908	8936675.132
TT51	928882.005	9044824.534
ZAMB	833332.148	8296920.043
ZP2	918475.127	8284187.711
ZP22	1099493.970	8401283.276
ZP23	1290803.751	8434384.160
ZP27	664268.254	8049212.979

ZP44	760569.403	8175602.794
ZP46	776324.861	8245408.184
ZP66	873618.577	8467421.469
ZP76	974174.123	8526340.911
ZP90	782617.418	8541906.437
ZP94	782957.576	8634363.494
ZP134	1185361.690	9000897.768
ZP141	1154410.586	8870397.134
ZP150	1055954.079	8903335.803
ZP164	926107.490	8843731.712
ZP170	891007.932	8830198.225
ZP180	1004093.195	8665715.592
ZP188	715042.923	8723470.298
ZP192	683371.152	8665910.116
ZP196	549646.591	8651037.447
ZP234	591064.196	8263131.541
ZP261	613354.116	8454769.526
ZP268	681862.392	8372772.776
ZP323	955259.114	8719484.502
ZP338	1313352.509	8941859.296
ZS44	1376977.207	8780884.566
ZS191	871814.940	8188212.300
ZS273	1388554.713	8877720.584