

SCHOOL OF NATURAL SCIENCES 2013

ACADEMIC YEAR

1	BIO 2302	BASIC MICROBIOLOGY
2	BIO3132	COMMUNITY ECOLOGY
3	BIO 3412	GENETICS
4	BIO 4182	WILDLIFE RANGELAND MANAGEMENT
5	BIO 4352	FOOD MICROBIOLOGY
6	BIO 4582	FOOD MICROBIOLOGY
7	BS4182	WILDLIFE AND RANGELAND MANAGEMENT
8	C492	ORGANIC INDUSTRIAL CHEMISTRY
9	CHE 1000	INTRODUCTION TO CHEMISTRY
10	CHE 2112	INTRODUCTION TO BIOCHEMISTRY
11	CHE 2522	FUNCTIONAL GROUP AND ARENE CHEMISTRY
12	CHE 2615	BASIC PHYSICAL CHEMISTRY
13	CHE 3122	ENERGY TRANSDUCTION SYSTEMS
14	CHE 3222	SPECTROSCOPIC METHODS OF ANALYSIS
15	CHE 3511	
16	CHE 4611	QUANTUM CHEMISTRY AND MOLECULAR SPECTROSCOPY
17	CHE 4822	INORGANIC INDUSTRIAL CHEMISTRY
18	CSC 3402	ARTIFICIAL INTELLIGENCE
19	CSC 3600	SOFTWARE ENGINEERING
20	CSC 3120	ANALOGUE AND DIGITAL ELECTRONICS
21	CSC 3612	IT PROJECT MANAGEMENT

22	CSC 4505	GRAPHIC AND VISUAL COMPUTING
23	CSC 4722	DISTRBUTED SYSTEMS
24	CS 4130	HARDWARE DESIGN AND IMPEMNTATION
25	GEO 482	ENVIRONMENT AND DEVLOPMENT II
26	GEO 492	NATURAL RESOURCE ECONOMICS
27	GES 2130	THE GEOGRAPHY OF AFRICA WITH SPECIAL REFERENCE TO ZAMBIA
28	GES 3342	ENVIRONMENTAL PLANNING AND MANAGEMENT
29	GES 4342	ENVIRONMENT AND NATURAL RESOURCES MANAGEMENT
30	GES 4372	TOURISM, ENVIRONMENT AND DEVELOPMENT
31	MAT 1100	FOUDATION MATHEMATICS
32	M II2	MATHEMATICAL METHODS II
33	MAT 1110	MATHEMATICS FOR SOCIAL SCIENCES
34	MAT 2100	ANALYTICAL GEOMETRY AND CALCULUS
35	MAT 2200	LINEAR ALGEBRA
36	MAT 2300	INTRODUCTION TO REAL ANALYSIS
37	MAT 2602	INTRODUCTION TO STATISTICS
38	MAT 3100	ADVANCED CALCULUS
39	MAT 3110	ENGINEERING MATHAMATICS
40	MAT 3300	REAL ANALYSIS
41	MAT 3612	LINEAR MODELS AND DESIGNS OF EXPERIMENTS
42	MAT 3702	DIFFERENTIAL GEOMETRY
43	MAT4600	MULTIVARIATE ANALYSIS
44	MSE 3060	CHEMISTRY TEACHING METHODS
45	PHY 1010	INTRODUCTORY PHYSICS

- 46 PHY 2522 ANALYTICAL MECHANICS AND SPECIAL RELATIVITY
- 47 PHY 3032 COMPUTATIONAL PHYSICS I
- 48 PHY 3242 STATISTICAL AND THERMAL PHYSICS
- 49 PHY 4222 SOLID STATE PHYSICS II

**THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES**

**2013 ACADEMIC YEAR
FINAL EXAMINATIONS**

**BIO 2302: BASIC MICROBIOLOGY
THEORY PAPER**

TIME: THREE HOURS

INSTRUCTIONS: ANSWER FIVE QUESTIONS; TWO FROM SECTION A AND TWO FROM SECTION B AND THE FIFTH FROM EITHER SECTION. USE ILLUSTRATIONS WHERE NECESSARY. USE SEPARATE ANSWER BOOKLETS FOR EACH SECTION.

SECTION A: BACTERIOLOGY

1. Describe structures of mesosomes, ribosomes and plasmids and their roles in bacteria.
2. Describe the structure, arrangement and function of flagella in prokaryotes.
3. (a) Compare and contrast cell wall structure in Gram positive and Gram negative bacteria.
(b) Describe the Gram staining procedure.
4. Summarize each of the following:
 - (a) Diauxic growth in bacteria
 - (b) Bacterial differentiation
 - (c) Fimbriae
 - (d) Prokaryotic cytoplasmic membrane.

SECTION B: VIROLOGY

5. Discuss transmission of viruses in plants.
6. Discuss different virus classification systems.
7. (a) State any six (6) types of enzymes coded for by named viruses.
(b) Describe the function or purpose of each respective virus enzyme which you stated in question (a) above.
8. Summarise each of the following:
 - (a) Antigenic shift and antigenic drift in influenza.
 - (b) Lysogeny conversion.
 - (c) Types of plant virus disease symptoms.

TURN OVER

(d) Non-genetic interactions between viruses.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR
FINAL EXAMINATIONS

BIO 3132: COMMUNITY ECOLOGY
THEORY PAPER

TIME: THREE HOURS

INSTRUCTIONS: ANSWER QUESTION ONE AND ANY **FOUR** OTHER QUESTIONS.
USE ILLUSTRATIONS WHERE NECESSARY.

1. (a) Discuss factors that affect productivity in terrestrial communities.
(b) Explain one way by which productivity is measured in aquatic communities.
2. Discuss facilitation and inhibition as modes of succession in communities.
3. Describe the vertical stratification of a tropical freshwater community with respect to:
 - (a) Light penetration.
 - (b) Profiles of temperature and oxygen.
4. Compare and contrast semelparity and iteroparity as species reproductive strategies in communities.
5. Summarise each of the following:
 - (a) Fecundity of a population.
 - (b) Disclimax.
 - (c) Allogenic succession.
 - (d) Age structure of a population.
6. Discuss community stability in relation to the Monoclimax hypothesis (Clements, 1916) and Polyclimax hypothesis (Tansley, 1939).
7. Discuss the species survival strategies of pioneer and late seral species.
8. Summarise each of the following:
 - (a) Positive community interactions
 - (b) Commercial applications of community interactions.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR
 FINAL EXAMINATIONS

BIO 3412 GENETICS

THEORY PAPER

TIME: THREE HOURS

INSTRUCTIONS: ANSWER FIVE QUESTIONS; TWO FROM EACH SECTION AND THE FIFTH QUESTION FROM EITHER SECTION. USE DIAGRAMS AND OTHER ILLUSTRATIONS WHEREVER NECESSARY. USE A SEPARATE ANSWER BOOKLET FOR EACH SECTION.

SECTION A

1. (a) Compare and contrast the origins of Turner and Down syndromes.

(b) The following pedigree shows a Zambian family affected by a common human trait called tooth agenesis, a dental malformation in which teeth are missing because they fail to develop. Study the pedigree carefully and answer the following questions.

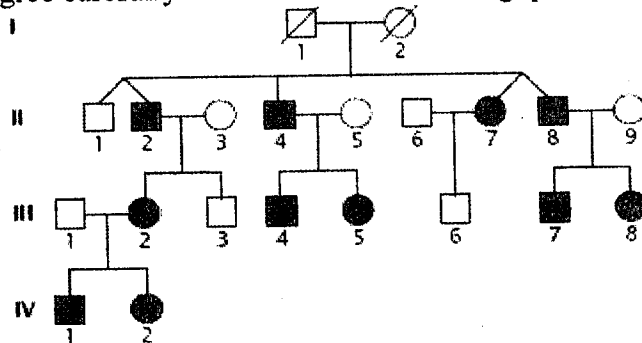


Fig. 1. Pedigree of a Zambian Family showing inheritance of tooth agenesis

- (i) Explain the most likely mode of inheritance for tooth agenesis in this family.
- (ii) Distinguish the two sets of twins in this family.
- (iii) If IV-1 married a woman who has a full set of teeth, determine the probability of their child having tooth agenesis.
- (iv) If III-4 and III-8 married and had a child, determine the probability of their child having tooth agenesis.

TURN OVER

2. (a) Explain the diagnostic features (genetic or cytological) used to identify each of the following chromosomal alterations:
 - (i) Deletions
 - (ii) Duplications
 - (iii) Inversions
 - (iv) Reciprocal translocations
 (b) Discuss how variation in chromosome number has been useful in agriculture.
3. Compare and contrast generalised and specialised transduction.
4. Distinguish between the following pairs of terms in bacterial genetics:
 - (a) Conjugation and transformation
 - (b) Auxotroph and prototroph
 - (c) Intergenic mapping and intragenic mapping
 - (d) Hfr strain and F^+ strain

SECTION B

5. (a) Describe the mapping function which relates RF to chiasma frequency.
 - (b) Explain how the mapping function in (a) above can be used to estimate genetic distance between two genes in centimorgans.
 - (c) Explain the advantage of estimating genetic distance using chiasma frequency compared to the method of using recombination frequency.
6. (a) Describe the Hardy Weinberg Equilibrium (HWE) and list the assumptions made.
 - (b) Explain four applications of the HWE.
7. A population has the following allele frequencies at the ABO blood group locus: I^A , 0.155; I^B , 0.127 and I^O , 0.718. The I^A and I^B alleles are codominant with respect to one another, and both alleles are dominant over the I^O allele. (Note the alleles I^A and I^B are codominant to each other and that both are dominant to allele I^O . Also assume that the frequency of $I^A = p$, frequency of $I^B = q$ and frequency of $I^O = r$)
 - (a) Determine the type of allelic condition and the Hardy-Weinberg formula for this kind of allelism.
 - (b) If you sampled nine hundred people from this randomly mating population, determine how many people would fall have blood types A, B, AB and O.
8. In the study of plant height in *Zea mays*, variance within parents and within F_1 and was estimated to be 7.18. Standard deviation within F_2 families was estimated to be 4.5.
 - (a) Estimated the expected variance within the F_3 families.
 - (b) Estimated the expected variance within the F_4 families.
 - (c) Estimate the heritability of plant height in the *Zea mays* population under this study.
 - (d) Explain the importance of heritability in crop and livestock improvement.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR
FINAL EXAMINATIONS

BIO 3712: ANIMAL PHYSIOLOGY
THEORY PAPER

TIME: THREE HOURS

INSTRUCTIONS: ANSWER **FIVE** QUESTIONS. USE ILLUSTRATIONS
WHERE NECESSARY.

1. (a) Describe the various types of ventilation in animals.
(b) Describe the principal means by which animals acclimatise to high altitude.
2. (a) Discuss the properties of a neurotransmitter.
(b) Compare and contrast electrical and chemical synaptic transmission.
3. (a) Explain how cardiac function is regulated in vertebrates.
(b) Explain the neuromuscular junctional events during skeletal muscle contraction and relaxation.
4. (a) Describe the mechanoreceptor cells of sound in vertebrates.
(b) Discuss sound transduction in vertebrates.
5. (a) Discuss the occurrence and nature of excretory organs in **three** named invertebrates.
(b) Describe the roles of **three** named hormones in the regulation of renal function in vertebrates.
6. Describe the roles of each of the following in animal reproduction and/or development:
 - (a) Oxytocin.
 - (b) Prothoracicotropic hormone.
 - (c) Activin.
 - (d) Aggregation pheromone.
7. Discuss the causes and consequences of energy intake exceeding energy expenditure in vertebrates.
8. (a) Discuss the various thermoregulatory responses to high and low temperatures in animals.
(b) Compare and contrast hibernation and torpor.

END OF EXAMINATION

**THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES**

2013 ACADEMIC YEAR SECOND SEMESTER
FINAL EXAMINATIONS

BIO 4182: WILDLIFE RANGELAND MANAGEMENT
THEORY PAPER

TIME: THREE HOURS

INSTRUCTIONS: ANSWER FIVE QUESTIONS. ANSWER QUESTION ONE AND ANY OTHER FOUR QUESTIONS. USE ILLUSTRATIONS WHERE NECESSARY.

1. Pemba Farms Ltd is considering establishing a game ranch in the Namwala District along the Kafue River. Initial investigations show that the range is suitable for Impala, Zebra, Wildebeest, Kudu and Buffalo. Nine per cent (9 %) of the range is a steep hill and in addition only 60% of the range is within 6.5 km from water. Based on the information from the Ministry of Agriculture and Livestock in Mpika, the soils are generally suitable for game ranching. Furthermore, results from your preliminary estimates indicate that the production of key forage species averages about 700kg/ha of dry matter per year. The proposed farm is 10,000 ha in size. Assuming that allowable use is 55% of the total biomass and daily dry matter intake is 2% of the animal body weight,
 - (a) Determine the number of 950 kg buffaloes you would stock as your base herd in the area.
 - (b) Determine Animal Units / ha/month of the buffalo.
 - (c) Determine number of hectares you would need to stock 160 Wildebeest per year in this ranch.
 - (d) Discuss limitations of this method in estimating stocking rate of wildlife species.

2. Discuss the management application of the following methods as used in wildlife and range management:
 - (a) Bitterlich method.
 - (b) King Census method.
 - (c) Point centered quarter method.
 - (d) Point- intercept method.

3. Summarise each of the following:
 - (a) *Ceratotherium simum*.
 - (b) Contribution by Aldo Leopold.
 - (c) Aerial census.
 - (d) Perissodactyla.

TURN OVER

4. (a) Compare and contrast the concepts of carrying capacity and stocking rate in the management of wildlife species.
(b) Describe methods for wildlife habitat improvement in a semi-arid environment
 5. Summarise each of the following:
 - (a) Wildlife protected area system of Zambia
 - (b) Implications of island biogeography theory in selecting an area for establishing a wildlife reserve.
 - (c) Harvesting strategies in wildlife management
 - (d) Threats to wildlife conservation
 6. Discuss each of the following:
 - (a) Characteristics of a wildlife habitat.
 - (b) Restoration methods of *Tragelaphus spekei* habitat.
 7. Discuss methods used to determine:
 8. $1 - e^{-H}$ in the exploitation of wildlife populations
 9. Quotas for harvesting wildlife species.
 10. Summarise each of the following:
 - (a) Kidney/Fat ratio.
 - (b) Limitation of Maximum Sustainable Yield.
 - (c) *Phacochoerus aethopicus*
 - (d) Limitations of Community Based Natural Resources Management
-

END OF EXAMINATION

ANIMAL SPECIES ADULT MALE MAXIMUM WEIGHT (KG) AND ANIMAL UNITS EQUIVALENT

Animal Species	Maximum Weight (Kg) Adult Males	Animal Units Equivalent Males(AUE)
Elephant	5400	11.89
White Rhino	1300	2.86
Hippo	1800	3.96
Black Rhino	1100	2.42
Giraffe	1400	3.10
Buffalo	700	1.54
Eland	900	1.98
Roan	270	0.59
Zebra	260	0.57
Sable	270	0.59
Waterbuck	270	0.59
Lichtenstein hartebeest	170	0.37
Greater Kudu	250	0.55
Blue Wildebeeste	250	0.55
Tsessebe	140	0.31
Nyala	108	0.24
Red Lechwe	100	0.22
Kafue Lechwe	100	0.22
Black Lechwe	100	0.22
Sitatunga	115	0.25
Reedbuck	70	0.15
Puku	74	0.16
Bushpig	50	0.11
Impala	50	0.11
Warthog	105	0.23
Sthn R'buck	40	0.09
Bushbuck	30	0.07
Oribi	16	0.03
Common Duiker	10	0.02
Klipspringer	10	0.02
Steenbok	11	0.02
Grysbok	11	0.02
Blue Duiker	4	0.01

One cow= 453.6

4. (a) Compare and contrast the concepts of carrying capacity and stocking rate in the management of wildlife species.
(b) Describe methods for wildlife habitat improvement in a semi-arid environment
 5. Summarise each of the following:
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Sthn R'buck	40	0.09
Bushbuck	30	0.07
Oribi	16	0.03
Common Duiker	10	0.02
Klipspringer	10	0.02
Steenbok	11	0.02
Grysbok	11	0.02
Blue Duiker	4	0.01

One cow= 453.6

SECTION B

5. Fermentation is one of the most ancient food processing technologies in many cultures. However, scientists and policy makers have neglected this simple technology. Discuss the benefits of fermenting dairy products and outline the basic principles of this process. (20 Marks)
6. Quality or excellence in our food supply should be an important concern to all food processors. Safety and wholesomeness are the most important attributes of food quality. The lack of quality as it relates to safety and wholesomeness can result in personal injury, sickness or death. Food-borne illness is an example of sickness which could even cause death when unsafe foods are produced and consumed. Discuss quality control as a model program for the food industry according to:
 - (i) Critical, major and minor attributes. (6 Marks)
 - (ii) Development of a quality control program. (9 Marks).
 - (iii) Basic tools of quality control. (5 Marks).
7. Over the last 80 years, many different methods have been developed for detecting pathogenic microorganisms or their toxins in food or in food products.
 - (a) Describe the necessity and importance of microorganism identification and characterization. (6 Marks)
 - (b) Discuss the main methods of detecting microbial contaminations in food or food products and list one technique of each method. (14 Marks)
8. Summarise any four (4) of the following:
 - (a) Significance of a working product-recall system. (5 Marks)
 - (b) Manufacturing processes of soy sauce. (5 Marks)
 - (c) Differences in the production of green and black olives. (5 Marks)
 - (d) The significance of using sweeteners, flavors and fruit preparations in yogurt. (5 Marks)
 - (e) Starter cultures. (5 Marks)

END OF EXAMINATION

**THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCE**

2013 ACADEMIC YEAR
FINAL EXAMINATIONS

BIO 4582: FOOD MICROBIOLOGY
THEORY PAPER

TIME: THREE HOURS

INSTRUCTIONS: ANSWER FIVE QUESTIONS; TWO FROM EACH SECTION AND THE FIFTH FROM EITHER SECTION. USE A SEPARATE ANSWER BOOKLET FOR EACH SECTION. USE ILLUSTRATIONS WHERE NECESSARY

SECTION A

1. (a) With necessary detail, yet brief, distinguish between pathogens and spoilage organisms, and list four types of microorganisms responsible for most food contamination. (10 Marks)
- (b) Briefly, but with necessary detail, describe food infections and intoxications. (10 Marks)
2. Bacteriocins are regarded as antibiotics; however, they clearly differ from antibiotics in a number of ways.
 - (a) Define the term bacteriocin.
 - (b) Explain in detail how bacteriocins differ from antibiotics and give an account of their basic classification.
 - (c) Explain how bacteriocins are used in food preservation industry and state the factors that limit their efficacy as food preservatives. (20 Marks)
3. Elaborate on the three ranges of low-temperature food preservation and describe low-temperature food preservation techniques and their benefits. (20 Marks)
4. (a) Describe dehydration food preservation techniques and their benefits. (10 Marks)
- (b) Describe high-temperature food preservation techniques and their benefits. (10 Marks)

TURN OVER

SECTION B

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 - (c) Differences in the production of green and black olives. (5 Marks)
 - (d) The significance of using sweeteners, flavors and fruit preparations in yogurt. (5 Marks)
 - (e) Starter cultures. (5 Marks)

END OF EXAMINATION

8. You are required to use the map (Figure 1) provided to answer this question.
- (a) Describe each of the following habitats as indicated by the vegetation types:
- Munga – Combretum thicket.
 - Mopane woodland.
 - Riverine.
 - Miombo woodland.
- (b) Identify an area in the map that is most suitable to tourism development, and give reasons.

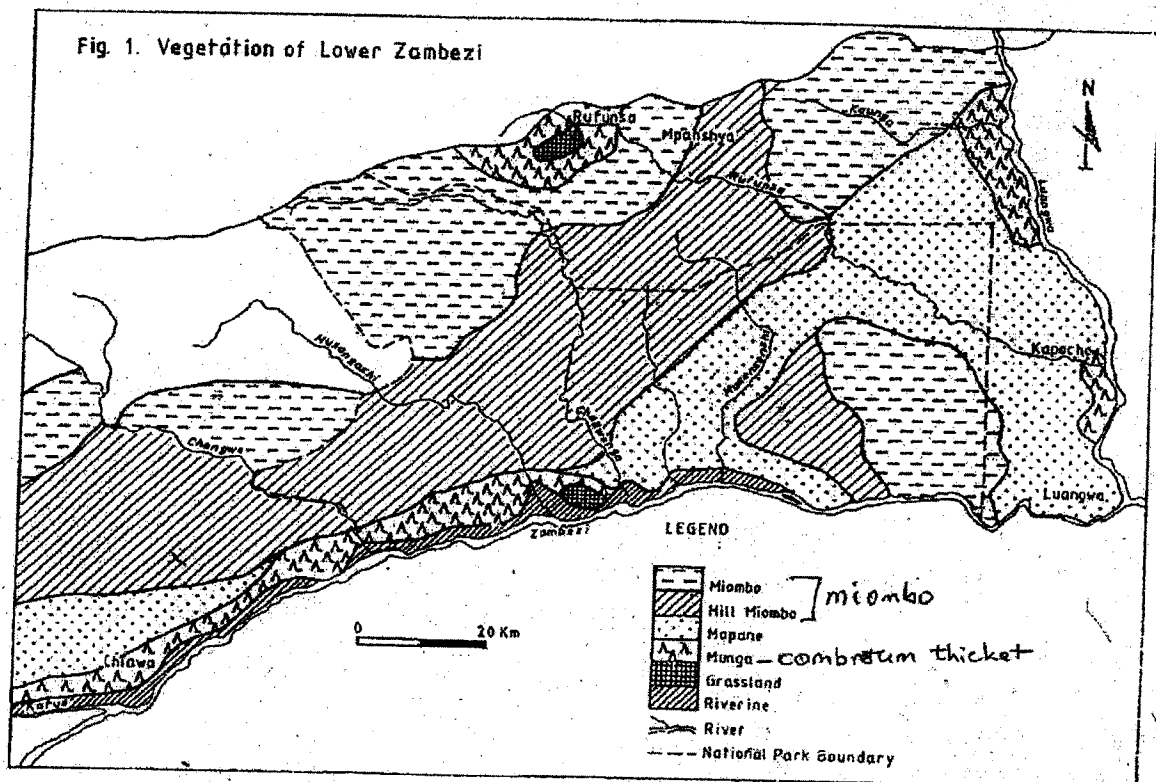


Figure 1: Lower Zambezi National Park showing habitat types

END OF EXAMINATION

UNIVERSITY OF ZAMBIA
2013 FINAL UNIVERSITY EXAMINATIONS, AUGUST 2014

C492: Organic Industrial Chemistry II

Time: Three (3) Hours

Instructions: Answer Question any other four (4) questions. All questions carry 15 marks.

Question 1

- (a) Describe/explain in brief three main transitions in polymers. Indicate in the answer the structural or morphological changes that occur.
- (b) (i) Describe the process of fibre spinning. Use an appropriate diagram to illustrate your answer.
- (ii) Explain how maximum strength in the fibres is attained following extrusion.

Question 2

- (a) Explain the meaning of the following terms: (i) chromophore (ii) auxochrome. Give the structure of one example in each case
- (b) Describe the chemical structure and mode of reaction of reactive dyes.
- (c) What are the essential features of a vat dye?

Question 3

- (a) Compare and contrast typical characteristics of initiatory and secondary explosives
- (b) Describe the manufacturing process of one initiatory and secondary explosive
- (c) Explain pressure changes that occur following an explosion with high explosives. Use suitable sketch diagrams to illustrate your answer.

Question 4

- (a) Explain how raw sugar is recovered from the molasses in two stages (A molasses and B massecuit).
- (b) Explain in detail the principle of decolonization process in sugar refinery.
- (c) Use a detailed Sketch/Diagram to describe the principle of heat transfer process of a five series evaporator system. Explain the basic functions of the unit in both Sugar production and Sugar refinery processes.
- (d) With the aid of a flow Diagram Describe the cane sugar production processes.

Question 5

- (a) Explain in detail the mechanism of operation for the following stages.
- (i) Hydrolysis of amide groups
 - (ii) Modification of guanide groups
 - (iii) Hydrolysis of keto-imide links in protein chains
- (b) Explain in detail the mechanism of operation for the following during leather manufacture;
- (i) Neutralizing.
 - (ii) Dyeing.
 - (iii) Fat Liquoring.
- (c) Briefly explain the significance of chromium and chromium-collagen compound during the Tanning process in leather manufacture.
- (d) With the aid of a flow Diagram describe to describe the process of Leather manufacture.

**THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES**

**2013 ACADEMIC YEAR
FINAL EXAMINATIONS**

CHE1000: INTRODUCTION TO CHEMISTRY

TIME: THREE (3) HOURS

INSTRUCTIONS TO THE CANDIDATES

1. Indicate your **student ID number** and **TG number** on **ALL** your answer booklets.
2. This examination paper consists of two (2) sections: **A** and **B**
3. Section **A** has ten (10) short answer questions. Questions carry equal marks. (Total marks = 40).
ANSWER ALL QUESTIONS IN SECTION A IN THE MAIN ANSWER BOOKLET
4. Section **B** has five (5) long answer questions. Questions carry equal marks. (Total marks = 60).
ANSWER QUESTION B1 and ANY THREE QUESTIONS, EACH IN A SEPARATE ANSWER BOOKLET.
6. **YOU ARE REMINDED OF THE NEED TO ORGANISE AND PRESENT YOUR WORK CLEARLY AND LOGICALLY.**
7. **ENSURE** that you have nine (9) printed pages and Periodic Table.

ADDITIONAL INFORMATION TO THE CANDIDATES:

Useful data is provided on page 9 and 10.

SECTION A

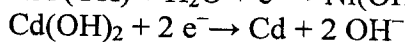
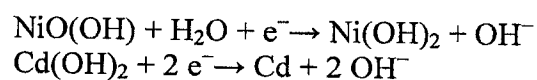
ANSWER ALL QUESTIONS

Question A1

- (a) Show using Lewis structures the reaction between fluorine and nitrogen leading to the formation of NF_3 .
- (b) Draw the molecular geometry of the resultant product in (a) and give the name of the shape.

Question A2

Nickel-cadmium cells are used to power electrical equipment such as drills and shavers. The electrode reactions are shown below.



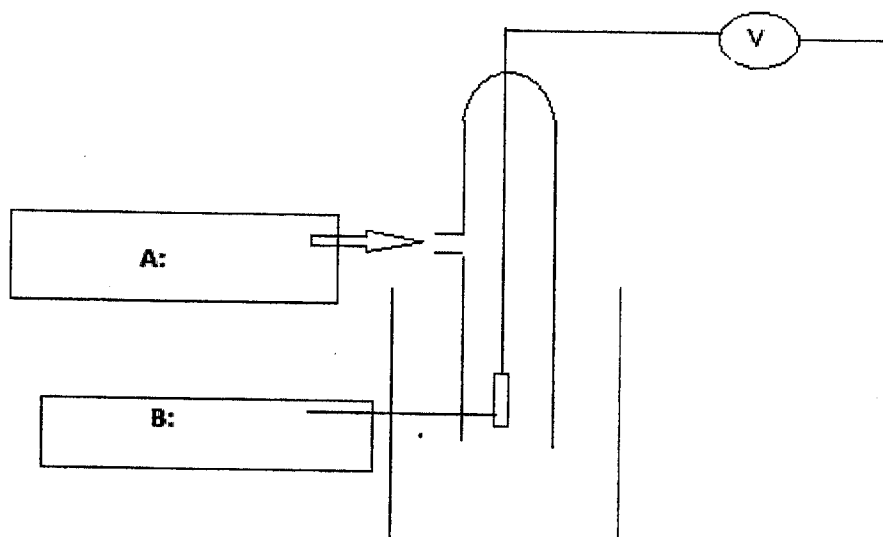
$$E = +0.52 \text{ V}$$

$$E = -0.88 \text{ V}$$

- (a) Calculate the e.m.f. of a nickel-cadmium cell.
- (b) Deduce an overall equation for the reaction that occurs in the cell when it is used.

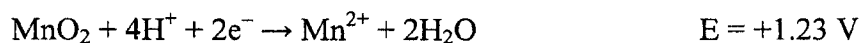
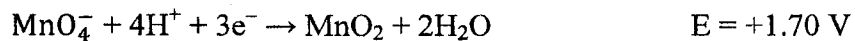
Question A3

- (a) Define the term **standard electrode potential**.
- (b) Identify A and B on the diagram of the **standard hydrogen electrode**.



Question A4

- (a) What is meant by disproportionation?
- (b) Use the following data to deduce whether MnO_2 will disproportionate in acidic solution.

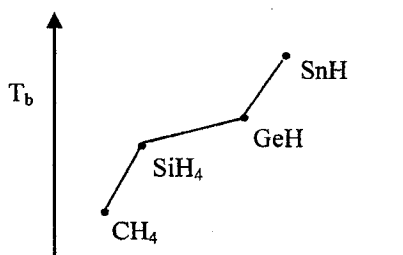


Question A5

- (a) Explain why carbonated (containing carbon dioxide) drinks must be stored in sealed container.
- (b) Once a beverage has been opened, why does it maintain carbonation when refrigerated than at room temperature?

Question A6

The diagram below shows the *boiling point trend of Group 14 hydrides*.



- (a) What is an instantaneous dipole?
- (b) Use the concept of polarizability to explain the trend observed in the diagram above.

Question A7

Consider a substance, X, which has the following properties

ΔH_{vap}	20 kJ/mol
ΔH_{fus}	5 kJ/mol
b.pt	75 °C
m.pt	- 15

Sketch a heating curve for the substance X starting at - 50 °C

Question A8

Some water is placed in a sealed glass container connected to a vacuum pump and the pump turned on reducing the pressure inside. The water appears to boil and then freezes.

- (a) Draw a phase diagram of water to explain these changes
- (b) What would happen if the vacuum pump was left on indefinitely?

Question A9

Compounds with representative formula C_nH_{2n} may be alkenes or cycloalkanes. Some of these compounds exhibit geometrical isomerism. Write down the geometrical isomers and give IUPAC names of:

- (a) 2-Butene
- (b) 1,3-Dimethylcyclobutane

Question A10

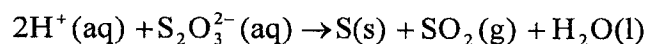
Draw structures of the compounds with the following descriptions

- (a) A straight chain primary alcohol with molecular formula $C_5H_{12}O$
- (b) A cyclic tertiary amine molecular $C_5H_{11}N$
- (c) An allylic alcohol with three carbons only. What is the molecular formula of this compound?

SECTION B**ANSWER QUESTION B1 AND ANY THREE QUESTIONS**

Question B1

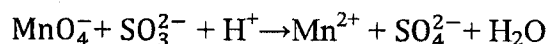
In CHE1000 laboratory you performed an experiment on the effect of temperature on the reaction rate. The reaction studied at different temperatures was of sulphuric acid with sodium thiosulphate as given in a reaction:



- (a) Write an expression of the rate of reaction involving all the species in the above reaction. **[2 marks]**
- (b) According to the experimental conditions, the initial rate of disappearance of sodium thiosulphate at 298.15 K was $1.29 \times 10^{-3} \text{ mol dm}^{-3} \text{ s}^{-1}$.
- (i) Calculate the initial rate of disappearance of H_2SO_4 . **[2 marks]**
- (ii) Determine the rate of production of sulphur dioxide. **[2 marks]**
- (iii) Calculate the rate of reaction. **[2 marks]**
- (c) A plot of $-\ln t$ versus $1/T$ gave a slope of $-3.9392 \times 10^3 \text{ K}$.
- (i) Make a sketch of a plot of $-\ln t$ versus $1/T$ **[2 marks]**
- (ii) What is the activation energy of the reaction? **[5 marks]**

Question B2

Pure sodium sulfite is difficult to get. One method of assessing its purity is by the reaction of potassium manganate (VII), KMnO_4 , in acidic solution according to the **unbalanced ionic** equation shown



Sodium sulphite, Na_2SO_3 , is slowly oxidized in air to sodium sulphate, Na_2SO_4 . 1.75 g of an impure sample of sodium sulphite was dissolved in water and made up to 250 cm^3 with distilled water. 25.0 cm^3 of this solution required 22.8 cm^3 of $0.0216 \text{ mol dm}^{-3}$ potassium manganate(VII) solution for complete oxidation. (Molar mass of sodium sulphite = 126.05 g/mol)

- (a) Balance the equation in acidic media **[4 marks]**
- (b) Calculate the change in oxidation number of sulphur in the reaction of sulphite ions with manganate(VII) ions. **[2 marks]**

- (c) Calculate the number of moles of manganate(VII) ions used in the titration. [2 marks]
- (d) Calculate the number of moles of sodium sulphite present in 25.0 cm³ of the solution. [3 marks]
- (e) Calculate the total mass of pure sodium sulphite in 250 cm³ of the solution. [2 marks]
- (f) Calculate the percentage purity of the sample of sodium sulphite. [2 marks]

Question B3

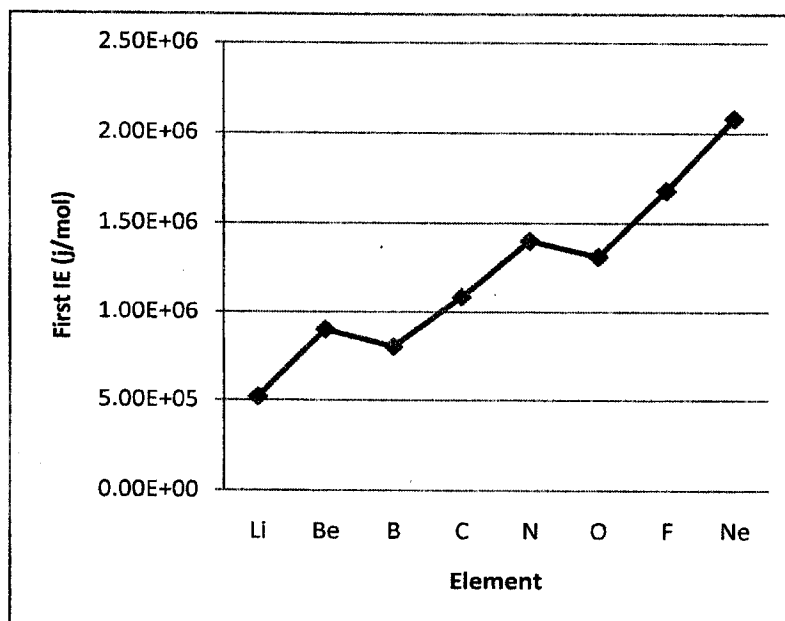
At elevated temperatures, SbCl₅ (g) decomposes into SbCl₃(g) and Cl₂(g) as shown by the following equation:



- (a) An 89.7 g sample of SbCl₅ (molecular mass 299.0 g mol⁻¹) is placed in an evacuated 15.0 dm³ container at 182 °C.
- (i) What is the concentration in moles dm⁻³ of SbCl₅ in the container before any decomposition occurs? [2 marks]
- (ii) What is the pressure in atmospheres of SbCl₅ in the container before any decomposition occurs? [2 marks]
- (b) If the SbCl₅ is 29.2 percent decomposed when equilibrium is established at 182 °C, calculate the value for equilibrium constant K_p and K_c, for this decomposition reaction. [11 marks]

Question B4

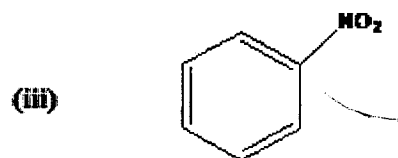
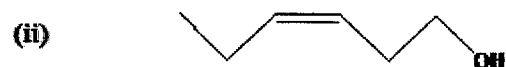
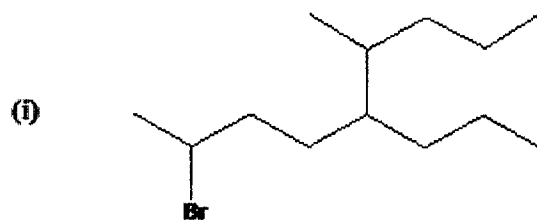
- (a) Define *Ionization Energy*. [1 mark]
- (b) Why do you think energy has to be added to pull an electron away from an atom? [2 marks]
- (c) An electron in H atom in 1s ground electronic state has an energy of -2.18×10^{-18} J. Calculate the ionization energy of H atoms in J mol^{-1} . [6 marks]
- (d) The first ionization energies of period 2 elements show a trend indicated in a graph below.
- (i) Write the ground state electronic configuration of Be and B. [2 marks]
- (ii) Draw an electron energy diagram of Be and B. [2 marks]
- (iii) Explain why ionization energy of B is lower than that of Be. [2 marks]



Question B5

(a) Write IUPAC names for the following given structures.

[7 marks]



(b) Draw the structures whose IUPAC names are given

[8 marks]

- (i) 2-Pentanone
- (ii) 5-Oxohexanenitrile
- (iii) Butyl butanoate
- (iv) N-methylbenzamide
- (v) Methylbenzene

USEFUL DATA

Physical constants

Avogadro constant, N_A	$6.022 \times 10^{23} \text{ mol}^{-1}$
Acceleration due to gravity	9.8 m s^{-2}
Faraday's constant, F	96485 C mol^{-1}
Mass of electron, m_e	$9.11 \times 10^{-31} \text{ kg}$
Planck's constant, h	$6.626 \times 10^{-34} \text{ J s}$
Rydberg constant, R_H	$1.097 \times 10^7 \text{ m}^{-1}$
Speed of light, c	$3.00 \times 10^8 \text{ m s}^{-1}$
Universal gas constant, R	$8.3145 \text{ J mol}^{-1} \text{ K}^{-1}$ $0.083145 \text{ L bar mol}^{-1} \text{ K}^{-1}$ $0.08206 \text{ L atm mol}^{-1} \text{ K}^{-1}$ $62.364 \text{ L torr mol}^{-1} \text{ K}^{-1}$ $62.364 \text{ L mmHg mol}^{-1} \text{ K}^{-1}$

Pressure conversions

$$1 \text{ atm} = 1.01325 \times 10^5 \text{ Pa} = 1.01325 \times 10^5 \text{ N m}^{-2} = 760 \text{ torr} = 760 \text{ mmHg} \\ = 1.01325 \text{ bar}$$

$$1 \text{ bar} = 1.00000 \times 10^5 \text{ Pa} \\ = 1.00000 \times 10^5 \text{ N m}^{-2}$$

Other conversion factors

$$V = J C^{-1} \quad 1 \text{ eV} = 1.602 \times 10^{-19} \text{ J} \\ 1 \text{ J} = 1 \text{ kg m}^2 \text{ s}^{-2}$$

PERIODIC TABLE OF THE ELEMENTS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
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KEY

Atomic number X Atomic mass Name of the element X

1 H 1.01 Hydrogen	2 He 4.00 Helium																	3 Li 6.94 Lithium	4 Be 9.01 Beryllium	5 B 10.81 Boron	6 C 12.01 Carbon	7 N 14.01 Nitrogen	8 O 16.00 Oxygen	9 F 19.00 Fluorine	10 Ne 20.18 Neon	11 Na 23.00 Sodium	12 Mg 24.31 magnesium	13 Al 26.98 Aluminum	14 Si 28.09 Silicon	15 P 30.99 Phosphorus	16 S 32.07 Sulphur	17 Cl 35.45 Chlorine	18 Ar 39.95 Argon	19 K 39.10 Potassium	20 Ca 40.08 Calcium	21 Sc 44.96 Scandium	22 Ti 47.88 Titanium	23 V 50.94 Vanadium	24 Cr 52.00 Chromium	25 Mn 54.94 Manganese	26 Fe 55.85 Iron	27 Co 58.93 Cobalt	28 Ni 58.69 Nickel	29 Cu 63.55 Copper	30 Zn 65.39 Zinc	31 Ga 69.72 Gallium	32 Ge 71.61 Germanium	33 As 74.92 Arsenic	34 Se 78.96 Selenium	35 Br 79.90 Bromine	36 Kr 83.80 Krypton	37 Rb 85.47 Rubidium	38 Sr 87.62 Strontium	39 Y 88.91 Yttrium	40 Zr 91.22 Zirconium	41 Nb 92.91 Niobium	42 Mo 95.94 Molybdenum	43 Tc 97.91 Technetium	44 Ru 101.07 Ruthenium	45 Rh 102.91 Rhodium	46 Pd 106.42 Palladium	47 Ag 107.87 Silver	48 Cd 112.41 Cadmium	49 In 114.82 Indium	50 Sn 118.71 Tin	51 Sb 121.76 Antimony	52 Te 127.60 Tellurium	53 I 126.90 Iodine	54 Xe 131.29 Xenon	55 Cs 132.91 Caesium	56 Ba 137.33 Barium	57-71 Lanthanum 89-103	57 La 138.91 Lanthanum	58 Ce 140.12 Cerium	59 Pr 140.91 Praseodymium	60 Nd 144.24 Neodymium	61 Pm 144.91 Promethium	62 Sm 150.36 Samarium	63 Eu 151.97 Europium	64 Gd 157.25 Gadolinium	65 Tb 158.93 Terbium	66 Dy 162.50 Dysprosium	67 Ho 164.93 Holmium	68 Er 167.26 Erbium	69 Tm 168.93 Thulium	70 Yb 173.04 Ytterbium	71 Lu 174.97 Lutetium	87 Fr (223.02) Francium	88 Ra 226.03 Radium	89-103	90 Th 232.04 Thorium	91 Pa 231.04 Protactinium	92 U 238.03 Uranium	93 Np 237.05 Neptunium	94 Pu 244.0 Plutonium	95 Am 243.06 Americium	96 Cm 247.07 Curium	97 Bk 247.07 Berkelium	98 Cf 251.08 Californium	99 Es 252.08 Einsteinium	100 Fm 257.10 Fermium	101 Md 260 Mendelevium	102 No 259.10 Nobelium	103 Lr 262.11 Lawrencium
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THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR
FINAL EXAMINATION

CHE 2112 INTRODUCTORY BIOCHEMISTRY

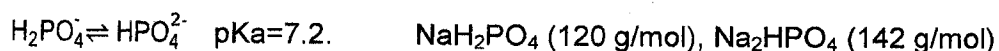
TIME: THREE HOURS

1. Answer any **five (5)** questions
2. Each question carries **20 marks**
3. There are **Four (4)** printed pages in this examination paper

Question 1

(a) **Explain** the term **buffer** solution. Use HA/A^- as an example of the acid/conjugate base pair in your explanation. **[5 marks]**

(b) **Calculate** the number of grams of solid NaH_2PO_4 and Na_2HPO_4 that are needed to prepare 1 dm^3 of a 0.1 M phosphate buffer, $pH\ 7.1$. Useful information is provided below:



[5 marks]

(c) Porins are proteins used as channels in bacteria and are almost 100 % β -barrels where the β -pleated sheets are arranged in an anti-parallel manner.

(i) Using 'R' as the side chain, **draw** the structure of the β -pleated sheets, **highlighting** the outstanding features. **[5 marks]**

(ii) The tripeptide, $^+H_3N\text{-Glu-Ser-Lys-COO}^-$, forms part of the N terminus of the porin extracted from *Salmonella typhimurium*. It was solubilized in the buffer in part (b) above.

Estimate the net charge of the tripeptide at the pH of the buffer in part (b) above. **[5 marks]**

pK_a values for amino acids

	$pK_a\ 1$ (carboxyl)	$pK_a\ 2$ (amino)	$pK_a\ 3$ (side chain)
Ser	2.2	9.2	
Glu	2.2	9.7	4.3
Lys	2.2	9.0	10.5

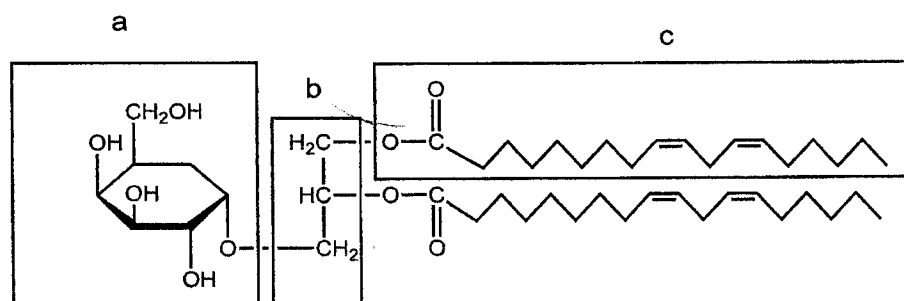
Question 2

(a) Starting with the **open chain-Fischer** structure of glucose, **show** the process of mutarotation. **[6 marks]**

(b) For each of the following, **explain in 3 sentences** the most prominent differentiating features.

- (i) Glycogen **[6 marks]**
- (ii) Cellulose **[2 marks]**

(c) The following is the structure of a glycolipid isolated from chloroplasts.



Name the

- (i) sugar "a" **[2 marks]**
- (ii) alcohol "b" **[1 mark]**
- (iii) fatty acid "c" using IUPAC nomenclature and short hand notation **[3 marks]**

Question 3

Provide the mechanism for hydrolysis of RNA under alkaline conditions using the full structure of the dinucleotide AU. **[20 marks]**

Question 4

Lactate dehydrogenase is an enzyme that catalyses the reduction of pyruvate to lactate with the help of NADH. Consider the following half reduction reactions



Given that temperature is 298 K, R is $8.314 \text{ J mol}^{-1}\text{K}^{-1}$, F is $96\,485 \text{ J V}^{-1} \text{ mol}^{-1}$

- (a) **Write** down the overall equation for the reduction of pyruvate **[3 marks]**
- (b) **Calculate** the ΔE° for the reaction in (a) **[3 marks]**
- (c) **Calculate** the ΔG° for the reaction in (a) **[3 marks]**
- (d) If the concentrations of pyruvate, lactate, NAD^+ and NADH are adjusted to 0.002 M, 10 M, 0.4 M and 0.003 M respectively, would you expect the reaction in (a) to be spontaneous? **Briefly** explain why. **[11 marks]**

Question 5

- (a) According to the Michaelis-Menten equation, **what** is the v/V_{\max} ratio when $[\text{S}] = 8 K_m$? **[5 marks]**
- (b) If $V_{\max} = 150 \text{ mmol/sec}$ and $K_m = 2 \text{ mM}$, **what** is the velocity of the reaction when $[\text{S}] = 15 \text{ mM}$? **[5 marks]**
- (c) For a Michaelis - Menten enzymatic reaction, $k_1 = 7 \times 10^7/\text{Msec}$, $k_{-1} = 1 \times 10^3/\text{sec}$, and $k_2 = 2 \times 10^4/\text{sec}$. **What** is the value of K_m ? **[5 marks]**
- (d) On the basis of the value obtained in (c) above, **does** this enzyme have a strong affinity for its substrate? In just one sentence, **explain** your answer. **[5 marks]**

Question 6

(a) **Draw** a sketch showing the effect of the following factors on enzyme action:

- (i) pH
- (ii) Temperature

[10 marks]

(b) **What** changes to the structure of the enzyme molecule account for the observed effects in (a) above? Be brief and to the point.

[10 marks]

END OF EXAMINATION

**THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES**

2013 ACADEMIC YEAR (SECOND HALF)

FINAL EXAMINATIONS

CHE2522: FUNCTIONAL GROUP AND ARENE CHEMISTRY

TIME: THREE HOURS

INSTRUCTIONS:

There are two sections A and B

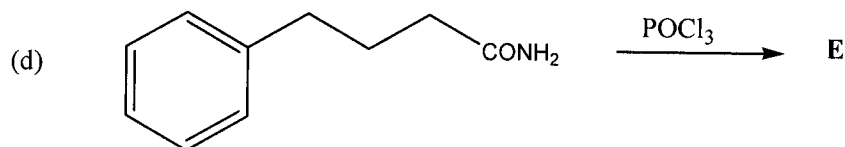
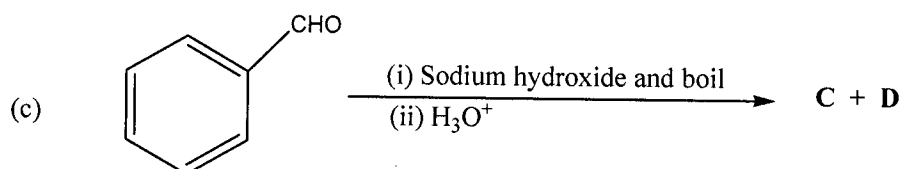
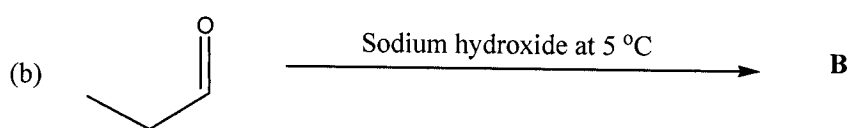
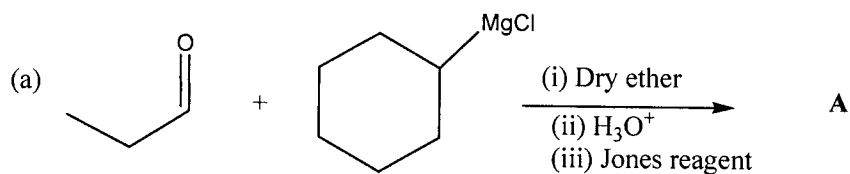
- 1. Answer any two questions from each section**
- 2. Each question is 30 marks**
- 3. Use separate booklets for each section**
- 4. Indicate your tutorial group on all your answer booklets**

SECTION A

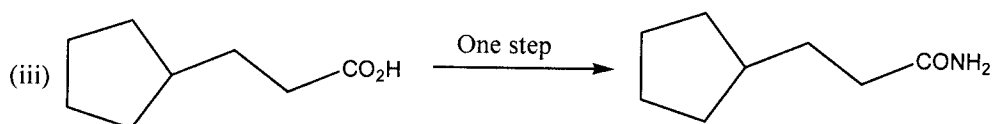
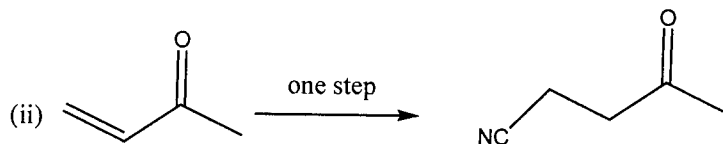
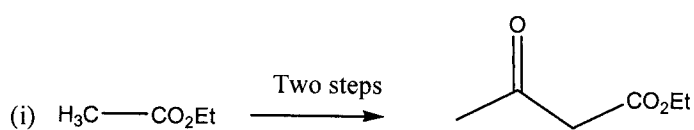
Answer any two questions from this section

A1

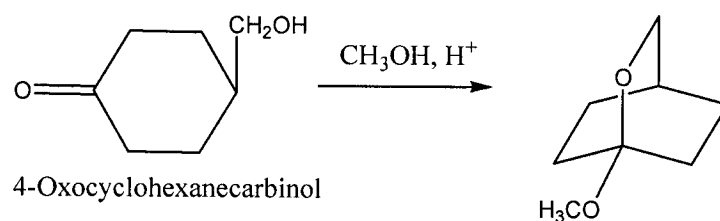
(a) Give the structures and IUPAC names obtained in the following reactions. (10 marks)



(b) What are the reagents (names and structures) and conditions needed to carry out the given transformations below? (7 marks)



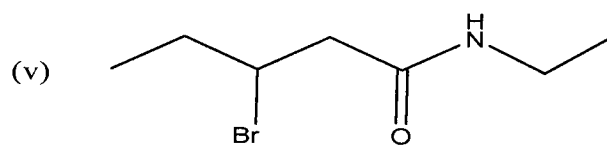
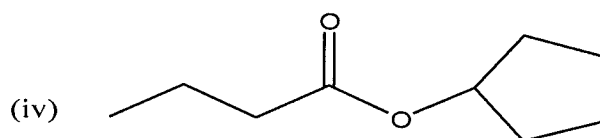
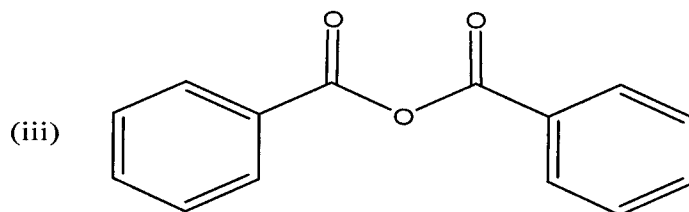
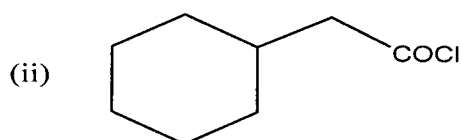
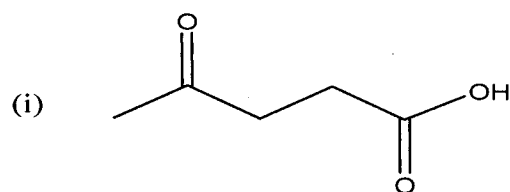
- (c) Write a reasonable mechanism for the reaction between 4-oxocyclohexanecarbinol and methanol in presence of catalytic sulphuric acid to give the oxabicyclic acetal. (7 marks)



- (d) Write a scheme for the synthesis of 2-butanone from ethanal and chloroethane. (6 marks)

A2

- (a) Give IUPAC names for the carboxylic acid and carboxylic acid derivatives given below. (10 marks)



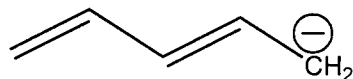
(b) Advise on the mechanism between benzoic acid and ethanol in presence of acid.

(7 marks)

(c) Draw the all the π -molecular orbitals of:

(i) 1,3-butadiene and appropriately fill in the electrons in the orbitals and hence label the HOMO and LUMO. (6 marks)

(ii) 2,4-pentadienyl anion and appropriately fill in electrons and hence label the HOMO and LUMO. (7 marks)



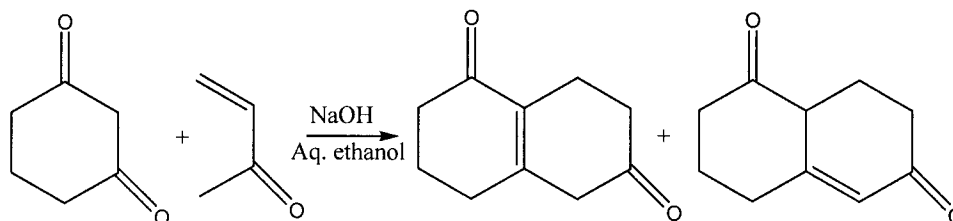
Pentadienyl anion

A3

(a) 1,3-cyclohexadione and 3-buten-2-one easily react in the presence of sodium hydroxide to give the bicyclic structures indicated below. (10 marks)

(i) Write a mechanism for this reaction.

(ii) What is the name of this reaction?



(b) Describe reactions that you would carry out to distinguish among classes of amines (primary, secondary and tertiary amines). State your observations in each reaction. (10 marks)

(c) Compounds of the form A-NH₂ react with aldehydes and ketones in acetic acid to give various derivatives. (10 marks)

(i) Give the structure of the product between cyclohexanone and A-NH₂ in presence of acetic acid.

(ii) Write a mechanism for the reaction in (i).

(iii) Give four types of compounds of the form A-NH₂.

SECTION B

Answer any two questions from this section

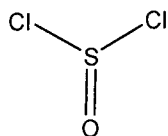
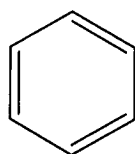
B1

(a) The acyclic conjugated multi-bonded hydrocarbons have better stability and reactivity compared to isolated multi-bonded hydrocarbons. (10 marks)

(i) Give the products and mechanisms for the reactions between 1,3-butadiene and HCl.

(ii) Draw the energy diagram for this reaction.

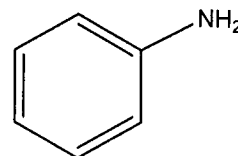
(b) Given benzene, thionyl chloride, hydrazine, sodium nitrite, hydrochloric acid and other reagents and solvents, show all the steps and intermediates to aniline. (13 marks)



Thionyl chloride



Hydrazine

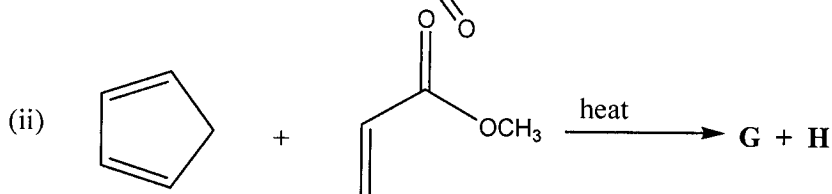
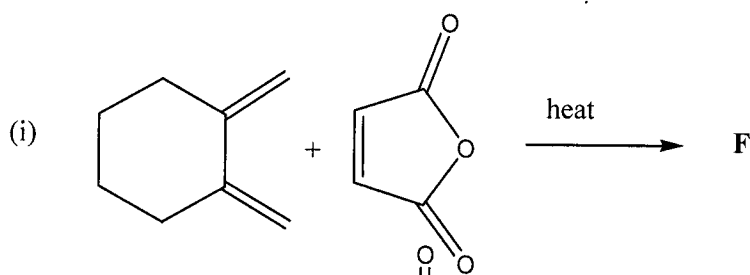


Aniline

NaNO_2
Sodium nitrite

and HCl

(c) Give the products arising from the following Diels-Alder reactions. (7 marks)



B2

(a) Provide the following information for 1,2dimethylbenzene (o-xylene) in the answer booklet. (6 marks)

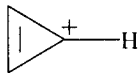

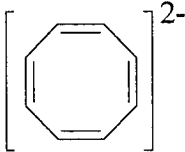
- (i) Draw two Kekule structures.
- (ii) Why are these structures not isomers? What are they?
- (iii) Give the carbonyl products formed on ozonolysis followed by treatment with Zn/CH_3COOH .

(b) What are the necessary conditions for: (4 marks)

- (i) Aromaticity?
- (ii) Anti-aromaticity?

(c) Reproduce the Table below and provide the solutions in your answer booklet.

For each of the structures given below, give the number of π -electrons; energy levels of π -MOs and electron distribution; and state the aromaticity. (6 marks)

Structure	Number of π Electrons	π MOs	Aromaticity
(i) 			
(ii) 			
(iii) 			

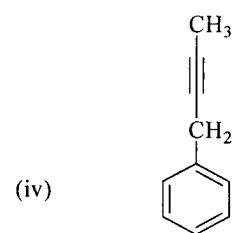
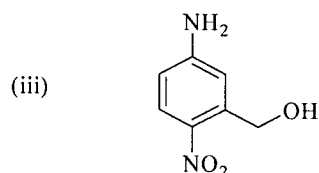
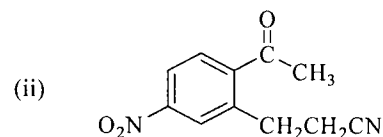
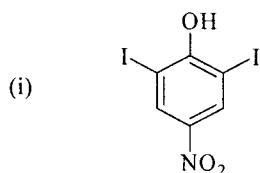
(d) Draw structures for the following compounds:

(6 marks)

- (i) 2,6-Dichloro-4-nitroaniline
- (ii) 1-(4-Bromo-3-nitro-phenyl)ethanone
- (iii) 2,4-difluoro-3-propylphenol

(e) Provide the systematic name for the following compounds:

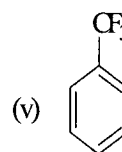
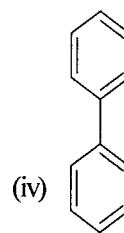
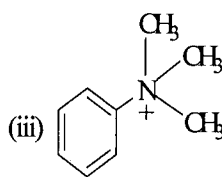
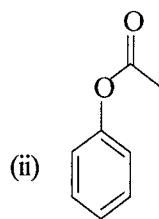
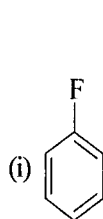
(8 marks)



B3

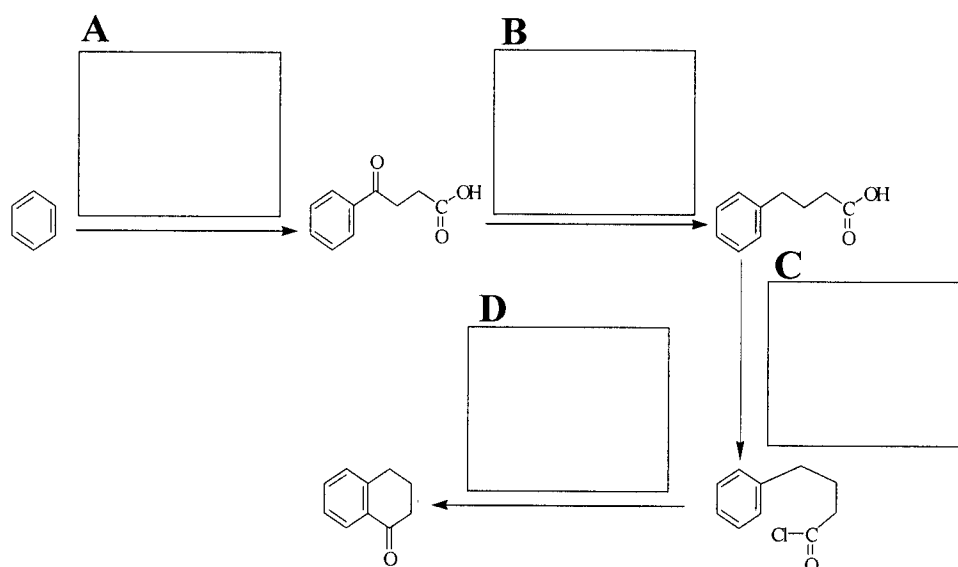
(a) For each of the following compounds, state the directing nature of the substituent and its reactivity compared to benzene:

(5 marks)

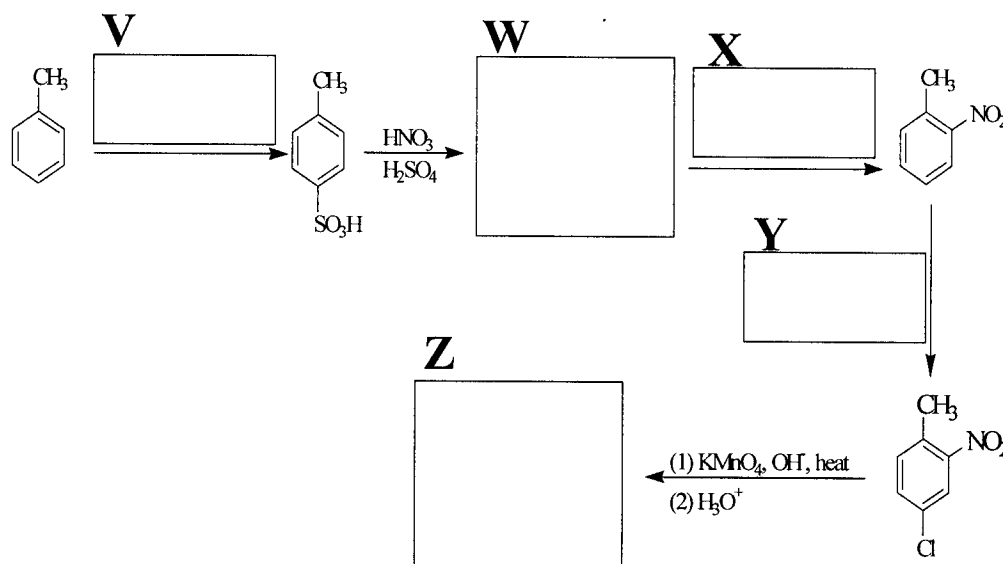


(b) Reproduce the schemes given below, and provide the required missing reagents and intermediates in your answer booklet.

- (i) A standard synthetic sequence for building a six-membered cyclic ketone onto an existing aromatic ring is outlined below. Specify the reagents necessary for each step. (5 marks)

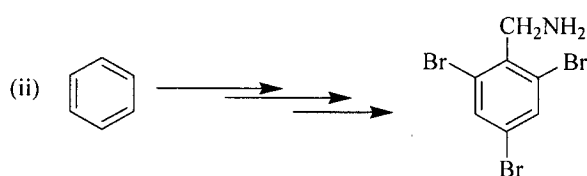
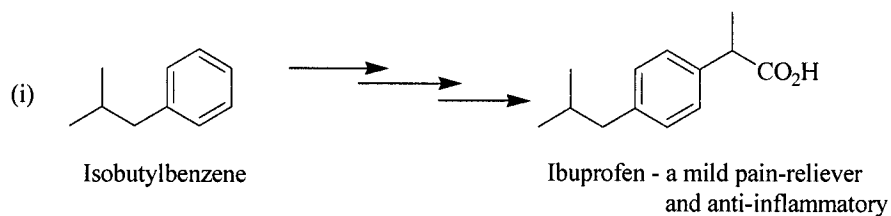


- (ii) Modifying toluene to a useful benzene derivative (5 marks)



(c) Explain why *m*-xylene undergoes nitration **100** times faster than *p*-xylene. (5 marks)

(d) Suggest a suitable series of reactions for carrying out each of the following synthetic transformations: (10 marks)



END OF EXAMINATIONS

UNIVERSITY EXAMINATION
2013 FINAL EXAMINATIONS

CHE2615 BASIC PHYSICAL CHEMISTRY

Duration: Three (3) Hour

Instructions:

This question paper is divided in two sections: A (40)&B (60)

Answer **all** questions in section A

Answer **4** questions in Section B

Answer Section A and B in **separate answer booklets**.

You are reminded to answer questions in a clear and logical manner

Useful Information and Constants:

$R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$, $h = 6.63 \times 10^{-34} \text{ Js}$, $N = 6.02 \times 10^{23} \text{ mol}^{-1}$,
Boltzmann constant = $1.381 \times 10^{-23} \text{ JK}^{-1}$, 1 mass unit = $1.6605 \times 10^{-27} \text{ kg}$.
Molar volume of gas at STP = $22.4 \text{ dm}^3 \text{ mol}^{-1}$, STP = 273 K and 1 atm
Avogadro's constant = $6.02 \times 10^{23} \text{ mol}^{-1}$, $A = 0.509$, $F = 96485 \text{ C mol}^{-1}$
 $\text{Cu} = 63.55$, $\text{He} = 4.00$

Useful equations

$$\frac{2.303RT}{v_e F} = \frac{0.05916 V}{v_e} ; V = JC^{-1}$$

$$PV = nRT$$

$$\left(P + \frac{n^2 a}{V^2} \right) (V - nb) = nRT$$

$$w = -nRT \ln \frac{V_f}{V_i}$$

$$\Delta S = nR \ln \frac{V_2}{V_1}$$

Question A1

What is the most probable speed of He atoms if the gas temperature is 264 K?

Question A2

From the conditions and the given definitions of the system, determine whether there is work done by the system, work done on the system, or no work done in the following.

- A balloon expands as a small piece of dry ice (solid CO_2) inside the balloon sublimates (balloon 5 system).
- The space shuttle's cargo bay doors are opened to space, releasing a little bit of residual atmosphere (cargo bay 5 system).

Question A3

Compound A is burned in a bomb calorimeter that contains 2.50 liters of water. If the combustion of 0.175 moles of this compound causes the temperature of the water to rise 45.0°C , what is the molar heat of combustion of compound A?

Question A4

What is the change in G for a process in which 0.022 mole of an ideal gas goes from 2505 psi (pounds per square inch) to 14.5 psi at a room temperature of 295 K?

Question A5

The most abundant isotope of uranium is ^{238}U ; 99.276% of the atoms in a sample of uranium are ^{238}U . Calculate the activity of the ^{238}U in 1 L of a 1.00 M solution of the uranyl ion, UO_2^{2+} . Assume that the rate constant for the decay of this isotope is 4.87×10^{-18} disintegrations per second.

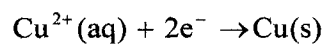
Question A6

The ionic molar conductance at infinity dilution of $\text{Ba}^{2+}(\text{aq})$ and $\text{F}^{-}(\text{aq})$ are 0.01273 and $0.00470 \Omega^{-1}\text{m}^2\text{mol}^{-1}$ respectively at 25°C . The solubility of saturated solution BaF_2 at 25°C is $1.82 \times 10^{-2} \text{mol L}^{-1}$.

- Calculate molar conductance, Λ_m° , at infinity dilution of BaF_2 at 25°C .
- What is the specific conductance, κ , of a saturated solution of BaF_2 ?

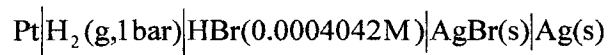
Question A7

A current of 0.473 A passed through a solution of copper sulphate for 5.00 minutes. Calculate the mass of copper deposited at the cathode. The reaction at the cathode is

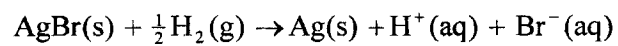


Question A8

Consider the cell below with E_{cell}° of +0.0711 V at 25 °C



The activity of HBr, a_{HBr} , in the is 1.559×10^{-7} . The cell reaction is



with $\nu_e = 1$.

- (a) Write the Nernst equation of the reaction.
- (b) Calculate the emf of the cell.

SECTION B**ANSWER ANY FOUR (4) QUESTIONS**

Question B1

- (a) Under what conditions would expect a real gas to exhibit ideal gas behavior?
- (b) Consider a 1.00 mole sample of sulfur dioxide, SO_2 , that has a pressure of 5.00 atm and a volume of 10.0 L. Predict the temperature of this sample of gas using the ideal gas law and the van der Waals equation. ($a = 6.714 \text{ atm L}^2/\text{mol}^2$, $b = 0.05636 \text{ L/mol}$)

Question B2

- (a) Differentiate between state and non-state functions. Give one example of each.
- (b) A piston filled with 0.0400 mole of an ideal gas expands reversibly from 50.0 mL to 375 mL at a constant temperature of 37.0°C . As it does so, it absorbs 208 J of heat. Calculate q , w , ΔU , and ΔH for the process.

Question B3

A 1.00-g sample of $\text{H}_2\text{O}(\text{g})$ at 100.0°C with a pressure of 1.00 atm has a volume of 1.70 L. It is isothermally compressed by an external pressure of 2.00 atm until it reaches a volume of 0.850 L. Determine ΔS_{sys} , ΔS_{surr} , and ΔS_{univ} . Assume ideal gas behavior.

Question B4

For a general reaction " $a\text{A} + b\text{B} \rightarrow \text{products}$ ", the following initial rates are determined experimentally when react ions are set up with the initial amounts indicated in units of molarity, M.

[A](M)	[B](M)	Initial rate (M/s)
1.44	0.35	3.37×10^{-3}
1.44	0.70	2.15×10^{-3}
2.89	0.35	2.69×10^{-3}

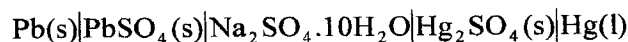
Assuming that the rate law can be written as

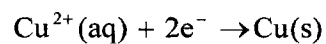
$$\text{rate} = k [\text{A}]^m [\text{B}]^n$$

determine the values of m , n , and k .

Question B4

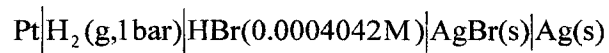
The electromotive force, emf, of the cell:



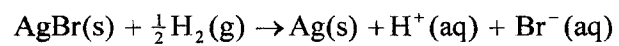


Question A8

Consider the cell below with E_{cell}° of +0.0711 V at 25 °C



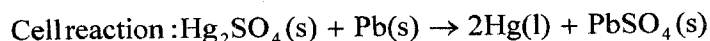
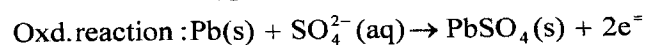
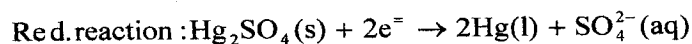
The activity of HBr, a_{HBr} , in the is 1.559×10^{-7} . The cell reaction is



with $\nu_e = 1$.

- (a) Write the Nernst equation of the reaction.
- (b) Calculate the emf of the cell.

is 0.9647 V, and temperature coefficient, $(\partial E / \partial T)_p$ is $+1.74 \times 10^{-4} \text{ V K}^{-1}$ at 25°C . The half-cell and cell reactions are:



- (a) Calculate ΔG of the reaction.
- (b) Determine ΔS of the reaction.
- (c) Calculate ΔH of the reaction.
- (d) The standard enthalpy of formation at 298 K, $\Delta H_{f,298\text{K}}^\circ$, of $\text{PbSO}_4(\text{s})$ and $\text{Hg}_2\text{SO}_4(\text{s})$ are $-920.0 \text{ kJ mol}^{-1}$ and $-743.1 \text{ kJ mol}^{-1}$ respectively.
 - (i) Calculate ΔH of the above cell reaction.
 - (ii) Compare the value calculated in d (i) and that of (c) above.

Question B5

The resistances of two aqueous solutions of the salt MX, a strong electrolyte, were measured at 25°C in a conductance cell having a cell constant equal to 18.72 m^{-1} are indicated below.

Solution	$c \text{ (mol L}^{-1}\text{)}$	$R \text{ (}\Omega\text{)}$
1	0.0005000	2622.0
2	0.005000	270.4

- (a) Calculate specific conductance κ of
 - (i) solution 1
 - (ii) solution 2
- (b) Calculate molar conductance, Λ_m , of
 - (i) solution 1
 - (ii) solution 2
- (c) Assume that the solutions of MX obey the Kohlrausch square-root law, use linear regression to determine the limiting molar conductance at infinity dilution, Λ_m° , of MX.
- (d) If the ionic molar conductance at infinity dilution for M^+ is $0.00755 \text{ }\Omega^{-1}\text{m}^2 \text{ mol}^{-1}$, use the value of Λ_m° , of MX calculated in (c) above to determine the ionic molar conductance at infinity dilution of X^- .

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR
FINAL EXAMINATIONS

CHE 3122: ENERGY TRANSDUCTION SYSTEMS

TIME: THREE HOURS

INSTRUCTIONS:

1. Answer any **FIVE (5)** questions
 2. Ensure that there are **TWO (2)** printed pages in this examination paper
 3. Each question carries **TWENTY (20)** marks
-

QUESTION 1

- a) **What types of molecules have difficulty crossing the plasma membrane? Why?**
[5 marks]
- b) **The Na⁺- K⁺ pump, the major electrogenic pump in animal cells, exchanges sodium ions for potassium ions, both of which are cations. How does this exchanger generate a membrane potential (voltage)?**
[5 marks]
- c) **In biological membranes, there is little or no (some say 'forbidden') flip-flop movement of lipids and yet there is an enzyme known as flipase. How do you explain this seemingly contradictory situation?**
[5 marks]
- d) **Name TWO types of proteins that are found in biological membranes. Describe an experiment that you would do to distinguish one type of membrane protein from another.**
[5 marks]

QUESTION 2

- a) **Explain the differences between skeletal, cardiac and smooth muscle.** [5 marks]
- b) **What does the term "muscle fatigue" refer to and how does the body recover from it?**
[5 marks]
- c) **Describe the mechanism of muscle contraction highlighting the role of ATP, myosin and actin.**
[10 marks]

QUESTION 3

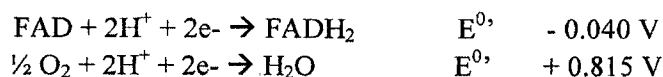
- a) What is **photorespiration** and why is it said to reduce photosynthetic efficiency? [5 marks]
- b) **Discuss the mechanisms (pathways)** that plants have evolved to ensure that photorespiration is minimised? [15 marks]

QUESTION 4

Using a **well labelled diagram** show the series of events that take place in the light-independent reactions (Calvin Cycle) and give the **OVERALL** equation for the Calvin Cycle. (Include all enzymes involved **EXCEPT** for the regeneration of Ribulose 1,5-bisphosphate.) [20 marks]

QUESTION 5

- a) **Write** a balanced equation for the reduction of oxygen by flavin adenine dinucleotide and **calculate** the standard free energy change, ΔG° . Reduction half reactions are given below. (Faraday's constant is $96,485 \text{ J.V}^{-1}.\text{mol}^{-1}$). [6 marks]



- b) The P/O ratio for FADH_2 is 1.5, **calculate** the volume of oxygen that is consumed when $150 \mu\text{mol}$ of ADP is added to a suspension of resting mitochondria in an oxygen chamber at 25°C when FADH_2 is the only oxidisable substrate available. Assume oxygen behaves as an ideal gas. (1 mole of gas occupies $24\,000 \text{ cm}^3$ at R.T.P). [10 marks]
- c) **Which** electron transport chain protein complex contains the heme b_{560} component? **Discuss** its main function. [4 marks]

QUESTION 6

- a) **Discuss** the main features of the chemiosmotic theory. [8 marks]
- b) **Describe** what happens when the following compounds are added to actively respiring mitochondria and **explain** the reason for their observed effect. [12 marks]
- Mixture of Rotenone and succinate
 - 2,4-Dinitrophenol (DNP)
 - Oligomycin

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR FINAL EXAMINATIONS

CHE3222 SPECTROSCOPIC METHODS OF ANALYSIS

TIME: THREE HOURS

INSTRUCTIONS

- 1 There are five questions in this examination paper.
 - 2 Answer any **four** questions. Questions carry equal marks.
 - 3 Questions carry equal marks
 - 4 Show your reasoning and/or calculations clearly.
-

Question 1

- (a) (i) What are liquid-junction potentials and how do they arise? How can they be eliminated or minimized?
- (ii) Given the E° for the following half- reactions:
- $$\text{Cu}^+ + e \longrightarrow \text{Cu} \quad E^\circ_{\text{red}} = 0.52\text{V}$$
- $$\text{Cu}^{2+} + 2e \longrightarrow \text{Cu} \quad E^\circ_{\text{red}} = 0.34\text{V}$$
- What is E° for the reaction:
- $$\text{Cu}^+ \longrightarrow \text{Cu}^{2+} + e ?$$
- (b) (i) Two samples of different alcohols were separated on a capillary GC column. Retention times recorded were 370 and 385seconds respectively with peak base widths of 16.0 s and 17.0 s while an un-retained air peak occurred at 10.0s. Calculate the resolution, R_s , for the two compounds.
- (ii) The height equivalent to a theoretical plate (HETP) is defined according to the equation:
- $$\text{HETP} = A + B/u + Cu;$$
- where u is the average velocity of the mobile phase. A , B , and C are factors which contribute to band broadening. What is the name of this equation which is used to ensure optimal operational conditions in GLC? Explain the nature and role of any one of the constants A , B and C in the fundamental equation for chromatography?

- (c) (i) Describe the differences in theory of operation and equipment needed for potentiometric and voltammetric measurements. List any guiding theoretical expressions that relate activity or concentration of an analyte to a measured electrochemical signal.
- (ii) Draw a well labeled diagram of the components of a High Performance Liquid Chromatography (HPLC). Indicate where band broadening could occur and how this can be minimized.

Question 2

- (a) (i) Compare the efficiency of two chromatographic separations whose retention factors (R_f values) of 0.90 and 0.25 respectively, with clear reference to the mobile and stationary phases of the separatory system.
- (ii) Substances A and B have retention times of 16.4 and 17.63 min respectively on a 30.0 cm column. An un-retained species passes through the column in 1.30 min. The peak widths at base for the two analytes are 1.11 and 1.21 min respectively. Using $N = 16 (t_R/w)^2$ calculate the individual residence times for either analyte A or B on the stationary phase.
- (b) Distinguish between:
- (i) Voltammetry and polarography.
 - (ii) A residual current and a diffusion current
 - (iii) conductance and conductivity
 - (iv) linear scan polarography and pulse polarography
 - (v) isocratic and gradient elution
- (c) Voltammetry was used to determine the zinc content of a breakfast cereal. A 2.314g sample was digested in boiling concentrated nitric acid. After the sample dissolved, it was diluted to 100ml. A 5.00ml portion of this solution was analyzed by differential pulse polarography, giving a current of 2.31 μ A. When 50.0 μ L of 100ppm zinc standard was added to this solution, the current was 2.99 μ A. What is the concentration of zinc in the cereal?

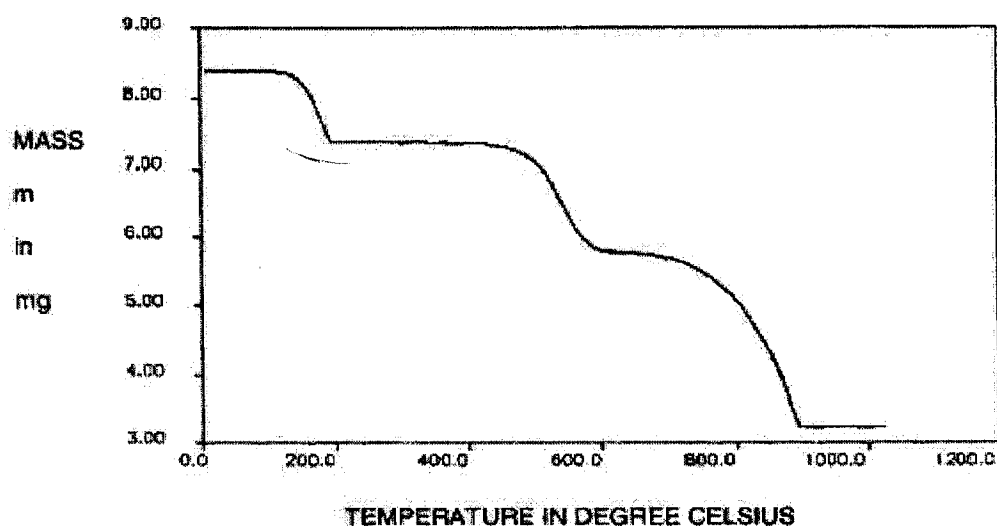
Question 3

- (a) An unknown amount of Co^{2+} ions in a food sample produces a faradic current of $12.3\mu\text{A}$ on a normal pulse voltammogram. After 0.100ml of $1.0 \times 10^{-3}\text{M}$ Co^{2+} is added to the original volume of 5.00ml , the new current is $28.2\mu\text{A}$. Calculate the original amount of cobalt in the food sample.
- (b) Solute is transported to an electrode by diffusion, convection and electrostatic attraction. In polarography, we want the current to be limited by diffusion. Explain how convective and electrostatic attractions are minimized.
- (c) (i) Define the term "retention time", and any name two types of chromatography in which retention time is used. Draw an illustrative schematic diagram to show the chromatogram for analytes x , y and z given that $t_{Ry} > t_{Rx} > t_{Rz}$.
- (ii) Calculate the retention factor, R_f , for a chromatography experiment during which one analyte was recorded as having travelled 2.1cm , while the solvent front travelled 2.8cm . What are the units of R_f ?

Question 4

- (a) (i) Explain how you would determine the molar conductivity at infinite dilution for a strong and weak electrolyte in a food sample
- (ii) In coulometric titration of Fe^{2+} with Ce^{4+} which were generated at the cathode. The resistance was $R = 150\ \Omega$, potential was 0.705V and the end point was reached after 352 seconds. Calculate the amount of iron in μg . Take $F = 96487$, $\text{RAM Fe} = 55.85$
- (b) (i) You want to determine A solution contains 1.0 g of iodine dissolved in 20.0 cm^3 of potassium iodide solution. If we shook this solution with 20.0 cm^3 of tetrachloromethane (assume $x\text{ g}$ of iodine was removed from the aqueous phase) how much iodine will be transferred into the tetrachloromethane, given that K_D for water/chloroform at 25°C is 85 ?
- (ii) The peak of one of many analytes on a GC chromatogram has a retention time of 52.3 mm , and width at peak base = 9.0 mm . Calculate the number of theoretical plates for the column.

- (c) (i) Describe the basic principles involved in Thermogravimetry (TG) and the type of information obtained by the application of this technique. What additional information, over that obtained by TG, may be obtained using Differential Scanning Calorimetry (DSC)?
- (ii) The thermogram below is for the stepwise decomposition of a sample of calcium oxalate monohydrate powder ($\text{Ca}(\text{COO})_2 \cdot \text{H}_2\text{O}$) as a function of temperature to yield CaO at temperatures above 900°C . From the thermogram, suggest a mechanism for each step in the decomposition.



Question 5

- (a) The determination of riboflavin in vitamin formulations can be carried out polarographically because it is more easily reduced than other vitamin B factors. A 250.1 mg vitamin tablet is dissolved to form a 100.00ml solution. A 10.0ml aliquot of this solution was added to a 0.1M phosphate supporting electrolyte at pH 7.2 and diluted to 50ml. The diffusion limited current was $0.28 \mu\text{A}$. A 50.0ml standard 4.10ppm riboflavin solution in the same electrolyte gave diffusion limited current of $0.45 \mu\text{A}$. Calculate the riboflavin concentration in the diet supplement.

- (b) Distinguish between X-ray fluorescence and X-ray absorption measurements. Calculate the wavelength limit for an x-ray operated at 30kV. Planck's constant, $h = 6.63 \times 10^{-34} \text{Js}$; $c = 3.00 \times 10^8 \text{ms}$.
- (c) (i) Name any three (03) key components of a gas chromatograph, then, give details of what occurs in the sample injection port.
- (ii) The efficiency of a chromatographic (GLC) column is expressed as (N) the number of theoretical plates. Calculate the retention time of a column for which one analyte peak has width at peak base = 20.0 s, and number of theoretical plates equal to 500 plates.
-

END OF EXAMINATION

PERIODIC TABLE OF THE ELEMENTS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
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KEY
 Atomic number
X
 Atomic mass
 Name of the element X

1 H 1.01 Hydrogen	2 He 4.00 Helium											17 Cl 35.45 Chlorine	18 Ar 39.95 Argon				
3 Li 6.94 Lithium	4 Be 9.01 Beryllium											13 Al 26.98 Aluminum	14 Si 28.09 Silicon	15 P 30.99 Phosphorus	16 S 32.07 Sulfur		
11 Na 23.00 Sodium	12 Mg 24.31 magnesium	21 Sc 44.96 Scandium	22 Ti 47.88 Titanium	23 V 50.94 Vanadium	24 Cr 52.00 Chromium	25 Mn 54.94 Manganese	26 Fe 55.85 Iron	27 Co 58.93 Cobalt	28 Ni 58.69 Nickel	29 Cu 63.55 Copper	30 Zn 65.39 Zinc	31 Ga 69.72 Gallium	32 Ge 71.61 Germanium	33 As 74.92 Arsenic	34 Se 78.96 Selenium	35 Br 79.90 Bromine	36 Kr 83.80 Krypton
19 K 39.10 Potassium	20 Ca 40.08 Calcium	39 Y 88.91 Yttrium	40 Zr 91.22 Zirconium	41 Nb 92.91 Niobium	42 Mo 95.94 Molybdenum	43 Tc 97.91 Technetium	44 Ru 101.07 Ruthenium	45 Rh 102.91 Rhodium	46 Pd 106.42 Palladium	47 Ag 107.87 Silver	48 Cd 112.41 Cadmium	49 In 114.82 Indium	50 Sn 118.71 Tin	51 Sb 121.76 Antimony	52 Te 127.60 Tellurium	53 I 126.90 Iodine	54 Xe 131.29 Xenon
37 Rb 85.47 Rubidium	38 Sr 87.62 Strontium	72 Hf 178.49 Hafnium	73 Ta 180.95 Tantalum	74 W 183.84 Tungsten	75 Re 186.21 Rhenium	76 Os 190.23 Osmium	77 Ir 192.22 Iridium	78 Pt 195.08 Platinum	79 Au 196.97 Gold	80 Hg 200.59 Mercury	81 Tl 204.38 Thallium	82 Pb 207.2 Lead	83 Bi 208.98 Bismuth	84 Po 208.98 Polonium	85 At 209.99 Astatine	86 Rn 222.02 Radon	
55 Cs 132.91 Cesium	56 Ba 137.33 Barium	89-103										87 Fr (223.02) Francium	88 Ra 226.03 Radium				

57 La 138.91 Lanthanum	58 Ce 140.12 Cerium	59 Pr 140.91 Praseodymium	60 Nd 144.24 Neodymium	61 Pm 144.91 Promethium	62 Sm 150.36 Samarium	63 Eu 151.97 Europium	64 Gd 157.25 Gadolinium	65 Tb 158.93 Terbium	66 Dy 162.50 Dysprosium	67 Ho 164.93 Holmium	68 Er 167.26 Erbium	69 Tm 168.93 Thulium	70 Yb 173.04 Ytterbium	71 Lu 174.97 Lutetium
89 Ac 227.03 Actinium	90 Th 232.04 Thorium	91 Pa 231.04 Protactinium	92 U 238.03 Uranium	93 Np 237.05 Neptunium	94 Pu 244.0 Plutonium	95 Am 243.06 Americium	96 Cm 247.07 Curium	97 Bk 247.07 Berkelium	98 Cf 251.08 Californium	99 Es 252.08 Einsteinium	100 Fm 257.10 Fermium	101 Md 260 Mendelevium	102 No 259.10 Nobelium	103 Lr 262.11 Lawrencium

(UV 1)

The University of Zambia

Chemistry Department

CHE3511, 2013

UV rules for predicting λ_{max} for conjugated systems

Many scientists have come up with procedures for predicting λ_{max} for various conjugated systems. Woodward and Fieser came out with first rules on conjugated dienes.

Table 1.2 Rules for diene and triene absorption

Value assigned to parent <i>s-trans</i> diene (like 1)	214 nm
Value assigned to parent <i>s-cis</i> diene (like 2)	253 nm
Increment for:	
(a) each alkyl substituent or ring residue	5 nm
(b) the exocyclic nature of any double bond	5 nm
(c) a double-bond extension	30 nm
(d) auxochrome:	
—OAcyl	0 nm
—OAlkyl	6 nm
—SAlkyl	30 nm
—Cl, —Br	5 nm
—NAlkyl ₂	60 nm
λ_{calc}	Total

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Table 1.3 Longest wavelength maxima of some simple polyenes

<i>n</i>	<i>trans</i> -Me(CH=CH) _{<i>n</i>} Me		<i>trans</i> -Ph(CH=CH) _{<i>n</i>} Ph	
	λ_{max} , nm	ϵ	λ_{max} , nm	ϵ
3	274.5	30 000	358	75 000
4	310	76 500	384	86 000
5	342	122 000	403	94 000
6	380	146 500	420	113 000
7	401	-	435	135 000
8	411	-	-	-

Table 1.4 Principal maxima for conjugated polyynes Me(C≡C)_{*n*}Me

<i>n</i>	Me(C≡C) _{<i>n</i>} Me		Me(C≡C) _{<i>n</i>} Me	
	λ_{max} , nm	ϵ	λ_{max} , nm	ϵ
2	-	-	250	160
3	207	135 000	306	120
4	234	281 000	354	105
5	260.5	352 000	394	120
6	284	445 000	-	-

Table 1.10 Rules for the principal band of substituted benzene derivatives RC_6H_4COX

Parent chromophore:			λ_{max} (EtOH) nm		
X					
alkyl or ring residue					
H					246
OH or Oalkyl					250
					230
Increment (nm) for each substituent:			Increment (nm) for each substituent:		
R	<i>o, m</i> or <i>p</i>	Increment	R	<i>o, m</i> or <i>p</i>	Increment
Alkyl or ring residue	<i>o, m</i>	3	Br	<i>o, m</i>	2
	<i>p</i>	10	NH ₂	<i>p</i>	15
OH, OMe, Oalkyl	<i>o, m</i>	7		<i>o, m</i>	13
	<i>p</i>	25	NHAc	<i>p</i>	58
O ⁻	<i>o</i>	11		<i>o, m</i>	20
	<i>m</i>	20	NHMe	<i>p</i>	45
	<i>p</i>	78		<i>p</i>	73
Cl	<i>o, m</i>	0	NMe ₂	<i>o, m</i>	20
	<i>p</i>	10		<i>p</i>	85

(Reprinted with permission from A. I. Scott, *Interpretation of the Ultraviolet Spectra of Natural Products*, Pergamon Press, Oxford, 1964.)

3.29 Tables of data

Table 3.4 Some parameters of magnetic nuclei

Isotope	NMR frequency (MHz) at 9.396T	Natural abundance (%)	Relative sensitivity	Spin (I) ^a
¹ H	400.00	99.98	1.00	1/2
² H	61.40	0.015	0.00965	1
³ H	426.80	0	1.21	1/2
⁷ Li	155.44	92.58	0.293	3/2
¹¹ B	128.32	80.42	0.165	3/2
¹³ C	100.56	1.11	0.0159	1/2
¹⁴ N	28.92	99.63	0.00101	1
¹⁵ N	40.52	0.37	0.00104	-1/2
¹⁷ O	54.24	0.037	0.0291	-5/2
¹⁹ F	376.32	100	0.833	1/2
²³ Na	105.80	100	0.0925	3/2
²⁷ Al	104.24	100	0.206	5/2
²⁹ Si	79.44	4.70	0.00784	-1/2
³¹ P	161.92	100	0.0663	1/2
³⁵ Cl	39.20	75.53	0.0047	3/2
³⁹ K	18.68	93.10	0.000508	3/2
⁴¹ K	10.24	6.88	0.000084	3/2
⁵¹ V	105.12	99.76	0.382	7/2
⁵³ Cr	22.60	9.55	0.000903	3/2
⁵⁵ Mn	98.64	100	0.175	5/2
⁵⁷ Fe	12.92	2.19	0.000034	1/2
⁵⁹ Co	94.44	100	0.277	7/2
⁶⁵ Cu	113.60	30.91	0.114	3/2
⁷⁹ Br	100.20	50.54	0.0786	3/2
⁸¹ Br	108.00	49.46	0.0985	3/2
⁸⁵ Rb	38.60	72.15	0.0105	5/2
¹¹³ Cd	88.72	12.26	0.0109	1/2
¹¹⁹ Sn	149.08	8.58	0.0518	-1/2
¹³³ Cs	52.48	100	0.0474	7/2
¹⁹⁵ Pt	86.00	33.80	0.00994	1/2
²⁰⁷ Pb	83.68	22.60	0.00916	1/2

^aA minus sign in the Spin column signifies that the magnetogyric ratio γ , and hence the magnetic moment, is negative.

Table 3.19 ¹H Chemical shifts in methyl, methylene and methine groups

	<i>Methyl protons</i>	δ_H	<i>Methylene protons</i>	δ_H	<i>Methine protons</i>	δ_H	
C	R—CH ₃	0.9	R—CH ₂ —R	1.4	R—CHR ₂	1.5	
	C=C—C—CH ₃	1.1	C=C—C—CH ₂ —R	1.7			
	O—C—CH ₃	1.3	O—C—CH ₂ —R	1.9	O—C—CHR ₂	2.0	
	N—C—CH ₃	1.1	N—C—CH ₂ —R	1.4			
	O ₂ N—C—CH ₃	1.6	O ₂ N—C—CH ₂ —R	2.1			
	C=C—CH ₃	1.6	C=C—CH ₂ —R	2.3			
	Ar—CH ₃	2.3	Ar—CH ₂ —R	2.7	Ar—CHR ₂	3.0	
	O=CC=C—CH ₃	2.0	O=CC=C—CH ₂ —R	2.4			
	O=CC(CH ₃)=C	1.8	O=CC(CH ₂ —R)=C	2.4			
	C≡C—CH ₃	1.8	C≡C—CH ₂ —R	2.2	C≡C—CHR ₂	2.6	
	RCO—CH ₃	2.2	RCO—CH ₂ —R	2.4	RCO—CHR ₂	2.7	
	ArCO—CH ₃	2.6	ArCO—CH ₂ —R	2.9	ArCO—CHR ₂	3.3	
	ROOC—CH ₃	2.0	ROOC—CH ₂ —R	2.2	ROOC—CHR ₂	2.5	
	ArOOC—CH ₃	2.4	ArOOC—CH ₂ —R	2.6			
	N—CO—CH ₃	2.0	N—CO—CH ₂ —R	2.2	N—CO—CHR ₂	2.4	
	N=C—CH ₃	2.0	N=C—CH ₂ —R	2.3	N=C—CHR ₂	2.7	
	N	N—CH ₃	2.3	N—CH ₂ —R	2.5	N—CHR ₂	2.8
		ArN—CH ₃	3.0	ArN—CH ₂	3.5		
		RCON—CH ₃	2.9	RCON—CH ₂ —R	3.2	RCO—N—CHR ₂	4.0
N ⁺ —CH ₃		3.3	N ⁺ —CH ₂ —R	3.3			
O ₂ N—CH ₃		4.3	O ₂ N—CH ₂ —R	4.4	O ₂ N—CHR ₂	4.7	
O	HO—CH	3.4	HO—CH ₂ —R	3.6	HO—CHR ₂	3.9	
	RO—CH ₃	3.3	RO—CH ₂ —R	3.4	RO—CHR ₂	3.7	
	C=CO—CH ₃	3.8	C=CO—CH ₂ —R	3.7			
	ArO—CH ₃	3.8	ArO—CH ₂ —R	4.3	ArO—CHR ₂	4.5	
	RCOO—CH ₃	3.7	RCOO—CH ₂ —R	4.1	RCOO—CHR ₂	4.8	
			(RO) ₂ CH ₂	4.8	(RO) ₃ CH	5.2	
Hal	F—CH ₃	4.3	F—CH ₂ —R	4.1	F—CHR ₂	3.7	
	Cl—CH ₃	3.1	Cl—CH ₂ —R	3.6	Cl—CHR ₂	4.2	
	Br—CH ₃	2.7	Br—CH ₂ —R	3.5	Br—CHR ₂	4.3	
	I—CH ₃	2.1	I—CH ₂ —R	3.2	I—CHR ₂	4.3	
S	RS—CH ₃	2.1	S—CH ₂ —R	2.4	S—CHR ₂	3.2	
	RSO—CH ₃	2.5	RSO—CH ₂ —R	2.7			
	RSO ₂ —CH ₃	2.8	RSO ₂ —CH ₂ —R	2.9			
		(RS) ₂ CH ₂	4.2				
P	R ₂ P—CH ₃	1.4	R ₂ P—CH ₂ —R	1.6	R ₂ P—CHR ₂	1.8	
Si	R ₃ Si—CH ₃	0.0	R ₃ Si—CH ₂ —R	0.5	R ₃ Si—CHR ₂	1.2	
Se	RSe—CH ₃	2.0					

R = alkyl group. These values will usually be within ± 0.2 p.p.m. unless electronic or anisotropic effects from other groups are strong. An obsolete scale used τ values; these are related to δ values by the equation $\tau = 10 - \delta$.

Estimation of ¹H chemical shifts in alkanes

For R¹R²R³C—H,







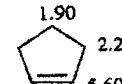
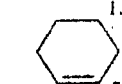


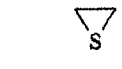
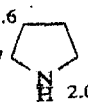
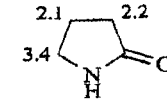
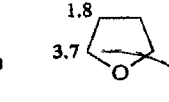
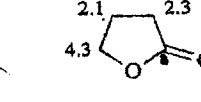
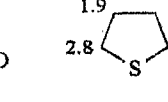
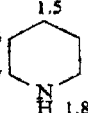
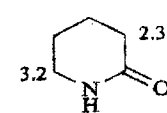
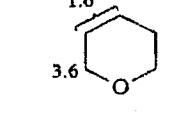
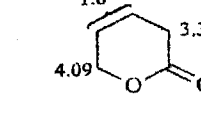
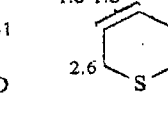
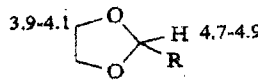
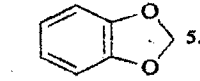
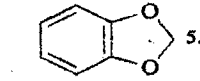
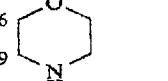
$$\delta_H = 1.50 + \sum z_i$$

(3.20)

Table 3.20 Substituent constants *z* for Eq. 3.20

R ⁱ	<i>z</i>	R ⁱ	<i>z</i>	R ⁱ	<i>z</i>
H—	-0.3	HC=C—	0.9	MeO—	1.5
Alkyl—	0.0	OHC—	1.2	PhO—	2.3
CH ₂ =CHCH ₂ —	0.2	MeCO—	1.2	AcO—	2.7
MeCOCH ₂ —	0.2	RO ₂ C—	0.8	Cl—	2.0
HOCH ₂ —	0.3	NC—	1.2	Br—	1.9
ClCH ₂ —	0.5	H ₂ N—	1.0	I—	1.4
CH ₂ =CH—	0.8	O ₂ N—	3.0	MeS—	1.0
Ph—	1.3	HO—	1.7	Me ₃ Si—	0.7

Table 3.21 ¹H Chemical shifts of -CH₂- and =CH- groups in some aliphatic cyclic compounds

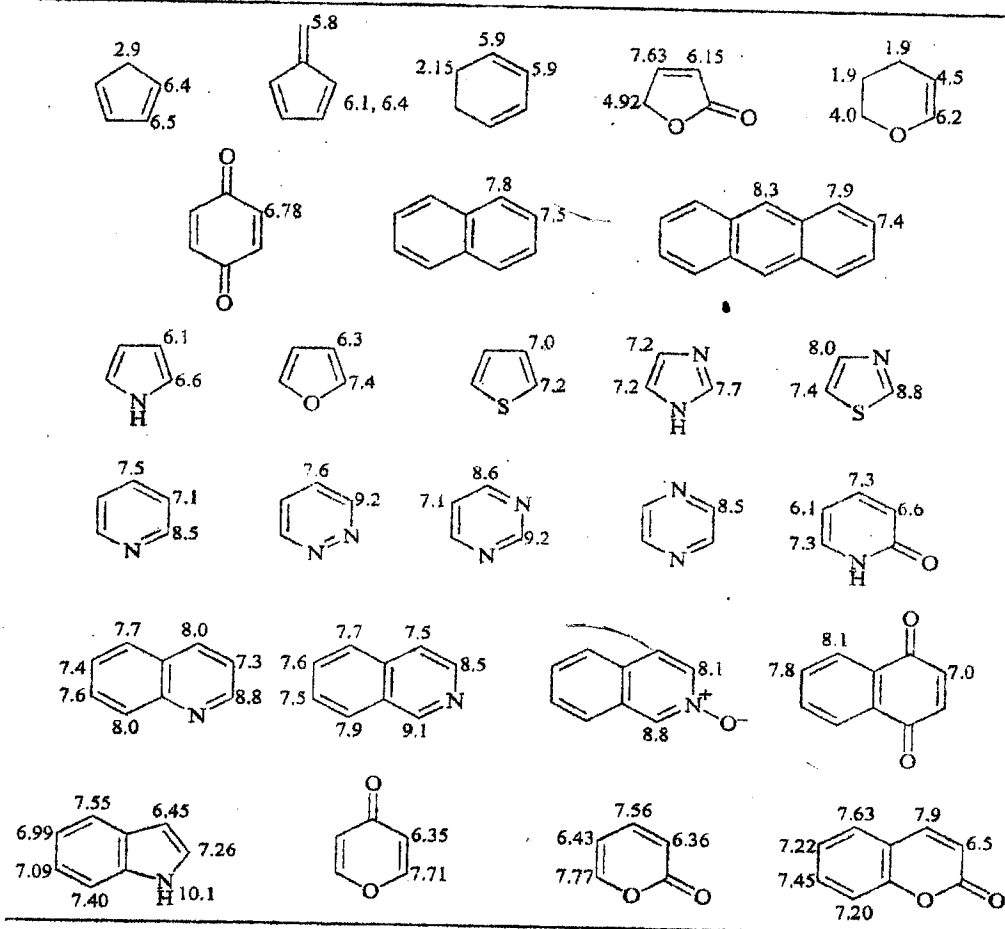
				at -100°					
				H _{ax}	H _{eq}				
	0.3		1.96		1.51		1.44		
	0.92 7.01		2.57 5.95		1.90 2.28 5.60		1.65 1.96 5.59		
	1.6 0.0		2.6		2.3				
	1.6 2.7 2.0		2.1 3.4 2.2		1.8 3.7		2.1 4.3 2.3		1.9
	1.5 2.7 1.8		1.5 3.2 2.3		1.6 3.6		1.6 4.09 3.31		1.6-1.8 2.6
	3.9-4.1		4.7-4.9		5.9		3.6 2.9 1.9		

Axial protons generally come into resonance at higher field than their equatorial counterparts

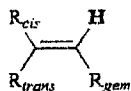
Table 3.23 ^1H Chemical shifts of protons attached to multiple bonds

Structure	δ_{H}	Structure	δ_{H}
RCHO	9.4-10.0	$\text{R}_2\text{C}=\text{CHR}$	4.5-6.0
ArCHO	9.7-10.5	$\text{R}_2\text{C}=\text{CH}-\text{COR}$	5.8-6.7
ROCHO	8.0-8.2	$\text{RHC}=\text{CR}-\text{COR}$	6.5-8.0
R_2NCHO	8.0-8.2	$\text{RHC}=\text{CR}-\text{OR}$	4.0-5.0
$\text{RC}=\text{CH}$	1.8-3.1	$\text{R}_2\text{C}=\text{CH}-\text{OR}$	6.0-8.1
$\text{R}_2\text{C}=\text{C}=\text{CHR}$	4.0-5.0	$\text{RHC}=\text{CR}-\text{NR}_2$	3.7-5.0
ArH	6.0-9.0	$\text{R}_2\text{C}=\text{CH}-\text{NR}_2$	5.7-8.0

Table 3.24 ^1H Chemical shifts (largely attached to double bonds) in some unsaturated cyclic systems (for simple cycloalkenes see Table 3.21)



Estimation of ¹H chemical shifts in alkenes



$$\delta_H = 5.25 + \sum z_{gem} + \sum z_{cis} + \sum z_{trans} \quad (3.21)$$

Table 3.22 Substituent constants *z* for Eq. 3.21

	Substituent <i>R</i>	<i>z</i> _{gem}	<i>z</i> _{cis}	<i>z</i> _{trans}	
H	H—	0	0	0	
C	Alkyl—	0.45	-0.22	-0.28	
	^a Ring-alkyl—	0.69	-0.25	-0.28	
	N=CCH ₂ — or RCOCH ₂ —	0.69	-0.08	-0.06	
	ArCH ₂ —	1.05	-0.29	-0.32	
	R ₂ NCH ₂ —	0.58	-0.10	-0.08	
	ROCH ₂ —	0.64	-0.10	-0.02	
	HalCH ₂ —	0.70	0.11	-0.04	
	RSCH ₂ —	0.71	-0.13	-0.22	
	^a Isolated RCH=CH—	1.00	-0.09	-0.23	
	^b Conjugated CH=CH—	1.24	0.02	-0.05	
	Ar—	1.38	0.36	-0.07	
	OHC—	1.02	0.95	1.17	
	Isolated RCO—	1.10	1.12	0.87	
	^b Conjugated RCO—	1.06	0.91	0.74	
	Isolated HO ₂ C—	0.97	1.41	0.71	
	^b Conjugated HO ₂ C—	0.80	0.98	0.32	
	Isolated RO ₂ C—	0.80	1.18	0.55	
	^b Conjugated RO ₂ C—	0.78	1.01	0.46	
	R ₂ NCO—	1.37	0.98	0.46	
	ClCO—	1.11	1.46	1.01	
	RC≡C—	0.47	0.38	0.12	
	N≡C—	0.27	0.75	0.55	
	N	(Alkyl)HN— or (Alkyl) ₂ N—	0.80	-1.26	-1.21
		^b (Conjugated alkyl or aryl) ₂ N—	1.17	-0.53	-0.99
		AcNH—	2.08	-0.57	-0.72
	O	O ₂ N—	1.87	1.30	0.62
		AlkylO—	1.22	-1.07	-1.21
^b Conjugated alkyl or arylO—		1.21	-0.60	-1.00	
Hal	AcO—	2.11	-0.35	-0.64	
	F—	1.54	-0.40	-1.02	
	Cl—	1.08	0.18	0.13	
	Br—	1.07	0.45	0.55	
Si	I—	1.14	0.81	0.88	
	R ₃ Si—	0.90	0.90	0.60	
S	RS—	1.11	-0.29	-0.13	
	RSO—	1.27	0.67	0.41	
	RSO ₂ —	1.55	1.16	0.93	

^aUse the 'ring-alkyl' values when the double bond and the alkyl group are part of a five- or six-membered ring. ^bUse the 'conjugated' values when either the substituent or the double bond is further conjugated.

(6)

Table 3.5 ¹³C Chemical shifts in some alkanes

Structure	¹³ C Chemical Shifts (ppm)
Ethane	15.9, 15.4
Propane	22.4, 13.5, 34.3
Butane	22.8, 29.5, 14.1, 32.4
Pentane	31.6, 21.8, 11.3, 29.7
Hexane	27.1, 36.0, 27.0, 23.1, 33.4
Cyclohexane	27.1, 36.0, 27.0, 23.1, 33.4
Norbornane	29.2, 37.3, 30.6
Bicyclo[2.2.1]heptane	24.6, 30.6
Bicyclo[2.2.2]octane	37.8, 28.5
Decalin	44.0, 34.6, 27.1
Bicyclo[2.2.2]octane	43.3, 34.3, 26.4
Bicyclo[2.2.2]octane	36.8, 24.5, 29.7

n	δ_C
3	-2.8
4	23.1
5	26.3
6	27.1
7	28.8

Axial methyl groups
-4.5 p.p.m. upfield
from equatorial methyl
groups

Estimation of ¹³C chemical shifts in aliphatic chains

$$\delta_C = -2.3 + \Sigma z + \Sigma S + \Sigma K \quad (3.16)$$

where -2.13 is the ¹³C chemical shift for methane, z is the substituent constant (Table 3.6), S is a 'steric' correction (Table 3.7), and K is a conformational increment for γ -substituents (Table 3.8).

Table 3.6 Substituent constants z for Eq. 3.16

Substituent	z				
	α	β	γ	δ	
H	H—	0	0	0	0
C	alkyl—	9.1	9.4	-2.5	0.3
	—C=C—	19.5	6.9	-2.1	0.4
	—C≡C—	4.4	5.6	-3.4	-0.6
	Ph—	22.1	9.3	-2.6	0.3
	OHC—	29.9	-0.6	-2.7	0.0
	—CO—	22.5	3.0	-3.0	0.0
	—O ₂ C—	22.6	2.0	-2.8	0.0
	N=C—	3.1	2.4	-3.3	-0.5
N	R ₂ N—	28.3	11.3	-5.1	0.0
	O ₂ N—	61.6	3.1	-4.6	-1.0
O	—O—	49.0	10.1	-6.2	0.0
	—COO—	56.5	6.5	-6.0	0.0
Hal	F—	70.1	7.8	-6.8	0.0
	Cl—	31.0	10.0	-5.1	-0.5
	Br—	18.9	11.0	-3.8	-0.7
	I—	-7.2	10.9	-1.5	-0.9
S	—S—	10.6	11.4	-3.6	-0.4
	—SO—	31.1	9.0	-3.5	0.0

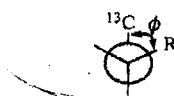
(7)

Table 3.7 'Steric' constants S for Eq. 3.16

Observed ^{13}C atom	Number of substituents other than H on the atoms directly bonded to the observed ^{13}C			
	1	2	3	4
Primary	0.0	0.0	-1.1	-3.4
Secondary	0.0	0.0	-2.5	-7.5
Tertiary	0.0	-3.7	-9.5	-15.0
Quaternary	-1.5	-8.4	-15.0	-25.0

†Except that CO_2H , CO_2R and NO_2 groups are counted as primary (column 1), Ph, CHO, CONH_2 , CH_2OH and CH_2NH_2 groups as secondary (column 2), and COR groups as tertiary (column 3).

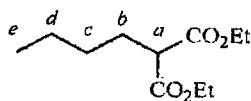
Table 3.8 Conformational correction K for γ substituents in Eq. 3.16



ϕ	0°	60°	120°	180°	Freely rotating
K	-4	-1	0	+2	0

Example of application of Eq. 3.16

Diethyl butylmalonate has ^{13}C signals at δ 13.81, 14.10, 22.4, 28.5, 29.5, 52.03, 61.12 and 169.32.



Take the methine carbon a :

Base value	-2.3	(methane)
1 α -alkyl group	9.1	(carbon b)
1 β -alkyl group	9.4	(carbon c)
3 γ -alkyl groups	-7.5	(carbon d and two Et groups of the OEt groups)
3 δ -alkyl groups	0.9	(Me and two Me groups of the OEt groups)
2 α - CO_2R groups	45.2	
S	-3.7	(a is a tertiary carbon bonded to two CO_2Et groups, which count as primary)
K	0	(open-chain compound with free rotation)
Calculated shift	51.5	Observed value 52.03

There is no difficulty in assigning this signal, because it is the only methine and is easily identified as such. However, the three methylenes at 22.4, 28.5 and 29.5 are less securely identifiable.

8

The corresponding calculation for carbon *b* is:

Base value	-2.3	(methane)
2 α -alkyl group	18.2	[carbon <i>c</i> and (EtO ₂ C) ₂ CH]
1 β -alkyl group	9.4	(carbon <i>d</i>)
2 β -CO ₂ R groups	4.0	
1 γ -alkyl groups	-2.5	(Me)
2 δ -alkyl groups	0.6	(CH ₂ groups of the OEt groups)
2 β -groups	4.0	
S	-2.5	(<i>b</i> is secondary and bonded to a carbon, namely <i>a</i> , with three groups on it other than hydrogen)
Calculated shift	28.9	

Similar calculations for carbons *c* and *d* give calculated values of 29.1 and 22.8. It is therefore likely, although not certain, that the signals at 22.4, 28.5 and 29.5 can be assigned to *d*, *b* and *c*, respectively. If you have access to a program for predicting chemical shifts you might do a little better. ChemNMR in ChemDraw®, using very similar protocols to Eq. 3.16 and the data in Tables 3.6-3.8, gives:

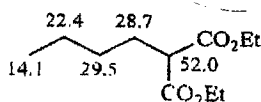


Table 3.9 ¹³C Chemical shifts in some alkenes, alkynes and nitriles

(9)

Estimation of ^{13}C chemical shifts in substituted alkenes



$$\delta_{\text{C}} = 123.3 + \Sigma z_1 + \Sigma z_2 + \Sigma S \quad (3.17)$$

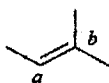
where 123.3 is the ^{13}C chemical shift for ethene, z_1 and z_2 are the substituent constants (Table 3.10) and S is a 'steric' correction for alkyl substituents:

For each pair of <i>cis</i> substituents	$S = -1.1$
For a pair of geminal substituents on C-1	$S = -4.8$
For a pair of geminal substituents on C-2	$S = 2.5$

Table 3.10 Substituent constants z for Eq. 3.17

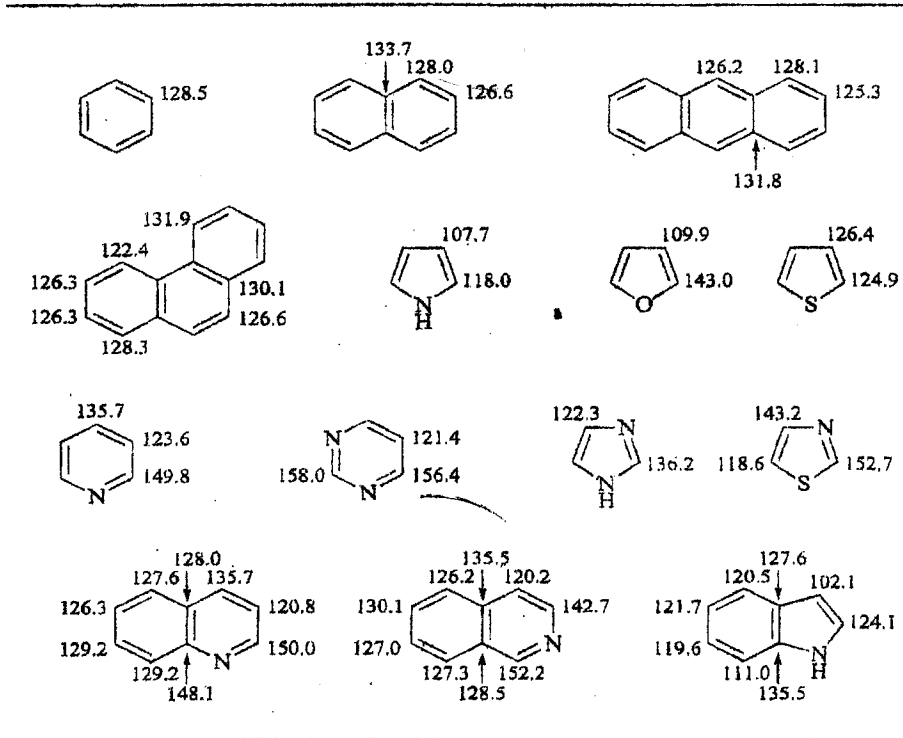
Substituent R		z_1	z_2
H	H—	0	0
C	Me—	10.6	-7.9
	Et—	15.5	-9.7
	Pr ⁿ —	14.0	-8.2
	Pr ⁱ —	20.4	-11.5
	Bu ^t —	25.3	-13.3
	ClCH ₂ —	10.2	-6.0
	HOCH ₂ —	14.2	-8.4
	Me ₃ SiCH ₂ —	12.5	-12.5
	CH ₂ =CH—	13.6	-7.0
	Ph—	12.5	-11.0
	OHC—	13.1	12.7
	RCO—	15.0	5.8
	RO ₂ C—	6.3	7.0
	N=C—	-15.1	14.2
	N	RAcN—	6.5
O	RO—	29.0	-39.0
	AcO—	18.4	-26.7
Hal	F—	24.9	-34.3
	Cl—	2.6	-6.1
	Br—	-7.9	-1.4
	I—	-38.1	7.0
Si	Me ₃ Si—	16.9	16.1
P	Ph ₂ P(=O)—	8.0	11.0
S	RS—	18.0	-16.0

Example of application of Eq. 3.17

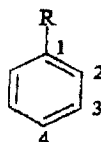


<i>a</i>	Base value	123.3	<i>b</i>	Base value	123.3
	1-Me	10.6		2 × 1-Me	21.2
	2 × 2-Me	-15.8		2-Me	-7.9
	1 <i>cis</i> pair	-1.1		1 <i>cis</i> pair	-1.1
	1 gem pair on C-2	2.5		1 gem pair on C-1	-4.0
Calculated		119.5	Calculated		130.7
Observed		118.5	Observed		131.8
Calculated(ChemNMR)		118.6	Calculated(ChemNMR)		132.8

Table 3.11 ¹³C Chemical shifts in some arenes



(11)

Estimation of ^{13}C chemical shifts in substituted benzenes

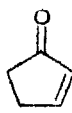
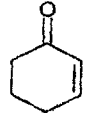
$$\delta_{\text{C}} = 128.5 + \sum z_i$$

(3.18)

Table 3.12 Substituent constants z_i for Eq. 3.18

	Substituent R	z_1	z_2	z_3	z_4
H	H—	0	0	0	0
C	Me—	9.3	0.6	0.0	-3.1
	Et—	15.7	-0.6	-0.1	-2.8
	Pr ⁿ —	14.2	-0.2	-0.2	-2.8
	Pr ⁱ —	20.1	-2.0	0.0	-2.5
	Bu ^t —	22.1	-3.4	-0.4	-3.1
	ClCH ₂ —	9.1	0.0	0.2	-0.2
	HOCH ₂ —	13.0	-1.4	0.0	-1.2
	CH ₂ =CH—	7.6	-1.8	-1.8	-3.5
	Ph—	13.0	-1.1	0.5	-1.0
	HC≡C—	-6.1	3.8	0.4	-0.2
	OHC—	9.0	1.2	1.2	6.0
	MeCO—	9.3	0.2	0.2	4.2
	RO ₂ C—	2.1	1.2	0.0	4.4
	N≡C—	-16.0	3.5	0.7	4.3
	N	H ₂ N—	19.2	-12.4	1.3
Me ₂ N—		22.4	-15.7	0.8	-11.8
AcNH—		11.1	-16.5	0.5	-9.6
O ₂ N—		19.6	-5.3	0.8	6.0
HO—		26.9	-12.7	1.4	-7.3
O	MeO—	30.2	-14.7	0.9	-8.1
	AcO—	23.0	-6.4	1.3	-2.3
	F—	35.1	-14.3	0.9	-4.4
Hal	Cl—	6.4	0.2	1.0	-2.0
	Br—	-5.4	3.3	2.2	-1.0
	I—	-32.3	9.9	2.6	-0.4
Si	Me ₃ Si—	13.4	4.4	-1.1	-1.1
P	Ph ₂ P—	8.7	5.1	-0.1	0.0
S	MeS—	9.9	-2.0	0.1	-3.7

Table 3.13 ¹³C Chemical shifts of carbonyl carbons

$\begin{array}{c} \text{O} \\ \parallel \\ \text{R}^1-\text{C}-\text{R}^2 \end{array}$	δ_{C}	$\begin{array}{c} \text{O} \\ \parallel \\ \text{R}^1-\text{C}-\text{R}^2 \end{array}$	δ_{C}
Me— —H	199.7	Me— —OH	178.1
Et— —H	206.0	Et— —OH	180.4
Pr ⁱ — —H	204.0	Pr ⁱ — —OH	184.1
		Bu ^t — —OH	185.9
CH ₂ =CH— —H	192.4	CH ₂ =CH— —OH	171.7
Ph— —H	192.0	Ph— —OH	172.6
Me— —Me	206.0	Me— —OMe	170.7
Et— —Me	207.6	Et— —OMe	173.3
Pr ⁱ — —Me	211.8	Pr ⁱ — —OMe	175.7
Bu ^t — —Me	213.5	Bu ^t — —OMe	178.9
ClCH ₂ — —Me	200.7	CH ₂ =CH— —OMe	165.5
Cl ₂ CH— —Me	193.6	Ph— —OMe	166.8
Cl ₃ C— —Me	186.3		
CH ₂ =CH— —Me	197.2	—(CH ₂) ₃ O—	177.9
Ph— —Me	197.6	—(CH ₂) ₄ O—	175.2
		Me— —NH ₂	172.7
—(CH ₂) ₃ —	208.2	CH ₂ =CH— —NH ₂	168.3
—(CH ₂) ₄ —	213.9	Ph— —NH ₂	169.7
—(CH ₂) ₅ —	208.8	—(CH ₂) ₃ NH—	179.4
—(CH ₂) ₆ —	211.7	—(CH ₂) ₄ NH—	173.0
	209.0	Me— —OAc	167.3
	198.0	Ph— —OAc	162.8
		Me— —Cl	168.6
		CH ₂ =CH— —Cl	165.6
		Ph— —Cl	168.0
		Me— —SiMe ₃	247.6
		Ph— —SiMe ₃	237.9
		Me— —SiPh ₃	240.1

(121)

IR TABLES (from Spectroscopic Methods in Organic Chemistry by D. Williams and I. Fleming)

2.14 Tables of data

Table 2.1 C—H stretching vibrations

Group	Band	Remarks
$\text{C}\equiv\text{C}-\text{H}$	~3300(s)	Sharp
$\begin{array}{c} \text{H} \\ \diagup \\ \text{C}=\text{C} \\ \diagdown \\ \text{H} \end{array}$	3095-3075(m)	Sometimes obscured by the stronger bands of saturated C—H groups, which occur below 3000 cm^{-1}
$\begin{array}{c} \text{H} \\ \diagdown \\ \text{C}=\text{C} \\ \diagup \end{array}$	3040-3010(m)	
Aryl—H	3040-3010(w)	Often obscured
Cyclopropane C—H		
Epoxide C—H	~3050(w)	
—CH ₂ —halogen		
—COCH ₃	3100-2900(w)	Often very weak
Unfunctionalised C—H stretching:		
$\begin{array}{c} \diagdown \\ \text{CH}_2 \text{ and } -\text{CH}_3 \\ \diagup \end{array}$	2960-2850(s)	Usually 2 or 3 bands
—CH	2890-2880(w)	
—CHO	2900-2700(w)	Usually 2 bands, one near 2720 cm^{-1}
—OCH ₃	2850-2810(m)	
$\begin{array}{c} \diagdown \\ \text{NCH}_3 \text{ and } \text{NCH}_2- \\ \diagup \end{array}$	2820-2780(m)	
—OCH ₂ O—	2790-2770(m)	

Table 2.2 C—H bending vibrations

Group	Band	Remarks
$\begin{array}{c} \diagdown \\ \text{CH}_2 \text{ and } -\text{CH}_3 \\ \diagup \end{array}$	1470-1430(m)	Asymmetrical deformations
—C(CH ₃) ₃	1395-1385(m) and 1365(s)	
—CH ₃	1390-1370(m)	Symmetrical deformations
—OCOCH	1385-1365(s)	The high intensity of these bands often dominates this region of the spectrum
$\begin{array}{c} \diagdown \\ \text{C}(\text{CH}_3)_2 \\ \diagup \end{array}$	~1380(m)	A roughly symmetrical doublet
—COCH ₃	1360-1355(s)	

(122)

Table 2.2 continued		
$\begin{array}{c} \text{H} \\ \diagup \text{C}=\text{C} \diagdown \\ \text{H} \quad \text{H} \end{array}$	995-985(s) and 940-900(s)	
$\begin{array}{c} \text{H} \\ \diagup \text{C}=\text{C} \diagdown \\ \text{H} \end{array}$	970-960(s)	C—H out-of-plane deformation. Conjugation shifts the band towards 990 cm^{-1}
$\begin{array}{c} \text{H} \\ \diagup \text{C}=\text{C} \diagdown \\ \text{H} \end{array}$	895-885(s)	
$\begin{array}{c} \text{H} \\ \diagup \text{C}=\text{C} \diagdown \\ \text{H} \end{array}$	840-790(m)	
$\begin{array}{c} \text{H} \quad \text{H} \\ \diagup \text{C}=\text{C} \diagdown \\ \text{H} \quad \text{H} \end{array}$	730-675(m)	C—H out-of-plane deformation
$\begin{array}{c} \diagdown \text{CH}_2 \end{array}$	~720(w)	Rocking

Table 2.3 O—H stretching and bending vibrations

<i>Group</i>	<i>Band</i>	<i>Remarks</i>
Water in dilute solution	3710	
Water of crystallisation in solids	3600-3100(w)	Usually accompanied by a weak band at 1640-1615 cm^{-1} ; residual water in KBr discs shows a broad but weak band at 3450 cm^{-1}
Free O—H	3650-3590(v)	Sharp
H-bonded O—H	3600-3200(s)	Often broad, but may be sharp for some intramolecular single-bridge H-bonds; the lower the frequency the stronger the H-bond
Intramolecularly H-bonded O—H of the chelate type and as found in carboxylic acid dimers	3200-2500(v)	Broad; the lower the frequency the stronger the H-bond; sometimes so broad as to be overlooked
O—H	1410-1260(s)	O—H bending
$\begin{array}{c} \diagdown \text{C}-\text{OH} \end{array}$	1150-1040(s)	C—O stretching

Table 2.4 N—H stretching and bending vibrations

Group	Band	Remarks
$\begin{array}{c} \text{H} \\ \\ -\text{N} \\ \\ \text{H} \end{array} \quad \begin{array}{c} \diagup \\ \text{N}-\text{H} \\ \diagdown \end{array} = \begin{array}{c} \text{H} \\ \\ \text{N} \end{array}$	3500-3300(m)	Primary amines show two bands: the unsymmetrical and the symmetrical stretching. Secondary amines absorb weakly, but pyrrole and indole N—H is sharp
Amino acids —NH ₃ ⁺ Amine salts —NH ₃ ⁺	3130-3030(m) ~3000(m)	Values for solid state; broad bands also (but not always) near 2500 and 2000 cm ⁻¹
$\begin{array}{c} \diagup \\ \text{NH}_2^+ \\ \diagdown \end{array} \quad \begin{array}{c} \diagup \\ -\text{NH}^+ \\ \diagdown \end{array} \quad \begin{array}{c} \diagup \\ \text{NH}^+ \\ \diagdown \end{array}$	2700-2250(m)	Values for the solid state; broad because of the presence of overtone bands, etc.
—CONH ₂	~3500(m) and ~3400(m)	Lowered by ~150 cm ⁻¹ in the solid state and when involved in H-bonding; often several bands 3200-3050 cm ⁻¹
—CONH—	3460-3400(m) 3100-3070(w)	Two bands; lowered in the solid state and when involved in H-bonding; only one band with lactams A weak extra band with H-bonded and solid state samples
$\begin{array}{c} \text{H} \\ \\ -\text{N} \\ \\ \text{H} \end{array}$	1650-1560(m)	N—H bending
$\begin{array}{c} \diagup \\ \text{N}-\text{H} \\ \diagdown \end{array}$	1580-1490(w)	Often too weak to be noticed
—NH ₃ ⁺	1600(s) and 1500(s)	Secondary amine salts have the 1600 cm ⁻¹ band

Table 2.5 Miscellaneous R—H stretching vibrations

Group	Band	Remarks
$\begin{array}{c} \text{O} \\ \\ \diagup \text{P} \diagdown \\ \\ \text{OH} \end{array}$	2700-2560(m)	Associated O—H
$\begin{array}{c} \diagup \\ \text{B}-\text{H} \\ \diagdown \end{array}$	2640-2200(s)*	
S—H	2600-2550(w)	Weaker than O—H and less affected by H-bonding
$\begin{array}{c} \diagup \\ \text{P}-\text{H} \\ \diagdown \end{array}$	2440-2350(m)	Sharp

Table 2.5 continued

Group	Band	Remarks
$\begin{array}{c} \diagdown \\ \text{Si}-\text{H} \\ \diagup \\ \text{Si} \\ \diagdown \\ \text{H} \end{array}$	2360-2150(s) and 890-860	Sensitive to the electronegativity of substituents
$\begin{array}{c} \text{H} \\ \\ \text{Si} \\ \\ \text{H} \end{array}$	~2135(s) and 890-860	
R—D	1/1.37 times the corresponding R—H frequency	Useful when assigning R—H bands; deuteration leads to a recognisable shift to lower frequency

Table 2.6 Stretching frequencies of triple bonds and cumulated double bonds

Group	Band	Remarks
Carbon dioxide O=C=O	2349(s)	Appears in many spectra because of inequalities in path length
Nitrile oxides $\text{—C}\equiv\text{N}^+\text{—O}^-$	2305-2280(m)	
Isocyanates $\text{—N}=\text{C}=\text{O}$	2275-2250(s)	Very intense; position little affected by conjugation
Internal acetylenes $\text{—C}\equiv\text{C—}$	2260-2150(v)	Strong and at low end of range when conjugated; weak or absent for nearly symmetrical substitution
$\text{R}_3\text{SiC}\equiv\text{CH}$	2040	
Nitriles $\text{—C}\equiv\text{N}$	2260-2200(v)	Strong; conjugated at the low end of the range; occasionally very weak; some cyanohydrins do not absorb in this region
Diazonium salts $\text{—N}^+=\text{N}$	~2260	
Thiocyanates $\text{—S—C}\equiv\text{N}$	2175-2140(s)	Aryl thiocyanates at the upper end of the range, alkyl at the lower end
Azides $\text{—N}=\text{N}^+=\text{N}^-$	2160-2120(s)	
Carbodiimides $\text{—N}=\text{C}=\text{N—}$	2155-2130(s)	Very intense; conjugation leads it to split into two bands of different intensity
Ketenes $\text{>C}=\text{C}=\text{O}$	2155-2130(s)	Very intense
Terminal acetylenes $\text{—C}\equiv\text{CH}$	2140-2100(w)	C≡C stretching ($\nu_{\text{C-H}}$ at ~3300 cm^{-1})

(12.5)

Table 2.6 continued		
<i>Group</i>	<i>Band</i>	<i>Remarks</i>
Isothiocyanates —N=C=S	2140-1990(s)	Broad and very intense
Diazoketones —CO·CH=N ⁺ =N ⁻	2100-2050(s)	
Diazoalkanes C=N ⁺ =N ⁻	2050-2010(s)	
Ketenimines C=C=N	2050-2000(s)	Very intense
Allenes C=C=C	1950-1930(s)	Two bands when terminal allene or when bonded to carbonyl and similar groups

Table 2.7 Stretching frequencies of C=O groups (all bands listed are strong)

<i>Group</i>	<i>Band</i>	<i>Remarks</i>
Acid anhydrides —CO—O—CO—		
Saturated	1850-1800 and 1790-1740	Two bands usually separated by about 60 cm ⁻¹ ; the higher-frequency symmetric band is more intense in acyclic anhydrides and the lower frequency asymmetric band is more intense in cyclic anhydrides
Aryl and α,β-unsaturated	1830-1780 and 1770-1710	
Saturated five-ring	1870-1820 and 1800-1750	
All classes	1300-1050	One or two bands: C—O stretching
Acid chlorides —COCl		
Saturated	1815-1790	COF higher, COBr and COI lower
Aryl and α,β-unsat.	1790-1750	
Acid peroxides —CO—O—O—CO—		
Saturated	1820-1810 and 1800-1780	
Aryl and α,β-unsat.	1805-1780 and 1785-1755	


(126)

Table 2.7 continued

Group	Band	Remarks
Esters and lactones		
—CO—O—		
Saturated	1750-1735	
Aryl and α,β -unsaturated	1730-1715	
Aryl and vinyl esters C=C—O—CO—	1800-1750	The C=C stretch is also shifted to higher frequency
Esters with electronegative α -substituents	1770-1745	e.g. <chem>ClC(=O)O</chem>
α -Keto esters	1755-1740	
Six-ring and larger lactones	Similar values to the corresponding open-chain esters	
Five-ring lactones	1780-1760	
α,β -Unsaturated five-ring lactones	1770-1740	When there is an α -C—H present, there are two bands, the relative intensity depending upon solvent
β,γ -Unsaturated five-ring lactones	~1800	
Four-ring lactones	~1820	
β -Keto ester in H-bonding enol form	~1650	Chelate-type H-bond causes shift to lower frequency than normal ester; the C=C is usually near 1630(s) cm^{-1}
Aldehydes	Values below are for solution spectra; lowered by 10-20 cm^{-1} in liquid film or solid state and raised in the gas phase. See also Table 2.1 for C—H	
Saturated	1740-1720	
Aryl	1715-1695	<i>o</i> -Hydroxy or amino groups shift this range to 1655-1625 cm^{-1} because of intramolecular H-bonding
α -Chloro or bromo	1765-1695	
α,β -Unsaturated	1705-1680	
$\alpha,\beta,\gamma,\delta$ -Unsaturated	1680-1660	
β -Keto aldehyde in enol form	1670-1645	Lowering caused by intramolecular H-bonding

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Table 2.7 continued

Group	Band	Remarks
Saturated	1725-1700	Monomer near 1760 cm ⁻¹ , rarely observed; occasionally both free monomer and H-bonded dimer can be seen in solution spectra; ether solvents give one band ~1730 cm ⁻¹
α,β -Unsaturated	1715-1690	
Aryl	1700-1680	
α -Halo	1740-1720	
Carboxylate ions —CO ₂ ⁻		For special features of amino acids, see text
Most types	1610-1550 and 1420-1300	Asymmetric and symmetric stretching, respectively
Amides and lactams	See Table 2.4 for N—H bands	
		
Primary —CONH ₂	~1690 and ~1600 ~1650 and ~1640	Solution state amide I and II Solid state amide I and II; sometimes overlap; amide I is generally more intense than amide II
Secondary —CONH—	1700-1670 and 1550-1510 1680-1630 and 1570-1515	Solution state amide I and II; amide II not seen in lactams Solid state amide I and II; amide II not seen in lactams; amide I is generally more intense than amide II
Tertiary —CONR ₂	1670-1630	Solid and solution spectra are little different
Five-ring lactams	~1700	Shifted to higher frequency when the N atom is in a bridged system in which overlap of the N lone pair with the C=O π -bond is diminished
Four-ring lactams (β -lactams)	~1745	
—CO—N—C=C		Shifted by +15 cm ⁻¹ from the corresponding amide or lactam
C=C—CO—N		Shifted by up to +15 cm ⁻¹ from the corresponding amide or lactam, unusual for the effect of α,β -unsaturation

(129)

Table 2.7 continued		
Group	Band	Remarks
Imides —CO—N—		
CO—		
Six-ring	~1710 and ~1700	Shifted +15 cm ⁻¹ with α,β -unsaturation
Five-ring	~1770 and ~1700	•
Ureas N—CO—N		
—NHCONH—	~1660	
Six-ring	~1640	
Five-ring	~1720	
Carbamates (=urethanes)		
—O—CO—N	1740-1690	Also shows an amide II band when non- or mono-substituted on N
—S—CO—N	1700-1670	
Thioesters and acids		
—CO—SH	~1720	Shifted ~25 cm ⁻¹ when aryl or α,β -unsaturated
—CO—S—alkyl	~1690	Shifted ~25 cm ⁻¹ when aryl or α,β -unsaturated
—CO—S—aryl	~1710	Shifted ~25 cm ⁻¹ when aryl or α,β -unsaturated
Carbonates		
—O—CO—Cl	~1780	•
—O—CO—O—	~1740	
Ar—O—CO—O—Ar	~1785	
Five-ring	~1820	
—S—CO—S—	~1645	
Ar—S—CO—S—Ar	~1715	
Acylsilanes —CO—SiR ₃		
Saturated	~1640	
α,β -Unsaturated	~1590	

Table 2.8 C=N: Imines, oximes, etc.

Group	Band	Remarks
$\begin{array}{c} \diagup \\ \text{C}=\text{N} \\ \diagdown \\ \text{H} \end{array}$	3400-3300(m)	N—H stretching
$\begin{array}{c} \diagup \\ \text{C}=\text{N} \\ \diagdown \end{array}$	1690-1640(v)	Difficult to identify because of large variations in intensity and the closeness of C=C stretching bands; oximes usually have weak absorptions
α, β -Unsaturated	1600-1630(v)	
Conjugated cyclic systems	1660-1480(v)	

Table 2.9 N=N; Azo compounds

Group	Band	Remarks
—N=N—	1500-1400(w)	Weak or inactive in IR, sometimes seen in Raman
$\begin{array}{c} \text{O}^- \\ \\ \text{N}=\text{N} \end{array}$	1480-1450 and 1335-1315	Asymmetric and symmetric stretching

Table 2.10 C=C; Alkenes and arenes

Group	Band	Remarks
$\begin{array}{c} \diagup \\ \text{C}=\text{C} \\ \diagdown \end{array}$	See Table 2.2 for =C—H bands	
Unconjugated	1680-1620(v)	May be very weak if more or less symmetrically substituted
Conjugated with aromatic ring	~1625(v)	More intense than unconjugated C=C
Dienes, trienes etc.	1650(s) and 1600(s)	Lower-frequency band usually more intense and may obscure the higher-frequency band
α, β -Unsaturated carbonyl compounds	1640-1590(s)	Usually weaker than the C=O band
Enol esters, enol ethers and enamines	1690-1650(s)	
$\begin{array}{c} \text{O} \\ \\ \text{C}=\text{C} \end{array} \quad \& \quad \begin{array}{c} \text{N} \\ \\ \text{C}=\text{C} \end{array}$		
Aromatic rings	~1600(m)	
	~1580(m)	Stronger when the ring is further conjugated
	~1500(m)	Usually the strongest of the three peaks

IR II

Table 2.11 Nitro and nitroso groups, etc.

Group	Band	Remarks
Nitro compounds C—NO ₂	1570-1540(s) and 1390-1340(s)	Asymmetric and symmetric N=O stretching; lowered by ~30 cm ⁻¹ when conjugated
Nitrates O—NO ₂	1650-1600(s) and 1270-1250(s)	
Nitramines N—NO ₂	1630-1550(s) and 1300-1250(s)	
Nitroso compounds		
C—N=O		
Saturated	1585-1540(s)	
Aryl	1510-1490(s)	
Nitrites O—N=O	1680-1650(s) 1625-1610(s)	s-trans conformation s-cis conformation
N-Nitroso compounds		
N—N=O	1500-1430(s)	
N-Oxides		
$\text{>N}^+\text{—O}^-$		
Aromatic	1300-1200(s)	Very strong bands
Aliphatic	970-950(s)	
Nitrate ions NO ₃ ⁻	1410-1340 and 860-800	

Table 2.12 Ethers

Group	Band	Remarks
>C—O—C<	1150-1070(s)	C—O stretching
>C=C—O—C<	1275-1200(s) and 1075-1020(s)	
C—O—CH ₃	2850-2810(m)	C—H stretching
Epoxides >C—O—C<	~1250, ~900 and ~800	

IR II

UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF CHEMISTRY
2013/2014 ACADEMIC YEAR HALF YEAR SESSIONAL EXAMINATIONS
CHE 4611: QUANTUM CHEMISTRY AND MOLECULAR SPECTROSCOPY

TIME: THREE HOURS

INSTRUCTIONS: ANSWER ANY FIVE QUESTIONS

DATA YOU MAY WISH TO USE IS PROVIDED ON PAGE 5

QUESTION 1 (20 MARKS)

a) (5 marks)

It can be shown that, in spherical coordinates, the square of the total angular momentum operator J^2 and its component J_z along the z-axis are:

$$J^2 = -\hbar^2 \left[\frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial}{\partial\theta} \right) + \frac{1}{\sin^2\theta} \frac{\partial^2}{\partial\phi^2} \right]; \text{ and}$$

$$J_z = -i\hbar \frac{\partial}{\partial\phi}$$

Calculate $[J^2, J_z]$ and comment on the significance of your result.

b) (5 marks)

Light of wavelength $\lambda = 4.28 \times 10^{-7}$ m interacts with a "motionless" hydrogen atom. During this interaction it transfers all its energy to the orbiting electron of the hydrogen atom. What is the velocity of this electron after interaction?

c) (5 marks)

Consider the angular function $\Phi_m(\varphi) = Ae^{im\varphi}$ for which $0 \leq \varphi \leq 2\pi$.

(i) What is the value of A?

(ii) Show that the function $\Phi_m(\varphi) = Ae^{im\varphi}$ is an eigenfunction of the operator $J_z = -i\hbar \frac{\partial}{\partial\varphi}$. What is the eigenvalue?

d) (5 marks)

Do you expect to experimentally observe the infra-red vibration frequency of the molecule $^{14}\text{N}^{16}\text{O}$? Explain your answer.

QUESTION 2 (20 MARKS)

According to quantum mechanical Postulate IV, the average value of or the expectation value of an observable F is

$\langle F \rangle = \int \psi^* F \psi d\tau$; where the wavefunction ψ is normalized. For the normalized wavefunction of a particle in a one-dimensional box

a) (4 marks)

What is the average value of the position, $\langle x \rangle$?

b) (5 marks)

What is the expectation value of the linear momentum, $\langle P_x \rangle$?

c) (5 marks)

What is the expectation value of the square of the linear momentum, $\langle P_x^2 \rangle$?

d) (4 marks)

Now if we define the uncertainty in the measurement of the linear momentum as the root-mean square deviation, that is

$$\Delta P_x = \sqrt{\langle P_x^2 \rangle}$$

and use the fact that the uncertainty in position x is of the order of the length of the one-dimensional box, l , and in fact $\Delta x =$ your answer in Question 2 a) of this problem. Show that the product, $\Delta P_x \Delta x$, for the lowest state, $n = 1$ of the particle in the box is just the Heisenberg Uncertainty Principle.

e) (2 marks)

Explain the seemingly contradictory results of your solutions to Questions 2b) and Question 2c).

QUESTION 3 (20 MARKS)

a) (12 marks)

Suppose that there is a set of eigenfunctions ψ_n of some Hermitian operator α . Show that according to quantum mechanical postulates, one obtains real answers if α represents an observable.

QUESTION 3 CONTINUES TO THE NEXT PAGE

b) (8 marks)

Now suppose further that α is also an Hermitian operator of another set of eigenfunctions ψ_k . Using the result obtained in Question 3 a), derive the orthonormalization relation for the functions ψ_n and ψ_k (i.e. show that $\int \psi_k^* \psi_n d\tau = \delta_{kn} \begin{cases} 1 & n = k \\ 0 & n \neq k \end{cases}$)

QUESTION 4 (20 MARKS)

Consider a two-dimensional harmonic oscillator with potential: $V(x,y) = \frac{1}{2}k_x x^2 + \frac{1}{2}k_y y^2$.

a) (4 marks)

Write the time independent Schrödinger equation for this harmonic oscillator.

b) (10 marks)

Use the *Separation of Variables Method* to show that the total energy is just the sum of the energy of one-dimensional harmonic oscillators. Show all work.

c) (6 marks)

- (i) Write an expression for the total energy of the oscillator. Define all terms in the expression .
- (ii) Write an expression for the total energy of the oscillator if it were isotropic, i.e. if $k_x = k_y$.

QUESTION 5 (20 MARKS)

In the near-infrared spectrum of carbon monoxide ($^{12}\text{C}^{16}\text{O}$) there is an intense band at 2144 cm^{-1} .

a) (3 marks)

State the selection rule which leads to the appearance of the vibration band.

b) (4 marks)

Calculate the fundamental vibration frequency of CO in units of hertz.

c) (2 marks)

Calculate the period of vibration τ .

d) (7 marks)

Calculate the force constant

e) (4 marks)

Calculate the zero-point energy of CO in kJ mol^{-1} .

QUESTION 6 (20 MARKS)

a) (5 marks)

Draw a picture sketch of the following (un-normalized) wavefunctions for a particle in a box: $\psi_+ = \phi_1 + \phi_2$ and $\psi_- = \phi_1 - \phi_2$ where ϕ_1 and ϕ_2 are the eigenfunctions of the particle in a box for $n = 1$ and $n = 2$ energy levels.

b) (5 marks)

(i) The wavelength of the photo-electric threshold for tungsten is 2300×10^{-10} m. Calculate the energy (in eV) of electrons ejected from the surface by ultraviolet radiation of wavelength 1800×10^{-10} m.

(ii) What is the velocity of the ejected electrons?

c) (5 marks)

In a particle experiment the system under study was prepared in a state described by the wavefunction

$$\Psi = \frac{3i}{\sqrt{12}}\psi_a + \frac{1}{2}\psi_b$$

The eigenfunctions ψ_a and ψ_b are normalized. A single measurement of the energy was then carried out. What is the per cent probability of obtaining the particular energy associated with the state ψ_a ?

d) (5 marks)

The one-dimensional linear harmonic oscillator wavefunctions for the energy levels $n = 0$ and $n = 1$ are: $\psi_0 = \left[\frac{\alpha}{\pi}\right]^{\frac{1}{4}} e^{-\alpha x^2/2}$ and $\psi_1 = \left[\frac{4\alpha^3}{\pi}\right]^{\frac{1}{4}} x e^{-\alpha x^2/2}$, where $\alpha = \frac{4\pi^2 \nu m}{h}$.

Show that the two functions are orthogonal.

END OF CHE 4611 EXAMINATION

UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF CHEMISTRY

2013/2014 ACADEMIC YEAR TERM I AND TERM II SESSIONAL EXAMINATIONS
CHE 4611: QUANTUM CHEMISTRY AND MOLECULAR SPECTROSCOPY

DATA

$$h = 6.63 \times 10^{-34} \text{ J s}; \quad R_H = 2.18 \times 10^{-18} \text{ J} = 1.0973731 \times 10^5 \text{ cm}^{-1}; \quad c = 3.00 \times 10^8 \text{ m s}^{-1};$$

$$N_A = 6.023 \times 10^{23} \text{ mol}^{-1}; \quad 1 \text{ amu} = 1.661 \times 10^{-27} \text{ kg}; \quad 1 \text{ J} = \text{m}^2 \text{ kg s}^{-2}; \quad 1 \text{ N} = \text{m kg s}^{-2} = \text{J m}^{-1};$$

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}; \quad \psi_n(0 \leq x \leq a) = \sqrt{\frac{2}{a}} \sin \frac{n\pi}{a} x; \quad \int \sin ax \cos ax dx = \frac{1}{2a} \sin^2 ax;$$

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a}; \quad \int x \sin^2 ax dx = \frac{x^2}{4} - \frac{x \sin 2ax}{4a} - \frac{\cos ax}{8a^2};$$

$$\int_0^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}; \quad \int_0^{\infty} x e^{-ax^2} dx = \frac{1}{2a}; \quad \int_0^{\infty} x^2 e^{-ax^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{a^3}}$$

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES DEPARTMENT OF CHEMISTRY

CHE 4822: INORGANIC INDUSTRIAL CHEMISTRY

TIME : 3 HOURS

EXAMINATION – JULY 2014

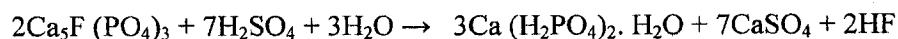
ANSWER : ANSWER ANY FIVE QUESTIONS.
: ALL QUESTIONS HAVE EQUAL MARKS.

QUESTION 1

- a) Explain the mechanisms by which heat is transferred from the combustion space into a glass melt (Glass tank furnace). How can heat transfer be enhanced? (8 marks)
- b) Sketch a simple diagram of the Glass tank furnace showing regenerative chamber. Explain the mechanism of operation. (6 marks)
- c) Explain briefly the basic beneficiation processes that are involved in the manufacturing of ceramics. (6 marks)

QUESTION 2

- a) The manufacturing process of Superphosphate depends on reacting phosphate rock with sulfuric acid and the fertilizer contains about (16 – 20 %) P₂O₅. The net reaction proceeds as follows:



The process can be divided into two stages, explain briefly what each stage represents.

(7marks)

- b) Describe the process recovery of Fluorine fumes in the manufacture of Superphosphate. (3 marks)
- c) With the aid of a flow Diagram describe the manufacturing process for the following.
- I. Single superphosphate Fertilizer.
 - II. Triple Superphosphate Fertilizer.

(10 marks)

QUESTION 3

- a) Briefly describe the Wet – process Phosphoric acid manufacture. (8marks)
- b) Use a sketch to illustrate the mechanism of operation for the Bird-Prayon Tilting – Pan Filter for the phosphoric acid manufacture. Indicate clearly the source of the Weak water liquor and explain its importance. (6 marks)
- c) With the aid of a flow Diagram, describe the manufacturing process of Phosphoric acid wet process. (6 marks)

QUESTION 4

- a) Describe the Manufacturing Process of Synthetic Ammonia. (5 marks)
- b) Briefly describe the following processing steps. (10 marks)
- I. Natural gas desulfurization.
 - II. Catalytic steam reforming.
- d) With the aid of a flow Diagram, describe the manufacturing process of Synthetic Ammonia (5 marks)

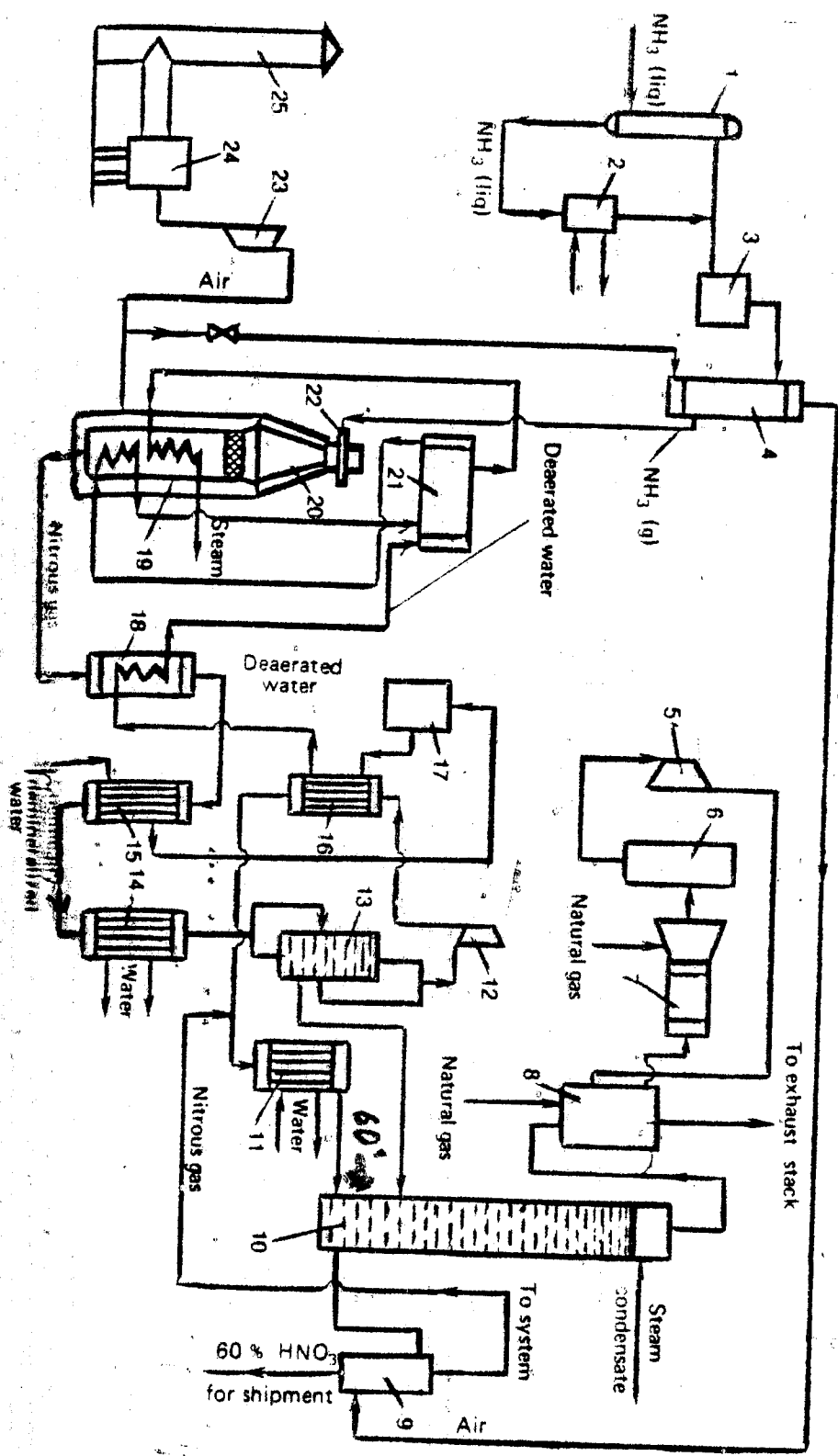
QUESTION 5

- a) Briefly describe the manufacturing process of Nitric Acid. Write the reactions involved in the Production process. (8 marks)
- b) Explain the major function of the Catalytic Scrubbing Unit in the Manufacturing process of Nitric Acid (6 marks)
- c) The AK-72 process for the Manufacture of Nitric acid is based on a closed – cycle power cogeneration scheme which includes two ammonia oxidation steps. With reference to the flow sheet of the AK-72 nitric acid plant, briefly outline the production process (6 marks)

QUESTION 6

- a) With the aid of a flow Diagram describe the Manufacturing Process of Sulphuric Acid. (8 marks)
- b) Briefly describe the oxidation of SO₂ to SO₃ process (Indicate Kindling and other temperatures if Vanadium catalyst is used). (6 marks)
- c) Describe the absorption process of SO₃ and explain why a second absorption stage is necessary. State the properties of 98.3% Sulphuric acid and why this acid is used for absorption of SO₃ containing gas? (6 marks)

Wish you well, The End





THE UNIVERSITY OF ZAMBIA

School of Natural Sciences

Department of Computer Studies

END OF YEAR - FINAL EXAMINATION

ARTIFICIAL INTELLIGENCE CSC 3402

Date: TUESDAY, 22ND JULY, 2014
Time: 09:00hrs – 12:00hrs
Duration: 3 Hours
Venue: GLT

Instructions

- a) *There are SEVEN (7) questions in this paper and you are required to answer ANY FIVE (5) OF THEM IN ANY ORDER.*
- b) *All questions have carry the weight of 20 marks each*
- c) *Each question should start on its separate page or booklet*

1.

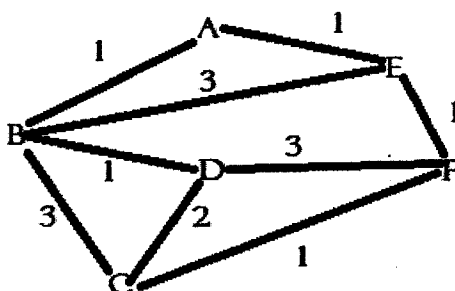
- a. Define the following
- i. An agent [4 marks]
 - ii. A rational agent [4 marks]
- b. Briefly describe the four agent architectures. [12 marks]

2.

- a. Define the following
- i. admissible heuristic [3 marks]
 - ii. consistent heuristic [3 marks]
- b. Briefly describe how the A* search functions [4 marks]
- c. Show that if A* uses admissible heuristic the A* tree-search is optimal. [6 marks]
- d. Prove that the average of two admissible heuristics is also admissible. [4 marks]

3.

- a. Explain what local-search algorithms are and give two examples. [8 marks]
- b. Consider the navigation problem represented in the graph below. You are currently in B and you need to be in F. Suppose you came up with a heuristic $h(n) = h^*(n)$ where h^* is the actual cost moving from n to the goal. [12 marks]

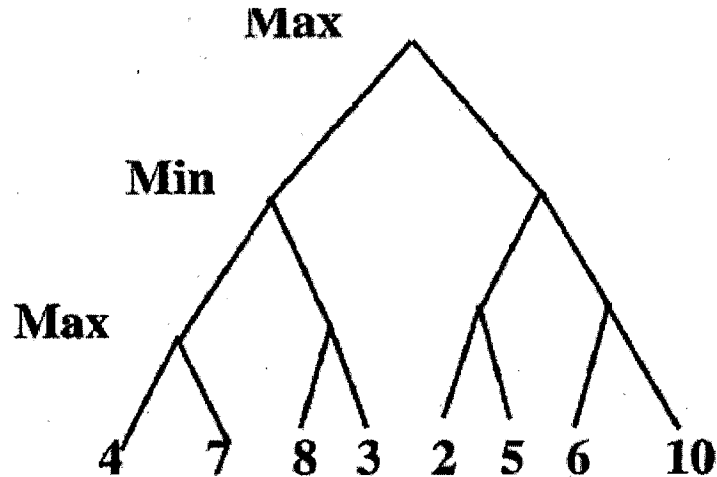


For each of the following search strategies,

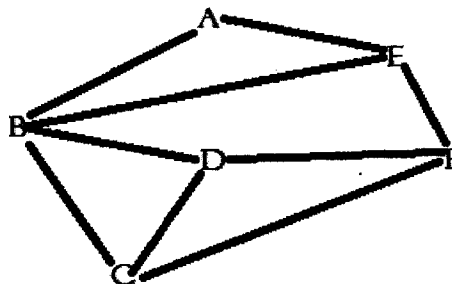
- Draw the search tree
 - Order of node visitation
 - Cost
- i. Greedy best-first search
- ii. A* search

4.

- a. Concisely explain how the Mini-max algorithm for game playing with alpha-beta pruning works. [10 marks]
- b. Given the game tree below. Show the move Max plays and the branches cut-off by the alpha-beta algorithm. [10 marks]



5. Consider the constraint graph given below representing a map colouring problem in which variables with constraints can't have the same colour.



- a. Draw an example of the sketch map with the constraint graph above. [4 marks]
- b. What is the minimum number of colours required for this problem to be solved? Give a brief explanation based on the graph. [4 marks]
- c. Suppose you have three colours red, green and blue, show the order in which the values are assigned using the arc-consistency and applying the relevant value-assigning heuristics. If there's tie use alphabetical order as final tie-breaker. [12 marks]

6.

- a. Define the following as applied in propositional logic
 - i. Entailment of a sentence β by α denoted by $\beta \models \alpha$ [3 marks]
 - ii. A valid sentence [3 marks]
- b. State how the validity of a sentence is related to $\text{KB} \models \alpha$ [4 marks]
- c. Show that $h \Rightarrow d \wedge \neg g \models g \Rightarrow \neg h$ using
 - i. Truth table [5 marks]
 - ii. A refutation tree [5 marks]

7. Given the following KB.

“For every test in a CS course, at least one person fails.”

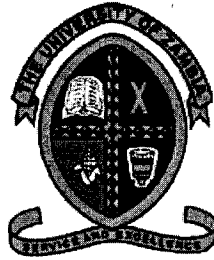
“Everyone passes an easy test in a course.”

“No one can both pass and fail the same test.”

“Class1 had an easy test.”

- a. Convert the KB into first-order predicates [10 marks]
- b. Use resolution to prove that “Class1 is not a CS course.” [10 marks]

*****END OF EXAMINATION*****



THE UNIVERSITY OF ZAMBIA
School of Natural Sciences
Department of Computer Studies

SOFTWARE ENGINEERING
CSC 3600

2013-2014 FINAL EXAM

Date: 31st July 2014
Venue: API
Time: 09:00hrs – 12:00hrs
Duration: 3 Hours

Instructions

- 1. This examination has two sections*
- 2. Answer all ten (10) questions in Section A. Each question carries 4 marks*
- 3. Answer any four (4) questions in Section B. Each question carries 15 marks*

Section A

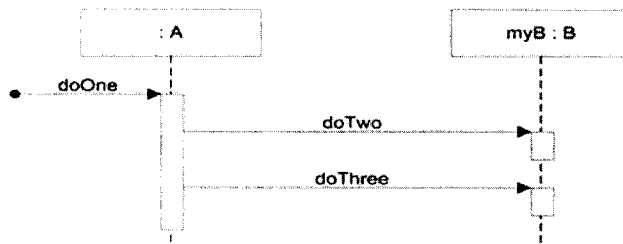
Answer all the questions. Each question carries 4 marks

1. Define software engineering? What are the essential attributes for good software?
2. Describe about the scrum and benefits of the scrum?
3. Using the technique suggested here, where natural language descriptions are presented in a standard format, write plausible user requirements for the following functions:
 - a) An unattended petrol (gas) pump system that includes a credit card reader. The customer swipes the card through the reader then specifies the amount of fuel required. The fuel is delivered and the customer's account debited. [2 Marks]
 - b) The cash-dispensing function in a bank ATM.[2 Marks]
4. Briefly explain the different architectural views?
5. Explain different types of application with examples?
6. Write short notes on performance testing and user testing?
7. Analyze the java code below and answer the questions that follows; [4 Marks]

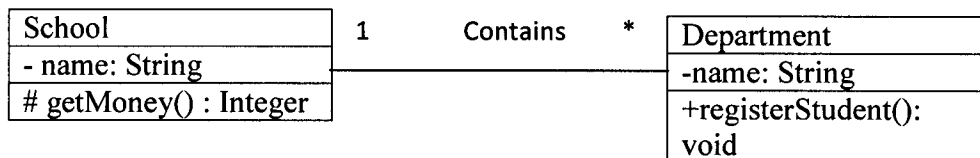
```
public class Student{  
    private int computerNumber;  
    protected String name;  
  
    public Student(String computerNumber, String name{  
        this.computerNumber = computerNumber;  
        this.name = name;  
    }  
  
    public void register(String password){  
    }  
  
    public String takeExam(){  
    }  
} //end class student
```

- a) How many attributes and how many behaviors does the above code contain? [1 Mark]
- b) Represent the above code in a class diagram using UML [3 Marks]

8. Map the following sequence diagram to java code. [4 Marks]



9. Map the following class diagram to java code. [4 Marks]



10. Explain the following OOAD artifacts. [4 Marks]

- a) Use Case Text [1 Mark]
- b) Domain Model [1 Mark]
- c) Class Diagram [1 Mark]
- d) Sequence diagram [1 Mark]

Section B

Answer any four (4) questions. Each question carries 15 marks

1. Explain in detail about the different types of software process model?
2. a) Explain different ways of writing system requirement specification? [10 Marks]
 b) Identify and describe the four types of requirement that may be defined for computer-based system?[5 Marks]
3. Write short notes on the following topic and give appropriate examples.
 - a) Pipe filter architecture[3 Marks]
 - b) Client server architecture [3 Marks]
 - c) Application Architecture [3 Marks]
 - d) Test-driven development [3 Marks]
 - e) Software inspection's and its advantages [3 Marks]

4. The University of Zambia (UNZA) has a record of information about students and employees. A person has a name, address, phone number and e-mail address. A student has a class status (fresher, matusa, masadi, mafosa or mafifi). An employee has an office, salary and date-hired. The University has hired you to computerize their information system.
- Design a domain model for the scenario given above. [5 Marks]
 - Design a static model from the domain model you designed in question a) above [5 Marks]
 - Propose a system architecture you would use to satisfy the UNZA information needs. Give reasons to support your choice. [5 Marks]

5. Analyze the use case given below;

Use case Name: Purchase Goods
Actors: Cashier
Pre - condition: Cashier is identified and authenticated
Post - condition: Sale is saved. Tax is correctly calculated. Accounting and Inventory are updated. Commissions recorded. Receipt is generated. Payment authorization approvals are recorded.
<p>Main Success Scenario</p> <ol style="list-style-type: none"> Customer arrives at POS checkout with goods and/or services to purchase. Cashier starts a new sale. Cashier enters item identifier. System records sale line item and presents item description, price, and running total. Price calculated from a set of price rules. <p>Cashier repeats steps 3-4 until indicates done.</p> <ol style="list-style-type: none"> System presents total with taxes calculated. Cashier tells Customer the total, and asks for payment. Customer pays and System handles payment. System logs completed sale and sends sale and payment information to the external Accounting system (for accounting and commissions) and Inventory system (to update inventory). System presents receipt. Customer leaves with receipt and goods (if any).

- Design a domain model from the use case diagram given above. [5 Marks]
- Design a Class diagram to satisfy the use case above. [5 Marks]
- Design a sequence diagram to satisfy the use case above [5 Marks]

6. Analyze the problem statement below and answer the questions that follow;

Problem Specification:

A new Palestinian news agency, called SRT, wishes to launch a news website on the internet. The site intends to provide Palestinians living in the Gaza strip, the west bank as well as other places with full coverage on news that is of interest to them.

Additionally, it shall inform its visitors of various Palestinian and International topics to help visitors understand and analyze news better. SRT also aims at giving its visitors a chance to collaborate through the site. They should also be able to customize the site as they like.

In summary, SRT management is well aware of the competition they have locally. Hence, they prefer to be unique and creative to maximize the number of their visitors.

They are aware that software of high quality could take a longer time. However, the general manager is keen to finalize the site as soon as possible, and he is not quite knowledgeable that requirements on its own could take a long time.

Notices:

_ Use your judgment to assume any specification you think you need to solve the problem.

_ Keep your answer short, and neat. Be short and to the point.

_ Justify your answers.

- a) Define stakeholder. [2 Marks]
- b) Identify all the stakeholders from the problem statement given above. [4 Marks]
- c) Describe all the stakeholders identified in question b) above. [4 Marks]
- d) Analyze the stakeholders identified in question b) above. [5 Marks]



The University of Zambia Examination 2014

School of Natural Science

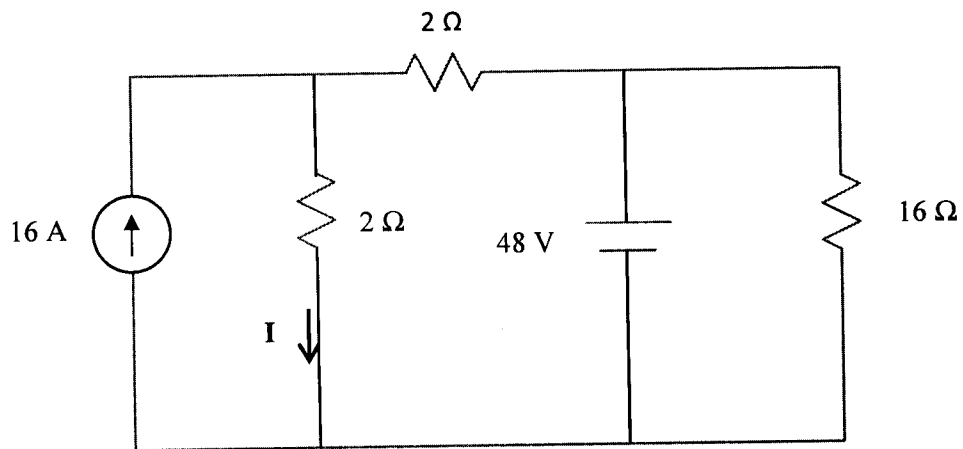
Analogue and Digital Electronics CS3120

Time: 3 Hours

Instructions: Answer any 5 questions

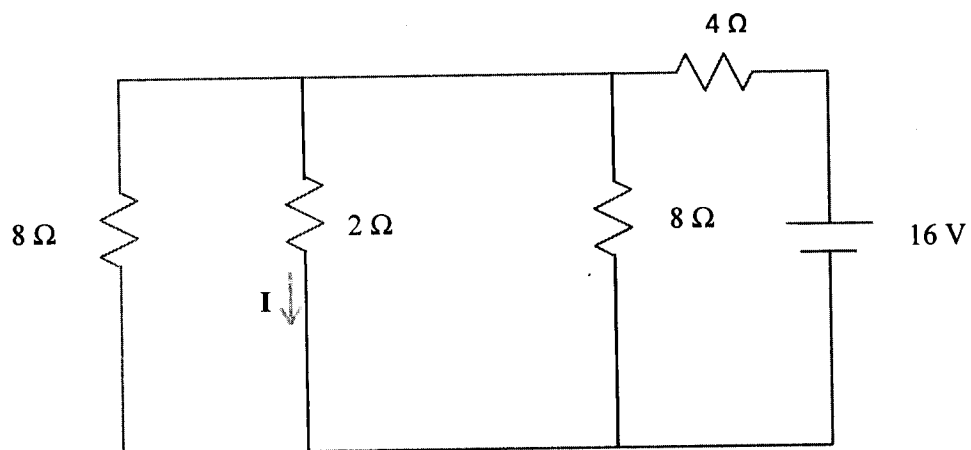
All questions carry 20 marks

1. Use Kirchoff's current and voltage laws to find the current I in the circuit.



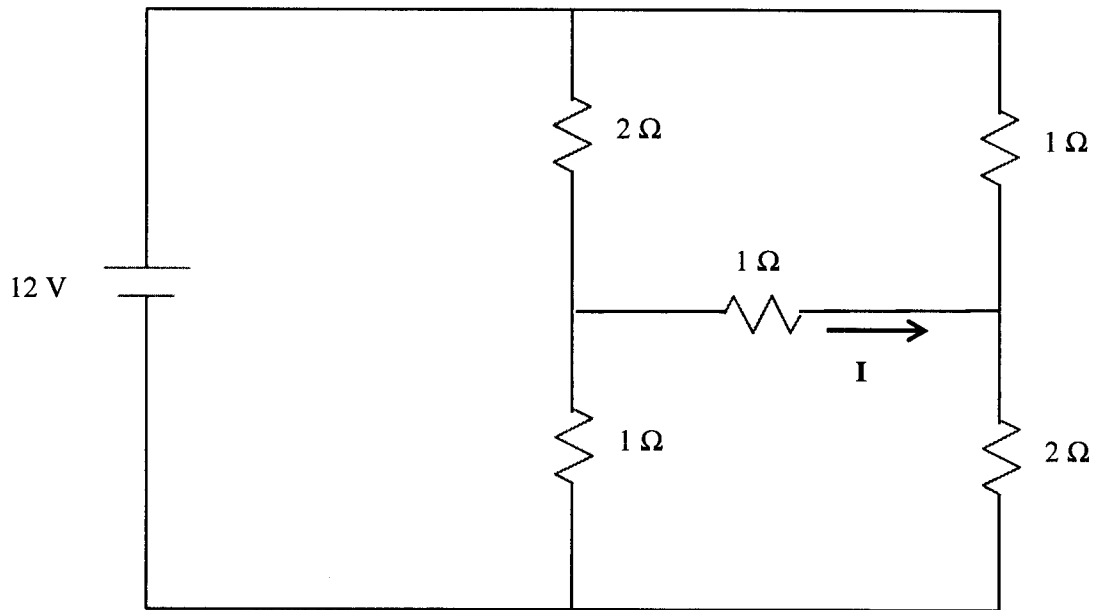
2.

- a. Use Norton's theorem to find the current I in the $2\ \Omega$ resistor.



- b. Convert the Norton equivalent circuit in part a. to a Thevenin equivalent circuit.

3. Use Nodal analysis to find the current **I** in the circuit.



4.

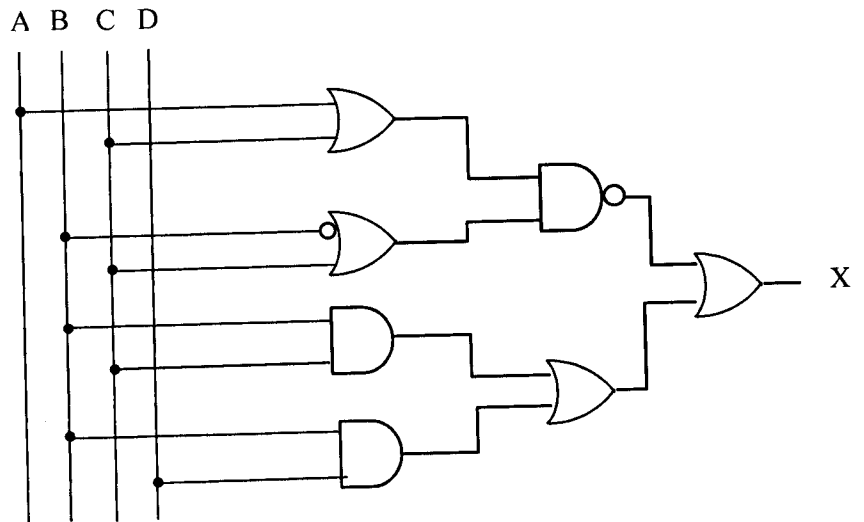
- a. Draw the logic circuit for the following equation.

$$Z = \overline{\overline{(B + C)(\bar{A} + \bar{C} + \bar{D})}} + \overline{(B + C)(\bar{A} + \bar{C})(\bar{A} + D)}$$

- b. Use De Morgan's theorem and Boolean algebra rules to reduce the equation to a SOP form.
 c. Simplify the circuit using Boolean rules and identities.
 d. Draw the simplified circuit.

5.

- a. Write the Boolean equation for the circuit below.

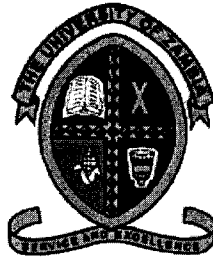


- b. Use De Morgan's theorem and Boolean algebra rules to simplify the equation.
 c. Draw the simplified circuit.
 d. Simplify the same circuit using a K-Map from the SOP. Draw the circuit.
6. The truth table below represents a voting system for a tribunal. A motion is accepted if at least two of the three judges vote yes (1). A, B and C represent each of the judge's votes and Y represents the output (1 for yes)

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

- a. Make a K-Map to represent this truth table.
 b. From the K-Map derive the simplified Boolean expression.
 c. Draw the circuit.

END OF EXAMINATION



THE UNIVERSITY OF ZAMBIA
School of Natural Sciences
Department of Computer Studies

IT PROJECT MANAGEMENT
CSC 3612

2013-2014 FINAL EXAM

Date: 23rd July 2014
Venue: UPPER DINING HALL
Time: 09:00hrs – 12:00hrs
Duration: 3 Hours

Instructions

1. *This examination has two (2) sections*
2. *Answer any five (5) questions in Section A. Each question carries 5 marks*
3. *Answer any five (5) questions in Section B. Each question carries 15 marks*
4. *Present your answer neat and understandable*

Section A

Answer any 5 questions. Each question carries 5 marks

1. Define Project Management? Explain briefly the different knowledge areas?
2. Explain the predictive life cycle models?
3. Describe the suggestion for performing integrated change control?
4. Discuss why project scope must be tied to WBS? What are some advantages using milestones in the WBS?
5. Explain the different processes in the project time management?
6. Explain the five cost categories related to quality?

Section B

Answer any 5 questions. Each question carries 15 marks

1. a) Describe the role and the suggestive skills for project manager?
b) Explain the project management process groups?
2. a) Perform a financial analysis for this project Assume the projected costs and benefits for this project are spread over four years as follows: Estimated costs are \$100,000 in Year 1 and \$25,000 each year in Years 2, 3, and 4. Estimated benefits are \$0 in Year 1 and \$80,000 each year in Years 2, 3, and 4. Use an 8 percent discount rate. Calculate the NPV, ROI, and year in which payback occurs. Give reasons explaining whether you would recommend investing in this project, based on your financial analysis.
b) Explain the common elements should be presented in the project management plan?
3. a) Develop a WBS that will allow Air Zambia to keep track of all scheduled maintenance for its chartered aircraft?
b) Explain the WBS dictionary and scope baseline? Give suggestions for creating a WBS dictionary and Scope baseline?

4. a) Develop a network diagram using the AON technique and calculate the critical path using the information in the table. What is the critical path?

Task/Activity	Estimated Duration	Predecessor
A	1 day	-
B	3 days	A
C	4 days	B
D	2 days	B
E	1 day	C
F	3 days	C,D
G	3 days	E
H	1 day	F
I	2 days	G,H
J	5 days	I

b) Why is it important to determine activity sequencing on projects? Discuss diagrams you have seen that are similar to network diagrams. Describe their similarities and differences

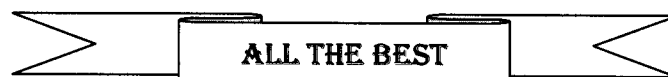
5. a) Explain the basic principles of cost management?

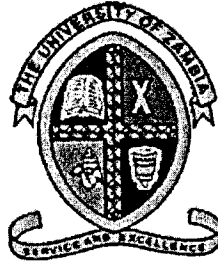
b) Given the following information for a one-year project, answer the following questions.

Planned Value (PV)	= \$45,000
Earned Value (EV)	= \$35,000
Actual Cost (AC)	= \$50,000
Budget at completion (BAC)	= \$200,000

- i) What is the cost variance, schedule variance, cost performance index (CPI), and schedule performance index (SPI) for the project
- ii) How is the project doing? Is it ahead of schedule or behind of schedule? Is it under budget or over budget?
- iii) Use the CPI to calculate the estimate at completion (EAC) for this project, Is the project performing better or worse than planned?
- iv) Use the SPI to estimate how long it will take to finish this project.
- v) Sketch the earned value chart based for this project.

6. Explain in detail about the tools and techniques used in the quality planning?





THE UNIVERSITY OF ZAMBIA
School of Natural Sciences
Department of Computer Studies

FINAL EXAMINATION

GRAPHIC AND VISUAL COMPUTING
CSC 4505

Date: 30th JULY 2014
Time: 14:00hrs – 17:00hrs
Duration: 3 Hours
Venue: Upper Dining Hall

Instructions

1. There are five (5) questions and two (2) sections in this paper.
2. *Answer all the questions in Section A and choose any one (1) question from Section B*

Section A

Q. 1

- a) Define Scan Conversion and name one other term you can use in its place. (2 marks)
- b) What are homogeneous coordinates? (2 marks)
- c) The emitted light by phosphor in CRT fades very quickly and the technique is needed to maintain the screen picture. Name and explain it? (2 marks)
- d) What does the display() function do in an OpenGL program? (1 mark)
- e) Illustrate the Raster display system. (4 marks)
- f) Explain how to eliminate a flicker in an OpenGL program. (2 marks)
- g) What is involved in Shearing about an arbitrary point P in space? (3 marks)

Total marks 16

Q. 2

- a) What type of the buffer is used in 3-dimensional graphics? (2 marks)
- b) What does the function called glutInit() do in an OpenGL program? (1 mark)
- c) What are we suppose to do if we want to apply a series of transformations T1, T2 and T3 to a set of points? (4 marks)
- d) Callbacks are used for two purposes, what are the purposes? (2 marks)
- e) What is the purpose of the following: geometric transformation, rotation, scaling and translation? (4 marks)
- f) What makes the DDA not a very good algorithm of drawing a line and how can we improve its performance? (3 marks)
- g) Define Computer graphics? (1 mark)

Total marks 17

Q. 3

- a) What do you know about Commutivity of Transformations? (4 marks)
- b) Name the two types of refresh. (2 marks)

- c) What is the function of the Evaluator in Opengl operation? (2 marks)
- d) Define Opengl. (1 marks)
- e) Explain how to light up the 3D world. (4 marks)
- f) What is involved in Rotation about an arbitrary point P in space rotation? (3 marks)
- g) What does the function gluOrtho2D do? (1 mark)

Total marks 17

Section B

Q. 4

- a) Explain briefly about beam-penetration and the shadow-mask methods. (4 marks)
- b) Line drawing is accomplished by calculating intermediate positions along the line path between two specified endpoint positions. Write three line drawings algorithms. (12 marks)
- c) Write an Opengl program which draws the Spinning Square. (7 marks)
- d) Explain briefly the use of computer graphics in computer art. (4 marks)
- e) What is involved in Reflection through an arbitrary line? (4 marks)
- f) Explain briefly the scan conversion of the simple polygon. (5 marks)
- g) Explain briefly on OpenGL operation. (5 marks)
- h) An ellipse is defined as the set of points such that the sum of the distances from two fixed positions is the same for all points. Write an ellipse drawing algorithm. (5 marks)
- i) Explain briefly about Raster Scan Display. (4 marks)

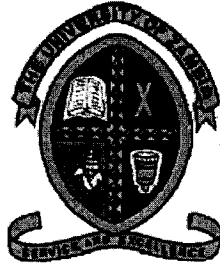
Total marks 50

Q.5

- a) Generally, any procedure that identifies those portions of a picture that is either inside or outside of a specified region of space is referred to as a clipping algorithm. Explain briefly about point and line clipping. (6 marks)
- b) Write an Opengl program which draws the Spinning Triangle. (7 marks)
- c) Explain briefly the use of computer graphics in Education and training. (5 marks)

- d) Explain briefly about the Co ordinate systems. (4 marks)
- e) The circle is a frequently used component in pictures and graphs, a procedure for generating either full circles or circular arcs is included in most graphics packages. Write two circle drawing algorithms. (7 marks)
- f) Applications of clipping include extracting part of a defined scene for view ing; identifying visible surfaces in three-dimensional views; creating objects using solid-modeling procedures; and drawing and painting operations that allow parts of a picture to be selected for copying, moving, erasing, or duplicating. Write Cohen and Sutherland clipping algorithm. (6 marks)
- g) Briefly explain on the following display technologies: direct-view storage tube (DVST), liquid- crystal display (LCD), electroluminescent (EL) display and electrophoretic display. (6 marks)
- h) We can correctly clip a polygon by processing the polygon boundary as a whole against each window edge. This could be accomplished by processing all polygon vertices against each clip rectangle boundary in turn. Write the Sutherland – Hodgeman clipping algorithm. (5 marks)
- i) Explain briefly about Random Scan Display (4 marks)

Total marks 50



THE UNIVERSITY OF ZAMBIA
School of Natural Sciences
Department of Computer Studies

FINAL EXAMINATION

DISTRIBUTED SYSTEMS - CSC 4722

Date: Tuesday, 29th July 2014
Time: 09:00hrs – 12:00hrs
Duration: 3 Hours
Venue: Upper Dining Hall

Instructions

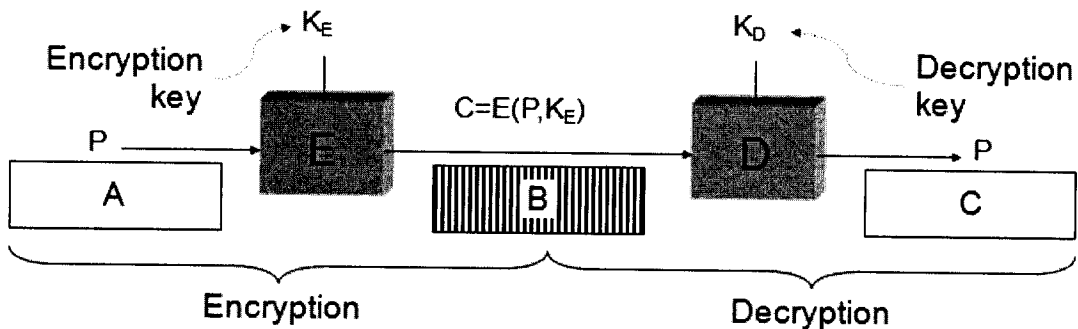
1. There are **six (6) questions** and **two (2) sections** in this paper.
2. Each question carries **20 marks**,
3. *You are required to answer a total of **Five (5) Questions***
4. *Answer **all the questions in Section A** and choose **any three (3) questions from Section B***

SECTION A

This section has Two Questions. Answer all the questions

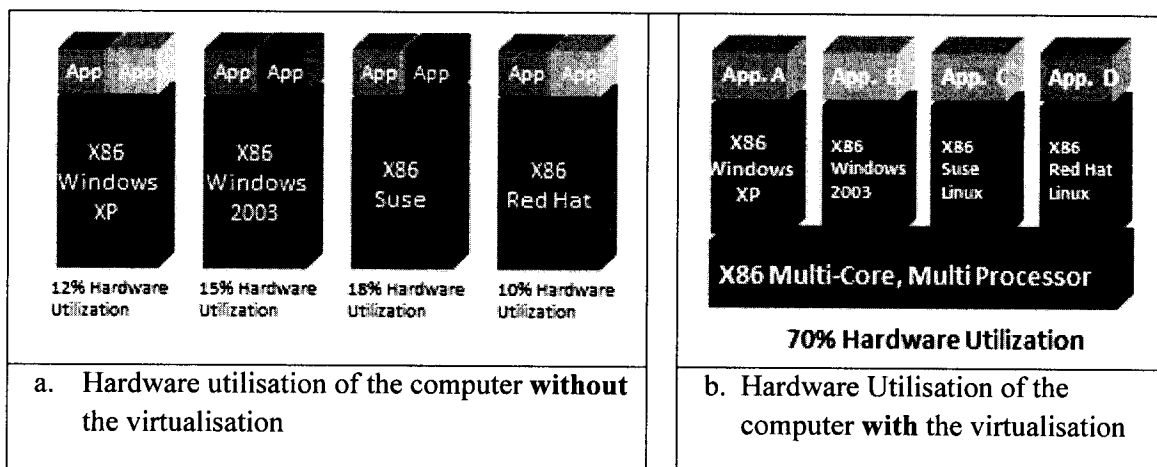
Question I

- a) Define each of the following terms [4]
- Distributed Computing
 - Cloud Computing
 - Virtual Machine Monitor (VMM)
 - Remote Procedure Call
- b) There have been bigger projects based on distributed systems in use today. Below is a list of three major ones distributed systems currently in use. Give a brief description for each one of them [6].
- Collaborative Knowledge Bases** – Wikipedia
 - Internet** – Gomez Distributed PEER Client (peerReview)
 - Life Sciences** - Compute Against Cancer® (CAC)
- c) Failure masking in distributed systems can be achieved through several ways. However **redundancy** is one of the common ways to mask faults. Briefly discuss the following three ways of masking errors [6].
- Information redundancy
 - Time redundancy
 - Physical redundancy
- d) The diagram below shows the basic process of cryptography [4].
- In the diagram, name the component represented by the letters A, B and C.
 - Using the diagram below, give the basic description of data cryptography.



Question II

- a) The current Enterprise Information Technology Centers support many service applications such as Microsoft Exchange, Oracle, SAP, Web servers and Citrix. However, each of these service application demands its own environment, specific version of operating system, multiple processors and disks and specialized configurations. This makes it difficult and expensive to run These Enterprise systems. One of the solutions is to implement virtualization. **Define** each of the following in relation to virtualisation[2]
- i. Virtualization
 - ii. Host Operating system
- b) **Give any four examples** of Server based virtualisation software and give any two advantages of using virtualisation when implementing servers [4]
- c) The diagram below shows the hardware utilisation of the computer hardware (a) without virtualisation and (b) with virtualisation implemented. Desktop virtualization also called client virtualization is a virtualization technology used to separate a computer desktop environment from the physical computer as shown in the diagram below (b).
- i. **Give a brief description of any four examples** of Desktop Virtualization Software [4]
 - ii. **Choose any one of the desktop virtualisation software** you are very familiar with and **briefly discuss** the following [10]
 1. Hardware and system requirements
 2. Supported operating systems
 3. Overview of installation procedure
 4. Creation and managing the virtual machine
 5. Limitation of the virtual machine



SECTION B

This section has four Questions. Choose any three questions

Question I

- a) Define each of the following [2]
- i. Stub
 - ii. CORBA
- b) One of the issues one has to deal with in distributed system is *Fault tolerance*. **Define failure tolerance**. In relation to fault tolerance, **briefly describe** each of the following terms [4];
- i. Fault
 - ii. Error
 - iii. Failure
- c) **With the aid** of a diagram give a brief description for each of the following [6]
- i. Centralized systems
 - ii. Grid Computing
 - iii. Cluster Computing
- d) With the aid of the **diagram**, give a **brief description** for each of the following distributed system architectures [8]
- i. Client-server
 - ii. 3-tier architecture
 - iii. Tightly coupled (clustered)
 - iv. Peer-to-peer

Question II

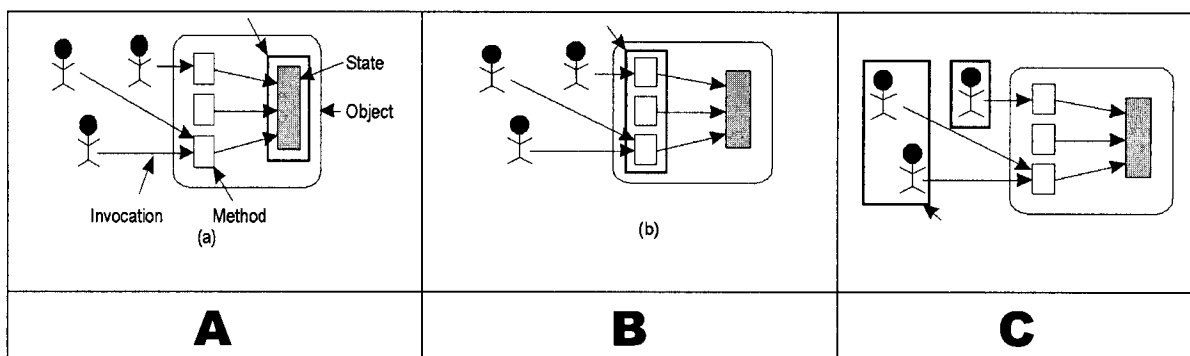
- a) Identifiers are names that are unique and location independent. Give three properties of identifiers [3]
- b) Give a brief description for each of the following in relation to naming in distributed system [4]
- i. Flat Naming
 - ii. Structured Naming
 - iii. Attribute-Based Naming
 - iv. Name Resolution
- c) Distribution transparency is an important feature in distributed systems. Good examples include **Location Transparency** which refers to the fact that in a true *distributed system*, users cannot tell where hardware and software resources such as CPUs, printers, files and databases are located. **List** and describe **four** other examples of distributed systems transparency [4].
- d) Give and briefly discuss **three characteristics** of **dependable** distributed systems [3]:
- e) Give a brief description for each of the following in relation to distributed system [6]
- i. Consistency Models
 - ii. Replication and Scaling
 - iii. Causal Consistency

Question III

- a) Draw and complete the table below that shows the goals and threats in relation to security environment [2].

#	GOAL	THREAT
1	Data confidentiality	
2		
3		Denial of service

- b) Security Play an important role in distributed systems. Therefore, distributed systems should provide mechanisms that allow several other security policies to be implemented. Give the three major mechanisms that need to be provided in order to secure distributed systems [3].
- c) Name and discuss any four kinds of IT related security intruders [4]
- d) Give three major reasons and for each reason give a detailed explanation as to why it is so difficult to implement security on distributed systems [6]
- e) Give a brief description of **Globus** in relation to security in distributed systems [2]
- f) There are generally three approaches for protection against security threats as shown the diagrams below. Discuss each of the three methods (A, B and C) [3].



Question IV

- a) Give two reasons why it would be important to replicate data in distributed systems [2]
- a) Assuming that the distributed system consists of a collection of servers that interact with each other and with client processes. Failures affect the ability of the system to provide the service it advertises. Describe in detail each of the failure types below in relation to distributed systems [10].
- i. Crash error
 - ii. Omission error
 - iii. Timing error
 - iv. Response error
 - v. Arbitrary error
- b) The concept of distributed computing is the most efficient way to achieve the optimization. Distributed computing is currently implemented anywhere including intranet, Internet or mobile ubiquitous computing. It deals with hardware and software systems that contain more than a single processing, storage and run in concurrently. Give and discuss **four advantages** and **four disadvantages** of implementing distributed systems [4]
- c) When designing distributed systems, one need to consider the issues and challenges that comes with distributed systems implementation. These include **Heterogeneity, Openness, Transparency** and **Scalability**. Give a description for each one of these in relation to distributed systems [4]

The University of Zambia Examination 2014
Hardware Design and Implementation CS4130

Time: 3 Hours

Instructions: Read instructions for each section

Section 1: Multiple Choice

(20 marks)

Answer all (select 1)

1. What is not true about firmware?
 - a. portion of the BIOS contained in ROM chips
 - b. found on the motherboard and in some adapter cards
 - c. it is volatile
 - d. contains basic routines to start up the PC
2. What is true about CMOS?
 - a. It is a form of BIOS
 - b. CMOS stores the system drivers
 - c. CMOS is non-volatile
 - d. CMOS contains the real time clock
3. A processor bus is sometimes called a...
 - a. FSB
 - b. AGP
 - c. PCI
 - d. ISA
4. The bus designed specifically for a video card is the
 - a. PCI
 - b. AGP
 - c. ISA
 - d. FSB
5. The PCI bus was first introduced in the
 - a. Intel 8086
 - b. Intel 386
 - c. Intel 486
 - d. AMD Athlon
6. Which bus is ideal for slow peripherals?
 - a. PCI
 - b. AGP
 - c. FSB
 - d. ISA
7. Which of the following is not a system resource?
 - a. Memory
 - b. IRQ channels
 - c. DMA channels
 - d. I/O port addresses

8. Which of these memory types is not built on the motherboard?
 - a. SRAM
 - b. ROM
 - c. EPROM
 - d. DRAM
9. Which of these statements is true about SRAM and DRAM?
 - a. SRAM is slower than DRAM
 - b. DRAM has a lower density than SRAM
 - c. DRAM is cheaper than SRAM
 - d. SRAM needs refreshing while DRAM does not
10. Which type of RAM is suitable for transferring a block of data from main memory to cache?
 - a. SDRAM
 - b. Page mode RAM
 - c. EDO RAM
 - d. Rambus RAM
11. How does DDR SDRAM transfer data twice as quickly as standard SDRAM?
 - a. Uses a twice as fast a clock speed SDRAM
 - b. Is twice more dense than SDRAM
 - c. Preloads some of the data before accessing it
 - d. Transfers data twice per transfer cycle
12. Which motherboard feature controls the amount of data that a CPU transfers at one time?
 - a. pin grid array
 - b. front side bus
 - c. MMX instructions
 - d. read-only memory
13. What laptop component converts DC power to AC so that the display can illuminate?
 - a. Inverter
 - b. Backlight
 - c. Battery
 - d. motherboard
14. What determines to which device the computer searches first to locate boot files?
 - a. partition size
 - b. BIOS boot sequence option
 - c. type of partition
 - d. the order in which the operating system or operating systems were installed
15. Which of the statements is true about X86 and X64 cpu architectures?
 - a. X86 processor is backward compatible with the x64 processor
 - b. X86 is faster than the X64
 - c. The X64 is 64 bit and the X86 is 32 bit
 - d. An x86 processor can support 32-bit and 64-bit operating systems.

16. Which of these is not a typical power supply output voltage?
 - a. 10 V
 - b. 3.3 V
 - c. 5 V
 - d. 12 V
17. What is the address bus size of the 8051 microprocessor?
 - a. 8 bit
 - b. 16 bit
 - c. 32 bit
 - d. 64 bit
18. The register banks in the 8051 processor occupy the addresses
 - a. 00h to FFh
 - b. 80h to FFh
 - c. 00h to 80h
 - d. 00h to 1Fh
19. The command MOV A, @R1 in an 8051 processor is an example of using
 - a. Immediate addressing
 - b. Indirect addressing
 - c. Indexed addressing
 - d. Relative Addressing
20. Which of these is not a flag in the PSW of the 8051 processor?
 - a. Carry flag
 - b. Auxiliary carry flag
 - c. Overflow flag
 - d. Odd parity flag

Section 2: Short answer questions

(20 marks, 2marks each)

Answer all

1. How does BIOS allow the same applications to run on different hardware?
2. What is an API and what is its function?
3. What is the POST and what are its major functions?
4. How does a PC user access system configuration and setup program?
5. How does the boot loader verify a valid MBR?
6. Define a bus?
7. What are the four factors that lead to disk latency?
8. What is the main factor that determines cache effectiveness?
9. What is the difference between multiprocessing and multithreading?
10. What are the four functions of an OS?

Section 3: Long answer questions

(20 marks, 10 marks each)

Answer any 2 questions

1. Briefly explain the how modern implementation of BIOS differs from the old implementation?
2. Briefly explain the booting process from switching on the PC to loading the OS?
3. Explain the difference between tracks and cylinders and hence how data is read from a hard disk.

Section 4

(40 marks , 20 marks each)

Answer 2 questions

1.

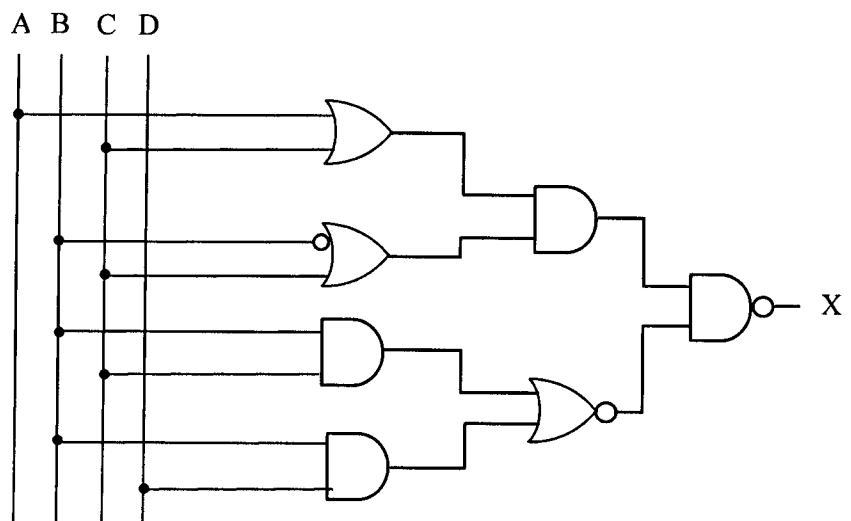
- a. Draw the logic circuit for the following equation.

$$X = \overline{(A + B + \bar{D})}(A + B + C + D) + (\bar{A} + B + A\bar{B})C$$

- b. Use De Morgan's theorem and Boolean algebra rules to reduce the equation to a SOP form.
- c. Simplify the circuit using Boolean rules and identities.
- d. Draw the simplified circuit.

2.

- a. Write the Boolean equation for the circuit below.



- b. Use De Morgan's theorem and Boolean algebra rules to simplify the equation.
 - c. Draw the simplified circuit.
 - d. Simplify the same circuit using a K-Map from the SOP. Draw the circuit.
3. a) Determine the value of the accumulator after the execution of instruction A:, B:, and C:

```
MOV A,#02H
MOV R0,#46H
A:  XRL A,R0      ; A = _____
B:  ORL A,#71H   ; A = _____
C:  ANL A,#0F6H  ; A = _____
```

- b. Write a program that will produce an output at port 1 that counts up from 50H to 90H repeatedly.

END OF EXAMINATION

**THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES**

**2012 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS
GEO 482: ENVIRONMENT AND DEVELOPMENT II**

TIME: **Three hours**

INSTRUCTIONS: **Answer any FOUR questions.**
 All questions carry equal marks.

1. Explain the potential social and environmental impacts of the UNZA Business Park.
 2. Since the advent of human civilization, nature has usually seemed unassailable, abundant with plant and animal life but during the course of the 20th century, this view changed. Discuss.
 3. Using the Waste Management Hierarchy Model, explain the steps that reduce or eliminate municipal, industrial and hazardous-waste output.
 4. Outline and discuss the major drivers of deforestation in Zambia and the general effects on the environment.
 5. Provide an elaborate exposition of the physical or hydrological functions of the Zambezi wetland.
 6. 'Population Growth and Urbanization are among the eight identified Global environmental problems and seen as one of the most serious problem of the century'. Elucidate
-

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2012 ACADEMIC YEAR SECOND SEMESTER FINAL EXAMINATIONS

GEO 492: NATURAL RESOURCES ECONOMICS

Time: Three hours

Instructions: Answer any four questions.
All questions carry equal marks. You are encouraged to use illustrations wherever appropriate.

1. Explain what discounting does and what it does not.
 2. Using an appropriate economic model, demonstrate how a non-renewable natural resource can be efficiently recycled over time.
 3. Describe the effects of extraction costs on Hotelling's model of resource depletion.
 4. Describe an economic approach that can be used to select a pollution control method by a regulatory body.
 5. Explain the reasons why the Maximum Economic Yield (MEY) is considered a social as well as a private optimum in fisheries management.
 6. Using an illustration, explain the problems of excluding externalities when explaining the problem of agriculture induced deforestation.
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END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR FINAL EXAMINATIONS

**GES 2130: THE GEOGRAPHY OF AFRICA WITH SPECIAL REFERENCE TO
ZAMBIA**

TIME: **Three Hours**

INSTRUCTIONS: **Answer any four questions. Candidates are advised to make use of
Illustrations and examples wherever appropriate. Use of a Philips
University Atlas is allowed.**

1. Discuss the types and importance of the lakes that are found on the African continent.
 - ✓ 2. Explain the threats to wild fauna conservation in Zambia.
 - ✓ 3. ‘Although Africans are greatly influenced by universal religions, African Traditional Religions and Philosophy are still beneficial in present times.’ Discuss.
 4. Describe the responses by Africans to the rapid change resulting from the colonial impact on societies according to Mbiti (2011) in terms of cultural dynamics .
 - ✓ 5. Describe the paths of socio-economic development that Zambia has followed since independence and explain the factors that have caused shifts in policies.
 - ✓ 6. Account for the existence of regional disparity in economic development in Ghana and Cote D’Ivoire (Ivory Coast) after their independence and suggest ways to address the challenges.
-

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2013 ACADEMIC YEAR FINAL EXAMINATIONS

GES 3342: ENVIRONMENTAL PLANNING AND MANAGEMENT

TIME: Three hours

INSTRUCTIONS: Answer any four questions. All questions carry equal marks. Use of appropriate illustrations and examples is encouraged.

1. Discuss the assertion that '*The Tragedy of the Commons*' thesis is an inaccurate description of how common property regimes work in reality.
2. The government of the Republic of Zambia is concerned about the high concentration of sulphur dioxide pollutants in the Copperbelt Province. A number of industrial units have been identified as sources of these pollutants. The government, in this regard, has decided to develop environmental standards to address this concern. The standards are targeting plant designs, production processes, atmospheric emissions from the units and the products distributed by the industrial units. Assuming you are working for the government as an environmental advisor, explain the type of standards you would advise the government to consider and provide justification for your choice of standards.
3. The University of Zambia Business Park will be a new focal point for commercial activities comprising among other things; a hotel, entertainment facilities and access roads. Discuss the potential social and environmental impacts of this project.
4. A major study commissioned to look into problems associated with biodiversity conservation at a nature reserve in Northern Province of Zambia shows that conservation problems are largely rooted in a dysfunctional conservation strategy that emphasises exclusive state control in access, use and management of resources in the reserve at the expense of other actors. As a response to the report, authorities have sought your services (as conservation consultant) to advise on a new conservation strategy that will be inclusive of other actors. In considering the choice of your strategy, you have been requested to draw on the principles of subsidiarity, participation and equity.

In accordance with the demands of your task, explain:

- (a) The type of conservation strategy you would choose including its core elements
 - (b) How your choice of strategy takes into consideration the three principles above
 - (c) What negative elements the reserve should guard against in employing the new strategy.
5. What are the major challenges to implementing an Environmental Impact Assessment (EIA) in a developing country like Zambia?
6. 'While in theory, Payments for Ecosystem Services (PES) systems resolve long standing challenges associated with the costs of environmental conservation to communities, in practice, PES systems have their own challenges that affect their ability to yield environmental and economic benefits'. Discuss.

END OF EXAMINATION

**THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES**

**2013 ACADEMIC YEAR FINAL EXAMINATIONS
GES 4342: ENVIRONMENT AND NATURAL RESOURCES MANAGEMENT**

TIME: **Three hours**

INSTRUCTIONS: **Answer any FOUR questions**
 All questions carry equal marks

1. Explain the values and functions of the Zambezi wetland in the South Central Africa region.
2. Population growth and Urbanization have become the greatest challenge causing increased environmental pollution and degradation for Zambia. Discuss.
3. Provide an exposition as to why cities in developing economies and economies in transition are experiencing a huge waste management problem.
4. The excessive deposition of chemical nutrients in water bodies created by sewage and industrial water pollution degrades the quality of water and increases the Biological and Chemical Oxygen Demand. Discuss the challenges of both Biological and Chemical Oxygen Demand to a water body.
5. "Definition of pollution is dependent on both some *physical effect* of waste on the environment and a *human reaction* to that physical effect". Explain pollution by applying a practical case of your choice.
6. One of the major challenges for Zambia is that a large percentage of people live below the poverty line and without any major policy and technology changes, the trend will threaten the stability of the communities and the environment. Elucidate

END OF EXAMINATION

**THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES**

2013 ACADEMIC YEAR FINAL EXAMINATIONS

GES 4372: TOURISM, ENVIRONMENT AND DEVELOPMENT

TIME: THREE (3) hours

INSTRUCTIONS: Answer Question 1 (40 %) and any other 3 Questions (20 % each)
Candidates should use illustrations wherever relevant
Use of **approved** calculator is allowed

1. Sanika Island on the Zambezi River is used for day trips, educational excursions and picnics. It is also a sighting area for the fish eagle and about 65 other bird species. It covers 5 hectares and received 357, 947 visitors in 2013 made up of both locals and foreigners. The Island has one main walking trail called '*the Loop*' which goes round the edges of the island covering a total of 375 metres. On average the trail is 1 metre wide and each visitor uses 1 square metre. Individual guides working in shifts take visitors in groups of 15 and the distance between groups is 50 metres for minimum disturbance. The walking trails are open from 6:30 hours to 17:30 hours every day and are open all year round. Basic Back-packers (BB) is located on the island and has a total of 20 bed spaces and is only open for 100 days in a year during the fishing season. In one fortnight it checked in 14 visitors of which 6 were staying for 4 days, 3 stayed for 5 days and the rest were booked to stay for one week. The island currently has 10 members of staff while the ideal number should be 27 members of staff.
 - a) Using the following correction factors;
Precipitation = 2.31%; Erosion = 33.62% and Slope = 33.62%.

Calculate the *Physical, Real* and *Effective* carrying capacities for '*the Loop*', giving both daily and annual visitor capacities (show all calculations).
 - b) Outline the disadvantages of using this method to attain sustainable tourism.
 - c) Calculate the Average length of stay for Basic Backpackers for that 2 week period (show all calculations).
2. Write short explanatory notes on **ALL** of the following :
 - (a) Cohen's typology of tourists
 - (b) Leakages in the tourism industry
 - (c) Doxey's IRRIDEX
 - (d) Value-for-money (VFM)
3. Evaluate the potential of tourism as a means of economic development in Zambia.

4. Explain the concept of a 'symbiotic relationship' existing between tourism and the environment.
5. Discuss the opportunities and limitations faced by the Zambia Tourism Board in marketing tourism.
6. Examine whether the concepts of 'demonstration effect' and 'acculturation' adequately explain the tourist impacts on host societies.

END OF EXAMINATION

The University of Zambia
School of Natural Sciences
Department of Mathematics & Statistics
2013/14 ACADEMIC YEAR FINAL EXAMINATIONS
MAT1100 – FOUNDATION MATHEMATICS
14th July, 2014

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- INSTRUCTIONS:**
- (1) Write down your **Computer number** and the **TG number** on each answer booklet used.
 - (2) Answer any **Five (5)** questions only.
 - (3) Show all essential working to avoid loss of marks.
 - (4) Write down the **question numbers** of all the questions attempted in the first column on the cover of the main answer booklet.
 - (5) Calculators and tables are **not allowed** in this paper

TIME ALLOWED: Three (3) hours.

Q1. (a) The sets A, B, C all intersect and U is the universal set. Shade the part described by the set $[B \cup (A' \cap C')]'$ in a Venn diagram.

(b) The functions f and g are defined by

$$f(x) = \frac{4}{x-2}, \quad x \in \mathbb{R}, x \neq 2$$

$$g(x) = x + 2, \quad x \in \mathbb{R}.$$

Find

(i) the domain of the function $g \circ f$

(ii) $(g \circ f)^{-1}(x)$.

(c) Use Cramer's rule to solve the system of equations:

$$\begin{array}{rcl} x + 2y + 3z & = & -2 \\ -2x & + & z = 3 \\ x - 3y & = & 5 \end{array}$$

(d) Find the equation of the tangent to the curve

$$y = x^2 - 9x^{-1},$$

at the point $(3, 6)$.

Q2. (a) Find the solution set of each of the following inequalities:

(i) $|3x - 2| > 4$

(ii) $\frac{2}{x-3} < \frac{3}{2x-1}$

(b) Find the equation of a circle which passes through the points (1,4), (7,5) and (1,8).

(c) (i) Find the period and phase shift for the function

$$f(x) = 1 - 3\sin(2x + \pi),$$

and hence or otherwise,

(ii) sketch the curve for $-\pi \leq x \leq \pi$.

(d) Differentiate the function $f(x) = \frac{2}{\sqrt{3x-1}}$ from first principle.

Q3. (a) Evaluate the limits:

(i) $\lim_{x \rightarrow 3} \frac{x^3 - 27}{x^2 - 9}$ (ii) $\lim_{x \rightarrow +\infty} \frac{2x^2 + 1}{6 + x - 3x^2}$

(b) The line $y = 5x - 13$ meets the circle $(x - 2)^2 + (y + 3)^2 = 26$ at the points A and B .

(i) Show that A has coordinates (1, -8) and B has coordinates (3, 2).

M is the midpoint of the line AB .

(ii) Find the equation of the line which passes through M and is perpendicular to the line AB . Write your answer in the form $ax + by + c = 0$, where a , b and c are integers.

(c) Express each of the following in the form $r(\cos\theta + i\sin\theta)$:

(i) $(3 - 3i\sqrt{3})(1 - i)$

(ii) $\frac{-2 - i\sqrt{3}}{-2 + i\sqrt{3}}$

(d) Show that $x = 0$ is a solution of the equation

$$\begin{vmatrix} x-1 & 4 & -1 \\ 1 & x+2 & 1 \\ 2x-4 & 4 & x-4 \end{vmatrix} = 0,$$

and find the other two roots.

Q4. (a) Given the sets $A = [-7, 3]$, $B = [-3, 3]$, $C = (-1, 6)$ and let \mathbb{R} be the universal set. Find each of the following sets and represent it on a number line:

(i) $A - B$

(ii) $A \cup (B \cap C)$

(b) Evaluate each of the following integrals:

(i) $\int \frac{3x}{\sqrt{1-2x^2}} dx$

(ii) $\int \frac{x^3 + 3}{x^2 - 1} dx$

(iii) $\int_0^{\pi/2} x \cos x dx$

(c) (i) Given that $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 6 \\ 1 & 2 & 3 \end{pmatrix}$, find A^{-1} .

(ii) Hence or otherwise, find X given that $Y = AX$ and $Y = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$.

(d) Solve the equation

$$|x^2 - 3x| = -4x + 6$$

Q5. (a) For each of the following, find $\frac{dy}{dx}$:

(i) $y = e^{4x}(3x^2 - 5)^6$

(ii) $x^2 + \sin y - y^2 = 7$

(b) The roots of the equation

$$9x^2 + 6x + 1 = 4kx,$$

where k is a real constant, are denoted by α and β .

(i) Show that the equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is

$$x^2 + 6x + 9 = 4kx.$$

(ii) Find the set of values of k for which α and β are real.

(c) Solve each of the following trigonometric equations for x , if $0 \leq x \leq 2\pi$:

(i) $\cos^2 x - \sin x - 1 = 0$

(ii) $\cos 2x + 3\sin x - 2 = 0$

(d) Use the principle of mathematical induction to prove that for $n \in \mathbb{Z}^+$,

$8^n - 1$ is divisible by 7.

Q6. (a) (i) Express $(1-i)^6$ in the form $p+iq$, where p and q are integers.

(ii) Expand $\frac{3}{1+3x}$ as a series in ascending powers of x , up to and including the term in x^3 , indicating the range of values of x for which the expansion is valid.

(b) Solve the equation for x :

$$3^{2x+1} + 5 = 16(3^x).$$

(c) A cylindrical can, with no lid, has a circular base of radius r cm. The total surface area of the can is 300π cm².

(i) Show that the volume V cm³ of the can is given by

$$V = \frac{\pi}{2}r(300 - r^2).$$

(ii) Find the value of r for which V is maximum.

(d) Prove each of the following identities:

(i) $\cot \theta + \tan \theta \equiv \sec \theta \csc \theta$

(ii) $\frac{\cos A + \tan A}{\sin A \cos A} \equiv \csc A + \sec^2 A$

Q7. (a) Given that $(3x+2)$ is a factor of $f(x) = 3x^3 + Ax^2 - 4x - 4$.

(i) Show that $A = 5$.

(ii) When $A = 5$, factorize $f(x)$ completely.

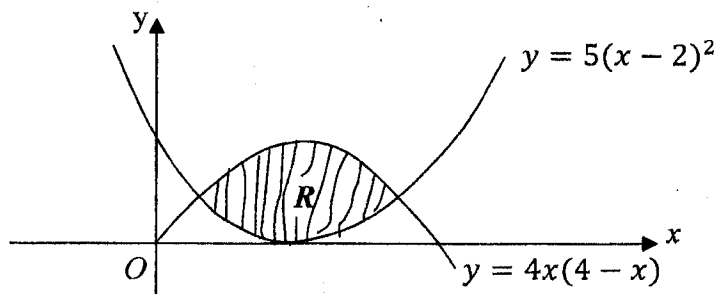
(iii) Hence, sketch the graph of $y = f(x)$, indicating the x - and y -intercepts and the turning points.

(b) Determine whether the function $f(x) = 2x - 5x^3$ is even or odd or neither.

(c) Solve the equation

$$z^3 + 32\sqrt{3} + 32i = 0$$

(d) The diagram shows the shaded region R which is bounded by the curves $y = 4x(4-x)$ and $y = 5(x-2)^2$. Find the area of the shaded region R .



END OF EXAMINATION

The University of Zambia
School of Natural Sciences
Department of Mathematics and Statistics
2012 Academic year
Second Semester Examinations
M112 Mathematical Methods II

Time Allowed: Three (3) Hours

19th August 2013

Instructions:

1. You must write your **Computer Number**, Your **TG Number** on each answer booklet used.
 2. Indicate the number of each question attempted in the first column on the right side of the main answer booklet
 3. There are **seven (7)** questions in this paper. You must answer any **five (5)** questions. All questions carry equal marks.
 4. **No Calculators** to be used
-

1. (a) Solve the following equations;

(i) $2^{2x} - 5(2^{x+1}) + 16 = 0$

(ii) $\log_2 x + 4 \log_x 2 = 5$

(b) (i) Find the term independent of x in the expansion of

$$\left(\frac{1}{3x} - \frac{3x^2}{2} \right)^9$$

(ii) Given that $y = e^x + x^2$. Prove that $\frac{d^2y}{dx^2} = y + 2 - x^2$

(c) Show that $\int_1^{\sqrt{3}} \frac{x+3}{\sqrt{4-x^2}} dx = \frac{\pi}{2} + \sqrt{3} - 1$

2. (a) (i) Express $\frac{x^2 + x + 1}{(x+1)(x^2 + 1)}$, in partial fractions

(ii) Show that $\int_0^1 \frac{x^2 + x + 1}{(x+1)(x^2 + 1)} dx = \frac{3}{4} \ln 2 + \frac{\pi}{8}$

(b) The point (6,3) divides the line segment PQ where P(4,5) and Q(x,y) in the ratio 2:5 internally. Find

(i) the coordinates of Q

(ii) the coordinates of the midpoint of PQ

(c) Use Mathematical induction to prove that $x^{2n} - y^{2n}$ is divisible by $x - y$ for every positive integer values of n .

3. (a) (i) Given that $A = \begin{pmatrix} 1 & 0 & 2 \\ t & 3 & 1 \\ -2 & -1 & 1 \end{pmatrix}$, find the value of t if the matrix A is singular.
- (ii) Given that $B = \begin{pmatrix} 0 & -1 & 1 \\ 2 & -1 & 2 \\ 1 & 0 & 1 \end{pmatrix}$. Show that $B^3 = I_3$
- (iii) Use (ii) above to find B^{-1}
- (b) (i) By using De Moivre's Theorem, or otherwise, show that $\sin 5\theta = 16\sin^5 \theta - 20\sin^3 \theta + 5\sin \theta$
- (ii) Find the roots of the equation $z^5 - 1 = 0$ in the form $r(\cos \theta + i \sin \theta)$ and show them on an argand diagram
- (c) (i) Find in Cartesian form an equation of a circle which passes through the points $(2,0)$, $(8,0)$, and $(10,4)$.
- (ii) Prove that the y -axis is a tangent to the circle in (c)(i) above.
- (iii) State the coordinates of the point of contact of the tangent in (ii) to the circle in (i) above.
4. (a) For the curve $y = \frac{x^2 + 9}{x^2 - 1}$, find
- (i) The turning point(s) of the curve y
- (ii) The vertical asymptotes
- (iii) The Horizontal asymptotes
- (iv) Sketch the curve.
- (b) (i) The angle between the vector $i + j$ and $2i + j + \lambda k$ is $\frac{\pi}{4}$, find the possible values of λ .
- (ii) Solve the equation $3\sinh^2 x - 2\cosh x - 2 = 0$
- (c) (i) Use Binomial theorem to expand $(3 + 10x)^4$, giving each coefficient as an integer.
- (ii) Use your expansion in (i) above with appropriate value of x , to find the exact value of $(1003)^4$

5. (a) The curve C is given by $y = \frac{15 - 2x - x^2}{4}$. The line with equation $x + 2y = 3$ meets the curve C at the points P and Q. Denote the region bounded by C and the line PQ by R.

- (i) Determine the coordinates of P and Q
 (ii) Sketch the curve C and the line PQ
 (iii) Find the area of region R.

- (b) Find (i) $\int \cos^3 x dx$
 (ii) $\int x(3x+1)^{-\frac{1}{2}} dx$
 (iii) $\int \frac{dx}{\sqrt{x}(3+\sqrt{x})^3}$

- (c) Using Mathematical Induction prove that

$$1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}, \text{ for all positive values of integer } n.$$

6. (a) Let $A = \begin{pmatrix} 1 & 0 & 1 \\ 3 & 1 & 1 \\ 4 & 2 & 7 \end{pmatrix}$

- (i) Find A^{-1}
 (ii) Hence, solve the system of equations

$$\begin{aligned} x + z &= 2 \\ 3x + y + z &= 3 \\ 4x + 2y + 7z &= 1 \end{aligned}$$

- (b) Find the equation of the circle whose centre is at $(-2, -5)$ and passes through the Point $(6, 1)$.

- (c) Given that $y = x^3 - x^2 - x + 1$, find

- (i) The stationary points
 (ii) State the nature of the stationary points
 (iv) Determine the intervals where the curve y is increasing and decreasing.
 (v) Sketch the curve of y

7. (a) The variable point $P(x,y)$ moves in such a way that $AP^2 = 4BP^2$ where A is the point (1,3) and B is the point (4,-3)
- (i) Show that P lies on the circle C with equation

$$x^2 + y^2 - 10x + 10y + 30 = 0$$
- (ii) The line OT is a tangent to the circle C, where O is the origin and T lies on the circle. Calculate the length OT.
- (b) (i) Find the modulus and argument of the complex number $\frac{5+i}{3-2i}$
- (ii) Solve the equation $Z^4 - \frac{5+i}{3-2i} = 0$, giving your answer in the form $r(\cos\theta + i\sin\theta)$.
- (a) (c) (i) USING CRAMER'S RULE.

$$x + y + 2z - 2 = 0$$

$$-3x + y + 2z - 1 = 0$$

$$6x + 2y + z + 4 = 0$$
- (ii) Given that $\mathbf{a} = -i + 2j + 5k$ and $\mathbf{b} = 5i - 2j + k$, find a unit vector which is perpendicular to both \mathbf{a} and \mathbf{b}

END of EXAMINATION

UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS
SESSIONAL EXAMINATIONS-2013/2014

MAT1110- Mathematics for Social Sciences

Instructions:

1. There are two sections in this paper.
2. Answer 5 questions in total.
3. Answer 4 questions from Section One and 1 question Section Two.
4. Show all your work to earn full marks.

Time Allowed: Three (3) Hours

SECTION ONE

1. (a) Given that $f(x) = \sqrt{2x}$
Find the derivative of $f(x)$ from the first principle's.
 - (b) (i) If $x - 4$ is a factor of $f(x) = 3x^3 + kx - 2$,
Find the value of k .
 - (ii) Find the remainder when the polynomial $x^{100} + 4x^{99} + 3$
is divided by $x + 1$
 - (iii) Find the range of values of x for which $2x(6x - 11) \geq (3x - 2)(4x + 3)$
 - (iv) Express $\frac{3-i}{6+2i}$ in the form $a + bi$, where $a, b \in \mathbb{R}$
 - (c) A colony of bacteria decays so that the population after t days since present
day ($t = 0$) is given by $p(t) = 1000 \left(\frac{1}{2}\right)^t$
 - (i) Find the population when $t = 0$
 - (ii) What will the population after 4 days?
 - (iii) Sketch the graph of $p(t)$
2. (a) Simplify
 - (i) $\sqrt{2} (\sqrt{2} + 1)^2$
 - (ii) $\left(\frac{16}{25}\right)^{-3/2}$
 - (iii) $(-4)^{-2}$
 - (b) (i) Express the rational function $\frac{x^2 + 1}{x^3 - 9x}$ as a sum of partial functions.

- (ii) Hence or otherwise find $\int \frac{x^2+1}{x^3-9x} dx$
- (c) Let A and B be any two subsets of a universal set E. Express the following as a single set
- (i) $(A' \cup E) \cap A$ (ii) $(A \cup B) \cap (A' \cap B')$
3. (a) Given the function $f(x) = -x^2 - 2x + 15$
- (i) Complete the square
(ii) Sketch the graph of $f(x)$, marking the vertex, axis of symmetry and the x and y intercepts on your graph.
- (b) Given the function
- $$F(x) = \begin{cases} x^2 + 1 & x \leq 1 \\ x & x > 1 \end{cases}$$
- (i) Find $\lim_{x \rightarrow 1} F(x)$
(ii) Find $F(1)$
(iii) Sketch the graph of $F(x)$
- (c) Find range of values of x for which $2x^3 + 32x < 0$
4. (a) Find real value(s) of x in the following expressions:
- (i) $\log_2(x) + \log_2(x-2) = 3$ (ii) $x = \sin\left(\frac{-3\pi}{4}\right)$
- (ii) $x = \frac{\sin^2\left(\frac{\pi}{3}\right)}{1 + \cos^2\left(\frac{\pi}{3}\right)}$ (iv) $x = \tan 300^\circ$
- (v) $\sin^2 x = \sin x$
- (b) Find the equation of the tangent line to the curve $x + \cos y = 0$ at point $(0, \pi/2)$
- (c) A manufacturer produces a metal box with a square base, no top and a volume of 4000 cm^3 .
- (i) If the base edges are x cm long, express the height h in terms of x .
- (ii) Show that the surface area of the metal used in the box is given by the function $A(x) = x^2 + \frac{16000}{x}$
- (iii) Find the values of x and h which minimize the amount of metal used to make the box.

5. (a) Find the derivatives of the following functions in the most simplified form:

(i) $f(x) = \left(x^2 - 2x + \frac{1}{x}\right)^3$

(ii) $g(x) = \sqrt{\frac{\sin x}{1 + \cos x}}$

(iii) $r(x) = \ln\left(\frac{x+2}{2x^3(x+1)}\right)$

(iv) $h(x) = e^{-2x}(x^2 + 1)$

(b) Given the function

$$f(x) = x^3 - 3x + 2$$

(i) Find the intervals on which $f(x)$ increases or decreases

(ii) Find the inflection points if there are any

(iii) Sketch the graph of the function

(iv) Find the area of the region under the curve $f(x)$ and enclosed by the x-axis, the ordinates $x=1$, $x=2$.

(c) Evaluate the following integrals

(i) $\int \frac{2x}{(x^2 + 1)^3} dx$

(ii) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

(iii) $\int_0^{\pi/2} \frac{\cos x}{1 + \sin x} dx$

SECTION TWO

6. (a) The prices (in Kwacha) of a share of BATA ZAMBIA at Lusaka Stock exchange over the 30 days of April 2014 were:

1.8	2	1.5	1.7	1.5	3.4	0.95
0.98	1.5	1.2	1.8	2.7	3.2	3.3
1.4	1.5	2	2.5	2.8	2.9	3.0
3.2	3.4	2.0	2.4	2.2	2.2	2.1
2.7	2.8					

(i) Group the data into class intervals of equal width with the first class interval as “ $0 - < 1$ ” or $[0, 1)$.

(ii) Draw a relative frequency histogram for these data

(iii) Find the percentage of values that are K3 or more.

- (b) Thirty samples of milk purchased at various stores in Lusaka were tested for bacteria. The bacteria count per millilitre in these 30 samples is given in the table below

Bacteria count	3	4	5	6	7	8
No. of Stores	1	3	6	9	7	4

Find the mean and variance of the data.

- (c) What does the following information tell you about two different basket ball teams?

Team A: mean height of players = 192 cm, standard deviation = 4 cm

Team B: mean height of players = 192 cm, standard deviation = 8 cm

7. (a) Mr and Mrs Banda each bought 10 lottery tickets. Each of their three children bought 4 tickets. If 4280 tickets were sold in all, find the probability that the grand prize winner is

- (i) Mr. or Mrs. Banda
 (ii) One of the 5 Bandas
 (iii) None of the Bandas

- (b) Of the 1260 households in a small town, 632 have dogs, 568 have cats and 114 have both types of pet. If a household is chosen at random, find the probability that the household has either type of pet.

- (c) Each student in a class of 30 students studies one foreign language and one science. The students' choices are shown in the table below:

	Chemistry	Physics	Biology
French	7	4	3
Spanish	1	6	9

- (i) Find the probability that a randomly chosen student studies Chemistry.
 (ii) Find the probability that a randomly chosen student studies Chemistry given that the student studies French.
 (iii) Are the events " Student studies Chemistry " and " Students studies French " independent?

END OF EXAMINATION

- (b) Thirty samples of milk purchased at various stores in Lusaka were tested for bacteria. The bacteria count per millilitre in these 30 samples is given in the table below

Bacteria count	3	4	5	6	7	8
No. of Stores	1	3	6	9	7	4

Find the mean and variance of the data.

- (c) What does the following information tell you about two different basket ball teams?

Team A: mean height of players = 192 cm, standard deviation = 4 cm

Team B: mean height of players = 192 cm, standard deviation = 8 cm

7. (a) Mr and Mrs Banda each bought 10 lottery tickets. Each of their three children bought 4 tickets. If 4280 tickets were sold in all, find the probability that the grand prize winner is

- (i) Mr. or Mrs. Banda
(ii) One of the 6 Bandas
(iii) None of the Bandas

- (b) Of the 1260 households in a small town, 632 have dogs, 568 have cats and 114 have both types of pet. If a household is chosen at random, find the probability that the household has either type of pet.

- (c) Each student in a class of 30 students studies one foreign language and one science. The students' choices are shown in the table below:

	Chemistry	Physics	Biology
French	7	4	3
Spanish	1	6	9

- (i) Find the probability that a randomly chosen student studies Chemistry.
(ii) Find the probability that a randomly chosen student studies Chemistry given that the student studies French.
(iii) Are the events " Student studies Chemistry " and " Students studies French " independent?

END OF EXAMINATION

The University of Zambia
Department of Mathematics & Statistics
2013 Academic Year, Examinations
MAT 2100 - ANALYTIC GEOMETRY AND CALCULUS

Time allowed : Three (3) Hours

Full marks : 100

Instructions:

1. Answer **any Three (3)** questions from **Section A** and **any Two(2)** questions from **Section B**.
2. All questions carry equal marks.
3. **Full credit** will only be given when **necessary work** is shown.
4. Indicate your **computer number** on all answer booklets.

This paper consists of 4 pages of questions.

SECTION A

ANSWER ANY THREE (3) QUESTIONS FROM THIS SECTION

1. The equation of a conic section is given by

$$2xy - 6y - 4x + 11 = 0.$$

- (a) (i) Use a suitable rotation of axes to transform the equation in its standard form.
Hence
(ii) identify the conic.

- (b) Find
- (i) the focus or foci and vertex or vertices
 - (ii) the directrix or directrices.

(c) Sketch the curve.

2. (a) Using the definition of $\lim_{x \rightarrow 1} f(x) = L$, prove that

$$\lim_{x \rightarrow 1} [x^2 - x - 2] = -2$$

(b) Evaluate each of the limits below:

(i) $\lim_{x \rightarrow 1^+} \left[\frac{1}{\ln x} - \frac{1}{x-1} \right]$.

(ii) $\lim_{x \rightarrow 1^+} \left[x^{\left(\frac{1}{x-1} \right)} \right]$.

(c) Given that $I_n = \int_0^{\frac{\pi}{2}} \sin^n x dx$, $n \geq 2$, show that

$$nI_n = (n-1)I_{n-2}.$$

Hence, evaluate I_4 .

3. (a) Determine whether each of the following series converges. If so, find its sum:

(i) $\sum_{n=1}^{\infty} \left(\frac{2}{3} \right)^n$.

(ii) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

(b) (i) State Rolle's theorem.

(ii) Show that the hypotheses of Rolle's theorem are satisfied by the function

$$f(x) = \frac{x^2 - 4x}{x + 2},$$

on the interval $[0, 4]$.

(iii) For the function $f(x) = \frac{x^2 - 4x}{x + 2}$, find the value of c which satisfies the conclusion of Rolle's theorem on the given interval above.

(c) Find the length of the arc of the curve $y = 3x^{\left(\frac{3}{2}\right)} - 1$ from $x = 0$ to $x = 1$.

4. (a) Evaluate each of the following integrals:

(i) $\int x^2 \tan^{-1} x dx.$

(ii) $\frac{d\theta}{2+\cos \theta}.$

(b) Find the equation of circle of curvature of the curve $y = x - \frac{1}{4}x^2$ at $x = 2.$

(c) The curve with parametric equations

$$x = t - \sin t, \quad y = 1 - \cos t,$$

from $t = 0$ to $t = \pi$ is rotated completely about the x - axis. Find the area of the surface of revolution generated.

SECTION B

ANSWER ANY TWO (2) QUESTIONS FROM THIS SECTION

1. (a) Suppose C is a curve given by the vector equation

$$\mathbf{r}(t) = t\mathbf{i} + \cos t\mathbf{j} + \sin t\mathbf{k}; \quad 0 \leq t \leq 2\pi.$$

Find the equation of the

(i) normal plane of the curve at the point $(\frac{\pi}{2}, 0, 1).$

(ii) osculating plane at the point $(\frac{\pi}{2}, 0, 1).$

(b) Find the local maximum, minimum, and saddle points of the function

$$f(x, y) = xy + 2x - 33y - 6$$

(c) i) Verify that $y = e^x$ is a solution of the homogeneous equation

$$(x - 1)y'' - xy' + y = 0.$$

ii) Hence, find the general solution of $(x - 1)y'' - xy' + y = e^x$ using the reduction of order method.

2. (a) (i) Find symmetric equations of the line which passes through the point $(-1, 1, 2)$ and whose direction vector is orthogonal to the direction vectors of the lines:

$$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-1}{3} \quad \text{and} \quad \frac{x-1}{5} = \frac{y-2}{2} = \frac{z+1}{-3}.$$

(ii) Discuss the continuity of the function

$$f(x, y) = \begin{cases} \frac{x^2 y^3}{2x^5 + y^5} & \text{if } (x, y) \neq (0, 0) \\ 1 & \text{if } (x, y) = (0, 0) \end{cases}$$

at the point $(0, 0)$.

(b) (i) Show that $f(x, y) = x^{-2} \sin(\frac{x}{y})$ is homogeneous and state the degree.

(ii) Show that the equation $x^2 - 3xy + y^3 = 7$ defines y implicitly as a function of x in the neighbourhood of the point $(4, 3)$. Hence, find $\frac{dy}{dx}$ at the point $(4, 3)$.

(c) Find the general solution of the equation

$$y'' - 4y = e^x \cos x.$$

3. (a) (i) Find the curvature of the curve

$$\mathbf{r} = \langle t^2, t^3, t \rangle$$

at a general point $(x(t), y(t), z(t))$ and at the point $(0, 0, 0)$.

(ii) A moving particle starts at an initial position $\mathbf{r}(0) = \langle 0, 1, 0 \rangle$, with initial velocity $\mathbf{v}(0) = \langle 1, 1, -1 \rangle$, and its acceleration is $\mathbf{a}(t) = \langle 1, 4t, 6t \rangle$. Find its velocity and position at time t .

(b) (i) Find the linear approximation of the function $f(x, y) = 1 - xy \cos \pi y$ at the point $(1, 2)$ and use it to approximate $f(1.02, 1.97)$.

(ii) Find the absolute minimum and maximum of $f(x, y) = x^2 + y^2 + 2y - 1$ on

$$D = \{(x, y) | x^2 + y^2 \leq 1\}$$

(c) (i) Solve the initial value problem

$$y' + y \tan x = \sin 2x; \quad y(0) = 1.$$

(ii) Assume that f and g have continuous second order partial derivatives, show that any function of the form $z = f(x + at) + g(x - at)$ satisfies the wave equation

$$\frac{\partial^2 z}{\partial t^2} = a^2 \frac{\partial^2 z}{\partial x^2}.$$

END OF EXAM!

The University of Zambia
Department of Mathematics & Statistics
2013 Academic Year Final Examinations
MAT2200 - Linear Algebra

Time allowed : **Three (3) hours**

Full marks : 100

Instructions: • Attempt **any five (5)** questions. **All** questions carry **equal** marks.

- Indicate your **computer number** on all answer booklets used.
- No help materials allowed.

This paper consists of 3 pages of questions.

1. (a) Define the following terms:

- i. An equivalence relation on a set S .
- ii. The inverse image of a set under a function.
- iii. A skew-symmetric matrix.

(b) i. Prove that the equivalence classes of an equivalence relation on a set X constitute a partition of X .

ii. Let $f : X \rightarrow Y$ be a function and $C, D \subset Y$. Prove that

$$f^{-1}(C \cap D) = f^{-1}(C) \cap f^{-1}(D).$$

iii. If A is a square matrix, show that $\frac{1}{2}(A + A^t)$ is symmetric and $\frac{1}{2}(A - A^t)$ is skew-symmetric.

(c) i. On the set \mathbb{Z} of integers, define a relation \mathcal{R} by $x\mathcal{R}y$ if and only if $x + y$ is even. Show that this defines an equivalence relation and determine the equivalence classes.

ii. Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$f(x) = 1 - 2x^2.$$

Find $f^{-1}(B)$, where $B = [-2, 1)$.

2. (a) What is meant by the following:
- A non-singular matrix.
 - An elementary matrix.
 - The normal form of a matrix.
- (b) Prove the following:
- If A and B are non-singular n by n matrices, then AB is non-singular and $(AB)^{-1} = B^{-1}A^{-1}$.
 - If E is an elementary matrix, then $\det(E) \neq 0$.
 - If E is an elementary matrix, then $\det(E^t) = \det(E)$.
- (c) i. Apply elementary row operations to find the inverse of the matrix $A = \begin{pmatrix} 2 & -1 & 0 \\ 1 & 1 & 3 \\ 0 & -2 & 4 \end{pmatrix}$.
- ii. Find the normal form of the matrix $\begin{pmatrix} 1 & -1 & 2 & 3 \\ 2 & 0 & -3 & 1 \\ 1 & 1 & -1 & 2 \end{pmatrix}$.
3. (a) Define the following:
- A subspace of a vector space.
 - A basis for a vector space.
 - The transition matrix from a basis B to a basis B' .
- (b) i. Prove that a subset U of a vector space V is a subspace of V if and only if $\alpha u + v \in U$ for all $u, v \in U$ and all scalars α .
- ii. Prove that any linearly independent subset of a finite dimensional vector space V can be extended to a basis for V .
- (c) i. Show that $U = \{(a, b, c, d) : a - 2b + c - 3d = 0\}$ is a subspace for \mathbb{R}^4 . Find a basis for U , and hence state the dimension of U .
- ii. Extend the set $S = \{(1, 1, 0, 0), (1, 0, 1, 0)\}$ to a basis for \mathbb{R}^4 containing the vectors $(1, 1, 0, 0)$ and $(1, 0, 1, 0)$.
- iii. Let $B = \{(1, 1, 0), (0, 1, 1), (1, 1, 1)\}$ and $B' = \{(1, 0, 0), (0, 2, 0), (0, 0, -3)\}$ be bases for \mathbb{R}^3 . Find the transition matrix from B to B' .
4. (a) When are two real symmetric matrices said to be orthogonally similar?
- (b) Determine whether the matrix $A = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$ is diagonalizable.
- (c) Sketch the graph of the conic section $x^2 - 8xy + 16y^2 - 34\sqrt{17}y = 68$.

5. (a) i. Suppose V and W are real vector spaces and $T : V \rightarrow W$ is a mapping. What are the two conditions that are necessary and sufficient for T to be linear.
- ii. Give an example of a mapping $T : \mathbb{R}^2 \rightarrow \mathbb{R}$ such that

$$T(av) = aT(v)$$

for all $a \in \mathbb{R}$ and all $v \in \mathbb{R}^2$ but T is not linear.

- (b) Consider the linear transformation $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ defined by

$$T(x, y, z, t) = (x + 2y + 3z + 2t, 2x + 4y + 7z + 5t, x + 2y + 6z + 5t).$$

Find an \mathbb{R} -basis and the dimension of

- i. the range T ,
- ii. the null T .
- (c) Let $\{x_1, x_2, x_3\}$ and $\{y_1, y_2\}$ be \mathbb{R} -bases for \mathbb{R}^3 and \mathbb{R}^2 respectively, and define the linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ by

$$\begin{aligned} Tx_1 &= y_1, \\ Tx_2 &= y_1 + y_2, \\ Tx_3 &= y_1 - 2y_2. \end{aligned}$$

Find the matrix of T relative to the \mathbb{R} -bases

$$\{x_1 + x_2, x_1, x_2 - x_3\} \quad \text{and} \quad \{-y_1 - y_2, y_1 + 2y_2\}$$

for \mathbb{R}^3 and \mathbb{R}^2 respectively.

6. (a) Let V be a finite dimensional inner product space, U a subspace of V and $x, y \in V$.
- i. Define the projection of x along y .
- ii. Prove that $U^\perp = \{0\}$ if and only if $U = V$.
- (b) Suppose $P(\mathbb{R})$ denotes the vector space of real polynomials with an inner product

$$(f, g) = \int_{-2}^2 (1 - x^{10}) f(x)g(x) dx.$$

Find $p \in \text{span}\{x^3, x^4\}$ that minimizes $\|p(x) - x^3\|$.

- (c) Suppose P is an invertible matrix and

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

By considering $(P^{-1}AP)^n$, find A^n for some positive integer n .

END OF PAPER

The University of Zambia
Department of Mathematics & Statistics
2013 Academic Year Final Examinations
MAT2300 - Introduction to Real Analysis

Time allowed : **Three (3) hours**

Full marks : 100

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- Instructions:**
- Attempt any five (5) questions. All questions carry equal marks.
 - Indicate your **computer number** on all answer booklets used.
 - No help materials allowed.

NOTATION:

\mathbb{N} is the set of natural numbers.

\mathbb{R} is the set of real numbers.

\mathbb{C} is the set of complex numbers.

\mathbb{Q} is the set of rational numbers.

This paper consists of 3 pages of questions.

1. (a) i. Give the definition of a statement.
ii. Which of the following statements is true?

$$\forall x, \exists y \ni (x + y = 0) \tag{1}$$

$$\exists x \ni \forall y, (x + y = 0) \tag{2}$$

- iii. Translate the following English sentence into a symbolic sentence with quantifiers:

For every positive real number x , there is only one y such that $2^y = x$.

- iv. Give the negation of the sentence in part (iii).
v. The "if ... then ..." connective is written as either "if P then Q " or " $P \implies Q$ ".
Give the truth table for this connective.

- (b) i. Give the definition of a set.
 ii. Let A, B and C be sets. Suppose $A \subset B$ and $B \subset C$. Prove that $A \subset C$.
 iii. Define a relation from a set A to a set B .
 iv. Define a function from a set A to a set B .
 v. Let $f : A \rightarrow B$. Prove that f^{-1} is a function if and only if f is a bijection.
 vi. What is a sequence?
2. (a) i. Let P be a non-empty set. When is P called partially ordered set.
 ii. Let P be a partially ordered set and $p, q \in P$. When is $p < q$?
- (b) i. Let P_1 and P_2 be partially ordered sets. When is P_1 said to be order isomorphic to P_2 ?
 ii. Let P be a partially ordered set and $a \in P$. Define the segment $S(a)$ of P determined by a .
- (c) i. Let W be a well ordered set. If $g : W \rightarrow W$ is an order preserving injective function, prove that
- $$\forall x \in W, x \leq g(x).$$
- ii. Let W_1 and W_2 be well ordered sets, and $g : W \rightarrow W$ be an order preserving injective function. Denote the range of g by \mathcal{R}_g . Show that
- $$g^{-1} : \mathcal{R}_g \rightarrow W_1$$
- is order preserving.
3. (a) i. Let L and U be non-empty subsets of the rational numbers \mathbb{Q} . When is the pair (L, U) called a section of rational numbers.
 ii. Define a real rational number.
 iii. Define an irrational number.
- (b) i. If a_1 and a_2 are two different real numbers, prove that either $a_1 < a_2$ or $a_2 < a_1$.
 ii. Let $a = (L_1, U_1)$ and $b = (L_2, U_2)$. Give the definition of the sum $a + b$.
 iii. Let $a = (L, U)$ be a real number. Give the definition of the negative, $-a$.
- (c) Prove that $\forall a \in \mathbb{R}, a + (-a) = 0^*$
4. (a) i. Give the definition of a mathematical structure called a field.
 ii. Define a totally ordered field $(\mathbb{F}, +, \cdot, P)$.
- (b) Let $(\mathbb{F}, +, \cdot, P)$ be a totally ordered field. Suppose $a, b \in \mathbb{F} \ni a \leq b$ & $b \leq a$, Prove that $a = b$.

- (c) Let $(\mathbb{F}, +, \cdot, P)$ be a totally ordered field and $S \subset \mathbb{F}$.
- If S has a least upper bound, prove that it is unique.
 - If S has a greatest lower bound, prove that it is unique.
5. (a) i. Let $a \in \mathbb{R}$. Define the absolute value $|a|$, of a .
- ii. Prove that $\forall a, b \in \mathbb{R}, |AB| = |A||B|$.
- (b) i. If $x \in \mathbb{R}$ and $x > 0$, prove that $\exists n \in \mathbb{N} \ni n > x$.
- ii. Let $\varepsilon \in \mathbb{R}, \varepsilon > 0$. Show that $\exists n \in \mathbb{N} \ni \frac{1}{n} < \varepsilon$.
- (c) i. When is a subset O of \mathbb{R} called an open set in \mathbb{R} ?
- ii. Let $F \subset \mathbb{R}$. Prove that F is a closed set in \mathbb{R} if and only if its complement F' is an open set in \mathbb{R} .
6. (a) i. Give the definition of a sequence.
- ii. Let $\{x_n\}_{n=1}^{\infty}$ be a sequence in \mathbb{R} and $l \in \mathbb{R}$. When is
- $$l = \lim_{n \rightarrow \infty} x_n; \quad \infty = \lim_{n \rightarrow \infty} x_n \quad \text{and} \quad -\infty = \lim_{n \rightarrow \infty} x_n.$$
- iii. When is l a limit point of $\{x_n\}_{n=1}^{\infty}$?
- iv. Give the definition of a subsequence of a sequence $\{x_n\}_{n=1}^{\infty}$ in \mathbb{R} .
- (b) Prove that a bounded sequence in \mathbb{R} contains a convergent subsequence.
7. (a) i. If $\{x_n\}_{n=1}^{\infty}$ is a sequence of real numbers, define the infinite series generated by the sequence.
- ii. The comparison test of the second kind states: Let $\sum_{n=1}^{\infty} c_n$ and $\sum_{n=1}^{\infty} d_n$ denote respectively a convergent and a divergent series of positive terms. If the terms of a given series $\sum_{n=1}^{\infty} a_n$, of positive terms satisfy that given $m \in \mathbb{N}$ and $\forall n > m$,
- The condition $\frac{a_{n+1}}{a_n} \leq \frac{c_{n+1}}{c_n}$, then the series $\sum_{n=1}^{\infty} a_n$ converges,
 - consistently, $\frac{a_{n+1}}{a_n} \geq \frac{d_{n+1}}{d_n}$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.
- Prove this comparison test.
- (b) Cauchy's ratio test states: If from some place onwards in the series $\sum_{n=1}^{\infty} a_n$, $a_n > 0$ and $\frac{a_{n+1}}{a_n} \leq a < 1$, then the series is convergent. Prove Cauchy's ratio test.

END OF PAPER

The University of Zambia

School of Natural sciences

Department of Mathematics and Statistics

Final Examination Time Allowed: Three (3) Hours

MAT 2602 Introduction to Statistics

Instructions:

- (1) This Examination has six (6) questions, answer any five (5) questions only.
- (2) All calculations should be shown in order to get full marks
- (3) Graph papers and statistical tables for Z, t, F, χ^2 will be provided, please ensure you have them before the examination start.

1. (a) The concentration of antibody in blood serum(g/l) are given below:

11.6 15.8 17.0 14.2 16.2 8.4 13.2 12.5 15.7 14.0 19.2 12.0 8.2 13.8 12.6 17.5 15.2
07.2 17.0 05.2 05.2 15.3 07.8 11.2 10.0 12.1 17.3 10.8 09.6 09.4 10.5 15.7 09.9 16.0
07.5 12.2 17.3 14.0 13.7 15.5 08.6 12.0 15.0 11.7 10.7 09.9 11.5 10.2 14.8 14.2 16.1

- (i) Using the data given above, construct a frequency table of class interval length of 1.9 starting from 4.0.

Using the frequency table constructed in (i) above, estimate

- (ii) The standard deviation
- (iii) The interquartile range

(b) The figures below shows the results of the extension of a spring(y in cm) and the load (load in newton)

X (load in Newton)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Y(length of spring in cm)	32.5	37.1	35.5	37.7	41.5	46.4	44.4	45.8	53.9	62.2

- (i) Draw a scatter diagram to show the length of spring against the load in Newton.
- (ii) Determine the equation of the least square estimates of the regression line of Y on X, using least squares methods.

- (iii) Use the equation in (ii) to predict the length of the spring for $x = 0.65$.
2. (a) (i) In order to calculate the confidence interval for the mean mass of packets of butter produced by the machine, a random sample of ten packets is taken. These have masses (measured in kg) of x_1, \dots, x_{10} , such that

$$\sum_1^{10} x_i = 2.57, \quad \sum_1^{10} x_i^2 = 0.6610. \text{ Calculate 95\% confidence limits.}$$

- (ii) If it is known that the standard deviation of the mass of a packet of butter is 0.008kg, what is the least number to be sampled in order to give a 95% confidence limits for the mean mass whose width is less than 0.002kg?
- (b) (i) Let Y_1, \dots, Y_n be a random sample with $E(Y_i) = \mu$ and $\text{var}(Y_i) = \sigma^2$

$$\text{Show that } S^2 = \frac{1}{n} \sum_1^n (Y_i - \bar{Y})^2 \text{ is a biased estimator of } \sigma^2.$$

- (ii) A sales manager in charge of sales in a big company claims that the salesmen are only averaging 15 sales contacts per week. (He would like to increase this figure). To check his claim, $n = 36$ salesmen are selected at random and the number of contacts recorded for a single randomly selected week. The sample reveals a mean of 17 contacts and a variance of 9. Does the evidence contradict the manager's claim at 5% level of significance?
3. (a) The regression line of Y on X is given by $Y = \alpha + \beta X + \varepsilon$, where α and β are the y-intercept and the slope of the least regression line, and ε are errors.
- (i) State the assumptions of ε on the regression line
- (ii) Derive the estimates of α and β .
- (iii) Hence or otherwise, using the method of least squares fit a line to the following $n = 5$ data points:

Y	-2	-2	0	1	2
X	0	0	1	1	3

- (b) Four groups of students were subjected to different teaching techniques and tested at the end of a specified period of time. Due to dropout from the experimental groups for various reasons, the number of students varied from groups to groups. The results of the test are given below:

	GROUP 1	Group 2	Group 3	Group 4
	6.5	7.5	5.9	9.4
	8.7	6.9	7.8	8.9
	7.3	8.3	6.7	8.0
	7.9	8.1	6.2	8.8
	8.1	7.2	8.3	-
	6.9	7.9	7.6	-
	-	9.0	-	-
Totals	45.4	54.9	42.5	35.1

$$\sum_{i=1}^4 \sum_{j=1}^{n_j} Y_{i,j}^2 = 13951.1$$

Do the data shown above present sufficient evidence to indicate a difference in the mean achievement for the four teaching techniques? Use 5 percent as your level of significance.

4. (a) The table below shows heights of pupils in a class in cm (to the nearest cm)

Height(cm)	120-129	130-139	140-149	150-159	160-169	170-179
frequency	2	12	17	18	7	1

- (i) Find the mean and standard deviation of the heights of pupils in the class.
- (ii) Find the relative frequency
- (iii) Construct a cumulative frequency curve

Using your cumulative frequency curve

- (iv) Estimate 90th percentile, 7th Decile, lower quartile and upper quartile of the heights of pupils.

- (b) Two brands of refrigerators, denoted by A and B, are each guaranteed for one year. In a random sample of 50 refrigerators of brands A and B , 12 were observed to fail before the guarantee period ended. A random sample of 60 brand B refrigerators also revealed 12 failures during the guarantee period.

Construct a confidence interval of the difference between proportions of failure during guarantee period at 2% level of significance.

5. (a) (i) Fifty measurements of the acceleration due to gravity, g has a mean value of 9.8m/s and s.d. of 0.75m/s . What are the 95% confidence limits of the Mean.
- (ii) A comparison of the wearing quality of two types of automobile tyres was obtained by road testing samples of $n_1 = n_2 = 100$ tires for each type. The number of km until wear-out was recorded, where wear-out was defined as specific amount of tire wear. The test results were as follows:

$$\bar{x}_1 = 26400\text{km} \quad \bar{x}_2 = 25100$$

$$s_1^2 = 1,440,000 \quad s_2^2 = 1,960,000$$

Estimate the difference in mean time to wear-out and place a bound on the error of estimation.

- (b) The table below shows a sample of houses classified by compound and type.

	Detached	Semi-Detached	Flats
Northmead	95	297	242
Kabulonga	175	417	362
Kabwata	703	994	861

Test the null hypothesis that the type of housing and the compound are independent, at 5% level of significance

6. (a) A teacher has marked large set of examination papers and has then done some statistical work on the marks, obtaining the following results.
 Mean mark = 36, Median mark = 42, variance = 144, interquartile range = 15, highest mark = 72. The teacher then decides to scale the marks by means of the formula:
 $y = cx + d$, where x is the original mark, y the new mark, and c and d are positive integers chosen so that the new marks y have a mean of 50 and variance of 8.
- (i) Calculate the values of c and d . Hence, find the new values for
- (ii) Median
- (iii) Interquartile range
- (iv) The highest mark

- (b) A stimulus response experiment involving three treatments A, B, C was laid out in a randomised block design showing four subjects. The response was the length of time to reaction measured in seconds. The data arranged in blocks are shown below. The treatment numbers is shown in square brackets above the observation.

Subject 1	Subject 2	Subject 3	Subject 4
[A] 1.7	[C] 2.1	[A] 0.1	[B] 2.2
[C] 2.3	[A] 1.5	[B] 2.3	[A] 0.6
[B] 3.4	[B] 2.6	[C] 0.6	[C] 1.6

- (i) Do the data present sufficient evidence to indicate a difference in mean response for treatment? Use $\alpha = 0.05$.
- (ii) Construct a 95% confidence interval for the difference between treatments A and B.

END OF EXAMINATION

The University of Zambia
Department of Mathematics & Statistics
2013 Academic Year Final Examinations
MAT3100 - Advanced Calculus

Time allowed : **Three (3) hours**

Full marks : 100

-
- Instructions:**
- Attempt **any five (5)** questions. **All** questions carry **equal** marks.
 - Indicate your **computer number** on all answer booklets.
 - **Full credit** will only be given when **necessary work** is shown.
 - **Calculators** are not allowed for this examination.

This paper consists of 3 pages of questions.

1. (a) Let $x^2 - y^2 + z^2 - 4x - 2y - 2z + 4 = 0$ be the equation of a surface.
 - i. Identify the surface and write down the centre.
 - ii. Sketch the surface.
 - (b) Let $f(x, y) = xy^2 + \ln(1 + xy)$.
 - i. Find the Taylor polynomial for f , upto the second degree terms about the point $(1, 1)$.
 - ii. Write down a cartesian equation for the tangent plane to the surface $z = f(x, y)$ at the point where $x = 1$ and $y = 1$.
 - iii. Find all the stationary points of f , and classify **only** the stationary point $(0, 0)$.
 - (c) Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y, z) = xyz$ subject to the constraint $x^2 + 2y^2 + 3z^2 = 6$.
2. (a) Let S be the solid enclosed by the parabolic cylinder

$$y = x^2$$

and the planes $z = 6y$, $z = 10 + y$. Express the volume of S as a difference of repeated integrals, integrating first with respect to y , and then with respect to x .

- (b) Evaluate the integral

$$\int_0^1 \int_{x^3}^1 x^5 \sin(y^3) dy dx.$$

Hint: Sketch the region over which you are integrating.

- (c) A solid E lies within the sphere $x^2 + y^2 + z^2 = 7$, above the xy -plane, and below the cone $z = \sqrt{x^2 + y^2}$. Use spherical coordinates to find the mass of E if the density function at the point (x, y, z) is $\frac{3}{\sqrt{x^2 + y^2}}$.
- (d) Use the change of variables $x = a$ and $y = 5b$ to evaluate

$$\iint_R y^2 dA$$

where R is the region bounded by the ellipse $25x^2 + y^2 = 100$.

3. (a) Investigate the existence of the Laplace transform of the function

$$f(t) = e^{(t-1)^2}.$$

- (b) Evaluate the integral,

$$\int_0^\infty \frac{e^{-2t} - e^{-8t}}{t} dt.$$

- (c) Use Laplace transform to solve the simultaneous ordinary differential equations

$$f'' + f' + g = 0$$

$$f' + g' = 0$$

subject to $f(0) = 0$, $f'(0) = 0$ and $g(0) = 1$.

- (d) Show that the origin of the system

$$x' = -2y + yz - x^3$$

$$y' = x - xz - y^3$$

$$z' = xy - z^3$$

is an asymptotically stable equilibrium point.

4. (a) Classify all the singular points of the equation

$$3xy'' + y' - y = 0.$$

- (b) Use the method of canonical forms to find the general solution to the equation

$$u_{xx} + 4u_{xy} + u_x = 0.$$

(c) Solve the heat flow problem

$$\begin{cases} u_t - 3u_{xx} = 0 & 0 < x < \pi, \quad t > 0, \\ u(0, t) = u(\pi, t) = 0 & t \geq 0, \\ u(x, 0) = 2 \sin 5x \cos x & 0 \leq x \leq \pi. \end{cases}$$

(d) Consider the equation

$$uu_x + u_y = 0.$$

Use the method characteristics to find a solution satisfying $u(x, 0) = x^2$.

5. (a) Evaluate the line integral, where C is the given curve.

i. $\int_C xy \, dx + (x - y) \, dy$, where C consists of the line segments from $(0, 0)$ to $(2, 0)$ and from $(2, 0)$ to $(3, 2)$.

ii. $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = e^y \mathbf{i} + xe^y \mathbf{j} + (z + 1)e^z \mathbf{k}$, $C: \mathbf{r}(t) = t \mathbf{i} + t^2 \mathbf{j} + t^3 \mathbf{k}$

(b) Verify the divergence theorem for the case where $\mathbf{F}(x, y, z) = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$ and B is the solid sphere of radius R centred at the origin.

(c) Use Green's theorem to find the work done by the force $\mathbf{F}(x, y) = x(x + y) \mathbf{i} + xy^2 \mathbf{j}$ in moving a particle from the origin along the x -axis to $(1, 0)$, then along the line segment to $(0, 1)$ and then back to the origin along the y -axis.

6. (a) Find the area of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$.

(b) Let

$$f(x) = \begin{cases} x, & 0 < x < 4 \\ 8 - x, & 4 < x < 8 \end{cases}$$

i. Sketch the given function.

ii. Expand $f(x)$ in a Fourier series of sines.

(c) Find the complex Fourier series of $f(x) = e^x$ if $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$ and obtain the usual Fourier series.

END OF PAPER

The University of Zambia
Department of Mathematics & Statistics
2013 Academic Year, Distance Examinations
MAT 3100 - ADVANCED CALCULUS

Time allowed : Three (3) Hours

Full marks : 100

Instructions:

1. Attempt **any Five (5)** questions out of the **Seven (7)** QUESTIONS.
2. All questions carry equal marks.
3. **Full credit** will only be given when **necessary work** is shown.
4. Indicate your **computer number** on all answer booklets.

This paper consists of 3 pages of questions.

1. (a) Use the transformation $x + y = u$ and $y = uv$ to evaluate the integral

$$\int_0^1 \int_0^{1-x} e^{\frac{y}{x+y}} dy dx.$$

- (b) Use Cylindrical coordinates to find the volume of the region above the xy -plane bounded by the paraboloid $z = x^2 + y^2$ and the cylinder $x^2 + y^2 = 9$
- (c) (i) Identify and sketch the surface represented by the equation

$$x = 4y^2 + z^2 - 4z + 4.$$

- (ii) Use the convolution theorem to solve the integral equation

$$y(x) = x^3 + \int_0^x \sin(x-t)y(t)dt.$$

2. (a) Given that $\mathbf{F} = yz\mathbf{i} + xz\mathbf{j} + (xy + 2z)\mathbf{k}$. Show that \mathbf{F} is irrotational, hence find a function f such that $\nabla f = \mathbf{F}$.

(b) (i) State the Fundamental Theorem of Line Integrals. Hence,

(ii) evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where \mathbf{F} is the vector field in (a) and C is the line segment from $(1, 0, -2)$ to $(4, 6, 3)$.

(c) Find the fourier series of the function,

$$f(x) = \begin{cases} 0 & \text{if } -\pi \leq x < 0 \\ x & \text{if } 0 \leq x \leq \pi \end{cases}$$

3. (a) (i) State Green's theorem.

(ii) Evaluate $\oint_C (3y - e^{\sin x})dx + (7x + \sqrt{y^4 + 1})dy$, where C is the circle $x^2 + y^2 = 9$.

(b) Evaluate $\int_C (2x + y)ds$, where C is the curve given by $x^2 + y^2 = 25$, $x \geq 0$ and s is the arc length parameter.

(c) Determine the minimum value of $f(x, y, z) = x^2 + y^2 + z^2$ subject the constraints:

$$g_1(x, y, z) = x + 2y + 3z - 6 = 0, \quad g_2(x, y, z) = x + 3y + 9z - 9 = 0$$

4. (a) Find the Laplace transform of the function, $g(t) = 3t^4 e^{5t} + t^2 \sin 4t$.

(b) Evaluate

$$\int_S x^2 dS,$$

where S is the the unit sphere $x^2 + y^2 + z^2 = 1$.

(c) Find the directional derivative of

$$f(x, y, z) = x^2 y^2 z^2$$

at the point

$(1, 2, -1)$ in the direction of $g(t)$ where $g(t) = (t^2 - t + 2, t, t + 2)$

5. (a) (i) State the divergence theorem.

(ii) Verify the divergence theorem for the vector field

$$\mathbf{F}(x, y, z) = z\mathbf{i} + y\mathbf{j} + x\mathbf{k}$$

and E is the solid ball $x^2 + y^2 + z^2 \leq 16$

(b) Discuss the continuity of the function

$$f(x, y) = \begin{cases} \frac{x^2 y^3}{2x^2 + y^2} & \text{if } (x, y) = (0, 0) \\ 1 & \text{if } (x, y) \neq (0, 0) \end{cases}$$

(c) Let f, g and h be differentiable functions of two variables. Show that the vector field

$$\mathbf{F} = f(y, z)\mathbf{i} + g(x, z)\mathbf{j} + h(x, y)\mathbf{k}$$
 is incompressible.

6. (a) Given the function

$$f(x) = x^2, \quad -2 \leq x \leq 2.$$

(i) Find the Fourier series of $f(x)$.

(ii) Use your result in (i) to find the sum $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2}$.

(b) Use the definition to find the Laplace transform $L = F(s)$ of the function

$$f(t) = e^{-t} \sin t$$

and state the range of values of s for which the transform is valid.

(c) (i) Find an equation of the tangent plane to the hyperboloid

$$z^2 - 2x^2 - 2y^2 = 12$$

at the point $(1, -1, 4)$.

(ii) Find the unit normal vector to the hyperboloid in (i) at the point $(1, -1, 4)$.

7. (a) Find the absolute maximum and minimum values of

$$f(x, y) = x^2 + y^2 - 2x,$$

on the closed triangular region with vertices $(2, 0)$, $(0, 2)$, $(0, -2)$

(b) Use the Laplace transform method to solve the problem

$$y'' + 4y' + 4y = e^{-4t}; \quad y(0) = 0 = y'(0).$$

(c) Find a set of symmetric equations for the normal line to the surface given by

$$xyz = 12$$

at the point $(2, -2, -3)$.

END OF EXAM!

The University of Zambia
Department of Mathematics & Statistics
2013 Academic Year Final Examinations
MAT 3110 - ENGINEERING MATHEMATICS II

Time allowed : Three (3) Hours

Full marks : 100

Instructions:

1. This examination paper consists of two sections, namely, section A and Section B.
2. Attempt **Four (4)** questions from section A and **One(1)** question from section B.
3. All questions carry equal marks.
4. **Full credit** will only be given when **necessary work** is shown.
5. Indicate your **computer number** on all answer booklets.

This paper consists of 5 pages of questions.

SECTION A

1. (a) Find the Laplace transform of
 - (i) $f(x) = \sinh ax$; by definition
 - (ii) $f(t) = \begin{cases} 3t - t^2, & \text{if } 0 < t < 3 \\ 0, & \text{elsewhere} \end{cases}$
- (b) Find the inverse Laplace transform of

$$F(s) = \frac{s - 2}{s^2 - 4s + \frac{17}{4}}$$

(c) Solve the following system by Laplace Transform method

$$x' + 2x + y = e^t$$

$$y' - x = 0;$$

with initial conditions $x(0) = y(0) = 0$.

2. a i) Solve the convolution equation $\int_0^t \cos(t - \tau)x(\tau)d\tau = t$.

ii) Find the general solution of $x^2y'' + 3xy' + 4y = 0$

b) (i) Find the Fourier series of the function

$$f(x) = \begin{cases} 0, & \text{if } -2 < t < -1 \\ \alpha, & \text{if } -1 < t < 1 \\ 0, & \text{if } 1 < t < 2 \end{cases}$$

(ii) Hence, show that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$.

(c) (i) Find the Fourier Integral representation of

$$f(t) = \begin{cases} 1, & \text{if } -1 \leq t < 1 \\ 0, & \text{otherwise} \end{cases}$$

(ii) Hence, evaluate the integral

$$\int_0^{\infty} \frac{\sin \omega}{\omega} d\omega.$$

3. (a) Find the deflection $u(x, t)$ of the vibrating string satisfying

$$u_{tt} = 4u_{xx}, \quad 0 < x < \pi, \quad t > 0,$$

$$u(0, t) = u(\pi, t) = 0, \quad t \geq 0, \quad u(x, 0) = 3(\sin x + 2 \sin 3x),$$

$$u_t(x, 0) = 4 \sin 2x + 16 \sin 4x, \quad 0 \leq x \leq \pi.$$

(b) Let $\mathbf{R} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $r = \sqrt{x^2 + y^2 + z^2}$, show that

$$\operatorname{div}(r^n \mathbf{R}) = (n + 3)r^n.$$

(c) (i) Evaluate

$$\int_0^1 \int_{\arcsin y}^{\frac{\pi}{2}} \cos x \sqrt{1 + \cos^2 x} dx dy$$

(ii) Find the equation for the tangent plane to the surface given by the equation $\arctan\left(\frac{y}{x}\right) - z = 0$ at the point $(1, 1, \frac{\pi}{4})$.

4. (a) (i) State Green's theorem in the plane.

(ii) Evaluate $\oint_C (3y - e^{\sin x})dx + (7x + \sqrt{y^4 + 1})dy$, where C is the circle $x^2 + y^2 = 9$, oriented counterclockwise.

(b) Given $I = \int_0^1 \int_{x^2}^1 \int_0^4 dz dy dx$.

(i) Sketch the volume represented by I .

(ii) Evaluate the integral

$$\int_0^1 \int_{x^2}^1 \int_0^4 x^3 \sin(y^3) dz dy dx.$$

(c) Use the transformation $x = 2u + v$, $y = u + 2v$ to evaluate the integral

$$\iint_R (x - y) dA,$$

where R is the region with vertices $(0, 0)$, $(2, 0)$, $(2, 1)$ and $(1, 2)$.

5. (a) (i) State the divergence theorem.

(ii) Sketch the region E bounded by the cylinder $z = 1 - x^2$ and the planes $z = 0$, $y = 0$, and $y + z = 6$. Hence, evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where,

$$\mathbf{F}(x, y) = xy\mathbf{i} + (y^2 + e^{xz})\mathbf{j} + \sin(xy)\mathbf{k},$$

and S is the surface of E .

(b) Evaluate $\iint_S y^2 dS$ where S is the unit sphere $x^2 + y^2 + z^2 = 1$.

(c) The distribution of voltage on a metal plate is given by

$$V = 50 - x^2 - 4y^2,$$

Describe the path of a particle that starts at the point $(1, -2)$ and moves in the direction of greatest voltage increase.

SECTION B

6. (a) (i) An experiment consists of flipping a coin and then flipping it a second time if a head occurs. If a tail occurs on the first flip, then a die is tossed once. List all possible outcomes and calculate the probability of obtaining at least one head.
- (ii) Events A and B are such that

$$P(A) = \frac{2}{3}, \quad P(A \cap B) = \frac{1}{6}, \quad P(A|B) = \frac{1}{3}.$$

Find $P(B)$ and determine whether or not, events A and B are independent.

- (b) An experiment consists of rolling two fair dice. Define the events A , and B by:

$A \equiv$ obtaining a sum between five(5) and eight(8), not including 5 and 8.

$B \equiv$ obtaining a sum which is a prime number.

- (i) List the elements of the sets A and B .
- (ii) Evaluate $P(A)$ and $P(B)$.
- (iii) Evaluate $P(B|A)$.
- (c) Three members of a private country club have been nominated for the office of president. The probability that Mr Akapelwa will be elected is 0.3, the probability that Mr Bunda will be elected is 0.5, and the probability that Ms Chama will be elected is 0.2. Should Mr Bunda or Ms Chama be elected, the corresponding probabilities for an increase in membership fees is 0.1 and 0.4, respectively. If Mr Akapelwa is elected the probability of having an increase in membership fees is 0.8.
- (i) What is the probability that there will be an increase in membership fees?
- (ii) If it is known that the membership fees have increased, what is the probability that Ms Chama was elected president of the club?

7. (a) A random variable X has probability function $f(x)$ with

$$f(1) = 0.2, \quad f(2) = y, \quad f(3) = 0.4, \quad f(4) = 0.2, \quad f(5) = 0.1.$$

Find

- (i) the value of y
- (ii) mean and variance of X .

(b) A small filling station is supplied with gasoline every Saturday afternoon. Assume that its volume X of sales in liters has the probability density

$$f(x) = kx(1 - x) \quad \text{if } 0 \leq x \leq 1, \quad \text{and } 0 \quad \text{otherwise.}$$

Find

- (i) the value of k .
- (ii) the mean of the random variable X .
- (iii) the variance of X .

(c) Let X be the life in hours of a light bulb with distribution function

$$F(x) = 1 - e^{-0.001x}, \quad x \geq 0.$$

- (i) Find its probability density function $f(x)$.
- (ii) Find $P(1 < X < 4)$ and $P(X > 10)$.

END OF EXAM!

The University of Zambia
Department of Mathematics & Statistics
2013 Academic Year Final Examinations
MAT3300 - Real Analysis

Time allowed : **Three (3) hours**

Full marks : 100

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- Instructions:**
- Attempt any five (5) questions. All questions carry equal marks.
 - Indicate your **computer number** on all answer booklets used.
 - No help materials allowed.

This paper consists of 3 pages of questions.

1. (a) Let $A \subset \mathbb{R}$ be a set. Define what is meant by
 - i. A is an open set,
 - ii. $f : A \rightarrow \mathbb{R}$ is a continuous function at a point $x_0 \in A$,
 - iii. A is a compact set.(b) Let A and F be subsets of E . Show that if A is open in E then $A \cap F$ is open in F .
(c) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a real valued function. Prove that f is continuous implies that if A is a closed subset of \mathbb{R} , then $f^{-1}(A)$ is closed.

2. (a) Let $A \subset \mathbb{R}$ be a set. Define what is meant by
 - i. separation of A ,
 - ii. $f : A \rightarrow \mathbb{R}$ is a Lipschitz function on A ,
 - iii. the uniform Cauchy property.(b) Let $A = [-\frac{1}{2}, \frac{1}{2}]$. The function $f : A \rightarrow \mathbb{R}$ is defined by $f(x) = \frac{x}{1-x^2}$. Show that f is a Lipschitz function on A .
(c) Suppose that $A \subset \mathbb{R}$ is a connected set and $f : A \rightarrow \mathbb{R}$ is continuous on A . Show that $f(A)$ is a connected set.

3. (a) Define the following:

- i. Pointwise convergence of a sequence of functions $f_n : A \rightarrow \mathbb{R}$.
- ii. Variation of a function $f : [a, b] \rightarrow \mathbb{R}$.
- iii. The Weierstrass M-test.

(b) Let f be a continuous function on a closed and bounded interval $[a, b]$, and let f' exist on (a, b) with $|f'(x)| \leq M$ for some positive real number M and for all $x \in (a, b)$. Show that f is of bounded variation on $[a, b]$.

(c) Let

$$f_n(x) = \frac{x^n}{2 + 3x^n}, \quad 0 \leq x \leq 1.$$

Show that the sequence $\{f_n\}$ converges pointwise on $[0, 1]$ but that it does not converge uniformly.

4. (a) i. State the Interior Maximum Theorem.

ii. Assume that a real function f is differentiable on $[a, b]$, and $f'(a) < \lambda < f'(b)$. Prove that there exists $x \in (a, b)$ such that $f'(x) = \lambda$.

iii. Assume that a real function f is continuous on $[a, b]$ and differentiable in (a, b) . Show that if $f > 0$ and $f' \neq 0$ on $[a, b]$, then there exists $c \in (a, b)$ such that

$$f(b) = f(a) + \ln \left(\frac{f(b)}{f(a)} \right) f(c).$$

Remark: No derivative appears on the right hand side!

(b) i. Suppose a real function g on \mathbb{R} has a bounded derivative. Fix $\varepsilon > 0$, and define

$$f(x) = x + \varepsilon g(x).$$

Prove that f is one-to-one if ε is small enough.

ii. Suppose f' is continuous on $[a, b]$ and $\varepsilon > 0$. Prove that there exists $\delta > 0$ such that

$$\left| \frac{f(y) - f(x)}{y - x} - f'(x) \right| < \varepsilon$$

whenever $0 < |y - x| < \delta$, $a \leq x \leq b$ and $a \leq y \leq b$.

Remark: That is, f is uniformly differentiable on $[a, b]$ if f' is continuous on $[a, b]$.

(c) Suppose that $f : \mathbb{R} \rightarrow \mathbb{R}$ is a C^2 function that satisfies $f \geq 0$ and $f'' \leq 0$ everywhere. Show that f is a constant function.

Hint: Use Taylor's theorem.

5. (a) Let $f : [a, b] \rightarrow \mathbb{R}$ be a bounded function. Define what is meant by

- i. P is a partition of $[a, b]$,
- ii. the upper and lower sums, $U(P, f)$ and $L(P, f)$ respectively,
- iii. the upper and lower integrals, $\overline{\int_a^b} f$ and $\underline{\int_a^b} f$ respectively,
- iv. f is Riemann integrable over $[a, b]$.

(b) i. State and prove the Riemann-Stieltjes integrability criterion theorem.

ii. Suppose $f : [0, 1] \rightarrow \mathbb{R}$ is a bounded function with upper and lower integrals

$$\overline{\int_0^1} f = 1 \quad \text{and} \quad \underline{\int_0^1} f = 0$$

respectively. Prove that for every $\varepsilon > 0$ there exists a partition P of $[0, 1]$ such that the difference between the upper and lower sums of f on P satisfies

$$1 \leq U(P, f) - L(P, f) < 1 + \varepsilon.$$

(c) i. Suppose f is continuous and nonnegative on $[0, 1]$, $0 \leq x_0 \leq 1$, and $f(x_0) > 0$. Prove that f is integrable on $[0, 1]$, and that $\int_0^1 f dx > 0$.

ii. Construct a nonnegative function f on $[0, 1]$ with $f(\frac{2}{7}) > 0$ but $\int_0^1 f dx = 0$.

6. (a) i. State the First Mean Value Theorem for integrals.

ii. Suppose f is a real, continuously differentiable function on $[a, b]$, $f(a) = f(b) = 0$, and $\int_a^b f^2(x) dx = 1$. Prove that

$$\int_a^b x f(x) f'(x) dx < 0, \quad \text{and that} \quad \int_a^b [f'(x)]^2 dx \cdot \int_a^b x^2 f^2(x) dx > 0.$$

(b) Let $f : \mathbb{R} \rightarrow [0, \infty)$ be a continuous function. Prove that

$$\lim_{n \rightarrow \infty} \frac{1}{n} \int_0^n x f dx = 0$$

if the integral $\int_0^\infty f dx$ is convergent.

(c) For $p > 1$, use Holder's inequality to find the values of the parameter λ for which

$$\lim_{\varepsilon \rightarrow 0^+} \frac{1}{\varepsilon^\lambda} \int_0^\varepsilon f dx = 0$$

for all $f \in L^p[0, 1]$.

Hint: A function f is in $L^p[0, 1]$ if it satisfies $\int_0^1 |f|^p dx < \infty$.

END OF PAPER

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS

2013 ACADEMIC YEAR
END OF YEAR EXAMINATIONS

MAT3612 : LINEAR MODELS AND DESIGN OF EXPERIMENTS

TIME ALLOWED: Three (3) Hours

INSTRUCTIONS: 1. Answer any **Four (4)** Questions
2. Show All Essential Working

1. (a) Define the following:
- (i) orthogonal matrix.
 - (ii) linearly dependent vectors x_1, x_2, \dots, x_k .
- (b) (i) If $Y = c^T X$, prove that $Var(Y) = c^T \Sigma c$, where $Var(X) = \Sigma$, X is a random vector and c is a vector of constants.
- (ii) Given the model $Y = X\beta + \varepsilon$, where $\varepsilon \sim N(0, \sigma^2 V)$, prove that the weighted least squares estimator of β is given by
- $$\tilde{\beta} = (X^T V^{-1} X)^{-1} X^T V^{-1} Y$$
- (c) Given the random vector $X = (X_1, X_2, X_3)^T$ follows a multivariate normal distribution with mean

$$\mu = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \text{ and variance } \Sigma = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

- (i) Find $Cov(2X_1 - X_3, X_2)$
 - (ii) Are $2X_1 - X_3$ and X_2 independent? Explain.
 - (iii) Find $Cov(X_1 + 2X_2, 2X_1 - X_3)$.
 - (iv) Find the distribution of $Y = (Y_1, Y_2)^T$ where $Y_1 = X_1 + X_2 - X_3$ and $Y_2 = X_1 - 2X_2 + X_3$
2. (a) Given a multiple regression model
- $$Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_p x_{ip} + \varepsilon_i, \quad i = 1, 2, \dots, n.$$
- (i) Explain how you would check whether the assumption $E(\varepsilon_i) = 0$ holds.
 - (ii) Define a studentized residual $d_i, i = 1, 2, \dots, n$.

- (i) What design was used in the experiment?
- (ii) Write down a model for the study. Explain all the terms in the model and state all the assumptions.
- (iii) Copy and complete the following ANOVA table:

Source	SS	df	MS	F*
A				
B				
C				
AB				
AC	3.5			
BC	4.667			
ABC	4.333			
Error	19			
Total	52			

- (c) A study was performed to determine if there was a difference in the strength of fiber produced by three different machines. The study involved measuring the strength (Y) and thickness (X) of the fiber in the sample. The following data were collected:

	Machine					
	1		2		3	
	Y	X	Y	X	Y	X
	36	20	40	22	35	21
	41	25	48	28	37	23
	39	24	39	22	42	26
	42	25	45	30	34	21
	49	32	44	28	32	16
Total	207	126	216	130	180	107

(Additionally $Y_{..} = 603$, $X_{..} = 363$, $SST_Y = 346.4$, $SST_X = 244.4$, $SST_{XY} = 274.4$)

- (i) Write down a model for the study. Explain all the terms in the model and state all the assumptions.
- (ii) Prepare an ANCOVA table for the study.
- (iii) Are the three machines significantly different? Test at 5% level of significance.

END OF EXAMINATION

Source	SS	df	MS	F*
Regression				
Residual			0.001	
Total	53.389			

- (iii) Test the hypothesis $H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$. Use $\alpha = 0.05$.
 (iv) Test the hypothesis that $\beta_3 < -0.45$. Use $\alpha = 0.05$.
 (v) Test the hypothesis $H_0: 2\beta_1 = \beta_2$. Use $\alpha = 0.05$.
 (vi) Copy and complete the following ANOVA table obtained after fitting the regression model $Y = \beta_0 + \beta_2x_2 + \beta_3x_3 + \varepsilon$

Source	SS	df	MS	F*
Regression				
Residual	12.563			
Total				

- (vii) Based on the model in (vi) above, find a 95% confidence interval for the mean response of Y when $x_2 = 5$ and $x_3 = 10$, given that the fitted regression equation is

$$\hat{Y} = 21.369 + 0.467x_2 - 0.741x_3$$

and

$$(X^T X)^T = \begin{pmatrix} 1.805 & -0.051 & -0.123 \\ -0.051 & 0.017 & -0.006 \\ -0.123 & -0.006 & 0.014 \end{pmatrix}$$

5. (a) Define the following:
 (i) observational study.
 (ii) confounding.
- (b) In a production of a particular material, three varieties were of interest: A, the operator effect (op1, op2, op3); B, the catalyst used in the experiment (ct1, ct2, ct3); and C, the washing time of the product following the cooling process (15 minutes or 20 minutes). Two runs were made at each combination of the factors. The following coded yields were obtained:

	Washing time (C)						Total
	15 minutes			20 minutes			
	Catalyst (B)			Catalyst (B)			
Operator (A)	ct1	ct2	ct3	ct1	ct2	ct3	
op1	11	10	11	11	11	12	135
	12	10	12	12	11	12	
op2	11	10	11	10	13	11	129
	12	11	11	11	8	10	
op3	14	12	11	11	10	12	144
	15	12	11	12	12	12	
Total	75	65	67	67	65	69	408

	Concentration					Total
	1	2	3	4	5	
	68	82	77	69	59	
	73	87	84	58	61	
	63	94	86	72	69	
	69	92	81	68	57	
	71	86	80	74	60	
Total	344	441	408	341	306	1840

(Additionally $\sum_{i=1}^5 \sum_{j=1}^5 Y_{ij}^2 = 138276$)

- (i) What design was used in the experiment?
 - (ii) Write down a model for the above study, explain all the terms in the model and state all the assumptions.
 - (iii) Test whether the growth rate is the same for all the levels of concentration. Use $\alpha = 0.05$.
 - (iv) Use the LSD method to compare the concentration levels. What are your conclusions? Use $\alpha = 0.05$.
4. (a) (i) Define replication.
(ii) State two reasons why blocking is important.
- (b) The following data resulted from 15 experimental runs made on four independent variables and a single response Y.

Subject	Y	x_1	x_2	x_3	x_4
1.	14.8	7.5	4.3	11.5	6.3
2.	12.1	6.9	3.9	14.5	7.4
⋮	⋮	⋮	⋮	⋮	⋮
15.	18.2	9.6	8.4	11.9	5.4

Fitting a regression model to the data gives the fitted regression equation

$$\hat{Y} = 19.985 + 0.304x_1 + 0.596x_2 - 0.497x_3 - 0.704x_4$$

and

$$(X^T X)^{-1} = \begin{pmatrix} 4.198 & -0.206 & -0.010 & -0.129 & -0.109 \\ -0.206 & 0.021 & -0.001 & 0.004 & -0.003 \\ -0.010 & -0.001 & 0.020 & -0.003 & -0.012 \\ -0.129 & 0.004 & -0.003 & 0.019 & -0.015 \\ -0.109 & -0.003 & -0.012 & -0.015 & 0.055 \end{pmatrix}$$

- (i) Explain the meaning of the regression coefficients associated with x_1 and x_4 .
- (ii) Copy and complete the following ANOVA table for the above fitted model:

- (b) Consider the simple linear regression model
 $Y_i = \beta_0 + \beta_1(x_i - \bar{x}) + \varepsilon_i, i = 1, 2, \dots, n$, where $\varepsilon_i \sim N(0, \sigma^2)$
- (i) Find the least squares estimator of β_0 .
- (ii) Show that the least squares estimator of β_1 is given by

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})Y_i}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

- (iii) Find $E(\hat{\beta}_1)$.
- (iv) Find $Var(\hat{\beta}_1)$.
- (c) To improve nutrition value for sheep, an experiment was conducted to compare four diet treatments; control (A), urea at feeding (B), ammonia-treated (C) and urea-treated straw (D). Twelve sheep were used in the experiment and were separated according to relative weight. For each of the 12 sheep, the percent dry matter digested was measured and the following data obtained:

Diet	Group by weight		
	1	2	3
A	33	36	36
B	36	39	38
C	49	54	52
D	47	43	45

- (i) What design was used in the experiment?
- (ii) Write down a model for the study. Explain all the terms in the model and state all the assumptions.
- (iii) Write down the model in the form $Y = X\beta + \varepsilon$, clearly stating Y, X, β and ε .
3. (a) Define the following:
- (i) variance inflation factors.
- (ii) randomization.
- (b) Given the model $Y = X\beta + \varepsilon$, where $\varepsilon \sim N(0, \sigma^2 I)$. Show that
- (i) the least squares estimator of β is given by $\hat{\beta} = (X^T X)^{-1} X^T Y$.
- (ii) $\hat{\beta} \sim N(\beta, \sigma^2 (X^T X)^{-1})$.
- (c) An experiment was conducted to study the effect of 5 levels of concentration of a certain chemical on growth (in mm) of a certain type of plant. Five plants were randomly assigned to each level of concentration level and growth for each plant was measured. The following data were recorded:

UNIVERSITY OF ZAMBIA
DEPARTMENT OF MATHEMATICS AND STATISTICS

MAT3702 - DIFFERENTIAL GEOMETRY

END OF YEAR EXAMINATIONS

TIME ALLOWED: 3 HOUR

DATE: 2014

INSTRUCTIONS

There are Six (6) questions in this paper.

Attempt any Five (5) questions.

1. (a) (i) Let M be an n -dimensional manifold. Define what is meant by an atlas on M .
(ii) Prove that the function $\varphi : \mathbf{R} \rightarrow \mathbf{R}$, given by $\varphi(t) = t^3$ defines a C^∞ -differentiable structure on \mathbf{R} and show that this structure is different from the one defined by the atlas $\{(\mathbf{R}, id_{\mathbf{R}})\}$ where $id_{\mathbf{R}}$ is the identity map.
(b) Let $\sigma : \mathbf{R} \rightarrow \mathbf{R}^2$ be defined by $\sigma(t) = (2 \cos(t - \frac{\pi}{2}), \sin 2(t - \frac{\pi}{2}))$
(i) Determine whether σ is an immersion.
(ii) Show whether or not σ is injective.
2. (a) (i) Define what is meant by an n -dimensional topological manifold M .
(ii) Let M_1 and M_2 be differentiable manifolds of dimensions n_1 and n_2 respectively. Show that $M_1 \times M_2$ is a manifold of dimension $n_1 + n_2$
(b) Let $\sigma : \mathbf{R} \rightarrow \mathbf{R}^2$ be the curve defined by $x = \cos t$, $y = \sin t$, $t \in (0, \pi)$. A function $f : \mathbf{R}^2 \rightarrow \mathbf{R}$ is defined by $f(x, y) = 2x + y^3$.
(i) Find the vector \mathbf{v} tangent to σ at $t = \frac{\pi}{4}$.
(ii) Calculate $\mathbf{v}(f)$ at $t = \frac{\pi}{4}$.
3. (a) (i) Let X be a vector field on \mathbf{R}^3 , U an open subset of \mathbf{R}^3 . Define an integral curve of the vector field X passing through the point $p \in U$
(ii) Let $f(x, y, z) = y^2$, $g(x, y, z) = x^2 + yz^3$, be functions and $X = 5x \frac{\partial}{\partial x} + \frac{\partial}{\partial y} - y \frac{\partial}{\partial z}$ a vector field. Find $X(fg)$.
(b) Find the integral curve $\gamma(t)$ to the vector field X passing through the point (x_0, y_0) when $t = 0$ in each of the following:
(i) $X = 2 \frac{\partial}{\partial x} + y \frac{\partial}{\partial y}$
(ii) $X = e^{-x} \frac{\partial}{\partial x} - \frac{\partial}{\partial y}$

4. (a) (i) Let $\omega = f(x, y)dx + g(x, y)dy$ be a 1-form on \mathbf{R}^2 . Show that
- $$d\omega = \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right) dx \wedge dy.$$
- (ii) Let $\omega = ydx - xdy$ be a 1-form on \mathbf{R}^2 and let $\psi : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ be defined by $\psi(u, v) = (u^2 - v^2, uv)$. Find $d(\psi^*\omega)$.
- (b) Let $\omega = 2dx + 7dy - 3dz$ and $\phi = dx - 4dy + 3dz$ be two 1-forms on \mathbf{R}^3 and let $\mathbf{v}_1 = (2, 1, 3)$, $\mathbf{v}_2 = (1, 1, 2)$ be two vectors;
- (i) Find the value of $\omega \wedge \phi(\mathbf{v}_1, \mathbf{v}_2)$.
- (ii) Compute $\omega \wedge \phi$.
5. (a) (i) Let $\{U_\alpha\}_{\alpha=1,2,\dots,N}$ be a finite covering of a space M by coordinate patches U_α . Define what is meant by the partition of unity on M .
- (ii) Let $\omega = x^2(y + z^2)dx \wedge dy + z(x^3 + y)dy \wedge dz$ be a 2-form on \mathbf{R}^3 . Calculate $d\omega$
- (b) Let $\omega = xdx \wedge dy \wedge dz$ be a 3-form over the region V of \mathbf{R}^3 where V is the region in the **first octant** bounded by the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ and the planes $z = 0$ and $z = 2$. Find the value of the integral $\int_V \omega$.
6. (a) (i) State the Stokes Theorem.
- (ii) Let ω be a differential form on \mathbf{R}^3 defined by $\omega = ydx + zdy + xdz$. If $\psi : \mathbf{R}^2 \rightarrow \mathbf{R}^3$ is given by $\psi(u, v) = (\sin u \cos v, \sin u \sin v, \cos u)$. Find the pullback of ω , $\psi^*\omega$
- (b) Use the Stokes Theorem to evaluate the integral $\int_C x^3 dx + (\frac{1}{3}x^3 + xy^2)dy$ where C is the circle of radius 2, centered about the origin.

End of examination.

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
2013 ACADEMIC YEAR FINAL EXAMINATIONS
MAT 4600: MULTIVARIATE ANALYSIS

INSTRUCTIONS:

1. Answer any FIVE (5) questions.
2. Calculators are allowed.
3. You may use statistical tables provided if necessary.
4. Show all your work to earn full marks.

TIME: THREE (3) Hours

Q1 Let $\underline{X} = (X_1, X_2, \dots, X_p)'$ be distributed as a multi-variate normal $f(\underline{X}) = N_p(\underline{\mu}, \Sigma)$ where the density function has the form below:

$$f(\underline{X}) = \frac{1}{(2\pi)^{p/2} |\Sigma|^{1/2}} e^{-\frac{(\underline{X}-\underline{\mu})' \Sigma^{-1} (\underline{X}-\underline{\mu})}{2}} \quad \text{where } -\infty < X_j < \infty, j = 1, 2, \dots, p$$

(a) Let $d(\underline{X}, \underline{\mu}) = (\underline{X} - \underline{\mu})' \Sigma^{-1} (\underline{X} - \underline{\mu})$

(i) Explain why $d(\underline{X}, \underline{\mu})$ is referred to as a *statistical distance*.

(ii) What property should Σ possess for $d(\underline{X}, \underline{\mu})$ to qualify as a distance metric?

(b) Let $(\lambda_j, \underline{e}_j)$ be an eigenvalue - eigenvector pair for Σ , where $j = 1, 2, \dots, p$

(i) Derive the eigen value - eigen vector pair for Σ^{-1} and hence,

(ii) Show that $d(\underline{X}, \underline{\mu})$ can be expressed as $d(\underline{X}, \underline{\mu}) = \sum_{j=1}^p a_j y_j^2$ for appropriate a_j and y_j .

(c) Let $\Sigma = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & -\sqrt{2} \\ 0 & -\sqrt{2} & 4 \end{pmatrix}$

(i) Find the eigen values and eigen vectors of Σ and hence,

(ii) Verify that $|\Sigma| = \prod_{j=1}^3 \lambda_j$, where λ_j 's are eigen values.

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Q2 Suppose that Y_1, Y_2, \dots, Y_n is a random sample from the distribution $f(\underline{Y}) \sim N_3(\underline{\mu}, \Sigma)$.

Let $\bar{Y} = \frac{\sum_{j=1}^n Y_j}{n}$ be the sample mean vector and $S = \frac{1}{n-1} \sum_{j=1}^n (Y_j - \bar{Y})(Y_j - \bar{Y})'$, the sample covariance matrix.

- (a) (i) Obtain the mean and variance for \bar{Y}
- (ii) State the distribution of \bar{Y} in full, i.e., give name and parameters.

(b) Given that $E(Y_j - \underline{\mu})(\underline{\mu} - \bar{Y})' = E(\underline{\mu} - \bar{Y})(Y_j - \underline{\mu})' = -\frac{1}{n}\Sigma$

(i) Show that:

$$\sum_{j=1}^n (Y_j - \bar{Y})(Y_j - \bar{Y})' = \sum_{j=1}^n (Y_j - \underline{\mu})(Y_j - \underline{\mu})' + \sum_{j=1}^n (Y_j - \underline{\mu})(\underline{\mu} - \bar{Y})' + \sum_{j=1}^n (\underline{\mu} - \bar{Y})(Y_j - \underline{\mu})' + \sum_{j=1}^n (\underline{\mu} - \bar{Y})(\underline{\mu} - \bar{Y})'$$

and hence,

(ii) Show that $E(S) = \Sigma$

(c) Let $\underline{X} = A\underline{Y}$, where A is a matrix of constants.

- (i) Obtain the mean and variance for \underline{X}
- (ii) Determine the matrix A such that $\text{Var}(\underline{X}) = I_p$

Q3 A property developer asked three real estate agents to assess the value of five houses in a neighborhood. The results, in thousands of dollars, are given in the table.

House	Agent		
	A	B	C
1	21.0	21.8	22.6
2	19.2	19.0	19.8
3	18.3	18.7	18.5
4	22.7	22.3	23.7
5	24.2	24.0	23.7

- (a) In what sense could we regard these data as coming from a repeated measures design?
- (b) Let μ_A, μ_B, μ_C be the true average rating of agents A, B and C, respectively. The developer would like to know whether the average ratings for three agents are equal or not. State the null hypotheses in the form $H_0: C\underline{\mu} = \underline{0}$ and its alternative.
- (c) Carry out the test of H_0 in (b) at 5% level of significance using Hotelling's test.

Q4. A first year class taking a mathematics course for social scientists was divided into 20 tutorial groups for purposes of weekly problem assignments. Each tutorial group was assigned a tutor, some were new graduates and others were old. There was concern that there may be variation in performance due to a tutor factor. Below are the relevant summary statistics on three quizzes; Q1, Q2 and Q3. In the table TG = Tutorial group and n = size of tutorial group.

TG	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	TOTAL
n	38	39	37	33	26	29	32	38	35	42	39	34	41	33	36	37	38	39	40	40	726

Between-Subjects SSCP Matrix

		Q1	Q2	Q3	
TG	Q1	553.153	113.247	-40.360	} B Matrix
	Q2	113.247	452.308	-2.349	
	Q3	-40.360	-2.349	92.094	
Error	Q1	5738.073	2029.275	2437.898	} W Matrix
	Q2	2029.275	3816.468	1934.297	
	Q3	2437.898	1934.297	2825.214	

Let $Y_{ij} = \mu + \tau_i + \epsilon_{ij}$ be the model associated with these data, where $i = 1, 2, \dots, 20$ and $j = 1, 2, \dots, n_i$.

- State the assumptions often imposed on the model.
- State the appropriate null and alternative hypotheses in relation to the model.
- Carry out the test of the null hypothesis in (b) using Wilk's Lambda at $\alpha = 0.05$, assuming large sample size. You may find the table below useful.

Distribution of Wilk's Lambda $\Lambda = W / B + W $		
No. of variables (p)	No. of groups (g)	Sampling distribution of multivariate normal data
$p = 1$	$g \geq 2$	$\left(\frac{\sum n_i - g}{g - 1}\right) \left(\frac{1 - \Lambda}{\Lambda}\right) \sim F_{g-1, \sum n_i - g}$
$p = 2$	$g \geq 2$	$\left(\frac{\sum n_i - g - 1}{g - 1}\right) \left(\frac{1 - \sqrt{\Lambda}}{\sqrt{\Lambda}}\right) \sim F_{2(g-1), 2(\sum n_i - g - 1)}$
$p \geq 1$	$g = 2$	$\left(\frac{\sum n_i - p - 1}{p}\right) \left(\frac{1 - \Lambda}{\Lambda}\right) \sim F_p, \sum n_i - p - 1$
$p \geq 1$	$g = 3$	$\left(\frac{\sum n_i - p - 2}{p}\right) \left(\frac{1 - \sqrt{\Lambda}}{\sqrt{\Lambda}}\right) \sim F_{2p, 2(\sum n_i - p - 1)}$
If $\sum n_i = n$ is large		$-\left(n - 1 - \frac{(p + g)}{2}\right) \text{Ln} \left(\frac{ W }{ B + W }\right) \sim \chi^2_{p(g-1)}$

Q5 The simple linear regression model, assuming normal errors, can be represented in matrix form as shown below.

$\underline{y} = X\underline{\beta} + \underline{\epsilon}$, where $\underline{\epsilon} \sim N_p(\underline{\mu}, \Sigma)$ and $(y_1, x_1), (y_2, x_2), \dots, (y_n, x_n)$ are n pairs of observations in X and Y.

- (a) (i) State the assumptions imposed on the error terms of the model.
 (ii) State the values of p , $\underline{\mu}$ and Σ associated with the model if assumptions in (i) hold.

(b) Let $\hat{\beta}_0$ and $\hat{\beta}_1$ be estimates of β_0 and β_1 , respectively, for the simple linear regression model.

- (i) Write down expression for both $\hat{\beta}_0$ and $\hat{\beta}_1$
 (ii) Show that both $\hat{\beta}_0$ and $\hat{\beta}_1$ are linear combinations of the y_j 's and hence,
 (iii) Obtain the covariance of $\hat{\beta}_0$ and $\hat{\beta}_1$, i.e., obtain $\text{cov}(\hat{\beta}_0, \hat{\beta}_1)$.

(c) A certain head of household was following the price (in Kwacha) of a particular food item for the first seven months of the year. The data he recorded are shown below and he was interested in fitting a simple linear regression of price on time such as the model stated above.

Month (X)	1	2	3	4	5	6	7
Price (Y)	116.6	117.1	117.8	118.9	119.5	120.3	120.6

- (i) Obtain the estimates $\hat{\beta}_0$ and $\hat{\beta}_1$ of β_0 and β_1 , respectively.
 (ii) How much should he expect to pay in month 8?
 (iii) If in month 8 the cost of the item is K120.8 what is the error incurred by the fitted model?

Q6. Consider the problem of modeling m responses Y_1, Y_2, \dots, Y_m on a single set of predictor variables X_1, X_2, \dots, X_m according to the models below.

$$Y_1 = \beta_{01} + \beta_{11}X_1 + \dots + \beta_{r1}X_r + \epsilon_1$$

$$Y_2 = \beta_{02} + \beta_{12}X_1 + \dots + \beta_{r2}X_r + \epsilon_2$$

...

$$Y_m = \beta_{0m} + \beta_{1m}X_1 + \dots + \beta_{rm}X_r + \epsilon_m$$

Assume that for each model there are n observations so that the model may be expressed as:

$$Y = X\beta + \epsilon, \quad \text{where}$$

Y is an $n \times m$ matrix

X is an $n \times (r + 1)$ design matrix

β is an $(r + 1) \times m$ matrix of parameters

ϵ is an $n \times m$ matrix of error terms.

Define the derivative of $f(\beta)$ with respect to β (an $n \times p$ matrix) as the matrix

$$\frac{\partial f(\beta)}{\partial \beta} = \left(\frac{\partial f(\beta)}{\partial \beta_{ij}} \right)$$

from which the following results follow:

$$(I) \quad \frac{\partial a' \beta}{\partial \beta} = a$$

$$(II) \quad \frac{\partial \beta' \beta}{\partial \beta} = 2\beta$$

$$(III) \quad \frac{\partial \beta' A \beta}{\partial \beta} = (A + A')\beta$$

$$(IV) \quad \frac{\partial \beta' A y}{\partial \beta} = A y$$

(a) Obtain the estimate of the matrix β in the model by solving the matrix equation $\frac{\partial S(\beta)}{\partial \beta} = 0$ where $S(\beta) = (Y - X\beta)'(Y - X\beta)$.

(b) Show that the estimate matrix $\hat{\beta}$ obtain in (a) is an unbiased estimate of β .

(c) Given the data below for a multivariate linear model with two responses Y_1 and Y_2 whose individual models are shown below.

$$Y_{j1} = \beta_{01} + \beta_{11}X_{j1} + \epsilon_{j1}$$

$$Y_{j2} = \beta_{02} + \beta_{12}X_{j1} + \epsilon_{j2} \quad j = 1, 2, \dots, 5.$$

x_1	0	1	2	3	4
y_1	1	4	3	8	9
y_2	-1	-1	2	3	4

(i) Obtain the estimate, $\hat{\beta}$, of the parameter matrix β

(ii) Obtain the estimate of the fitted matrix \hat{Y}

(iii) Obtain the estimate of the matrix of errors $Y - \hat{Y}$

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
2013/2014 ACADEMIC YEAR FINAL EXAMINATION
MSE 3060: CHEMISTRY TEACHING METHODS

Time: Three (3) hours

Marks: 100

Information

1. There are seven (7) questions in this paper
2. The number of marks is given in brackets [] at the end of each question or part question
3. This question paper has **three** (3) pages only

Instructions

1. Answer **ALL** questions from section A [80 marks]
 2. Choose **TWO** questions from section B [20 marks]
-

Section A: Attempt ALL Questions in this section

Question 1

- a) Write one behavioural objective in organic chemistry (sub-topic esterification) and underline the three components. [4]

 - b) What are the main characteristics of:
 - i. A lesson plan? [4]
 - ii. Schemes of work? [2]

 - c)
 - i. List four (4) ways you can use to sequence chemistry topics. [4]
 - ii. What is the importance of sequencing instructions? [2]

 - d) There are a variety of teaching strategies that teachers can use to improve student learning. One such method is the whole class strategy.
 - i. What is 'whole class teaching'? [1]
 - ii. Give one example of a whole-class teaching strategy. [1]
 - iii. Give one (1) advantage and one (1) disadvantage of this strategy. [2]
- [20]

Question 2

- a) Distinguish between a syllabus and a curriculum. [4]
- b) Distinguish between Criterion – referenced and Norm – referenced tests. [4]
- c) There are several self-regulatory processes that students instigate, modify, and sustain. Explain the processes involved in self-regulated learning. [4]
- d) Give a learning-teaching situation in which the following are applied:
 - i. Deductive reasoning [4]
 - ii. Inductive reasoning [4]

[20]

Question 3

- a) List any four (4) steps necessary for conducting successful micro teaching. [4]
- b) Discuss the advantages of improvised chemistry apparatuses in the absence of standard laboratory (commercially available) apparatuses. [4]
- c) State four (4) ways in which modern technology, such as computers, is useful for teachers and pupils. [4]
- d) List any four (4) general rules of storing laboratory chemicals. [4]
- e) Outline the first aid steps you would take in the event of burns arising from skin contact with sulphuric acid. [4]

[20]

Question 4

- a) Explain four (4) of the strategies you can employ to instil discipline in a chemistry class. [4]
- b) List four (4) characteristics of an effective teacher. [4]
- c) Why do some teachers in general and chemistry teachers in particular struggle with classroom management? [4]
- d) List four (4) categories of disciplinary interventions that can be used to provide a balance of positive and negative consequences. [4]
- e) One of the effects generated by the Cognitive Load Theory (CLT) is the goal-free effect. Describe a classroom situation in which you can apply the goal-free effect to effectively teach a given chemistry lesson. [4]

[20]

Section B: Answer TWO (2) questions in this section

Question 1

Constructivism is a theory which explains the nature of knowledge and learning.

- Compare and contrast between a traditional chemistry classroom and a constructivist chemistry classroom. [5]
- What aspects of the constructivist and traditional lessons can you combine to effectively come up with a lesson which will inspire learners to appreciate science beyond the classroom? [5]

[10]

Question 2

Substance X oxidizes potassium iodide to iodine. The rate of this reaction was studied as follows. To each test tube were added: 20 ml (excess) aqueous potassium iodide, 3 ml aqueous sodium thiosulphate, and 1 ml of starch. A 20 ml aliquot of solution X was then added to one tube and the time t for the blue colour to appear was observed (the potassium iodide is oxidised to iodine which then reacts with sodium thiosulphate, when all of the thiosulphate has been used up the iodine then formed gives the usual blue colour with starch). The experiment was repeated with 20 ml of the solution X of various concentrations. The results were:

Table 1: Results of the experiment

Concentration of X (M)	0.1	0.3	0.4	0.6	0.7
Time t for colour to appear (s)	120	40	30	20	15

- Write two (2) behavioural objectives relevant to this practical activity. [2]
- Explain why the rate of reaction is proportional to $1/t$. [2]
- Plot $1/t$ (vertical axis) against concentration of X. [2]
- Deduce from the graph how the rate of reaction varies with concentration of X. [2]
- What is the practical application of this experiment? [2]

[10]

Question 3

Most students know the formula for water to be H_2O . However, very few learners are able to cite any evidence for why they believe this to be the case.

- How can you reduce the gap between the students' words that the formula of water is H_2O and how they perceive matter at the microscopic level? [2]
- Describe an experiment you would conduct to reduce the misconception. [8]

[10]

END OF EXAMINATION



The University of Zambia
School of Natural Sciences
Department of Physics
2013/2014 Academic Year
Final Examinations
PHY1010: Introductory Physics

Question 1 is compulsory. Attempt only four more questions.

Clearly indicate on the answer script which questions you have attempted.

All questions carry equal marks. The marks are shown in brackets.

Show your working clearly. Omission of essential work will lead to loss of marks.

Time: Three Hours.

Maximum Marks: 100.

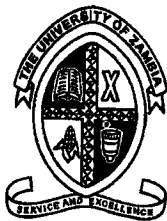
Write only your computer number clearly on the answer book as well as on the answer sheet for Question 1. Tie them together. Do not write your name.

Wherever necessary use:

$$G = 6.67 \times 10^{-11} \text{ N.m}^2/\text{kg}^2 : g = 9.8 \text{ m/s}^2 : P_A = 1.013 \times 10^5 \text{ N/m}^2 : \rho_{\text{water}} = 1000 \text{ kg/m}^3 :$$

$$\text{specific heat capacity of water} = 4200 \text{ J/kg.K} : 1 \text{ pascal} = 1 \text{ N/m}^2 : 1 \text{ cal.} = 4.2 \text{ J} :$$

$$R = 8314 \text{ J/kmol.K} :$$



**The University of Zambia
Physics Department
University Examinations 2013/2014
PHY1010 Introductory Physics**

Answer sheet for Question 1

Computer Number only

**Q1. Put a cross (x) or tick mark (√) in the appropriate box.
If it is on the dividing line, it will not be counted.**

	a	b	c	d
A				
B				
C				
D				
E				
F				
G				
H				
I				
J				

**Do NOT write here.
For official use only:**

	Number of parts N	Factor f	Marks f × N
Correct		2	
Wrong		-(0.67)	
Net Marks:			

Attach this sheet firmly with the main answer book. If you lose this sheet, you will lose the marks for Question 1!!

Question 1: Sample answers: F(a), G(d).... etc. For each correct answer, 2 marks.

For each wrong answer, 0.67 will be deducted. No answer, zero mark.

Minimum total mark for Question 1 is zero. **It is worth trying!!** [10 × 2 = 20]

(A) A body will gain in G.P.E. if it:

- (a) moves up
- (b) moves down
- (c) moves horizontally
- (d) does not move.

(B) A shell fired from a cannon explodes in mid-air. Then:

- (a) its total momentum increases
- (b) its total momentum decreases
- (c) its kinetic energy increases
- (d) its momentum and kinetic energy remain unchanged.

(C) All rotating objects at sea level that have the same mass and angular velocity also have the same:

- (a) moment of inertia
- (b) angular momentum
- (c) gravitational potential energy
- (d) kinetic energy.

(D) The center of gravity of an object:

- (a) is sometimes arbitrary
- (b) may be outside the object
- (c) is always at its geometrical center
- (d) is always at the interior of the object.

(E) Good absorbers of heat are also:

- (a) poor emitters (b) non-emitters
- (c) good emitters (d) highly polished.

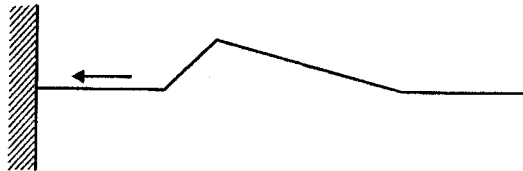
(F) Air in a cylinder is suddenly compressed by a piston, which is then maintained at the same position. With the passage of time:

- (a) the pressure decreases
- (b) the pressure increases
- (c) the pressure may remain constant
- (d) the pressure may increase or decrease, depending on the nature of the gas.

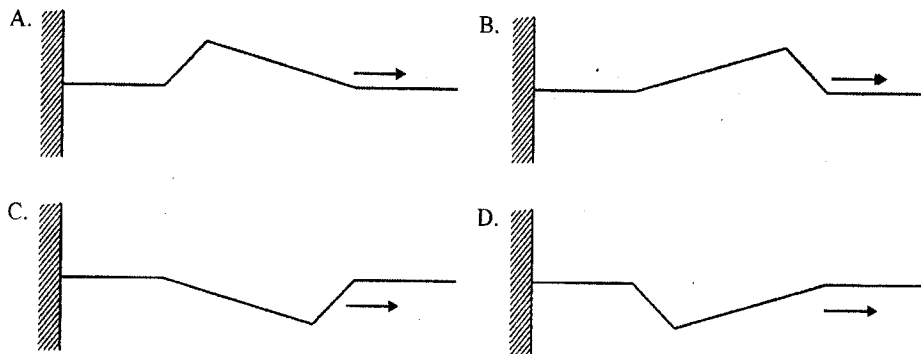
(G) Two waves are given by the equations $y = a \sin(\omega t - kx)$ and $y = a \cos(\omega t - kx)$. The phase difference between the two waves is:

- (a) π
- (b) $\pi/4$
- (c) $\pi/2$
- (d) zero.

(H) A pulse is sent down a string fixed at one end.



Which **one** of the following diagrams best represents the reflected pulse?



(I) Which of the following statements is false?

- (a) change in air temperature has no effect on the speed of sound,
- (b) change in air pressure has no effect on the speed of sound,
- (c) speed of sound in water is higher than that in air
- (d) two persons on moon cannot hear the sound of each other.

(J) Doppler's effect is applicable for:

- (a) light waves only
- (b) sound waves only
- (c) both light and sound waves
- (d) neither light nor sound waves.

Attempt any four questions only from below. If you attempt more than that, we may choose the first four attempted.

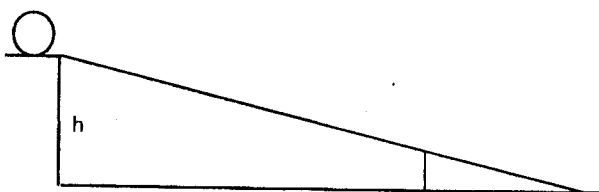
Q2(a). A vector **A** of magnitude 6N has a direction 30° anti-clockwise from the +y-direction. Another vector **B** magnitude 15 N has a direction 180° clockwise from the +x-direction. A third vector **C** magnitude 9 N has a direction 45° anti-clockwise from the +x-direction. They all start out from the origin of the axes. Find **A - B + C**. [7]

(b) A police siren appears to change pitch from 850 Hz to 770 Hz as it rushes past a person standing by the road side. How fast is the police car moving past the person?

The speed of sound in air is 343 ms^{-1} .

[7]

(c) A uniform disk and a uniform sphere of the same mass and radius roll down an inclined plane from the same point. Find the ratio of the disk's speed to that of the sphere at the bottom of the incline. [For the sphere, $I = (2/5) MR^2$ and for the disk $I = (1/2) MR^2$] [6]



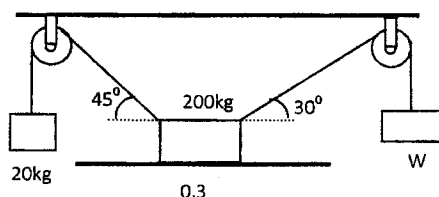
Q3(a). If a freely falling body travels half its total path in the last second, calculate the time and height of its fall. [8]

(b) An iron bar 400 cm long is clamped 100 cm from one end. Find the lowest resonance frequency for longitudinal waves in the bar. Draw the figure.

(Given, velocity of the wave in the bar = 4000 m/s)

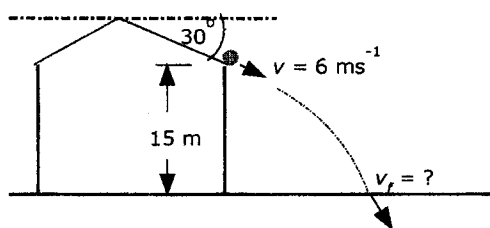
[5]

(c) Two weights are hung over two frictionless pulleys as shown in the figure below. What weight W will cause the 200kg block to just start moving to the right. Given, coefficient of static friction between the table and the block is 0.3 [7]



Q4(a). A sloping roof of a building makes an angle of 30° with respect to the horizontal and the edge of the roof is 15 m above the ground. A ball rolling from the top of the roof attains a velocity of 6 ms^{-1} as it leaves the edge of the roof. See figure below.

- i) How long does it take for the ball to hit the ground?
 - ii) How far horizontally from the bottom of the building does the ball strike the ground?
- [7]**



(b) A glass rod and a steel rod are of equal length at 0°C . At 100°C they differ in length by exactly 0.001m. What were the lengths at 0°C ?

[7]

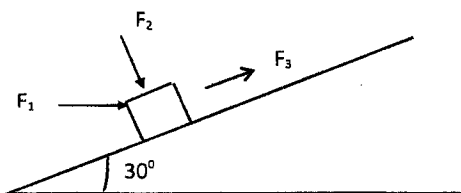
Given $\alpha_{\text{glass}} = 0.000008/^\circ\text{C}$ and $\alpha_{\text{steel}} = 0.000012/^\circ\text{C}$.

(c) Find the maximum load in kg which may be hung from a steel wire of diameter 1mm if the permitted strain may not exceed $1/1000$. Young's modulus for steel is $2.00 \times 10^{11} \text{ N/m}^2$.

[6]

Q5(a). A block moves up a 30° incline under the action of certain forces, three of which are shown in the figure. F_1 is a horizontal force of magnitude 40N. F_2 is normal to the plane having a magnitude of 20N, while F_3 is parallel to the plane of magnitude 30N.

Determine the work done by each force as the block moves 80cm up the incline. **[6]**



(b) A swimmer has 0.820 liter of dry air in his lungs when he dives into a lake. Assuming the pressure of the dry air inside his lungs is 95% of the external pressure at all times, what is the volume of the dry air in his lungs at a depth of 10.0 m?

Assume that atmospheric pressure at the surface is 1.013×10^5 Pa.

[7]

(c) Two guitar strings are tuned to the same frequency of 294 Hz. The tension in one string is then decreased by 2%. What will be the beat frequency when the two strings are played together?

[7]

Q6(a). A billiard ball moving at 4 m/s strikes another identical billiard ball at rest and moves off at 3.46 m/s in a direction 30° from the original direction. Find the velocity of the target ball.

[7]

(b) A small balloon partly filled with helium has a radius of 18.0cm (spherical shape). Internal pressure is 1.05 atm at 20°C . Find the number of moles of helium in the balloon, and the mass of the helium. ($M = 4$ for He_2)

[7]

(c) As shown in the figure below, consider the following two-step process. Heat is allowed to flow out of an ideal gas at constant volume so that its pressure drops from 2.2 atm to 1.4 atm. Then the gas expands at constant pressure, from a volume of 6.8 litres to 9.3 litres, where the temperature reaches its original value.

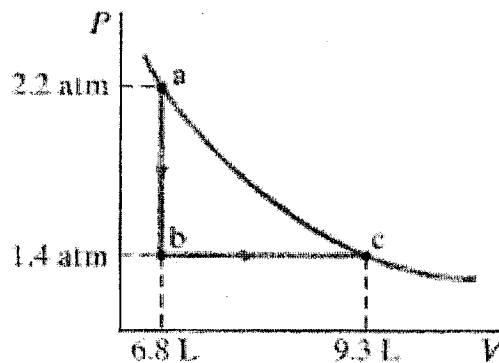
Calculate

(i) the total work done by the gas in the process,

(ii) the change in internal energy of the gas in the process, and

(iii) the total flow of heat into or out of the gas.

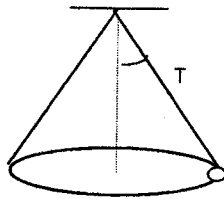
[6]



Q7(a). As shown in the figure below,

- (i) calculate the force necessary to keep a mass of 0.8kg revolving in a horizontal circle of radius 0.7m with a period of 0.5s,
- (ii) calculate the direction of this force,
- (iii) calculate also the tension in the string.

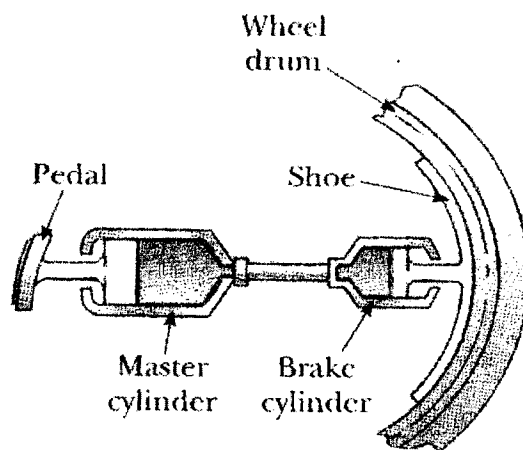
[7]



(b) Figure below shows the essential parts of a hydraulic brake system. The area of the piston in the master cylinder is 6.4cm^2 , and that of the piston in the brake cylinder is 1.8cm^2 . The incompressible brake fluid is pumped from the Master cylinder to the Brake cylinder when the brake is applied. The coefficient of friction between shoe and wheel drum is 0.5.

If the wheel has a radius of 34cm, determine the frictional torque about the axle when a force of 44N is exerted on the brake pedal.

[6]



(c) The efficiency of a reversible engine working between two temperatures is 0.5. If on increasing the temperature of the cold reservoir by $100\text{ }^\circ\text{C}$, the efficiency becomes 0.333, find the temperatures of the source and the cold reservoir.

[7]

Q8(a). A glass contains 0.25 kg of water initially at 25°C . How much ice, initially at -20°C , must you add to obtain a final temperature of 0°C with all the ice melted?

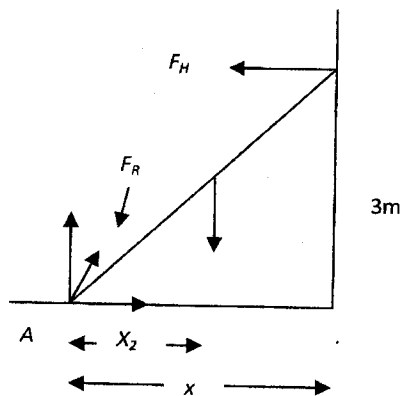
Neglect the heat capacity of the glass.

Given, $c_w = 4190\text{J/kg}\cdot\text{K}$, $c_i = 2100\text{J/kg}\cdot\text{K}$, $L_f = 3.34 \times 10^5\text{J/kg}$

[10]

(b) As shown in the figure below, a uniform ladder 5m long is leaning against a frictionless wall with its upper end 3m above the ground level. If the ladder weighs 150N, what forces does it exert on the wall (F_H) and on the ground (F_R)?

[10]



==End of PHY1010 Final Exam 2014==

Some Equations

Uniformly accelerated motion:

$$x = \bar{v}t \quad \bar{v} = \frac{1}{2}(v_f + v_i) \quad v_f = v_i + at \quad v_f^2 = v_i^2 + 2ax \quad x = v_i t + \frac{1}{2}at^2$$

Projectile motion:

$$v_x = v_i \cos \theta_i = \text{constant} \quad v_y = v_i \sin \theta_i - gt \quad y = (v_i \sin \theta_i)t - \frac{1}{2}gt^2$$

$$y = (\tan \theta_i)x - \left[\frac{g}{2v_i^2 (\cos^2 \theta_i)} \right] x^2 \quad R = \frac{v_i^2}{g} \sin 2\theta \quad t = \frac{2v_i \sin \theta}{g}$$

Force and motion:

$$F = ma \quad w = mg \quad F_{AB} = -F_{BA} \quad F_f = \mu F_N$$

Energy:

$$PE = wh = mgh \quad KE = \frac{1}{2}mv^2 \quad W = Fx \cos \theta \quad P = \frac{W}{t} = Fv \cos \theta$$

Linear momentum:

$$p = mv \quad F\Delta t = \Delta mv$$

Circular motion and gravitation:

$$T = \frac{2\pi r}{v} \quad a_c = \frac{v^2}{r} \quad F_c = \frac{mv^2}{r} \quad F_{grav} = G \frac{m_A m_B}{r^2} \quad 1 \text{ rev} = 360^\circ = 2\pi \text{ rad}$$

Rotational motion and angular momentum:

$$\theta = \frac{s}{r} = \left(\frac{\omega_i + \omega_f}{2} \right) t \quad \omega = \frac{\theta}{t} \quad \theta = \omega_i t + \frac{1}{2}\alpha t^2 \quad \omega_f = \omega_i + \alpha t$$

$$v = \omega r \quad \omega_f^2 = \omega_i^2 + 2\alpha\theta \quad \alpha = \frac{\Delta\omega}{\Delta t} = \frac{a_T}{r} \quad I = \sum mr^2 \quad I = mk^2$$

$$KE_{rot} = \frac{1}{2}I\omega^2 \quad \tau = FL = I\alpha \quad W = \tau\theta \quad P = \tau\omega \quad L = I\omega$$

Properties of matter:

$$\rho = \frac{m}{V} \quad F = -kx \quad \frac{\Delta L}{L_i} = \frac{1}{Y} \frac{F}{A} \quad \phi = \frac{s}{d} = \frac{1}{s} \frac{F}{A} \quad B = \frac{\Delta P}{\Delta V / V_0} \quad W_{app} = W \left(1 - \frac{\rho_{fluid}}{\rho} \right)$$

Thermal Properties of matter:

$$PV = nRT : \Delta Q = mc\Delta T = nC\Delta T : \Delta L = \alpha L\Delta T : L_t = L_0(1 + \alpha t) : \Delta V = \gamma V\Delta T : \Delta W = P\Delta V$$

$$(\Delta Q / \Delta t) = (kA\Delta T) / \Delta L \quad \Delta Q = \Delta U + \Delta W \quad \text{volume of a sphere} = (4/3)\pi r^3$$

Thermodynamics:

$$\Delta Q = \Delta U + \Delta W : P_1V_1^\gamma = P_2V_2^\gamma \quad PV = nRT \quad W = p\Delta V \quad COP = \frac{Q_c}{W} \quad COP = \frac{Q_H}{W}$$

$$e = 1 - \frac{T_c}{T_h}$$

Waves and Sound:

$$f = \frac{1}{\tau} \quad v = \pm \sqrt{\frac{k}{m}(x_0^2 - x^2)} \quad f' = f \frac{v \pm v_L}{v \mp v_S} \quad (\text{dB}) = 10 \cdot \log \frac{I}{I_0} \quad v = \sqrt{\frac{T}{m/L}}$$

$$\tau = \frac{1}{f} = 2\pi \sqrt{\frac{m}{k}} \quad a = -\left(\frac{k}{m}\right)x \quad v = \sqrt{\frac{Y}{\rho}} \quad v = \sqrt{\frac{B}{\rho}}$$



THE UNIVERSITY OF ZAMBIA
DEPARTMENT OF PHYSICS
SCHOOL OF NATURAL SCIENCES
2013/14 ACADEMIC YEAR

END OF YEAR UNIVERSITY EXAMINATIONS

PHY-2522: ANALYTICAL MECHANICS AND SPECIAL RELATIVITY

Time allowed: 3 Hours

Instructions

- This examination paper contains 7 questions. Each question carries 20 marks. Attempt any 5 questions out of the 7 questions given.
- This paper has a total of 100 marks. All questions carry equal marks
- Show all your work clearly. Omission of essential work will result in loss of marks
- Write your computer number clearly on the answer sheet

Where necessary, you may use the following:

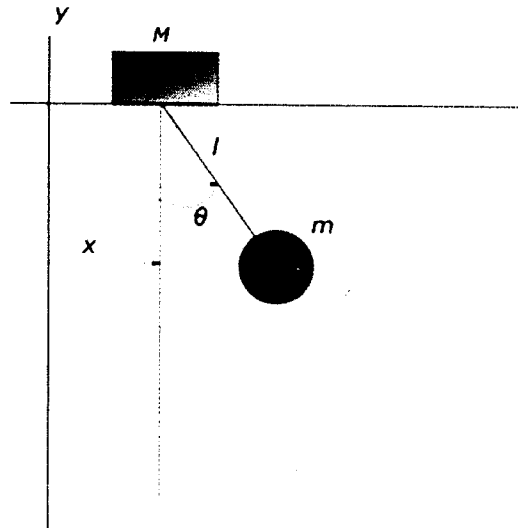
$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_k} \right) - \frac{\partial L}{\partial q_k} = Q_k, \quad \dot{q}_k = \frac{\partial H}{\partial p_k}, \quad \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}, \quad -\dot{p}_k = \frac{\partial H}{\partial q_k}, \quad x' = \gamma(x - vt),$$

$$t' = \gamma \left(t - \frac{Vx}{c^2} \right), \quad y' = y, \quad z' = z, \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad \text{speed of light } c = 3 \times 10^8 \text{ m/s}$$

$$\frac{\partial^2 u(x,t)}{\partial t^2} = \frac{T}{\mu} \frac{\partial^2 u(x,t)}{\partial x^2}, \quad \gamma L = L_0, \quad I_{\text{sphere}} = \frac{2}{5} Ma^2$$

$$A_n = \frac{2}{L} \int_0^L u_0(x) \sin\left(\frac{n\pi x}{L}\right) dx, \quad B_n = \int_0^L \dot{u}_0(x) \sin\left(\frac{n\pi x}{L}\right) dx,$$

- Q1** Consider a pendulum of mass m and length ℓ , which is attached to a support with mass M , which can move along a line in the x -direction as shown in the figure below. Let x be the coordinate along the line of the support, and the position of the pendulum is denoted by the angle θ from the vertical.



(a) How many degrees of freedom does the system have? **[1 mark]**

(b) Show that the Lagrangian of the system is

$$L = \frac{1}{2}(M + m)\dot{x}^2 + m\dot{x}l\dot{\theta}\cos\theta + \frac{1}{2}ml^2\dot{\theta}^2 + mgl\cos\theta \quad \text{[9 marks]}$$

(c) Identify the ignorable coordinate in the system and state its physical significance. **[2 marks]**

(d) Show that the equations of motion arising from the Lagrangian formalism are:

$$ml^2\ddot{\theta} + ml(\ddot{x}\cos\theta + g\sin\theta) = 0 \quad \text{and} \quad \text{[4marks]}$$

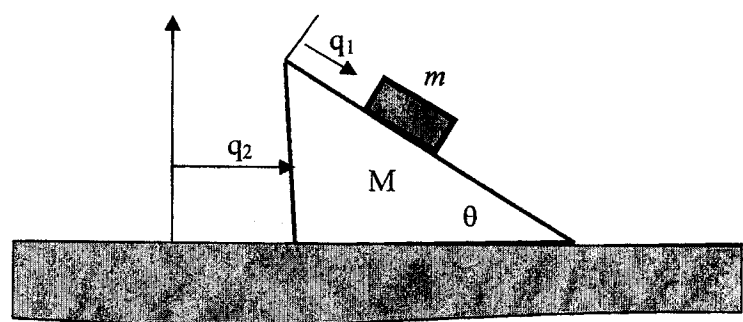
$$(M + m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = 0 \quad \text{[4 marks]}$$

- Q2(a)** Show that the general solution of a damped harmonic oscillator for which $\gamma = \omega_0/4$ acted on by a driving force $F = F_0 \cos \omega t$ is given by

$$x(t) = e^{-\omega_0 t/4} (A_1 \cos 0.97 \omega_0 t + A_2 \sin 0.97 \omega_0 t) + B_1 \cos \omega t + B_2 \sin \omega t \quad [18 \text{ marks}]$$

- (b) Explain briefly how you would obtain the constants A_1 and A_2 . [2 marks]

- Q3(a)** A block of mass m is held motionless on a frictionless plane of mass M and angle of inclination θ (see Fig. below). The plane rests on a frictionless horizontal surface. The block is released. Using the Euler-Lagrange equations, find the expression for the horizontal acceleration of the plane? [Hint: use the generalized coordinates shown in the figure]



[16 marks]

- (b) Show that the resulting two equations in (a) above lead to the conservation of momentum in the x - direction, that is

$$(M\dot{q}_2 + m(\dot{q}_2 + \dot{q}_1 \cos \theta)) = \text{constant} \quad [4 \text{ marks}]$$

- Q4(a)** Use the Hamiltonian formalism to obtain the equation of motion for a projectile of mass m which is launched near the surface of the earth at an angle of 30° to the horizontal with a velocity of 50 ms^{-1} [6 marks]

- (b) Solve the equation of motion and find the conserved quantity. [6 marks]

- (c) Deduce the maximum height the projectile attains. [8 marks]

- Q5** A string of length L and linear mass density μ is fixed tight at both ends. The string is set into vibration under the following initial conditions:

$$u(x, t) = u_0(x), \quad \left(\frac{\partial u}{\partial t} \right)_{t=0} = \dot{u}_0(x)$$

Show that in general, the resulting normal modes of vibration are given by

$$u(x, t) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{L}\right) \cos\left(\frac{n\pi v}{L}t\right) + B_n \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{n\pi v}{L}t\right)$$

where A_n and B_n are constants for the n^{th} mode of vibration **[20 marks]**

- Q6** The trajectory of a particle moving parallel to O_x with a uniform velocity v in the stationary inertial reference frame S is given by

$$x = x_0 + vt$$

- (a) Show that in the reference frame S' in standard configuration with S and moving with a velocity V relative to S , the trajectory of the particle is given by

$$x' = \frac{x_0}{\gamma \left(1 - \frac{vV}{c^2}\right)} + \frac{(v - V) t'}{\left(1 - \frac{vV}{c^2}\right)} \quad \text{[4 marks]}$$

- (b) Show that the equation describing a spherical electromagnetic wave propagating with constant speed c is not invariant under Galilean transformations. **[5 marks]**
- (c) The relativistic expression for the kinetic energy of a moving particle is given by

$$T = mc^2 - m_0c^2$$

Show that for velocities v much lower than the speed of light c , this expression reduces to the Newtonian one, that is

$$T = \frac{1}{2} m_0 v^2 \quad \text{[5 marks]}$$

- (d) Starting from the equation of motion for the one dimensional damped harmonic oscillator of mass m given by

$$m\ddot{X} + C\dot{X} + kX = 0$$

- (i) Show that the characteristic equation is

$$m\alpha^2 + \alpha C + k = 0$$

if a solution of the form $X = e^{\alpha t}$ is assumed.

[4 marks]

- (ii) Find the two values of α in terms of m , C and k .

[2 marks]

- Q7 (a) Obtain Hamilton's equations for a particle of mass m moving in a plane with potential energy function $V(r)$.

[8 marks]

- (b) Relative to the inertial frame S two events are observed to occur simultaneously on the x -axis. Relative to the inertial frame S' (in standard configuration with S), the two events occur with a time interval of 0.25s, a distance 2.0×10^8 m apart. Calculate the speed of S' relative to S .

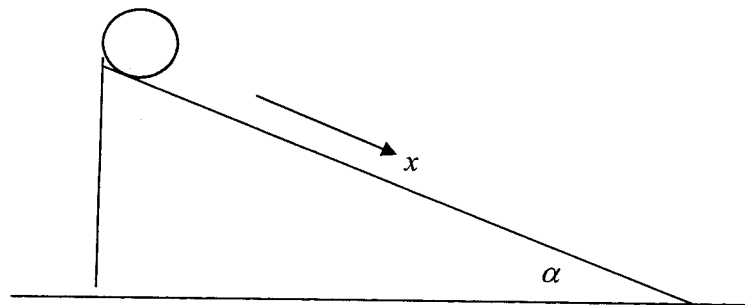
[6 marks]

- (c) A solid sphere of mass M and radius a is rolling without slipping down the incline as shown in the figure below. Using Lagrange's method, show that its acceleration down the plane is given by

$$\ddot{x} = \frac{5}{7} g \sin \alpha$$

where g is the acceleration due to gravity.

[6 marks]



*****END OF PHY 2522 EXAMINATION*****



The University of Zambia
School of Natural Sciences
Department of Physics
2013/2014 Academic Year
End of Year University Examination
Computational Physics I-PHY 3032

Instructions

Total Marks 100

- **Time Allocation:** Three (3) Hours.
 - **All questions carry equal marks.**
 - Marks for each question are shown in the square brackets [].
 - Whenever necessary, use the information given in the **appendix**
 - **Answer:**
 - i) Question one (1) and
 - ii) Any three (3) questions from 2, 3, 4, 5 and 6.
-

Q.1 (a) In the biased floating-point representation, if 8 bits are used for the exponent, typical of a 32-bit processor:

- i) what will be the biased range for the values of the exponent?
- ii) what is the biased binary exponent for -99?

[5]

(b) What will be the output of the following program

```
PROGRAM output
IMPLICIT NONE
INTEGER :: i,j
DO j=0,12,2
  DO i=1,4
    IF((j>5).AND.(MOD(i,2)==0))PRINT*,j*i
  END DO
END DO
STOP
END PROGRAM output
```

[8]

(c) A 1 liter volume of 4 moles of methane gas expands to 5 liters at a constant temperature of 25°C. Using the Trapezoid integration method and a function subprogram, approximate the work, in joules, done by the gas on the environment. Assume that the gas follows the Van der Waals equation with $a = 0.23 \text{ Pa m}^6/\text{mol}^2$ and $b = 4.3 \times 10^{-4} \text{ m}^3/\text{mol}$. Note that

$$W = \int_{v_1}^{v_2} p dv$$

where

$$p = \left(\frac{nRT}{v - nb} \right) - \left(\frac{n^2 a}{v^2} \right)$$

Take $R = 8.314 \text{ J/mol K}$ and number of nodes $N = 5$.

[12]

Q.2 (a) Given the following matrix,

$$\mathbf{X} = \begin{pmatrix} 2 & 3 & 6 & 10 & 24 \\ 4 & 8 & 12 & 16 & 20 \\ 6 & 9 & 12 & 15 & 18 \\ 5 & 7 & 9 & 11 & 13 \\ 18 & 12 & 8 & 4 & 1 \end{pmatrix}.$$

Write a program that will initialize \mathbf{X} and find the sum of its main diagonal elements.

[5]

- (b) A physics professor developed a theory that describes the efficiency of a revolutionary semiconductor laser his lab has invented. Once the current through his device is raised above some threshold, his theory predicts that every milliamp increase in current will increase the output laser power by about a microwatt. In mathematical terms this is expressed as,

$$P = aI - I_0$$

To test his theory, the professor and his students construct one of the new lasers and took some data. Examining the data, the professor concludes that the threshold current must be about 0.3 mA. Using the data from 0.4 mA and up, given below, the professor obtains the fit that gives a large value for I_0 .

$I(\text{mA})$	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
$P(\mu\text{W})$	0.02	0.06	0.12	0.2	0.3	0.52	0.56	0.82	0.78	1.2	1.5	1.7	2.0	2.3	2.9	2.9

After reexamining the original theories, the professor's protege found an aspect of their gain/loss equation that had not been accounted for correctly and he develops the correct power versus current formula:

$$P = a(I - I_0)^2$$

- i) what is the value for a and I_0 ?
- ii) Write a program that finds and prints, to the standard output, the value of a and I_0 .

[20]

- Q.3** (a) Show that the number 2.718 cannot be stored exactly in the memory of a processor that has a total of 8 bits for real number storage with 4 bits reserved for the integer and fraction parts. What decimal value will be stored instead?

[8]

- (b) Suppose that a projectile is fired from the origin with an angle of elevation θ and initial velocity u . In elementary physics, air resistance is neglected in the mathematical models for the height $y = f(t)$ and distance traveled $x = r(t)$ and tend to give too high an altitude and too long a range for the projectile's path. If air resistance is taken into account with the assumption that it is proportional to the velocity, the equations of motion become

$$y = f(t) = (Cu_y + 32C^2) \left(1 - e^{-t/C}\right) - 32Ct, \quad \text{and} \quad x = r(t) = Cu_x \left(1 - e^{-t/C}\right)$$

where $C = m/k$ and k is the coefficient of air resistance and m is the mass of the projectile. u_x and u_y are the horizontal and vertical components of the initial velocity respectively.

Suppose that a projectile is fired with an angle of elevation $\theta = 45^\circ$, $u = 70 \text{ ms}^{-1}$, and $C = 10$. Write a program that uses the Newton-Raphson method to find the elapsed time (t) until impact and the range (x). What are the approximate values of t and x after two iterations? *Hint: $f(2.0) > 0$ and $f(3.0) < 0$.*

[17]

- Q.4 (a) The following Fortran 90 program intends to print the time t and position x of a 8 kg mass, for a period of 8 minutes at 0.4 minutes intervals, on a frictionless horizontal surface attached to a spring of force constant $\kappa = 5$ N/m pulled aside a distance of 0.2 m and let go. The position x is given by

$$x = A \cos \omega t \quad \text{where} \quad \omega = \sqrt{\kappa/m}, \quad A = 0.2 \text{ m} \quad \text{and} \quad m = 8 \text{ kg.}$$

```

PROGRAM shm
IMPLICIT NONE
INTEGER, PARAMETER :: m=8, k=5, p=8
REAL, PARAMETER :: A=0.2, dt=0.4
REAL :: x,t,omega
omega=k/m
omega=SQRT(omega)
WRITE(*,FMT='('' Time'',8X,'Position'')')
t=0
DO
x=A*COS(omega*t)
WRITE(*,FMT='(F6.2,6X,F6.2)')t,x
t=t+dt
IF(t>p)EXIT
END DO
STOP
END PROGRAM shm

```

The program compiles without errors but produces undesirable results because of two bugs. Identify the bugs and rewrite the program correctly.

[10]

- (b) In electrical field theory, it is proved that the magnetic field induced by a current flowing in a circular loop of wire has intensity

$$H(x) = \frac{4Ir}{r^2 - x^2} \int_0^{\pi/2} \left[1 - \left(\frac{x}{r} \right)^2 \sin^2 \theta \right]^{1/2} d\theta$$

where I is the current, r is the radius of the loop, and x is the distance from the center to the point where the magnetic intensity is being computed ($0 \leq x \leq r$). Write a program that uses the Simpson method, with $n = 20$, to approximate the magnetic intensity accurate to nine decimal places if $I = 15.3$, $r = 120$, and $x = 84$. Using the Simpson method with $n = 8$, find the approximate value for the magnetic intensity.

[15]

- Q.5 (a) Which of the following are valid and invalid FORTRAN 90 variable names? If invalid, give a reason why.

- | | | |
|------------|------------------|---------------|
| i) A123BC, | iii) first name, | v) f2i, |
| ii) 5mass, | iv) _velocity, | vi) β . |

[9]

(b) The law of radioactive decay is given by

$$-dN = \lambda N dt, \quad N(t = 0) = N_0$$

which is a first order ordinary differential equation, where N is the number of radioactive nuclei (not decayed nuclei), $-dN/dt$ is the decrease of the this number per unit time and $\lambda = 0.693/T$ is the decay constant (T is the half-life). N_0 is the number of nuclei at the beginning, $t = 0$.

A certain radioactive material has a decay constant of 0.05 and if initially there is 1000 atoms of the radioactive material, write a program that uses the Euler's method to approximate how much of the material is remaining every second for a period of 1000 seconds.

[16]

Q.6 (a) Write FORTRAN 90 expressions for the following equations:

i) $z = ab^{\frac{1}{2}}e^{-x}$

iii) $y = \frac{1}{1 + e^{ax+b}}$

ii) $\tau = \frac{\ln 2}{\lambda}$

iv) $a = -cn^2 \sin(nt + \epsilon)$

[8]

(b) The normal probability function ϕ is defined as:

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

Write a program to evaluate $\phi(x)$ for values of x from -3.0 to +3.0 in steps of 0.2, and store these in an array. Display the results, to two decimal places, in a table saved to a file called `normal.dat`, with columns for values of x and ϕ .

[17]

***** End of Examination *****

Appendix

Commonly Used Fortran Intrinsic Functions:

function	Type	Purpose
ABS(x)	INTEGER or REAL	returns the absolute value of x
COS(x)	REAL	returns the cosine of x in radians
SIN(x)	REAL	returns the sine of x in radians
TAN(x)	REAL	returns the tangent of x
LOG10(x)	REAL	returns the logarithm of x to base 10
SQRT(x)	REAL	returns the square root of x
LOG(x)	REAL	returns the natural logarithm of x
EXP(x)	REAL	returns e raised to the power x
MOD(x,y)	INTEGER or REAL	returns the remainder of the division of x by y

Newton-Raphson:

Iteratively approximates the root of a non-linear function $f(x)$ starting with an initial guess x_i

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Linear Least Squares Fit:

Finds the best fitting line, $f(x) = a + bx$ through a set of data points (x_1, y_1) , $(x_2, y_2), \dots, (x_n, y_n)$, where x is the independent variable and y is the dependent variable. The constants a and b are obtained using

$$a = \frac{\left(\sum_{i=1}^n y_i\right) \left(\sum_{i=1}^n x_i^2\right) - \left(\sum_{i=1}^n x_i\right) \left(\sum_{i=1}^n x_i y_i\right)}{n \left(\sum_{i=1}^n x_i^2\right) - \left(\sum_{i=1}^n x_i\right)^2}$$
$$b = \frac{n \left(\sum_{i=1}^n x_i y_i\right) - \left(\sum_{i=1}^n x_i\right) \left(\sum_{i=1}^n y_i\right)}{n \left(\sum_{i=1}^n x_i^2\right) - \left(\sum_{i=1}^n x_i\right)^2}$$

Numerical Integration:

A definite integral

$$I = \int_a^b f(x)dx$$

can be approximated using the following numerical methods

$$\int_a^b f(x)dx \approx \frac{h}{2} [f(a) + 2 \sum_{i=1}^{N-1} f(x_i) + f(b)]$$

known as the composite Trapezoidal rule, where $h = (b-a)/N$, N is the number of evenly spaced sub-intervals between a and b . The composite Simpson rule is given by

$$\int_a^b f(x)dx \approx \frac{h}{3} \left[f(x_0) + 2 \sum_{i=1}^{(n/2)-1} f(x_{2i}) + 4 \sum_{i=1}^{n/2} f(x_{2i-1}) + f(x_n) \right]$$

where $x_0 = a$ and $x_n = b$ for even n and $h = (b-a)/n$.

Euler's Method:

The most basic technique for finding the approximate solution of a first-order ODE $y' = f(x, y)$ with initial condition $y(x_0) = y_0$ on an interval $[x_0, x_n]$. The interval $[x_0, x_n]$ is divided into n equally spaced points (nodes) and the distance between each node $h = (x_n - x_0)/n$ is called the step-size. This method approximates solution values y_1, y_2, \dots, y_n at the node points x_1, x_2, \dots, x_n .

$$y_{i+1} = y_i + hf(x_i, y_i)$$

THE UNIVERSITY OF ZAMBIA
Department of Physics
School of Natural Sciences
 2013/14 Academic Year
 End of Year University Examinations
 PHY3242 - Statistical and Thermal Physics

DURATION: THREE (3) HOURS
 TOTAL MARKS: 100

Instructions

- Answer any four (4) questions.
- All questions carry equal marks. Marks for each question are indicated in square brackets [].
- Show your working clearly. Omission of essential work will lead to loss of marks.

Formulae you may find useful.

$$TdS = dE + dW, \quad C_V = \left(\frac{\partial \bar{E}}{\partial T} \right)_V, \quad \beta = \frac{\partial \ln \Omega}{\partial E}, \quad \bar{X} = \frac{1}{\beta} \frac{\partial \ln Z}{\partial x}, \quad \bar{p} = \frac{1}{\beta} \frac{\partial \ln \Omega}{\partial V}$$

$$\bar{E} = -\frac{\partial \ln Z}{\partial \beta} \quad \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad \bar{y} = \frac{\sum y_r P_r}{\sum P_r}, \quad \bar{n} = Np, \quad \sigma^2 = Npq$$

Binomial distribution $W_N(n_1) = \frac{N!}{n_1!n_2!} p^{n_1} q^{n_2}$,

Gaussian distribution $W(n) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(n - \bar{n})^2}{2\sigma^2}\right)$

Maxwell relations

$$\left(\frac{\partial T}{\partial V} \right)_S = - \left(\frac{\partial P}{\partial S} \right)_V, \quad \left(\frac{\partial S}{\partial V} \right)_T = \left(\frac{\partial P}{\partial T} \right)_V, \quad \left(\frac{\partial V}{\partial S} \right)_P = \left(\frac{\partial T}{\partial P} \right)_S,$$

$$\left(\frac{\partial S}{\partial P} \right)_T = - \left(\frac{\partial V}{\partial T} \right)_P$$

1. (a) Two true 6-sided dice are thrown 24 times in succession.
- (i) What is the probability of obtaining at least one pair of sixes? [3]
 - (ii) What is the probability of obtaining only one pair of sixes in the 24 throws. Give numerical answers. [3]

(b) The number of states of an ideal diatomic gas consisting of N identical molecules in a volume V in the energy range from E to $E + \delta E$ is

$$\Omega(E) = BV^N E^{5N/2}$$

where B is a constant.

- (i) Obtain the equation of state of such a gas. [3]
 - (ii) Obtain the heat capacity of such a gas. [3]
- (c) A box is divided into two equal volumes by a partition that has a small hole in it. N atoms are randomly distributed in the box. Occasionally, one atom may pass through the hole from one side to the other causing the number of atoms on a side to fluctuate about its mean value.
- (i) Use the Gaussian distribution to calculate the ratio of probabilities for finding 24 and 25 atoms on one side when $N = 50$. (To save work, find an analytical expression for the ratio and evaluate it, rather than evaluating the two probabilities separately and taking their ratio.) [4]
 - (ii) Now calculate the ratio of probabilities for finding 2400 and 2500 atoms on one side in a system with $N = 5000$ total atoms. (See the work-saving hint in (i).) [4]
 - (iii) Compare these two ratios. How does the system size, i.e., the value of N , affect the chance of observing the same relative fluctuation? Explain the reasons for the difference. [5]

2. A simple harmonic one-dimensional oscillator has energy levels given by $E_n = (n + \frac{1}{2})\hbar\omega$, where ω is the characteristic (angular) frequency of the oscillator and where the quantum number n can assume the possible integral values $n = 0, 1, 2, \dots$. Suppose that such an oscillator is in thermal contact with a heat reservoir at temperature T low enough so that $kT/(\hbar\omega) \ll 1$.

- (i) Find the ratio of the probability of the oscillator being in the first excited state to the probability of its being in the ground state. [4]
- (ii) Assuming that only the ground state and first excited state are appreciably occupied, find the mean energy of the oscillator as a function of the temperature T [6]

(b) Define briefly ensemble and then distinguish microcanonical, canonical and grand canonical ensembles. [4]

(c) For the microcanonical ensemble, the entropy is given by

$$S = k \ln \Omega$$

whereas for the canonical ensemble the entropy can be expressed as

$$S = -k \sum_r P_r \ln P_r$$

Show that the two expressions for the entropy are equal. [5]

(d) The latent heat of melting of ice is L per unit mass. A bucket contains a mixture of water and ice at the ice point (absolute temperature T_0). It is desired to use a refrigerator in order to freeze an additional mass m of water in the bucket. The heat rejected by the refrigerator goes to warm up a body of constant heat capacity C and, initially, also at temperature T_0 . What is the minimum amount of heat which the refrigerator must reject to this body in the process? [6]

3. (a) Show that for a particle in equilibrium with a heat reservoir at absolute temperature T , the probability that it is in a state ϵ_r is

$$P_r = \frac{e^{-\beta\epsilon_r}}{\sum_r e^{-\beta\epsilon_r}}$$

[8]

(b) Now consider a collection of N particles in contact with a heat reservoir at absolute temperature T . If only the states with energy $\epsilon_r = 0$ and $\epsilon_r = \epsilon$ are accessible to the particles, show that the heat capacity at constant volume of the particles is

$$C_V = \frac{Nkx^2e^{-2x}}{(1 + e^{-x})^2}$$

[10]

where $x = \beta\epsilon$.

(c) Two systems, A and A' are in weak thermal interaction. The quantum energies of one system are E_r and the energies of the other system are E_s where r and s are quantum numbers.

(i) Prove that if the systems' respective partition functions are Z_1 and Z_2 then the partition function for the combined system, is

$$Z = Z_1 Z_2$$

[2]

(ii) Show that the entropy of the combined system is the sum of the entropies of the two sub-systems. [5]

4. (a) The nuclei of the atoms in a certain crystalline solid have spin 1. Thus each nucleus can be in any one of a number of three quantum states labelled by the quantum numbers ($m = 0, \pm 1$). This quantum number measures the projection of the nuclear spin along the crystal axis of the solid. Since the electric charge distribution in the nucleus is not spherically symmetric but ellipsoidal, the energy of a nucleus depends on its spin orientation with respect to the internal electric field existing at its location. Thus a nucleus has the same energy $E = \epsilon$ in the states $m = \pm 1$, compared with an energy $E = 0$ in the state $m = 0$.

- (i) What is the nuclear contribution to the molar internal energy of the solid at absolute temperature T ? [6]
- (ii) Use $S = k(\ln Z + \beta \bar{E})$ to obtain the nuclear contribution to the molar entropy at absolute temperature T ? [3]
- (b) Monoatomic molecules absorbed on a surface are free to move on this surface and can be treated as classical ideal two-dimensional gas. At absolute temperature T , what is the heat capacity per mole of molecules thus absorbed on a surface of fixed size? [6]
- (c) A 100g piece of metal of constant specific heat capacity $c = 5.0 \times 10^5 \text{ Jkg}^{-1}\text{K}^{-1}$ at temperature 25°C is placed in thermal contact with a heat reservoir at 0°C . Calculate the entropy change of the metal and reservoir. [10]
5. (a) Distinguish between Maxwell-Boltzmann, Bose-Einstein and Fermi-Dirac statistics. [6]
- (b) Consider a system of two weakly interacting identical particles. Each particle can be in one of three quantum states of energy 0, ϵ , or ϵ (The upper energy level is doubly degenerate.) The system is in contact with a heat reservoir at absolute temperature T .
- (i) Evaluate the canonical partition function Z treating the particles as fermions and as bosons. [4]
- (ii) Differentiate Z to find an expression for the mean energy \bar{E} of each system. What are the limiting values of these mean energies as $T \rightarrow 0$ and as $T \rightarrow \infty$? (You should find the low temperature limits to be different and the high temperature limits to be the same.) [6]
- (iii) Find an expression for the constant volume heat capacity of each system [4]
- (c) A solid at absolute temperature T is placed in an external magnetic field $H = 30,000$ gauss. The solid contains weakly interacting paramagnetic atoms of spin- $\frac{1}{2}$ so that the energy of each atom is $\pm\mu H$. If the magnetic moment μ is equal to one Bohr magneton, i.e., $\mu = 0.927 \times 10^{-20}$ ergs/gauss, below what temperature must one cool the solid so that more than 75 percent of the atoms are polarized with their spins parallel to the external magnetic field? [5]
6. (a) Explain what is meant by an external parameter, a mean generalized force and an equation of state mean. [6]
- (b) The only external parameter of a system is the volume V . Prove the following results
- (i) $\bar{E} = -\frac{\partial \ln Z}{\partial \beta}$ [5]
- (ii) $dW = \frac{1}{\beta} \frac{\partial \ln Z}{\partial V} dV$ [6]

(c) A container of volume V contains N molecules of dilute gas at a temperature T . The mass of a molecule is denoted m . You can use classical mechanics to describe the motion of the molecules. The density of gas molecules at position \vec{r} and velocity \vec{v} is given by the Maxwell Boltzmann distribution,

$$n(\vec{r}, \vec{v}) = \frac{N}{V} \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-mv^2/2kT}$$

Show that the respective mean values of the $\overline{|v_z|}$ and $\overline{v_z^2}$ of the z-component are given by

$$\overline{|v_z|} = \sqrt{\frac{2kT}{\pi m}}$$

and

$$\overline{v_z^2} = \frac{kT}{\pi m}$$

[8]

THE UNIVERSITY OF ZAMBIA
PHYSICS DEPARTMENT
EXAMINATIONS 2014
PHY4222: SOLID STATE PHYSICS II

TIME: THREE HOURS
ANSWER: ANY FOUR QUESTIONS
MAXIMUM MARKS: 100

$$\mathbf{b}_i = 2\pi \frac{\mathbf{a}_j \times \mathbf{a}_k}{\mathbf{a}_i \cdot (\mathbf{a}_j \times \mathbf{a}_k)}, \quad i, j, k \text{ in cyclic order}$$

$$\int_0^\infty x^{1/2} e^{-x} dx = \left(\frac{\pi}{4}\right)^{1/2}$$

$$\cos \theta = \frac{hh' + kk' + ll'}{(h^2 + k^2 + l^2)^{1/2} (h'^2 + k'^2 + l'^2)^{1/2}}$$

$$\int y e^{xy} dy = \frac{1}{x} \left[y e^{xy} - \frac{1}{x} e^{xy} \right] + C$$

1. (a) (i) What is a lattice and what is special about a Bravais lattice? [2marks]

(ii) Explain the importance of the Wigner-Seitz unit cell and its significance in reciprocal space. [2marks]

(iii) What is a lattice translation vector and how is it expressed in terms of the fundamental lattice vectors? [2marks]

(b) A certain space lattice has the lattice vectors

$$\mathbf{a}_1 = \frac{a}{2} \hat{\mathbf{i}} + \frac{a\sqrt{3}}{2} \hat{\mathbf{j}}, \quad \mathbf{a}_2 = -\frac{a}{2} \hat{\mathbf{i}} + \frac{a\sqrt{3}}{2} \hat{\mathbf{j}} \quad \text{and} \quad \mathbf{a}_3 = a \hat{\mathbf{k}}$$

(i) Determine the volume of the unit cell corresponding to this lattice. [4marks]

(ii) Determine the shortest translation vector in the lattice defined by these lattice vectors and give the integers corresponding to it. [3marks]

(iii) Determine the reciprocal lattice vectors corresponding to this system. [6 marks]

(c) (i) Show that the direction indices of the main diagonal in a simple cubic system are

$$[hkl] = [111]$$

and that those of the floor diagonal in the xy plane are

$$[h'k'l'] = [110]$$

(ii) Determine the angle which these two diagonals make with one another [6 marks]

2 (a) Derive the Laue condition

$$\mathbf{R} \cdot (\mathbf{k} - \mathbf{k}') = 2\pi m, \quad m = 0, 1, 2, 3, \dots$$

for X-ray diffraction in a crystal

[9 marks]

(ii) Explain the connection between this formula and the reciprocal lattice. [3 marks]

(b) Show that the Laue condition is equivalent to the Bragg diffraction condition

$$2d \sin \theta = n\lambda$$

(c) Explain the following:

[5 marks]

(i) Intrinsic semiconductors

[2 marks]

(ii) The distinction between p-type and n type-semiconductors

[4 marks]

(iii) The conduction band

[2 marks]

3. (a) (i) What is a crystal plane?

[2 marks]

(ii) A plane makes the intercepts 1, 2 and 0.5 Å on the crystallographic axes of an orthorhombic crystal with fundamental vectors \mathbf{a} , \mathbf{b} and \mathbf{c} such that $a : b : c := 3 : 2 : 1$. Determine the Miller indices of this plane. [4 marks]

(b) (i) Prove that in a cubic crystal system of lattice constant a , the distance between members of a family of planes with Miller indices (hkl) is given by

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

[13 marks]

(ii) Determine the distance between the planes belonging to the family with Miller indices (100) and explain your answer. [2marks]

(c) In a certain simple-cubic crystal, first-order Bragg reflection corresponds to a certain angle θ when X rays of wavelength 1.37 Å are used. It is known that a family of lattice planes with the Miller indices (122) is responsible for the diffraction. If the lattice constant is $a = 5.4 \times 10^{-10}$ m, find θ . [4 marks]

4 (a) (i) Show that in the classical Langevin theory, the paramagnetic susceptibility is

$$M = N \frac{\mu \int_0^\pi \cos \theta \sin \theta \exp\left(\frac{\mu B \cos \theta}{k_B T}\right) d\theta}{\int_0^\pi \sin \theta \exp\left(\frac{\mu B \cos \theta}{k_B T}\right) d\theta}$$

[8 marks]

(ii) Show that with the substitutions

$$x = \frac{\mu B}{k_B T} \text{ and } y = \cos \theta$$

the magnetisation can be written as

$$M = N\mu \left[\coth x - \frac{1}{x} \right] = N\mu L(x)$$

[8 marks]

(iii) At ordinary temperatures, $x \rightarrow 0$. Given that

$$L(x)|_{x=0} = \frac{x}{3}$$

derive the Curie law

$$\chi = \frac{C}{T}$$

and determine the Curie constant C .

[3 marks]

(b) Briefly outline the difference between ferromagnetism, ferrimagnetism and anti-ferromagnetism. [6 marks]

5. (a) (i) Prove that if the potential $V(x)$ is periodic, the Hamiltonian

$$H = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x)$$

is also periodic.

[3marks]

(ii) Assuming that the Hamiltonian is periodic, prove Bloch's theorem

$$\psi_{\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u_{\mathbf{k}}(\mathbf{r})$$

[10 marks]

(iii) Determine the eigenvalue equation for $u_{\mathbf{k}}(\mathbf{r})$ for the one-dimensional case. [4 marks]

(b) In the Kronig-Penney model, the energy quantisation condition is

$$\cos kl = \begin{cases} ac \cosh \beta b - \frac{\alpha^2 - \beta^2}{2\alpha\beta} \sin \alpha c \sinh \beta b, & E < 0 \\ \cos \alpha c \cos \lambda b - \frac{\alpha^2 + \lambda^2}{2\alpha\lambda} \sin \alpha c \sin \lambda b, & E > 0 \end{cases}$$

where $\alpha = \alpha(E)$ and b, c, λ and β are constants.

Use this to demonstrate that

(i) The energy levels occur in bands. [4 marks]

(ii) The energy is an even function of k . [2 marks]

(iii) The energy is periodic in k . [2 marks]

6. The Fermi-Dirac distribution function is

$$f(E) = \frac{1}{e^{(E-E_F)/k_B T} + 1}$$

the magnetisation can be written as

$$M = N\mu \left[\coth x - \frac{1}{x} \right] = N\mu L(x)$$

[8 marks]

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[3marks]

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[4 marks]

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$$\cos kl = \begin{cases} \alpha c \cosh \beta b - \frac{\alpha^2 - \beta^2}{2\alpha\beta} \sin \alpha c \sinh \beta b, & E < 0 \\ \cos \alpha c \cos \lambda b - \frac{\alpha^2 + \lambda^2}{2\alpha\lambda} \sin \alpha c \sin \lambda b, & E > 0 \end{cases}$$

where $\alpha = \alpha(E)$ and b, c, λ and β are constants.

Use this to demonstrate that

(i) The energy levels occur in bands.

[4 marks]

(ii) The energy is an even function of k .

[2 marks]

(iii) The energy is periodic in k .

[2 marks]

6. The Fermi-Dirac distribution function is

$$f(E) = \frac{1}{e^{(E-E_F)/k_B T} + 1}$$

and the density of electronic states in the energy range dE is

$$D(E) = \frac{4\pi}{\hbar^3} (2m)^{3/2} E^{1/2}$$

(i) Show that the number of electrons per unit volume in the conduction band of a semiconductor is

$$n = 2 \left(\frac{2\pi m_e^* k_B T}{\hbar^2} \right)^{3/2} \exp \left[- \left(\frac{E_C - E_F}{k_B T} \right) \right] = N_C \exp \left[- \left(\frac{E_C - E_F}{k_B T} \right) \right]$$

You may assume that

$$\frac{|E - E_F|}{k_B T} \gg 1$$

Note that m_e^* is the effective mass of the electron, \hbar is Planck's constant, T is the absolute temperature, E_F is the Fermi energy, k_B is Boltzmann's constant and E_C is the conduction-band edge. [15marks]

(ii) Given also that the number of holes carriers per unit volume in the valence band is

$$p = 2 \left(\frac{2\pi m_h^* k_B T}{\hbar^2} \right)^{3/2} \exp \left[- \left(\frac{E_F - E_V}{k_B T} \right) \right] = N_V \exp \left[- \left(\frac{E_F - E_V}{k_B T} \right) \right]$$

and that $N_C = N_V$, show that the Fermi level for an intrinsic semiconductor is in the middle of the band gap when $T = 0$ or when m_p^* , the hole effective mass, is equal to the electron effective mass m_e^* . [4 marks]

(iii) Show that the carrier concentration can be written as

$$n = 2 \left(\frac{2\pi k_B T}{\hbar^2} \right)^{3/2} (m_e^* m_p^*)^{3/4} \exp \left(- \frac{E_g}{2k_B T} \right)$$

and explain how this can be used to measure the band gap E_g . [6 marks]

*****END OF EXAMINATION*****