

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

THE GRADUATE COLLEGE

May 15, 1979

WE HEREBY RECOMMEND THAT THE THESIS BY

Zachariah Simfukwe

ENTITLED Efficiency of Resource Allocation on Selected Illinois Farms

BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR

THE DEGREE OF Master of Science

John T. Scott, Jr.

Director of Thesis Research

D. Pedberg

Head of Department

Committee on Final Examination†

Chairman

† Required for doctor's degree but not for master's.

**EFFICIENCY OF RESOURCE ALLOCATION
ON SELECTED ILLINOIS FARMS**

BY

ZACHARIAH SIMFUKWE

B.A., University of Zambia, 1976

THESIS

**Submitted in partial fulfillment of the requirements
for the degree of Master of Science in Agricultural Economics
in the Graduate College of the
University of Illinois at Urbana-Champaign, 1979**

Urbana, Illinois



ACKNOWLEDGEMENTS

I am deeply indebted to Professor John T. Scott, Jr., for his guidance and counsel during the course of my study, and especially during the preparation of the thesis.

Faculty members and fellow students who provided helpful suggestions towards completion of this thesis are to be commended also.

TABLE OF CONTENTS

Chapter	Page
I. INTRODUCTION	1
Objectives	2
Hypothesis	6
Literature Review.	6
II. THEORETICAL CONSIDERATION	9
The Marginal Principle	11
Production Surface and Isoquants	13
The Firm's Optimal Production Decisions.	17
III. SOURCES OF DATA AND TECHNIQUE OF ANALYSIS	22
Sources of Data.	22
Characteristics of Samples.	22
Technique of Analysis.	23
Residual Imputations vs. Production Functions	23
The Production Function	25
The Cobb-Douglas Production Function.	26
Analysis.	30
Marginal Productivities and Market Prices	31
IV. RESULTS	38
1973 Results	38
1977 Results	44
Comparison of Efficiency of Resource Use Between 1973 and 1977.	47
V. CONCLUSIONS	53
LIST OF REFERENCES	58

LIST OF TABLES

Table	Page
1.1 Indexes of Prices Received by Farmers, Annual Averages, United States, January 1966-77. (1967=100) (selected products)	3
1.2 Indexes of Prices Paid by Farmers for Selected Items Used in Production, Annual Averages, United States, January 1966-77. (1967=100).	4
1.3 Realized Net Farm Income and Total Net Farm Income: Illinois, 1970-77	5
4.1 Cost per Tillable Acre - Grain Farms - No Livestock, Central Illinois Standards, 1973 and 1977	39
4.2 Means and Ranges of All the Variables Used in the Production Function Estimation for 1973 and 1977	41
4.3 Variables Correlation Matrix - 1973	42
4.4 Regression Coefficients, Standard Errors, R^2 and Sum of Coefficients - 1973 and 1977.	42
4.5 Estimated Elasticities of Production, b_1 , and Calculated Elasticities of Production, b_1^* , to Equate $\frac{MVPx_1}{Px_1}$ to Unity in 1973	44
4.6 Variables Correlation Matrix - 1977	45
4.7 Estimated Elasticities of Production, b_1 , and Calculated Elasticities of Production, b_1^* , to Equate $\frac{MVPx_1}{Px_1}$ to Unity in 1977	46
4.8 Marginal Value Productivities of the Resources Estimated from Production Functions	49
4.9 Elasticities Estimated from the 1977 Production Function Results and the Calculated Elasticities for 1977 which would have been Necessary to Equate the Marginal Value Productivities of Each Resource in Both Periods	51

LIST OF FIGURES

Figure		Page
1	Production Surface for Two Variable Inputs.	15
2	Isoquants for Two Variable Inputs	15

CHAPTER I
INTRODUCTION

The production decisions of farmers involve answering three basic questions: What commodities should I produce? How much of these will I produce? And how will I produce them? How farmers arrive at these answers is not readily discerned from observing the myriad of farm jobs they undertake.

One response to the above questions, and one not completely without validity, is tradition. But despite the tendency on the part of farmers to do things as they have in the past, major changes have occurred. Data of Illinois agriculture, which is based largely on crop production, show that corn production in Illinois has increased from 3.7 million bushels in 1928 to 1.2 billion in 1977. Soybean production, which was relatively small at 3 million bushels in 1928, was in excess of 327 million bushels in 1977. Continued adoption of new and advanced technology and large decreases in farm numbers illustrate some of the major changes in the way farm products are produced.

If a survey was carried out to find why farmers have made changes in levels of production of a given crop or product, the majority of them would likely respond that they changed because it increased profits. Most farmers are profit maximizers within their constraints. To be able to maximize profits productive resources should be employed efficiently. While recognizing the fact that optimal allocation of resources for the firm is dependent upon the defined ends or goals of production, this study is done within the framework of the predetermined goal of profit maximization.

Objectives

The main objective of this study is to compare the efficiency of grain production on some selected group of Illinois farms for the years 1973 and 1977 with respect to resource allocation. Another objective, assuming that there were significant differences in the degree of resource use between the two periods, will be to see whether the departures were towards equilibrium or away from it. A third objective is to see whether there was any significant change in the proportion of resources used or products produced. That is, did increases in production occur along a scale line from 1973 to 1977 or did the expansion path deviate from a scale line and if so towards what resources?

Tables 1.1 and 1.2 give indexes of agricultural prices. From these tables one sees that prices of items used by farmers in production increased more than prices received by farmers for their products. While the index for the former went up by 54 percentage points between 1973 and 1977, the index for the latter increase only by 17 percentage points during the same period, (the 54 percentage points include prices for items used in livestock production as well, the 17 percentage points is just for crops). Table 1.2 shows that prices of items used by farmers in production did not change by the same proportion between 1973 and 1977. While the index of farm wages rose by 75 percentage points, the index of machinery prices went up 101 percentage points for the same period. The index of land cash rent shows an upward shift of 99 percentage points while this figure is 79 percentage points for fertilizers in the period under consideration.

The years 1973 and 1977 are chosen for this study because, as shown in Table 1.3, the realized net farm income in Illinois displayed an

TABLE 1.1 Indexes of Prices Received by Farmers, Annual Averages, United States, January 1966-77. (1967=100) (selected products)

Year	All Farm Products	All Crops	Feed Grains	Food Grains	Oil Bearing Crops
1966	106	106	105	105	109
1967	100	100	100	100	100
1968	102	100	90	91	96
1969	107	97	96	88	93
1970	110	100	103	92	99
1971	113	108	108	95	111
1972	125	114	107	109	122
1973	179	175	163	215	226
1974	192	224	249	300	232
1975	185	201	232	242	197
1976	186	197	214	202	205
1977	183	192	174	156	243

SOURCE: Crop Reporting Board, U.S. Department of Agriculture, June, 1978 pr. 1-3 (78).

TABLE 1.2 Indexes of Prices Paid by Farmers for Selected Items Used in Production, Annual Averages, United States, January 1966-77. (1967=100)

Year	All Items Used in Production	Fertilizers	Wage Rates	Cash Rents	Fuels & Energy	Tractors and Self-propelled Machinery
1966	100	102	93		98	96
1967	100	100	100		100	100
1968	100	94	109		101	104
1969	104	87	119		102	111
1970	108	88	127		104	116
1971	113	91	133	113	107	122
1972	121	94	142	123	108	128
1973	146	102	156	136	116	137
1974	166	167	180	166	159	161
1975	182	217	192	199	177	195
1976	193	185	213	214	187	217
1977	200	181	231	235	202	238

SOURCE: Crop Reporting Board, U.S. Department of Agriculture, June, 1978. pr. 1-3 (78).

TABLE 1.3 Realized Net Farm Income and Total Net Farm Income:
Illinois, 1970-77

Calendar Year	Realized Net Income	
	Total All Farms (million dollars)	Average per farm (dollars)
1970	775.9	6,015
1971	594.5	4,644
1972	810.9	6,335
1973	1,481.9	11,668
1974	1,683.8	13,364
1975	840.9	6,893
1976	1,594.1	13,284
1977	1,159.0	9,822

SOURCE: Illinois Agricultural Statistics, Illinois Cooperative Crop Reporting Service, Annual Summary, 1978.

increasing trend from 1970 through 1974.¹ After 1974 this trend did not continue. In 1975 realized net farm income decreased by 34 percent from the 1974 level. It was down 17 percent from the 1974 level in 1976. And in 1977 it was down 27 percent from 1976. We want to see whether the level of resource use contributed to high realized net farm income in 1973 and the low realized net income in 1977.

¹Realized net farm income is the amount left after deducting farm production expenses from gross farm income.

Hypothesis

The hypothesis to be tested is that there had not been any significant change in the efficiency of grain production with respect to the levels of inputs used between the years 1973 and 1977. Another hypothesis also to be tested is that increase in production between the two years occurred along a scale line.

The author would like to make the assumption that during both periods, that is 1973 and 1977, production had not been at equilibrium, where equilibrium is to be understood as that level of input use that equates the marginal value productivity of each input category to its marginal cost. In other words, there is an assumption of resource maladjustment in both periods. This assumption is justified because it is only under "riskless" production conditions and no technological change that all competitive firms would equate the marginal value productivity of each resource category to its cost.

Literature Review

The review of literature reveals that a very large number of studies have been done in the area of resource allocation in farming.²

Using data from samples drawn in 1951 for the Piedmont area of Alabama, north central Iowa, southern Iowa, and a dry-land wheat area of Montana, Heady and Shaw used marginal techniques to compare the efficiency of labor and capital used in the four areas.³ Among their conclusions was one that

² Because of the large number of studies done only a few are summarized here. Some of the studies are indicated in the references chapter at the end of the study.

³ E. O. Heady and R. Shaw, "Resource Returns and Productivity Coefficients in Selected Farming Areas," Journal of Farm Econ., 36: 243-57, 1954.

revealed that while the level of resource returns was different, the relative degree of efficiency would be just as great as in areas conventionally classified as "highly efficient."

In 1957 in an article entitled, "Resource Adjustment on 146 Commercial Corn-Belt Farms, 1936-1959," Swanson used regression analysis to study how farmers allocated their inputs of labor, power machinery, purchased feed, land investment, building and soil improvement, and livestock investments.⁴ He estimated elasticities of production for each of the six resource categories in the periods 1936-39 and 1950-53 from which he derived marginal products. His results, in part, showed significant differences (one-percent level) for categories of land and labor. No other resource exhibited a difference in adjustment between the time periods significant at less than the 10 percent probability level.

Miller's work involved studying the allocative efficiency of farmers under different tenure systems.⁵ His hypothesis was that tenure classes were different populations, with different patterns of resource allocation and levels of efficiency. He fitted a Cobb-Douglas type function to cross-sectional data of farms in Iowa and Illinois. Data of each tenure class was fitted to a separate equation. The finding showed that the differences between tenure classes in the overall efficiency were not significant in a probability sense.

⁴E. R. Swanson, "Resource Adjustment on 146 Commercial Corn-Belt Farms, 1936-53," Journal of Farm Econ., 39: 502-505, May, 1957.

⁵See Walter C. Miller, "Comparative Efficiency of Farm Tenure Classes in the Combination of Resources," Ag. Econ. Research, 11: 6-16, 1959.

While the three studies referred to above involved comparison of allocative efficiency, among regions for the Heady and Shaw study, between periods for the Swanson study, and among different tenure classes, this fourth study is an example in which allocative efficiency was studied for one area and for same kinds of farmers. This study was done in India by Hopper.⁶ Hopper tested the hypothesis that Indian cultivators who used traditional technology made rational profit-maximizing allocations of factors. He tested this hypothesis by observing the allocation of four major inputs to four production alternatives for 43 farmers in north central India. Production functions were determined and the implicit marginal productivity of each factor and the implicit value of each product. The calculated relative prices implicit in the factor-factor-product allocations conformed, within error limits, to expectations of profit maximizing behavior derived from a static model.

These are just a few of the many studies that have been done in the area of resource allocation efficiency. The common characteristic of these studies is that they are all concerned with the way in which farmers behave as they decide on the levels of input to use in the production process.

⁶See W. D. Hopper, "Allocation Efficiency in a Traditional Indian Agriculture," AJAE 47: 611-624, 1965.

CHAPTER II

THEORETICAL CONSIDERATION

Economic efficiency refers to the combinations of inputs that maximize profit. Profit is rarely if ever maximized in a global sense. Thus economic efficiency must occur in the more restricted setting of profit maximization given individual or social objectives. Economic efficiency is defined in terms of two conditions: necessary and sufficient.

The necessary condition is met in a production process when there is: (a) no possibility of producing the same amount of product with fewer inputs and (b) no possibility of producing more product with the same amount of inputs. In production function analysis, this condition is met in Stage II; that is, when the elasticity of production is equal to or greater than zero and equal to or less than one ($0 \leq E_p \leq 1$). The necessary condition refers only to the physical relationship. It is universal because it is applicable in any economic system. It is a natural objective. No one would knowingly produce in Stage III because the same or larger output could be obtained by moving to Stage II and using less of at least one or more inputs. In a given input-output relationship, many input-output combinations will satisfy the necessary condition. For this reason, an additional condition is needed to single out one alternative from the many that meet the necessary condition.

Unlike the necessary condition which is objective in nature, the sufficient condition for efficiency encompasses individual or social goals and values. Because of its subjectivity it varies in nature among individual producers. The sufficient condition for a producer striving for high yields per acre will be different from that of an individual whose objective is

maximization of profits per acre. In subsistence agriculture, a family preferring potatoes to turnips will place greater emphasis on potato production. For profit maximization the sufficient condition is the combination of inputs in Stage II which equates the marginal product of each input with the inverse price ratio of the product and the input, that is,

$$\frac{\Delta Y}{\Delta X_1} = \frac{PX_1}{P_y} ; \frac{\Delta Y}{\Delta X_2} = \frac{PX_2}{P_y} ; \dots ; \frac{\Delta Y}{\Delta X_n} = \frac{PX_n}{P_y}$$

or multiplying both sides of each equation by P_y to give the relationship

$$MVPx_1 = Px_1, MVPx_2 = Px_2; \dots ; MVPx_n = Px_n$$

or

$$\frac{MVPx_1}{Px_1} = \frac{MVPx_2}{Px_2} = \frac{MVPx_n}{Px_n} = 1^1$$

In the United States commercial farming is big business. Farmers have become increasingly dependent on the market system to obtain inputs and sell outputs. Farming is also a highly competitive business. A farmer, like any businessman who wants to survive, has to be mindful of costs of inputs and prices of products that these inputs produce. So that in order to remain in business farmers must be efficient in the way they employ their productive inputs. As stated in the introductory chapter profit maximization is the assumed sufficient condition for efficiency for this study.

Maximization of profit is a basic assumption of production theory. The static theory of the competitive firm is a useful starting point for construction of a structural model since: (a) in some respects agriculture is best represented by the purely competitive market structure and (b) the firm is a logical beginning point for analysis of more general, dynamic market

¹This relationship will be derived in detail later in the chapter.

phenomena. We begin with the assumptions that the decision maker maximizes profits in an environment of known physical input/output relationships and price ratios, instantaneous adjustments divisibility of commodities (inputs or outputs) and unlimited capital. Furthermore, prices are given, individual decisions are assumed to have no influence on price of either inputs or products under these competitive conditions. Given this setting, marginal analysis will be used to estimate and analyze the most economically efficient use of productive resources.

The Marginal Principle

The margin refers to an added unit, or the last unit, either of output or input. In both cases it can be measured either in physical or financial terms. Thus, as regards output, in referring to the return obtained from one more unit of input, or one more unit of output, this can be either the marginal physical product or the marginal revenue (or marginal return). That is, the addition to total physical product or total revenue made by one more (or the last) unit of input, or output is the marginal unit. As regards inputs, reference can be made to the additional physical quantity of resources needed to produce one more (or the last) unit of output, or the marginal cost of producing one more unit of output, or of supplying one more unit of input.

A marginal change is normally symbolized by Δ or by the partial derivative as used in calculus. In mathematical terms this need not necessarily refer to an additional unit, but possibly to any point on a continuous function, that is, the rate of change on any particular point. Marginal product, for example, can therefore be depicted as

$$\frac{\Delta \text{ product, and}}{\Delta \text{ input}}$$

marginal cost as:

$$\frac{\Delta \text{ cost}}{\Delta \text{ output}}$$

If the level of output has no effect on the price per unit received, as is usually the case in farming where the individual producer is usually supplying only an insignificant part of the total production of a commodity, the marginal revenue (or return) of additional (equal units of) output is constant. This is in contrast to the marginal return to additional (equal) units of input, which usually declines after a point when added to one or more fixed factors of production due to diminishing returns production functions.

Similarly, the marginal cost of additional units of input is constant if the amount required does not affect the price per unit of input. The marginal cost of additional (equal units of) output, by contrast, usually rises.

Average relationships are quite different. The average (physical) product, revenue or return is simply the total production or revenue divided by the number of units of either output or input. In the former case, the average return on the individual farm is normally constant, being equal to the price per unit of output.

Average input, or cost, is the total input or cost, divided by the total number of units of either input or outputs. In the former case, the average cost on the individual farm is normally constant, being equal to the price per unit of input.

Production Surface and Isoquants

Unlike the factor product relationship where output is assumed to be dependent upon the amount of a single variable input that is combined with certain other fixed resources, the factor-factor relationship is the concern of this study. The factor-factor relationship is relevant when several factors of production are used to produce a product. Here the entrepreneur decides which inputs he will use in the production of which product, and what amounts of each he will use, given their prices and the price of the product. For example, a corn producer may vary plant population and fertilizer used in the production of corn, a dairy farmer may vary the combination of grain and hay that he uses in feeding his dairy cows depending upon the prices of each and the amount of milk production he desires.

Suppose two variable inputs - labor and nitrogen fertilizer - are applied to a given piece of land to produce corn. Let us denote this functionally as:

$$Y = f(X_1, X_2), \text{ where}$$

Y = corn yield,

X_1 = labor,

X_2 = nitrogen fertilizer.

Unlike the single-variable situation that is easily represented by a single curve, this two-variable input function cannot be so represented. It depicts, not a curve, but a surface in the three-dimensional space with axes X_1 , X_2 and Y . Two diagrammatic possibilities exist. Either we can show the production function as a surface in three dimensions, or we may depict it by a series of curves in two dimensions. The latter is to be

preferred for analytic purposes. Figure 2.1 exemplifies the three dimensional alternative. The production surface ABCD is merely the surface of the "output-hill" traced out as X_1 and X_2 increase. Changes in the specification of the function would cause changes in the shape of the surface.

Figure 2.2 shows the preferred alternative of depicting $Y = f(X_1, X_2)$ by a series of output contours to give a bird's-eye view of the surface. These contours are known as ISOQUANTS. Each isoquant is the locus of all combinations of X_1 and X_2 that produce the same level of output, Y .

Associated with the isoquants are other concepts in the realm of factor-factor relationships. These are:

- (i) the marginal rate of substitution of X_1 for X_2 , written $MRS_{1,2}$;
- (ii) the elasticity of substitution of X_1 for X_2 , written as $ES_{1,2}$;
- (iii) the family of isocline equations
- (iv) the ridge-line equations.

Marginal rate of substitution of X_1 for X_2 is given by the slope of the isoquant equation. Algebraically, at any point on an isoquant we have:

$$\begin{aligned} MRS_{12} &= \partial X_1 / \partial X_2 \\ &= 1 / MRS_{21} \end{aligned}$$

The marginal rate of substitution of X_1 for X_2 tells us the rate at which we have to substitute X_1 for X_2 if we decrease X_2 by an infinitesimal amount and wish to maintain output unchanged. It can range from minus to plus infinity. Being a rate, MRS_{12} is measured in units of X_1 per unit of X_2 even though it is evaluated at any point on the isoquant.

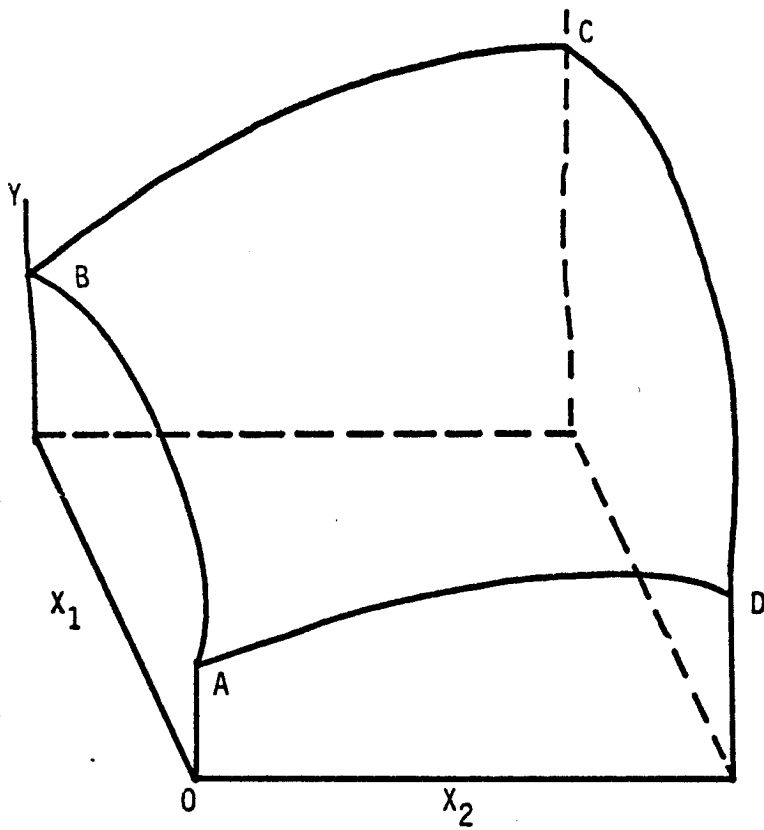


FIGURE 2.1 Production Surface for Two Variable Inputs

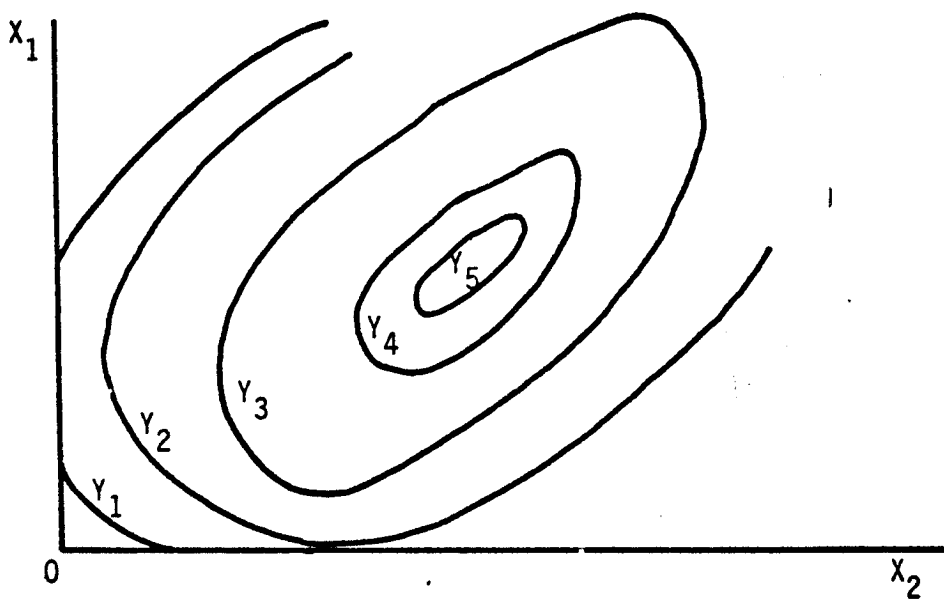


FIGURE 2.2 Isoquants for Two Variable Inputs

Elasticity of substitution of X_1 for X_2 is defined as the relative change in X_1 divided by the relative change in X_2 if we substitute X_1 for X_2 while keeping output unchanged. In incremental units we have:

$$ES_{12} = (\Delta X_1 / X_1) / (\Delta X_2 / X_2) \text{ which,}$$

estimated at a particular point on the isoquant, can be written as:

$$\begin{aligned} ES_{12} &= (\partial X_1 / \partial X_2) (X_2 / X_1) \\ &= (MRS_{12}) (X_2 / X_1) \\ &= 1 / ES_{21} \end{aligned}$$

Being an elasticity, ES_{12} is a pure number. It can range from minus to plus infinity; and is conveniently interpreted as the percentage change in X_1 needed to maintain Y unchanged if we change X_2 by 1 per cent.

Isoclines are defined as the loci of all combinations of X_1 and X_2 which have the same marginal rate of substitution. Hence they constitute paths up or down the response surface joining points of equal curvature on the isoquants. The family of isocline equations is derived by solving:

$$\partial X_1 / \partial X_2 = k$$

to obtain X_1 as a function of X_2 ; k being the value of MRS_{12} which specifies a particular isocline. For example, in Figure 2.2 the isocline of a MRS_{12} of -0.5 could be drawn by connecting the points where the isoquants have a slope of -0.5 .

For every possible isoquant a least-cost combination of inputs exists. The isocline that connects points on the isoquants at which the least-cost combination of resources occurs is a special one and is referred to as the expansion path.

Ridge-lines are those two special isoclines for which MRS_{12} is equal to zero or infinity, as implied respectively by their equations:

$$\partial X_1 / \partial X_2 = 0$$

$$\partial X_2 / \partial X_1 = 0$$

The ridge-lines mark the boundary between rational and irrational combinations of inputs, since in the areas outside the ridge lines to move along the isoquant (the same level of output) more of both X_1 and X_2 would need to be used rather than a substitution of one for the other.

The Firm's Optimal Production Decisions

Having thus far considered the various factor-factor relationships that occur between two variable inputs in the production of an output let us now derive the best combination of the inputs in producing an output. Later in the chapter the conclusion will be generalized to problems involving more than two variable resources.

On the surface, the solution to the problem of determining the best combination of resources is a very simple one. Common sense would tell us that the most profitable combination of a given amount of product is that which is produced for minimum cost. The manager would say this was a completely satisfactory answer if he knew the conditions that would have to be met in order to have the least-cost combination. He would like to know what to do when it is possible to substitute some of one resource for another without having an effect on output. Also, he will want to know what to do if such substitution is not possible.

With two variable resources it is necessary that:

$$MVPx_1 = Px_1 \text{ and } MVPx_2 = Px_2 \quad (2.1), (2.2)$$

if both equations are divided by their respective prices, we have:

$$\frac{MVP_{x_1}}{P_{x_1}} = 1 \text{ and } \frac{MVP_{x_2}}{P_{x_2}} = 1 \quad (2.3), (2.4)$$

Since both ratios are equal to 1, we can combine them to have:

$$\frac{MVP_{x_1}}{P_{x_1}} = \frac{MVP_{x_2}}{P_{x_2}} = 1 \quad (2.5)$$

This not only tells us the way the resources should be combined, but also the best level at which to produce the product. For any given level of production it is necessary merely that:

$$\frac{MVP_{x_1}}{P_{x_1}} = \frac{MVP_{x_2}}{P_{x_2}} \quad (2.6)$$

Further generalizations can be made from equation (2.6). We know that the marginal value product equals the marginal physical product multiplied by the constant price of the product. Therefore, substituting this knowledge into equation (2.6), we have:

$$\frac{MPP_{x_1} \cdot P_y}{P_{x_1}} = \frac{MPP_{x_2} \cdot P_y}{P_{x_2}} \quad (2.7)$$

Dividing both ratios by P_y we obtain:

$$\frac{MPP_{x_1}}{P_{x_1}} = \frac{MPP_{x_2}}{P_{x_2}} \quad (2.8)$$

indicating that the best combination of resources is independent of the price of the product for any given level of production as long as P_y is a constant. This makes sense. If it is profitable to produce a given amount of product at all, it will be most profitable to produce this amount for the least possible cost regardless of the product price. Continuing, we recall that the marginal physical product is defined as the change in physical product obtained from the marginal change in resource used or $\frac{\Delta Y}{\Delta X}$.

Therefore, the equation (2.8) may be written:

$$\frac{1}{P_{x_1}} \cdot \frac{\Delta Y}{\Delta X_1} = \frac{1}{P_{x_2}} \cdot \frac{\Delta Y}{\Delta X_2}, \quad (2.9)$$

We can assume that the two ΔY 's are equal, making it possible to divide both sides of the equation by ΔY to obtain:

$$\frac{1}{P_{x_1} \Delta X_1} = \frac{1}{P_{x_2} \Delta X_2} \quad \text{or} \quad P_{x_1} \Delta X_1 = P_{x_2} \Delta X_2 \quad (2.10) \quad (2.11)$$

If we divide both sides of the equation (2.11) by $P_{x_1} \Delta X_2$ it becomes:

$$\frac{P_{x_1} \Delta X_1}{P_{x_1} \Delta X_2} = \frac{P_{x_2} \Delta X_2}{P_{x_1} \Delta X_2} \quad (2.12)$$

which reduces to:

$$\frac{\Delta X_1}{\Delta X_2} = \frac{P_{x_2}}{P_{x_1}} \quad (2.13)$$

Equation (2.13) is an alternative way of writing

$$\frac{MVP_{x_1}}{P_{x_1}} = \frac{MVP_{x_2}}{P_{x_2}},$$

the necessary condition for the optimum combination of two resources for a particular level of production. But $\frac{\Delta X_1}{\Delta X_2}$ refers to the marginal rate of

substitution of X_2 for X_1 . So, another way of stating this necessary condition is that the marginal rate of substitution of the resources must equal their price ratio.

When more involved production problems using more than two resources in the production of one product are considered, the same basic economizing principles are followed. Added costs are matched against added returns, subject to the law of diminishing returns, as a condition for maximizing

profits. Geometry is limited to three dimensions. We must now rely entirely on algebraic symbols and their verbal meaning.

When three inputs are used, we merely extend our algebraic definition of the high profit point to include another term for the third resource.

Thus we have:

$$\frac{MPP_{x_1}^P}{P_{x_1}} = \frac{MPP_{x_2}^P}{P_{x_2}} = \frac{MPP_{x_3}^P}{P_{x_3}} = 1$$

This reasoning can be extended to cover any number of inputs, n , merely by adding terms and equality signs as follows:

$$\frac{MPP_{x_1}^P}{P_{x_1}} = \frac{MPP_{x_2}^P}{P_{x_2}} = \frac{MPP_{x_3}^P}{P_{x_3}} = \frac{MPP_{x_4}^P}{P_{x_4}} = \dots = \frac{MPP_{x_n}^P}{P_{x_n}} = 1$$

Such an extended algebraic statement would be read in English as follows: The most profitable quantities and combinations of variable resources are being used to produce a product when their respective marginal value products are all exactly equal to the cost of acquiring another unit of each resource.

In terms of this study the marginal value productivity for each of the chosen resources will be compared to its own marginal factor cost (price) in each of the two periods, 1973 and 1977. Under most efficient production we would expect to have the ratio of $\frac{MVP_{x_1}}{P_{x_1}}$ to be equal to one. Depending

on how far this ratio is from one, some conclusions will be drawn. For example, if we take one resource, say labor and obtain for this labor resource the ratio $\frac{MVP_L}{P_L}$ in 1973 and a similar ratio will be obtained

for 1977. If the ratio is 1.6 in 1973 and 1.9 in 1977 we would conclude

that there was a difference in the efficiency of labor used on the farms studied during the two periods. We would further conclude that the movement in efficiency of labor use was away from equilibrium since 1.6 is closer to one than is 1.9. If, however, the ratios are 1.6 and 1.2 for 1973 and 1977, respectively, we would conclude that there was a difference in the efficiency of labor use on the farms studied but in this case we would add that the movement was towards equilibrium.

To conclude whether or not there was an increase in production from 1973 to 1977 will just be a matter of comparing total production in both periods. However, if there was an increase, to ascertain whether or not production increased along a scale line from 1973 to 1977 we would have to compare the figures for the returns to scale in both periods. If the returns to scale is the same for both periods then the conclusion would be that the expansion path did not deviate from a scale line. But if different returns to scale are obtained in the two periods, then we would conclude that there was a deviation of the expansion path from the scale line. In case of a significant deviation, we would be able to tell towards which resources this deviation occurred by looking at the MVP's of each resource in both periods.

As was stated earlier in the chapter, assumptions of perfect competition in both factor and product markets have been used here. To make the analysis simpler, risk and time factors will not be included in the analysis of the data.

CHAPTER III

SOURCES OF DATA AND TECHNIQUE OF ANALYSIS

Sources of Data

The data on indexes of prices paid by farmers for items used in production and indexes of prices received by farmers for their products are obtained from appropriate USDA publications. The farm data used in this study are from farmers cooperating with the University of Illinois Cooperative Extension Service, the Department of Agricultural Economics, and the Illinois Farm Business Farm Management Association. These data, which are compiled annually, are currently stored on computer tapes at the University of Illinois. In 1977 about one out of every five commercial farmers in Illinois was enrolled in this service. In Illinois, farms that are more than 180 acres per farm are often referred to as "commercial farms."¹

Characteristics of Samples

From both of the 1973 and 1977 records a number of farms was selected to include in the study. Farms included in the sample for each year had the following characteristics:

- (a) soil productivity rating 86-100
- (b) tillable land between 189 and 650 acres
- (c) ratio of the value of feed fed to feed and grain returns
less than 0.01

¹Farms that sell \$40,000 or more of farm products per year are also considered commercial.

Using the above sampling criteria 534 farms were obtained from the 1973 records and 647 farms from the 1977 records. Characteristic (a) ensures that all farms sampled had highly productive soils so that variations in soil productivity are minimal. Characteristic (b) was chosen because for both years under consideration more than 70 percent of all farms that satisfied criterion (a) were in this size range. Characteristic (c) means that essentially only grain farms were included in the samples.

For each farm that was included data on amount of farm production and amount of selected resources were obtained.²

Technique of Analysis

Given our assumptions of pure competition in both product and factor markets and a timeless environment, efficiency of input use may be estimated by the methods of linear programming, budgeting, residual imputation or from production functions. The nature of the data available makes use of linear programming and budgeting difficult.³

Residual Imputations vs. Production Functions

Resource productivity may be estimated by means of the method of "residual imputation." If it is desired, for instance, to compute the productivity of the factor land, the quantity or value of all the resources (other than land) that are employed is ascertained, and the cost, at current

²Details of the exact resource items chosen to include in the analysis are given later in this chapter.

³For linear programming and budgeting techniques of estimating efficiency more elaborate data are needed than can be obtained from the FBFM records.

market prices, of these resources is deducted from the total income of the enterprise or enterprises. (These costs include both cash costs and opportunity costs.) The income remaining, i.e., the residual amount is then regarded as the contribution of land to income. This imputed contribution of land may be compared to the price of land. In efficiency studies, the costs of land, labor and capital are frequently deducted, and the residual quantity is then imputed to the factor of management. This method of assessing resource productivity is extremely arbitrary. The arbitrariness of the residual imputation method becomes apparent when it is realized that any surplus over the "cost of production" may be attributed to each factor in turn, as the remaining factors are assumed to return only their cost.⁴

Whereas the residual imputation method assesses the productivity of each resource keeping the others constant, when a production function is used the marginal value productivities are assessed simultaneously. No assumption is made that marginal productivity equals price regarding any factor. Also, the residual imputation procedures give average resource productivities whereas the production functions give marginal productivities. Although both measures are useful in comparing the efficiency with which resources are used, marginal productivities give a better idea of what is possible by changing the quantities of factors used.

Therefore, in this study the production function procedure will be used to compare the efficiency with which resources were used.

⁴ It is assumed that for all factors that are held constant the marginal productivity is equal to price.

The Production Function

The relationship between the inputs of resources and the output of product may be written, in algebraic terminology as:

$$Y = f (X_1, X_2, \dots, X_n)$$

where Y is the quantity of output and X_1, X_2, \dots, X_n are quantities of the various inputs. The relationship is called the production function. The exact mathematical formulation of the production function for almost all production processes is highly complex, to which any manageable mathematical equation is likely to be, at best, only a reasonable approximation. In choosing an appropriate function one is guided by one's conception of the production process involved. For example, in the case of the relationship between fertilizer applications and yields, a simple linear relationship (i.e., that equal increments of fertilizer result in equal increments of yield) may be assumed over some small range of input and output. Such a relationship has the mathematical form

$$Y = a + bX,$$

where Y is yield, X is fertilizer input and a and b are constants. However, a hypothesis more in keeping with our knowledge of biological science for a more extended range of yield per acre is that as fertilizer is added in units of uniform size successive increments in yield diminish at a constant rate. This hypothesis can be formulated in mathematical terms as

$$Y = M - ar^X$$

where Y and X have the same meaning as before, M is the theoretical maximum yield and a and r are the constants. This formulation is known

as the "Spillman" function.⁵

The Cobb-Douglas type of production function has been used more frequently in attempts to express mathematically the input-output relationship of firms or industries and will be used in this study.

The Cobb-Douglas Production Function

The Cobb-Douglas function is of the form

$$Y = aX_1^{b_1} \cdot X_2^{b_2} \cdot \dots \cdot X_n^{b_n} U$$

which if transformed into logarithm, reduces to the simple linear equation:

$$\log Y = \log a + b_1 \log X_1 + b_2 \log X_2 + \dots + b_n \log X_n + \log U$$

Y is the output measured in value or physical terms and the $X_{1,s}$ are the inputs. The U is the error term. In its linear form the Cobb-Douglas function can be solved by the familiar means of least squares. This property simplifies the computational work involved in the use of the function. The function has a number of other properties, too, which, since they correspond to well-known economic axioms, enhance its value in an economic context.

1. The coefficient (b_1, b_2, \dots) associated with each factor input (X_1, X_2, \dots) corresponds to what is known in economic terminology as the elasticity of production of the factor. That is, it expresses the percentage change in output which results from a 1 percent change in the input of the factor.

⁵The Spillman function is also known as the Mit-Scherlich function. For its properties see Earl O. Heady and J. Dillon. Agricultural Production Functions, 1961, Ames, Iowa: Iowa State University Press, pp. 77-78 and 86-88.

Elasticities are independent of the unit of measurement of input and output and hence are directly compared one with another.

2. By adding together the elasticities associated with all the factors, it is possible to determine whether the production process as a whole yields constant, diminishing or increasing returns to scale. A sum of coefficients equal to unity indicates constant returns to scale, less than unity, diminishing returns to scale, and greater than unity, increasing returns to scale.
3. The marginal rate of substitution is a linear function of the ratio in which X_1 and X_2 are combined and is given by the equation:

$$\frac{\partial X_1}{\partial X_2} = \frac{-b_2 X_1}{b_1 X_2}$$

If inputs of X_1 and X_2 are increased in constant proportions, the marginal rate of substitution remains constant at the ratio b_2/b_1 even though the level of output changes. This condition is unrealistic for certain classes of inputs.

4. The marginal productivity of a resource, (say X_1), can be obtained directly from any production function by partially differentiating with respect to the factor concerned. In the case of the Cobb-Douglas function, this partial derivative is of the form

$$\frac{\partial Y}{\partial X_1} = a, b_1 X_1^{b_1-1} X_2^{b_2} \dots X_n^{b_n}$$

This relationship has two important properties:

- (a) It allows for diminishing marginal productivity of any factor i.e., it allows successive equal increments of a factor to give rise to successively smaller increments of output. This occurs when the elasticity of production, b_1 , of factor X_1 , is less than unity: then the coefficient (b_1-1) associated with X_1 in the equation above becomes negative, and hence the larger the value assumed by X_1 , the smaller the increment in Y associated with a given increment in X_1 .
- (b) The marginal productivity of any single factor is influenced by the level of input of the other factors. This is indicated by the presence in the partial derivative of terms $X_2, X_3,$
 $\dots X_n$.

Both these properties are in accord with the assumption of economic theory.

The Cobb-Douglas function also has a number of restrictive features. For instance, it does not allow for changing elasticities of production or of substitution to accompany changes in the size and ratio of factor inputs. These restrictions can be overcome by the use of general equations in the second degree but these involve much more computational work than the Cobb-Douglas type. For example, with six factors, six regression coefficients must be estimated for a Cobb-Douglas; 27 must be estimated for a function involving linear squared and cross-product terms for each factor.

Other problems associated with firm-level production function estimation are those of specification, identification, classification and measurement. These problems will not be discussed in detail here except

to define what they are.⁶

The specification problem (bias) occurs when some important input factor is omitted from the production function. When important variables are omitted from the function the estimates of included variables are biased. The bias is greater when the included and the omitted variables are related. The most commonly omitted variable is management.

The identification bias is mainly because of the choice of the functional form. The estimates of the coefficients will differ depending on what mathematical formulation is used to describe the relationship between the inputs and the output(s).

The classification problem is encountered when different inputs have to be aggregated. The problem is which input categories should be grouped together and which ones should not.

And finally, the measurement problem concerns what units of measurement should be used for inputs and output(s). For some inputs, especially land and labor measurements are given either in physical or monetary terms. Which one to use will depend upon the assumptions made about the system.

Perhaps also the last problem could be an accounting problem where farm records are used. The farmers do not gather the data for the purpose of research. So use of such data may result in errors of estimation.

All the problems described above have a general characteristic that they result in biased estimates. In spite of all the problems cited above

⁶For details on problems associated with the use of economic data for production function analysis see P. H. Parish and J. L. Dillon, "Recent Applications of the Production Function in Farm Management," Review of Marketing and Agricultural Economics, Vol. 23, No. 4, pp. 221-30, December, 1955.

we will make the bold assumption that the regression coefficients in the functions fitted for this study provide unbiased estimates of the population coefficients.⁷

Analysis

The Cobb-Douglas function was chosen to analyze the data. For each of the two periods, 1973 and 1977, the following function was used:

$$Y = aX_1^{b_1} X_2^{b_2} X_3^{b_3} X_4^{b_4} U$$

where

Y = value of farm production in dollars, defined as total cash sales of products plus or minus change in inventory.

a = the intercept

X_1 = total soil fertility cost in dollars. This includes the total outlay for chemical fertilizer.

X_2 = total labor cost in dollars. Total labor includes hired labor, the operator's labor and unpaid family labor.

X_3 = total machinery costs in dollars. These include utilities, repairs, machine hire, gas and oil, auto expense, and depreciation.

X_4 = total tillable land in acres.

The Least-Squares single equation was used to estimate the coefficients. As explained earlier in this chapter the coefficients, b_1 's, are the elasticities of production of the factors, X_1 's.

⁷This assumption, of course, implies the further assumption that such coefficients in fact do exist, that is, that all farms in the samples are operating on the same production function.

Marginal Productivities and Market Prices

From the Cobb-Douglas function, the marginal value products of resources entering the function are computed at the geometric mean level of output of the sample and at the geometric mean level of the particular resource being considered.⁸ That is, the MVP of a particular resource is computed at its geometric mean, all other resources being held constant at their geometric means. Computationally, this is very simple. If we have a function:

$$Y = aX_1^{b_1} X_2^{b_2} \dots X_n^{b_n}$$

where Y is production in money value, X_1 and X_2 are the individual resources measured either in money value or physical units, b_1 and b_2 are the elasticities of production of X_1 and X_2 respectively, then the marginal value product of X_1 , denoted MVP_{x_1} is found as follows:

$$MVP_{x_1} = b_1 \frac{\bar{Y}}{\bar{X}_1}$$

where \bar{Y} and \bar{X}_1 are the geometric means of Y and X_1 respectively. For X_2 ,

$$MVP_{x_2} = b_2 \frac{\bar{Y}}{\bar{X}_2}$$

Having obtained MVP_{x_i} the next step is to compare it to the market price of the i^{th} input, P_{x_i} . If $MVP_{x_i} \geq P_{x_i}$, the use of the input X_i

⁸The geometric mean is used because the estimating equations were derived after logarithmic transformation of the variables. The arithmetic mean could be used as well, but would, of course, be subject to larger errors of estimate. Normally the arithmetic mean is used in the case of linear production functions.

should be expanded, and conversely contracted if $MVP_{xi} \leq P_{xi}$.⁹ This is done successively for all resources entering the function.

For the present study the marginal value productivities are not going to be compared to the prices directly. If we assume that for all inputs $MVP_{xi} = P_{xi}$, then an elasticity b_i^* will be computed to satisfy this condition of equality between MVP_{xi} and P_{xi} at the geometric mean input and output levels. This b_i^* will be computed as follows:

Let us assume a production function where we are concerned with only one variable input. If this input is measured in money value and output is also in money value then the marginal productivity of this one input will refer to the return to one additional dollar of expenditure on the input concerned. If the output is denoted Y and the input X then the marginal value productivity is represented by MVP_x . From production function analysis

$$MVP_x = \frac{dY}{dX} \cdot P_y \quad (1)$$

where P_y is the price of output. Under the assumption of efficient use of the input X, that is $P_x = MVP_x$ equation (1) may be written as

$$P_x = \frac{dY}{dX} \cdot P_y \quad (2)$$

But from the Cobb-Douglas function $b_i = \frac{dY}{dX} \cdot \frac{\bar{X}}{\bar{Y}}$

substituting this knowledge into (2) gives

$$P_x = \frac{dY}{dX} \frac{\bar{X}}{\bar{Y}} \cdot \frac{\bar{Y}}{\bar{X}} P_y$$

⁹ $MVP_{xi} \leq P_{xi}$ may also mean some other resource is limited.

which then becomes

$$P_x = b \frac{\bar{Y}}{\bar{X}} \cdot P_y \quad (3)$$

Under our assumption of pure competition in both the factor and product markets efficiency of resource use is achieved where marginal factor cost (where the price of the input X , P_x) is equal to the marginal value product (where the price of the product P_y). Because in our work Y is always in money value and when most X variables are in money value P_x and P_y are both equal to one. In that case the elasticity, b^* , necessary to make equation (3) hold is easily computed by

$$b^* = \left(\frac{\bar{X}}{\bar{Y}} \right) \quad (4)$$

However, when an input is not measured in money value such as acres of land

$$b^* = P_x \left(\frac{\bar{X}}{\bar{Y}} \right) \quad (5)$$

Equation (4) will be used to obtain a b^* for soil fertility, labor, and machinery. Money value is used for these three resources. Land is given in acres so for land b^* will be obtained by equation (5). Equations (4) and (5) are solving for the elasticity value, b^* , that would equate the marginal value productivity of a resource to its price. After obtaining a b^* for each resource considered in this study it will be compared to the estimated empirical elasticity, b . The statistical significance of the difference between the ratio MVP_{xi}/P_{xi} and unity can then be tested using the t-test. The test statistic is obtained as follows:

$$t = \frac{|b_i - b_i^*|}{SE_i}$$

where SE_i is the standard error of the coefficient of the i^{th} resource. This will show based on statistical probability whether or not the i^{th} resource was efficiently used.

Apart from testing the equality between the marginal value productivity and the price of each resource individually, the null hypothesis that

$$\frac{MVP_{x_1}}{P_{x_1}} = \frac{MVP_{x_2}}{P_{x_2}} = \frac{MPV_{x_3}}{P_{x_3}} = \frac{MPV_{x_4}}{P_{x_4}} = 1$$

will also be tested. This allows for the resources to be considered together instead of separately. The F-statistic for the test is

$$F_{v_1, v_2} = \frac{(b_i - b_i^*)' X'X (b_i - b_i^*)}{ns^2}$$

where

$(b_i - b_i^*)'$ is a 1×4 row vector and is the transpose of the 4×1 column vector $(b_i - b_i^*)$.

$X'X$ is a 4×4 cross product matrix

n is the number of variables in the function, including the dependent variable.

s^2 is the sample variance.

To compare the efficiency of resource use between the two years studied the following procedure will be used.

Let us assume for the moment that only one resource is important out of the four to be estimated and denote this input simply as X . Define $b_i \frac{\bar{Y}_1}{\bar{X}_1}$ as the MVP of this resource determined from the production function

in 1973, and $b_2 \frac{\bar{Y}_2}{\bar{X}_2}$ as the MVP of this resource determined from the

production function in 1977.

If the resource was used to produce the same marginal value product in both periods then the relationship below, equation (6), would be expected to hold. Also since the price of the product is the same at one dollar, this would also denote the same efficiency of resource use in the two years.

$$b_1 \frac{\bar{Y}_1}{\bar{X}_1} = b_2 \frac{\bar{Y}_2}{\bar{X}_2} \quad (6)$$

Considering usual empirical differences the two MVP's are not likely to be exactly equal. Given the value of production and the total input at their geometric means for each year, \bar{Y}_1 , \bar{Y}_2 , \bar{X}_1 , \bar{X}_2 , and the elasticity of production of the resource in 1973, b_1 , an elasticity b_2^{**} can be computed to have the equality relationship in (6) hold. This can be represented as:

$$b_2^{**} \cdot \frac{\bar{Y}_2}{\bar{X}_2} = b_1 \frac{\bar{Y}_1}{\bar{X}_1} \quad (7)$$

Re-arranging (7) gives:

$$b_2^{**} = b_1 \frac{\bar{Y}_1 \bar{X}_2}{\bar{Y}_2 \bar{X}_1} \quad (8)$$

Given the product and the input of the particular resource as averages in 1977 we thus find what elasticity of production would have been necessary in 1977 to equate marginal value productivity of the resource in 1977 to the marginal productivity of the same resource in 1973.

Having computed b_2^{**} it will be compared to the estimated elasticity, b_2 . The t-test will be used, where

$$t = \frac{b_2 - b_1 \frac{\bar{Y}_1 \bar{X}_2}{\bar{Y}_2 \bar{X}_1}}{\sqrt{SE_{b_2}^2 + \frac{\bar{Y}_1 \bar{X}_2}{\bar{Y}_2 \bar{X}_1}^2 SE_{b_1}^2}}$$

SE_{b_1} and SE_{b_2} are the standard errors of b_1 and b_2 respectively and b_1 and b_2 are assumed to be independent since they come from cross section data of two different years.

If the difference between b^2 and b_2^{**} is statistically significant that would mean that the resource in question was used at statistically different levels of efficiency in 1973 and in 1977.

Where the efficiency of input use is significantly different in the two periods, we want to see whether the change in efficiency was toward or away from equilibrium from 1973 to 1977. This will involve comparing $b_i - b_i^*$ for the particular resource in both periods (b_i is the estimated elasticity of the i^{th} resource and b_i^* is the calculated elasticity of the same resource that would equate MVP_{xi} to P_{xi}).

Suppose that the first input factor was used at statistically different efficiency levels in both periods. If we let b_{11} and b_{11}^* be the estimated and calculated elasticities of this first input factor X_1 in 1973 and b_{12} and b_{12}^* be the estimated and calculated elasticities respectively for the same input factor in 1977, then we can define d_{11} and d_{12} as

$$b_{11} - b_{11}^* = d_{11}$$

$$b_{12} - b_{12}^* = d_{12}$$

If $d_{11} > d_{12}$ that would mean that the change in efficiency was toward

equilibrium but if $d_{11} < d_{12}$ it indicates that the adjustment from 1973 to 1977 was away from equilibrium with respect to the first input factor.

Whether production increased between 1973 and 1977 will involve direct comparison of total production in both periods.

The sum of the elasticities will give the nature of returns to scale that occurred in each period. Here a test of significance of constant returns to scale will be used for both periods the null hypothesis of $\sum b_i = 1$ against the alternative hypothesis of $\sum b_i \neq 1$ will be tested. The statistic for this test is

$$t_{n-k} = \left[\frac{b_1 + b_2 + b_3 + b_4 - 1}{S(b_1 + b_2 + b_4)} \right]^{10}$$

where $n-k$ are the degrees of freedom and b_1, \dots, b_4 are the estimated elasticity coefficients.

¹⁰For a two variable production function

$$S(b_1 + b_2) = \sqrt{S_{b_1}^2 + S_{b_2}^2 + \text{COV}(b_1, b_2)}$$

This formula may be generalized to include more than two variables.

CHAPTER IV

RESULTS

In this chapter the results of the data analyses are presented. These results are given in three parts. The first part gives results for the 1973 period, the second part gives results for the 1977 period, and in the third part the two periods are compared with respect to efficiencies of resource use and whether or not the subject farms moved toward greater efficiency of resource use and consequent equilibrium.

1973 Results

The 534 farms whose data were analyzed for the 1973 period ranged in size from 189 to 645 acres. The average size was 386 acres. Most of these farms produced both corn and soybeans. The results indicate that on the average these farms had about two-thirds of all tillable land per farm under corn and one-third under soybeans. The mean corn:soybean ratio was 2.75:1. The mean yields were 123 bushels per acre for corn and 40 bushels per acre for soybeans. The average prices received by these farmers were \$1.58 per bushel for corn and \$4.44 per bushel for soybeans, a price ratio of 1:2.81. Table 4.1 shows the cost per tillable acre on grain farms in 1973 and 1977. These figures are classified by the accounting procedures of the Illinois Farm Business Record Association.

The estimated production function for the 1973 period was:

$$Y = 94,072 X_1^{0.0696} X_2^{0.0844} X_3^{0.0553} X_4^{0.8412}$$

where again Y is the value of farm production in dollars. The value of production used is the one reported by the farmer.

TABLE 4.1 Cost per Tillable Acre - Grain Farms - No Livestock,
Central Illinois Standards, 1973 and 1977.

	1973	1977
Variable Costs:		
Soil Fertility	\$ 17.02	\$ 27.82
Seed and Crop	13.82	25.23
Machinery, Repairs & Supplies	7.01	9.30
Machinery Hire	2.07	2.02
Gas and Oil	<u>5.12</u>	<u>7.68</u>
Total Variable Costs	\$ 45.04	\$ 72.05
Other Costs:		
Labor	\$ 18.76	\$ 24.72
Buildings	4.07	6.28
Machinery Depreciation	14.50	25.75
Taxes	11.64	16.02
Interest on investment	55.05	124.13
Overhead (Ins., Misc., Lstk., Auto, Utilities)	<u>4.63</u>	<u>8.51</u>
Total Other Costs	\$108.15	\$205.41
Total All Costs	<u>\$153.69</u>	<u>\$277.46</u>

SOURCE: Illinois Farm Business Farm Management Records - Unpublished data from Western Sangamon Valley, Eastern, East Central and Pioneer Associations.

In another approach a value of production was obtained by multiplying the yields by the prices of each crop and then adding the total but this procedure gave inappropriate results and was not used.

X_1 is the cost of soil fertility in dollars,

X_2 is total labor costs in dollars,

X_3 is machinery expenditure in dollars,

X_4 is tillable land in acres.

In Table 4.2 the means and ranges of the five variables are presented for 1973 and 1977.

The variables correlation matrix is given in Table 4.3. The regression coefficients as well as other statistics are given in Table 4.4. The coefficient of multiple determination (R^2) was 0.8666 indicating that approximately 87 percent of the variation in the value of production was explained by the variations in the four explanatory variables used. The standard error of the estimate was 0.071. All the elasticities were significant at the 1 percent level. According to these elasticity figures a 1 percent increase in the land input would account for 0.84 percent of increase in the value of production, the largest contribution. And a 1 percent increase in machinery input would have increased the value of production by only 0.05 percent, the smallest contribution.

The sum of the coefficients was 1.050. The one-tailed test showed that this sum was significantly greater than one indicating increasing returns to scale. Increasing returns to scale means that returns would be increased more than costs at larger levels of operations, giving a larger profit per unit of input.

TABLE 4.2 Means and Ranges of All the Variables Used in the Production Function Estimation for 1973 and 1977.

Variable	Mean		Minimum		Maximum	
	1973	1977	1973	1977	1973	1977
Value of Production Y	\$98,610	\$96,241	\$19,723	\$9,090	\$218,621	\$257,372
Soil Fertility X_1	\$ 6,660	\$10,887	\$ 317	\$ 122	\$ 25,148	\$ 38,500
Labor X_2	\$ 8,206	\$10,796	\$ 536	\$ 450	\$ 17,525	\$ 47,605
Machinery X_3	\$28,408	\$55,471	\$ 4,676	\$4,742	\$ 81,854	\$227,530
Land X_4	386 acres	391 acres	189 acres	189 acres	645 acres	639 acres

TABLE 4.3 Variables Correlation Matrix - 1973.

		X_1	X_2	X_3	X_4
Soil Fertility	X_1	1.00000			
Labor	X_2	0.38854	1.00000		
Machinery	X_3	0.57554	0.49871	1.00000	
Land	X_4	0.67759	0.61377	0.75606	1.00000

TABLE 4.4 Regression Coefficients, Standard Errors, R^2 and Sum of Coefficients - 1973 and 1977.

			1973	1977
Soil Fertility	X_1	b_1	0.0696**	0.0415**
		SE_1	0.0154	0.0135
Total Labor	X_2	b_2	0.0844**	0.0110 ^{n.s.}
		SE_2	0.0221	0.0163
Machinery	X_3	b_3	0.0553**	0.0470**
		SE_3	0.0206	0.0157
Land	X_4	b_4	0.8412**	0.9194**
		SE_4	0.0322	0.0272
Sum of Coefficients			1.0505	1.0188
R^2			0.8666	0.8741
SE			0.0715	0.0686
Sample Size			534	642

* Significant at the 1 percent level.

n.s. non-significant at 10 percent level.

In Table 4.5 the estimated elasticity of each resource, b_1 , is given together with the calculated elasticity, b_1^* , that would have been necessary to equate the ratio of marginal value productivity and marginal factor cost to unity. When tested simultaneously the hypothesis that

$$\frac{MVPx_1}{PX_1} = \frac{MVPx_2}{PX_2} = \frac{MVPx_3}{PX_3} = \frac{MVPx_4}{PX_4} = 1$$

was rejected at the 10 percent level. However, when taken separately, there was no significant respective difference between the estimated b_1 and b_2 with the calculated b_1^* and b_2^* necessary for the relationship $\frac{MVPx}{Px} = 1$ to hold. This means that for 1973 we cannot reject the hypothesis that the input factors of soil fertility and labor were used in an economically efficient way. But for machinery and land the difference between the estimated elasticities, b_3 and b_4 , and their respective calculated elasticities, b_3^* and b_4^* , necessary for equating $\frac{MVPx}{Px}$ to unity were highly significant, (at the 10 percent level). Because of the significant negative difference between the estimated elasticity for the machinery input, b_3 , and the calculated elasticity, b_3^* , this suggests that in 1973 too much machinery was used for the optimum combination. But for land too little was used for the optimum combination since $b_4 - b_4^*$ is positive and significant. This large difference for land between b_4 and b_4^* may be explained by the relative fixicity of the land input. The farmer may be aware of increasing returns to land but additional land may not be available to the farmer, at least in the short run.

TABLE 4.5 Estimated Elasticities of Production, b_i , and Calculated Elasticities of Production, b_i^* , to Equate $\frac{MVPx_i}{Px_i}$ to Unity in 1973.^a

Variable	b_i	b_i^*	$(b_i - b_i^*)$
Soil Fertility X_1	0.0696	0.0619	0.0077 (0.4951)
Total Labor X_2	0.0844	0.0853	-0.0009 (-0.0009)
Machinery X_3	0.0553	0.2782	-0.2229 (-10.8189)
Land X_4	0.8412	0.3745	0.4667 (14.4837)

^aCalculated t-values are in parentheses.

1977 Results

The 642 farms analyzed for the 1977 period ranged in size from 189 to 639 acres. The average size was 391 acres. Just like in 1973 the farms included in the study produced corn and soybeans. And the results show that in 1977 the ratio of corn to soybeans produced was lower on the average, that is, the proportion of soybeans increased from 1973 to 1977 - about two-fifths of all tillable land, on the average, was planted in soybeans in 1977 as compared to one-third in 1973. Considering the change in the price ratio of corn and soybeans and also the fact that soybean yield per acre increased between these two periods one would expect an increase in soybean production and a decrease in corn production. The mean yields were 122 bushels per acre for corn, the same as in 1973, and 43 bushels per acre for soybeans, an increase of 3 bushels per acre above the

mean yield of soybeans in 1973. The average prices received during this period were \$2.01 per bushel for corn and \$6.42 per bushel for soybeans, a price ratio of 1:3.19. Costs of production per acre are shown in Table 4.1.

The estimated production function for 1977 was

$$Y = 145.549 X_1^{0.0415} X_2^{0.0110} X_3^{0.0470} X_4^{0.9194}$$

where Y, X_1 , X_2 , X_3 and X_4 have the same definitions as in the 1973 function.

The means and ranges of the five variables are presented in Table 4.2 along with similar figures for 1973. The variable correlation matrix is given in Table 4.6.

TABLE 4.6 Variables Correlation Matrix - 1977

	X_1	X_2	X_3	X_4
Soil Fertility X_1	1.00000			
Total Labor X_2	0.41724	1.00000		
Machinery X_3	0.55569	0.46147	1.00000	
Land X_4	0.67949	0.58247	0.72845	1.00000

The regression coefficients as well as other statistics are shown in Table 4.4. The coefficient of multiple determination (R^2) was 0.8741, indicating that approximately 0.87 percent of the variation in the value of production was explained by variations in the explanatory variables considered. The standard error of the estimate was 0.069. All the

elasticities expect for the labor input were significant at the 1 percent level. The elasticity of production with respect to labor was insignificant at the 10 percent level.

The sum of the coefficients was 1.0188. This sum was not significantly different from 1 at the 10 percent level, and therefore the null hypothesis that returns to scale were constant in 1977 is not rejected.

As was done for 1973, Table 4.7 gives the estimated elasticity of each input factor together with the corresponding calculated elasticity that would have been necessary to equate the ratio of marginal value productivity and marginal factor cost to unity.

TABLE 4.7 Estimated Elasticities of Production, b_i , and Calculated Elasticities of Production, b_i^* , to Equate $\frac{MVPx_i}{Px_i}$ to Unity in 1977.^a

Variable	b_i	b_i^*	$(b_i - b_i^*)$
Soil Fertility X_1	0.0415	0.1041	-0.0626 (-4.6271)
Total Labor X_2	0.0110	0.1131	-0.1022 (-6.3115)
Machinery X_3	0.0470	0.5424	-0.4953 (-31,5653)
Land X_4	0.9194	0.2968	0.6226 (28.4769)

^a Calculated t-values are in parentheses.

Here also the null hypothesis of equality of $\frac{MVPx_i}{Px_i}$ to unity was rejected

when the inputs were considered together. But for this year when taken

separately all of the ratios of marginal value productivities to marginal factor costs were significantly different from one as indicated by the large t-values in Table 4.7. For all the input factors considered other than the land input it seems too much was used for the optimum combination with respect at least to the amount of land available. But as in 1973 too little land was used for optimum combination in 1977 by the subject farms. This is evidenced by the significant negative differences of $b_i - b_i^*$ for soil fertility, labor, and machinery input factors and a significant positive difference of $b_4 - b_4^*$ for the land factor.

Comparison of Efficiency of Resource Use Between 1973 and 1977

At this stage comparisons in efficiency of resource use on the subject farms will be made between the two periods in question.

Before making comparisons it is worthwhile mentioning at this point that these comparisons would be more meaningful if exactly the same decision makers were analysed during both periods. The farm records used identify farms by (serial) numbers rather than by names of individual operators. Because of individual confidentiality, names are not available with the farm identification number. Also the number assigned to a farm remains the same even when the farm comes under a different operator. This makes it difficult to tell whether or not one is dealing with the same group of decision makers in both periods. However, because of the large samples used there is a high probability that most of the same decision makers operated the same farms in both periods under consideration.

As Table 4.2 shows the farms under consideration had the same size ranges in both periods. The average size per farm in 1977 was 391 acres -

only 5 acres above the average size of 386 acres per farm in 1973. From Table 4.2 it can also be seen that all the input factors studied were used at a higher level in 1977 than in 1973, with the machinery input factor almost doubling between the two periods from \$28,408 in 1973 to \$55,471 in 1977.¹ The soil fertility input was increased by 63 percent and the labor input increased by 32 percent. However, when taken in total months available the labor input increased only by a mere 0.8 percent between the two periods.² Although the maximum value of production per farm was higher in 1977, see Table 4.2, on the average the value of production was not much different in the two periods, \$98,610 in 1973 and \$96,241 in 1977 showing in fact that on the average the value of production did increase slightly between the two periods.

The average prices received by the farmers studied went up 27 percent per bushel for corn and 45 percent for soybeans between 1973 and 1977. The average yield per acre achieved was essentially the same in the two periods.³ The results also show that more soybeans were produced in 1977 than in 1973, the average proportion of tillable land per farm under soybeans was 40 percent in 1977 compared to 36 percent in 1973.

Let us now turn to the evaluation of the efficiency using the marginal concept as described in Chapter III.

¹Even when deflated by the appropriate indexes results still show that more of these inputs were used in 1977 than in 1973.

²Total labor available averaged 12.71 months per farm in 1973 and 12.82 months per farm in 1977.

³The mean yield for corn was 123 bu./A. in 1973 and 122 bu./A. in 1977. For soybeans the mean yield was 40 bu./A. in 1973 and 43 bu./A. in 1977.

In Table 4.8 the marginal value productivities of the resources considered are given for both periods. The individual marginal productivities were obtained using the formula

$$MVP_{ij} = b_{ij} \frac{\bar{Y}_j}{\bar{X}_{ij}} \quad (i = 1, 2, 3, 4)$$

$$(j = 1, 2)$$

MVP_{ij} = marginal value product of the i^{th} resource in period j .

b_{ij} = estimated elasticity of the i^{th} resource in period j .

\bar{Y}_j = geometric mean of the value of production in period j .

\bar{X}_{ij} = geometric mean of the i^{th} resource in period j .

TABLE 4.8 Marginal Value Productivities of the Resources Estimated from Production Functions.

Resource	Marginal Value Productivity	
	1973	1977
Soil Fertility X_1	0.640	0.395
Total Labor X_2	0.863	0.122
Machinery X_3	0.152	0.113
Land X_4	117.945	121.885

Both functions were re-estimated using constant dollar values for variables that were measured in monetary values.

As explained in Chapter III if we assume only one variable resource in both periods then we can drop the subscript i and define $b_1 \frac{\bar{Y}_1}{\bar{X}_1}$ as the

marginal value product of this resource in 1973 and $b_2 \frac{\bar{Y}_2}{\bar{X}_2}$ as the marginal

value product of the same variable resource in 1977. If this one variable resource was employed with the same efficiency in both periods then as explained in Chapter III we would expect $b_1 \frac{\bar{Y}_1}{\bar{X}_1}$ to be equal to $b_2 \frac{\bar{Y}_2}{\bar{X}_2}$ or

$MVP_1 = MVP_2$. Making this assumption of the same efficiency of resource use in both periods and given \bar{Y}_1 , \bar{Y}_2 , \bar{X}_1 , \bar{X}_2 and b_1 , an elasticity value b_2^{**} was computed to have the relationship below hold, that is:

$$b_2^{**} \frac{\bar{Y}_2}{\bar{X}_2} = b_1 \frac{\bar{Y}_1}{\bar{X}_1} .$$

The b_2^{**} may be defined as the elasticity of production with respect to the variable resource which would have been necessary in 1977 to equate the marginal value productivity of the resource in 1977 to the marginal value productivity of the same resource in 1973. For each of the resources considered in this study a b^{**} was calculated and was subsequently compared to the elasticity which actually was estimated by regression from the data obtained in 1977. Table 4.9 shows the results of these comparisons.

From the results in Table 4.9, we see that the soil fertility and the labor inputs were used at statistically different levels of efficiency in 1973 and 1977, whereas machinery and land inputs were used with statistically similar efficiencies in both periods. For the input factors that were utilized at different efficiency levels in both periods we want to see whether the change in efficiency from one period to the next was toward or away from equilibrium.

TABLE 4.9 Elasticities Estimated from the 1977 Production Function Results and the Calculated Elasticities for 1977 which would have been Necessary to Equate the Marginal Value Productivities of Each Resource in Both Periods.

Resource	Estimated Elasticity (b_{12})	Calculated Elasticity (b_{12}^{**})	t-value for $b_{12} - b_{12}^{**}$
Soil Fertility	0.0415	0.0673	1.746 ^c
Total Labor	0.0110	0.0806	1.669 ^d
Machinery	0.0470	0.0635	0.579 ^e
Land	0.9194	0.8896	0.681 ^e

^cSignificant at 10 percent level.

^dSignificant at 1 percent level.

^eNon-significant at 10 percent level.

Letting $d_{ij} = b_{ij} - b_{ij}^*$ denote the difference between the estimated elasticity of production with respect to the i^{th} input in the j^{th} period, b_{ij} , and the computed elasticity of production with respect to the i^{th} input in the j^{th} period, b_{ij}^* , that would be necessary to allow $\frac{MVP_{ij}}{Px_{ij}} = 1$.

d_{11} , d_{12} , d_{21} and d_{22} were computed for the soil fertility and labor input factors. It was found that $d_{11} < d_{12}$ and also that $d_{21} < d_{22}$. This indicates that the change in the efficiency with which these two input factors were used between 1973 and 1977 was away from equilibrium. This is evident also from the fact that in 1973 the test of $\frac{MVP_x}{Px} = 1$ showed

that for these two input factors $\frac{MVP_x}{P_x}$ was not significantly different from one. But in 1977 this relationship did not hold showing that the change in efficiency was away from equilibrium. As explained earlier too much of these input factors was used for an economically efficient combination in 1977. Although the differences in the efficiency of machinery and labor use were not statistically significant as indicated by the low t-values in Table 4.9, these differences showed a movement away from equilibrium as well. Too much machinery and too little land were used for economically efficient combination.

Although many of the subject farms might have been operated at a profitable level in both periods, if the total costs of production per acre are taken as those shown in Table 4.1, from the average farm sizes and the figures for the average value of farm production, it can be shown that on the average an income of \$101.78 was realized over and above the costs per acre in 1973 while a loss of \$31.32 was incurred in 1977.⁴ This also serves to indicate that there must have been a movement away from equilibrium resource combination between 1973 and 1977.

⁴These numbers were arrived at by subtracting the average cost per acre shown in Table 4.1 from the average return per acre obtained by dividing the mean value of production in each year by the corresponding mean farm size.

CHAPTER V
CONCLUSIONS

The conclusions made here may not be generalized to all farm operations in Illinois. This study considered only a selected group of farms as described in Chapter II. It should be borne in mind also that throughout the study the assumption of perfect competition in both the factor and product markets was made and the time variable as well as risk did not go into the analysis of the data.

The first objective was to compare the efficiency of production with respect to resource allocation between the periods 1973 and 1977. The results obtained indicate the rejection of the null hypothesis that:

$$\frac{MVPx_1}{Px_1} = \frac{MVPx_2}{Px_2} = \frac{MVPx_3}{Px_3} = \frac{MVPx_4}{Px_4} = 1 \text{ for both periods. This means that}$$

with respect to these inputs the allocation was different than the economically optimum combination in both periods. However, as evidenced by low t-values in Table 4.5 the labor and soil fertility inputs were used at a level not significantly different from economic optimum in 1973. Furthermore the results show that resource use was farther away from the economically optimum level in 1977 than in 1973 for all resources considered in this study. Hence it may be concluded that our subject farms were economically less efficient in 1977 than in 1973. It is a well-known fact, however, that farm managers likely seek to maximize returns over some period greater than one year. Thus, observed combinations of resources at any one given time may represent attempts to maximize returns over some period greater than the time observed (one year in this case). This is



most likely to occur in the case of longer term investment items which typically must be added in lumps. An example of such a possibility is the case of the purchase of additional land. The results have shown that in both periods too little land was used for economically optimum combination. While the subject farmers might have been aware of the high marginal productivity of land, perhaps a longer time than the four years from 1973 to 1977 considered here might have been required for them to re-organize their businesses or to combine inputs in an economically optimum manner. The high marginal value productivity could also be due to the limited size of the farms considered. Since the study was limited to farms of not more than 650 acres in size it is possible that farms analyzed for 1973 could not have been exactly the same ones analyzed for the 1977 period. Some farms included in the 1973 sample might have acquired additional land by 1977 so that they were omitted from the analysis in 1977. In which case the 1977 sample might have included new farmers.

For the machinery factor the results indicate that too much machinery was used in both periods. It is probable that these farms showed higher machinery costs because they had acquired additional machines with the hope of expanding their operations. Farm operators tend to believe that they have a better chance of acquiring control of more land if they have more machinery and equipment. Landlords also tend to favor leasing their land to operators who are well equipped. Another reason for higher machinery costs could be related to the relative lumpiness of this resource. Most farmers would rather have too much than too little. Timeliness of operations is also an important factor especially when the farmer's income

depends on the level of his production. To avoid losses in yields due to late operation a farmer might prefer to have higher machinery costs if such a situation will ensure timely operations. In this country tax laws encourage investment in durable assets. So that the high values for machinery reported by the subject farmers could be a reflection of tax credits and depreciation allowances for income tax purposes.

For the labor input the low marginal value productivity calculated for 1977 could be attributed solely to the increase in the wage rates. On the average the subject farms reported more or less the same amount of available labor (12 months). And this amount of labor signifies that these farms were one-man operations so that even if wages went up they could not reduce labor accordingly. From this it may be concluded that with the machinery costs reported, the average farm size was a little too small for a one-man operation.

Because the fertilizer input was quoted in value terms its higher cost reported in 1977 may just be due to the increase in the price level. In 1977 on the average more soybeans were produced than in 1973 and since soybeans do not have as high a requirement for fertilization as corn one would have expected this cost to be lower in 1977 than in 1973. It may then be concluded that on the average the soil fertility input factor was used at a higher level than required for optimum allocation in 1977.

The second objective was to see whether production did increase between the two periods. Although on the average less corn was grown in 1977 than in 1973 and the price of soybeans went up more than corn prices, the result reported show a slight decrease in the value of production between 1973 and 1977 where 1977 prices are deflated to 1973 dollars.

As for the returns to scale, the increasing return that occurred in 1973 meant that an increase in the level of the input use would have increased production more than proportionally. But it appears as if while farmers increased the level of the other three input except the land input during this period production did not increase significantly. This is because the land input did not accompany the increase in the level of other inputs considered here. If all inputs had been increased in the same proportion the value of production would have shown a more than proportionate increase. So the same may be said of the constant returns to scale that occurred in 1977. Increasing the level of input use by a certain proportion would result in an equal increase in the value of farm production. But it is clear that as long as farm size is restricted as in this study such increases in the value of farm production may not be observed.

Finally it should be noted that this method of computing marginal value productivity of resources at some observed level of all resources, and comparing these with the market prices (here done indirectly) can lead to incorrect conclusions about the direction of resource shifts required to bring about an optimum combination of resources. Generally to bring resource use to an optimum, fairly large, and certainly non-marginal, adjustments are necessary.

Summarizing, it can be stated that if all resources are considered variable, and the changes in resource allocation required or implied are non-marginal, then, from a given function, recommendations can be made for one resource only, using the conventional approach to these recommendations. Where the above circumstances hold (all resources

variable) and more than one resource is considered, an error may be introduced. This may be so because the marginal value productivity of any one resource is a function, not only of the level of that resource, but of the level also of all other resources in the production relationship.

LIST OF REFERENCES

- Anderson, J. R. "A Note on Some Difficulties in Response Analysis." Australian Journal of Agricultural Economics, Vol. 12, No. 1, pp. 46-53, June, 1968.
- Baumol, William J. Economic Theory and Operation Analysis. Prentice-Hall, 1965.
- Clarke, J. W. "Production Functions in Farm Management Research." Canadian Journal of Agricultural Economics, Vol. 2, No. 1, pp. 36-41, Spring, 1954.
- Dillon, John L. The Analysis of Response in Crop and Livestock Production. Oxford: Pergamon, 1967.
- Doll, John and Orazem, F. Production Economics, Theory with Applications. Grid Incorporated, 1978.
- Dulloy, J. H. "Resource Allocation and a Fitted Production Function." Australian Journal of Agricultural Economics, Vol. 3, No. 2, pp. 75-85, December, 1959.
- Griliches, Zvi. "Specification Bias in Estimates of Production Functions." Journal of Farm Economics, Vol. 39, No. 1, pp. 8-20, August, 1957.
- _____. "Measuring Inputs in Agriculture: A Critical Survey." Journal of Farm Economics, Vol. 42, No. 5, pp. 1411-1426, December, 1980.
- _____. "Research Expenditure, Education and the Aggregate Agricultural Production Function." The American Economic Review, Vol. 54, No. 6, pp. 961-974, December, 1964.
- Heady, Earl O. Economics of Agricultural Production and Resource Use. Prentice-Hall, 1952.
- _____. "Use and Estimation of Input-Output Relationships or Productivity Coefficients." Journal of Farm Economics, Vol. 34, No. 5, pp. 775-786, December, 1952.
- Heady, E. O. and Dillon, J. R. Agricultural Production Functions. Ames: Iowa State University, 1961.
- Hopper, D. W. "Allocation Efficiency in a Traditional Indian Agriculture." American Journal of Agricultural Economics, Vol. 47, No. 3, pp. 611-624, August, 1965.

- Jarrett, F. G. "Resource Productivities and Production Functions." Australian Journal of Agricultural Economics, Vol. 1, No. 1, pp. 65-78, February, 1957.
- Johnson, Glen L.; Heady, E. O.; and Hardin, L. S. Resource Productivity, Returns to Scale, and Farm Size. Ames: Iowa State College Press, 1956.
- Lawrence, Klein R. An Introduction to Econometrics. Prentice-Hall, 1965.
- Miller, C. M. "Comparative Efficiency of Farm Tenure Classes in the Combination of Resources." Agricultural Economics Research, Vol. 11, No. 1, pp. 6-16, January, 1959.
- Parish, P. H. and Dillon, J. L. "Recent Applications of the Production Function in Farm Management." Review of Marketing and Agricultural Economics, Vol. 23, No. 4, pp. 215-236, December, 1955.
- Plaxico, James C. "Problems of Factor-Product Aggregation in Cobb-Douglas Value Productivity Analysis." Journal of Farm Economics, Vol. 37, No. 4, pp. 664-675, November, 1955.
- Rueben, Buse C. and Bromley, Daniel W. Applied Economics: Resource Allocation in Rural America. Ames: Iowa State University Press, 1975.
- Shaw, R. and Heady, O. E. "Resource Returns and Productivity Coefficients in Selected Farming Areas." Journal of Farm Economics, Vol. 36, No. 2, pp. 243-257, May, 1954.
- Swanson, Earl R. "Resource Adjustment on 146 Commercial Corn-Belt Farms, 1936-53." Journal of Farm Economics, Vol. 39, No. 2, pp. 502-505, May, 1957.
- Uvleling, Edwin F. and Fletcher, L. B. "A Cobb-Douglas Production Function with Variable Returns to Scale." American Journal of Agricultural Economics, Vol. 52, No. 2, pp. 332-326, May, 1970.
- University of Illinois Co-operative Extension Service, Summary of Illinois Farm Business Records, 1973, Circular No. 1097 and 1977, Circular No. 1162.
- USDA, Crop Reporting Board, pr. 1-3 (78), pp. 28-29, December, 1977.