PRIMARY SCHOOL TEACHERS' SUBJECT MATTER KNOWLEDGE AND PEDAGOGICAL CONTENT KNOWLEDGE OF NUMBER BASES: A CASE STUDY OF LUSAKA DISTRICT

By

George Mulenga

(2015130647)

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DECLARATION

I, Mulenga George, do declare that the thesis hereby submitted is my own work and it has not previously been submitted for a degree, diploma or other qualification at the University of Zambia or any other learning institution, and that all sources are acknowledged.

Signed…………………………………..

Date…………………………………….

APPROVAL

This thesis by George Mulenga is approved as a fulfillment of the requirements for the award of degree of Doctor of Philosophy in Mathematics Education of the University of Zambia

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ABSTRACT

The study was inspired by the necessity to advance professional learning for primary school teachers that might support them to proficiently teach the mathematics topic of number bases. This was after a base line study conducted at National In-Service Teachers College (NISTCOL) in 2014 by the College Lectures in the Department of Mathematics amongst primary school teachers who came from all provinces of the country revealed that Grade 7 primary school teachers encounter challenges when teaching number bases. This qualitative case study evaluated the mathematics Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK) of Grade seven primary school teachers with a focus on Number Bases, a topic which consolidates the concept of place value, the basis for all arithmetic algorithms (Wilson, 2006). Dimensions of the Knowledge Quartet: foundation, transformation, connection and contingency (Rowland, Turner, Thwaites, & Huckstep, 2009) and the strands of mathematical proficiency (Kilpartick, 2001) were applied to code, interpret and discuss the findings. Thematic analysis of lesson observations, questionnaire, concept maps and interviews was done by applying the pre-specified categorisation scheme of the Subject Audit Instrument (knowledge Quartet). Initially thirty participants took part in the questionnaire. Out of these nineteen took part in the concept maps. Then based on the performance in the questionnaire and concept maps two high performers, two moderate performers and two low achievers were selected to take part in lesson observations and post-lesson observation interviews. The study was guided by four research questions:

1. What foundation knowledge of Number Bases do primary school teachers have?
2. How do primary school teachers transform the content knowledge they have into forms that are pedagogically convenient to their pupils when teaching Number Bases?
3. How do the primary school teachers connect instructions when teaching Number Bases?
4. What are the primary school teachers' contingency actions when teaching Number Bases?

With regard to the nature of primary school teachers' foundation knowledge the findings indicated that most primary school teachers were uncertain as to the status or importance of topic of number bases and when making instructional decisions they mainly relied on imitation of one another or trial and error, rather than relying on the theoretical foundation required to guide instructional decisions. On the nature of primary school teachers' transformation knowledge the findings indicated that in most cases examples were mathematically correct, selected intentionally, and matched the purpose of the lesson and that participants exhibited knowledge of various forms of mathematical representations such as tabular, symbolic and tally marks. With regard to the nature of primary school teacher's connection knowledge the findings revealed that during lesson observation participants mainly used a single method; they rarely used several methods when solving problems on number bases. Moreover, they could not change tack when teaching number bases, amidst pupils facing challenges in comprehending the number bases concepts. On the nature of primary school teachers' contingency knowledge of number bases the findings indicated that participants responded to student ideas mainly in two ways: *ignore* or *acknowledge but sideline.* In addition the findings revealed that most participants could not reflect in action of either example or representation being used during lesson enactment even when faced with challenges. In line with the research findings, a reference manual that could enhance primary school teachers' SMK and PCK in the context of number bases has been proposed. Practically, the study has made contributions to areas in which primary school teachers need assistance such as the aspect of relational approaches to teaching mathematics.

Recommendations for primary school teachers, Institutions training primary school teachers and for further research have been made. For instance, in order to enhance professionalism in the teaching of number bases primary school teachers should be sensitized on how to notice and interpret key features of classroom interaction as well as ensuring that they acquire skills of stopping to reflect-in-action and change tack (example or representation being used) amidst challenges.

Key words: *Knowledge Quartet, Foundation, Transformation, Connection, Contingency, Number bases, Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK)*

DEDICATION

To my late wife (Edith) who is always in my mind as well as to my children Bwalya, Chanda, Mapalo, Subilo and Kachimfya, I say I love you all very much.

Also, to my beloved parents, Petronella Musabanga Mulenga and Brighton Mulenga, I say thank you very much for your great contribution to my education.

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I gratefully acknowledge the support of all the Head teachers of the visited primary schools who welcomed me into their schools and allowed me to administer the research instruments. I am thankful to all the primary school teachers who participated in this study: without whom this research would not have been possible. The high level of sincerity and commitment I encountered with participants encouraged me throughout this study. Thank you all for your precious contributions.

Special thanks to my late wife (Edith) and my Children Bwalya, Chanda, Mapalo, Subilo and Kachimfya, without their love and support, I would not have been able to do this. More importantly, I would like to thank the almighty Lord, for it is written: *"Giving thanks always and for everything to God the Father in the name of our Lord Jesus Christ," Ephesians 5: 20***.**

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ABBREVIATIONS AND ACRONYMS

CPD	Continuing Professional Development
KQ	Knowledge Quartet
MKT	Mathematical knowledge for teaching
MOE	Ministry of Education
NISTCOL	National In-service Teachers' College
PCK	Pedagogical Content Knowledge
PK	Pedagogical Knowledge
PST	Primary School Teachers
SACMEQ	Southern and Eastern Africa Consortium for Monitoring Education Quality
SMK	Subject Matter Knowledge
BESSIP	Basic Education Sub-Sector Plan

OPERATIONAL DEFINITION OF TERMS

The following terms have been used in the research as follows:

Pedagogical Content Knowledge (PCK)*:* In this study, Pedagogical Content Knowledge refers to the knowledge of teaching and learning that spans the topic of number bases (Rowland et al., 2005).

Subject Matter Knowledge: In this study, Subject Matter Knowledge (SMK) refers to the depth and breadth of knowledge in the topic of number bases (Rowland et al., 2005).

Number bases is a mathematical topic which refers to positional representation system in which a real number is represented by an ordered set of characters where the value of a character depends on its position (Ashley, 2015).

Knowledge Quartet (KQ) is a theoretical framework for the analysis and development of mathematics teaching (Stevenson, 2013).

Primary School Teachers refers to someone who teaches grades from grade one to grade seven

CHAPTER ONE

INTRODUCTION

Introduction

This study explored the Mathematics Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK) of Grade seven primary school teachers with a focus on the topic of Number Bases. Thus, the participants in my study were the Grade seven primary school teachers. The key theory that notified my study is the Knowledge Quartet (KQ).

This chapter presents the context and background of the study, specifies the problem of the study, describes its significance, and presents the research objectives, outlines the research questions that this study sought to answer, gives key definitions and provides the theoretical and conceptual framework.

1.1 Context and background to the study

According Carmody (2020) teacher education by Christian missionaries in Zambia started in 1890. Carmody (2020) points out that teacher education has evolved from being a form of apprenticeship in the villages in the days now past to being included in the university curriculum today. Carmody (2020) explains that historically various attempts were made to professionalise teaching in Zambia from the dawn of Christianity when the teacher saved as an evangelist or catechist of the Christian missionaries, to the colonial days when the teacher became the civil servant through, to the present days of heightened calls to professionalise teaching. Therefore, in Zambia teacher education has translated from the initial Christian missionaries' evangeristic agenda to fulfilling the national human capital needs especially after independency when Zambia was in urgent need of human resource (Carmody, 2020). For someone to fully understand how the teaching and learning of topics such as Number Bases is done it is important to understand how the teaching and learning of mathematics in Zambia has evolved from evangelism days to the present professionalism days. According to the Ministry of Education (1996) after independency the government embarked on a vigorous programme of training teachers. Phiri (2011) notes that at the end of the 1990s, the Government of Zambia implemented an ambitious plan, the Basic Education Sub-Sector Plan (BESSIP) for 1999 -2002 to improve access and quality of basic education. According to Phiri (2011) under BESSIP, the Ministry of Education recruited and trained large numbers of new primary school teachers using a new teacher training package called the Zambia Teacher Education Programme (ZATEC). Phiri (2011) notes that under ZATEC students used to learn for one year and then go for teaching practice for the whole of the second year. Primary School Certificates were offered after a student teacher successfully completes ZATEC. Phiri (2011) notes that subject learnt under certificate course were:

- Mathematics, which was learnt every day and comprised Content and Methodology
- Science (Environmental and Agriculture),
- Technology studies (Home craft Subject, Industrial Arts Expressive arts)
- Languages,
- Social Spiritual and Moral Education,
- Social studies (geography, history, Religious Education, Civics)
- Expressive Arts(Physical arts, Music, Art and Design)

In Zambia, previously all primary school teachers had Primary School Certificates (MOE, 1996). However, according to the Ministry of Education, Science, Vocational Training and Early

Education (2013) the trend has changed and currently all primary school teachers are expected to have a diploma as a minimum qualification. Therefore, training institutions for primary school teachers are offering either a primary teacher diploma and/or degree. Training institutions for primary school teachers offering diploma include Kasama, Malcolm Moffat, Solwezi, Mongu, Chipata, Chiles Lwanga, Kitwe and Mansa colleges of Education. Training institutions for primary school teachers offering degree include the University of Zambia, Malcolm Moffat college of Education and Chalimbana University. According to Phiri (2011) candidates for primary school teacher training are required to have five credits or better including English and Mathematics. Thus, Mathematics is one of the key subjects and it is important that every primary school teacher has sound Subject Matter Knowledge and Pedagogical Content Knowledge of all the mathematics topics taught in primary schools.

The Curriculum Development Centre (2013) articulates six content domains in the area of learning Mathematics in primary schools. Table 1 indicates the six content domains and their respective topics:

Table 1: Mathematics strands and their respective topics

S/N	STRANDS	TOPICS
1	Number and Calculations	Numbers and Notation, Addition, Subtraction, Multiplication, Division, Combined operations, Fractions, Decimals, Factors and Multiples, Social and Commercial Arithmetic, Index notation, Approximation, Ratio and Proportion, Percentages, Integers, Number Bases and Number Sequences
2	Algebra	Equations, Inequalities, ratio and propotion and pattern and relationships
3	Geometry	Angles, Plane Shapes, and Solid shapes
4	Measures	Measures
5	Probability and Statistics	Statistics
6	Relations	Relations

Source: Curriculum Development Centre (2013)

Thus, from Table 1 we see that most areas of mathematics are covered and that the topic Number Bases is placed in the Number and Calculations strand.

In addition to mathematics other subjects taught at primary schools are:

- Science,
- English
- Zambian Languages,
- Social Development Studies (R.E, History, Geography),
- Creative and technology Studies (Art and Design, Home Economics and Needle work)

Thus, a variety of subjects are taught at primary schools which might produce pupils who are well vested in many areas.

In a week generally the timetable for government primary schools is as follows:

- 7 periods per day
- Each period is 40 minutes
- There are 35 periods per week for all subjects
- Mathematics is taught every day and it has 7 periods per week

The Ministry of Education (2013) notes that despite Zambia recording significant success in access since the 1990s there have been several concerns raised about the quality of education. As reported by MOE (2013) concerns on the quality of education have mostly been based on the findings from the Ministry of Education's bi-annual national assessment surveys, which have been conducted since 1999. The surveys indicate that there has been very little progress in quality, as measured by the examination scores of learners. Moreover, MOE (2013) reveals that the Southern and Eastern Africa Consortium for Monitoring Education Quality (SACMEQ) assessments of Grade 6 learners indicate that the performance of Zambian learners lies close to the bottom regionally. In the 2007 assessment, the average reading performance of Zambian learners (434) was below the SACMEQ overall average (512) and the average mathematics performance (435) was below the SACMEQ overall average (510), (MOE, 2013). As pointed out by MOE (2013) generally the quality of education in Zambia is low. The teaching and learning of Mathematics is not an exception. Despite the fact that one of the aims of the mathematics curriculum at primary school level is to provide pupils with mathematical knowledge and skills necessary for terminal and further education (Curriculum Development Centre, 2000), research has revealed that most pupils have low levels of self-confidence in learning mathematics (Phiri, 2011). Similarly, the Research Triangle Institute (RTI) (2015) reports that results from the Early Grade Reading Assessment (EGRA) and Early Grade Mathematics Assessment (EGMA) in Zambia strongly suggest that the teaching of mathematics is focused on the memorization of facts, rules, and formulas. According to the Research Triangle Institute (2015) report EGMA in Zambia has shown that pupils are unable to apply their memorized knowledge, and therefore, they are not well prepared to learn more advanced mathematics in the higher grades. Pupils perform well on the more procedural tasks (basic addition and subtraction facts), yet they struggled to apply this procedural knowledge to solve tasks that are more conceptual in nature (RTI, 2015). Correspondingly, as shown in Table 2, the 2018 Examinations Council of Zambia report reveals that results of the surveys conducted amongst grade five pupils over the years indicate that mean performance in mathematics remains below 40 percent mark.

Table 2: Grade Five Pupils Mean Performance across the Surveys – (1999 to 2017)

Years	Average mark (%)
1999	34.3
2001	35.7
2003	38.5
2006	38.5
2008	39.3
2012	38.3
2014	35.8
2015	35.5
2017	36.7

Source: Examination Council of Zambia (2018) Report.

Therefore, there is need to build a firm foundation in mathematics education at an early stage (Primary school level) through conducting studies like this one, which evaluates primary school teachers' SMK and PCK.

The major goal of mathematics education is to improve pupils' understanding of mathematics, and the main factor in achieving this goal is the mathematics teacher (Dabiri, 2003; Green, 2014).

"The single factor which seems to have the greatest power to carry forward our understanding of the teacher's role is the phenomenon of teachers' knowledge," (Green, 2014). Similarly, McDonough (2007) indicates that poor subject matter knowledge in mathematics has a negative impact on the learning and teaching of mathematics. Also, Dabiri (2003) notes that the education community recognises that subject matter content knowledge and pedagogical content knowledge form the basis for effective teaching. Therefore, mastery of the subject matter by all teachers is an absolute necessity for effective teaching and they must possess a basic qualification in the subject (Sidhu, 2013).

Nonetheless, the baseline study conducted at National In-service Teachers College (NISTCOL), in 2014 revealed that most of the in-service primary school teachers considered the topic of Number bases to be mystifying. They had challenges with teaching the mathematics topic of Number Bases. Table 3 indicates number of in-service primary school teachers who had difficulties with some topics.

Table 3: Number of in-service primary school teachers who had difficulties with some topics

Topic	**Number of in-service primary school teachers who considered a particular topic to be difficult**	**Percentages (%)**
Number bases	216	66.26
Number line	53	16.26
Fractions	21	6.44
Long Division	15	4.6
Angles	9	2.76
Other topics	12	3.68
TOTAL	**326**	**100**

As can be seen in Table 3, most of the in-service primary school teachers who took part in the survey considered the topic of number bases to be very difficult to teach. Majority of the primary school teachers interviewed expressed keen interest in a well- researched guidance about how they could improve their own SMK and PCK in the area of Number Bases in an effort to enhance their pupils' standard of achievement and performance. Definitely, such a research would be in line with the American National Council for Curriculum and Assessment (2006: 29) consultation document which indicates that "students need their best teachers at a young age, teachers who really know what they are doing and really understand the simplicity of what they are doing. Similarly, both the Ministry of General Education (2020) midline survey report and Ministry of General Education (2018) baseline study affirm that number bases is among the topics primary school teachers find difficult to teach. Figure 1 shows the topics primary school teachers find difficult to teach according to MOGE (2020) report.

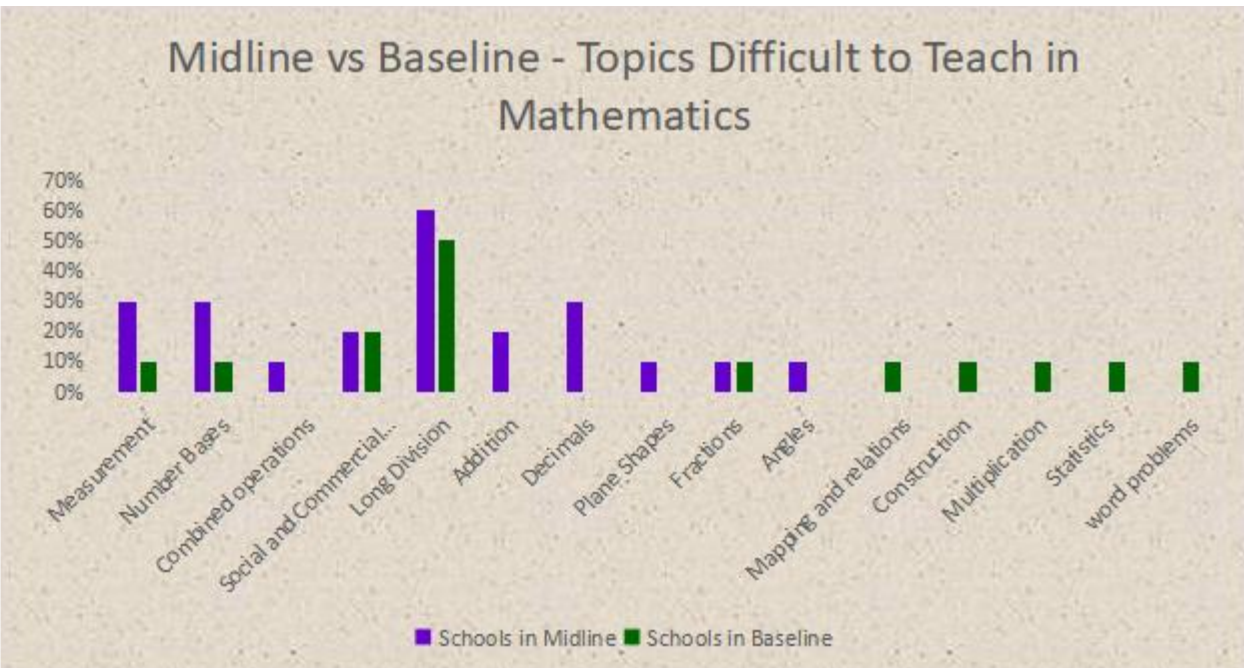


Figure 1: Topics difficult to teach in Mathematics in primary schools (MOGE, 2020)

The difficulties experienced by most primary school teachers when teaching Number Bases (a topic taught in grade seven) is a pity since, as contended by Hill et al., (2005), primary school mathematics is a very important gateway to secondary school mathematics and college enrollment. This view is supported by Sidhu (2013) who said that the primary school years can be a predictor of success. Such a dilemma about primary school teachers SMK and PCK is not only in Zambia, but also in other countries. For instance, Green (2014) highlights that internationally discontent exists regarding the low standard of mathematics subject matter knowledge amongst both qualified and prospective primary school teachers. Sidhu (2013) notes that problems associated with teachers' mathematical content knowledge are particularly common in the Primary and early Child sector where generalist teachers often lack confidence in their own mathematical ability. Owing to the challenges of teaching the topic of Number Bases, primary school teachers may not fully impact the expected mathematical knowledge on their learners. Consequently, this might affect the actualization of the 1996 Ministry of Education Mission Statement which states that:

> *The mission of the Ministry of Education is to guide the provision of education for all Zambians so that they are able to pursue knowledge and skills, manifest excellence in performance and moral uprightness, defend democratic ideals, and accept and value other persons on the basis of their personal worth and dignity, irrespective of gender, religion, ethnic origin, or any other discriminatory characteristic (Ministry of Education, 1996, p. 1*).

Accordingly, with such unresolved dilemma, it would be difficult to manifest excellence in performance despite it being an integral component of the mission statement.

Green (2014) advises that the sufficiency of Primary school and early Child school teachers' knowledge of mathematics, for their own professional purposes, cannot by any means be taken for granted. To add up all, as Dabiri (2003) notes that the implication of the everyday thinking studies, and studies on teachers' pedagogical content knowledge in mathematics is that teachers ought to be conversant with the subject matter knowledge they intend to teach. This should include appropriate understanding of the content, to an extent that enables the teachers to unpack the content during their teaching practice (Kilpatrick, et al, 2001). Moreover, just as Sidhu (2013) contends, this study like any other study with a similar focus on teachers' SMK and PCK, do not only bring content back to the center of primary school teachers' knowledge, but also bridges content knowledge and the practice of teaching.

1.2 Problem Statement

The base line study conducted at National In-service Teachers College (NISTCOL) in 2014 by the College Lecturers in the Department of Mathematics amongst in-service primary school teachers who came from all provinces of the country revealed that, for some time several in-service primary school teachers have been facing challenges with regard to teaching the topic of Number bases. Three hundred twenty six primary school teachers took part in the survey and out of these the majority (216) that is 66.26 % indicated that they faced challenges with the topic of Number Bases (see Table three, page 7 for detail). Similarly, the Ministry of General Education (2020) midline survey report affirms that number bases is one of the prominent topics primary school teachers find difficult to teach. This situation is very unfortunate especially in the light of Hodgen (2011) who posit that students need their best teachers at a young age. That is teachers who really understand the simplest of what they are doing (Hodgen, 2011). It is in this regard that by conducting this study the researcher opt not to be a mere spectator but to be part of a problem solver to what is happening to the teaching and learning of mathematics, especially with regard to the mathematics topic of number bases, in most primary schools in Zambia. To the researcher's knowledge, very little is known on the primary school teachers' Subject Matter Knowledge and Pedagogical Content Knowledge on number bases.

1.3 Purpose of the study

The purpose of this study was to evaluate the mathematics Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK) of Grade Seven primary school teachers with a focus on number bases. This would enable the development of a proposed reference manual that could enhance primary school teachers' knowledge of number bases and that might support them to proficiently teach the mathematics topic of number bases.

1.4 **Objectives of the study**

The objectives of this study were as follows:

1. To determine primary school teachers' foundation knowledge on number bases
2. To assess the primary school teachers' capacity to transform the content knowledge they have on number bases into forms that are pedagogically convenient to their pupils when teaching.
3. To assess how the primary school teachers' connect instructions when teaching number bases
4. To determine the primary school teachers' contingency actions when teaching number bases

1.5 Research questions;

To address the above research objectives this study sought answers to the following questions which had been framed in line with the four broad dimensions of the knowledge quartet, the theoretical framework for my study.

General question:

What are primary school teachers' Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK) of Number Bases?

Sub questions:

The first sub question is based on the foundation dimension, the second sub question is based on the transformation dimension, the third sub question is based on the connection dimension, and the fourth sub question is based on contingent dimension. For more explanation of the four Knowledge Quartet (KQ) dimensions see Table 5, page 75.

1. What foundation knowledge of Number Bases do primary school teachers have?
2. How do primary school teachers transform the content knowledge they have on Number Bases into forms that are pedagogically convenient to their pupils?
3. How do the primary school teachers connect instructions when teaching Number Bases?
4. What are the primary school teachers' contingency actions when teaching Number Bases?

1.6 Significance of the study

It is anticipated that the findings of this study might be beneficial: In providing necessary and vital information which primary school teachers can apply to create an environment that is conducive for each individual primary school pupil to demonstrate an understanding of the topic of Number Bases.

To come up with ways of making Number Bases teaching both more purposeful and more interesting for pupils in grade seven.

To provide information to researchers and educators to adequately look at effective preparation and support programmes for primary school teachers to efficiently and effectively teach the topic Number Bases.

To come up with empirically based findings for productive discussion of mathematics content knowledge, between teacher educators, trainees and primary school teachers.

To contribute to the research literature, especially to the local and international research literature, on primary school teachers' subject matter knowledge and pedagogical content knowledge of Number Bases.

1.7 Theoretical framework for the study

The theoretical framework for the study is the Knowledge Quartet (KQ). The KQ is a theoretical framework for the analysis and development of mathematics teaching (Stevenson, 2013). The Knowledge Quartet was developed by Rowland, Thwaites, and Huckstep in 2003 (Stevenson, 2013). The purpose of the research in which the KQ emerged was to develop empirically based conceptual framework for lesson review discussions with a focus on the mathematics content of the lesson and the role of the trainees in mathematical subject matter knowledge as well as the pedagogical content knowledge (Rowland et al, 2005). The KQ has been used as a theoretical framework because, for the KQ the categories of knowledge are not the focus; instead how this knowledge can potentially be observed in practice is the motivation (Adams, 2015). Besides, Rowland et al., (2003) claim that to gain a full picture of a teacher's knowledge, it is not sufficient merely to ask the teacher to complete questionnaire items, but researchers must also observe the teacher in action. They claim that different features of a teacher's knowledge come together "in the teaching moment" and are not fully accessible outside of this context (Rowland et al., 2005, p.20). Rowland et al., (2003) contends that the knowledge quartet can be used as a framework for lesson observation and for mathematics teaching. Therefore, since this study includes lesson observation, the knowledge quartet theory perfectly suits this study. In supporting this view Watson (2008) contends that mathematical knowledge is something which is not passive but active. Perhaps, lesson observation provides a better picture on mathematical knowledge of a teacher. For the same reason even in Zambia, as part of training, student teachers go for school experience where they are assessed on their levels of competence with regard to PCK. The connection between this study and the knowledge quartet is twofold i) They both address/assess teachers' knowledge in mathematics (ii) They both look at teachers who are in elementary levels. More importantly, the knowledge quartet is intended as a tool to support teacher development, with a sharp and structured focus on the impact of their SMK and PCK on teaching (Rowland et al., 2005). This study intends to achieve a similar objective. Rowland et al. (2005) found that episodes within a lesson can be understood in terms of two or more of the four units; for example, a contingent response to a pupil's suggestion might helpfully connect with ideas considered earlier. The four units: Foundation, Transformation, Connection and Contingency with their corresponding codes have been used as constructs for this study.

Despite, choosing the knowledge quartet as the theoretical framework for this study, the conceptual framework (see Section 1.8, page 15) has also included aspects of the MKT and Mathematical Proficiency for Teaching Framework. This is in order to include some aspects of expertise to the study which are found in these frameworks but not in the Knowledge Quartet. For instance, the strands from the Mathematical Proficiency for teaching framework assisted in the construction of the questionnaire items. This is an area which the Knowledge Quartet on its own could not effectively be applied as the focus of the KQ framework is on lesson observation and not questionnaire or other study instruments such as concept maps.

1.8 Conceptual Framework

This Section presents the conceptual framework.

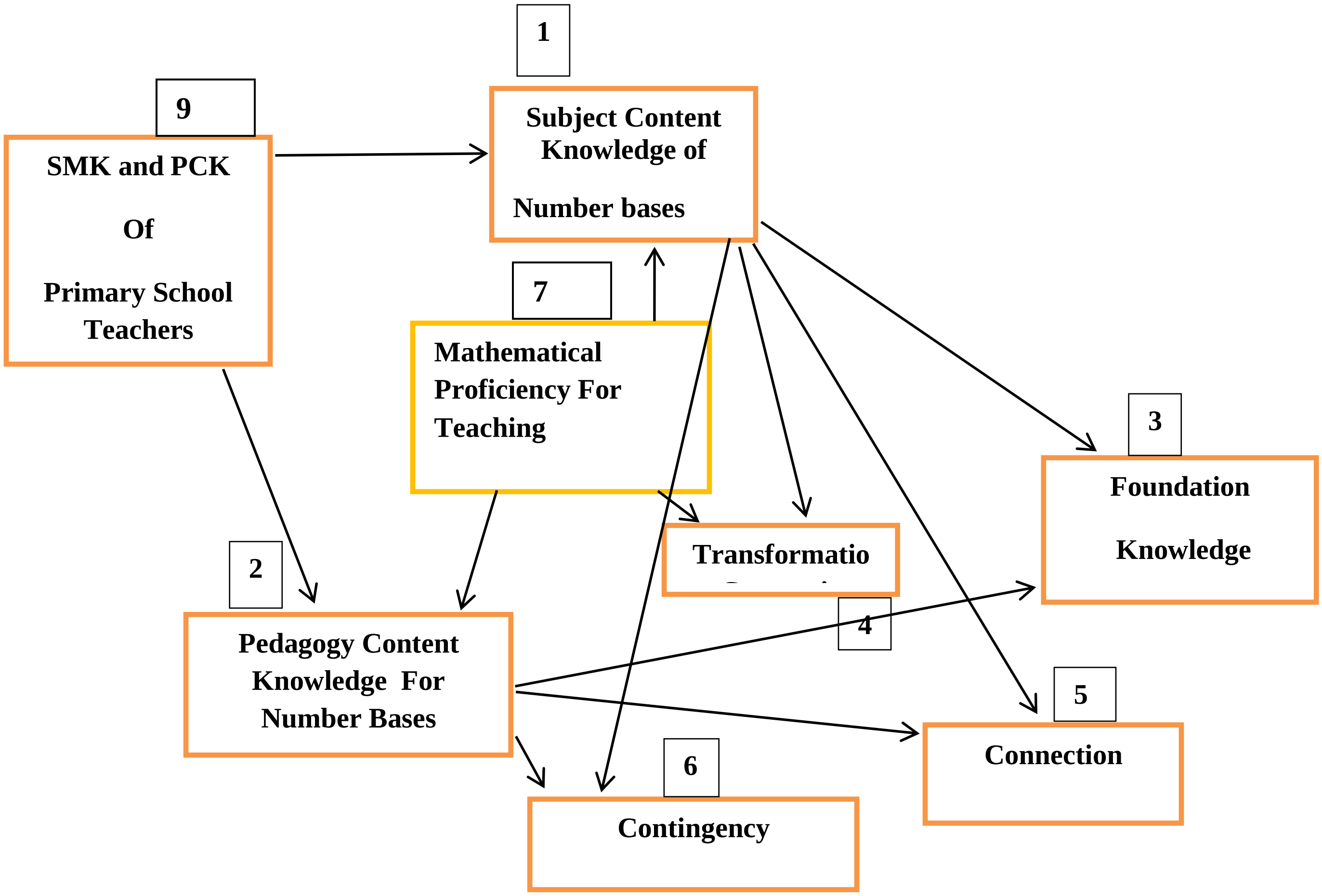


Figure 2: Conceptual framework for the study

The conceptual framework indicated in Figure 2 has emerged from the literature reviewed in Chapter two. The conceptual framework highlights proposed relationships between main variables related to SMK and PCK of Grade 7 primary school teachers as they engage with Number Bases. The arrow from construct 1 to construct 3 shows that construct 3 has some aspects of 1 (Foundation knowledge has some aspects of subject matter knowledge), similarly the arrow from construct 2 to construct 6 shows that construct 6 has some aspects of construct 2. The arrow from construct 2 to construct 5 means that construct 5 has some aspects of construct 2.

Thus, for this conceptual framework if the arrow moves from a given construct let us say from construct A to Construct B it means that construct B has some aspects of construct A. The notion of SMK and PCK came from the works of Shulman (1986) and O' Meara (2010). Constructs 1 and 2 were obtained from the literature in particular from the works of Hawkins (2012). Construct 3, 4, 5 and 6 were obtained from the theoretical framework, that is, the Knowledge Quartet. Construct 7 was obtained from Kilpatrick, Blume and Allen (2006) framework entitled '*Mathematical Proficiency for Teaching'*. The conceptual framework served as a guide to realise the study's intent. These constructs in the conceptual framework were used in the interpretation of data. All these constructs were key in exploring primary school teachers PCK and SMK of number bases as they contain aspects of teachers' knowledge and skills.

1.9 Delimitation of the study

This study was restricted to eight primary schools of Lusaka province. I used eight primary schools in order to have an in-depth understanding of primary school teachers' SMK and PCK in the context of number bases. All these eight schools had grade seven primary school teachers as the topic of number bases is taught in grade seven.

1.10 Limitations of the study

One way of data collection was through lesson observations, though this has the advantage of not relying on people's willingness or ability to provide information as well as allowing the researcher to directly see what people do rather than relying on what people say they did, nonetheless, it has a weakness of being susceptible to what Powell (1996, p. 2) refers to as the

"Hawthorne effect," that is, people usually perform better when they know they are being observed. This is the limitation inherent in the method.

Another limitation was owing to the fact that this study is an example of a small-scale study therefore the quality of psychometric properties of the data collected by a questionnaire and lesson observations might be affected (Powell, 1996). In addition, interviewing the primary school teachers on the basis of their responses on the questionnaires would have enriched the focus of the study in that it could have helped to get a better idea of why a particular participant could have come up with a given answer. Nonetheless, doing so would have been a drainer on the researcher and as well as on the participants who were also expected to take part in post lesson observation interviews, questionnaire, concept maps construction and lesson observations.

1.11 Structure of the thesis

This thesis is structured into six chapters as follows:

Chapter One introduces the study by giving the background and context of the study. The chapter also presents key areas such as the statement of the problem, the purpose of the study, research objectives and research questions, significance of study, theoretical frameworks, conceptual framework, delimitations and limitations of the study including operational definitions of terms.

Chapter Two discusses review of related literature, structured into Sections as follows: Meaning and importance of Number bases, Aspects of Subject Matter Knowledge and Pedagogical Content Knowledge, models and theories of teaching mathematics and the Literature summary and Illuminating the gap.

Chapter Three presents information on the research paradigms, research approaches, research designs, sampling procedure, data collection, data analysis and reflection on ethical issues.

Chapter Four presents the findings of the research which sought to explore the mathematics subject matter knowledge (SMK) and pedagogical content knowledge (PCK) of primary school teachers with a focus on Number Bases. The presentation of findings is guided by the following research questions: What foundation knowledge of number bases do primary school teachers' have? How do primary school teachers transform the content knowledge they have on number bases into forms that are pedagogically convenient to their pupils? How do the primary school teachers connect instructions when teaching number bases? What are the primary school teachers' contingency actions when teaching number bases? and How do primary school teachers' foundation, transformation, connection and contingency knowledge relate to each other towards the development of a model that could render them competent teachers in number bases?

Chapter Five discusses the findings of the study. It starts by discussing the foundation knowledge of number bases that primary school teachers do possess. This is followed by a discussion of how primary school teachers transform the content knowledge they possess into forms that are pedagogically convenient to their pupils when teaching number bases. The third discussion is on how the primary school teachers connect instructions when teaching number bases. The fourth discussion is on the nature of the primary school teachers' contingency actions when teaching number bases, The last discussion is on how primary school teachers' foundation, transformation, connection and contingency knowledge relate to each other towards the development of a model that could render them, competent teachers in number bases?

Chapter Six presents the conclusions and the recommendations based on the findings of the study.

1.12 Summary

This Chapter presented the context and background related to the teaching of number bases in primary schools. Among other items the Chapter presented, the statement of the problem, purpose of the study, research objectives and research questions, significance of the study, theoretical frameworks which is based on the Knowledge Quartet, conceptual framework, delimitations and limitations of the study, operational definitions of terms, and structure of the thesis. The next Chapter focuses on reviewing of the related literature that provided gaps and background to the study.

CHAPTER TWO

REVIEW OF RELATED LITERATURE

2.1 Introduction

The style to this literature review is as suggested by Maxwell (2006): presenting a Section of literature chosen because of its relevance to the study, rather than presenting a comprehensive overview of the whole field. This review of literature is divided into four Sections. The first Section of the chapter is titled "Meaning and importance of Number bases". In order to examine and evaluate teachers' knowledge on a mathematical topic, it is important to address crucial mathematical ideas for the topic and literature on teaching and learning of it (Koklu & Aslan, 2012). Therefore, since my study is assessing primary school teachers' knowledge about number bases it is in this regard that this chapter begins by highlighting mathematical meaning and importance of the topic 'number bases'. In order to fully explore the meaning and importance of Number bases under this Section I have included the following Subsections: 1) the Meaning of the concept 'Number Bases, 2) importance of teaching and learning number bases and 3) teaching and learning of Number bases in Zambia. Thus, this Section fully articulates the significance of teaching Number bases.

The second Section is titled "Aspects of Subject Matter Knowledge and Pedagogical Content Knowledge". The purpose of my study is to evaluate the Mathematical Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK) of primary school teachers with a focus on the topic of Number Bases. It is in this respect that this section has been included in order to discuss various issues related to Pedagogical Content Knowledge as well as to Subject Matter Knowledge of teachers. In order to fully explore the aspects of Subject Matter

Knowledge and Pedagogical Content Knowledge the section encompasses the following subsections: 1) Meaning of Pedagogy and Content Knowledge, 2) Why primary school teachers should have sufficient Mathematics Subject Matter Knowledge, 3) Teacher Knowledge, Pedagogic Practice and Classroom Environments, 4) Some studies done on pedagogy content knowledge, and 5) The practices of highly effective mathematics teachers. In the third section I have discussed models and theories of teaching mathematics. There are several models and theories relating to teachers' professional and pedagogical knowledge and skills. My aim in presenting these is to provide a theoretical and conceptual base to explore the SMK and PCK of the primary school teachers in my study about number bases. The last Section of the Chapter is the Literature summary, where I have illuminated the gap which my study intends to make a contribution to.

2.2 Meaning and importance of the topic 'Number Bases'

As highlighted in the introduction of this chapter in this Section I now discuss in detail: Meaning of the concept Number bases, importance of teaching and learning number bases, difficulties of teaching and learning number bases and the teaching and learning of Number Bases in Zambia.

2.2.1 Meaning of the concept 'Number Bases'

Zazkis and Khoury (1994) explain that Number Bases is a mathematical topic which refers to positional representation system in which a real number is represented by an ordered set of characters where the value of a character depends on its position. This entails that number bases encompass different ways of writing the same number (Wilson et al., 2005). Michele (2006) explains that:

> *the base of a system of numerals is the number that determines the place values for all of the numerals in that system. For example, in Base 10, 234 equals 2 times 100 (10*

squared), 3 times 10 (10 to the first power), and 4 times 1 (10 to the zero power). In Base 8, 234 equals 2 times 64 (8 squared), 3 times 8 (8 to the first power), and 4 times 1 (8 to the zero power). Any positive integer greater than one can be used as a base. You can express any number in any base (p.18).

The radix of a given number base system is the number of unique digits, including zero, used to represent numbers in the positional numeral system. For instance, in the decimal system (the most common number base system used today) the radix is 10. The ten distinct symbols used in the decimal system are: 1, 2, 3, 4, 5, 6, 7, 8, and 9. Similarly, the base two system has exactly two symbols 0 and 1. Thus from here we see that for a given number base system, let's say base *b* the number of digits are *b* and the largest digit has a value of *b-1*. For my study I observed how primary school teachers bring in all these notions to the teaching of Number Bases.

The symbols of a given base let's say base ten are known as digits. Though base two symbols are also known as bits (short for binary digits). A bit has a single binary value, either 1 or 0. All numbers of a given base are built as strings of digits. For instance, the binary number 101 is said to be built from three digits.

Numbers in a given base system are written as x_b where x denotes a given number and *b* represents the number base system in which *x* is written. For instance, in base eight numbers could be written as $567_{8,}$ $124_{8,}$ 562_8 and so on.

Weissteinnat (2015) notes that common number bases systems are given special names based on the value of *b*. The most common number bases are binary, hexadecimal and decimal. A summary of common bases and their respective special names is given in Table 4.

Table 4: Common bases and their respective special names

Base	Special name
2	Binary
3	Ternary
4	Quaternary
5	Quinary
6	Senary
7	Septanary
8	Octal
9	Nonary
10	Decimal
11	Undenary
12	Duodecimal
16	Hexadecimal
20	Vigesimal
60	Sexagesimal

Source: Weissteinnat (2015)

Primary school teachers in Zambia are expected to know some of the special names especially those of base two, five and eight which are part of the Zambian primary school mathematics syllabus. They are also expected to proficiently explain to their pupils the meaning of some of the notations (such as x_b) used in number bases. Therefore, it was important to assess them on their conceptual knowledge of the number bases system. I believe this would help to come up with ways of making number bases teaching both more purposeful and more interesting for pupils in grade seven in particular and in Zambian primary schools in general.

Numbers in base 10 can be converted to other bases and vice versa. For instance, let us say you are given the quinary number 123. Then you want to convert it to base ten. You notice that in the number 123 in base ten the digit three represents ones, the digit two represents tens and finally the digit one represents hundreds. Thus, the number can be broken into a series of values like this. 123 = three *x* ones + two *x* tens + one *x* hundred. Using this as a model, it is possible to move into base five. This time instead of ones, tens, and hundreds, we have ones, fives and twenty-fives. Therefore, to convert the quinary number 124 to base ten you proceed as follows.

$124_5 = 4\,(1) + 2\,(5) + 1(25)$

$= 39_{10}$

Similarly, you can convert the quinary 39 to base ten. To do that you use repeated division. Each time you divide by 5 you write down the remainder

$$\frac{39}{5} = 7r4$$
$$\frac{7}{5} = 1r2$$

Then the remainders become part of the answer. In this case the answer is 124_5. Knowledge of how to convert number bases from one system to another is significant as it consolidates the concept of place value (NCTM, 2000). The development of place value understanding is directly relevant to understanding number and operations, developing number sense and gaining fluency in arithmetic and form the core of mathematics education for elementary grades (NCTM, 2000, p.32). Therefore, it is very important that all primary school teachers know and utilize the appropriate algorithms when converting number bases from one system to another. Thus, whilst observing primary school teachers I carefully, checked how they converted numbers from one base to another and how they incorporated and explained the concept of place value.

2.2.2 Importance of teaching and learning Number Bases in Primary Schools

Michele (2006) posit that the learning of Number Bases for a primary school pupil creates certain ways of thinking, which is largely about how ideas can lead to notations, and how those notations can lead to new ideas, which lead to new notations, and so on. Peterson (2002) points out four reasons why apart from learning base 10 number system pupils in primary schools should also learn other bases such as base eight and base two. Firstly, Peterson (2002) notes that just as another language will be useful if you go to a country where it is spoken, other number bases are used in certain places where base ten cannot. For instance, binary and hexadecimal (bases 2 and 16) are used a lot in computers, whereas the decimal base is not. Therefore, learning about these bases in both Zambian primary and secondary schools would greatly help to equip pupils with the necessary knowledge they require as prospective computer programmers.

Secondly, Peterson (2002) contends that just as learning a foreign language can help you understand your own language better, knowing how bases in general work can help to clarify the meaning of place value and other concepts. Therefore, primary school teachers who are conversant with number bases will easily teach their pupils how to internalize the concept of place values. It is in this regard, that I have taken keen interest to establish how primary school teachers apply their SMK and PCK of number bases to effectively teach the concept of place value (which includes three ideas: Ordering, Position and Amount). Ordering refers to comparing numbers with each other, Position refers to understanding how the place of a digit affects its value in any particular number and Amount refers to knowing what the digits represent (Woodham & Pennant, 2014).

Thirdly, Peterson (2002) points out that just as learning other languages can keep you from the common American view that America is all there is (off course this is analog to the Zambian Proverb that a child who does not move thinks that the mother is the best cook), learning other bases can prevent what Peterson (2002) refer to as "basism," which is the false view that whatever is true in base ten is true of numbers themselves. For example, a number does not have a certain number of digits; its representation in base ten does. To illustrate this point, in base two the number three is written as 101 while in base ten it is written as 3. Thus, we see that in base two the number three has three digits but in base ten it has only one digit. Peterson (2002) argues that working with other number bases reveals that base ten is not even the best base there is. Peterson (2002) defends this view by revealing that the Babylonians used base 60, particularly for fractions as in our hours and minutes, and it worked very well. Even up to date we still rely on these powerful Babylonians mathematical findings of base 60 when dealing with time and angles. Therefore, stopping to think about what a problem looks like in another base can sometimes keep you from making false assumptions (Peterson, 2002). Fourthly, Peterson (2002) contends that number bases are just a good exercise for the mind just like physical exercise is good for the body.

The main point I take from Peterson (2002) is that number bases is a coherent body of knowledge, it is multipurpose, it is malleable, and thus no primary school teacher can effectively teach mathematics without appropriate notion of number bases. It is therefore, important for my study to explore if primary school teachers are aware of the relevance of teaching number bases.

2.2.3 Difficulties of teaching and learning Number Bases

Garz and Garcia (2015) note that special pedagogy is needed to teach number bases in primary schools. This is because in primary schools teachers focus on base 10 and often use base ten blocks to support the development of operations and number structure (Garz & Garcia, 2015). Learning other base systems forces the pupils deconstruct a system that they do not always structurally conceptualise (Garz & Garcia, 2015). Thus, several primary school pupils face difficulties in understanding number bases because of this situation (Garz & Garcia, 2015.) It is therefore, important for my study to explore whether teachers are able to both deconstruct and structurally conceptualise number basis in their teaching.

Fuadiah and Suryadi (2017) contend that some primary school pupils face difficulties in understanding number bases simply because their teachers do not go through the prerequisites such as place value and indices as they introduce number bases. Therefore, for this study it was important to take note of how primary school teachers in Zambia go through the prerequisites as they introduce number bases.

Ashley (2015) states that many pupils in primary schools have difficulty with number bases because it is abstract. Pupils in primary schools are concrete learners, meaning they need sensory experiences to develop their learning (Ashley, 2015). Using manipulatives, such as base-ten blocks, can help students connect the abstract with more concrete representations (Ashley, 2015).

Ashley (2015) notes that most pupils in primary schools do not see how number bases concepts apply to their individual lives especially when primary school teacher does not use real world examples when teaching number bases. This acts as a source of difficulties in understanding number bases (Ashley, 2015). Therefore, it is important for this study to evaluate how primary school teachers in Zambia employ manipulatives when teaching Number Bases.

Ashley (2015) notes that in order to explain the number bases concepts to children in a manner that makes sense to them; primary school teachers must have a thorough understanding of place value and the number system. However, some primary school teachers have difficulties in teaching number bases because they do not personally understand number bases concepts (Ashley, 2015). Therefore, it is important for this study to evaluate primary school teachers understanding of Number Bases concepts.

Here, Ashley (2015) has raised very important points, therefore for this study when observing primary school teachers teaching number bases it was important to take note of the real word examples primary school teachers use, their productive disposition and their overt display of subject knowledge.

2.2.4 The Teaching and learning of Number Bases in Zambia

In Zambia number bases are taught both at primary and secondary school level. At primary school level number bases are learnt in grade seven whilst at secondary school level number bases are learnt in grade nine. The objectives of teaching the mathematical topic of number bases in primary school are as follows:

1.1 Illustrate base ten numeration system

1.2 Describe other number bases

1.3 Convert from Base 10 to Bases 2, 5 and 8.

1.4 Convert from Bases 2, 5 and 8 to Base 10.

1.5 Convert from base 2 to base 5 and vice versa

1.6 Add and subtract in Bases 2, 5 and 8.

Source: Ministry of Education, Science, Vocational Training and Early Education (MESVTEE) (2013)

Thus, these are objectives which should be attained as far as the teaching of number bases is concerned. Questions for class exercises, tests or final examinations are set in line with the given objectives. Therefore, it is cardinal in my study to establish if primary school teachers have the knowledge of number bases in relation to the concepts, skills, facts, and mathematical process necessary for them to teach effectively.

This Section has articulated the significance of teaching Number Bases, unfortunately, several primary school teachers face difficulties to proficiently teach Number Bases as evidenced in the base line study conducted at Chalimbana University in 2014. Therefore, there is need to find an empirically based approach of how to assist them possibly by conducting studies such as this one.

2.3 Aspects of Subject Matter Knowledge and Pedagogical Content Knowledge

As earlier explained in the introduction of this chapter, in this section I now fully discuss: the meaning of pedagogy and Content Knowledge, why primary school teachers should have sufficient Mathematics Subject Matter Knowledge, Teacher Knowledge and Pedagogical practices, some studies done on pedagogical content knowledge, the practices of highly effective mathematics teachers, and templates of teaching mathematics

2.3.1 Meaning of Subject Matter Knowledge and Pedagogical Content Knowledge

This Section reveals various definitions of subject matter knowledge and pedagogical content knowledge as advanced by different scholars in the field of mathematics education. Various scholars have defined subject matter knowledge and pedagogical content knowledge differently. Cogill (2008) notes that what constitutes pedagogy is complex and not easily defined. Cogill (2008) further contends that even the definition of pedagogy appears to be hard to perceive.

Whilst some researchers in the field of mathematics education such as Hodgen (2011) and Sherin et al., (1999) have distinguished subject matter knowledge and pedagogical content knowledge, others such as McNamara (1991) and Anghileri (2006) have strongly argued that there is no clear distinction between subject matter knowledge and pedagogical content knowledge. Similarly, McEwan and Bull (1991) have contended that there is no difference between SMK and PCK, and they have claimed that for teachers, all knowledge is pedagogic.

Nonetheless, Petrou and Goulding (2011) claim that regarding all mathematical knowledge in teaching as pedagogic may not be helpful in teacher preparation and development programmes. Petrou and Goulding (2011) further argue that there is need for some specific attention to be paid to SMK. For instance, those with challenges of converting numbers from base ten to base two may need opportunities to work on this at their own level as learners of mathematics. In line with Petrou and Goulding (2011) the important thing here would be that the examples and activities chosen for this work would not be far removed from the school curriculum. Furthermore, according to Petrou and Goulding (2011) the unpacking and deepening of SMK can be seen as part of the process of transformation required for robust PCK to be developed. .

Shulman (1987) states that he introduced the term pedagogical content knowledge (PCK) during his presidential address to the American Educational Research Association. Shulman argued that for a long-time research on teaching and teacher education had undeservedly ignored questions dealing with the content of the lesson taught (Driel & Berry, 2010). In his address Shulman presented a strong case for PCK as a specific form of knowledge for teaching which refers to the transformation of subject matter knowledge in the context of facilitating student understanding (Driel & Berry, 2010). Additionally, Shulman contended that teachers need PCK to structure the content of their lessons, to choose or develop specific representations or analogies to understand and anticipate particular perceptions or learning difficulties of their students and so on (Driel & Berry, 2010). Shulman (1987, p. 8) defined pedagogical content knowledge (PCK) as "that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding."

O' Meara (2010) distinguishes Pedagogical Content Knowledge (PCK) from Subject Matter Knowledge (SMK) by explaining that Pedagogical Content Knowledge (PCK) refers to knowing how to teach while Subject Matter Knowledge (SMK) refers to knowing the subject. Berry et al., (2009) define Pedagogy as any conscious activity by one person aimed at promoting the learning of another. Reys (2018) states that pedagogy is what one needs to know, and the skills one needs to command in order to make and justify the many different kinds of decisions of which teaching is constituted.

Leach and Moon (1999) contributed to the definition of pedagogy by coming up with the term known us 'Pedagogical Setting'. They argue that Pedagogical Setting is the practice that a teacher, together with a particular group of learners creates, enacts and experiences. They also contend that pedagogy is a joint activity in which the learner and teacher have an active role. Moreover, Leach and Moon (1999) differentiated the mathematical question from a pedagogy question by stating that a *mathematical* question involves how to show the statement is true, but a *pedagogical* question involves how to enable others to see that the statement is true. Therefore, in line with Leach and Moon (1999) it was interesting in this study to explore the types of questions primary school teachers ask in the context of number bases.

Rowland, Huckstep and Thwaites (2005) assert that content or subject matter knowledge is the depth and breadth of knowledge in a specific content area. This implies that teachers are expected to be extremely knowledgeable in mathematics concepts such as perimeter, area, fractions, factorization, quadratic equation, differentiation, number bases and so on. These are some of the things which the average person does not know. Additionally, Rowland et al. (2005) contends that pedagogical knowledge is the knowledge of teaching and learning that spans content areas. Rowland et al. (2005) adds that components of pedagogical knowledge encompass formative assessment, classroom management, and motivation strategies. Accordingly, Rowland et al. (2005) argues that teachers make pedagogical decisions about teaching and learning based on their content area. For example, a mathematics teacher when teaching construction of a 60° angle may choose demonstration method, however when teaching factorization of quadratic expression of the form ax^2+bx+c may choose lecturing or discussion method (Rowland et al, 2005). Therefore, in line with Rowland et al. (2005) it is important through this study to assess the suitability of the methods applied by primary school teachers in Zambia when teaching the topic of number bases.

Kulm and Wu (2004, p. 146) define PCK as "the knowledge of effective teaching, which includes three components: knowledge of content, knowledge of curriculum, and knowledge of teaching." Corbin and Campbell (2001) argue that pedagogical content knowledge is most useful as a metaphor that locates teacher knowledge as embedded within the complex and unpredictable practice of teaching.

This Section has revealed that many scholars have taken keen interest to define SMK and PCK, though they have defined them differently. Some claim that there is a distinction between SMK and PCK but others contend that there is no difference. Based on what different scholars have written, it is clear that both SMK and PCK are important ingredients of primary school mathematics professional knowledge. They both interact with effective teaching. However, there is a distinction between SMK and PCK in that SMK is the material teachers teach and PCK is a tool or vehicle for teachers to deliver the content knowledge in their minds to pupils in a comprehensive manner. Therefore, my choice of description is adopted from Rowland *et al*. (2005) who have explained that Pedagogical Content Knowledge (PCK) refers to the knowledge of teaching and learning that spans content areas while Subject Matter Knowledge (SMK) refers to the depth and breadth of knowledge in a specific content area. Applying this definition helped my study to effectively achieve its goal of exploring the PCK and SMK of primary school teachers about Number Bases. It fully enabled me to explore how primary school teachers teaching of Number bases relate to their knowledge of Number bases.

2.3.2 Why Primary School Teachers should have sufficient Mathematics Subject Matter Knowledge?

As revealed by Hill *et al.* (2004, p.11) it is only “In the past two decades that teachers' knowledge of mathematics has become an object of concern”. Wall (2001) and Thompson (1994) note that the main reason for this was that many nations especially the United States and United Kingdom, following discontent with their pupils' relatively lowly mathematical performance in international comparative studies when compared to their Eastern peers, were eager to identify the ‘causes’ of this unsatisfactory scenario. The aspiration to seek the causes resulted in increased status and attention being assigned to the issue of teachers' mathematics subject matter knowledge (Wall, 2001). I believe it is important that as a nation we also actively get involved in such type of studies to enhance the standard of performance in mathematics in general, and on topics such as Number Bases in particular which are perceived to be difficult, hence the significance of my study.

However, as Hodgen (2011) and Posner (1978) note that there is little agreement on what constitutes the mathematical knowledge required in order for effective teaching and learning to take place. Additionally, Hodgen (2011) discloses that some researchers in the field of mathematics education have argued that improving teachers' SMK of mathematics will lead to better teaching and learning of mathematics. On the contrary, other studies such as the one conducted by Askew *et al.* (1997) found no relationship between teachers' mathematical knowledge, as measured in terms of academic mathematical qualifications, and effective teaching. What is certain is that the relationship between teacher knowledge and teaching outcomes is neither simple nor straightforward (Green, 2014). Therefore it is important in this study to explore how primary school teachers' subject matter knowledge relates to their teaching outcomes in the context of number bases.

Additionally, Hodgen (2011) reveals that there is now widespread agreement that the quality of primary and secondary school mathematics teaching depends crucially on the subject-related knowledge that teachers are able to bring to bear on their work. Besides, Hodgen (2011) contends that there is a widespread proposition that effective teaching calls for distinctive forms of subject-related knowledge and thinking. Moreover, Hodgen (2011) reveals that practitioners and researchers have come to appreciate the need for teachers to command varied and different forms of knowledge. According to Shulman (1997) knowing the content, the what and the why, is not enough for teachers to be able to teach effectively. Teachers must also possess pedagogical knowledge that is the know how to teach (Shulman, 1987). In other words, effective teachers utilize both content knowledge and pedagogical knowledge, and understand and appreciate how the two are interrelated (Shulman, 1987). Here we see that Shulman (1987) and Hodgen (2011) present very important ingredients of an effective teacher, therefore in view of these it is significant to explore if primary school teachers have the 'the what, the why and the know how to teach about number bases in an effort to improve the standard of performance as well as their self-efficacy.

Accordingly, Iacono et al. (2011) and McNamara's (1991) argue that many issues of concern and interest involving primary school teachers and their pupils are related to the success or failure of the pedagogies and practices employed in the classroom when learning mathematics. In a supportive view, Kagan (1992) claim that "teacher personal philosophies of mathematics lie at the very heart of teaching". Hence, teachers "approach to teaching are commensurate with their conceptions of teaching mathematics" (Kagan, 1992, p. 85). For this study, it is therefore important to find out primary school teachers' views of teaching number bases.

Contributing to the concept of SMK, Rowland et al. (2005) contend that mathematics SMK for mathematics teachers is crucial in the choice and application of illustrations, explanations, analogies, instructional materials and demonstrations. Rowland *et al.,* (2005) further asserts that mathematics SMK impacts on the teacher of mathematics concerning sequencing between and within lessons. Therefore it is interesting in this study to explore the various types of representations primary school teachers apply in the context of number bases. In a supportive view, Ball, Hill and Bass (2005) suggest that the nature of a teacher's subject matter knowledge affects his/her ability to make quick decisions regarding the most appropriate instructional materials, presentation, emphasis, and sequence of instruction. Moreover, Ball *et al.,* (1990) argues that mathematics SMK inspires and motivates the teacher to respond proficiently to students' contingency actions such as unexpected questions and answers. Ball *et al.* (1990) makes a very important comment here, which necessitates the need for this study to also inquire the nature of primary school teachers' contingent actions when teaching number bases as part of exploring their PCK. Moreover, Ball *et al.,* (1990) says that although there is a general agreement that practicing primary school teachers require deep and rich mathematics subject matter knowledge, it is essential to point out what constitutes suitable mathematical knowledge for teaching. According to Ball *et al.,* (1990) Mathematical Knowledge for teaching (MKT) includes abilities such as analyzing the learners thinking that led to an incorrect response, identifying the mathematical understanding a leaner does not yet have, as well as deciding how to best represent a mathematical idea so that it can be understood by learners. Similarly, the United Kingdom's Training and Development Agency for Schools (2006) asserts that a teacher requires a high level of knowledge and understanding in order to confidently and effectively develop pupils' mathematical knowledge and understanding. I agree with this assertion advanced by United Kingdom's Training and Development Agency for Schools (2006), that is why it is very important to find ways of raising teachers levels of knowledge and understanding of topics such as number bases for the benefit of learner learning.

As noticed by Goulding (2003) it is universally accepted that subject matter knowledge beyond a certain 'threshold' is not associated with greater pupil achievement. This implies that primary school teachers do not need to study mathematics to degree level in order for them to teach mathematics proficiently. I consider this situation to be an analogy to the elasticity principal found in physics. The principle of elastic limit states that for any elastic object the extension is directly proportional to the expansion provided the elastic limit is not exceeded.

Rowland, Huckstep and Thwaites (2005) note that in the past, there was a perception from some quarters that elementary teachers need very little mathematics subject matter knowledge (a 'minimalist' view). The minimalist view also assumes that any well-educated adult possesses the subject matter knowledge required to teach at primary school level. This reflects the belief that 'He who knows mathematics, knows how to teach it' (Boero, 1996). However, Prestage and Perks (1999) argue that this position is challenged by the revelation that teachers require more than learner knowledge given that pupils can ask questions that extend beyond the formal curriculum. My study is in line with the views advanced by Prestage and Perks (1999) that is why it is exploring both SMK and PCK of primary school teachers.

Hill *et al.*, (2005) and Schwartz and Riedesel (1994) suggests that a certain kind of mathematics subject matter knowledge is needed to teach the subject effectively at primary school level, additional to that required by those pursuing other mathematically intensive careers notably accountants, engineers and so on. Hodgen (2003, p. 38) expresses a similar view referring to "the need for school teachers to know mathematics differently". Ball *et al*. (2005) refer to this knowledge as 'specialised'. Hill *et al.* (2005, p. 373) suggest that "the 'specialised' mathematical knowledge required for the work of teaching is vast given that this 'work' includes explaining terms and concepts to students, interpreting students' statements and solutions, judging and correcting textbook treatments of particular topics, using representations accurately in the classroom, and providing students with examples of mathematics concepts, algorithms and proofs". As for my study, whilst exploring SMK and PCK of primary school teachers in the context of number bases, I also explored all these terms and concepts advanced by Hill *et al.*, (2005).

Kessel and Ma (2000) point out that in order to teach effectively, a teacher must possess conceptual understanding of the various mathematical concepts and procedures as well as recognising and understanding the interconnections between them. This belief receives support from Schulman (1987) who highlighted that "subject matter knowledge required for teaching do not only include facts and concepts in a domain but also why facts and concepts are true and how knowledge is generated and structured in the discipline" (Hill *et al*., 2005, p. 376).

Contributing to the notion of SMK, Barber and Heal (2003) note that teachers with weak SMK rely on coping strategies. They revealed that coping strategies employed by such teachers include avoiding topics altogether, over dependence on the text, limitation of interaction and a focus on rules and procedures as isolated facts. In such context's pupils must depend on memorization rather than understanding which in turn leads to the "failure to lay the groundwork for future development of student understanding" (Leavy & O' Loughlin, 2006 p. 54). Here, Barber and Heal (2003) highlight very important teaching and learning aspects that is why for my study, I employed lesson observations as one of the data collection instruments in order to take note of the coping strategies practiced by primary school teachers when teaching number bases. Worse off, in the Zambian context where pupils during final grade seven examinations are tested on all the topics covered in mathematics employing copying strategies by a teacher can have adverse effects on the performance of pupils.

This Section has highlighted that sufficient SMK is crucial in enhancing teachers' conceptual understanding of the various mathematical concepts and procedures as well as recognising and understanding the interconnections between the concepts of SMK. It is also crucial in the choice of illustrations, explanations, analogies, instructional materials and demonstrations. Therefore, in line with the focus of my study all these elements are taken as part of the conceptual framework to assist in the analysis of data.

2.3.3 Teacher Knowledge, Pedagogical Practice and Classroom Environments

Stigler and Hiebert (1999) note that different teachers' beliefs about the nature of mathematics, the nature of learning, the role of the teacher, the structure of the lesson, and teacher responses to individual student differences lead to different modes of instruction in the United States and Japan. Admittedly, teachers' philosophy of mathematics in Zambia has the same implications on the teaching and learning of mathematics just like in the United States and Japan. That is why it is interesting to research how it affects the teaching of topics such as number bases.

Meyer *et al.,* (1992) and Markham *et al.,* (1994) point out that there is a globalisation effect in terms of moderation across models of schooling in national education systems. Nonetheless, Delaney *et al*., (2008) and Anghileri (2006) argue that international comparisons of teachers' mathematical knowledge need to be considered in line with the variations that may exist in the knowledge that teachers use in each country.

Similarly, Putnam and Borko (2000) note that knowledge in and for teaching is 'situated', that is knowledge is adapted to particular contexts. Putnam and Borko (2000, p. 13) further contend that "this professional knowledge is developed in context, stored together with characteristic features of classrooms and activities, organised around the tasks that teachers accomplish in classroom settings, and accessed for use in similar situations." Definitely the implication of

Putnam and Borko's (2000) assertions to my study is that in order to fully assess primary school teachers' PCK and SMK lesson observation should be one of the instruments to use. For lesson observations can help to show how primary school teachers organise their activities when teaching number bases. Moreover, this is also supported by Boaler (2002), in her work on pupil knowledge and identity construction. Boaler (2002, p. 1) concluded that knowledge should be viewed "not as an individual attribute, but as something that is distributed between people and activities and systems of their environment".

Similarly, Lave and Wenger (1991) note that there is a shift from a focus on only upon knowledge, to one that attends to the inter-relationships of knowledge, practice and identity. That is a shift of focus from teacher or pupil knowledge to knowledge that is constituted through the course of mutual engagement and interactions. One of the key issues raised in this section is that teachers in different circumstances apply varying modes of instructions. Therefore, in this study during lesson observations I took keen interest in observing the lesson structures primary school teachers employ when teaching number bases.

2.3.4. Some studies done on Subject Matter Knowledge and Pedagogical Content Knowledge in Mathematics Education

In this Section, I refer to previous research, related to SMK and PCK that I see as relevant to my study. Safi (2009) explored the development of prospective primary school teachers' subject matter knowledge of whole number concepts and operations in base eight. Safi (2009) employed a qualitative approach with a case study design. Safi (2009) investigated whether prospective primary school teachers with varying initial content knowledge developed differently through instructional sequence. Safi (2009) gave a test to 15 prospective primary school teachers. The test assessed prospective primary school teachers understanding of whole number concepts and number operations in base ten. Based on the test results, Safi (2009) selected two participants one with low content knowledge and the other with high content knowledge who then took part in a course pack that was entirely designed in base eight. After sometime the two prospective primary school teachers were given another test. Results of the second test indicated that both prospective primary school teachers demonstrated equal conceptual understanding of counting strategies and number operations (Safi, 2009). Safi (2009) concluded that teacher educators and educational policy makers can revisit and possibly revise instructional practices and sequences in order to develop teachers with greater conceptual understanding of concepts vital to elementary mathematics. Despite, Safi (2009) study being concerned with base ten and base eight only while my study also includes base five and base two I have learnt important lessons from him. For instance, before selecting primary school teachers to who took part in lesson observations and interviews I first asked them to complete questionnaire and concept maps. Then based on their performance in the questionnaire and concept maps I picked two low achievers, two medium achievers and two high achievers. These are the ones who took part in lesson observations and interviews.

Stacey et al. (2001) conducted a study on pre service elementary school teachers' pedagogy knowledge of decimal numeration number system. In their study they asked the teacher participants to complete a decimal comparison questionnaire, mark items they thought would be challenging to pupils, and explain why. The study revealed the need for teacher education to emphasize content knowledge that integrates different aspects of number operations knowledge, and pedagogical content knowledge that includes a thorough understanding of common difficulties in number operations. The difference with Stacey et al. (2001) study is that it did not observe pre service elementary school teachers to get first hand information of how they actually teach the topic. However, here I learn that Stacey et al. (2001) administered a questionnaire as a research instrument in order to assess pre service elementary school teachers' SMK and PCK of decimal numeration number system. The implication of Stacey et al. (2001) study to mine is that it gave me insight on the constructed questionnaire to include items that demands primary school teachers to explain why they follow a particular order when sequencing Number Bases (See Item 17, part f on Appendix A for details).

Roy (2014) explored the prospective primary school teachers' subject matter knowledge of number concepts and operations. The study focused on prospective primary school teachers' understanding of addition and subtraction with whole numbers in base eight and ten (Roy, 2014). Thirty three Participants took part in the study. Roy (2014) collected data by: firstly presenting a base-eight mathematical task in the form of a word problem, picture, or both to the prospective teachers. Secondly, the prospective teachers solved the problems in ways that made sense to them mathematically. Thirdly, prospective teachers worked either individually or in groups of two to four individuals. Finally, the whole-class discussion was conducted in order to allow the prospective teachers an opportunity to examine different solutions and strategies. The class sessions were videotaped to record the whole class dialogue that occurred (Roy, 2014). The video records were then transcribed for analysis (Roy, 2014). The findings revealed that base-eight allows prospective teachers to reason about addition and subtraction with whole numbers in similar ways that elementary aged students' reason in base-ten (Roy, 2014). Roy (2014) study differs from mine in that it only involves aspects of SMK whilst my study involves both aspects of SMK and PCK. Nonetheless, Roy (2014) study is important to me and what I applied from his study was the recording of the lessons on number bases and then transcribing them for analysis. The findings revealed that base-eight allows prospective teachers to reason about addition and subtraction with whole numbers in similar ways that elementary aged students' reason in base-ten (Roy, 2014). In line with his study findings I evaluated whether or not primary school teachers were aware of the productive disposition of learning alternate number systems.

Archer (2021) conducted a study that investigated how the multi-base blocks could be used effectively to address pupil's challenges in adding and subtracting in base two in the Junior High Schools in Ghana. Archer (2021) conducted an action research which consisted of sixteen (16) pupils. Archer (2021) purposively sampled the participants. The study followed a pre-test, intervention and post-test design (Archer, 2021). The pretest results revealed poor performance by pupils in terms of addition and subtraction in base two which could be attributed to pupil's lack of understanding of place value which also resulted in disarrangements of addends or subtrahend in a vertical form (Archer, 2021). Also pupils were found adding in base ten instead of base two, since they did not understand the concept of base two and therefore pupils performed poorly (Archer, 2021). Upon a series of intervention using the multi-base blocks a post-test was conducted which revealed a tremendous improvement in their performance (Archer, 2021). Archer (2021) concluded that the high achievement by pupils is evidently clear that the multi-base blocks is an effective teaching and learning material which can aid pupils understanding in number bases and also motivate them to learn. Archer (2021) study differs from mine in that it only involved base two whilst mine also includes base five and base eight. However, two aspects I learnt from Archer (2021) study was the application of purposive sampling when selecting participants and that of observing the nature of concrete materials primary school teachers were using when teaching Number Bases.

Harshman (2020) conducted a study in Zambia entitled "The Influence of Instruction in Base 8 on Prospective Teachers' Mathematical Knowledge for Teaching". The focus of the research was to extend existing research literature by providing insight on how prospective teachers with differing levels of mathematical knowledge developed their conceptual understanding of whole number concepts and operations (Harshman, 2020). A mixed method approach was employed (Harshman, 2020). A Mathematical Knowledge for Teaching (MKT) model was used to measure teachers' SMK and PCK about base eight (Harshman, 2020). The researcher focused on two specific constructs: Common Content Knowledge (CCK) and Specialized Content Knowledge (SCK). The qualitative portion of the study involved carefully constructed student interviews which allowed the researcher to deeply explore how prospective teacher conceptual understanding changed as a result of taking the unit in base 8 (Harshman, 2020). Four participants with varying levels of knowledge were selected to be interviewed based upon initial scores on the MKT: 1) low CCK and low SCK, 2) low CCK and high SCK, 3) high CCK and low SCK, 4) high CCK and high SCK. Results of the interviews were used to help explain results from the MKT (Harshman, 2020). Quantitative and qualitative results showed that participants did not significantly increase their CCK, but did experience an increase in their conceptual understanding (SCK) as a result of taking the unit in base 8 (Harshman, 2020). Prospective teachers all showed deeper conceptual understanding of whole number concepts and operations at varying levels by the end of the base 8 unit (Harshman, 2020). Harshman (2020) noted that increased PCK does not necessarily lead to increased SMK. Harshman (2020) only focused on base eight. The teaching and learning of number bases in Zambia concerns base ten, base eight, base five and base two therefore, we cannot rely on Harshman (2020) to deduce the SMK and PCK of primary school teachers in Zambia. In spite of that one key lesson I took from Harshman (2020) study was that before observing and interviewing primary school teachers, they first took part in the questionnaire. Then based on their scores in the questionnaire, I selected those who took part in the classroom observation and interviews in line with Harshman (2020) strategy.

Livy (2014) used the Knowledge quartet model (Rowland et al, 2011) to investigate how primary school pre-service teachers prepared and taught subtraction of numbers in base ten. Livy (2014) collected data from field notes, audio recording of part of a lesson, and an interview with the pre-service teacher after the lesson. The study revealed that contributing factors that assisted primary school pre-service teachers to develop MCK during teaching included program structure providing breadth and depth of experience (Livy, 2014). Both Livy (2014) and my study apply the knowledge quartet, nonetheless they differ in that Livy's (2014) study drew on pre-service teachers whilst mine is drawing on primary school teachers who are already serving teachers. Moreover, Levy observed a teacher who was teaching the topic of 'Subtraction in base ten', but mine focuses on several number bases (base ten, base eight, base five and base two). Therefore, we cannot fully rely on Livy's (2014) study to deduce the SMK and PCK of Primary school teachers about Number Bases. However, one important aspect I drew from Livy's (2014) study was the incorporating in my study the aspect of audio recording lessons of primary school teachers.

Ghazali and Saleh (2010) explored criteria for primary school teachers' teaching and instructional practices that support and develop student's number sense in the Malaysian Primary Schools. Ghazali and Saleh (2010) collected data through interview of primary school teachers, classroom observation of teachers' lessons and examination of other artifacts such as teacher's lesson plan, teaching aids, relevant resources used in teaching. All the classroom observations were videotaped for analysis (Ghazali & Saleh, 2010). Findings indicated the emergence of certain aspects of teaching such as good teacher characteristics and effective student involvement (Ghazali & Saleh, 2010). However, other aspects of teaching such as teacher's pedagogical content knowledge and connection of the mathematics to the content were observed as posing a challenge to the teachers (Ghazali & Saleh, 2010). Therefore, in line with Ghazali and Saleh (2010) study focused on how numbers work and on how numbers relate to each other, it did not include aspects of converting numbers from one number system to another, therefore we cannot rely on it to deduce SMK and PCK of primary school teachers in Zambia on number bases. Nonetheless, important aspects I drew from Ghazali and Saleh (2010) study was the collection of data through lesson observations. Furthermore, in line with Ghazali and Saleh (2010) study findings I took note of certain aspects of primary school teachers' such as observing how they encouraged their students' participation and how they explained the link between Number Bases and other mathematical topics when teaching Number Bases.

A study by Yang (2007) evaluated PCK and SMK of 15 pre-service primary school teachers from a university in southern Taiwan in the context of number sense. In order to examine the strategies used by pre-service primary school teachers when responding to number sense-related items Yang (2007) collected data through interviews. Yang (2007) defines number sense as consisting of the following four categories: (a) understanding the meanings of numbers, operations, and their relationships; (b) recognizing relative number size; (c) judging the reasonableness of a computational result by using strategies of estimation; and (d) developing and using benchmarks appropriately. Yang (2007) found that for each category, about two thirds of participants relied on rule-based methods to answer the questions. Based on the finding Yang (2007) concluded that pre-service primary school teachers, especially those in the low-ability group, tended to reason procedurally. The study by Yang (2007) is helpful for my study and one thing I drew from his study was that whist observing primary school teachers' lessons on number bases I took note of how they concentrated on procedure when changing numbers from one number system to another number system. The difference in the study by Yang (2007) is that he did not observe the pre-service primary school teachers to obtain first-hand information on how they actually teach number sense. Instead he relied on getting information from pre-service primary school teachers by means of interviews only.

A study by McClain (2003) inquired primary school teachers' conceptions of an alternate base eight. McClain (2003) asked 24 primary school teachers enrolled in the second of two methods courses in the United States to work in the Candy Factory context in which eight candies were packed into a roll of candies and eight rolls were packed into a box of candies. McClain (2003) found that the primary school teachers were distracted by being asked to use base-eight notation and focused more on that than on the mathematics of quantifying, adding, and subtracting numbers. However, with the Candy Factory context, McClain (2003) found that primary school teachers easily understood place value and the multiplicative structure of the system. The study by McClain (2003) is helpful for my study and one thing I drew from her study was that whist observing primary school teachers lessons on number bases I took note of how they incorporate manipulatives in their lessons. Nonetheless, the study by McClain (2003) only focused on base eight therefore it differs from mine which also includes base two and base five.

A study by Thanheiser et al., (2010) investigated prospective primary school teachers' interpretations of regrouped digits. Thanheiser et al., (2010) employed a qualitative study. Data was collected through interviews from 15 prospective primary school teachers in the United States before their first content course for teachers (Thanheiser et al., 2010). The interview data allowed for the identification and categorization of prospective primary school teachers conceptions of multidigit whole numbers into *reference units* and *groups of ones* (Thanheiser et al., 2010). Thanheiser et al., (2010) found that two thirds of the prospective primary school teachers in the study saw the digits in a number incorrectly in terms of ones, at least some of the time. Thanheiser et al., (2010) noted that for a number such as 100 prospective primary school teachers may struggle to explain that the 1 represents 10 *tens.* Although the study by Thanheiser et al., (2010) is helpful for my study, in that it also explores elements of teachers' mathematical knowledge in the context of number bases, it differs from mine in that it only focused on interpretations of regrouped digits it did not encompass other operations on number bases such as addition and changing of numbers from one number system to a different number system. Nonetheless, based on Thanheiser et al. (2010) study findings during lesson observations I took keen interest to observe how primary school teachers incorporate the concept of place value in the teaching and learning of Number Bases.

Fasteen (2015) investigated pre-service mathematics teachers' SMK and PCK in the context of alternate numeration number systems. Fasteen (2015) employed a qualitative approach. Data was analysed by conducting thematic analysis of pre-service mathematics teachers' textbooks and by conducting a teaching experiment to analyze pre-service mathematics teachers' mathematical activity as they engaged with a base five task sequence to reinvent an algorithm for multiplication (Fasteen, 2015). The findings indicated that alternate numeration systems can be leveraged to create opportunities for pre-service mathematics teachers to engage in guided reinvention of an algorithm and improve understanding of base ten by comparing it to other numeration systems (Fasteen, 2015). Fasteen (2015) study focuses on number bases just like this study. The difference is that Fasteen (2015) did not include the base ten numeration systems instead only focused on the alternate numeration systems. Therefore we cannot fully rely on her study to deduce the SMK and PCK of primary school teachers' about number bases in Zambian primary schools. However, what I drew from Fasteen (2015) study was the application of a qualitative approach. Furthermore, based on Fasteen's (2015) study findings during lesson observations I took keen interest to observe how primary school teachers apply the alternate numeration systems to help their pupils understand better the base ten numeration system.

Yee (2020) explored the nature of Mathematics Pedagogical Content Knowledge (MPCK) concerning arithmetic word problems and number operations amongst grade 3 primary school pre-service teachers in Hong Kong. Interviews were used to collect data from the sample (Yee, 2020). The results revealed that a deep understanding of elementary number theory seems to be a precondition for developing pre-service teachers' MPCK in teaching arithmetic word and number operations (Yee, 2020). The difference with Yee (2020) study is that the pre-service teachers were not observed to ascertain their actual classroom practices. Yee (2020) study concerns Grade 3 primary whilst this study concerns Grade 7 primary school teachers, nonetheless one thing I drew from Yee (2020) study was the application of interviews to collect data. However, in addition also applied classroom observation and questionnaires in order to obtain detailed data about primary school teachers SMK and PCK in the context of number bases. Furthermore, based on Yee's (2020) study findings during lesson observations I took note of how primary school teachers explained the necessary pre-requisite knowledge for number bases.

Maugesten (2019) conducted a study entitled "exploration of what constitutes a good Norwegian primary school teachers' mathematics PCK. Maugesten (2019) study involved several primary school topics such as number bases, fractions, angles, shapes and so on. Maugesten (2019) employed a qualitative approach. The data used in the study was collected through focus group interview with seven lower primary school teachers who just completed a two-year professional development program for mathematics teachers (Maugesten, 2019). The findings revealed that a good teacher must give students time to think after asking questions and that they should not feel that the students should rather be solving written math problems (Maugesten, 2019). Both Maugesten (2019) study and mine concerns aspects of primary school teachers' PCK in mathematics. Maugesten (2019) study did not focus on single topic learnt in primary schools unlike my study which focuses on number bases, however based on her study findings one thing I applied from her study was that whilst observing primary school teachers as they taught number bases, I took keen interest to observe whether or not primary school teachers were giving their students time to think after asking questions.

Mntunjani (2015) explored how primary school teachers use mathematical resources to teach number concepts in Western Cape of South Africa. Mntunjani (2015) employed an interpretive qualitative research paradigm and used a case study design. The participants in the study were five primary school teachers at two schools in the Western Cape (Mntunjani, 2015). Data was collected through lesson plan analysis, lesson observations and semi-structured interviews (Mntunjani, 2015). The data collected was then analysed through the lens of Vygotsky's socio-cultural theory which states that that knowledge is first acquired interpersonally, then intrapersonally, as learners first learn from others, then internalise or individualise knowledge while going through the four stages of the Zone of Proximal Development (ZPD). The findings of the study revealed that teaching for understanding was often compromised by teaching to enable learners to pass assessments (Mntunjani, 2015). Teachers understood the importance of using resources to teach number concepts, but were inclined to rote teaching with work drills in preparation for assessments (Mntunjani, 2015). The study recommended the need of having more research on the use of resources when teaching and learning of number resources. Mntunjani (2015) study differs from this study in that it was only concerned with the teaching of number concepts in base ten only whilst this also involves base eight, base five and base two. One thing I draw from Mntunjani (2015) study is the possibility of also employing lesson observation and sem-structured interviews. Furthermore, based on his (2015) study findings during lesson observations I took note of how primary school teachers were incorporating aspects of rote teaching when teaching Number Bases.

Naukushu (2016,p1) conducted a study in Namibia entitled, *"A Critical Theory enquiry in the development of number sense in Namibian first year pre-service secondary mathematics teachers,"*. Naukushu (2016) study used a convenient sample of sixty (60) pre-service secondary school mathematics teachers. The study employed both qualitative and quantitative methods with a pre-test-post-test control design to draw data from the participants (Naukushu, 2016). Data was collected through test, an in-depth focus group interview, document analysis as well as a questionnaire with both open ended and closed ended questions (Naukushu, 2016). Both the qualitative and quantitative findings revealed that the number sense of the pre-service mathematics secondary teachers was below basic before the intervention. The study also found out that the changes in academic performance could be attributed to number sense up to 23% and vice-versa (Naukushu, 2016). Both Naukushu (2016) study and my study involve the number and calculations strand. However, the difference with (Naukushu, 2016) study is that he used convenience sampling which does not produce a representative result (Creswell, 2014). Nonetheless, what I drew from (Naukushu, 2016) study was that when selecting primary school teachers I included questionnaires as one of the research instruments. In line with Naukushu's

(2016) study findings I also took note of how primary school teachers applied the concept of number sense to the teaching of Number Bases.

Using the knowledge Quartet as a theoretical framework Mohammed (2015) conducted a study entitled “The relationship between mathematics teachers’ content and pedagogical knowledge and their handling of student contributions: the case of Saudi trainee primary school teachers”. Mohammed (2015) used lesson observation and interviews to collect data. The Knowledge Quartet framework was used to analyse the data in order to highlight the knowledge of the trainees and to direct attention towards the incidents where the teachers responded to the students’ answers Mohammed (2015). The findings indicated that the trainees responded to the students’ answers usually with one of two types of response. *Confirmation* actions in which the teachers confirmed the correctness or the fault of a given answer and *questioning* actions where the teachers asked further questions usually before making a decision about the answer (Mohammed, 2015). My study just like the one conducted by Mohammed (2015) is also using the Knowledge Quartet as a theoretical framework. Nonetheless, I cannot rely on Mohammed (2015) study to deduce primary school teachers’ SMK and PCK in the context of number bases because Mohammed (2015) did not focus on a specific topic such as number bases, but his study included a number of topics such as linear equations, number bases, quadratic equations and transformation. However, what I drew from his study was the exploration of how primary school teachers respond to their students’ answers as well as the using of lesson observation and interviews to collect data.

Sitrava (2018) conducted a qualitative case study that investigated prospective mathematics teachers’ subject matter knowledge of the underlying concepts of standard and nonstandard algorithms used to solve the problems with whole numbers. According to Sitrava (2018) participants of the study were the twenty three prospective mathematics teachers enrolled in the Elementary Mathematics Education Program of one of the most successful universities in Turkey. The data was collected through tasks containing basic algorithms (Sitrava, 2018). Data was analysed by using content analysis (Sitrava, 2018). The results of the study revealed that more than half of the prospective mathematics teachers had knowledge about the use of place value concept in addition and multiplication. However, most of the prospective teachers could not explain the underlying principle and the meaning of the nonstandard algorithm in subtraction. Similar to their knowledge on subtraction, prospective teachers' knowledge on division was limited. Sitrava (2018) involves aspects of place value therefore one important lesson I still drew from his study was the taking note of how primary school teachers apply the concept of place value to solve problems on Number Bases as they are teaching. However, we cannot rely on Sitrava (2018) study to deduce the SMK and PCK of primary school teachers since his study did not involve elements of PCK, it only involved aspects of SMK.

Salma and Uwamahoro (2021) used mixed-methods to assess the level of mathematics teachers' pedagogical content knowledge (PCK) in selected secondary schools of Zanzibar. Salma and Uwamahoro (2021) study included a number of topics such as mensuration, number bases, fractions, vectors and trigonometry. A Likert-scale questionnaire related to PCK self-assessment was administered to 69 teachers, 12 of whom were observed three times during lesson delivery in their respective mathematics classrooms (Salma & Uwamahoro, 2021). The study based on a questionnaire revealed that the level of mathematics teachers' PCK was moderate (Salma and Uwamahoro, 2021). Salma and Uwamahoro (2021) did not focus on a single mathematical topic such as number bases. However, in line with Salma and Uwamahoro (2021) study, I also used questionnaires as one of the instruments in order to evaluate primary school teachers' SMK and PCK in the context of number bases.

Manirano (2017) conducted a study entitled "The pedagogical content knowledge (PCK) of Rwandan grade six mathematics teachers and its relationship to student learning"**.** The research tools Manirano (2017) used included a teacher test, a teacher questionnaire, video recording of lessons, learner questionnaires and learner pre- and post-tests. Though, my study and that of Manirano (2017) both involve PCK of primary school teachers in mathematics, I cannot rely on Manirano (2017) to deduce the SMK and PCK of primary school teachers in the context of number bases as he did not focus on a specific topic instead he focused on several topics which included fractions, shapes, number bases, and percentages among others. However, one thing I learnt from Maniranos' (2017) study was the application of teacher questionnaire and video recording of lessons during data collection.

Chikiwa (2017) conducted a study in South Africa entitled "An investigation into the Mathematics Knowledge for Teaching required to develop grade 2 learners' number sense through counting". Chikiwa (2017) employed a qualitative approach with a case study design to investigate Mathematics Knowledge for Teaching (MKfT) enacted in the teaching of number sense to Grade Two learners. Chikiwa (2017) used convenience sampling to select the participants. The study found that Foundation Phase teaching requires employment of all the domains of the MKfT to develop number sense to Grade 2 learners (Chikiwa, 2017). The study by Chikwa (2017) is important to my study as it involves both aspects of teacher knowledge. Nonetheless, the impotency with Chikwa (2027) study is that convenience sampling which was used runs a high risk that the sample may not represent the population (Creswell, 2014). The topic of number sense which Chikiwa (2017) investigated is in the same strand as number bases.

Therefore, since Chikiwa (2017) used a case study one thing I drew from his study was the using of a case study to evaluate the teaching and learning of Number Bases. In line with Chikwa's (2017) study findings I also took note of how primary school teachers employed the domains of the MKfT to the teaching of Number Bases.

Ayasile (2020) investigated the opportunities for learning number concepts and operations in mathematics that are available to learners in upper primary classes in Malawi. The study used mixed methods approach (Ayasile, 2020). Textbook analysis was used to collect data (Ayasile, 2020). Learners' textbooks for grade 5 to grade 7 were analyzed using the Mathematical Discourse of Instruction (Ayasile, 2020). The analysis was based on exemplification as one of the interacting components of a mathematics lesson that help to illuminate what is made available to learn. Ayasile (2020) points out that the analysis of text books was in line with the following questions: What opportunities to learn number concepts and operations are provided through the examples and task in the textbooks? How do the tasks enable enactment of the learning objects that are stated in the textbooks? And To what extent do tasks allow learners to apply and connect critical features of the mathematical content? Findings suggest that the textbooks do not provide a balanced range of examples and tasks as such; the textbooks offer few high-level thinking examples and few high-level cognitively demanding tasks (Ayasile, 2020). Ayasile (2020) investigation of opportunities for learning number concepts and operations in mathematics was limited to text books analysis only; therefore, we cannot rely on it to fully explore SMK and PCK of primary school teachers. Nonetheless, based on Ayasile's (2020) study findings during lesson observations I took note of how the primary school teachers use their text books when teaching Number Bases.

Wessels and Courtney (2014) evaluated the skills, knowledge, strategies and confidence related to number sense of Namibian final year pre-service primary school teachers. Concurrent mixed methods design was used and 47 purposively sampled final-year primary school pre-service teachers participated in the study (Wessels & Courtney, 2014). Questionnaires and tests were administered to collect quantitative data on pre-service primary school teachers' number sense (Wessels & Courtney, 2014). Qualitative data was collected through semi-structured interviews (Wessels & Courtney, 2014). To enhance the reliability and validity of the adapted instruments and the semi-structured interview Wessels and Courtney (2014) conducted a pilot study with local lower primary school teachers. The study showed that the final year pre-service primary school teachers are not proficient in number sense themselves and are therefore ill-equipped to promote number sense in the learners that they will teach (Wessels and Courtney, 2014). The weakness in the study by Wessels and Courtney (2014) is that they did not observe the pre-service primary school teachers to obtain first-hand information on how they actually teach number sense. Instead they relied on getting information from pre-service primary school teachers by means of questionnaires, tests and semi-structured interviews. However, what I learn from Wessels and Courtney (2014) study is that I should also conduct a pilot study in order to refine the instruments and the data collection process. Furthermore, based on their study findings during lesson observations I evaluated the proficiency of primary school teachers in Number Bases.

Azuka and Kurumeh (2013, p.1) conducted a study in Nigeria entitled "Use of Counters for Simplifying the Teaching of Number Bases in Secondary Schools". This was after they noted that the notion of the pedagogical sequence of "concrete to abstract" is violated in the teaching of Number Bases by many teachers in Nigeria (Azuka & Kurumeh, 2013). As a result many teachers teach number bases abstractly and this has created difficulties for students in trying to study and understand the topic (Azuka & Kurumeh, 2013). Azuka and Kurumeh (2013) sensitized teachers on how to use counters during a workshop. With counters in teaching number bases in workshops, many teachers were excited and also confessed that they never understood number base before the workshop (Azuka and Kurumeh, 2013). The study by Azuka and Kurumeh (2013) and my study both concern the teaching and learning of number bases. Nonetheless, the study by Azuka and Kurumeh (2013) focused on one asepect: use of instructional materials when teaching number bases but my study includes other aspects in addition such as choice of examples, teacher insight, and representations. Based on Azuka and Kurumeh (2013) study findings during lesson observations I took note of how proficiently the primary school teachers were using concrete teaching aids.

Mbonabi (2020) explored strategies used by Grade 6 primary school teachers when teaching learners in the multiplication of whole numbers in five selected primary schools in the Kavango East and West Regions of Namibia. A mixed method that complements qualitative and quantitative approaches was used to collect data (Mbonabi, 2020). Mbonabi (2020) selected ten mathematics teachers (two from each school) in the study from five different primary schools. Convenient sampling was used to select the teachers (Mbonabi, 2020). The ten teachers teaching grade 6 mathematics were requested to complete a questionnaire which required them to indicate the strategies that they employed in class when teaching multiplication of whole numbers. In addition a total of 200 learners' mathematics exercise books were analysed in order to identify the commonly used strategies by learners in multiplying whole numbers. The teachers indicated that they used a variety of strategies including repeated addition, complete-number, partitioning and compensation to teach multiplication of whole numbers (Mbonabi, 2020). Both Mbobani

(2020) study and my study involve primary school teachers as well as the number and calculations strand. However, the impotence with (Mbonabi, 2020) study is that he used convenience sampling which does not produce a representative result (Creswell, 2014). Nonetheless, one aspect I drew from Mbonabi (2020) study is that when selecting primary school teachers I ensured that the primary school teachers were not all sampled from the same school. Based on Mbonabi's (2020) study findings during lesson observations I took note of the various strategies primary school teachers were employing when teaching Number Bases.

Tabakamulamu (2010) used concurrent nested strategy (this involves simultaneous collection of qualitative and quantitative data) to conduct a study that assessed the extent to which teachers in Zambia could learn how to foster the use of strategies for mental calculation relating to double-digit whole number addition and subtraction in base ten in early primary mathematics; and to determine the corresponding impact of this on pupils' performance in numeracy. His study revealed that after the ten-week implementation period, participants who were part of the experimental schools changed their existing beliefs about mathematics teaching and learning and to some extent their classroom practices as well, to support the use of strategies for mental calculation for double-digit addition and subtraction (Tabakamulamu, 2010). Though Tabakamulamu (2010) study involved primary schools in Zambia just like my study, the focus for his study (on mental calculations of primary school pupils) is completely different from my study therefore we cannot rely on his study to deduce the SMK and PCK of primary school teachers about number bases in Zambia. Nonetheless, there are certain things that I should drew from Tabakamulamu's (2010) study notably the application of interviews during data collection.

Using a quantitative approach Kandjinga (2018) investigated the Mathematics teachers' Subject Matter Knowledge and Pedagogical Content Knowledge in some selected public schools in the Khomas Education Region of Namibia. A closed-ended questionnaire and a test were used to collect data from the sample (Kandjinga, 2018). The findings of the study revealed that Grade 12 mathematics teachers have satisfactory Subject Matter Knowledge but insuficient Pedagogical Content Knowledge of Mathematics. Most of the mathematics teachers acknowledged that they had challenges to explain some of the topics such as number bases, statistics, probability and trigonometry included in the Namibian Secondary school sylabus (Kandjinga, 2018). The study by Kandjinga (2018) is important to this study in that it also explored teachers' aspects of SMK and PCK. However, the difference in the study by Kandjinga (2018) is that the teachers were not observed to get first-hand information on how they teach mathematics. Instead the study relied on getting information from teachers by means of questionnaires only. Based on Kandjinga's (2018) study findings during lesson observations I evaluated both the Subject Matter Knowledge and Pedagogical Content Knowledge of primary school teachers in Number Bases.

Watanabe *et al.*, (2020) investigated how Zambian Grade 1 to Grade 4 children at primary school see a group of 10 (in base ten) as an effective pattern and structure with the given concrete material. Watanabe et al. (2020) employed a qualitative approach and data was collected through interviews. Watanabe et al. (2020) study consisted of three phases consisting of two pilot studies and the main study, during which a total of 146 children from Grades 1-4 were asked during interviews. Watanabe et al. (2020) asked children to perform tasks that focused on number competencies related to counting objects, expressing numbers of concrete objects, and composing and decomposing numbers. Watanabe et al. (2020) study results showed that, in form addition with two-digit numbers, all children counted without identifying 'groups of 10' while some used concrete materials, some recognized numbers in a pattern and identified a 'group of 10', which has not been previously observed in Zambia. The children were able to manipulate concrete objects and to recognize a 'group of 10' in the given 10-frames (Watanabe et al., 2020). Watanabe *et al*. (2020) study was only concerned with base ten unlike my study which also involves base eight, base two and base five. Therefore we cannot rely on Watanabe et al. (2020) study to deduce the SMK and PCK of primary school teachers about number bases in Zambia. Nonetheless, based on Watanabe *et al*. (2020) study findings during lesson observations I took keen interest to check if primary school teachers were giving their learners opportunities to manipulate concrete objects.

2.3.5 The practices of effective mathematics teachers

The Australian Association of Mathematics Teachers, (2006) contends that with regard to mathematics, there is widespread concern for good teaching practices. However, Frank et al (2007) state that the process of teaching and learning of mathematics is complex and that what constitutes good teaching is controversial. Dabiri (2003) acknowledge that teaching is a complex enterprise. It is neither a hard science nor an abstract art (Dabri, 2003).

Cooney (1999) note that there are four ways to view teachers: Isolationists, Naïve idealists, Native connectionists and Reflective connectionists. Isolationists' teachers think they know the right way to teach and bother minimally to incorporate new ideas. Naive idealist teachers integrate outside knowledge without much reflection. Native connectionists teachers engage in reflection, but compartmentalize contradictions or conflicts that arise among theories without any attempt to resolve the apparent contradictions. Reflective connectionists' teachers are reflective and they endeavor to resolve conflicts among theoretical perspectives (Cooney, 1999). As far as the teaching of number bases is concerned this implies that primary school teachers are supposed to embrace new progressive ideas. Turnuklu and Yesildere (2007) and Sebesta and Martin (2004) posit that though a number of factors may influence the teaching of mathematics, however, teachers play a key role in the teaching process. Besides, Turnuklu and Yesildere (2007) note that the common belief in society is that mathematics teachers who know mathematics very well are the best to teach mathematics. Though they acknowledge that, this on its own is not enough for someone to be a good teacher of mathematics. Another key aspect is the 'knowing to teach mathematics' ingredient (Turnuklu & Yesildere, 2007). It is in this regard that various researchers have explored different ways of enhancing the performance of mathematics teachers. For instance, during the proceedings of the second national conference in mathematics education held on 14th and 15th September, 2007 at St. Patrick's College, Dublin, McDonough (2007) revealed that for many years' researchers have sought to describe teacher behaviors that correlate positively with growth in student achievement in Mathematics. In addition, McDonough (2007) states that as early as the 1970s and 1980s, the so-called *process-product* research sought to describe behavior's that correlated positively with student achievement in mathematics. I hope my study may contribute to the enhancement of the performance amongst primary school teachers with regard to the teaching of mathematics in general and number bases in particular.

Fennema and Franke (1992) determined the components of proficient mathematics teachers' knowledge as;

i) Knowledge of mathematics, which includes

- Content knowledge
- The nature of mathematics
- The mental organization of teacher knowledge

ii) Knowledge of mathematical representations

iii) Knowledge of students

iv) Knowledge of teaching and decision making

The first item is about having conceptual understanding of mathematics. Fennema and Franke (1992) argue that if a teacher has conceptual understanding of mathematics, this influences classroom instruction in a positive way; therefore, it is important to have mathematics knowledge for teachers. Teachers' interrelated knowledge is very important as well as procedural rules. The second item "mathematical representations" refers to the mathematics teachers' ability to translate those abstractions into a form that enables learners to relate mathematics to what they already know. The third item "Knowledge of students" refers to knowledge of students' cognitions. This is seen as one of the most important components of teacher knowledge, because, according to Fennema and Franke (1992), learning is based on what happens in the classroom. Besides, Fennema and Franke (1992) and Thompson (1994) claim that it is not only what students do that matter, but also the learning environment.

The last component of teacher knowledge is knowledge of teaching and decision making. Fennema and Franke (1992) posit that teachers' beliefs, knowledge, judgments, and thoughts have an effect on the decisions they make which influence their plans and actions in the classroom. Fennema and Franke (1992) note that the first two components that is knowledge of mathematics and knowledge of mathematical representations are related to content knowledge, whilst knowledge of students and knowledge of teaching are related to pedagogical content knowledge. Fennema and Franke (1992) have outlined very important variables which I have used to form part of my conceptual framework, used to assess the PCK and SMK of primary school teachers on number bases.

Similarly, O' Meara (2010) note that in order to be able to teach Mathematics effectively teachers require three categories of subject knowledge: content knowledge (*know the mathematics*), subject specific pedagogical content knowledge *(know how to teach)* and the curriculum knowledge *(primary school Mathematics curriculum).* Kilpatrick et al (2001) claim that key ingredients of effective teaching include, conceptual understanding, procedural fluency, strategic competence (problem solving), adaptive reasoning (proof and justification) and productive disposition (perseverance and thinking mathematically). As for Shulman (1987) effective teaching begins with a teacher's understanding of what is to be learned and how it is to be taught. It proceeds through a series of activities during which the students are provided specific instruction and opportunities for learning, though the learning itself ultimately remains the responsibility of the students. Teaching ends with new comprehension by both the teacher and the student. Therefore, in line with Shulman (1987) assertions during lesson enactments of the topic of number bases, I keenly observed how primary school teachers incorporate all these important aspects.

Wilson, et al (2005) conducted a study on the qualities of an effective mathematics teacher with a sample size of nine secondary school teachers. Their study results indicated that qualities of an effective teacher include: connecting mathematics between topics and to the real world, helping students see mathematics through the use of computers or calculators, drawings, or concrete materials, frequent assessment of students, engaging and motivating students, managing effectively and reflecting on teaching.

Using a survey as a strategy of inquiry Mulenga (2012) conducted a study which aimed at identifying and describing the factors that inspire and motivate upper basic and high school student teachers in Zambia to choose mathematics as their teaching subject. The study revealed that on average teachers' knowledge of the subject seemed to have been the most important motivating quality of a mathematics teacher, followed by the quality of encouraging pupil participation, and the ability to teach at the high school level. Therefore, it would be necessary to establish primary school teachers' levels of SMK and PCK in relation to number bases.

McDonough (2007) conducted a case study in which a test involving aspects of the number system, computation, and problem solving, was aurally administered by teachers, with children writing their answers. The study revealed that highly effective teachers: connected different ideas of mathematics and different representations of each idea by means of a variety of words, symbols and diagrams and they also encouraged students to describe their methods and emphasised the development of mental skills. As for Askew (1998) a good teacher is the one who creates an environment for success by ensuring that students know that the teacher is interested in their success. The high expectations teachers hold for all their pupils send clear messages of how teachers feel about their education (Askew, 1998).

In a supportive view Fennema (2000) outlines that an effective teacher raises expectations for students' learning, develops effective methods of supporting the learning of mathematics by all learners and provides the learners with resources they need such as mathematics textbooks, mathematical instruments and any other materials needed for the learning of mathematics. A good mathematics teacher also addresses each student as an individual with instructional opportunities (Phiri, 2003).

Kilpatrick (2001) argue that teachers who improve students' mathematical achievements are those with the ability to unpack the content by making basic underlying concepts become visible. In a supportive view Wilson (2006) and Anghileri (2006) contend that when teachers are given the opportunities to learn in ways that address the needs of students and the ways of unpacking the content of mathematics that agree with the way's students think, the teachers tremendously improve students' mathematical achievement. (Askew , 1998; Fennema, 2000; Kilpatrick, 2001; & McDonough, 2007) have raised very important issues, hence the importance for my study in finding out how primary school teachers show concern, interest and provide support to pupils' success.

Based on review of literature in mathematics education, the philosophy of mathematics, the philosophy of education and research on Teaching and Learning, Kuhs and Ball (1986) identified "at least four dominant and distinctive views of how mathematics should be taught. These are

- Learner focused
- Content focused with emphasis on conceptual understanding
- Content focused with an emphasis on performance
- Classroom focused

Learner focused involves Focusing on learner's personal construction of mathematical knowledge (Kuhs & Ball, 1986), Content focused with emphasis on conceptual understanding is driven by content that is focused on conceptual understanding (Thompson, 1992), Content focused with an emphasis on performance refers to student performance, mastery of mathematical rules and procedures, combined with use of exact, rigorous, mathematical language and Classroom focused refers to how mathematical teaching activities are structured and organized (Thompson, 1992). Kuhs and Ball (1986) have raised very important pedagogical issues and I have taken these as part of my conceptual framework to assist me in exploring primary school teachers' SMK and PCK about number bases. Gillard (2007) reveals that the Cockcroft Committee which was set up in England to investigate alleged lack of basic computational skills, the lack of qualified mathematics teachers and a variety of other issues recommended that mathematics teaching at all levels should include opportunities for:

- exposition by a teacher
- discussion between teacher and pupils and between pupils themselves
- appropriate practical work
- consolidation and practice of fundamental skills and routines
- problem solving including the application of mathematics to everyday situations
- investigational work

Gillard's (2007) revelations, implies that when conducting my study, I should take into consideration the teaching methods used by primary school teachers as they teach number bases. The reviewed literature in this section clearly outlines the requirements for effective teaching of mathematics. It is imperative therefore that primary school teachers in Zambia possess the attributes discussed in this section. One way to achieve this is by conducting researches like this one which explores primary school teachers SMK and PCK about Number Bases.

2.4 Models and theories of teaching mathematics

In this section I have discussed Aristotle's, Shulman's, Mathematical Knowledge for Teaching (MKT) model, Knowledge quartet, Lappan and Theule-Lubienski, and Leinhardt's model of agenda.

2.4.1 Aristotle theory

As Mayer (1992) reveals, beginning theories on how humans think and learn are traceable to the Greek philosopher Aristotle. Aristotle argued that learning and memory takes place in three stages, notably *contiguity, similarity,* and *contrast.* The doctrine of contiguity posits that events or objects that occur in the same time or space are associated in memory, so that thinking of one will cause thinking of the other. The doctrine of similarity claims that events or objects that are similar tend to be associated in memory. And finally, the doctrine of contrast stipulates that events and objects that are opposites tend to be associated in memory (Mayer, 1992). Aristotle's theory is crucial to my study it as it lays a firm foundation as far as theories of how human beings think and learn are concerned. Moreover, my study is exploring the teaching and learning of number bases, Aristotle theory also covers aspects of learning and teaching. Aristotle's doctrine of contrast implies that when conducting my study, I should take into consideration aspects of how primary school teachers relate mathematical operations such as that of changing numbers from base five to base ten to that of changing numbers from base ten back to base five.

2.4.2 Shulman's Model of teacher knowledge

Shulman's (1987) theoretical framework consists of seven knowledge categories (the categories are presented in Figure 2 below). These seven categories have become the foundation for describing the knowledge base for teaching (Livy, 2014). Among them, the first four are content-free and the last three are on content-specific knowledge (Rowland & Turner, 2007). Rowland and Ruthven (2011) point out that nearly all studies on mathematics teacher knowledge adopt Shulman's categorization of teachers' knowledge. Nonetheless, internationally, there is no universal prearrangement on widely-accepted framework for describing teachers' mathematical knowledge in teaching (Graeber & Tirosh, 2008).

- General pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organisation that appear to transcend subject matter
- Knowledge of learners and their characteristics
- Knowledge of educational contexts, ranging from workings of the group or classroom, the governance and financing of school districts, or the character of communities and cultures
- Knowledge of educational ends, purposes, and values, and their philosophical and historical grounds
- Content knowledge
- Curriculum knowledge, with particular grasp of the materials and programs that serve as "tools of the trade" for teachers understanding
- Pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding

Figure 3: Shulman's Major Categories of Teacher Knowledge.
Source: (Shulman, 1987, p. 8)

Commenting on Shulman model Ball et al (2008) reveals that the category which caused the greatest excitement amongst researchers was PCK. Graeber and Tirosh (2008) argue that though Shulman (1987) came up with different types of pedagogical knowledge he does not reflect in detail on the interrelationship between them or influences that may affect teachers' pedagogy. In addition, Graeber and Tirosh (2008) reveal that despite its pre-eminence, Shulman's notion of

PCK has been challenged and the concept has been expanded and modified by a number of other researchers to come up with other different types of models.

Despite Shulman's models having some weakness "it has been highly accepted because it was the manifestation of the paradigm shift in teacher education from a separate focus on only content and only pedagogical skills to combination of these two elements" (Tutak, 2009, p.14). It is in this regard that I have included it in my study, whose focus is on both primary school teachers' knowledge of number bases and its teaching.

2.4.3 Mathematical Knowledge for Teaching (MKT) model

Ball et al (2008) expanded Shulman's (1986) notion of Pedagogical Content Knowledge to establish the concept of Mathematical Knowledge for Teaching (MKT). Adams (2015, p.2) reports that "MKT is the mathematical knowledge needed to carry out the work of teaching mathematics".

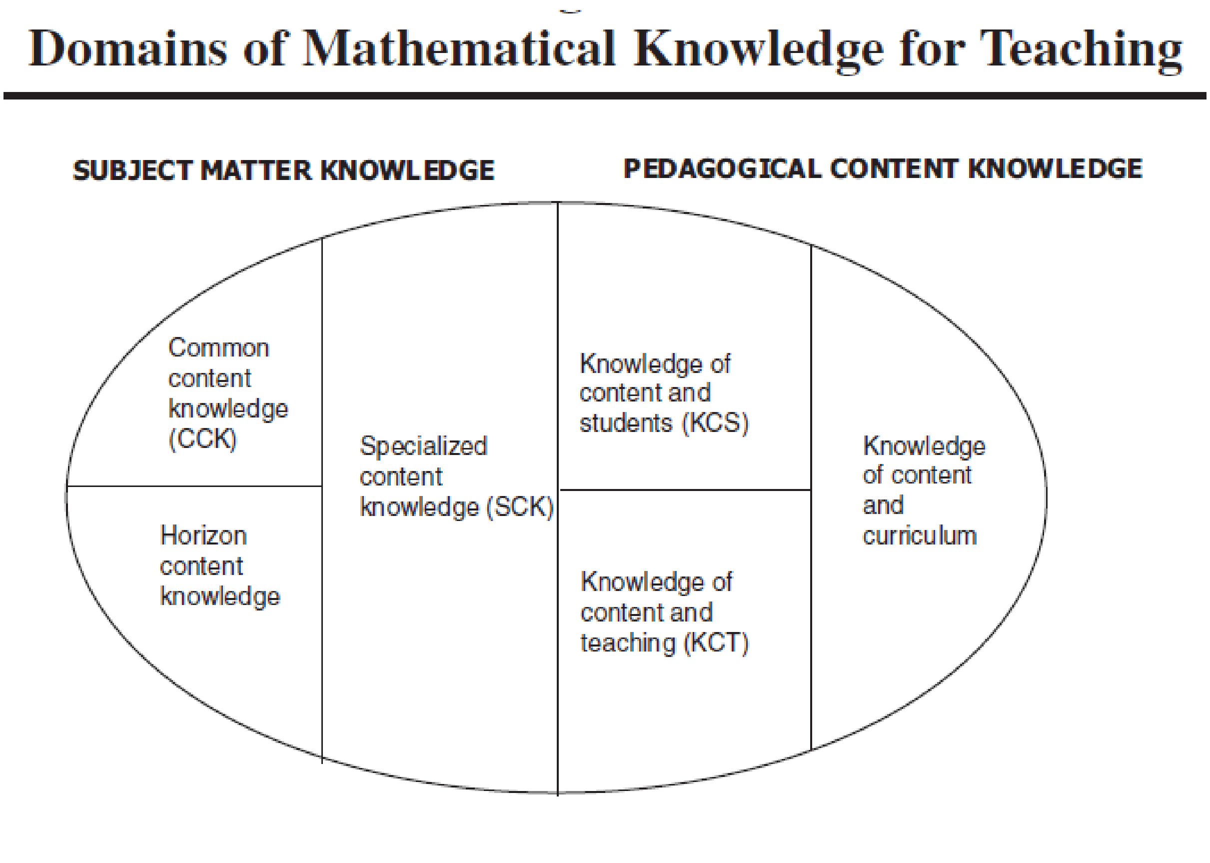


Figure 4: Domains of mathematical knowledge for teaching (MKT) framework (Ball et al., 2008, p. 403).

According to Adams (2015) the MKT framework deals with two key domains of teacher knowledge discussed by Shulman (1986) notably PCK and SMK. The empirically established model of Mathematical Knowledge for Teaching is based on observations of USA third grade mathematics classes (Adams, 2015).

Ball *et al.* (2008) explains that in this model the SMK domain is comprised of three sub-domains: Common Content Knowledge (CCK), Horizon Content Knowledge (HCK) and Specialized Content Knowledge (SCK). CCK refers to mathematical knowledge and skills not unique to teaching. The link with teaching comes from the need for the teacher to be able to do the mathematical work assigned to students (Adams, 2015). HCK refers to knowledge of how different topics are related over the span of mathematics included in (and just beyond) the curriculum. This knowledge is important for the appropriate sequencing of taught content. SCK refers to mathematical knowledge and skills unique to teaching.

Ball et al (2008) explain that SCK includes the ability to carry out such tasks as looking for patterns in student errors and determining if nonstandard approaches are valid and generalizable. Ball et al. (2008) have also devised a decomposition of Shulman's PCK domain. In the MKT framework, this domain comprises Knowledge of Content and Teaching (KCT), Knowledge of Content and Students (KCS) and Knowledge of Content and Curriculum (KCC). KCT combines knowing about teaching and knowing about mathematics. This comes into play in various ways, but perhaps most importantly when mathematical knowledge and choices in relation to instructional options and purposes come together (Adams, 2015). As pointed out by Ball et al (2008) KCT is also likely to be involved in contingent teaching actions, where, for example, a teacher decides which student contributions to pursue and which to put on hold. KCS involves knowledge that combines knowing about students and knowing about mathematics in a way that enables teachers to (for example) anticipate what students may think and what they will find confusing, interesting and motivating, and to interpret students' (not fully coherent) spoken words and written work (Ball et al, 2008). As revealed by Adams (2015) the MKT model has been found useful in a variety of ways such as the provision of a framework for the discussion of teachers' mathematical knowledge, and informing the development of teacher education programmes and the design of support materials for teachers. I strongly believe it is important for Primary school teachers in Zambia to be familiarized with Mathematical Models such as this one.

The sub-domains of MKT are very crucial to my study which focuses on establishing primary school teachers PCK and SMK about number bases. This is because they address various aspects of PCK and SMK such as the appropriate sequencing of a taught topic, looking for patterns in student errors and contingent actions (Ball et al, 2008). Therefore, the sub-domains of MKT are part of my conceptual framework to assist with the analysis of data. Nonetheless, this does not mean that the model has never been critiqued. For instance, Hurrel (2013, p.59) notes that the reservations against the model are that: the first 'line' between the Common Content Knowledge (CCK) and the Specialised Content Knowledge (SCK) is blurred, SSK appear to employ a larger area than KCC which might mean that SSK is more important than KCC, the third reservation is in using the term Pedagogical Content Knowledge (PCK) to describe the domains regarding pedagogical concerns as there is a strong argument to be stated that PCK is actually the link between the knowledge bases of content, pedagogy and context. For my study, this critique implies that it is significant to note how primary school teachers incorporate various aspects of the MKT framework model in their teaching of number bases without necessarily assigning greater importance to any one of them.

2.4.4 The Knowledge Quartet

According to the National Council of Teachers of Mathematics (2000) the Knowledge Quartet is defined as an empirically grounded theory of knowledge for teaching in which the distinction between different kinds of knowledge is of lesser significance than the classification of situations in which mathematical knowledge surfaces in teaching. Koklu and Aslan (2012) note that the Knowledge Quartet (KQ) framework provides studying teachers' content knowledge during instruction. It emerged from the work of the UK SKIMA project (Subject Knowledge in Mathematics). The researchers wanted to identify and to understand better ways in which the elementary teachers should teach mathematics content knowledge. The knowledge quartet has been widely used as an analytical tool to support the development of early-career primary school teachers' mathematics content knowledge.

The framework suggests four broad units in investigating teachers' mathematical knowledge in teaching: (i) foundation, (ii) transformation, (iii) connection, and (iv) contingency.

Figure 5 also summarizes the main theoretical views in my study.

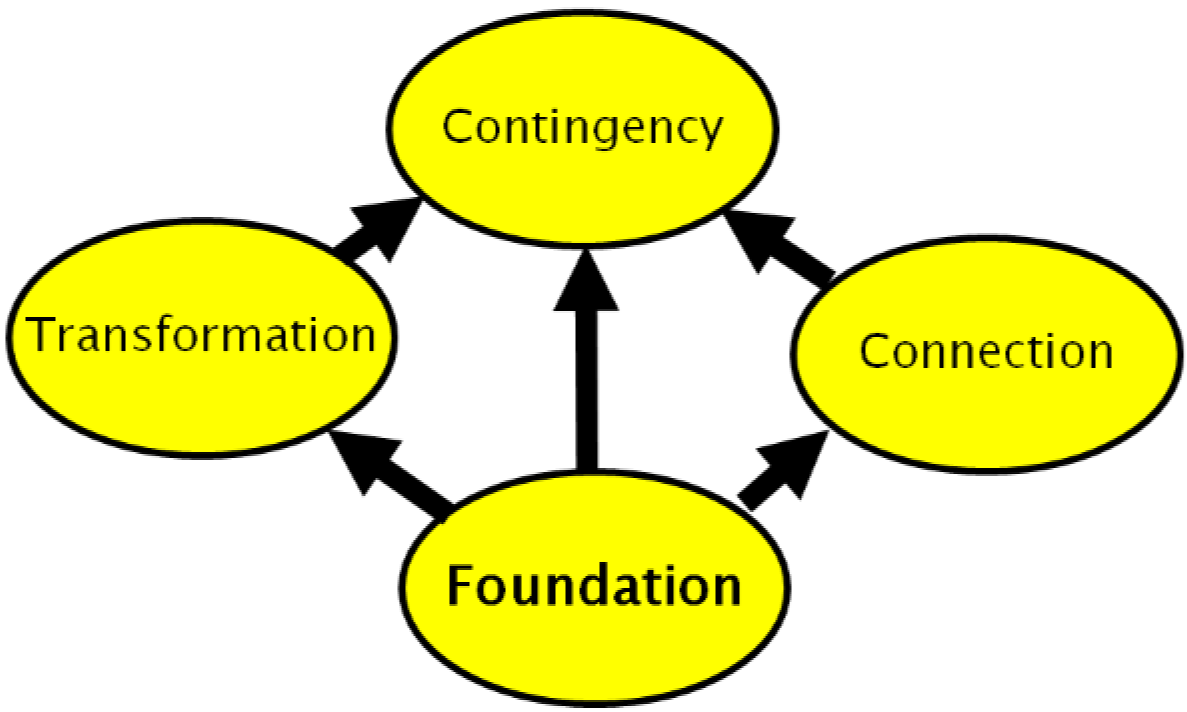


Figure 5: Knowledge quartet model **(**Tim Rowland et al., 2012).

The first unit of the KQ is rooted in the foundation of teachers' knowledge, beliefs and understanding of mathematics and teaching. Stevenson (2013) compares foundation stage with Shulman's subject matter knowledge (SMK).

Foundation differs from the other three units in the sense that it is about knowledge possessed irrespective of whether it is being put to purposeful use or not whilst the remaining three units

(Transformation, Connection and Contingent) refer to ways and context in which knowledge is brought to bear on the preparation and conduct of teaching (Rowland et al, 2005). They focus on knowledge in action as demonstrated both in planning to teach and in the act of teaching itself. This distinction relates directly to Aristotle's account of potential and actual knowledge (Rowland, 2000).

The second unit of the KQ, transformation, is knowledge-in-action which is visible throughout planning and teaching. The unit includes a teacher's capacity in transforming the content knowledge into pedagogically powerful forms. Stevenson (2013) asserts that this is where Shulman's pedagogical content knowledge PCK may be applied. As Shulman (1987) indicates, the presentation of ideas to learners entails their representation in the form of analogies, illustrations, examples, explanations and demonstrations.

The third unit 'Connection' concerns the depth, breadth and coherence of relationships demonstrated in an episode, lesson or series of lessons. It binds together certain choices and decisions that are made for the more or less discrete parts of mathematical content, perhaps of a concept or procedure (Rowland et al, 2005). Connections are important in mathematics in that mathematics is not, after all, a subject that contains discrete topics and also connections enable pupils sequence experiences, anticipate what they likely find 'hard' or 'easy' and understand typical misconceptions in a given topic. Stevenson (2013) links this stage to Shulman's curricular knowledge. The fourth unit of the framework, contingency, encompasses responding appropriately to the events and ideas which occur in classroom during instruction. It is about contingent action of teachers in the classroom (Rowland et al, 2005). The fourth unit takes the model beyond what we might think of as 'knowledge', and into the domain of professional skills (Stevenson, 2013). It requires an experience and connected level of knowledge on the part of the teacher to be able to do this effectively and efficiently. Novice teachers are much more likely to need to adhere to a prepared plan. Each of these KQ units is represented with codes. For example, there are three codes for contingency (responding to students' ideas, use of opportunities, and deviation from agenda). According to Rowland and Ruthven (2011) there are eight-teen codes in total for four units. Table five indicates the four dimensions of the knowledge quartet with their corresponding contributory codes.

Table 5: The Knowledge Quartet – dimensions and contributory codes

Dimension	**Contributory codes**
Foundation**:** Knowledge and understanding of mathematics per se and of mathematics specific pedagogy, beliefs concerning the nature of mathematics, the purposes of mathematics, and the conditions under which students will best learn mathematics	Awareness of purpose; Adheres to textbook; Concentration on procedures; Identifying errors; Overt display of subject knowledge; Theoretical underpinning of pedagogy; Use of mathematical terminology
Transformation**:** The presentation of ideas to learners in the form of analogies, illustrations, examples, explanations and Demonstrations	Choice of examples; Choice of representation; Use of instructional materials; Teacher demonstration (to explain a procedure)
Connection**:** The sequencing of material for instruction, and an awareness of the relative cognitive demands of different topics and tasks	Anticipation of complexity; Decisions about sequencing; Making connections between procedures; Making connections between concepts; Recognition of conceptual appropriateness
Contingency**:** The ability to make cogent, reasoned and well-informed responses to unanticipated and unplanned events	Deviation from agenda; Responding to students' Ideas; Use of opportunities; Teacher insight during instruction

Source: Rowland et al., (2009).

Rowland et al., (2005) says that the knowledge quartet has widely been used to investigate the mathematics content knowledge of primary trainee teachers, and the ways that this knowledge becomes visible both in their planning and in their teaching in the classroom. Therefore, the Knowledge Quartet is related to my study in that they both focus on establishing SMK and PCK of primary school teachers about particular topics in mathematics. Rowland *et al.*, (2005) note that KQ has been grounded in classroom practice, and the findings have been open to enhancement and revision in the case of new research data which I hope my study would do. The Knowledge Quartet is a very useful tool. Nonetheless, this does not mean that the Knowledge Quartet has never been critiqued. For instance, Warburton (2015) notes that the reservation against the model is that since it is intended to be used to discuss how teachers can improve their lessons some positive aspects of the teachers' lessons may be overlooked during lesson observations.

2.4.5 Lappan and Theule-Lubienski (1994) Model

Lappan and Theule-Lubienski (1994) presented a model of the domain of teachers' knowledge that incorporates three spheres of knowing: Pedagogy of mathematics, Students, and Mathematics. The three spheres of teacher knowledge represent knowledge of the mathematics content; knowledge of students' cognition, knowledge of students' difficulties with concept domains, and how to motivate and facilitate learning; and finally, knowledge of how to orchestrate pedagogy of mathematics that empowers learning and students' involvement. Lappan and Theule-Lubienski (1994) characterization fits within the model established by Shulman (1986, 1987). Though Lappan and Theule-Lubienski (1994) model deals with pedagogy of mathematics, it has not fully explained how to assess the three spheres of knowing embedded in it, therefore, I cannot fully rely on it to establish the PCK and SMK primary school teachers hold about number bases. However, one thing I learn from the model is that possibly during lesson observation I should take note of how primary school teachers take note of learners' difficulties in the context of numbers bases.

2.4.6 Leinhardt's model of agenda

Leinhardt's model consists of three key areas: agenda, scripts, and routines. The three areas are perceived to make claims about the forms of teacher knowledge as perceived through observation of classroom practice. According to this model:

> "*Agenda is the master plan that teachers impose on the mathematical content to facilitate pedagogy. Scripts are specific plans for dealing with specific topics that allow teachers to unpack the mathematical content for pedagogy. Routines are scripted sets of behaviors that allow teachers to carry out some activities in a relatively automated manner and with minimum cognitive load" (Sherin et al, 1999, p. 361).*

Sherin et al., (1999) note that Leinhardt's model is a cognitive one that explains teachers' behaviors from data gathered, possibly, from videotaping of teachers teaching in classrooms, viewing the videotapes, and inferring from the videotapes teachers' cognitions and then using the inferred cognitive structures of teacher knowledge to explain the scripts and routines of the teachers. Moreover, the model is more suitable for analyzing teachers' actual classroom practice (Sherin et al, 1999). Therefore, one important aspect that I draw from Leinhardt's model of agenda for my study in order to fully establish primary schools' teachers PCK is that I should possibly collect data by videotaping of primary school teachers teaching in classrooms. This would provide an opportunity to explore how primary school teachers apply the three key areas found in the Leinhardt's model: agenda, scripts, and routines in the teaching of number bases.

2.4.7 Mathematical Proficiency for Teaching Framework

Kilpatrick, Blume and Allen (2006) proposed a framework entitled '*Mathematical Proficiency for Teaching'*. It has two main parts: *mathematical proficiency with content* (MPC) and *mathematical proficiency in teaching* (MPT). The *mathematical proficiency with content* (MPC) includes conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), strategic competence (ability to formulate, represent, and solve mathematical problems), adaptive reasoning(capacity for logical thought, reflection, explanation and justification), productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy), cultural and historical knowledge, knowledge of structure and conventions, and knowledge of connections within and outside the subject. The *mathematical proficiency in teaching* (MPT) consists of knowing students as learners, assessing one's teaching, selecting or constructing examples and tasks, understanding and translating across representations, understanding and using classroom discourse, knowing and using the curriculum, and knowing and using instructional tools and materials (Kilpatrick et al., 2006). Groves (2012) compares this framework to Shulman's (1986) subject matter knowledge and pedagogical knowledge. The Kilpatrick et al. (2006) framework has been widely used in several countries to enhance the teaching and learning of mathematics (Groves, 2012). For instance, the new Australian Curriculum which was implemented in 2011 includes four strands of *mathematical proficiency with content* (MPC): conceptual understanding, procedural fluency, strategic competence and adaptive reasoning (Groves, 2012). The framework has also been used by several researchers as a guide to come up with questionnaire items based on the strands of mathematical proficiency (Groves, 2012). Therefore, to achieve similar goals of producing a research questionnaire that addresses important aspects of learning and teaching mathematics, I have also applied the Kilpatrick et al (2001) framework in the construction of the questionnaire items.

In this Section I have discussed Aristotle's theory, Shulman's model, Mathematical Knowledge for Teaching (MKT) model, the Knowledge quartet model, Lappan and Theule-Lubienski model, and Leinhardt's model of agenda. In line with the teaching and learning aspects raised by Aristotle during lesson observations of number bases Aristotle's theory has been applied to explore how primary school teachers relate mathematical operations such as that of changing numbers from base five to base ten to that of changing numbers from base ten back to base five, Shulman's models has been vital in this study exploring primary school teachers' SMK and PCK in that it was the manifestation of the paradigm shift in teacher education from a separate focus on only content and only pedagogical skills to combination of these two elements. From Ball et al (2008) I applied the notion of HCK knowledge (which is key for the appropriate sequencing of taught content) to explore the criteria primary school teachers use to sequence number bases lessons, The Knowledge quartet and the Mathematical Proficiency for teaching has been used as part of the theoretical framework. Lappan and Theule-Lubienski (1994) model has helped to evaluate how primary school teachers take note of learners difficulties during lesson observation in the context of numbers bases. From the Leinhardt's model I obtained and applied the notion of videotaping of primary school teachers as they taught number bases in order to fully explore their PCK. The next Section provides literature summary and illuminates the gap.

2.5 Literature summary and Illuminating the gap

In this Chapter, the researcher presented a review of literature related to this study. The review of literature included: the meaning and importance of number bases, aspects of Subject Matter Knowledge and Pedagogical Content Knowledge and models and theories of teaching mathematics. Table 6 shows some of the authors reviewed and their main findings related to the SMK and PCK of primary school teachers

Table 6: Summary of the main findings

Author	Main finding
Archer (2021)	Multi-base blocks is an effective teaching and learning material which can aid pupils understanding in number bases and also motivate them to learn
Wessels and Courtney (2014)	Grade 12 mathematics teachers have satisfactory Subject Matter Knowledge but insufficient Pedagogical Content Knowledge of Mathematics
Roy (2014)	Base-eight allows prospective teachers to reason about addition and subtraction with whole numbers in similar ways that elementary aged students' reason in base-ten
Watanabe et al. (2020)	In the absence of concrete materials pupils counted without identifying 'groups of 10' but in the presence of concrete materials some recognized numbers in a pattern and identified a 'group of 10.
Tabakamulamu (2010).	Through training pupils can change their existing beliefs about mathematics teaching and learning and to some extent their classroom practices as well.
Mbonabi (2020).	Teachers indicated that they used a variety of strategies including repeated addition, complete-number, partitioning and compensation to teach multiplication of whole numbers
Azuka and Kurumeh (2013)	With counters in teaching number bases in workshops, many teachers were excited and also confessed that they never understood number base before the workshop
Fasteen (2015)	Alternate numeration systems help pupils to understand better the base ten numeration system
Ayasile (2020).	Textbooks do not provide a balanced range of examples and tasks as such, the textbooks offer few high-level thinking examples and few high-level cognitively demanding tasks
Chikiwa (2017)	Foundation Phase teaching requires employment of all the domains of the MKfT to develop number sense to Grade 2 learners.
Uwamahoro and Salma (2021).	The study based on a questionnaire revealed that the level of mathematics teachers' PCK was moderate
Sitrava (2018)	Most of the prospective teachers could not explain the underlying principle and the meaning of the nonstandard algorithm in subtraction.
Mohammed (2015)	Trainees responded to the students' answers by *Confirmation* actions or *questioning*
Naukushu, 2016).	The number sense of the pre-service mathematics secondary teachers was below basic before the intervention.
Mntunjani, (2015).	Teachers understood the importance of using resources to teach number concepts, but were inclined to rote teaching with work drills in preparation for assessments
Maugesten (2019)	Good teacher must give students time to think after asking questions and that they should not feel that the students should rather be solving written mathematics problems.
Fasteen (2015)	Alternate numeration systems can be leveraged to create opportunities for pre-service mathematics teachers to improve their understanding of base ten .
Safi (2009)	Educators and educational policy makers can revisit and possibly revise instructional practices and sequences in order to develop teachers with greater conceptual understanding of concepts vital to elementary mathematics.
Harshman (2020	Increased PCK does not necessarily lead to increased SMK

Stacey et al. (2001)	Need for teacher education to emphasize content knowledge that integrates different aspects of number operations knowledge

Review of related literature has shown that some of the studies on numbers bases were focused on a single alternate base only such as base eight whilst my study includes four number bases (base ten, base five, base eight and base two). Review of literature related has also shown that some of the studies on number bases focused on a single attribute such as "concrete materials used in the teaching of Number Bases" whilst my study in addition to instructional materials used in the teaching of Number Bases includes many other attributes such as "choice of examples, overt display of subject knowledge and teacher insight among many others. Some studies on the teaching of mathematics in primary schools involved several topics such as fractions, number bases and angles but my study specifically focuses on one topic 'number bases'. Some studies done on Number Bases were conducted in secondary schools while this one is being conducted in primary schools were most of the learners mainly depend on concrete materials to understand certain concepts. In some studies done on Number Bases data was collected using interviews and questionnaires only without including lesson observations. This makes it difficult to know what really happens in the classroom. This study is using the Knowledge Quartet, but other studies conducted about number bases to my knowledge used different theoretical frameworks, it is not clear if same results can be obtained. The reviewed literature is also limited on account of being based mainly on studies conducted outside Zambia. The few studies that were conducted in Zambia did not focus on Grade seven primary school teachers and never used the Knowledge Quartet as their theoretical frameworks. For instance the study by Watanabe *et al*. (2020) which was conducted in Zambia differs from my study in that my study involves both base ten and the alternate bases whilst that of Watanabe *et al*. (2020) focused on base ten only. Another study on Number Bases conducted in Zambia was done by

Harshman (2020) and it focused on base eight only whilst my study in addition to base eight also includes base five, base ten and base two. Therefore, my research which explores the mathematics subject matter knowledge (SMK) and pedagogical content knowledge (PCK) of primary teachers with a focus on the breadth and depth of number bases is warranted and would contribute to our understanding of primary school teachers' knowledge of Mathematics as well as contributing positively to impacting the valuing of primary school teachers' professional knowledge and practice in Zambia. The next chapter describes the methodology employed to carry out the study.

CHAPTER THREE

METHODOLOGY

3.1 Introduction

The purpose of this study is to evaluate the mathematics subject matter knowledge (SMK) and pedagogical content knowledge (PCK) of Grade 7 primary school teachers with a focus on the topic of number bases. This Chapter presents the research methodology. Methodology is the ‘strategy or plan of action which lies behind the choice and use of particular methods’ (Crotty, 1998. p. 3). Precisely this Chapter presents information on the research paradigms, research approaches, research designs, research methods and reflection on ethical issues.

3.2 Research Paradigm

The study employed the interpretivism paradigm. This is because interpretivism usually seeks to understand a particular context, and the core belief of the interpretive paradigm is that reality is socially constructed (Willis, 2007). Besides, interpretivism paradigm is appropriate for education researchers whose purpose is to investigate a phenomenon in a group of students or a particular school (Creswell, 2014). In conformity with the interpretivism paradigm, my study took place in the school environment, a natural setting for the primary school teachers. Therefore, the choice of interpretivism paradigm is suitable for my study; evaluating primary school teachers’ SMK and PCK on Number Bases. Applying this philosophical view in my study enabled me to collect rich and deep data on the primary school teachers subject matter knowledge (SMK) and pedagogical content knowledge (PCK) about Number Bases as researchers using interpretivist paradigm often seek experiences, understandings and perceptions of individuals for their data to uncover reality rather than rely on numbers of statistics (Creswell, 2014).

3.3 Ontological assumptions

The ontological position of interpretivism is that there are multiple socially constructed realties (Creswell, 2014). This implies that, reality is subjective and differs from person to person (Guba & Lincoln, 1994, p. 110). Therefore, in line with Guba & Lincoln, (1994) the ontological assumptions for my study is that there are multiple realities of evaluating primary school teachers' SMK and PCK in the context of number bases. This entails that I had to use quotes and themes in words of participants and provide evidence of different perspectives (Kimberly, 2008).

3.4 Epistemological assumptions

The epistemological stance on interpretive approaches is that researchers interact with what is being researched (Clarke, 2005). This entails that knowledge of reality is gained only through social constructions such as language, shared meanings, tools, documents and so on (Walsham, 1993). Thus, in line with this epistemological stance my study embedded interactive methods such as interviews, questionnaires, lesson observations and concept maps in order to proficiently explore primary school teachers SMK and PCK in the context of number bases.

3.5 Research Design and Approach

I used a qualitative research approach. This is because according to Denzin and Lincoln (2005) proponents of interpretive paradigm use qualitative research methods. Denzin and Lincoln (2005) explain that a qualitative research is an approach to the study of human behavior that relies on the analysis of narrative data to create an interpretation of the meaning of these behaviors from the perspective of the participants themselves, within their own social context. Willis (2007) states that examples of qualitative study designs include: *case studies* (in-depth study of events or processes over a prolonged period), *phenomenology* (the study of direct experience without allowing the interference of existing preconceptions), *hermeneutics* (deriving hidden meaning from language), and *ethnography* (the study of cultural groups over a prolonged period). Amongst, these study designs I used the case study design. Yin (1994, p. 13) defines a case study as “an empirical inquiry that investigates a contemporary phenomenon within its real-life context, especially when the boundaries between phenomenon and context are not clearly defined”. Despite having the limitation of being subjective and not objective, I still employed a case study design because I wanted to shade more light on the Grade 7 primary school teachers' SMK and PCK in the context of number bases. This is because a case study allows an investigation to shade more light on a phenomenon by studying in depth a single case example of a phenomenon (Yin, 1994,p. 13). In addition, the application of case study design in this study helped me to provide a deep and logical way of evaluating the SMK and PCK of Grade 7 primary school teachers in the real life context for the topic of number bases (Creswell, 2014).

3.6 Research Methods

Research methods involve the forms of data collection, analysis, and interpretation that researchers propose for their studies (Creswell, 2014). Therefore, in this Section I will discuss sampling procedure, research unit, research site, target population, sample size, sampling procedure, data collection and data analysis

3.6.1 Research unit

According to Easton and Mc Coll (2009) a research unit is a person, animal or thing which is actually studied by a researcher; the basic objects upon which the study or experiment is carried out. In this study the research units were the individual Grade 7 primary school teachers who took part in the study.

3.6.2 Research site

The study was conducted in eight primary schools based in Lusaka district. An explanation on the nature of the schools and how they were selected is provided in Section 3.6.6, page 88.

3.6.3 Target Population

The target population included all Grade 7 primary school teachers teaching in primary schools in Lusaka Province.

3.6.4 Sample size

A sample size is a part of the population chosen for a survey to gain information about the whole (Ary & Razavieh, 1996; Kumar, 1999; Creswell, 2014). A sample of thirty Grade 7 primary school teachers, selected from eight primary schools took part in the study. I used a sample of thirty primary school teachers in line with Karania (2017) contentions that in a case study sample size are typically less than 30. All the 30 participants were Grade 7 teachers because the topic of number bases is taught in grade 7. Besides, Karania (2017, p. 1) notes that 'qualitative research methods are about developing *richness* in data based on an in-depth understanding rather than *representativeness* which is the preserve of quantitative research methods, hence sample size required for qualitative research methods is small in number (relative to the sample required for quantitative research methods).

3.6.5 Sampling Procedure

Sampling procedure involved two phases or stages namely: purposive sampling and nested relationship sampling design. First I applyed purposive sampling and then nested relationship sampling design. An elaboration of how I applied purposive sampling and nested relationship sampling design is provided in sub Section 3.6.5.1 and sub Section 3.6.5.2, respectively.

3.6.5.1 Purposive Sampling

Purposive sampling was used to select the eight primary schools from different zones. For a school to be purposively selected I had to ensure that it had Grade 7 pupils, since the topic of number bases is taught in Grade 7. Each of the selected primary schools had at least three grade seven primary school teachers. School A, B, C, D, E, and F had four grade seven teachers each, while school G and H had three grade seven teachers each. Thus, eight schools in total were selected. These eight schools provided adequate number of Grade 7 primary school teachers (thirty in total) who would provide rich and detailed information for a qualitative study (Ary & Razavieh, 1996; Cresswell, 2003). The thirty selected primary school teachers took part in the questionnaire and concept map.

Purposive sampling is a type of sampling that involves the use of judgment by the researcher as to who can provide the best information to achieve the objectives of the study (Kumar, 1999; Bless & Smith, 1995). Thus, sample elements judged to be typical, or representative, are chosen from the population, using purposive sampling (Ary & Razavieh, 1996; Cresswell, 2003). Purposive sampling is very useful when you want to construct a historical reality, describe a phenomenon or develop something about which only a little is known (Kumar, 1999). I used purposive sampling because of its power which lies in selecting information rich-cases for in-depth analysis related to the central issues being studied (Kombo & Tromp, 2006).

3.6.5.2 Nested relationship sampling design

Out of the thirty primary school teachers who took part in the questionnaire and concept map, a subset consisting of six primary school teachers was selected. These six are the ones who were observed and interviewed. The criteria for the selection of these six primary school teachers depended on their performance in the questionnaire and concept maps. The first two were the high achievers, the next two were the middle achievers and the last two were the low achievers. This was done in order to come up with a heterogeneous group that would take part in the lesson observations and interviews; a group whose proportional composition would mirror the population. A nested relationship implies that the sample members selected for one phase of the study represent a subset of those participants chosen for the other facet of the investigation (Onwuegbuzie & Collins, 2007, p. 292).

3.6.6 Data Collection Instruments

In studying events in their natural setting, the case study makes use of multiple methods of data collection such as interviews, documentary reviews, archival records, and direct and participant observations (Karania, 2017). Therefore, in line with the assumptions of the case studies as contented by (Karania, 2017) multiple methods of data collection were applied in this study, namely: concept maps, questionnaires, lesson observations and interviews. In the next sub-section, I highlight how each instrument was used.

3.6. 6.1 Concept Mapping

Ministry of Education, Science, Vocational Training and Early Education (2013) categorically states that one of the AIMS of teaching primary school mathematics is to enrich learners' understanding of mathematical concepts. One useful tool that can be used to enrich learners' understanding of mathematical concepts is the concept map (Baralos, 2007). According to Boogaard (2021) a concept map is a visual tool or diagram that consists of two main elements:

Concepts and relations. Boogaard (2021) contends that concepts are typically represented by circles, ovals or boxes while relationships are represented by arrows that connect the concepts. The arrows connecting relationships are called “cross-links” (Novak &Canas, 2008).

Concept map are used to measure conceptual knowledge which includes knowledge of facts, meanings of symbols, concepts and principles (Boogaard, 2021) of a particular field of mathematics. It is in this regard that this study applied concept maps for measuring Grade 7 primary school teachers’ conceptual knowledge on Number Bases. Thus, as noticed by Boogaard (2021) the use of Concept maps also helped to evaluate how Grade 7 primary school teachers organize their knowledge of Number Bases as a subject matter and gave an observable record of their understanding.

Boogaard (2021) note that concept map organizes knowledge into categories and sub-categories so that it can be easily remembered and retrieved especially if these concept maps are drawn by the students themselves. It is in this step that for my study concept maps were drawn by the individual Grade 7 primary school teachers themselves. Prior to administering the concept maps, a workshop aimed at familiarizing primary school teachers with concept maps as well as explaining the significance of concept maps in the teaching and learning of mathematics was conducted. The workshop included information on types of concept maps and steps in constructing concept maps. For more details on instructions to construct concept maps see Appendix B on page 214 and Figure 6, page 90 provides a sample of a concept map.

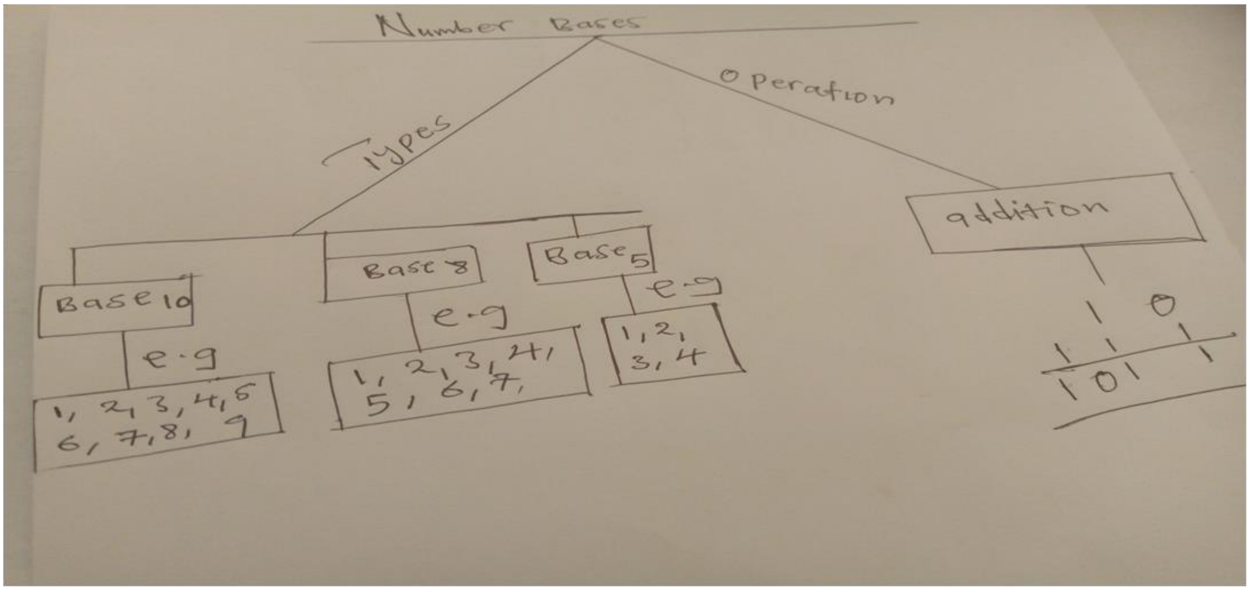


Figure 6: Sample of concept map constructed by T10

3.6. 6. 2 Questionnaire on number bases knowledge

A questionnaire on number bases was developed to explore Grade 7 primary school teachers' conceptual and procedural knowledge of number bases. For detail, see Appendix A on page 204. Thus, the questionnaire was designed to assess primary school teachers' knowledge of rules, formulas, algorithms, mathematical principles, ideas and representations of mathematical concepts used in the teaching of number bases. Thus, it differed from the conventional tests which normally assess behavioral skills such as rules, formulas and algorithms, which concern the procedural knowledge, but are not functional for the students' conceptual structure detection on certain topics (Baralos, 2007). The inclusion of the questionnaire was in line with Bhat (2020) who notes that in a qualitative research questionnaires are created to have a better understanding of a particular topic or to inspect a new subject to understand the nerve of respondent experiences. The questionnaire covered the following aspects of number bases:

1. Illustration of base ten numeration system

2. Description of other number bases

3. Conversion from Base 10 to Bases 2, 5 and 8.

4. Conversion from Bases 2, 5 and 8 to Base 10.

5. Conversion from base 2 to base 5 and vice versa

6. Addition and subtraction in Bases 2, 5 and 8.

7. Definitions, terminology, and conventions

The choice of these concepts for the questionnaire on Number bases was in accordance with the 2013 Zambian primary school syllabus. The questionnaire consisted of both items that assessed primary school teachers' subject matter knowledge (SMK) and pedagogical content knowledge (PCK). For instance for item 3 of the questionnaire part i assessed SMK while part ii and part iii assessed the PCK of the primary school teachers. After developing the questionnaire in line with the 2013 Zambian primary school syllabus, I gave the sample questionnaire to my supervisor to check whether or not the questions were phrased clearly to enhance validity (Best & Khan, 2008) and also in order to solicit new, relevant and important issues to include in the questionnaire. The pilot study was then carried out at one of the primary schools. The purpose of the pilot study was to detect difficult sentences, concepts and wordings as well as challenges in the collection of data as suggested by Karania (2017). A total of five primary school teachers took part in the pilot study. Karania (2017) contended that you rarely need to include more than 20 subjects in a pilot study. The outcomes of the pilot study lead to some minor modifications such as correction of spellings, grammar and layout. For instance, item 5 initially read: state the meaning of the digit 5 in the number 1523_8 and was changed to read: state the *value* of the digit 5 in the number 1523_8, in base ten, item 7 read: express 125_8 as a number of base two and was changed to read: express 125_8 as a number in base two? and item 10 read: why can 7+ 7 = 16? and was changed to: when does 7+ 7 = 16? In all the cases the small changes were done in order to use the correct mathematical terminology which would convey the appropriate mathematical meaning.

3.6. 6. 3 Lesson observations

Lesson observations are a way of gathering data by watching behavior, events, or noting physical characteristics in the classroom (Compbell & Ronfeldt, 2018). Observations have weaknesses such as susceptibility to observer's bias and failure to increase the observers understanding of why people behave as they do (Compbell & Ronfeldt, 2018). Nonetheless, I still used them because of their strength to enabling the observer to collect data where and when an event or activity is occurring and they enable the observer to directly see what people do rather than relying on what people say they did (Compbell & Ronfeldt, 2018). In a supportive view Kelly et al., (2020) and Hodgen (2011) unanimously agree that the knowledge employed in teaching is dynamic and should be studied in actual classroom settings. Also, Rowland and Ruthven (2011) argue that conceptualizing mathematical knowledge for teaching would unlikely be successful unless it carefully takes the classroom context of teachers' professional work into account. Kelly et al., (2020) point out that classroom observation can also be either direct or indirect. Direct observations is when you watch interactions, processes, or behaviors as they occur in the classroom; for example, observing a teacher teaching a mathematics lesson from a written lesson plan to determine whether they are delivering it with fidelity (Compbell & Ronfeldt, 2018). Indirect observations is when you watch the results of interactions, processes, or behaviors; for example, measuring the amount of plate waste left by students in a school cafeteria to determine whether a new food is acceptable to them. For my study I employed direct observations, in order to fully explore the PCK of the Grade 7 primary school teachers as they interacted with learners with focus on number bases. Lessons were audio recorded and transcribed for the key issues I intended to observe pertaining to PCK and SMK. These included aspects such as choice of representations, use of mathematical terminology and concentration on procedures. For detail, see Appendix C on page 219.

3.6. 6. 4 Post Lesson Semi-Structured Interviews

The interviews were semi-structured in nature. This means the interviews were planned in advance but contingent upon the performance of the interviewee's during post lesson observation and deviations dependent upon the interviewee's responses during the actual interview (Compbell & Ronfeldt, 2018). Kelly et al., (2020) contend that clinical interview questions: are open-ended, and this enhances chances for discussion and allows interviewees and interviewer to reflect on responses. The lesson post-observation interviews were conducted as soon as possible after lesson observations, so that the lesson would be fresh for both myself and in the observed primary school teachers' minds. Each interview was audio-recorded and later transcribed. The interviews were audio recorded in order to enhance accuracy and efficiency in data collection. The interviews were guided by the knowledge quartet. For detail, see Appendix D on page 221. The post-lesson interviews provided an opportunity to question primary school teachers on any aspects of the lesson that would require clarification or explanation.

3.4.6.5 Interviews based on the proposed Reference Manual

After analysing the results I proposed a reference manual that could enhance Primary School Teachers' SMK and PCK in the context of number bases (see Section 5.7, page 173 for details of the proposed reference manual). Six primary school teachers who took part in all the research instruments (the questionnaire, concept maps, lesson observations and post lesson observation interviews) were sensitised about the key areas that emerged from the study as well as on the possible ways of addressing the respective key areas in line with the findings. The following year (2019) out of these six primary school teachers, three were interviewed after they had taught the topic of Number Bases. This was done in order to explore the impact of the reference manual on primary school teachers SMK and PCK in the context of number bases (see Section 5.7, page 173 for the nature of questions asked and responses the participants provided). Though evaluation through interviews may suffer from the fact that participants may not come up with honest replies, however I still used interviews because they enabled me to gain valuable insights on how primary school teachers were teaching number bases after having participated in the study (Kelly et al, 2020).

3.6.7 Data Collection Procedure

After getting approval of the University of Zambia Ethics Committee I went to District Education Board Secretary Office to obtain permission to conduct my research in schools located in Lusaka district. Then I approached the head teachers for permission to conduct research in their schools. When getting permission, from the Head teachers, I showed them a letter from the District Education Board Secretary Office, indicating that I had been granted permission to conduct research in schools located in Lusaka district. After, permission had been granted by the head teachers, I then proceeded to meeting the primary schools. Once in contact with the primary school teachers, I explained the relevance and usefulness of the study as suggested by Karania (2017). Participants received Informed Consent Forms (Appendix F), see page 226 for details. Data collection was done by first using questionnaires followed by Concept maps, lesson observations, conducting post-lesson observation interviews and finally by conducting interviews based on the proposed reference manual. This was done in order to first asses Primary school teachers' conceptions and structure of the domain of Number bases, then evaluate their pedagogical content knowledge with respect to Number bases and finally monitor their application of facts and concepts in a real class room situation. As stated in Section 3.6.8.4 on page 94, conducting of interviews after lesson observations helped to shade more light on any aspect of the lesson that required clarification or explanation. Finally, conducting interviews based on the proposed model helped to explore the effectiveness of the measures taken to address the identified areas of concern.

3.6.8 Data analysis

This section discusses the ways in which the data collected from questionnaires, concept maps, lesson observations and post-lesson interviews was analysed. The data was analysed by applying thematic analysis. Thematic analysis is a qualitative analytic method for: *'identifying, analysing and reporting patterns (themes) within data. It minimally organises and describes your data set in (rich) detail. However, frequently it goes further than this, and interprets various aspects of the research topic.*' (Braun & Clarke, 2006, p.79)

Thematic analysis emphasizes pinpointing, examining, and recording patterns (or "themes") within data. Since the theoretical framework for my study (the knowledge quartet) contains four dimensions each with its own themes, therefore the choice of thematic analysis perfectly suits my study as it is in line with the demands of the theoretical framework. Thematic analysis was done by applying the constructs in the conceptual framework which were extracted from the Mathematical Proficiency for Teaching framework and Subject Audit Instrument (Knowledge Quartet). To achieve this, the audible and visual data obtained from classroom observations and interviews was transcribed into written form. The transcription of the observed lessons was coded and matched with Rowland's et al., (2009) four categories from the 'knowledge quartet'.

The four categories; foundation, connection, transformation and contingency were discussed with reference to Grade 7 primary school teachers' lessons in order to assess what SMK and PCK had been demonstrated during the mathematics lessons. Finally, findings from different teachers were compared and then similarities and differences were given. Table 7 indicates the description of the codes.

Table 7: Description of the codes

DIMENSION	CODE	SMK or PCK	INDICATOR
Foundation	Productive disposition	PCK	Habitual inclination to see mathematics as useful
	Identifying pupil errors	SMK	Good when a teacher Identifies errors
	Use of mathematical terminology	SMK	Demonstrate knowledge of the correct mathematical terms and their precise meanings
	Theoretical underpinning of pedagogy	PCK	Use of a theoretical foundation to guide instructional decisions, rather than relying on imitation of another teacher or trial and error.
	Overt display of subject knowledge	SMK	Teachers demonstrating that they know themselves the subject matter that has been selected for the class to learn.
	Adherence to text books	PCK	Using textbooks as a starting point and a guide for teaching with no rigid adherence to given materials
	Concentration on procedures	PCK	Focusing on instrumental approach in mathematics when explaining number bases
Transformation	Use of instructional materials	PCK	Good use of instructional materials results in simplifying students work,
	Choice of representations	SMK	Ability to configure number bases in form of symbols, images, or concrete objects
	Choice of examples	SMK	High cognitive demand, mathematically correct, numbers are selected intentionally, match the purpose of the lesson
Connection	Conceptual understanding	SMK	The teacher not facing difficulties when explaining the strategies or operations on number bases
	Anticipation of complexity	PCK	To “introduce ideas and strategies in an appropriately progressive order
	Decission about sequencing	SMK	To “introduce ideas and strategies in an appropriately progressive order
	Recognition of conceptual appropriateness	SMK	deciding to use another method to check if they would find the same answer
Contingency	Responding to students' ideas	PCK	Ability to make cogent, reasoned and well-informed responses to unanticipated learners ideas or suggestions
	Deviation from lesson agenda	PCK	Displaying deeper subject matter knowledge in taking a pupil’s remark as a starting point for deeper enquiry, probing pupils wrong answers
	Teacher insight	PCK	Teacher stops to reflect-in- action and changes tack

Source: Adapted from Rowland, et al. (2009) and Kilpartrick et al. (2006).

These are the themes I used to analyse data. Before, giving an example of how I exactly used these themes, I first state the steps that I followed. The following steps as suggested by the proponents of the knowledge Quartet (Rowland et al, 2004) were followed in order to analyse the lesson observations:

i) Write a detailed account of the lesson soon after the observation.

ii) Add to this detailed account any codes from the Knowledge Quartet which relate to the episodes. Not all episodes will have a code.

iii) Select the more useful/ interesting episodes from the lesson description and write a more fluid description of what happened to complete with some background/ context for the reader.

iv) Add to this fluid account screen shots from the lesson and/ or transcript sections from the interviews where relevant.

v) Finally, cut down on the information and add references from other authors where applicable which may help to account for the behavior observed.

For my study the codes from the knowledge quartet which relate to the episode have been italised for identification purposes.

Next, I now give an example of one of the lessons that I observed as well as the themes that emerged from the same lesson as a way of demonstrating how I was actually extracting the themes from the primary school teachers' observed lessons. For the proceedings of other lessons see Appendix G, on page 227.

Teacher 15, Lesson 1, Date: 07-06-17. Time: 08:40 – 09:20

Lesson overview

The focus of the lesson was on converting numbers from base two to base ten. T15 started the lesson by asking pupils on the digits used in base two and base five. This was followed by a few examples on the operation of changing numbers from base two to base ten. Finally T15 gave pupils an exercise which she marked during the same period. The lesson template is what Sullivan (2007) refer to us *active teaching.* Both the focus and pacing were determined by the teacher with limited learners' choice. In terms of the Kilpatrick et al. (2001) dimensions, this lesson template was useful for building conceptual understanding and developing procedural fluency strands. In this lesson T15 demonstrated instrumental approach to understanding mathematics.

Knowledge Quartet Analysis

In this lesson the first transcript was related to the *'use of mathematical terminology' theme* of the Knowledge Quartet. After expanding 2222 to obtain 2^3, 2^2, 2^1, 2^0, T15 referred to the expanded form as place value. So when coding before I could identify this as *'use of mathematical terminology'* and then make respective analysis on how T15 fared in relation to this theme, I first compared my accounts with the descriptions of each of the 17 themes indicated in Table 6. After finding out that the *'use of mathematical terminology'* theme was well related to this transcript, that is why I chose it.

The next theme in this lesson that emerged: *'decision about sequencing'* was related to the connection knowledge dimension of the Knowledge Quartet. In this lesson T15 demonstrated connection knowledge of making good *'decision about sequencing'* as identified by the knowledge quartet as can be seen in the following exchange:

T15: When you are looking at base two which digits do you use?

Pv: Zero and one

T15: Ok, what about five

Pz: five

T15: Is he correct?

Px: No

Here by highlighting the pre-requisite knowledge necessary for converting numbers in base two and base five to base ten T15 demonstrated ability to make sound *'decision about sequencing'*. Just like in the first transcript when coding before I could identify this theme as *'decision about sequencing'* and then make respective analysis on how T15 fared in relation to this code, I first related my accounts with the descriptions of each of the 17 themes indicated in Table 6 on page 93.

Another theme that emerged during this lesson was '*choice of examples'.* These emerged through the following example T15 provided on the board:

Change 2222 from base two to base ten

An analysis of the problem reveals that T15 used wrong digits as binary numbers only have two digits, zero and one. After comparing my accounts with the descriptions of each of the 17 themes indicated in Table 6, this exchange was found to be related to the '*choice of examples'* theme of the Knowledge Quartet.

As for questionnaire, the items were prepared in line with the Knowledge quartet. For instance as indicated in Appendix A, page 200 item 1 part i of the questionnaire assessed 'Overt display of subject knowledge', item 2 part ii evaluated 'theoretical underpinning of pedagogy', and item 3 part ii evaluated 'Conceptual understanding'. For example Item three read as follows: Using the binary base, construct one addition problem and solve it? Figure 7 shows sample of a response for item 3

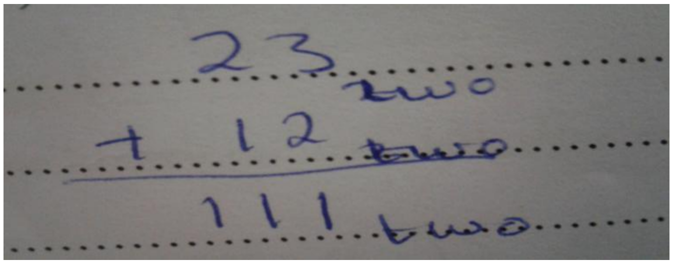


Figure 7: Sample of a response for item three

Since this item was under *Overt display of subject knowledge*, I would respectively place the response as part of *Overt display of subject knowledge.*

As for the concept maps since they were evaluating how primary school teachers connect concepts, all responses were categorised as part of connection knowledge.

3.7 Trustworthiness

According to Olivia (2017) for qualitative studies, trustworthiness is all about establishing research study's findings that are credible, transferable, confirmable, and dependable. *Credibility* is the how confident the qualitative researcher is in the truth of the research study's findings (Johnson, 2014). To ensure credibility for my study four data collection methods were used: Lesson observations, Concept maps, questionnaires and Post lesson observation interviews. Lesson observations were used to explore aspects such as contingent actions of a teacher which the questionnaires or concept map could not assess. Similarly, concept maps were used to assess conceptual knowledge which lesson observation would not fully explore. *Transferability* is how the qualitative researcher demonstrates that the research study's findings are applicable to other contexts. To ensure *Transferability* for the study, as suggested by Olivia (2017) thick description was used to show that the research study's findings can be applicable to other contexts, circumstances, and situations. Thick description was achieved by taking notes and recording with a tape recorder during lesson observation and post-lesson observation interviews (Olivia, 2017).

According to Johnson (2014) *confirmability* is the degree of neutrality in the research study's findings. To establish *confirmability*, for this qualitative study, I had to keep records of every step of data analysis and evidence was provided by using participants own words (Olivia, 2017). In addition, I ensured that evidence was provided in participants own words or writing. Olivia, (2017) explains that *dependability* is the extent to which a study could be repeated by other researchers and that the findings would be consistent. To establish dependability for my study, I ensured that the logic used for selecting primary schools, and participants to take part in the questionnaire, concept maps, lesson observation and post-lesson observation interviews, is clearly presented in the methodology chapter in line with Olivia's (2017) assertions.

3.8 Ethical Considerations

Munhall (1988) explains that research ethics are the norms or standards for conduct that distinguish between right and wrong. Ethical consideration is important in research because it avoids the fabrication or falsifying of data and therefore, promotes an atmosphere of trust, accountability, and mutual respect among researchers (Munhall, 1988). American Psychological Association (2017) Ethics Code contend that researchers who conduct research should inform participants about: 'the purpose of the research, expected duration and procedures, participants' rights to decline to participate and to withdraw from the research whenever they feel they no longer want to continue participating, as well as the anticipated consequences of doing so, any prospective research benefits and incentives for participation'. Researchers should also respect confidentiality and privacy of participants, (American Psychological Association, 2017). Therefore, in line with the American Psychological Association (2017) assertions I took into consideration the following ethical issues: Prior to collecting data, I got approval from the University of Zambia Ethics Committee. See Appendix

H for detail. I also communicated the aims of the research to the Head teachers of Primary schools in which the study was conducted. The Primary school teachers were not compelled to participate in the research. I also explained that the participants might benefit by acquiring more expertise necessary to teach number bases. I also assured them that the information they would provide would be treated confidentially. For detail see Appendix E on page 224. I used synonyms in order to protect the identity of the participants as well as that of their respective schools. For instance, the symbols T1, T2, T3, T4, T5 and T6 were used to represent the names of the primary school teachers who took part in the lesson observations. The primary school teachers were also assured that their participation will not affect their job position in any way.

3.9 Summary

This chapter focused on the research methodology. For my study, I employed interpretivism paradigm, qualitative research approach and a case study design. Purposive sampling was used to select 30 primary school teachers. Data collection instruments included questionnaires, concept maps, observations and interviews. Trustworthiness was achieved by ensuring that the findings were credible, transferable, confirmable, and dependable. Ethical considerations were done by obtaining permission from various authorities and from participants. The next Chapter presents the research findings.

CHAPTER FOUR

FINDINGS OF THE STUDY

4.1 Introduction

This Chapter presents the findings of the research which sought to evaluate the mathematics subject matter knowledge (SMK) and pedagogical content knowledge (PCK) of primary school teachers with a focus on Number Bases.

The presentation of findings is guided by the following research questions:

1. What foundation knowledge of Number Bases do primary school teachers have?
2. How do primary school teachers transform the content knowledge they have on Number Bases into forms that are pedagogically convenient to their pupils?
3. How do the primary school teachers connect instructions when teaching Number Bases?
4. What are the primary school teachers' contingency actions when teaching Number Bases?

4.2 Research question one: What foundation knowledge of Number Bases do primary school teachers have?

Research question one was intended to explore primary school teacher's knowledge and understanding of number bases in general and in particular of number bases specific pedagogy, beliefs concerning the nature of number bases, the purposes of number bases, and the conditions under which students will best learn number bases.

I have presented question one along the following themes (codes): Theoretical underpinning of pedagogy, Productive disposition, Identifying pupil errors, Overt display of subject knowledge, Use of mathematical terminology, Adherence to textbook and Concentration on procedures. In the above, apart from Productive disposition which has been taken from the Mathematical Proficiency theoretical framework advanced by Kilpartrick *et al*. (2006), the rest of the attributes have been identified by Rowland, *et al*. (2009) as part of the foundation dimension. The themes Theoretical underpinning of pedagogy, Productive disposition, Adherence to textbook and Concentration on procedures fall under PCK whilst the themes Identifying pupil errors, Overt display of subject knowledge, Use of mathematical terminology fall under SMK. For each of these major themes subthemes emerged from the data. In order to address question one fully the following instruments were used: questionnaire, concept maps, classroom observations and interviews.

4.2.1 Productive disposition

Under productive disposition the major emerging subthemes were (i) enhancing learners counting system and (ii) laying a solid foundation

4.2.1.1 Enhancing learners' counting skills

This subtheme was evidenced during the interviews by all the six participants. For example, when asked to state the importance of teaching and learning number bases, T13 said the essence of teaching number bases is:

> *To help learners know how to count."* and T1 said:
>
> *To help learners* so *that they learn other counting systems*

However, during lesson observations none of the six observed primary school teachers explained to their pupils the essence of teaching and learning number bases with regard to enhancing learners' counting skills. Moreover, none of the 19 participants who took part in concept maps provided a link for enhancing learners' counting skills. This might negatively affect the teaching of number bases.

4.2.1.2 Laying a solid foundation

This subtheme was evidenced during the interviews by only one participant, T12 who demonstrated *awareness of purpose* as can be seen in the following exchange:

> *To expose learners to different number systems …lay a foundation on which they are going to work on when they go on higher education….to enhance their counting skills to enable them count*

This shows that T12 had good subject matter knowledge (*awareness of purpose)* as far the teaching of number bases is concerned. However, one surprising thing is that T12 never mentioned or explained this to her pupils when teaching number bases. This might negtively affect the learners' performance.

From this analysis it is clear that primary school teachers' level of awareness of purpose in the context of number bases is quite low. They have not fully developed productive disposition to see number bases as functional, beneficial, and worthwhile.

4.2.2 Theoretical underpinning of pedagogy

Under theoretical underpinning of pedagogy the major emerging subthemes were (i) teaching methods, teaching aids and pre-requisite knowledge and (ii) use tabular representation

4.2.2.1 Teaching methods, teaching aids and pre-requisite knowledge

This subtheme emerged during the interviews. During the interviews when asked to mention what was considered when planning to teach the lesson on number bases, all but T17 failed to state any concrete measures that were put in place. T17 was the only one who said:

> *"I consider teaching methods, teaching aids and prerequisite knowledge*
>
> *when planning to teach the lesson on number bases."*

Here T17 made important remarks as far as the teaching of number bases is concerned. This shows that T17 has good foundation knowledge (*theoretical underpinning of pedagogy)*. Nonetheless when constructing concept maps no participant provided a link for teaching materials and pre-requisite knowledge necessary for the topic of number bases, an indication that perhaps participants faced challenges on the use of a theoretical foundation to guide instructional decisions. This implies that very few primary school teachers use concrete teaching materials when teaching number bases.

4.3.2.2 Using tabular representation

During lesson observations the subtheme *using tabular representation* was only evidenced in the lesson taught by T17. This happened when T17 gave the following example:

> '*Convert 352_8 to base ten?'*

T17 decided to work out the problem using tabular representation as shown in Figure 8.

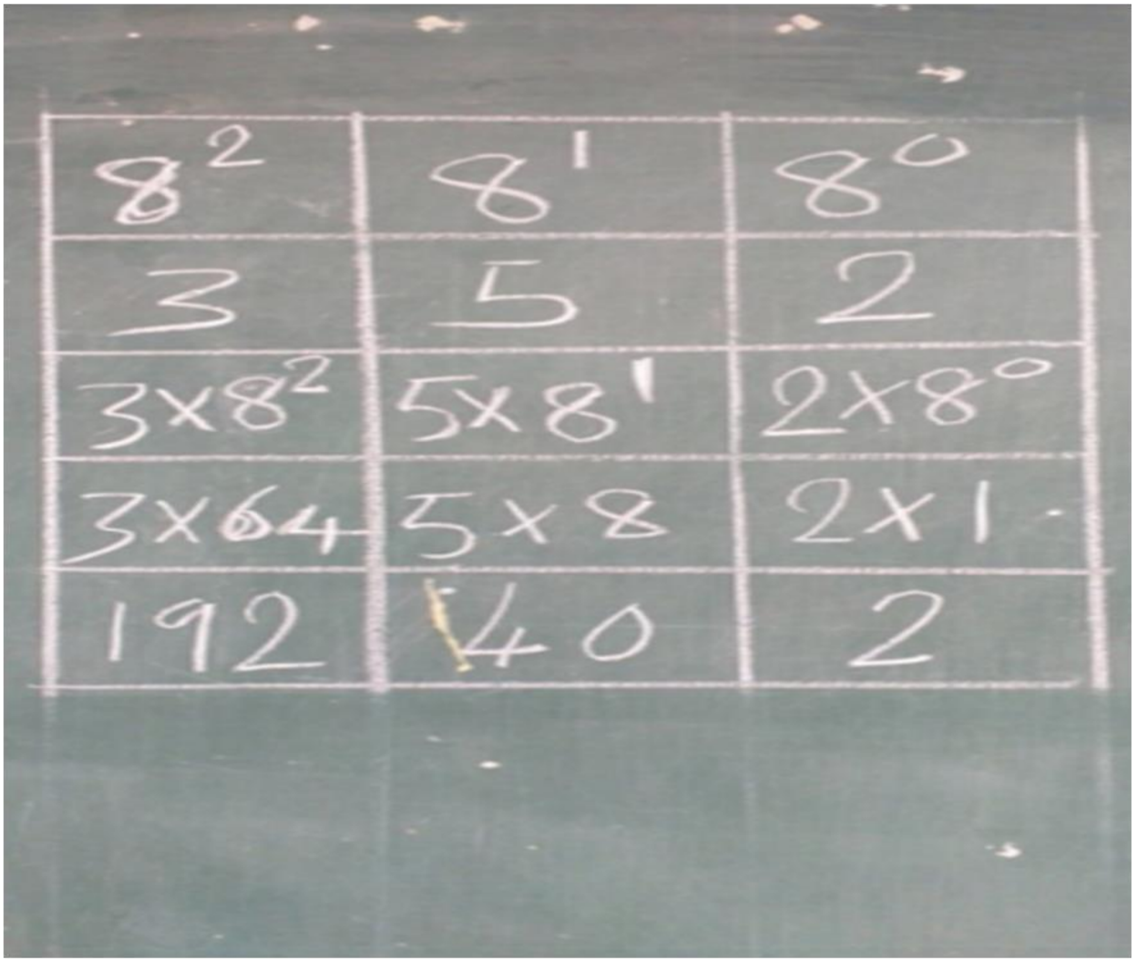


Figure 8: Changing 352_8 to base ten

T17: why have I used three columns?

Px: because it is in base eight

T17: who else can try?

Pv: I think because it has three digits

T17: Very good.

T17: As you can see on top of 2 the first digit, I have written 8^0, on top of 5 the second digit I have written 8^1 and on top of 3 I have written 8^2

T17: Then you have to multiply, 2 by 8^0, 5 by 8^1 and 3 by 8^2, to get 192, 40 and 2.

T17: Finally, you add 192, 40 and 2 to get 234 as the final answer

Here, T17 demonstrated foundational knowledge (*theoretical underpinning of pedagogy)* when she asked the pupil why she used three columns. This is an indication that T17 takes into consideration a theoretical foundation to guide instructional decisions, rather than relying on imitation of another teacher or trial and error.

4.2.3 Use of mathematical terminology

In relation to mathematical terminology the major emerging subthemes were (i) reading numbers in other bases as if they are in base 10 and (ii) reference to expanded form as place value.

4.2.3.1 Reading numbers in other bases as if they are in base 10

With regard to reading numbers in other bases as if they are in base 10, lesson observations revealed that all observed primary school teachers did not demonstrate sound knowledge of the correct mathematical terms and their precise meanings in the context of number bases. For example, T1 read 33_{five} as *thirty-three* and not as three fives and three ones, T11 read 23_{five} as *twenty-three* instead of two fives and three ones while T13 read *314_5 as three hundred fourteen* instead of three twenty fives, one five and four ones. This might negatively affect their learners in terms of reading alternate number bases.

4.2.3.2 Reference to expanded form as place value

One out of the six teachers *referred to expanded form as place value.* This was T15 who after expanding 2222 obtained 2^3, 2^2, 2^1, $2^{0.}$

T15: we call the expanded form as place value.

T15 is supposed to know that place value is a value represented by a digit in a number on the basis of its position in the number. Similarly, Figure 9 indicates that T5 did not indicate the special names for bases ten, eight and two. This has revealed that the average ability of primary school teachers to understand number bases principles and apply appropriate terminology and notations is not high.

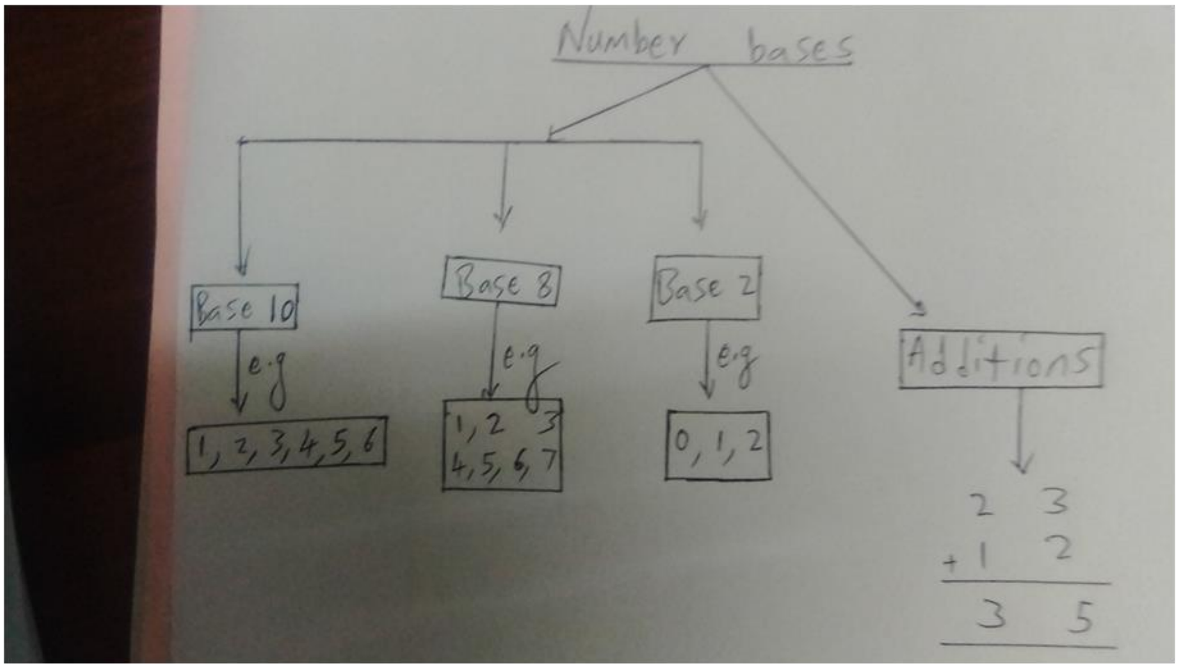


Figure 9: Sample of a concept map drawn by T5

4.2.4 Overt display of subject knowledge

In relation to overt display of subject knowledge the major emerging subthemes were: (i) Criteria for choosing digits used in a given number system, (ii) Place value and value of each digit and (iii) Formulating questions using some digits beyond the required base.

4.2.4.1 Criteria for choosing digits used in a given number system

This subtheme was evidenced in the questionnaires and concept maps. For example, Figure 10 shows a sample response for item eleven of the questionnaire by T 22.

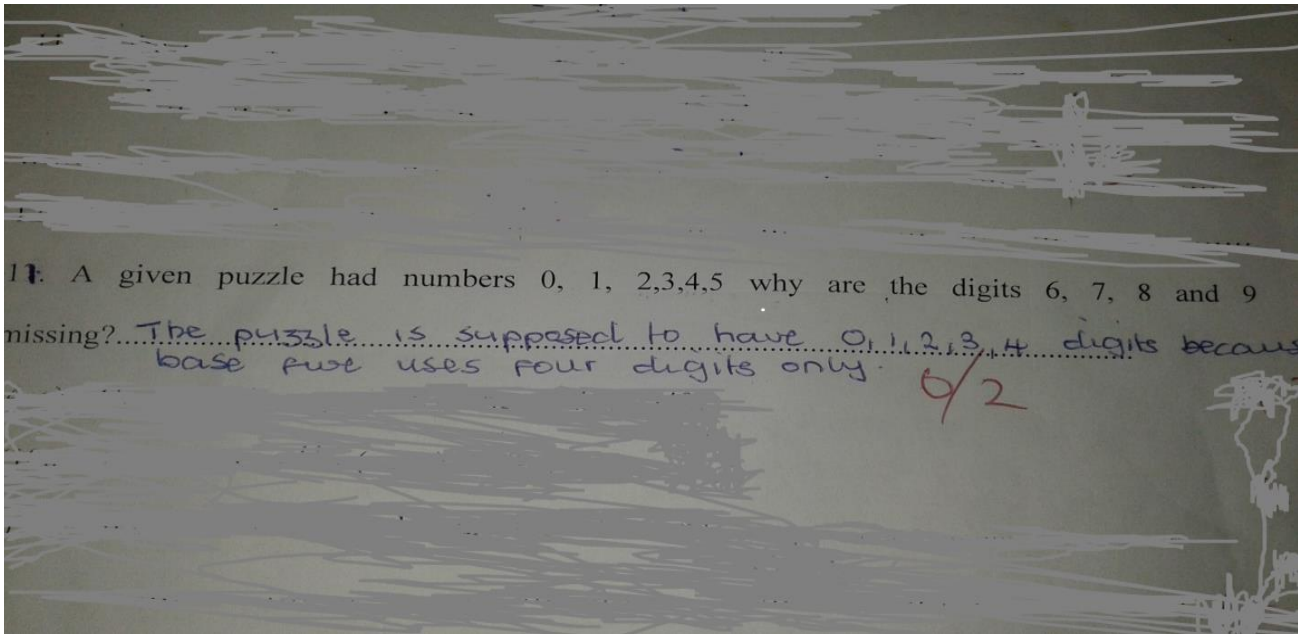


Figure 10: Sample response to item 11 by T 22

As can be seen in Figure 8 participant 22 explained that "the given puzzle had the numbers 0, 1, 2, 3, 4 without the digits 6, 7, 8 and 9 because "*base five uses four digits only*". Surprisingly, 17 out of the 30 questionnaire participants did this.

Likewise, findings from the concept map indicated a similar trend. For instance, as can be seen in Figure 7 (see Section 4.3.3.2 for Figure 7) in which T5 provided a link for base two which contained zero, one and two as digits found in the binary system. 10 out of the 19 concept map participants did this. Again, this is a sign that majority of the participants did not have well developed understanding foundation knowledge *(overt display of subject knowledge)* of the correct digits used in a particular number system.

Additionally, findings from lesson observations revealed that four out of six participants did not have well-grounded understanding of the digits used in a particular number system. For instance, during lesson observation T15 asked pupils to solve the following problem, as part of home work to be done over the weekend:

Convert 2222 from base two to base ten?

This was against the mathematical principle that the binary system only uses two digits, notably one and zero. As for T12, understanding of the appropriate digits used in a given number system '*overt display of subject knowledge'* was revealed during the following exchange:

T12: How many digits have the base ten systems?

Px; Ten

T12: No, it is nine.

Here T12 wrongly told the pupils that there are nine digits in base ten instead of ten. T12 was not aware that the number of digits in a given base system corresponds to the base system under consideration. That is base ten has ten digits, base five has five digits and so on.

4.2.4.2 Place value and value of each digit

This subtheme was evidenced during the lessons taught by two out of six of the observed teachers. Figure 11 was created by T12 to demonstrate the concept of place value and value of each digit in the number 753.

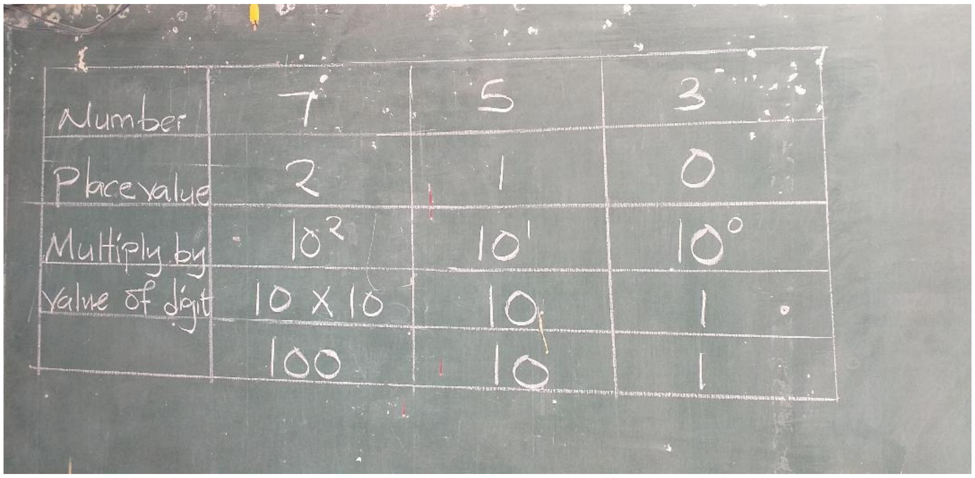


Figure 11: Finding of place value and value of digits by T12

As can be seen in Figure 11 T12 indicated that for the number 753, the place value for the digit 7 is two, the place value of the digit 5 is one and the place value of the digit 3 is zero. When in fact the place value for the digit 7 is 700, the place value of the digit 5 is 50 and the place value of the digit 3 is 3.

4.2.4.3 Formulating questions using some digits beyond the required base

Findings from those who took part in interviews revealed that five out of the six (T1, T11, T12, T17 and T15) did not have well developed ability to formulate questions in the context of number bases. Examples of responses indicating the nature of their strategic competence included:

T11: [*I have never formulated my own questions; it is not possible to do so*]

and T17: [*I always rely on textbooks and past examination questions as I give an exercise. I cannot remember when I wrote my own questions*].

This entails that they did not have well developed strategic competencies to either formulate or represent their own questions in the context of number bases. However, only T15 indicated having sound strategic competence and said:

> *[Yes, sometimes I prepare my own questions. For example, today when I saw that some pupils finished faster than what I expected, I was forced to prepare two more questions. One on converting 101_2 to base 10 and another one on converting 11_2 to base five].*

Results from questionnaire indicated that 22 out of 30 participants were not well grounded in formulating questions in the context of number bases. Figure 12 shows a sample response for item three of the questionnaire that was given by T14. Item three read as follows: Using the binary base, construct one addition problem and solve it?

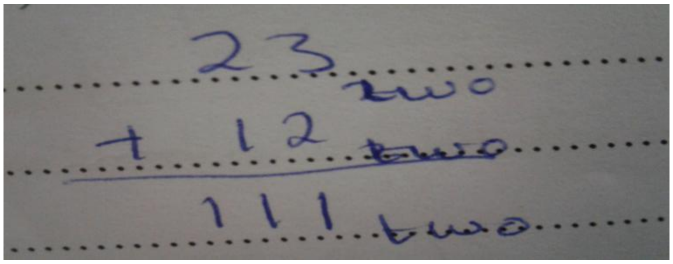


Figure 12: Sample of a response for item three

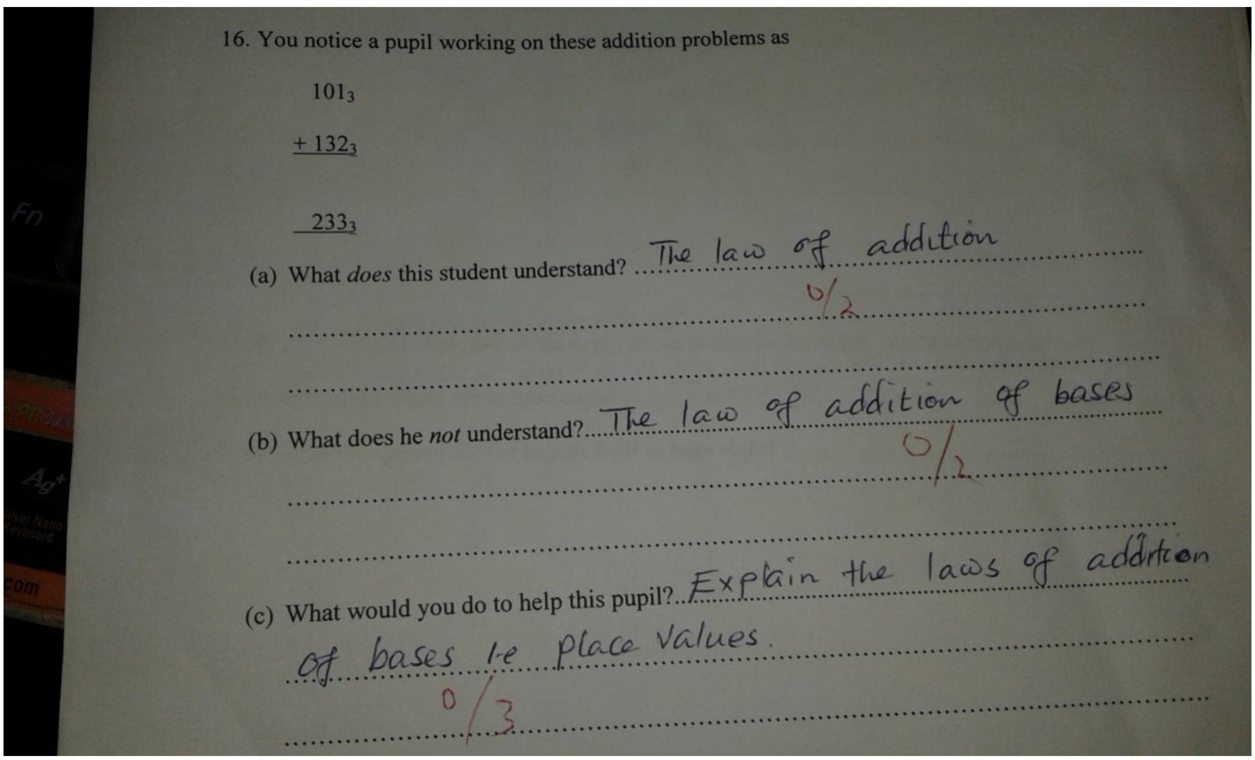
As can be seen in Figure 12, T14 wrote 23_{two} and 12_{two}, when it is very clear that base two has only two digits which are one and zero. This depicts that understanding the rules related to the representation of number in a particular base system by the participant are not well-grounded. Hence challenges in formulating appropriate questions in the context of number bases.

4.2.5 Identifying pupils' errors

In relation to identifying pupils' errors the major emerging subthemes were: (i) errors on identifying digits, (ii) errors on counting and place value and (iii) errors on adding and dividing number bases.

4.2.5.1 Errors on identifying digits

Item 16 of the questionnaire helped to explore primary school teachers' ability to identify errors on digits and address learners' errors with regard to number bases Figure 13 shows a sample response for item 16, given by T 23.



16. You notice a pupil working on these addition problems as

$$\begin{array}{r} 101_3 \\ +\ 132_3 \\ \hline 233_3 \end{array}$$

(a) What *does* this student understand? The law of addition

0/2

(b) What does he *not* understand? The law of addition of bases

0/2

(c) What would you do to help this pupil? Explain the laws of addition of bases i.e place values.

0/3

Figure 13: Sample response to item 16 given by T23

As can be seen from the sample response, T23 could not identify that the pupil only understood working problems in base ten. This pupil was not familiar with the digits used in base three. T23 was supposed to explain that in base three you only use three digits notably zero, one and two. T23 was also supposed to state that this pupil committed an error by using the digit three in base three.

Table 8 shows the statistics of the participants' performance with regard to item 16.

Table 8: Statistics for item sixteen

Mean	3.1
Median	3.5
Mode	0
Minimum	0
Maximum	7

As can be seen in Table 8 the mode score was zero despite the maximum score being seven. This clearly shows that generally participants' ability to identify and address learners' errors with regard to the digits used for a given number system was not well developed.

4.2.5.2 Errors on counting and place value

The subtheme was evidenced during the interviews among one out of the six participants. For example, T17 when asked to respond to the questions: *i) Are you able to identify patterns in pupils' errors on number bases?*

T17 responded as follows:

> "*Yes, I'm able to identify errors in pupils working…. For example, where the pupils do not follow rules for indices but just multiples…When counting others could write zero remainder one instead of one remainder zero…… when changing numbers from base ten to five, some were starting from the top when writing answers."*

Here T17 highlighted very important remarks regarding pupils' errors on number bases. Thus, it is clear in this excerpt that T17 possessed some elements of foundation knowledge *(identifying pupils' errors*). Sample of errors made by pupils are presented in Figure 14.

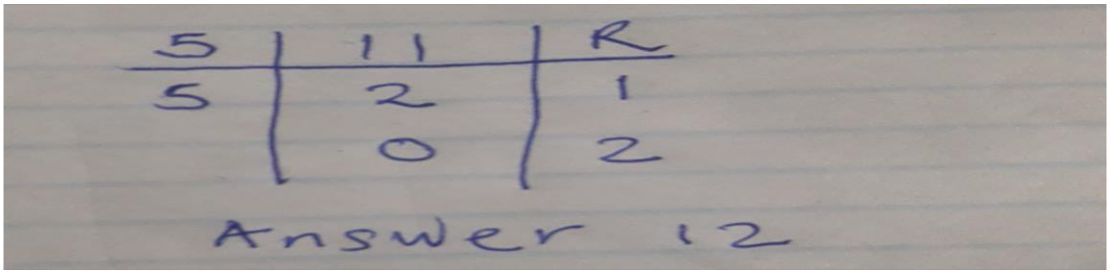


Figure 14: sample of errors made by pupils on writing answers

4.2.5.3 Errors on adding and dividing number bases

This subtheme was evidenced during the interviews. For example, T15 when asked to respond to the questions: *i) Are you able to identify patterns in pupils' errors on number bases?*

T15 responded as follows:

"Yes…. Sometimes when adding they write the number base in their answer

…. For examples when adding in base two…. In their answer they would have 121."

Sample of errors made by pupils on adding are presented in Figure 15.

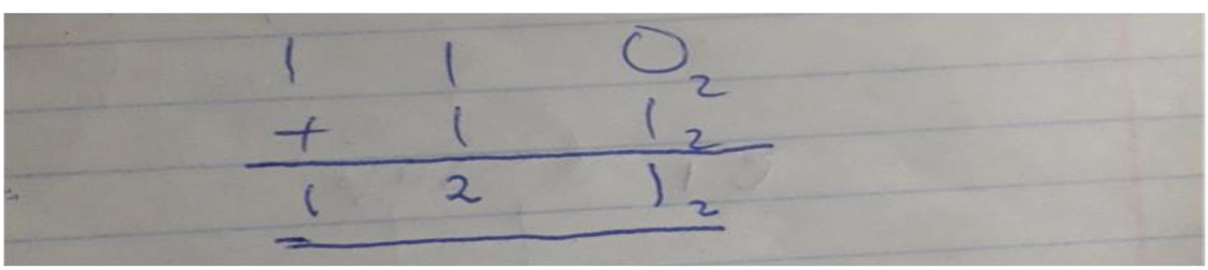


Figure 15: sample of errors made by pupils on adding

It is clear in this excerpt that T15 possessed some elements of foundation knowledge (*identifying pupils' errors).*

Also, T1 when asked to respond to the question: *What are the common errors pupils commit on number bases?* During the interview T1 explained that:

> *Most of them fail to divide; they divide but fail to tell there is a remainder…. For example twenty divide by four equals five remainder zero, but pupils find it difficult to say the remainder is zero, since there is need to write zero for a place value holder.*

It is clear in this excerpt that T1 possessed some elements of foundation knowledge (*identifying pupils' errors*).

4.2.6 Concentration on procedure

Concentration on procedure occurred mainly when changing numbers from: base ten to base five, base ten to base two and base five to base ten.

4.2.6.1 Changing numbers from base ten to base five

Findings, from the classroom observations indicated that all the six teachers focused on the algorithmic steps to reaching an answer. For example, T11 tendency to '*concentrate on procedures'* is demonstrated in the following exchange where she was changing 13 from base ten to base five:

> *T11: Writes 13 zeroes [0 0 0 0 0 0 0 0 0 0 0 0 0]. Then asks pupils: "How many groups of five can be formed from 13?"*
>
> *Px : Two*
>
> *T11: What is the remainder?*

Py: Three

T11: Therefore, the answer is two three base five

As can be seen T11 focuses on the algorithmic steps to reaching an answer. This entails that T11 concentrates on the rules and procedures without pupils actually understanding why these rules and procedures work.

4.2.6.2 Changing numbers from base ten to base two

An example of concentrating on procedure when changing from base ten to base two took place when T12 gave pupils the following instructions:

Consider both base ten and base two where you are converting from base ten to base two. Then divide the base into the number we are converting. We divide the base into the number we are converting, we keep on dividing and dividing till we reach zero.

Here T12 emphasised on the steps for changing numbers from base ten to base two.

4.2.6.3 Changing numbers from base five to ten

An example of concentrating on procedure when changing numbers from base five to base ten took place when T13 gave pupils the following instructions:

T13: Writes on board [Convert 231_5 to base ten]?

Px: [Moves in front and writes on the board] 5 5 5

T13: The first five is five power what?

Py: Five power zero

T4: What about the next five

Pz: Five power one

T4: What about the third five

Pk: Five power two

T13: What shall we do next?

Px: We shall multiply

Here T13 concentrated on mechanical steps to arriving at the answer as opposed to conceptual understanding.

4.2.7 Adherence to textbook

In relation to adherence to text book the major emerging subthemes were: (i) strict adherence to text books and (ii) non-adherence to textbooks.

4.2.7.1 Strict adherence to textbook

This subtheme was evidenced during the interviews. For example, Interview findings indicated that four out of six participants strictly adhered to the textbook. For example, when asked how confident he was to formulate his own questions on number bases T1 replied:

> *"I always use a text book for fear of doing what is not in the syllabus. In case a pupil finishes the work, we always use three or two textbooks."*

This response clearly shows that T1 strictly adhered to textbooks. This could be as a result of not having fully developed o*vert display of subject matter knowledge***.** Correspondingly, when asked to explain the criteria followed to choose and sequence examples when teaching numbers bases T17 said:

> "*I followed the ones given in the pupils' books."*

This shows that T17 strictly adhered to the textbooks.

4.2.7.2 Non-adherence to textbook

This subtheme was evidenced during the interviews by one of the six interviewed participants, that is, T15 to be specific. During interviews T15 said:

> *[Even without textbooks I can prepare my own questions. This is because I have taught number bases for 12 years. So I find it easy]*

Here T15 indicated that she could prepare questions without using a text book a sign of having sound foundation knowledge of non-adherence to textbook in the context of number bases.

4.3 Research question two: How do the primary school teachers transform the content knowledge they have on number bases into forms that are pedagogically convenient to their pupils when teaching?

This question aimed at exploring primary school teacher's presentation of ideas to learners in the form of analogies, illustrations, examples, explanations and demonstrations. I have presented question two along the following main themes: *Choice of examples, Use of instructional materials, and Choice of representation.* These attributes have been identified by Rowland, et al (2009) as part of the transformation dimension. The themes *Choice of examples and Choice of representation* fall under SMK whilst the theme *Use of instructional materials* fall under PCK. Subthemes to represent the major themes emerged from the data. In order to address question two fully the following instruments were used: questionnaire, concept maps, classroom observations and interviews.

4. 3.1 Choice of examples and exercises

Under choice of examples and exercises the major subthemes that emerged were (i) differentiated demand, (ii) similar demand and (iii) use of incorrect digits.

4.3.1.1 Differentiated demand

This subtheme was evidenced during the lesson observations and interviews among 2 out of the 6 participants. For instance, T15 gave the following two examples on addition in base five:

Example *1: Work out* $131_5 + 113_5$

Example 2: *Work out* $314_5 + 123_5$.

The choice of these examples was good as can be seen that for the first example the sum of all pairs is less than five but in the second example the sum of pairs is more than five (the base under consideration). This could help learners to determine when to divide.

Interviews findings revealed that five out of six participants had sound transformation knowledge *(Choice of examples).* Example of responses indicating good differentiated demand on *choice of example* included:

> T12 [*The criteria I used were first to work with smaller numbers that are easier to divide or group, then go to b*igger *ones*] and T15 [*The criteria I used were first to work with smaller numbers that are easier to convert from one number system to another*]

Here both T15 and T12 demonstrated sound knowledge of choice of examples.

4.3.1.2 Similar demand

This subtheme was evidenced during the lesson observations. The sample of a similar demand *choice of examples* I present was given by T1

T1: Writes on board: *Convert 13 from base ten to base five?*

T1: Draws thirteen strokes as shown in Figure 16:

Figure 16: Thirteen strokes

T1: Groups the strokes in groups of five as shown in Figure 17. *Two complete groups consisting of five strokes each and the incomplete group consisting of three strokes.*

Figure 17: Two complete groups and an incomplete one

T1: Tells pupils: *Since two groups each consisting of five strokes were made with three strokes remaining, the answer is two three base five*.

The second example given by T1was as follows:

T1: Writes on board: *Convert 17 from base ten to base five?*

T1: Draws seventeen strokes as shown in Figure 18:

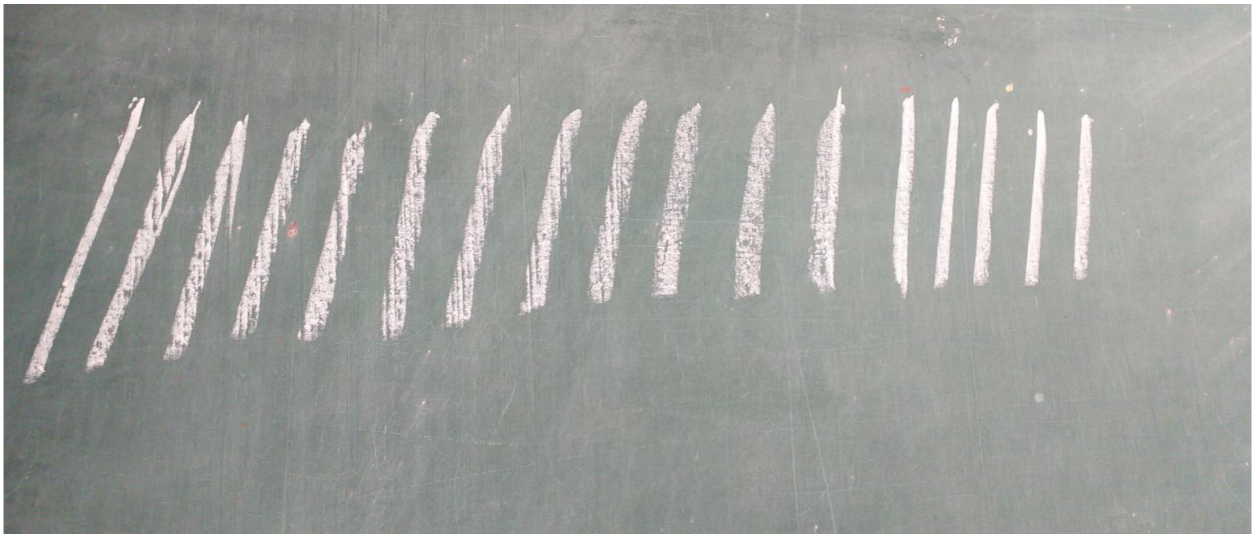


Figure18: Seventeen strokes

T1: Groups the strokes in groups of five as shown in Figure 19. *Three complete groups consisting of five strokes each and the incomplete group consisting of two strokes.*

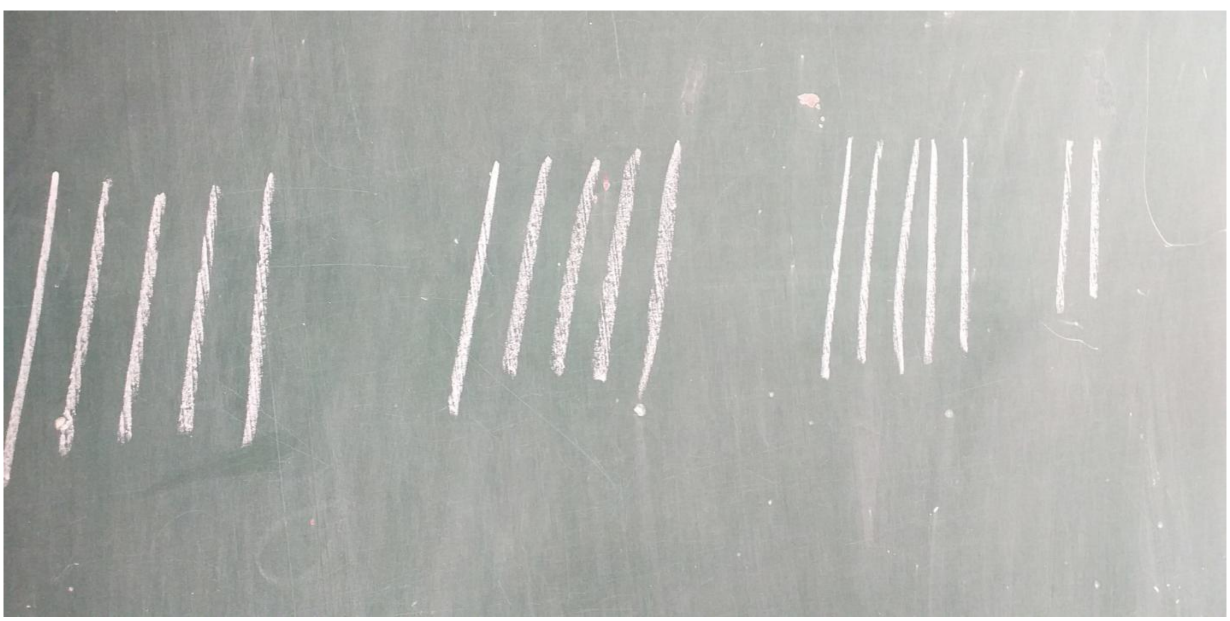


Figure 19: Three complete groups and one incomplete group

T1: "*Since there are three groups each consisting of five strokes with two strokes remaining, the answer is* 32_5*.*"

As can be seen the examples given were progressively becoming more challenging in that the first example had two complete groups while the second example had three complete groups, an indication of good *choice of examples.*

4.3.1.3 Use of incorrect digits

This subtheme was evidenced during the lesson observations by T13 (one of the six lesson observation participants) who demonstrated faintness in transformation knowledge '*choice of examples'* as can be seen through the example provided:

T13: Writes on board: *[Example: Change 2222 from base two to base ten]*

An analysis of the example reveals that T13 uses the digit 2 which is not part of binary numbers.

4.3.2 Use of instructional materials

Under use of instructional materials the major subthemes that emerged were (i) use of an abacus in addition to chalk and textbooks and (ii) use of chalk and textbooks only.

4.3.2.1 Use of an abacus in addition to chalk and textbooks

This subtheme was evidenced during the interviews and lesson observations. Out of the six observed primary school teachers only one (T17) used a concrete teaching aid (an "abacus" to be specific) when teaching number bases. On the other hand, the only example of response indicating use of an abacus was given by T17 who said (and as shown in Figure 19):

[*The teaching aid I use is an abacus. I use it because it helps learners to count accurately. I also consult different mathematics textbook*s *and fellow teachers before teaching*].

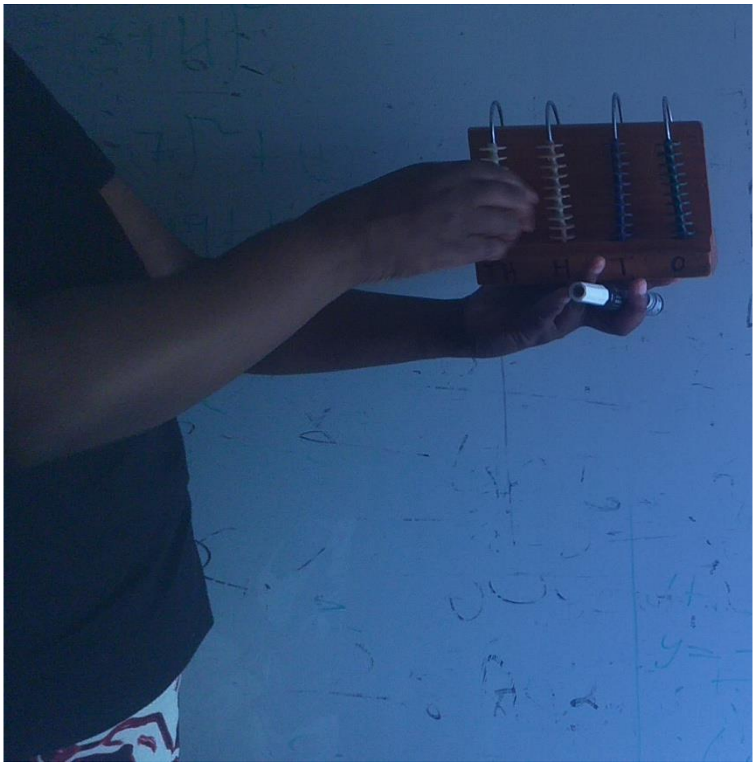


Figure 20: T17 demonstrating to pupils on how to use an abacus

During the lesson T17 explained:

> *The abacus is used to teach mathematics concepts such as counting, addition, subtraction, multiplication and division. If you are counting in base ten, starting from the left the beads in the first vertical wire represent thousands, those in the second vertical wire represents hundreds, those in the third vertical wire represents tens and those in fourth vertical wire represents ones. If you are counting in base five, starting from the left the beads in the first vertical wire represent five five fives, those in the second vertical wire represents five fives, those in the third vertical wire represents fives and*

those in fourth vertical wire represents ones

Then T17 gave the following example as shown in Figure 21:

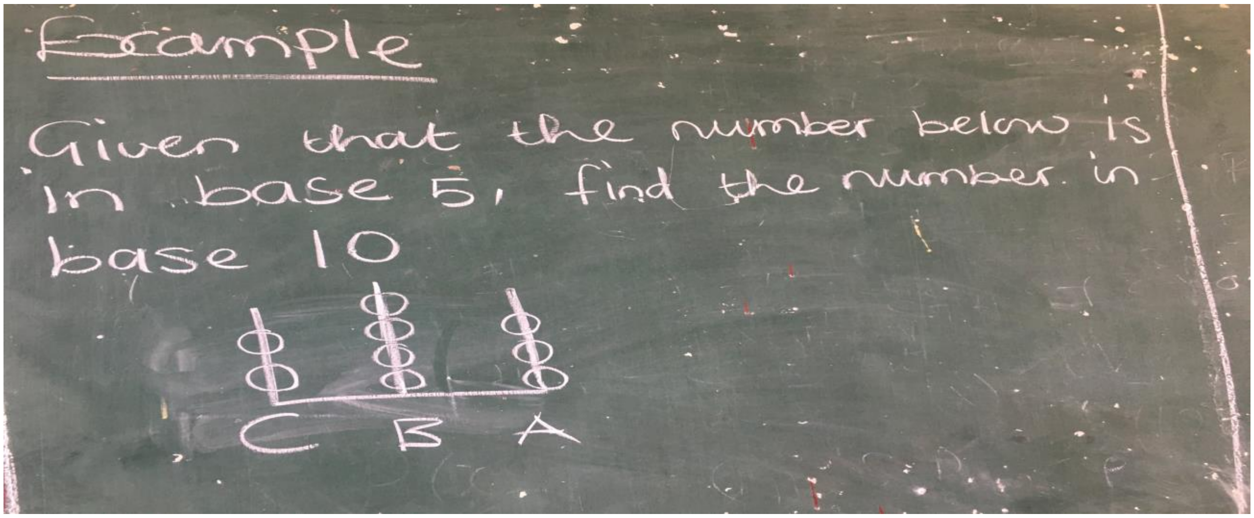


Figure 21: An example based on an abacus

Then T17 explained that

> *Column A represents 3 ones, Column B represents 4 fives and column C represents 2 five fives. So the answer is* 3 x 1 + 4 x 5 + 2 x 5 x 5 = 3 + 20 + 50 = 73

This is a clear indication that T17 had sound transformation knowledge *(use of instructional materials)* with regard to number bases.

4.3.2.1 Use of chalk and textbooks only

This subtheme was evidenced during the interviews, concept maps and lesson observations. During lesson observation five out of the six observed participants relied on chalk, text book and board only. They never used any teaching aid even when it was possible to do so. The following exchange shows what took place when T11 was introducing the concept of base five. In this scenario T11 said:

"When in base ten we group in groups of tens but in base five we group in groups

of fives".

Here T11 was supposed to be practical by grouping stones, sticks or any concrete objects rather than merely talking to the learners.

Similarly, concept map findings revealed the trend of low pedagogical content knowledge of *instructional materials.* For example, none of the nineteen participants that took part in concept maps provided a link for *instructional materials*. This is a clear indication that most of them do not know the *instructional materials* required to teach number bases effectively. Correspondingly, Interview findings revealed that five out of six participants did not have a well-developed understanding of transformation knowledge *(use of instructional materials).* For example, when responding to the question: *What do you think are some suitable teaching aids you should use when teaching number bases*?

> T1 said: '*I don't know any suitable teaching aid that I should use to teach number bases* and T12 said: '*I only use text books like Longman*'.

This could entail why both T1 and T12 never used any teaching aid during the lesson observation session apart from the obvious ones notably the chalk and board when they could have used so many of them such as stones, sticks, and base ten blocks.

4.3.3 Choice of representations

Under choice of representations the major subthemes that emerged were (i) tabular, and (ii) symbolic representations.

4.3.3.1 Tabular representations

This subtheme was evidenced during the lesson observations taught by two out of six participants. An example of the applied tabular mathematical representation of number bases is demonstrated in the following exchange:

T12: *We are going to change 15 from base ten to base two*.

T12: *To change 15 from base ten to base two you have to*

make the table I will make. [Then she drew the outline as shown in Figure 22]

T12: The table has three columns

T12: In the first column we will write the number 15 [then she wrote 15 as

Indicated in Figure 22]

T12: In the second column we put the base we are changing to [then she wrote 'Base'

as indicated in Figure 22]

T12: In the third column we are going to write the remainder [then she wrote 'remainder' as indicated in Figure 22]

T12: What is 15 divided by 2?

Pz: It is 7 remainder 1

T12: Good, so we are going to write 7 below 15 and the remainder 1 below

Remainder [Then T11 wrote 7 and 15 in their respective positions as shown

in Figure 22].

T12: We are going to continue to divide

T12: What is 7 divided by 2?

Pv: It is 3 remainder 1

T12: Good, so we are going to write three below seven and the remainder one in the remainder column below the first remainder one [Then T12 wrote three and one in their respective positions as shown.

T12: We are going to continue to divide

T12: What is 3 divided by 2?

Pz: It is one remainder one

T12: Correct, so we are going to write one below three and the remainder one in the remainder column below the second remainder one [Then T12 wrote three and one in their respective positions as shown.

T12: We are not done yet, what do you think is remaining?

Pk: To divide one by two

T12: Very good

T12: So, what is one divided by 2?

Pz: It can't

T12: So, what do we do?

Pt: We write zero below one and one below one in the remainder column

T12: Can you clap for him [Then the classmates clapped for Pt]

Then T12 completed drawing the table indicated in Figure 22.

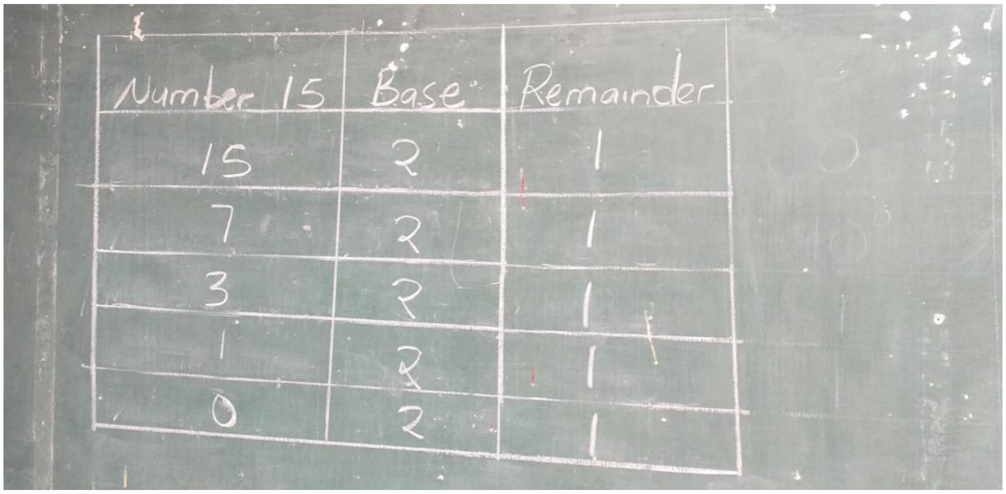


Figure 22: T12 representation for changing numbers from base ten to base two

Px: Teacher so this is the table we are going to use (referring to Figure 22)

T12: Yes, it is the one and it is important that you know it very well.

Here we see that T12 used the tabular representation to explain the concept of changing from base ten to base two. Another example of application of tabular representation was seen when T11 gave pupils an example of converting 13 from base ten to base five. T11 gave tabular representation of the number bases as indicated in Figure 22.

T11: *Today you are going to learn how to change numbers from base ten to base five*

T11: *Who can come in front and change 13 from base ten to base two.*

Pupils: Madam no one knows

T11: *Why are you answering for your friends?*

T11: Ok to *change 13 from base ten to base five you start by making a table of three columns [then she drew a table with three columns as shown in Figure 21]*

T11: In the first column we will write five [then she wrote 5 as Indicated in Figure 21*]*

T11: since 13 is number we are changing we put it in the next column on top [then she wrote '13' as indicated in Figure 23*]*

T11: in the third column we are going to write the remainder [then she wrote 'R' as indicated in Figure 23

T11: what is 13 divided by 5?

Py: It is 2 remainder 3

T11: Good, so we are going to write 2 below 13 and the remainder 3 below R

T11: What is two divide by five

Pz: Madam it can't

T11: Since you can't divide, you write zero below two and the two in the remainder column below three [then T11 wrote zero and two in their respective positions as shown in Figure 23*]*

T11: I expect everyone to know this procedure of drawing a table

Then T11 completed drawing the table as indicated in Figure 23.

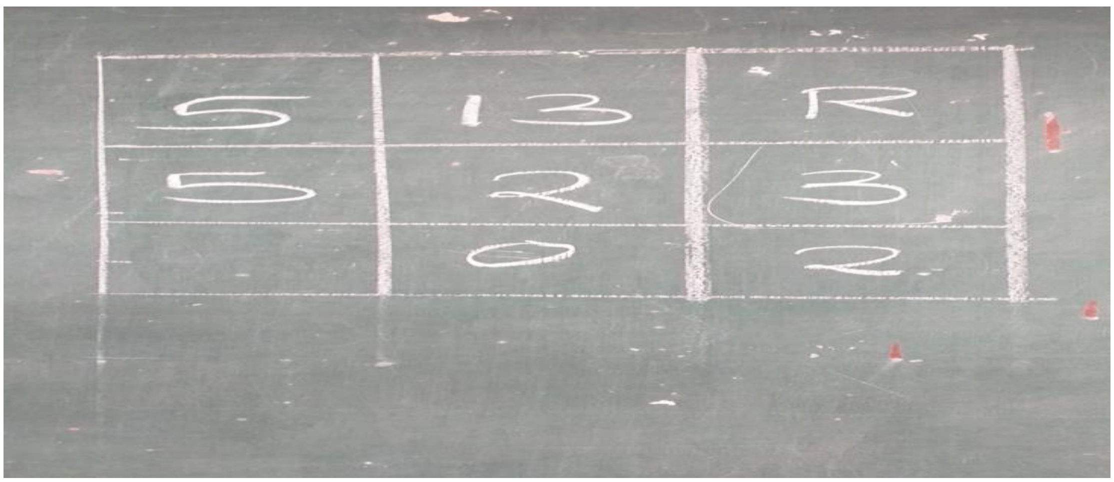


Figure 23: T11 representation for changing numbers from base ten to base five

Here we see that the tabular representation was applied to explain the concept of changing from base ten to base two.

4.3.3.2 Symbolic representations

Among the six observed primary school teachers only one used symbolic representation. An example of symbolic representation occurred when T11 demonstrated how to change 17 from base ten to base eight:

> *T11*: *Writes 17 zeroes. [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]. Then she said we have two groups of eight and remainder ……..*
>
> *Pv: shouts [one]*
>
> *T11: So, the answer is 21 in base eight. Who can come and use the other method?*
>
> *Pupils: Madam just show us*

As can be noted, here T11 applied symbolic representation of number bases when changing 17 from base ten to base eight

4.3.3.3 Tally marks representations

An example of tally marks representation of number bases occurred during lesson observations when T11 demonstrated how to change 17 from base ten to base five: See Figures 15, 16 and 17 presented earlier in Section 4.4.1.2 for an example of tally marks representations as well as for details of what happened.

Research question three: How do the primary school teachers connect instructions when teaching number bases?

Research question three was intended to explore primary school teacher's sequencing of material for instruction, and awareness of the relative cognitive demands of different concepts and tasks

.

I have presented question three along the following themes: *recognition of conceptual appropriateness, anticipation of complexity, decision about sequencing and conceptual* *understanding.* Apart from *conceptual understanding* which has been taken from the Mathematical Proficiency theoretical framework advanced by Kilpartrick et al. (2006), these attributes have been identified by Rowland, et al (2009) as part of the connection dimension. The themes *recognition of conceptual appropriateness, decision about sequencing and conceptual understanding* fall under SMK and the themes *anticipation of complexity* fall under PCK. Subthemes to represent the major themes emerged from the data. In order to address question three fully the following instruments were used: questionnaire, concept maps, classroom observations and interviews.

4. 4.1 Recognition of conceptual appropriateness

In relation to recognition of conceptual appropriateness the major emerging subthemes were: (i) use of several methods and (ii) use of single method.

4. 4.1.1 Use of several methods

Findings from lesson observations revealed that out of the six participants only one (T11) demonstrated strong knowledge with regard to pedagogical content knowledge *recognition of conceptual appropriateness.* After using the tabular representation method presented above in Figure 20 to convert 13 from base ten to base five, T11 went on to apply another method as demonstrated in the following exchanges (and Figure 24):

T11: Writes 13 zeroes

Figure 24: Thirteen zeroes

Then asks pupils: "How many groups of five can be formed from 13?"

Px : Two

T11: What is the remainder?

Py: Three

T11: Therefore, the answer is Two, three base five

Here T11 demonstrated connection knowledge of '*recognition of conceptual appropriateness'* as identified by the knowledge quartet by deciding to use another method to check if they would find the same answer.

4. 4.1.2 Use of single method

Interviews findings revealed that five out of six participants did not fully make use of the pedagogical content knowledge *(recognition of conceptual appropriateness)* when teaching number bases. For instance, when they were asked the following question: *Would you like to* *have used other methods but you were prevented from using them for some reason?* Some of the responses to this question included:

> T13: [*Off course, some pupils did not understand but there is nothing I could do, since the method of making a table is the only one I know*] and T12: [*Yes, some learners had challenges but changing using a table is the only method I know, I wish I had known other methods]*

It appears both T12 and T13 knew only the tabular representation method when changing numbers from base ten to base five or two with no choice of using alternative methods. See Figure 22 presented earlier in Section 4.3.3.1 for an example of tabular representation as well as for details of what happened.

4.4.2 Anticipation of complexity

In relation to anticipation of complexity the major emerging subthemes were: (i) reading of final answer from the table, (ii) converting from base five to base ten.

4.4.2.1 Reading of final answer from table

Findings from lesson observations revealed that five out of the six participants did not demonstrate anticipation of complexity. For instance, the following exchange reveals what happened in the case of T12 as he demonstrated how to convert 17 from base ten to base two by applying the tabular representation indicated in Figure 25.

> *T12: Yesterday we looked at how to change numbers from base ten to base two.*
>
> *T12: Today we are going to look at how to change numbers from base ten to base five*
>
> *T12: So, let's change 17 from base ten to base five*
>
> *T12: How many columns should we make?*

Pv: Three columns

T12: Very good. [Then he drew a table with three columns]

T12: Who can come and fill in the first row

Px: Moved in front and filled in as indicated in Figure 25

T12: Very good. So, from here like in the previous examples we continue dividing by five and write the remainder in the remainder column.

T12: Finally, he completed the table as indicated in Figure 25

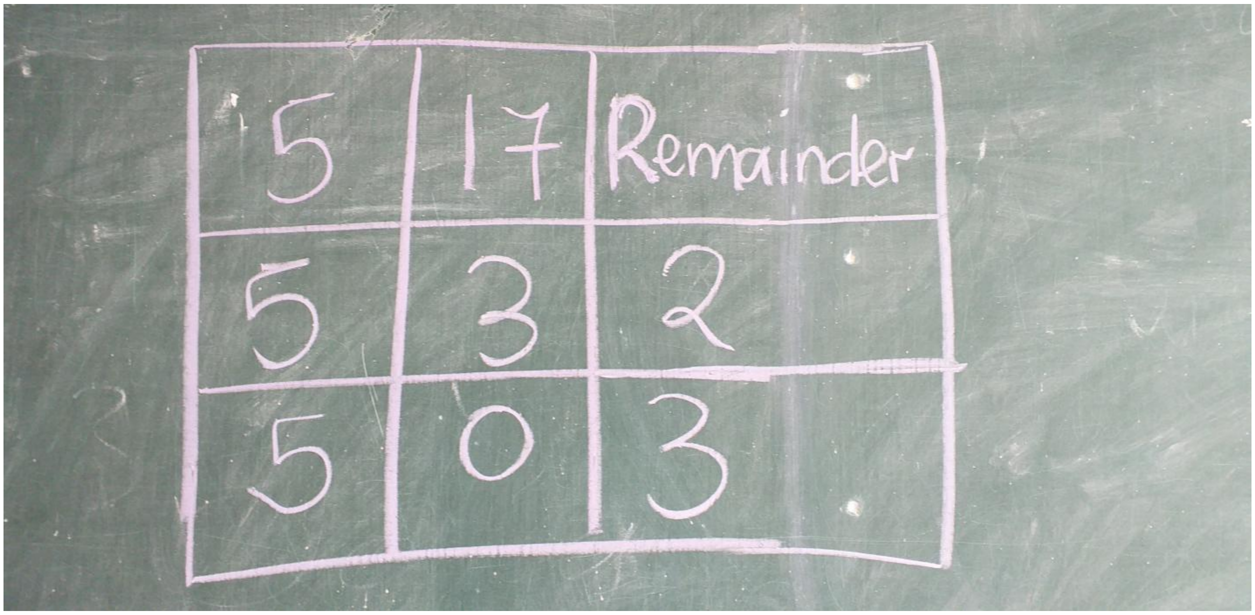


Figure 25: T12 representation for converting 17 from base ten to base five

Upon finishing working out the problem T12 just said the answer is 32; she did not explain why the answer had to start with three and not two. Not attending to *anticipation of complexity* by T12 was a source of confusion amongst pupils as evidenced in the following exchange:

Px: Why is the answer 32?

T12: That is the formula.

Py: Teacher but the answer is 23.

T12: No, it is 32.

Pv: Teacher how come?

T12: We start from the bottom.

Px: But why?

T12: Just know that, let us do the next example.

Here T12 failed to fully explain why 32_5 was considered to be the answer and not 23_5 possibly because of not *anticipating complexity*. T12 did not anticipate that pupils would not understand why the answer should start with three and not two (starting with the digit at the bottom and ending with the digit at the top and not vice versa).

4.4.2.2 Converting from base five to base ten

Another example of a scenario during lesson observations that involved 'anticipation of complexity' is the one that happened when T1 was converting 33 from base five to base ten as can be seen in the following exchange:

T1: What do we get when we change three, three base five to base ten

Px: Teacher after converting three, three base five we get 18 as an answer [Which of course was the correct answer]

T1: three, three base five is the answer

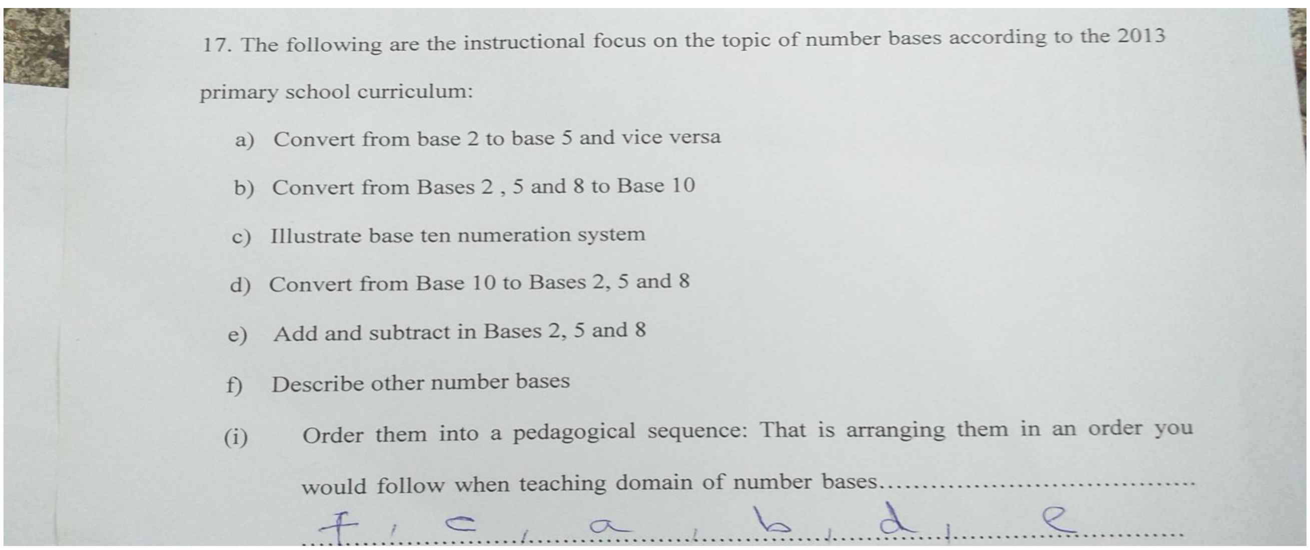
Here T1 failed to find the correct answer by multiplying the first 3 by five and the second three by one and then add the result to obtain 18 as the answer.

4.4.3 Decision about sequencing

Effective connection Knowledge (*decision about sequencing)* involves introducing ideas and strategies in an appropriately progressive order. As a way of exploring the connection knowledge *(decision about sequencing)* primary school teachers were given the instructional focus on the topic of number bases according to the 2013 primary school curriculum and asked to order them into a pedagogical sequence. That is arranging them in an order they would follow when teaching the domain of number bases. In relation to overt decision about sequencing the major emerging subthemes were (i) inappropriate sequencing and (ii) prerequisite knowledge for number bases.

4.4.3.1 Inappropriate sequencing

Figure 26 shows a sample response by T24 with regard to inappropriate sequencing of number bases.



17. The following are the instructional focus on the topic of number bases according to the 2013 primary school curriculum:

a) Convert from base 2 to base 5 and vice versa

b) Convert from Bases 2 , 5 and 8 to Base 10

c) Illustrate base ten numeration system

d) Convert from Base 10 to Bases 2, 5 and 8

e) Add and subtract in Bases 2, 5 and 8

f) Describe other number bases

(i) Order them into a pedagogical sequence: That is arranging them in an order you would follow when teaching domain of number bases.....................................

f, c, a, b, d, e

Figure 26: Sample response on sequencing instructional focus on the topic of number bases

As can be seen in Figure 26, T24 when writing the suitable pedagogical sequence started by writing part f (which is about describing other number bases) followed by part c (which is about illustrating the base ten numeration system). In fact, 23 out of 30 of the participants who took part in the questionnaire failed to come up with an appropriate pedagogical sequence. The appropriate order would be: Illustrate base ten numeration system, describe other number bases, Convert from Base 10 to Bases 2, 5 and 8, Convert from Bases 2, 5 and 8 to Base 10, Convert from base 2 to base 5 and vice versa and Add and subtract in Bases 2, 5 and 8.

4.4.3.2 Prerequisite knowledge for number bases

Interviews findings revealed that only one out of six participants had full understanding of connection knowledge *(decision about sequencing).* For instance, when responding to the question on pre-requisite knowledge pupils need to possess for them to effectively learn number bases, the following response was given by T17.

> T17: [*They have to know to count in tens, fives, eights, and twos*].

Indeed, by highlighting all the necessary prerequisites T17 demonstrated sound pedagogical content knowledge on the *decision about sequencing* with regard to number bases. On the other hand, sample of responses from other participants were as follows. :

> T13: [*No, I do not know any pre-requisite knowledge pupils need to possess for them to effectively learn number bases*] and T12
> [*All that I know is to follow the textbook*]

There was need for T12 and T13 to effectively, explain what informed their *decision about sequencing.*

4.4.4 Conceptual understanding

In relation to conceptual understanding the major emerging subthemes were: (i) types of bases and (ii) digits used in a particular base.

4.4.4.1 Types of bases

This subtheme was evidenced in the concept maps constructed by the participants. Only two out of nineteen participants provided links for the *types of bases* (base eight, two, five and ten) learnt in primary school. The fact that 17 out of 19 participants did not provide the link for the *types of bases.* Figure 27 shows sample of the concept map drawn by T17.

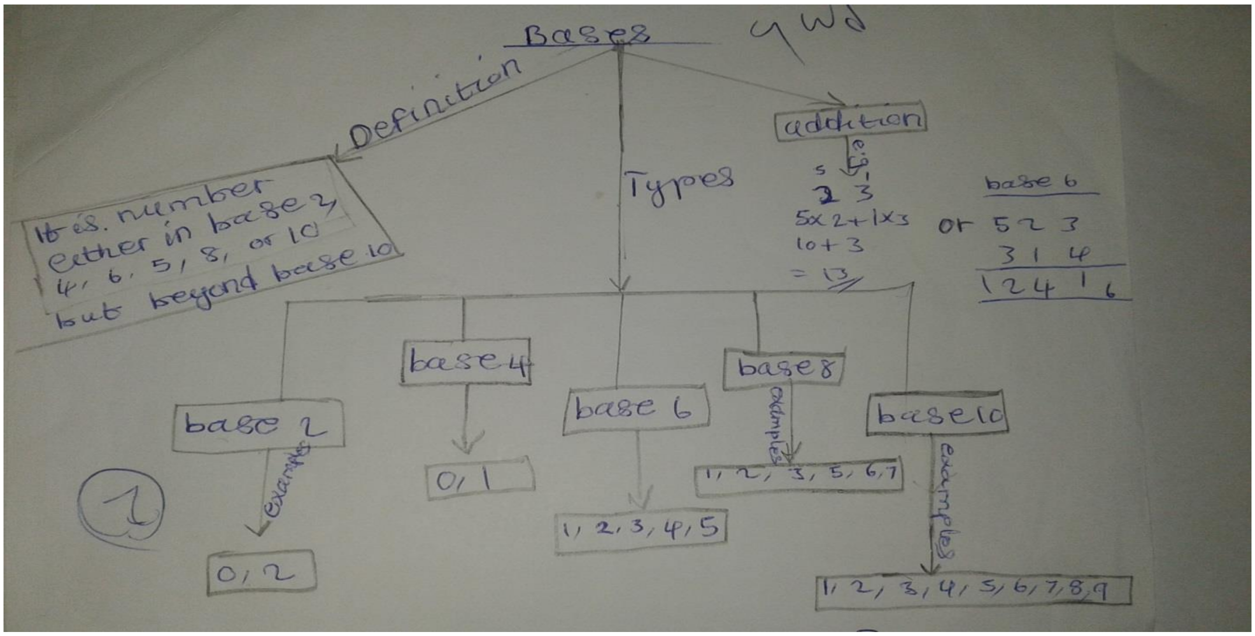


Figure 27: Sample of a concept map drawn by T17

This participant was one of the only two participants out of nineteen who provided the link for the types of bases. As can be seen in Figure 27, T17 included base 2, base 4, base 6, base 8 and base 10. Under each number system T17 tried the best to include the appropriate digits. For base 10, and base 6, T17 only missed the digit zero. For base 8, T17 missed the digits 4 and 0. For base 2 T17 missed digit 1 and wrongly added 2 as part of the binary number system. For base 4 T17 missed the digits 2 and 3. This shows that T17 has some understanding of the connection knowledge *(making connections between concepts*).

4.4.4.2 Digits used in a particular number system

This subtheme emerged during lesson observation. For instance, in the following exchange T11 demonstrates good ‘*connection between concepts’*:

T11: Today we are going to start a new topic

Pupils: Yes, teacher

T11: We count in base ten because we group things in base ten. [Then writes the digits used in base ten].

T11: Can any explain why we count in base ten

Px: It is because we have ten fingers

T11: These are the digits used in base ten, and we can see that in base ten we do not have a single digit representing ten.

This was good demonstration of ‘*connection between concepts’ as* pupils need to know the digits used in a particular base before carrying out calculations in that particular base.

However, contrary to the earlier exchange, in the following exchange T11 demonstrates missing links in *‘connection between concepts’.*

T11: We are going to do an example on how to change from base ten to base five

Pupils: Yes, teacher

Here, before providing an example of how to change from base ten to five, T11 was expected to explain the concept of place value, indices and the digits used in base five.

4.5 Research question four: What are the primary school teachers' contingency actions when teaching number bases?

Research question four was intended to explore primary school teachers' ability to make cogent, reasoned and well-informed responses to unanticipated and unplanned events in the context of number bases. I have presented question four along the following themes: *Responding to students' ideas, Deviation from lesson agenda and Teacher insight.* These attributes have been identified by Rowland, et al (2009) as part of the contingency dimension. All the three themes fall under PCK. For these major themes, subthemes emerged from the data. In order to address question four fully the following instruments were used: classroom observations and interviews.

4.5.1 Responding to students' ideas

In relation to responding to students ideas the major emerging subthemes were: (i) affirming learner's answer without clarification, (ii) out rightly dismissing wrong answers, and (iii) re-directing question to another learner.

4.5.1.1 Affirming learner's answer without clarification

This subtheme was evidenced during the lesson taught by one out of the six participants. The following exchange by T1 (and as indicated in Figure 28) shows an example of what happened in one of the lessons.

> *T1: [Writes 16 on the board and asks] who can change 16 from base ten to base five*
>
> *Pupil Y: [Moves in front to convert 16_{ten} to 31_{five}.]*
>
> *Pupil Y: [Carried out the conversion as follows.]*

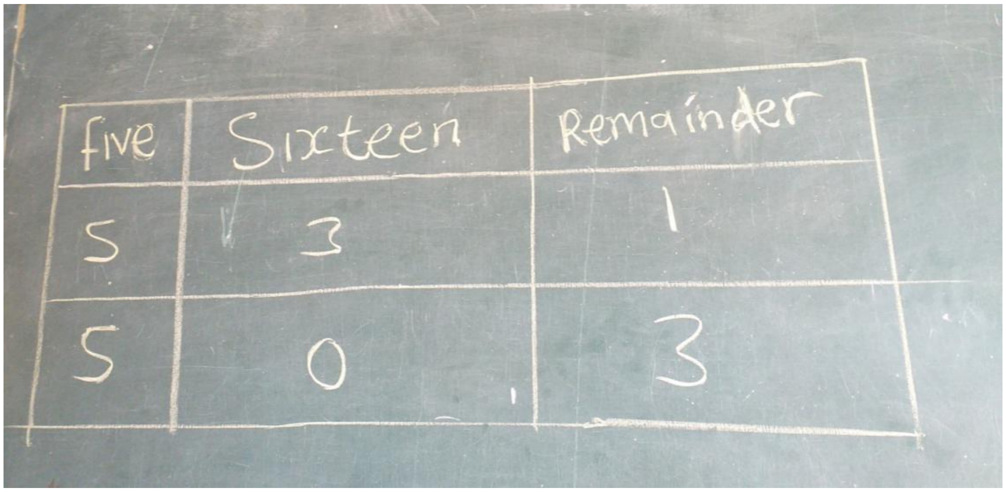


Figure 28: Changing 16 from base ten to base five

Pupil y: So, the answer is 31.

T1: Alright. Clap for him

Here T1 '*responds to pupils' ideas'* by simply affirming the correctness of the answer without making clarifications.

4.5.1.2 Out rightly dismissing wrong answers

This subtheme was evidenced during the lesson taught among two out of the six participants. The following exchange by T11 shows an example of what happened in one of the lessons.

T11: Yesterday we started a new topic. Who can remember what we discussed?

Px: We looked at changing from base ten to base five

T11: Yes. How many digits are in base five?

Py: 4

T11: No

T11 just dismissed wrong answers. She seemed not to be concerned about the sources of pupils' mistakes.

4.5.1.3 Re-directing question to another learner.

This subtheme was evidenced during the lesson taught by one out of the six participants. An example of re-directing question to another learner took place as T15 was teaching and is demonstrated in the following exchange:

T15: What is 8^2

Pv: 16

T15: Is he correct?

Pupils: No

T15: Who can try?

Px: 64

T15: Very good

T15 did not just dismiss the answer given but intervened by re-directing. T15 way of '*responding to pupils' ideas* could enable more pupils to participate as well as allowing a pupil to correct another pupil's incorrect statement.

4.5.2 Teacher insight

In relation to teacher insight the major emerging subthemes were: (i) Affirming method used and (ii) Probing learner thinking.

4.5.2.1 Affirming method used

This subtheme was evidenced during the lesson taught by one out of the six participants. An instance of such a scenario is demonstrated in the following exchange in which T12 worked out the following problem: Add 110_2 and 1011_2 as follows and shown in Figure 29:

T12: To solve this question we first put the place values on top

T12: Then, we add zero and one to get 1

T12: In the next column we add one plus one to get two

T12: Since we cannot write two, we write zero and carry one

T12: And in the first column we add one and zero plus the one we carried forward. So again, we write zero and carry one forward

T12: So, this is how you work out this question

Figure 29: T12 Working for the problem: Add 110_2 and 1011_2

After T12 had finished working out the problem, there was mixed reaction amongst the pupils:

Px: Teacher I am completely lost out

Py: Can you repeat or is there another way we can use to find the answer?

T12: That is the only way….so you have no choice but to know it

Thus, we see in this scenario the teacher could not stop and reflect-in-action and consequently change tact. This shows that T12 did not exhibit a developed understanding of pedagogical knowledge '*teacher insight'*.

4.5.2.2. Probing learner thinking

This subtheme was evidenced during the lesson taught by one out of the six participants. An instance of such a scenario is demonstrated in the following exchange (and as indicated in Figure 29) in which T17 exhibited contingency knowledge '*teacher insight'.*

> *T17: Who can work out this problem [Writes:* $321_5 + 423_5$*]*
>
> *Px: [Moves in front and added 1 and 3 and wrote 4]*.
>
> *T17: Why was it necessary to write 4 straight without dividing?*
>
> *Pt: It is because we are adding in base five and the answer four is less than five.*

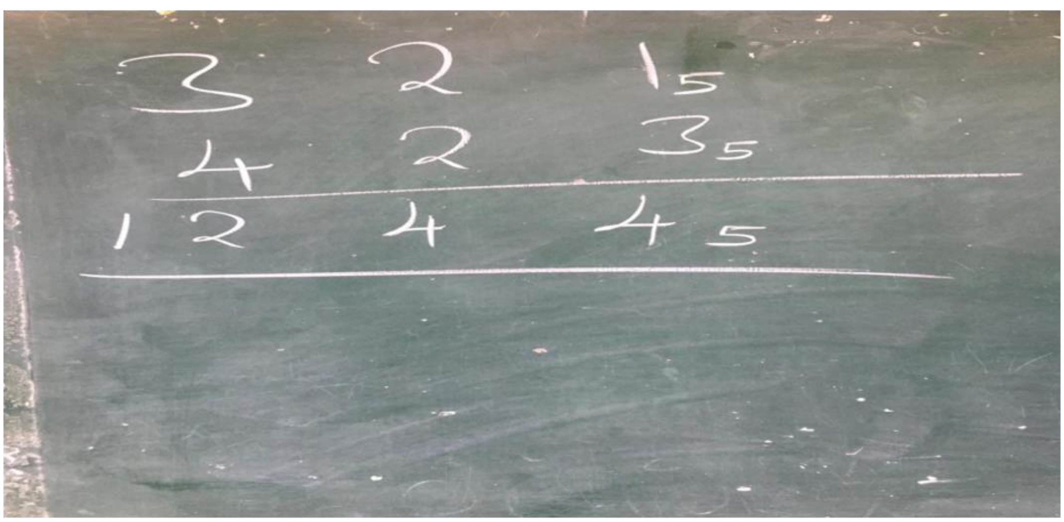


Figure 30: Working for the problem: Add 321_5 and 423_5

Here T17 demonstrated contingency knowledge (*teacher insight*) by reflecting in action through asking probing questions to try to elicit where the pupil's thinking was coming from. This was a good demonstration of contingency knowledge (*teacher insight).*

4.5.3 Deviation from lesson agenda

In relation to deviation from lesson agenda the major emerging subthemes were: (i) Introducing indices and (ii) Maintaining the status quo.

4.5.3.1 Introducing indices

This subtheme was evidenced during the lesson taught among three out of the six participants. An instance of such a scenario happened when T12 demonstrated elements of contingency knowledge *(deviation from agenda)* as can be seen in the following exchange (and as indicated in Figure 31:

> "*First, lesson did turn out differently from how I had planned. I thought learners were100% familiar with indices, yet they were not ……..Yes, children had challenges in grouping. So I adjusted to suit the unanticipated, ….So I had to deviate indirectly and start explaining ……I had to bring in indices to familiarize them with indices before proceeding."*

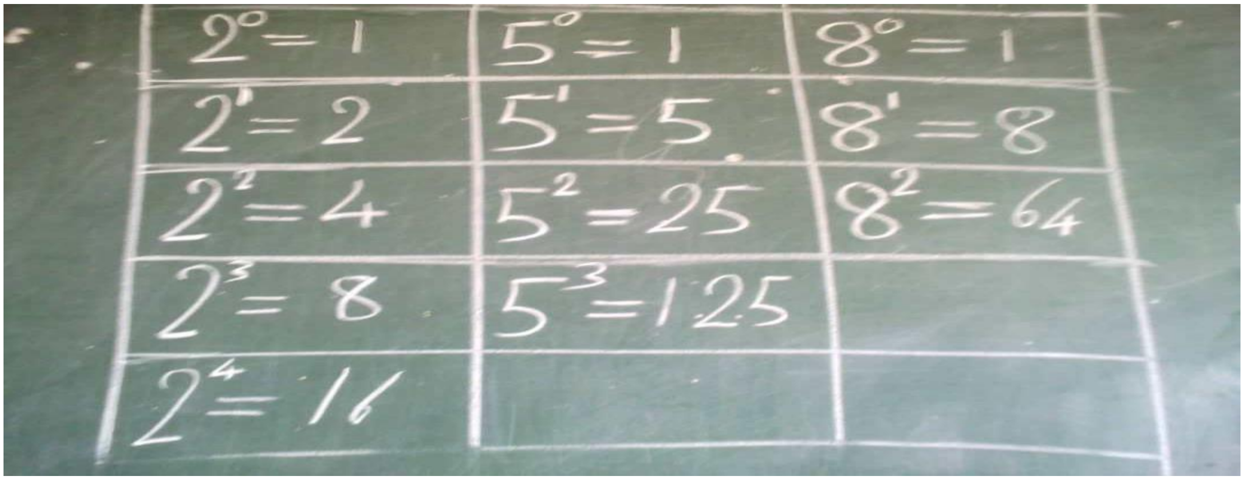


Figure 31: Indices introduced by T12

This is what transpired when T12 exercised elements of deviation

> *T12: Today we are going to convert numbers from base five to base ten*
>
> *T12: To express a number such as 101_5 in base ten we need to find five power two, five power one and five power zero.*
>
> *Px: Teacher what do you mean by five power two, five power zero?*
>
> *T12: I thought every one already knows what we mean by five power two*
>
> *Pupils: No we don't know Teacher*
>
> *T12: So, before we go a heard with converting numbers from base five to base ten, let us revise indices.*

Here T12 exercised some elements of deviation based on conceptual focus to address the unanticipated changes which was a good basis for deviation.

4.5.3.2 Maintaining the status quo

Interviews findings revealed that five out of six participants were not well grounded in contingency knowledge *(deviation from agenda).* When faced with a challenge in most cases they failed to deviate and if they did their deviation was not characterised by conceptual focus. For instance, when asked to explain if the lesson had turned out differently from what was planned, T13 and T1 had this to say:

T13: [*Yes, the lesson turned out differently from what I had planned at first, but,*

I did not know how to address the situation. I hope next time it would be different]

and T1: [*Yes, children had challenges in grouping tens problems. To adjust to suit the unanticipated changes I had to involve the pupils by using question and answer method]*.-

Here is an excerpt (and as indicated in Figure 32) of what exactly happened in the case of T13

T13: How to we solve the problem: $110_2 + 1011_2$*?*

T13: What should we do?

Pupil x: We write the bases.

T13: Ok, we add [Writes 0 + 1 =1]. Then we add [Writes 1+1 = 2].Then divide 2 by 2 and you get one with remainder zero. So write the remainder. Then in the third column add [Writes 1+ 0 = 1] and add the remainder, then divide by two and get one remainder zero, so write the remainder zero. Carry one to the front column

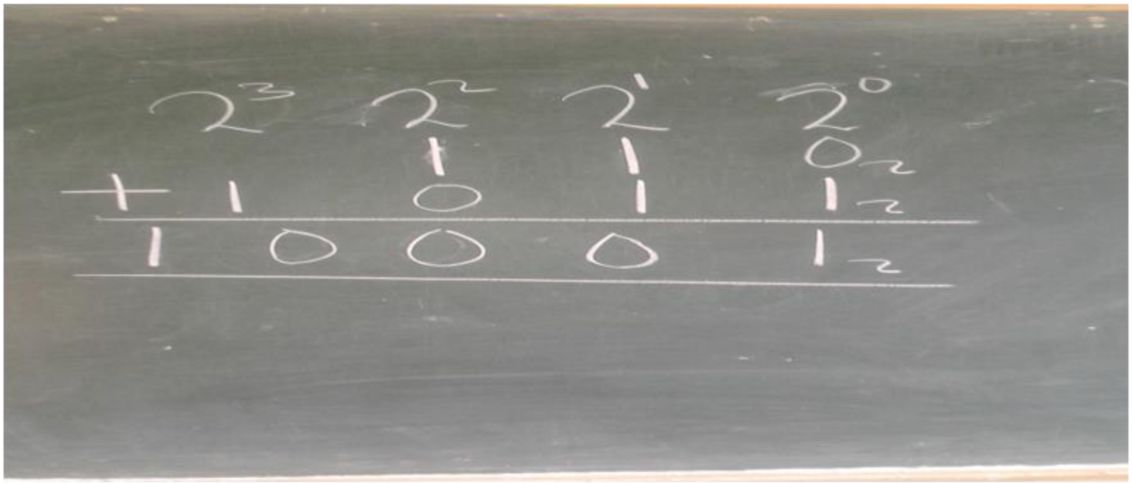


Figure 32: The format used by T13 to work the problem: $110_2 + 1011_2$

After T13 had finished working out the problem, there was mixed reaction amongst the pupils. Some said they were clear about the whole process, others said that they were completely lost. For instance, Pt and Px had this to say:

> Pt: [*Madam, I have followed very well. The secret is to divide if the answer is equal or greater than two] and* Px: [*Madam, I am lost. Why are we not writing two if we obtain two after adding.]*

Here T13 acknowledged that the lesson turned out differently, but failed to make appropriate pedagogical deviations, especially with regard to meeting the needs of those who were lost. For instance during interviews T13 had this to say:

> T13: [*I did not expect that pupils would meet so many challenges. Many pupils did not know when to divide. But I did not know what to do. I hope next time it will be better*]

4.6 Cross analysis of Question 1, Question 2, Question 3, and Question 4.

In this Section I have presented a cross analysis of the findings in relation to Question 1, Question 2, Question 3, and Question 4. A scoring guide was constructed in order to evaluate primary schools teachers' level of foundation, transformation, connection and contingency. Table 9 shows the scoring guide and part of the observation schedule used during lesson observations. For a complete lesson observation schedule see Appendix C.

Table 9: Scoring guide and part of the observation schedule used during lesson Scoring Guide: 4 = very high, 3= High, 2 = Moderate, 1 = Low, 0 = very low

Areas	0	1	2	3	4

Part of Observation schedule

awareness of purpose: Explaining the significance of number bases, clarifies the purpose and learning goals					
Identifying pupil errors, reaction to errors					
Demonstrating overt subject knowledge; specifying number of digits for a given base, Position, ordering, Amount, discussing the use of number bases other than base ten, explaining the symbol X_{b},					
theoretical underpinning of pedagogy					
Use of mathematical terminology: including terms such as number bases, octet, binary,					
adheres to textbook: Adapting, text book examples, procedures					
Concentrates on procedures, uses appropriate algorithms, frequencies of mistakes committed when giving examples or explaining concepts					
Foundation total					

In this study primary school teachers' foundation, transformation, connection and contingency knowledge have been explored fully by applying the findings from question one, question two, question three and question four, respectively. Therefore, the cross analysis is dependent on the findings from the other four preceding research questions. Therefore, prior to presenting the cross analysis findings summaries on the description of participants performance in each of the four preceding research questions are presented. Description of performance in each of the Foundation codes, Transformation codes, Connection codes and Contingency codes are presented in Table 10, Table 11, Table 12 and Table 13 respectively.

Table 10: Description of performance in each of the Foundation codes

Teacher	**Description of performance in the code**					
Code	*Theoretical underpinning of pedagogy*	*Awareness of purpose*	*Overt display subject knowledge*	*Identifying pupils errors*	*Use of mathematical terminology*	*Concentration on procedures*
T1	Low	Very Low	Very Low	Moderate	High	Low
T11	Low	Low	Moderate	High	Low	Low
T12	Very Low	Moderate	Low	High	Low	Low
T13	Low	Very Low	Strong	High	moderate	Moderate
T15	Moderate	Strong	Strong	High	Low	Moderate
T17	Moderate	Very Low	Strong	Very strong	High	High

Table 11: Description of performance in each of the Transformation codes

Teacher	Description of performance in the code		
Code	*Choice of examples*	*Use of instructional materials*	*Choice of representations*
T1	High	Low	High
T11	High	Low	High
T12	Moderate	Low	Low
T13	Low	Low	Low
T15	Moderate	High	Moderate
T17	Moderate	High	High

Table 12: Description of performance in each of the Connection codes

Teacher	**Description of performance in the code**			
Code	***Recognition of conceptual appropriateness***	***Anticipation of complexity***	***Decision about sequencing***	***Making connection between concepts***
T1	Low	Moderate	Moderate	Very Low
T11	Low	Low	Low	Moderate
T12	Moderate	Low	Strong	High
T13	Low	Low	Moderate	High
T15	High	Moderate	Moderate	moderate
T17	High	High	Moderate	moderate

Table 13: Description of performance in each of the Contingency codes

Teacher	**Description of performance in the code**		
Code	***Responding to pupils ' ideas***	***Deviation from lesson agenda***	***Teacher insight***
T1	Moderate	Strong	Low
T11	Low	High	High
T12	Very High	High	High
T13	Moderate	Moderate	Moderate
T15	Moderate	High	High
T17	High	High	Moderate

Based on the data presented in Tables 10, 11, 12 and 13 a summary of how each participant performed in each dimension is drawn and displayed in Table 14.

Table 14: Overall performance by each of the six participants in each dimension

Participant	**Description of performance in the dimension**			
dimension	**Foundation**	**Transformation**	**Connection**	**Contingency**
T1	Low	Moderate	Low	Moderate
T11	Low	Moderate	Low	Moderate
T12	Low	Low	Moderate	High
T13	Moderate	Low	Moderate	Moderate
T15	High	Moderate	Moderate	High
T17	High	High	High	High

As can be seen in Table 14, the findings have shown that generally, for each participant the knowledge levels in foundation dimension is almost the same as that in the other three knowledge quartet dimensions. That is, if in foundation dimension the knowledge level is low then in the other dimensions the knowledge levels should also be low, if not then the knowledge levels should either be very low or moderate. Similarly, if in foundation dimension the knowledge level is strong then in the other three dimensions the knowledge levels should also be strong, if not then the knowledge levels should either be very strong or moderate. For example, as can be seen in Table 14, For T1 and T11 foundation knowledge and connection knowledge were both weak, while their transformation and contingency knowledge levels were both moderate. In the case of T12 foundation and transformation knowledge were both low while connection was moderate and surprisingly contingency was High. T13 had moderate foundation, connection and contingency knowledge levels but low transformation knowledge. T17 had high knowledge levels in all the four knowledge quartet dimensions. T15 had high foundation and contingency knowledge levels but moderate transformation and connection knowledge levels.

Therefore, it follows that for a particular participant the knowledge level in one dimension is either equal to that in the other three dimensions or is the same as that of successive knowledge levels in the other three dimensions.

4.7 Summary

This Chapter has presented findings from the participants on the primary school teachers' PCK and SMK in the context of Number Bases. Table 15 presents the summary of the findings with regard to the research questions:

Table 15: Summary of the Chapter (Meaning of the findings)

Question	SMK / PCK	Summary
Question 1	**SMK**	With regard to question one results related to classroom enactment revealed that participants strictly adhered to the textbook, they read 23_{five} as twenty-three. Interview results indicate that participants consider the purpose of teaching and learning number bases to be important in enhancing learners' counting skills, participants never formulate their own questions in the context of number bases and participants have noted that learners make errors on identifying digits, counting and when adding and dividing number bases. Questionnaire results indicate that when constructing problems in base two participants could write numbers such as 23_{two} and 12_{two} despite base two only having two digits one and zero. Concept maps results indicate that when writing digits that are part of base two participants could include the digit 2, thogh base two has only two digits one and zero.
	PCK	With regard to question one results related to classroom enactment revealed that teachers focused on the algorithmic steps to reaching an answer. Interview results indicate that primary school teachers consider teaching methods, teaching aids and prerequisite knowledge when planning to teach the lesson on number bases.

Question 2	**SMK**	Classroom enactment revealed that the examples given were progressively becoming more challenging such that if the first example had two complete groups then the second example would have three complete groups, an indication of good choice of examples. Participants used various forms of representations tabular tally marks and symbolic representation. Interview results indicate that when choosing examples the criteria participants employed was first to work with smaller numbers that are easier to divide or group, then go to bigger ones.
	PCK	With regard to question two results related to classroom enactment revealed that five out of six participants used chalk and text books as the only instructional materials. Interview results indicate that five out of six participants did not know any teaching aid to use apart from chalk and text books when teaching number bases.
Question 3	**SMK**	With regard to question one results related to classroom enactment revealed that participants strictly adhered to the textbook, they read 23_{five} as twenty-three and explained to pupils that we count in base ten because we group things in base ten. Interview results indicate that when teaching number bases participants just follow the order which is in the textbooks, without taking into consideration any pre-requisite knowledge. Questionnaire results indicate that when asked to come up with an appropriate order for the instructional focus on the topic of number bases according to the 2013 primary school curriculum only 7 out of 30 participants came up with the correct pedagogical sequence. Concept maps results show that participants indicated bases four, bases five, base six, bases eight, base ten and bases two as examples of types of bases.
	PCK	With regard to question three results related to classroom enactment revealed that participants when reading final answer from table only one out of the six participants demonstrated anticipation of complexity after converting numbers from base ten to base two using the tabular

		representation. Interview results indicate that primary school teachers consider teaching methods and teaching aids when planning to teach the lesson on number bases.
Question 4	**PCK**	With regard to question four results related to classroom enactment revealed participants demonstrated their contingency knowledge by affirming learner's answer without clarification, out rightly dismissing wrong answers, re-directing question to another learner, and/or by probing learner thinking. Interview results indicate that participants when faced with a challenge in most cases they failed to deviate and if they did their deviation was not characterised by conceptual focus
Cross Analysis of Research Questions	**PCK /SMK**	For a particular participant the knowledge level in one dimension is either equal to that in the other three dimensions or is the same as that of successive knowledge levels in the other three dimensions.

CHAPTER FIVE

DISCUSSION OF THE FINDINGS

5.1 Introduction

This chapter discusses the findings of the research which sought to evaluate primary school teachers' subject matter knowledge and pedagogical content knowledge of number bases. While the previous Chapter presented the findings, this Chapter explains how the findings relate to the literature, theoretical framework and any implications they might have for future use. The discussion is guided by the following five research questions:

1. What foundation knowledge of number bases do primary school teachers have?
2. How do primary school teachers transform the content knowledge they have on number bases into forms that are pedagogically convenient to their pupils when teaching?
3. How do the primary school teachers connect instructions when teaching number bases?
4. What are the primary school teachers' contingency actions when teaching number bases?

5.2 Discussion of research question one

This Section, discusses question one which was used to explore the primary school teachers' foundation knowledge along the following themes: Theoretical underpinning of pedagogy, Productive disposition, Identifying pupil errors, Overt display of subject knowledge, Use of mathematical terminology, Adherence to textbook and Concentration on procedures. The themes Theoretical underpinning of pedagogy, Productive disposition, Adherence to textbook and Concentration on procedures fall under PCK whilst the themes Identifying pupil errors, Overt display of subject knowledge, Use of mathematical terminology fall under SMK

Findings on the theme (*productive disposition)* were presented in Section 4.2.1. The findings from interviews revealed that participants suggested that number bases help in enhancing learners counting system and laying a solid foundation. However, during classroom observations and construction of concept maps none of the participants displayed subject matter knowledge on the *productive disposition* of number bases, (See Section 4.2.1 for details). Interviews findings contradict with those of Wong *et al.,* (2016) who conducted a study on the teaching of number sense. Wong *et al.,* (2016) study found out through interviews that all the three teachers who took part in the study were not aware of the role of number sense on longer term mathematical learning. Accordingly, Wilson *et al.,* (2005) note that place value plays a key role in:

> *"A solid understanding of the place value system, and how it is used, is the foundation for both arithmetic and algebra. The place value system is the foundation of our numbering system. (p. 4).*

Contrary, to Wilson *et al*., (2005) assertions during classroom observations none of the participants explained the *productive disposition* of number bases. This, could be one of the reasons why the general performance in mathematics is low as revealed by Examination Council of Zambia (2018) report (see Table 2, p 6). One possible threat to the learning of mathematics which can emanate from primary school teachers' low foundation knowledge (*productive disposition)* is that the topic of number bases which is the mathematical back bone for many topics in the curriculum may not be accorded the time and attention it requires.

Findings on the theme *theoretical underpinning* were presented in Section 4.2.2. The Findings from classroom observations and interviews revealed that only one out of the six participants mentioned important attributes under theoretical underpinning such as teaching methods, pre-requisite knowledge and the use of tabular representation. This suggests that they are not well grounded in the foundation knowledge *theoretical underpinning* in the context of number bases (see Section 4.2.2) for details). They mainly rely on imitation of one another or trial and error, rather than relying on the theoretical *foundation* required to guide instructional decisions. These findings tally with those of Mntunjani (2015) study results which revealed that most primary school teachers use the traditional approach to mathematics teaching. Mntunjani (2015) note that their teaching was characterised by transmission of mathematical knowledge as enshrined in official textbooks and teachers' guides into the minds of the pupils without any room for pupils' maneuvers. These findings imply that teachers need to draw on knowledge of well-established results in mathematics education research regarding the way pupils learn number bases to underpin the delivery of the lesson (Rowland et al., 2017)

Findings on the theme *(Use of mathematical terminology)* were presented in Section 4.2.3. The findings from classroom observations, concept maps and interviews all revealed that the participants had challenges in the reading of numbers in bases other than base 10 and one participant incorrectly referred to the expanded form as place value (see Section 4.2.3 for details). My study findings are consistent with Ball's (2008) and Sitrava (2018) studies conducted amongst pre-service primary teachers which revealed that many prospective primary school teachers did not have an explicit understanding of mathematical terms such as place value. Poor usage of mathematical terminologies amongst primary school teachers is likely to be transmitted to their pupils (Sitrava, 2018). This seems to suggest that primary school teachers should take into consideration the importance of internalizing mathematical language in the context of number bases, during activities such as Continuing Professional Development (CPD).

This might help them to acquire knowledge of the correct usage of number bases terms and their actual meanings.

Findings on the theme *(overt display of subject knowledge)* were presented in Section 4.2.4. The findings from questionnaire, lesson observation, concept maps, and interviews indicated that most of the participants did not have a well-developed understanding of the appropriate digits used in a particular number system, incorrectly used terms such as *place value* and *value* of each digit and lacked the ability to formulate questions in the context of number bases, (see Section 4.2.3 for details). My findings on *overt display of subject knowledge* corroborates with those of Wessels and Courtney (2014) who conducted a research on place value amongst primary school pre-service teachers in Namibia. Their study showed that the final year pre-service primary school teachers are not proficient in number sense themselves and are therefore ill-equipped to promote number sense in the learners that they would teach (Wessels and Courtney, 2014). A teacher with sound demonstration of (*overt display of subject knowledge*) is likely to make swift decisions concerning the most appropriate instructional materials, presentation, emphasis, and sequence of instruction (Ball *et al.*, 2005). Therefore, it is not good to note that most of the primary school teachers do not have a well-developed understanding (*overt display of subject knowledge)* in the context of number bases. This shows that there is need for the primary school teachers during activities such as Continuing Professional Development (CPD) to find ways of sharpening their foundation knowledge (*overt display of subject knowledge*) in the context of number bases. Findings on the theme *(identifying pupils' errors)* were presented in Section 4.2.5. During interviews participants claimed to possess some elements of foundation knowledge in *identifying pupils' errors* on counting, adding and dividing. However, lesson observations and questionnaire findings revealed that generally participants' had challenges in identifying and addressing learners' errors with regard to the digits used for a given number system (see Section 4.2.5 for details). The challenges by most primary school teachers in identifying pupils' errors could be as a result of lack of overt display of subject knowledge (Rowland, *et al*., 2012). These findings support those of Ball et al., (2008) study which revealed that most primary school teachers are unable to find and explain the errors in children's mathematics problems. Kelly *et al*., (2020) note that one of the qualities of proficient teachers of mathematics is the ability to use their knowledge of mathematics in ways that provide the tools to instill understanding and help pupils with mathematical misunderstandings. This finding implies that primary school teachers should be sensitised to be creating situations in which pupils tell their errors so that teachers are able to systematically correct them.

Results on the theme *concentration on procedure* presented in Section 4.2.6 indicate that all the observed participants applied instrumental approach to teaching mathematics with little or no focus on relational approach. This means that they focused on the algorithmic steps to reaching an answer without understanding why these rules and procedures work (Kilpatrick et al.,2001). These findings resonate with Tabakamulamu (2010) who contended that most primary school teachers emphasize drilling pupils in factual knowledge that promotes passive acceptance and reproduction of information as the ideal. Besides, these findings are consistent with Schwartz and Riedesel (1994) study which focused on primary school teachers' teaching of long division. Their study revealed that although the primary school teachers knew the algorithm for long division, they did not make out why the procedures worked. They are also in agreement with Sitrava (2018) study findings that most of the primary school teachers do not explain the underlying principle and the meaning of the nonstandard algorithm in subtraction or division of numbers. This situation needs agent attention especially in the light of Kelly *et al.*, (2020) assertions that when conceptual knowledge and procedural knowledge are not equilibrated, students do not have a full facility with the topics. Besides, without conceptual knowledge, students cannot comprehend the significance of mathematical concepts and related procedures (Kelly *et al.*, 2020). The concentration on procedure revealed in this study may inhibit learners in primary schools from acquiring a solid grasp of number bases. Possibly this could be the reason why most primary school pupils in Zambia perform poorly as evidenced in Chapter one on page six. This finding of the current study shows that primary school teachers should be acquainted with and apply both instrumental and relational approaches to understanding mathematics. Such measures would capacitate or enhance their ability to follow and assess mathematical reasoning. Findings on the theme *(adherence to textbook)* were presented in Section 4.2.7. The findings revealed that five out of the six participants' strictly adhered to textbooks. The participant could not extend mathematics lessons on number bases beyond the textbook (see Section 4.2.7 for details). For all the participants the examples provided were literally taken from the textbooks with no adjustments. The teachers rigidly followed what was in the textbooks. These findings agree with Tabakamulamu (2010) who observed that Grade 7 examination emphasised book knowledge at the expense of conceptual knowledge, consequently primary school teachers did likewise when teaching number bases. Nonetheless, these practices are against Ayasile (2020) arguments that the textbooks do not provide a balanced range of examples and tasks as such, the textbooks offer few high-level thinking examples and few high-level cognitively demanding tasks, and thus they should not be strictly adhered to. In addition according to Rowland (2003) textbooks being major teaching resources, they must be used creatively to be helpful. This finding suggests that primary school teachers must use textbooks to function as a guide for teaching, with no strict adherence.

5.3 Discussion of research question two

This section discusses question two which was used to explore the primary school teachers' transformation knowledge along the following codes: *Choice of examples, Use of instructional materials, and Choice of representation.* The themes *Choice of examples and Choice of representation* fall under SMK whilst the theme *Use of instructional materials* fall under PCK

Findings on the theme (*choice of examples)* were presented in Section 4.3.1. The findings from this study indicate that participants' *choice of examples* was mainly directed by differentiated demand. That is in most cases examples were mathematically correct, selected intentionally, and matched the purpose of the lesson (see Section 4.3.1 for details). These findings are in line with what Rowland et al., (2009) considers being a good criterion for *choice of examples.* That is *choice of examples* should be of high cognitive demand, mathematically correct, realistic context and/or numbers; no misconceptions, and numbers are selected intentionally (Rowland et al, 2009). This result suggests that it is important for primary school teachers to be aware of various ways of providing intentional *choice of examples.*

Findings on the theme (*use of instructional materials)* were presented in Section 4.3.2. The findings from this study indicate that during classroom observations five out of the six participants only used chalk and text books without concrete manipulatives to aid their teaching (see Section 4.3.2 for details). These findings contradict those of Sebasta and Martin (2004) who conducted a study on the use of instructional materials when teaching fractions, a topic which is in the number and calculation strand just like number bases. Sebasta and Martin (2004) study revealed that primary school teachers were applying *instructional materials* such as base ten blocks when teaching fractions. Nonetheless, they are in agreement with Azuka and Kurumeh

(2013) who sensitized teachers on how to use counters in teaching number bases during a workshop after noticing that many had challenges in applying appropriate *instructional materials*. These findings need attention especially in the light of Berry et al., (2009) and Archer (2021) whose research results indicate that instructional materials such as base ten blocks provide hands-on ways to learn place value, number concepts, operations, and measurement. Besides, their use provides something “concrete” about which teacher and students can talk (Kelly et al., 2020). This finding entails that primary school teachers through activities such as Continuous Professional Development (CPD) should be encouraging one another on the usage of instructional materials such as an abacus in teaching number bases. Teachers should also develop a skill of how to use improvised locally made materials to enable the teaching of number basis with understanding.

Findings on the theme (*choice of representation)* were presented in Section 4.3.3. The Findings from this study indicate that participants exhibited knowledge of various forms of mathematical representations such as concrete, tabular, symbolic and tally marks (see Section 4.3.3 for details). These findings resonate with those of Hill *et al.*, (2005) who contend that various forms of representation such as diagrams, graphical displays and symbolic expressions have long been part of school mathematics. These findings are very good especially in the light of Reys (2018) who have emphasized the importance of knowledge of mathematical representations. They have argued that ‘mathematics is a composition of a large set of highly related abstractions and if teachers do not know how to translate those abstractions into a form that enables learners to relate the mathematics to what they already know, they will not learn with understanding’ Reys (2018). This finding shows that primary school teachers should be aware of the important role knowledge of mathematical representations plays in the provision of quality instruction on number basis through enabling the choice of appropriate representations (Hill *et al*., 2005).

5.4 Discussion of question three

This Section discusses question three which was used to explore the primary school teachers' connection knowledge along the following codes: *recognition of conceptual appropriateness, anticipation of complexity, decision about sequencing, making connections between concepts.* The themes *recognition of conceptual appropriateness and conceptual understanding* fall under SMK and the themes *anticipation of complexity* and *decision about sequencing* fall under PCK.

With regard to *recognition of conceptual appropriateness* findings from this study indicate that during lesson observation only one out of the six participants was in position to use several methods when solving a problem (see Section 4.4.1 for details). These findings which have revealed primary school teachers failure to change tack when teaching number bases, amidst pupils facing challenges in comprehending the number bases concepts contradicts (Hill *et al*., 2005) endorsements that in order to mitigate mathematical errors and enhance learners self-confidence, teachers need to be conversant and consequently communicate with their pupils on various ways of ascertaining solutions. This finding was unexpected especially that when solving a problem of converting from one number system to another one can easily check the accuracy of their working by simply grouping concrete objects such as stones or sticks in line with the given number system. This indicates that primary school teachers must be knowledgeable about various methods of converting numbers from one base system to another through activities such as Continuing Professional Development (CPD). Doing so would make a positive impact on the self-confidence and achievement of the pupils in the context of number bases.

Findings on the theme (*anticipation of complexity)* were presented in Section 4.4.2. The findings from this study indicate that during classroom observation participants encountered challenges such as failure to explain why the answers should start with the digit at the bottom and ending with the digit at the top and not vice versa when converting numbers from one system to another system and in some cases failure to convert numbers from base ten to base five (see Section 4.4.2 for details). This finding is in agreement with Reys (2018, p. 9) claims that 'too many teachers don't know the mathematics they need to know, in the way they need to know it, to help students learn'. This result suggests that primary school teachers need to be making anticipation guides in the context of number bases. This might challenge them to think about number bases before actually teaching them and consequently, propel them to be more focused on the main ideas in the number bases lessons. Moreover, primary school teachers should pay attention to and focus their time on the identified areas in order to optimize their opportunities to effectively teach number bases.

Findings on the theme (*decision about sequencing)* were presented in Section 4.4.3. The findings from this study revealed that 23 out of 30 of the participants who took part in the questionnaire failed to come up with an appropriate pedagogical sequence they would follow when teaching number bases (see Section 4.4.3 for details). They lack what Kim *et al.*, (2018) call breadth. Kim *et al*., (2018) described breadth as the capacity to connect one topic with others of similar conceptual understanding. These findings are consistent with the available literature on primary school teachers' connection knowledge of *decision about sequencing* which suggests that their ability to introduce mathematical ideas and strategies in an appropriately progressive order is not robust (Anghileri, 2006). This shows that primary school teachers should familiarize themselves with making effective links between different number bases terminologies (Anghileri, 2006).

Findings on the theme *(conceptual understanding)* were presented in Section 4.4.4. The findings indicate that primary school teachers scarcely demonstrated the ability to make connections between concepts. For instance, only two out of nineteen participants provided the link for the *types of bases* when constructing concept maps (see Section 4.4.4 for details). According to Boogaard (2021) low levels of conceptual knowledge is a sign of weak mental organization of a particular topic (in this case number bases). These findings are consistent with the available literature on preservice teachers' and primary school teachers' knowledge of mathematical concepts which suggests that their knowledge of mathematical concepts is not as robust and as connected as the mathematics education profession would prefer (Kim, 2018). Besides, Thornton (2016) contend that specific difficulties in terms of syntactic knowledge, connections and progression exist among primary school teachers. These are not pleasing findings especially in the light of Thornton (2016) who contended that content knowledge (the mental organization of teacher knowledge) influences classroom instruction in a positive way. Besides, Kim (2018) notes that for primary school teachers to teach number bases effectively they must possess high level of conceptual knowledge and be able to facilitate to promote learning by making explicit connections of mathematical topics. This result suggests that there is need to find ways of helping primary school teachers to develop their understanding of number bases structure and enhance their capacity to deconstruct content to key components and make connections between number bases concepts (Rowland *et al*., 2017). The application of concept maps in this study has also effectively reinforced the validity of Boogaard (2021) assertion that conceptual knowledge (the knowledge of facts, the meanings of symbols and the concepts and principles) of a particular field of mathematics cannot be measured by tests but by concept mapping. This suggests that primary school teachers should develop the skill of constructing concept maps in mathematics.

Such educational practices might potentially boost the teaching and general performance in mathematics.

5.5 Discussion of question four

This section discusses question four which was used to explore the primary school teachers' contingency knowledge along the following codes: *Responding to students' ideas, Deviation from agenda and teacher insight.* All the three themes fall under PCK.

Findings on the theme (*responding to pupils' ideas)* were presented in Section 4.5.1. The findings from this study indicate that participants responded to student ideas mainly in two ways: *ignore* or *acknowledge but sideline* (see Section 4.5.1 for details). These findings echo (Rowland *et al*, 2017) suggestions that with regard to the scenarios where pupils offer unexpected responses, there are three types of responses – *ignore*; *acknowledge but sideline*; and *respond and incorporate.* Knowledge of students' ideas is held as pedagogical content knowledge (Shulman, 1987). Moreover, Ball *et al*. (2008) argues that mathematics SMK inspires and motivates the teacher to respond proficiently to students' contingency actions such as unexpected questions and answers. Hence, considering the way most of the participants responded to pupils' ideas *(ignore*; and *acknowledge but sideline*) it appears that their pedagogical content knowledge in the sphere of *responding to pupils' ideas* is not fully developed. This finding shows that there is need to ensure that primary school teachers fully acquire skills of *responding to pupils' ideas*. For instance, when teaching number bases, they should not only be paying attention to whether an answer is correct, but also to the pupil's alternative mathematical thinking. Besides, there is need for primary school teachers to be allowing sufficient time for pupils to explore responses in depth and by asking for explanation and understanding (Maugesten, 2019). Such acts of spending time in questioning or probing pupils in order to find out why a pupil comes up with a wrong answer would really raise the standard of performance. Findings on the theme (*teacher insight)* were presented in Section 4.5.2. The findings from this study indicate that participants could not reflect in action and change tact of either example or representation being used during lesson enactment even when faced with challenges (See Section 4.5.2 for details). These findings do not resonate well with the work of Sherin *et al*., (2014) who contend that one type of teaching expertise is the ability "to notice and interpret key features of classroom interaction" (p. 156). This entails that primary school teachers should develop the skill of stopping to reflect-in-action and change tack (example or representation being used) amidst challenges (Rowland *et al.*, 2017) to enhance the teaching and learning of number basis.

Findings on this theme (*deviation from agenda)* were presented in Section 4.5.3. The findings from this study indicate that generally participants when faced with challenges in most cases they failed to deviate and if they did their deviation was not characterized by conceptual focus (See Section 4.5.3 for details). These findings are contrary to what Rowland *et al*., (2017) considers being a good criterion for *deviation from agenda.* That is *deviation from agenda* should be characterized by conceptual focus, presentation of the concept in a new way and illuminating with an everyday example. This implies that primary school teachers should be aware of the important elements with regards to deviation from agenda.

5.6 Discussion of cross analysis of Question 1, Question 2, Question 3, and Question 4 findings

Cross analysis of findings has revealed that for the Knowledge Quartet theoretical framework generally for the participants with higher level of foundation knowledge their corresponding levels of transformation, connection and contingency were higher. Identically, for the participants with lower level of foundation knowledge, generally their corresponding levels of transformation, connection and contingency were low. Similarly, for the participants with higher level of transformation knowledge their corresponding levels of foundation, connection and contingency were higher. Identically, for the participants with lower level of transformation knowledge, generally their corresponding levels of foundation, connection and contingency were low. Also for the participants with higher level of connection knowledge their corresponding levels of transformation, foundation and contingency were higher. Identically, for the participants with lower level of connection knowledge, generally their corresponding levels of transformation, foundation and contingency were low. Similarly, for the participants with higher level of contingency knowledge their corresponding levels of transformation, connection and foundation were higher. Identically, for the participants with lower level of contingency knowledge, generally their corresponding levels of transformation, connection and foundation were low. This is an indication that in order to raise professionalism of primary school teachers with regard to the teaching and learning of Number Bases all the four dimension of the knowledge Quartet must equally be taken into consideration. These results add credence to Rowland et al. (2009) assertion that teachers' foundation knowledge (beliefs and understanding acquired in the academy, in preparation for their role in the classroom) inform pedagogical choices and strategies in a fundamental way. The results also resonate with McNamara's (1991) claims that teachers with strong foundation knowledge may teach in a more interesting and dynamic way whilst those with little foundation knowledge may shy away from the more difficult aspects of the subject, or approach their teaching in a didactic manner. This is an indication that in order to raise the transformation, connection and contingency knowledge levels of primary school teachers, during activities such as Continuing Professional Development they should be starting on focusing on how to sharpen their own foundation knowledge.

5.7 Need for a reference manual that could enhance Primary School Teachers' SMK and PCK in the context of number bases

The findings from this study indicate that participants encounter several challenges when teaching number bases. For instance, in the area of identifying appropriate teaching aids, reading of numbers in bases other than base 10 and identifying pupils' errors. This suggests that there is need to come up with a mechanism of sharpening primary school teachers PCK and SMK in these areas in the context of number bases. One way of coming up with such a mechanism is through developing a reference manual on the effective teaching of number bases. The next Section presents the proposed reference manual.

5.8 Proposed reference manual that could enhance Primary School Teachers' SMK and PCK in the context of Number Bases

This Section presents the proposed reference manual. As contended by Loeb *et al*., (2017) when coming up with the proposed reference manual I considered the most salient features of the findings. Table 16 shows the salient features of the findings and the corresponding possible measures to address the salient features of these findings. With regard to the salient features emerging from the findings based on question one: prerequisite knowledge when planning to teach the lesson on number bases, Identifying and addressing learners' errors, understanding number bases principles and apply appropriate terminology and notations and digits used in a given number system and place value and value of each digit fall under SMK whilst the salient features explaining the productive disposition of number bases and proficient usage of textbooks fall under PCK. With regard to the salient features emerging from the findings based on question two: use of incorrect digits when giving examples, choice of examples and forms of representations fall under SMK whilst concrete instructional materials fall under PCK. All the three salient features emerging from the findings based on question three: complexity of reading of final answer from table, Converting numbers from base eight, two or five to base ten, Providing the link for important concepts (like *types of bases)* during the construction of concept maps all fall under SMK. All the three salient features emerging from the findings based on question four: affirming learners answer without clarification, out rightly dismissing wrong answers, and re-directing question to another learner and deviation from agenda fall under PCK.

Table 16: Proposed reference manual that could enhance Primary School Teachers' SMK and PCK in the context of number bases

RESEARCH QUESTION	MOST SALIENT FEATURES OF FINDINGS	MEASURES TO ADRESS THE SALIENT FEATURES OF THE FINDINGS
Question 1	• Prerequisite knowledge when planning to teach the lesson on number bases. • Identifying and addressing learners' errors • Understanding number bases principles and apply appropriate terminology and notations and digits used in a given number system, (ii) place value and value of each digit • Explaining the productive disposition of number bases • Proficient usage of textbooks	• Primary school teachers should acquaint their pupils with knowledge of Place value, and indices before teaching number bases • Primary school teachers to be aware of the moments when pupils make errors: errors on identifying digits, (ii) errors on counting and (iii) errors on adding and dividing number bases. • Primary school teachers to be aware of the terminologies such as binary, quinary and octet. To know that when writing numbers in base two, five and eight the base is used as a subscript (110_2, 231_5, 256_8) • Primary school teachers to always explain to their learners the usefulness of the topic so that learners can see the utility value of learning number bases • Primary school teachers to use textbooks to function as a guide for teaching, with no strict adherence
Question 2	• Use of incorrect digits when giving examples • Concrete instructional materials • Forms of representations • Choice of examples	• Primary school teachers to be aware what to consider as appropriate digits for a particular number system • To encourage use of locally available materials such as stones, sticks in order to provide hands-on ways to learn place value, number concepts, operations, and measurement • To apply several forms of number bases representation (tabular, concrete, symbolic) in order to accommodate differently abled learners: symbolic, tabular, Tally marks representations • To intentionally select examples in line with the lesson purpose
Question 3	• Complexity of reading of final answer from table. When changing from base ten to base eight or to base five using tabular representation the answer should start with the digit at the bottom and ending with the digit at the top and not vice versa. • Converting numbers from base eight, two or five to base ten • Providing the link for important concepts (like *types of bases)* during the construction of concept maps.	• Incorporating aspects of relational approach. For instance learners should be taught that we follow the place value • Familiarisation of learners with indices prior to teaching them number bases • Providing awareness on the importance of concept maps
Question 4	• Affirming learners answer without clarification. • Out rightly dismissing wrong answers, and re-directing question to another learner. • Deviation from agenda	• Primary school teachers to make clarifications after affirming the correctness of the answer • Affirming method used and probing learner thinking • Deviation from agenda to be characterized by conceptual focus, presentation of the concept in a new way and illuminating with an everyday example

5.6.1 Interview responses of the Primary School Teachers who engaged with the reference manual

After developing the frame work it was field tested. Six primary school teachers were interviewed after they had been sensitised about the model and consequently taught number bases. It was revealed that the frame work advances primary school teachers SMK and PCK for the topic of number bases. For example during interviews primary school teachers' responses on the impact of the model on their SMK and PCK in the context of number bases included:

> T11: [*" I no longer struggle to prepare for the lesson of number bases for I know that before I start teaching number bases, I have to acquaint my pupils,. with the prerequisite knowledge which includes knowledge of Place value".*
> and T17: [*from the time I came across the model I use correct terminologies such as binary for base two"*

With regard to instructional materials T1 said

Previously I used to use only chalk, board and text books when teaching but because of the model I now use a variety of materials. Some of them include those outlined in the model for number bases such as stones and sticks. The time I used concrete material to demonstrate how to change numbers from base ten to base five pupils were very excited" and T13 said *" in my ten years of teaching number bases this is the first time I have known that you can use locally obtained materials like stones to teach number bases which makes the work easier).*

On handling of mathematical errors committed on number bases T15 said

> *before I start teaching the lesson I first sensitise the pupils on the common sources of errors for the topic of number bases. I explain to them how to identify digits appropriate for a given number system and how to count numbers*

that are in base two, five and eight.

With regard to contingency knowledge T15 said *"previously I used to dismiss wrong answers. This time because of what I have learnt from the model I make clarifications. I don't just dismiss or affirm the correctness of the answer"*

The overall impact of the reference manual is that primary school teachers who participated demonstrated developed knowledge and skills in the teaching of number bases. They appreciated the model and were able to apply it proficiently in their classroom enactments. For instance T3 who took part in the model had this to say

I used to be scared of teaching number bases, but now I really enjoy the lessons. I am happy that I am teaching the grade 7 and T4 had this to say *it is now interesting to teach number bases and I even facilitate during CPD*

This shows that the implementation of the reference manual could be a success. Given that primary school teachers have been facing challenges with regard to teaching number bases, it seems even more important that a framework has been proposed upon which they can base their teaching of number bases. Though there is need for further interrogation of the reference manual so that it can further be clarified and refined. To enhance In future studies the primary school teachers should be observed after being sensitised about the reference manual. The primary school teachers should also take part in other activities such as questionnaires and concept maps. This could help to obtain deep information about the effectiveness of the reference manual. However, doing so now could have been much more work considering that they were earlier on observed, interviewed and participated in the questionnaires and construction of concept maps.

5.7 Summary

This Chapter discussed the findings of the research which explored primary school teachers' subject matter knowledge and pedagogical content knowledge of number bases. The discussion was along the constructs from the conceptual framework. The discussion included issues of how the findings are supported by other studies on the subject matter knowledge and pedagogical content knowledge of primary school teachers. The next chapter looks at the recommendations and conclusions of the study based on the research findings.

CHARPTER SIX

CONCLUSIONS AND RECOMMENDATIONS

6.1 Introduction

This Chapter presents the conclusions and the recommendations based on the findings of the study. The aim of this research was to explore Grade 7 primary school teachers' subject matter knowledge and pedagogical content knowledge of number bases, in order to comprehend and find ways of resolving the challenges Grade 7 primary school teachers' encounter when teaching number bases. This study arose from the base line study conducted at NISTCOL in 2014 by the College Lectures in the mathematics department amongst primary school teachers who came from all parts of the country. The aim of the base line study was to find out the topics in the primary school mathematics syllabus which primary school teachers perceive to be difficult to teach. The base line study revealed that; several primary school teachers face challenges with regard to teaching the concept of Number bases. Therefore, in order to develop professional learning for primary school teachers that might support them to proficiently teach number bases; I decided to embark on this study.

6.2 The main research findings and conclusions

The findings of this study have provided insight into the position and organization of primary school teachers' subject matter knowledge and pedagogical content knowledge in the context of number bases.

The research aimed to evaluate primary school teachers' subject matter knowledge (SMK) and pedagogical content knowledge (PCK) of Grade Seven primary school teachers with a focus on the topic of number bases. Based on the qualitative analysis various ways of how primary school teachers demonstrate their PCK and SMK about the topic of number bases have been established.

With regard to pedagogical content knowledge (PCK) of Grade Seven primary school teachers several interesting findings have been revealed. For instance, during lesson enactments when making instructional decisions Grade Seven primary school teachers mainly relied on imitation of one another or trial and error, rather than relying on the theoretical foundation required to guide instructional decisions. Moreover, the study found that during lesson observations primary school teachers applied instrumental approach to teaching number bases. This means that they focus on the algorithmic steps to reaching an answer without understanding why these rules and procedures work. Concentration on procedure occurred mainly when changing numbers from: base ten to base five, base ten to base two and base five to base ten. In addition the study has shown that when teaching number bases primary school teachers mainly use chalk and text books as the only instructional materials. In rare cases they use an abacus in addition to chalk and textbooks. Additionally, the findings have indicated that Grade seven primary school teachers strictly adhere to textbooks when teaching number bases. In most cases the examples provided are literally taken from the textbooks with no adjustments. It was found during lesson observations that Grade seven primary school teachers responded to student ideas mainly by (i) affirming learner's answer without clarification, (ii) out rightly dismissing wrong answers, and (iii) re-directing question to another learner. Additionally, the findings from this study indicate that generally participants when faced with challenges in most cases they could either maintain the status quo or deviate from lesson agenda by reintroducing a prerequisite topic to number bases such as indices. Findings have also shown that when teaching number bases primary schools teachers' instructional decisions depend on the teaching methods, teaching aids and pre-requisite knowledge.

With regard to subject matter knowledge (SMK) of Grade seven primary school teachers about the topic of number bases the study has revealed several important findings that are worth to take note. For instance, the study has revealed that during lesson observation in most cases examples provided by primary school teachers were mathematically correct, selected intentionally, and matched the purpose of the lesson. With regard to mathematical terminology it was evidenced that primary schools teachers read numbers in alternate bases as if they are in base 10 and others refer to expanded form as place value. For instance they read numbers like 23_5 as "twenty three base five". The study has also shown that primary school teachers consider number bases to be crucial in enhancing learners counting system and laying a solid foundation. The study has also revealed that primary school teachers possess some elements of foundation knowledge in *identifying pupils' errors* on counting, adding and dividing. The study has shown that primary school teachers exhibit knowledge of various forms of mathematical representations such as concrete, tabular, symbolic and tally marks when teaching number bases. In relation to recognition of conceptual appropriateness the study has revealed that primary school teachers when solving a problem on number bases either use several methods or a single method. The study has also shown that when sequencing the lessons for the topic of number bases primary schools teachers take into consideration the prerequisite knowledge for number bases.

6.3 Contributions

Practically, the researcher has proposed the reference manual for the effective teaching of number bases.

6.4 Recommendations for policy, practice and further research

Based on the findings of my study, I have made the following recommendations for practice, policy and further research that might be valuable for helping Grade 7 primary school teachers to proficiently teach number bases.

6.4.1 Recommendations for practice

Results indicate that in the context of number bases primary school teachers do not have a well-developed understanding in areas such as awareness of purpose, use of instructional materials and teacher insight therefore during Continuous Professional Development (CPD) primary school teachers should be encouraged to discuss these identified areas. Most participants demonstrated deficiencies in basic mathematical knowledge such as the number of digits present in a particular number system, hence there is need for writers of primary school mathematics textbook to explain the connection between such concepts. Accordingly, to potentially boost the performance of pupils in number bases in particular and in mathematics in general the teacher educators should ensure that primary school teachers are acquainted with and apply both instrumental and relational approaches to understanding mathematics during their classroom enactments. The teacher educators should also ensure that all primary school teachers are trained and are conversant with the construction of concept maps in number bases in particular and in mathematics in general. Institutions training primary school teachers should incorporate the knowledge quartet and mathematical proficiency for teaching mathematics frameworks in their mathematics education courses. Mathematics educators in primary colleges of education should aim to acquaint primary school teachers with a deeper understanding of basic ideas in mathematics, whereby prospective primary school teachers should be in position to call upon a richly integrated understanding of all the topics in the Number and Calculations strand (this is the strand containing number bases) as an effective and efficient way to bring about sound mathematical preparation of primary school teachers. In order to enhance professionalism in the teaching of number bases lecturers training primary school teachers should be ensuring that primary school teachers are sensitized on how to notice and interpret key features of classroom interaction as well as ensuring that they acquire skills of stopping to reflect-in- action and change tack (example or representation being used) amidst challenges.

6.4.2 Recommendation for further research

I have enumerated the following recommendation for further research:

Future research should replicate this study to be extended to other provinces of the country. This will increase the sample size and enhance the generalisability of the results. Also, future research should also be extended to other topics in the ‘Number and Calculation strand so that Primary school teachers’ mathematics subject matter knowledge (SMK) and pedagogical content knowledge (PCK) in the different topics of the ‘Number and Calculation strand can be described and compared. In future studies the primary school teachers should be observed after being sensitised about the reference manual. The primary school teachers should also take part in other activities such as questionnaires and concept maps. This could help to obtain deep information about the effectiveness of the reference manual. However, doing so now could have been much more work considering that they were earlier on observed, interviewed and participated in the questionnaires and construction of concept maps.

6.4.1 Recommendation for policy

There is need for the Ministry of General Education (MOGE) to take note of the key areas outlined in the proposed model as well as to affect the identified measures of addressing the key areas for primary school teachers’ effective teaching of number bases. MOGE should embrace the aspects of knowledge quartet and start using them to monitor the teaching and learning of mathematics in primary school.

6.5 Reflection on the study

This Section reflects the lessons and challenges encountered during the period I was writing this thesis. The process of conducting this study began with the desire to explore ways of enhancing primary school teachers' SMK and PCK for the topic of number bases, after learning that many had challenges in this area. This research process has expanded my own research abilities as I am now more equipped to conduct research. I feel a sense of relief and achievement after successfully completing this thesis on exploring primary school teachers' SMK and PCK in the context of number bases. Owing to the intricacy of the task I required a good deal of time to read, think and write. Therefore, working and at the same time being a full time student was a serious challenge; however, I remained focused and dedicated to the undertaking. After successfully coming up with my thesis topic, I struggled with choice of an appropriate theoretical framework. I took some time thinking on what could be the appropriate theoretical framework. After reading more journals and books I learnt about the Knowledge Quartet and I finally adopted it as part of my theoretical framework. The other challenging part of the research was choosing an appropriate research paradigm. After reading several books among them Creswell (2014) and Tashakkori and Teddlie (2008), I gained deeper understanding of the different types of research paradigms and I settled on interpretivism paradigm and the case study approach. In fact, one of the key lessons I have learnt through this study is how to align the research paradigm to the research approach and methods. I have also learnt that for the Knowledge Quartet theoretical framework generally for the participants with higher level of knowledge in one dimension their corresponding levels of knowledge in other dimensions were also higher. Identically, for the participants with lower level of knowledge in one dimension their corresponding levels of knowledge in other three dimensions were also lower. This is a clear indication that in order to raise proffessionalism of primary school teachers with regard to the teaching and learning of Number Bases all the four dimension of the knowledge Quartet must be must be intenalised. Applying four research instruments to collect the data was costly and time consuming, however for the sake of achieving research trustworthiness I held on. Getting permission to various schools was very smooth; I was warmly welcomed by head teachers in the various schools that I visited. However, I had some few missed opportunities owing to the nature of structured interviews the study used. For instance in Section 4.3.2.1 I did not interrogate further on the type of teaching aids and teaching methods T17 was conversant with. During the period of writing this dissertation I encountered both joyful and sad moments. I had happy moments when my articles were accepted to be published in journals. Another happy moment was when I came up with the proposed model. I hope many primary school teachers will benefit from the proposed model which has highlighted key suggestions which if effected could enhance their SMK and PCK in the context of number bases. I also had sad moments especially the demise of my late wife. I thank the most high God for strengthening me during this journey of writing my thesis.

6.6 Summary

This Chapter highlighted the main research findings on the mathematics subject matter knowledge (SMK) and pedagogical content knowledge (PCK) of Primary school teachers with a focus on Number Bases. Based on the research findings, practical contributions were presented. Recommendations for primary school teachers, mathematics teacher educators in institutions training primary school teachers, policy makers and for further research to contribute to the gap in our understanding were also presented.

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8. APPENDICES

8.1 Appendix A

Primary school Teachers' Conceptual and procedural knowledge questionnaire on Number Bases

1. i) Express 343_5 as a base 10 numeral… ………………………………………………..

………………………………………………………………………………………………

…………………………………………………………………………………………………

.(**Overt display of subject knowledge**)

ii) What are some of the important mathematical ideas which pupils might use to answer this question correctly? …………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………(**Theoretical underpinning of pedagogy)**

2. i) The length of a wall fence in base ten is 40m. What is its length in base five? …………………………………………………………………………………………

…………………………………………………………(**Overt display of subject knowledge**)

ii) What mathematical ideas would pupils use to answer this question correctly?..

....

....(**Theoretical underpinning of pedagogy)**

3.i) Using the binary base, construct one addition problem and solve it…………………………………………………………………………………………………

………………………………………………………………………………………………………

…………………………………………………………(**Overt display of subject knowledge**)

ii) Explain how you would go about explaining to the pupils how you arrived at the answer.

………………………………………………………………………………………………………

………………………………………………………………………………………………………

……………………………………………………………(**Conceptual understanding**) iii) What pre-requisite knowledge do pupils need to have before learning addition of numbers in base two……………………………………………………(**Theoretical underpinning of pedagogy)**

4. Subtract 1100_5 from 1101_5 ....

……………………………………………………………(**Overt display of subject knowledge**)

5. State the value of the digit 5 in the number 1523_8, in base ten……………………………

....………

………………………………(**Overt display of subject knowledge**)

6. Given $3 \times 5^3 + 4 \times 5^2 + 5p = 3420_5$, find the value of *p?*……………………………

....

....(**Overt display of subject knowledge**)

7. Express 125_8 as a number in base two?..

………………………………………………………………………………………

…………………………………………………………(**Overt display of subject knowledge**)

8. In the binary number 101 what does each digit represent?...

……………………………………………………………………………**Choice of representations**

9. In the number 123_5 what does each digit represent? ………...

……………………………………………………………………………………………

……………………………………………………………………………**Choice of representations**

10. When does 7+ 7 = 16? ………………………… …………………………………………

11. A given puzzle had numbers 0, 1, 2, 3, 4, 5 why are the digits 6, 7, 8 and 9 missing?...(**Overt display of subject knowledge**)

12. a) Complete the expansion of the number 7548?

7(1000) + 5( ) + ------ ( ) + ----- ( )

(b) What do you notice about the numbers in brackets?..

………………………………………………………………(**Decission about sequencing)**

13. Write down the first six numbers in base two?..……………………...**Decission about sequencing**

14. Peter was born in 1990, how old is he this year? Express your answer in base five………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………(**Overt display of subject knowledge**)

15. Ms Phiri who is a grade seven teacher at Tupase Primary school, has 13_8 blue sweets, 5_8 green and 43_8 yellow sweets.

i) Find the total number of sweets that she has, in base eight?..

……………………………………………………………………………………………..

……………………………………………………………………………………………

………

………

………………………………………………………(**Overt display of subject knowledge**)

ii) What number of sweets would that be in Base 10?...

…………………………………………………………………………..

………………………………………………………………………………………………………

………1

6. You notice a pupil working on these addition problems as

$$\begin{array}{r} 101_3 \\ +\ 132_3 \\ \hline 233_3 \\ \hline \end{array}$$

(a) What *does* this student understand? ……………………………………………………...

………………………………………………………………………………………………

……………………………………………………………………**(Identifying pupils' errors)**

(b) What does he *not* understand?...

………………………………………………………………………………………………

…………………………………………………………**(Identifying pupils' errors)**

(c) What would you do to help this pupil?...

………………………………………………………………………………………………

………………………………………………………………………………**(Teacher insight)**

17. The following are the instructional focus on the topic of number bases according to the 2013 primary school curriculum:

a) Convert from base 2 to base 5 and vice versa

b) Convert from Bases 2, 5 and 8 to Base 10

c) Illustrate base ten numeration system

d) Convert from Base 10 to Bases 2, 5 and 8

e) Add and subtract in Bases 2, 5 and 8

f) Describe other number bases

g) Order them into a pedagogical sequence: That is arranging them in an order you would follow when teaching domain of Number Bases……………………………………….

…………………………….

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………… (**Decission about sequencing)**

(i) Explain why you chose to put the activities in the order, you have come up with? (**Decission about sequencing)**

18. i) How man digits are in base eight nuimber system?……………….....(**Overt display of subject knowledge**)

ii) Write the largest digit in base eight?..(**Overt display of subject knowledge**)

8.2 Appendix B:

Introduction to concept mapping

One useful tool for explicitly stressing mathematical connections is concept mapping. Concept mapping (Novak & Canas, 2008) is a visual representation of an individual's knowledge structure on a particular topic. This representation takes the form of a finite graph with nodes that depict the mathematical concepts and links (lines or arcs) which in turn represent the relationships among them (Baralos, 2007).

They are often represented in circles or boxes, concepts are linked by words and phrases that explain the connection between the ideas. Most concepts maps represent a hierarchical structure, whereby the overall, broad concept is written first with connected subtopics, more specific concepts, following.

Steps in Constructing a Concept Map

1. Start with a main idea or topic to focus on**:** Once a topic is selected, that will help with the hierarchical structure of the concept map.

2. Then write down all you know about the topic on sheets of paper.

3. Then determine the key concepts for the topic: Come up with the main concepts that connect your topic.

4. Rank the concepts: most general, inclusive concepts come first, then link to smaller, more specific concepts.

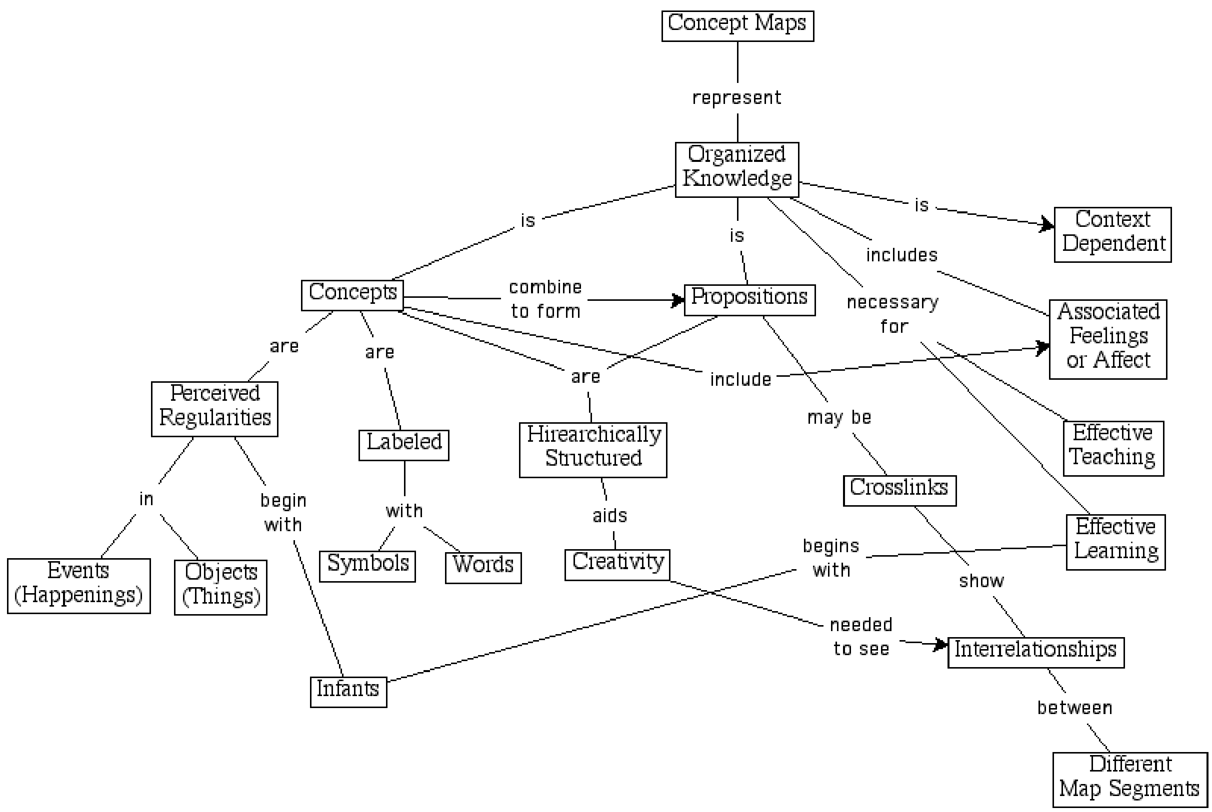
5. End by connecting concepts: Once the basic links between the concepts are created, add cross-links, which connect concepts in different areas of the map.

6. If there are ideas and concepts that appear in two or more clusters, then include those ideas as members of one cluster and use cross-links to connect the prolific concepts to the other clusters.

7. Finally check your work, to correct some errors.

Now let us look at figure 2 and for an example. For the exercise that we will do, we will try to draw a concept map of the topic of number bases.

Sample of Concept map about concept mapping



Source: Novak (2008)

The following are the objectives of teaching the topic of number bases according to the 2013 Zambian primary school syllabus:

1. Illustrate base ten numeration system

2. Describe other number bases

3. Convert from Base 10 to Bases 2, 5 and 8.

4. Convert from Bases 2, 5 and 8 to Base 10.

5. Convert from base 2 to base 5 and vice versa

6. Add and subtract in Bases 2, 5 and 8.

7. Definitions, terminology, and conventions

Activity

In this activity you are expected to construct hierarchical concept map on the topic of Number bases. Use these objectives to construct a concept map for the topic of number bases. The concept map should incorporate all the aspects of these objectives. Include as many branches as possible. The purpose is to determine your breadth, depth, and knowledge of the structure of the domain of Number base.

Table 17: Concept Map Rubric

Concept map element	**High 3**	**Medium 2**	**Low 1**	**Below standard 0**
Organization	Well organized Logical format Give added meaning	Easy to follow most of concepts covered in a meaningful way and are thoughtfully organized.	Somewhat organized Somewhat incoherent Contains only a few of the main concept	Choppy and confusing the unit are missing.
Terminology	Shows an	Makes some	Makes many	Shows no

	understanding of the topic ' s concepts and principles and uses appropriate terminology and notations	mistakes in terminology or shows a few misunderstandings of concepts	mistakes in terminology and shows a lack of understanding of many concepts	understanding of the topic's concepts and principles
Propositional structure	Identifies all the important concepts and shows an understanding of the relationships among them	Identifies all important concepts but makes some incorrect connections	Identifies a few Important concepts with some incorrect connections	Fails to use any appropriate concepts or appropriate connection
Cross – links	At least three cross-links	At least 2 cross-links	One cross-link	No cross-link
Linking words	Links succinctly and accurately describe all relationships	Links are descriptive and valid for most relationships	Some links unclear or vague; some invalid or unclear	Links are vague; show inconsistent relationships
Efficient links	Each link type is distinct from all	Most links are distinct from	Several links are	Most links synonymous or

	others, clearly describes relationship; used consistently	others; discriminate concepts; present variety of relationships; used fairly consistently	synonymous; don't discriminate concepts well; don't show a variety of relationships; used inconsistently	vaguely describe relationships and aren't distinct from other links
Total number of links	At-least 10 links	From 7 to 9	From 4 to 6	Below 4

8.3 Appendix C: Observation schedule

Part A

Date ………………   Participant Identification number……………………

Gender……………….

Scoring Guide: 4 = very high, 3= High, 2 = Moderate, 1 = Low, 0 = very low

Areas	0	1	2	3	4

Part B

Foundation

awareness of purpose: Explaining the significance of number bases, clarifies the purpose and learning goals					
Identifying pupil errors, reaction to errors					
Demonstrating overt subject knowledge; specifying number of digits for a given base, Position, ordering, Amount, discussing the use of number bases other than base ten, explaining the symbol X_{b},					
theoretical underpinning of pedagogy					
Use of mathematical terminology: including terms such as number bases, octet, binary,					
adheres to textbook: Adapting, text book examples, procedures					
Concentrates on procedures, uses appropriate algorithms, frequencies of mistakes committed when giving examples or explaining concepts					
Foundation total					

Transformation

Teacher Demonstration, explaining procedure					
Use of instructional material, Posing mathematical questions that are productive for students’ learning					
Choice of representation, manipulates					
choice of examples, from simple to complex, adapted or not; uses appropriate algorithms, pace of giving examples					
Creation of interactive activities					
Transformation total					

Connection

making connections between procedures					
making connections between concepts, consecutive lessons, discussion of pupils' prior knowledge					
Anticipating, with given activities and problems in a lesson, what students are likely to do and get confused about					
decisions about sequencing of activities, representations, examples, explanations, questions					
recognition of conceptual appropriateness					
Link between lesson template and topic					
Connection total					

Contingency

responding to pupil's ideas, utility of prompts, level of communication, explore reasoning					
use of opportunities: formatively assess learners, creating time and space for pupils to provide feedback					
deviation from lesson agenda					
teacher insight: Deciding what is most important for students to know and understand about the provided tasks and problems in a lesson					
responding to the (un)availability of tools and resources					
Contingency total					

8.4 Appendix D: Post-lesson Observation Interview Protocol

Opening remarks

Thank you very much for the chance you gave me to observe your class. I have a few questions for you. The questions are about your experiences as a primary school teacher in general and about the teaching of number bases in particular, and specifically about today's lesson. This in order for me to understand better some of the pedagogic decisions you took during the enactment of the lesson. You are free to skip any question if it causes any challenge. Remember that your comments are strictly confidential and none of your remarks will be associated with you by name

A. Teacher background

1. How many years have you been working as a primary school teacher?
2. For how long have you been teaching this class
3. For how long have you taught number bases
4. How many pupils are in your class?

B. Today's lesson

1. Tell me the main reason for teaching number bases to primary school pupils **(Foundation/ Productive disposition)**
2. Tell me how long you have been teaching number bases
3. i) Tell me the reasons for the choice of the teaching aids you used. ii) What sources did you consult before teaching? **(Transformation dimension)**
4. What pre-requisite knowledge do pupils need to know for them to effectively learn number bases? **(Connection dimension)**
5. When planning to teach this topic, what teaching pre-requisites did you consider? (**theoretical underpinning of pedagogy**

8. How does today's lesson connect to previous and the next lessons? **(Connection dimension)**

9. How confident are you to formulate your own questions on number bases? **(Foundation)**

10. How did this lesson turn out differently from what you planned? How did you adjust to suit the unanticipated changes? Would you like to have used other methods but you were prevented from using them for some reason? **(Contingent actions)**

11. What pre-requisite knowledge do pupils need in order to introduce this topic of number bases? **(Foundation dimension)**

12. i) Are you able to identify patterns in pupils' errors on number bases? ii) What are the common errors pupils commit on number bases? **(Identifying pupils' errors)**

13. When teaching number bases what are some of the challenges you anticipated pupils to face? **(Contingent action)**

18. When you were teaching number bases what criteria did you follow to choose and sequence examples? (**Connection dimension**)

19. You have just taught number bases. i) What were the strengths of the lesson? ii) How would you change the lesson next time? iii) If you think the lesson catered for all learners how do you know this? Before teaching what challenges, did you expect would occur during the lesson? **(Anticipation of complexity)**

20. What do you think these students learned from this lesson? (**Teacher insight)**

21. (**Free Question)** *I will ask on any key issue that would occur during classroom observation. For instance, issues related to,* Choice and use of representations, dealing with incorrect answers and explanations)

C. Closing remarks:

Is there anything else that you would like to share?

Thank you for contributing to the promotion of teaching and learning of primary school mathematics.

8.5 Appendix E: Letter of Information to the Primary School Teachers

26 May, 2017

Dear Teacher

You are invited to take part in this research project being conducted by George Mulenga a student in the Department of Mathematics and Science Education at the University of Zambia. The purpose of this study will be to assess the mathematics subject matter knowledge (SMK) and pedagogical content knowledge (PCK) of Primary school teachers with a focus on the topic of Number bases. The research may be valuable in the future in providing necessary and vital information which educators researchers can apply to create an environment that is conducive to each Primary School teachers and be able to understand their pupils' achievements in mathematics in general and Number bases in particular.

What you will be asked to do?

If you decide to participate you will be asked to fill in a questionnaire on Number bases, to come up with a concept map on number bases, and to be observed as you teach the topic of number bases.

Risks

Except for your time and inconvenience there are no foreseeable risks to you participating in this study.

Benefits

The study may help individual participating primary School teachers to professionally reflect and improve upon the way they teach the topic of Number bases especially that many Primary School Teachers find teaching this topic to be a very big challenge as evidenced by the base line study conducted at NISTCOL in 2014.

Confidentiality

Your name will not appear on any of the documents. Please do not put your name on the test paper or concept map. It will not be possible to link the data to an individual primary school teacher. The data that you provide may be used to provide scholarly works. This may include, but is not limited to, Journal article and Conference presentations. Scholarly works produced will neither identify individual Primary School Teachers nor schools.

Yours faithfully,

Mulenga George

8.6 Appendix F: Participant consent form

I ________________________________(name of a teacher) here by consent to participate in the research study focusing on the teaching of Number bases in Primary Schools and it is entitled Primary school teachers' knowledge of number bases: subject matter content knowledge and pedagogical content knowledge. The study is being conducted by George Mulenga, for his doctoral research study. I have been provided with all the necessary information about the study and I have been granted the chance to ask questions. I will understand that I will take part in constructing a concept map, filling a questionnaire, interviews and in the teaching of Number bases during my normal Mathematics class room lesson. During this process my teaching will be observed.

I am satisfied that the aims of study and my role in the study have been explained at the beginning of the process. I agree that that the research data gathered for this study may be published provided neither my name nor that of my school is identified.

I am also aware that I can withdraw from participating in the study at any stage if I wish to do so.

________________________________ ____________________________

Signature of Teacher **Date**

8.7 Appendix G: ANALYSIS OF LESSON OBSERVATION

4.3.1 Teacher 1 (T1) Observation 1 (11/05/2017). Time: 10:20 – 11:00

Lesson overview

The lesson was on how to convert base ten numbers to base five. The teacher stated the lesson by reminding the pupils what they did the previous day (which was topic of Roman/Arabic numerals). The teacher then called a pupil in front to demonstrate an example of how to represent five in Roman numerals. After that the teacher introduced the topic of number bases. Without explaining the significance of teaching and learning number bases the teacher went straight to examples. Examples were on how to convert numbers from base ten to base five (such as 13 from base ten to base five). Then the teacher gave an exercise and concluded the lesson after marking the pupils work.

Knowledge Quartet Analysis

In this lesson T1 '*use of mathematical terminology'* as identified by the knowledge quartet frame work was good. When introducing base five he told pupils:

> Base ten is also known as decimal while base five is also known as quinary.

This enabled pupils to be aware of the other terms used to denote base ten and five. With regard to *choice of examples* the examples T1 seem to be good. The first example is as follows:

Question: Convert 13 from base ten to base five?

Working: The teacher drew thirteen strokes as shown:

/////////////

Then he grouped the strokes in groups of five. Two groups consisting of five strokes each were made with three strokes remaining.

/////   /////   ///

Then he told pupils that since two groups each consisting of five strokes were made with three strokes remaining, the answer is 23_5.

The second example was as follows:

Convert 17 from base ten to base five?

Working: The teacher drew seventeen strokes as shown:

////////////////

Then he grouped the strokes in groups of five. Three groups consisting of five strokes each were made with two strokes remaining.

///// ///// ///

Then he told pupils that since there are three groups each consisting of five strokes with two strokes remaining, the answer is 32_5.

As can be seen the examples given were progressively becoming more challenging an indication of good *choice of examples.* In example one when the teacher found 23_5 as the answer he emphasised to the pupils that this is not read as twenty three but as two fives and three ones. Thus, demonstrating that he had some element of' o*vert display of subject knowledge'*. T1 used drawn strokes when demonstrating how convert numbers from base ten to base five. It could be better if *use of instructional materials* included concrete objects such as block ten blocks or locally improvised materials such as stones or sticks. Such manipulatives could have helped to concretise the concepts consequently reducing on rote memorization.

When T1 wrote 23_5 as the answer to example one he did not explain the essence this type of representation whereby at the end of 23 the subscript five. It is important for T1 to always explain c*hoice of representations.* Pupils should be aware that apart from the decimal system we put a subscript to indicate the base of a given number system under consideration.

T1 demonstrated lack of *awareness of purpose*. He went started talking about base ten without explaining why pupils in primary schools should learn number bases. He started the lesson as follows:

> T1: To day we are going to look at number bases
>
> T1: In Zambia we count things in units
>
> T1: We call this base ten
>
> T1: It is easier to understand base ten when we group things
>
> T1: For us to understand base ten we have to look at the digits used in base ten

T1 was supposed to explain the utility of number bases to real life situations, prior to start teaching the lesson. This could have enabled pupils to see the value and consequently appreciate the learning of number bases.

Throughout this lesson there was evidence that T1 way of *responding to pupils ideas* was very calm. This is evidenced in the following exchange when he asked a question about the quinary system:

> T1: Having done examples on how to convert numbers from base ten to base five you are going to do an exercise.
>
> Pupil c: Teacher I am still not sure about the quinary system.

T1: It uses four digits (which was actually a wrong response as the quinary system has five digits)

Pupil b: Teacher what is a base

T1: I gave an example in Zambia we count things in groups of ten (T1 did not explain why we count in base ten)

In all these instances T1 amicably responded to pupils' questions, he never ignored a single question. Though T1 failed to make cogent and well informed responses to some of the unanticipated questions. For instance he said the quinary system has four digits instead of five.

4.1.2 Teacher one-Lesson two (12-05-17). Time: 08:20 – 09:00

Lesson overview

The lesson was on changing numbers from base five to base ten. The teacher started the lesson by asking the pupils' questions based on the previous lesson of converting numbers from base ten to base five. The he gave some examples on how to convert numbers from base five to base ten. The mathematics taught was mainly instrumental in nature (Mathematics with a focus on getting the correct answer without paying attention to conceptual understanding). At the end of the lesson T1 honestly admitted that he had challenges in converting numbers from base five to base ten (gap in *overt knowledge*).

Knowledge Quartet Analysis

At the beginning of the lesson there was evidence of T1 making sound 'decision *on sequencing'* as identified by the Knowledge Quartet framework. When introducing the lesson, T1 asked:

T1: Is there anyone who can tell us what we did yesterday

Pupil X: We looked at number bases

This demonstrates T1's sound knowledge of the '*connection'* between successive lessons. T1 also demonstrated '*awareness of purpose' in* the lesson discussion:

T1: To day we shall look at changing from base five to base ten.

Thus, T1 effectively communicated what the pupils were about to learn.

T1 '*Responded to students ideas'* as identified by the Knowledge Quartet framework was poor as can be seen in the following exchange:

T1: [Writes 16 on the board and asks] who can change 16 from base ten to base five

Pupil Y: [Moves in front and converts 16_{ten} to 31_{five}.]

T1: Alright

Here T1 '*responds to pupils' ideas'* by simply affirming the correctness of the answer without making clarifications.

This lesson demonstrated T1's limited foundation knowledge of converting from base five to base ten as can be seen in the following example he provided of converting 33_{five} to base ten. T1 worked out the problem as follows:

T1 Working: The teacher drew thirty three strokes which he grouped in groups of tens as shown:

////////// ////////// ////////// ///

Upon interpreting that he had obtained 33_{ten} after converting 33_{five} to base ten, T1 looked stranded:

T1: hmm. We are going to have the same answer.

T1: [Asks the pupils] what are we missing here?

T1: It looks there is something missing.

Px: We can multiply by the digits then we add

T1: Another, who has an idea

Py: We can get five as the answer

T1: Actually we have the answer. We shall just say 33_{five} =33.

Pt: No teacher

T1: Actually it is correct since numbers in base five should be less than five. It is correct since numbers in base five should be less than five.

This classroom interaction clearly shows that T1 had limited '*overt display of subject knowledge*' with regard to how to convert numbers from base five to base ten.

As a further matter, T1way of '*responding to pupils ideas'* became rigid with no room at all for new ideas as can see in the following exchange:

Px: Teacher after converting 33_{five} we get 18 as an answer [Which off course was the correct answer]

T1: 33 is the answer

Py: Teacher I have my self a different way.

T1: No! Don't use a different way but the remainder is 3

Perhaps T1 need to realize that learners construct their own mathematical ideas. T1 could have prompted questions to check where the learners thinking was emanating from This also demonstrates lack of *'teacher insight'* as identified by the knowledge quartet.

T1 also demonstrated poor 'u*se of mathematical terminology'*. He read 33_{five} as thirty three and not as three fives and three ones.

With regard to '*anticipation of complexity'* it appears that T1 had not anticipated complexity of converting from base five to base ten. It was these challenges had caught him unaware and almost lead to paralysis of the lesson.

Table 21; Summary of T1, Observation

Part B

Foundation

Areas	**1**	**2**	**3**	**4**
Awareness of purpose: Explaining the significance of number bases, clarifies the purpose and learning goals	x			
Identifying pupil errors, reaction to errors	x			
Demonstrating overt subject knowledge; specifying number of digits for a given base, Position, ordering, Amount, discussing the use of number bases other than base ten, explaining the symbol $X_{b,}$	x			
Theoretical underpinning of pedagogy		x		
Use of mathematical terminology: including terms such as number bases, octet, binary,			X	
Adheres to textbook: Adapting, text book examples, procedures		X		
Concentrates on procedures, uses appropriate algorithms, frequencies of mistakes committed when giving examples or explaining concepts	x			
Foundation total	**11**			
Foundation percentage score	**39%**			

Transformation

Areas	1	2	3	4
Teacher Demonstration, explaining procedure		x		
Use of instructional material, Posing mathematical questions that are productive for students' learning	x			
Choice of representation, manipulates			x	
Choice of examples, from simple to complex, adapted or not; uses appropriate algorithms, pace of giving examples			x	
Transformation total	**9**			
Transformation total percentage score	**56**			

Connection

Areas	1	2	3	4
Making connections between procedures	x			
Making connections between concepts, consecutive lessons, discussion of pupils prior knowledge		x		
Anticipation of complexity, with given activities and problems in a lesson, what students are likely to do and get confused about	x			
Decisions about sequencing of activities, representations, examples, explanations, questions			x	
Recognition of conceptual appropriateness	x			
Link between lesson template and topic			x	
Connection total	**11**			

Connection percentage	46%

Contingency

Areas	1	2	3	4
Responding to pupil's ideas, utility of prompts, level of communication, explore reasoning		x		
use of opportunities: formatively assess learners, creating time and space for pupils to provide feedback		x		
Deviation from lesson agenda			x	
Teacher insight: Deciding what is most important for students to know and understand about the provided tasks and problems in a lesson		x		
Responding to the unavailability of tools and resources or availability	x			
Contingency total	**10**			
Contingency percentage	**50%**			

Teacher 11, Lesson 1, Date: 10-05-17. Time: 11:40 – 12:20

Lesson overview

The lesson objectives were to: "Identify the numbers used in base ten and change numbers from base ten to base five". T11 started the lesson by explaining the concept of number bases. The teacher explained that "when we count in base ten, it means we count in base ten". Then she gave examples on how to convert numbers from base ten to base five. Finally she gave an exercise to pupils which she marked.

Knowledge Quartet Analysis

In this lesson, T11 made explicit '*connections between concepts'* at the beginning but latter as the lesson proceeded demonstrated poor *'connection of concepts'*.

For instance in the following exchange T11 demonstrates good '*connection between concepts'*:

> T2: Today we are going to start a new topic
>
> Pupils: Yes, teacher
>
> T11: We count in base ten because we group things in base ten. [Then writes the digits used in base ten].

This was good demonstration of '*connection between concepts' as* pupils need to know the digits used in a particular base before carrying out calculations in that particular base.

However, contrary to the earlier exchange in the following exchange T11 demonstrates poor *'connection between concepts'*

> T11: We are going to do an example on how to change from base ten to base five
>
> Pupils: Yes, teacher

Here T11 made poor '*connection between concepts'*. Before, introducing the concept of converting from one number system to a different one T11 was supposed to explain the concept of place value, indices and the digits used in base five. All the aspects are key promoting procedural fluency and conceptual understanding amongst pupils.

As identified by the knowledge quartet T11 demonstrated lack of appropriate use of '*instructional materials'*. For instance T11 verbally explained: When in base ten we group in groups of tens but in base five we group in groups of fives. Here T11 was supposed to be practical by grouping stones or sticks.

In this lesson T11 exhibited lack of 'awareness of purpose'. T11 went straight to teaching number bases without discussing the significance of learning number bases or the application of number bases to real life situations. However T11 demonstrated 'overt *display of subject knowledge'* when she explained that:

> 11_{two} means the number is in base two and 4_{five} means that the number is in base five. But if it is in base ten no need to put the prefix

This was good on the part of the teacher. This also demonstrated sound knowledge of *'choice of representation'* about number bases.

Within the lesson, there was evidence that T11 '*concentrated on procedures'* as demonstrated in the following exchange when she was changing 13 from base ten to base five:

> T2: Writes 13 zeroes [0 0 0 0 0 0 0 0 0 0 0 0 0 0]. Then asks pupils: "How many group of five can be formed from 13?"
>
> Px : Two
>
> T2: What is the remainder?
>
> Py: Three
>
> T2: Therefore, the answer is 23

As can be seen T11 focuses on the algorithmic steps to reaching an answer. T11 also demonstrated '*choice of representation'* and '*recognition of conceptual appropriateness'* as identified by the knowledge quartet by deciding to use another method to check if they would find the same answer. This time T11 decided to use tabular representation. T11 gave instructions on how to convert 13 from base ten to base five. Such instructions included:

> When changing thirteen from base ten to base five, five would be the divisor and thirteen would be the dividend. The remainders obtained each time after the division operation would be written in the remainder column till we cannot divide any longer.

5	13	R
5	2	3
	0	2

However, T11 just said the answer is 23 did not explain why the answer had to start with two and not three. This lack of '*anticipation of complexity'* by T11 was a source of confusion amongst pupils as was revealed during the interviews. Also, T11 lacked '*overt display of mathematical knowledge'* as evidenced in the '*use of mathematical terminology'*. T11 read 23_{five} as twenty three instead of 2 fives and three ones. Nonetheless, T11 way of *'responding to pupils questions'* was well done:

Pv: Teacher where did you get the remainder three?

T11: We divided five into thirteen and obtained two remainder three

The teacher paid attention and amicably '*responded to the pupil's question'* and this motivated the pupils.

Teacher 2, Lesson 2, Date: 11-05-17. Time: 08:20 – 09:00

Lesson overview

The focus of the lesson was on converting numbers from base ten two base eight. T11 started the lesson by revising the previous lesson. The she gave some examples on how to convert numbers from base ten system to base eight system. Finally T11 gave pupils an exercise which she marked during the same period. The lesson template is what Sullivan (2007) refer to us *active teaching.* In this lesson template, there was limited learners' choice. Both the focus and pacing were determined by the teacher. In terms of the Kilpatrick et al. (2001) dimensions, this lesson template was useful for building conceptual understanding and developing procedural fluency strands. However, there was no element of strategic competence, adaptive reasoning or productive disposition.

Knowledge Quartet Analysis

In this lesson T11 demonstrated '*making connections between concepts'* as identified by the knowledge quartet:

> T2: Yesterday we started a new topic. Who can remember what we discussed?
>
> Px: We looked at changing from base ten to base five
>
> T2: Yes. How many digits are in base five?
>
> Py: 4
>
> T2: No
>
> Pv: 5
>
> T2: Yes, what are the digits?
>
> Pz: 0, 1, 2, 3, 4, 5
>
> T2: Yes and in base eight we use the following digits: 0, 1, 2, 3, 4, 5, 6, 7,

Here T11 made explicit connection between this lesson and the previous one. However T11 way of *'responding to pupils ideas'* revealed that that she had some pedagogical inadequacies. T11 just dismissed wrong answers. She seemed not to care about the sources of pupils mistakes. Such acts can make pupils look dump. Besides, there would be limited potential for prompting communication. Within the lesson, there was evidence that T11 '*concentrated on procedures'* as demonstrated in the following exchange when she was changing 17 from base ten to base eight:

> T11: Writes 17 zeroes. [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]. Then she

said we have two groups of eight and remainder ……..

Pv: shouts [one]

T2: So the answer is 21 in base eight. Who can come and use the other method?

As can be noted, these statements focus on the mechanical steps to finding an answer. Thus there was a lot of 'concentration *on procedures'* with little or no emphasis on the reasons for those algorithms '*making connections between concepts'*. There is need for T11 to promote '*awareness of purpose'* by incorporating aspects of relational mathematics for pupils to appreciate the essence of those mathematical algorithms.

As identified by the knowledge quartet in this lesson there was no '*deviation from the agenda'*.

Part B

Table 22: Summary of T11, Observation

Foundation

Areas	**1**	**2**	**3**	**4**
awareness of purpose: Explaining the significance of number bases, clarifies the purpose and learning goals	x			
Identifying pupil errors, reaction to errors			X	
Demonstrating overt subject knowledge; specifying number of digits for a given base, Position, ordering, Amount, discussing the use of number bases other than base ten, explaining the symbol $X_{b,}$		x		
Theoretical underpinning of pedagogy		x		
Use of mathematical terminology: including terms such as number bases, octet, binary,		x		
Adheres to textbook: Adapting, text book examples, procedures			X	
Concentrates on procedures, uses appropriate algorithms, frequencies of mistakes committed when giving examples or explaining concepts			X	
Foundation total score	**15**			
Foundation percentage	**54%**			

Transformation

Areas	1	2	3	4
Teacher Demonstration, explaining procedure			X	
Use of instructional material, posing mathematical questions that are productive for students' learning		x		
Choice of representation, manipulates			X	
Choice of examples, from simple to complex, adapted or not; uses appropriate algorithms, pace of giving examples			X	
Transformation total score	**11**			
Transformation percentage	**69%**			

Connection

Areas	**1**	**2**	**3**	**4**
Making connections between procedures			x	
Making connections between concepts, consecutive lessons, discussion of pupils prior knowledge		x		
Anticipating, with given activities and problems in a lesson, what students are likely to do and get confused about	x			
Decisions about sequencing of activities, representations, examples, explanations, questions		x		
Recognition of conceptual appropriateness		x		
Link between lesson template and topic			x	
Connection total score	**10**			
Connection percentage	**50%**			

Contingency

Areas	1	2	3	4
Responding to pupil's ideas, utility of prompts, level of communication, explore reasoning		x		
Use of opportunities: formatively assess learners, creating time and space for pupils to provide feedback		x		
Deviation from lesson agenda			x	
Teacher insight: Deciding what is most important for students to know and			x	
Responding to the unavailability or availability of tools and resources		X		
Contingency total	**12**			
Contingency percentage	**50%**			

Teacher 12, Lesson 1, Date: 12-05-17. Time: 08:40 – 09:20

Lesson overview

The focus of the lesson was on illustrating the base ten numeration system and describing other number bases. T12 started the lesson by informing the learners that they were starting a new but very interesting topic of number bases. Then she went on explaining the concept of place value which was followed by some examples on how to determine the place value of a digit in a given number. Finally T12 gave pupils an exercise which she marked during the same period. The lesson template is what Sullivan (2007) refer to us *active teaching.* In this lesson template, there was limited learners' choice. Both the focus and pacing were determined by the teacher. In terms of the Kilpatrick et al. (2001) dimensions, this lesson template was useful for building conceptual understanding and developing procedural fluency strands. T12 demonstrated relational approach to understanding mathematics. Relational understanding is having a mathematical rule, knowing how to use it, and knowing why it works.

Knowledge Quartet Analysis

From the beginning of the lesson T12 demonstrated sound *'decision about sequencing' and 'making right connections between procedures'* as can be seen in following exchange.

T3: To day we are going to look at a new topic: Number bases. It is a very important topic.

T3: How do we use numbers in our everyday life?

Px: In calculating bills

T3: Where else do we use numbers?

Pv: When counting

T3: How?

Py: We use numbers when counting

T3: Ok, count from one to nine. [Then pupils counted as instructed]. So we have counted

` from one to nine. What type of number system do we use when counting?

Pv: Place value

T3: Thank you. What else?

Px: It is the base ten number system. This is the system we use in everyday life.

By appropriately applying proper *'decision about sequencing' and 'making right connections between procedures'* T12 effectively introduced base ten numeration system ' awareness of purpose'. *'Responding to students' ideas'* was nicely done. In some instances T12 appreciated learners' ideas. This is a good way of motivating and encouraging learners' participation.

However, T12 did not demonstrate '*awareness of purpose'* with regard to the essence of teaching and learning number bases. As she introduced the topic of Number Bases, she should have told the pupils the use of number bases in real life situations.

T3's foundation knowledge of number bases ('*overt display of subject knowledge')* was weak as demonstrated through the following discussion:

T12: How many digits have the base ten systems?

Px; Ten

T3: No it is nine.

Here T12 three wrongly told the pupils that there are nine digits in base ten instead of ten. T12 should be aware that the number of digits in a given base system corresponds to given base system under consideration. That is base ten has ten digits, base five has five digits and so on.

T12 demonstrated making connections between concepts as can be seen in the following exchange:

T12: Mention any three digit number?

Px: 345

T12: Do we know the place value for each digit

Pupils [shouted] the place value for three is hundred, for two it is ten and for five it is one.

T12: What is the value for three? [Writes: value of 3 = 3 x 100 = 300]

Here T3 also made '*good decision about sequencing'*. She started teaching the topic by explaining the concept of place value before teaching how to convert numbers from base ten to other bases. This was a good way of introducing number bases. Such pre- requisites (concept of place value) could be used in the subsequent lessons such as the one of converting numbers from base five to base ten.

Throughout this lesson T12 made an effort to '*identify pupil errors'* as can be seen in the following exchange:

T12: I gave you a task to determine the place value and value of each digit in the number 753. Who got everything correct?

Pupils: [some pupils put up their hands]

T3: Who made some mistakes?

The above utterances show T12 determination to '*identify pupil errors'* and desire to enquire learners understanding. Such positive interventions would help pupils to be well vested with the topic.

T12's contingency knowledge (*responding to pupil ideas*) was well done as can be seen in the following exchange.

T12: [writes: Show the place value place value of 4 for the number 945].

Px: Teacher you have written the word place value twice

T12: okay [she rubs one]

She shows that her contingency knowledge (*responding to pupil ideas*) was pedagogically powerful as she could accommodate learners, views.

Teacher 12, Lesson 2, Date: 15-05-17. Time: 08:40 – 09:20

Lesson overview

The focus of the lesson was on changing numbers from base ten to base two and base five. She started the lesson by revising the previous lesson which was illustrating the base ten numeration system. She then went on explaining the base two and base five number systems. This was followed by a few examples on how to convert numbers from base ten to the base two and base five number systems. Finally, T12 gave pupils an exercise which she marked during the same period. The lesson template is what Sullivan (2007) refer to us *active teaching.* In this lesson template, there was limited learners' choice. Both the focus and pacing were determined by the teacher. In terms of the Kilpatrick et al. (2001) dimensions, this lesson template was useful for building conceptual understanding and developing procedural fluency strands. In this lesson T12 demonstrated instrumental approach to understanding mathematics. Instrumental understanding is having a mathematical rule and being able to use and manipulate it without necessarily knowing how it works.

Knowledge Quartet Analysis

T12 did not explain her '*choice of representation'* when she wrote 15_{10}. There was need for her to explain why she was writing the subscript 10 especially that it was for the first time.

T12 *'concentrated on procedures'* when teaching pupils how to change numbers from base ten system to base two system. She gave pupils the following instructions:

> *Consider both base ten and base two where you are converting to. Then divide the base into the number we are converting. We divide the base into the number we are converting, we keep on dividing and dividing till we reach zero.*

Here T12 emphasised on the steps for changing numbers from base ten to base two. The emphasis was on procedural fluency and not conceptual understanding. Then, she illustrated by changing fifteen from base ten to base two using the tabular form as a *'choice of representation'.*

She explained that:

when changing fifteen from base ten to base two, two would be the divisor and fifteen would be the dividend. Each time after the division operation the quotient becomes the new dividend but the divisor is constant it remains two. The remainders obtained each time after the division operation would be written in the remainder column

This scenario shows T12 weakness on '*choice of representation'* and *'decision about sequencing'*. Prior to using the tabular form representation form T12 could have used fifteen stones or sticks and then group them in groups of two's. This could have helped to concreticise the topic of number bases.

T12 exhibited inappropriate usage of mathematical terminology with respect to number bases as can be seen in the following exchange:

T12: So what is the answer?

Px: From the remainder

T12: Clap for him

Pupils: [clap]

T12: So what do we do?

Py: We add the ones

T12: So we do not need do anything, multiply or add. Just gets the number starting with the digit at the bottom. So our answer is 1111_2 read as one, one, one, one.

Here T12 exhibited inappropriate usage of '*mathematical terminology'* with respect to number. For 1111_2 is read as one eight one four, one two, and one one. However T3's way of '*responding to pupils ideas'* was good as she appreciated the learners answers which could motivate them and consequently enhance their level of participation.

T12 displayed lack of '*anticipation of complexity'* as she demonstrated how to convert 13 from base ten to base two as shown in Table 36.

Table 23: T12 representation for changing 13 from base ten to base two

5	13	R
5	2	3
	0	2

Just like T12 (who gave the same example), T12 just said the answer is 23 did not explain why the answer had to start with two and not three. This lack of '*anticipation of complexity'* by T12 was a source of confusion amongst pupils.

Table 24: Summary of T12, Observation

Foundation

Areas	**1**	**2**	**3**	**4**
awareness of purpose: Explaining the significance of number bases, clarifies the purpose and learning goals	x			
Identifying pupil errors, reaction to errors			x	
Demonstrating overt subject knowledge; specifying number of digits for a given base, Position, ordering, Amount, discussing the use of number bases other than base ten, explaining the symbol $X_{b,}$	x			
Theoretical underpinning of pedagogy	x			
Use of mathematical terminology: including terms such as number bases, octet, binary,	x			
Adheres to textbook: Adapting, text book examples, procedures		x		
Concentrates on procedures, uses appropriate algorithms, frequencies of mistakes committed when giving examples or explaining concepts			x	
Foundation total score	**12**			
Foundation percentage	**43%**			

Transformation

Areas	**1**	**2**	**3**	**4**
Teacher Demonstration, explaining procedure			x	
Use of instructional material, Posing mathematical questions that are productive for students' learning		x		
Choice of representation, manipulates		x		
Choice of examples, from simple to complex, adapted or not; uses appropriate algorithms, pace of giving examples			x	
Transformation total score	**10**			
Transformation percentage	**63%**			

Connection

Areas	1	2	3	4
Making connections between procedures			X	
Making connections between concepts, consecutive lessons, discussion of pupils prior knowledge			X	
Anticipation of complexity, with given activities and problems in a lesson, what students are likely to do and get confused about	x			
Decisions about sequencing of activities, representations, examples, explanations, questions			x	
Recognition of conceptual appropriateness		x		
Link between lesson template and topic			x	
Connection total score	**15**			
Connection percentage	**63%**			

Contingency

Areas	1	2	3	4
Responding to pupil’ s ideas, utility of prompts, level of communication, explore reasoning				x
Use of opportunities: formatively assess learners, creating time and space for pupils to provide feedback		x		
Deviation from lesson agenda			x	
Teacher insight: Deciding what is most important for students to know and understand about the provided tasks and problems in a lesson				X
Responding to the unavailability or availability of tools and resources		x		
Contingency total score	**13**			

Contingency percentage	65%

Teacher 13, Lesson 1, Date: 16-06-17. Time: 08:40 – 09:20

Lesson overview

The focus of the lesson was on changing numbers from base five to base ten. T13 started the lesson by revising the previous lesson which was on converting numbers from base two to base ten. This was followed by a few examples on how to convert numbers from the base five systems to the base ten number systems. Finally, T13 gave pupils an exercise which she marked during the same period. The lesson template is what Sullivan (2007) refer to us *active teaching.* Both the focus and pacing were determined by the teacher with limited learners' choice. In terms of the Kilpatrick et al. (2001) dimensions, this lesson template was useful for building conceptual understanding and developing procedural fluency strands. In this lesson T13 demonstrated instrumental approach to understanding mathematics. Instrumental understanding is having a mathematical rule and being able to use and manipulate it without necessarily knowing how it works.

Knowledge Quartet Analysis

In this lesson T13 demonstrated *'making connections between procedures'* as identified by the knowledge quartet. Prior to teaching learners how to convert numbers from base five system to base ten the T13 revised the previous lesson which was on converting numbers from base two to base ten.

T13: Who can come and convert 101_{two} to base ten?

Px: [Moves in front and works it out as follows]

1	0	1
1 x 2^2	0 x 2	1 x 1
1 x 4	0 x 2	1 x 1
4	0	1

Answer = 401

Then T13 went through to emphasis the main points. Here T13 connected the previous lesson of converting base two numbers to base ten to the one which was under consideration (converting base five numbers to base ten).

In this lesson this lesson there was evidence of T13 '*concentrating on procedures*' as identified by the Knowledge Quartet framework. This is demonstrated in the following exchange:

T13: Writes on board [Convert 231_5 to base ten]. Then draws a table and asks pupils what to do.

Px: [Moves in front and writes on the board] 5 5 5

T13: The first five is five power what?

Py: Five power zero

T13: What about the next five

Pz: Five power one

T13: What shall we do next?

Px: We shall multiply

Here T13 concentrates on mechanical steps to arriving at the answer as opposed to conceptual understanding. In this lesson T13 exhibited lack of proper ' *use of mathematical terminology'*. She read 314_5 as three fourteen instead of three twenty fives one fives and four ones. However, T13 demonstrated ' *overt display of subject knowledge'* when she explained that 5^1 means five should appear once. T13 '*choice of examples'* demonstrated both elements of strength and weakness. The strength was exhibited in the numbers used as examples. First T13 converted 231_5 to base ten, then demonstrated how to convert 314_5 to base ten. Hence the examples increased in size, consequently enhancing the complexity of the example. The weakness in the *'choice of examples'* was that all the numbers used (231_5 and 314_5) were both three digits numbers. A variation in the number of digits could have been better especially in enforcing the concept of place value.

Teacher 13, Lesson 2, Date: 07-06-17. Time: 11:00 – 11:40

Lesson overview

The focus of the lesson was on addition of number bases in base five and base two. T4 started the lesson by discussing the previous lesson which was converting numbers from base five to base ten. This was followed by a few examples on the operation of adding of numbers in base two and base five. Finally T13 gave pupils an exercise which she marked during the same period. The lesson template is what Sullivan (2007) refer to us *active teaching.* Both the focus and pacing were determined by the teacher, with limited learners' choice. In term s of the Kilpatrick et al. (2001) dimensions, this lesson template was useful for building conceptual understanding and developing procedural fluency strands. In this lesson just like in the previous one T13 demonstrated instrumental approach to understanding mathematics.

Knowledge Quartet Analysis

In this lesson T13 demonstrated good *'decision about sequencing* as identified by the knowledge quartet when she discussed with pupils the previous lesson before starting the new lesson.

When discussing the problem 110_2 + 1011_2, T4 reflects the '*concentration on procedures'* approach that she has taken:

T13: What should we do?

Pupil x: We write the bases.

T13: Ok, we add [Writes 0 + 1 =1].Then we add [Writes 1+1 = 2].Then divide [Writes $\frac{2}{2}$] and you get one with remainder zero. So write the remainder. Then in the third column add [Writes 1+ 0 = 1] and add the remainder, then divide by two and get one remainder zero, so write the remainder zero. Carry one to the front column

The format for working appeared as follows:

$$\begin{array}{rrr} 2^2 & 2^1 & 2^0 \\ 1 & 1 & 0 \\ \underline{10} & \underline{1} & \underline{1} \\ \underline{100} & \underline{0} & \underline{1} \end{array}$$

Here, T13 demonstrated foundation knowledge *(concentration on procedure*).

After T13 had finished working out the problem, there was mixed reaction amongst the pupils (some said they were clear about the whole process, others said that they were completely lost. This shows '*lack of teacher insight'*. Perhaps there is need for T13 to change tack. This shows that T13 lacked *recognition of conceptual appropriateness.*

T13 called a pupil to solve the following problem: $214_5 + 312_5$

Px moved in front and work out as follows:

$$\begin{array}{r} 214_5 \\ + \underline{312_5} \\ \underline{1031_5} \end{array}$$

This was an indication that the majority of the learners did not follow the first example. This problem was in base in base five but the pupil decided to divide by two upon getting a sum above two. This is T13 did not explain the reason for dividing by two each time after getting a sum equal or greater than two in example one. Since this problem was in base five the pupil should have divided by five.

This seem to confirm that T13 did not *'anticipate the complexity'* of the strategies needed to carry out the addition the problem.

T4 demonstrated foundation knowledge of '*identifying pupil errors'* after asking pupils to work out the following problem: $111_2 + 111_2$. Most of the pupils got 1000_2. Here T4 discovered that the source of error for those who got 1000_2 was lack of carrying one forward after dividing by two.

Table 25: Summary of T13, Observation
Foundation

Areas	**1**	**2**	**3**	**4**
Awareness of purpose: Explaining the significance of number bases, clarifies the purpose and learning goals	x			
Identifying pupil errors, reaction to errors			x	
Demonstrating overt subject knowledge; specifying number of digits for a given base, Position, ordering, Amount, discussing the use of number bases other than base ten, explaining the symbol $X_{b,}$			x	
Theoretical underpinning of pedagogy		x		
Use of mathematical terminology: including terms such as number bases, octet, binary,		x		
Adheres to textbook: Adapting, text book examples, procedures		x		
Concentrates on procedures, uses appropriate algorithms, frequencies of mistakes committed when giving examples or explaining concepts			X	
Foundation total score	**16**			
Foundation percentage	**57%**			

Transformation

Areas	1	2	3	4
Teacher Demonstration, explaining procedure			x	
Use of instructional material		x		
Choice of representation, manipulates		x		
Choice of examples, from simple to complex, adapted or not; uses appropriate algorithms, pace of giving examples		x		
Transformation total score	**9**			
Transformation percentage	**56%**			

Connection

Areas	1	2	3	4
Making connections between procedures			x	
Making connections between concepts, consecutive lessons, discussion of pupils prior knowledge				x
Anticipating of anticipation of complexity: with given activities and problems in a lesson, what students are likely to do and get confused about	x			
Decisions about sequencing of activities, representations, examples, explanations, questions			X	
Recognition of conceptual appropriateness	x			
Link between lesson template and topic			X	
Connection total score	**15**			
Connection percentage	**63%**			

Contingency

Areas	**1**	**2**	**3**	**4**
Responding to pupil’ s ideas, utility of prompts, level of communication, explore reasoning		x		
Use of opportunities: formatively assess learners, creating time and space for pupils to provide feedback		x		
Deviation from lesson agenda			X	
Teacher insight: Deciding what is most important for students to know and understand about the provided tasks and problems in a lesson		x		
Responding to the unavailability or availability of tools and resources		x		
Contingency total score	**11**			
Contingency percentage	**55%**			

Teacher 17, Lesson 1, Date: 05-06-17. Time: 08:40 – 09:20

Lesson overview

The focus of the lesson was on converting numbers from base two to base five number system and vice versa. T17 started the lesson by discussing the previous lesson which was converting numbers from base two to base ten. This was followed by a few examples on the operation of adding of numbers in base two and base five. Finally T17 gave pupils an exercise which she marked during the same period. The lesson template is what Sullivan (2007) refer to us *active teaching.* Both the focus and pacing were determined by the teacher with limited learners’ choice. In terms of the Kilpatrick et al. (2001) dimensions, this lesson template was useful for building conceptual understanding and developing procedural fluency strands. In this lesson just like the other participants T17 demonstrated instrumental approach to understanding mathematics.

Knowledge Quartet Analysis

In this lesson T17 demonstrated good *'decision about sequencing* as identified by the knowledge quartet when she started the lesson by asking pupils to state what they learnt the previous day as can be seen in the following exchange:

T17: It is now time for mathematics. What were we doing yesterday?

Px: Converting numbers from base two to base eight

T17: Who can convert 352_8 to base ten?

Pv: [moves in front and works it as follows]

8^2	8^1	8^0
3	5	2
3×8^2	5×8	2×8^0
3×64	5×8	2×1
192	40	2

Answer $192 + 40 + 2 = 234$

T17: How did you know that you should use 3 columns?

Pv: We have three digit number

T17: Good

This classroom enactment here, prior to discussing how convert numbers from base two base five decision shows T17 strength in her '*decision of sequencing'* of activities. Since, the mathematical concepts discussed here are a pre-requisite to the lesson T17 was about to introduce. T17 demonstrated foundational knowledge; *'overt display of subject knowledge'* and *theoretical underpinning of pedagogy* when she asked the pupil why she used three columns. This is an indication that T17 takes into consideration all that pupils need to know. Throughout the lesson the teacher relied on the chalk and board when giving examples. Perhaps this is an indication of T17's limitation of transformation knowledge (*choice of representation* and *use of instructional materials*). Perhaps T17 could have grouped sticks or stones in base two and then regroup them in base five. This could have helped learners to conceptualise and concretises the process of converting number bases from one system to another system.

T17 seemed to be lacking in the '*use of mathematical terminology'* despite teaching on base two and base five she never used terms such as binary and quinary for base two and base five respectively.

T17 seem to be lacking transformation knowledge (*choice of examples*) in that all the numbers in the examples she had chosen had three digits. Perhaps there was need to use numbers with two or four digits and then check how pupils would determine the place values of each digit in the given number. However, T17 demonstrated her strength foundation knowledge (*identifying pupil error*) when she discovered that most pupils were failing to evaluate 5^2.

Teacher 17, Lesson 2, Date: 06-06-17. Time: 08:40 – 09:20

Lesson overview

The focus of the lesson was entitled 'addition in different bases'. T17 five started the lesson by telling learners to count in tens. With pupil input T17 worked out two examples on addition of numbers in base five. Finally T17 gave pupils an exercise which she marked during the same period. The lesson template is what Sullivan (2007) refer to us *active teaching.* Both the focus and pacing were determined by the teacher with limited learners' choice. In terms of the Kilpatrick et al. (2001) dimensions, this lesson template was useful for building conceptual understanding and developing procedural fluency strands. In this lesson just like in the previous one T5 demonstrated instrumental approach to understanding mathematics.

Knowledge Quartet Analysis

In this lesson T17 demonstrated poor decision about *'making connection between concepts'* when she told pupils to count by tens up to two hundred. Since the lesson focus was on addition of numbers in base five, perhaps T17 could have told the learners to count by fives. The main purpose of number bases is to internalize the concept of place value and T17 demonstrated '*awareness of purpose'* when she said;

> "We are going to use place values columns to help us add easily"

T17 demonstrated foundation knowledge '*use of instruction materials'* when she called a pupil to add two and four using an abacus and then demonstrated how to change numbers from base five to base ten. Such hands on activities would help pupils develop authentic understanding of place value. T17 exhibited contingency knowledge '*teacher insight'*. This was in an instance when Px was working out the problem: $321^5 + 423^5$. Px added 1 and 3 and wrote 4. T5 asked Px why it was necessary to write 4 straight without dividing. Here T17 was asking probing questions to try to elicit where the pupil's thinking was coming from. T17 also demonstrated *anticipation of complexity'* when she gave an opportunity to those pupils who were not clear to ask. This was an indication that T17 had anticipated pupils would encounter some challenges and that T5 was prepared to address those challenges.

Table 26: Summary of T17, Observation
Foundation

Areas	**1**	**2**	**3**	**4**
Awareness of purpose: Explaining the significance of number bases, clarifies the purpose and learning goals	x			
Identifying pupil errors, reaction to errors				x
Demonstrating overt subject knowledge; specifying number of digits for a given base, Position, ordering, Amount, discussing the use of number bases				x
Theoretical underpinning of pedagogy			X	
Use of mathematical terminology: including terms such as, octet, binary,			X	
Adheres to textbook: Adapting, text book examples, procedures		x		
Concentrates on procedures, uses appropriate algorithms			X	
Foundation total score	**20**			
Foundation percentage	**71%**			

Transformation

Areas	**1**	**2**	**3**	**4**
Teacher Demonstration, explaining procedure			X	
Use of instructional material, Posing mathematical questions that are productive for students' learning			X	
Choice of representation, manipulates			X	
Choice of examples, from simple to complex, adapted or not; uses appropriate algorithms, pace of giving examples		x		
Transformation total score	**11**			
Transformation percentage	**69%**			

Connection

Areas	**1**	**2**	**3**	**4**
Making connections between procedures		x		
Making connections between concepts, consecutive lessons, discussion of pupils prior knowledge		x		
Anticipation of complexity, with given activities and problems in a lesson, what students are likely to do and get confused about			X	
Decisions about sequencing of activities, representations,			X	
Recognition of conceptual appropriateness		x		
Link between lesson template and topic			X	
Connection total score	**15**			
Connection percentage	**63%**			

Contingency

Areas	**1**	**2**	**3**	**4**
Responding to pupil' s ideas, utility of prompts, level of communication, explore reasoning			X	
use of opportunities: formatively assess learners, creating time and space for pupils to provide feedback			X	
Deviation from lesson agenda			X	
Teacher insight: Deciding what is most important for students to know and understand about the provided tasks and problems in a lesson			X	
Responding to the (un)availability of tools and resources		x		
Contingency total score	**14**			
Contingency percentage	**58%**			

Knowledge Quartet Analysis

In this lesson T15 demonstrated connection knowledge of making good *'decision about sequencing* and '*making connection between concepts'* as identified by the knowledge quartet as can be seen in the following exchange:

T15: When you are looking at base two which digits do you use?

Pv: Zero and one

T15: Ok, what about five

Pz: five

T15: Is he correct?

Px: No

Here by highlighting the pre-requisite knowledge necessary for converting numbers in base two and five to base ten T15 demonstrated ability to make sound *'decision about sequencing* and '*making connection between concepts'* . T15 way of '*responding to pupils'* wrong answers was by throwing the question out to the class. Though, this was a constructive response, the teacher could have gone further to inquire the reasoning behind the answer.

T6 shows weakness in foundation knowledge '*overt display of subject knowledge'* and transformation knowledge '*choice of examples'* as can be seen through the example T6 provides on the board:

Change 2222 from base two to base ten

An analysis of the problem reveals that T15 uses wrong digits as binary numbers only have two digits, zero and one. T15 also exhibits weakness in the *'use of mathematical terminology'*. After expanding 2222 to obtain 2^3, 2^2, 2^1, 2^0 she referred to the expanded form as place value.

When working out the problem T15 appear to concentrate on procedure. The tendency of 'concentrating on procedure' was evidenced when T15 called on a pupil to convert the problem: 131_5 to base ten. The pupil tried to change but failed. This could perhaps as a result of lack of '*anticipation of complexity'* in T15's connection knowledge.

Teacher 15, Lesson 2, Date: 08-06-17. Time: 08:40 – 09:20

Lesson overview

The focus of the lesson was on addition of numbers in base two. T6 started the lesson by asking pupils sing. As the pupils were singing t6 randomly gave some pupils papers. Each paper had one question written on it. This was followed by a discussion of the questions that were written on the papers. Then T15 gave some examples on the operation of addition in base two. Finally T6 gave pupils an exercise which she marked during the same period. The lesson template is what Sullivan (2007) refer to us *active teaching.* Both the focus and pacing were determined by the teacher with limited learners' choice. In terms of the Kilpatrick et al. (2001) dimensions, this lesson template was useful for building conceptual understanding and developing procedural fluency strands. In this lesson T15 demonstrated instrumental approach to understanding mathematics.

Knowledge Quartet Analysis

The introduction depicted T15 sound connection knowledge of *'decision about sequencing* and '*making connection between concepts'* as identified by the knowledge quartet. At the start of the lesson T15 provided pupils with papers which contained the following questions:

Question 1: The base which uses 0, 1, 2, 3 and 4?

Question 2: Simplify 2^2?

Question 3: Find 8^2?

Question 4: Which base uses 0 and 1?

Question 5: What is the other name for base two?

Q6: What is the other name for base eight?

The discussion of these questions (based on what had already been covered on the same topic) was vital in laying firm foundation, as they provided pre-requisite knowledge for addition of number bases in base two, five and eight.

T15 way of '*responding to pupils ideas'* was mainly by re-directing as can be seen in the following exchange:

T15: What is 8^2

Pv: 16

T15: Is he correct?

Pupils: No

T15: Who can try?

T15 way of '*responding to pupils responses'* could enable more pupils to participate as well as allowing a pupil to correct another pupil's incorrect statement.

T15 '*concentrates on procedures'* when teaching the pupils to add numbers in base two. This is demonstrated when working out the problem: $110_2 + 1111_2$

T15: They in the same base, so it is easier to add. We arrange them as follows;

$$\begin{array}{r} 1\;\;1\;\;0_2 \\ \underline{1\;\;1\;\;1\;\;1_2} \\ \underline{1\;\;1\;\;2\;\;1_2} \end{array}$$

T6; where do we start from?

Pv: [Moves in front and solves the problem as follows]

$$\begin{array}{r} 1\ \ 1\ \ 0_2 \\ \underline{1\ \ 1\ \ 1\ \ 1_2} \\ \underline{1\ \ 1\ \ 2\ \ 1_2} \end{array}$$

T15: Is the answer correct?

Pupils: Yes

T15: But there is a two. Do we write two in base two? To add you follow the following instructions. We start by adding one and zero in the fourth column and obtain ones [Writes 1]. Then move to the third column where you add one plus one to obtain two. Since we got two which is not used in base two we divide it by two to obtain one remainder zero. [Writes zero below one]. Then we move to the second column where we add the one we carry forward to the one which is already there. Again we obtain two. So again we divide by two to obtain one remainder zero [Writes 0].

$$\begin{array}{r} 1\ \ 1\ \ 0_2 \\ \underline{1\ \ 0\ \ 1\ \ 1_2} \\ 1\ \ 0\ \ 0\ \ 0\ \ 1_2 \end{array}$$

Here, the procedure of adding numbers in base two is the focus. T15 provides no detailed explanation why we divide by two as evidenced in the confusion which emanated amongst learners when pupils were asked to solve the problem: $141_5 + 201_5$. Despite this being in base five pupils were dividing by base two. However, T15 demonstrated foundation knowledge of *'identifying pupils' errors'* when she corrected the pupil who had written the digit two as part of the answer.

T15 ‘*choice of examples’* was good. The second and third examples she provided were on addition in base five: $141_5 + 201_5$ and $314_5 + 123_5$. The choice of these examples was good as can be seen for the second example the sum of all pairs is less than five but in the third example the sum of sum pairs is more than five (the base under consideration). This could help learners to master when to divide.

T15 demonstrated ‘*conceptual appropriateness’* when on the exercise she included a question which was in base eight. This was despite the fact that had taught pupils how to add in base two and base five only.

Table 28: Summary of T15 Observation

Foundation

Areas	1	2	3	4
Awareness of purpose: Explaining the significance of number bases, clarifies the purpose and learning goals			x	
Identifying pupil errors, reaction to errors			x	
Demonstrating overt subject knowledge; specifying number of digits for a given base, Position, ordering, Amount, discussing the use of number bases other than base ten, explaining the symbol $X_{b,}$				x
Theoretical underpinning of pedagogy			x	
Use of mathematical terminology: including terms such as number bases, octet, binary,	x			
Adheres to textbook: Adapting, text book examples, procedures		x		
Concentrates on procedures, uses appropriate algorithms, frequencies of mistakes committed when giving examples or explaining concepts			x	

Foundation total score	**19**
Foundation percentage	**68%**

Transformation

Areas	**1**	**2**	**3**	**4**
Teacher Demonstration, explaining procedure		x		
Use of instructional material, Posing mathematical questions that are productive for students' learning			X	
Choice of representation, manipulates		x		
Choice of examples, from simple to complex, adapted or not; uses appropriate algorithms, pace of giving examples			X	
Transformation total score	**10**			
Transformation percentage	**63%**			

Connection

Areas	**1**	**2**	**3**	**4**
Making connections between procedures		x		
Making connections between concepts, consecutive lessons, discussion of pupils prior knowledge		x		
Anticipation of complexity, with given activities and problems in a lesson, what students are likely to do and get confused about	x			
Decisions about sequencing of activities, representations, examples, explanations, questions			X	
Recognition of conceptual appropriateness			X	

	1	2	3	4
Link between lesson template and topic			X	
Connection total score	**14**			
Connection percentage	**58%**			

Contingency

Areas	**1**	**2**	**3**	**4**
Responding to pupil's ideas, utility of prompts, level of communication, explore reasoning		x		
Use of opportunities: formatively assess learners, creating time and space for pupils to provide feedback			X	
Deviation from lesson agenda			X	
Teacher insight: Deciding what is most important for students to know and understand about the provided tasks and problems in a lesson		x		
Responding to the unavailability or availability of tools and resources		X		
Contingency total score	**12**			
Contingency percentage	**60%**			

8.8 Appendix H: Ethical clearance

THE UNIVERSITY OF ZAMBIA

SCHOOL OF EDUCATION

DEPARTMENT OF LANGUAGE AND SOCIAL SCIENCES EDUCATION

REPORT ON RESEARCH PROPOSAL REVIEW ON ETHICAL ISSUES

TITLE OF RESEARCH PROPOSAL: *PRIMARY SCHOOL TEACHERS'KNOWLEDGE OF NUMBER BASES: SUBJECT MATTER KNOWLEDGE AND PEDAGOGICAL CONTENT KNOWLEDGE'*

PRINCIPAL INVESTIGATOR: MR GEORGE MULENGA

This is a detailed proposal and has made a mention of ethical issues to be considered in order to safeguard the privacy and safety of all the participants. For example, the researcher has stipulated simple steps to be taken into account such as obtaining permission to conduct the interviews in schools; briefing all the participants what the research is all about. However, the ethical consideration should indicate in very simple language and clear terms on how study intends to do to ensure that what has been proclaimed takes place. It is not enough to simply say confidentiality, respect etc. will be maintained at all times without stating how this is to be achieved neither is it enough to state that participants will be assured of protection physically and emotionally from those who may feel injured by their responses without stating how this is to be achieved. Anybody can say what is given under the ethical consideration section but can still do nothing at the end. Better state what will be done, e.g. no name will be disclosed nor written on the report. The test to be given to teachers is so detailed and elaborate that correct data will be collected though could be embarrassing in an event that teachers score lowly on those aspects being tested. Though not labeled as a test, the exercise has all the qualities of a test and may cause a lot of stress on the part of the teachers, for sure. It may help to explain that this is not a test and results will not be presented as though it were a test

I propose that the research is allowed to proceed but must attend to observations stated in the report and are not plenty.

8.9 Appendix I: Publications

8.9.1 Exploration of Grade seven primary school teachers' contingency knowledge of number bases

International Journal of Education (IJE)
Volume 6, Issue 1, January-December 2019, pp. 34–41, Article ID: IJE_06_01_003
Available online at
http://www.iaeme.com/IJE/issues.asp?JType=IJE&VType=6&IType=1

EXPLORATION OF GRADE SEVEN PRIMARY SCHOOL TEACHERS' CONTINGENCY KNOWLEDGE ON NUMBER BASES

George Mulenga and Patricia P Nalube
The University of Zambia

ABSTRACT

This is a qualitative case study that explored grade seven primary school teachers' contingency knowledge on number bases. Thematic analysis of lesson observations, and interviews was done by applying the pre-specified categorisation scheme of the Subject Audit Instrument (knowledge Quartet). Six purposively sampled primary school teachers took part in lesson observation and post-lesson observation interviews. Results indicate that when faced with a challenge in most cases primary school teachers failed to deviate. They failed to reflect in action and change tack of either example or representation being used during lesson enactment even if learners are not clear of the example given. Therefore, during professional meetings primary school teachers should be addressing these identified areas.

Key words: *Knowledge Quartet* and *Contingency.*

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http://www.iaeme.com/IJE/issues.asp?JType=IJE&VType=6&IType=1

1. INTRODUCTION

Green and Ollerton (1999) have observed that problems associated with teachers' mathematical content knowledge are particularly common in the Primary and early Child sector where generalist teachers often lack confidence in their own mathematical ability. In a supportive view base line study conducted in Zambia at Chalimbana University in 2014 revealed that majority of the in-service primary school teachers considered the topic of Number bases to be mystifying. They had challenges with teaching the mathematics topic of Number Bases. Table 1 indicates number of in-service primary school teachers who had difficulties with some topics.

Table 1 Number of in-service primary school teachers who had difficulties with some topics

Topic	Number of in-service primary school teachers who considered a particular topic to be difficult (Out of 300)	Percentages (%)
Number bases	216	72
Number line	53	19
Fractions	21	7
Long Division	15	5
Angles	9	3
Other topics	12	4

As can be seen in Table 1, most of the in-service primary school teachers who took part in the survey considered the topic of number bases to be very difficult to teach.

This situation is very unfortunate especially in the light of Hodgen (2011) who posit that students need their best teachers at a young age. That is teachers who really understand the simplest of what they are doing. According to Dabiri (2003), to produce such proficient teachers, researchers and teacher educators need to device effective preparation and support programmes for teachers. Briton et al (2000; 57) note that, "What we so often do is work more on describing a problem than actually trying to solve it". "In fact, we are very good at describing problems, very good especially in describing the achievement gaps". "We understand this gap in lots of different ways, with increasing statistical power and sometimes with even nicer slides. In other words, we have increasingly elegant solutions that do not have a chance of happening except in the most unusual or special problem". It is in this regard that by exploring the nature of classroom interactions primary school teachers experience when teaching number bases we opt not to be mere spectators but to be part of problem solvers to what is happening to the teaching and learning of mathematics in Zambia.

1.2. Objectives of the study

- To determine how grade seven primary school teachers, respond to students on number bases
- To determine the nature of grade seven primary school teacher's knowledge about teacher insight on number bases.
- To explore grade seven primary school teacher's knowledge on deviation from agenda on number bases

2. REVIEW OF RELATED LITERATURE

This review of literature is divided into three Sections. The first Section has literature on the meaning of number bases, the second section has literature on the learning of mathematics in Zambian primary schools. The third section covers literature on the aspects of Contingency knowledge.

2.1. Meaning and importance of the topic 'Number Bases'

Farlex (2012) explains that number bases is a mathematical topic which refers to positional representation system in which a real number is represented by an ordered set of characters where the value of a character depends on its position. This entails that number bases encompass different ways of writing the same number. Michele (2006) explains that:

the base of a system of numerals is the number that determines the place values for all of the numerals in that system. For example, in Base 10, 234 equals 2 times 100 (10 squared), 3 times 10 (10 to the first power), and 4 times 1 (10 to the zero power). In Base 8, 234 equals 2 times 64 (8 squared), 3 times 8 (8 to the first power), and 4 times 1 (8 to the zero power). Any *positive integer greater than one can be used as a base. You can express any number in any base (p.18).*

2.2. Learning of mathematics in Zambian Primary Schools

This section elaborates on how the pupils learn mathematics in Zambian primary schools with reference to the number of periods, main topics covered as well as the other subjects learnt in addition to mathematics.

In a week generally the timetable for government primary schools is as follows:

- 7 periods per day
- Each period has 40 minutes
- 35 periods per week for all subjects
- Mathematics is taught every day and it has 7 periods per week

The Curriculum development Centre (2013) articulates six content domains in the area of learning Mathematics. Table 2 indicates the six content domains and their respective topics:

Table 2: Mathematics strands and their respective topics

S/N	STRANDS	TOPICS
1	Number and Calculations	Numbers and Notation, Addition, Subtraction, Multiplication, Division, Combined operations, Fractions, Decimals, Factors and Multiples, Social and Commercial Arithmetic, Index notation, Approximation, Ratio and Proportion, Percentages, Integers, Number Bases and Number Sequences
2	Algebra	Angles, Plane Shapes, Solid Shapes
3	Geometry	Angles, Plane Shapes, and Solid shapes
4	Measures	Measures
5	Probability and Statistics	Statistics
6	Relations	Relations

Source: Curriculum Development Centre (2013)

Thus, from Table 2 we see that most areas of mathematics are covered and that the topic Number Bases is placed in the Number and Calculation strand.

2.3. Contingency Knowledge

Contingency knowledge is one of the four dimensions of the Knowledge Quartet (Rowland et al, 2005). According, to the National Council of Teachers of Mathematics (2000) the knowledge quartet is defined as an empirically grounded theory of knowledge for teaching in which the distinction between different kinds of knowledge is of lesser significance than the classification of situations in which mathematical knowledge surfaces in teaching. Koklu and Aslan (2012) note that the Knowledge Quartet (KQ) framework provides studying teachers' content knowledge during instruction. Contingency knowledge encompasses the teacher's ability to responding appropriately to the events and ideas which occur in classroom during instruction. It is about contingent action of teachers in the classroom (Rowland et al, 2005). It requires an experience and connected level of knowledge on the part of the teacher to be able to do this effectively and efficiently. Novice teachers are much more likely to need to adhere to a pre-prepared plan. According to Rowland and Turner (2007) contingency knowledge has three contributory codes: (responding to students' ideas, teacher insight and deviation from agenda). There are eight-teen codes in total for four units (Rowland & Turner, 2007). Table two indicates the meaning of each of the three contributory codes.

Responding to students' ideas	This code includes the ability to make cogent, reasoned and well-informed responses to unanticipated ideas or suggestions from students
Deviation from lesson agenda	Good when the teacher displays deeper subject matter knowledge in taking a pupil's remark as a starting point for deeper enquiry, probing pupils in order to find out why a pupil comes up with a wrong answer or deviation characterized by conceptual focus.
Teacher insight	Teacher stops to reflect-in- action and changes tack (example or representation being used)

3. RESEARCH METHODOLOGY

A qualitative research approach was used to explore the contingency knowledge on number bases. Six grade seven primary school teachers were observed and interviewed. The participants were purposively sampled. The data was analysed by applying thematic analysis. Thematic analysis of lesson observations and interviews was done by applying the Subject Audit Instrument (knowledge Quartet). Trustworthiness was ensured by applying multiple data collection methods, taking notes and recording with a tape recorder during lesson observation and post-lesson observation interviews. Ethical considerations included informing participants about the purpose of the research, expected duration and procedures, participants' rights to decline to participate and to withdraw from the research whenever they felt that they no longer wanted to continue participating.

4. FINDINGS OF THE STUDY

The findings are presented according to the study objectives.

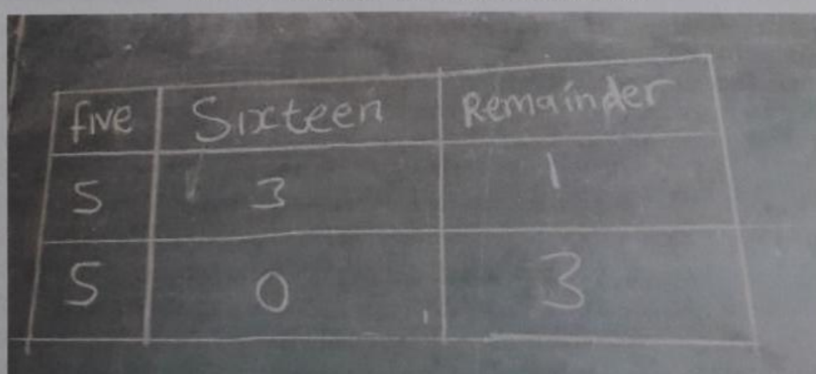
Findings from lesson observations revealed that four of out the six participants were not fully grounded with regard to contingency knowledge (***responding to students' ideas***). The following exchange by T1 shows an example of what happened in one of the lessons.

T1: [Writes 16 on the board and asks] who can change 16 from base ten to base five

Pupil Y: [Moves in front to converts 16_{ten} *to* 31_{five}*]*

Pupil Y: Carried out the conversion as follows.

Table 3: Changing 16 from base ten to base five



Pupil y: So, the answer is 31.

T1: Alright. Clap for him

Here T1 '*responds to pupils' ideas*' by simply affirming the correctness of the answer without making clarifications. Another instance, in which the contingency knowledge (*responding to students' ideas)* was not fully expressed, took place as T11 was teaching and is demonstrated in the following exchange:

T11: Yesterday we started a new topic. Who can remember what we discussed?

Px: We looked at changing from base ten to base five

T11: Yes. How many digits are in base five?

Py: 4

T11: No

T11 just dismissed wrong answers. She seemed not to care about the sources of pupils' mistakes.

An example of a sound application of contingency knowledge (*responding to pupil ideas*) took place as T12 was teaching and is demonstrated in the following exchange:

T12: [writes: Show the place value place value of 4 for the number 945].

Px: Teacher you have written the word place value twice

T12: okay [she rubs one]

She shows that her contingency knowledge (*responding to pupil ideas*) was pedagogically powerful as she could accommodate learners' views.

Another, example of a well-developed contingency knowledge (*responding to pupil ideas*) took place as T15 was teaching and is demonstrated in the following exchange:

T15: What is 8^2

Pv: 16

T15: Is he correct?

Pupils: No

T15: Who can try?

T15 did not just dismiss the answer given but intervened by re-directing. T15 way of '*responding to pupils' ideas* could enable more pupils to participate as well as allowing a pupil to correct another pupil's incorrect statement.

4.2. Teacher insight

Findings from lesson observation revealed that five out of the six participants were not fully developed in the contingency knowledge *(teacher insight)*. They did not have the ability to reflect in action and change tack of either example or representation being used during lesson enactment even if learners are not clear of the example given. An instance of such a scenario is demonstrated in the following exchange in which T12 worked out the following problem: Add 110_2 and 1011_2 as follows:

T1: To solve this question we first put the place values on top

T12: Then, we add zero and one to get 1

T12: In the next column we add one plus one to get two

T12: Since we cannot write two, we write zero and carry one

T12: And in the first column we add one and zero plus the one we carried forward. So again, we write zero and carry one forward

T12: So, this is how you work out this question

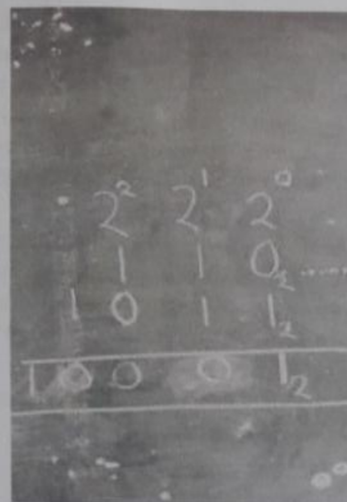


Figure 17 Working for the problem: Add 110_2 and 1011_2

After T12 had finished working out the problem, there was mixed reaction amongst the pupils:

PX: Teacher I am completely lost out

Py: can you repeat or is there another way we can use to find the answer?

T12: that is the only way….so you have no choice but to know it

Thus, we see in this scenario the teacher could not stop and reflect-in-action and consequently change tact. This shows that T12 did not exhibit a developed understanding of pedagogical knowledge '*teacher insight*'.

T17 was the only one who exhibited contingency knowledge '*teacher insight*' and this is demonstrated in the following exchange:

T5: Who can work out this problem [Writes: $321_5 + 423_5$]

Px: [Moves in front and added 1 and 3 and wrote 4].

T5: Why was it necessary to write 4 straight away without dividing?

Pt: It is because we are adding in base five and the answer four is less than five.

Here T17 demonstrated contingency knowledge (*teacher insight*) by reflecting in action through asking probing questions to try to elicit where the pupil's thinking was coming from. This was a good demonstration of contingency knowledge (*teacher insight).*

4.3. Deviation from lesson agenda

Interviews findings revealed that five out of six participants were not well grounded in contingency knowledge *(deviation from agenda)*. When faced with a challenge in most cases they failed to deviate and if they did their deviation was not characterised by conceptual focus. For instance, when asked to explain if the lesson had turned out differently from what was planned:

T13: [Yes, the lesson turned out differently from what I had planned at first, but, I did not know how to address the situation. I hope next time it would be different] and T1 [Yes, children had challenges in grouping tens problems. To adjust to suit the unanticipated changes I had to involve the pupils."].

Here T13 acknowledged that the lesson turned out differently but, failed to make appropriate pedagogical deviations despite facing challenges. Similarly, though T1 made some deviation they were not characterised by conceptual focus. T12 is the only one who demonstrated elements of contingency knowledge *(deviation from agenda)* as can be seen in the following exchange:

"First, lesson did turn out differently from how I had planned. I thought learners were 100% familiar with indices, yet they were not .... Yes, children had challenges in grouping. So, I adjusted to suit the unanticipated, ... So, I had to deviate indirectly and start explaining ......I had to bring in indices to familiarize them with indices before proceeding."

Here T12 exercised some elements of deviation based on conceptual focus to address the unanticipated changes which was a good basis for deviation.

5. DISCUSSIONS

The findings from this study indicate that participants responded to student ideas mainly in two ways: *ignore* or *acknowledge but sideline*. These findings echo (Rowland et al, 2009) suggestions that with regard to the scenarios where pupils offer unexpected responses, there are three types of responses – *ignore*; *acknowledge but sideline*; and *respond and incorporate*. Knowledge of students' ideas is held as pedagogical content knowledge (Shulman, 1986). Moreover, Ball et al (1990) argues that mathematics SMK inspires and motivates the teacher to respond proficiently to students' contingency actions such as unexpected questions and answers. Hence, considering the way most of the participants responded to pupils' ideas *(ignore*; and *acknowledge but sideline*) I can claim that their pedagogical content knowledge in the sphere of *responding to pupils' ideas* is not fully developed. Teacher educators training primary school teacher should be ensuring that primary school teachers fully acquire skills of *responding to pupils' ideas*. For instance, when teaching number bases, they should not only be paying attention to whether an answer is correct, but also to the student's alternative mathematical thinking. Besides, there is need for primary school teachers to be allowing sufficient time for students to explore responses in depth and by asking for explanation and understanding (Rowland et al, 2009). Such acts of spending time in questioning or probing pupils in order to find out why a pupil comes up with a wrong answer would really raise the standard of performance.

Findings on the nature of *teacher insight* indicate that participants could not reflect in action and change tact of either example or representation being used during lesson enactment even when faced with challenges. These findings do not resonate well with the work of Sherin (2008) who contend that one type of teaching expertise is the ability "to notice and interpret key features of classroom interaction" (p. 156).

Findings on *deviation from agenda* indicate that generally participants when faced with challenges in most cases they failed to deviate and if they did their deviation was not characterized by conceptual focus. These findings are contrary to what Rowland et al (2009) considers being a good criterion for *deviation from agenda*. That is *deviation from agenda* should be characterized by conceptual focus, presentation of the concept in a new way and illuminating with an everyday example.

6. CONCLUSIONS AND RECOMENDATIONS

The study has also revealed that in order to enhance professionalism in the teaching of number bases primary school teachers should be sensitized on how to notice and interpret key features of classroom interaction. Primary school teachers should acquire skills of stopping to reflect-in- action and change tack (example or representation being used) amidst challenges. It is therefore crucial that lecturers training primary school teachers sensitise prospective primary school teachers on these important elements of deviation from agenda.

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8.9.2 Primary school teachers subject matter knowledge and pedagogical knowledge of number bases: A case study of Lusaka province

Article

Primary School Teachers' Subject Matter Knowledge and Pedagogical Content Knowledge of Number Bases: A Case of Lusaka Province

George Mulenga, Patricia P Nalube

University of Zambia, School of Education, Zambia

gcmulegachanda@yahoo.com, pphirinalube@yahoo.co.uk

Abstract

This is a qualitative case study that explored the mathematics subject matter knowledge (SMK) and pedagogical content knowledge (PCK) of Grade 7 primary school teachers with a focus on Number Bases, a topic which consolidates the concept of place value; the basis for all Arithmetic algorithms (Wilson, 2006). Dimensions' of the Knowledge Quartet: foundation knowledge and transformation were applied to code, interpret and discuss the findings. Thematic analysis of lesson observations, questionnaire, concept maps and interviews was done by applying the pre-specified categorization scheme of the Subject Audit Instrument (knowledge Quartet). Initially thirty participants took part in the questionnaire. Out of these nineteen took part in the concept maps. Then based on the performance in the questionnaire and concept maps two high performers, two moderate performers and two low achievers were selected to take part in lesson observation and post-lesson observation interviews. The findings indicate that primary school teachers face challenges in teaching number bases mainly due to weakness in use of mathematical terminology, inability to identify pupils' errors and limited knowledge in concrete mathematical representations on number bases. The study recommends that: Lectures training prospective primary school teachers should sensitise their students on the importance of internalizing mathematical language in the context of number bases, they should also familiarize their students on various ways of concrete mathematical representations on number bases. Also primary school teachers should be familiarized with common pupils' errors in the context of number bases.

Key words: Knowledge Quartet, Foundation, Transformation

1. Introduction:

Dabiri (2003) notes that the education community recognises that subject matter content knowledge and pedagogical content knowledge form the basis for effective teaching. Therefore, mastery of the subject by all teachers is an absolute necessity for effective teaching and they must possess a basic qualification in the subject (Sidhu, 2013). Nonetheless, a base line study conducted at Chalimbana University in 2014 revealed that majority of the in-service primary school teachers considered the topic of Number bases to be mystifying. They had challenges with teaching the mathematics topic of Number Bases. Table 1 indicates number of in-service primary school teachers who had difficulties with some topics.

Table 1: *Number of in-service primary school teachers who had difficulties with some topics*

Topic	**Number of in-service primary school teachers who considered a particular topic to be difficult (Out of 300)**	**Percentages (%)**
Number bases	216	72
Number line	53	19
Fractions	21	7
Long Division	15	5
Angles	9	3
Other topics	12	4

As can be seen in Table 1, most of the in-service primary school teachers who took part in the survey considered the topic of number bases to be very difficult to teach. Majority of the primary school teachers interviewed expressed keen interest in a well- researched guidance about how they could improve their own SMK and PCK in the area of Number Bases in an effort to enhance their pupils' standard of achievement and performance. Definitely, such a research would be in step with the American National Council for Curriculum and Assessment (2006: 29) consultation document which indicates that "students need their best teachers at a young age, teachers who really know what they are doing and really understand the simplicity of what they are doing.

1.1 Statement of the Problem

This research was based around the need to enhance the teaching and learning of the central topic of number bases. It emanated as a result of the base line study conducted at NISTCOL in 2014 by the College Lectures in the mathematics department amongst in-service primary school teachers who came from all parts of the country. The base line study revealed that, for some time several in-service primary school teachers have been facing challenges with regard to teaching the concept of number bases. This situation is very unfortunate especially in the light of Hodgen (2011) who posit that students need their best teachers at a young age. That is teachers who really understand the simplest of what they are doing. According to Dabiri (2003), to produce such proficient teachers, researchers and teacher educators need to device effective preparation and support programmes for teachers. Teachers need more support in knowing how to reach diverse students' through their teaching career during preparation program, particularly during their induction period. Briton et al (2000; 57) note that, "What we so often do is work more on describing a problem than actually trying to solve it". "In fact, we are very good at describing problems, very good especially in describing the achievement gaps". "We understand this gap in lots of different ways, with increasing statistical power and sometimes with even nicer slides. In other words, we have increasingly elegant solutions that do not have a chance of happening except in the most unusual or special problem". It is in this regard that by conducting this study we opt not to be mere spectators but to be part of problem solvers to what is happening to the teaching and learning of mathematics, especially with regard to the mathematical topic of number bases, in most primary schools in Zambia. To our knowledge, very little is known on the primary school teachers' subject matter content knowledge and pedagogical content knowledge on number bases.

1.2 Purpose of the Study

The purpose of this qualitative study was to explore the mathematics Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK) of Grade 7 primary school teachers with a focus on number bases: identifying contributing factors why Primary school teachers face challenges with regard to teaching the concept of Number bases.

1.3 Objectives of the Study

The objectives of this study were as follows:

1. To determine primary school teachers foundation knowledge on number bases
2. To assess the primary school teachers capacity to transform the content knowledge they possess into forms that are pedagogically convenient to their pupils when teaching number bases

2. Methodology

A qualitative research approach was used to explore the mathematics Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK) of Grade 7 primary school teachers with a focus on the topic of number bases. The case study design was used. The application of case study design in this study helped to provide a deep and logical way of exploring the SMK and PCK of Grade 7 primary school teachers in the context of number bases. The target population included all Grade 7 primary school teachers teaching in primary schools in Zambia. A sample of thirty Grade 7 primary school teachers, selected from eight primary schools took part in the study. Purposive sampling was used to select the eight primary schools. Each of the selected primary school had at least three grade seven teachers. School A, B, C, D, E, and F had four Grade 7 teachers each, while school G and H had three Grade 7 teachers each. Thus, eight schools in total were selected with a total number of thirty participants. Out of the thirty primary school teachers who took part in the questionnaire and concept map a subset consisting of six primary school teachers was selected. These six are the ones who were observed and interviewed. The criteria for the selection of these six primary school teachers depended on their performance in the questionnaire and concept maps. The first two were the high achievers, the next two were the middle achievers and the last two were the low achievers. This was done in order to come up with a heterogeneous group that would take part in the lesson observations and interviews; a group whose proportional composition would mirror the society. Multiple methods of data collection were applied in this study, namely: concept maps, questionnaires, lesson observations and interviews. The data was analysed by applying thematic analysis. Thematic analysis of lesson observations, questionnaire, concept maps and interviews was done by applying the Subject Audit Instrument (knowledge Quartet). To insure trustworthiness for my study I used multiple data collection methods: Lesson observations, Concept maps, questionnaires and Post lesson observation interviews and took notes and recorded with a tape recorder during lesson observation and post-lesson observation interviews. Ethical considerations included informing participants about the purpose of the research, expected duration and procedures, participants' rights to decline to participate and to withdraw from the research whenever they felt that they no longer wanted to continue participating.

3. Findings

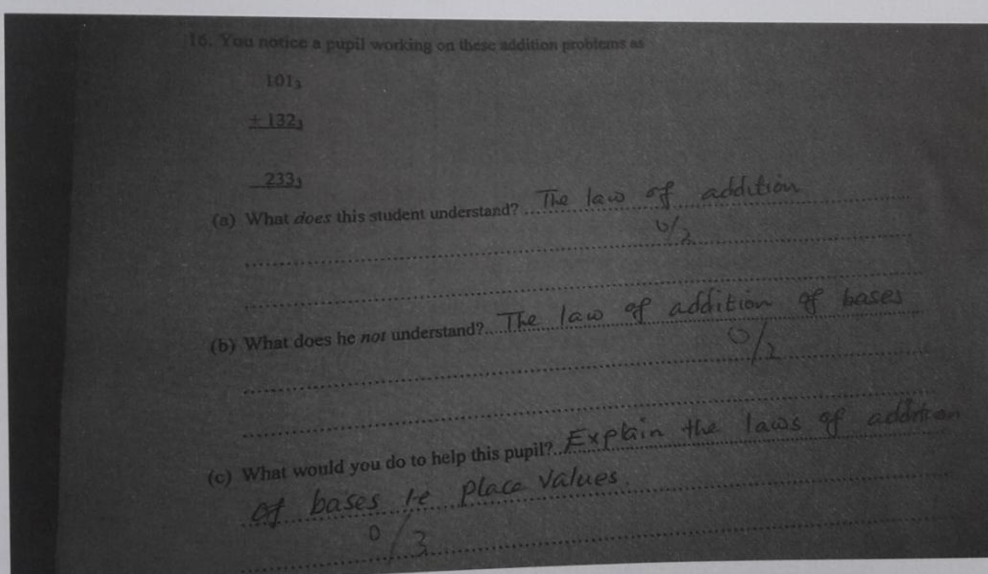
Findings of the study are presented according to the research questions. This first part gives findings on primary school teachers' foundation knowledge in the context of number bases and the second part gives findings on primary school teachers' transformation knowledge in the context of number bases.

3.1 What is the Primary School Teachers' Foundation Knowledge of Number Bases?

3.1.1 Use of Mathematical Terminology

Five out of the six participants observed had weak foundation knowledge on use of mathematical terminology. The following exchange shows what happened in the case of T1: After expanding 2222 to obtain 2^3, 2^2, 2^1, 2^0. T1then said: "*We refer to this expanded form as place value.*"Clearly, the teacher failed to state what a teacher with sound knowledge of mathematical terms ought to state. For T1 had merely expressed the numbers in index form.

3.1.2 Identifying Pupils' Errors


16. You notice a pupil working on these addition problems as

101_3
$+ 132_3$
233_3

(a) What *does* this student understand? The law of addition 0/2

(b) What does he *not* understand? The law of addition of bases 0/2

(c) What would you do to help this pupil? Explain the laws of addition of bases i.e. place values 0/3

As can be seen on the sample response the participant lacked the foundation knowledge necessary to identify and address such errors.

3.1.3 Concentration on Procedure

Findings, from the classroom observation indicated that all the teachers focused on the algorithmic steps to reaching an answer. For example, T2

> T2: Writes 13 zeroes [0 0 0 0 0 0 0 0 0 0 0 0 0]. Then asks pupils: "How many groups of five can be formed from 13?"

Px: Two
T2: What is the remainder?
Py: Three
T2: Therefore, the answer is 23

As can be seen T2 focuses on the algorithmic steps to reaching an answer. Hence, only focusing on instrumental way of understanding number bases.

3.2 How Do Primary School Teachers Transform the Content Knowledge They Possess into Forms that Are Pedagogically Convenient to Their Pupils When Teaching Number Bases? This question was answered by applying the transformation codes: (Choice of examples and Choice of representations)

3.2.1 Choice of Examples

With regard to choice of examples all the participants performed well. For instance, T6 gave the following two examples she provided on addition in base five: Example 1: Work out $141_5 + 201_5$ Example 2: Work out $314_5 + 123_5$.The choice of these examples was good as can be seen for the first example the sum of all pairs is less than five but in the second example the sum of pairs is more than five (the base under consideration). This could help learners to master when to divide and the examples ere progressively becoming more complex.

3.2.2 Choice of Representations

With regard to Choice of representations, findings from Lesson observations revealed that the primary school teachers who took part in lesson observations exhibited knowledge of various forms of mathematical representations such as, tabular and symbolic, tally mark, though only one participant demonstrated concrete mathematical representations.

Example for tabular representation

Number 15	Base	Remember
15	2	1
7	2	1
3	2	1
1	2	1
0	2	1

An example of symbolic representation

T2: Writes 17 zeroes. [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0].

Then she said we have two groups of eight and remainder ……..

foundation knowledge (*identifying pupils' errors*) evidenced in this study as well as in view of Schoenfeld and Kilpatrick (2008) assertions, prospective primary school teachers should be trained to be creating situations in which pupils tell their errors so that teachers are able to systematically correct them.

4.1.3 Concentration on Procedure

Results indicate that all the observed participants applied instrumental approach to teaching mathematics with little or no relational way of understanding number bases mathematics. This means that they focused on the algorithmic steps to reaching an answer without understanding why those rules and procedures work. These findings resonate with Kelly (1991) who contend that most primary school teachers emphasize drilling pupils in factual knowledge and that promotes passive acceptance and reproduction of information as the ideal. Besides, these findings are consistent with Schwartz and Riedesel (1994) study which focused on primary school teachers teaching of long division. Their study revealed although the primary school teachers knew the algorithm for long division, they did not make out why the procedures worked. This finding is a sad one especially in the light of Aydın and Soylu (2006) assertions that when conceptual knowledge and procedural knowledge are not equilibrated, students do not have a full facility with the topics. Besides, without conceptual knowledge, students cannot comprehend the significance of mathematical concepts and related procedures (Lamon, 1999). Thus, the concentration on procedure revealed in this study may inhibit learners in primary schools from acquiring a solid grasp of number bases. Possibly this could be a reason why most primary school pupils in Zambia perform poorly as evidenced in Chapter one on page five. One implication of this finding is that, educational standards officers should ensure that primary school teachers are acquainted with and apply both instrumental and relational approaches to understanding mathematics. Such measures would capacitate or enhance primary school teachers' ability to follow and assess mathematical reasoning.

4.2 Discussion of Question Two

We now discuss question two which explored the primary school teachers' transformation knowledge along the following codes: Choice of examples, Use of instructional materials, and Choice of representation.

4.2.1 Choice of Examples

Findings on this theme (choice of examples) indicate that participants had strong transformation knowledge of choice of examples. These findings are in line with what Rowland et al (2009) considers being a good criterion for *choice of examples*. That is *choice of examples* should be high cognitive demand, mathematically correct, realistic context and/or numbers; no misconceptions, and numbers are selected intentionally (Rowland et al, 2009). Therefore, it is important for primary school teachers to continue providing examples that become progressively more challenging.

4.2.2 Choice of Representation:

Findings on this theme (*choice of representation)* indicate that participants exhibited knowledge of various forms of mathematical representations. These findings resonate with those of (Hill et al., 2008b; Hill, Rowan & Ball, 2005) who contend that various forms of forms of representation such as diagrams, graphical displays, and symbolic expressions have long been part of school mathematics. These findings are very good especially in the light of Fennema

- *Pv: shouts [one]*
- *T2: So the answer is 21 in base eight. Who can come and use the other method?*

Surprisingly despite, exhibiting knowledge of various forms of mathematical representations, only one participant demonstrated concrete mathematical representations.

4. Discussions

The discussion is guided by the following research questions:

1. What is the primary school teachers' foundation knowledge of Number base?
2. How do primary school teachers transform the content knowledge they possess into forms that are pedagogically convenient to their pupils when teaching Number Bases?

4.1 Discussion of Question One.

I have discussed question one which was used to explore the primary school teachers' foundation knowledge along the following codes: Identifying pupil errors, Use of mathematical terminology, and Concentration on procedures.

4.1.1 Use of Mathematical Terminology

On this theme *(use of Mathematical Terminology)* findings from classroom observations, and interviews both revealed that there is a considerable lack of *use of mathematical terminology* in the context of teaching of number bases, on the part of the primary school teachers. My study findings are consistent with (Ball, 1988) study conducted amongst pre-service primary teachers which revealed that many prospective primary school teachers did not have an explicit understanding of mathematical terms such as place value. Poor usage of mathematical terminologies amongst primary school teachers is likely to be transmitted to their pupils (Ball, 1988). Therefore, in line with these findings, perhaps there is need for lectures training prospective primary school teachers to sensitize their students on the importance of internalizing mathematical language in the context of number bases. This might help primary school teachers to acquire knowledge of the correct usage of number bases terms and their exact meanings.

4.1.2 Identifying Pupils' Errors

Findings on this theme *(identifying pupils' errors)* revealed that primary school teachers commit several types of errors in the context of number bases. The challenges by most primary school teachers in identifying pupils' errors could be as a result lack of overt display of subject knowledge (Booker, 1989). These findings support Ball (1988) study which involved nineteen pre-service primary school teachers. Ball (1988) study revealed that most primary school teachers are unable to find and explain the errors in children's mathematics problems. Also, these findings add credence to (Booker, 1989) study which investigated common errors pupils in primary school grades commit. Schoenfeld and Kilpatrick (2008) note that one of the qualities of proficient teachers of mathematics is the ability to use their knowledge of mathematics in ways that provide the tools to instill understanding and help pupils with mathematical misunderstandings. Thus in line with the primary school teacher deficiencies in and Franke (1992: 153) who have emphasized the importance of knowledge of mathematical representations. They have argued that 'mathematics is a composition of a large set of highly related abstractions and if teachers do not know how to translate those abstractions into a form that enables learners to relate the mathematics to what they already know, they will not learn with understanding'(Fennema and Franke, 1992: 153). Therefore, teacher educators should continue sensitizing their learners on the important role knowledge of mathematical representations plays in the provision of quality instruction through enabling the choice of appropriate representations.

5. Conclusion

Primary teachers face challenges in teaching number bases mainly due to weakness in *use of mathematical terminology, inability to Identifying pupils' errors and limited knowledge in concrete mathematical representations on number bases.*

6. Recommendations

Lectures training prospective primary school teachers should be sensitizing their students on the importance of internalizing mathematical language in the context of number bases. This will help primary school teachers to acquire knowledge of the correct usage of number bases terms and their exact meanings. Lectures training prospective primary school teachers should also familiarize their student on various ways of *concrete mathematical representations on number* bases. This would promote both Instrumental and Relational understanding of number bases.

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