

**MATHEMATICS TEACHERS' PEDAGOGICAL PRACTICES FOR LEARNERS'
PROFICIENCY IN ALGEBRAIC CONCEPTS: A CASE OF SELECTED SECONDARY
SCHOOLS IN LIVINGSTONE DISTRICT, ZAMBIA**

BY

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**A Dissertation submitted to the University of Zambia in fulfillment of the requirements for
the award of the degree of Master of Education in Mathematics Education.**

THE UNIVERSITY OF ZAMBIA

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DECLARATION

I, **Stanslous Malubila**, do hereby declare that this dissertation is my own work and the works of other people have been appropriately acknowledged. I further declare that this work has never in part or whole been submitted to the University of Zambia or any other institution for the award of any academic qualification.

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APPROVAL

This dissertation by Stanslous Malubila is approved as fulfilling the requirements for the award of the Master of Education Degree in Mathematics Education by the University of Zambia.

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ABSTRACT

This was a qualitative case study which investigated mathematics teachers' pedagogical practices in the context of learners' mathematical proficiency with a focus on selected algebraic concepts. Three secondary schools were purposively selected as well as four mathematics teachers who participated in the study. Further, six learners were purposively selected from each of the four teachers' classes giving a total of twenty-four learners. Observation and semi-structured interview schedules were used to collect data preceded by document analysis of lesson plans, schemes of work and records of work. Data from observation and documents were analysed using a rubric by Christiansen and Ally relating to mathematical proficiency while data from interviews was analysed thematically. The findings were that most of the mathematics teachers did not adequately prepare for learners' proficiency in line with the strands for mathematical proficiency espoused by Kilpatrick, Swafford, and Findell (2002). Most predominant among the practices of teachers were aspects of revising prerequisite topics and giving the standard definition of concepts. While these were necessary, they did not adequately provide for learners' mathematical proficiency. Learners had challenges to solve quadratic equations using completing the square method and solving questions that involved real life situations. It would appear that they lacked conceptual understanding. It is recommended that Senior Education Standards Officers should routinely organise Continuous Professional Development (CPD) trainings to empower mathematics teachers with adequate knowledge on how to teach algebra, especially the topics like 'Quadratic Functions and Equations' and how to effectively use visual teaching aids. Through CPD trainings, mathematics teachers should be enlightened on the disadvantages of their learners using the pseudo-methods when solving equations.

DEDICATION

I sincerely dedicate this study to my Parents Mr. and Mrs. Malubila. Their genuine love, unwavering support and belief in my ability have always spurred me on. I also dedicate this work to my sisters Ireen and Barbara for coming in to assist whenever I fell short.

And to my dear wife Maggie, for her love and support which inspired me to reach this far.

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ABBREVIATIONS AND ACRONYMS

CDC	Curriculum Development Centre
ECZ	Examinations Council of Zambia
HOD	Head of Department
MKT	Mathematical Knowledge for Teaching
MOE	Ministry of Education
MoGE	Ministry of General Education
MP	Mathematical Proficiency
NAEP	National Assessment of Educational Progress
NCTM	The National Council of Teachers of Mathematics
OTDAR	Opportunity to Develop Adaptive Reasoning
OTDCU	Opportunity to Develop Conceptual Understanding
OTDMP	Opportunities to Develop Mathematical Proficiency
OTDPD	Opportunity to Develop Productive Disposition
OTDPF	Opportunity to Develop Procedural Fluency
OTDSC	Opportunity to Develop Strategic Competence
PCK	Pedagogical Content Knowledge
SESO	Senior Education Standards Officer
U. S	United States
ZPD	Zone of Proximal Development

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CHAPTER ONE

INTRODUCTION

1.1 Overview

This chapter contains the background of the study, statement of the problem, the purpose of the study, research objectives and research questions. Furthermore, it contains the significance of the study, a theoretical framework, the conceptual framework, operational definitions of key terms, delimitations, limitations, ethical considerations, rigour or trustworthiness and the structure of the dissertation.

1.2 Background

Today the world is increasingly becoming advanced in terms of economy, technology and science, and mathematics remains the bedrock in the study of these disciplines. Mathematics has a vital role in the classroom not only because of direct application of the syllabus material but also the reasoning processes that learners can develop. The problem of knowledge for teaching mathematics is of growing concern to educators and researchers in ongoing efforts to improve mathematics education. One of the greatest mathematicians of all time, Carl Frederick Gauss, expressed his sentiments through a famous statement, “Mathematics is the queen of the subjects” (Dudley, 2014). To seek a plausible justification to such a claim of mathematicians to the field is by no means straightforward. What actually constitutes mathematics is a matter of elaborate elucidation and even the best accounts fall short of a completely acceptable answer (Franke, & Kazemi, 2001).

In the United States, it has been observed that elementary and middle school teachers possess a limited knowledge of mathematics, including the mathematics they teach (Bowen, 2011). The mathematical education received, both as K-12 learners and in teacher preparation, has not provided them with appropriate or sufficient opportunities to learn mathematics. As a result of that education, teachers may know the facts and procedures that they teach but often have a relatively weak understanding of the conceptual basis for that knowledge (Christou, Vosniadou, & Vamvakoussi, 2007). Many have difficulty clarifying mathematical ideas or solving problems that involve more than routine calculations. For instance, teachers can multiply multi-digit numbers,

but several researchers have found that many prospective and practicing elementary school teachers cannot explain the basis for multi-digit multiplication using place-value concepts and the underlying properties for adding and multiplying (Stone, Corinne, Donna, Morgan, & Susan, 2006)

Mathematics is one of the core subjects in the Zambian curriculum for both the academic and practical career pathways. In a study carried out by Changwe (2017), he found out that mathematics is taught in schools in order to build up understanding and appreciation of mathematical concepts and computational skills in learners so that they can apply them in other subject areas and everyday life. According to Koji, Mulenga, and Mukuka (2016), mathematics develops a person's competence in logical reasoning, spatial visualisation, analysis and abstract thought. Roitman (1981) states that mathematics forms the basis under which science and technology are based. Therefore, there is greater pressure on learners to succeed in mathematics than in other subjects.

Although mathematics is one of the most important subjects in the Zambian curriculum, the performance of learners has been poor in the past years. This scenario can be attributed to various factors. The study on factors contributing to poor performance in mathematics undertaken by Michael (2015) revealed that poor performance of learners in mathematics is due to overcrowded classes, negative attitudes, beliefs by learners towards mathematics and lack of appropriate teaching and learning materials among others. Other studies have revealed that poor performance of learners in mathematics is also caused by teachers' qualifications and methods, ill prepared teachers, teachers' poor attitudes and lack of readiness to teach appropriately (Changwe, 2017; Okafor & Anaduaka, 2013). According to Kelly (1991) one third of boys and two third of girls have registered total failure in mathematics since 1987 while only half of the boys and one fifth of the girls have obtained an equivalent of an O-level pass in mathematics. In addition, the performance of learners who proceed to higher institutions of learning for science and mathematics programs is mixed. Some learners do extremely well, while the performance of the majority is average and a sizeable proportion cannot cope.

In the past most secondary school mathematics teachers in Zambia were not suitably qualified. Majority of these teachers in secondary schools had diplomas. The government came up with various programs such as BEDMAS and fast-track in order to generate large numbers of graduate teachers within a short period of time (MOE, 2010). However, the number of teachers holding

university degrees has been steadily increasing in secondary schools but this seems not to have improved the performance of learners in mathematics to the desired outcome (MoGE, 2016). This is because the quality of teachers produced as a result of teacher education curricula lacked knowledge and skills necessary for effective teaching (MoGE,2016). Mathematics comprises many branches like arithmetic, algebra and geometry among others.

Algebra is of particular importance since it functions as a basis for later mathematics courses. Thus, algebra is fundamental in all areas of mathematics because it provides tools or the language for representing and analysing quantitative relationships, for modeling structure, for solving problems and for stating and proving generalisations. This implies that algebra permeates all areas of mathematics and there is no way a learner can easily pass mathematics without first being proficient in algebraic concept (Dudley, 2014).

The Examinations Council of Zambia has registered repeated failure in algebra at grade twelve final examinations (ECZ, 2018). The chief examiner's annual reports on mathematics in the Joint Examinations for the School Certificate and General Certificate of Education Ordinary Level have revealed that most learners perform poorly in algebra regardless of measures put in place and this is mainly attributed to inadequate pre-requisite knowledge. Generally, learners face various problems when solving questions in algebra which include difficulties in grouping like terms, difficulties in manipulating algebraic signs and symbols and that teachers were too fast in explaining algebraic concepts (Koji, Mulenga & Mukuka 2016). As a result, mathematics teachers are now desperately struggling with learners' lack of algebraic proficiency. The knowledge of algebra for most learners is deficient and not deep-rooted. However, most learners are expected to have high ability skill in handling algebra.

1.3 Statement of the Problem

The problem of poor performance in algebra is experienced in many countries the world over. For instance, educators in secondary schools and higher institutions of Netherlands have complained of learners' failure to solve simple problems to do with linear equation (Van Stiphout, Drijvers & Gravemeijer, 2013).

In Zambia, the overall performance of pupils in school certificate is generally unsatisfactory and this can largely be attributed to poor performance in mathematics and science subjects. However,

the poor performance in mathematics is partly due to lack of proficiency in algebra which is exhibited in the inability of learners to master basic concepts in algebra. Koji et al.(2016) investigated the challenges faced by secondary school teachers and pupils in the teaching and learning of algebraic linear equations respectively in selected schools of Mufulira district. The study revealed that some learners were unable to solve algebraic linear equations due to lack of pre-requisite knowledge, lack of conceptual understanding, procedural knowledge and skills required for solving algebraic linear equations and inappropriate methods used in teaching of algebraic linear equations. Such as grouping like terms and manipulating algebraic symbols and signs (ECZ, 2018). However, some of the learners who have passed mathematics and proceeded to institutions of higher learning to pursue mathematics programmes tend to have problems in mathematics related courses (Kelly, 1991). Therefore, in Zambia, the repeated failure in algebra at grade twelve final examinations has been a source of concern to mathematics teachers and educators for some years now (ECZ, 2018). This study assessed how mathematics teachers' pedagogical practices affect the learners' proficiency in algebraic concepts.

1.4 General Objective

Assessment of mathematics teachers' pedagogical practices for learners' proficiency in algebraic concepts.

1.5 Specific Objectives

Below are the three main specific objectives that the researcher came up with and which guided him throughout the study.

- (i) To ascertain teachers' ability to prepare their lesson plans for learners' proficiency in algebraic concepts.
- (ii) To identify mathematics teachers' pedagogical practices when teaching for learner proficiency in algebraic concepts.
- (iii) To determine learners' algebraic conceptual difficulties despite teachers' pedagogical practices for algebraic proficiency.

1.6 The Research Questions

Below are the three main research questions that the researcher answered after interpretation of the findings.

- (i) How do teachers of mathematics prepare their lesson plans for attainment of learner proficiency in algebraic concepts?
- (ii) What pedagogical practices do teachers of mathematics engage in as they teach for learners' attainment of proficiency in algebraic concepts?
- (iii) What algebraic conceptual difficulties do learners exhibit despite teachers' pedagogical practices for algebraic proficiency?

1.7 The Significance of the Study

The findings of the study are significant in that they may provide insight to mathematics educationists on pedagogical practices of teachers and how these could be used to come up with appropriate practices to uplift the algebraic proficiency of learners. These findings may also provide knowledge to teacher educators and teachers on the best practices that would influence learners' algebraic proficiency. The findings may also add to the body of literature on mathematics education.

1.8 The Theoretical Framework of the Study

The study was informed by the ideas of Piaget (1969) and Vygotsky (1978) on the revolutionary progressive educational theories on learning. According to the revolutionary progressive educational theories on learning and teaching, modern educationists today are deviating from the traditional behaviourist approach to constructivist approach. According to Vygotsky, the traditional approach perceived the mind as a blank slate that had to be fed with knowledge. However, the constructivist learning theory states that all knowledge is constructed from a base of prior knowledge. Children are not a blank slate and that knowledge cannot be imparted without the child making sense of it according to his or her current conceptions. At the heart of constructivist philosophy is the belief that knowledge is not given but gained through real experiences that have purpose and meaning to the learner and the exchange of perspectives about the experience with others (Piaget, 1969).

In order for teachers to teach mathematics for understanding, they should ensure that learners learn by doing and not as passive recipients of already-made mathematics. They are supposed to be treated as active participants in the learning process in which they themselves, with the guidance from the teacher, develop all sorts of mathematical tools as well as insight. In most pedagogies based on constructivism, the teacher's role is not only to observe and assess but also engage with the students while they are completing activities, wondering aloud and posing questions to the learners for promotion of reasoning (Van Stiphout et al., 2013). Thus, constructivism acknowledges the learner's active role in the personal creation of knowledge, the importance of experience (both individual and social) in this knowledge creation process, and the realisation that the knowledge created will vary in its degree of validity as an accurate representation of reality. Constructivism is divided into three broad categories: cognitive constructivism, radical constructivism and social constructivism. However, this study adopted social constructivism because it emphasises interactive learning or collaborative learning which is the basis for the revolutionary progressive learning method that puts the learner at the centre of learning process.

Social constructivism is a variety of cognitive constructivism that emphasizes the collaborative nature of much learning. Social constructivism was developed by post-revolutionary soviet psychologist Lee Vygotsky. Vygotsky was a cognitivist but rejected the assumption made by cognitivists like Piaget and Perry that it was possible to separate learning from its social context. He argued that all cognitive functions originate in, and must therefore be explained as products of social interactions and that learning must not be simply the assimilation and accommodation of new knowledge by learners. It must be a process by which learning integrated into knowledge community. According to Vygotsky (1978), every function in the child's culture development appears twice, first, on the social level and, later on the individual level, first between people and then inside the child. Social constructivism, structure the researcher decided to adopt this in the study. Adittionally since social constructivism deals with social interactions that are collaborative as learners learn, it was appropriate because the study dealt with pedagogical practices at the social level and also at the individual level of the learner. Unlike cognitive and radical constructivism which emphasises all four of the epistemological tenets. Like radical constructivism, social constructivism would be considered a "strong" form of constructivism.

Below is the Constructivist Theoretical Framework proposed by Vygotsky (1978). The theoretical framework informed the study by providing the main structure of theories of learning that the research study is based on.

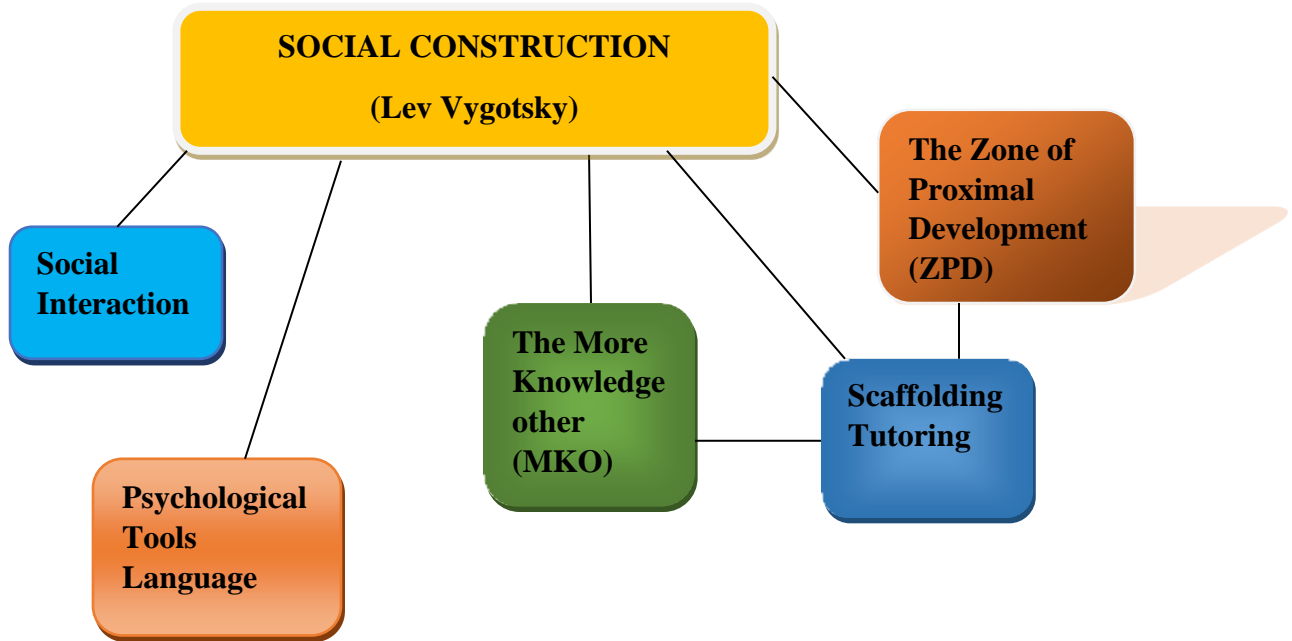


Figure 1: Model of Social Constructivist Theoretical Framework on Learning and Teaching by Vygotsky (1978)

Source: (Pinterest) 2020

1.9 The Conceptual Framework

A Conceptual Framework represents a synthesis of literature on how to explain a phenomenon (Creswell, 2007). According to Creswell, Hanson and Clark (2007), a conceptual framework is a plan that shows or explains, either diagrammatically or in narrative form, the main things to be studied. It spells out the key factors, concepts or variables to use and the presumed relationship among them.

After reviewing the literature on conceptual frameworks of other researchers and adopting some concepts from the theoretical framework the researcher designed his own conceptual framework through which he answered the research questions.

The study was influenced by the works of several scholars, firstly, the researcher looked at Hill, Ball, and Schilling (2008) on how mathematical knowledge for teaching could influence how teachers prepare for learner proficiency in algebraic concepts. Hill, et al. (2008) following Shulman's conceptualisation of Pedagogical Content Knowledge (PCK), came up with the notion of Mathematical Knowledge for Teaching (MKT) in which the knowledge for teaching mathematics was also considered. Haas (2005) in their research on the practices of teaching for algebraic proficiency came up with a list of practices that he recommended would enhance algebraic proficiency. He recommended direct instruction, problem-based learning and multiple representations as tools to incorporate in any lesson. Also of importance is Schoenfeld (2008) who produced a different list of requirements for proficiency in learning mathematics, but not directly linked to traditional teaching of mathematics. It included general competencies such as crafting and managing learning environment, developing classroom norms, and supporting classroom discourse as part of teaching for understanding. Learner Proficiency in Algebra and the meaning of algebraic concepts from Kilpatrick's mathematical proficiency framework, based on the five strands of mathematical proficiency, helped answer questions on how the teaching practices provided the opportunity to develop algebraic proficiency in learners and in answering research question 3 on the difficulties learners have in learning algebra respectively. The socio-cultural theory by Vygostky was adopted as the general theory that supported the study. This theory advocates that learners need to construct their own understanding of each mathematical concept. The primary role of teaching is not to teach, explain or otherwise try to transfer mathematical knowledge but create situations for learners that will foster their making of necessary mental constructions.

Figure 2 shows the Conceptual Framework that the researcher came up with after adopting from other researchers' conceptual frameworks and the theoretical framework.

The Conceptual Framework diagram below shows how the algebraic proficient levels of learners are influenced by teachers' pedagogical practices on preparation and teaching. It also shows how teachers' pedagogical practices on preparation and teaching are influenced by teachers' mathematical knowledge for teaching and knowledge for algebraic proficiency and algebra. The intervening factors, like the standard of schools researched on and the qualification of teachers, will also have a bearing on the quality of preparation and teaching.

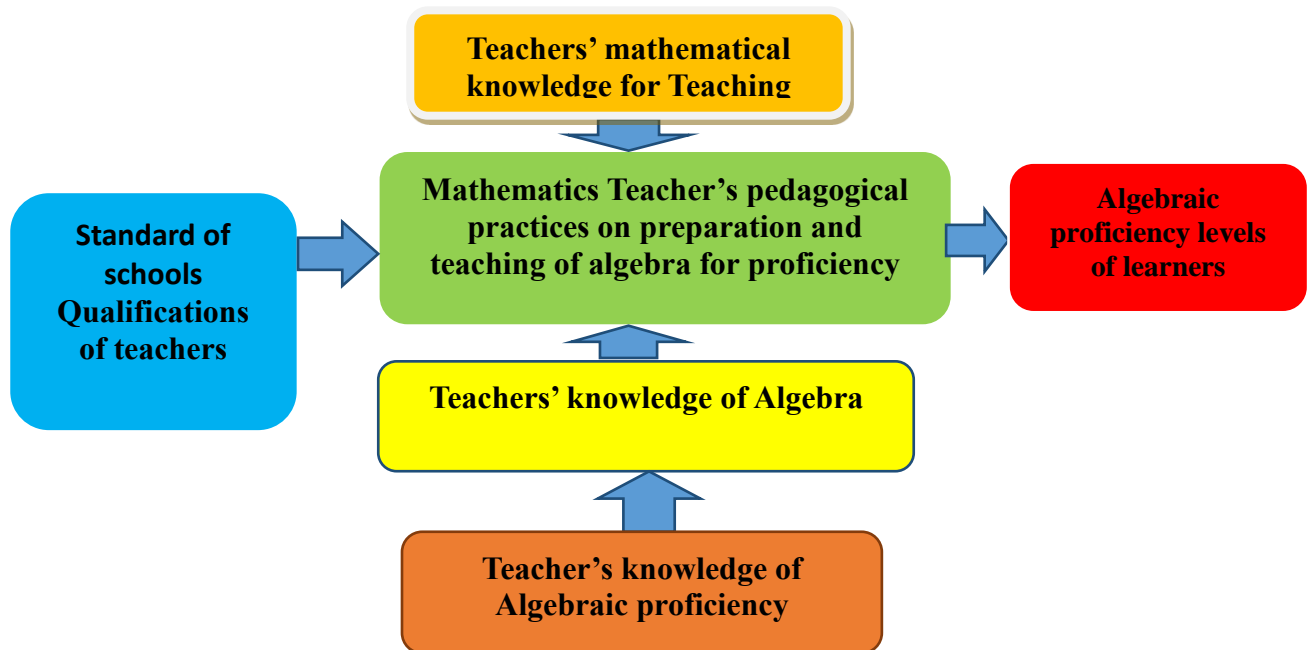


Figure 2: Conceptual Framework

Source: Author (2020)

1.10 Operational Definitions of Terms

Algebraic Proficiency:	refers to having skill to correctly and efficiently carry out an algebraic task with full understanding (Van Stiphout et al. 2013).
Lesson Plan:	refers to the primary organisational structure of the instructional process (Yara & Otieno, 2010).
Mathematical Proficiency:	refers to having skill to correctly and efficiently carry out a mathematical task with full understanding (Killpartrick, Swafford & Findell.,2001).
Meta-analysis:	this refers to a procedure by which researchers combine data from multiple studies of other researchers to come with findings that are more credible (Walker, Hernandez, & Kattan, 2008).
Pedagogical practices:	refers to various types of tasks, ways of working or types of activities and practices, which guide effective teaching and learning. Such practices include preparing well in advance schemes of work, lesson plans, lesson notes, teaching aids, prompt setting of written and practical exercises, evaluation of all written and practical exercises, provision of feedback to learners on assessments, and undertaking of remedial teaching among others (Massey, 2013).
Pedagogy:	refers to the art of teaching (Richardson, 2003).
Pseudo- method:	refers to a sub-standard method that does not conform to the stipulated norms of working (Dudley, 2014).

1.11 The Structure of the Dissertation

In this section of the dissertation the researcher has described the outline of the entire dissertation from chapter one to six.

Chapter One introduced the study by giving a brief history of the problem at hand. The problem was discussed from the international level to the national level. In the second section, the 'statement of the problem' was clearly given and explained problem that prompted the researcher to carry out the study. The next section is the 'General Objectives' and in it the broad aims of the research that gave the researcher the general direction of how to go about the study. The next section consisted of 'Specific Objectives' that gave the researcher how to go about the study. In the next section the corresponding research questions were given which were a mirror of the specific objectives and are the questions that the researcher answered in the study. The next section is the 'Significance of the study' and in it the researcher explained why the findings were important to the study. The next section was the 'Theoretical Framework' and in it the researcher explained the formal theories that guided the study. The section of 'Conceptual Framework' followed and the researcher used it as an analytical tool to explain the concepts in the study. The section that followed was the 'Operational Definitions of key Terms' and in it the key terms used in the study were clarified.

Chapter Two is a review of Literature and in it the researcher reviewed the literature by surveying books, scholarly articles and any other sources relevant to his area of research and by so doing he provided a description and critical evaluation of these works in relation to the research problems being investigated and identified the gaps in the literature.

Chapter Three presents the 'Research Methodology' and in it the researcher explained the research approach and design that he used and justified their use. He also gave the population from which he derived the sample and the sampling procedure that he used and justified their use. He also described the three instruments that he used to collect data and the procedure he used to analyse data. The 'Delimitation' section was then given and in it the researcher stipulated the scope or where the research was confined to. The 'Limitation' section followed in it the researcher outlined the constraints that he encountered in the study. The next section was the 'Ethical Considerations' and in it the researcher dealt with the sensitivity of the rights and dignity of others that were

involved in the research study. The last section was the ‘‘Rigour or Trustworthiness’ of the Study and in it the researcher explained how credibility and validity was ensured in the study.

Chapter Four presents the ‘Findings’ of the study and in it the researcher clearly and systemically related the findings from all the three instruments namely document analysis, interviews and observation.

Chapter Five is the ‘Discussion of the Findings’ and in it the findings were discussed in relation to the reviewed literature and the theoretical framework.

Chapter Six gives the ‘Conclusions’, ‘Recommendations’ and ‘Future Research’ and in it the researcher clearly gave the conclusion, then listed the recommendations and finally suggested the future research.

1.12 Summary of Chapter One

In chapter one, the brief history of the problem of low-level algebraic proficiency was discussed from an international level in the United States to a national perspective in Zambia. The problem at international level in the Netherlands and national level was then briefly explained. The general objective was also stated as a guide to the general direction of the study. The three specific objectives that looked at teachers’ preparation, teaching and difficulties learners encounter in algebra followed, and were a specific guide to the direction of the study. The corresponding research questions were listed and were the questions that the study answered. The significance of the study to mathematics educationist, policy makers, teacher educators and teachers were highlighted. The theoretical framework provided the theory of learning on which to base the study as social constructivism. The researcher came up with his conceptual framework that looked at how algebraic proficiency levels of learners are affected by teachers’ pedagogical practices from reviewing literature and from theoretical framework. The operational definitions of key terms were given to clarify the key terms used in the study. Finally, the outline of the whole dissertation was given chapter by chapter.

CHAPTER TWO

LITERATURE REVIEW

2.1 Overview

In this chapter, the researcher critically reviewed the relevant literature related to the study. He firstly looked at what other researchers and scholars say on the 'Meaning of Algebraic Concepts in Mathematics' and then 'Teachers' Pedagogical Practices of Preparing for Learners' as they are engaged in teaching mathematics. He also looked at literature on 'Pedagogical Practices of Direct Teaching of Learners' and he also reviewed literature on 'Pedagogical Practices on Indirect Teaching of Algebra' and lastly he reviewed literature on 'Learner Proficiency in Algebra'. Literature Review helps the researcher to understand what other researchers have already done and also helps the researcher to identify the gaps that remain to be filled (Creswell , 2007). It also helps to some extent to clarify the meaning of some concepts that would be used in the study. Therefore, literature review was critical for the researcher to clearly understand what other researchers had done and also to identify the gaps in the literature. The independent variable in this study was 'teachers' pedagogical practices' and the dependent variable was 'learner proficiency in algebraic concepts. The variables were the main headings under the objectives.

2.2 Meaning of Algebraic Concepts in Mathematics

2.2.1 Definition of Concept of Algebra

Before giving the definition of Algebraic Concepts it would suffice to first define what algebra is and concepts are. Algebra is a branch of mathematics that deals with relations and their combinations (Dudley, 2014). Algebra is a branch of mathematics dealing with symbols and the rules for manipulating these symbols (Kilpatrick et al., 2002). In Stacey and Chick (2006), school algebra is seen as a way of expressing, generalising; a study of symbol manipulations. For example, we can use algebra to generalise as: $a + b = 12$, where a and b are integers. Or we can use algebra to solve linear equations as: $3a + 4 = 10$, simultaneous equations as: $3a + 4b = 16$, $2a + 3b = 12$ and even quadratic equations as: $2x^2 + 9x + 3 = 0$.

What about the meaning of concept? Concepts are defined as abstract ideas or general notions in the mind, in speech or in thought (Stacey & Chick, 2006). According to Dummett, (2010) concepts

are abilities that are peculiar to cognitive agents. Then Algebraic Concepts can be defined as ideas or general notions that deal with symbols and the rules for manipulating these symbols. For example there is the concept of adding algebraic terms is by adding like terms together as: $a + 3b + 4a - 5b = a + 4a + 3b - 5b = 5a + -2b = 5a - 2b$ Or the concept of solving equations involves finding the value of the variable by simplifying through doing whatever is done to one side of the equal sign to be done to the other side as well as: $4a + 7 = 19$, then $4a + 7 - 7 = 19 - 7$ then $4a = 12$ then $\frac{4a}{4} = \frac{12}{4}$ therefore $a = 3$.

2.2.2 Importance of Algebra in Secondary Schools

Then what is the importance of algebra in secondary schools? Algebra in secondary school is the fundamental course for learners' access to higher level mathematics and for access to our increasing technological society (Batista & Baptista, 2014). The importance was also highlighted by (Arcavi, 1994), who defined the aims of school algebra as including expressing, generalizing, establishing relations, solving problems, exploring properties, proving theorems and calculating. Batista and Baptista (2014), also came up with a very interesting definition of algebra for he was able to easily connect it to arithmetic and referred it as generalized arithmetic. It is therefore essential that instruction gives the learners opportunities to make sense of general procedures. (Usiskin, 1997), who called algebra as a language and that to comprehend this language one must understand the concept of a variable and variable expression and the meaning of solution. Stacey and Chick (2006) school algebra is seen as a way of expressing, generalising; a study of symbol manipulation.

According to Hill, Ball and Schilling. (2008), the main purpose of algebra is to learn how to represent general relationships and procedures; through these representations, a wide range of problems can be solved and new relationships can be developed from those known. However, learners tend to view algebra as little more than a set of arbitrary manipulative techniques that seem to have little, if any, purpose to them. Perhaps the typical algebra curriculum focuses too heavily on simplification and manipulation, rather than the generalised ideas that create the basis of algebra.

Immediately pupils have been taught how to operate on the set of integers they are introduced to algebra. Findel (2000), algebra is fundamental in all areas of mathematics because it provides tools

(i.e. language and structure) for representing and analyzing quantitative relationships, for modeling structure, for solving problems and for stating and proving generalizations. In simple terms it is a branch of mathematics that involves the use of letters and hence it is very suitable to use in solving a variety of problems in mathematics. Kilpatrick et al. (2002) further explains that an important aspect of algebra is to provide general and unifying mathematical concepts. This capacity is a powerful resource for building coherence and connectivity in the school curriculum, across grade levels and across mathematical settings. Algebra is said to systematize the construction and analysis of the formulas, equations and functions that make much of mathematics and its applications. By the time most pupils graduate from secondary schools they would have not mastered algebra and are far from developing algebraic proficiency including solving problems that require the use of algebra (Kilpatrick et al., 2002). He further emphasises that without proficiency in algebra learners cannot access a full range of educational and career options, and they have limited chances of success.

Stacey and Chick (2006), came up with recommendations that are expectations related to mathematical proficiency and therefore algebraic proficiency: (i) the ability to work flexibly and meaningfully with formulae and algebraic relations, to use them to represent situations, to manipulate them and to solve the equations they represent. (ii) a structural understanding of basic operations of arithmetic and to solve the equations they represent. (iii) a structural understanding of basic operations of arithmetic and of notational representations of numbers and mathematical operations (for example place value, fraction notation expectation). (iv) a robust understanding of the notion of function, including representing function (for example tabular, analytic and graphical forms), having a good repertoire of the basic functions (linear and quadratic, polynomials and exponentials, rational and trigonometric functions) and using functions to study the change of one quantity in relation to another. (v) Knowing how to identify and name significant variables to model quantitative contexts, recognising patterns and using symbols, formulae and functions to represent contexts.

For the learners to acquire all the above abilities it takes teachers who are well equipped with content knowledge for teaching.

The above literature informed the study in that the researcher outlined and discussed the importance of algebra in the curriculum of the learners as it is the foundational course to higher level mathematics.

2.2.3 Teachers' Pedagogical Practices for Learner Proficiency in Algebraic Concepts

Research on the 'Teachers' pedagogical practices' is very extensive with many researchers agreeing and disagreeing on many issues of pedagogy. According to Kotari (2004), pedagogical practices for teachers are practices educators use that facilitate and promote children's learning. Hill et al., (2008), defined Teachers' Pedagogical Practices as the learning activities that support the unit of content, the instructional approach such as active learning, learner to learner engagement, teaching to multiple learning styles and variety of assessment. Lakkala, Ilomäki, and Kantosalo (2011), state that pedagogical practices refer to the various types of tasks, ways of working or types of activities and practices, which guide effective teaching and learning.

Similarly, in this study teachers' pedagogical practices were considered as different types of practices teachers engage in, that directly impinge on actual teaching and those that do not involve actual teaching. These include among others, pedagogical approaches to teaching preparing well in advance relevant schemes of work, lesson plans, lesson notes and teaching aids; prompt setting of exercises, prompt evaluation of all exercises provision of feedback to learner and assessments, and undertaking of remedial teaching to ensure learning. These pedagogical practices can be divided into two, those that directly involve teaching like revision of prerequisite topics and those that do not directly involve actual teaching like learners working in groups.

2.2.4 Assessment of Teachers' Ability to Prepare for Learners' Proficiency in Algebraic Concepts

Studies were carried out to find out the possible explanation for the low achievement in algebra. Kafata and Mbetwa (2016) found out that there was little thoroughness in teachers' competency when preparing and using instructional tools. Preparation naturally leads to a clear link between utilising instructional products and effective teaching. Gibbs and Simpson (2005) note that teachers' competence in instructional design helps the teachers to develop expertise in teaching that topic. The effort put in the preparation opens up the teachers' mind to various possibilities that

can arise during the actual teaching and learning encounter. Consequently, this process allows the understanding of the various problems that the learners may have. This understanding makes the teacher to come up with the best instructional plans. Franke et al. (2001) has shown that a close relationship exists between teaching techniques and learners' achievement.

Whilst Gibbs and Simpson (2005) in their study showed the value of competence in the preparation of lessons and Franke et al. (2001) identified the link between teaching techniques and learner achievement in algebra. Gurganus (2018) in his study found that learners mainly learn mathematics through the experiences their teachers provide. The teaching episodes encountered shape the students' understanding of concepts and skills. Improvement of mathematics learning for all learners requires effective mathematics teaching in all classrooms. Christou et al. (2007) notes that teaching involves giving reasons, showing and weighing evidence, acting according to principles, and drawing conclusions on relevant evidence to justify learning action. This view is further echoed by Gibbs and Simpson (2005) who found that lessons that are well structured will be learnt smoothly and be more satisfying for all. Such lessons will give the less able mathematics learners in a class a sense of achievement. Similarly, such lessons are capable of stretching the more able learners in ways that will open up even more interesting avenues for them.

According to Kilpatrick et al. (2002), if learners are to develop algebraic proficiency, teachers must have a clear vision of the goals of instruction and what proficiency means for the specific mathematical content they are teaching. They need to know the mathematics they teach as well as the horizons of that mathematics where it can lead and where their learners are headed with it. They need to be able to use their knowledge flexibly in practice to appraise and adapt instructional materials, to represent the content in honest and accessible ways, to plan and conduct instruction, and to assess what learners are learning. Teachers need to be able to hear and see expressions of learners' mathematical ideas and to design the instructional method.

A teacher must interpret learners' written work, analyse their reasoning, and respond to the different methods they might use in solving a problem. Teaching requires the ability to see the mathematical possibilities in a task, sizing it up and adapting it for a specific group of learners. Familiarity with the trajectories, along which fundamental mathematical ideas, is crucial if a teacher is to promote learners' movement along those trajectories. In short, teachers need to master

and deploy a wide range of resources to support the acquisition of algebraic proficiency (Stein, Kaufman, Sherman, & Hillen, 2011).

Much research has been done on how teachers can adapt to different situations and deploy different techniques in their practice of lesson preparation but so far little research has been done to explore how much mathematics teachers use varied methods as they plan to teach algebraic concepts in Zambia.

2.2.5 Pedagogical Content Knowledge in Preparation

Booth, Lange, Koedinger and Newton (2013) revealed that teachers certainly need to be able to understand concepts correctly and perform procedures accurately, but they also must be able to understand the conceptual foundations of that knowledge. In the course of their work as teachers, they must understand mathematics in ways that allow them to explain and unpack ideas in ways suited for anyone who wishes to learn mathematics. The mathematical sensibilities they hold matter in guiding their decisions and interpretations of learners' mathematical efforts. Teachers need to know mathematics in ways that enable them to help learners learn. The specialised knowledge of mathematics that they need is different from the mathematical content contained in most college mathematics courses, which are principally designed for those whose professional uses of mathematics will be in mathematics, science, and other technical fields. The difference matters in considering the mathematical education of teachers (Graham & Fennell, 2001).

Study by Rukangu (2014), showed that the possible explanations for low learners' achievement in mathematics were teaching and learning strategies, student's attitude and teacher characteristics. Since the essence of teaching is to make the content easily understood by the learners, the teacher needs to identify what preparations need to be made for the learners to understand the content being taught. This calls for making notes for himself to look at when working through the examples and checking the answers before asking the class to do them (Gibbs & Simpson, 2005). It may be necessary to list all the relevant pieces of information that might be required such as definitions, formulae, possible discussions routes, questions to be asked at each stage of the lesson or activity. This makes the teacher to become an expert at teaching because the learners' problems are clearly understood and anticipated beforehand. Thus, the teachers should know how to prepare and plan lessons that will reveal learners' prior knowledge and then design experiences and lessons that will

respond to and build on that knowledge. The above literature informed the current study by showing what is involved in preparing lessons.

2.3 Teachers' Pedagogical Practices of Teaching for Learner proficiency in algebraic concepts

2.3.1 Pedagogical Practices on Direct Teaching of Learners

Learner achievement in algebra has become a matter of increased focus in the recent years and the researcher looked at other researches on the pedagogical practices involving the teaching of algebra in secondary schools. A meta-analysis of thirty-five independent experimental studies carried out in the Netherlands shows that six teaching method categories have positive effect on learner achievement in the secondary school algebra classroom (Haas, 2005). Teaching method categories are defined and ranked according to their effect sizes.

Meta-analyses studies suggested two very successful practices: concept mapping and advanced organiser (Hattie, 2003). Concept mapping involves graphs clearly showing the connection between concepts to facilitate learning and in this context, they are a clear explanation of concepts. Advanced organiser is a way of clearly showing the connection between the previous lesson and the present using graphs or otherwise and they are no different to revision of prerequisite topics in this context. The other strategies to engage learners more in learning mathematics were: small-group learning. For example Peer instruction and according to Yara & Otieno (2010) peer instruction is whereby a teacher periodically poses confusing or difficulty questions during a lesson. Another one is problem-based learning and it is a teaching method that gives learners challenging questions for the learners to solve in groups. Another instructional practice for learner active engagement involves teachers linking new contents and materials to the learners' prior knowledge (Gersten, Beckmann,& Witzel.,2009), In another meta analysis carried out by hope, the most effective method of teaching algebra was deemed to be explicit or direct teaching which involves the teacher modeling for learners how to start and succeed on a task and then giving them ample time to practice. Its effectiveness was attributed to the focus on appropriate pacing and both guided and independent practice. It would suffice to say that what the researchers found out were very effective practices because each is an attribute that contributes to algebraic proficiency. Concept Mapping and Advanced Organiser provided the opportunity for learners to develop

conceptual understanding, whereas Explicit Instruction provided the opportunity for learners to develop procedural fluency since a lot of practice was required.

Findings based on meta-analysis conducted by Gersten et al. (2009), converged to suggest that in addition to formative assessment evaluation after a lesson, the following three instructional practices in order of convincing evidence support learning algebra in a secondary school setting. They are (a) explicit and systematic instruction (b) visual representation which are simply illustrations depicting a concept and (c) co-operative learning which is also called collaborative learning where learners help each other on working out tasks. Makewa, Elizabeth, Too and Kiplagat (2012), recommended that teachers should continue to help learners connect familiar arithmetic processes with new algebraic ones and make the connections explicit. This should be supported by all teachers. Haas (2005), analysed algebra teaching methods for secondary schools. He located thirty-five of published sources published between 1980 and 2002 and found out that direct instruction (explicit instruction) had the largest impact for learners who were considered to be high ability and low ability alike. Haas (2005), concluded that educators should focus on direct instruction, problem-based learning and multiple representations which are different ways of referring to same entity as tools to be incorporated into any learning process.

The problem-based method already mentioned above is another method that was recommended by many researchers to be very effective in the teaching of algebra in secondary schools. With problem-based learning (PBL), learning begins with a problem to solve and this problem is posed in such a way that the learners need to gain new knowledge before they can solve the problem. Beklemishev and Visser (2005) found that learners who were taught algebra through problem-based learning achieved better than those taught by traditional method. This corroborated with other researchers (Arcavi, 2003) who worked on the implementation of problem-based learning and agreed that it is an effective teaching and learning method of algebra in class.

The problem with teaching algebra through problem solving is with teachers themselves (Booth et al., 2013). They found out that teachers tend to teach in the same manner as they have been taught. The researcher agrees with them in that most teachers tend to be conservative in their teaching and teach in the same manner as they were taught themselves. Based on reliable evidence from meta-analyses a number of researchers claimed to have come up with some of the best methods to teach algebra in secondary schools. The research on the best teaching methods for algebra is wide and

inexhaustible; and therefore, the only thing the researchers should try to find is how these different methods complement each other.

Many researchers have come up with different instructional methods that they have recommended as yielding higher achievement in the teaching of algebra but so far, no researcher has investigated the practices that teachers use in the Zambian scene.

This literature informed the study by exploring a wide range of teaching methods and how they are graded for effectiveness by different researchers.

2.3.2 Environmental Factors Influencing Mathematical Proficiency

Of late some researchers have also looked at other factors that influence mathematical proficiency other than the traditional direct teaching method (Schoenfeld, 2008). These factors tend to create an environment that supports more learning and facilitates teaching than the traditional teaching methods and are called environmental factors. The researcher therefore, wanted to identify the environmental factors that teachers employ in their quest to influence mathematical proficiency and algebraic proficiency in particular.

The influence of the conducive learning environment upon knowledge development has received relatively little attention in the field of mathematics teaching and learning (Schoenfeld, 2008). This was propounded by Cobb and Yackel (1996), who explained that what happens in the classroom has an impact on learners' opportunity to learn. The activities in the classroom, the repeated actions in which learners and teachers engage in as they learn and teach are important because they constitute the knowledge that is produced. This assertion was supported by Anthony and Walshaw (2009) who posit that caring classroom communities that are focused on mathematical goals help develop students' mathematical identities and proficiencies. They have called this as an 'ethic of care'. Schoenfeld (2008), also found out that teachers can also develop students' mathematical identities by attending to the differing needs that derive from home environment, languages, capabilities and perspectives. He then concluded that effective teachers allow learners to derive a positive attitude to mathematics. Wang (2015) also found out that effective teachers provide learners with opportunities to work both independently and collaboratively to make sense of the given idea. Wang (2015) pointed out that effective teachers plan mathematics learning experiences that enable students to build on their existing proficiencies, interests and experiences.

Schoenfeld (2008) also produced a different list of requirements for proficiency in learning mathematics, but also not directly linked to the actual teaching. It included general competencies such as crafting and managing learning environment, developing classroom norms, and supporting classroom discourse as part of “teaching for understanding” and so on.

What has been described above are some of the environmental factors outside actual learning that facilitate teaching and learning of mathematics and particularly algebra.

2.3.3 Interactive Teaching Approaches

In their study, Amadalo, Wasike, and Wambua (2011), found that use of interactive teaching approaches contributes significantly to learners’ achievement in mathematics. This is because such approaches involve learners in active learning processes and provide them with reasonable control over their learning. Michael (2015), showed that if learners were exposed to strategies that promote interactions, then this usually led to high achievement scores in mathematics. This is in support of Vygotsky (1978) theoretical framework as a guiding theory of this study. In a study carried out by Graham and Fennell (2001), who found out that teachers who spent most of their teaching time demonstrating how to solve questions, asked questions, and gave lectures usually ended up with passive mathematics students. Therefore, meaningful mathematics interactions can only be achieved when the learners take centre stage and become the active drivers and participants in the mathematics lessons.

Cooperative learning allows learners the opportunity to work collaboratively under the guidance of teachers (Yara & Otieno, 2010). There is research on the advantages of cooperative learning (Bossert, 2013). First, cooperative learning strengthens learners’ reasoning through stimulating higher order thinking skills. Second, cooperative learning strengthens learners’ tolerance of other opinions. Third, cooperative learning increases the amount of time for learners to rehearse information. Fourth, cooperative learning increases learner engagement and socialisation skills. The researcher agrees with Bossert (1988) and others on the many advantages of cooperative learning and more especially when learners are more open with fellow learners than with the teacher to express themselves and ask questions.

2.3.4 Visual Teaching Aids in Influencing Algebraic Proficiency

Much research has been done on the advantages of displaying visual teaching aids on walls although Schoenfeld (1989), did not bring it out when discussing on crafting and managing environment for learner proficiency. The value of mounting the visual teaching aids on walls cannot be under estimated as they facilitate teaching and learning in a natural way. In supporting Schoenfeld on crafting environment, Rasul, Bukhsh, and Batool (2011), pointed out that visual aids can be used in the classroom to encourage students' learning process and make it easier and interesting, This is supported by Majidi and Aydinlu (2016) who stated that visual aids arouse the interest of learners and helps the teachers explain the algebraic concepts easily. Shabiralyani, Hasan and Iqbal (2015), also found out that training learners through visual representations improves learners' mathematical proficiency and particularly algebraic proficiency. Shabiralyani et al. (2016) converged with other researchers on this by stating that the use of visual teaching aids to teaching and learning process has multifarious values. The researcher agrees with the above assertions on the value of the visual teaching aids in teaching algebra and, especially the visual aids that are stuck on walls for learners to see and learn from them any time of the day and at their own pace.

Other researchers like Hattie (2003) did not want to be swayed by advocates of environmental factors and argued that actual teaching has the most profound effect on the development of algebraic proficiency because a good teacher has the ability to quickly adapt to different situations and still find ways of how to teach in any such unfavorable environments. This is supported by the researcher that although the presence of instructional materials and so on may contribute to development of algebraic proficiency in learners, actual teaching by a teacher is still the most important in-school factor in promoting learning.

This literature informed the study by showing how teachers influenced algebraic proficiency by manipulating the classroom environment to facilitate the learning of algebra in their practice of teaching mathematics. Also, how these environmental factors can indirectly help to create opportunities for developing in learner's mathematical proficiency and algebraic proficiency in particular.

2.3.5 Influence of Mathematical Knowledge for Teaching on Teachers' Pedagogical Practices

2.3.5.1 Effective Teaching through Mathematical Knowledge for Teaching

Mathematical Knowledge for Teaching has a direct bearing on Teachers' Pedagogical Practices as teachers need the mathematical knowledge in order to effectively plan and execute their practices. Shulman (1987), proposed seven different categories of teacher knowledge. These are (i) general pedagogical knowledge. (ii) Knowledge of learners' characteristics. (iii) Knowledge of educational context. (iv) Knowledge of educational purposes and values. (v) Content knowledge. (vi) Curriculum knowledge and (vii) Pedagogical content knowledge. All the above seven categories complement each other for effective teaching and learning to take place. As educational scholars struggled to come up with a definition of what an effective teacher or good teacher is, they realised that it is not only mathematical knowledge as listed above that matters but that every good teacher should develop some kind of natural talent to be flexible so as to easily adjust and teach in whatever circumstance (Shulman, 1987).

An effective teacher should be able to make the learners know that mathematics is not a strict corpus of knowledge but on the contrary a living and expanding discipline. In recent years, reformers have stressed that priority should be placed on teaching mathematics better rather than teaching better mathematics (Franke & Kazemi, 2001).

Following Shulman's conceptualization of pedagogical content knowledge (PCK), the notion of mathematical knowledge for teaching (MKT) rose to address several issues around PCK, a term that has been coined to capture the mathematical knowledge needed by teachers to perform the demands of teaching mathematics. Mathematical Knowledge for Teaching was theorised out of the proposition to shift the emphasis of the area of research from understanding how teacher knowledge develops to how this knowledge is used in and for teaching (Hill et al., 2008).

2.3.5.2 Mathematical Knowledge for Teaching Vs Disciplinary Knowledge

Larbi (2014), explained that if all the attributes of proficient learners in mathematics and particularly algebra are to be developed overtime, the classroom environment also needs to be constructed in such a way that all the attributes are promoted. This is in line with the notion of

Mathematical Knowledge for Teaching (MKT) as defined by Hill et al. (2008) who regard MKT to be above disciplinary knowledge with a view to assisting learners in their development of mathematical proficiency. What this means is that the knowledge that teachers have to teach mathematics effectively, also called MKT, is of profound importance than the content or disciplinary knowledge on its own.

Every teacher who has the sound knowledge base for teaching mathematics and particularly algebra should know what constitutes mathematical proficiency and algebraic proficiency in particular. Makewa et al. (2012), on developing proficiency in teaching mathematics did not engage indicators of teaching for each attribute of mathematical proficiency. Instead they discussed the knowledge base for teaching mathematics including three types, namely mathematical knowledge, knowledge of learners and knowledge of instructional practice. A teacher who is well equipped with mathematical knowledge for teaching, especially knowledge of instructional practice will realise that a variety of teaching methods can help a great deal in developing algebraic proficiency in learners. Kroesbergen and Johannes (2003), emphasised that a variation of teaching methods is important because different teaching methods draw attention to different competencies in mathematics.

In the planning of a mathematics lesson one of the important tasks a teacher must be able to do is to select and decide the sequencing of examples or performance tasks that would allow the learners to gain understanding of the topic. How well the teacher is able to do this reflects his or her MKT. While a teacher who has attained greater proficiency in the domain of Knowledge of Content and Students (KCS) would know beforehand the level of difficulty an item has and what prior example may help prepare the class for the rigour demanded by that item. A lesser knowledgeable teacher would find greater challenge in doing the same work.

Research on mathematics teaching indicates that instruction often focuses on applying mathematics procedures with little attention to the underlying concepts (Gurganus, 2018). As standards for learner learning that emphasise conceptual understanding and procedural fluency are put into place, attention has turned to new forms of instruction. Research suggests that instruction that explicitly provides learners with opportunities to make sense of symbols and procedures, focuses on mathematical connections, emphasises the big ideas in mathematics, and allows

learners to struggle with mathematical ideas promotes the development of conceptual understanding. This form of instruction also promotes procedural fluency.

The above literature informed the study by showing how mathematical knowledge for teaching can influence algebraic proficiency by equipping teachers with the knowledge for teaching algebra and mathematics in general.

2.4 Learners Difficulties despite Teachers' Pedagogical Practices for Algebraic Proficiency.

2.4.1 Learner Proficiency in Algebra

Kamina and Iyer (2009), found that despite the significance of algebra in students' education, the algebra achievement of U.S. students on the National Assessment of Educational Progress (NAEP) is poor. In fact, 53.8 percent of all responses given on Remedial Intermediate Algebra examinations by a group of freshman college students were incorrect. Heddens categorized an alarming 40.2 percent of these incorrect responses as a result of errors caused by lack of prerequisite knowledge. Could poor algebra achievement be due to learners not being properly prepared for algebra courses? If lack of preparation is the problem, then it is essential to identify content whose mastery is required for the learning of algebra. What do learners need to know prior to entering an algebra course in order to be successful.

Mwamba (2019), who carried out a research on the effectiveness of the problem-based learning on assessment of algebra in Mafinga district of Zambia found that there was improvement when learners were taught algebra using problem-based learning approaches. The study undertaken by Koji, Mulenga and Mukuka (2016), revealed that both teachers and learners faced challenges as they dealt with algebra. This was attributed to poor transition from arithmetic to algebra and lack of exposure to formulation of equations from situations with which they are familiar.

Mbewe (2019), who carried out a study on mathematical knowledge for teaching algebra in Katete district of Zambia, found out that poor performance of learners in algebra was as a result of mathematics teachers inadequate subject matter knowledge consisting of memorised facts and procedures to enable them teach effectively. It was revealed that the teachers did not come up with the strategies to teach effectively but simply explained how to carry out the procedures or apply rules according to algorithms in order to solve equations.

2.4.2 Difficulties in Learning Algebra by Beginners

Algebra can be difficult to learn because it is often taught without recognisable meaning. Rasul et al. (2011), provide an interesting analogy. They claim that learning a language is easy because it means words generally represent something touched or experienced. If one looks at a word like “mango” or “computer” one will obviously visualise an object” mathematics on the hand can be difficult because, it is often taught with no recognisable meaning $2x$ or x^2 cannot be visualised to stand for anything. Algebra is taught as an integral part of mathematics which involves, among others, set theory, number of its operations, algebra, geometry, measurement, matrices and trigonometry to develop reasoning and logical thinking among learners (Ministry of Education of Pakistan, 2006).

The National Council of Teachers of Mathematics (NCTM) (2000) recommended introducing algebra and algebraic reasoning in elementary and middle grades throughout the courses of mathematics. Studies reveal that learners do not understand algebraic concepts properly because the teachers mostly deal with algebraic variables mechanically without explaining real meaning in social context (NCTM, 2000). This obviously results in poor learning, pedagogical knowledge, and knowledge of student thinking.

Christiansen and Ally (2013) conducted a study to investigate the effect of algebra tiles (algebra tiles are square and rectangle shaped tiles that represent numbers and variables) on learners’ performance in algebra. Two intact classes, from two schools, were selected for the study and were assigned experimental and control group. The experimental group was taught using algebra tile manipulative while the control group received instruction using the traditional method, a treatment which lasted for three weeks. It was concluded that the use of tile manipulative promotes students understanding in the learning of algebra. Findings from the study showed that there was significant difference between the two groups. The group that received instruction using algebra tiles scored better than the group without the tiles. It was concluded that the use of tile manipulative method promotes learner understanding in the learning of mathematics.

In a similar study, Franke et al. (2009), found statistically significant differences between two groups in favour of the experimental group, those who were taught using the algebra tiles. The above literature informed the study in that it highlighted on the various difficulties learners have

in learning algebra. In a study carried out by Mbewe (2019), it was found out that through lesson observations a variety of possible learner difficulties and misconceptions on solving quadratic equations, which included computational problems, problems with square root, and problem with negative sign were common. It was observed that such possible misconceptions committed by learners were related to learners' insufficient knowledge and skills in arithmetic.

2.4.3 The Five Strands of Mathematical Proficiency

According to Booth et al.(2013), the main purpose of algebra is to learn how to represent general relationships and procedures; for through these representations, a wide range of problems can be solved and new relationships can be developed from those known. However, students tend to view algebra as little more than a set of arbitrary manipulative techniques that seem to have little, if any, purpose to them (Booth et al., 2013). Perhaps the typical algebra curriculum focuses too heavily on simplification and manipulation, rather than the generalised ideas that create the basis of algebra.

Kilpatrick et al. (2002) contend that:

The term mathematical proficiency (MP) was chosen to capture what was believed to be necessary for anyone to learn mathematics'since algebra is part of mathematics, mathematical proficiency can also mean algebraic proficiency. Proficiency in mathematics was characterized in terms of five strands. These are conceptual understanding, procedural fluency, adaptive reasoning, productive disposition and strategic competency (p.46).

Conceptual understanding simply means comprehension of mathematical concepts, operations and relations and procedural fluency means skill in carrying out procedures flexibly, accurately and appropriately. Then there is also Strategic Competence which means ability to formulate, represent and solve mathematical problems. Adaptive Reasoning is another strand which means capacity for logical thought, reflective explanation and justification.

Last and not the least is Productive disposition which means habitual inclination to see mathematics as sensible, useful and worthwhile in everyday life. This is what has come to be called

real life situation questions. The expectations are that a successful mathematics learner is proficient if he possesses all the components of the five strands in such a way that they can come to bear on different situations. These strands are interwoven and interdependent meaning that they tend to enhance one another. Anthony and Walshaw (2007), go on to explain that while many teacher educators may regard conceptual understanding as superior in contrast to procedural fluency, other researchers and scholars still value computational capabilities. This is supported by van Stiphout et al. (2013) who have revealed that educational research indicates that structured practice leads to the development of accurate procedural fluent skills which are also highly important to mathematical proficiency and they cannot be acquired in any other way possible. This left the researcher wondering as to how much the mathematics teachers strove to reach a balance in teaching on the five strands of mathematical proficiency Advisory Committee on Mathematics Education (ACME, 2011).

National Mathematics Advisory Panel (NMAP, 2008), explained that teaching for conceptual understanding promotes procedural fluency in algebra, including for struggling learners. However, because teachers did not experience instruction that promotes conceptual understanding as learners and curricular materials and tests emphasise procedural fluency, algebra instruction tends to focus on using procedures to solve traditional algebra problems. Instructional practices that emphasise sense making, promote conceptual understanding and procedural fluency in Algebra. Examples include instructional approaches that provide opportunities for learners to make predictions, conduct algebra exploration, justify their thinking, and compare and contrast algebra concepts and approaches to solving problems.

A growing body of literature shows that learners who experience skills focused instruction tend to master the relevant skills but do not do well on tests of problem solving and conceptual understanding. On the other hand, learners who study more broad-based curricula tend to do normally well on tests of skill but do much better on assessment of conceptual understanding and problem solving. What this entails is that the teacher has the responsibility to select the strand of mathematical proficiency he intends to develop. Then Christiansen and Ally (2013) looked for empirical evidence for the promotion of the five strands of mathematical proficiency. What they found out was that 90 percent of the 243 video-recorded five-minute lesson segments from 50 lessons conferred opportunities for developing procedural fluency with only 17% for conceptual

understanding, 8 percent for adaptive reasoning and less than 2 percent for strategic competence and 20 percent for productive disposition. This 90 percent for procedural fluency means that the teacher was able to show clearly why, when and how a procedure was used. It also means that he was able to show coherent sequencing in the development of a procedure.

It is known that effective instruction depends on coherent connection overtime among lessons designed collectively to achieve important mathematical goals. It is also difficult to try and develop the isolated five strands of mathematical proficiency in one lesson and therefore, the development of mathematical proficiency takes place gradually over months rather than the minutes (Kilpatrick et al., 2002). What he meant was that in one lesson a teacher may concentrate on developing one isolated strand say, conceptual understanding and in another lesson, he may concentrate on strategic competency. As a learner advances to senior grades in secondary schools the more strands he has to deal with at a time. By the time a learner reaches grade eleven or twelve he should have developed his proficiency levels to the extent of being able to tackle mathematics questions that require solving higher order algebraic equations.

The above literature informed the study by exploring how teachers struggle to develop all the five strands of mathematical proficiency and algebraic proficiency in particular.

2.4.4. Opportunities to Develop Mathematical (algebraic) Proficiency (OTDMP)

As some kind of innovation in the study of mathematical proficiency Christiansen and Ally (2013), came up with a list of Opportunities to Develop Mathematical Proficiency (OTDMP). The presence of an opportunity does not imply that the opportunity is realised in learning. It simply means that there is a prospect of learners engaging mathematically in such a way that one or more strands of mathematical proficiency could be furthered. Obviously there could be barriers to learning. Hattie (2003), making a distinction between mathematical proficiency and opportunities to develop mathematical proficiency allows us to distinguish between opportunities presented through teaching and the extent to which such opportunities are realised in learning, thus expanding our possibilities of researching links between teaching and learning.

Christiansen and Ally (2013), postulate three necessary conditions of opportunities to develop mathematical proficiency: Opportunities must exist, occur regularly and occur with degree of strength and of course influenced by the learners' personal attributes, circumstances and their

current understanding of the topic being dealt with. He developed a rubric which is a form of a table for assessing the teachers' provision of opportunities to develop mathematical proficiency in the classroom which has three ratings as indicators of the five strands of mathematical proficiency. The third rating of each strand is an indicator describing the highest level of proficiency for each strand. The third rating of the opportunity to develop conceptual understanding (OTDCU) is described as clear explanations of concepts, connections to other concepts indicated and at least two representations explicitly connected. The indicators for third rating for the opportunity to develop productive disposition (OTDPD) was that teachers demonstrated sensitivity, respect and interest in the learners' responses and questions. Learners were encouraged to persevere and learn. Opportunities were fostered to develop links between real world situations and mathematics. The indicators for the third rating for the opportunity to develop procedural fluency (OTDPF) were: why, when and how a procedure was applied was clear.

There was also coherent sequencing in the development of a procedure. Different procedures were also compared. The indicators of third rating for an opportunity to develop adaptive reasoning (OTDAR) were that justifications by teachers occurred frequently and learners were encouraged to justify. The indicators of third rating for the opportunity to develop strategic competence (OTDSC) were that there were multiple heuristics to solve problems and there were also opportunities for those engaged to chat flexibly. It is very likely that the above indicators of the five strands of mathematical proficiency have been used by many researchers as standard ones and they can facilitate in the assessment of mathematical proficiency by those doing a study on mathematical proficiency

The rubric was the tool that facilitated for the researcher to categorise themes and the extent to which the teachers provided the opportunity to develop each strand of mathematical proficiency and particularly algebraic proficiency in the learners (Christiansen & Ally, 2013).

Table 1 is the rubric with indicators for assessing the provision of the opportunity to develop mathematical proficiency with the three ratings as developed by Christiansen and Ally (2013).

Table 1: A rubric for Opportunity to Develop Mathematical Proficiency (OTDMP)

Components of OTDMP	Rating 1 (Low)	Rating 2 (Medium)	Rating 3 (High)
OTDCU	<p>No link between concepts or representations.</p> <p>Representations do not ‘capture’ central aspects of concepts.</p>	<p>Some links between concepts and/or representations.</p>	<p>Clear explanations of concepts. Connections to other concepts indicated.</p> <p>At least two representations explicitly connected.</p>
OTDPD	<p>Inconsistent messages of ability, effort or performance.</p> <p>Real-world situations described, but not explicitly related to mathematics.</p>	<p>Occasional positive reinforcement of effort, performance or ability.</p> <p>Encouragement of interest. Some attempts to stress sense-making.</p> <p>Real-world situations mentioned and used to motivate mathematics, but connections are only partially explicit.</p>	<p>Demonstrated sensitivity, respect and interest in learners’ responses and questions.</p> <p>Learners encouraged to persevere and learn.</p> <p>Opportunities fostered to develop links between real-world situations and mathematics.</p>
OTDPF	<p>Only one procedure shown, with no justification.</p> <p>Procedures may not be performed fluently by teacher.</p>	<p>Opportunities offered to perform procedures appropriately and fluently. Some reasons for the procedure given.</p> <p>Alternate procedures not explored.</p>	<p>Why, when and how a procedure is applied is clear.</p> <p>Coherent sequencing in development of a procedure.</p>

Components of OTDMP	Rating 1 (Low)	Rating 2 (Medium)	Rating 3 (High)
			Different procedures may be compared.
OTDAR	Justifications given, but invalid, with reference to authority or through inappropriate analogies.	Reasoning explicit and valid. Justifications sometimes given by teacher.	Justifications occur frequently. Learners encouraged to justify.
OTDSC	Inappropriate heuristics; problem-solving reduced to algorithm.	A single heuristic appropriate to the topic.	Multiple heuristics to solve problems. Opportunities to choose flexibly among these engaged.

Source: Christiansen and Ally (2013)

Above is the table with the three ratings for the opportunity for a learner to develop the five strands of mathematical proficiency and therefore of algebraic proficiency as well.

The recommendations made earlier by Kilpatrick et al. (2002), as expectations to higher algebraic proficiency are just the same as the third rating for the rubric above. Working flexibly and meaningfully simply means a third rating indicator for conceptual understanding. Also, the structural understanding of basic operations simply means third rating indicator for procedural fluency. Then a robust understanding simply represents a third rating for adaptive reasoning. Additionally, using symbols, formulae and functions to represent context is simply the same as the indicator for the third rating for strategic competence on the rubric.

The above literature informed this study on what is required for a learner to attain algebraic proficiency and what is required for a teacher to influence algebraic proficiency in a learner. It is very important that the learners in secondary schools develop algebraic proficiency gradually and

at a steady pace over the five years they are in secondary schools. This is not an easy task for it would otherwise require fundamental changes to be made concurrently in the curriculum, instructional materials, assessments, classroom practice, teacher education and Continuous Professional Development. All these changes would therefore require coordinated action on the part of policy makers, curriculum specialists, teacher educators, teachers and parent as wells (ACME, 2011).

2.5 Identified Research Gaps based on the Reviewed Literature

Several gaps were identified in the course of reviewing literature. Although much research had been done on the value of good lesson preparation by researchers like Gibbs and Simpson (2005), and Schettino (2016), no research had so far been done on the preparation habits of mathematics teachers on algebra topics in Zambia.

Much research had been done on how teachers of mathematics adapt to different situations to use different techniques to prepare for lesson, little research has so far been done to explore how much mathematics teachers used varied methods to teach algebra.

From reviewing of literature many researchers had recommended different methods as the best methods for teaching algebra. Notable among these researchers is Haas (2005), who recommended direct instruction and AlBuali and Khan (2018), who recommended problem-based method. Yet no research had so far been done on how these different methods could complement each other to enable learners attain algebraic proficiency.

After reviewing much literature on algebraic proficiency, the researcher concluded that no research has so far been done to investigate how much the use of pseudo-methods affect the learner's ability to attain algebraic proficiency.

In the current study the researcher was primarily concerned with the major gap as explained below and it involved the other three gaps mentioned above. Generally, a lot of research had been undertaken on algebraic proficiency but almost none on practices teachers use in their quest to create opportunities for developing in grade eleven learners algebraic proficiency in their practice of teaching and learning. This was especially when teaching on algebraic concepts that require solving higher order equations like quadratic equations. For example, asking pupils to formulate a

quadratic equation from a problem on real life situation questions, and then asking them to solve it by using ‘completing the square method’. This had never been attempted by any researcher so far. It required many skills like formulating equations and a good grasp on the concepts on solving quadratic equations and fluency in solving them. The focus on ‘Quadratic functions and Equations’ was deemed appropriate for it would test on all the five strands of mathematical proficiency and algebraic proficiency in particular.

2.6 Summary of the Chapter 2

In summary, this literature informed the study by showing how teachers influenced algebraic proficiency by manipulating the classroom environment so that the minds of learners were more receptive to algebra in their practice of teaching mathematics. It also showed how these environmental factors can help to create opportunities for developing in learner’s mathematical proficiency and algebraic proficiency in particular by facilitating learning.

The literature also informed this study in that there was need for the researcher to observe how the teachers used their mathematics knowledge for teaching with respect to content, learners and pedagogy in their practice of teaching and learning algebra. The researcher also showed that the six domains of knowledge have a bearing on how these teachers come up with practices that directly involve teaching. The researcher also identified several gaps and notable among these was that he had not so far come across a research study that had clearly described the teachers practices and their effectiveness in creating learners’ opportunities for developing algebraic proficiency and particularly on a topic of “Quadratic Functions and Equations”.

Table 2 Summarises the main themes in the Literature Review

Table 2: Summary of the Themes in Literature Review

<u>Pedagogical Practices</u>	<u>Descriptions</u>
(a) Meaning of Algebraic Concepts (b) Pedagogical Practices of Preparing for Learners (c) Pedagogical Practices of Direct Teaching of Learners (d) Pedagogical Practices of Indirect Teaching of Algebra (e) Influence of MKT on Learner Proficiency (f) Learner Proficiency in Algebra	<ul style="list-style-type: none"> - Definition of Algebra - Importance of Algebra - Value of proper preparation - Teachers become experts in preparation - Talking walls with mathematical charts as teaching aids/ - MKT vs Disciplinary knowledge - Problem of learning algebra - Interactive teaching method - Value of the Visual teaching aids - Preparation makes smooth teaching and learning. - Preparation makes teachers experts - five strands of mathematical proficiency - 2013 revised curriculum and its problems
Rubric for the opportunity to develop mathematical proficiency	<ul style="list-style-type: none"> - Three ratings on the rubric with indicators to develop each strand of mathematical proficiency.
Identified Gaps in the Reviewed Literature	<ul style="list-style-type: none"> - So far no researcher has explored mathematics teachers' pedagogical practices as they prepare and teach an algebraic topic of 'Quadratic Functions and Equations' -

Source: Author (2020)

Table 2 has clearly defined themes in the literature review by listing all the themes of pedagogical studies with their corresponding descriptions in order to have a clear understanding of the topics.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 Overview

In this Chapter, the researcher describes the methodology that was used in the study. The chapter comprises the following sections: research approach, research design, study population, study sample, sampling procedure, data collection instruments, data collection procedure and data analysis. In this chapter, the researcher will clarify on the research methods chosen and how they addressed the research questions.

3.2 Research Approach and Design

A research approach is a plan and procedure that consists of the steps of broad assumptions to detailed method of data collection, analysis and interpretation (Creswell ,Hanson & Clark,2007). Creswell, Shope and Green 2006), state that the qualitative research approach is suitable for documenting people's beliefs and interpretation of reality as well as their actions. Since some of the data was obtained from teachers and learners' interviews and observation on what was happening in class to allow for in-depth understanding, qualitative research approach was found to be appropriate.

Since the study focused on getting primary data concerning pedagogical practices mathematics teachers engage in to attain learners' proficiency in algebraic concepts, a descriptive case study was deemed appropriate. It was a multiple case study design as it involved four different teachers and this increased the validity of the study as compared to a single case study. The multiple case studies allow wider exploring on research questions and theoretical evolution (Eisenhardt & Graebner, 2007). In this multiple case study 4 cases were selected to develop a more in-depth understanding of the phenomenon than a single would. It is also appropriate in that it integrates different components of the study in a coherent and logical way, thereby ensuring that the research problem is effectively addressed.

The purpose for the use of this multiple case study methodology in this research was to establish how teaching practices affect the learners' proficiency in algebra. In addition, a multiple case study provides an in-depth look on test subjects and how data is collected from various sources and

compiled using the details to create a bigger conclusion (Kotari, 2004). The critics of the single case study method believe that the study of a single cases can offer no ground for establishing objectivity or generality of findings (Kotari, 2004). Therefore, the researcher chose the multiple case study to try and establish objectivity and generalise the findings.

3.3 Population

Population refers to a group from which the sample is drawn (Cohen & Bennie 2006). Population is defined as a subset of the target population to which the researcher wishes to generalise his findings of the test sample (Cohen & Bennie, 2006). Therefore, the population is meant to comprise people with characteristics relevant to the study. In this study the population comprised all mathematics teachers and grade eleven learners in secondary schools of Livingstone district, Southern Province of Zambia. These are twenty-seven (27) mathematics teachers and about eight hundred (800) grade eleven learners in nine (9) private, grant-aided and public secondary schools. Cresswell (2006) posits that the more experienced researchers start by selecting the population before the sample. Therefore, researcher made sure to start with selecting the population before selecting the sample.

3.4 Sample

According to Cohen and Bennie (2006) a sample is a subset of the population that is selected for a particular study. The sample should be representative of its population. The sample for this study consisted of four mathematics teachers and 140 grade eleven learners from three secondary schools. Of the 140 grade eleven learners six learners comprising three boys and three girls in each of the four grade eleven classes and bringing the total to twenty-four learners participated in the interviews. All the 140 learners were those in the four grade eleven classes whom the researcher observed. The sample was selected from Livingstone district.

3.5 Sampling Procedure

A sampling procedure is a method by which a researcher determines their sample without being extremely biased against the prevailing variables. It involves selection of individuals to take part in the study so that they represent the large group from which they were selected (Creswell, 2006).

In this study, the researcher used purposive sampling to select participants. Purposive sampling relies on the judgment of the researcher to select the participants Creswell, et al.,(2007).

First the researcher purposively selected three schools in Livingstone district as part of the study to ensure that private, public and grant aided secondary schools were part of the study. The three categories of schools represented all the schools found in the district. Next, the researcher purposively selected two grade eleven mathematics teachers from a public school and one from each of the other two selected schools, grant-aided and private, to take part in the study with the help of the HODs. The teachers were purposively selected to represent teachers of high ability learners and also of low ability learners, and gender was considered to have equal numbers of male teachers and female teachers. Thereafter, six learners (three females and three males) were purposively selected from each secondary school, apart from the public school which had twelve learners, to take part in the study. The learners were purposively selected to ensure representation from high ability learners to low ability learners, who consistently attended classes.

3.6 Research Instruments

Qualitative methods of data collection used by the researcher included semi-structured interviews, document analysis and observation.

3.6.1 Semi Structured Interviews

The study used semi-structured interview guides to collect data from teachers and learners. Adejimi, Oyediran and Ogunsanmi (2010) described semi-structured interview as a general structure by deciding in advance what ground is to be covered and what main questions are to be asked, even addition of more questions deemed necessary. The semi-structured interviews were appropriate because they allowed more questions to be added on where necessary. Using the interview guides, one-on-one interviews were conducted to collect data in line with research questions. Open ended questions were used in the interviews to collect in-depth information in order to have a detailed understanding of the subject matter. Interview guides (see Appendix B and C) were used so that the researcher could gain insights into the subject matter and also an audio recorder was used for the purpose of verbatim. Generally, interviews provided an advantage since information was collected there and then.

3.6.2 Observations

According to Chishimba and Kasonde-Ng'andu (2018), observation is a method of data collection which allows the researcher to put behaviour in context in order to understand it better. The researcher engages in careful observations of behaviour without interacting directly with the subjects (Creswell, Shope, Plano Clark, & Green, 2006). During the period of research, the researcher video-recorded the observed lessons conducted by mathematics teachers in the selected secondary schools. Afterwards he used an observation schedule (see Appendix A) to grade what he saw in the video and then transcribed the video clips. This helped to contextualise the realities of the study area.

3.6.3 Document Analysis

According to Bowen (2011), document analysis is a systematic procedure for reviewing or evaluating documents. It requires that data be examined and interpreted in order to elicit meaning, gain understanding and come up with themes. In this regard, the researcher analysed documents from the Examination Council of Zambia containing relevant information on the performance of learners in algebra in school Certificate and General certificate Examinations. Other documents relevant to the study were also examined to collect data which included official reports, lesson plans, records of work and schemes of work

3.7 Data Collection Procedure

After all the formal procedures of getting access to the sites like getting permission from the gatekeepers was done within three days, the HOD's then helped the researcher to purposively select mathematics teachers that were part of the study in the three selected schools. However, a female and male teacher were selected from the public school while the other two, female and male, from a private and grant-aided school respectively. Thereafter, the researcher purposively chose six grade eleven learners (three female and three male) from the class of each teacher that participated in the interview bringing the total to twenty-four. This was followed by data collection through interviews, observations and analysis of documents such as lesson plans, schemes of work and records of work among others. The researcher was also able to video record all the lessons at the three schools as well as tape record the interviews with teachers and learners

3.8 Data Analysis

Creswell, et al. (2007), argue that data analysis is the process of bringing order, structure and meaning to the mass of collected data. Data was analysed using thematic analysis. The method below was used to analyse data.

3.8.1 Thematic Analysis

The qualitative data analysis was used on primary data collected from teachers and learners. Qualitative data analysis is defined as an interpretation of the collected data for drawing conclusions that reflect on the interests, ideas and theories that initiated the research (Clark, 2007). According to Creswell (2006), interpretation is carried out by looking at data from different angles to identify the major themes. In this study the researcher read through interview data, lesson plans, schemes of work and learners work before categorising it according to themes. The researcher's observations and information gathered from secondary sources were also used in the data analysis. In addition, data analysis of research objective one and two was facilitated by indicators of the rubric designed by Christiansen and Ally (2013) on assessing the opportunity to develop a mathematical proficiency. (see Table 1).

3.9 Delimitations

Delimitations are set so that your goals do not become impossibly large to complete (Hodkinson & Hodkinson, 2001). The study was therefore confined to three secondary schools in Livingstone District of Zambia. The first one was a public or government school, the second one was a private school and the third one was a grant- aided mission school.

3.10 Limitations of the Study

Limitations are factors which the researcher foresees as restrictions, problems and such other elements which might affect the objectivity and validity of research findings (Hodkinson & Hodkinson, 2001). The limitations that the researcher encountered in the study were that the researcher was not informed of sudden changes in the time table. The participants interviewed could not have been the most suitable ones since prior information about them was not given. The learners were not followed into universities and colleges to check if they had developed algebraic

proficiency. The study focused only on three selected secondary schools of Livingstone district of Zambia. Therefore, the findings of the study could not be generalised since the research did not cover the whole country.

3.11 Ethical Considerations

In this section of the study, the researcher made an outline of the steps he undertook to ensure ethical considerations for his respondents.

Ethics is defined as a “matter of principled sensitivity to the rights of others and that while truth is good, respect for human dignity is better”(Benatar, 2002,p52). Therefore, the researcher ensured that he abided by the research ethical norms by firstly being cleared by the Ethical Clearance Committee from the University of Zambia, School of Humanities and Social Sciences. The researcher got permission from the gate keepers to access the sites by using a letter from his school (see Appendix E). It was also important to ask the respondents to sign a ‘letter of consent’ (see Appendix D) to accept to participate in the research or withdraw at their own will. He then read out to learners and teachers the introductory information that informed them what the research was all about and that he was not to use force or his status or power in eliciting information from the respondents (See Appendix B and C). Also, from the introductions, he also ensured to tell the respondents what the information was to be used for and he ensured to give them the feedback. Also, from the introduction he indicated that he was not to use their real names and that everything that they were to say was to be kept confidential. He also kept the names of the sites very confidential and used letters to name them. Lastly, he made sure to establish trust and protect the vulnerable in the research.

3.12 Rigour or Trustworthiness

In the study, the researcher used a model proposed by Creswell (2007), which is based on the identification of four aspects of trustworthiness that are both relevant to quantitative and qualitative studies: (a) truth value (b) applicability (c) consistency and (d) neutrality.

He achieved truth value in the interviewing process by reframing questions, repetitions of questions and exposure of questions on different occasions. Applicability is the ability to generalise from the findings to large populations. He achieved this by making sure that his choice of secondary schools covers broad spectrum of the quality of schools from high standard to mediocre and low standards to ensure that every standard was represented for generalization to be

scaled up to larger populations. Consistency is whether the findings would be consistent if the inquiry were replicated with the same subjects and in a similar context (Eisenhardt & Graebner, 2007). He achieved this through repetitions in the interviews and observations and he would then check if there was consistency by seeing if codes were applied consistently. Neutrality is the freedom from biases in the research procedure and results. He achieved this through lengthy periods of observation to ensure that the situation remained the same. He also used reflexivity and triangulation. Reflexivity refers to assessment of the influence of the investigators own background, perceptions and interests on the qualitative research process. He ensured that his background, perceptions and interests did not have any bearing on his research process.

Triangulation is based on the idea of convergence of multiple perspectives for trustworthiness of data to ensure that all aspects of a phenomenon have been investigated (Korstjens & Moser, 2018). In his research triangulation of data methods was achieved by using three data methods namely observation, Interviews and document analysis. In this study the researcher used triangulation of data sources by observing a variety of teachers and using a variety of interviewees. He did not use triangulation of research investigators in his research because it was him alone who carried out the research. He also used triangulation of the theories by considering a number of concepts from mathematical knowledge for teaching, algebraical proficiency and constructivism as in the conceptual frame work.

3.13 Summary of Chapter Three

This chapter has clearly explained the methods used in this research. The chapter has explained and justified the case research design as the method used in the collection of data. Population, sample size as well as sampling procedures used were also discussed in this chapter. Sampling procedures used was purposive sampling. In addition, the chapter has outlined sources of data, data collection instruments and data analysis procedures that were used in this research. The chapter has further brought out ethical issues considered, delimitation used and limitations encountered in the study were well stated. How the trustworthiness was ensured was discussed.

CHAPTER FOUR

PRESENTATION OF THE FINDINGS

4.1 Overview

In the previous chapter the research methodology was described and used to come up with the findings presented in this chapter. The themes that were presented in this chapter emerged from the data collected from document analysis, lesson observation and semi-structured interviews. Data analysis of research question one and two was facilitated by indicators of the rubric designed by Christiansen and Ally (2013) on the Opportunity to Develop Mathematical Proficiency (OTDMP). Data analysis of research question two and three was also based on themes that emerged from lesson observations and semi-structured interviews.

Vignettes or snapshots were widely used in this chapter as a technique to elicit perceptions of what transpired in the teaching and learning of mathematics. According to Hill, et al. (2008) vignettes are short scenarios in written or pictorial form intended to elicit responses to typical scenarios.

In this study, four teachers of mathematics from three different schools had their documents analysed, were observed whilst teaching and were interviewed, and a total of 140 learners in four different grade eleven classes amongst which six learners per class were also interviewed. Gender was considered and every effort was made to have equal number of females and males in the study.

The following codes were used for identification of the participants: Teacher A and B (at HODI secondary school), Teacher C (at GODI secondary school) and Teacher D (at TODI secondary school); The male learners for teacher A were AM-1, AM-2 and AM-3 and female learners were AF-1, AF-2 and AF-3. The male learners for teacher B were BM-1, BM-2 and BM-3 and female learners were BF-1, BF-2 and BF-3. The male learners for teacher C were CM-1, CM-2 and CM-3 and female learners were CF-1, CF-2 and CF-3. The male learners of teacher D were DM-1, DM-2 and DM-3 and female learners were DF-1, DF-2 and DF-3. The data analysis procedure was guided by the following research questions:

- (i) How do teachers of mathematics prepare the lesson plans for learner proficiency in algebraic concepts?

- (ii) What pedagogical practices when teaching do teachers of mathematics engage in for learner proficiency in algebraic concepts?
- (i) What algebraic difficulties do learners exhibit despite teachers' pedagogical practices for algebraic proficiency?

The planning and practices involving lesson preparation and teaching were analysed separately under different headings for clarity. The researcher used the rubric by Ally & Christiansen (2013) (see Table 1) to categorise the extent to which the teachers provided the opportunity to develop each strand of mathematical proficiency in learners. This is what developed into categories of themes.

The environmental factors that were not directly involved with teaching were categorised under the research question two because they were more inclined to actual teaching.

Refer to the rubric, Table 1, which has indicators that aided the researcher to categorise into themes the items of teaching that propped up during the analysis of the data from lesson observations and document analysis of lesson plans.

4.2 Research Question 1: How do teachers of mathematics prepare their lesson plans for learner proficiency in algebraic concepts?

The lesson plans were analysed in three episodes namely introduction, development of the lesson and the evaluation of the lesson.

4.2.1 Introductions of Lesson Plans

The analysis of the introductions in the lesson plans revealed the teachers' desire to open up the topics by making learners connect with the previous prerequisite topics and by giving the standard definitions of the main concepts.

4.2.1.1 OTDCU- Clear explanation of concepts

Clear explanation of concepts was one of the components of conceptual understanding used in the analysis of the data and by giving the standard definitions of a function and an equation the teachers

were trying to introduce concepts. According to the rubric of Christiansen and Ally (2013) trying to clarify a concept lies under opportunity to develop conceptual understanding.

4.2.1.1.1 A Quadratic Function and Equation

All the 4 teachers gave the same standard definition of the quadratic function in their lesson plans but did not explain why 'a' should not be equal to 0 except teacher A. The standard definition that the teachers gave was: 'A quadratic function is an algebraic expression of the form $f(x) = ax^2 + bx + c$, where a, b and c are constants or real numbers and $a \neq 0$.

Teacher C also gave the standard definition of a quadratic equation. She wrote: The standard form of the quadratic equation is $ax^2 + bx + c = 0$. Where a, b and c are real numbers and $a \neq 0$

Page 47 below shows an excerpt of teacher A and page 48 shows teacher C who explained why a should not be 0 and also teacher C. (lesson plans, 2A and 1C)

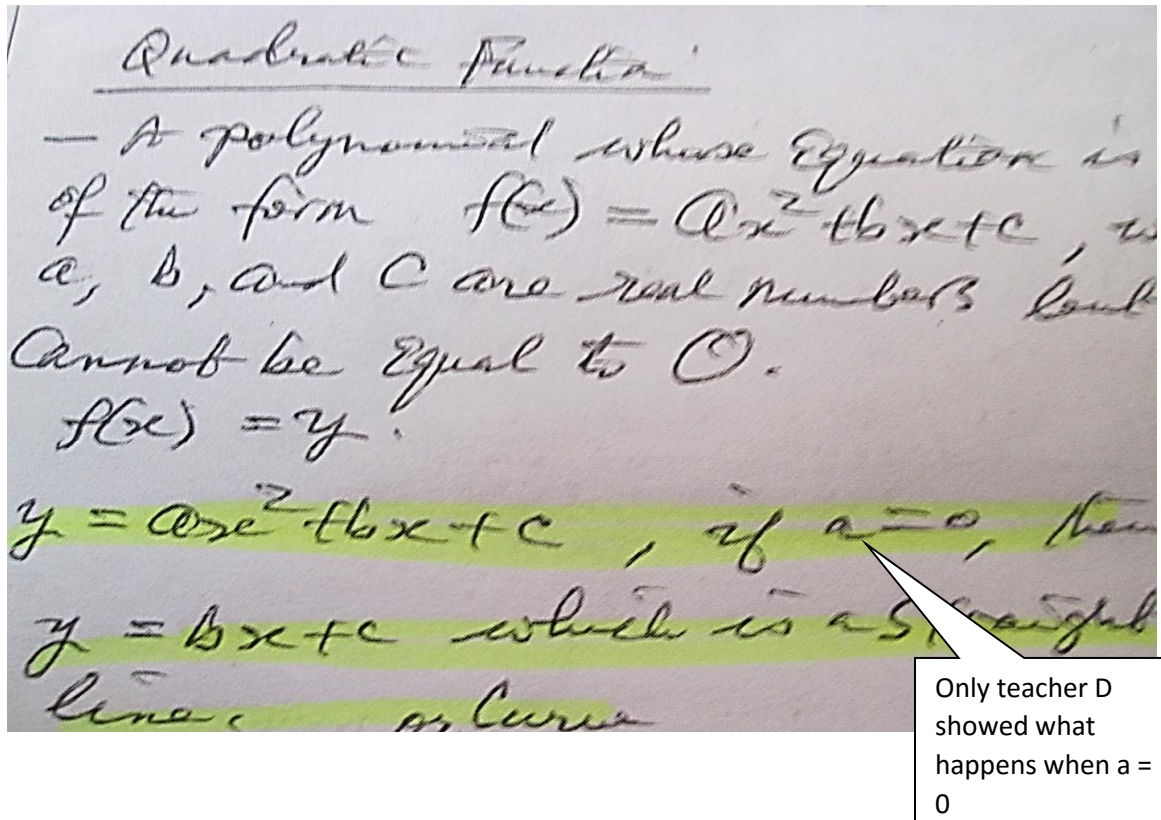


Figure 3: Teacher A stating the standard definition of a quadratic function in his lesson plan

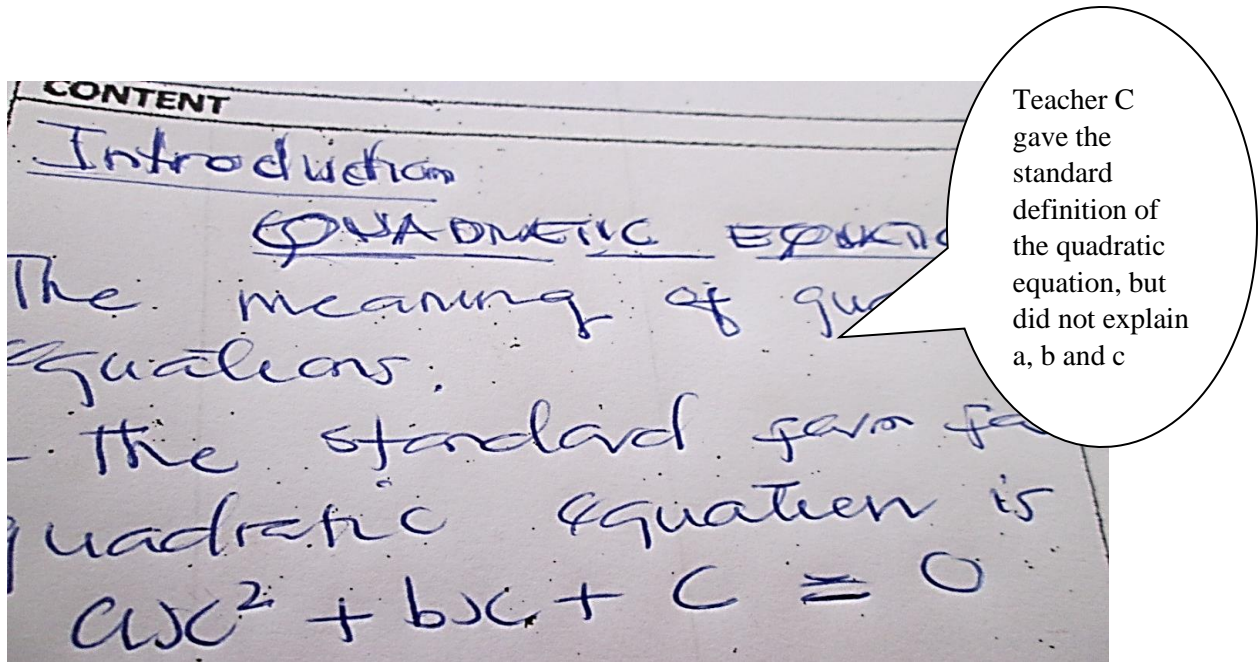


Figure 4: Teacher C stating the standard definition of a quadratic equation in his lesson plan

4.2.1.2 OTDCU- Connections to other concepts

Connections to other concepts was another component used under the Opportunity to Develop Conceptual Understanding, according to the rubric of Christiansen and Ally (2013) revision of the previous prerequisite topic lies under the Opportunity to Develop Conceptual Understanding as it connects the previous prerequisite topic to the present one.

4.2.1.2.1 Revision of Previous Prerequisite Topics

All the four teachers wrote in at least one of their lesson plan introductions that they wanted to introduce the lesson by revising the previous topic. They all did this in a similar manner. On page 49 is an excerpt from lesson plan 3A,

CONTENT	TEACHER ACTS
<p><u>INTRODUCTION:</u></p> <p><u>Previous work covered:</u></p> <ul style="list-style-type: none"> - y - quadratic form - y - intercept - x - intercept - Axis of Symmetry - Turning point - Scale of the graph. 	

Figure 5: Teacher A listed previous topics taught as an introduction

4.2.2 Lesson Plan Development

4.2.2.1 OTDCU- Clear explanation of Concepts and Connections to other Concept

According to the rubric of Christiansen and Ally (2013) clear explanation of concepts and connections to other concepts lie under the Opportunity to Develop Conceptual Understanding

4.2.2.1.1 Explanation of the Parabolic Curve and its connection to Solving Quadratic Equations

In the development stage of their lesson plan one all the four teachers tried to achieve a clear explanation of what a parabolic curve is by identifying its main features.

All the four teachers did explain clearly that if 'a' is < 0 , then the parabola has the maximum turning point. If it has a > 0 , then it has the minimum turning point, but they all did not clearly explain why it is called the turning point.

All the four teachers also explained what the y-intercepts and x-intercepts are on the graph of a parabolic curve and even connected it to the topic of solving quadratic equations using the graphical method. They also explained the axis of symmetry as the straight line that divides the parabola into two equal parts and even how to name the line.

All the four teachers did it in a similar way and below is an excerpt of the lesson plan of Teacher A.

Teacher A gave a clear explanation of the parabolic curve and connected it to solving quadratic equations by graphical method

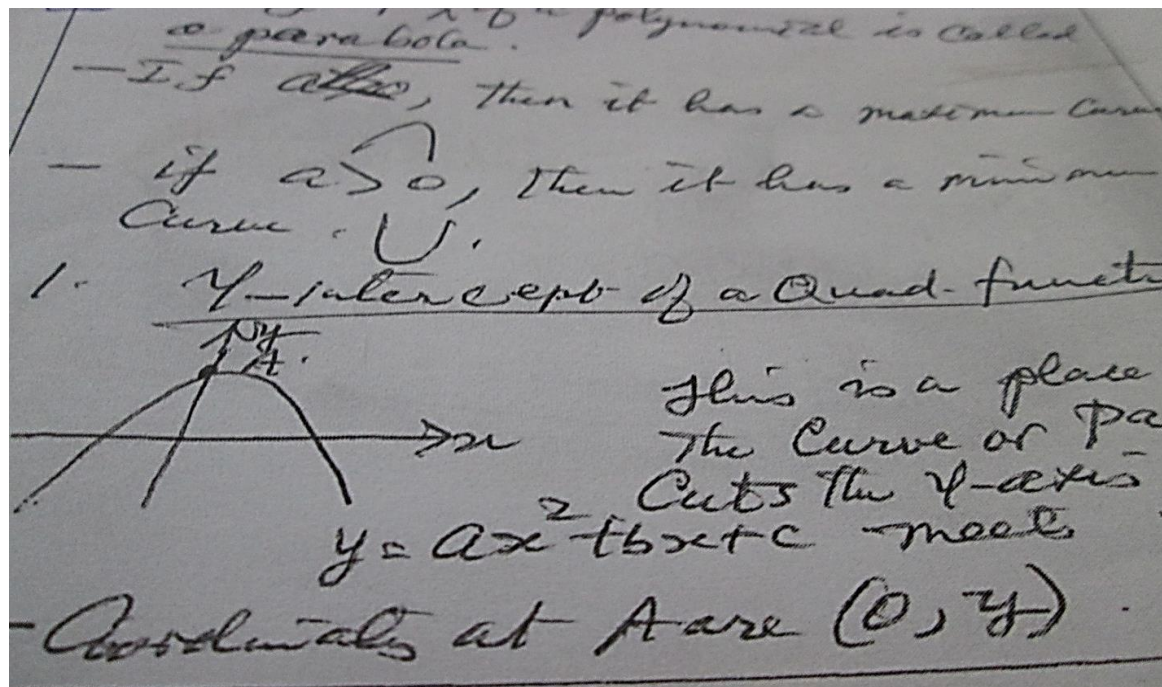


Figure 6: Teacher A gave clear explanation of parabolic curve in his lesson plan

4.2.2.1.2 OTDCU- Solving Quadratic Equation Using Completing the Square Method

All the four teachers had difficulties explaining all the concepts and steps involved in solving quadratic equations using “completing the square method” in their lesson plans.

Firstly, all the four teachers did not explain in their lesson plans what “completing the square method is all about. They merely laid out all the steps but the critical steps like the dividing the coefficient of x by 2, were not very well explained.

They all did it in a similar way and below is an excerpt from teacher B showing the steps without explaining the critical ones. (Lesson Plan 3D)

Teacher D laid out all the steps but did not give conceptual explanation of any of the critical steps involved.

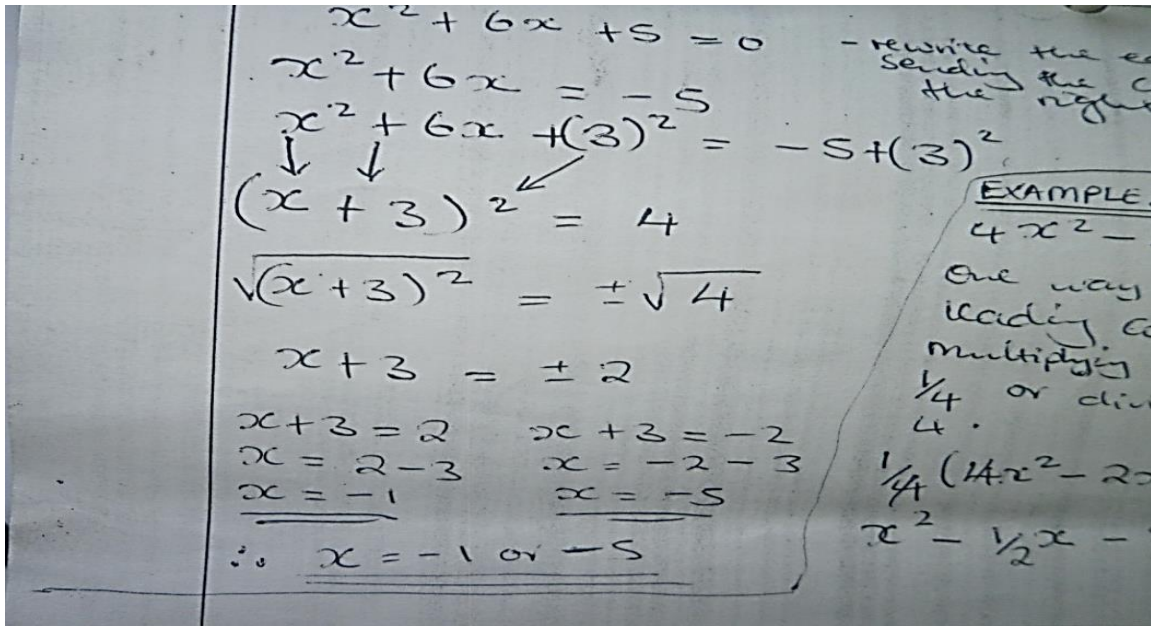


Figure 7: Teacher D listed down the steps in solving quadratic equations using completing the square method

4.2.2.1.3 Planning of Lesson Plans Promoting the Opportunities to Develop Procedural Fluency

4.2.2.1.3.1 OTDPF- Why, When and How a Procedure is Applied

The procedures were mostly in the development stage of the lesson plans. The procedures found in the introductions were about the revisions on the prerequisite topics. All the four teachers explained very well the procedures involved in finding the x and y-intercepts of the parabol and also the factorisation and the formula methods in their lesson plans. However, they all had some problems with completing the square method.

They all did it in a similar way and below are the excerpts of the lesson plans of Teacher D and C.

$$\begin{aligned}
 &x^2 - 6x + 2 = 0 \\
 &a = 1 \quad b = -6 \quad c = 2 \\
 &\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &\frac{6 \pm \sqrt{36 - 4(2)}}{2(1)} \\
 &\frac{6 \pm \sqrt{36 - 8}}{2} \\
 &\frac{6 \pm \sqrt{28}}{2} \\
 &x = \frac{6 + \sqrt{28}}{2} \text{ or } \frac{6 - \sqrt{28}}{2} \\
 &x = \underline{11.291} \text{ or } \underline{0.709}
 \end{aligned}$$

Figure 8: Teacher D showing steps in formula method

Example

$$\begin{aligned}
 &1) \quad 6x^2 + x - 12 = 0 \\
 &\begin{array}{r|l} P & -12 \\ S & 1 \\ \hline \end{array} \quad 6x^2 - 8x + 9x - 12 = 0 \\
 &\begin{array}{r|l} S & 1 \\ \hline \end{array} \quad 2x(3x - 4) + 3(x - 4) = 0 \quad \text{Q + A} \\
 &\begin{array}{r|l} P & 9, -8 \\ \hline \end{array} \quad (2x + 3)(3x - 4) = 0 \\
 &2x + 3 = 0 \quad 3x - 4 = 0 \\
 &\frac{2x}{2} = \frac{-3}{2} \quad \frac{3x}{3} = \frac{4}{3} \\
 &x = \underline{\frac{-3}{2}} \text{ or } x = \underline{\frac{4}{3}}
 \end{aligned}$$

Figure 9: Teacher C showing factorization method in her lesson plan.

4.2.2.2 OTDPF- Coherent sequencing in the development of the procedure

According to the rubric by Christiansen and Ally (2013), above coherent sequencing of steps is an Opportunity to Develop Procedural Fluency.

4.2.2.2.1 Completing the Square Method

All the four teachers did not have the problems showing the coherent steps in the development of the procedure in solving quadratic equations using completing the square method.

All the teachers did it in a similar way and below is an excerpt of Teacher D showing the layout of steps in solving quadratic equations using completing the square method.

Expected Answers

1. $2x^2 - 4x - 30 = 0$
 $\frac{2}{2}x^2 - \frac{4}{2}x = \frac{30}{2}$
 $x^2 - 2x = 15$
 $x^2 - 2x + (-1)^2 = 15 + 1$
 $(x-1)^2 = 16$
 $\sqrt{(x-1)^2} = \pm \sqrt{16}$
 $x-1 = \pm 4$
 $x-1 = 4 \text{ or } x-1 = -4$
 $x = 5 \text{ or } x = -3$

Teacher D did not give conceptual explanation to the critical step of dividing b by 2

Figure 10: Teacher D laid out steps of how to solve quadratic equation using completing the square method

4.2.2.3 OTDPF-Different Procedures Compared

According to the rubric by Christiansen and Ally (2013), different procedures compared lie under Opportunity to Develop Procedural Fluency. The researcher found out that all the four teachers did not clearly explain in their lesson plans the situation where each method of solving quadratic equation would be most appropriate to use as compared to the others. This seems to have been left out in their lesson plans.

4.2.2.3.1 Lesson Planning Promoting the Opportunities to Develop Adaptive Reasoning

4.2.2.3.1.1 OTDAR-Justifications occur frequently by teacher or learner

According to the rubric by Christiansen and Ally (2013) when justifications occur frequently from a teacher or a learner is an indication of an Opportunity to Develop Adaptive Reasoning. Justifications on the maximum and minimum turning point with respect to ' a ' > 0 or ' a ' < 0 lie under opportunity to develop Adaptive Reasoning.

4.2.2.3.1.1.1 Maximum or minimum turning point and ' a ' less or greater than 0

There were justifications mostly when the teachers were writing in their lesson plans on the maximum and minimum turning point. The justification was that since $a < 0$, therefore the graph of the quadratic function had a maximum turning point or otherwise. The justification could also come about by having the values of x and y in a particular function.

All the four teachers did it in a similar way and below is an excerpt of the lesson plan of teacher A making some justification.

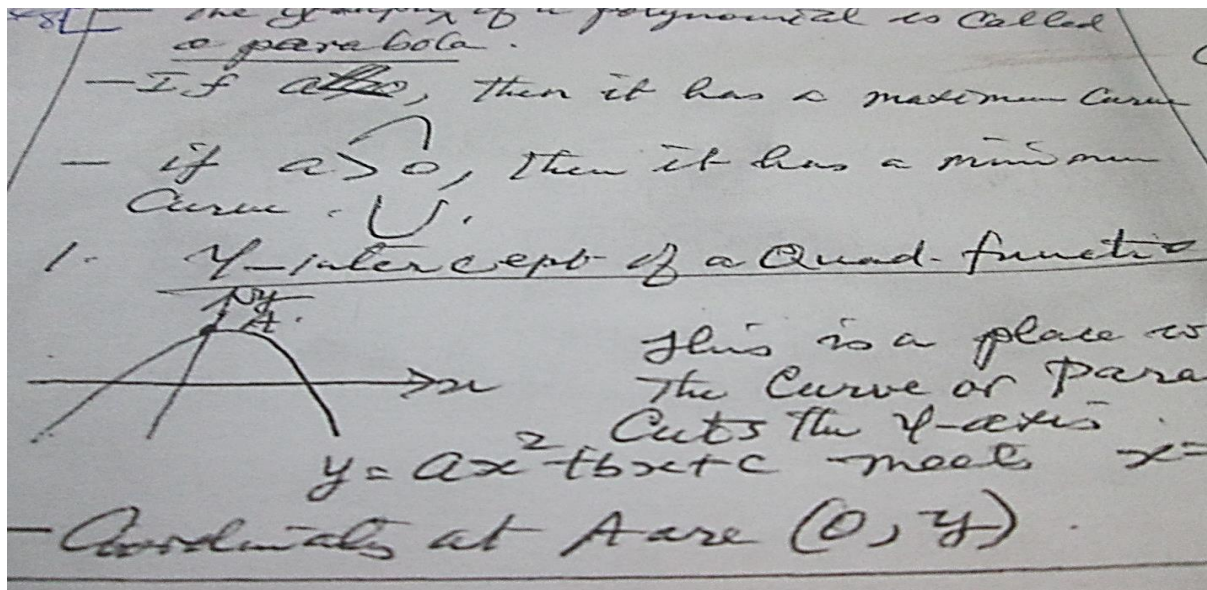


Figure 11: Teacher A showing how he made some justifications in his lesson plan

4.2.2.3.2 Lesson Planning that Promote the Opportunities to Develop Strategic Competence

4.2.2.3.2.1 OTDSC- Multiple Heuristics to Solving Problems

According to the rubric by Christiansen and Ally (2013) multiple heuristics to solving problems is a component of Opportunity to Develop Strategic Competence. Clear explanation of appropriate methods for solving particular quadratic equations lies under Opportunity to develop Strategic Competence.

4.2.2.3.2.1.1 Appropriate Methods to Solving Particular Quadratic Equations

All the four teachers did not clearly explain in their lesson plans that for each particular quadratic equation there should be certain questions for which it is appropriate to use.

Teacher B and C actually recommended that the most suitable method for solving any type of quadratic equation is the formula method which is not correct.

4.2.2.3.3 Solving Real Life Situation Problems

According to the rubric by Ally and Christiansen (2013) solving real life situation questions is an attribute of promoting the Opportunity to Develop Strategic Competence. Only Teacher D had lesson plans teaching real life situation problems and therefore he was the only one who had to promote the Opportunity to Develop Strategic Competence by teaching how to solve them.

On page 55 is an excerpt of the lesson plan for Teacher D to do with solving real life situation problems.

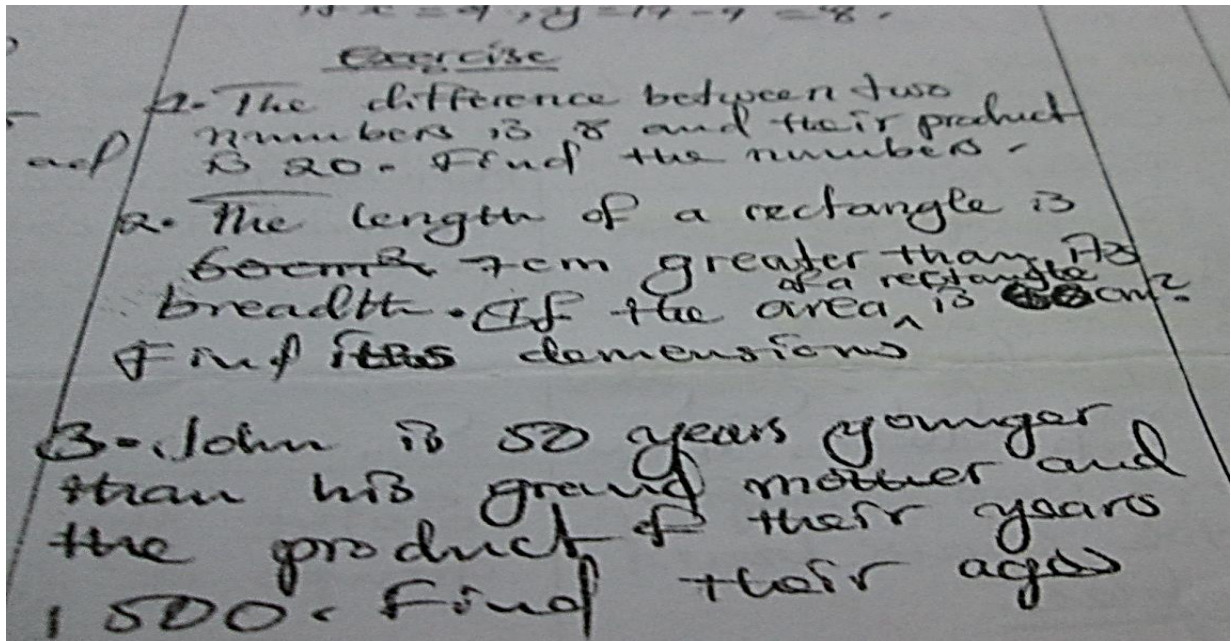


Figure 12: Teacher D's set of questions on real life situation question that require strategic competence to solve in his lesson plans.

4.2.2.4 Lesson Planning that Promote the Opportunities to Develop Productive Disposition

4.2.2.4.1 OTDPD- Opportunities fostered to develop links between real world situations and mathematics

According to the rubric by Christiansen and Ally (2013) when many opportunities are fostered to develop links between real life situations and mathematics, it is a component of Productive Disposition. Solving of real-life situation questions also lies under productive disposition as it fosters links between real world situations and mathematics.

4.2.2.4.1.1 Real Life Situation Questions

Only Teacher D taught on real life situation questions and also gave questions on real world situations.

On page 56 is an excerpt of the lesson plans of Teacher D showing the real - life situation questions.

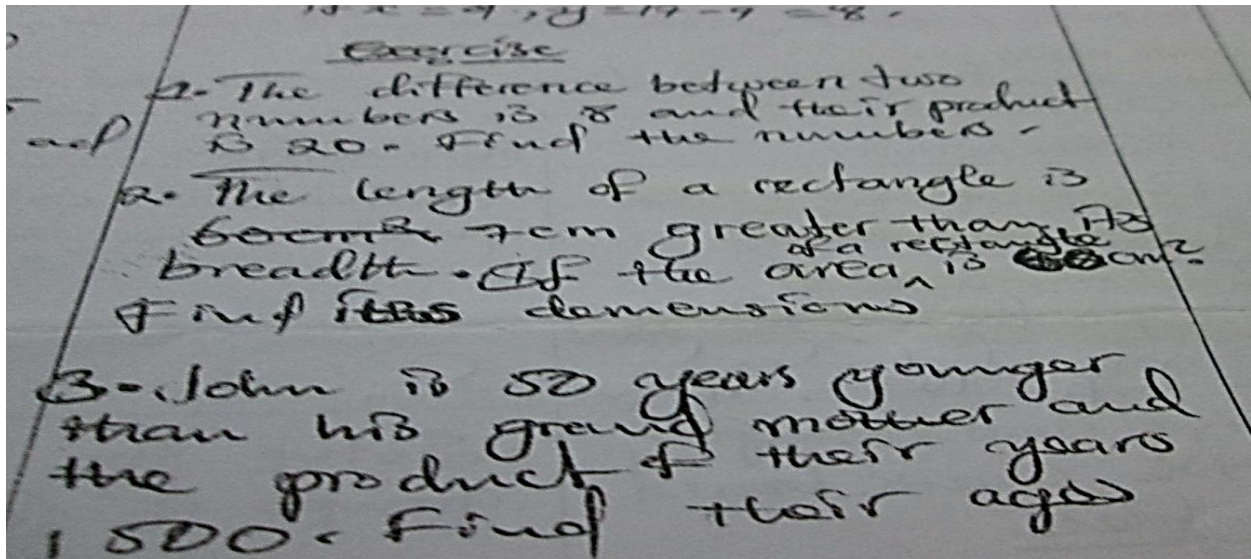


Figure 13: Teacher D's lesson plan showing real life situation questions

4.2.3 Evaluation Exercises in the Lesson Plans

The researcher observed that despite the evaluation exercises set not being of very high standard, all the four teachers managed to give the exercises for evaluation of the lessons in their lesson plans.

All the four teachers did it in a similar way and below is an excerpt from the lesson plan of Teacher B (Lesson Plan 1B).

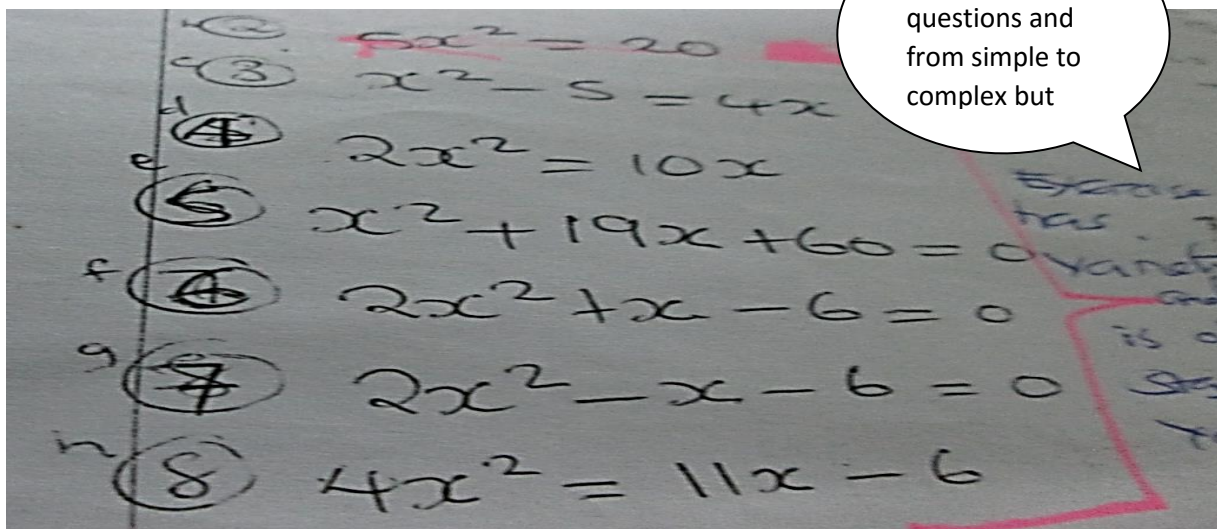


Figure 14: Teacher B's set of questions in an evaluation exercise in her lesson plan

The researcher also asked the teachers, *“What are the major challenges do you have in preparing your lesson plans.”*

Teacher A answered, *“As for me, I do not have any problem except that the new text books have errors and the standard is not very good.”* (Interview 2, Teacher A) which was exactly what Teacher B replied, *“Preparing lesson plans can be hectic, but I do manage and the old text books have proved useful than the new ones.”* (Interview 3, Teacher B)

Teacher C answered, *“As long as the text books are there, preparing lesson plans is not an issue, but this new curriculum has come with text books that are not as good as the old ones.”* (Interview 7, Teacher C) which was the same complaint as Teachers A and B and also Teacher D replied, *“As for me preparing of lesson plans has always been easy but with the coming of new books as the new curriculum was put in place it has been a challenge because the text books seem to be of low standard.”* (Interview 10, Teacher D)

All the four teachers had problems in the preparation of lesson plans because the new text books for the new curriculum were not of a very high standard.

4.3 Research Question 2: What pedagogical practices of teaching do teachers of mathematics engage in for learner proficiency in algebraic concepts?

4.3.1 Introductions when Teaching

The analysis of the transcripts of the video clips of the observed lessons revealed that all the four teachers started the lessons by first trying to explain the main concepts involved in the lesson as depicted in their lesson plans. This was usually done by giving the standard definition of the concepts, like that of a quadratic function.

4.3.1.1 OTDCU- Clear Explanation of Concepts when Teaching

All the four teachers tried to make clear explanation of the main concepts during lessons in order to make sure that learners had full conceptual understanding. This is in conformity with the rubric by Christiansen and Ally (2013) that clear explanation of concepts is an Opportunity to Develop Conceptual Understanding.

4.3.1.1.1 OTDCU- A Quadratic Function and Equation

When teaching, as in the lesson plans, all the four teachers gave the standard definition of a quadratic function when teaching as an expression in the form of: $ax^2 + bx + c = f(x)$, where a , b and c are constants or real numbers and $a \neq 0$. And for a quadratic equation as: $ax^2 + bx + c = 0$, where a , b and c are constants or real numbers and $a \neq 0$.

When teaching all the four teachers did explain why 'a' should not be equal to 0.

All the four teachers did it in a similar manner and below is an excerpt of Teacher D in form of a snap-shot.

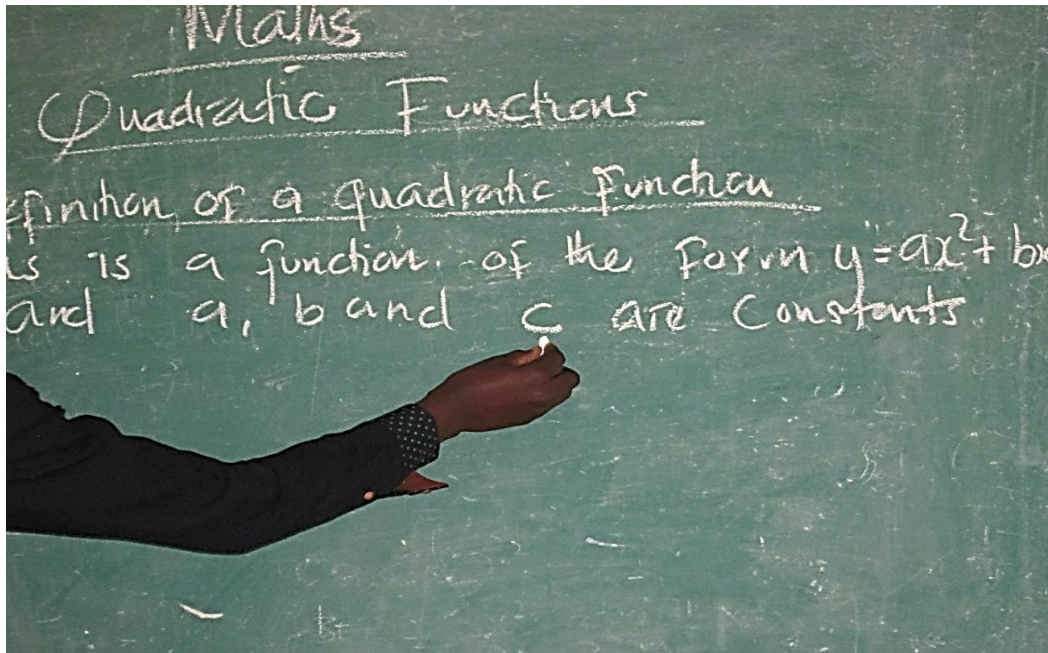


Figure 15: Teacher D giving a standard definition of a quadratic function during teaching.

The researcher also asked the learners how they would want the introductions by the teachers to be presented. four out of twenty-four of the learners interviewed said that they were contented with the way their teachers introduced the topics yet some had complaints to make as the following:

Learners BF-1, BM 1 both said almost the same thing "*She should first of all go back to the basics and prerequisites before introducing anything new.*" (Interview 5, Learner BF 1) which was similar to what Learner BM 1 said, "*She should research more because introductions are just written.*"(Interview 4, Learner BM 1) which also almost carried the same meaning when Learner

CF-3 complained, “*Introductions usually start with too much writings and unclear explanations then follow up.*”(Interview 9, Learner CF 3)

Learner BT-3 from Mrs. B’s class said something totally opposite but a complaint as well: “*Introductions are usually prolonged and not to the point or clear. We end up being bored at the end*” (Interview 5, Learner BF 3)

All the four learners complained about how the introductions were conducted.

4.3.1.2 OTDCU- Connections to Other Concepts

According to the rubric by Christiansen and Ally (2013) connections to other concepts is a component of conceptual understanding. As already explained “Revision of Prerequisite topics” lies under connections to other topics and is therefore under Conceptual Understanding.

4.3.1.2.1 Revision of prerequisite topics

The analysis of the video clip transcripts showed that all the four teachers made sure to make connection to other concepts by revising the previous prerequisite topics. All the teachers did in a similar way. Below is a snap-shot of teacher B teaching on the prerequisite topic of solving quadratic equations using factorisation method.

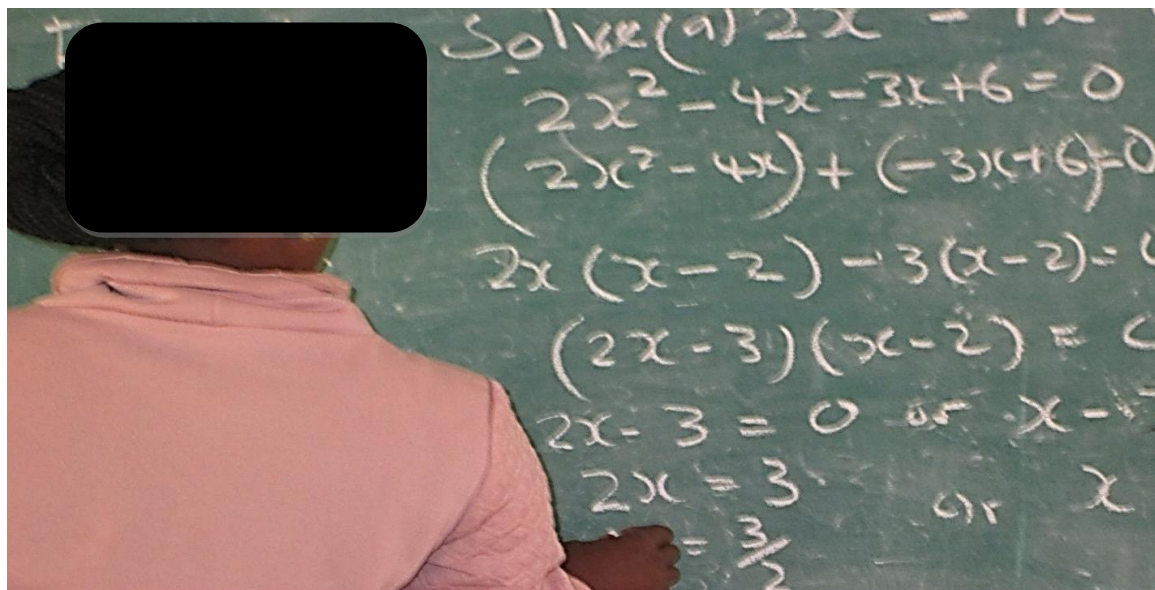


Figure 16: Teacher B teaching on a prerequisite topic of solving quadratic equations using factorization method

4.3.2 Lesson Development when teaching

4.3.2.1 OTDCU-Clear Explanation of Concepts and Connection to Other Concepts

According to the rubric of Christiansen and Ally (2013) clear explanation of concepts and connections to other concepts are a component of conceptual Understanding.

4.3.2.1.1 OTDCU- Explanation of the parabolic curve and its connection to solving quadratic equations

All the four teachers went into the development of their lesson or by explaining clearly the main features of the parabolic curve.

All the four teachers explained clearly that if $a < 0$, then the parabola has the maximum turning point. If it has $a > 0$, then it has the minimum turning point, but they all did not clearly explain why it is called the turning point.

All the four teachers also explained what the y-intercepts and x-intercepts are on the graph of a parabola and even connected it to the topic of solving quadratic equations. They also explained the axis of symmetry.

They all did it in a similar way and below is a snap shot, an excerpt from Teacher A.

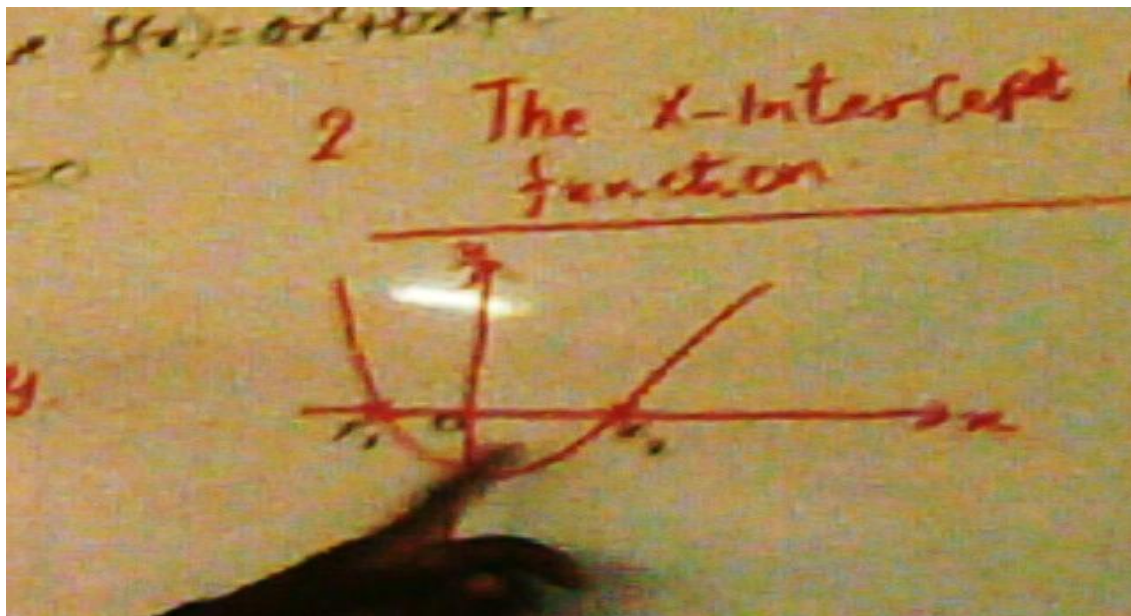


Figure 17: Teacher A teaching on the turning points and x and y intercepts.

4.3.2.1.2 OTDCU- Solving quadratic equations using completing the square method

The analysis of video-clip transcripts revealed that the teachers had problems on explaining clearly all the concepts and steps involved in the completing the square method.

Firstly, they did not clearly explain what the term ‘completing the square method’ really meant. They should have explained that it is like solving for x or making x the subject of the formula. What all the teachers actually did was to lay out the steps involved without clearly explaining the concepts behind the steps, especially the critical step of dividing the coefficient of x by 2.

All the four teachers did it in a similar way and below is an excerpt of Teacher D laying down the steps involved.

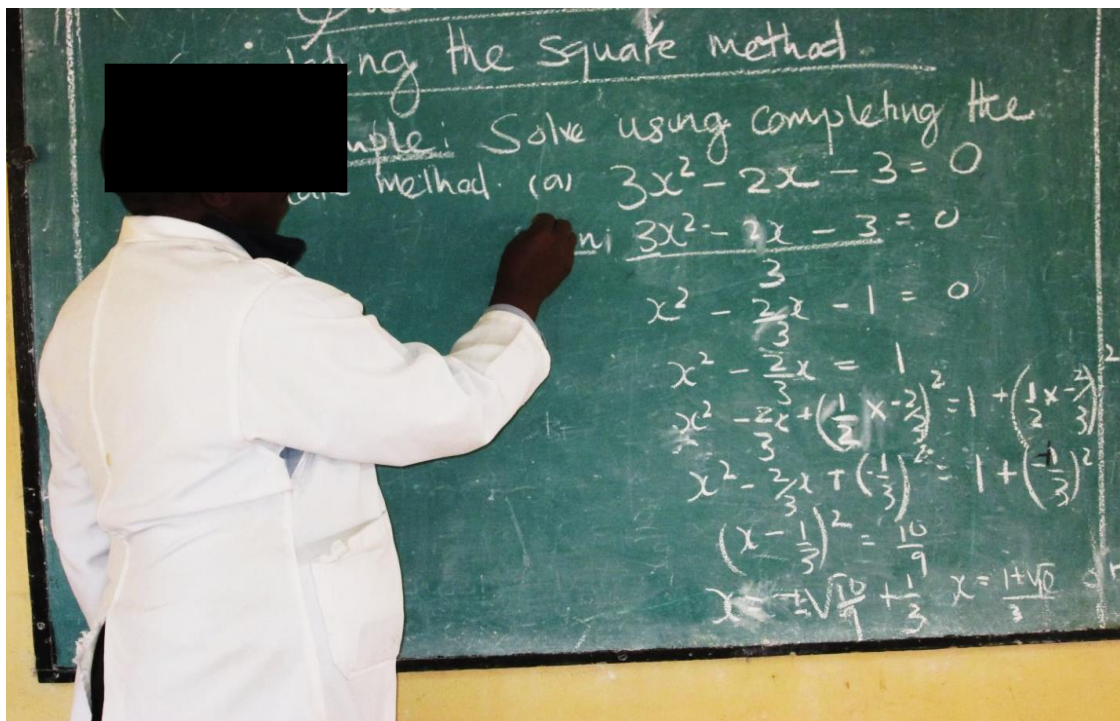


Figure 18: Teacher D teaching on solving quadratic equations using completing the square method

The researcher asked teachers A, B, C and D, “*What is mathematical manipulation (concept) do you find easiest to teach?*”

Teacher A answered, “*I think I do not have any challenge with any concept and I allow learners with different concepts to bring them all out for discussion.*” (Interview 2, Teacher A) which was contrary to what Teacher B answered, “*I find it easy teaching procedures and not concepts.*” (Interview 3, Teacher B)

Teacher C did not understand the question very well and answered, “*Take a recent topic on quadratics. I find it easy to teach formula method. I find teaching computations very easy.*” (Interview 7, Teacher C)

Teacher D understood the question very well and his answer matched that of Teacher A and B , “*Concept is that thing that you want the learners to understand. For me I find teaching concepts easy. If I find concepts that are difficult, I make sure that I consult other teachers who can help me understand them more.*” (Interview 10, Teacher D)

Only Teacher A and D found teaching of concepts just as easy as teaching on procedures.

4.3.2.2 Teaching Lessons that promote the Opportunities to Develop Procedural Fluency

4.3.2.2.1 OTDPF- Why, when and how a procedure is applied as compared to other methods.

According to Christiansen and Ally (2013), why, when and how a procedure is applied as compared to other methods is a component of Procedural Fluency.

All the teachers did show all the four methods of solving quadratic equations but did not explain clearly where each method was most appropriate to use. Teacher B and C actually explained that the formula method was the most suitable method for solving quadratic equations of all types. The ‘why’ and ‘when’ the procedure was to be applied were not clear.

Below is an excerpt of teacher A explaining the ‘completing the square method’

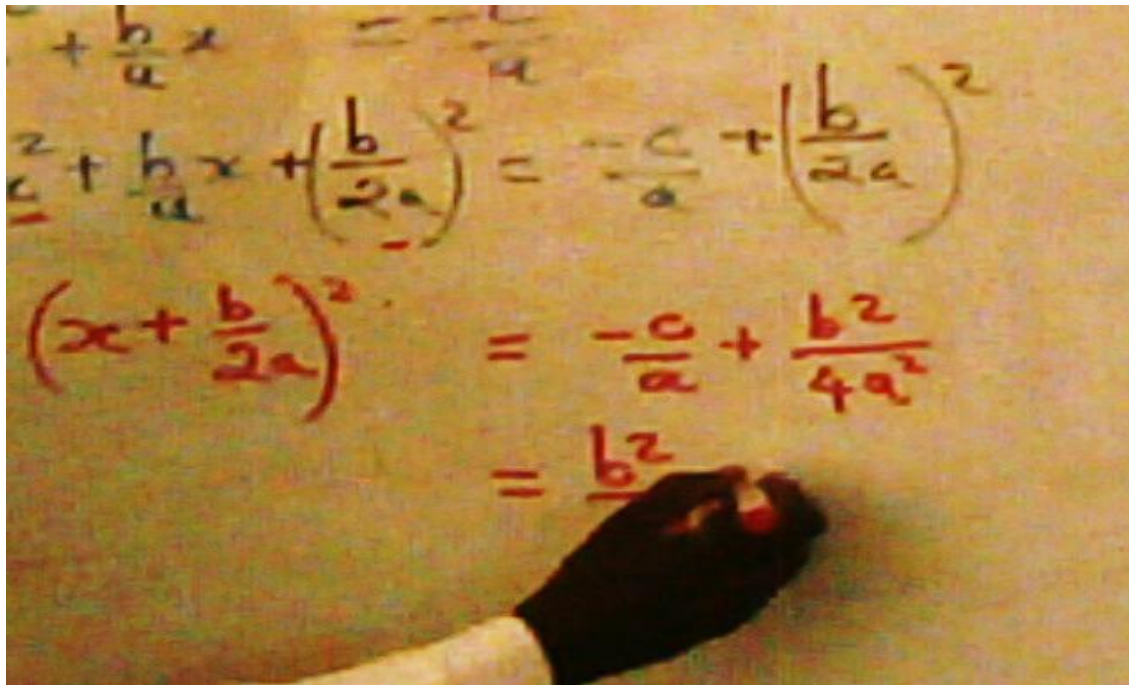


Figure 19: Teacher A teaching on ‘completing the square method’ of solving quadratic equation but did not explain where it was most suitable

The researcher also asked teachers A, B, C and D “What stage in the working procedure of problem solving do you find easiest to teach?”

Teacher A answered, “Solving equations in the problem.” (Interview 2, Teacher A) which was similar to what Teacher B replied, “Following mathematics rules is easiest to teach.” (Interview 3, Teacher B) and yet Teacher C did not understand the question very well and replied, “If it involves calculations, then I can simply tell them to use a calculator.” (Interview 7, Teacher C)

Teacher D also did not understand the question very well and answered, “It is easy and again it is not easy. If a concept is understood, then it is easy to calculate. Understanding concepts makes calculations easy.” (Interview 10, Teacher D)

Only Teachers B and D understood the question very well and Teacher B found calculations easier to teach whilst Teacher D found that teaching on conceptual understanding and procedures went hand in hand.

4.3.2.3 Teaching Lessons Promoting the Opportunities to Develop Adaptive Reasoning

4.3.2.3.1 OTDAR- Justifications Frequently Made by Teacher or Learner

According to the rubric by Christiansen and Ally (2013) justifications made frequently by teacher or learner is a component of Adaptive reasoning.

4.3.2.3.2 The maximum and the minimum turning point and when 'a' is greater or less than 0

The justifications of the maximum and minimum turning point when 'a' is greater or less than 0 lies under justifications frequently made by the teacher and learner which is also under Adaptive Reasoning. Most justifications occurred frequently when the teachers were teaching on the minimum and maximum turning point.

Below is a vignette of Teacher D's learner being taught on the maximum and minimum turning point.

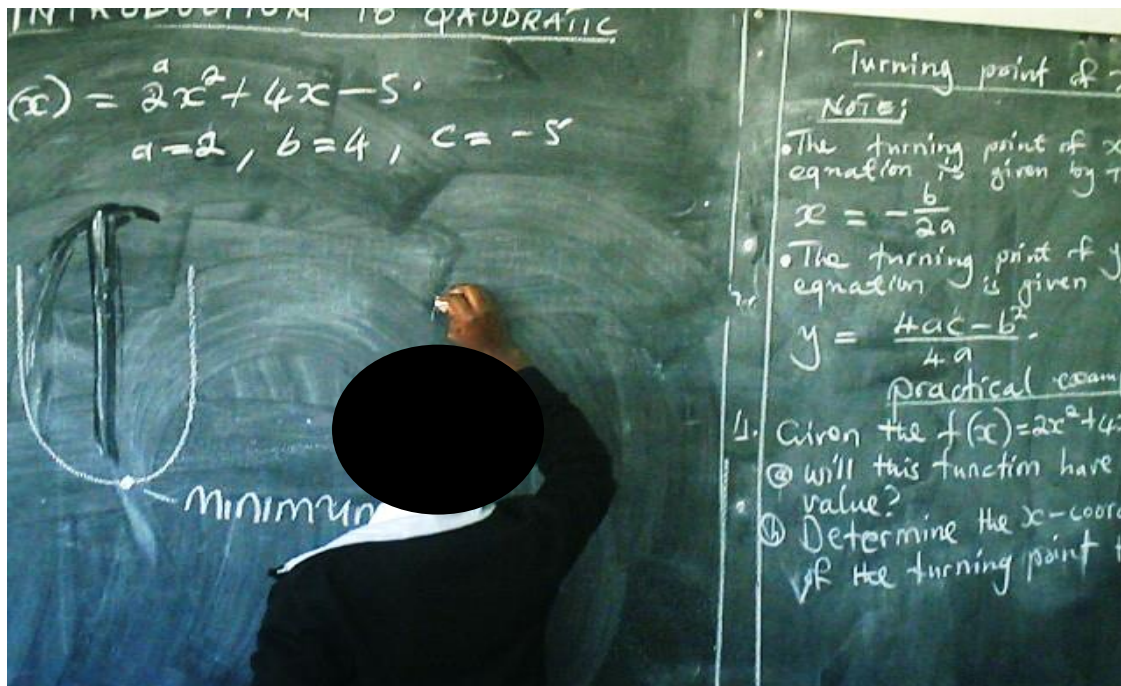


Figure 20: Learner from a class of Teacher D explaining about the minimum and maximum turning point to friends on the board.

I also asked teacher A and B, “How do you train your learners to use logic or common sense in reasoning?”

Teacher A answered, “*My students do not to overuse the calculators. They have to first figure out from their mind*” (Interview 2, Teacher A)

Teacher B answered, “*I always encourage them to use common sense.*” (Interview 3, Teacher B)

The 2 teachers did not clearly understand the concept of justification and logic when teaching.

4.3.2.4 Teaching Lessons Promoting the Opportunities to Develop the Strategic Competence

4.3.2.4.1 OTDSC- Multiple Heuristics to solving Problems

According to the rubric by Christiansen and Ally (2013) above multiple Heuristics to solving problems is a component of Opportunities to Develop Strategic Competence.

4.3.2.4.1.1 Appropriate Method for Solving Particular Quadratic Equations

Coming up with an appropriate method for solving a particular quadratic equation is part of multiple Heuristics to solving problems which is under strategic competence. The analysis of the video-clips transcripts revealed that the four teachers did not clearly indicate exactly where each quadratic equation is most appropriate to use. Teachers B and C indicated to the learners that the best method to use for any quadratic equation was the formula method.

Below is an excerpt of Teacher A teaching on Formula method but not explaining where it was most appropriate to use.

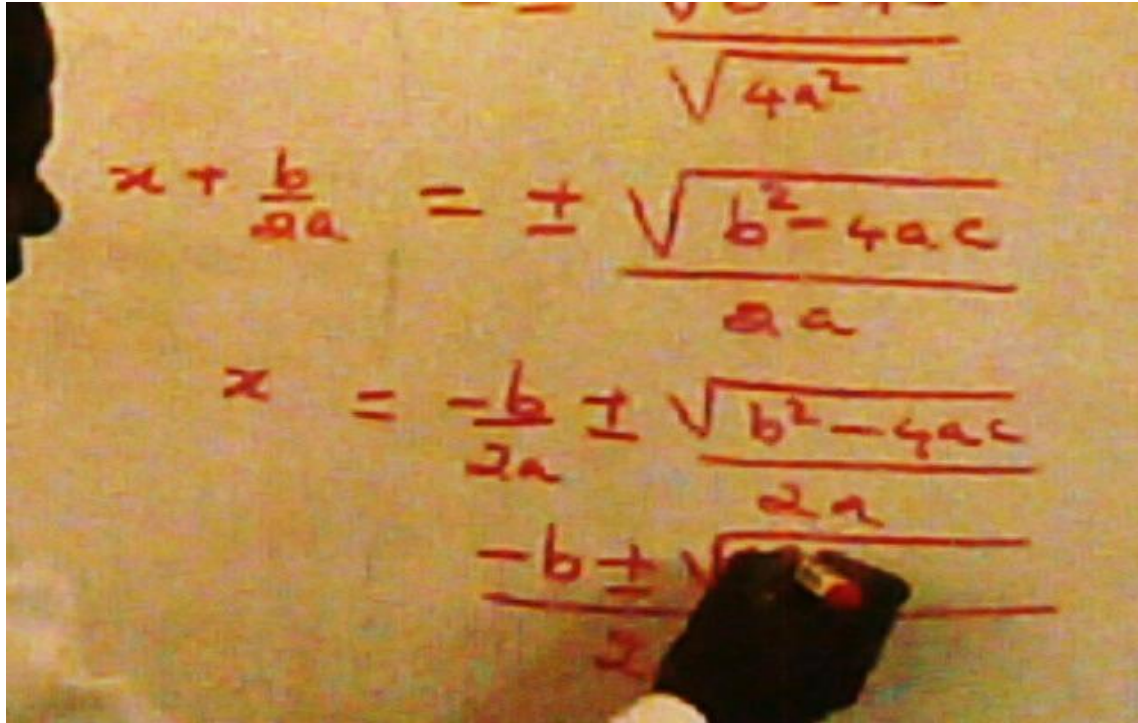


Figure 21: Snap-shot of Teacher A teaching on solving quadratic equations using the formula method

4.3.2.4.1.2 Solving real life situation problems

According to the rubric by Christiansen and Ally (2013), ability to solve real life situation questions requires strategic competence.

Only Teacher D was able to teach on real life situation questions. It is these problem questions that require the learners to use their strategic competence in order to solve them. A learner has to strategically plan how to solve such problem questions.

Below is an excerpt from the video-clips of Teacher D teaching on real life situation problems.

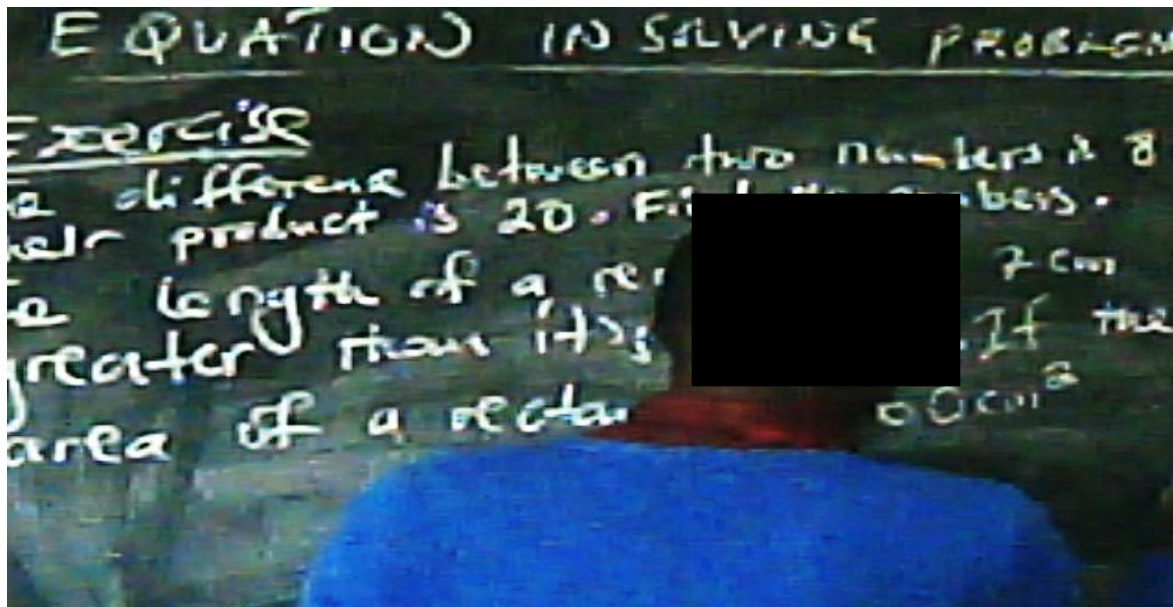


Figure 22: A snap-shot of Teacher D teaching on real life situation question that require strategic competence.

4.3.2.5 Teaching Lessons Promoting the Opportunities to Develop Productive Disposition

4.3.2.5.1 OTDPD- Opportunities fostered to develop links between real life situations and mathematics

According to the rubric of Christiansen and Ally (2013, when opportunities are fostered to develop links between real life situation and mathematics it is a component of Productive disposition.

4.3.2.5.1.1 Solving Real life situation Questions

Real life situation questions are under opportunities fostered to develop links between real life situation and mathematics which are under the opportunities to develop productive disposition.

Only Teacher D taught on real life situation questions. He gave the learners several real life situation questions after solving some examples. Below is an excerpt from the video clips of Teacher D teaching on real life situation questions.

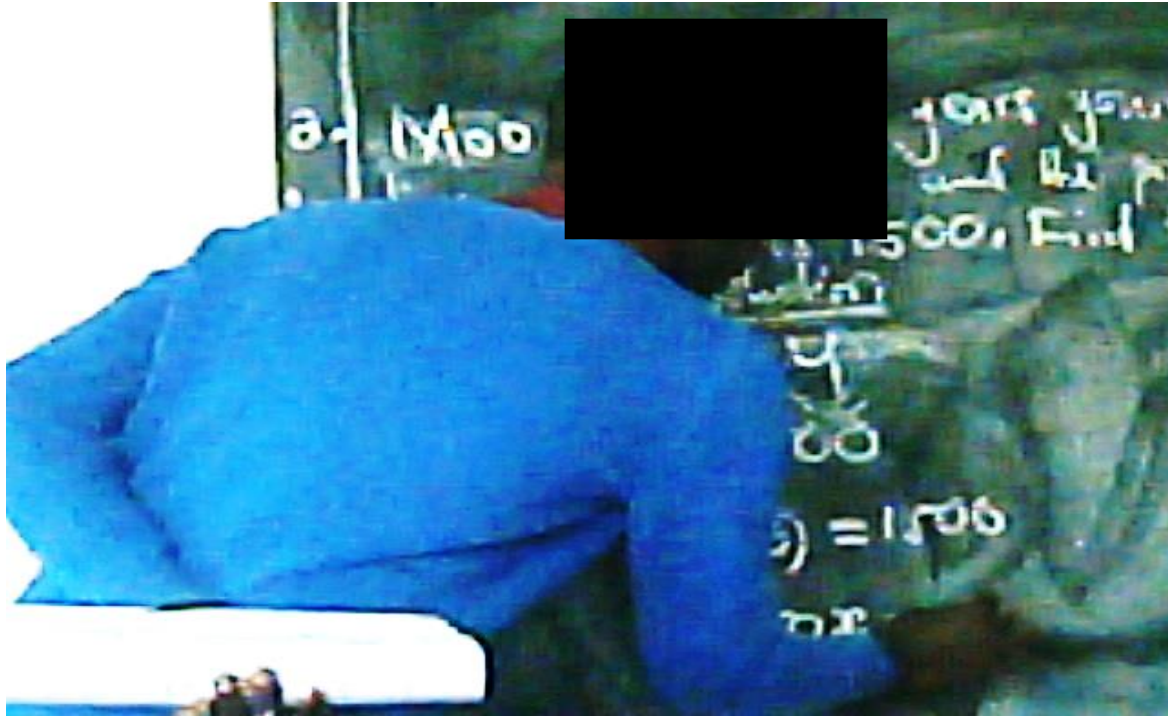


Figure 23: A snap-shot of: Teacher D solving problems on the board based on real life situation.

The researcher also asked the teachers how often they gave questions on real-life situations.

Teacher B said, “*Sometimes I start with a real-life story and then afterwards I tell them that everything involves mathematics in real life.*” (Interview 3, Teacher B) which was similar to what Teachers A and C said.

Teacher D also said the same thing, “*Every now and then I ensure that I bring in questions that deal with real life situations.*” (Interview 10, Teacher D)

Although Teachers A, B and C claimed to teach on real life situation questions, it was only Teacher D who taught on real life situation questions.

4.3.2.6 Environmental Factors Influencing Mathematical Proficiency

According to Schoenfeld (2000), environmental factors are factors other than actual teaching that influence algebraic proficiency of learners.

4.2.2.6.1 Calling of Learners to the Board

All the four teachers made an effort to call the learners to the board to work out some questions which was one way of involving the learners in the learning process and by so doing creating an environment suitable for learning mathematics.

Below are excerpts of learners from classes of Teachers A and B called to the board.

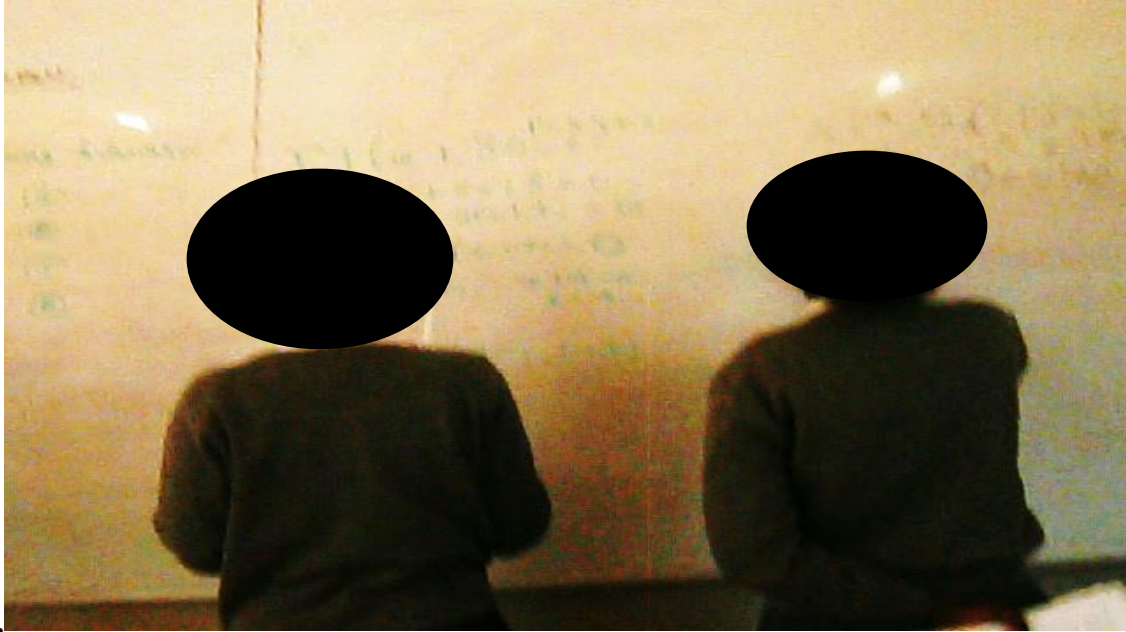


Figure 24: A snap-shot of Teacher A's learners called to the board to work out questions



Figure 25: A snap-shot of Teacher D’s learner called to the board to work out questions

4.3.2.6.2 Putting of Learners into groups

All the four teachers managed to put the learners into groups for interactive learning and by so doing an environment suited for learning mathematics was created. Below is an excerpt from a class of Teacher B.



Figure 26: Teacher B's learners put in groups to work in collaboration

The researcher also asked teachers, "How do you use variation of methods in your teaching practice?"

Teacher A answered, "*If I find that they do not understand then I go to discussions. I do involve various teaching methods.*" which was very similar to what Teacher B answered, "*I use group work. I let them investigate a problem. I also call them to the board to demonstrate.*" (Interview 3, Teacher B), (Interview 2, Teacher A)

Teacher C also supported them when she answered, "*Sometimes I use class discussions. Sometimes I use question and answer. Sometimes I probe and add more information.*" (Interview 7, Teacher C)

Putting learners into groups created an environment where all the learners got involved in the learning of mathematics by interacting with each other and all the teachers managed to do it.

4.3.2.6.3 Display of Visual Teaching Aids on Walls

All the four teachers did not display the visual teaching aids on the walls although two claimed to have removed them during new painting.

The researcher also asked the teachers, "*Do you ensure that the classroom walls are donned with charts of mathematics teaching aids?*"

Teacher A answered, "*That is not being done with me but it is one of the things that we should do.*" Which was totally the opposite of what Teacher B said, "*Yes, only that the walls were recently painted and we had to remove charts. I presently have teaching aids stuck in almost all the classrooms I teach.*"

Teacher C replied, "*By involving pupils to draw these charts, although not all the topics require teaching aids.*" She also claimed to have been studying teaching aids in the walla which was the same with Teacher D when he answered, "*Currently, I do not. Previously we have been doing that but some learners have been removing them from the walls.*"

Only Teacher A admitted not having been using the teaching aids whilst other teachers claimed to have been using them but were removed of late.

Table 3: Summary of Pedagogical Practices

S. No	Practice	Description of practice	Findings
1.	Clear Explanation of concept	Standard definition of a quadratic function and equation. Conceptual understanding achieved	Teachers gave standard definition of a quadratic function and equation to provide opportunity to develop conceptual understanding
2.	Connections to other concepts	Revision of previous prerequisite topics.	Teachers revised prerequisite topics to provide opportunity to develop conceptual understanding.
3.	Clear explanation of concepts and connection to other concepts	Clear explanation of the parabolic curve and connected to graphical method of solving quadratic equation.	Teachers clearly explained that if $a < 0$, then maximum turning point (T. Pt) and if $a > 0$, then minimum T.Pt. But did not explain why they are called turning pts. Also clearly explained x and y intercepts and connected them to graphical method of solving quadratic equations to provide opportunity to develop conceptual Understanding
4.	Clear explanation of concepts.	Clear explanation of all the steps in the completing the square method of solving quadratic equations.	Teachers failed to clearly explain the critical steps in the completing the square method in order to provide the opportunity to develop conceptual understanding.
.5.	Why, when and how a procedure is applied.	Revision of prerequisite topics to achieve procedural fluency.	Teachers revised the prerequisite topics to provide the opportunity to develop procedural fluency

6.	Coherent sequencing in procedure development	Laid down the coherent steps in solving quadratic equations using completing the square method.	Teachers managed to give coherent steps of completing the square method of solving quadratic equations. In order to provide opportunity to develop procedural fluency
7.	Different procedures compared.	Showing where each method could be appropriately used.	Teachers failed to indicate where each method could be appropriately used in order to provide the opportunity to develop procedural fluency
8.	Justifications occur frequently by teachers and learners	Justifications on maximum and minimum turning point of the parabola when $a < 0$ and when $a > 0$.	Teachers made many justifications when explaining on the maximum and minimum turning point in order to provide the opportunity to develop adaptive reasoning,
9.	Multiple heuristics to solving problems	Clear explanation of strategies to solving quadratic problems on real life situations.	3 out of 4 teachers did not give real life situation questions so as to teach on strategies to solving them. Opportunity to develop strategic competence is not achieved.
10.	Links between real life situation and mathematics	Solving real life situation questions.	Teachers rarely gave real life situation questions in order to provide the opportunity to develop productive disposition.
11.	Using a Pseudo-method to solve equations.	Using of the common Pseudo method to solve linear and quadratic equations.	Teachers allowed the learners to use the pseudo-method to solve equations and this compromised their ability to solve quadratic equations using completing the

			square method. They were not provided with the opportunity to develop conceptual understanding
12.	Environmental factors not directly connected to actual teaching	Calling learners to the board and putting them in groups to work collaboratively.	All four teachers called learners to the board to work out questions and put them in groups to work collaboratively. This was aimed at providing the opportunity to develop different strands of mathematical proficiency.
13	Environmental factors not directly connected to actual teaching	Putting up visual teaching aids on the walls to facilitate learning.	Teachers did not put up visual teaching aids on the walls to facilitate teaching and learning.

Source: Author (2019)

4.4 Research Question 3: What algebraic difficulties do learners exhibit despite teachers' pedagogical practices for mathematical proficiency?

4.4.1 OTDCU-Learners Difficulties with Solving Quadratic Equations using 'Factorisation' Method and Formula Method

In all the four classes there were around seventeen percent of the learners that had not yet mastered how to solve quadratic equations using the factorization and formula methods as they had problems solving quadratic equations using these methods in their exercises.

Apart from the difficulties the learners had in the exercises in class, all the four teachers gave a test on solving quadratic equations. Below are the score tables of all the classes.

Table 4: Score table for learners of Teacher A

Marks	100 – 80	79 – 70	69 – 60	59 – 50	49 – 40	39 – 0
No of Learners	3	2	5	1	2	20

Source: Author (2020)

Table 5: Score table of learners of Teacher B

Marks	100 – 80	79 – 70	69 – 60	59 – 50	49 – 40	39 – 0
No of Learners	0	1	3	2	8	22

Source: Author (2020)

Table 6: Score table of learners of Teacher C

Marks	100 – 80	79 – 70	69 – 60	59 – 50	49 – 40	39 – 0
No of Learners	0	0	1	2	8	20

Source: Author (2020)

Table 7: Score table of learners of Teacher D

Marks	100 – 80	79 – 70	69 – 60	59 – 50	49 – 40	39 – 0
No of Learners	1	1	2	3	7	24

Source: Author (2020)

From the Tables 4, 5, 6 and 7 of scores it is evident that more than half of learners in all the classes failed the test on quadratic equations. Class of Teacher A recorded a 61 percent fail and only 3 got 80 percent and above. Class of Teacher B recorded a 65percent fail and no one got above 80 percent. Class of Teacher C recorded a 65% fail and no one got 80% and above. Class of Teacher D recorded a 63% fail and only one got 80% and above.

On page 77 and 78 are some of the marked scripts of the learners indicating some of the difficulties the learners had.

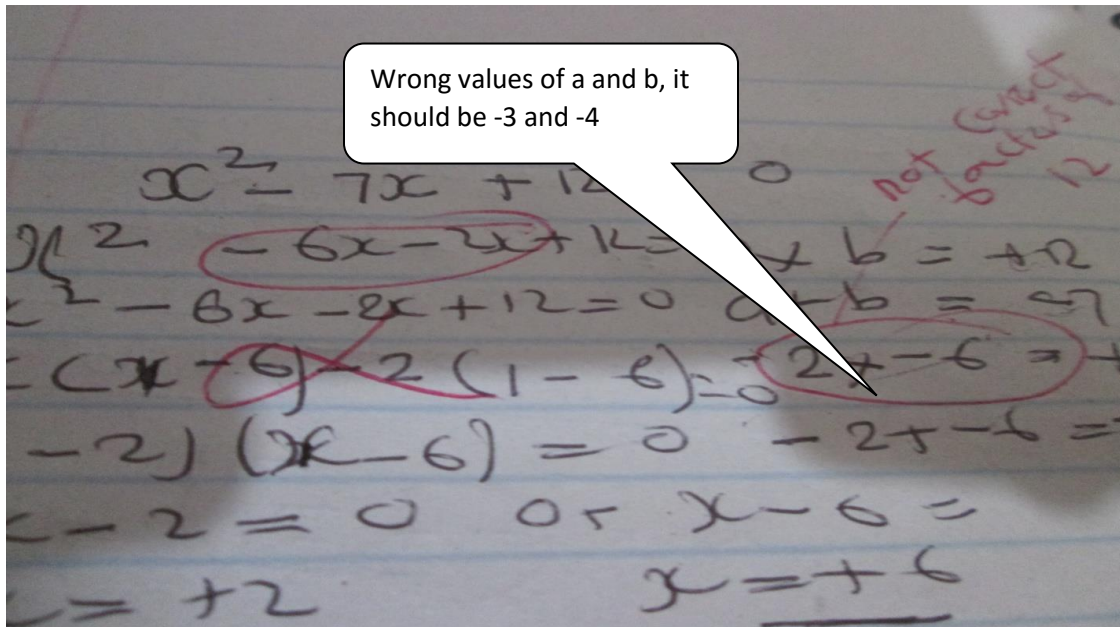


Figure 27: A marked scrip of a learner of Teacher C failing to find the correct values of a and b such that if multiplied they give +12 and if added they give -7.

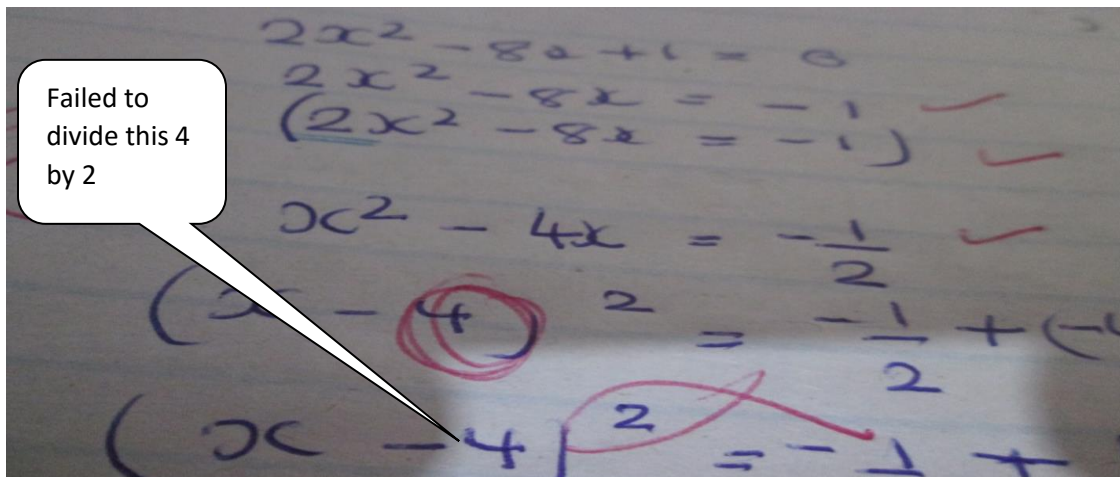


Figure 28: A marked script of a learner of Teacher B failing to divide 4 by 2 which was one of the critical steps in solving quadratic equations using completing the square method.

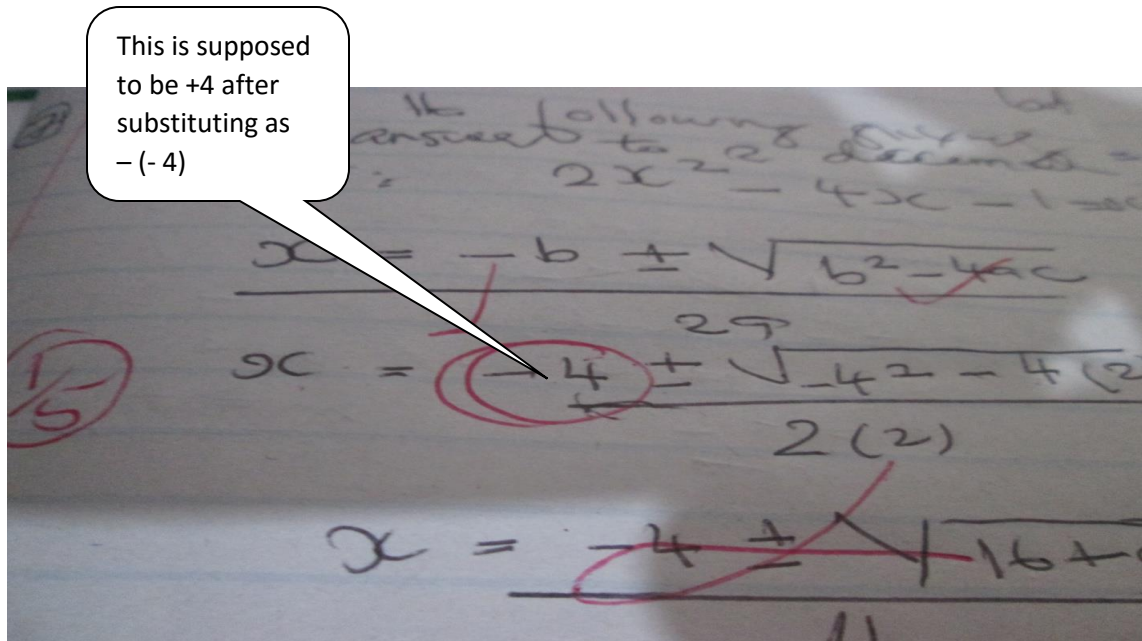


Figure 29: A marked script of a learner of Teacher D failing to correctly substitute b with - 4 and as a result the answer was wrong

Below is an excerpt of learners in a classes of Teacher C and. Teacher B having difficulties solving quadratic equations.

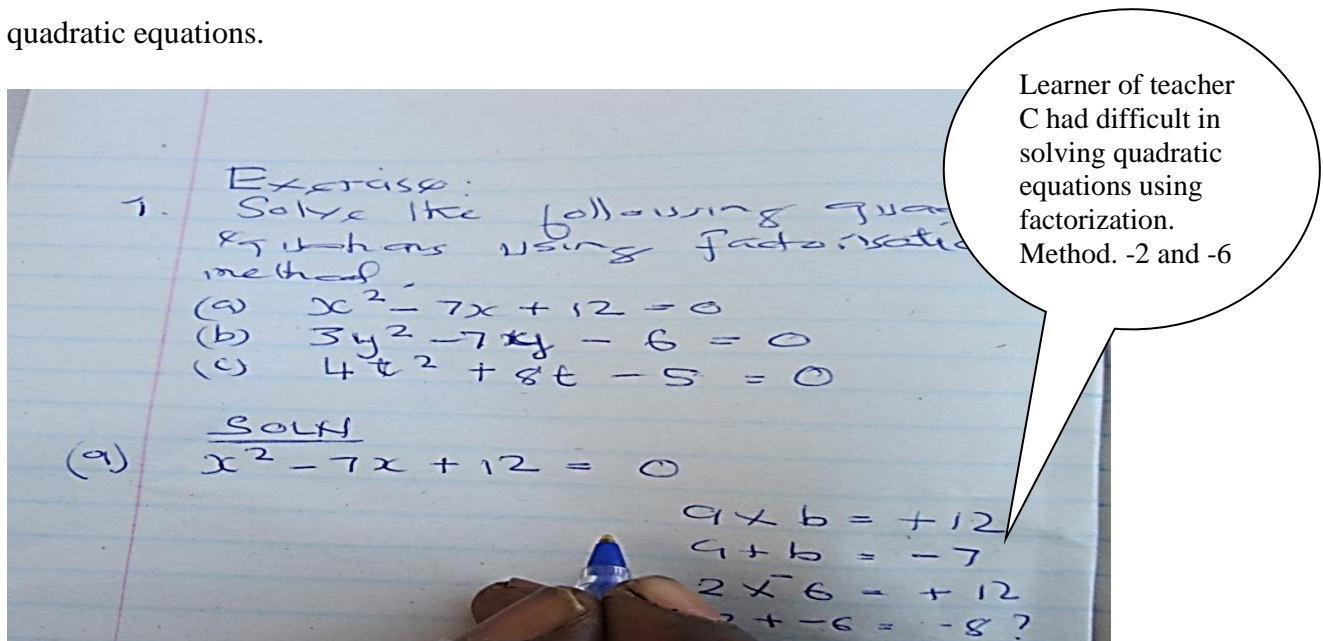
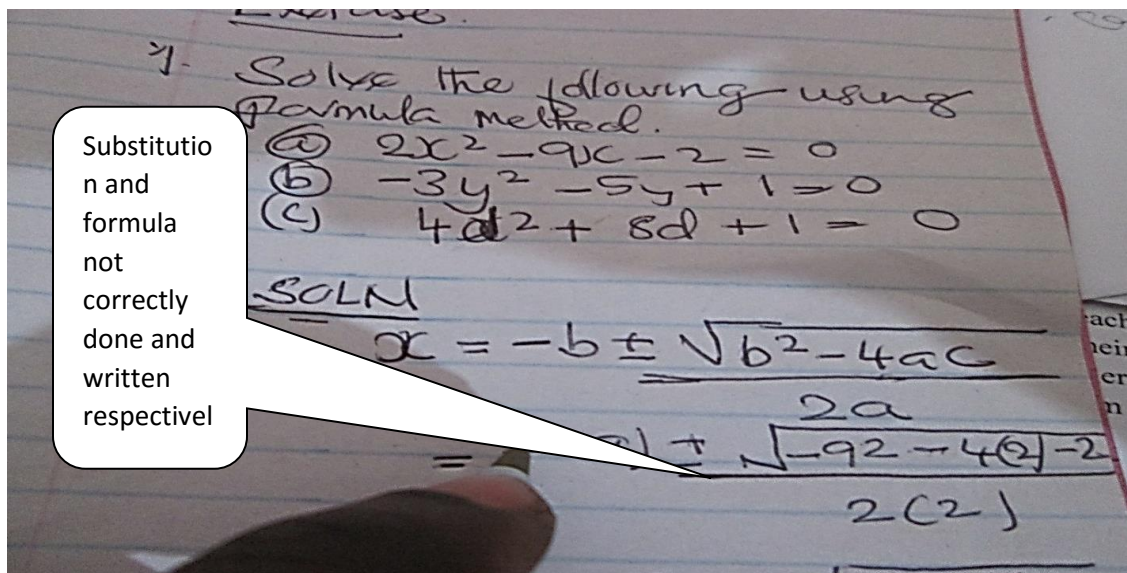


Figure 30: A Learner from a class of Teacher C having problems solving using factorization method



Substitution and formula not correctly done and written respectively

Figure 31: A learner from a class of Teacher B having problems solving using the formula method

The researcher also asked the learners "How good are you at managing the methods of working (procedures) in mathematics?"

Learner AF-1 said, "No, I do not have problems with calculating and computations." (Interview 4, Learner AF-1) which was contradictory to what Learner BM-3 said when he confessed, "Not good, because I mess with negatives and positives." (Interview5, Learner BM-3)

Learner CM-1boasted, "I am good, it is just a matter of practice." (Learner CM-1, Interview 4) which is exactly whatLearner DF-3 meant when he said "I am good at that if I concentrate more." (Learner DF-3, Interview 11)

Only one learner seemed to have problems with procedures as she had not mastered how to operate on the integers.

4.4.1.1 The Pseudo-method Used by Most Learners

This was not a difficulty as such but it was the source of most of the difficulties learners had in solving higher order equation and failing touse a standard method could be regarded as a difficulty.

In all the four classes of the four teachers there were many learners who had got used to using the pseudo-method for solving linear equations. According to the pseudo-method, when a term goes

to the other side it changes the sign to the opposite of what it is. They call this collecting like terms which is a misapplied concept. All the four teachers never used the pseudo-method and emphasised the standard method but did not emphasise to learners to stop using it. Using a pseudo-method implies having difficulties using a standard method. Not being able to use the standard method implies that the learners would face challenges in solving higher order equations in future.

Below is an excerpt from a class of Teacher D of a learner using a pseudo-method.

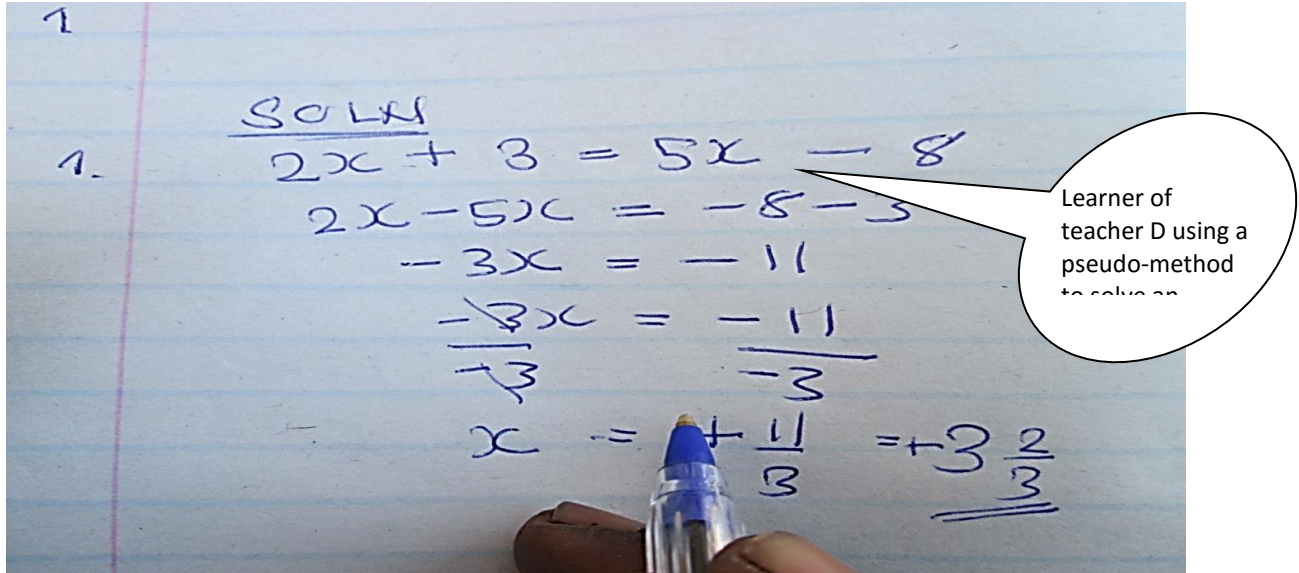


Figure 32: Learner in a class of Teacher D using a pseudo-method to solve equations

4.4.2 Difficulties Faced by Learners in Using ‘Completing the Square Method’ to Solve Quadratic Equations

Around twenty five out of thirty five of the learners from the four classes had a lot of difficulties solving quadratic equations using ‘completing the square’ method.

Below is an excerpt from a learner from a class of Teacher C having problems solving quadratic equations using completing the square method.

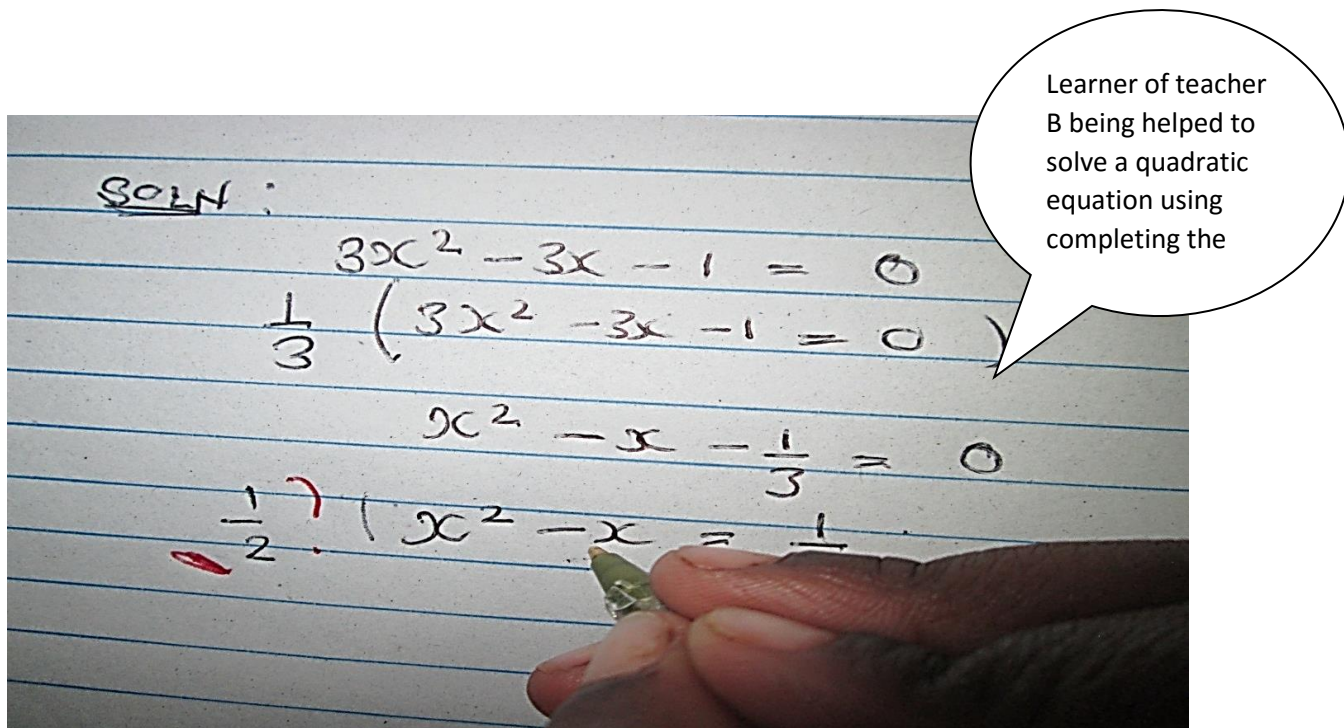


Figure 33: Learner from a class of Teacher B having problem solving using ‘completing the square method

The researcher also asked the learners, “How good are you at grasping the main underlying idea (concept) in algebra?”

Learner CF-3 answered, “*I have problems with that.*” (Interview 9, Learner CF-3) which was just the same as what Learner DF-3 answered, “*I am not really good, but I always ask if I do not understand.*” (Interview 11, Learner DF-3)

Learner DM-1 answered, “*I am very close to good.*” (Interview 11, Learner DM-1) which was tantamount to what Learner BM-2 answered, “*Yes, at times I am very good, but at times I need someone to help me grasp the concept.*” (Interview 5, Learner BM-2)

All the learners expressed concern on their problems with grasping the main concepts. Some managed somehow whilst others utterly failed.

4.4.3 Learners Difficulties in Solving Real Life Situation Questions

Only teacher D taught on real life situation questions and most of learners, especially from the classes of teachers A, B and C where the teachers did not teach on real life situation questions had some difficulties solving problems on real life situation questions.

Below is an excerpt of learner from class of Teacher D having difficulties solving real life problems and is being assisted by his teacher.

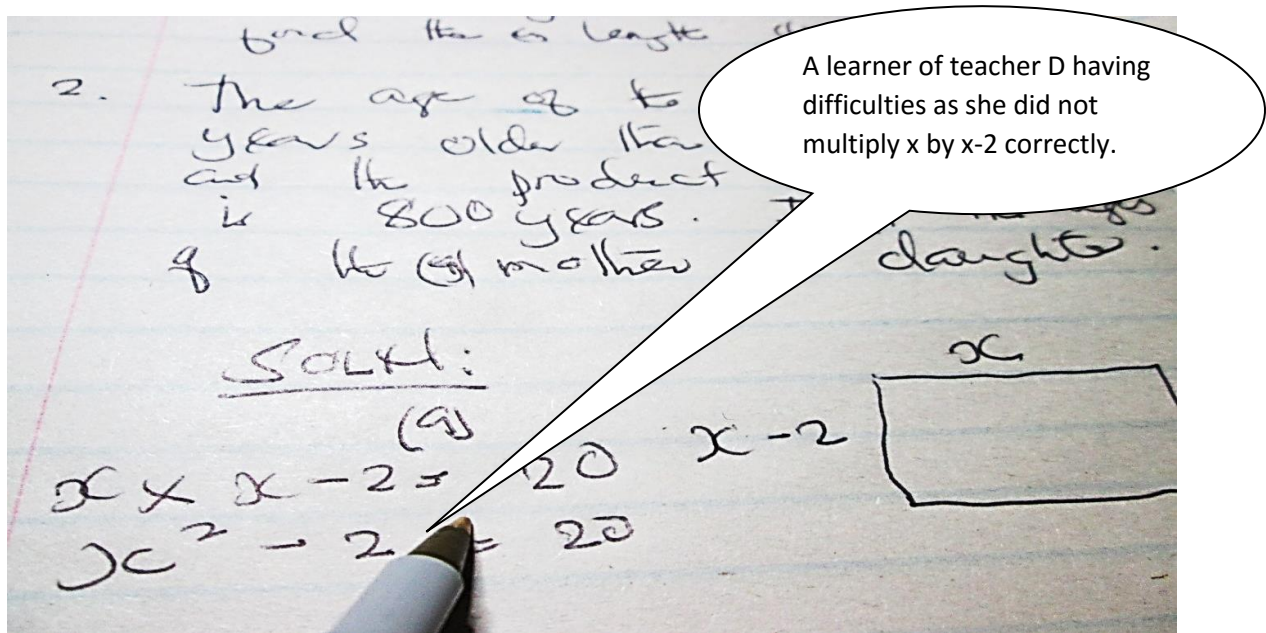


Figure 34: A snap-shot of a learner of Teacher D having difficulties solving a real life situation question

The researcher also asked the learners, “How often do you tackle questions on real life situation?”

Learner CF -1 answered, “Yes, I do sometimes.” (Interview 8, Learner CF-1) which matched with Learner AM -1 when he said, “Not very often.” (Interview 4, Learner AM-1)

Learner DF-3 replied, “I usually see these types of questions when revising for exams.” (Interview 11, Learner DF-3) which also matched with Learner BM-2 when he answered, “As soon as I have the materials containing such, but we do see them in tests and exams.” (Interview 5, Learner BM-2)

Teachers A, B and C seemed not realise the value of teaching on real life situation questions and as a result their learners would never come to appreciate the value of mathematics in real life.

Table 8: Summary of the Difficulties of Learners

S.No	Constraint	Description of Constraint
1.	Solving quadratic equations using factorization and formula method	Around 6 out of 35(17%) learners in each class had difficulties using factorization method because they had difficulties coming up with the correct factors. Those that had problems with the formula method either did not write the correct formula or did wrong substitution or were unable to operate fluently on integers.
2.	Ability to solve quadratic equations was compromised by the use of the pseudo-method. As it was not in conformity with the standard method.	Around 25 out of 35 (71%) learners in each class had difficulties solving quadratic equations because when it came to multiplying or dividing both sides of the equal sign, they got confused and failed to solve the equations.
3.	Solving quadratic equations using completing the square method.	Around 25 out of 35(71%) learners in each class had difficulties solving quadratic equations using completing the square method. Because they did not understand what the method was all about, why 'b' was divided by 2 and why 'c ² ' was added both sides. They lacked conceptual understanding of critical steps.
4.	Solving real life situation questions.	Around 25 out of 35 learners in each class had difficulties solving real life situation questions because they rarely dealt with them in class and only saw them in examinations and therefore, they had not developed their strategic competence to solve them.

Source: Author (2020)

4.5 Themes from Data Analysis

The qualitative data was collected from the document analysis, observation by video-recording and interviews by audio-recording. In the afternoons the same day this data was transcribed and then coded and highlighted to try and come up with themes. Below is a table summarising the themes that were emerging in the research study.

Table 9: Shows the themes that emerged in the data analysis the strand of the OTDMP that was provided

- Description of teaching action, spoken words in interview or analysed item in document	Description of Themes that Emerged	Strand of the OTDMP provided(Christiansen &Ally 2013)
<ul style="list-style-type: none"> - Defining of major concepts and revision of pre-requisite topics - giving of variety of questions in the exercises 	<ul style="list-style-type: none"> - Trying to make new concepts clearer and easy to understand - solving variety enhanced learners understanding 	<ul style="list-style-type: none"> - Conceptual understanding was fostered.
<ul style="list-style-type: none"> - Coherent laying down steps in solving quadratic equations - Giving many questions in the exercises 	<ul style="list-style-type: none"> - Trying to make learners master how to calculate very well when solving quadratic equations 	<ul style="list-style-type: none"> - Procedural fluency was fostered-
<ul style="list-style-type: none"> - Expressing equations in different forms that are equivalent - , esp. on the turning point of parabola 	<ul style="list-style-type: none"> - Trying to make learners see logic and develop the ability to justify 	<ul style="list-style-type: none"> - Adaptive reasoning was fostered.

<ul style="list-style-type: none"> - Giving a variety of questions in the exercises - Giving real life situation questions 	<ul style="list-style-type: none"> - Trying to make the learners come up with plans or strategy on how to solve such questions 	<ul style="list-style-type: none"> - Strategic competence was fostered.
<ul style="list-style-type: none"> - Teachers giving examples on real life situations. - Teachers giving real life situation questions 	<ul style="list-style-type: none"> - Trying to make learners appreciate mathematics in real life 	<ul style="list-style-type: none"> - Productive disposition was fostered

Source: Author (2019)

4.6 Summary of Chapter Four

In this chapter the findings were presented which the researcher used to answer the following research questions:

- (i) How do teachers of mathematics prepare for learner proficiency in Algebraic concepts/?
- (ii) What pedagogical practices do teachers of mathematics engage in for learner proficiency in algebraic concepts?
- (iii) What algebraic difficulties do learners exhibit despite teachers' pedagogical practices for algebraic proficiency?

The rubric meant for the opportunity to develop mathematical proficiency by Christiansen and Ally (2013) was used to facilitate the analysis of data and come up with the themes.

The environmental factors that influenced mathematical proficiency but were not directly involved with actual teaching were analysed under question two as they were more inclined to actual teaching.

The data was analysed under the three episodes of lesson introduction, lesson development and lesson evaluation.

Under research question one the findings revealed that lesson plans were generally not adequately prepared because in the introductions the teachers concentrated on providing the opportunity to develop conceptual understanding. This was evidenced by giving standard definition of concepts and revision of prerequisite topics. In the lesson developments, the teachers concentrated on providing the opportunity to develop procedural fluency, little of conceptual understanding and very little of adaptive reasoning and strategic competence and rarely anything on productive disposition. In most cases, the evaluation exercises were substandard in that they lacked sufficient variety, questions were not extended and real-life situation questions were almost always absent. The learners were also tolerated to use the pseudo-method in solving equations which was not in conformity with the standard method which rendered them incapable to solve higher order equations like 'completing the square method of solving quadratic equations'.

Under research question two, the scenario was more or less the same as the teachers concentrated on providing the opportunity to develop conceptual understanding in introductions and procedural fluency in the development of the lesson but little of the other strands of mathematical proficiency. Under research question two there were teaching methods that were under environmental factors that influenced mathematical proficiency being provided like calling learners to the board to work out questions and putting the learners in groups to work collaboratively. The teachers did not put up visual teaching aids on the walls to provide the opportunity to develop different strands of mathematical proficiency and particularly algebraic proficiency.

Under research question three, only few learners had difficulties with solving quadratic equations using the factorisation and formula method but most of them had difficulties with the completing the square method and real-life situation questions. Their consistent use of the pseudo method by learners compromised their understanding and subsequently their ability to fluently solve some equations. A table that summarised the themes that emerged during data collection was constructed.

CHAPTER FIVE

DISCUSSIONS OF THE FINDINGS

5.1 Overview

The researcher in the previous chapter presented the findings of the study which explored the pedagogical practices that secondary school mathematics teachers use to influence the attainment of algebraic proficiency by learners. According to Simon and Goes (1990) the discussion of findings is an important part of the dissertation which brings the findings to reality by giving reference to the existing knowledge. The discussion should be organised from the general to the specific linking your findings to the literature review, then to the theoretical framework and to practice. Through the use of related literature presented in chapter two, research objectives and the theoretical framework, the researcher will, in this chapter, discuss the research findings. It is the researcher's view that the research objectives are mentioned so as to easily observe whether or not they were met. The research objectives of the study were as follows:

- (i) To ascertain teachers' ability to prepare lesson plans for learners' proficiency in algebraic concepts.
- (ii) To identify mathematics teachers' pedagogical practices when teaching for learner proficiency in algebraic concepts.
- (iii) To determine learners' algebraic conceptual difficulties despite teachers' pedagogical practices for mathematical proficiency.

The findings of this study from all the three categories of instruments namely document analysis, semi-structured interviews and observation were simultaneously discussed based on the themes. This was to avoid repetition and to maintain a steady flow of ideas. This helped in reaching a conclusion when all similar findings were presented at the same time.

5.2 Objective 1: To ascertain teachers' ability to prepare lesson plans for learners' proficiency in algebraic concepts.

The lessons were discussed in three episodes namely the lesson introductions, lesson developments and lesson evaluation. The letters 'a', 'b' and 'c' are with reference to the general quadratic expression: $ax^2 + b x + c$ in the discussion.

5.2.1 Lesson Plans Introductions

The study revealed that all the four mathematics teachers did not adequately prepare their lesson plans to give learners the opportunity to develop proficiency in algebraic concepts. This conforms with what Makewa et al. (2012) found out in his study that there was little thoroughness with teachers' competence when preparing. Teachers blamed lack of text books while others thought the quality of the new text books were the cause for poor lesson plan preparation. This can be testified by the actual words of Teacher C: "*The new books seem to be shallow and we have resorted to using the old books.*" The new books that were produced as a result of the revised curriculum of 2013 were a source of concern as all the four teachers complained about the quality. Yet it could be also that they did not just seem to see the value or did not put in enough effort to prepare lesson plans of standard quality with a view to providing the learners with the opportunity to develop algebraic proficiency. The knowledge of algebraic proficiency is important to a teacher and this is affirmed by Kilpatrick et al. (2002) who contend that mathematical proficiency was what was believed to be attributes necessary for anyone to learn algebra successfully.

The introductions were rather too brief although they were more inclined to promoting the Opportunity to Develop Conceptual Understanding by giving standard definitions and referring to revision of pre-requisite topics.

5.2.1.1 Standard Definition of Concepts and Revision of Pre-requisite Topics

The study revealed that the four teachers provided the Opportunity to Develop Conceptual Understanding in their introductions by defining important concepts and revising previous pre requisite topics. The important definitions that all the four teachers managed to give were of the quadratic functions and equations. All the four teachers were able to state: "A quadratic function is an algebraic expression that can be expressed in the form $ax^2 + bx + c$, where a , b and c are real numbers or constants and $a \neq 0$ and similarly for the quadratic equation as: it is an equation of the form $ax^2 + bx + c = 0$, where a , b and c are constants or real numbers and $a \neq 0$. In the introductions all the teachers gave the standard definition and revised prerequisite topics so as to provide the learners with the opportunity to develop conceptual understanding. This is in line with Kilpatrick et al. (2002), who argued that while many teachers may regard conceptual understanding as superior in contrast to procedural fluency, some scholars still value computational capabilities

highly. Other researchers like Booth et al.(2013) with their conceptual mapping (conceptual understanding) and Hattie (2003) with his advanced organiser (revision of prerequisite topics) converge on supporting the proponents of conceptual understanding being superior to procedural fluency. But the researcher did not agree with both and believed that the two complemented one another to give the best results. But Haas (2005), argues that according to a meta-analysis he carried out explicit instruction (also called direct instruction) is the most effective method of teaching algebra. But there was a tendency by three of the four teachers to value conceptual understanding more than computations. Computational capabilities should not be underestimated as Li and Moschkovich (2013) argues that educational research indicates that structured practice leads to the development of accurate procedural fluency skills which are also highly important to mathematical proficiency and they cannot be acquired in any other way possible. Kilpatrick et al. (2002) emphasised that these strands are interdependent and interwoven for them to provide the best results.

The findings also revealed that most of the times all the four teachers were only able to provide opportunities for the development of only one or two of the strands of algebraic proficiency in particular in any given lesson. This means that the learners could, therefore, only attain algebraic proficiency over time. This is in line with what was discussed in literature review by Kilpatrick et al.(2002) that it is difficult to try and develop the isolated five strands of proficiency in one lesson and therefore the development of algebraic proficiency takes place gradually over months rather than minutes. This also supported the actual words of learner 3-DM when he said, *“If the teacher was to start introductions with simple parts and also show how the equations were derived. And also refer to prerequisite topics then learners would understand more.”* However, a good teacher should endeavor to develop as many strands as he can in a single lesson.

5.2.2 Lesson Plans Developments

The findings revealed that all the four teachers started the development stage of their lesson plans by trying to achieve a clear explanation of what a parabolic curve is by first identifying its main features. This conforms to Christiansen and Ally (2013) who suggested that achieving a clear explanation is providing the opportunity to develop conceptual understanding.

5.2.2.1 The Maximum and the Minimum Turning Point and When ‘a’ is Greater or Less than 0

As the teachers identify the parts of the parabola there are usually frequent justifications such that if ‘a’ is less than 0 ($a < 0$), then the parabola has a maximum turning point and vice versa. By so doing they are actually providing the opportunity to develop adaptive reasoning in learners. This conforms to Christiansen and Ally (2013), who stated in their rubric that frequent justifications made are an Opportunity to Developing Adaptive Reasoning.

Justifications are also made in the arithmetic calculations in solving equations such that if a number is operated on one side of the equal sign therefore it should also be similarly operated on the other side.

5.2.2.2 Completing the Square Method of Solving Quadratic Equations

From the findings, it was revealed that all of the four teachers were able to lay down all the steps involved in solving quadratic equations by completing the square method but did not clearly explain the critical steps in the procedure. Firstly, they did not clearly explain what ‘completing the square method’ entailed. The critical steps of dividing ‘b’ with 2 and also adding the squared ‘c’ to the other side of the equal sign in their lesson plans were not clearly explained.

Around twenty-five out thirty-five of the learners in each class ended up confused and therefore had problems grasping the concept behind this method because the critical steps were not clearly explained. According to the rubric of Christiansen and Ally (2013) the teachers concentrate on only providing the Opportunity to Develop Procedural Fluency, by laying down the steps, could be that some valued procedures more or some lacked some content knowledge on the topic.

5.2.2.3 Factorisation and Formula Methods of Solving Quadratic Equations

The study revealed that all the four teachers in their lesson plans clearly showed how to solve quadratic equations using the factorisation and the formula method. According to the rubric of Christiansen and Ally (2013), there was coherent sequencing of the steps which is providing the Opportunity to Develop Procedural Fluency.

However, all the four teachers did not clearly explain in their lesson plans where each method of solving quadratic equations could appropriately be used in order to provide the Opportunity to Develop Strategic Competence. Some teachers even recommended in their lesson plans that the most suitable method to use for all types of quadratic equations was the formula method. This is really not true because each method is most appropriate for a particular quadratic equation. This is what Gibbs & Simpson (2005), meant when he said that teacher competence in instructional design helps them develop expertise.

5.2.2.4 Real Life Situations

The study also revealed that one out of four teachers prepared lessons on real life situation questions but others did not. They seemed to be in a hurry to finish their work and always left this out in their lesson plans, especially that it always came at the end of every topic. This is consistent with the argument advanced by Gurganus (2018) who contends that teachers seem to be in a hurry to finish the syllabus and scheme of work and as result leave out some important work. It could be that these teachers trivialise this work that they leave out as it comes at the end of the topics.

It is not a surprise that only one teacher provided the Opportunity to Develop Productive Disposition and this is supported by Christiansen and Ally (2013) who found out that from his video-clips recorded of observed lesson only 20 percent conferred to providing the opportunity to develop productive disposition.

5.2.3 Lesson Plan Evaluations

The findings revealed that although all the four teachers gave the evaluation exercises in their lesson plans, not all exercises were of a standard quality. In case of teacher C and D the exercises lacked sufficient variety in questions, the types left out were of the form $ax + \frac{b}{x} + c = 0$ and sometimes they were not in the standard order of simple to complex questions. It was also discovered that in their entire lesson plan evaluations, some type of quadratic equations ‘in disguise’ were even left out. One typical example was one of the type: $ax + \frac{b}{x} + c = 0$. This is in contrast to Kilpatrick et al. (2002), who argued it is very important to ensure that the questions in the exercises are extended to show how much learners have understood of the concept.

The real-life situation questions are almost always in the last section of evaluation exercise. The sheer fact that this sub topic comes at the end of the topic of “Quadratic functions and Equations’, it is left out by most of the teachers.

5.3 Objective 2: To identify mathematics teachers’ pedagogical practices for learner proficiency in algebraic concepts

5.3.1 Introductions in Lesson Teaching

The study findings revealed that all the four teachers started the lessons by first trying to explain the main concepts in the lesson and also to revise the pre-requisite topics. These main concepts are the standard definition of the quadratic function and equation.

5.3.1.1 Standard Definition of the Quadratic Function or Equation

The study revealed that explanation of the main concepts by all the four teachers was usually done by giving the standard definition of the quadratic function or equation. This is in line with the rubric of Christiansen and Ally (2013) by giving the clear explanation of a concept the teachers are providing the opportunity to develop conceptual understanding. The standard definition of a quadratic function most teachers give is: “an expression in the form of: $ax^2 + b x + c = f(x)$, where a, b and c are constants and $a \neq 0$. And for a quadratic equation as: $ax^2 + b x + c = 0$, where a, b and c are constants and $a \neq 0$ ”.

Unlike in the lesson plans most teachers were able to explain why ‘ a ’ is not supposed to be 0. They explain that if $a = 0$, then the expression ceases to be a quadratic function or equation but it becomes a linear function or equation of the form $f(x) = b x + c$ or $b x + c = 0$. The concepts which were not clear in the lesson plan were usually clarified when teaching.

5.3.1.2 Revision of Pre-requisite Topics

The study revealed that all the teachers revised the pre-requisite topics. By so doing they tried to make connections with other concepts previously taught and they were therefore promoting the opportunity to develop conceptual understanding and this is in line with Christiansen and Ally (2013) who stated that connection to other concepts provided an Opportunity to Develop Conceptual Understanding.

The main prerequisite topic that was revised was the factorisation method of solving quadratic equations that was previously introduced in grade ten. Next the learners were then taught that there were other quadratic equations whose roots or values of 'x' were not whole numbers and therefore can only be solved using 'completing the square' method or the formula method. In this case a calculator had to be used to find the square roots of the discriminant.

5.3.2 Developments in Lesson Teaching

All the teachers went into the development stage of the first lesson by explaining clearly the main features of the parabolic curve (refer to snap-shot 1A). By doing so they are actually providing the opportunity to develop two strands at the same time namely conceptual understanding and adaptive reasoning.

5.3.2.1 The Maximum and the Minimum Turning Point and when 'a' is Greater or Less than zero

The study revealed that all the four teachers explained clearly that if 'a' is < 0 , then the parabola has the maximum turning point. If it had $a > 0$, then it has the minimum turning point, but they all did not clearly explain why it is called the turning point (video clip 1A). The study also revealed that when the teachers reached the development stage, they tended to concentrate more on providing the opportunity to develop procedural fluency and tended to leave out some important conceptual explanations and this is what could have happened. This was in support of the study done by Christiansen and Ally (2013) who found out that 90 [percent of the 243 video-recorded five minute lesson segments from fifty lessons conferred opportunities for developing procedural fluency with only 17 percent for conceptual understanding, 8 percent for adaptive reasoning and less than 2 percent for strategic competence and 20 percent for productive disposition.

This showed that it was possible to promote more than one strand from one concept under study. The teachers were supposed to explain that part was called the turning point by because as one moved along the parabolic curve one reached the tip (turning point) before one turned and started moving in the opposite direction.

5.3.2.2 Axis of Symmetry, x and y-Intercepts Connected to Solving Equations

The study revealed that all the four teachers also explained what the y-intercepts and x-intercepts were on the graph of a parabola and even connected it to the topic of solving quadratic equation by using the graph. They were also able to draw a parabolic curve that cuts through the x-axis and they-axis and explained that the x-coordinates of the points where the graph cuts that x-axis are the roots and the y-coordinate of the point where the curve cuts the y-axis was the y-intercept. They were also able to explain the axis of symmetry. Here there was a connection to other concepts which were a promotion of the strand of conceptual understanding as elaborated in the rubric of Christiansen and Ally (2013) in the literature review.

The graphical method is actually the fourth method of solving the quadratic equations and therefore requires drawing the graph of a parabolic curve. The x-intercepts or roots are the values of x that we get when we solve the quadratic equations. This method is not ideal in most cases because it is long as it requires drawing the table of values and the parabolic curve as well.

5.3.2.3 Completing the Square Method

The study revealed that all the four teachers had problems in explaining clearly all the concepts and steps involved in ‘completing the square method’. Firstly, they did not clearly explain what ‘completing the square method’ really involved. They should otherwise have explained that by completing the square it meant solving for x or making x the subject of the formula. What the teachers actually did was only to lay out the steps involved without clearly explaining the steps, especially the critical step of dividing the coefficient of x by 2 and adding the squared ‘c’ to the other side of the equal sign.

The main reason most learners had some difficulties was because most of them used the pseudo method when solving the equations. They had not yet grasped the main concept of solving equations from the fundamental law of solving equations which states that whatever you do to one side of the equal sign you should do to the other side as well. The learners who understood very well the concept of the fundamental law did not have problems solving quadratic equations using completing the square method.

5.3.2.4 Appropriate Method for Solving Particular Quadratic Equations

The study further revealed that all the four teachers did not clearly indicate exactly where each method was most appropriate to use. Coming up with an appropriate method for solving a particular quadratic equation is part of multiple Heuristics to solving problems which is under strategic competence as discussed in literature review by Christiansen and Ally (2013) in their rubric for OTDMP. Teachers B and C even indicated to their learners that the formula method was the best method to use for all types of quadratic equations.

The four methods for solving the quadratic equations are: factorisation method, completing the square method, formula method and graphical method. Each method is most suitable for a particular type of a quadratic equation. For example, if the two factors can easily be found such that if they are multiplied, we get 'c' and if added we get 'b' then the best method would be the factorisation method. Then, there is no need to go on and use the formula method which would be longer and tedious. Shulman (2019), clearly explained that mathematics teachers ought to be equipped with pedagogical content knowledge to be able to understand the content very well and also how to teach it.

5.3.3 Evaluation of Lesson Exercises

At the end of each lesson, all the teachers set the exercises and went around the classroom marking and correcting the learners' work. Most of the times the learners were given the home work. Just as in the lesson plans, the exercises set were usually below standard and notably the real-life situation questions were missing most of the times.

5.3.3.1 Real life Situation Questions

The study revealed that only Teacher D taught on real life situation questions. These were questions that require the learners to use their strategic competence because a learner had to strategically plan on how to solve such problem questions. For one reason or another teachers A, B and C had left out this part of the topic. Maybe because it was at the end of the topics and they regarded it as trivial, and therefore a sheer waste of time. This agrees with Berry & Thunder (2012), who says that teachers seem to be in a hurry to cover the scheme of work and as a result they leave out some work.

According to Christiansen and Ally (2013), the real-life situation questions not only provide the Opportunity to Develop Strategic Competence but also productive disposition. By giving them the real-life situation questions, the learners learn to appreciate mathematics in real life. They also come to realise that mathematics is not just a boring discipline that deals with meaningless numbers but a discipline that is very real.

5.3.3.2 Environmental Factors Influencing Mathematical Proficiency

According to Schoenfeld (2008), environmental factors are factors other than actual teaching that influence mathematical proficiency. The environmental factors simply prepare the mind of the learner to be more receptive to mathematics before, during and even after the lesson. This can be done by calling the learners to the board, putting them in groups and even putting up the visual teaching aids on the walls among others.

5.3.3.2.1 Calling of Learners to The Board

The study revealed that all the four teachers made an effort to call the learners to the board to work out some questions. Teacher B related, *“I call the learners to the board so that I can involve them in learning and put the learner at the centre of learning.”* This in line with Edelman (2017), who declared that ‘to teach mathematics for understanding, students should learn it by doing and not as passive recipients of already-made mathematics’. The learners are supposed to be treated as active participants in the learning process in which they themselves, with the guidance from the teacher, develop all sorts of mathematical tools as well as insight. Most of the teachers put in effort to try and adapt to the revolutionary method of teaching by putting learners at the centre of learning. This is supported by what teacher B said, *“I think you saw me call the learners to the board so as to involve them and make them the centre of learning”*.

The other reason was that if their fellow learners were able to work out the questions, it gave them confidence to work out the question as well. It is also important to ensure that not only the same learners were chosen to come to the board but all the learners had been given the opportunity to work on the board.

5.3.3.2.2 Putting of Learners into Groups

The study revealed that all four teachers managed to put the learners into groups. When teacher C was asked whether she put the learners into groups to work collaboratively, she said, “*We sometimes let them work in groups so that they can learn from each other where they fail to learn from the teacher. Myself I just guide them through.*” This is supported in theoretical framework by Vygotsky (1978), who says that in most pedagogies based on constructivism, the teacher’s role is not only to observe and assess but also engage with the students while they are completing activities, wondering aloud and posing questions to the learners for promotion of reasoning.

Mathematics is a subject that requires doing in order to learn effectively. The teachers have come to learn that learners do not have to be passive learners but active and taking part in the learning process. Some learners feel very free to ask fellow friends questions instead of teachers thus promoting work.

5.3.3.2.3 Display of Visual Teaching Aids on Walls

The study also revealed that all the four teachers did not display the visual teaching aids on the wall. They all seemed to under rate the value of visual teaching aids on the walls. This is in contrast to Rasul et al. (2011), who declared that visual aids are devices which are used in the classroom to encourage students’ learning process and make it easier and interesting.

The visual teaching aids on the walls are very important because the learners can learn from them any time of the day. They are also very convenient to the slow learners who take time to digest what has been taught in order to understand. According to Rasul et al. (2011), many people in their life time have learnt some things from the teaching aids on the walls without the assistance of a teacher.

5.4 Objective 3: To investigate learners' algebraic conceptual difficulties despite teachers' pedagogical practices for mathematical proficiency

5.4.1 Solving Quadratic Equations Using Factorisation and Formula Method

The study revealed that there were always few learners in each class who had not yet mastered how to solve quadratic equations using the factorisation method. These were usually the same learners who also had problems with operating fluently on the integers.

Actually, the main reason why some learners had difficulties with the factorisation method was because they could not easily find the two integers such that if they added them up, they get 'b' and if they multiplied them, they get 'c'. These were usually learners who could not fluently operate on the integers. It was a phenomenon that the researcher felt should be investigated where by ten to twenty five percent of learners in some classes could not operate fluently on the integers. Learners who had not mastered how to operate on the integers could not manage to solve quadratic equations using factorisation method. This is contrary to Kilpatrick et al. (2002) who says that immediately learners are taught how to operate on the integers they are introduced to algebra because the learners seem not to have been taught how to operate on the integers.

The main problem the learners had with the formula method was not writing the formula correctly when the formula was introduced, but later on after some practice almost all of them managed to write it correctly.

At this stage the researcher was compelled to suggest that a look at how teachers developed a good foundation in learners, to influence mathematical proficiency, would be worth investigating in future. This could be a prospect for future research.

5.4.2 Solving Quadratic Equations Using 'Completing the Square' Method

The study revealed that around twenty-five out of thirty-five learners in each class had many difficulties solving quadratic equations using 'completing the square' method. The learners were usually given the procedures in terms of the steps involved without a backing from conceptual explanations. So, when the learners were given the exercises for evaluation, most of them did not manage to solve them using completing the square method. Therefore, the teachers tried to adjust to this situation and went around assisting learners by explaining, prompting and correcting

individual learners (video clip 2C). This is supported by Shulman (1987), who suggested that every good teacher should develop some kind of natural talent to be flexible so as to easily adjust and teach in whatever circumstance.

As explained earlier, the teachers did not clearly explain the critical steps involved in this method. Firstly, they should have explained what completing the square method was just from the title itself. Secondly, they should have explained the critical step of dividing 'b' by 2. All the teachers should make sure to explain this part clearly with several examples until they were sure that the learners had grasped the concepts.

5.4.3 The Pseudo-Method for Solving Equations

The findings also revealed that there were some learners who had got used to using the pseudo-method for solving equations. This had an adverse effect on them in that later they could not solve equations that were slightly complex as the pseudo-method did not conform with the standard way of solving equations. According to the pseudo-method, when a term goes to the other side it changes the sign to the opposite of what it is. The pseudo method affected the learners' procedural fluency in the long run. Higher order equations that required more manipulations to both sides of the equal signs became difficult for them to solve. The reason was that they had not grasped the concept of solving equations by following the 'Fundamental law of Solving Equations', which states that 'whatever you do to one side of the equal sign do to the other as well'.

The pseudo method has proved to be very popular with most learners in secondary schools. This is because the pseudo method seems to be a swifter and quicker way of solving equations than the standard one of adding additive inverses. However, the pseudo-method seems to foster procedural fluency only and not conceptual understanding. It is also important that those using it should be aware that it is a pseudo method that cannot take them very far.

Below is an example of a typical equation that was solved using a pseudo-method in A and then in B the standard method was used. The equation is as follows:

$-2x + 5 = 4x + 17$ (A) The pseudo- method goes thus: when $4x$ goes to the other side (but how?) it becomes $-4x$ and when $+5$ goes to the other side it becomes -5 . The equation then looks like this:

$-2x-4x = +17-5$.5.3.3 (5.1) (But one may ask as to how it goes to the other side, the answer one usually gets is by collecting like terms. There is no collecting like terms when solving equations. Otherwise this method creates even more confusion about the concept.) And then $-6x=12$ and the finally dividing by -6 : (This method only supports procedural fluency and not conceptual understanding.)

$$\frac{-6x}{-6} = \frac{12}{-6} = -2 \quad (5.2)$$

Despite the answer being correct the method only supports procedural fluency and not conceptual understanding.

In (B) The second method also supports the conceptual understanding and seems to be a standard method and is shown below. In this method the equal sign is traditionally regarded as a balance and anyone solving an equation should maintain the balance by doing whatever operation is done on one side should also be done on the other side.

$$-2x + 5 = 4x + 17 \quad (5.3)$$

$-4x$ $-4x$ (conceptual understanding= subtracting $4x$ on both sides of the equal sign)

$$-6x + 5 = 0 + 17$$

(Conceptual understanding requires subtracting 5 from both sides of equal sign)

$$-6x + 5 = +17$$

$$\underline{\hspace{1cm} - 5 \quad - 5} \quad (5.4)$$

$$-6x + 0 = +17 - 5$$

$\frac{-6x}{-6} = \frac{12}{-6} = -2$: $x = -2$ (conceptual understanding requires dividing by -6 on both sides of the equal sign.)

$$(5.5)$$

The first method does not follow the concept of solving equations as a balance by manipulating in both sides always and lacks logic and conceptual understanding. This lack of conceptual understanding renders one incapable of solving more complex equations.

5.4.4 Solving Real Life Situation Questions

The findings also revealed that only Teacher D taught on real life situation questions and around thirty out of thirty five of the learners had some difficulties solving real life situation problem questions. As we had seen real life situation questions were important in that they promoted two strands of mathematical proficiency, namely strategic competence and productive disposition. Unfortunately, three mathematics teachers tended to leave out this part of the topic probably because it came at the end of the topic and they regarded it as trivial. From the literature review this was similar to what Amirali (2010) found out that in attempt to get through the book in basal programs resulting concerns are a lack of adequate practice review, unsound sequence of problems and an absence of strategic and step by step procedures for backing problem solving.

Learners who had only encountered real life situation questions in the tests and examinations had difficulties solving them because they had no prior experience to solving such problems. When the researcher asked whether she was taught on real life situation questions, Learner BF-2 complained: *“We only see such questions only in the tests and examinations and since we they are not given to us to practice in class, they are very difficult.”* This is supported by van Stiphout et al. (2013) who argue that educational research indicates that structured practice results in development of accurate procedural fluent skills which are also highly crucial for mathematical proficiency and they cannot be acquired in any other way possible.

Even some learners who were taught on real life situation questions had problems solving them as they required using strategic competence ability, which was lacking because they rarely solved such type of questions in class.

5.5 Summary of Chapter 5

In this chapter, an effort was made to interpret the findings obtained and it also demonstrated why the findings were relevant to the research and to other researches carried out by other researchers.

CHAPTER SIX

CONCLUSIONS AND RECOMMENDATIONS

6.1 Introduction

In chapter four the researcher presented the findings which were later discussed in chapter five to ensure that the research questions raised in chapter one was addressed. In this chapter, the researcher presents the summary of the main findings which answered the research questions and finally recommendations and prospects for future research are given.

6.2. Conclusion

This is the final chapter of the study. It is important to note that the study explored the pedagogical practices of secondary school mathematics teachers in Livingstone district and how they influenced the ability of the learners to grasp knowledge of algebra. The researcher wanted to establish if pedagogical practices used by teachers contributed to high algebraic proficiency in learners. The researcher in the subsections below presented the main findings as mirrored by the research questions.

6.2.1 Teachers' Preparation for Learners Proficiency in Algebraic Concepts.

Generally, the study showed that lesson plan preparations were not of very high standard to promote all the attributes of mathematical proficiency in algebraic concepts. This was because the introductions were too brief and mostly provided the opportunity to develop conceptual understanding. The lesson developments of the lesson plans mostly provided the opportunity to develop procedural fluency with little backing from conceptual understandings. In the lesson developments there was little or no provision of the opportunity to develop other three strands of mathematical proficiency namely adaptive reasoning, strategic competence and productive disposition. The evaluation exercises were also below standard in that the formulated questions lacked variety and were not extended although most of the teachers managed to prepare the questions in the order of the simplest to the most complex. Generally, teachers did include real life situation questions in their evaluation exercises.

6.2.2 Teachers' Pedagogical practices for Learners' Proficiency in Algebraic Concepts

Overall, the study showed that there are two types of teaching practices. There were teaching practices that were directly connected to teaching and those that were not directly connected and were included in a category of environmental factors. The environmental factors included calling learners to the board and also putting them into groups to work collaboratively. One important environmental factor that most teachers did not utilise was the use of the visual teaching aids on walls to facilitate their learning of algebraic concepts. It appears that teachers underestimated the value of wall visual teaching aids in influencing algebraic proficiency.

The teaching practices, that involved actual teaching, included giving standard definitions of certain concepts and revision of pre-requisite topics when introducing the topics and these promoted the development of conceptual understandings. In the development stage there was providing the opportunity to develop procedural fluency. There was usually very little promotion of the other three strands of mathematical proficiency namely, adaptive reasoning, strategic competence and productive disposition. At the end of the lessons, the evaluation exercises were always set, although not of very high standard. The four teachers then managed to go around marking, correcting and prompting. Home work was usually given as well.

6.2.3 Conceptual Difficulties Encountered by Learners

There were several difficulties that the learners had despite the teachers' pedagogical practices for mathematical proficiency. Only around ten out of thirty five learners had problems with solving quadratic equations using factorisation method because these learners had not mastered how to operate on integers. Generally, learners had no difficulties solving using the formula method except for a few who initially made mistakes in writing the general quadratic formula correctly. Apart from that, most learners had difficulties with completing the square method as they usually failed to grasp the critical concepts involved.

The use of the pseudo method by most learners when solving the equations compromised the learners' conceptual understanding of solving the equations and therefore the quadratic equations. This was because it did not conform to the standard way of solving equations. The teachers seemed to tolerate the use of pseudo-methods by the learners instead of sticking to the 'Fundamental law

of Solving Equations'. Most learners also had difficulties in solving the real-life situation questions because they rarely learnt such in class and only saw them in the tests and examinations.

6.3 Recommendations

The findings of the study pin–pointed some issues that can be looked at so as to improve the quality of teaching and learning algebra. The study therefore makes the following recommendations.

6.3.1 On Policy

The government through the Ministry of Education should enhance the existing policy on CPD programs on algebra teaching at zonal, district and provincial levels.

6.3.2 On Teaching Practice

The Senior Education Standards Officer (SESO) of mathematics in the province should make sure that all the secondary school teachers study critical topics in algebra and more especially the topic of 'Quadratic functions and Equations' in the CPDs and workshops to address the problems learners have in algebra.

Through the CPDs the SESO should ensure that teachers are enlightened on the disadvantages of their learners using the pseudo-methods when solving equations.

The SESO should also advocate for the effective use of wall teaching aids through CPDs. The wall teaching aids should display concepts, working procedures and sketches of parabolic curves to facilitate learning.

6.3.3 Prospects for Future Research

After the findings and recommendations, the following suggestions were made for future research.

1. Mathematics Teachers Practices in Developing Sound Foundation for Learners to Achieve Algebraic Proficiency.
2. Effects of Interactive Methods on the Teaching of Algebra for Proficiency

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APPENDICES

Appendix A: Observation Schedule

NAME OF SCHOOL.....

DATE:.....

CODE FOR TEACHER:.....

TIME:.....

SUBJECT:.....

TOPIC:.....

The following letters will be used to grade an observation: A-very good, B- good, C-average, D- below average.

	Environmental and Personal Representation of the Teacher	A	B	C	D
1.	Was the teacher dressed casual smart?				
2.	Was the mood of the teacher cordial to learners?				
3.	Did the teacher greet the learners?				
4.	Is the classroom donned with mathematics teaching aids?				
5.	Did the teacher put up on the board a “Maths” heading?				
6.	Was the blackboard neat and well divided before the lesson?				
7.	Did the teacher write the title of the topic and the subheading properly?				
8.	Were the pupils quiet and receptive before the lesson started?				
9.	Did the teacher give some compliments to learners on cleanliness or good manners?				

10.	Did teachers provide learner with opportunities to work independently and collaborately?				
11.	Did teachers attend to differing needs of learners?				
12.	Did teachers allow the learners to derive a positive attitude to mathematics?				
13.	Did teachers plan mathematics learning experiences that enabled learners to build on their existing proficiencies, interests and experiences?				
	Introduction of the Topic and use of the Lesson Plan	A	B	C	D
1.	How good was the introduction?				
2.	If it is a continued topic was there reference to previous topic?				
3.	Was the introduction motivating to the learners?				
4.	If examples and teaching aids were used in the examples, how suitable were they?				
5.	How well did the learners respond to introduction?				
6.	How well prepared was the introduction from the lesson plan?				
7.	How well written was the introduction summary on the board?				
8.	Did the learners seem to have fully understood the introduction?				
9.	Were there learners who seemed confused and had not grasped the concept?				
10.	How clear was the concept to the learners?				

	Strive for Mathematical Proficiency at the Lesson Development stage.	A	B	C	D
1.	Did the teacher try to touch all the strands of mathematical proficiency as he was teaching?				
2.	Did the teacher manage to concentrate on one strand for the learners to grasp?				
3.	How good was the teacher in explaining the ideabehind (concept)?				
4.	How good was the teacher in teaching the working format(procedure)?				
5.	Did the teacher manage to bring in questions on how to solve problems using equations (strategic competence)?				
6.	Did the teacher manage to test the learners on logic or common sense (adaptive reasoning)?				
7.	Were the questions on real life situations (productive disposition) considered in the lesson?				
8.	Was the teacher able to touch more than two strands of mathematical proficiency in a lesson?				
9.	From the class exercises how well did the learners grasp the proficiency strand the teacher aimed to teach?				
10.	How much of the content did the learners grasp from what the teacher had taught as indicated in the performance in the exercises?				

Appendix B: Semi- Structured Interview Schedule for A Teacher

CODE NO:.....

LOCATION/SCHOOL:.....

SEX OF INTERVIEWEE:.....

DATE:.....

Introduction

In this interview with you I would like to learn from you about many problems you encounter as you strive to teach at best of your ability. Firstly, I would like to know your learners attitudes towards mathematics. What is the major problem you have in teaching these learners? I would also like to find out the methods you use to enable the learners readily grasp the concepts. I would like to know the most important points you find easiest to teach? I would like find out the main techniques you employ to ensure that the learners grasp all the main points of mathematical proficiency. Your name will not be revealed to any one and what you talk about will only be shared with government officials from the Ministry of Education, the University of Zambia and other people interested in the betterment of education. You are also free to withdraw from the study at any time. Before we proceed, I would like to get your permission for me to conduct and record the conversation with you (read Consent of agreement)

Learners Attitudes Towards Mathematics (*these are only leading questions*)

1. Do most of your pupils enjoy learning mathematics?
2. What is the main complaint that these learners have about learning mathematics?
3. What can you say are the problems for those learners who say that mathematics is a difficult subject?
4. What do you think should be done to make learners develop a positive attitude towards the learning of mathematics?
5. Do you think a positive attitude towards mathematics can be the first step towards developing mathematical proficiency?
6. Can you describe the type of learners who usually have negative attitudes towards learning of mathematics?

7. How do you ensure that the caring class that is focused on mathematical goals?
8. Do you ensure that the classroom walls are donned with charts of mathematics teaching aids?
9. Do you take time to attend to differing needs of the learners?
10. How do you allow learners derive a positive attitude to mathematics?
11. How do you plan mathematics learning experiences that enable them build on their existing proficiencies, interests and experiences?
12. Do you attend to learners' differing needs, and if so how?
13. Do you sometimes allow your learners to work collaborately or in groups?
14. What are the major challenges do you have in preparing your lesson plans?

Teachers' practices with the aim of attaining Mathematical Proficiency

1. What is the main point or item of the idea behind a mathematical manipulation do you find easiest to teach?
2. What is the main point or item of working format or procedure do you find easiest to teach?
3. What is the main point or item of solving problems using equations do you find easiest to teach?
4. What is the main point or item of using common sense do you find easiest to teach?
5. What is the main point or item of real life situation questions do you find easiest to teach?
6. How good are the learners in grasping the ideas behind mathematical manipulations (concepts)?
7. How good are the learners in being able to carry out the working procedures or processes?
8. How often do you test your learners on solving problems by first formulating equations? (strategic competence)
9. How do you train your learners to use logic or common sense in reasoning? (Adaptive reasoning).
10. How often do you train your learners to use mathematics to solve problems that they encounter in real life? (Productive Disposition).
11. How do you use variation of methods in your teaching practice?
12. How has your participation in CPDs helped improve your teaching abilities?

Appendix C: Semi-Structured Interview Schedule for Learners

CODE NO:.....

LOCATION/SCHOO:.....

SEX OF INTERVIEWEE:.....

DATE:.....

Introduction

In this interview with you I would like to learn from you about how much you like learning mathematics. I would also like to find out if you find mathematics easy going. I would also like to find out the main problems that you have in mathematics. Do you have particular problems in grasping the ideas behind each mathematical manipulation the teacher does? Are you good at calculating and working out mathematics questions? How often do you tackle questions that require solving problems that require formulating equations? I also want to find if you are able to manage all what is required for standard learning of mathematics (the mathematical proficiency strands) including common and logic Reasoning and real life situation questions. Your name will not be revealed to anybody and what you talk about will only be shared with government officials from the Ministry of Education, the University of Zambia and other people interested in the betterment of education. You are also free to withdraw from the study at any time. Before we proceed, I would like to get your permission for me to conduct and record the conversation with you (read Consent of Agreement)

Attitudes Toward Mathematics

1. Tell me about your experiences of learning mathematics?
2. What is it that you find easy in the learning of mathematics?
3. What is it that you find very difficult in the learning of mathematics? Do you think mathematics is a difficult or easy subject and why?
5. What do you think should be done to make many learners enjoy learning mathematics?
6. Why do you think people dislike mathematics?

What Opportunities for the Development of Mathematical Proficiency are made available?

1. How good are you at grasping the main ideas behind (concepts) in mathematics?
2. How good are you at managing the methods of working (procedures) in mathematics?
3. Are you able to solve problems by formulating equations? (carry out the strategic competence questions)
4. How often do you tackle questions on logic or common sense (Adaptive) Reasoning?
5. How often do you tackle questions on everyday life situations? (Productive Reasoning)
6. Do you think your teacher is striving hard at making you understand mathematics very well (reach mathematical proficiency)?
6. How do you think your strive to understand mathematics more with standard ability (mathematical proficiency) has helped you learn mathematic better?
7. Do you think someone who understands mathematics better with standard ability (is proficient in mathematics) enjoys mathematics more?
8. How do you think understanding mathematics better with standard ability (being proficient in mathematics) will help you in future?

Learners Recommendations on how the Teacher can help provide opportunities for developing mathematical proficiency?

1. What is it that you feel the teacher can do to help learners like mathematics?.
2. Is there anything that you would want changed on the way the teacher introduces the topics?
3. Is there any recommendation on the way you want the teacher explain meanings behind a mathematical idea? (concepts in mathematics)
4. Do you feel the teacher teaches well on methods of solving complex problems using formulation of equations?(strategic competence)
5. Do you think teachers are doing enough on teaching logic or common sense in problem solving? (adaptive reasoning)
6. Do you think teachers give you question on real life situations to make you appreciate mathematics? (productive disposition questions)

Appendix D: Consent of Agreement Signed by Participants in the Study

Instructions and Conditions

Your name will not be revealed to anybody and what you talk about will only be shared with government officials from the Ministry of Education, the University of Zambia and other people interested in the betterment of education. You are free to withdraw from the study at any time and the records of our conversations will be destroyed at the end of the study.

Tick your choice.

I Accept

I Refuse

I have understood the instructions and conditions concerning the study by Mr S. Malubila and I agree to participate as he asks. I also understood that I am free to withdraw from the study at any time and that the records of our conversations will be destroyed at the end of the study.

Sign:.....

Date:.....

Appendix E: Permission Letter

MACHA GIRLS SECONDARY SCHOOL,
P.O. BOX 630316,
CHOMA,

10TH January, 2017

THE DISTRICT EDUCATION BOARD SECRETARY,
MINISTRY OF GENERAL EDUCATION,
P.O BOX 630035,
CHOMA.

U.F.S
THE HEADTEACHER,
MACHA GIRLS SECONDARY SCHOOL,
P.O. BOX 630316,
CHOMA.



Dear Sir/Madam

REF: PERMISSION TO GO TO LIVINGSTONE TO ARRANGE FOR MY RESEARCH PROGRAM

With reference to the subject stated above. I am a teacher at Macha Girls Secondary School, TS Number 805106. I am planning to go to Livingstone town to arrange for my research program at some two schools. I plan to go on 10th January, 2017 and come back the same day.

I am kindly asking you for permission to allow me to go to Livingstone.

Your positive response will be highly appreciated,

Yours Faithfully,

Stanslous Malubila.