THE UNIVERSITY OF ZAMBIA

UNIVERSITY EXAMINATIONS - 1998/99
FIRST SEMESTER
SCHOOL OF NATURAL SCIENCES

1.	BS 211-		Molecular Biology And Genetics - Theory - Def/sup
2.	BS 331-		Theory - Def/sup
3.	M 111	-	Mathematics - Def/sup
4.	M 111	-	Mathematics Methods 1
5.	M 161	-	Mathematics
6.	M 211	-	Mathematical Methods 111
7.	M 211	-	Mathematical Methods 111 - Def/sup
8.	M 221	-	Linear Algebra 1
9.	M 231	-	Real Analysis 1
10.	M 241	-	Introduction to Computer Programming
11.	M 261	-	Introduction to Statistics
12.	M 335	-	Topology
13.	M 361	-	Mathematical Statistics
14.	M 361	-	Mathematical Statistics - Def/sup
15.	M 411	-	Theory of Functions of A Complex Variable 1
16.	M 411	-	Theory of Function of A Complex Variable 1 - Def.
17.	M 421	-	Structure and Representation of Groups
18.	M 421	-	Structure And Representation of Groups - Def/Sup
19.	M 431	-	Real Analysis V

20.	M 431	-	Real Analysis V - Def/Sup
21.	M 461	-	Multivariate Analysis
22.	M 465	-	Non Parametric Methods
23.	M 911	-	Mathematical Methods V
24.	M 981	-	Numerical Analysis 1

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF BIOLOGICAL SCIENCES

1998/99 SEMESTER I DEFERRED/SUPPLEMENTARY EXAMINATIONS

BS 211: CELL MOLECULAR BIOLOGY AND GENETICS

THEORY PAPER

INSTRUCTIONS:

TIME: THREE HOURS

ANSWER: FIVE QUESTIONS, TWO FROM SECTION A AND TWO FROM SECTION B, AND ONE FROM EITHER SECTION A OR SECTION B.

ANSWER SECTION A AND SECTION B IN SEPARATE BOOKLETS.

SECTION A : CELL MOLECULAR BIOLOGY

1. With the help of clearly labeled diagrams, explain briefly what led researchers from the Unit Membrane Hypothesis to the Fluid Mosaic Model of the biological membrane.

2. Write short concise notes on the following topics :

   (a) Difference between prosthetic groups and coenzymes.

   (b) Forces involved in the binding of the substrate to the active site of its enzyme.

3. Explain in detail how external factors influence the activity of an enzyme.

4. Explain in detail how the inherited condition of sickle-cell anemia exemplifies the significance of the protein primary structure.

5. With the help of clearly labeled diagrams, explain in detail why cell walls have been rather aptly compared to reinforced concrete.

SECTION B : GENETICS

D/S 98/99

1. (a) Describe chromosome behaviour in Prophase of meiosis I.
(b) Discuss the genetic consequences of meiosis.

2. Discuss the genetic and environmental issues surrounding obesity in humans.

3. Discuss the double helical structure of DNA. Show its significance in the inheritance of genetic information.

4. Discuss diversity, evolution and species formation.

5. Discuss assumptions related to the Hardy-Weinberg (H-W) model. Demonstrate how the H-W law can be used to measure gene frequencies or genotype frequencies.

-END-

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF BIOLOGICAL SCIENCES

1998/99 SEMESTER I DEFFERED/SUPPLEMENTARY EXAMINATIONS

BS 331 THEORY PAPER

TIME : THREE HOURS

INSTRUCTIONS : ANSWER FIVE QUESTIONS

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1. (a) By means of an equation, express the relationships between volume flow density, Jv, of water passing through the stem and the water potential gradient, $\Delta\psi$, the hydraulic resistance, R, of the conducting tissue and the cross section area, A, of the conducting tissue.

   (b) In an experiment to study coupling of leaf water potential to transpiration in maize seedlings, hydraulic resistance of the root, Rr, was found to be 1.3×10^9 MPa S m^{-3} for a root surface area of 7.5×10^{-3} m^2.
   Calculate the hydraulic conductivity , Lpr, of the root.

2. What is the significance of electrical potentials in the ionic relations of plant roots? Support your answer with appropriate equations.

3. Light drives photosynthesis and also regulates it. What mechanisms are involved in the regulation of the pentose reduction cycle by light?

4. One method by which the solute potential of plant cells may be estimated is that of "limiting plasmolysis".

   (a) Under what circumstances is this method applied?

   (b) What are the limitations of this method?

   (c) Briefly describe how you would use the method to estimate the solute potential of plant cells.

5. Outline the major evidence which established the involvement of two light reactions in photosynthesis.

6. Write on the physiological role of calcium (Ca^{2+}) ions in plant cells.

7. How does light affect the growth of plants other than through its roles in photosynthesis?

8. How do chloroplast antenna pigments capture light energy in photosynthesis and transfer it to the reaction centres?

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
UNIVERSITY EXAMINATION FIRST SEMESTER DEFERRED / SUPPLEMENTARY - 1998 /99
DISTANCE EDUCATION

MATHEMATICS M111

INSTRUCTIONS : Answer any FIVE (5) questions .

TIME ALLOWED : Three (3) Hours .

1. a) Given that x is an integer value, solve the following:

   i) $\sqrt{x+5} - 2 = \sqrt{x-7}$

   ii) $\left(x - \frac{2}{x}\right)^2 + 4\left(x - \frac{2}{x}\right) = 5.$

   b) Solve for $x \in \mathbf{R}$:

   i) $x^3 + 2x^2 - 3x > 0$

   ii) $|x + 3| - 2 < 1.$

   c) i) Find the integer values a and b given that

   $$\frac{a}{b} = \frac{1}{2+\sqrt{3}} + \frac{1}{2-\sqrt{3}}$$

   ii) Show that, when $x > 1$,

   $$\frac{1}{\sqrt{x-1}\ -1} - \frac{1}{\sqrt{x-1}+1} = \frac{2}{x}.$$

2. a) Show that the polynomial $p(x) = 2x^4 + x^3 - 11x^2 - 4x + 12$ is divisible by $x^2 + x - 2$. Find the four linear factors of $p(x)$. Hence solve for $p(x) = 0$.

   b) Let $f(x) = x^2 + 1,\ x \geq 0$ and $g(x) = \sqrt{x-1},\ x \geq 1$.

   Verify that f and g are inverses of each other.

c) If $f(x) = 3x - 4$ and $g(x) = ax + b$, find conditions on a and b such that $(f \circ g)(x) = (g \circ f)(x)$.

3. a) i) Determine the nature of the roots of the equation

$$(3k + 2)x^2 - (6k + 1)x + (3k - 1) = 0.$$

ii) Find the set of values of k for which the equation

$$(3k + 2)x^2 - (6k + 1)x + (3k - 1) = 0$$

has real roots.

b) The function $f(x) = ax^2 + bx + c$ takes the value 10 when $x = -2$ and when $x = 4$. Its minimum value is -8.

Find the values of a, b and c. Hence sketch the graph.

c. If one of the roots of the equation $27x^2 + kx - 8 = 0$ is the square of the other, find k.

4. a) Find the constants A, B and C in the identify

$$\frac{x^2}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

b) Given that a and b are real numbers solve the equation

$$(a + bi)^2 = -5 + 12i$$

c) The equation $ax^2 + bx + c = 0$ has roots α and β.

Find the values of:

i) $\frac{1}{\alpha} + \frac{1}{\beta}$ ii) $\alpha^2\beta + \alpha\beta^2$.

5. a) i) Find the value of θ between 0^{0} and 360^{0} for which

$$2\sin^2\theta + \cos\theta = 1.$$

ii) Show that

$$\frac{1}{1-\cos\alpha} + \frac{1}{1+\cos\alpha} = 2\csc^2\alpha.$$

b) i) Define a rational number.
Hence show that $0.4\bar{1}$ is a rational number.

ii) Given that $p = x^{1/2} + x^{-1/2}$ and $q = x^{1/2} - x^{-1/2}$, find $p^2q^2 + 2$ in terms of x, giving your answer in its simplest form.

c) If $\log x + \log y = 2$, find xy. Hence solve the simultaneous equations

$$\log x + \log y = 2,\ x + y = 25.$$

6. a) Sketch the following function:

$$f(x) = \begin{cases} 8, & x < 3 \\ x^2 - 1, & -3 \le x \le 2 \\ 3, & x \ge 2. \end{cases}$$

b) Find the limit of each of the following:

i) $\lim_{x \to 3} \frac{x^2 + 2x - 15}{x^2 - 9}$

ii) $\lim_{x \to +\infty} \frac{x^3 + 4x - 1}{4x^3 + x^2 + 7}$

c) i) Define the derivative of the function f(x).

ii) Using the definition in (i), find y' given that $y = \frac{1}{x^2}$.

7. a) The curve $y = ax^2 + bx + c$ has a maximum point at (2,18) and passes through the point (0,10). Evaluate a,b, and c.

b) i) Consider the functions

$f(x) = 3x^2 + 2x + 5$ and
$g(x) = x^3 - 4x^2 - 3x + 6$.

Show that there is one value of x for which f(x) and g(x) both have a stationary value.

ii) A cube of metal is heated, and each side expands by 2%. Find the percentages in its surface area and volume.

c) Find the coordinates of the turning points of the curve $y = x^3 - 3x$.

Show that the tangent to the curve at the origin does not meet the curve again.

END OF EXAMINATION.

THE UNIVERSITY OF ZAMBIA

UNIVERSITY EXAMINATIONS – SEMESTER ONE, 1998/99

M111 – MATHEMATICAL METHODS I

INSTRUCTIONS: . Attempt ANY **Five** (5) Questions
. Write down your computer numbers on all answer scripts used

TIME ALLOWED: Three (3) Hours

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1. a) For each of the following, rationalize the denominator and express in the simplest possible form:

(i) $\dfrac{\sqrt{x}}{\sqrt{x}+\sqrt{y}}$ (ii) $\dfrac{\sqrt{5}+1}{\sqrt{5}-\sqrt{3}}$

b) If $Q(x) \equiv x^4 + hx^3 + gx^2 - 16x - 12$ has factors $(x + 1)$ and $(x - 2)$, find

(i) the constants h and g;

(ii) the remaining factors of Q.

c) Two opposite sides of a square are each increased by 3 cm and the other two sides are each decreased by 2 cm. If the area is increased by 8 cm^2, find the length of the side of the square.

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2. a) Given that $f(x) = \dfrac{1}{x}$ and $g(x) = 2x + 3$, find

(i) $f \circ g$ and determine its domain;

(ii) whether $f \circ g$ is even, odd or neither.

b) Verify that $f(x) = x^2 + 1$ for $x \geq 0$ and $g(x) = \sqrt{x-1}$ for $x \geq 1$ are inverse functions of each other.

(c) Given the function $f(x)=\sqrt{x^2-3x-40}$,

(i) determine its domain; and

(ii) sketch its graph.

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3. a) Solve the equation $\sqrt{3-t}-\sqrt{3+t}=\sqrt{t}$, for real values of t.

b) Solve the inequality $\left|\frac{x-2}{x+3}\right|\leq 4$, for real values of x.

c) At the start of an experiment, a culture of bacteria is found to contain 10,000 individuals. The growth of the population was observed, and it was found that at any subsequent time t (in hours) after the start of the experiment, the population size $p(t)$ could be expressed by the formula $p(t)=2500(2+t)^2$.

i) Determine the formula for the rate of growth of the population at any time t.

ii) Calculate the growth rate after 15 minutes.

iii) Calculate the growth rate after 2 hours and interpret the result.

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4. a) Given the equation $x^2-2x-k=0$,

(i) find the range of values of k for which this equation has real roots.

(ii) If the roots of this equation differ by one, find the value of k.

b) The roots of the equation $2x^2-7x+4=0$ are α and β.

(i) Find the values of $\frac{1}{\alpha}+\frac{1}{\beta}$ and $\frac{1}{\alpha\beta}$.

(ii) Determine the equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

c) Find the derivative of $f(x)=\sqrt{x-1}$ from first principles.

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5. a) Differentiate the following functions with respect to x:

(i) $f(x)=\sqrt{\dfrac{2x-3}{x-1}}$ (ii) $g(x)=x^2\,(4x^2-5x)^{10}$

b) (i) Determine whether $g(x)=\begin{cases}2x+5 & \text{if } x\le -1\\ -x^2+2 & \text{if } x>-1\end{cases}$ is continuous at $x = -1$.

(ii) Find a value for the constant k, that will make the function $h(x)=\begin{cases}kx^2 & \text{if } x\le 2\\ x+k & \text{if } x>2.\end{cases}$ be continuous.

c) Let $f(x)=\frac{1}{2}\sin(2x+\pi)$. Find its

(i) period,

(ii) amplitude,

(iii) phase shift, and

(iv) sketch its graph.

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6. a) Prove that $\dfrac{\cos x+\tan x}{\sin x\cos x}=\csc x+\sec^2 x$.

b) Find the general solution for the equation

$$\sin x\tan x=\sin x.$$

c) Without using tables, solve each of the following equations, expressing your answers as simple as possible:

(i) $9\log_x 5 = \log_5 x$, (ii) $\log_8 \frac{x}{2} = \frac{\log_8 x}{\log_8 2}$.

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7. a) Let $y = x^4 + 2x^3 - 3x^2 - 4x + 4$. Find the intervals in which y is

(i) increasing, and

(ii) decreasing.

Also, determine

(iii) the minimum and maximum values of y;

(iv) sketch the graph of y.

b) Solve $|2x+1| = |3-4x|$, for all values of x.

END OF EXAMINATION

UNIVERSITY OF ZAMBIA

UNIVERSITY EXAMINATIONS - SEMESTER ONE 1998/99

M161 - MATHEMATICS

INSTRUCTIONS: Answer ANY Five(5) Questions.
All questions carry equal marks.
Calculators, tables are not allowed.

TIME ALLOWED: Three (3) hours.

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1. (a) (i) Represent the following set on the real line

$$A = (-\infty, -1) \bigcup \left[-\frac{1}{2}, \frac{1}{2}\right] \bigcup (1, \infty)$$

(ii) Find the set of values of x for which

$$f(x) = \frac{x^2 - 6x + 1}{(x+1))(x-2)(x-7)}$$

does not exist.

(b) (i) Given that

$$\frac{x^3 - 25x}{x^4 - 3x^3 - 10x^2} = \frac{x + A}{x(x + B)}$$

find the constants A and B.

(ii) Find the set of values of x for which

$$|x+1| > 2|x - 1|$$

(c) In a class of 30 students, all students take Economics or Political Science. If 12 students take Economics, 8 students take Economics but not Political Science,

1. (c) (i) find the number of students who take Economics and Political Science

(ii) find the number of students who take Political Science but not Economics.

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2. (a) (i) Prove that $-1 + 2i$ is a root of the equation

$$3x^3 + 5x^2 + 13x - 5 = 0$$

(ii) Find the sum and product of the roots of the equation

$$(2- i)x^2 + (4 + 3i)x + (-1 + i) = 0$$

(b) Solve for x and y:

(i) $(1 - i)^4 = x + iy$

(ii) $(1 + 3i)x + (3 - 4i)y - 5 + 2i = 0.$

(c) Find the quotient and the remainder by using the synthetic division when

$8x^3 + 6x^2 - 2x + 1$ is divided by $2x - 1$

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3. (a) Solve the inequality

(i) $\dfrac{x+1}{x-1} > 3$

(ii) $x(x - 1)(x - 2) < 0.$

(b) The roots of the quadratic equation $x^2 - 3px + p^2 = 0$ are α and β. Find the value of :

(i) $\alpha^2 + \beta^2$ (ii) $(\alpha - \beta)^2$

(c) Solve for x given the equation $2x^2 - 4x - 3 = 0$

(i) by completing the square

(ii) using the quadratic formula

4. (a) The function f is defined by

$$f(x) = \frac{x+3}{x-1}, \quad x \neq 1.$$

Find: (i) $f^{-1}(x)$
(ii) the range of f(x).
(iii) (fof)(2)

(b) For the function

$f(x) = x^2 - 4x + 8$

(i) find the y intercept
(ii) find the turning point of the graph of f(x).
(iii) Sketch f(x), showing the line of symmetry.

(c) One number is 60 more than twice the second number.

(i) Express the function P(x) as a product of the two numbers
(ii) Show that this product has a smallest value.

Find this smallest value and the two numbers for which this product is smallest.

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5. (a) (i) Factorise completely

$$f(x) = x^3 - x^2 - 9x + 9$$

Hence solve f(x) = 0.

(ii) Solve for $x \in \Re$:

$$\sqrt{x+9} - \sqrt{x-3} = 2$$

(b) (i) If the points A(5, -2) B(-5,7) and C(-1,t) are collinear, find t.

(ii) The function $f(x) = x^3 + kx^2 - 2x + 1$ gives a remainder k when divided by x - k. Find the possible values of k.

5. (c) Show that AB is parallel to DC and perpendicular to BC when A,B,C,D are the four points (-1,3), (1,0), (4,2) and (0,8) respectively.

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6. (a) (i) Compute and simplify

$$\frac{2x}{x^2-4}+\frac{1}{x(x-2)}-\frac{1}{x-2}$$

(ii) Find the values of A,B and C for which

$$A(x-1)(x-2)+B(x+2)(x-2)+C(x+2)(x-1)=x^2-5x-2$$

(b) (i) Solve for $x \in \Re$:

$$|2x^2-x+1|=7$$

(ii) Show that $x+3$ is a factor of $x^3+7x^2+10x-6$ and $x=-3$ is a root of $x^3+7x^2+10x-6$.

(c) Resolve into partial fractions

(i) $\dfrac{5x+2}{(x+1)(x+3)}$ (ii) $\dfrac{x^2-1}{(x-2)^2}$

7. (a) (i) Prove that

$\log_a x + \log_a y = \log_a xy$.

(ii) Evaluate:

$2\log 25 - 3\log 5 + \log 20$

(b) (i) Show that the equation

$$2^{2x} - 7.\ 2^{x} - 8 = 0$$

has only one real root

(ii) Solve the equation

$$4\log_x 3 - \log_3 x = 3.$$

(c) (i) Expand and simplify

$$\left(a+\sqrt{b}\right)^3 + \left(a-\sqrt{b}\right)^3$$

(ii) Find the a^9b^2 in the expansion of $(a + b)^{11}$.

END OF EXAMINATION

UNIVERSITY OF ZAMBIA

UNIVERSITY EXAMINATIONS - SEMESTER ONE 1998/99

M211- MATHEMATICAL METHODS III

INSTRUCTIONS: Answer ANY Five(5) Questions.

TIME ALLOWED: Three (3) hours.

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1. (a) Find the equation of the ellipse with foci at (1,-2) and (7,-2) and minor axis joining the points (4,0) and (4,-4)

   (b) Describe the conic obtained from the graph of $2x^2 + xy + 2y^2 = 10$.

   Find the centre, focus (foci), and eccentricity of the conic.

2. (a) A racing car on the elliptical race track $\frac{x^2}{400} + \frac{y^2}{100} = 1$ went out of control at the point (16,6) and there after continued on the tangent line till it hit a tree at (14,k). Sketch the track of the car and determine k.

   (b) State the focus directric equation of a conic which has e as its eccentricity. Using this equation, derive a polar equation for the parabola with focus at the origin and directrix the line $r\cos\left(\theta - \frac{\pi}{2}\right) = 2$

3. (a) State the Mean value theorem and apply it to the function $f(x) = \sin x$ on the interval [a,b]. Hence show that $|\sin b - \sin a| \le |b - a|$

   (b) Evaluate the following limits

   (i) $\lim_{x \to \pi/2} (\tan x \ln \sin x)$

   (ii) $\lim_{x \to 1^+} \left(\frac{x}{x-1} - \frac{1}{\ln x}\right)$

   (iii) $\lim_{x \to 0} \frac{\sin x - x}{x^3}$

4. (a) Write a definite integral which represents the area under the curve $y = \frac{1}{\sqrt{x}}$ between $x = 0$ and $x = 1$. Discuss if it is an improper integral. Hence evaluate it.

(b) Use the Taylor polynomial of order 4 to approximate ln(1.1) and find a bound on the error of the approximation.

5. (a) Evaluate the following integrals

(i) $\int_{0}^{\pi/2} \sqrt{1+\sin x}\ \cos x\, dx$

(ii) $\int x \sec^2 x\, dx$

(iii) $\int_{-2}^{-1} \frac{x dx}{x^2 + 4x + 5}$

(b) Given that $I_n = \int_0^1 x^n e^{-x} dx,$

show that, for $n \geq 1,$

$I_n = nI_{n-1} - e^{-1}$

Hence evaluate $\int_0^1 x^6 e^{-x}\ dx$

6. (a) Sketch the graphs of $y = \cos x$ and $y = \sin x$ for x between 0 to $\frac{3\pi}{2}$ in the same co-ordinate plane. Hence find the area enclosed between curves $y = \sin x$ and $y = \cos x$ for $0 \leq x \leq \frac{5\pi}{4}$.

(b) Find the volume generated by revolving the region bounded by the curve $y = \sqrt{x},$ the line $y = 2$ and the y - axis about the line $y = 2$.

END OF EXAMINATION

The University of Zambia
Deferred/Supplementary Examinations
First Semester - 1998/1999
M211 - Mathematical methods III

Time allowed : Three (3) hours.

Answer any five questions.

1.a Find the equation of the hyperbola that goes through the point (2,3) and has foci at $(\pm 2,0)$.

b. Identify the conic described by the equation $2x^2+3xy+2y^2=7$ and find its focus (foci) and directrix (directrices) as the case may be. Hence sketch it.

2.a Find the maximum and minimum value of

$$f(x) = \begin{cases} x+1, & 0 \le x \le 1 \\ -x^2+3, & 1 < x \le 3 \end{cases}$$

on the interval $0 \le x \le 3$.

b. Derive the polar equation of a circle with radius 2 units and whose centre has polar coordinates $(5, \frac{\pi}{3})$.

3a. Discuss the validity of the theorem of the Mean for the function

$f(x) = \dfrac{x^2-4x+3}{x-2}$ on the interval [1,3]. If the theorem of the Mean is valid for the given function on the given interval, then find all possible values of x_0 in the interval $1 < x < 3$ such that

$$f'(x_0) = \frac{f(3)-f(1)}{3-1}$$

b. Evaluate the following limits

4.a Compute $\int_{1}^{3} \frac{dx}{(3-x)^2}$

b. Find the fifth order Taylor polynomial of the function e^x about the base point 0. Hence approximate $\sqrt{e}$. (you need not simplify your answer). Find a bound on the error of approximation.

5.a Evaluate the following integrals

(i) $\int \frac{(2x+41)dx}{x^2+5x-14}$

(ii) $\int_{-1.5}^{1.5} \frac{x^2 dx}{\sqrt{9-x^2}}$

(iii) $\int \tan^4 x \sec^6 x dx$

b. Given that $I_n = \int_0^{\frac{\pi}{2}} \sin^n x dx$, show that $I_n = \frac{n-1}{n} I_{n-1}$, $n \geq 1$

Hence evaluate $\int_0^{\frac{\pi}{2}} \sin^8 x dx$

6.a. Find the area bounded by the curves $y^2 = x-1$ and $y = x - 3$

b. The region R, bounded by the positive x – axis, the y –axis and the curve $y = \sqrt{a^2 - x^2}$ is revolved about the y-axis. Find the volume of the solid generated.

UNIVERSITY OF ZAMBIA

UNIVERSITY EXAMINATIONS - SEMESTER ONE 1998/99

M221-LINEAR ALGEBRA I

INSTRUCTIONS: Attempt Any Five (5) Questions.

TIME ALLOWED: Three (3) hours.

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1. (a) What is meant by the following:

   (i) an *equivalence relation* in a set A? [*2 marks*]
   (ii) the *composition of two functions*? [*2 marks*]

   (b) (i) Let $\boldsymbol{Z}$ be the set of all integers. Given $a, b \in \boldsymbol{Z}$, define $a \sim b$ if $a - b$ is an even integer. Then show that this defines an equivalence relation of $\boldsymbol{Z}$. [*3 marks*]

   (ii) Let R be an equivalence relation in a set A; and let Ra denote the equivalence class of $a \in A$. Then prove that if a is not related to b, $Ra \cap Rb = \phi$; and that if a is related to b, $Ra = Rb$. [*5 marks*]

   (c) (i) Prove that the composition of two bijective functions is a bijective function. [*5 marks*]

   (ii) Let $f: \boldsymbol{R} \to \boldsymbol{R}$ be defined by the formula $f(x) = x^3$. Then show that f is bijective, and hence find the inverse function of f. [*3 marks*]

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2. (a) Define the following terms

   (i) a *symmetric matrix* A; [*2 marks*]
   (ii) *row equivalent matrices.* [*2 marks*]

   (b) (i) Let A and B be invertible n-square matrices. Then prove that
   (1) the inverse of A is unique; [*2 marks*]
   (2) AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$. [*3 marks*]

   (ii) Let A and B be n-square symmetric matrices. Then, prove that AB is symmetric if and only if A and B commute. [*4 marks*]

(c) (i) Show that the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{pmatrix} \text{ is row equivalent to}$$

the matrix $B = \begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ [*3 marks*]

(ii) If $A(\alpha) = \begin{pmatrix} 1 & \alpha & \alpha^2/2 \\ 0 & 1 & \alpha \\ 0 & 0 & 1 \end{pmatrix}$, show that $A(\alpha)A(\beta) = A(\alpha+\beta)$,

and hence, find the inverse of $A(\alpha)$. [*4 marks*]

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3. (a) Evaluate the determinant

$$\begin{vmatrix} -1 & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 \\ 0 & -1 & 1 & 1 \\ 1 & 2 & 5 & 3 \end{vmatrix}.$$

[*5 marks*]

(b) Show that

$$\begin{vmatrix} y+z & x & x^3 \\ z+x & y & y^3 \\ x+y & z & z^3 \end{vmatrix} = (x+y+z)^2(y-x)(z-x)(z-y).$$

[*7 marks*]

(c) Show that if $k \neq 1$, there is always a solution to the equations

$$x - 2y + z = p$$
$$x + ky - 2z = p^2$$
$$2x - y - z = 6$$

whatever the value of p. Find the solution when $k = 2$ and $p = 1$. [*8 marks*]

4. (a) Briefly explain the meaning of each of the following terms:

(i) the *row reduced echelon* form of a matrix A; [*3 marks*]

(ii) a *consistent system* of linear equations. [*2 marks*]

(b) Reduce the matrix

$$\begin{pmatrix} 1 & 2 & -1 & 2 & 1 \\ 2 & 4 & 1 & -2 & 3 \\ 3 & 6 & 2 & -6 & 5 \end{pmatrix}$$

to its row reduced echelon form.

Hence, find all the solutions to the linear system of equations

$$\begin{aligned} x + 2y - z + 2w &= 1 \\ 2x + 4y + z - 2w &= 3 \\ 3x + 6y + 2z - 6w &= 5. \end{aligned}$$

[*7 marks*]

(c) Determine the values of k so that the following system.

(i) is inconsistent;

(ii) has more than one solution;

(iii) has a unique solution:

$$\begin{aligned} x + y - z &= 1 \\ 2x + 3y + kz &= 3 \\ x + ky + 3z &= 2. \end{aligned}$$

[*8 marks*]

---

5. (a) Let V be a vector space over the field K.
What is meant by the following:

(i) W is a *subspace* of V? [*2 marks*]

(ii) V is the *direct sum* of its subspaces V and W? [*2 marks*]

(b) (i) Let U and W be subspaces of the vector space V over K. Then, show that the intersection $U \cap W$, of U and W, is also a subspace of V. [*4 marks*]

(ii) Let $V = \mathbf{R}^3$ and let $W = \{(x, y, z): x + y + z = 0\}$ be a subset of V. Show that W is a subspace of V. [*4 marks*]

5. (c) (i) Let U and W be subspaces of $V = \boldsymbol{R}^3$ defined by

$$U = \{(a, b, c): a = b = c\} \text{ and } W = \{(0, b, c)\}.$$

Then, show that V is the direct sum of U and W. [*5 marks*]

(ii) Let U and W be subspaces of à K-space V. Prove that U and W are contained in $U + W$. [*3 marks*]

---

6. (a) What is meant by each of the following terms:

(i) a *linearly dependent* subset of a K-space V? [*2 marks*]
(ii) the *dimension* of a K-space V? [*2 marks*]

(b) (i) Let V be a vector space over the field K; and let u, v, and w be linearly independent vectors in V. Then, show that $u + v$, $u - v$ and $u - 2v + w$ are also linearly independent in V. [*4 marks*]

(ii) Let $V = \boldsymbol{R}^3$. Find the coordinate vector of

$u = (3, 1, -4)$, relative to the *R-basis*
$v_1 = (1, 1, 1)$, $v_2 = (0, 1, 1)$, $v_3 = (0, 0, 1)$. [*3 marks*]

(c) Let U and W be the following subspaces of $\boldsymbol{R}^4$:

$U = \{(x, y, z, w): y + z + w = 0\}$,

$W = \{(x, y, z, w): x + y = 0, z = 2w\}$.

Find a basis and the dimension of

(i) U (ii) W, (iii) $U \cap W$. [*9 marks*]

END OF EXAMINATIONS

UNIVERSITY OF ZAMBIA

UNIVERSITY EXAMINATIONS - SEMESTER ONE 1998/99

M231-REAL ANALYSIS I

INSTRUCTIONS: Answer Any FIVE questions.

TIME ALLOWED: Three (3) hours.

Q1. If $f: A \to B$ and if $X \subset B$, $Y \subset B$, then prove the following:

(i) $f^{-1}(\phi) = \phi$ and $f^{-1}(B) = A$

(ii) $f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$

(iii) $f^{-1}(X') = [f^{-1}(X)]'$.

Q2. (a) Let $A \subset R$. Define the least upper bound of A and greatest lower bound of A.

$$\text{Let } A = \{x \in R : x = \frac{4n+3}{n},\ n \in N\}.$$

Find the lub and glb of A.

(b) (i) Let $A \subset R$. If u is the lub of A, then prove that there exists $x \in A$ such that

$$u - \varepsilon < x \leq u \quad \text{for every } \varepsilon > 0.$$

(ii) If v is the glb of A, then prove that there exists $x \in A$ such that

$$v \leq x < v + \varepsilon.$$

Q3. (a) If x and y are real numbers and if $y > 0$, then prove that there exists a natural number n such that $ny > x$.

(b) If x and y are real numbers, then prove that

(i) $|x + y| \leq |x| + |y|$

(ii) $-|x-y| \leq |x| - |y| \leq |x - y|$

Q4. (a) Let (S_n) be a sequence of real numbers. If $\lim_{n\to\infty} S_n$ exists, then prove that it is unique.

(b) If $a_n \leq b_n \leq c_n$ for every n, and

$\lim_{n\to\infty} a_n = A = \lim_{n\to\infty} c_n$, then prove that $\lim_{n\to\infty} b_n = A$.

Q5. (a) Define Cauchy sequence

If (S_n) is a convergent sequence of real numbers, then prove that it is a Cauchy sequence.

(b) Applying Cauchy's criterion of convergence, prove that the sequence (S_n) defined by

$$S_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} \text{ diverges.}$$

Q6. (a) If (s_n) is a sequence of real numbers, then prove that $\lim_{n\to\infty} \inf s_n \leq \lim_{n\to\infty} \sup s_n$.

(b) If (s_n) and (t_n) are bounded sequences of real numbers, and if $s_n \leq t_n$ for $n \in N$, then prove that

(i) $\lim_{n\to\infty} \sup s_n \leq \lim_{n\to\infty} \sup t_n$

(ii) $\lim_{n\to\infty} \inf s_n \geq \lim_{n\to\infty} \inf t_n$

Q7. (a) If $\limsup_{n\to\infty} |x_n / x_{n+1}| < 1$, prove that $x_n \to 0$ as $n \to \infty$.

Applying this result find the limit of $S_n = \frac{n}{2^n}$ as $n \to \infty$.

(b) Find $\limsup_{n\to\infty} S_n$ and $\liminf_{n\to\infty} S_n$ where

(S_n) is defined by $\sin n\pi/2 + (-1)^n$.

END OF EXAMINATION.

UNIVERSITY OF ZAMBIA

UNIVERSITY EXAMINATIONS - SEMESTER ONE 1998/99

M241-INTRODUCTION TO COMPUTER PROGRAMMING

INSTRUCTIONS: 1. Answer Question one(1) and any four questions from the remaining five.

TIME ALLOWED: Three (3) hours.

---

1. a) Define the following terms as applied in Computer Science

   (i) Program
   (ii) Auxiliary Memory
   (iii) Hardware
   (iv) Algorithm
   (v) Label

   b) Name two kinds of program errors that can't be detected at compile time. Give an example of an error that can't be detected either at run-time or compile time.

   c) A computer has two major components, the software and the hardware. Name two other categories composing software. In the categories mentioned, in which of them do each of the following fall:

   (i) Pascal Programs
   (ii) Operating System
   (iii) Programs for game playing
   (iv) Compiler

   d) Draw a diagram of the elements of a computer, showing its 'logical structure'. Label all parts, distinguishing clearly between data and signal flows.

2. (a) In Pascal, the repeat and the while are termed as repetitive control structures. State their differences.

(b) Rewrite this program segment as a single repeat statement.

```
:
read(First, Last);
While(First=Start) and (Finish <> Last)do
read(First, Last)
:
```

(c) Assume the variables x,y, quotient and remainder are declared as integers. Write down the output from the program segment below

```
:
begin
read(x,y);
remainder:=x; quotient:=0;
while remainder >=y do
begin
   quotient:=quotient + 1;
    remainder:=remainder - y;
end
writeln(x,y, quotient, remainder)
end.
```

if the user enters the following values

(i) 4,5
(ii) 5,4
(iii) 23, 7
(iv) Rewrite the above program segment, using the goto statement to obtain the same result.

3. (a) (i) What is a bug?
(ii) The following program segment inspects fifty characters and counts those that aren't 'T' or '5'. Find the bug:

```
count:=0;
for i:=1 to 50 do begin
  read(ch);
if (ch<>'T') or (ch<> '5') then begin
   count:=count + 1
  end;
end;
```

(ii) Classify the bug as either syntactic or semantic

(b) H,T,U are variables declared as follows:

H,T,U:'O'...'9'
Write down an assignment statement which will assign to an integer variable I the decimal number denoted by the characters

htu
in that order.

(c) Write a complete Pascal Program that computes the equation

$$y = x^n - 1$$

where x ia a real and n an integer. Show output of the values y, x and n.

4. (a) When would a repeat statement be better to use than a for statement?

(b) What's the output of the program segment

```
   :
var counter : integer;
begin
   for counter:=1 to 9 do
     case counter mod 5 of
     0:write('Often');
     1:write('What');
     2,4:write('is');
        3:write('not')
     end;
    writeln
end.
```

(d) What is the output of the program segment below if the limit is 3 (i,j,N, Sum and total declared as integers)

```
:
begin
writeln('Enter the limit');
readln(N);
Total:=0;
For i:=1 to N do begin
    sum:=0;
for j:= 1 to i do
   sum:=sum + j;
   Total:=Total + sum
end;
writeln('The sum of the subtotals is', Total:1)
end.
```

(c) Write a Pascal code that prints the first letter of the first line of input, the second letter of the second line, etc. stop on the last character of line Last.

5. (a) What is a compound statement?

(b) (i) What is the output of this program segment?
writeln(1=2,2=2, (2+3)=7);

(ii) Suppose that the two statements below appear, as shown, in a program. What single if statement could you replace them with?

```
if n >2 then n:=3*n+1;
if n>=7 then n:=n-7;
```

(c) What are the possible outputs of this program segment? Assume that First and Second are char variables and that the only possible inputs are AB, BA, AA and BB.

```
readln(First, Second);
Case First of
    'A':Case Second of
        'A':write('It');
        'B':write('is')
      end;
    'B':begin
        write('an');
        Case Second of
            'A':write('Ancient');
            'B':write('Mariner')
         end
         end
  end;
  writeln
   ⋮
```

d) Rewrite the above program segment using if statement.

6. (a) Write Pascal statements that would let you know:

(i) The number that follows zero
(ii) The whole part of a real variable called GPA
(iii) The character that follows the zero digit character
(iv) How far away from the letter 'A' the letter 'T' is

(b) Given the following sequence of PASCAL definitions and declarations

```
CONST
    linemax=64;
    printwanted=true;
TYPE
   lineposition=1..linemax;
   spacing = (single, double, treble);
VAR
   thischar, lastchar:char;
            thisposition:lineposition;
            spacingnow:spacing;
```

List all the constant-identifiers, all the type-identifiers which occur.

(c) What is a base type?

(d) Write a program to read in three integers a,b, and c assumed to be the coefficients of a quadratic

$$ax^2 + bx + c = 0$$

and calculate and output the roots using the formula

$$\text{root} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Account for the three possibilities
(i) two different roots
(ii) one (double) root
(iii) a pair of complex conjugate roots

END OF EXAMINATION

UNIVERSITY OF ZAMBIA

UNIVERSITY EXAMINATIONS - SEMESTER ONE 1998/99

M261- INTRODUCTION TO STATISTICS.

INSTRUCTIONS:

1. Attempt **Five (5)** questions in the following way:
   - **two** questions should come from section **C**
   - one question should come from section A
   - one question should come from section B
   - the last question may come from any section

2. You may use calculators and tables, tables have been provided.

TIME ALLOWED: **Three (3) hours**

SECTION A

POPULATION AND SAMPLES

QUESTION 1

For each of the following questions or statements provide a **brief** but complete answer:

(a) There are several definitions of **statistics as a field** mention **two.**

(b) (i) In statistics we often use random samples, what does the phrase 'random sample' mean?

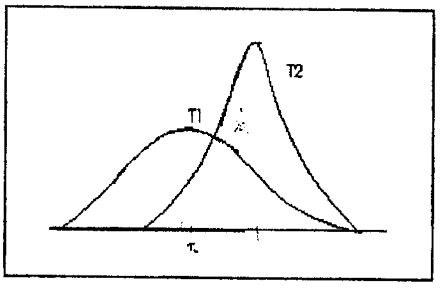
(ii) Himonga, Bwalya, Phiri, and Ndhlovu are being interviewed to fill three positions in a company. After the interview the panel decides to randomly select three among the four of them because they all were equally qualified. They assign numbers 1 to Himonga, 2 to Bwalya, 3 to Phiri, and 4 to Ndhlovu. If they choose the second row of the partial random number table below, who gets selected?

88618	19161
71299	23853
27954	58909
80863	00514
33564	60780

(iii) If the last column had been chosen who could have been selected?

(iv) One of the people interviewing them comes from the Eastern Province, he is not familiar with this procedure of random selection. Before selection began he was concerned that people from the East; Phiri and Ndhlovu were given the last numbers 3 and 4. Would these numbers affect their selection? Explain your answer briefly.

(c) Two estimators T1 and T2 have been developed to estimate maize yield at a certain farm. Assume maize yield at the farm is τ bags/hectare. The behaviour of the two estimators is depicted in the figure below.



(i) What can you say about the estimator T1 in terms of precision and accuracy?

(ii) What can you say about the estimator T2 in terms of precision and accuracy?

(iii) For planning purposes, the farmer would like accurate estimate of the yield, which estimator should she choose?

(iv) Which estimator would require more data to get a good estimate of τ

(v) If the sample sizes were the same, which estimator would give a wider confidence interval for τ

QUESTION 2

(a) The Government has embarked on a housing scheme for ordinary Zambians who did not benefit from the recent sale of Council houses. The scheme is called the 'Presidential Housing Initiative'(PHI) and the goal is to provide affordable housing. We may consider the *target population* as 'all ordinary Zambians who did not benefit from the sale of council houses', where the word 'ordinary' means 'not rich'. Suppose the Government decides to carry out a study to assess demand for low cost houses and puts up advertisement in various media for the purchase of an initial number of houses.

(i) Suppose the advertisement is sent to the population of *'all Zambians who can afford to buy the houses'*. Identify **two differences** between this population and the target population.

(ii) Is the population defined in (i) above a *study population?*

(iii) If your answer to (ii) was **Yes**, explain why it qualifies as a study population. If your answer to (ii) was **No**, suggest a modification that would qualify it as a study population.

(b) In order to determine a reasonable price for the low cost houses, the Government decides to invite Zambians to send their bids (price each person was comfortable buying a house). Suppose the initial five bids in millions of Kwacha were as follows:

14, 13, 16, 15, and 37

(i) Obtain the median for the bids

(ii) Obtain the sample mean

(iii) Obtain the sample variance

(iv) What is best measure of location? Explain why it is the best.

(v) How do the mean and median compare?

(vi) From this initial sample of bids what is a reasonable price for a house under PHI?

(c) Give a better example of a study population for the PHI scheme different from the one mentioned above. Give one reason why your choice is better than the one above.

SECTION B

VARIABLES AND DESCRIPTIVE STATISTICS

QUESTION 3

(a) Mention **three** ways in which data may be summarized.

(b) Mention **three** advantages a stem and leaf plot has over a histogram

(c) A private clinic has been in operation for two years and does admit patients. A nurse going through patient's files records the following ages with the corresponding cost for care:

Age of patient	Number of days admitted
18	4
25	3
20	4
38	1
33	0
39	2
44	1
46	0
42	3
42	0
47	7
51	4
51	2
53	2
52	1
59	3
69	0
63	4
74	1

(i) Obtain a stem and leaf plot of the patients ages

(ii) What is the median age?

(iii) What is the mean age?

(iv) How do the mean and median compare?

The nurse decides to classify patients who have the *median age or less* as **'young clients'**, and those *older than the median age* as **'older clients'**

(v) Obtain the average number of days of admission for each group of clients i.e., obtain the mean for number of days admitted for **young clients** and also for **older clients.**

(vi) If the average cost of one day of admission is K50,000 Kwacha, which group of clients paid more on average?

QUESTION 4

(a) To analyze data it is essential to know the type of variables involved. Social Scientists classify variables into four measurement scales given below. For each measurement scale give **one example** of a random variable associated with that scale:

	Measurement Scale	Give **one example of a random variable** associated with the scale
(i)	Nominal	
(ii)	Ordinal	
(iii)	Interval	
(iv)	Ratio	

(b) Suppose $\bar{x}$ and s_x^2 are the mean and variance of a random sample $X_1, X_2, \ldots X_n$.
Let $Y_j = CX_j$, where $j = 1, 2, \ldots, n$.

Showing all the work show that:

(i) $\bar{y} = C\bar{x}$, where $\bar{y}$ is the sample mean for y's

(ii) $s_y^2 = c^2 s_x^2$

(iii) What value(s) of C are required for s_y^2 to equal s_x^2?

(iv) Obtain two possible values of $\overline{y}$ in terms of $\overline{x}$ when $S_y^2 = s_x^2$

(c) A salesperson has sold 10 items, the mean of the 10 sales is K165,000 with a standard deviation of K7,000.

Find:

(i) Her average gain in commission if she receives 10% commission on every item sold.

(ii) The standard deviation of gains in commissions if she receives 10% commission on every item sold.

HINT: use knowledge from part (b) to do part (i) and (ii).

(iii) Her average gain in commission if she receives 5% commission on every item sold plus a fixed amount of K10,000 on each item sold.

SECTION C

INFERENCE

QUESTION 5

(a) For each of the following statements, state whether it is **true** or **false**:

(i) A Type I error of a hypothesis testing occurs when a true null hypothesis is not rejected.

(ii) The significance level of a statistical test is the same as the probability of a type I error.

(iii) If the p-value of a statistical test is less than the significance level α, then the null hypothesis is rejected.

(iv) A significance level can be changed after the analysis

(v) the significance level $\alpha = 0.05$ is arrived at theoretically

(b) The normal rainfall season in Zambia has an average of 1043 Millimeters (mm). A sample of 16 rainfall amounts were obtained from some weather stations in Zambia for the 1994/95 rainfall season. The data are given below together with necessary statistics.

PROVINCE	1994/95 RAINFALL (mm)
NORTHWESTERN	1244
	1057
LUAPULA	1143
	955
NORTHERN	1384
	726
	1310
COPPERBELT	833
EASTERN	757
	684
WESTERN	623
	611
CENTRAL	602
LUSAKA	413
SOUTHERN	419
	399

Summary for 1994/95 rainfall data

$\bar{X} = 822.5$mm

$s = 324.73$mm

(i) Test the null hypothesis that rainfall for 1994/95 reason was normal versus the alternative that the rainfall was not normal.

Use $\alpha = 0.05$ and be sure to show the following:

(a) H_0 and H_1 in terms of $\mu_{1994/95}$ (the mean rainfall for 1994/95)

(b) Test statistic

(c) Decision rule

(d) Conclusion

(ii) Report a p-value or p-value range.

QUESTION 6

A proprietor of a certain company in town is concerned about her workers reporting late for work after lunch-break. A supervisor has been advised to record the time 10 randomly selected workers report for work after lunch. He is to record two times for each of the 10 randomly selected workers; one record is when the proprietor is *present* and then other is when the proprietor is *absent*. The times (in minutes) are given below together with the difference between the two times, also shown are some statistics you may find useful. The proprietor is interested to know whether workers take longer reporting for work after lunch when she is absent.

Worker ID	Proprietor Absent	Proprietor Present	Difference
1	3.21	2.71	0.50
2	2.88	2.62	0.27
3	3.62	1.40	2.22
4	2.31	2.43	-0.13
5	3.41	1.42	1.99
6	4.42	0.85	3.56
7	3.45	1.76	1.69
8	0.73	1.76	-1.03
9	5.23	0.10	5.14
10	3.09	2.54	0.55
$\sum x$	32.35	17.59	14.76
$\sum x^2$	117.54	37.47	52.54

(a) What type of design was used in this experiment?

(b) Does it matter how we take a difference if the alternative hypothesis is two-sided?

(c) Which test is appropriate, the One-tailed or Two-tailed test?

(d) Perform the test at 5% level of significance stating:
 (i) H_0 and H_1 clearly
 (ii) The test statistic
 (iii) Decision rule
 (iv) Your conclusion using either the p-value or critical value approach.

Assume the differences are normally distributed.

QUESTION 7

D. C. Montgomery describes an experiment in which the tensile strength of synthetic fiber is of interest to the manufacturer. It is suspected that strength is related to the percentage of cotton in the fiber. Five levels of cotton percentage are used, and five replications are run in random order resulting in the data below.

	Cotton Percentage				
Observation	15%	20%	25%	30%	35%
1	7	12	14	19	7
2	7	17	18	25	10
3	15	12	18	22	11
4	11	18	19	19	15
5	9	18	19	23	11

Source: *Design and Analysis of Experiment*, 2[nd] Edition (John Wiley & Sons, 1984)

We would like to find out whether cotton percentage affects breaking strength. Let us define the following terms:

	Cotton Percentage				
Quantity	15%	20%	25%	30%	35%
Sample mean	$\bar{X}_1$	$\bar{X}_2$	$\bar{X}_3$	$\bar{X}_4$	$\bar{X}_5$
Parameter	μ_1	μ_2	μ_3	μ_4	μ_5

$$SST = \sum_{i=1}^{5}\sum_{j=1}^{5}(x_{ji} - \bar{x})^2 = 636.96$$

$$SSB = \sum_{i=1}^{5}\sum_{j=1}^{5}(\bar{x}_i - \bar{x})^2 = 475.76$$

(a) The incomplete ANOVA table is given below:

Source	Sum of Squares	Df	Mean Square (MS)	F value
Between	475.76	(ii)	(iii)	(iv)
Within	(i)	20	8.06	
Total	636.96	24		

Find the values for the following parts indicated in the ANOVA table:

(i)

(ii)

(iii)

(iv)

(b) In order to test whether cotton percentage affects breaking strength two versions of the null hypothesis are given to you, against H_1 that not all mean breaking strength are equal:

Version 1 H_0: $\bar{X}_1 = \bar{X}_2 = \bar{X}_3 = \bar{X}_4 = \bar{X}_5$ and

Version 2 H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$

(i) Of the two versions, which one is correct? Why?

(ii) Obtain the **p-value or p-value range** for the correct version of H_0 at 5% level of significance using your answer from (a) (iv).

(iii) From the value obtained in (a) (iv) what conclusion can you make regarding H_0?

(iv) Is it necessary to do pair-wise comparisons? Why?

END OF EXAM

UNIVERSITY OF ZAMBIA

UNIVERSITY EXAMINATIONS - SEMESTER ONE 1998/99

M335- TOPOLOGY

INSTRUCTIONS: Answer ANY Four(4) out of six questions.

TIME ALLOWED: Three (3) hours.

1] a) Define the following terms:

(i) A metric space.
(ii) A discrete metric space.
(iii) An open set in a metric space.

b) Prove the following:

(i) Let (A,d) be a metric space, and x_0 be a limit point of E a subset of A, then every neighbourhood of x_0 contains infinitely many points of E.

(ii) A necessary and sufficient condition that two metric spaces (A,d) and (B,d ′) to be metrically equivalent is that there exists a function $f: A \to B$ such that f is one-one, onto and for each pair $x,y \in A$ $d'(f(x), f(y)) = d(x,y)$.

c) (i) Let $A = [a,b]$ a subset of the real line, and let X denote the set of all real-valued continuous functions on A. For $f,g \in X$, define

$$d(f,g) = \max_{x \in A}\{|f(x) - g(x)|\}.$$

Show that d is a metric on X.

(ii) Let (X,d) and (Y,d ′) be two metric spaces, define $d^*(x,y) = d(x_1,y_1) + d'(x_2,y_2)$, where $x = (x_1, x_2)$ and $y = (y_1,y_2)$ is d* a metric on XxY? Justify your answer.

2] a) Define the following terms:

(i) A neighbourhood of a point in a metric space.

(ii) A limit point of a set in a metric space.

(iii) Metrically equivalence of two metric spaces.

2. b) Prove the following:

(i) Let $f:(X,£_x) \rightarrow (Y,£_y)$ be a function and let $A \subset X$, the function $g:(A,£_a) \rightarrow (Y, £_y)$ such that $g(a) = f(a)$ $\forall$ $a \in A$ is called a restriction of g to A and is denoted by $g = f/A$.
Then f/A is continuous if f is continuous.

(ii) Let (A,d) be a metric space, a subset of A is closed if and only if it contains all its limit points.

c) Let $X = \{a,b,c,d,e\}$ and

$£ = \{\phi, \{a\}, \{b,c\}, \{d,e\}, \{a,b,c\}, \{a,d,e\}, \{b,c,d,e\}, X\}$

(i) List all the closed sets

(ii) List all the non-open neighbourhoods of the element d.

(iii) Find the interior of $A = \{a,d,e\}$.

(iv) Find the closure of $B = \{c,d,e\}$.

(v) Is the topological space (X,£) connected? Justify your answer.

3] a) Define the following terms:

(i) Topological space.

(ii) Interior point of a set.

(iii) Closure of a set.

b) Prove the following:

(i) If $(X,£_X)$ is a topological space and Z is a non-empty subset of X, then the collection $£_z$ of subsets of Z defined by

$£_z = \{ Z \cap E: E \in £_x\}$ is a topology on Z.

(ii) A subcollection β of a topology £ is a base for £ if and only if for any point x belonging to an open set u, there exists $B \in \beta$ such that $x \in B \subset U$.

3. c) Let $A = \{x \in \mathbf{R}: 1 < x < 2\}$, and £ to consist of the sets

$E_k = \{x \in \mathbf{R}: k < x < 2\}$, where $K \in [1,2]$.

(i) Prove that £ is a topology on A.

(ii) What are the closed sets in the topological space (A,£).

(iii) Show that the intersection of any collection of the closed sets in (ii) is indeed closed.

(iv) Is the topological space (A,£) connected? Justify your answer.

4] a) Define the following

(i) Boundary point of a set.
(ii) Continuous function at a point.
(iii) Homeomorphism.

b) (i) Prove that if A is a subset of a topological space $(X, £_x)$, then the set of boundary points A is empty if and only if A is both open and closed.

(ii) State the Heine-Borel theorem.
(iii) Prove the Heine-Borel theorem.

c) Let $f:(\mathbf{R}^+, £_1) \rightarrow (\mathbf{R}^-, £_2)$, where $f(x) = -\sqrt{x}$ with $£_1$ and $£_2$ the topologies on $\mathbf{R}^+$ and $\mathbf{R}^-$ respectively from the usual metrix on **R**. Is f a homeomorphism? Justify your answer.

5. a) Define the following

(i) Subspace of a topological space.
(ii) A base for a topology.
(iii) Open covering.

b) (i) Prove that if the topological spaces $(X, £_x)$ and $(Y, £_y)$ are homeomorphic then X is compact if and only if Y is compact.

(ii) State the intermediate-value theorem.
(iii) Prove the intermediate-value theorem.

c) Let (**R**, £) be the topological space with £ the usual topology on the real line **R** and $Z = [a, b] \cup (c, d)$ with $a < b < c < d$. Show that $A = [a, b]$ and $D = (c,d)$ are both open and closed in the subspace $(Z, £_z)$.

6] a) Define the following:

(i) Compact topological space.

(ii) Connected topological space.

(iii) Uniformly continuous.

b) Prove the following:

(i) A topological space (X,£) is compact if and only if whenever a family $\{F_\alpha : \alpha \in \Omega\}$ of closed sets is such that $\bigcap_{\alpha \in \Omega} F_\alpha = \phi$ then there exists a finite subset of indices $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ such that

$$\bigcap_{j=1}^{n} F_{\alpha_j} = \phi.$$

(ii) Let f: (X, £$_x$) → (Y,£$_y$) be a continuous function. If A is a connected subset of X, then f(A) is a connected subset of Y.

c) (i) State the Fixed-Point theorem.

(ii) Prove the Fixed-Point theorem.

END OF EXAMINATION

UNIVERSITY OF ZAMBIA

UNIVERSITY EXAMINATIONS - SEMESTER ONE 1998/99

M361- MATHEMATICAL STATISTICS

INSTRUCTIONS: Answer ANY Five(5) Questions.
All essential working must be shown.
Calculators are allowed.

TIME ALLOWED: Three (3) hours.

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1. (a) Define the following terms:

   (i) joint moment generating function of $Y_1, Y_2, \ldots, Y_k$.

   (ii) Jacobian of the transformation $(x,y) \to (u,v)$.

   (b) Prove the following:

   (i) If $X_1, X_2, \ldots, X_n$ are independent and identically distributed with common distribution function $F_X(.)$, then

   $$f_{Y_1}(y) = n(1 - F_X(y)^{n-1} f_X(y)$$

   where $Y_1 = \min(X_1, X_2, \ldots, X_n)$.

   (ii) If X and Y are jointly distributed with density $F_{X,Y}(x,y)$, then

   $$f_z(z) = \int_{-\infty}^{\infty} f_{X,Y}(x, z-x)dx$$

   where $Z = X + Y$.

   (c) Suppose X and Y are independent random variables, each uniformly distributed over the interval $(0,1)$. Let $Z = XY$ and $U = \frac{X}{Y}$.

   Find
   (i) the density function of Z.
   (ii) the density function of U.

2. (a) Define the following terms:

(i) t-distribution with n degrees of freedom.
(ii) F-distribution with m and n degrees of freedom.

(b) Prove the following:
(i) Let $X_1, X_2,...,X_n$ be independent random variables with moment-generating functions

$M_{X_1}(t), M_{X_2}(t),..., M_{X_n}(t)$, then

$M_Y(t) = M_{X_1}(t)M_{X_2}(t)...M_{X_n}(t)$

where $Y = X_1 + X_2 + ... + X_n$

(iii) If $X_i \sim N(\mu_i, \sigma_i^2)$, then

$$U = \sum_{i=1}^{n}\left(\frac{X_i - \mu_i}{\sigma_i}\right) \sim \chi^2(n)$$

(c) Derive the probability density function for a t-distribution with n degrees of freedom.

3. (a) Define the following terms:

(i) order statistics of a random sample $X_1, X_2,..., X_n$.
(ii) mean squared error of an estimator T of θ.

(b) Prove the following:

(i) If $X_1, X_2,...,X_n$ is a random sample from a normal distribution,

$X_i \sim N(\mu,\sigma^2)$, then

$$\frac{\overline{X} - \mu}{\frac{S}{\sqrt{n}}} \sim t(n-1), \quad \text{where } S^2 = \text{sample variance.}$$

(ii) The mean-squared error of an estimator is equal to the sum of the variance of the estimator and its bias squared.

3. (c) Suppose that X_1, X_2, and X_3, represent a random sample of size 3 from a population with density

$$f(x) = \frac{x}{2}, \quad 0 < x < 2.$$

Find

(i) the joint density of the order statistics Y_1, Y_2 and Y_3.

(ii) the marginal density functions of Y_1, Y_2 and Y_3.

4. (a) Define the following terms:

(i) a sufficient statistic.
(ii) relative efficiency of two estimators.

(b) Prove the following:

(i) If $X_1, X_2, \ldots, X_n$ is a random sample from a Bernoulli distribution, $X \sim B(1, \theta)$, then

$$T_n = \sum_{i=1}^{n} X_i \text{ is sufficient for } \theta.$$

(ii) A sequence $\{T_n\}$ of estimators of $\tau(\theta)$ is mean squared error consistent if and only if it is asymptotically unbiased and $\lim_{n \to \infty} \text{var}(T_n) = 0$

(c) Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the uniform distribution on the interval $(0, \theta)$. Two estimators of θ are

$$T_1 = 2\bar{Y} \text{ and } T_2 = \left(\frac{n+1}{n}\right) Y_n$$

where $Y_n = \max(Y_1, Y_2, \ldots, Y_n)$ and $\bar{Y}$ is the sample mean.

(i) Show that T_1 and T_2 are unbiased estimators of θ.
(ii) Find the efficiency of T_1 relative to T_2.

5. a) Define the following terms:

(i) asymptotically unbiased estimator.

(ii) uniformly minimum-variance unbiased estimator (UMVUE).

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from $f(x;\theta) = (\theta+1)x^{-\theta-2}$, $x > 1$. Find

(i) the method of moments estimator for θ.

(ii) the maximum likelihood estimator for θ.

(c) Construct a most powerful test for testing

$H_o : \mu = \mu_o$ against $H_1 : \mu = \mu_1$ (where $\mu_1 > \mu_o$)

where $X_1, X_2, \ldots, X_n$ is a random sample from $X \sim N(\mu, \sigma^2)$ with σ^2 known.

6. (a) Define the following terms:

(i) the power function of a test.
(ii) most powerful test.

(b) (i) State the Neyman- Pearson Lemma.

(ii) Prove the Neyman - Pearson Lemma.

(c) Consider a random sample of size n from a Bernoulli distribution, $X \sim B(1, \theta)$.
Find
(i) the Cramer-Rao lower bound for the variances of unbiased estimators of θ.

(ii) a UMVUE of θ.

7. (a) Define the following terms:

(i) an unbiased test.
(ii) generalized likelihood ratio test.

(b) (i) Suppose X has the pareto density $f_X(x) = \theta x^{-\theta-1}$, $x \geq 1$. Find the density of $Y = \ln X$.

(ii) Let X and Y be independent random variables each having an exponential distribution with parameter λ. Find the density function of $Z = X + Y$.

(c) Construct a generalized likelihood ratio test for $H_o : \mu \neq \mu_o$ against $H_1 : \mu \neq \mu_o$ where $X_1, X_2, \ldots, X_n$ is a random sample from $X \sim N(\mu, \sigma^2)$ with σ^2 unknown.

END OF EXAM - GOOD LUCK

UNIVERSITY OF ZAMBIA

SEMESTER ONE DEFERRED/SUPPLEMENTARY EXAMINATION

1998/99

M361- MATHEMATICAL STATISTICS

INSTRUCTIONS: Answer ANY Five(5) Questions.
All essential working must be shown.
Calculators are allowed.

TIME ALLOWED: Three (3) hours.

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1. (a) Define the following terms:

    (i) an unbiased test.
    (ii) generalized likelihood ratio test.

   (b) (i) Suppose X has the pareto density $f_X(x) = \theta x^{-\theta-1}$, $x \geq 1$. Find the density of $Y = \ln X$.

    (ii) Let X and Y be independent random variables each having an exponential distribution with parameter λ. Find the density function of $Z = X + Y$.

   (c) Construct a generalized likelihood ratio test for $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$ where $X_1, X_2, \ldots, X_n$ is a random sample from $X \sim N(\mu, \sigma^2)$ with σ^2 unknown.

2. (a) Define the following terms:

    (i) the power function of a test.
    (ii) most powerful test.

   (b) (i) State the Neyman- Pearson Lemma.
    (ii) Prove the Neyman - Pearson Lemma.

   (c) Consider a random sample of size n from a Bernoulli distribution, $X \sim B(1, \theta)$.
   Find
    (i) the Cramer-Rao lower bound for the variances of unbiased estimators of θ.

    (ii) a UMVUE of θ.

3. a) Define the following terms:

(i) asymptotically unbiased estimator.

(ii) uniformly minimum-variance unbiased estimator (UMVUE).

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from $f(x;\theta) = (\theta + 1)x^{-\theta-2}$, $x > 1$. Find

(i) the method of moments estimator for θ.

(ii) the maximum likelihood estimator for θ.

(c) Construct a most powerful test for testing

$H_o : \mu = \mu_o$ against $H_1 : \mu \neq \mu_1$ (where $\mu_1 > \mu_o$)

where $X_1, X_2, \ldots, X_n$ is a random sample from $X \sim N(\mu, \sigma^2)$ with σ^2 known.

4. (a) Define the following terms:

(i) a sufficient statistic.
(ii) relative efficiency of two estimators.

(b) Prove the following:

(i) If $X_1, X_2, \ldots, X_n$ is a random sample from a Bernoulli distribution, $X \sim B(1, \theta)$, then

$$T_n = \sum_{i=1}^{n} X_i \quad \text{is sufficient for } \theta.$$

(ii) A sequence $\{T_n\}$ of estimators of $\tau(\theta)$ is mean squared error consistent if and only if it is asymptotically unbiased and $\lim_{n \to \infty} \operatorname{var}(T_n) = 0$

(c) Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the uniform distribution on the interval $(0, \theta)$. Two estimators of θ are

$$T_1 = 2\overline{Y} \text{ and } T_2 = \left(\frac{n+1}{n}\right) Y_n$$

where $Y_n = \max(Y_1, Y_2, \ldots, Y_n)$ and $\overline{Y}$ is the sample mean.

(i) Show that T_1 and T_2 are unbiased estimators of θ.
(ii) Find the efficiency of T_1 relative to T_2.

5. (a) Define the following terms:

(i) order statistics of a random sample $X_1, X_2, \ldots, X_n$.

(ii) mean squared error of an estimator T of θ.

(b) Prove the following:

(i) If $X_1, X_2, \ldots, X_n$ is a random sample from a normal distribution,

$X_i \sim N(\mu, \sigma^2)$, then

$$\frac{\overline{X} - \mu}{\frac{S}{\sqrt{n}}} \sim t(n-1), \quad \text{where } S^2 = \text{sample variance.}$$

(ii) The mean-squared error of an estimator is equal to the sum of the variance of the estimator and its bias squared.

(c) Suppose that X_1, X_2, and X_3, represent a random sample of size 3 from a population with density

$$f(x) = \frac{x}{2}, \quad 0 < x < 2.$$

Find

(i) the joint density of the order statistics Y_1, Y_2 and Y_3.

(ii) the marginal density functions of Y_1, Y_2 and Y_3.

6. (a) Define the following terms:

(i) t-distribution with n degrees of freedom.

(ii) F-distribution with m and n degrees of freedom.

(b) Prove the following:

(i) Let $X_1, X_2, \ldots, X_n$ be independent random variables with moment-generating functions

$M_{X_1}(t), M_{X_2}(t), \ldots, M_{X_n}(t)$, then

$$M_Y(t) = M_{X_1}(t) M_{X_2}(t) \ldots M_{X_n}(t)$$

where $Y = X_1 + X_2 + \ldots + X_n$

(iii) If $X_i \sim N(\mu_i, \sigma_i^2)$, then

$$U = \sum_{i=1}^{n}\left(\frac{X_i - \mu_i}{\sigma_i}\right) \sim \chi^2(n)$$

(c) Derive the probability density function for a t-distribution with n degrees of freedom.

7. (a) Define the following terms:

(i) joint moment generating function of $Y_1, Y_2, \ldots, Y_k$.

(ii) Jacobian of the transformation $(x,y) \to (u,v)$.

(b) Prove the following:

(i) If $X_1, X_2, \ldots, X_n$ are independent and identically distributed with common distribution function $F_X(.)$, then

$$f_{Y_1}(y) = n(1 - F_X(y))^{n-1} f_X(y)$$

where $Y_1 = \min(X_1, X_2, \ldots, X_n)$.

(ii) If X and Y are jointly distributed with density $F_{X,Y}(x,y)$, then

$$f_z(z) = \int_{-\infty}^{\infty} f_{X,Y}(x, z-x)dx$$

where $Z = X + Y$.

(c) Suppose X and Y are independent random variables, each uniformly distributed over the interval $(0,1)$. Let $Z = XY$ and $U = \dfrac{X}{Y}$.

Find

(i) the density function of Z.

(ii) the density function of U.

END OF EXAM - GOOD LUCK

UNIVERSITY OF ZAMBIA

UNIVERSITY EXAMINATIONS - SEMESTER ONE 1998/99

M411- THEORY OF FUNCTIONS OF A COMPLEX VARIABLE I.

INSTRUCTIONS: Answer ANY Five(5) Questions.

TIME ALLOWED: Three (3) hours.

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1. a) Define **uniform continuity** of a complex valued function f(z).

Hence show that if $f(z) = z^2 + 2z$

$$\lim_{z \to i} f(z) = 2i - 1$$

uniformly in the complex plane.

b) Prove that $f(z) = |z|^2$ is **continuous** at all points $z_0 \in$ **C**, but the derivative exists *only* at the origin.

c) Using **definition of a limit**, show that

$$\lim_{z \to 1} \frac{z^3 - z}{z^2 - 3z + 2} = -2.$$

2. a) Define **absolute convergence** of an infinite complex series.

b) Show that

$$z - \frac{z^3}{3!} + \frac{z^5}{5!} - \ldots$$

is absolutely convergent.

c) Discuss *convergence* of the following series

(i) $\sum_{n=0}^{\infty} n! z^n$ (ii) $\sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!}$

3. a) State Cauchy's theorem

b) Verify Cauchy's theorem given

$$f(z) = e^z$$

and the path C is the triangle with vertices at 0, 1, 1 + i.

c) Evaluate the *complex integral*

$$\int_C \bar{z}^2 dz \qquad C: |z| \leq 1$$

Can Cauchy's theorem be applied to this integral? Give reason(s) for your answer.

4. (i) Find **principal** value of

$$(1 - i)^{2-3i}$$

(ii) Find all *possible* solutions of

$$e^z = i$$

(iii) Obtain an *upper bound* for the integral

$$\frac{1}{2i}\int_C \frac{e^{2z}}{1+z^2}dz, \quad \text{where } C: |z| = 3$$

5. a) Define a **conformal** mapping. If f(z) = u + iv is analytic prove that

$$\frac{\partial(u.v)}{\partial(x,y)} = |f'(z)|^2$$

b) State *necessary condition* for f(z) to be conformal. Determine points at which

$$f(z) = z + \frac{1}{z}, \quad z \neq 0$$

fails to be conformal.

c) Find a bilinear mapping which maps the interior of a unit circle onto the lower half plane so that points 1, i, -i are mapped into 1, -1, ∞, respectively.

6. a) From ***first principles***, if

$$f(z) = \frac{2z - i}{z + 2i}, \quad z_o = i$$

show that $f'(z_o) = \frac{-5}{9}i$

b) Compute $\int_o^{1+i} \bar{z}dz$ along the arc $y = x^2$.

c) Show that $u = 2x(1 - y)$ is harmonic and find its harmonic conjugate.

Express $f(z) = u + iv$ in terms of z.

END OF EXAMINATION.

UNIVERSITY OF ZAMBIA
UNIVERSITY SUPPLEMENTARY/DEFERRED EXAMINATIONS
SEMESTER - ONE 1998/99

M411-THEORY OF FUNCTION OF A COMPLEX VARIABLE I.

INSTRUCTIONS : **Attempt Any Five(5) Questions.**

TIME ALLOWED: **Three(3) Hours**

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1. (a) Find a linear fractional transformation which maps the left half-plane onto the interior of the unit circle so that points (-i,0,i) are mapped into (i,-i,1)

   (b) Find all possible values

$$\left[\frac{5}{2}(1+i\sqrt{3})\right]^{i}$$

   Hence find the principal value.

2. (a) Show that $u = x^3 - 3xy^2$ is harmonic. Find its harmonic conjugate.

   (b) Prove that $u = x^2 - y^2$ and $v = \dfrac{y}{x^2+y^2}$ are both harmonic.
   However, show that the function $f(x,y) = u + iv$ is **not** analytic.

3. (a) Prove that $\sec^{-1}z = \dfrac{1}{i}\ln\left(\dfrac{1+\sqrt{1-z^2}}{z}\right)$

   (b) Find all solutions of the equation

$$\log z = 1 + \pi i$$

4. (a) Define a **conformal mapping.**

(b) Determine whether

$$f(z) = \frac{z-1}{z^2+1}$$

is analytic and conformal at $z = -1$.

(c) Using definition of limit (or from first principles) find

$$\lim_{z \to -2i} \frac{z^2 - 4}{z + 2i}$$

5. (a) Find an upper bound for the integral

$$\int_C z^2 dz \qquad C: 0 \text{ to } 2 + 2i$$

(b) Evaluate $\int_C \frac{z+4}{z^2 + 2z + 5} dz$ along $C \ |z+1-i| = 2$

6. (a) Define conditional convergence.

(b) Determine whether the following series converges

(i) $\sum_{n=1}^{\infty} \frac{(z-i)^n}{n^2}$ (ii) $\sum_{n=1}^{\infty} z^{2n}$

where applicable find the radius of convergence.

END OF EXAMINATION

UNIVERSITY OF ZAMBIA

UNIVERSITY EXAMINATIONS - SEMESTER ONE 1998/99

M421- STRUCTURE AND REPRESENTATIONS OF GROUPS.

INSTRUCTIONS: Attempt *four(4)* questions in all and *atleast two*(2) questions from section *A* and *atleast one* from section B.

TIME ALLOWED: Three (3) hours.

SECTION A (STRUCTURE OF GROUPS)

(Attempt atleast two(2) questions from this section)

1. Define each of the following terms:

   (i) A *normal series* of a finite group
   (ii) A *commutator (derived)* subgroup G' of a group G.

(a) Prove that:

   (i) if a group G has a normal series

$$G = G_0 \supseteq G_1 \supseteq \dots \supseteq G_{r-1} \supseteq G_r = \{e\}$$

   then every subgroup N of G possesses a normal series.

   (ii) if G has a normal subgoup N such that the factor group G/N is abelian, then N contains the commutator (derived) subgroup G' of G.

(b) (i) Show that if N is a normal subgroup of a group G such that $N \cap G' = \{e\}$, then N contains the centre $Z(G)$ of G.

   (ii) Given that H is a subgroup of G such that the square of every element of G is contained in H then show that G/H is abelian. Hence deduce that $G' \subseteq H$.

2. Define each of the following terms

(i) A *solvable group* (ii) A *nilpotent group*

(a) Prove that

(i) if a solvable group G contains a normal subgroup N, then G/N is also solvable

(ii) if G is a nilpotent group which contains a subgroup H, then H is also nilpotent. Hence deduce that in this case H is solvable.

(b) Show that:

(i) the symmetric group S_5 of degree 5 is not solvable.

(ii) A group G of order p^{α}, where p is a prime, is nilpotent.

Hence by using the fact that a direct product of nilpotent groups is nilpotent, or otherwise, confirm that a group G of order $p_1 p_2$, where p_1 and p_2 are primes such that $p_1 > p_2$ and p_2 doesnot divide $p_1 - 1$, is nilpotent.

3. Let G be a permutation group acting on a set Ω. Then explain the meaning of each of the following terms:

(i) the *stabilizer* G_α of $\alpha \in \Omega$ in G

(ii) G is a *primitive permutation* group.

(a) Show that:

(i) the stabilizer G_α is a subgroup of G for each $\alpha \in \Omega$; and that if G is transitive on Ω then the stabilizers G_α of $\alpha \in \Omega$ are all conjugate in G.

(ii) if G is a primitve permutation group on Ω, then G_α is maximal in G for all $\alpha \in \Omega$.

(b) (i) Show that a nontrivial normal subgroup N of a primitive permutation group G is transitive.

(ii) Show that if G is a primitive regular permutation group then it is of prime order.

4. What is meant by each of the following terms

(i) The map θ is the action of a group H on a group K?

(ii) a semidirect product of K by H?

(a) (i) Let θ be the action of H on K and let

$$G = \{(h,k)/h \in H,\ k \in K\}.$$

Show that G forms a group under the product

$$(h_1,k_1)(h_2,k_2) = (h_1\theta(k_1)h_2,\ k_1k_2)$$

(ii) Let G be a semidirect product of K and H. Show that if H is normal in G then G is a direct product of K and H.

(b) Let $H_1, H_2, \ldots, H_n$ be normal subgroups of a group G such that $G = H_1H_2\ldots H_n$ and $H_1H_2\ldots H_{i-1} \cap H_i = \{1\}$ for each i.

Then show that

(i) every element x of G has a unique expression of the form $x = x_1x_2\ldots x_n$, where $x_i \in H_i$

(ii) The expression for $x \in G$ in b(i) above implies that G is isomorphic to a direct product of its normal subgroups $H_1, H_2, \ldots, H_n$.

SECTION B (REPRESENTATIONS OF GROUPS)

(Attempt atleast one(1) question from this section)

5. Give the meaning of each of the following terms:

(i) a completely reducible representation of a group G.

(ii) an irreducible representation of a group G.

(a) (i) Prove that if K is a field whose characteristic does not divide the order of the group G, then every representation of G over K is completely reducible.

(ii) State and prove Schur's Lemma.

(b) Let T be a representation of G of degree 2 over the field Q of rationals such that for some element z in the centre of G,

$$T(z) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Then show that T is a reducible representation of G. Is it completely reducible? Justify your answer.

6. Give the meaning of each of the following terms:

(i) The *character* of a representation T of a group G.

(ii) The *first orthogonality relations* for the characters of a group G.

(a) Let χ and θ be any characters of a group G and

$\chi_1 = 1_G, \chi_2, \ldots, \chi_s$ be the irreducible characters of G. If

$$(\chi,\theta)_G = \frac{1}{|G|} \sum_{g \in G} \chi(g)\, \theta(g^{-1}),$$

Prove that

(i) $(\chi_i, \chi_j) = \delta_{ij}$

(ii) two representations of G are equivalent if and only if they have the same character.

(b) Determine the character table of the dihedral group D_3 of degree 3, which is given by

$$D_3 = \langle a, b / a^3 = b^2 = 1;\ ba = a^2 b \rangle.$$

END OF EXAMINATION

UNIVERSITY OF ZAMBIA
UNIVERSITY SUPPLEMENTARY/DEFERRED EXAMINATIONS
SEMESTER - ONE 1998/99

M421-STRUCTURE AND REPRESENTATIONS OF GROUPS

INSTRUCTIONS : **Attempt four(4) questions in all and atleast two(2) questions from section A and atleast one(1) from section B.**

TIME ALLOWED: **Three(3) Hours.**

SECTION A (STRUCTURE OF GROUPS)
(Attempt atleast two(2) questions from this section)

1. Let G be a finite permutation group on the set Ω. Then give the meaning of each of the following terms:

   (i) The *stabilizer* G_α of $\alpha \in \Omega$ in G.
   (ii) G is a *regular permutation group*.

   (a) (i) Show that in a transitive permutation group, all the stabilizers are conjugate to each other.

   (ii) Show that if G is an imprimitive transitive group, then no stabilizer G_α is maximal in G.

   (b) Show that if G is a regular primitive group on Ω then it is of prime order.

2. Define each of the following terms:

   (i) A normal series of a finite group
   (ii) A commutator (derived) subgroup G' of a group G

   (a) Prove that:

   (i) If a group G has a normal series $G=G_0 \supseteq G_1 \supseteq \ldots \supseteq G_{r-1} \supseteq G_r = \{e\}$ then every subgroup N of G possesses a normal series.
   (ii) If G has a normal subgroup N such that the factor group G/N is abelian, then N contains the commutator (derived) subgroup G' of G.

(b) (i) Show that if N is a normal subgroup of a group G such that $N \cap G' = \{e\}$, then N contains the centre $Z(G)$ of G.

(ii) Given that H is a subgroup of G such that the square of every element of G is contained in H then show that G/H is abelian. Hence deduce that $G' \subseteq H$.

3. What is meant by the saying "The group G is a direct product of its normal subgroups $H_1, H_2, \ldots, H$"?

(a) Given that $H_1, H_2, \ldots, H_n$ are normal subgroups of a group G such that

(i) $G = H_1H_2H_3 \ldots H_n$, and

(ii) $H_1H_2H_3 \ldots H_{i-1} \cap H_i = \{e\}$.

Then show that each element x of G has a unique expression of the form

$x = h_1h_2 \ldots h_n$, where $h_i \in H_i$.

(b) Show that if G is a group of order p_1p_2, where p_i are primes such that p_2 is greater than p_1 and p_1 does not divide $p_2 - 1$, then G is a direct product of its sylow subgroups.

Hence deduce that a group of order 35 is a direct product of its sylow subgroups.

4. Define each of the following terms

(i) A solvable group. (ii) A nulpotent group.

(a) Prove that

(i) if a solvable group G contains a normal subgroup N, then G/N is also solvable;

(ii) if G is a nilpotent group which contains a subgroup H, then H is also nilpotent. Hence deduce that in this case H is solvable

(b) Show that:

(i) The symmetric group S_5 of degree 5 is not solvable;
(ii) A group G of order p^{α}, where p is a prime is nilpotent.

Hence deduce that a group G of order $p_1 p_2$, where p_1 and p_2 are primes such that $p_1 > p_2$ and $p_2 + p_1 - 1$ is nilpotent.

SECTION B(REPRESENTATIONS OF GROUPS)
(Attempt atleast one(1) question from this section)

5. Give the meaning of each of the following terms:

(i) A *reducible representation* of a group G;

(ii) A *group character* χ of a group G.

(a) Prove that a group character of a group G is a class function on G

(b) State Schur's lemma.

Given that a matrix representation T of a group G over the field Q of rationals is such that for a certain element x in the centre $Z(G)$ of G,

$$T(x) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Then

(i) show that the matrix A given by $A = I2 + T(x)$ satisfies the condition

$$T(g)\, A = A\, T(g), \text{ for all } g \in G,$$

(ii) deduce that T is a reducible representation of G over Q.

6. Give the meaning of each of the following terms:

(i) The *character* of a representation T of a group G;

(ii) The *first orthogonality relations* for the characters of a group G.

(a) Let χ and θ be any character of a group G and

$$\chi_1 = 1_G, \chi_2, \chi_3, \ldots, \chi_s$$

be the irreducible characters of G. If

$$(\chi, \theta)_G = \frac{1}{|G|} \sum_{g \in G} \chi(g)\theta(g^{-1}),$$

prove that

(i) $(\chi_i, \chi_j) = \delta_{ij}$;

(ii) two representations of G are equivalent if and only if they have the name character .

(b) Determine the character table of the dihedral group D_3 of degree 3, which is given by

$$D_3 = \langle a, b \; / a^3 = b^2 = 1; \; ba = a^2 b \rangle.$$

END OF EXAMINATION

UNIVERSITY OF ZAMBIA

UNIVERSITY EXAMINATIONS - SEMESTER ONE 1998/99

M431 - REAL ANALYSIS V.

INSTRUCTIONS: Answer ANY Five(5) Questions:

TIME ALLOWED: Three (3) hours.

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1. (a) (i) Define a partial order relation in a non-empty set.
   (ii) Define a well ordered set.

   (b) Let W_1 and W_2 be well ordered sets. For $(m_1,n_1), (m_2,n_2) \in W_1 \times W_2$, write $(m_1,n_1) \leq (m_2,n_2)$ if any of the following hold:

   (i) $m_1 < m_2$, and (ii) $m_1 = m_2$ and $n_1 \leq n_2$.

   Show that $\leq$ is a partial order on $W_1 \times W_2$.

2. (a) (i) Let $1 \leq p < \infty$. Define the l^p set. If $a \in l^p$, define $\|a\|_p$.

   (ii) Define the l^∞ set. If $a \in l^\infty$ define $\|a\|_\infty$.

   (b) Let $1 \leq p < \infty$. Define d: $l^p \times l^p \to \mathbf{R}$ by

   $d(x,y) = \|x - y\|_p$. Let $N \in \mathbf{N}$ and

   $E = \left\{\{x_n\}_{n=1}^{\infty} \in l^p : x_n = 0, \forall n > N\right\}$.

   Prove that $(\overline{E})^\circ = \phi$.

   (c) Define d: $l^\infty \times l^\infty \to \mathbf{R}$ by $d(x,y) = \|x - y\|_\infty$. If A is any countable subset of l^∞, prove that $\overline{A} \neq l^\infty$.

3. (a) Let (X,d) be a metric space and $f:X \to \mathbf{R}$. Let $x_0 \in X$. When is f said to be

(i) continuous at x_0,

(ii) upper semi continuous at x_0?

(b) By assuming that if $a \geq 1$ and $0 < h \leq 1$ then $a^h - 1 \leq 2(a-1)h$, prove that, if $a > 1$, the function $f:[0,\infty) \to [0,\infty)$ defined by $f(x) = a^x$ is continuous on $[0,\infty)$.

(c) Let (X,d) be a metric space. Prove that, if f is upper semi continuous at $x_0 \in X$, then for any sequence $\{x_n\}_{n=1}^{\infty}$ in X converging to x_0, $\limsup f(x_n) \leq f(x_0)$.

4. (a) Let (X,d) be a metric space and $A \subset X$. When is A said to be compact?

(b) Let (X,d) be a metric space If X is compact, $A \subset X$ is closed, show that A is compact.

(c) Let (X,d) be a metric space and $A, K \subset X$. Suppose A is closed in X, K is compact and $A \cap K = \phi$. Prove that $\inf\{d(x,y): x \in A \text{ and } y \in K\} > 0$.

5. (a) Define a countable set

(b) Let A be an infinite countable set and $f: A \to B$ be surjective. Prove that B is countable.

(c) Let A be an infinite countable set and

$$C = \left\{ \{a_k\}_{k=1}^{n} : n \in N, a_k \in A, 1 \leq k \leq n \right\}.$$

Prove that C is countable.

6. (a) Define the cantor set

(b) Give the first four (4) terms of the ternary expansion of $\frac{3}{4}$.

7. (a) (i) Let X be a set. Define a metrix on X.

(ii) Let (X,d) be a metrix space, $x \in X$ and $r \in \mathbf{R}$, $r > 0$. Define an open ball of radius r and centre. x.

(b) Let d: **R** x **R** $\rightarrow$ **R** be defined by

$d\left((x_1, x_2), (y_1, y_2)\right) = \max\left\{|x_1, -y_1|\right\}$. Show that d is a metrix on **R** x **R** and sketch the open ball of radius 1 and centre (0,0).

END OF EXAMINATION.

UNIVERSITY OF ZAMBIA
UNIVERSITY SUPPLEMENTARY/DEFERRED EXAMINATIONS
SEMESTER - ONE 1998/99

M431-REAL ANALYSIS V

INSTRUCTIONS : **Attempt Five(5) questions.**

TIME ALLOWED: **Three(3) Hours.**

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1. a) Define the following

 i) a finite set
 ii) a countable set.

 b) If a set A is countable and if $B \subset A$ prove that B is countable.

---

2. a) Define the following

 i) The Cantor set
 ii) A nowhere dense subset of a metric space.

 b) Prove that the Cantor set is nowhere dense in (**R**,d), where $d(x,y)=|x-y|$.

---

3. a) Define the following

 i) limit of a sequence in a metric space.
 ii) limit point of a sequence in a metric space.
 iii) limit point of a subset of a metric space.

 b) Let (X,d) be a metric space. Let $\{x_n\}_{n=1}^{\infty}$ be a sequence X and $x \in X$. Prove that x is limit point of $\{x_n\}_{n=1}^{\infty}$ if and only if $\exists$ a subsequence $\{x_{n_k}\}_{k=1}^{\infty}$ such that x is the limit of the subsequence.

 c) Let (X,d) be a metrix space, $A \subset X$. Prove that if x is a limit point of A then there exists a sequence $\{y_n\}_{n=1}^{\infty}$ such that x is the limit of this sequence.

---

4. a) Define the following

i) A compact metric space

ii) A uniformly continuous function from on metric space to another.

b) Let (X,d_1) be a compact metric space and (Y,d_2) be a metric space. Prove that if $f:X\to Y$ is continuous, then it is uniformly continuous.

---

5. a) Let (X,d) be a metric space. Define the following

i) an open set in X,

ii) a closed set in X.

iii) an interior point of a subset of X

iv) interior of a subset of X.

b) Let (X,d) be a metric space and $x\in X$. Let $r>O$. Prove that $\{y\in X:d(x,y)\leq r\}$ is a closed set in X.

c) In (**R**,d) where $d(x,y)=|x-y|$ show that the interior of the closed interval [0,1] is the open interval (0,1).

---

6. a) i) Let $1\leq p$. Define l^p, l^∞, $\|\cdot\|_p$, $\|\cdot\|_\infty$.

ii) Define a point of closure of a subset of a metric space.

iii) The closure of a subset of a metric space.

iv) a dense subset of a metric space.

b) Let (X,d) be a metric space and $A\subset X$. Let $B=\{x\in X:x$ is a point of closure of $A\}$.
Assuming that both B and the closure of A are closed sets in x, prove that the closure of A equals B.

c) Let $p\geq 1$. $\forall N\in \mathbf{N}$ let $E_N=\left\{\{x_n\}_{n=1}^{\infty}\in l^p:x_n=0,\forall n>N\right\}$ and $E=\bigcup_{N\in N} E_N$. Prove that E is dense in (l^p,d) where $d(x,y)=\|x-y\|_p$.

---

7. a) Define the following

i) a partially ordered set.
ii) a well ordered set.

b) Let α and β be ordinal numbers.
Let $W_\alpha \in \alpha$, $W_\beta \in \beta \ni W_\alpha \cap W_\beta = \phi$. Let $W = W_\alpha \cup W_\beta$ be ordered by $x \leq y$ if any of the following holds:

1) $x, y \in W_\alpha$ and $x \leq y$ in W_α,
2) $x, y \in W_\beta$ and $x < y$ in W_β.
3) $x \in W_\alpha$, $y \in W_\beta$.

Prove that W is a well ordered set.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

UNIVERSITY EXAMINATIONS - SEMESTER ONE 1998/98

M461 - MULTIVARIATE ANALYSIS

INSTRUCTIONS: Answer ANY Five(4) questions.
All essential working must be shown.

Calculators are allowed.
Statistical Tables will be provided.

TIME ALLOWED: Three (3) hours.

1. (a) Define the following terms:

   (i) sample correlation coefficient.
   (ii) quadratic form.

   (b) Prove the following:

   (i) If V is a vector random variable with mean vector $E(V) = \mu_v$ and covariance matrix $E(V - \mu_v)(V - \mu_v) = \Sigma_v$, then $E(VV^T) = \Sigma_v + \mu_v \mu_v^T$

   (ii) $\bar{X}$ is an unbiased estimator for μ and has covariance $\frac{1}{n}\sum$, where $X_1, X_2, \ldots X_n$ is a random sample such that each X_i has mean μ and covariance matrix Σ.

   (iii) $\frac{n}{n-1} S_n$ is an unbiased estimator for Σ, where $X_1, X_2, \ldots X_n$ is as given in (ii) above.

   (c) Given the five measurements on variables X_1, X_2 and X_3.

x_1	9	2	6	5	8
x_2	12	8	6	4	10
x_3	3	4	0	2	1

Find the

   (i) mean vector $\bar{x}$.
   (ii) covariance matrix S.
   (iii) correlation matrix R.

2. (a) Define the following terms:

   (i) trace of a matrix.
   (ii) spectral decomposition of a matrix.

(b) Prove the following:

   (i) If X is a p-dimensional random vector and c a p-dimensional vector of constants, then $Y = c^T X$ has mean $a^T \mu$ and covariance $c^T \Sigma c$.

   (ii) If Σ is a positive definite matrix and (λ, e) is an eigenvalue-eigenvector pair of Σ, then (λ^{-1}, e) is an eigenvalue-eigenvector pair of Σ^{-1}.

   (iii) $(X-\mu)^{\Gamma} \Sigma^{-1} (X-\mu) \sim \chi_p^2$, where $X \sim N_p(\mu, \Sigma)$.

(c) Let

$$A = \begin{pmatrix} 9 & -2 \\ -2 & 6 \end{pmatrix}$$

   (i) Determine the eigenvalues and eigenvectors of A.

   (ii) Write the spectral decomposition of A.

   (iii) Find A^{-1}.

   (iv) Find the eigenvalues and eigenvectors of A^{-1}.

3. (a) Define the following terms:

(i) random matrix.
(ii) independence of two random vectors.

(b) (i) State and prove the cauchy-shwarz inequality.
(ii) Show that if

$$\Sigma = V^{\frac{1}{2}} \rho V^{\frac{1}{2}} \text{ then}$$

$$\rho = \left(V^{\frac{1}{2}}\right)^{-1} \Sigma \left(V^{\frac{1}{2}}\right)^{-1}$$ where $V = \text{diag}(\sigma_{11}, \ldots, \sigma_{pp})$ hence or

otherwise find ρ given

$$\Sigma = \begin{pmatrix} 4 & 1 & 2 \\ 1 & 9 & -3 \\ 2 & -3 & 25 \end{pmatrix}$$

(c) Let $X \sim N_3(\mu, \Sigma)$ with $\mu^T = (-3,1,4)$
and

$$\Sigma = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

(A) Which of the following random variables are independent? Explain.

(i) X_1 and X_2

(ii) X_2 and X_3

(iii) (X_1, X_2) and X_3

(iv) $\frac{X_1 + X_2}{2}$ and X_3

(B) Find the conditional distribution of X_2 given $X_1 = x_1$ and $X_3 = x_3$.

4. (a) Define the following terms:

(i) Hotelling's T^2 statistic for $H_o: \mu = \mu_o$ versus $H_1: \mu \neq \mu_o$.

(ii) a $100(1-\alpha)\%$ confidence region for θ.

(b) Prove the following:

(i) The T^2 statistic is invariant under transformations of the form

$$Y = CX + d$$

with C nonsingular

(ii) If $\begin{pmatrix} X_1 \\ \hline X_2 \end{pmatrix} \sim N_p\left[\begin{pmatrix} \mu_1 \\ \hline \mu_2 \end{pmatrix}, \left(\begin{array}{c|c} \Sigma_{11} & \Sigma_{12} \\ \hline \Sigma_{21} & \Sigma_{22} \end{array}\right)\right]$, then the

conditional distribution of X_1, given $X_2 = x_2$ is normal with

mean $= \mu_{1/2} = \mu_1 + \Sigma_{12} \Sigma_{22}^{-1} (x_2 - \mu_2)$ and

covariance $= \Sigma_{1/2} = \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21}$

(c) Let the data matrix for a random sample of size $n = 4$ from a bivariate normal population be

$$X = \begin{pmatrix} 2 & 12 \\ 8 & 9 \\ 6 & 9 \\ 8 & 10 \end{pmatrix}$$

(i) Evaluate T^2 for testing $H_o : \mu^T = (7,11)$.
(ii) Specify the distribution of T^2.
(iii) Using (i) and (ii), test $H_o : \mu^T = (7,11)$ versus $H_1 : \mu^T \neq (7,11)$ at the $\alpha = 0.05$ level. What conclusion is reached?
(iv) Evaluate the Wilk's lambda.

5. (a) Find the maximum likelihood estimates of the 2x1 mean vector μ and the 2x2 covariance matrix Σ based on the random sample

$$X = \begin{pmatrix} 3 & 6 \\ 4 & 4 \\ 5 & 7 \\ 4 & 7 \end{pmatrix}$$

from a bivariate normal population.

(b) The various costs associated with transporting milk from farms to dairy plants were studied for 25 gasoline trucks. Three variables X_1 = fuel, X_2 = repair and X_3 = capital, were measured with the following results

$$\bar{x} = \begin{pmatrix} 12.56 \\ 8.1612 \\ 10.5444 \end{pmatrix} \text{ and } S = \begin{pmatrix} 28.96593 & 17.21536 & 2.694567 \\ 17.21536 & 21.45285 & 6.044528 \\ 2.694567 & 6.044528 & 13.59904 \end{pmatrix}$$

(i) construct the 95% Bonferroni intervals for the individual cost means.
(ii) construct the 95% T^2 - intervals for the individual cost means.

(c) Samples of waste water were divided and sent to two labs for testing. One-half of each sample was sent to a state lab and the other half to a private lab. Measurements of biochemical oxygen demand (BOD) and suspended solids were obtained for n = 11 sample splits, from the two labs. The data is displayed below.

Sample i	Private lab x_{i11}(BOD)	x_{i21}(SS)	State lab x_{i12}(BOD)	x_{i22}(SS)
1	6	27	25	15
2	6	23	28	13
3	18	64	36	22
4	8	44	35	29
5	11	30	15	31
6	34	75	44	64
7	28	26	42	30
8	71	124	54	64
9	43	54	34	56
10	33	30	29	20
11	20	14	39	21

Do the two lab's chemical analyses agree?
Use $\alpha = 0.05$.

END OF EXAMINATION.

UNIVERSITY OF ZAMBIA

UNIVERSITY EXAMINATIONS - SEMESTER ONE 1998/99

M465- NON-PARAMETRIC METHODS.

INSTRUCTIONS:

1. Attempt **Five (5)** questions, ***at least one question from each section***
2. You may use calculators and tables, table have been provided.

TIME ALLOWED: Three (3) hours

SECTION A

CONCEPTS

QUESTION 1

(a) Most of the statistical procedures assume the data are distributed according to some specified family of distributions such as the normal.

(i) Mention two situations when it becomes necessary to use nonparametric procedures.

(ii) Mention **one advantage** and **one disadvantage** nonparametric procedures have over parametric ones.

(iii) Mention one graphical tool you would use to check normality assumptions for a quantitative variable.

(iv) A sign test and *t* test were performed, a sign test failed to reject the null while the t test rejected the null. You checked normality and found it satisfactory. Explain briefly why a t test would reject where the sign test fails to reject.

(b) Inference is about saying something about a population based on results from a sample, provide brief comments or answers to the following statements or questions:

(i) A statistician just performed a test in which she failed to reject the null hypothesis. What type of error could she have committed?

(ii) The researcher who brought her the data wanted to know what chances she had of detecting a difference if it really did exist. What is the special name given to the probability the researcher was asking?

(v) Explain why a p-value is a conditional probability, you only need to define it in full.

(C) (i) A sign test uses the binomial distribution on the number of positive or negative signs, why is it referred to as a nonparametric test?

(ii) What extra information does the signed rank test carry that the sign test does not?

(iii) Mention one assumption one needs to make in order to use the sign test?

QUESTION 2

(a) Describe briefly your understanding of:

(i) a Chi-square test of independence

(ii) a Chi-square test of homogeneity

(iii) the major difference between the two tests?

(b) A student has been given a project involving data analysis. Part of the analysis involves the following 2X2 table:

	VARIABLE B Yes	No	TOTAL
VARIABLE A Yes	n_{11}	n_{12}	$n_{1.}$
No	n_{21}	n_{22}	$n_{2.}$
TOTAL	$n_{.1}$	$n_{.2}$	n

When he inquired how the data was collected, he discovered VARIABLE A was a group variable.

(i) How many samples does the 2X2 table represent?

(ii) He would like to estimate:
Pr(Variable A = yes) and
Pr(Variable B = yes)
Which one among the two probabilities is he able to estimate?

(iii) If he performs a Chi-square test, what test is performing? See (a).

(c) Table 2.1 below displays the results of a study of depression following the loss of a spouse. In this study, 92 subjects were assessed for depression twice; shortly after the death of his or her spouse (first time) and then again approximately one year later (second time). At each of the two time points the subject was found to be either ***depressed*** or ***not depressed.***

Shortly After death	One Year Later: Depressed	Not Depressed	TOTAL
Depressed	12	24	36
Not Depressed	4	52	56
TOTAL	16	76	92

(i) What type of design is this?

(ii) Define the following:
Concordant pair
Discordant pair

(iii) Using the 5% level of significance, test if the probability of being depressed is the same at the two time points? Use the McNemar's test

(iv) Report the p-value or p-value range.

SECTION B

INFERENCE

QUESTION 3

Company Executives in a certain firm have noticed that if the median revenue for the firm is around K5 million every quarter, the firm is able to pay its workers without undue costs. A random sample of 10 revenues realized by the firm in the immediate past is as follows:

5.1, 4.0, 3.0, 3.8, 5.5, 5.4, 4.8, 2.8, 5.2, 4.5, in millions of Kwacha.

(a) The firm would like to know from you whether the median revenue is more than K5 million.

Let X be the revenue of the firm of a randomly chosen revenue period. Further, let Y = X - 5. Using the Binomial Test carry out the following:

Be sure to show every step to earn full credit.

(i) State the null and alternative hypotheses in terms of the median of Y.

(ii) State the decision rule at 5% level of significance.

(iii) Carry out the test and state your conclusion in words.

(iv) Report a p-value or p-value range.

(c) An improvement on the above test is the Wilcoxon signed rank test.

Be sure to show every step to earn full credit.

(i) Carry out the same test at 5% level of significance using the Wilcoxon signed rank test.

(ii) Report your conclusion

(iii) How do the two tests compare?

QUESTION 4

In a study to determine if there are sex differences in response to heat stress, a random sample of 10 men and a random sample of 8 women were put through a vigorous exercise program that involved the use of the treadmill (monotonous task). The environment was hot, and only minimal amounts of water were made available to the subjects. The percentage of body weight lost by each subject is given below arranged in ascending order:

Males		Females	
Subject	Body weight Loss (%)	Subject	Body weight Loss (%)
1	2.4	1	2.5
2	2.7	2	2.6
3	2.8	3	3.0
4	2.9	4	3.1
5	3.2	5	3.3
6	3.5	6	3.4
7	3.6	7	3.7
8	3.8	8	4.0
9	3.9		
10	4.1		

Be sure to show every step to earn full credit.

Using the Wilcoxon rank sum test carry out the following:

(a) State the null and a **two-sided** alternative hypotheses in terms of the percentage of body weight lost.

(b) State the decision rule at 5% level of significance.

(c) Carry out the test and state your conclusion in words.

(d) **state in full** the parametric test you would use to carry out the test, do not perform the test.

(e) Mention **two** things that make it difficut to check normality in this data set.

QUESTION 5

Cholera is an infectious intestinal disorder. It is common in most parts of Southern Asia and Africa. Cholera is caused by a comma-shaped bacterium called Vibrio cholerae. The microorganism is transmitted by water or food that has been contaminated with the feces (solid body wastes) of people who have the disease. Zambia has experienced the disease annually. Assume the following cases of the disease were reported from the regions shown, also shown are the ranks of the observations:

REGION							
Lusaka	(Rank)	**Kabwe**	(Rank)	**Chipata**	(Rank)	**Kitwe**	(Rank)
42	(17)	28	(6)	16	(4)	35	(13)
39	(15)	31	(9)	18	(5)	30	(8)
48	(18)	36	(14)	14	(2)	40	(16)
54	(20)	29	(7)	15	(3)	34	(12)
50	(19)	32	(10)	10	(1)	33	(11)

Be sure to show every step to earn full credit.

(a) Using Kruskal-wallis test , test the null hypothesis that the mean number of Cholera cases does not differ between the four regions, against the alternative that they differ. Use $\alpha = 0.05$.

(b) We would like to compare the cases of Lusaka versus Kabwe, and Kabwe versus Kitwe at 5% level of significance. What should be the adjusted α to ensure a level of 5%?

(c) Contrary to (b) above, we would like to compare just two cities Lusaka versus Kitwe i.e., $\alpha = 0.05$ will not be adjusted. Perform a test to determine whether the Cholera cases differ between the two cities.

SECTION C

THOERY

QUESTION 6

Consider a matched pair experiment with observations $(X_1, Y_1), \ldots, (X_n, Y_n)$. Let N equal the number of pairs such that $X_i = Y_i$ and S be the sign statistic.

By carrying out every step show that:

(a) The conditional distribution of S given $N = n_0$ is $Bin(n - n_0, P[Z > 0 | Z \neq 0])$, where $Z = X - Y$. i.e., $Z_i = X_i - Y_i$ for $i = 1, \ldots, n$.

(b) Deduce that under the hypothesis of no treatment effect the conditional distribution of S is $Bin(n - n_0, ½)$.

(c) When is it impossible to perform this test?

(d) When n is large and n_0 is relatively small, a large sample size test could be used. **State** in terms of n and n_0 the following:

(i) The mean for the large sample size test
(ii) The variance for the large sample size test
(iii) The test statistic you would use.

QUESTION 7

Consider a paired design in which x_i is the measurement on the first time point and y_i is the measurement on the second time point on the ith subject. Assume we have a random sample of N subjects i.e., $i = 1, 2, \ldots, N$.

Let:

- $D_i = X_i - Y_i$ be the difference of the measurements on the ith subject. $i = 1, 2, \ldots, N$. Assume the D's are independently identically distribution random variables from a symmetric distribution about zero.
- R_i be the rank of D_i among $|D_1|, |D_2|, \ldots, |D_N|$

The rank of D_i has N possible values, all of them equally likely.

Given:

- $Pr(R_i = r_i) = \frac{1}{N}$, $r_i = 1, 2, \ldots, N$.
- $Pr(R_i = r_i\ ,\ R_j = r_j) = \frac{1}{N(N-1)}$, $r_i \neq r_j$,

  $r_i = 1, 2, \ldots, N$
  $r_j = 1, 2, \ldots, N$

Be sure to show every step to earn full credit.

(a) Show that $E(R_i) = \frac{N+1}{2}$

(b) Show that $Cov(R_i, R_j) = -\frac{N+1}{12}$.

(b) Show that $var(R_i) = \frac{N^2-1}{12}$

(c) State a large sample size test statistic for testing $H_0: \mu_{Ri} = E(R_i) = \frac{N+1}{2}$ versus $H_1: \mu_{Ri} \neq E(R_i)$

END OF EXAM

UNIVERSITY OF ZAMBIA

UNIVERSITY EXAMINATIONS - SEMESTER ONE 1998/99

M911 - MATHEMATICAL METHODS V

INSTRUCTIONS: (i) Attempt any Five(5) Questions.
(ii) Use of calculators not allowed.

TIME ALLOWED: Three (3) hours.

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1. (a) Let $f(x,y) = 4xy + \lambda(x^2 + 2y^2 - 1)$, λ = constant.

   (i) Find **critical points** of f

   (ii) Determine values of λ for f to have

   - saddle point(s)
   - maximum point(s)
   - minimum point(s)

   (b) Determine critical points which lie in the ***first quadrant*** for the function

   $$x^3y^2(6 - x - y)$$

2. (a) Given $5z^2 + 4x^2y - 6xz^2 = 3$ and P(1,1,1)

   (i) Find equation of *tangent plane* at P
   (ii) *Normal line* at P
   (iii) If normal at p meets the surface again at points A and B and C is the midpoint of AB find length of PC.

   (b) Write down the *parametric representation* of the surface

   $$z = \sqrt{3x^2 + 3y^2}$$

   (i) Find the *normal* to the surface at any point
   (ii) Write down the *first fundamental* form of the surface

3. (a) Using *differentials*

(i) compute approximate value of $\left(\sqrt{15} + \sqrt{99}\right)^2$

(ii) obtain $\Delta f = f(Q) - f(P)$ if

$$f = \frac{xy}{x + y}$$

P(2,3) and Q(1.98, 3.03)

(b) The period of oscillation of a pendulum of length L is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where g is acceleration due to gravity. Suppose L and g are measured with errors of at most 0.5% and 0.1% respectively. Find percentage error in calculated value of T.

4. (a) Obtain Taylor series expansion of

$$2xy + z^2$$

at the point (1,-1,3).

(b) Find directional derivature of $f = 2xy - z^2$ at (2,-1,1) in the direction towards the point (3,1,-1)

(c) Find *maximum* and *minimum* values of

$$f = x^2 - 3y^2 - 2x + 6y$$

on the square region whose vertices are (0,0) (0,1) (2,1) (2,0)

5. (a) Let $\mathbf{r} = x\mathbf{i} + y\mathbf{i} + z\mathbf{k}$ and $f(r)$ be scalar function of r.

(i) compute $\operatorname{div}\left(\frac{\mathbf{r}}{r}\right)$ in terms of r.

(ii) show that $\operatorname{curl}[f(r)\mathbf{r}]$

(b) Show that $\nabla^2 r^n = n(n+1)r^{n-2}$

6. (a) Determine whether the following functions *are linearly independent*

$$x = u + v + w,$$
$$y = u + v,$$
$$z = -2uw - 2vw - w^2$$

(b) Minimize the function

$$x^2 + y^2 + z^2$$

subject to two constraints: $2x + y + z = 2$ and $x - y - 3z = 4$

(c) A closed cylindrical can has volume Vm^3. Use *Lagrange multipliers* to show that the minimum surface area is achieved when the height is equal to the diameter.

END OF EXAMINATION

UNIVERSITY OF ZAMBIA

UNIVERSITY EXAMINATIONS - SEMESTER ONE 1998/99

M981 - NUMERICAL ANALYSIS I

INSTRUCTIONS: Attempt ANY Five(5) Questions.

TIME ALLOWED: Three (3) hours.

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1. The area A inside the closed curve $y^2 + x^2 = \cos x$ is given by

$$A = 4\int_{o}^{\alpha} (\cos x - x)^{1/2}\, dx$$

where α is the positive root of the equation $\cos x = x^2$

a) Compute α to three correct decimals.

b) Hence, use simpson's Rule over 8 intervals to compute the area A.

2. Given the following data

x	1.0	1.3	1.6	1.9	2.2
f(x)	0.7651977	0.6200860	0.4554022	0.2818186	0.1103623

(a) Use appropriate divided differences formulae to approximate

$f(1.1)$ and $f(2.0)$

(b) Hence or otherwise obtain $f'(1.9)$

3. A person runs the same track for five consecutive days and is timed as follows:

day(x)	1	2	3	4	5
time(y)	15.30	15.10.8	15.00	14.50	14.00

Suppose you want a least squares fit for the above data using the function

$a + b/x + c/x^2$:-

(a) Determine the normal equations for the solution

(b) Hence determine a least squares approximation for the given data.

4. (a) (i) Find the iterative methods based on the Newton-Raphson method for finding $\sqrt{N}$, 1/N, $N^{1/3}$ where N is a positive real number.

(ii) Apply the methods in (i) to N = 18 to obtain the results correct to two decimal places For the iterative methods in (a) use the initial values $x_0 = 4$, $x_0 = 0.1$ and $x_0 = 2$ respectively.

(b) One major difficulty with the Newton-Raphson method is that we need to know the value of the derivative of f at each approximation. Derive the Secant Method to show how this problem can be circumvented.

5. (a) (i) The natural logarithm function of a positive x is defined by

$$\ln x = -\int_x^1 \frac{dt}{t}$$

Calculate ln(0.75) by estimating the integral using the Composite Trapezoidal rule, with n = 5.

(ii) Give the maximal stepsize h to get the truncation error bound 0.5×10^{-3}

(b) Use a Taylor polynomial about $\frac{\pi}{4}$ to approximate cos42° to an accuracy of 10^{-6}.

6. (a) For $x \in [-1,1]$, Chebyshev polynomials are given by

$$T_n(x) = \cos(n \arccos x) \quad \text{for } n \geq 0$$

Derive the recurrence relation

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x) \text{ for each } n \geq 1$$

(b) (i) The function $f(x) = e^x$ is to be approximated on the interval $[-1,1]$ by the fourth Maclaurin polynomial $p_4(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$ with truncation error

$$|R_4(x)| = \frac{|f^{(5)}(\xi(x))||x^5|}{120} \quad \text{for } -1 \leq x \leq 1.$$

Use the Chebyshev economization to obtain a polynomial of degree 3 that best uniformly approximates $p_4(x)$ on $[-1,1]$

(i) What is the new error bound for this approximation?

7. (a) (i) Write an algorithm to find a solution to $f(x) = 0$ given the continuous function f on the interval $[a,b]$ where $f(a)$ and $f(b)$ have opposite signs.

(ii) Use the algorithm in part (ii) to find an approximation to $f(x) = x^3 + 4x^2 - 10 = 0$. Determine the number of iterations necessary to obtain an accuracy of 10^{-3} on the interval $[1,2]$

(b) A car travelling along a straight road is clocked at a number of points. Use methods of $O(h^2)$, the following times and positions to predict the speed at each time listed.

Time	0	3	5	8	10	13
Distance	0	225	383	623	742	993

END OF EXAMINATION