A THEORETICAL INVESTIGATION OF HEAT TRANSFER IN A LADLE OF MOLTEN STEEL DURING POURING

By

STEPHEN CHIPETA

A THEORETICAL INVESTIGATION OF HEAT TRANSFER IN A LADLE OF MOLTEN STEEL DURING POURING

BY

STEPHEN CHIPETA

A DISSERTATION SUBMITTED TO THE UNIVERSITY OF ZAMBIA IN PARTIAL FULFILMENT OF THE REQUIREMENTS OF THE DEGREE OF MASTER OF SCIENCE IN MATHEMATICS.

THE UNIVERSITY OF ZAMBIA

LUSAKA

1991

DECLARATION

I, Chipeta Stephen, declare that this dissertation has been written by me and that the work recorded is my own. The sources of all materials used have been acknowledged. To the best of my knowledge the dissertation has not been previously submitted for a degree at this or another University.

Signature : ....

(Stephen Chipeta)

Date : 13th Sept. 1991

DEDICATION

TO MY WIFE AND CHILDREN

APPROVAL

This dissertation of **Stephen Chipeta** is approved as fulfilling part of the requirements for the award of the degree of Master of Science in Mathematics by the University of Zambia.

Signature:	Date:
....	22 June 1992
....	12-10-92
....	....
....	....

ACKNOWLEDGEMENT

This dissertation could not have been successfully completed without a close supervision of Dr. W. Kunda. I thus take this opportunity to thank him for suggesting the topic to me, his supervision and the encouragement he rendered to me while I was studying. Special thanks go to Prof. Bartholomeusz, E.F. and Mrs Jain, S. for the lectures they administered to me in the appropriate units and for their encouragement. Thanks are also due to the Head of Mathematics Department Dr. A. M. Ngwengwe and his staff for the services rendered.

I also thank M. C. Musongole for having assisted me in so many ways during my studies. Thanks also go to Miss R. Mweendo for typing the dissertation.

Finally, I owe special debt to my wife (Anne Chipeta) and children (Panji, Steve, Simeon and Suzyo) for 3 years of loving patience while I was studying away from them.

CONTENTS

Page

DECLARATION i

APPROVAL ii

ACKNOWLEDGEMENT iV

INTRODUCTION 1

CHAPTER ONE - FORMULATION OF THE MATHEMATICAL MODEL 4

1.1 General Structure of a Teeming Ladle containing liquid steel 4

1.2 Assumptions made in the formulation of the Mathematical Model 4

1.3 Formulation of the Mathematical Model...... 8

CHAPTER TWO - SEPARATION OF THE ENERGY EQUATION INTO TWO EQUATIONS 15

2.1 Separation of the Energy equation into two equations (radial and axial variation equations) 15

2.2 The Analytical Solution of the Energy equation in the Radial variation 18

Page

CHAPTER THREE - FINITE DIFFERENCES OF GOVERNING ENERGY EQUATION IN THE AXIAL VARIATION 23

3.1 Transformation of the equation in terms of y and τ* 23

3.2 Discretization of the equation 24

CHAPTER FOUR - NUMERICAL SOLUTIONS 35

4.1 Computer Program 35

4.2 Numerical Solutions 35

4.3 Accuracy of the computer solutions 36

APPENDICES 41

REFERENCES 68

INTRODUCTION

Steel is an alloy of iron and other elements manufactered in molten form. Molten steel at temperature of about 1600°C is transferred into a refractory lined vessel called a ladle which is used for pouring the molten steel into solidification moulds, a process normally referred to as casting. The quality of steel for specific purposes is stipulated before the manufacturing process. Molten steel let it be of excellent quality, pure from non-metallic inclusions and gases, well oxidized, and accurately within the specified analysis, may be spoiled during casting. The quality of steel ingots may be affected by cleanliness of the ladle, moulds and places around moulds, lubrication of moulds, method of casting, quality of refractories and patterns of ingots colling, as well as temperature and rate of casting [see Edneral (1)].

During pouring of molten steel from a ladle heat transfer from the steel and temperature stratification occur. To produce steel of high quality, the molten steel temperature during the pouring sequence (termed "teem") must fall within a closely specified temperature range (as little as ±5°C for certain steels). [see Egerton et al (2)]. The pouring process for a single ladle can take up to 60 minutes. The optimum pouring time for a given steel casting would be established on the basis of weight and shape of the mould, temperature of the molten steel and its solidification characteristics, and thermal stability of the mould. [see

Metals Handbook (3)]. It is very important to devise a method for predicting temperature variation throughout the cast period because quantitative information on the average pouring temperature and its variation can have significant benefits for the steel plant. Egerton et al have devised a method for this purpose. In their analysis they are concerned with the common method of pouring, through an off centre nozzle built into the ladle base. Their paper examines theoretically the temperature distribution and flow of steel within a ladle during transfer and pouring.

This dissertation examines theoretically the temperature distribution and flow of steel within a ladle during pouring. The mathematical model used is that of Egerton et al and is thus an independent validation of their results. In the first chapter of this dissertation is where the formulation of the Mathematical model of the ladle pouring and heat transfer has been done using the assumptions of Egerton et al. The reduction of the governing equations in this chapter is shown explicitly and the reduced equations agree with those of Egerton et al. The main governing equation is the equation of energy which is put in dimensionless form. The separation of the energy equation into two equations to facilitate solution is done in the second chapter. The analytical solution of one of the equations is also given. The second equation is treated numerically in the third chapter. The equation is put in finite difference form.

Chapter One

FORMULATION OF THE MATHEMATICAL MODEL

1.1 General Structure of a Teeming Ladle

Teeming ladles are usually of conical shape with greater diameter at the top so as to facilitate removal of metal skulls which often remain in the ladle upon teeming. The height of a ladle is made equal to or slightly greater than, its diameter in order to lower the pressure of falling metal jet. The working surface of a ladle is laid with fire clay brick which serves as an insulator. A shaped brick with a hole (called the seat block) is inserted into the ladle bottom and a pouring nozzle, inside it. The hole of the nozzle is closed by a stopper end which is screwed onto a steel rod clad with refractory stopper sleeves. A schematic diagram of a ladle is given in figure 1.

1.2 Assumptions made in the Formulation of the Mathematical Model

The following are the assumptions made in the formulation of the mathematical model of ladle pouring and heat transfer:

(i) To predict the heat transfer within the ladle during pouring, it is assumed that the flow of the molten steel is inviscid and irrotational.

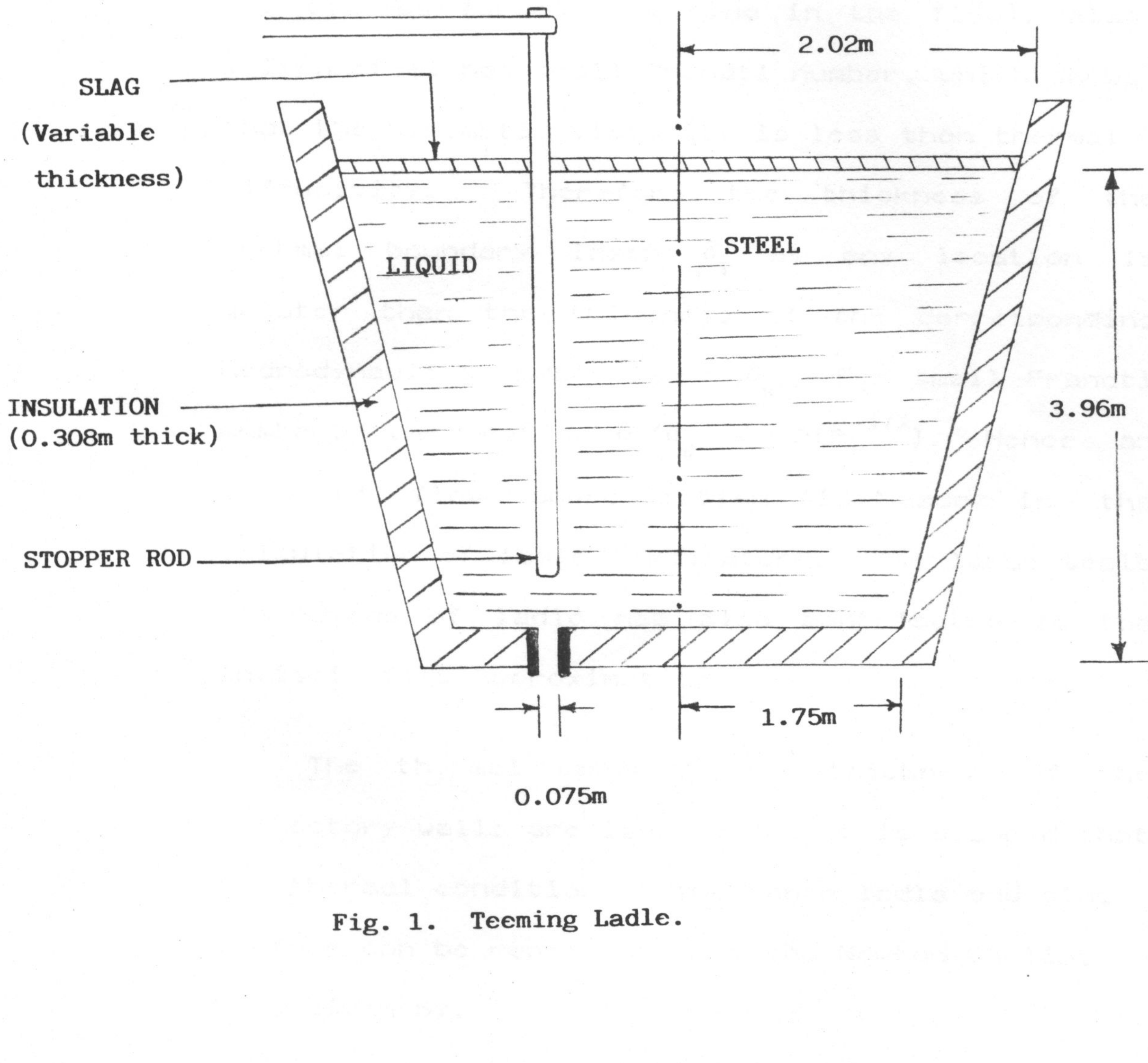

Fig. 1. Teeming Ladle.

During the process of ladle emptying, transient viscous and thermal boundary layers are attached to the inner walls of ladle. But during the same process, molten steel has high Reynolds number (meaning, that the relative contribution of inertia forces effects is much greater than that the viscous forces occurring in the flow). Also molten steel has small Prandtl number, which shows that the kinematic viscosity is less than thermal diffusivity. Therefore the thickness of the thermal boundary layer δ_T at any location is greater than the thickness of the corresponding hydrodynamic boundary layer δ_H. For small Prandtl number, the ratio $\delta_T/\delta_H = O(P^{-1/2})$. Hence an inviscid flow approximation is made in the calculation of ladle temperature. The large scale dimensions of ladle has also contributed to the inviscid flow approximation.

(ii) The thermal capacity and thickness of the refractory walls are ignored and it is assumed that the thermal condition at the inner ladle and slag surface can be represented by the Newton Cooling Law, given by,

$$-K\partial T/\partial n = \bar{h}(T-T_o). \qquad (1.2.1)$$

T is the external surface temperature of molten steel, T_o is the surrounding air temperature, $\bar{h}$ is the heat transfer coefficient at the molten steel boundary and n is measured outwards along the normal from the steel volume. K is the thermal conductivity.

(iii) It is assumed that the effect of ladle taper on heat transfer is small. Thus the ladle is represented by a circular cylinder having a mean radius $(a_T + a_B)/2$, where a_T and a_B are respectively the top and bottom inner ladle radii.

The ladle taper is neglected because of the following reasons:

(a) The approximation of an inviscid flow, directs the velocity of the molten steel vertically downwards and is equal in magnitude to the rate at which the top (slag covered) surface falls. Thus it is a function of time only, and independent of the height z measured upwards from the ladle base.

(b) Because of this idealization and assumption of the Newton cooling law, the thermal field can be separated into radial and axial variations to allow analytical and numerical solution to be obtained.

(iv) Finally it is assumed that the physical properties of the molten steel are independent of the temperature.

The actual variation in density, specific heat viscosity, conductivity, heat transfer coefficient(ladle walls, bottom), heat transfer coefficient(slag cover), diffusivity, and fusion temperature of steel is small over the relevant range of temperature range, of between 1550 and 1600°C. The assumption makes the problem more tractable in the mathematical sense. It, however, implies that the motion induced by the gravitational body force is neglected. Buoyancy forces may not be negligible in the region adjacent to the (vertical) ladle walls. However this region of thermal stratification is small compared to the radius of ladle, and dimensional arguments show that any buoyancy driven up flow has little effect on the overall down flow.

1.3 Formulation of the Mathematical Model.

A teeming ladle (shown in fig.1) pouring, with the assumptions in (1.2) is considered in the formulation of the mathematical model.

Consider first the inviscid flow of molten steel within the ladle during pouring. Denote the cross-sectional area of the cylindrically shaped ladle by A_L and that of the nozzle by A_N, where A_N is very small compared to A_L. Let the initial head of molten steel be denoted by H and during the pouring sequence let the height of the top surface be located at $z = h(t)$. During the pouring sequence denote the rate of flow of molten steel in the axial direction, in the ladle, by V_L and the rate of out-flow from the nozzle by V_N. Let t_f denote the total final emptying time for the ladle.

The total final emptying time t_f for the ladle is found by using the law of conservation of mass (equation of continuity), in fluid dynamics, and Bernoulli's equation. In this case, the equation of continuity is given by

$$A_L V_L = A_N V_N. \tag{1.3.1}$$

The Bernoulli's equation is given by

$$\frac{V_L^2}{2g} + H = \frac{V_N^2}{2g}, \tag{1.3.2}$$

where g is the gravitational acceleration, due to the fact that the pressure is atmospheric and the gauge pressure is zero, and the pressure is also atmospheric at the place where it becomes parallel sided at the nozzle outlet if streamlined [see Francis (4)].

Using (1.3.1) and (1.3.2)

$$V_L = -\left\{2gH/\left[\left(A_L/A_N\right)^2 - 1\right]\right\}^{1/2}$$

from which

$$\frac{1}{H^{1/2}}\int_H^0 dh = -\left\{2g/\left[\left(A_L/A_N\right)^2 - 1\right]\right\}^{1/2}\int_0^{t_f} dt,$$

since $V_L = dh/dt$. Evaluating the integrals yields

$$2H^{1/2} = \left\{2g/\left[\left(A_L/A_N\right)^2 - 1\right]\right\}^{1/2} t_f$$

so that

$$t_f = \left[\left(A_L/A_N\right)^2 - 1\right]^{1/2} \surd\left(2H/g\right). \qquad (1.3.3).$$

Considering the fact that A_N is very much smaller than A_L, the value of $[(A_L/A_N)^2 - 1]$ is approximated to $(A_L/A_N)^2$, and also taking into account for the inward bending of the streamlines towards the vertical axis through the nozzle during pouring t_f will be taken as

$$t_f = \left(A_L/A_N C_N\right)\surd\left(2H/g\right), \qquad (1.3.4)$$

where $C_N=D_J/A_N$, is the contraction coefficient and D_J is the jet diameter of molten steel from the nozzle. Assuming that the nozzle is sharp-edged with a 45^o level on it, and with smooth, well-finished surfaces everywhere, C_N is usually taken as 0.63.[see Francis (4)].

The radial flow of molten steel near the bottom of the ladle is approximated to zero because the value of $\frac{1}{2}[V_L+(A_L/A_N)V_L]$ is small enough to be ignored. Therefore in terms of cylindrical polar co-ordinates (r,θ,z) the inviscid irrotational flow of the molten steel has velocity components

$$\underline{v} = (v_r, v_\theta, v_z) = \left(0, 0, \frac{dh}{dt}\right), \qquad (1.3.5)$$

given that

$$\frac{dh}{dt} = \frac{-2H}{t_f}\left(1 - \frac{t}{t_f}\right). \tag{1.3.6}$$

The temperature distribution in the flow satisfies the equation of energy for the Newtonian fluid with constant physical and thermal properties. The energy equation in the cylindrical polar coordinates (r, θ, z) is given by

$$\rho c\left[\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + v_\theta\frac{\partial T}{\partial \theta} + v_z\frac{\partial T}{\partial z}\right] =$$

$$\kappa\left[\frac{1}{r}\frac{\partial}{\partial r}\left\{r\frac{\partial T}{\partial r}\right\} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right]$$

$$+ 2\mu\left[\left\{\frac{1}{r}\left(\frac{\partial v_\theta}{\partial \theta} + v_r\right)\right\}^2 + \left\{\frac{\partial v_r}{\partial r}\right\}^2 + \left\{\frac{\partial v_z}{\partial z}\right\}^2\right]$$

$$+ \mu\left[\left\{\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right\}^2 + \left\{\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right\}^2 + \left\{\frac{1}{r}\frac{\partial v_r}{\partial \theta} + r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right\}^2\right]$$

.... (1.3.7)

in which T is the temperature, c is heat capacity at constant volume per unit mass, κ is thermal conductivity of fluid, ρ is density of fluid, and μ is viscosity of fluid.

By consideration of the motion equations (1.3.5) – (1.3.6) and the assumption that the flow of molten steel in a ladle is inviscid, the energy equation (1.3.7) reduces to

$$\frac{\partial T}{\partial t} - \frac{2H}{t_f}\left(1 - \frac{t}{t_f}\right)\frac{\partial T}{\partial z} = k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right) \tag{1.3.8}$$

in which $k = \kappa/\rho c$ is the thermal diffusivity.

boundary conditions (1.3.10) − (1.3.12), the dimensionless variables are introduced as follows:

$$Z = z/H, \text{ so that } \frac{\partial T}{\partial z} = \frac{\partial T}{\partial Z} \cdot \frac{\partial Z}{\partial z} = \frac{1}{H}\frac{\partial T}{\partial Z}$$

and

$$\frac{\partial^2 T}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right) = \frac{\partial}{\partial Z}\left(\frac{1}{H}\frac{\partial T}{\partial Z}\right)\frac{\partial Z}{\partial z} = \frac{1}{H^2}\frac{\partial^2 T}{\partial Z^2};$$

$$R = r/a, \text{ so that } \frac{\partial T}{\partial r} = \frac{\partial T}{\partial R}\frac{\partial R}{\partial r} = \frac{1}{a}\frac{\partial T}{\partial R},$$

where a is the radius of the ladle;

$$\tau = t/t_f, \text{ so that } \frac{\partial T}{\partial t} = \frac{\partial T}{\partial \tau} \cdot \frac{\partial \tau}{\partial t} = \frac{1}{t_f}\frac{\partial T}{\partial \tau};$$

and

$$\theta = (T - T_o)/(T_1 - T_o), \text{ so that } (T_1 - T_o)\partial\theta = \partial T.$$

This transforms the energy equation (1.3.8) to

$$\frac{\partial \theta}{\partial \tau} - 2(1-\tau)\frac{\partial \theta}{\partial Z} = \frac{kt_f}{H^2}\left(\frac{\partial^2 \theta}{\partial Z^2} + \frac{H^2}{a^2 R}\frac{\partial}{\partial R}(R\frac{\partial \theta}{\partial R})\right) \qquad (1.3.13)$$

Put $\varepsilon = kt_f/H^2$ and the height to radius aspect ratio $\alpha = H/a$ into (1.3.13). And from the boundary conditions, a dimensionless group, $Bi = \bar{h}H/K$, called the Biot number emerges. With these dimensionless variables, the governing equation (1.3.13) is now

$$\frac{\partial \theta}{\partial \tau} - 2(1-\tau)\frac{\partial \theta}{\partial Z} = \varepsilon\left[\frac{\partial^2 \theta}{\partial Z^2} + \frac{\alpha^2}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \theta}{\partial R}\right)\right], \qquad (1.3.14)$$

with an initial condition

$$\theta = 1, \quad \tau = 0, \quad 0 \le Z \le 1, \quad 0 \le R \le 1, \qquad (1.3.15)$$

and boundary conditions

$$\frac{\partial\theta}{\partial Z} = -Bi_2\theta \qquad \text{at } Z = (1 - \tau)^2, \quad 0 \leq R \leq 1, \qquad (1.3.16)$$

$$\frac{\partial\theta}{\partial Z} = Bi_1\theta \qquad \text{at } Z = 0, \quad 0 \leq R \leq 1, \qquad (1.3.17)$$

$$\frac{\partial\theta}{\partial R} = -\frac{Bi_1\theta}{\alpha} \qquad \text{at } R = 1, \quad 0 \leq Z \leq (1-\tau)^2, \qquad (1.3.18)$$

in which

$$Bi_1 = \frac{H\bar{h}_1}{K} \quad \text{and } Bi_2 = \frac{H\bar{h}_2}{K},$$

are the surface Biot numbers.

Equation (1.3.14) together with the conditions (1.3.16)-(1.3.18) is the equation of energy that will determine the temperature distribution and flow of steel within the ladle during pouring.

Chapter Two

SEPARATION OF THE ENERGY EQUATION INTO TWO EQUATIONS

2.1 Separation of the energy equation into two equations

Each term of the boundary conditions (1.3.16) – (1.3.18) contains either the dependent variable or one of its derivatives. This means that the boundary conditions (1.3.16) – (1.3.18) are homogeneous in θ. Therefore the thermal field is separable in the form:

$$\theta(R, Z, \tau; \varepsilon) = U(R, \tau; \varepsilon)V(Z, \tau; \varepsilon). \qquad (2.1.1)$$

The first partial derivative of θ in (2.1.1) with respect to τ gives

$$\frac{\partial\theta}{\partial\tau} = V\frac{\partial U}{\partial\tau} + U\,\frac{\partial V}{\partial\tau}\,. \qquad (2.1.2)$$

Partial differentiation of θ in (2.1.1) with respect to Z, gives

$$\frac{\partial\theta}{\partial Z} = U\,\frac{\partial V}{\partial Z}\,. \qquad (2.1.3)$$

Differentiating θ in (2.1.1) with respect to Z again, yields

$$\frac{\partial^2\theta}{\partial Z^2} = U\,\frac{\partial^2 V}{\partial Z^2}\,. \qquad (2.1.4)$$

Also, the partial differentiation of θ in (2.1.1) with respect to R, gives

$$\frac{\partial\theta}{\partial R} = V\,\frac{\partial U}{\partial R}\,. \qquad (2.1.5)$$

Substitution of the equations (2.1.2) - (2.1.5) into the governing energy equation (1.3.14) yields

$$V\frac{\partial U}{\partial \tau} + U\,\frac{\partial V}{\partial \tau} - 2(1 - \tau)\,U\frac{\partial V}{\partial Z} = \varepsilon\left[U\,\frac{\partial^2 V}{\partial Z} + \frac{\alpha^2}{R}\,\frac{\partial}{\partial R}\,\left(RV\frac{\partial U}{\partial R}\right)\right].$$

Hence

$$\frac{1}{U}\,\frac{\partial U}{\partial \tau} - \frac{1}{U}\,\frac{\varepsilon\alpha^2}{R}\,\frac{\partial}{\partial R}\left(R\,\frac{\partial U}{\partial R}\right) = \frac{1}{V}\,\varepsilon\,\frac{\partial^2 V}{\partial Z^2} - \frac{1}{V}\left[\,\frac{\partial V}{\partial \tau} - 2(1-\tau)\frac{\partial V}{\partial Z}\,\right]$$

.... (2.1.6)

The boundary conditions (1.3.15) - (1.3.18) can now be written as follows:

$$\frac{\partial\theta}{\partial Z} = U\,\frac{\partial V}{\partial Z} = -\,Bi_2 UV \quad \text{at } Z = (1-\tau)^2, \quad 0 \leq R \leq 1$$

or

$$U\left[\frac{\partial V}{\partial Z} + Bi_2 V\right] = 0 \quad \text{at } Z = (1 - \tau)^2, \quad 0 \leq R \leq 1.$$

If $U(R,\tau;\varepsilon) = 0$, then $\theta(R,Z,\tau;\varepsilon) = 0$, which is not the case. Hence $U \neq 0$, and

$$\frac{\partial V}{\partial Z} = -\,Bi_2 V \quad \text{at } Z = (1-\tau)^2, \; 0 \leq R \leq 1.$$

At $Z = 0$, $\quad 0 \leq R \leq 1$,

$$\frac{\partial\theta}{\partial Z} = U\,\frac{\partial V}{\partial Z} = Bi_1 UV,$$

and by the same argument

$$\frac{\partial V}{\partial Z} = Bi_1 V.$$

At $R = 1$, $\quad 0 \leq Z \leq (1-\tau)^2$,

$$\frac{\partial\theta}{\partial R} = V\frac{\partial U}{\partial R} = -\frac{Bi_1}{\alpha} UV$$

or

$$V\left[\frac{\partial U}{\partial R} + \frac{Bi_1}{\alpha} U\right] = 0.$$

If $V(Z,\tau;\varepsilon) = 0$, then $\theta(R,Z,\tau;\varepsilon) = 0$, which is not the case. Hence $V \neq 0$, and

$$\frac{\partial U}{\partial R} = -\frac{Bi_1}{\alpha} U.$$

The initial condition (1.3.15) in terms of U and V is

$$UV = 1 \quad \text{at} \quad \tau = 0, \qquad 0 \leq Z \leq 1, \qquad 0 \leq R \leq 1.$$

Let $U(R,\tau;\varepsilon)$ be the solution of

$$\frac{\varepsilon\alpha^2}{R} \frac{\partial}{\partial R}\left(R \frac{\partial U}{\partial R}\right) = \frac{\partial U}{\partial \tau}, \qquad 0 \leq T \leq 1, \quad 0<R<1, \qquad (2.1.7)$$

with boundary conditions

$$\frac{\partial U}{\partial R} = -\frac{Bi_1}{\alpha} U \qquad \text{at} \quad R = 1 \qquad (2.1.8)$$

and as ε becomes very much less than 1,

$$U \longrightarrow 1 \qquad \text{as } R \longrightarrow 0 \qquad (2.1.9)$$

and the initial condition

$$U = 1 \qquad \text{at} \quad \tau=0, \quad 0 \leq R \leq 1. \qquad (2.1.10)$$

Also let $V(Z,\tau;\varepsilon)$ be the solution of

$$\frac{\partial V}{\partial \tau} - 2(1-\tau) \frac{\partial V}{\partial Z} = \varepsilon \frac{\partial^2 V}{\partial Z^2}, \quad 0 \leq \tau \leq 1, \; 0 \leq Z \leq 1 \qquad (2.1.11)$$

with boundary conditions

$$\frac{\partial V}{\partial Z} = -Bi_2 V \qquad \text{at} \quad Z = (1-\tau)^2, \qquad (2.1.12)$$

$$\frac{\partial V}{\partial Z} = Bi_1 V \qquad \text{at } Z = 0. \qquad (2.1.13)$$

and initial condition

$$V = 1 \qquad \text{at} \quad \tau=0, \quad 0\leq Z\leq 1. \qquad (2.1.14)$$

Then

$$\theta(R,Z,\tau;\varepsilon) = U(R,\tau;\varepsilon).V(Z,\tau;\varepsilon)$$

is the solution of the equation

$$\frac{\partial\theta}{\partial\tau} - 2(1-\tau)\frac{\partial\theta}{\partial Z} = \varepsilon\left[\frac{\partial^2\theta}{\partial Z^2} + \frac{\alpha^2}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\theta}{\partial R}\right)\right]$$

subject to the following boundary conditions:

$$\frac{\partial\theta}{\partial Z} = -Bi_2\theta \qquad \text{at } Z = (1-\tau)^2, \qquad 0\leq R\leq 1;$$

$$\frac{\partial\theta}{\partial Z} = Bi_1\theta \qquad \text{at } Z = 0, \qquad 0\leq R\leq 1;$$

$$\frac{\partial\theta}{\partial R} = -\frac{Bi_1}{\alpha} \qquad \text{at} \quad R = 1, \qquad 0\leq Z\leq (1-\tau)^2;$$

with initial condition

$$\theta = 1 \qquad \text{at} \quad \tau = 0, \quad 0\leq Z \leq 1, \qquad 0\leq R\leq 1.$$

2.2 The analytical solution of the energy equation in the radial variation.

Equation (2.1.7) with boundary conditions (2.1.8) and (2.1.9) and initial condition (2.1.10) is the governing equation of energy, in the radial variation. Its analytical solution, here, is found using the asymptotic expansion of Bessel functions.

Applying the Laplace transformation on the variable τ to the equation (2.1.7), gives the subsidiary equation

$$\frac{d^2\bar{U}}{dR^2} + \frac{1}{R}\frac{d\bar{U}}{dR} = \frac{1}{\varepsilon\alpha^2}\left[-U(0) + p\bar{U}\right], \quad 0 < R < 1 \qquad (2.2.1)$$

where $\bar{U}$ is the Laplace transform of U. Using the initial condition (2.1.10) in equation (2.2.1), yields

$$\frac{d^2\bar{U}}{dR^2} + \frac{1}{R}\frac{d\bar{U}}{dR} = \frac{1}{\varepsilon\alpha^2}\left[-1 + p\bar{U}\right],$$

which upon rearrangement of the terms becomes

$$\frac{d^2\bar{U}}{dR^2} + \frac{1}{R}\frac{d\bar{U}}{dR} - \frac{p}{\varepsilon\alpha^2}\bar{U} = -\frac{1}{\varepsilon\alpha^2}, \quad 0 < R < 1 \qquad (2.2.2)$$

The boundary conditions (2.1.8) and (2.1.9) treated in the same way give

$$\frac{d\bar{U}}{dR} = -\frac{Bi_1}{\alpha}, \quad R = 1 \qquad (2.2.3)$$

and

$$\bar{U} \to \frac{1}{p} \quad \text{as } R \to 0. \qquad (2.2.4)$$

Therefore, with boundary conditions (2.2.3) and (2.2.4), the solution of equation (2.2.2) is

$$\bar{U} = \frac{1}{p}\left\{1 - \frac{Bi_1 I_0(qR)}{\alpha q I_1(q) + Bi_1 I_0(q)}\right\} \qquad (2.2.5)$$

where $q = \sqrt{(p/\varepsilon\alpha^2)}$, and I_o and I_1 are Bessel functions [see Carslaw and Jaeger (5)].

To generate meaningful practical results, relatively small values of time steps are assumed. Therefore using the asymptotic expansion of the Bessel function $I_\nu(z)$, for large values of z, given by

$$I_\nu(z) = \frac{e^z}{\sqrt{(2\pi z)}}\left\{1 - \frac{4\nu^2 - 1^2}{1!8z} + \frac{(4\nu^2-1^2)(4\nu^2-3^2)}{2!(8z)^2} + O\left(\frac{1}{z^3}\right)\right\} +$$

$$\frac{e^{-z\pm(\nu+\frac{1}{2})\pi i}}{\sqrt{2\pi z}}\left[1 + O\left(\frac{1}{z}\right)\right], \tag{2.2.6}$$

yields

$$I_o(qR) = \frac{e^{qR}}{\sqrt{(2\pi qR)}}\left[1 + \frac{1}{8qR} + \frac{9}{128q^2R^2} + \ldots\right] \tag{2.2.7}$$

$$I_o(q) = \frac{e^q}{\sqrt{(2\pi q)}}\left[1 + \frac{1}{8q} + \frac{9}{128q^2} + \ldots\right] \tag{2.2.8}$$

and

$$I_1(q) = \frac{e^q}{\sqrt{(2\pi q)}}\left[1 - \frac{3}{8q} - \frac{15}{128q^2} + \ldots\right] \tag{2.2.9}$$

Using the above expansions of I_o and I_1, give

$$\alpha q I_1(q) + Bi_1 I_o(q) = \frac{\alpha q e^q}{\sqrt{(2\pi q)}}\left[1 - \frac{3}{8q} - \frac{15}{128q^2} + \ldots + \frac{Bi_1}{\alpha q} + \frac{Bi_1}{8\alpha q^2} + \frac{9Bi_1}{128\alpha q^3} + \ldots\right]$$

which simplifies to

$$\alpha q I_1(q) + Bi_1 I_o(q) = \frac{\alpha q e^q}{\sqrt{(2\pi q)}} \left[1 + \left(\frac{Bi_1}{\alpha} - \frac{3}{8}\right)/q + \left(\frac{Bi_1}{\alpha} - \frac{15}{16}\right)/8q^2 + \ldots\right], \qquad (2.2.10)$$

and

$$Bi_1 I_o(qR) = \frac{Bi_1 e^{qR}}{\sqrt{(2\pi qR)}} \left[1 + \frac{1}{8qR} + \frac{9}{128q^2R^2} + \ldots\right]. \qquad (2.2.11)$$

Thus substitution of (2.2.10) and (2.2.11) into (2.2.5), gives

$$\bar{U} = \frac{1}{p}\left\{1 - \frac{Bi_1}{\alpha q\sqrt{R}}\, e^{-q(1-R)} \frac{1+1/8qR+9/128(qR)^2+\ldots}{1+\left(\frac{Bi_1}{\alpha} - \frac{3}{8}\right)/q + \left(\frac{Bi_1}{\alpha} - \frac{15}{16}\right)/8q^2+\ldots}\right\}$$

which is approximated to

$$\bar{U} = \frac{1}{p} - \frac{Bi_1}{\alpha\sqrt{R}} \frac{e^{-q(1-R)}}{pq} . \qquad (2.2.12)$$

Using the Inversion formula of the Laplace transforms the solution of the governing equation in radial variation is found to be

$$U = 1 - \frac{Bi_1}{\alpha\sqrt{R}} \left\{2\left(\frac{\varepsilon\alpha^2\tau}{\pi}\right)^{1/2} \exp\left[-\left(1-R\right)^2/4\alpha^2\varepsilon\tau\right] - (1-R)\ \mathrm{erfc}\left[\left(1-R\right)/2\sqrt{\varepsilon\alpha^2\tau}\ \right]\right\}$$

[see Carslaw and Jaeger (2)]. Therefore,

Chapter Three

FINITE DIFFERENCES OF THE GOVERNING ENERGY EQUATION IN THE AXIAL VARIATION

3.1 Transformation of the equation in terms of y and τ^*

Equation (2.1.11) with boundary conditions (2.1.12) and (2.1.13) and initial conditions (2.1.14) is the governing equation in the axial variation. To make equation (2.1.11) with its boundary and initial conditions adaptable to numerical study introduce new variables

$$y = Z/(1-\tau^*)^2 \qquad \text{and } \tau^* = \tau$$

so that

$$V = V(y, \tau^*). \tag{3.1.1}$$

Then the total differential

$$dV = \frac{\partial V}{\partial y}\, dy + \frac{\partial V}{\partial \tau^*}\, d\tau^* \tag{3.1.2}$$

gives

$$\frac{\partial V}{\partial \tau} = \frac{\partial V}{\partial y}\frac{\partial y}{\partial \tau} + \frac{\partial V}{\partial \tau^*}\frac{\partial \tau^*}{\partial \tau}$$

which reduces to

$$\frac{\partial V}{\partial \tau} = \frac{2y}{1-\tau^*}\frac{\partial V}{\partial y} + \frac{\partial V}{\partial \tau^*}\ . \tag{3.1.3}$$

Also from the total differential (3.1.2)

$$\frac{\partial V}{\partial Z} = \frac{\partial V}{\partial y}\frac{\partial y}{\partial Z} + \frac{\partial V}{\partial \tau^*}\frac{\partial \tau^*}{\partial Z} = \frac{1}{(1-\tau^*)^2}\frac{\partial V}{\partial y}, \tag{3.1.4}$$

and

$$\frac{\partial^2 V}{\partial z^2} = \frac{\partial}{\partial y}\left(\frac{1}{(1-\tau^*)^2}\frac{\partial V}{\partial y}\right)\frac{\partial y}{\partial z} + \frac{\partial}{\partial \tau^*}\left(\frac{1}{(1-\tau^*)^2}\frac{\partial V}{\partial y}\right)\frac{\partial \tau^*}{\partial z}$$

which reduces to

$$\frac{\partial^2 V}{\partial z^2} = \frac{1}{(1-\tau^*)^4}\frac{\partial^2 V}{\partial y^2}. \qquad (3.1.5)$$

Replacing (3.1.3) – (3.1.5) into the governing equation (2.1.11), transforms it to

$$\varepsilon\frac{\partial^2 V}{\partial y^2} = (1-\tau^*)^4\frac{\partial V}{\partial \tau^*} + 2(y-1)(1-\tau^*)^3\frac{\partial V}{\partial y}, \qquad (3.1.6)$$

subject to the boundary conditions

$$\frac{\partial V}{\partial y} = -Bi_2(1-\tau^*)^2 V \qquad \text{at } y = 1, \qquad (3.1.7)$$

$$\frac{\partial V}{\partial y} = Bi_1(1-\tau^*)^2 V \qquad \text{at } y = 0 \qquad (3.1.8)$$

and initial condition

$$V = 1 \qquad \text{at} \quad \tau^* = 0, \qquad 0 \leq y \leq 1. \qquad (3.1.9)$$

3.2 Discretization of the governing equation in the axial variation.

The solutions to the problem are to be approximated using the Crank-Nicolson method. The problem is as set out in figure 2.

Select the mesh constant h such that

$h = 1/m$ for an integer $m > 0$.

To generate a physically meaningful numerical solution, a small time step $k = 1/N$, for some integer $N > 0$, is chosen.

For the $y\tau^*$-plane in figure 3, the grid points are (y_i, τ^*_j), where

$$y_i = ih, \quad \text{for} \quad i = 0,1,2,\ldots,m \tag{3.2.1}$$

and

$$\tau^*_j = jk, \quad \text{for} \quad j = 0,1,2,\ldots,N \tag{3.2.2}$$

Crank-Nicolson uses the finite differences to approximate the differential operators. Using the forward difference operator,

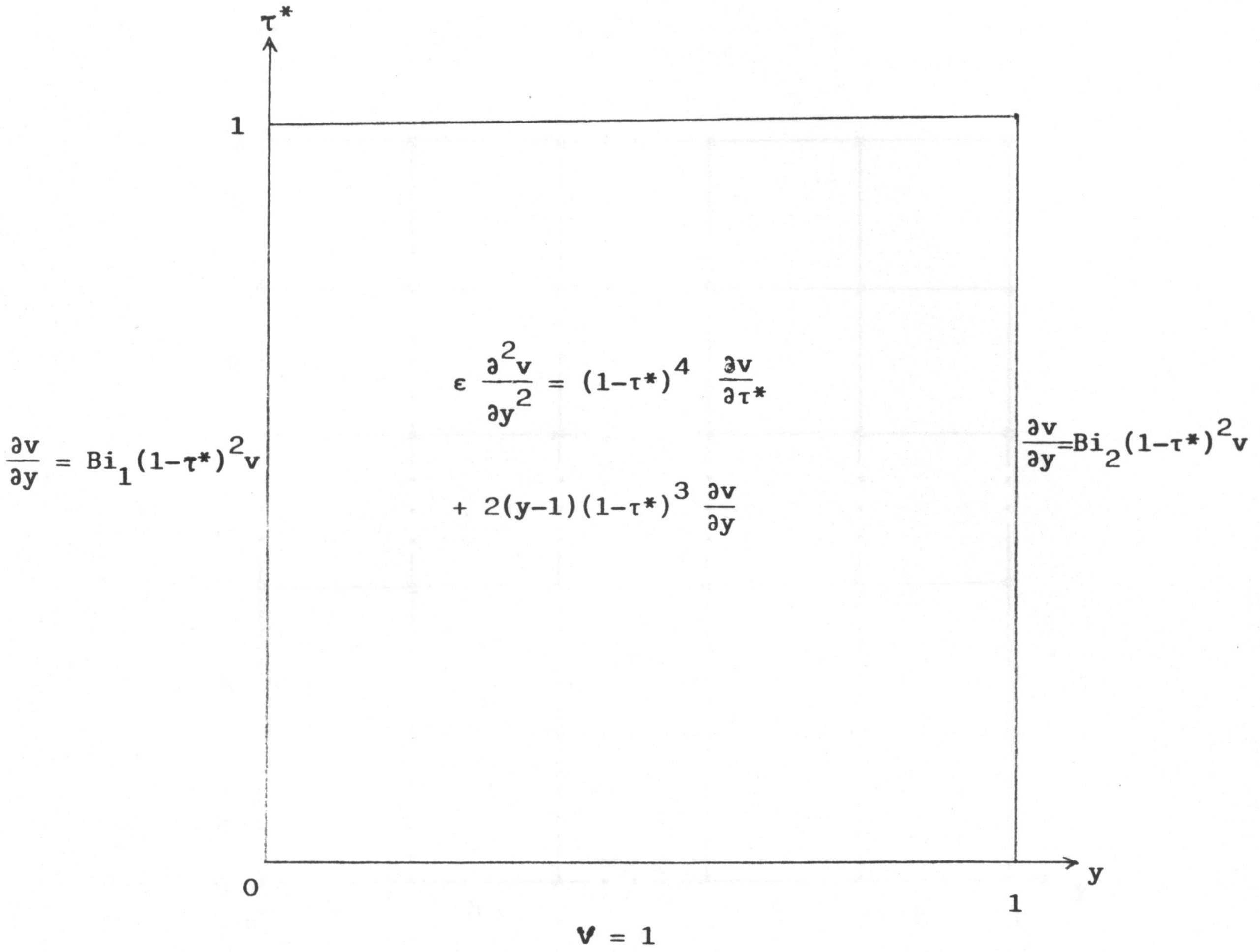
τ^*
1
$\varepsilon \frac{\partial^2 v}{\partial y^2} = (1-\tau^*)^4 \frac{\partial v}{\partial \tau^*}$
$+ 2(y-1)(1-\tau^*)^3 \frac{\partial v}{\partial y}$
$\frac{\partial v}{\partial y} = Bi_1 (1-\tau^*)^2 v$
$\frac{\partial v}{\partial y} = Bi_2 (1-\tau^*)^2 v$
0
1
y
$V = 1$

Fig. 2.

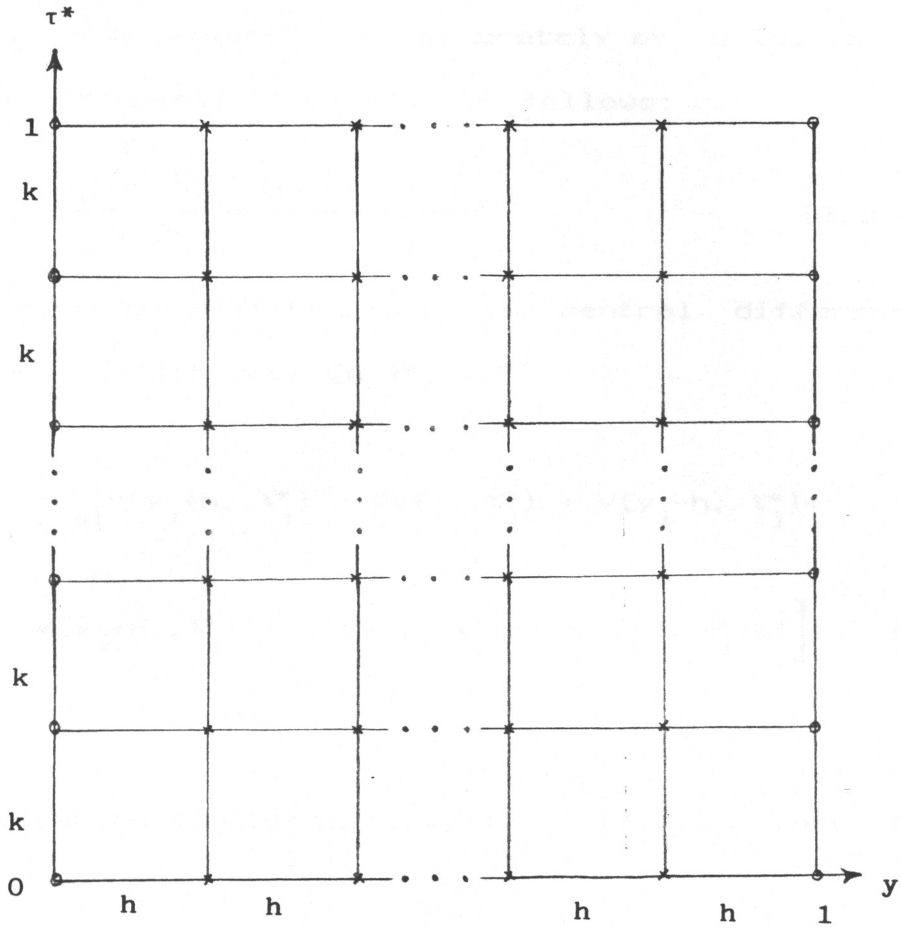

Fig. 3

$$\frac{\partial V}{\partial \tau^*} = \frac{V(y_i, \tau^*_j + k) - V(y_i, \tau^*_j)}{k} \qquad (3.2.3)$$

$\partial V/\partial y$ can be approximated more accurately by using a central difference differential operator as follows:

$$\frac{\partial V}{\partial y} = \frac{V(y_i+h,\ \tau^*_j) - V(y_i-h,\ \tau^*_j)}{2h}. \qquad (3.2.4)$$

Using the averaged differences of the central differences, at jth step and (j+1)th step in τ^*,

$$\frac{\partial^2 V}{\partial y^2} = \frac{1}{2h^2}\Big[V(y_i+h,\ \tau^*_j) - 2V(y_i, \tau^*_j) + V(y_i-h,\ \tau^*_j) +$$

$$V(y_i+h\ , \tau^*_j+k) - 2V(y_i, \tau^*_j+k) + V(y_i-h, \tau^*_j+k)\Big]. \qquad (3.2.5)$$

Substituting equations (3.2.3) – (3.2.5) into (3.1.6), yields

$$\frac{\varepsilon k}{2h^2}V_{i+1,j} - \frac{\varepsilon k}{h^2}V_{i,j} + \frac{\varepsilon k}{2h^2}V_{i+1,j+1} - \frac{\varepsilon k}{h^2}V_{i,j+1} + \frac{\varepsilon k}{2h^2}V_{i-1,j+1}$$

$$= (1-jk)^4 V_{i,j+1} - (1-jk)^4 V_{i,j} + \frac{k}{h}(ih-1)(1-jk)^3 V_{i+1,j}$$

$$- \frac{k}{h}(ih-1)(1-jk)^3 V_{i-1,j}. \qquad (3.2.6)$$

Letting $k/h^2 = \lambda$ in (3.2.6) and rearranging the terms gives

$$\frac{-\varepsilon\lambda}{2} V_{i-1,j+1} + \Big[(1-\lambda)^4 + \varepsilon\lambda\Big]V_{i,j+1} - \frac{\varepsilon\lambda}{2} V_{i+1,j+1}$$

$$= \lambda\Big[\frac{\varepsilon}{2} + h(ih-1)(1-jk)^3\Big]V_{i-1,j} + \Big[(1-jk)^4 - \varepsilon\lambda\Big]V_{i,j} + \lambda\Big[\frac{\varepsilon}{2} - h(ih-1)(1-jk)^3\Big]V_{i+1,j}.$$

.... (3.2.7)

for $i = 1, 2, \ldots, m-1$ and $j = 0, 1, \ldots, N-1$.

The boundary condition (3.1.8), in terms of the central differences, can be represented as

$$V_{1,j} - V_{-1,j} = 2hBi_1(1-jk)^2 V_{0,j},$$

which gives

$$V_{-1,j} = V_{1,j} - 2hBi_1(1-jk)^2 V_{0,j}, \qquad (3.2.8)$$

and similarly,

$$V_{-1,j+1} = V_{1,j+1} - 2hBi_1(1-jk)^2 V_{0,j+1}. \qquad (3.2.9)$$

[see Smith (6)].

Using equations (3.2.8) and (3.2.9), $V_{-1,j}$ and $V_{-1,j+1}$ can be eliminated from the equation obtained by putting $i=0$ in (3.2.7). This yields the result

$$-\frac{\varepsilon\lambda}{2}\left[V_{1,j+1} - 2hBi_1(1-jk)^2 V_{0,j+1}\right] + \left[(1-jk)^4 + \varepsilon\lambda\right]V_{0,j+1} - \frac{\varepsilon\lambda}{2}V_{1,j+1}$$

$$= \lambda\left[\frac{\varepsilon}{2} - h(1-jk)^3\right]\left[V_{1,j} - 2hBi_1(1-jk)^2 V_{0,j}\right] +$$

$$\left[(1-jk)^4 - \varepsilon\lambda\right]V_{0,j} + \lambda\left[\frac{\varepsilon}{2} + h(1-jk)^3\right]V_{1,j}.$$

Expanding and rearranging the terms of the above equation, gives

$$\left\{\left[(1-jk)^4 + \varepsilon\lambda\right] + \varepsilon\lambda h Bi_1(1-jk)^2\right\}V_{o,j+1} - \varepsilon\lambda V_{1,j+1} =$$

$$\left\{\left[(1-jk)^4 - \varepsilon\lambda\right] - 2\lambda h\left[\frac{\varepsilon}{2} - h(1-jk)^3\right]Bi_1(1-jk)^2\right\}V_{o,j} + \varepsilon\lambda V_{1,j},$$

...... (3.2.10)

for $j = 0,1,\ldots,N-1$

The boundary condition (3.1.7) can also be put in terms of the central differences as

$$V_{m+1,j} - V_{m-1,j} = -2hBi_2(1-jk)^2V_{m,j}$$

from which the following can be deduced:

$$V_{m+1,j} = V_{m-1,j} - 2hBi_2(1-jk)^2V_{m,j}; \qquad (3.2.11)$$

and

$$V_{m+1,j+1} = V_{m-1,j+1} - 2hBi_2(1-jk)^2\, V_{m,j+1}\,. \qquad (3.2.12)$$

[see Smith (6)].

Using equations (3.2.11) and (3.2.12), $V_{m+1,j}$ and $V_{m+1,j+1}$ can be eliminated from the equation obtained by putting $i=m$ in (3.2.7). This gives

$$-\frac{\varepsilon\lambda}{2}V_{m-1,j+1} + \left[(1-jk)^4 + \varepsilon\lambda\right]V_{m,j+1} -$$

$$\frac{\varepsilon\lambda}{2}\left[V_{m-1,j+1} - 2hBi_2(1-jk)^2V_{m,j+1}\right]$$

$$= \lambda\left[\frac{\varepsilon}{2} + h(mh-1)(1-jk)^3\right]V_{m-1,j} + \left[(1-jk)^4 - \varepsilon\lambda\right]V_{m,j} +$$

$$\lambda\left[\frac{\varepsilon}{2} - h(mh-1)(1-jk)^3\right]\left[V_{m-1,j} - 2hBi_2(1-jk)^2V_{m,j}\right].$$

which upon expansion and rearrangement of the terms, gives

$$-\varepsilon\lambda V_{m-1,j+1} + \left\{\left[(1-jk)^4 + \varepsilon\lambda\right] + \varepsilon\lambda hBi_2(1-jk)^2\right\}V_{m,j+1} = \varepsilon\lambda V_{m-1,j}$$

$$+ \left\{\left[(1-jk)^4 - \varepsilon\lambda\right] - 2\lambda h\left[\frac{\varepsilon}{2} - h(mh-1)(1-jk)^3Bi_2(1-jk)^2\right]\right\}V_{m,j} \qquad \ldots\ldots\ldots \quad (3.2.13)$$

for $j=0,1,\ldots,N-1$.

The initial condition at $\tau^* = 0$ is expressed as

$$V_{i,o} = 1, \quad 0 \leq y \leq 1 \quad , \quad \tau^* = 0 \qquad (3.2.14)$$

for $i = 0,1,2,\ldots,m$.

<u>Setting up the finite difference equations.</u>

To visualize the finite difference system of linear equations, the system of equations (3.2.7), (3.2.10) and (3.2.13) are represented in matrix form, by letting

$$a_o = \left[\left(1-jk\right)^4 + \varepsilon\lambda\right] + \varepsilon\lambda hBi_1\left(1-jk\right)^2, \quad b = \varepsilon\lambda,$$

$$c_o = \left[\left(1-jk\right)^4 - \varepsilon\lambda\right] - h\left(1-jk\right)^3\Big]Bi_1\left(1-jk\right)^2,$$

$$a = \left(1-jk\right)^4 + \varepsilon\lambda, \quad c = \left(1-jk\right)^4 - \varepsilon\lambda, \quad d = \lambda\left[\frac{\varepsilon}{2} - h\left(ih-1\right)^4\left(1-jk\right)^3\right],$$

$$e = \lambda\left[\frac{\varepsilon}{2} + h\left(ih-1\right)\left(1-jk\right)^3\right],$$

$$a_m = \left[\left(1-jk\right)^4 + \varepsilon\lambda hBi_2\left(1-jk\right)^2, \text{ and}\right.$$

$$c_m = \left[\left(1-jk\right)^4 - \varepsilon\lambda\right] - 2\lambda h\left[\frac{\varepsilon}{2} - h\left(mh-1\right)\left(mh-1\right)^3\right]Bi_2\left(1-jk\right)^2,$$

as follows:

$$\begin{pmatrix} a_o & -b & 0 & & & & 0 \\ -\frac{b}{2} & a & -\frac{b}{2} & & & & \\ 0 & -\frac{b}{2} & a & -\frac{b}{2} & & & \\ & & & & & & 0 \\ & & & & -\frac{b}{2} & a & -\frac{b}{2} \\ 0 & & & & 0 & -b & a_m \end{pmatrix} \begin{pmatrix} V_{o,j+1} \\ V_{1,j+1} \\ V_{2,j+1} \\ \\ V_{m-1,j+1} \\ V_{m,j+1} \end{pmatrix} =$$

$$\begin{pmatrix} c_o & b & 0 & & & 0 \\ e & c & d & & & \\ 0 & e & c & d & & \\ & & & & & 0 \\ & & & e & c & d \\ 0 & & & o & b & c_m \end{pmatrix} \begin{pmatrix} V_{0,j} \\ V_{1,j} \\ V_{2,j} \\ \\ V_{m-1,j} \\ V_{m,j} \end{pmatrix}, \quad (3.2.15)$$

for $j = 0,1,\ldots,N-1$.

In the discretization of the governing equation the following are the exceptional points:

(i,j)

for $i = 0,1,\ldots,m$ and $j = 0,1,\ldots,N-1$ (see fig.3)

Putting $i = 0$ and $j = 0$, in equations (3.2.10) and (3.2.14), yields the equation

$$\left\{1 + \varepsilon\lambda + \varepsilon\lambda h Bi_1\right\} V_{0,1} - \varepsilon\lambda V_{1,1} =$$

$$\left\{1 - \varepsilon\lambda - 2\lambda h\left(\frac{\varepsilon}{2} - h\right)Bi_1\right\} + \varepsilon\lambda \qquad (3.2.16)$$

For $1 \leq i \leq m-1$ and $j = 0$, the use of (3.2.7) and (3.2.14) yields

$$-\frac{\varepsilon\lambda}{2} V_{i-1,1} + \left[1+\varepsilon\lambda\right]V_{i,1} - \frac{\varepsilon\lambda}{2} V_{i+1,1}$$

$$= \lambda\left[\frac{\varepsilon}{2} + h(ih-1)\right] + \left[1-\varepsilon\lambda\right] + \lambda\left[\frac{\varepsilon}{2} - h(ih-1)\right]. \qquad (3.2.17)$$

Combining (3.2.13) and (3.2.14), for $i = m$ and $j = 0$, gives the equation

$$-\varepsilon\lambda V_{m-1,1} + \left[(1+\varepsilon\lambda) + h\varepsilon Bi_2\right]V_{m,1} = \varepsilon\lambda + \left[(1-\varepsilon\lambda) - \varepsilon\lambda h Bi_2\right]. \qquad \ldots\ldots\ldots\ldots\ (3.2.18)$$

For $i = 0$ and $1 \leq j \leq N-1$, using (3.2.10), gives

$$\left\{\left[(1-jk)^2 + \varepsilon\lambda\right] + \varepsilon\lambda h Bi_1 (1-jk)^2\right\}V_{0,j+1} - \varepsilon\lambda V_{1,j+1} =$$

$$\left\{\left[(1-jk)^4 - \varepsilon\lambda\right] - 2\lambda h\left[\frac{\varepsilon}{2} - h(1-jk)^3\right]Bi_1(1-jk)^2\right\}V_{0,j} + \varepsilon\lambda V_{1,j}. \qquad \ldots\ldots\ (3.2.19)$$

For $1 \leq i \leq m-1$ and $1 \leq j \leq N-1$, using (3.2.7), gives

$$-\frac{\varepsilon\lambda}{2} V_{i-1,j+1} + \left[(1-jk)^4 + \varepsilon\lambda\right]V_{i,j+1} - \frac{\varepsilon\lambda}{2} V_{i+1,j+1}$$

$$= \lambda\left[\frac{\varepsilon}{2} + h(ih-1)(1-jk)^3\right]V_{i-1,j} +$$

$$\left[(1-jk)^4 - \varepsilon\lambda\right]V_{i,j} + \lambda\left[\frac{\varepsilon}{2} - h(ih-1)(1-jk)^3\right]V_{i+1,j}. \quad (3.2.20)$$

For i = m and $i \leq j \leq N-1$, using (3.2.13), gives

$$-\varepsilon\lambda V_{m-1,j+1} + \left\{\left[(1-jk)^4 + \varepsilon\lambda\right] + \varepsilon\lambda h Bi_2(1-jk)^2\right\}V_{m,j+1} =$$

$$\varepsilon\lambda V_{m-1,j} + \left\{\left[(1-jk)^4 - \varepsilon\lambda\right] - \varepsilon\lambda h Bi_2(1-jk)^2\right\}V_{m,j}. \quad (3.2.21)$$

The system of equations (3.2.16) - (3.2.21) are solved for $V_{i,j+1}$ where $i = 0,1,\ldots,m$ and $j = 0,1,\ldots N-1$, using the Gauss Elimination method.

Description of the Crank-Nicolson method.

The Crank-Nicolson method uses the mean of the forward - difference method at the jth step in τ^* and the backward - difference method at the (j+1)st step in τ^*. Thus, it has local truncation error of order $O(k^2+h^2)$. This means that the solution obtained by Crank-Nicolson method are of greater accuracy than those by explicit (forward and backward - difference) methods which have truncation error of order $O(k+h^2)$. Crank-Nicolson is also preferred to other methods like the Richardsons's method of truncation error of order $O(k^2+h^2)$ because the Crank-Nicolson equations have unrestricted stability.

Chapter Four

NUMERICAL SOLUTIONS

4.1 Computer Program

The separation of the energy equation (1.3.14) with conditions (1.3.15) – (1.3.18) into two equations, (2.1.7) with conditions (2.1.8) – (2.1.10) and (2.1.11) with conditions (2.1.12) – (2.1.14), has simplified the whole problem to that of determining V, a function of one space coordinate and time. Therefore, the computer program that computes the numerical solutions has been written in FORTRAN from the finite difference equations (3.2.16) – (3.2.21) and it is given in Appendix I.

4.2 Numerical Solutions

Numerical solutions have been obtained for the dimensionless emptying times ϵ=0.002, 0.001 and 0.004. The effect of ladle surface heat transfer was simulated by the value Bi_1=5.0, and in the case of insulating slag surface with no initial thermal stratification, Bi_2=0.0 was used. The temperatures of molten steel on the vertical axis, obtained from the computer program, are displayed in Appendix II for various values of the dimensionless time τ^*, from which the graphs in figures 4, 5 and 6 have been extracted.

The thermal properties of liquid steel are given in table 1 below:

Table 1. Thermal properties of liquid steel

Density	$7510 kg\ m^{-3}$
Specific heat	$700 J kg^{-1} K^{-1}$
Viscosity	$48 \times 10^{-4} Nsm^{-2}$
Conductivity	$29 Wm^{-1} K^{-1}$
Heat transfer coefficient (Ladle walls, bottom)	$10-30 Wm^{-2} K^{-1}$
Heat transfer coefficient (Slag cover)	$1-15 Wm^{-2} K^{-1}$
Diffusivity $(K/\rho c)$	$5.17 \times 10^{-6} m^2 s^{-1}$
Fusion temperature of steel	$1470-1530\,^{\circ}C$

4.3 Accuracy of the computer solutions.

The correctness and accuracy of the computer solutions were established by comparing the graphs of the numerical results of the above computer program and those of Egerton et al (2), using the same set of values ($Bi_1=5.0$, $Bi_2=0.0$, $\epsilon=0.002$) and same spatial step $\Delta y = 0.01$ and time step $\Delta\tau^*=0.001$ as Egerton et al used. The graphs showing the variation in temperature of the molten steel on the vertical axis are displayed, for $\tau^*=0.7$, $\tau^*=0.8$ and $\tau^*=0.9$, in figure 4. The computed solutions are almost the same as those found by Egerton et al. The graphs, like those of Egerton et al, show that appreciable changes in temperature, below the initial temperature, only occur for τ^* greater than 0.8.

Figure 5 shows the variation in temperature of the molten steel on the vertical axis for $\epsilon=0.001$ and various

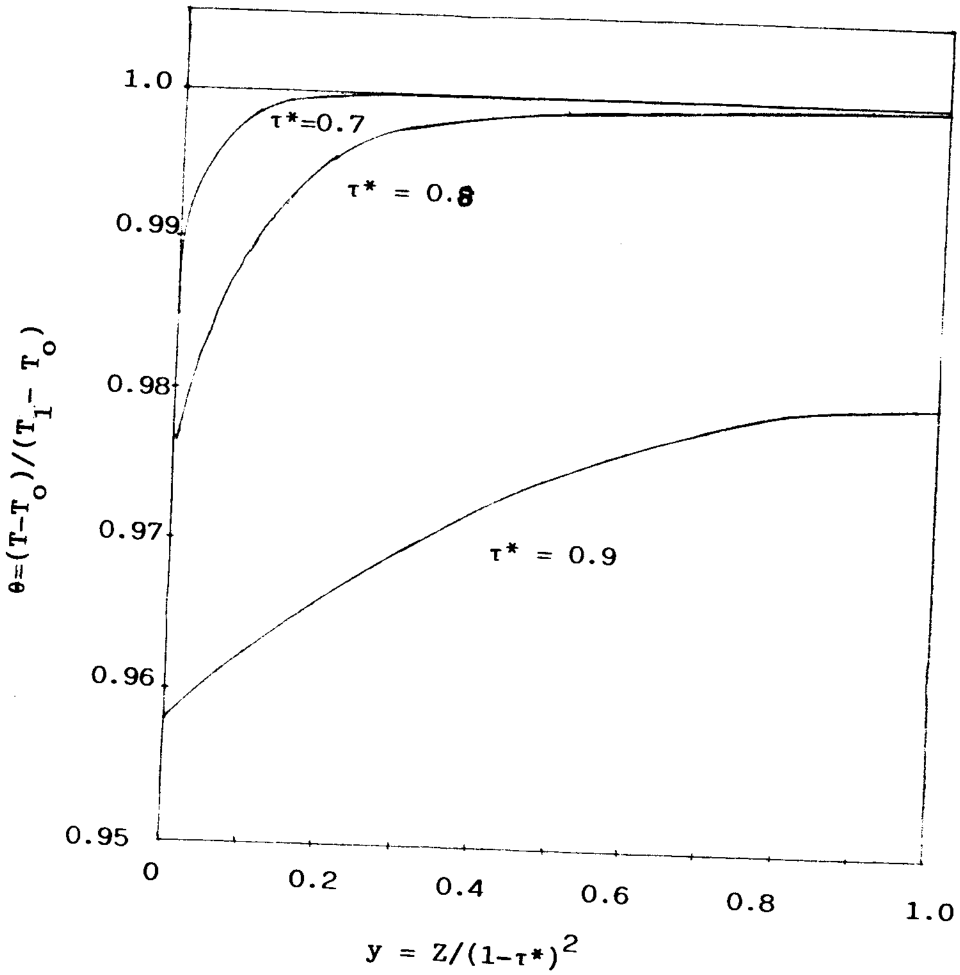

Fig. 4. Dimensionless temperature distribution for various times (Bi_1=5.0, Bi_2=0.0, ϵ=0.002)

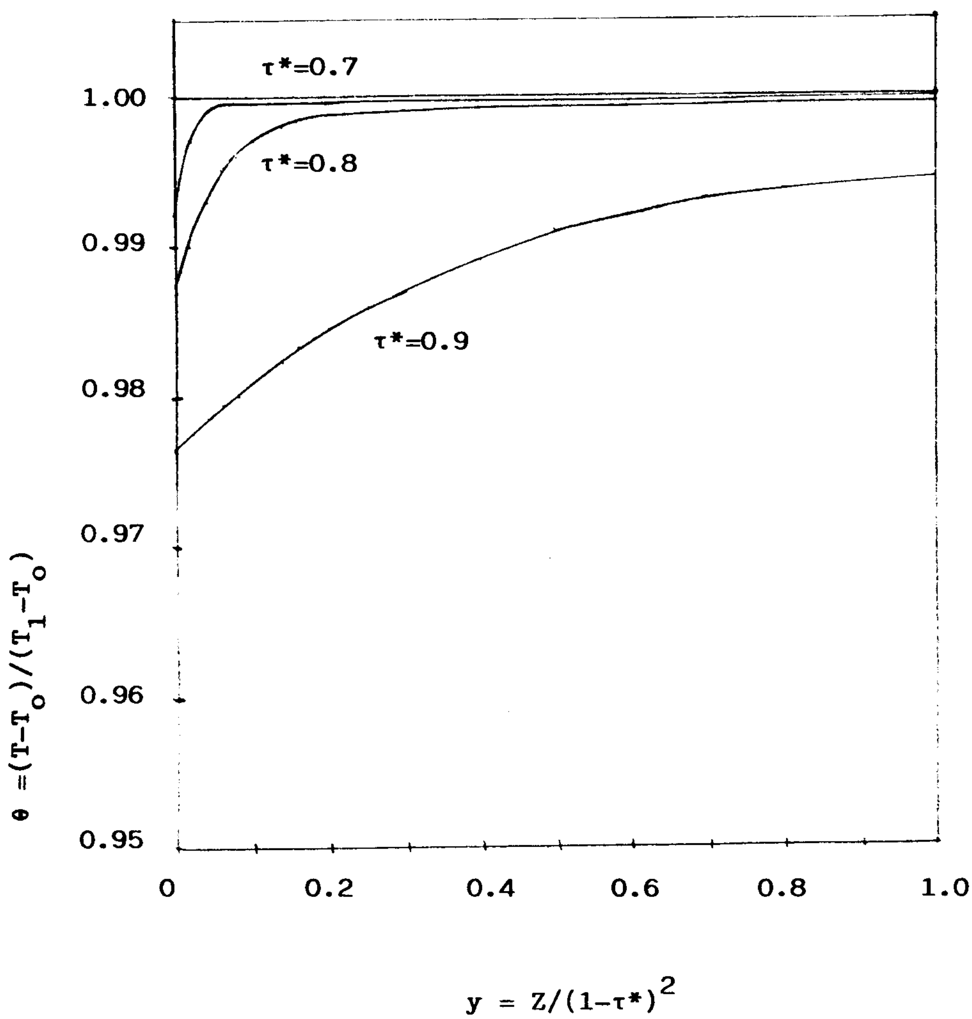

Fig. 5. Dimensionless temperature distribution for various times (Bi_1=5.0, Bi_2=0.0, ε=0.001)

values of the dimensionless time τ^* with same $Bi_1=5.0$ and $Bi_2=0.0$. The results, here, show that the significant changes in temperature, below the initial temperature, only occur as τ^* approaches 0.9.

Figure 6 shows the effect of the molten steel temperature on the vertical axis against time for various ε. This is for the case $Bi_1=5.0$ and $Bi_2=0.0$. From the graphs in the figure, a decrease in ε implies a slowing down of the cooling,and these results are in agreement with those found by Egerton et al. It is, therefore concluded that the computer solutions of this dissertation are accurate.

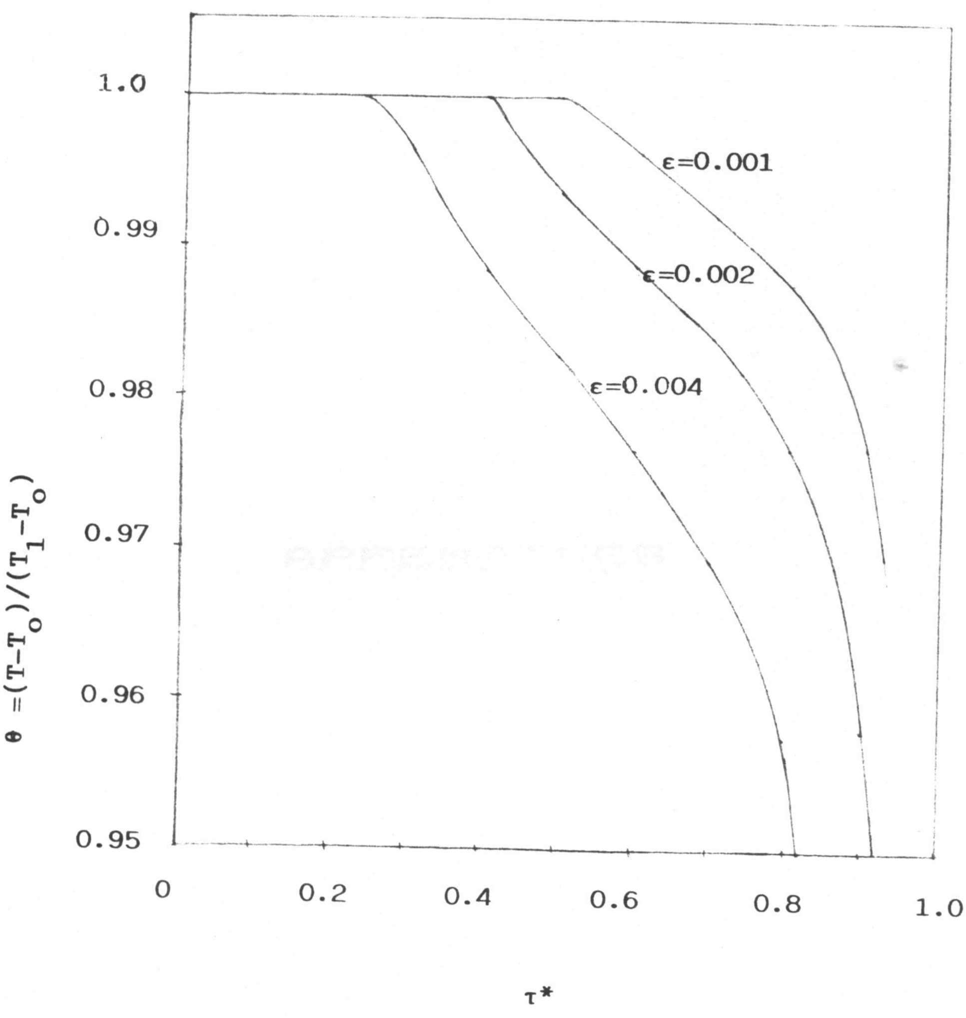

Fig. 6. Dimensionless molten steel temperature on the vertical axis against time for various ε (Bi_1=5.0, Bi_2=0).

APPENDICES

APPENDIX I

This appendix gives the computer program which finds the numerical solutions displayed in appendix II.

FILE: HEATEQ1 FORTRAN A1 UNIVERSITY OF ZAMBIA

```
      PROGRAM  HEATEQ
C        E DENOTES EPSILON
C        T DENOTES TAL=TIME
C        BI1 DENOTES BIOT NUMBER 1
C        BI2 DENOTES BIOT NUMBER 2
C        H DENOTES THE SPATAIL STEP
C        K DENOTES THE TIME STEP
C        LMDA DENOTES LAMDA
      REAL A(101,101),B(101,1),H,K,BI1,BI2,Y,T,E,TEMP(100,1000),
     & LMDA,Z(101),L(101),V(101,1000),W(1000),U,TS,
     & ALPHA,TEMPOJ(1000)
      INTEGER I,J,KK,K1,K2,M,N,K3
      COMMON / SIMEON / H
      COMMON / STEVE / K
      PRINT *,'                 NUMERICAL  SOLUTIONS'
      PRINT *,'                 --------------------'
      PRINT *
      PRINT *,'VALUES USED : M=100, N=1000, EPSILON=0.002, BI1=5.0,'
      PRINT *,'-----------'
      PRINT *,'                   BI2=0.0, '
      PRINT *
1     M=100
      N=1000
      ALPHA=2.0
      E=0.002
      BI1=5.0
      BI2=0.0
      U=1.0
      H=1.0/REAL(M)
      K=1.0/REAL(N)
      LMDA=K/H**2
C        WE GENERATE THE ELEMENTS OF MATRIX 'A' AND MATRIX 'B' FOR J=0
      A(1,1)=(1.0+LMDA*E)+E*LMDA*H*BI1
      A(1,2)=-1.0*LMDA*E
      B(1,1)=((1.0-LMDA*E)-2.0*LMDA*H*(E/2.0-H)*BI1)+E*LMDA
      DO  2  I=1,M-1
          KK=I+1
          K1=I
          K2=I+2
          A(KK,K1)=-1.0*(E*LMDA)/2.0
          A(KK,KK)=1.0+E*LMDA
          A(KK,K2)=-1.0*(LMDA*E)/2.0
          B(KK,1)=LMDA*(E/2.0+H*(I*H-1.0))+(1.0-LMDA*E)+
     &               LMDA*(E/2.0-H*(I*H-1.0))
2     CONTINUE
      A(M+1,M)=-1.0*LMDA*E
      A(M+1,M+1)=(1.0+LMDA*E)+E*LMDA*H*BI2
      B(M+1,1)=E*LMDA+((1.0-E*LMDA)-E*LMDA*H*BI2)
      GO TO 8
4     CONTINUE
C        WE GENERATE THE ELEMENTS OF MATRIX 'A' AND MATRIX 'B'
C         FOR J=1,2,...1000
         A(1,1)=((1.0-J*K)**4+LMDA*E)+E*LMDA*H*BI1*(1.0-J*K)**2
         A(1,2)=-1.0*LMDA*E
         B(1,1)=(((1.0-J*K)**4-LMDA*E)-2.0*LMDA*H*(E/2.0-H*
```

```
     &            (1.0-J*K)**3)*BI1*(1.0-J*K)**2)*V(1,J)+
     &             E*LMDA*V(2,J)
        DO  6   I=1,M-1
              KK=I+1
              K1=I
              K2=I+2
              A(KK,K1)=-1.0*(E*LMDA)/2.0
              A(KK,KK)=(1.0-J*K)**4+LMDA*E
              A(KK,K2)=-1.0*(LMDA*E)/2.0
              B(KK,1)=LMDA*(E/2.0+H*(I*H-1.0)*(1.0-J*K)**3)*V(I,J)+
     &                ((1.0-J*K)**4-E*LMDA)*V(I+1,J)+
     &                LMDA*(E/2.0-H*(I*H-1.0)*(1.0-J*K)**3)*V(I+2,J)
        CONTINUE
        A(M+1,M)=-1.0*LMDA*E
        A(M+1,M+1)=((1.0-J*K)**4+LMDA*E)+E*LMDA*H*BI2*(1.0-J*K)**2
        B(M+1,1)=E*LMDA*V(M,J)+
     &            ((((1.0-J*K)**4-LMDA*E)-LMDA*H*E*
     &             BI2*(1.0-J*K)**2)*V(M+1,J)
        GO TO 10
        CONTINUE
        J=0
0       CONTINUE
          WE SOLVE THE SYSTEM OF EQUATIONS FOR V BY GAUSS ELIMINATION
           METHOD
        L(1)=A(1,1)
        DO   12   I=1,M
              W(I)=A(I,I+1)/L(I)
              L(I+1)=A(I+1,I+1)-A(I+1,I)*W(I)
2       CONTINUE
        Z(1)=B(1,1)/L(1)
        DO   14   I=2,M+1
              Z(I)=(B(I,1)-A(I,I-1)*Z(I-1))/L(I)
4       CONTINUE
        N=M
        V(M+1,J+1)=Z(M+1)
6       CONTINUE
              V(N,J+1)=Z(N)-W(N)*V(N+1,J+1)
              N=N-1
         IF (N.GE.1) GO TO 16
         J=J+1
      IF (J.LE.999) GO TO 4
         WE COMPUTE THE TEMPERATURES
          TEMPOJ IS THE TEMPERATURE AT Y=0.0 FOR T=0.001,0.002,....,1.0
          TEMP IS THE TEMPERATURE AT EACH Y=0.01,0.02,....,1.0 FOR
           T=0.001,0.002,....,1.0
      N=1000
      DO   20   J=1,N
           TEMPOJ(J)=V(1,J)*U
           IF (TEMPOJ(J).GT.1.0) TEMPOJ(J)=1.0
           DO   18   I=2,M+1
                TEMP(I-1,J)=V(I,J)*U
                IF (TEMP(I-1,J).GT.1.0) TEMP(I-1,J)=1.0
18         CONTINUE
20    CONTINUE
      WRITE(6,25) T(900)
```

FILE: HEATEQ1 FORTRAN A1 UNIVERSITY OF ZAMBIA

```
25    FORMAT(7X,'TIME=',F8.2//)
      PRINT *
      PRINT *,'      Y       V(Y,TIME)        U(0, TIME)  TEMPERATURE'
      PRINT *,'     ----     ---------        ----------  -----------'
      PRINT *
      WRITE (6,30) Y(0),V(1,900),U,TEMPOJ(900)
      WRITE (6,30) (Y(I),V(I+1,900),U,TEMP(I,900),I=1,M)
30    FORMAT(101(2X,F8.2,G20.10,2G16.10/))
31    PRINT *
      PRINT *,'     TIME     TEMPERATURE'
      PRINT *,'     ----     -----------'
      K3=100
32    WRITE(6,34) T(K3),TEMPOJ(K3)
34       FORMAT(2X,F8.2,G20.10//)
         K3=K3+100
         IF (K3.LE.900) GO TO 32
36    STOP
      END

CC      SUBPROGRAMS

      REAL FUNCTION Y(I)
      REAL H
      INTEGER I
      COMMON / SIMEON / H
      Y=REAL(I)*H
      RETURN
      END

      REAL FUNCTION T(J)
      REAL K
      INTEGER J
      COMMON / STEVE / K
      T=REAL(J)*K
      RETURN
      END
```

A P P E N D I X   II

The appendix provides the numerical solutions from which the graphs given in figures 4, 5 and 6 were extracted.

FILE: HEATEQ1 OUTPUT7 A1 UNIVERSITY OF ZAMBIA

NUMERICAL SOLUTIONS

VALUES USED : M=100, N=1000, EPSILON=0.002, BI1=5.0,

BI2=0.0,

TIME= 0.70

Y	V(Y,TIME)	U(0, TIME)	TEMPERATURE
0.00	0.9840772150	1.000000000	0.9840772150
0.01	0.9879062770	1.000000000	0.9879062770
0.02	0.9908009170	1.000000000	0.9908009170
0.03	0.9929870370	1.000000000	0.9929870370
0.04	0.9946367740	1.000000000	0.9946367740
0.05	0.9958806040	1.000000000	0.9958806040
0.06	0.9968174700	1.000000000	0.9968174700
0.07	0.9975224730	1.000000000	0.9975224730
0.08	0.9980527160	1.000000000	0.9980527160
0.09	0.9984510540	1.000000000	0.9984510540
0.10	0.9987498520	1.000000000	0.9987498520
0.11	0.9989739060	1.000000000	0.9989739060
0.12	0.9991417530	1.000000000	0.9991417530
0.13	0.9992672200	1.000000000	0.9992672200
0.14	0.9993609790	1.000000000	0.9993609790
0.15	0.9994311930	1.000000000	0.9994311930
0.16	0.9994834660	1.000000000	0.9994834660
0.17	0.9995223880	1.000000000	0.9995223880
0.18	0.9995511770	1.000000000	0.9995511770
0.19	0.9995725150	1.000000000	0.9995725150
0.20	0.9995883110	1.000000000	0.9995883110
0.21	0.9995998740	1.000000000	0.9995998740
0.22	0.9996083380	1.000000000	0.9996083380
0.23	0.9996144770	1.000000000	0.9996144770
0.24	0.9996188880	1.000000000	0.9996188880
0.25	0.9996220470	1.000000000	0.9996220470
0.26	0.9996243120	1.000000000	0.9996243120
0.27	0.9996260400	1.000000000	0.9996260400
0.28	0.9996271730	1.000000000	0.9996271730
0.29	0.9996280070	1.000000000	0.9996280070
0.30	0.9996282460	1.000000000	0.9996282460
0.31	0.9996284840	1.000000000	0.9996284840
0.32	0.9996283050	1.000000000	0.9996283050
0.33	0.9996282460	1.000000000	0.9996282460
0.34	0.9996280070	1.000000000	0.9996280070
0.35	0.9996279480	1.000000000	0.9996279480
0.36	0.9996277690	1.000000000	0.9996277690
0.37	0.9996274110	1.000000000	0.9996274110
0.38	0.9996271730	1.000000000	0.9996271730
0.39	0.9996270540	1.000000000	0.9996270540

FILE: HEATEQ1 OUTPUT7 A1 UNIVERSITY OF ZAMBIA

0.40	0.9996268150	1.000000000	0.9996268150
0.41	0.9996265170	1.000000000	0.9996265170
0.42	0.9996264580	1.000000000	0.9996264580
0.43	0.9996261000	1.000000000	0.9996261000
0.44	0.9996258620	1.000000000	0.9996258620
0.45	0.9996258020	1.000000000	0.9996258020
0.46	0.9996255640	1.000000000	0.9996255640
0.47	0.9996253850	1.000000000	0.9996253850
0.48	0.9996253250	1.000000000	0.9996253250
0.49	0.9996252060	1.000000000	0.9996252060
0.50	0.9996252060	1.000000000	0.9996252060
0.51	0.9996251460	1.000000000	0.9996251460
0.52	0.9996250270	1.000000000	0.9996250270
0.53	0.9996247290	1.000000000	0.9996247290
0.54	0.9996248480	1.000000000	0.9996248480
0.55	0.9996247890	1.000000000	0.9996247890
0.56	0.9996250270	1.000000000	0.9996250270
0.57	0.9996247290	1.000000000	0.9996247290
0.58	0.9996244910	1.000000000	0.9996244910
0.59	0.9996243120	1.000000000	0.9996243120
0.60	0.9996242520	1.000000000	0.9996242520
0.61	0.9996244310	1.000000000	0.9996244310
0.62	0.9996241930	1.000000000	0.9996241930
0.63	0.9996243120	1.000000000	0.9996243120
0.64	0.9996239540	1.000000000	0.9996239540
0.65	0.9996238350	1.000000000	0.9996238350
0.66	0.9996237160	1.000000000	0.9996237160
0.67	0.9996234180	1.000000000	0.9996234180
0.68	0.9996231790	1.000000000	0.9996231790
0.69	0.9996230600	1.000000000	0.9996230600
0.70	0.9996230600	1.000000000	0.9996230600
0.71	0.9996231200	1.000000000	0.9996231200
0.72	0.9996231790	1.000000000	0.9996231790
0.73	0.9996231200	1.000000000	0.9996231200
0.74	0.9996232390	1.000000000	0.9996232390
0.75	0.9996234770	1.000000000	0.9996234770
0.76	0.9996232990	1.000000000	0.9996232990
0.77	0.9996232990	1.000000000	0.9996232990
0.78	0.9996233580	1.000000000	0.9996233580
0.79	0.9996235370	1.000000000	0.9996235370
0.80	0.9996234180	1.000000000	0.9996234180
0.81	0.9996232390	1.000000000	0.9996232390
0.82	0.9996232390	1.000000000	0.9996232390
0.83	0.9996232390	1.000000000	0.9996232390
0.84	0.9996231790	1.000000000	0.9996231790
0.85	0.9996233580	1.000000000	0.9996233580
0.86	0.9996234180	1.000000000	0.9996234180
0.87	0.9996235970	1.000000000	0.9996235970
0.88	0.9996240140	1.000000000	0.9996240140
0.89	0.9996242520	1.000000000	0.9996242520
0.90	0.9996244910	1.000000000	0.9996244910
0.91	0.9996246100	1.000000000	0.9996246100
0.92	0.9996249680	1.000000000	0.9996249680
0.93	0.9996250270	1.000000000	0.9996250270
0.94	0.9996251460	1.000000000	0.9996251460

FILE: HEATEQ1 OUTPUT7 A1 UNIVERSITY OF ZAMBIA

0.95	0.9996255040	1.000000000	0.9996255040
0.96	0.9996258620	1.000000000	0.9996258620
0.97	0.9996261000	1.000000000	0.9996261000
0.98	0.9996265770	1.000000000	0.9996265770
0.99	0.9996270540	1.000000000	0.9996270540
1.00	0.9996274710	1.000000000	0.9996274710

FILE: HEATEQ1 OUTPUT8 A1 UNIVERSITY OF ZAMBIA

NUMERICAL SOLUTIONS

VALUES USED : M=100, N=1000, EPSILON=0.002, BI1=5.0,

BI2=0.0,

TIME= 0.80

Y	V(Y,TIME)	U(0, TIME)	TEMPERATURE
0.00	0.9768142100	1.000000000	0.9768142100
0.01	0.9786942010	1.000000000	0.9786942010
0.02	0.9804209470	1.000000000	0.9804209470
0.03	0.9820063110	1.000000000	0.9820063110
0.04	0.9834615590	1.000000000	0.9834615590
0.05	0.9847972990	1.000000000	0.9847972990
0.06	0.9860236050	1.000000000	0.9860236050
0.07	0.9871482250	1.000000000	0.9871482250
0.08	0.9881798030	1.000000000	0.9881798030
0.09	0.9891251330	1.000000000	0.9891251330
0.10	0.9899916050	1.000000000	0.9899916050
0.11	0.9907857180	1.000000000	0.9907857180
0.12	0.9915136100	1.000000000	0.9915136100
0.13	0.9921800490	1.000000000	0.9921800490
0.14	0.9927901030	1.000000000	0.9927901030
0.15	0.9933492540	1.000000000	0.9933492540
0.16	0.9938599470	1.000000000	0.9938599470
0.17	0.9943274860	1.000000000	0.9943274860
0.18	0.9947553870	1.000000000	0.9947553870
0.19	0.9951471690	1.000000000	0.9951471690
0.20	0.9955051540	1.000000000	0.9955051540
0.21	0.9958319660	1.000000000	0.9958319660
0.22	0.9961313610	1.000000000	0.9961313610
0.23	0.9964048860	1.000000000	0.9964048860
0.24	0.9966541530	1.000000000	0.9966541530
0.25	0.9968819020	1.000000000	0.9968819020
0.26	0.9970898630	1.000000000	0.9970898630
0.27	0.9972797630	1.000000000	0.9972797630
0.28	0.9974532130	1.000000000	0.9974532130
0.29	0.9976121190	1.000000000	0.9976121190
0.30	0.9977563620	1.000000000	0.9977563620
0.31	0.9978879690	1.000000000	0.9978879690
0.32	0.9980078940	1.000000000	0.9980078940
0.33	0.9981177450	1.000000000	0.9981177450
0.34	0.9982180600	1.000000000	0.9982180600
0.35	0.9983090760	1.000000000	0.9983090760
0.36	0.9983924630	1.000000000	0.9983924630
0.37	0.9984675650	1.000000000	0.9984675650
0.38	0.9985361100	1.000000000	0.9985361100
0.39	0.9985979200	1.000000000	0.9985979200

FILE: HEATEQ1 OUTPUT8 A1 UNIVERSITY OF ZAMBIA

0.40	0.9986544250	1.000000000	0.9986544250
0.41	0.9987053280	1.000000000	0.9987053280
0.42	0.9987524150	1.000000000	0.9987524150
0.43	0.9987943770	1.000000000	0.9987943770
0.44	0.9988328220	1.000000000	0.9988328220
0.45	0.9988682870	1.000000000	0.9988682870
0.46	0.9989007710	1.000000000	0.9989007710
0.47	0.9989299180	1.000000000	0.9989299180
0.48	0.9989567400	1.000000000	0.9989567400
0.49	0.9989807610	1.000000000	0.9989807610
0.50	0.9990028140	1.000000000	0.9990028140
0.51	0.9990223650	1.000000000	0.9990223650
0.52	0.9990407230	1.000000000	0.9990407230
0.53	0.9990565780	1.000000000	0.9990565780
0.54	0.9990717170	1.000000000	0.9990717170
0.55	0.9990851280	1.000000000	0.9990851280
0.56	0.9990973470	1.000000000	0.9990973470
0.57	0.9991087910	1.000000000	0.9991087910
0.58	0.9991192220	1.000000000	0.9991192220
0.59	0.9991288190	1.000000000	0.9991288190
0.60	0.9991372230	1.000000000	0.9991372230
0.61	0.9991453290	1.000000000	0.9991453290
0.62	0.9991517660	1.000000000	0.9991517660
0.63	0.9991580840	1.000000000	0.9991580840
0.64	0.9991641640	1.000000000	0.9991641640
0.65	0.9991694690	1.000000000	0.9991694690
0.66	0.9991740580	1.000000000	0.9991740580
0.67	0.9991782900	1.000000000	0.9991782900
0.68	0.9991820450	1.000000000	0.9991820450
0.69	0.9991858010	1.000000000	0.9991858010
0.70	0.9991891980	1.000000000	0.9991891980
0.71	0.9991923570	1.000000000	0.9991923570
0.72	0.9991952180	1.000000000	0.9991952180
0.73	0.9991976620	1.000000000	0.9991976620
0.74	0.9991997480	1.000000000	0.9991997480
0.75	0.9992018340	1.000000000	0.9992018340
0.76	0.9992038010	1.000000000	0.9992038010
0.77	0.9992056490	1.000000000	0.9992056490
0.78	0.9992072580	1.000000000	0.9992072580
0.79	0.9992085700	1.000000000	0.9992085700
0.80	0.9992101190	1.000000000	0.9992101190
0.81	0.9992117290	1.000000000	0.9992117290
0.82	0.9992129210	1.000000000	0.9992129210
0.83	0.9992144110	1.000000000	0.9992144110
0.84	0.9992147090	1.000000000	0.9992147090
0.85	0.9992163780	1.000000000	0.9992163780
0.86	0.9992176890	1.000000000	0.9992176890
0.87	0.9992185830	1.000000000	0.9992185830
0.88	0.9992195370	1.000000000	0.9992195370
0.89	0.9992204900	1.000000000	0.9992204900
0.90	0.9992212060	1.000000000	0.9992212060
0.91	0.9992219210	1.000000000	0.9992219210
0.92	0.9992226360	1.000000000	0.9992226360
0.93	0.9992234110	1.000000000	0.9992234110
0.94	0.9992241260	1.000000000	0.9992241260

FILE: HEATEQ1 OUTPUT8 A1 UNIVERSITY OF ZAMBIA

0.95	0.9992241260	1.000000000	0.9992241260
0.96	0.9992237090	1.000000000	0.9992237090
0.97	0.9992238280	1.000000000	0.9992238280
0.98	0.9992247220	1.000000000	0.9992247220
0.99	0.9992246030	1.000000000	0.9992246030
1.00	0.9992244840	1.000000000	0.9992244840

FILE: HEATEQ1 OUTPUT9 A1 UNIVERSITY OF ZAMBIA

NUMERICAL SOLUTIONS

VALUES USED : M=100, N=1000, EPSILON=0.002, BI1=5.0,

BI2=0.0,

TIME= 0.90

Y	V(Y,TIME)	U(0, TIME)	TEMPERATURE
0.00	0.9581648710	1.000000000	0.9581648710
0.01	0.9586454630	1.000000000	0.9586454630
0.02	0.9591180090	1.000000000	0.9591180090
0.03	0.9595845340	1.000000000	0.9595845340
0.04	0.9600431920	1.000000000	0.9600431920
0.05	0.9604955910	1.000000000	0.9604955910
0.06	0.9609409570	1.000000000	0.9609409570
0.07	0.9613802430	1.000000000	0.9613802430
0.08	0.9618123170	1.000000000	0.9618123170
0.09	0.9622371200	1.000000000	0.9622371200
0.10	0.9626569750	1.000000000	0.9626569750
0.11	0.9630684850	1.000000000	0.9630684850
0.12	0.9634752870	1.000000000	0.9634752870
0.13	0.9638755920	1.000000000	0.9638755920
0.14	0.9642696980	1.000000000	0.9642696980
0.15	0.9646574850	1.000000000	0.9646574850
0.16	0.9650377630	1.000000000	0.9650377630
0.17	0.9654146430	1.000000000	0.9654146430
0.18	0.9657835960	1.000000000	0.9657835960
0.19	0.9661475420	1.000000000	0.9661475420
0.20	0.9665052890	1.000000000	0.9665052890
0.21	0.9668575530	1.000000000	0.9668575530
0.22	0.9672037960	1.000000000	0.9672037960
0.23	0.9675449730	1.000000000	0.9675449730
0.24	0.9678815600	1.000000000	0.9678815600
0.25	0.9682116510	1.000000000	0.9682116510
0.26	0.9685341720	1.000000000	0.9685341720
0.27	0.9688514470	1.000000000	0.9688514470
0.28	0.9691635370	1.000000000	0.9691635370
0.29	0.9694716930	1.000000000	0.9694716930
0.30	0.9697716830	1.000000000	0.9697716830
0.31	0.9700683950	1.000000000	0.9700683950
0.32	0.9703617100	1.000000000	0.9703617100
0.33	0.9706469180	1.000000000	0.9706469180
0.34	0.9709289070	1.000000000	0.9709289070
0.35	0.9712064860	1.000000000	0.9712064860
0.36	0.9714787600	1.000000000	0.9714787600
0.37	0.9717441200	1.000000000	0.9717441200
0.38	0.9720074530	1.000000000	0.9720074530
0.39	0.9722644090	1.000000000	0.9722644090

FILE: HEATEQ1 OUTPUT9 A1 UNIVERSITY OF ZAMBIA

0.40	0.9725176100	1.000000000	0.9725176100
0.41	0.9727636580	1.000000000	0.9727636580
0.42	0.9730063680	1.000000000	0.9730063680
0.43	0.9732437130	1.000000000	0.9732437130
0.44	0.9734743240	1.000000000	0.9734743240
0.45	0.9737016560	1.000000000	0.9737016560
0.46	0.9739249940	1.000000000	0.9739249940
0.47	0.9741420750	1.000000000	0.9741420750
0.48	0.9743561740	1.000000000	0.9743561740
0.49	0.9745652080	1.000000000	0.9745652080
0.50	0.9747707250	1.000000000	0.9747707250
0.51	0.9749715920	1.000000000	0.9749715920
0.52	0.9751678710	1.000000000	0.9751678710
0.53	0.9753590230	1.000000000	0.9753590230
0.54	0.9755446910	1.000000000	0.9755446910
0.55	0.9757264850	1.000000000	0.9757264850
0.56	0.9759064320	1.000000000	0.9759064320
0.57	0.9760809540	1.000000000	0.9760809540
0.58	0.9762511850	1.000000000	0.9762511850
0.59	0.9764152770	1.000000000	0.9764152770
0.60	0.9765764470	1.000000000	0.9765764470
0.61	0.9767330880	1.000000000	0.9767330880
0.62	0.9768849610	1.000000000	0.9768849610
0.63	0.9770322440	1.000000000	0.9770322440
0.64	0.9771756530	1.000000000	0.9771756530
0.65	0.9773167370	1.000000000	0.9773167370
0.66	0.9774535300	1.000000000	0.9774535300
0.67	0.9775823350	1.000000000	0.9775823350
0.68	0.9777073860	1.000000000	0.9777073860
0.69	0.9778289200	1.000000000	0.9778289200
0.70	0.9779481890	1.000000000	0.9779481890
0.71	0.9780648950	1.000000000	0.9780648950
0.72	0.9781773090	1.000000000	0.9781773090
0.73	0.9782836440	1.000000000	0.9782836440
0.74	0.9783853890	1.000000000	0.9783853890
0.75	0.9784843330	1.000000000	0.9784843330
0.76	0.9785795810	1.000000000	0.9785795810
0.77	0.9786710740	1.000000000	0.9786710740
0.78	0.9787582760	1.000000000	0.9787582760
0.79	0.9788413050	1.000000000	0.9788413050
0.80	0.9789195660	1.000000000	0.9789195660
0.81	0.9789968730	1.000000000	0.9789968730
0.82	0.9790669080	1.000000000	0.9790669080
0.83	0.9791337850	1.000000000	0.9791337850
0.84	0.9791970850	1.000000000	0.9791970850
0.85	0.9792563320	1.000000000	0.9792563320
0.86	0.9793121810	1.000000000	0.9793121810
0.87	0.9793655280	1.000000000	0.9793655280
0.88	0.9794139270	1.000000000	0.9794139270
0.89	0.9794575570	1.000000000	0.9794575570
0.90	0.9794979690	1.000000000	0.9794979690
0.91	0.9795359970	1.000000000	0.9795359970
0.92	0.9795686600	1.000000000	0.9795686600
0.93	0.9795974490	1.000000000	0.9795974490
0.94	0.9796230200	1.000000000	0.9796230200

FILE: HEATEQ1 OUTPUT9 A1 UNIVERSITY OF ZAMBIA

0.95	0.9796437030	1.000000000	0.9796437030
0.96	0.9796616440	1.000000000	0.9796616440
0.97	0.9796751740	1.000000000	0.9796751740
0.98	0.9796855450	1.000000000	0.9796855450
0.99	0.9796924590	1.000000000	0.9796924590
1.00	0.9796949030	1.000000000	0.9796949030

FILE: HEATEQ1 OUTPUT6 A1 UNIVERSITY OF ZAMBIA

NUMERICAL SOLUTIONS

VALUES USED : M=100, N=1000, EPSILON=0.001, BI1=5.0,

BI2=0.0,

TIME= 0.70

Y	V(Y,TIME)	U(0, TIME)	TEMPERATURE
0.00	0.9920684700	1.000000000	0.9920684700
0.01	0.9953170420	1.000000000	0.9953170420
0.02	0.9971600170	1.000000000	0.9971600170
0.03	0.9982044700	1.000000000	0.9982044700
0.04	0.9987961050	1.000000000	0.9987961050
0.05	0.9991302490	1.000000000	0.9991302490
0.06	0.9993192550	1.000000000	0.9993192550
0.07	0.9994258280	1.000000000	0.9994258280
0.08	0.9994854330	1.000000000	0.9994854330
0.09	0.9995185730	1.000000000	0.9995185730
0.10	0.9995369910	1.000000000	0.9995369910
0.11	0.9995465280	1.000000000	0.9995465280
0.12	0.9995514750	1.000000000	0.9995514750
0.13	0.9995538000	1.000000000	0.9995538000
0.14	0.9995549320	1.000000000	0.9995549320
0.15	0.9995549320	1.000000000	0.9995549320
0.16	0.9995542760	1.000000000	0.9995542760
0.17	0.9995535610	1.000000000	0.9995535610
0.18	0.9995526670	1.000000000	0.9995526670
0.19	0.9995518920	1.000000000	0.9995518920
0.20	0.9995508190	1.000000000	0.9995508190
0.21	0.9995501640	1.000000000	0.9995501640
0.22	0.9995495080	1.000000000	0.9995495080
0.23	0.9995488520	1.000000000	0.9995488520
0.24	0.9995479580	1.000000000	0.9995479580
0.25	0.9995472430	1.000000000	0.9995472430
0.26	0.9995464680	1.000000000	0.9995464680
0.27	0.9995461110	1.000000000	0.9995461110
0.28	0.9995455740	1.000000000	0.9995455740
0.29	0.9995451570	1.000000000	0.9995451570
0.30	0.9995445610	1.000000000	0.9995445610
0.31	0.9995442630	1.000000000	0.9995442630
0.32	0.9995439050	1.000000000	0.9995439050
0.33	0.9995433690	1.000000000	0.9995433690
0.34	0.9995430110	1.000000000	0.9995430110
0.35	0.9995425940	1.000000000	0.9995425940
0.36	0.9995420580	1.000000000	0.9995420580
0.37	0.9995416400	1.000000000	0.9995416400
0.38	0.9995411040	1.000000000	0.9995411040
0.39	0.9995408060	1.000000000	0.9995408060

FILE: HEATEQ1 OUTPUT6 A1 UNIVERSITY OF ZAMBIA

0.40	0.9995405670	1.000000000	0.9995405670
0.41	0.9995405670	1.000000000	0.9995405670
0.42	0.9995405080	1.000000000	0.9995405080
0.43	0.9995402100	1.000000000	0.9995402100
0.44	0.9995401500	1.000000000	0.9995401500
0.45	0.9995397930	1.000000000	0.9995397930
0.46	0.9995395540	1.000000000	0.9995395540
0.47	0.9995391960	1.000000000	0.9995391960
0.48	0.9995388390	1.000000000	0.9995388390
0.49	0.9995384810	1.000000000	0.9995384810
0.50	0.9995381830	1.000000000	0.9995381830
0.51	0.9995377660	1.000000000	0.9995377660
0.52	0.9995371100	1.000000000	0.9995371100
0.53	0.9995365140	1.000000000	0.9995365140
0.54	0.9995357990	1.000000000	0.9995357990
0.55	0.9995348450	1.000000000	0.9995348450
0.56	0.9995341300	1.000000000	0.9995341300
0.57	0.9995330570	1.000000000	0.9995330570
0.58	0.9995318060	1.000000000	0.9995318060
0.59	0.9995304350	1.000000000	0.9995304350
0.60	0.9995288250	1.000000000	0.9995288250
0.61	0.9995273950	1.000000000	0.9995273950
0.62	0.9995263810	1.000000000	0.9995263810
0.63	0.9995251300	1.000000000	0.9995251300
0.64	0.9995239380	1.000000000	0.9995239380
0.65	0.9995229840	1.000000000	0.9995229840
0.66	0.9995219110	1.000000000	0.9995219110
0.67	0.9995212550	1.000000000	0.9995212550
0.68	0.9995199440	1.000000000	0.9995199440
0.69	0.9995192290	1.000000000	0.9995192290
0.70	0.9995183350	1.000000000	0.9995183350
0.71	0.9995175600	1.000000000	0.9995175600
0.72	0.9995170830	1.000000000	0.9995170830
0.73	0.9995164280	1.000000000	0.9995164280
0.74	0.9995157120	1.000000000	0.9995157120
0.75	0.9995154740	1.000000000	0.9995154740
0.76	0.9995154740	1.000000000	0.9995154740
0.77	0.9995156530	1.000000000	0.9995156530
0.78	0.9995153550	1.000000000	0.9995153550
0.79	0.9995151760	1.000000000	0.9995151760
0.80	0.9995151760	1.000000000	0.9995151760
0.81	0.9995151760	1.000000000	0.9995151760
0.82	0.9995151760	1.000000000	0.9995151760
0.83	0.9995151760	1.000000000	0.9995151760
0.84	0.9995151760	1.000000000	0.9995151760
0.85	0.9995151760	1.000000000	0.9995151760
0.86	0.9995152350	1.000000000	0.9995152350
0.87	0.9995153550	1.000000000	0.9995153550
0.88	0.9995157120	1.000000000	0.9995157120
0.89	0.9995155330	1.000000000	0.9995155330
0.90	0.9995157720	1.000000000	0.9995157720
0.91	0.9995158310	1.000000000	0.9995158310
0.92	0.9995159510	1.000000000	0.9995159510
0.93	0.9995163680	1.000000000	0.9995163680
0.94	0.9995170240	1.000000000	0.9995170240

FILE: HEATEQ1 OUTPUT6 A1 UNIVERSITY OF ZAMBIA

0.95	0.9995176790	1.000000000	0.9995176790
0.96	0.9995182750	1.000000000	0.9995182750
0.97	0.9995189910	1.000000000	0.9995189910
0.98	0.9995197650	1.000000000	0.9995197650
0.99	0.9995202420	1.000000000	0.9995202420
1.00	0.9995208980	1.000000000	0.9995208980

FILE: HEATEQ1 OUTPUT5 A1 UNIVERSITY OF ZAMBIA

NUMERICAL SOLUTIONS

VALUES USED : M=100, N=1000, EPSILON=0.001, BI1=5.0,

BI2=0.0,

TIME= 0.80

Y	V(Y,TIME)	U(0, TIME)	TEMPERATURE
0.00	0.9875646830	1.000000000	0.9875646830
0.01	0.9893838760	1.000000000	0.9893838760
0.02	0.9909233450	1.000000000	0.9909233450
0.03	0.9922253490	1.000000000	0.9922253490
0.04	0.9933249950	1.000000000	0.9933249950
0.05	0.9942543510	1.000000000	0.9942543510
0.06	0.9950391050	1.000000000	0.9950391050
0.07	0.9957017300	1.000000000	0.9957017300
0.08	0.9962601660	1.000000000	0.9962601660
0.09	0.9967302680	1.000000000	0.9967302680
0.10	0.9971274140	1.000000000	0.9971274140
0.11	0.9974614380	1.000000000	0.9974614380
0.12	0.9977424740	1.000000000	0.9977424740
0.13	0.9979791640	1.000000000	0.9979791640
0.14	0.9981777670	1.000000000	0.9981777670
0.15	0.9983448980	1.000000000	0.9983448980
0.16	0.9984856250	1.000000000	0.9984856250
0.17	0.9986034630	1.000000000	0.9986034630
0.18	0.9987025860	1.000000000	0.9987025860
0.19	0.9987847810	1.000000000	0.9987847810
0.20	0.9988546370	1.000000000	0.9988546370
0.21	0.9989125730	1.000000000	0.9989125730
0.22	0.9989610310	1.000000000	0.9989610310
0.23	0.9990016220	1.000000000	0.9990016220
0.24	0.9990357760	1.000000000	0.9990357760
0.25	0.9990642070	1.000000000	0.9990642070
0.26	0.9990872740	1.000000000	0.9990872740
0.27	0.9991073010	1.000000000	0.9991073010
0.28	0.9991240500	1.000000000	0.9991240500
0.29	0.9991378190	1.000000000	0.9991378190
0.30	0.9991493230	1.000000000	0.9991493230
0.31	0.9991586800	1.000000000	0.9991586800
0.32	0.9991658930	1.000000000	0.9991658930
0.33	0.9991725680	1.000000000	0.9991725680
0.34	0.9991787080	1.000000000	0.9991787080
0.35	0.9991832970	1.000000000	0.9991832970
0.36	0.9991871120	1.000000000	0.9991871120
0.37	0.9991898540	1.000000000	0.9991898540
0.38	0.9991925360	1.000000000	0.9991925360
0.39	0.9991936680	1.000000000	0.9991936680

FILE: HEATEQ1 OUTPUT5 A1 UNIVERSITY OF ZAMBIA

0.40	0.9991950990	1.000000000	0.9991950990
0.41	0.9991958740	1.000000000	0.9991958740
0.42	0.9991968870	1.000000000	0.9991968870
0.43	0.9991971250	1.000000000	0.9991971250
0.44	0.9991980790	1.000000000	0.9991980790
0.45	0.9991985560	1.000000000	0.9991985560
0.46	0.9991995690	1.000000000	0.9991995690
0.47	0.9991998670	1.000000000	0.9991998670
0.48	0.9992002840	1.000000000	0.9992002840
0.49	0.9992002250	1.000000000	0.9992002250
0.50	0.9992004630	1.000000000	0.9992004630
0.51	0.9992004040	1.000000000	0.9992004040
0.52	0.9992006420	1.000000000	0.9992006420
0.53	0.9991998670	1.000000000	0.9991998670
0.54	0.9991999270	1.000000000	0.9991999270
0.55	0.9992001650	1.000000000	0.9992001650
0.56	0.9991998080	1.000000000	0.9991998080
0.57	0.9991996290	1.000000000	0.9991996290
0.58	0.9991995100	1.000000000	0.9991995100
0.59	0.9991994500	1.000000000	0.9991994500
0.60	0.9991993900	1.000000000	0.9991993900
0.61	0.9991993310	1.000000000	0.9991993310
0.62	0.9991992710	1.000000000	0.9991992710
0.63	0.9991991520	1.000000000	0.9991991520
0.64	0.9991996290	1.000000000	0.9991996290
0.65	0.9991993900	1.000000000	0.9991993900
0.66	0.9991999860	1.000000000	0.9991999860
0.67	0.9991999860	1.000000000	0.9991999860
0.68	0.9992001650	1.000000000	0.9992001650
0.69	0.9992005830	1.000000000	0.9992005830
0.70	0.9992005830	1.000000000	0.9992005830
0.71	0.9992010590	1.000000000	0.9992010590
0.72	0.9992012980	1.000000000	0.9992012980
0.73	0.9992014170	1.000000000	0.9992014170
0.74	0.9992014170	1.000000000	0.9992014170
0.75	0.9992012980	1.000000000	0.9992012980
0.76	0.9992010590	1.000000000	0.9992010590
0.77	0.9992013570	1.000000000	0.9992013570
0.78	0.9992022510	1.000000000	0.9992022510
0.79	0.9992026090	1.000000000	0.9992026090
0.80	0.9992033240	1.000000000	0.9992033240
0.81	0.9992038610	1.000000000	0.9992038610
0.82	0.9992043970	1.000000000	0.9992043970
0.83	0.9992050530	1.000000000	0.9992050530
0.84	0.9992052910	1.000000000	0.9992052910
0.85	0.9992058870	1.000000000	0.9992058870
0.86	0.9992063050	1.000000000	0.9992063050
0.87	0.9992067220	1.000000000	0.9992067220
0.88	0.9992065430	1.000000000	0.9992065430
0.89	0.9992071990	1.000000000	0.9992071990
0.90	0.9992080930	1.000000000	0.9992080930
0.91	0.9992096420	1.000000000	0.9992096420
0.92	0.9992100600	1.000000000	0.9992100600
0.93	0.9992102380	1.000000000	0.9992102380
0.94	0.9992102380	1.000000000	0.9992102380

FILE: HEATEQ1 OUTPUT5 A1 UNIVERSITY OF ZAMBIA

0.95	0.9992100600	1.000000000	0.9992100600
0.[illegible]	0.9992096420	1.000000000	0.9992096420
0.97	0.9992096420	1.000000000	0.9992096420
0.98	0.9992100000	1.000000000	0.9992100000
0.99	0.9992101190	1.000000000	0.9992101190
1.00	0.9992100600	1.000000000	0.9992100600

FILE: HEATEQ1 OUTPUT4 A1 UNIVERSITY OF ZAMBIA

NUMERICAL SOLUTIONS

VALUES USED : M=100, N=1000, EPSILON=0.001, BI1=5.0,

BI2=0.0,

TIME= 0.90

Y	V(Y,TIME)	U(0, TIME)	TEMPERATURE
0.00	0.9767837520	1.000000000	0.9767837520
0.01	0.9772706030	1.000000000	0.9772706030
0.02	0.9777451160	1.000000000	0.9777451160
0.03	0.9782071710	1.000000000	0.9782071710
0.04	0.9786595700	1.000000000	0.9786595700
0.05	0.9791029690	1.000000000	0.9791029690
0.06	0.9795331360	1.000000000	0.9795331360
0.07	0.9799556140	1.000000000	0.9799556140
0.08	0.9803711770	1.000000000	0.9803711770
0.09	0.9807730910	1.000000000	0.9807730910
0.10	0.9811680910	1.000000000	0.9811680910
0.11	0.9815526600	1.000000000	0.9815526600
0.12	0.9819297190	1.000000000	0.9819297190
0.13	0.9822952150	1.000000000	0.9822952150
0.14	0.9826530810	1.000000000	0.9826530810
0.15	0.9830043320	1.000000000	0.9830043320
0.16	0.9833436610	1.000000000	0.9833436610
0.17	0.9836780430	1.000000000	0.9836780430
0.18	0.9840031860	1.000000000	0.9840031860
0.19	0.9843227860	1.000000000	0.9843227860
0.20	0.9846329090	1.000000000	0.9846329090
0.21	0.9849373700	1.000000000	0.9849373700
0.22	0.9852312800	1.000000000	0.9852312800
0.23	0.9855194090	1.000000000	0.9855194090
0.24	0.9857997890	1.000000000	0.9857997890
0.25	0.9860734340	1.000000000	0.9860734340
0.26	0.9863368270	1.000000000	0.9863368270
0.27	0.9865960480	1.000000000	0.9865960480
0.28	0.9868498440	1.000000000	0.9868498440
0.29	0.9870994090	1.000000000	0.9870994090
0.30	0.9873387220	1.000000000	0.9873387220
0.31	0.9875739220	1.000000000	0.9875739220
0.32	0.9878050090	1.000000000	0.9878050090
0.33	0.9880266790	1.000000000	0.9880266790
0.34	0.9882451890	1.000000000	0.9882451890
0.35	0.9884614940	1.000000000	0.9884614940
0.36	0.9886687400	1.000000000	0.9886687400
0.37	0.9888708000	1.000000000	0.9888708000
0.38	0.9890695210	1.000000000	0.9890695210
0.39	0.9892591830	1.000000000	0.9892591830

FILE: HEATEQ1 OUTPUT4 A1 UNIVERSITY OF ZAMBIA

0.40	0.9894441370	1.000000000	0.9894441370
0.41	0.9896241430	1.000000000	0.9896241430
0.42	0.9898023610	1.000000000	0.9898023610
0.43	0.9899752140	1.000000000	0.9899752140
0.44	0.9901428820	1.000000000	0.9901428820
0.45	0.9903059600	1.000000000	0.9903059600
0.46	0.9904655810	1.000000000	0.9904655810
0.47	0.9906198980	1.000000000	0.9906198980
0.48	0.9907706380	1.000000000	0.9907706380
0.49	0.9909172060	1.000000000	0.9909172060
0.50	0.9910589460	1.000000000	0.9910589460
0.51	0.9911987190	1.000000000	0.9911987190
0.52	0.9913350340	1.000000000	0.9913350340
0.53	0.9914647940	1.000000000	0.9914647940
0.54	0.9915876390	1.000000000	0.9915876390
0.55	0.9917099480	1.000000000	0.9917099480
0.56	0.9918289780	1.000000000	0.9918289780
0.57	0.9919427630	1.000000000	0.9919427630
0.58	0.9920544030	1.000000000	0.9920544030
0.59	0.9921603200	1.000000000	0.9921603200
0.60	0.9922648670	1.000000000	0.9922648670
0.61	0.9923650620	1.000000000	0.9923650620
0.62	0.9924622180	1.000000000	0.9924622180
0.63	0.9925538300	1.000000000	0.9925538300
0.64	0.9926440720	1.000000000	0.9926440720
0.65	0.9927340750	1.000000000	0.9927340750
0.66	0.9928190110	1.000000000	0.9928190110
0.67	0.9928991790	1.000000000	0.9928991790
0.68	0.9929792280	1.000000000	0.9929792280
0.69	0.9930555820	1.000000000	0.9930555820
0.70	0.9931298490	1.000000000	0.9931298490
0.71	0.9932020900	1.000000000	0.9932020900
0.72	0.9932738540	1.000000000	0.9932738540
0.73	0.9933391210	1.000000000	0.9933391210
0.74	0.9934014680	1.000000000	0.9934014680
0.75	0.9934614900	1.000000000	0.9934614900
0.76	0.9935182330	1.000000000	0.9935182330
0.77	0.9935750960	1.000000000	0.9935750960
0.78	0.9936258200	1.000000000	0.9936258200
0.79	0.9936736820	1.000000000	0.9936736820
0.80	0.9937204720	1.000000000	0.9937204720
0.81	0.9937670230	1.000000000	0.9937670230
0.82	0.9938069580	1.000000000	0.9938069580
0.83	0.9938448670	1.000000000	0.9938448670
0.84	0.9938799740	1.000000000	0.9938799740
0.85	0.9939145450	1.000000000	0.9939145450
0.86	0.9939477440	1.000000000	0.9939477440
0.87	0.9939817790	1.000000000	0.9939817790
0.88	0.9940121170	1.000000000	0.9940121170
0.89	0.9940387010	1.000000000	0.9940387010
0.90	0.9940628410	1.000000000	0.9940628410
0.91	0.9940865040	1.000000000	0.9940865040
0.92	0.9941079020	1.000000000	0.9941079020
0.93	0.9941253070	1.000000000	0.9941253070
0.94	0.9941397910	1.000000000	0.9941397910

FILE: HEATEW1 OUTPUT4 A1 UNIVERSITY OF ZAMBIA

0.95	0.9941523670	1.000000000	0.9941523670
0.96	0.9941655990	1.000000000	0.9941655990
0.97	0.9941752550	1.000000000	0.9941752550
0.98	0.9941867590	1.000000000	0.9941867590
0.99	0.9941906930	1.000000000	0.9941906930
1.00	0.9941915870	1.000000000	0.9941915870

FILE: HEATEQ1 OUTPUT3 A1 UNIVERSITY OF ZAMBIA

NUMERICAL SOLUTIONS

VALUES USED : M=100, N=1000, EPSILON=0.002, BI1=5.0, BI2=0.0,

TIME	TEMPERATURE
0.10	1.000000000
0.20	1.000000000
0.30	1.000000000
0.40	1.000000000
0.50	0.9939053650
0.60	0.9888939860
0.70	0.9840772150
0.80	0.9768142100
0.90	0.9581648710

FILE: HEATEQ1 OUTPUT2 A1 UNIVERSITY OF ZAMBIA

NUMERICAL SOLUTIONS

VALUES USED : M=100, N=1000, EPSILON=0.001, BI1=5.0,
BI2=0.0,

TIME	TEMPERATURE
0.10	1.000000000
0.20	1.000000000
0.30	1.000000000
0.40	1.000000000
0.50	1.000000000
0.60	0.9961150290
0.70	0.9920684700
0.80	0.9875646830
0.90	0.9767837520

FILE: HEATEQ1 OUTPUT1 A1 UNIVERSITY OF ZAMBIA

NUMERICAL SOLUTIONS

VALUES USED : M=100, N=1000, EPSILON=0.004, BI1=5.0,
BI2=0.0,

TIME	TEMPERATURE
0.10	1.000000000
0.20	1.000000000
0.30	0.9965170620
0.40	0.9884685280
0.50	0.9824169280
0.60	0.9767506720
0.70	0.9693863990
0.80	0.9576728940
0.90	0.9167167540

REFERENCES

1. EDNERAL, F, P. Electrometallurgy of Steel and Ferro-alloys, Vol. 2, Mir Publishers, Moscow (1979).

2. EGERTON, P., HOWARTH, J.A., POOTS, G. and TAYLOR-REED, S. A Theoretical Investigation of Heat Transfer in Ladle of Molten Steel During Pouring, Int. J. Heat Mass Transfer Vol. 22. Pergamon Press Ltd, 1525-1532 (1979).

3. AMERICAN SOCIETY FOR METALS, Metals Handbook, Forging anc Casting, Vol. 5, Metals Park, Ohio 44073, (1970).

4. FRANCIS J.R.D., Fluid Mechanics for Engineering Students, Edwarc Andre (Publishers) Ltd (1975).

5. CARSLAW, H.S. and JAEGER, J,C. Conduction of Heat in Solids, enc ed. Clarendon Press, Oxford (1986).

6. SMITH, G,D. Solutions of Partial Differential Equations, Oxforc University Press, Oxford (1965).

7. HUNSAKER, J,C, and RIGHTMIRE, B,G. Engineering Application of Fluid Mechanics, McGraw-hill Book Company, Inc. New York (1974).

8. VENNARD, J,K. Elementary Fluid Mechanics, 4th ed, John Wiley & Sons, Inc. (1961).

9. FEYNMAN, R,P., LEIGHTON, R, B., and SANDS, M. The Feymar Lectures on Physics, Addison-Wesley Publishing Company, Inc. London (1964).

10. GREENSPAN, D. Discrete Numerical Methods in Physics anc Engineering, Academic Press, Inc. London (1974).

11. DAFFY, D,G. Solutions of Partial Differential Equations I Books, Inc. USA (1986).

12. BUGGY, K., BYCHKOV, Y., KANDVALOV, Y., KOVALENKO, V. anc TRETYAKOV, E. Iron and Steel Production, Mir Publishers, Moscow (1971).

13. STEPHEN, G. Partial Differential Equations for Scientists anc Engineers, Longman, London (1985).

14. BURDEN, R, L., FAIRES, J.D., and REYNOLDS, A, C. Numerical Analysis, Prince, Weber & Schmidt, Boston (1978).