USE OF CONSTRUCTIVE SOCIAL ACTIVITY MODELS WHEN LEARNING PROBABILITY IN SELECTED SECONDARY SCHOOLS OF MBALA DISTRICT-NORTHERN PROVINCE

BY

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DECLARATION

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APPROVAL

This Dissertation of **Doye Chikola** has been approved as fulfilling the requirements for the award of the Degree of Master of Education in Mathematics Education (M. Ed-Mathematics Education) by the University of Zambia.

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ABSTRACT

This study was based on, "Use of constructive social activity models when learning Probability in selected secondary schools of Mbala District." The study focused on the use of observable concrete activity tools (constructive social activity models) attributed to real life practices that call for learner participation and influence learner's own knowledge construction leading to understanding. The purpose of the study was to explore how learners use constructive social activity models when learning probability concepts as well as the assistance given to them by their teachers in clarifying concepts from the model perspective. The study used a qualitative approach in its data collection and analysis proceedures. The sample size comprised of ten(10) grade eleven(11) learners and two(2) grade 11 teachers who were sampled purposively. Data was collected through semi-structured interviews, lesson observations, focus group discussions and document analysis and was analysed thematically. The findings revealed that only the learners in one of the two sampled schools used a dice, coin and marbles as concrete tools although when it came to the aspect of conceptual mathematisation in the ZPD, some concepts were mis-interpreted by both teachers and learners. It was established that teachers were unable to appropriately link the abstract concepts with the constructive social activity models. Moreover, learners were not engaged fully in their own learning by use of concrete tools, teachers did not adequately probe learners in order to determine their prior knowledge on modelling tools and did not correct errors or even allow learners justify their thinking on written tasks. Learners were not given chance to come up with their own modelling tools as it was established by this research study in the sampled schools. Generally, it was concluded that there was no appropriate use of concrete constructive social activity models and even when the dice, coin, and marbles were used, their link to abstract concepts in probability was misinterpreted and the process of learning in one of the two schools was purely abstract without use of any concrete tools. Therefore, it is recommended that, school based continuous professional development activities could be strengthened on the use of constructive social activity models when learning probability concepts for conceptual relevance of learners' diverse real-life experiences.

Keywords: *Probability, use, constructive social activity models, learning, conceptual mathematisation, conceptual relevance*.

DEDICATION

This dissertation is dedicated to my two daughters (NGOSA & ENECHELA) and my husband (GEORGE MARTIN CHIBUYE) who granted me the time to come and pursue this course, leaving behind my duties of a wife and a mother. With their insightful vision, patience and understanding, I have accomplished the dream!

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ABBREVIATIONS AND ACRONYMS

AIEMS	Action to Improve English, Mathematics and Science
CRUM	Computational Representational Understanding of Mind
DEBS	District Education Board Secretaries
ECZ	Examination Council of Zambia
EV	Expected Value
FGD	Focus group Discussion
JICA	Japan International Cooperation Agency
MESVTEE	Ministry of Education, Science, Vocational Training and Early Education
NAS	National Assessment Surveys of Learning Achievement
PISA	Program for International Student Achievement
RME	Realistic Mathematics Education
SACMEQ	Southern and Eastern Africa Consortium of Monitoring Education quality
SMASTE	Strengthening of Mathematics, Science and Technology Education
STEM	Science, Technology, Engineering and Mathematics
SNDP	Seventh National Development Plan
ZAMSTEP	Zambia Mathematics and Science Teachers Education Project
ZECF	Zambian Education Curriculum Framework
ZEEP	Zambia Education Enhancement Project
ZPD	Zone of Proximal Development

CHAPTER ONE

INTRODUCTION

1.1 Overview

The first chapter presents the background to the study, statement of the problem, purpose of the study, study objectives, research questions, significance of the study, scope of the study (delimitations and limitations), theoretical framework and operational definitions of key terms.

1.2 Background of the study

There are different perspectives that motivated the researcher to carry out this study. These are; Curriculum revision and localisation by the Ministry of Education, Science, Vocational Training and Early Education (MESVTEE), STEM education, poor learner performance in mathematics and the Baseline Survey Report (Examination Council of Zambia ECZ; 2018), the process of knowledge acquisition being constrained as elaborated by Hatano (1996), Modeling tools as mediators in cognitive reorganization, and the aspect of conceptual mathematisation as described by De Lange (1987), just to mention but a few. These aspects are now discussed in detail.

1.2.1 Curriculum revision and localisation.

The Ministry of Education, Science, Vocational Training and Early Education (MESVTEE) implemented a competency-based localised curriculum in 2014. The Curriculum emphasizes Mathematics and Science in supporting Zambia's aspirations of becoming a middle-income country by 2030 apart from other higher investments. In order for learners to acquire the needed long-life skills and competencies as desired by the curriculum, it emphasised on the integration of indigenous knowledge, values and practical skills that leaners would have acquired in their home environment that act as a basis for formal learning of scientific concepts (ZECF 2013). The localisation of a competence based curriculum implied anticipating for higher performance achievement results for learners, which would emanate from the classroom use of real-life applications and concrete tools described as constructive social activity models in the process of learning scientific concepts, mathematics in particular, (ZECF 2013; ECZ, 2018). This is because

Mathematics curriculum must be an extension of society. The social background and classroom learning conditions should provide the context within which to interpret the learners' use of many concrete representations from real life experiences. Thus, the Zambian Education Curriculum Framework (ZECF) stipulates that all teachers and teacher educators must use methods that promote active learner participation and interaction to cater for learners' range of needs and experiences. They should take into account the available local resources from which learners can draw conceptual understanding of the subject matter (MESVTEE, 2013). Therefore, a lot of questions were raised as to what kind of local resources would be used in the learning of scientific concepts that are written objectively in text books such that learners would accurately draw conceptual understanding of scientific concepts, hence the interest to carry out the study.

Furthermore, with reference to the argument raised by Cobb (1991) and Gravemeijer (1994) demand that learners must be actively engaged rather than being passive and they should work on meaningful (constructive) social activity projects or models to answer questions of interest to them, the researcher was curious to check on how this could be achieved. The process of knowledge construction is also seen to be critical to scholars such as Bruning, Schraw, Norby, and Ronning (2004) who identified the following set of cognitive themes that resonate with integration of indigenous knowledge in the learning process: Learning is constructive, not a receptive, process. Motivation and beliefs are integral to cognition, Social interaction is fundamental to cognitive development, and Knowledge, strategies, and expertise are contextual. Thus, the curriculum aspirations and demands motivated the researcher to develop insight into use of constructive social activity models learners use when learning Mathematical concepts particularly probability. The use of local resources from learners' real life and integration of indigenous knowledge in bridging the gap that exists between scientific concepts written in textbooks and those spontaneous concepts that learners would have developed from their social background, is cardinal to the ZECF of 2013. Hence the title, 'use of constructive social activity models' learners use when learning Probability in selected secondary schools in Mbala district.'

Another aspect of the curriculum revision which inspired the researcher to conduct this study, was the inclusion of topics such as computer and calculator, probability, and matrices to the junior secondary school syllabus. Calculus, arithmetic and geometric progression, computer, composite functions, inverse functions, graphs of cubic functions, standard deviation in statistics, and trigonometric equations to the senior secondary school syllabus (ECZ, 2016; MESVTEE, 2013). This was done in order to increase learner's lifelong competences, through continuity or progression in knowledge strands or domains and coherency (Allfrey, 2001; Chernoff, 2012; MESVTEE, 2013). Thus, the need to build up progression in knowledge strands is as a result of having realised the significance of those topics among other things. For instance, Probability concepts are fundamental to the life of a child and as such its significance was realised and cannot be overemphasized. It is practical and applicable to all fields of knowledge in a real world. It is virtually ubiquitous and so the researcher was motivated to check on how those learners in grade 11 who would have progressively built up the knowledge strands in probability concepts at junior secondary from a model perspective would be able to relate it to other real-life experiences at senior secondary when they are almost graduating (Brase, 2012; ZECF, 2013)

Furthermore, the importance of Probability to real-life and human activities motivated the researcher to make an informed decision and chose it as the area of study in the context of this research study. De Jager (1966) argues that there is no topic in Mathematics that is so important to a child in his present everyday life and in adult life experiences as probability and so its inclusion to Junior Secondary School Curriculum is justified. Probability concepts underpins almost all the sciences and social sciences, such as its prevalent use in statistical testing, confidence intervals, regression methods, music information retrieval, style identification (Chai & Vercoe, 2001) and transcription (Klapuri & Davy, 2006), Technology, Industry, Business, Finance, Economics, Sociology, Psychology, Education and Medicine just to name but a few. The very elements of uncertainty embedded in the concept of probability are applicable to; costs, availability of raw materials, labour, production, reliability of components, safety issues, interests, markets, development of new drugs, trials, outcomes of an election and games of chance (Banda, 2012; Brase 2012; Mendnhall et al 1986; Newbold et al 2010).

Therefore, developing Probability concepts at early age would help create independent thinkers with skills that would answer to many forms of life experiences of present and future (Redclift, 1987). In addition, learners' life experiences as well as adult life revolve around chance, the decisions, choices are based on life experiences and predict the future phenomena or occurrences, and incorrect decisions can ruin one's life. Winning any game or election, going to school, recovering from an illness, selling a product and realising profit and many more happen by chance. The interpretation of probability is one of the most important such foundational problems (Brase, 2012). Thus, with the importance of probability, and curriculum review, the researcher was motivated and curious to check the process of knowledge construction of probability concepts with linkage to a real world of learners' own experiences in a natural setting, by use of constructive social activity models or tools.

1.2.2 STEM education

The need to champion the implementation of STEM Education in Zambian schools is a current concern by all educationists. Dugger (2016, p.5) stipulates that STEM education is "the integration of science, technology, engineering, and mathematics into a new transdisciplinary subject in schools." These fields of STEM are deemed essential components of education curriculum because of their significance in society, they are major drivers of any nation's economy, hence the need to implement them in the Zambian Education Curriculum and to be taught in an integrative manner and with approach to real world. Torlakson's (2014, p.7) definition of STEM education, "is much more than a convenient integration of science, technology, engineering, and mathematics; it is an interdisciplinary and applied approach that is coupled with real-world, problem-based learning. STEM education integrates the four disciplines through cohesive and active teaching and learning approaches" (Bybee, 2010; Dugger, 2016; Sanders, 2009).

Drawing from the key defining terms of STEM in which Mathematics is also coined, the significance of STEM lies in not only approaching the subject content matter integratively, but also with real-world, problem-based learning and social cohesion. A real world is a world of continuous interaction with physical, social phenomena that encompasses indigenous knowledge, life experiences of a learner in the context of own culture of which is in line with the demands of the ZECF of 2013. Since Mathematics is also concerned with practical problem solving of daily lives, it implies intellectual imagery interpretation and representation of that real world of such things as situations, space, time, shapes (geometry), classifications, numbers, motions, social relations (budgeting, profit/loss), just to name a few. For this reason, the researcher was concerned with how learners would be equipped with concepts that are abstract in an integrative manner from their real-life experiences by use of concrete situations or tools which the study calls 'the constructive social activity models.' Questions such as how do learners from a classroom learning process develop skills needed to interpret and analyse information, simplify and solve real life problems, assess risks, make informed decisions and further understand the world around through modelling in both abstract and concrete problems? Hence the title, 'Use of constructive social activity models when learning probability in selected secondary schools' (Weldon & Mitchell, 2018).

1.2.3 Poor learner performance and the Baseline Survey Report

The government of the Republic of Zambia has over the years actively engaged in various activities to support the teaching and learning of Mathematics and Science in Zambia as they are seen to be the most critical subjects in terms of concept acquisition and delivery. Various programs or interventions such as Zambia Mathematics and Science Teachers Education Project (ZAMSTEP), Action to Improve English, Mathematics and Science (AIEMS), and Strengthening of Mathematics, Science and Technology Education (SMASTE) among others were initiated. Carmody (2004), states that ZAMSTEP was a programme initiated in 1980 meant to equip the diploma holders with pedagogical knowledge so that they could confidently and efficiently teach mathematics and science in secondary schools. This intervention was mainly concerned with improving the instructive skills and knowledge of teachers. In trying to ensure that the country has quality mathematics and science teachers, the government through the Ministry of Education Science, Vocational Training and Early Education (MESVTEE) sponsored Mathematics teachers among others for further studies through Fast Track Teacher Education Programme in 2012 who are now graduates. Furthermore, a collaborative workforce with Japan International Cooperation Agency (JICA) has provided technical cooperation in improving the quality of mathematics and science education in Africa for about half a century. They are engaged in improving teaching and learning approaches especially in mathematics and science. They also offer a sustainable system for continuous in-service teacher education and training (Takahashi, et al 2017).

However, learners have continued performing poorly but they are not blank slates (Ernest, 1991). It could be that their experiences, misconceptions and conceptions about the world are diverse, subjective and continuous as compared to the formal concepts that are objective independent entities and universally agreed upon and written in textbooks which may constrain knowledge construction as they link to real world. It could also be that even when teachers are well trained with current skills but what and how they use the concrete situations to link abstract concepts in mathematics with learners own cultural experiences in order to challenge learners own misconception, is a challenge. Jaworski (1989) argues that a teacher may have a clear, coherent mathematical story that she/he wishes to convey to learners, and she may provide them with a good explanation as it is. Nevertheless, it is only good for learners if it fits their experience and needs. Mason (1987) also states that experience is fragmentary –learners come to the mathematics classroom with a diversity of experiences to which they attempt to relate what they do and hear, constructing stories which form the basis of their understanding. Mathematics meaning varies from one individual to another and communication depends upon the ability to share each other's meanings. Now, assuming that, all teachers were equipped with instructive skills and formal content knowledge, but the question which still remained unanswered was how do teachers assist learners from different social backgrounds integrate particular scientific concepts to learners' indigenous knowledge, experiences from a model perspective so that learners find meaning and appreciate scientific concepts in real life? Thus, the researcher was motivated to conduct a research and check how the concepts in Probability that are embedded in textbooks are negotiated and communicated to learners by use of concrete models (constructive social activity models) in the actual classroom learning process, among the grade 11 learners.

Furthermore, it is reported that the Government of the Republic of Zambia also requested technical and financial support from the World Bank for the design and implementation of programs to improve the quality of education in the context of its Seventh National Development Plan (SNDP). This resulted in formation of the Zambia Education Enhancement Project (ZEEP), which rides on the World Bank's country partnership strategy that focusses on reducing poverty and inequality in the country. The project aims at ensuring that teacher education system is feedback basis and that teachers have the necessary content knowledge, competencies and skills in Mathematics and Science. However, during the Baseline Survey for 2018, it was discovered that teachers lack various forms of representations to link mathematical concepts due to low competence and their limited views on mathematics lessons (ECZ, 2018). Unfortunately, the Diagnosis Report based on the 2018 ZEEP Baseline Survey reviewed that Probability was among other topics in Mathematics where both teachers and learners had difficulties that included the learners' learning difficulties and teacher's content limitations (ECZ, 2018). This motivated the researcher to carry out the study specifically developing insight on the use of constructive social activity models that are used in learning probability concepts. Some models of probability concepts are embedded in word problems, others may be provided by the teacher or the learners. Therefore, the purpose of this study was to develop insight into constructive social activity models that learners use when learning probability concepts as well as the assistance given to learners by their teachers in clarifying concepts from the model perspective in which various issues had to unfold.

Despite the relatively high levels of public investment in the education sector by government, learner performance in Mathematics and Science has been persistently low as demonstrated by the National Assessment Surveys of Learning Achievement (NAS), Southern and Eastern Africa Consortium of Monitoring Education quality (SACMEQ) and Program for International Student Achievement (PISA) for Development Studies.

According to ECZ (2017, p.27), "Mathematics is still posing a challenge to candidates even under the revised curriculum. It had the highest proportion of failure rate at 41.33 percent in 2016." It was also consistently noted in examiners' reports that the performance of many candidates in mathematics was generally poor and some candidates scored zero at grade 9 and grade 12 final examinations (Examinations Council of Zambia, 2015, 2016, and 2017). For instance, in Probability which also contributes about 8 to 15% at grade 9 and 12 of the final examination in mathematics (ECZ 2017, 2018), the Chief Examiner (2017, p. 59) reported, "Learners could draw correct tree diagrams but failed to indicate correctly the probabilities when the 2^{nd} or 3^{rd} selection is made after no replacement." Hence, the title, 'Use of constructive social activity models when learning probability in selected secondary schools, particularly in Mbala district with the following evidenced statistics on learner performance in the table below;

Table 1: *Examination results for school certificate in mathematics (2017-2019) comparison between Northern Province and Mbala district*

		2017	**2018**	**2019**
Distinction	Northern Province	13.3%	9.8%	9.6%
	Mbala District	10.1%	4.7%	7.3%
Merit	Northern Province	18.2%	13.9%	13.7%
	Mbala District	19.1%	10.0%	11.3%
Credit	Northern Province	21.7%	20.7%	17.5%
	Mbala District	19.3%	18.3%	13.4%
Pass	Northern Province	14.3%	16.9%	16.6%
	Mbala District	11.2%	16.0%	14.7%
Fail	Northern Province	32.6%	38.7%	42.5%
	Mbala District	40.3%	51.0%	53.3%

		2017	**2018**	**2019**
Distinction to Pass	Northern Province	67.4%	61.3%	57.5%
	Mbala District	59.7%	49.0%	46.7%

Source: *ECZ, 2017-2019*

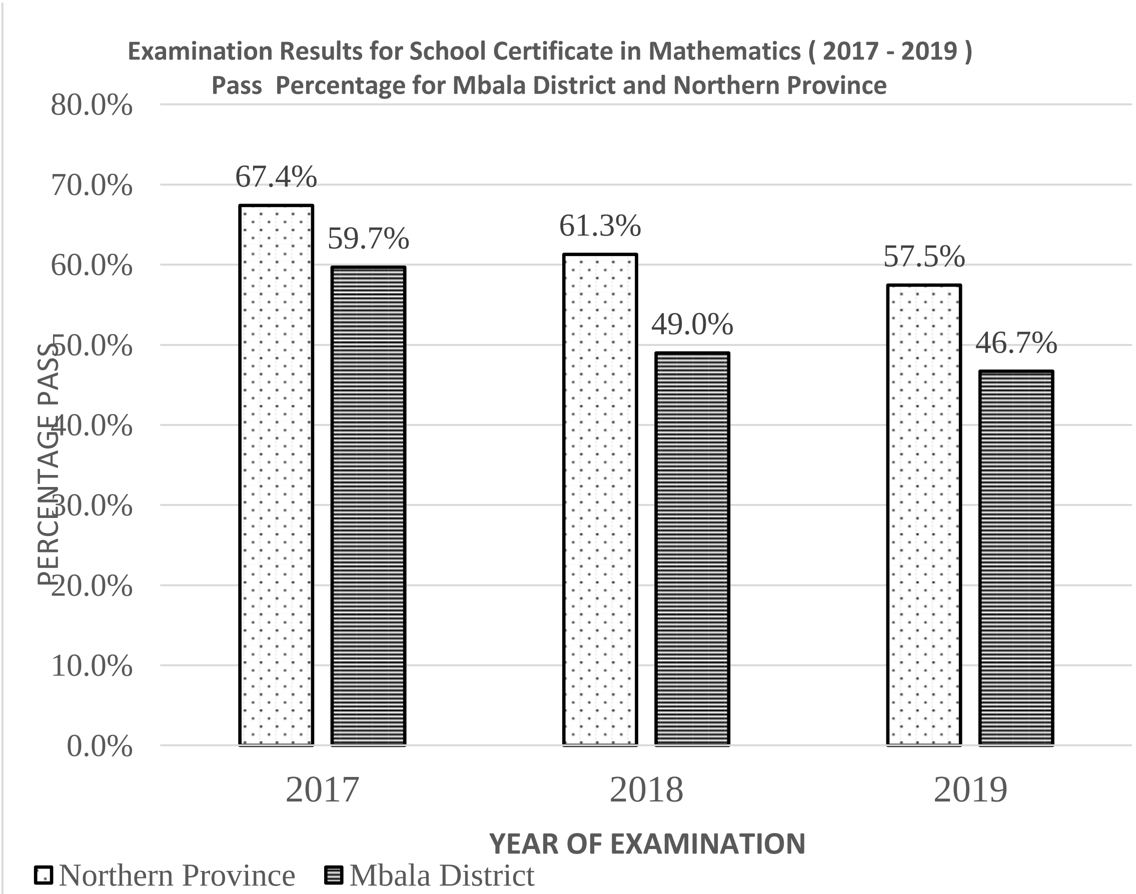


Figure 1: *School certificate in Mathematics (2017-2019) pass percentage for Mbala District and Northern Province. Source: ECZ, 2017-2019*

The pass rate in Table 1 and Figure 1 for Mbala District has been consistently below average pass rate for Northern Province for the period indicated in both sections. This simply showed that the performance in Mathematics at School Certificate was consistently below the Provincial Average Performance for the past years. This prompted the researcher to look into one of the possible causes of the poor performance in Mbala District in Northern Province.

1.2.4 The process of knowledge acquisition being constrained

The process of knowledge acquisition being constrained is among the five characterisations of knowledge acquisition processes as elaborated by Hatano from a constructivist lens which was also a basis for this study. The following piece of illustration outlines these characterisations as of the five issues of current concern in mathematics education that was elaborated by Hatano (1996); A class of 12 years old children saw a given trapezium differently. One boy said the figure was a square and the teacher gave him chance to explain why he thought so. According to Jaworski (1989), the teacher looked puzzled, as if he could not see a square either. He invited the boy to come out to the board to explain his square. The teacher in the illustration found it hard to see a square but he was prepared to let the boy explain, and from the boy's stumbling explanation the teacher and the class came to see the figure as if it were a square (Jaworski, 1989). The process of knowledge construction being constrained here is that there are many factors which may foster understanding or non-understanding as learners view objects differently. Among these factors which influence positively or negatively in the process of knowledge acquisition are; Innate or Early cognitive constraints; Prior knowledge as cognitive constraint; Shared artifacts; and Social constraints that results from interactions with seniors and peers. Therefore, this research focused on the use of shared artifacts in which the tools or objects of a culture are developed and shared among generations particularly in the formal learning process. How learners use kits or tools what tools challenge or constrain the link, or mediate, exemplify and clarify particular scientific concepts that are written in textbooks was the researcher's motivation to carry out the research. This was in order to establish what tools (constructive social activity models) learners use when learning scientific concepts in which the researcher preferred to look at probability concepts as stated earlier.

1.2.5 Modeling tools as mediators in cognitive reorganisation

The use of concrete modeling tools (constructive social activity models) in the learning process is cardinal for a leaner to be able to make sense of the world around them. According to De Lange (1987) the process of extracting the appropriate mathematical concept from a concrete situation is described as conceptual mathematisation. Therefore, a learning situation should not aim at transmitting all abstract concepts as fixed as they are, well defined independent entities that can only be explored in prototypical examples and exercises in reference books. This is because neither such exemplification can provide the important insights into the culture nor the authentic activities of that culture where learners' daily needs are met (Baise, 2013; Carraher & Schliemann 1985; Dowd, 2013; Lave, 1988; Saxe, 1991; Scribner, 1984). Thus, this research explored the contextual, structured, designed social activities from learners' diverse real-life experiences that follow mathematical structures or patterns which may be used in drawing conceptual understanding of probability (Freudental, 1991). It was assumed that either teachers or learners would set models in relation to particular task that would influence action in order to reflect the continuing practical reality of Probability and fulfil the demands of the curriculum. In so doing, learners would learn-long life skills and retain more as they use constructive social activity models that require them to think and process information rather than to passively listen to the teacher (Basu and Debnath 2015, Cobb 1994, Konold, 1991).

1.3 Summary

In conclusion, poor performance, the revised curriculum aspirations and the government efforts outlined above were a motivation to conduct the study in Mbala district. This is due to the district having portrayed the poorest performance ranking consistently as reviewed by the Examination council of Zambia reports, (ECZ 2014-2018). In addition, the examiners' reports that provide feedback to teachers, learners, policy makers, curriculum designers and other stakeholders on learners' performance in the examination recommended on how issues of poor performance may be addressed. One major pronouncement was that, learners lacked real life applications of concepts and teachers needed to provide concrete objects and physical manipulation of modelling tools from their environment in classroom learning for conceptual relevance (ECZ, 2014). Therefore, the questions that came to the researcher's mind was if the performance of learners is still low despite all government efforts, could it also be that the ways in which the scientific concepts are written in books may constrain knowledge construction with the use of concrete objects as mediators? What constructive social activity models do learners use as they learn scientific concepts, for instance Probability? Since probability is practical in nature and more of real-life situations, then how do the use of constructive social activities model tools foster conceptual understanding from learners' real world and their own experiences? Thus, the title, ‘use of constructive social activity models when learning Probability in selected secondary schools in Mbala district.’

1.4 Statement of the problem

Mathematics is an essential field in education curriculum as its components equip humans with the skills needed to interpret and analyse information, simplify and solve problems, assess risks, make informed decisions and further understand the world around through modelling both abstract and concrete problems (Timms, Moyle, Weldon & Mitchell, 2018). However, poor performance in mathematics has long been recognised and probability is one of the topics where learners have learning difficulties and teachers have content limitations (ECZ; 2016, 2017, 2018). In order to increase learners’ long-life competencies, the Ministry of General Education calls for all educationists to ensure that learning involves integrative approaches and use of concrete models (ZECF, 2013). Using integrative approaches and constructive social activity models in particular, involves constructing an intellectual imagery interpretation and representation of that real world to represent that which is in abstract during the learning process, while taking into account learners’ diverse social background. Therefore, learning concepts in Probability with reference to diverse learners’ real-life experiences and use of appropriate constructive social activity models would help learners acquire well-founded personal attributes in decision making by being independent thinkers and apply it to other fields of knowledge. Studies done concerning the teaching and learning of probability did not focus on the use of constructive social activity models when learning probability (Batanero & Díaz, 2012; Brase, 2012; De Kock, 2015; Dowd, 2013; Garfield & Ahlgren, 1988; Newbold et al 2010). Therefore, this study explored how learners use constructive social activity models when learning probability in Mbala District.

1.5 Purpose of the study

The purpose of the study was to explore the use of constructive social activity models when learning probability concepts, how learners use the models to draw clarity of concepts and the assistance given to them by their teachers in clarifying concepts from the model perspective.

1.6 Study Objectives

The objectives of this study were to:

i. Explore the constructive social activity models that learners use when learning probability concepts.

ii. Ascertain how learners use the constructive social activity models in drawing clarity for conceptual understanding.

iii. Assess the ways in which learners are assisted by the teachers in understanding the probability concepts from the constructive social activity models perspective.

1.7 Research questions

The research questions that guided the study included the following:

i. What constructive social activity models do learners use when learning probability concepts?

ii. How do learners engage in the use of constructive social activity models for conceptual understanding of Probability?

iii. How do teachers assist learners in enhancing the clarity of concepts in Probability from the constructive social activity models perspective?

1.8 Significance of the study

The study focused on exploring the 'use of constructive social activity models or tools when learning probability concepts.' Its significance lies in the fact that the study emphasizes the use of mediated activity tools within a particular social historical context and how the use of appropriate tools (models) in a social setting addresses the relationship between the mathematical concepts of probability in relation to their different cultural context in which learners exist. Thus, the teachers, Resource Centre personnel, Senior Education Standard Officers the District Education Board Secretaries (DEBS), Examination Council of Zambia (ECZ), teacher educators, the National Science Centre and curriculum designers could pay attention to particular concepts embedded in probability and how they develop in relation to their appropriateness in constructive social activity models from learners' real life experiences/ background and include in content for conceptual change. This is in order to provide the conditions under which effective learning takes place, active learning with hands on activities as learners share different interpretation of the constructive activity modelling tools on a social plane during the learning process of probability concepts, would increase knowledge retention.

Thus the study would inform policy makers on the effectiveness of what aspects of teaching aids to consider and review when making educational policies regarding teachers' pedagogical and social skills, learners own culture as well as STEM education. The findings could contribute significantly to research in mathematics education on various perceptions and perspectives on modelling tools that could be embedded in Mathematics content with their time of relevance locally, globally and would eventually contribute to improvement of learner performance.

1.9 Scope of the study

The scope of the study includes delimitation and the limitations of this study. These are discussed in the subsections below.

1.9.1 Delimitation

The study was carried out in Mbala District in Northern Province and was restricted to two government secondary schools because the schools in the district have poorest performance ranking reviewed consistently in the Examination Council of Zambia reports, (ECZ 2014- 2017). In addition, in order to have an in-depth understanding and accurate information on the constructive social activity model tools learners use when learning probability, the study was restricted to only learners who were learning Probability at that time and who had successfully completed junior secondary education and who were in grade 11. This is how the study was narrowed in scope (Creswell, 1994).

1.9.2 Limitations

There could be several limitations which may have affected the study's outcome. Limitations are the potential weaknesses in the study which are out of control of the researcher (Simon, 2011). Therefore, this section discusses factors that affected the study and which could have made it more informing than it may be.

- The theoretical framework: This is a limiting mirror of focus as it precedes the cognitive theorists; in this study, the theoretical framework (social cultural theory) posits that cognitive development is as a result of social construction of knowledge through the use of tools, objects of contextual relevance on a social plane (Wertsch, 1985). Using the cognitive theories or any other would have enriched this study as it would have helped to analyse the individual learners' mental process through test scores. Even so, the researcher's focus relied on the exploration of how those models or tools in the learners' reference books or otherwise (coin, jack port, dice, spinning wheels, deck of playing cards, marbles, chess etc.), in the context (learners' culture) and the existing particular knowledge concepts of probability were linked in the learning process. Therefore, the process of developing robust knowledge while manipulating model tools was more important in this study than the product of a correct or a wrong answer through mental processes.

- Absence of certain variables such as test scores determined through individual mental activity processing in independent problem solving would have reviewed whether or not there is a significant difference in performance as a result of either or not engaging in constructive social activity models. However, the constructive social activity models which are already embedded in the Mathematics learners' books or otherwise needed to be checked on how they are used, their linkage, culturally bound, consistency and their appropriateness right in their actual classroom learning process of developing probability concepts.
- Considering the fact that the study used qualitative study design methodologies which are subjective in nature, it may have resulted in biases due to focusing on objects of interest overlooking those that could have been of value and more informing especially on the choice of the study participants. However, the researcher was guided by the focus of the study and study objectives.
- The study sample in this study was small hence cannot be used to generalise the findings. However, this cannot overshadow the fact that the findings were informative and could easily provoke further research on a larger scale within Zambia and elsewhere.

1.10 Theoretical framework

The study is anchored on Vygotsky's (1978) social cultural theory and his notion of the Zone of Proximal Development (ZPD). Firstly, the social cultural theory of constructivism posits that knowledge construction emanates as a result of learner's personal life experiences and social interaction with physical phenomenon or tools in their daily lives as members of a culture (Vygostky 1979). Vygotsky places more emphasis on the social environment as a facilitator of development and learning (Schunk, 2012). Social Constructive Theory which is Vygotsky's approach stated that the human mind is constructed through a subject's interactions with the world and is an attribute of the relationship between subject and objects (Verenikina, 2010). The objects in this study are called constructive social activity models and the subjects are the teachers and the leaners including the learning material embedded in textbooks which leaners interact with.

In his sense, Vygotsky finds a significant role in humans' understanding of the world and of themselves and their understanding is attributed to 'tools' which the study calls constructive social activity models (Turuk, 2008). Furthermore, Vygotsky advocates that humans do not act directly on the physical world without the intermediary of tools (constructive social activity models). These tools can be any artifacts, whether symbolic or signs, created by human under specific cultural and historical conditions carrying with them the characteristics of the culture in question (Turuk, 2008). Hence, the theory views mathematics learning as a social activity of human creation of which in all its structures, rules, postulations, laws, formulas, theories and symbols have reference to the physical world of reality from which they are created (Cobb 1994; Freudental, 1991; Wertsch, 1984).

The social cultural theory backs this study because Karpov and Haywood (1998) and Moll (2001) believe that Vygotsky's ideas lend themselves to many educational applications marking the field of self-regulation as the strongly influenced (Schunk, 2012). One of such ideas in applications reflecting Vygotsky's theory is reciprocal teaching which involves interactive dialogues between teacher and small group of learners. It informs this study well in that firstly, the teacher models the activities after which, learners take turns in being the teacher and lead the discussions. Secondly, during performance on a task setting, learners are encouraged to ask questions, in order to determine their level of understanding. Implying also that a teacher can include a question-asking strategy in the instructional sequence. Thus, learners gradually develop skills, as reciprocal teaching comprises the principle of social interaction and ZPD of the Vygotskian perspective (Schunk, 2012). Peer collaboration is another area where Vygotsky's ideas fit well in this study. Bruner (1984), Ratner et al., (2002) state that it reflects the notion of collective activity (Schunk, 2012). The shared social interactions when peers work on tasks cooperatively serve an instructional function. This method, Schunk argues, is mainly used in learning mathematics, science, and language arts which attests to the recognised impact of the social environment during learning.

Furthermore, Vygotsky's own articulation of the notion of ZPD in Sociocultural theory is described as the distance between actual development level determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers' (Vygotsky, 1978). This links to this study in the following way; in the classroom context of learning, a teacher should be aware that learners are not completely blank as they have prior conceptions about probability from their cultural events in their usual life experiences which they have to share as peers on a constructive social activity model tools set as a representation of reality as they engage in collaboration with one another (Vygostky, 1979; Wertsch, 1984). The researcher based the study on Wertsch's (1984) elaborations on what 'problem solving under adult guidance or in collaboration with more capable peers,' means in the ZPD, by using his concepts of situation definition and intersubjectivity in application.

The two concepts define this study in that the concept of situation definition is a way in which a setting or context is represented or defined by those operating on it (Brodie1996) which the researcher called a constructive social activity model. The constructive social activity model/ models being operated on by learners, may be defined differently by learners and even different from the teachers own interpretation (Brodie, 1996; Wertsch, 1984). Learners' situation definitions as described by (Wertsch 1984) represents their individual actual level of development and through collaboration with peers; ideas are tested, refined and developed producing a common interpretation (intersubjective knowledge) of a constructive social activity model. When they have a common interpretation of the phenomenon then each learner makes meaning from the shared knowledge on a social plane which is then internalized (learners own meaning making).

Thus, the notion of ZPD gives a lens to this study so much that before new concepts in probability are introduced, a teacher does not begin a lesson by giving definitions but rather sets a constructive social activity model which is a real representation of those concepts which are in abstract and intended. According to Freudental (1991) constructive social activity models are mathematical models that are developed by either teachers or the pupils themselves and further says, 'a model is a model of a situation that is familiar to the pupils.' This means that real life situations are presented through models that are meaningful and interactive to challenge learners' prior experiences on the social task setting when learners are put in small groups. Their experiences and verbal interpretations of a phenomenon or a situation are shared through verbal communication to confer meaning. They are seen to interact by conjecturing, agreements, disagreements, counteractions, reaction, counter examples and contradictions before they reach a common understanding of a situation (Konold 1991). According to Puntambekar & Hübscher (2005), cited by Schunk (2012) the ZPD represents the amount of learning possible by a learner given the appropriate instructional conditions. The main reason to the introduction of the notion of the ZPD by Vygotsky was due to his dissatisfaction with two practical issues in educational psychology (Turuk, 2008). The first issue is assessing a child on intellectual abilities and the second is the evaluation of the instructional practices. He believes that testing should not be based only on the current level of a child's achievements but it is more important to include the child's potential development as well (Verenikina, 2010). Vygotsky's Social Constructivists Theory of Learning claims that the actual level of development which is considered as the level of independent performance does not sufficiently describe development. It rather indicates what is already developed or achieved, it is a 'yesterday of development'. The level of assisted performance indicates what a person can achieve in the near future, what is developing (potential level, ‘tomorrow of development’, what a person ‘can be’) (Verenikina, 2010). Hence ZPD can be described as the distance between what a person can do with and without help. According to Verenikina (2010), the term ‘proximal’ means ‘nearby,’ indicating that the assistance provided goes slightly beyond learners’ current competence complementing and building on their existing abilities. It is largely a test of a student’s developmental readiness or intellectual level in a specific domain, and it shows how learning and development are related (Schunk, 2012).

Furthermore, this study was influenced by the social cultural theory and the notion of the ZPD because the concepts elaborated by Wertsch (1984) as earlier alluded to, inform the researcher that during learner interaction with the set constructive social activity model/ models, some learners are likely to face challenges such as their inability to manipulate, interpret appropriately which in turn limit their constructs and result in making errors. They are likely to raise hands to seek clarity from the teacher or peers. Their limited interpretation of the given situation is dependent on each learner’s prior knowledge and may be able to construct that which is limited to their experience. As a result, they may need assistance from their peers or the teacher in order to be assisted in the construction of meaning to reach the potential level of development. If their prior knowledge is congruent to the subject content matter in the model, then new representational models must be introduced to reconstruct new knowledge (Lerman 1989). The ZPD enables the teacher to note individual differences, learner capabilities from their own articulation of beliefs and gradually creates conceptual shifts in individual learners by mediating those concepts which learners have not yet fully developed as they interact. This process brings about clarity and conceptual understanding of scientific concepts and long-life skills acquisition.

Another application of Vygotsky’s theory as a mirror in this study, is apprenticeships as they occur in cultural institutions like schools and agencies which helps in transforming learners’ cognitive development. Apprentices operate within a ZPD as mainly their work depend on tasks beyond their capabilities. Apprentices develop a shared understanding of important processes by working with experts and integrate this with their current understandings (Schunk, 2012). The theory is a link to the study as it emphasizes the importance of what the learner brings to any learning situation as an active meaning-maker and problem-solver (Turuk, 2008). It acknowledges the dynamic nature of the interplay between teachers, learners and tasks and provides a view of learning as arising from interactions with others. Ellis (2000, as cited by Maturuk, 2008) states that Vygotsky's theory assumes that learning arises not through interaction, but in interaction. Learners first succeed in performing a new task with the help of another person and then internalise this task so that they can perform it on their own. In this way, social interaction is advocated to mediate learning. According to Ellis, the theory goes further to say interactions that successfully mediate learning are those in which the learners scaffold the new tasks (Turuk, 2008). There are a number of prominent leading theoretical perspectives today developed from the theory of Vygotsky. Daniels (2001) calls these concepts and approaches which are associated with Vygotsky's theory as 'post-Vygotskian studies. The most common among them is the concept of scaffolding by Jerome Bruner. In an educational context which this study was aimed at exploring scaffolding is an instructional structure whereby the teacher models the desired learning strategy or task then gradually shifts responsibility to the learners (Turuk, 2008) and this fits well within the ZPD of Vygotsky (Schunk, 2012). Verenikina (2010) highlights further more post-Vygotskian studies which may have been referred to in this study. Having discussed the theoretical framework, the researcher presents the operational definitions of terms to this study.

1.11 Operational definitions of key terms

Constructive: a meaningful activity carried out by learners when learning probability.

Social activity: a phenomena that influences and provokes direct actions, it involves individual expression of feelings, desires, ambitions, thoughts, abilities, talents in order to influence action and performance on it in relation to probability concepts.

Model (an aid, tool, object, symbol, or diagram): something that can be used as an example to follow or imitate due to its structural representations or pattern when learning probability concepts.

Probability: the extent measure of how likely something is to happen.

Conceptual understanding: to grasp mathematical ideas from a model perspective in the contexts from which they are applicable in probability.

1.12 Summary of the chapter

The first chapter has presented the background to the study, statement of the problem, purpose of the study, study objectives, research questions, and significance of the study, scope of the study (delimitations and limitations), theoretical framework and the operational definition of terms. Therefore, in conclusion, Vygotsky's (1978, 1979) social cultural theory and Wertsch's (1984, 1985) elaborations influenced this study. Basing on this theory, the researcher's area of concern was to provide insight into the questions on; what constructive social activity models are being used in learning probability? How are learners being engaged in the constructive social activity models for conceptual understanding of probability? How are they assisted? This study therefore, provided insight into constructive social activity models which learners use when learning Probability concepts and how the teachers assist them.

1.13 Organisation of the dissertation

Chapter 1 introduced the study by giving the background of the study, statement of the problem, purpose of the study, study objectives, research questions, and significance of the study, scope (delimitation and limitation), the theoretical framework, the operational definition of terms and lastly the summary of the chapter.

Chapter 2 reviews the related literature to the study under the following sub-headings the concept of probability, characteristics of model tools and implications of using models in learning, studies done related to the topic and gaps identified.

Chapter 3 provides the methodology which includes the research paradigm, research design and approach, study area, study population, study sample, sampling techniques, data collection methods and instruments, data collection procedures and time line, data analysis, credibility and trustworthiness of the research, and ethical considerations. Lesson observation schedule was accompanied by video recordings. Semi-structured interviews, document analysis, and FGDs were used to supplement the lesson observations.

Chapter 4 presents the analysis of qualitative findings. The chapter ends with a summary of the findings.

Chapter 5 provides the discussion of the findings presented in the preceding chapter (chapter 4) in line with the research objectives. The findings are further discussed in view of the related literature and theoretical foundations of the study.

Chapter 6 gives the conclusion and recommendations based on the findings.

CHAPTER TWO

REVIEW OF RELATED LITERATURE

2.1 Introduction

The chapter presents literature which is related to this study as an important aspect (Creswell, 2009), under the following sub-headings: the concept of probability, Charecterisations of knowledge construction, characteristics of model tools, implications of using models in learning and studies done related to the topic and gaps identified. The literature reviewed was subdivided in order to give a clear lens through which the study is situated. Understanding the concept of probability gives diverse important hints on how the concepts developed and not by mere discovery or transmission, but socially constructed, communicated among humans and its applicability to real life situations cannot be overlooked. Characteristics of model tools features in the subheading in the sense that as an educator, one must understand the physical characteristics of any tool before it is used as an aid to learning so that it accurately represents that which is in abstract in the process of knowledge construction. Implications of using models in learning; this is also another important review in literature as it explains what scholars say about the benefits of using a concrete representation of abstract concepts by using constructive social activity models. Studies done related to the topic and gaps identified; this focus helped the researcher to identify areas which were not given attention on the part of the learners' own knowledge construction and where to narrow the study by looking at the use of constructive social activity models learners use when learning probability.

2.2 The concept of probability

Batanero and Díaz (2012) and Batanero (2005) outline the evolving conception of probability since the 1600s; from Pascal's objective probability versus probability as intuitive belief. Probability has shaped modern science; transformed our ideas of nature, mind and society, altered our values and as assumptions about matters as diverse as legal fairness to human intelligence. The significance of these transformations and their influence on the structure of knowledge, issues of opportunity and equity in modern society, has prompted the underlying questions of how to support people's ability to develop a coherent understanding of probability (Basu & Debnath 2015). In ancient times, Plato (428-348 BC) and his famous student, Aristotle (384-322 BC) used to discuss the word chance philosophically. In 324 BC, a Greek person, Antimenes (530-510 BC) first developed the system of insurance which guaranteed a sum of money against wins or losses of certain events. In view of many uncertainties of everyday life such as health, weather, birth, death and game that led to the concept of chance or random variables as output of an experiment (for example, the length of an object, the height of people, the temperature in a city in a given day). Almost all measurements in mathematics or science have the fundamental property that the results vary in different trials. In other words, results are, in general, random in nature. Thus, a quantity sought to be measured is called a random variable. In ancient times, the concept of probability arose in problems of gambling dealing with winning or losing of a game (Albert, 2003).

During the fifteenth century, the pragmatic approach to problems of games of chance with dice began in Italy (Basu & Debnath, 2015). During that time, references to games of chance were more numerous, but no suggestions were made on how to calculate probabilities of events until the first mathematical treatment of probability dealing with problems of mathematical expectation (or mean), addition of probability, frequency tables for throwing of a dice, n successes in n independent trials, and the law of large numbers. Cardano introduced the idea of probability p between 0 and 1 to an event whose outcome is random, and then applied this idea to games of chance. He also developed the law of large numbers that states that when the probability of an event is p, then after a large number of trials n, the number of times it will occur is close to np (Basu & Debnath, 2015).

Thus, the classroom implications of discussing the concept of probability are firstly to enable an educator understand that a topic such as this could be well communicated to learners through practical applications and hands on as seen through the origins and its developments. Understanding the concepts from which it developed enables learners to view the concepts not as abstract and fixed as they are written in textbooks but as intellectual imagery interpretation and representation of that real world of play. Secondly, learners are able to contextualise their own games or think of other constructive social activity models within their environment from which the concepts are applicable and exemplified. Therefore, teachers should ensure that learners explain loudly their thinking in order to examine both oral and written responses to tasks (Mubanga, 2020). Having reviewed the development of the concept of probability which developed as a result of games of chance, the researcher was motivated to carry out the study in order to develop insight into constructive social activity models used when learning probability concepts.

2.3 Characteristics of model tools

Characteristics of a modeling tool describes a way in which someone who understands the development of a particular scientific concept would accurately relate it with concrete representations of such concepts in the very context in which it is designed. Stansfield and Carlton (2011) argue that models are tentative schemes or structures that correspond to real objects, events, or classes of events and that have explanatory power. Models help scientists and engineers understand how things work. Models take many forms, including physical objects, plans, mental constructs or cognitive modeling, mathematical equations, and computer simulations. Scientific explanations incorporate existing scientific knowledge and new evidence from observations, experiments, or models into internally consistent, logical statements. The main concern of this sub heading is to describe the characteristics of a good concrete model for it to be worthy of use in representing abstract concepts.

Several concrete modeling tools in use for any particular culture reflect a cumulative wisdom of that culture in which learners come from. The learners' insights and experiences about tool uses as individual members of that culture lies in their knowledge of the concept, purpose for which the tool is developed and used (Berger 1998; Cockcroft, 1982; Freudenthal, 1973, 1991; Kirshner & Whiteson 1997, 1998; Shaughnessy, 1992; Vygotsky, 1979, 1986; Zahner & Corter, 2010). For this reason, as learners go to school they are not blank slates (Vygotsky 1979). Knowledge of a tool or an object or model implies the following; firstly, the moment it appears learners must identify (verbal interpretations by use of familiar language, imagery, symbolic, signs etc.), secondly is appropriate action on it which implies physical manipulations in the very context in which it is designed. The more an individual engages in physically actions on the tools, the more they gain experience and the more they learn. Therefore, tools that man uses to transform culture are situated and progressively developed, refined, validated and shared among generations (Vygotsky, 1979). If tool uses possess challenges during the activity, it may need checking and refinement or transformations or improvisations. For instance, if in the process of cutting vegetables you experience the knife is blunt then it requires either sharpening by rubbing against its edge back and forward on a rough solid rock or metal. Doing so, may increase the likelihood of its meaningful application in problem solving during the activity. This is what this research was also concerned about; to develop insight into the tools that learners use when learning probability. Are the tools culturally rooted? Are they contextual? How do learners interpret them? These were some of the questions which were being raised by the researcher.

Moreover, the activity, concept and culture (modeling tools) are interdependent and any formal learning of scientific concepts must involve all the three (Cobb, 1994; Kirshner & Whiteson 1997, 1998). In addition, the activity for which the model is used to socially construct knowledge through negotiations of meanings (past and present) must be rooted and culturally acceptable to real life problem solving (Vygotsky 1978, 1979). For instance, models like a dice or coin were used in an activity context of gambling from which the concept of probability first developed. However, gambling maybe regarded as unlawful in some societies due to its social implications. While certain activities of games of chance in certain communities such as playing cards, jack port, chess maybe gender biased and mainly played in Casinos. Even so, reflecting on the activities of learners' community of practice can be helpful in which tossing a coin for instance could be situated accurately.
For example, a teacher may say "I have a number of sweets in my bag but I intend to give to each one of you the number of sweets depending on how many coat of arms will land up after you try tossing a coin 5 times," because the features of the coin are reflective in the activity and authentic as the coin appears in present day life.

According to Freudental (1991) the constructive social activity models are developed by teachers or the learners themselves and further say, A model is a model of a situation that is familiar to the learners. The implication of this statement is that, given a physical model, its accurate interpretations depend on how familiar it is to those operating on it, as a result of prior experiences. The physical characteristics of a model which are observable, should illustrate the abstract concepts so that learners take probability as a physical property than abstract (Schlottmann, 2001). Freudental's (1991) theory encourages that learners should develop mathematical tools or models from which they develop insights by themselves. This implies that even learners themselves are capable of creating models of reference in their own situation and own understanding. Self-developed (or emergent) models of the pupils serve to bridge the gap between informal and formal knowledge (Gravemeijer, 1994).

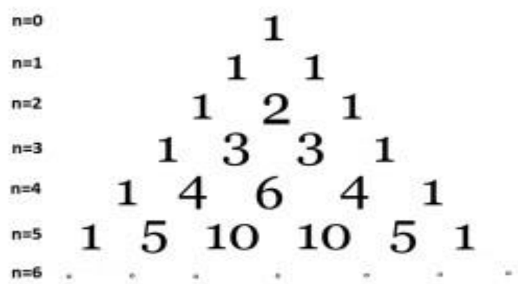
Furthermore, learning is a continuous life –long process with full of cultural dynamics resulting from situations and affects greatly how to relate inert knowledge concepts (learners reference books) in abstract to physical models and situated activities. Neglecting this part may not help well to improve and promote robust knowledge (Whitehead, 1929). Moreover, Voigt (1992), Rogoff (1990) and Saxe (1991) argue that the prominent objective of mathematics education is not that, "learners produce correct answers but that they do it insightfully and by reasonable thinking." Indeed, the focus as a practicing educationist is to develop insight into how the tension and gaps between the daily life experiences of a child and the knowledge of the experts that is in the learner reference books is being bridged. It is therefore necessary to observe learners from different cultural settings settle in natural classroom learning in small groups to share their knowledge of model tools together so that they gradually and consciously develop scientific concepts of probability. A learning situation should not aim at transmitting all abstract concepts as fixed as they are, well defined independent entities that can only be explored in prototypical examples and exercises in reference books. This is because such exemplification cannot provide the important insights into either the culture or the authentic activities of that culture where learners' daily needs are met (Baise, 2013; Carraher and Schliemann 1985; Dowd, 2013; Lave, 1988; Saxe, 1991; Scribner, 1984).

The constructive social activity model tools may share several significant features with knowledge concepts but to fully understand them, is through use. Thus, using those tools entails both changing the user's view of the world and adopting the belief system of a culture in which they are used and created (Cobb & Bowers 1998; Whitson & Kirshner 1997, 1998). Therefore, learning with constructive social activity models or tools, requires that learners contextualise definitions and algorithms in their own understanding and experiences in order to apply to other domain equivalent (Cobb & Bowers, 998). These assumptions therefore, have stimulated the researcher's focus on the constructive social activity models that learners engage in when learning probability concepts in Mbala district. It is likely that the inert models embedded in Mathematics learners' books may have little contextual relevance in daily life applications. For example, concepts of probability begun with gambling (Wagenaar, 1988) of which to some context was considered illegal and some even today may associate it to the standard deck of playing cards.

2.4 Implications for using models in the classroom learning

Drawing from Fredental's (1991) use of contexts in phenomenological exploration who states that the starting point of instruction should be experientially real to the learners by allowing them to become immediately engaged in the contextual situation. This means that instruction should not start with the formal systems of definitions. The phenomena by which mathematics concepts appear in reality should be the source of concept formation. The process of extracting the appropriate mathematical concept from a concrete situation is described by de Lange (1987) as conceptual mathematisation. This process forces learners to be fully engaged in exploring the concepts represented in a constructive social activity model. Learners are seen to engage by finding and identifying the relevant mathematical elements schematise and visualise in order to discover patterns/ rules in the model.

One of the important such illustrations of using a model to schematise and visualise patterns and rules is that which was developed by Pascal (1623-1662) and Fermat (1601-1665). Basu and Debnath, (2015) give an insight on Pascal (1623-1662) and Fermat (1601-1665) use of a coin in developing further concepts in probability which clearly links to Binomial and the famous Pascal's triangle. Theoretically, when an ideal coin is tossed, a priori probability of two possible outcomes, head or tail, is 1/ 2. According to Pascal, when a coin is tossed twice, there are four possible outcomes: HH, HT, TH and TT where H stands for heads and T for tails, a solution of which was closely related to the binomial coefficients of $(H + T)^2 = H^2 + HT + TH + T^2 = H^2 + 2HT + T^2$ below.



Similarly, when an ideal coin is tossed three times, there are eight possible outcomes: HHH, HHT, HT H, THH, HTT, THT, TTH, TTT. It provides further link to solution of the binomial coefficients (collecting all terms of H and T together) of $(H + T)^3 = H^3 + 3H^2T + 3HT^2 + T^3$. The binomial coefficients in this example led to the famous Pascal triangle. In connection with the Pascal triangle as shown above, Pascal observed the famous formula for the binomial coefficients as follows;

${}^{n}C_{r} = \binom{n}{r}$ involved in the coefficient of $a^r b^{n-r}$ in the binomial expansion of $(a + b)^n$ for any integer $n \geq 0$ and $0 \leq r \leq n$. He also linked binomial coefficients with the combinatorial coefficients that arose in probability, and used these coefficients to solve the problem of points in the case of one player require r points and the other n points to win a game, (Debnath, 2009; Debnath, 2010; Efron, 2013; Kolmogorov,1956). Therefore, the use of constructive social activity models in the learning process of probability concepts that this research explored was inspired by this literature.

The literature reviewed also inspired the researcher in that it identifies the roles of a teacher in the process of learning scientific concepts from a model perspective. The teachers' activities in the classroom setting are interventions portrayed in promoting thought process and reflection on the part of a learner with request for argument in support of their assertions. By using models learners can develop further models resulting in a mathematical concept and by the process of reflecting and generalising; they will develop more complete concepts. It is therefore expected that learners would subsequently apply probability concepts to other aspects of their daily life and as such the concepts are reinforced and strengthened (De Kock, 2015; Dowd, 2013; Drijvers et al 2014; Driver et al, 1994; De Lange, 1998; Freudental, 1991; Garfield and Ahlgren, 1988; Gravemeijer, 1994; Konold, 1991; Shaughnessy, 1992).

For instance, the activity of having items being taken from the container at random and returned to the container is also a constructive social activity model from which concepts of compound events could be exemplified. In this experiment, learners should list all possible outcomes and their associated probabilities as Probability is associated with games of chance which imply that the participants in the games are always actively involved and not passive. These models exemplify experiments or activities, which can be carried out with observable results. The set of all possible outcomes of an experiment is the sample space of the experiment. An event is any subset of a sample space that is of the observers' interest. The likelihood or chance, possibility of an event to occur is probability as it is expressed as a ratio. The objective view of probability is essentially based on the statistical concept of relative frequencies of events from observations and experiments. Based on games of chance with finite number of equally likely outcomes, the objective probability is measured by the ratio of the number of favorable outcomes to the total number of outcomes (Michael, 2003; Konold 1991, 1994).

However, there are some situations of chance that may not demand physical manipulation or experiment like other aspects of models but are explained verbally through cognitive modeling. According to Jaworsky(1996), cognition is the mental action or process of acquiring knowledge and understanding through thought, experience, and the senses. It encompasses processes such as knowledge, attention, memory and working memory, judgment and evaluation, reasoning and "computation", problem solving and decisionmaking, comprehension and production of language, just to mention but a few.

Human cognition is conscious and unconscious, concrete or abstract, as well as intuitive (like knowledge of a language) and conceptual (like a model of a language). Cognitive processes use existing knowledge and generate new knowledge. Polya (2014) has envisaged that to solve a problem, the first thing is to understand the problem and the second stage is to devise a plan. One of Polya's plans is to draw a picture that is a visual or make a model. Probability problems are often abstract to learners that is why making visual representations is one of the ways in which learners can make sense of the problems and as way to an accurate solution. Their cognitive and visual representations would help stoke their actual level of development in probability knowledge.

Single-case probabilities assumed in models may well be mere conveniences that do not correspond directly to anything physical in the world such as expectations, anxieties, hopes or predictions but those can still be modeled from a fantasy world of fairy tales. Cognitive models should exemplify probability problems from real world, from the fantasy world of fairy tales as long as the problems are experientially real in the learners' mind. For example, when one is struck with an illness, there are two possible outcomes, recovery or death and this cannot be exemplified or represented by a physical activity but through fairy tale. Levels of models have been described in Realistic Mathematics Educetion (RME) (Gravemeijer, 1994) as situational level, where domain-specific, situational knowledge and strategies are used within the context of the situation; referential level or the level 'model of,' where models and strategies refer to the situation described in the problem; general level or the level 'model for,' where a mathematical focus on strategies dominates over the reference to the context; and formal level of mathematics, where one works with conventional procedures and notations.

Therefore, this means that learners are free to make discoveries at their own level, build on their own experiential knowledge and perform shortcuts at their own pace. This is however time consuming following the school timetable and the syllabus assessment demands. Even so, the teacher should be seen to organise and stimulate the learners to compare their solutions in a class discussion. Freudental (1973, 1991), stated that a discussion refers to the interpretation of the situation presented in a constructive social activity model of the contextual problem and to focus on the adequacy and the efficiency of various solution procedures. The teacher should ask the learners to communicate, argue and justify their solutions as they manipulate the model (Broddie, 1996; De Lange, 1996; Freudental, 1991; Gravemeijer,1994; Konold,1991). Thus, literature reviews that any learning that is formally organised to be experienced by learners has history to refer to through the use of models (Freudental, 1991; Wertsch 1984, 1985). Therefore, the researcher's intention in this study was to explore how the use of appropriate tools (models) in a social setting address the relationship between the mathematical concepts of probability to their different cultural context in which learners exist.

2.5 Studies done related to the topic and gaps identified

Temperley (2014) did a study in California on, 'the Probabilistic Models of Melodic Interval' the study was an experimental design. The aim of the study was to compare two models reflecting alternative approaches to the modeling of melodic interval of which one model was called the Markov and the other Gaussian model. In the Markov model, the probability of an interval pitch was determined by its observed frequency in a corpus, conditioned on the previous one or more intervals. In a multiple viewpoint system, the probabilities for individual features of a note (perhaps conditioned on the features of previous notes) are combined to yield a single probability for the note. It included intervals and scale degree as features, along with numerous others such as absolute pitch, contour, rhythmic attributes, and more complex features such as interval to the first note of the measure. The second, Gaussian, model, is also probabilistic. In this model, however, the probabilities of intervals were defined not by their frequency in a corpus, but rather, by the general principle of pitch proximity: each pitch in a melody tends to be close to the previous pitch, so that small intervals are more frequent than large ones. One of the goals of the study was to investigate how well the distribution of intervals in tonal music can be explained by the principle of pitch proximity in combination with a preference for certain scale degrees over others. Participants heard two-note contexts followed by a continuation note that could be anywhere within an octave of the second context note (thus 25 different continuation notes were possible for each context); eight different contexts were used, creating 200 stimuli in all. Participants rated the expectedness of the continuation note given the two-note context on a scale of 1 to 7 with excitement (Temperly, 2014).

Similar reasoning is often applied to other irregularities in the interval distributions of random variables and human expectations. A model that assigns a probability to a sequence of notes can also be used to model human expectation judgement (Temperle & De Clercq, 2013). The use of melodic models provided the following cognition concepts in probability some of which informs the current study; expected outcome events such as mean, mode, median, amplitudes, interval, human expectations, Probability Density Functions, of continuous random variables to mention but a few. Temperly (2014)'s findings review that the use of a tools help to mediate conceptual mathematization and increase cognition through active participation. For instance, Probability Distribution relates to melodic musical model notes in that it is a statistical function that describes all possible values and likelihoods that a random variable can take within a given range bounded by the maximum and minimum values and thus in this case is similar to the case of the first note and the last note interval of a melody. Thus probability Density function can be plotted taking into account similar factors as in the Temperly (2014)'s musical experiment which are distribution mean (average), standard deviations and skewness. However, the current study focused on the use of constructive social activity models when learning probability while Temperly's study was an experimental design with focus on investigating how well the distribution of intervals in tonal music can be explained by the principle of pitch proximity in combination with a preference for certain scale degrees over others, hence the gap.

Dowd (2013) did a study in Washington among her learners based on, the circle model for multiplying probabilities. She was determined to test whether a circle model would be used to help her learners understand why compound probability scenarios involve multiplying probabilities. The circle's sectors model (chocolate and marbles, a circular pizza) represented the compound probabilities of random events on two occasions. The circle's inner area was divided into fourths (like how a pizza would be divided) to show that the first draw of a chocolate could be one of these four chocolates and so were the marbles. The circle model helped her learners to understand why the calculation method for two-stage probability (with and without replacement) involves multiplication. They also clarified the concepts of sample space, event and outcome through the physical manipulation of the model's activity. Dowd (2013) states that the circle model has offered an additional representation to assist in learners' analysis and development of schema about the two probability situations (drawing with and without replacement).

However, Dowd (2013) study does not guarantee that the same results that this study shows would be the same as in the Zambian contexts. This is due to environmental disparities and as such, activities of a knowledge domain are formed by ordinary practices of its culture, hence the gap. In addition, Dowd's study informs this research study in that the circular pizza model which was used was familiar and contextual to the learner's community. So, it is appropriate to use the representative model which learners are familiar with in order to make the learning of scientific concepts experientially real to the minds of the learners. Hence, the interpretation of the circular area of the pizza in relation to the concepts of Probability was accurate and relevant to learners' background experiences.

In addition, when learning compound events, learners are told to master the phrases such as 'both,' 'and,' to imply multiplication of probabilities but Dowd's study used a circle model to show why probabilities are multiplied in compound events. This is a guide to this study in that a concrete model can be used to show why a multiplication sign is used rather than just merely mastery of phrases. It also informs the study to relate multiplying proper fractions in such situations as multiplying 2/3 by 3/4, to a circle model: For example, from 3/4 pizza, eat 2/3 of this amount of pizza it is $2/3 \times 3/4$. The portion eaten is less than the whole amount of 3/4 pizza, or 3/4 pizza less than one time. It is eating 2/3 of the whole amount and if it's the whole one pizza eaten, then there will be no pizza left which in this case relates to the ranging of probability between zero to 1 as probability of any event (E) happening thus Pr (E) = $0 \leq p(E) \leq 1$. Therefore, this study focuses on exploring the use of constructive social activity models in drawing conceptual understanding of Probability concepts in relation to learners' background experiences in Mbala District.

Liu and Thompson (2007) conducted a study on Teachers' understanding of Probability, in Singapore which showed consistent documentation of poor understanding of probability concepts. The purpose of the study was to develop a theoretical framework for describing teachers' understandings of probability. The study implored mixed method designs in its data collection and analysis. The study focus was on teachers' understanding of probability. The data collected through videotapes sessions and interviews, teachers' written work, and researchers' field notes, revealed that there was a complex mix of conceptions and understandings of probability, both within and across the teachers, which were situationally triggered and incoherent. When the teachers tried to reflect on them it did not support their attempts to develop coherent pedagogical strategies regarding probability. It did not look at the social activity models from which probability concepts developed through demonstrations. Thus, it was also necessary for this study to engage in examining those very models which were used for evidence in determining probabilities in other cultural contexts as Konold (1991) argues that probability is developed with reference to the tendency of certain phenomena like flipping coins or die rolls and so on.

In addition, Liu and Thomson study gives an impression that lack of understanding of probability concepts by the teachers is not only due to intuitive thinking but also as a result of its own representative models (Konold, 1991; Dowd, 2013). This could be so because models are likely to cause difficulty in understanding probability concepts especially when taken to formal instructions according to (Konold 1991). Thus, it gave the researcher a similar perspective but with focus on the use of constructive social activity models from which probability concepts develop through demonstrations, physical manipulations and representations, hence the gap.

Mutara and Makonye (2016) conducted a study on, "learners' use of probability models in answering probability tasks," in South Africa. The purpose of the study was to explore the errors and misconceptions associated with grade 10 learners' different representations in solving probability problems. This study involved learners representing the sample spaces by use of Venn diagrams, contingency tables, tree diagrams, two-way tables and outcome listings. The study involved mixed methods in which it was generally reviewed that probability being a new teaching topic in South African schools, both teachers and learners were facing problems in representing the sample spaces by use of Venn diagrams, contingency tables, tree diagrams, two-way tables and outcome listings. The study recommended that in order to reduce the intuitive misconceptions, leaners should engage in activities while the teacher scaffolds their limited constructs.

The study also is a lens for this research in that giving tasks to learners is a way of assessing their prior knowledge, establishing understanding and non-understanding, correcting their errors as learners extract concepts from a given model. The literature is a guide to this research as it revealed learners' behavioural patterns related to the different types of representations used in solving probability tasks. Learners gain confidence in the efficiency of their work with the teacher's help such as when marking, checking their work and correcting their errors. For instance, this literature shows that for the tree diagram and despite the fact that learners experienced difficulties in constructing correct ones, about 90% of correct answers resulted from tree diagrams that were partially completed by their teachers. Therefore, the study concluded that though learners liked the tree diagrams, they often experienced difficulties in constructing them from the scratch; but they were quite competent in completing partially constructed tree diagrams and proceed to calculate the required probabilities. This literature review's findings inform the researcher's study that learners need scaffolding in modelling probabilistic problems using tree diagrams as this is in their zone of proximal development (Vygotsky, 1986). Starting with partially completed representations by the teacher or a more capable peer, would boost the success rate of other learners in solving the probability problems as revealed by the literature. Once completing partially completed representations is mastered it will boost their confidence in learning and more complex representation models would be developed. Therefore, this literature informs the researcher further that, the teacher has a role to play in helping learners reach their potentials, work out misconceptions by allowing learners to actively engage in activities rather than being passive.

However, Mutara and Makonyes' study did not examine the constructive social activity models which learners were engaging in learning probability concepts before solving the tasks, hence the knowledge gap. There was need to carry out this study focusing on the constructive social activity models of learning probability in Zambia. Even though, several research studies have been conducted around the world to investigate the ways in which people understand probability concepts, they have not examined the models from which probability concepts develop in relation to particular cultural experiences of the learners, hence the gap (Basu & Debnath 2015; De Kock, 2015; Dowd, 2013; Fischbein & Schnarch, 1997; Kahneman & Tversky, 1973; Konoid, 1989; Konoid et. al. 1993; Liu and Thompson 2007; Nisbett et. al. 1983).

De kock's (2015) research on probability which focused on, 'relating teachers' content knowledge with learner performance' indicate that learners do possess intuition about probability but that instruction needs to be changed to take advantage of this aptitude and to facilitate learning. In order to increase mental maturity of learners, do not use verbal or definitions to describe probabilistic situations. Therefore, this literature became necessary to this study in establishing whether instructional strategies such as verbal or definitions were used to build on learners' intuition in describing probabilistic situations other than constructive social activity models.

Schlottmann (2001) did a study titled "Understanding the Expected Value of Complex Gambles." It implored quantitative research methods and the purpose of the study was to determine how children judge expected value of complex gambles in which alternative outcomes have different prizes. Two experiments were carried out using Information Integration Theory to study Six-year-olds, 9-year-olds and adults (N = 73 in Study 1, N = 28 in Study 2) the chance games that involved shaking a marble in a bicolored tube had the conditions that one prize was won if the marble stopped on blue, another if it stopped on yellow. Children judged how happy a puppet playing the game would be, with the prizes and probability of the blue and yellow outcomes varied factorially. Three main results appeared in both studies: First, participants in all age groups used the normatively pre-scribed multiplication rule for integrating probability and value of each individual outcome-a striking finding because multiplicative reasoning does not usually appear before 8 years of age in other domains. Second, all age groups based judgment of overall expected value meaningfully on both alternative outcomes, but there were individual differences--many participants deviated from the normative addition rule, `showing risk seeking and risk averse patterns of judgment similar to the risk attitudes often found with adults. Third, even the youngest children took probability to be an abstract rather than physical property of the game. Overall, in contrast to the traditional view, the present results demonstrate functional understanding of probability and expected value in children as young as 5 or 6 years.

These results contribute to the growing evidence on children's intuitive reasoning competence. This intuition also contributes to the biases evident in adults' judgments. Even very young children know that people act to fulfill their desires (Wellman, 1990). Nevertheless, people do not act blindly to get what they want. Their action also depends on how likely the desired outcome is and this informs the study in that psychological modeling can also be established in the process of learning probability apart from use of concrete objects. It would help to interprete non concrete probabilistic situations in terms of decision making and making logical judgements about a certain occurrence. This literature was also relevant for this research in that it enabled the researcher to understand how humans evaluate their options for a given action, and that probability is a game of chance which is interactive and practical. The notion of expected value (EV), is also a concept basic to understanding goal-directed action in an uncertain world.

This study used quantitative methods to determine how children judge expected value of complex gambles in which alternative outcomes have different prizes. Marbles in a bicolored tube had the conditions that one prize was won if the marble stopped on blue, another if it stopped on yellow. The constructive social activity model in this case was the use of marbles to communicate the notion of (EV), as a concept in probability. However, the focus of this study reflects a knowledge gap that was identified in terms of its focus, methods and the study sample. The focus of my study was to establish learners' use of constructive social activity models when learning probability concepts in the classroom learning process. It implored qualitative approach and with 12 participants as sample size, hence the gap.

Mudenda (2018) did a study on the 'use of information and communication technologies (ICT) in the teaching of mathematics.' The study explored the use of ICTs, teachers' Technological Pedagogical and Content Knowledge (TPCK) in using technological tools, opportunities and challenges in the use of ICTs in teaching of mathematics in Zambia. The study utilized qualitative approach in its data collection strategies and procedures and 4 participants were included as a study sample size. It explored the integration of technological tools and equipment with appropriate instructional plans and strategies that would foster effective teaching and learning. Findings showed that teachers were using less sophisticated ICT tools that did not promote higher order thinking skills to strengthen the instructional transmission of knowledge and software that excluded the beneficial aspects of what technology could offer through collaboration.

While Mudenda's study focused on use of technological tools in learning mathematics, the researcher focused on the use of constructive social activity models in learning probability, hence the gap. However, the researcher was not only guided to focus on the use of tools (constructive social activity models) in the learning of probability but also using this literature, the researcher was able to interpret that technological tool use can mediate and enhance learners' learning opportunities by choosing and generating Mathematical tasks that take advantage of what technology can offer as a tool. Secondly, choosing an appropriate constructive social activity model that learners can lay hands on and be able to manipulate, draw mathematical concepts, can be more appealing and real to the minds of the learners. In addition, TPCK on technological tool use, can constrain knowledge construction as indicated in the findings and limit the choice of a tool to help mediate conceptual mathematization. Thus, this informs the study in that the use of modeling tools depends not only on the learners but also on the teachers' ability to use it, and how familiar they are. Therefore, the literature informs the researcher that teachers need to go beyond the general uses of the tools and integrate them into mathematical instructions for effective teaching and learning. Teachers should have knowledge of how to use tools in pedagogically appropriate ways that support instruction authentically, Mudenda (2018).

The United National Council of Teachers of Mathematics (UN NCTM), (2014) argued that use of tools, is an essential element in the classroom, to aid students in making sense of mathematics concepts and in developing their ability to reason and communicate. The researcher also believes that sense making is connected to what Skemp called relational understanding. Skemp (1976) stated that there are two kinds of understanding instrumental and relational. Instrumental understanding is the kind of learning the rule or method or algorithm which gives quicker results for the teacher while relational understanding is understanding how and why the procedures work in reality. The researcher believes that meaningful use of tools could help learners develop relational understanding, hence the study in titled, 'use of constructive social activity models in learning probability,' in selected schools of Mbala District.

2.6 Summary of the chapter

Considering the evolution of probability concepts and its many applications to practical real-life experiences and the studies done related to probability, there are gaps as well as key issues which informed this study. The literature reviewed about the evolution of probability informs my study about use of concrete activity models and that probability developed as social activity, and so it should not be learned in abstract. There should be accurate representation of abstract concepts by use of tools, tool must cause action, hands on, structural pattern or rules to follow, must be interesting, manipulated, and must be contextualized. Some gaps identified were for example, the use of circle model to multiply probabilities was based on a single concept of multiplication only while this study focused on how learners use constructive social activity models when learning probability. Use of marbles to communicate the notion of (EV). It implored quantitative approach the focus was on a single concept of probability while this study is based the entire probability concepts hence the gap. Thus, Table 2 is a summary of the themes and key issues reviewed from the literature which are related to the study.

Table 2: *Summary of themes and key issues related to the study*

Themes	Key issues	Related to the current study
Concept of probability	Practical, relevant, created, applicable to all fields of knowledge	Informs my study about use of concrete activity models, develops as social activity, it should not be learned in abstract
Characteristics of model tools	Culturally rooted, easy to use, fit with concepts, familiar to learners	Informs my study in that there should be accurate representation of abstract concepts by use of tools, tool must cause action, hands on, structural pattern or rules to follow, must be interesting, manipulated, must be contextualized.
Implications of using models in learning	Robust knowledge, knowledge retention, active participation, real world representation,	Informs my study in that tool use generates interest, represents reality, helps to correct misconceptions, and generate knew knowledge for future problem solving and develops more complex representative models.
Studies done related to the topic and gaps identified	The circle model for multiplying probabilities. Use of probability melodic models Teachers' understanding of probability Learners' use of probability models in answering probability tasks. Understanding the expected value of complex gambles.	Multiplication of probabilities can be shown using a representative concrete model and not just by use of phrases such as 'both,' 'and' Directed a focus of the study that both teachers and learners have teaching and learning difficulties and use of models in the process of learning probability may constrain knowledge construction. Use of marbles to communicate the notion of expected value (EV). It implored quantitative approach the focus was on a single concept of probability hence the gap. Use of tools mediate conceptual mathematization, active participation, increased cognition and retention.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1. Introduction

This chapter discusses the methodology employed in this study. It aims at highlighting various strategies used in the research such as research paradigm, design, study area, population, study sample, sampling techniques, data collection methods and instruments, data collection procedure, data analysis, credibility and trustworthiness of the study and ethical considerations. According to Kazdin (1992), research methodology implies principles, procedures, and practices that govern the entire research.

3.2 Research paradigm

The, "Use of constructive social activity models when learning Probability," focused on observable activities attributed to real life practices that call for learner participation and might influence learner's own knowledge construction which could lead to understanding. This involves interpreting the meanings, relevance, and appropriateness of reality of those models in application and how they foster or constrain knowledge construction process. The use of tools when learning scientific concepts is situated within a constructivist epistemological paradigm. Constructivism is a social theory that underpins qualitative research and it suggests the view that the realities explored by researchers are social products of the actors, of interactions and institutions (Flick, 2006). It argues that the realities of the world are created by individuals' understandings and experiences (Bryman, 2008; Flick, 2007) and as such scientific concepts can not be learned without the integration of physical tools in reality. Therefore, the researcher adopted constructivism paradigm because of the understanding that knowledge is constructed by individual's continuous interaction with the physical and social phenomena from their environment. Having discussed the research paradigm, the next section discusses the research design used.

3.3. Research design

This research study implored descriptive phenomenological design. This was because the research focus was based on mediated activity tools within a particular social historical context of learning. The study sought to establish insight into how the use of appropriate tools (models) in a social setting address the relationship between the mathematical concepts of probability in relation to their different cultural context in which learners exist. The conditions under which effective learning takes place as posited by the theory used are; discussions, negotiations and sharing of learners' different interpretation of the constructive activity modelling tools on a social plane during the learning process of probability concepts. Thus, the learners' reference books such as Progress in Mathematics for grade 9 and 11 on pages 154 and 147-151, respectively have various models to mediate the cognition of probability concepts which may need to be explored on how learners engage in drawing conceptual understanding of probability and their relevance to the particular dynamic cultural setting. Patton (2002) defines descriptive phenomenological design as one which describes phenomena as they exist, and that it is used to identify and obtain information on the characteristics of a particular problem and ask questions that are not based on no or yes as answers. This design was appropriate because the phenomena was the constructive social activity models which exist in the environment and learners. engaged when learning probability and their relevance for cognition in the learners' own culture. It explored even those models from learners' own perceptions, experiences and the quality of their application for conceptual change. The researcher only focused on the experiences from the participants' perspectives more especially the learners. As a result, the information was in turn used to answer questions that were raised.

The next section discusses the study area and the reasons for choosing it.

3.4. Study area

The study was conducted at two secondary schools in Mbala district, (i.e one class from each of the two sampled schools). There are many motivating factors that influenced the researcher's choice of the study site, such as; the nature and incidence of the problem, research time frame, and data accessibility, clients' interest and instructions, resource availability, performance in particular field, goals and objectives of the study (Msabila & Nalaila, 2013). The researcher picked secondary schools in Mbala district because the schools in the district have poor performance ranking in mathematics reviewed consistently in the Examinations council of Zambia reports (ECZ 2014- 2017). Population is discussed in the next section.

3.5. Population.

The targeted population was classes of mathematics learners in each of the selected secondary schools of Mbala district in Northern Province of Zambia. At the time of the research, only two schools with grade eleven learners were learning Probability in the district. Therefore, it was appropriate to target a population that the researcher had in mind from which information could be obtained and draw conclusions (Franken and Wallen, 2000). Thus, a small sample was taken from a large group of people and only targeted a minimum and acceptable number of participants. In the next section, the study sample is discussed.

3.6. Study sample

White (2003) defines a sample as a segment of the population that is selected for investigation. Hence, in this study, the sample size comprised of 10 grade eleven learners (10 selected learners in two secondary schools that is 5 from each school) and 2 grade eleven mathematics teachers (1 teacher from each school) were included in the sample. Each teacher who was included in the study sample had 91 and 15 learners in each of their own classes where they taught probability. This was a reasonable sample size for a qualitative study because it provided data needed to fulfil the requirements of efficiency, representativeness, reliability and flexibility (Merriam, 1998). According to Creswell (2014) qualitative research is context bound and uses a small sample size. Having discussed the study sample, the next section outlines the sampling techniques used in the study.

3.7. Sampling techniques

The study employed non-probability sampling techniques of participants. Purposive sampling was used to select regular classrooms which study mathematics and were scheduled to learn Probability in the first term of the year in Mbala district. The teachers were also selected because they were teaching probability to the grade elevens at that time. According to Tongco (2007), purposive sampling technique, also called judgment sampling, is the deliberate choice of an informant due to the qualities the informant possesses. Purposive sampling helped to select the participants to the convenience of the researcher. This study employed the homogeneous type of purposive sampling of learners who were included in the sample. They were selected on purpose as the researcher had the rights to select participants for having a shared characteristic of having been consistently attending lessons, being able to actively participate in group conversations. The researcher found out from the teacher in charge of the particular class of mathematics department on which classes could be observed during the learning of probability. The Head of Department guided on the names of teachers who were teaching probability then. The next section discussed the data collection instruments.

3.8. Data collection instruments and methods

The data was collected using lesson observations protocols, semi-structured interview guides, document analysis schedules and FGDs guides (Denscombe, 2003; Kothari, 2004). Instruments which were used to record data were; video cameras, cameras, tape recorders and a diary. A tape recorder was mostly used when conducting semi-structured interviews and during FGDs. According to Kothari (2004), under observation method, the information is sought by way of investigator's own direct observation without asking from the respondent. The interview method of collecting data involves presentation of oralverbal stimuli and reply in terms of oral-verbal responses. Dawson (2002) advised that when using a tape recorder, it is important to hear your own voice as well as that of the interviewee so that you know what answers have been given to which questions. The following subsections provide more details about data collection methods and procedures.

3.8.1 Lesson observations

Lesson observation protocols were used to collect data during the learning process of probability concepts in each of the two grade eleven class. Lesson observations were firstly conducted in order to see what constructive social activity models would be used to explicate probability concepts in the process of learning. Secondly, how learners were engaged in the manipulation of the constructive social activity models in the process of knowledge construction and thirdly, the assistance given to learners in drawing conceptual understanding from the model perspective ***(see Appendix 4 for details, p.150)***. According to McMillan and Schumacher (1993), lesson observation is a tool which provides information about the actual behaviour. Creswell (2014), added that, direct observation is useful because some behaviour involves habitual routines of which people are hardly aware. Patton (2002) also observed that, observation is a data collection procedure whereby researchers try to understand what is happening in a given setting by paying attention, video recording, watching and listening carefully.

Hence in this study, video recordings were used to capture lesson observations in totality. According to Merriam (1998), one of the advantages of using a video camera to record field observations is that it allowed the researcher not only to capture the physical environment but also to revisit and make reference to the images later and recall the experiences during the analysis.

3.8.2 Focus group discussions

FGDs were used in order to collect data from grade eleven learners on the use of constructive social activity models when learning probability. This was purely meant for the purpose of triangulation, in order to see if what was observed during lesson observations and semi-structured interviews with teachers was consistent with what the learners said. For example, FGDs were conducted after class observations of each sub topic was completed so that learners' feelings, opinions, and attitudes towards the use of models which would have constrained knowledge construction could be shared in detail and compared with what was observed. Cohen, Manion and Morrison (2007) described triangulation as the use of two or more methods of data collection in the study of some aspects of human behaviour. For this purpose, FGD were used ***(see Appendix 7 for details, p.154).*** FGDs with five learners each were held after lesson observations and semistructured interviews were conducted with the teachers. In order to enhance trustworthiness of these qualitative results, audio tape was used to record the proceedings of FDGs and this also helped to capture data in totality.

3.8.3 Semi-structured interviews

Semi-structured interviews were conducted with the teachers after lesson observations in probability. The reason for this was to have an in-depth understanding, opinions, and views pertaining to their choice of models, what model representations may have constrained knowledge construction. The researcher used semi-structured interview guides for this purpose ***(see Appendix 8 for details, p.156).*** According to Kvale (1996), researchers across the sphere contended that interviews are the best way to collect data because it helps the researcher to establish feelings, opinions, attitudes, views, and are useful in gathering in-depth data.

3.8.4 Document analysis

In this study, document analysis was done by looking at the learners' worksheets in order to find out if these documents reflected how much learners were able to internalise by checking through the tasks they were given. The imagery or diagrammatical representation of the models on their worksheets, their procedures in answering the tasks and how the teachers guided their understanding of the tasks given from the model perspective was important for this research study. For this purpose, document analysis was used ***(see Appendix 5 for details, p.158)***.

3.9. Data collection procedure and timeline

The researcher planned to collect data during the first term of the school calendar from February to April 2020. Data was first collected through lesson observations, which are classroom practices for the participants towards the end of January, February and Macrh 2020 for both schools A and B. Both schools had scheduled to teach probability in the period mentioned on different days selected according to the departmental timetable. The research lasted for about 8 weeks. The researcher observed and documented 16 lessons; that is two lessons for each week. Lesson observation was done every time there was a probability lesson right from the introduction to the conclusion because the researcher was given the timetable as a guide. The lessons were observed in order to see the constructive social activity model/ models for a particular task setting and learner engagement. After each lesson, the researcher then conducted FGDs with the five learners who were sampled from each class A or class B depending on the school observed and this was done with the help of the teachers. FGDs assisted in giving details to certain activities, practices, actions, responses and documented tasks, performances, and to see if they were consistent with what was observed as regards the use of constructive social activity models in the learning process.

After that, the researcher conducted semi-structured interviews with the teacher in order to check for consistency in what could have constrained the choice of modelling tools, learner participation, and mainly as regards the challenges and opportunities in the process of knowledge acquisition from constructive social activity model perspective. The researcher followed this order because she wanted to observe the learners' natural way of learning probability-using tools in the process of learning. FGDs were conducted after the lesson because it was assumed that learners would have learned the concepts and be able to discuss questions or issues of the researcher‘s interest with their knowledge of probability.

Conducting FGDs before being observed would have limited the ability of learners to express themselves in a way due to limited knowledge about probability. Cohen, Manion and Morrison (2004) advised that a focus group usually comprises five to eight individuals who share certain characteristics that are relevant for the study. For lesson observations, the researcher captured videos and snapshots while for semi-structured interviews and FGDs only some audio recordings were done. This helped the researcher during data analysis as it was easy to revisit data each time clarity was needed. The videos were transcribed and any recordings were coded accordingly. The discussions were rooted in the challenges, successes, and opportunities that were experienced during the teaching and learning process. Willig (1999) acknowledges the ways individuals make meaning of their experience and in turn, the ways the broader social context impinges on those meanings, while retaining focus on the material and other limits of reality. Having looked at the data collection and procedure for this study, in the next section data analysis is discussed.

3.10. Data analysis

The researcher started by arranging the field notes that are data collected from the lessons observed, focus group discussions, semi structured interview and learners exercise books and worksheets(documents) each time data gathering was done. The researcher familiarized herself with each data set from each instrument used. Video taped data was transcribed and captured only that which was familiar to research questions from which preliminary codes were assigned across all data sets. The initial coding was done accross all data set collected from the instruments used. For instance, data from document analysis sheet were analysed by searching for meanings on how learners used tools to conceptualize the concepts of probability individually through the written tasks and how teachers assited them as individuals in the comments they made to learners while marking the tasks. Having searched different data sets from the instruments used, initial codes were assined from which themes were generated across different data set. Themes were reviewed named and re-defined as major themes and sub-themes and these were general or common ideas that related to each of the answers that could respond to the research questions under the study. Dawson (2002) states that, for qualitative data, the researcher might analyse as the research progresses, continually refining and reorganising in light of the emerging results. Therefore, data was analysed thematically (McMillan & Schumacher, 1993) whereby data is identified or examine using underlying ideas and how they connect to each other from all the research instruments that were used as well as making assumptions, conceptualisations- and ideologies (Braun & Clarke 2006). Thus, the development of themes themselves involved interpretive work and the analysis that was produced could not just be descriptive, but also already theorised (Braun & Clarke 2006). **Table 3** is an example of how themes were generated from coded and categorised data during data analysis.

Table 3: Example of how themes were generated during data analysis

Lesson observation	Document analysis	FGDs	Semi-structured interview
TA. Using die, coin, and marbles. (1) **TA.** assist learners to connect a model tool to the concept.(3) **LB.** Made judgement and conclusions based on their mental processes.(1) **TB** what comes into your mind when you hear the word Probability(1) **TB.** going round marking learners work (3), **TA.** allowing learners to attempt questions on the board,(3) **TB.** giving learners homework,(3) **LA.** Carried out experiments. (2)	**TB.** Strictness and nonstrictness to learners' solutions. (3) **(L_1A, L2A, L_3A, L_4B)** only know chess, coin, die, that's all. (1) **L_4A**, Our sir is not helping us madam because some of us even homework we even feel lazy to work on it and submit for marking, he keeps books for days and days.(3)	**L_5A** my experience is using objects makes it easy to follow the concepts well, you cannot even doze. (2) **L_2A** The use of tools/objects help to have a clear picture these really helped me to understand probability. (2) **L_7B** "even I don't understand what I was writing our teacher was just too fast, especially when just one person answers it correctly him takes it that everyone has understood. (3)	**TB**…madam, you have seen for yourself how big this class is if they were to start demonstrating everything. We cannot finish the syllabus.(1) **TA.** am usually very busy, am acting senior teacher and head of Mathematics department and I have more than 5 classes with big numbers just look at these heaps of books for the other classes waiting for me to mark and these are examination classes. Anyway, I will try to mark them soon when I have time.(3)

For example, after coding data as shown in Table 2, the two themes for research question two on how learners engaged in the use of constructive social activity model for conceptual understanding of probability were generated as; verbal reasoning and physical manipulation. Whereas the codes **TA** imply **Teacher A** in **School A** and **TB** for **Teacher B in School B, LA** implies learners in **School A and LB** implying learners in school **B,** for those included in focus group discussions **L_1A….L_5A and L_6B….L_7B .** Also **1, 2, 3** represent each research questions (e.g. the codes **1, 2, 3** relates for **research question 1, 2** and **3** respectively).

3.11. Credibility and trustworthiness of the study

In order to ensure credibility and trustworthiness, participants' own words, and snapshots of learners have been used in the presentation of findings. During repeated observations, the researcher-built trust with participants, found gatekeepers to allow access to people and sites, established rapport so that participants could be comfortable in expressing themselves, and reciprocate by giving back to them what was being studied. The researcher also ensured collaboration; this means that the participants were involved in the study as co-researchers or in less formal arrangements. Moreover, the themes that emerged after data analysis were subjected to expert view to see whether they were in line with recordings and recognisable (Merriam, 1998). Furthermore, in order to ensure credibility, my supervisor and colleagues cross examined the data to ensure that they aligned with the study, and if they did not make sense alternative themes were suggested. According to Shenton (2004) credibility and trustworthiness can be attained by adopting research methods that are well established in qualitative approach such as observations, focus group discussions, semi-structured interviews and document analysis and these methods were used in data collection. Moreover, the use of lesson observations, semistructured interviews, FGDs and document analysis helped the researcher to triangulate data thereby reducing the effect of investigator bias (Maxwell, 1992; Shenton, 2004).

3.12. Ethical considerations

Before embarking on this study, the researcher obtained ethical clearance from the ethics committee of the University of Zambia (***See Appendix 9 for details, p.157***). The researcher also observed originality, quality and ownership of data as well as honesty (Creswell, 2014) by obtaining primary data in the actual learning process of probability. After being given ethical clearance by the committee to go out for research, the researcher then got permission from the District Education Board Secretary (DEBS) for Mbala district to gain entry into schools. The researcher also got permission from the head teachers of the sampled schools (***See Appendix 3 for details, p.149***). The participants were assured of their protection, anonymity and dignity (Henning 2004). The researcher explained the purpose of the study to the participants in advance and told them that their participation in the study was on voluntary basis, and that they could withdraw from participating at any stage of the study if they feel they could no longer continue participating. No names of participants and schools are mentioned in the dissertation instead pseudonyms such as in **Section 3.10** are used.

3.13 Summary of chapter

This chapter looked at the methodology on how the research was carried out. The research used the constructivist paradigm and the research design was descriptive phenomenological and a qualitative approach. Homogeneous purposive sampling was used to select the two teachers who were teaching probability to grade eleven learners at the time of data collection, and ten learners to include in the FGDs making a total of twelve participants. The data collection instruments that were used were lesson observations, focus group discussions, semi-structured interviews for teachers and document analysis of learners' worksheets. The data was then analysed thematically. The researcher also put into consideration the research ethics which included confidentiality of information obtained from the participants, anonymous identity of participants and informing participants of the aim of the study. Credibility and trustworthiness of the data and the research process were not overlooked, these included; giving excerpts and snapshots from the data, member checking whereby follow up questions were made to affirm the statements that were earlier made by participants, expert review and disconfirming evidence. In the following chapter, the researcher presents the findings of the study by linking with the research objectives as given by the respondents from the field.

CHAPTER 4

FINDINGS

4.1 Overview

In the previous chapter, the methodology that was adopted for this study was discussed. Hence, this chapter is a presentation of the findings from lesson observations, semi-structured interviews, document analysis and focus group discussions (FGDs). The themes that emerged from lesson observations, interviews, document analysis as well as FGDs were put under any of the three research questions according to how they were deemed suitable.

i. What constructive social activity models do learners use when learning probability?
ii. How do learners engage in the use of constructive social activity models for conceptual understanding of Probability?
iii. How do teachers assist learners in enhancing the clarity of concepts in Probability from the constructive social activity models perspective?

Findings for research question one revealed the following: it was either the use of some of the physical models embedded in the content of their reference books, or use of psychological models (cognitive modeling). Findings revealed that learners were engaged in the use of constructive social activity models in any of the following ways; physically performing the task under the directions of the constructive social activity model's instructions (overt guidance), whispering instructions to himself or herself while going through the constructive social activity model/task. They used conventional procedures and notations set in the content/task, copying from a fellow or from the worked examples, and self-reinforcement. Regarding research question three, teachers assisted learners in enhancing the clarity of concepts in Probability from the constructive social activity model perspective in either one of the following ways; setting the tasks or concepts with the required tools (modeling tools), probed learners' prior knowledge of the constructive social activity model (situation definition/ problem definition) and their experiences, asked learners to carry out experiment, giving learners chance to explain the link between the constructive social activity model to the concepts under discussion, going round marking learners work, allowing learners to attempt questions on the board, allowed learners to work in groups, giving learners homework, and strictness and non-strictness to learners' solutions. Snapshots, excerpts and written work, are provided as evidence to these findings.

4.2 Research question 1: What constructive social activity models do learners use when learning probability?

The first research question focused on observing what was used as a constructive social activity model/s in the classroom learning process of Probability. The constructive social activity models either could be from the learners' own reference books, word problems, or embedded in the content or improvised by the teacher or the learners. Findings of research question one revealed the following two themes; it was either the use of some of the physical or concrete models embedded in the content of their reference books and, or use of psychological models (cognitive modeling) which was exhibited in the classroom learning process of Probability.

4.2.1 Use of concrete models

The concrete models which were used were; dice, marbles and a coin.

4.2.1.1 Dice

TA linked the six-sided unbiased dice shown in Figure 2 to introduce probability. He said, *These dice have dots on each side and so we will use these as our outcomes*.

Figure 2*: TA displaying dice on the table in view of learners*

4.2.1.2 Coin

A coin is also another concrete constructive social activity model that learners from School A used to develop the concept of probability.

> So TA...*Can you come in front, here are the coins, will use the side for the coat of arm as 'Head' and the side where there is an animal as 'Tail' alright! Yes sir*! Learners in school A (LA) responded at once.

The coins in Figure 3 had a coat of arm on one side and a bird on the other side.

***Figure 3**: TA displaying coins on the table in view of learners*

4.2.1.3 Marbles

In addition to other concrete models which were used to present probability concepts by learners in school A during lesson observations were; marbles of different colours (green, blue, purple and red).

> TA..*I have marbles of different colours which I want us to use right now, I will ask you to come in front to pick a marble from this bag.*

The marbles were used to introduce the concepts of picking randomly with or without replacement, mutually exclusive events, compound events and theoretical probability. The snapshot in Figure 4 highlights the physical or concrete constructive social activity models (marbles) which learners used.

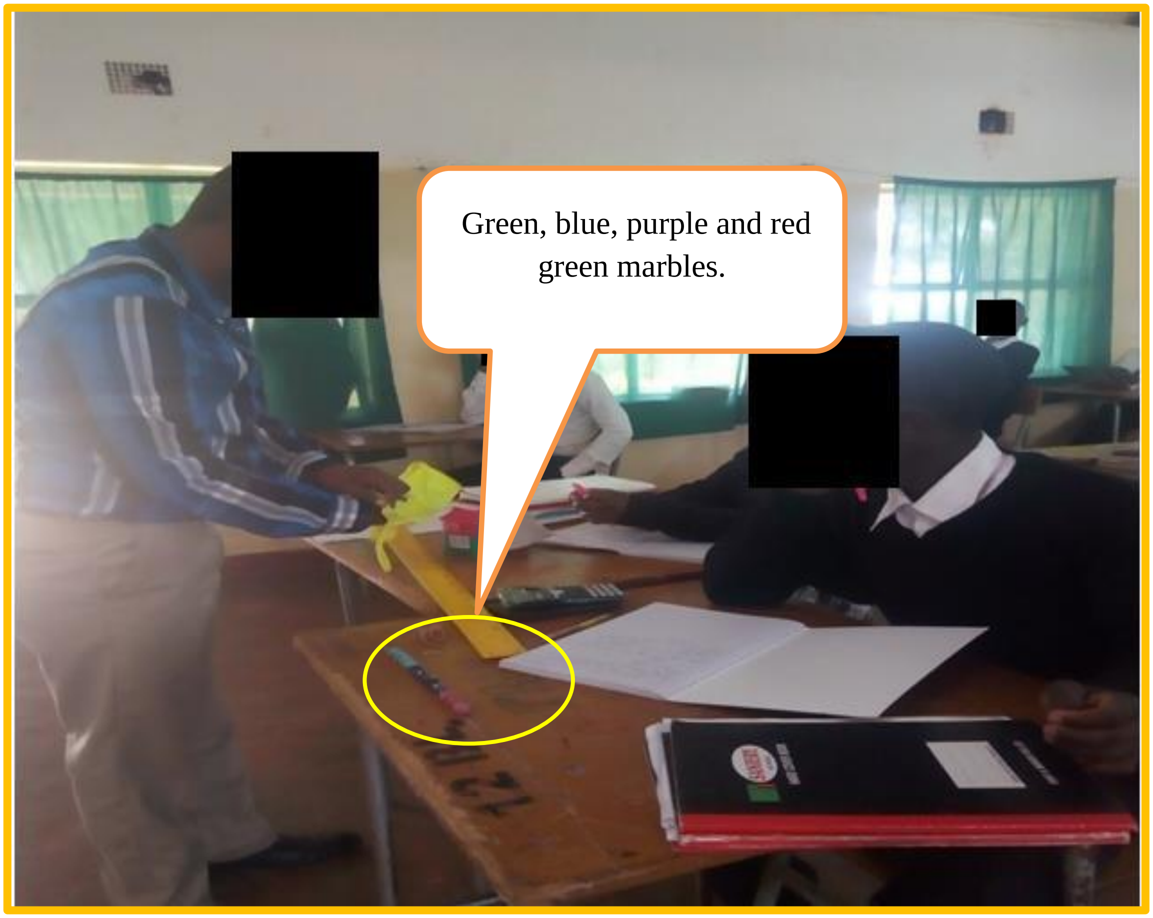


Figure 4: *TA displaying marbles in view of learners*

4.2.2 Use of psychological models (Cognitive modelling)

Research results from lesson observations showed that the teacher tasked the learners to use cognitive modelling during the lessons without use of tools. The following themes emerged; learners tried to use cognition concept from a model perspective, making logical judgement, copying, images or diagrams, and memorisations to present probability concepts.

4.2.2.1 Cognition concept from a model perspective

In an instance where learners in School A used a concrete model, the teacher assumed that the mind of the learners had then mental representations analogous to data structures and would be able to compute a given task following the procedures or algorithms demonstrated previously from a model perspective. For example, after the learners participated in the chance of picking marbles of different colours at random from a bag that the teacher had come with, L_2A had to use cognitive modelling or psychological modelling to work out task 2 from the board, as there were no concrete pens. The other example was to do with letters of the English alphabet that did not have concrete models too. The following extracts are typical examples showing the development through cognitive modelling for the learners in School A with TA:

> ***TA****...I want someone to try this on the board, it's the same thing with the one for picking marbles, I will write two examples here....anyone to try?*

TA proceeded to write the two questions on the board as examples. Learners put their hands up and the teacher pointed at L_1A first and then L_2A to go in front and work out on the board as shown by the snapshots in Figure 5 A and B:

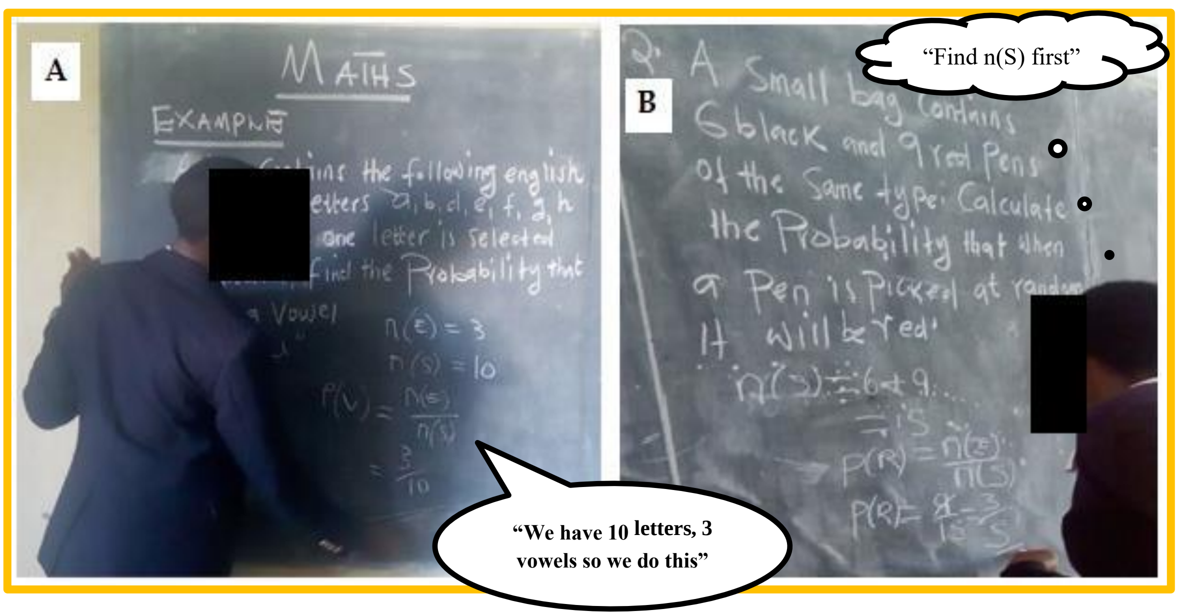


Figure 5: *L_1A and L_2A using cognitive modelling in part A and part B respectively*

In Figure 5, a non-concrete modelling example which L_1A worked out on the board read as follows; *A box contains the following English alphabet letters; a, b, d, e, f, g,h, i, j, k. if one letter is selected at random what is the probability that it is a vowel*?

4.2.2.2 Making judgement

Learners in School B used cognitive modeling or psychological models from the introduction of Probability to the development and conclusion. They did not use any concrete constructive social activity models but made judgement and conclusions based on their mental processes. For example, the following are sampled excerpts illustrating how introduction to Probability was done and how they proceeded with the lesson development from School B with TB.

> ***TB****...what comes into your mind when you hear the word Probability? You already learnt these things in Grade 9.*
>
> ***L_6B****…mmm in grade 9? I can't remember.* While whispering.
>
> ***L_7B****...Probability is the likelihood of an event happening, sir.*
>
> ***T_2B****...Alright, pay attention Probability is divided into two main branches which are; experimental and theoretical Probability, in my case I want us to start with* ***1. Experimental Probability…****so rolling die what are the expected outcomes? .... and what about tossing a coin?*
>
> ***L_7B****...the outcomes for rolling a die are 1, 2, 3, 4, 5, 6…. mmmm I think that's all and coin it has head and tail.*

Furthermore, the lesson was then developed with a word problem as a worked example on the board by TB which learners copied in their exercise books as shown in the snapshot in Figure 6 part A and was immediately followed by an exercise in Figure 6 part B:

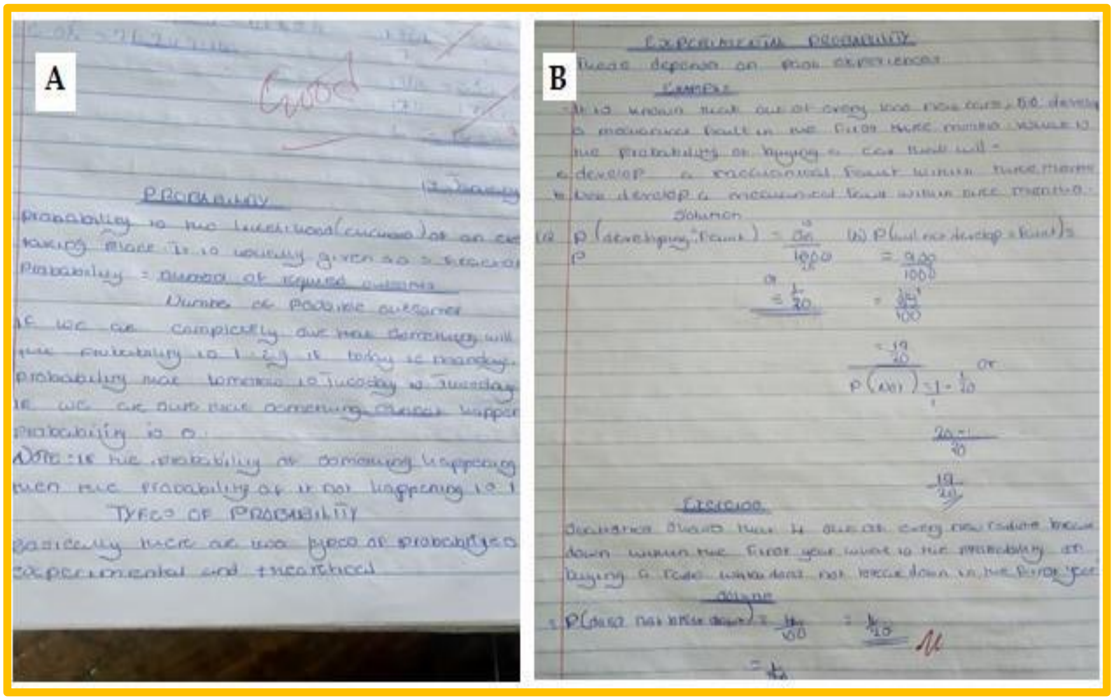


Figure 6: *Cognitive modelled notes, part A, cognitive modelled example and exercise, part B*

Learners from school A actively used the cognitive modeling by judging a situation in most instances more than the learners in school B. For instance, when introducing the concept of impossible probability, the following were a typical example of excerpts of the conversation;

> ***TA***..*consider the letters of the English alphabet and vowels, what is the sample space?*
>
> ***L_3A***..*English alphabet sir is the sample space and the favorable outcomes are all the vowels?*
>
> ***TA***..*OK, good what is the probability that today any one of you will eat pizza?*
>
> ***L_1 A..****mmm sir, it is not possible because they don't make pizza here in Mbala where can you even get it, so the probability is zero sir.*
>
> ***TA:..****you are right and this is what we call* ***impossible probability****. Impossible probability is always equal to zero… for example they may bring in the exam something like this….E. g 2 A bag contains 6 white*

pieces of chalk, 3 green, 5 red and 2 yellow. Find the probability of picking a Brown chalk. This is common sense, there is no brown….so it is impossible probability.

The teacher proceeded to write summary notes as shown in Figure 7:

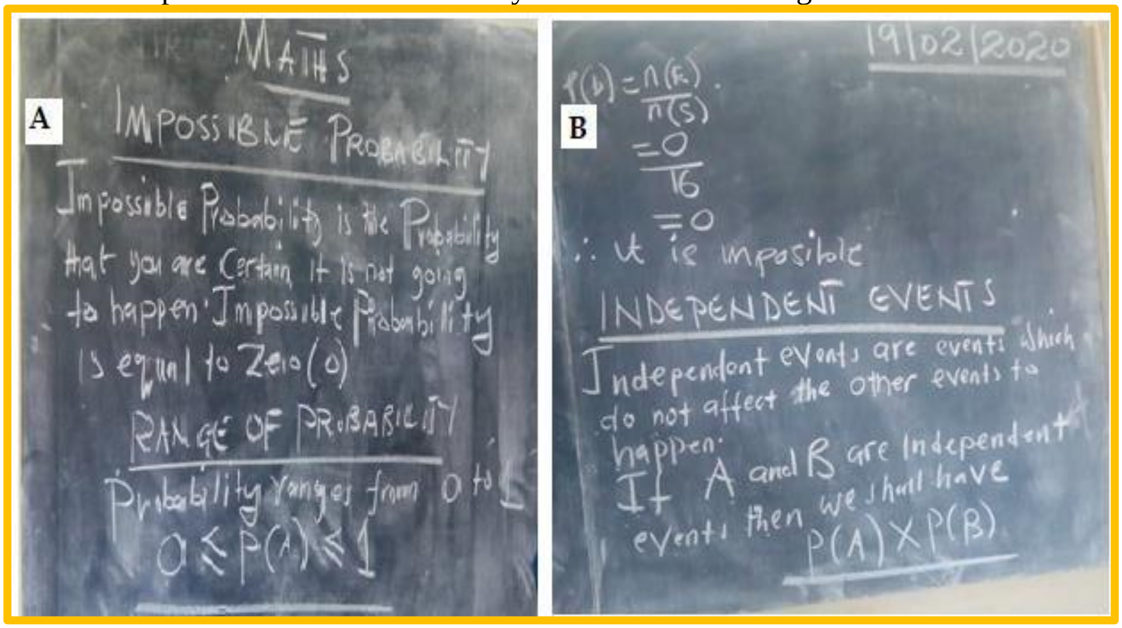


Figure 7: *Cognitive modelling summary notes in both parts A and B*

4.2.2.3 Memorisations

Learners in school B used cognitive modeling without any physical manipulation of the constructive social activity models, learners tried to memorise the procedures especially on tree diagrams and how to assign probabilities. For example, TB introduced tree diagrams by giving it verbal definition and wrote on the board then gave an example when introducing tree diagrams as follows;

Example 1. A bag contains 3 black and 2 white balls. A ball is taken out at random from the bag and then replaced. The bag is well shaken and second ball is taken.

a) Draw a tree diagram to illustrate the possible outcomes.
b) What is the probability that;
 i. Both balls are black
 ii. Balls are of different colours

TB worked out the example on the board which learners copied as well as the notes as shown in the snapshot of Figure 8.

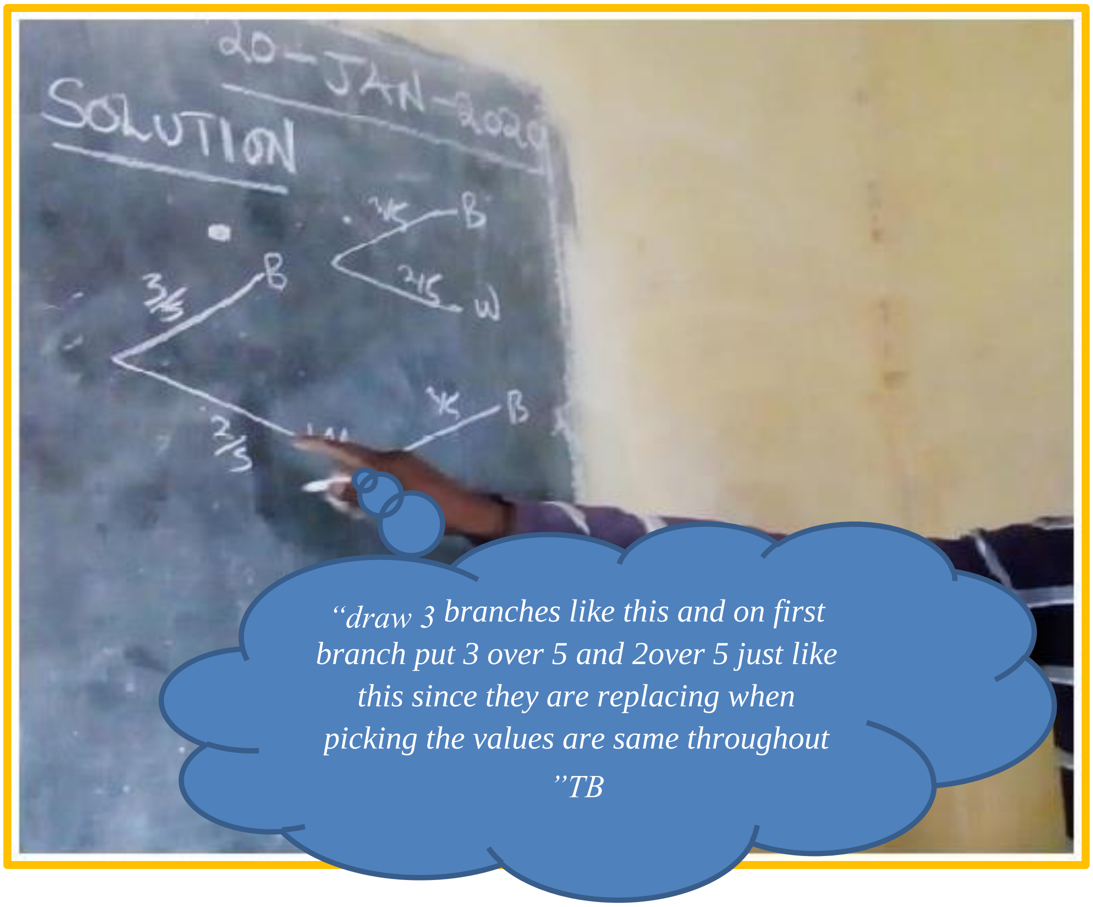


Figure 8: *TB's solution to illustrate a tree diagram to learners*

TB worked out all probabilities assigned to the tree diagram as shown in Figure 8 and in his explanations to answer (i) and (ii) are in the following excerpts below while he also continued solving on the board as shown in Figure 9;

> ***TB*** *“Just assume that you have 5 balls of which 3are black and 2 are white the chances of picking (i) two black is 3 over 5 times 3 over 5 which is equal to 9 over 25 as the probability for picking two black balls. Then two balls of different colours is you work out like this; showing the working on the board while learners listened.”*

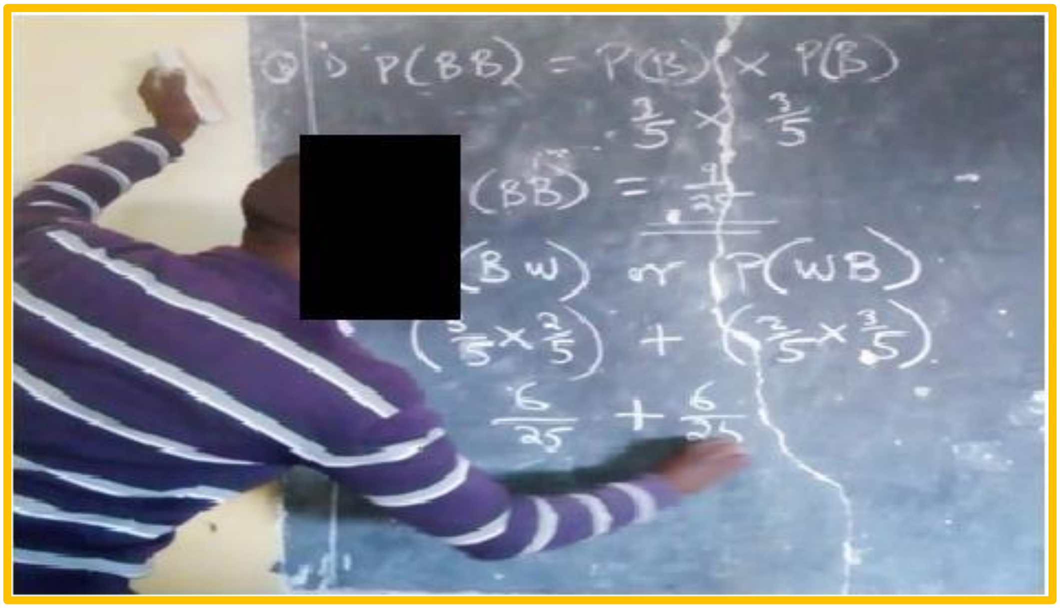


Figure 9: *TB calculating probabilities for b (i) and (ii)*

Learners in school B thereafter copied the notes and the worked examples as shown in the snapshot below in Figure 10.

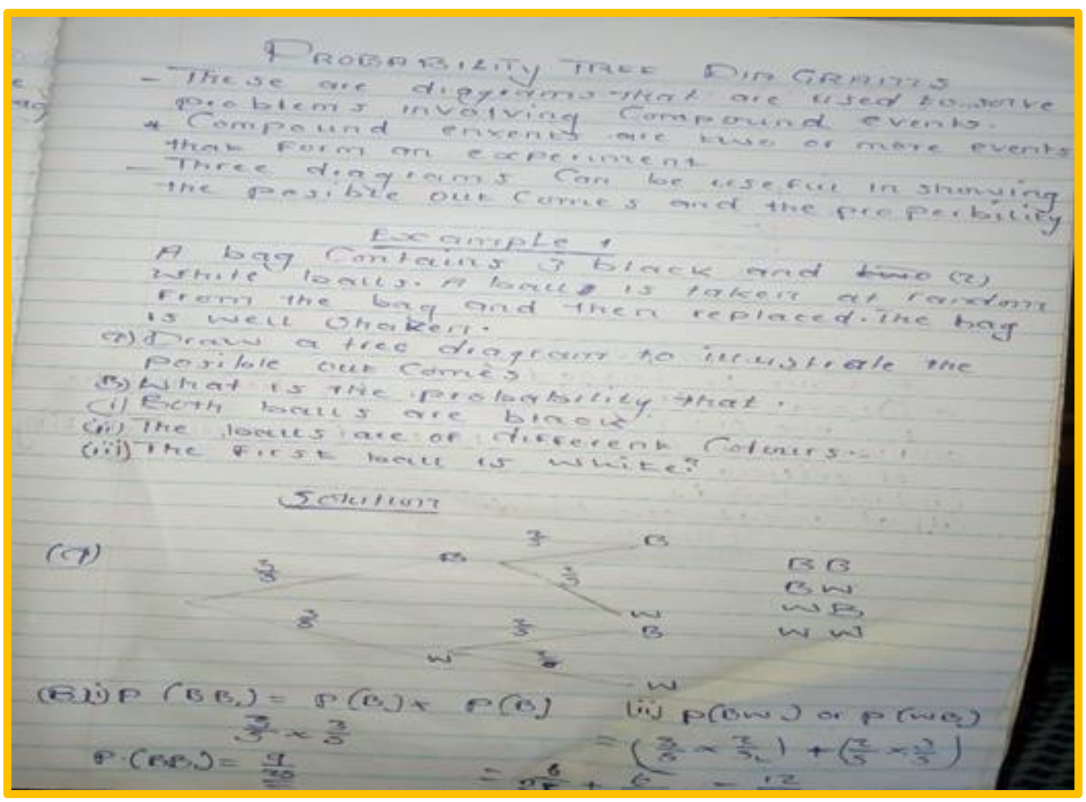


Figure 10: *Example of learners' copied notes and examples in TB's class*

TB gave learners in school B example 2 to work out from the board and an exercise. The researcher noticed that learners were trying to recall a certain procedure on how to construct tree diagrams and assign probabilities by just memorizing what the teacher did in Figure 8 and 9. They were working out at once and were both silent while working out as shown in Figure 11.

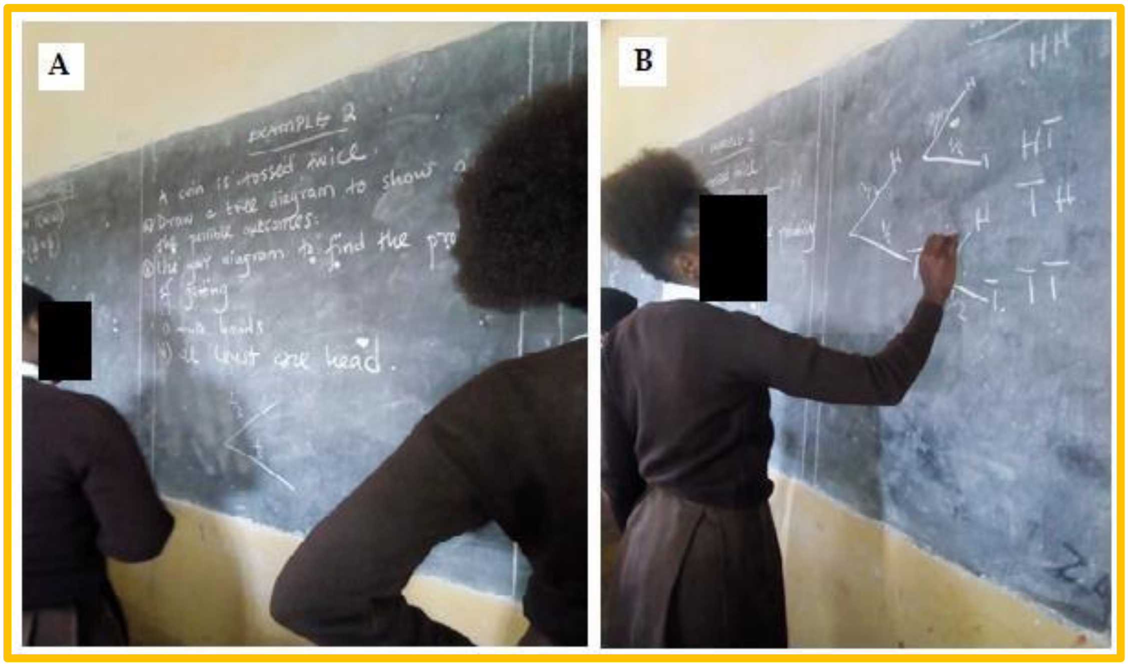


Figure 11: *Learners in school B trying to figure out how to write a tree diagram in both parts A and B*

During the exercise which stated: *A bag contains 3 red and 4 white marbles. A marble is picked at random from the bag then replaced.*

a) Draw the tree diagram to show all possible outcomes.

Find the probability that;

i. *Both marbles are red*

ii. *ii.The marbles are of different colours.* Other learners took time trying to recall the teacher's working from the previous examples. The following are

excerpts from the learners'
conversations;

L_6B *"what did he say we should?"*

L_7B *"what? You mean about drawing the tree diagram?"*

L_6B *"yes, hey how did he do his examples?"*

L_7B *"mm let me check first."*

The snapshot on Figure 12 shows learners as they were trying to consult each other in school B during the class exercise, without a formal discussion.

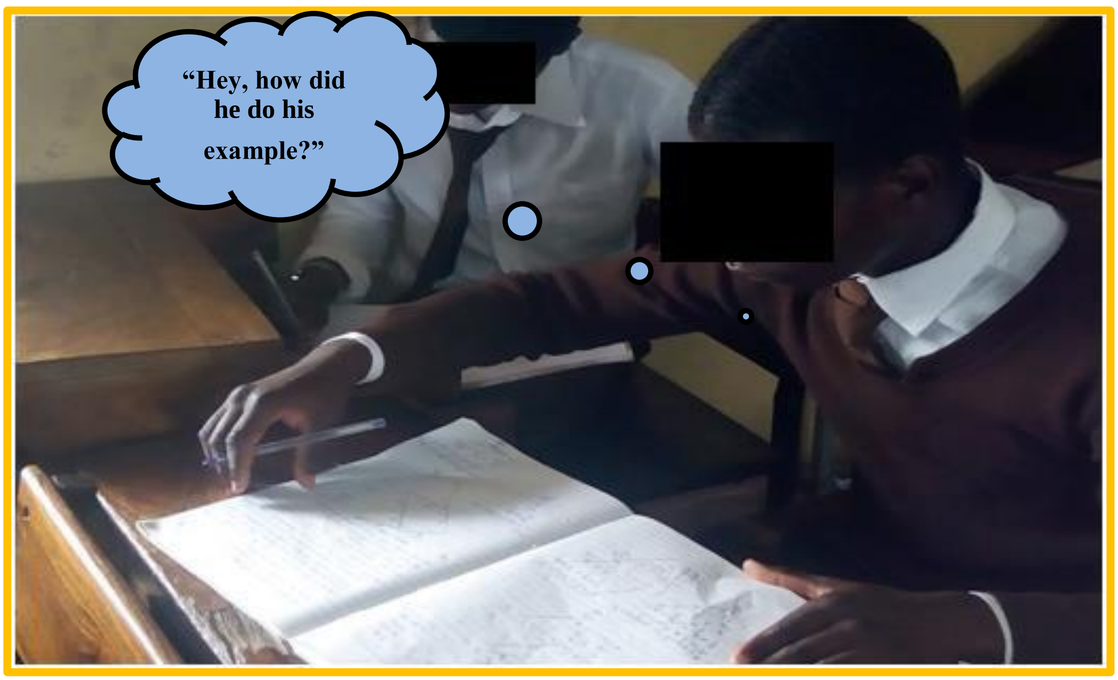


Figure 12: *L_6 B and L_7 B inquiring from each other when answering the exercise*

4.2.2.4 Copying

Some learners could leave their own desks in search of solutions and copy from those whose work was marked. Figure 13 shows a snapshot of learners as they were trying to copy from one another in order to have the working in their exercise books before taking for marking.

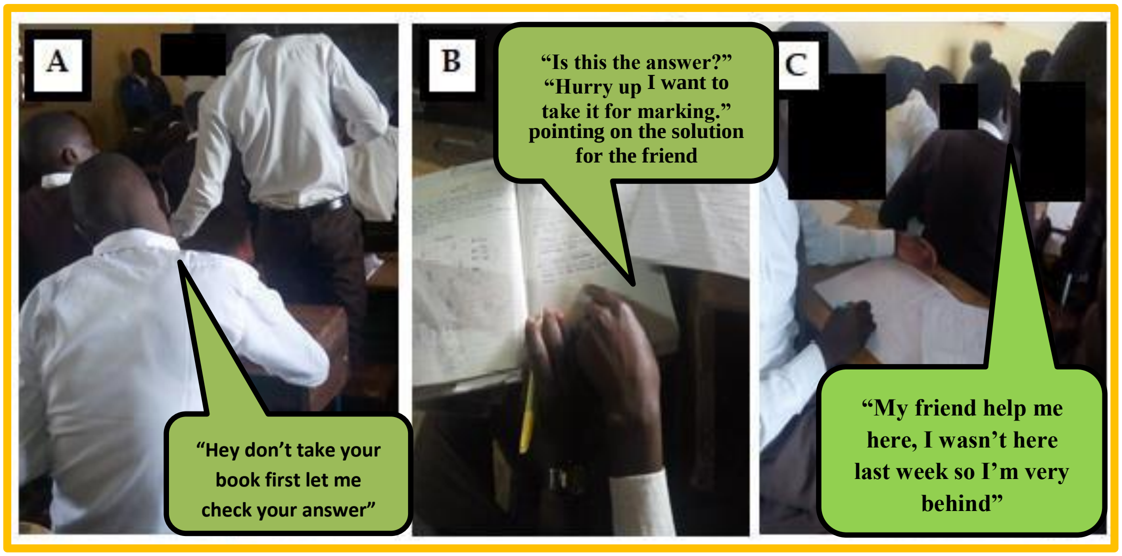


Figure 13: *Learners in school B seeking answers from their friends in all the parts A, B and C*

One of the tasks which learners were asked to work out in which there was a lot of learners copying as illustrated in the picture above was; *A bag contains 3 red and 4 white marbles. A marble is picked at random from the bag then replaced.*

b) Draw the tree diagram to show all possible outcomes.

Find the probability that;

(i) Both marbles are red

(ii) The marbles are of different colours

4.2.2.5 Images or diagrams

Through document analysis the researcher noted learners’ different computational representations of the mind by using the procedural imagery or diagrams which they wrote in their various worksheets. The snapshots were taken in variations that is from those who worked independently, those who worked in collaboration; either those who worked in collaboration by copying or inquiring. Below are the snapshot of various tasks which learners were given during the lessons and the images of their computational procedures. Figure 14 shows the image sample of the procedures for one of the learners who worked independently.

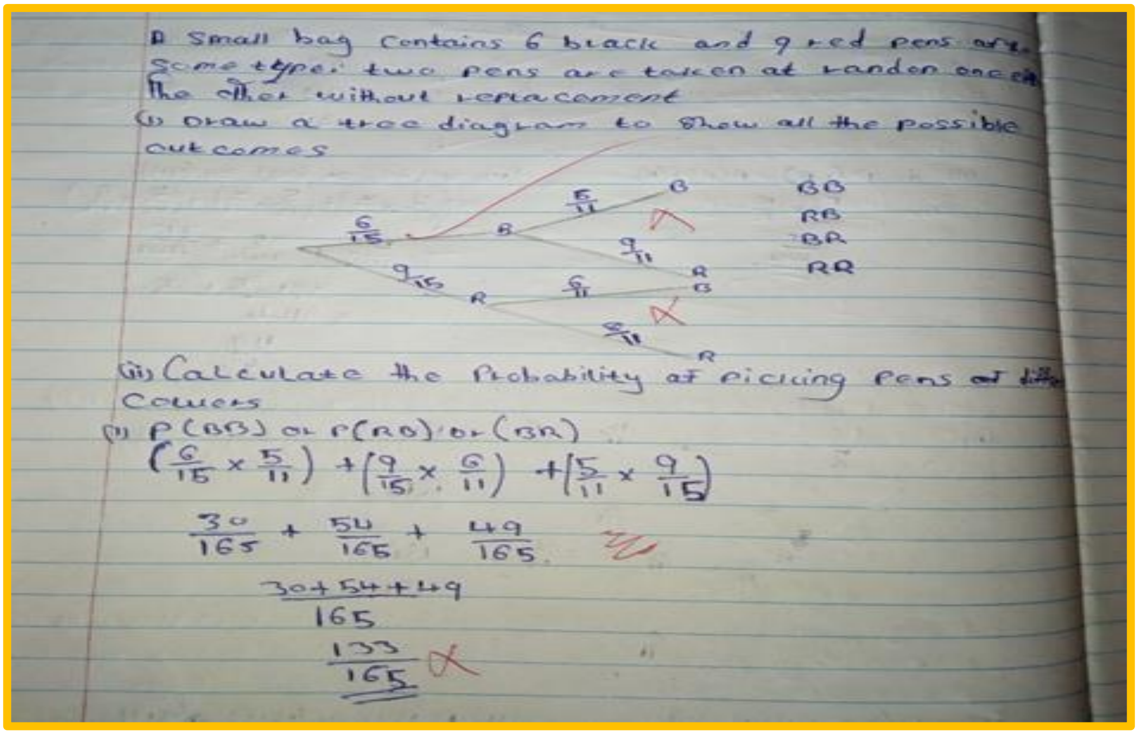


Figure 14: *Learner's independent computational procedures in school B.*

The Figure 15 also is an imagery illustration of cognitive modeling processes of a snapshot taken among the learners who worked in collaboration with each other in school B.

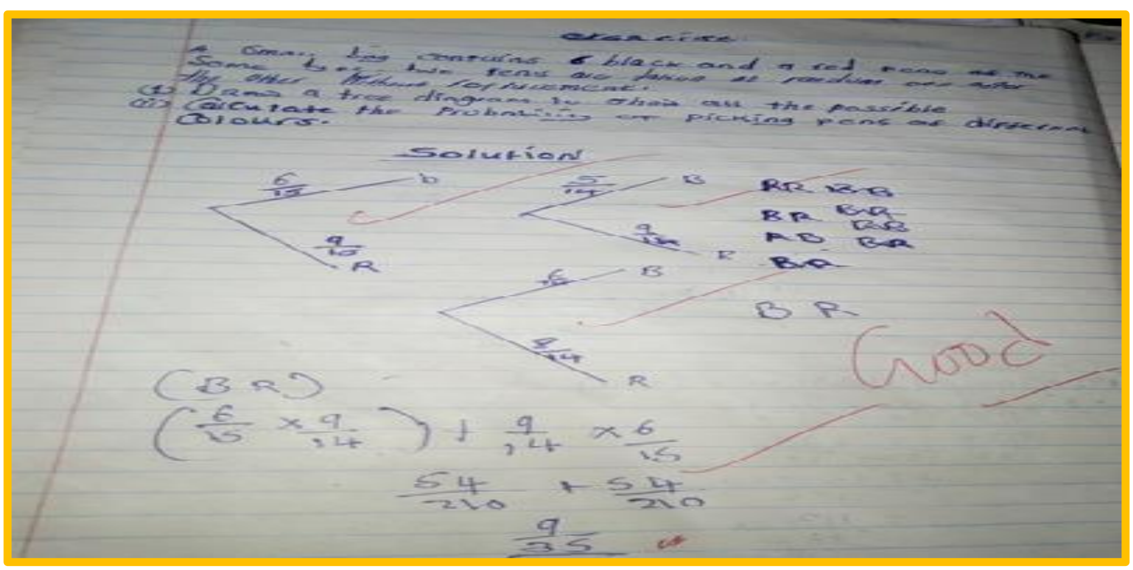


Figure 15: *Learner's independent computation procedures in school B*

Figure 16 also is an image computational procedure of another learner in school B who could only copy the exercise and not compute anything and waited until the lesson was done and wrote down questions for homework.

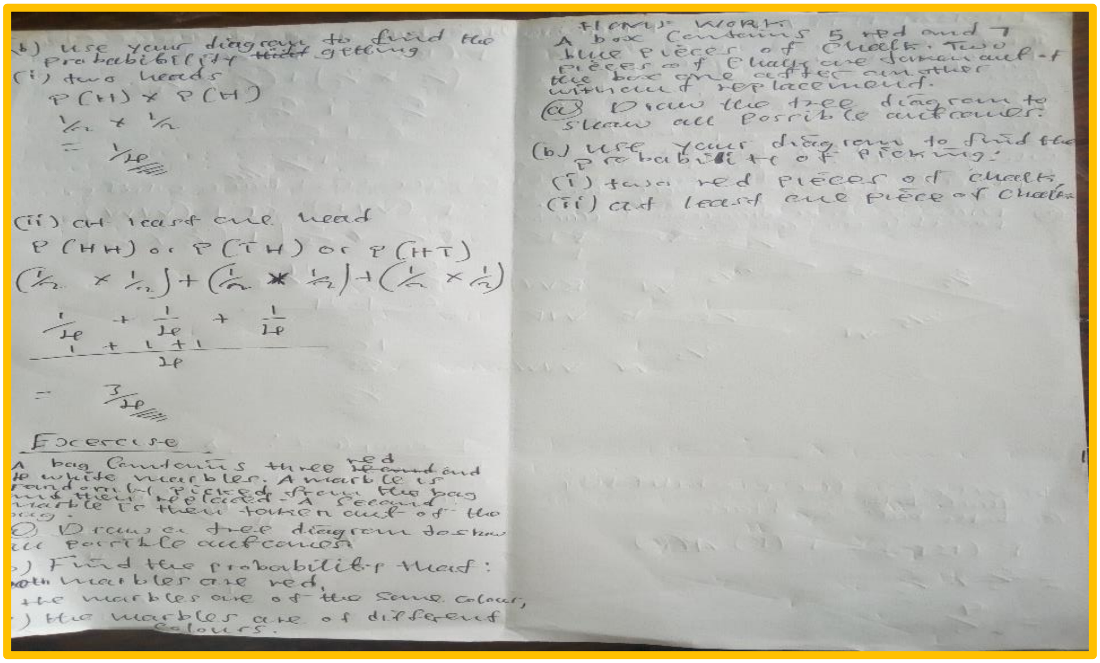
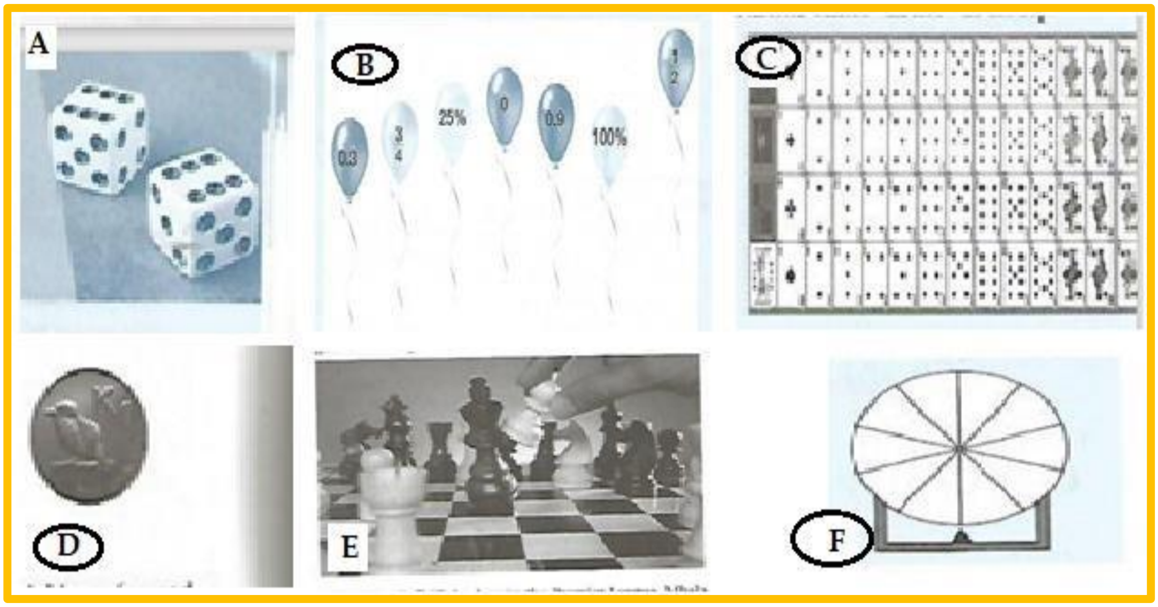
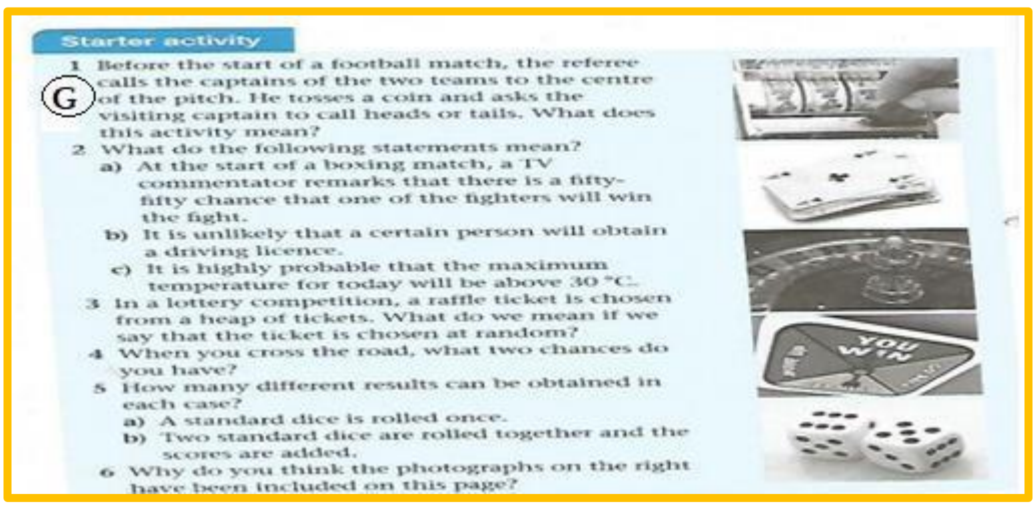


Figure 16: *Image computational procedures*

In order to develop more insight into constructive social activity models used, the researcher also used focus group discussion to establish if there were any constraints with regards to what constructive social activity models to use. Knowledge on how or if any of the concrete modeling tools could affect knowledge construction was vital for the researcher. Therefore, the researcher referred the learners to some of the pictured constructive social activity models from their own textbooks such as Progress in Mathematics Pupils Book, Grades 9 and 11, Zambia Senior Secondary School (ZSSS), Grades 11 and 12, pages 154, 147-157 and 315 and 236 respectively as shown in the Figure 17:




Starter activity

G

1 Before the start of a football match, the referee calls the captains of the two teams to the centre of the pitch. He tosses a coin and asks the visiting captain to call heads or tails. What does this activity mean?
2 What do the following statements mean?
 a) At the start of a boxing match, a TV commentator remarks that there is a fifty-fifty chance that one of the fighters will win the fight.
 b) It is unlikely that a certain person will obtain a driving licence.
 c) It is highly probable that the maximum temperature for today will be above 30 °C.
3 In a lottery competition, a raffle ticket is chosen from a heap of tickets. What do we mean if we say that the ticket is chosen at random?
4 When you cross the road, what two chances do you have?
5 How many different results can be obtained in each case?
 a) A standard dice is rolled once.
 b) Two standard dice are rolled together and the scores are added.
6 Why do you think the photographs on the right have been included on this page?

Figure 17: SOURCE: *Extracts from Progress in Mathematics Pupils Books, for grades 9, 11 and 12.*

The following excerpts are a typical example of the learners' responses when the researcher asked them to identify the pictured models from the textbooks and comment on them:

Researcher: 'Can we identify any of these pictures? Any comment on how we know them?"

I only know chess, coin, die,.mm..that's all (L_1A)

For me, I do play chess am in the school chess club, it also has chances of winning or losing. L_2B

...I know these cards we call them injuka in our language, there are 52 cards altogether but mm it's been long since I played. L_6B

let me help you, there are aces, flowers kings queens spades, something like that and what L_6B is saying injuka yes it is.. L_7B

me what I can say is I know "injuka' but not playing...us girls mm.. when you ask questions in probability I can't know how many Ace or kings are there, maybe if our teacher brings them to show us. L_8B

.... I know a coin and a die, I can give an example on a coin, I have seen this coin used when you want to have equal chances of choosing a goal post in a football game where there are two teams. They use a coin, like if you toss it and it lands head up then you either pick a goal post or if lands tail up, you start the ball possession depending on the conditions. L_9 ***B***

Me am not just into games, that's just what I can say. L_{10} B

These pictures cited in textbooks are for probability concept, some am familiar with while others am not. But most of them are everywhere and I use them in one way or the other. For example, work cards, marbles of different colors placed in one basket and so forth are used in probability, and I'm very familiar with only these objects I have mentioned as they made me clearly understand the concept of probability in general. L_{10}A

..and this is chess, it is just a game also, you can play win and or you lose. L_1A

It was also assumed by the researcher that through conceptual mathematisation during the learning process, learners could have developed a deeper understanding of Probability and link of further constructive social activity models resulting in a mathematical concept through the process of reflection and generalisation. It was therefore expected that learners would subsequently apply probability concepts to other aspects of their daily lives and as such, the concepts would be reinforced and strengthened. Thus, in the FDGs learners were also asked to identify those models from their environment that could be used to explicate probability concepts apart from those they used during the lessons or from the textbook. Some of excerpts from their discussions are cited below:

Researcher: From what you have learned in the classroom about probability concept for example(likelihood), what things or tools in life can demonstrate a chance? (either from world fairy tales or physical models)

> *Wheel of lucky; this is a game on betpawa that has six numbers (2, 4, 5, 10, 20, and 40) and for this there is the chance of winning and of losing. Now, how do I play this game? Firstly, I choose the amount of money I need to place on each number and after placing the amount, there is an option down that says 'SPIN' and when I spin, these numbers will start rolling and where it stands, that is the total amount you have won. E.g. if it stands at 5 and you placed a k5 on it, it gives k25. P (of winning) = 1- p (of losing).* L_4A

> *To me I come from a typical village called Kaka, but even if I come from poor background and others not, these concepts of probability demonstrate that there are more than 50% chances of changing who we are currently, through education.* L_3B

During the semi-structured interview with the teachers, the researcher wanted to know why learners to develop concepts of probability during the classroom learning process did not use some of the concrete models in the learners' reference (textbooks). They gave different perspectives as follows to the researcher's question:

Researcher: Why didn't your learners use some other models/tools from their reference books to explicate Probability concepts?

> *...some models are not familiar to some of the learners here, like what I can say is I have not used anything else in class before because even I don't know some of those things. So it's better for me to choose or suggest a coin, a dice or marbles or sweets or other things familiar to the learners because what I know, will not be challenging even for my learners. For the deck of playing cards sometimes it is difficult because most learners especially the girls have no idea of what contains the pack of playing cards (Standard deck of cards), and moreover the schools of nowadays don't even purchase any of these teaching aids. Anyway it's important to use a teaching aid.* ***TA***

…madam, you have seen for yourself how big this class is if they were to start demonstrating everything. We cannot finish the syllabus. I know learners know coins, dice, marbles because even in grade 9 they leaned. Except some of these things like lottery, chess, jackpot, spinning wheels, routledge, those which are very complicated we just skip them in order not to complicate ourselves. So even in word problems, which involve such we just skip, and pick or create examples which learners are familiar….and things like playing cards I don't remember the last time a question came in the final exam about anything to do with that. ***TB***

The researcher also wanted to find out from *TB* why either learners or the teacher could illustrate no model from probability concepts. The following were the excerpts from the responses:

Researcher: "Why were learners and you not able to use any concrete tool in the process of learning probability concepts then why don't you give chance to learners to come up with models or tools to use when learning probability?"

> *…this topic is a continuation and I take it that they already have ideas on probability since they learnt in grade 9 because these are almost the same things. This class is not mine am just standing in for someone who is on leave because l noticed that they can be very behind, so am just trying to rush them through. I have to also speed up a bit so that they catch up with the other classes because the other grade 11 classes have already done probability. If we had enough textbooks in the department we could give them yes to study on their own and maybe make some teaching aid. TB*

> *….mm giving them chance to come up with their own models before you even teach them! Mmm I don't think so because the language of probability is usually difficult for learners to interpret and what more to even make a model, but it's a good idea anyway, I think with time they can, though it has never crossed my mind. You know sometimes when you are overloaded with so many classes to teach, time is limited to even think about some of those things. TA*

4.3 Research Question 2: How do learners engage in the use of constructive social activity models for conceptual understanding of Probability?

Research question two focused on observing how learners engaged in the use of constructive social activity models for conceptual understanding of Probability. Being engaged in the use of constructive social activity models for conceptual understanding of Probability implied to the researcher, observing appropriate action on it (model tool) which involves learners' physical manipulations in the very context in which it is designed. It was assumed that the more an individual engages in physical actions on the tools, the more they gain experience on the object and the more they learn. Knowledge or situation definition of a tool or an object or model implies the following; firstly, the instance it appears learners must identify (verbal interpretations by use of familiar language, imagery, symbolic, signs and so on. In order to establish how learners, engaged in the use of constructive social activity model, the researcher observed grade 11 learners learning probability alongside document analysis of their individual worksheets in order to see if they were able to retain the concepts. Thereafter, focus group discussions were conducted to have an in-depth understanding on what may or may not have challenged their participation or retention of concepts from a model perspective. The themes that emerged were; verbal reasoning, physical manipulation and visual presentations.

4.3.1 Verbal reasoning

Verbal reasoning emerged as a result of what was observed where learners tried to show understanding and reasoning using concepts framed in words. It was one of the means that was used to evaluate the ability to think constructively, rather than at simple fluency or vocabulary recognition. This was noted either through the questions which learners asked or through responses which learners gave in the process of learning. In this way the researcher was able to note understanding and non-understanding among learners through verbal reasoning.

Research results from lesson observations also showed that learners explained verbally in their own words, the models used in Probability word problems. Learners in school A were more engaged and consistent in interpreting the models and situations of fancy world of fairy tales in order to draw concepts of probability than learners in school B. In their various explanations, some learners who could not do well their teacher could do it for them without redirecting the situation to others. In school A, when the teacher noted some errors he asked capable learners to explain for those who could not explain properly and did not ask for justification. Hence, learners' certain misconceptions in situation definitions were noted and corrected by either the teacher or the learners. The snapshots below Figure 18 highlight examples where the learners were asked to describe the following situations;

> *If you are 15 learners in here and among whom 2 are girls, what are the chances of picking a girl?* ***T$_2$B***

.... sir, it is 1 over 15 chances. ***L_8 B TB***

Ok, can someone else try?

....mm if we were 15 sir, then it will be 2 out of 15 chances. ***L_7 B***

TB*....yes, that's correct 2 out of 15 chances.*

The following were excerpts from teacher A's class in school A;

A bag contains 4 green, 3 red, and 6 blue marbles. A marble is picked at random, find the probability that it is a green marble . ***TA.***

...here the probability of picking a green marble is 4 out of 13 because there are 13 marbles and of which 4 are green, and by the way if this was reduceable we would reduce it but as it is we live it. ***L_4 A.***

Another learner counter reacted to L_4 A

....no it is already in its lowest terms so it is reduceable. L_3A

*Ok. Can anyone describe a die..**TA.***

*....a die has six sides and six numbers..**L_2 A.***

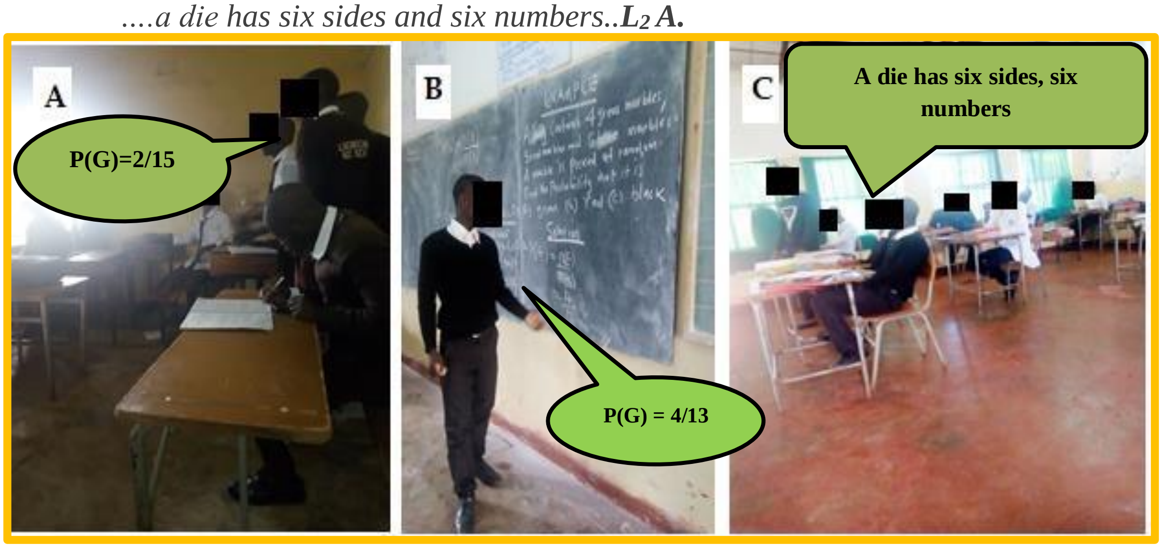


Figure 18: *Learners explaining the meaning of Probability word problems in their own words in TB's and TA's classes in all the parts A, B and C*

4.3.2 Physical Manipulation

Class observation also showed that learners physically manipulated the constructive social activity models following the rules in school A. The models they used and manipulated were coin, die and picking marbles from a bag. For example, TA put up a challenge for his learners by putting green, blue and red marbles in a bag and said,

> *If anyone picks a green marble without looking into the bag by trying two chances I will give you a coin to buy a fritter at break time.*

Learners started rushing in front with excitement in order to be the first to win if they picked a green marble. First participant attempted and picked a red marble, the teacher shook the marbles and tried the second time and a learner again picked blue marble. The second and third participant could not pick any green marble in their first and second trials. They asked the teacher… "*Sir, are you sure there is any green marble in this bag*?" said L_3A. Figure 19 shows how learners engaged in physical manipulation of the constructive social activity models of trying to pick a green marble without looking into the bag.

Figure 19: *Learner engaged physically in picking a green marble*

The TA then called on anyone to roll a fair die which he brought to the classroom such that a 3 is obtained. Five (5) learners tried rolling a die one after another and only one of them obtained a 3. Learners found it so interesting that they were rushing and anxiously waiting to role a fair die. The snapshot below in Figure 20 shows learners engaging in rolling the die one after another. The researcher noted the following outcomes from the experiment for each trial on the excerpts below;

L_1A...Oh no I rolled and scored a 1 sir!

L_2A,..Hey even me, it's a 4! haa!

L_3A,.. Aa my friend I rolled and have scored a 1 too oh no!

L_4A..Yes I won sir my coin, it's a 3!

TA,..Very good boy.

L_5A, ..No, let me also try…oh no I have scored a 4…

TA ..*can you list the outcomes of your experiment*? L_3A listed as "{1, 2, 3, 4,5, 6}

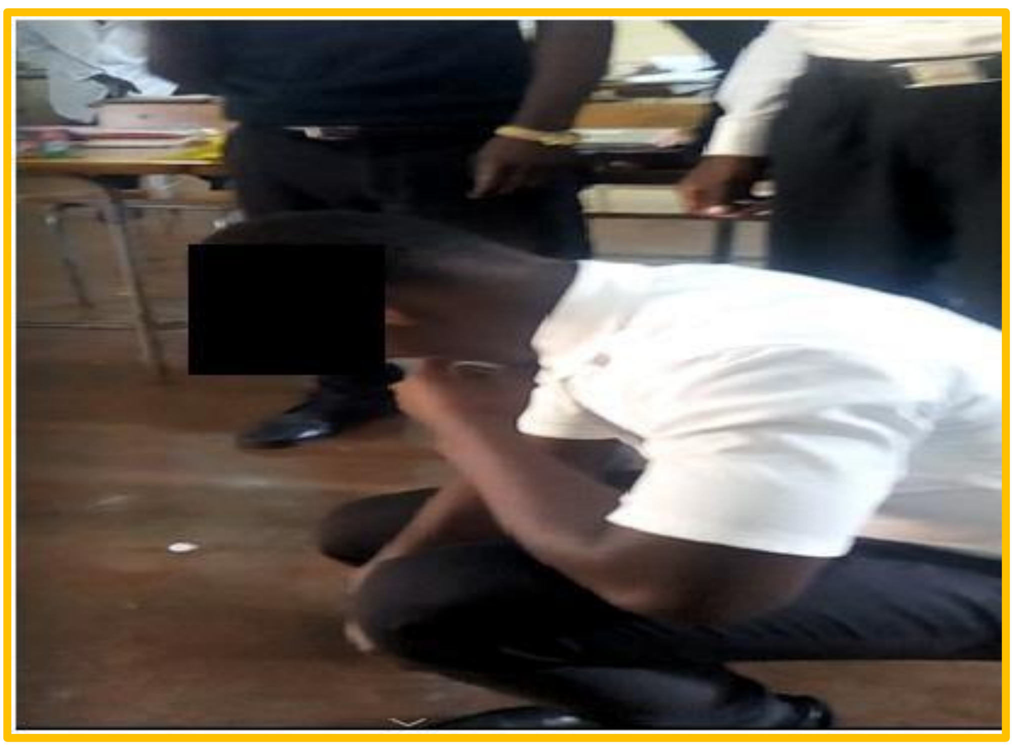


Figure 20: *A learner rolling a die*

Learners in school A also engaged themselves in trying to toss a coin in order to have a head land up as shown in Figure 21. TA asked if anyone would toss a coin and have a tail land up and three learners were given chance to try on the floor. Their outcomes were as follows in the following excerpts;

> *L_3A,..I will start first, ooh I have a head, ah no wait which one is representing a head sir? TA,..I said where there is a coat of arms, yes that should be a head you have scored. L $_1$A, ..oh yes! A tail sir, I have scored. L $_4$A, ..I have also scored a tail. TA,..Let's see, Ok, good.*

The researcher noted from the learners' experiments the following outcomes {H, T, T}.

Figure 21: *A learner tossing a coin*

During focus group discussion the researcher wanted to find out from the learners about what they thought as regards the use of tools in the learning process.

Researcher: How does the use of tools/ objects help or challenge your understanding of probability concepts?

> ***L_5A****, ..my experience is using objects makes it easy to follow the concepts well, you cannot even doze.*

L_6B...Probability is a topic that needs clear understanding of the question before answering the question and due to this, the challenges that I encounter is the language they use to some of the questions i.e. the way some questions are phrased. The reason why this was a challenge to me is because of the language barrier as I was used with the language we mostly use at home.

L_2A ...The use of tools/objects when learning probability helped me to have a clear picture of what the teacher is trying to say when explaining the concept of probability in class. And these really helped me to understand probability.

$L_{10}B$ ...Me the only challenge is when they pick without replacement, then constructing tree diagram that really is a problem for me.

4.3.3 Visual presentations

Some learners made visual presentations without giving verbal explanations and TB did not probe their thinking. TB asked, "*There is an example on the board saying a coin is tossed twice can I have two people to show all possible outcomes on the tree diagram.*" as shown in Figure 22. This was the same situation as in Figures 11,14 and 16 in this chapter.

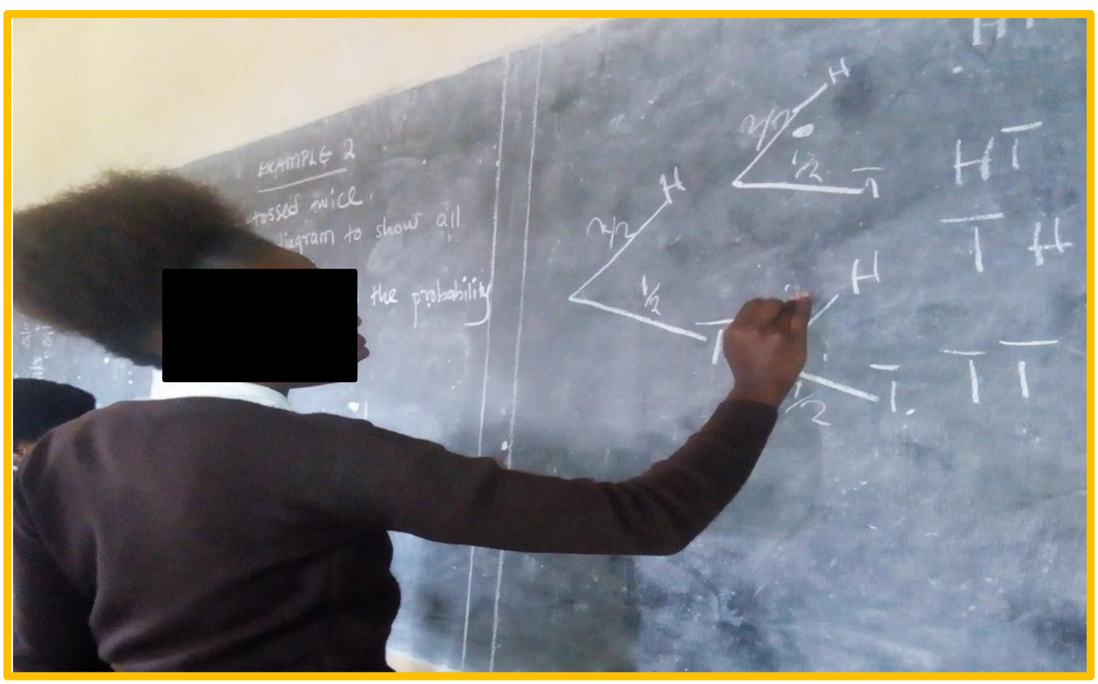


Figure 22: *L_9B and L_7B visually presenting outcomes by constructing the tree diagram*

4.4 Research question 3. How do teachers assist learners in enhancing the clarity of concepts in Probability from the constructive social activity models' perspective?

Research question three focused on observing what teachers did to ensure that the tension and gaps between the daily life experiences of a child and the knowledge of the experts which is in the learners' reference books is bridged. It was therefore necessary to observe teachers settle in a natural classroom learning and how they practiced their knowledge of model tools together with their learners so that they gradually and consciously develop scientific concepts of probability from a model perspective. The teacher was expected to note individual differences, learner capabilities from their own articulation of beliefs and gradually create conceptual shifts in individual learners by mediating those concepts that learners have not yet fully developed as they interact.

Data in this regard was collected through lesson observations, document analysis to check what learners were writing, focus group discussion; and semi-structured interviews were conducted with each particular teacher. The following themes emerged; setting concepts with appropriate physical tools, going round marking learners work, allowing learners to attempt questions on the board, allowed learners to work in groups, giving learners homework, strictness to learners' solutions.

4.4.1. Setting concepts with the appropriate physical tools

TA came to the classroom carrying a yellow bag in which there were marbles, coins and dice to introduce Probability concepts that made learners curious. During lesson observations, only TA set the concepts in the introduction and lesson development by use of appropriate tools such as die, coin, and marbles. This helped in assisting learners' meaning making, and clarity of concepts from the model tool perspective. The researcher noted that in trying to develop the lesson, from a constructive social activity model perspective (conceptual mathematisation) TA did not relate well the outcomes of the experiments that were done earlier in Figure 20 and 21 in this chapter to introduce experimental probability due to lack of clarity. Although the verbal explanations were clear the visual presentation on the board was incorrect as can be seen on Figure 23 in this chapter, learners connected the concepts with the teacher's assistance. The following were excerpts of TA and his learners before writing on the board.

> ***TA****,...you remember this die in my hands which you just rolled?* when learners said yes they were referring to their attempts shown earlier in figure 11*. The teacher said, "rolling a die once, the expected* **outcome scores** *are; {1, 2, 3, 4, 5, 6} which is also called a* **sample space** *and scoring atleast a 1 happens by* **chance***, so a topic that looks at games of chance or looks at chances of something happening is called* ***probability****. The probability of an* **event** *A happening is written as* ***P(A), or p(A****), e.g. the marbles we have among the many other different colours and* ***P (picking a green marble) = P(G) = n(G)/n (all marbles in this bag****) and picking any given colour of a marble is by chance****. So probability is the numerical measure of the ratio of all favourable outcomes to all possible outcomes.*** *When you tossed a coin you noticed it landed on either head or tail up. So again if I ask how many are you in this class? When learners said 15 in a chorus response, the teacher further asked, What would be the probability of picking a girl?* ***L_1A****.. 2/15 since there are two girls in this class. TA, ..very good. So there are two main branches of probability which are experimental like the one you were doing by rolling a die and recording the outcomes and theoretical probability. Now in my case I want us to start with experimental probability.*

In order to clarify the concepts further and link the previous constructive social activity model to Experimental probability concept, TA failed to differentiate between experimental and theoretical probability. As he developed, a lesson with reference to the learners' attempts made in rolling a die in Figure 20 and tossing coin in Figure 21, he wrote summary notes on the board as shown in Figure 23.

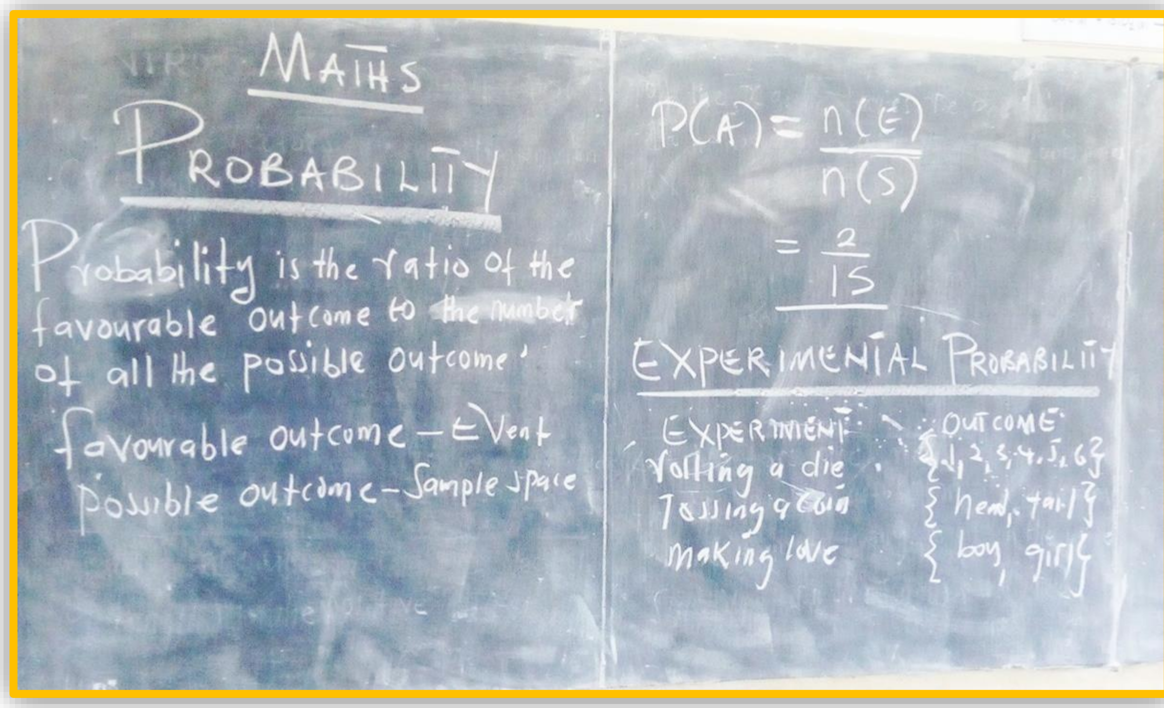


Figure 23: *Shows the concepts clarified by TA from the constructive activity model perspective*

The concept of experimental probability was introduced while referring to the constructive social activity model with clarity in the following manner by TA.

> ***TA*** *...can you list the outcomes of our previous experiment when we rolled a die?*
>
> ***L_2A***..*{1, 2, 3, 4, 5, 6}*
>
> *TA ...From this experiment what is the probability of scoring a 1 in the next trial?*
>
> ***L_4A***...*(scoring a 1) =1/6, that's what is there on the board sir.*
>
> ***L_1A*** *...(scoring a 1) =1/6, yes it is there on the board.* The

class was quiet for a moment.

> ***TA*** *....Recall that this experiment was done by you all when you rolled a die, and this is the die with six sides.*

The learners just paid attention and continued their usual anxious mood.

TA ...And this brings us to what we call a successful trial, where in an experiment, each attempt you were making in rolling a die to get a 3 you note that scoring a 3 is the probability of success and not being able to score a 3 is a probability of failure.

The teacher assisted learners to connect the constructive social activity model of picking marbles of different colours from the bag from Figure 19 as stated earlier in this chapter, to theoretical probability as shown in the example given in Figure 24.

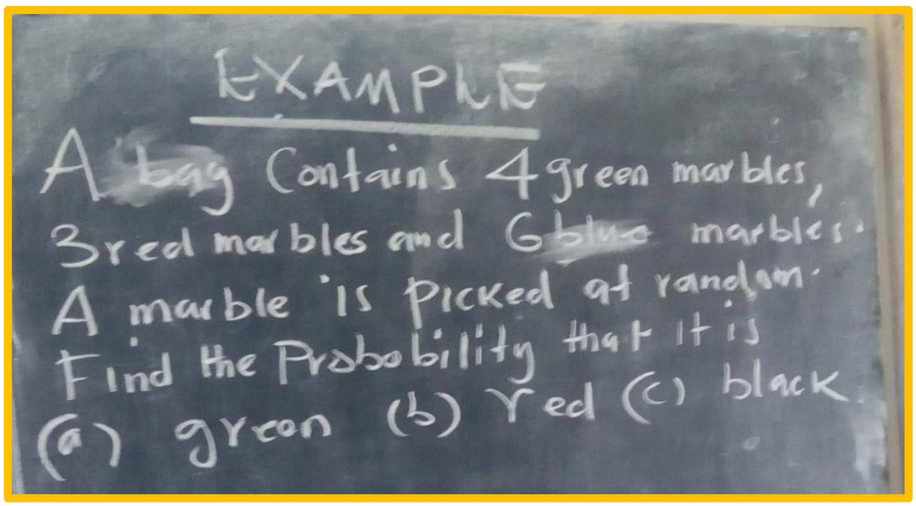


Figure 24: *Example set by TA to assist learners to connect a model tool to the concept*

Independent events and use of a **tree diagram** were developed in the following excerpts examples.

T_1A *..Now the way you were picking marbles randomly and noting the colour again you pick, or the way you were tossing a coin making trials in succession…we say whenever one or more events take place in succession, use a tree diagram to represent those trials in succession and their outcomes for each trial… Before you draw a tree diagram find the sample space, the branches show the outcomes of each trial.*

Figure 25 shows the example on the independent events in succession which was set by TA in order to assist learners' conceptual understanding of probability from a model perspective. TA,..*I want someone to do this example on the board.*

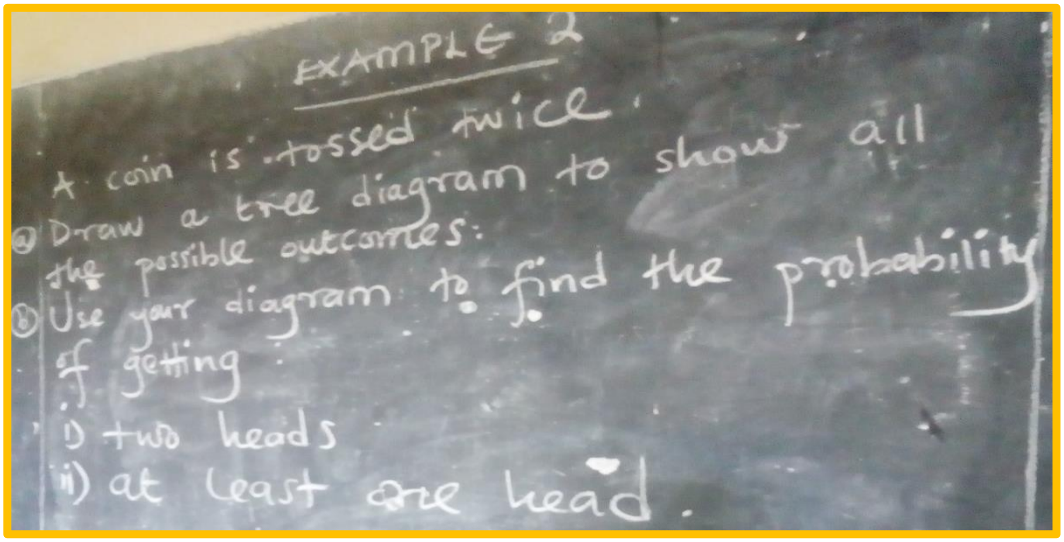


Figure 25: *An example set by TA in relation to the constructive social activity model*

During focus group discussion with learners as shown in Figure 26 in this chapter, the researcher wanted to find out what learners' experiences were as regards teachers' assistance in seeking clarity of probability concepts from a model perspective. Learners referred to their notes and tried to reflect on through each of the sub topics in which some of them performed well while others did not. The ones who had no challenges explained to their colleagues why they did not have challenges and they shared experiences.

Researcher: What guidance do you think the teacher should give or should have given you in the learning process in order that you answer the questions correctly? How did you answer these questions correctly?

> **L_4A**...*In order to make sure that all of us answer the questions correctly on probability, the teacher should have given us clear rules to follow when answering probability questions, I didn't even understand which one is theoretical and experimental probability.*
>
> **L_7B**...*As for me, the teacher needed to give all the conditions that are being applied on probability as this would have helped me to answer almost all the questions correctly and not only this but also to give me clear instructions on how to come up with a tree diagram of any type as questions come differently with different conditions.*
>
> **L_8B**...*for me I just find challenge in the word problems because the teacher can give just one example using marbles but the next question*

will be using other tools which we don't even know. L8B… turned and asked a friend, *Let's see your tree diagram..mmm this is also a challenge for me.. when it comes to picking without replacement.. I need help. L7B..even I don't understand what I was writing our teacher was just too fast, especially when just one person answers it correctly him takes it that everyone has understood.*

Figure 26 below is a snapshot showing one of FGD sessions with learners.

Figure 26: *Learners' session during focus group discussion*

4.4.2 Going round marking learners' work and non-strictness to learners' solutions

Checking and marking learners' work is a way of enhancing confidence in the learners' mental processes of knowledge construction from a model perspective. Teacher A gave tasks and examples relating to a model perspective but did not at any single time mark or check learners' individual working during and after lessons. Figure 27 is a snapshot of learners working which is evident that TA lacked consistency in helping learners get assistance and clarity of concepts even when the tool was withdrawn. There is an unchecked solution for the previous class work, the homework, and the next sub heading indicating Law of probability-mutually exclusive events.

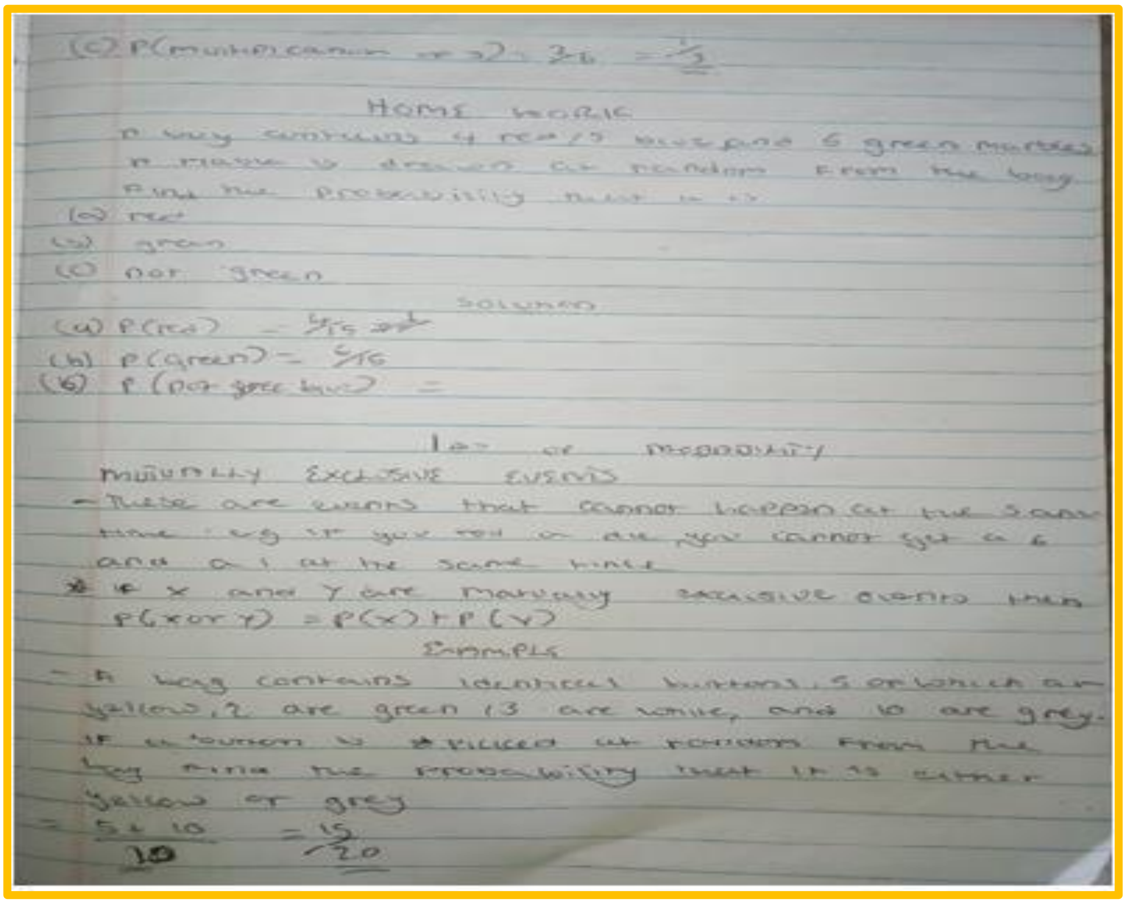


Figure 27: *Snapshot of unchecked working*

No tasks were checked and during semi-structured interviews, the researcher asked TA, why learners working or solutions were not checked.

> ***TA..*** *am usually very busy, am acting senior teacher and head of Mathematics department and I have more than 5 classes with big numbers just look at these heaps of books for the other classes waiting for me to mark and these are examination classes. Anyway, I will try to mark them soon when I have time.*

When learners were asked the same question by the researcher during focus group discussion pertaining to how they think about the situation, the following were excerpts of their responses:

Researcher: why is your work unchecked?

> ***L2A...***he teaches us even Additional Mathematics, so sometimes he has a lot of books to mark.* ***L3A...****it's just that he is always busy but he knows how to teach and he is good.*
>
> ***L4A..****Our sir is not helping us madam because some of us even homework we even feel lazy to work on it and submit for marking, he keeps books for days and days.*
>
> ***L5A..****I don't know if it can be okay to change our teacher because he is very busy though he teaches us well but he doesn't mark our work most of the times.*

TB as stated earlier did not use any of the concrete modeling tools but was consistent with marking learners work. However, he was not strict with assisting in correcting learners' misconceptions and work accurately as shown in Figure 28. The Figure 28 shows a marked solution in which the question stated;

> *The probability that Chakupalesa will go for remedial work on a particular day is 7/10. Find the probability that she will not go for remedial work on that particular day.*

The misconception in the learner's working was p= 7/10 -1 and the teacher did not correct it to P(Not going for lesson) = 1-7/10. The teacher marked the work without reminding the learner of the difference between p = 7/10 – 1 as well as the representation of the mathematical equation in terms of *p* and the symbol for probability. In addition, the second question shows learners misconception as P (die) P (coin) which needed to be noted and corrected but was not due to teacher’s non strictness.

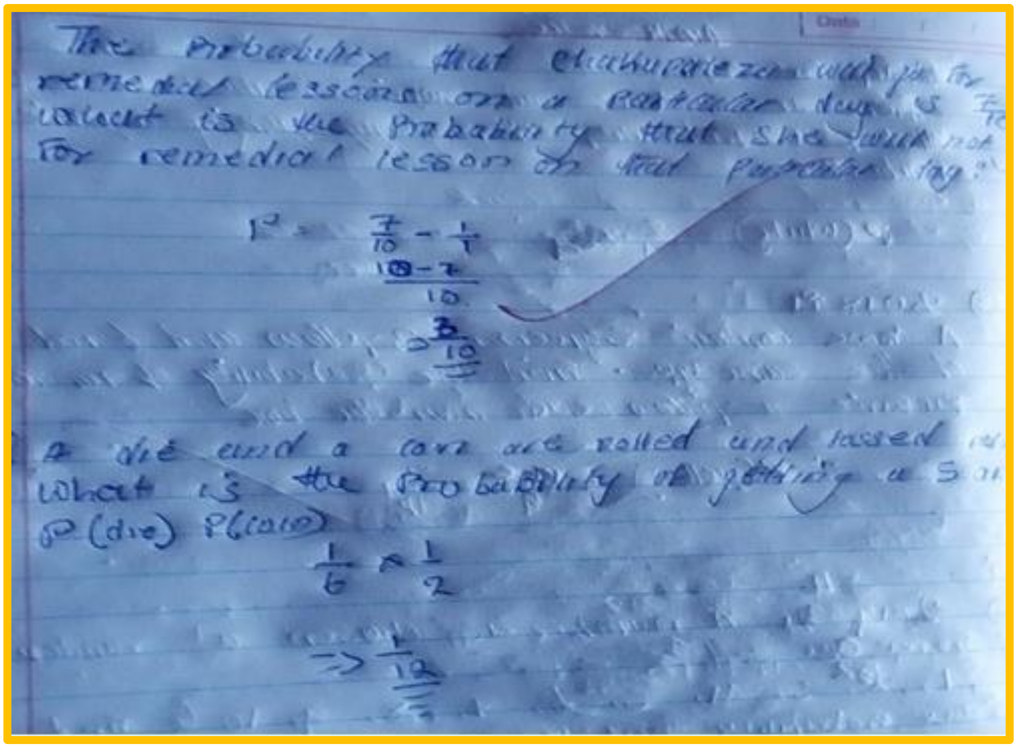


Figure 28: *Shows the snapshot of non-strictness in assisting learners with solutions*

The understanding of tree diagrams construction especially when it involved picking without replacement also showed that none of the teachers indicated any follow up commentary to guide the learners where they needed help, for example in Figures 11, 14 and 31 to mention but just a few, in this chapter.

4.4.3 Allowing learners to attempt questions on the board

Both Teachers A and B gave opportunities to learners to attempt tasks while showing their working on the board but did not remind learners to be loud or to justify their workings as shown in Figure 29:

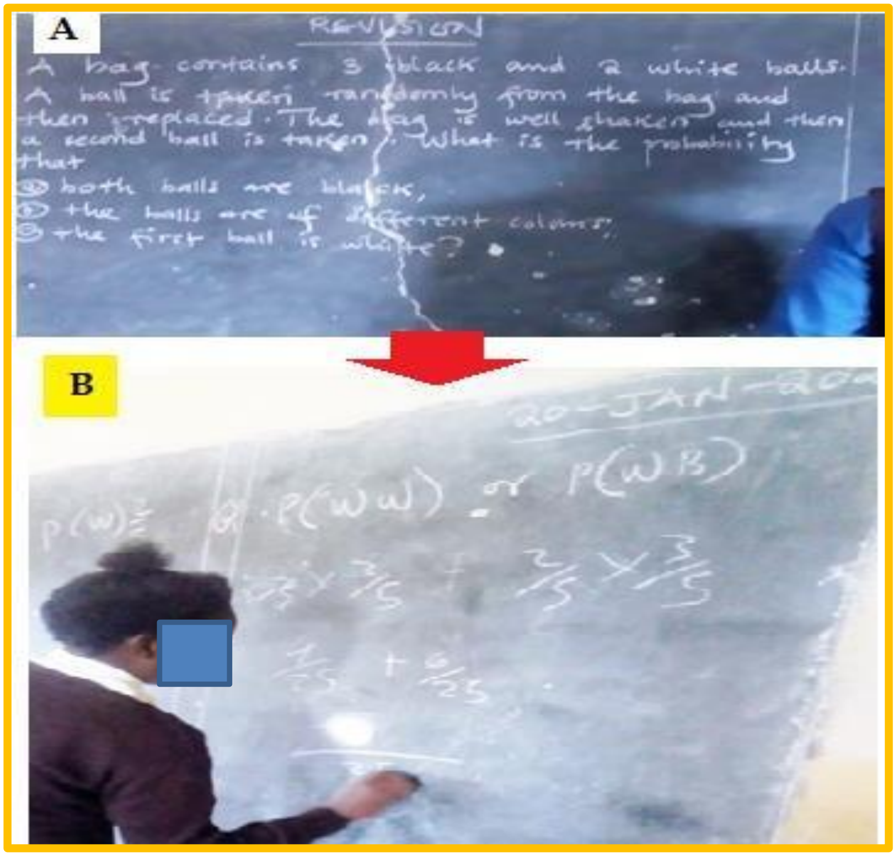


Figure 29: *Revision questions set by TB (Part A) and L6B working out the solution to part c of revision question (Part B)*

4.4.4: Attention to Individual differences

Document analysis reviewed that not much assistance was given to learners who were struggling such as the snapshot in Figure 30.

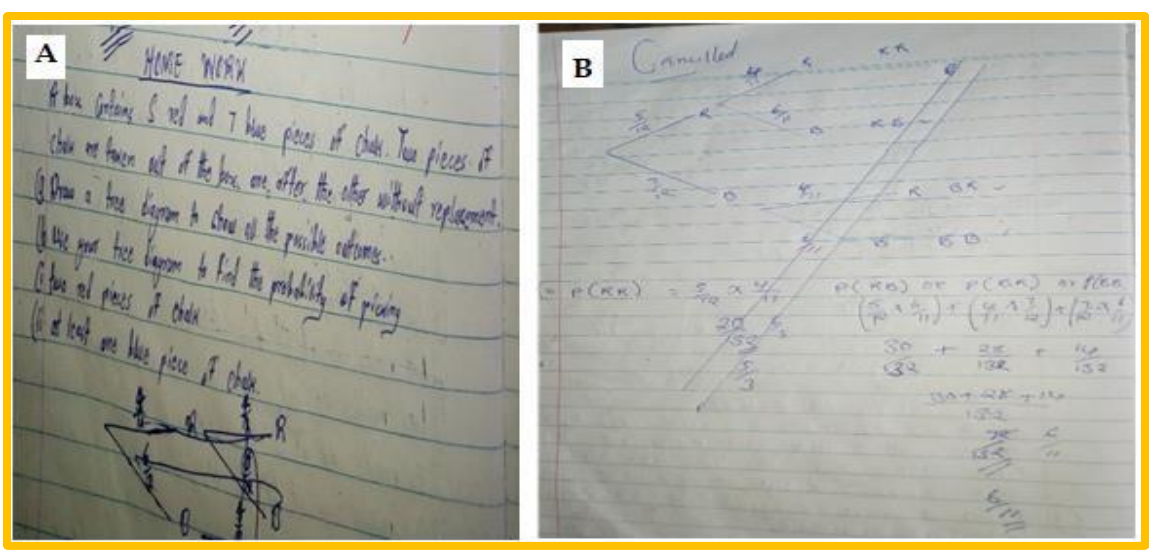


Figure 30: *learners' working sheets in school B*

Failure to note individual differences by TB was confirmed during FGDs where the researcher wanted to find out why there was so much fidgeting, copying and movements of learners from one desk to another to look for solutions in School B without any order. The following were excerpts from learners' responses;

> ***L_6B***..*the problem with members of this class is most of us are very behind, missing school days is our culture, so some even when they come they don't write anything.*

> ***L_7B..****Our teacher is just too fast and he will say something once and move to the next.*

Figures 31 also are snapshots of some learners who needed assistance as seen from their documents in School B in the process of knowledge construction.

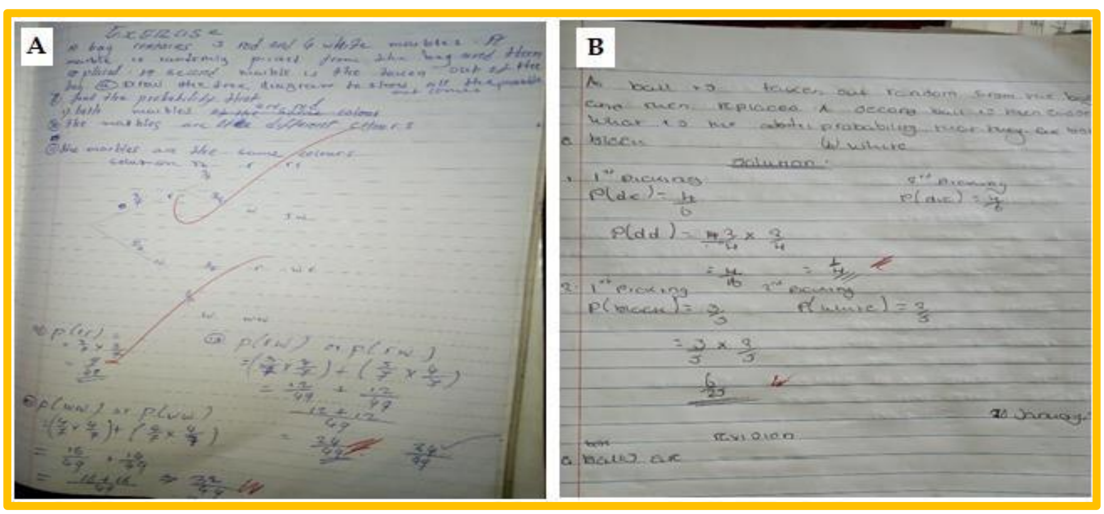


Figure 31: *L_{10} B's cognitive processes (Part A) and L_7 B's processes (Part B)*

4.5 Summary of findings for research question 1

Table 4: *Constructive social activity models learners used to learn probability.*

Models used / processes	The help the models provided to learners	Findings
▯ Dice ▯ Coin ▯ Marbles ▯ Psychological models (cognitive modelling) ▯ Cognitive concept from a model perspective	• Making judgement • Memorizing concepts • Copying • Creating images or diagrams • Linking concepts to real life experiences	▯ Learners in school A were exposed to constructive social activity models such as those listed in the first column in the learning of probability. ▯ Learners in school B were not exposed to the constructive social activity models listed in the first column physically, but theoretically through cognitive modelling.

4.6 Summary of findings for research question 2

Table 5: *How learners engaged in the use of constructive social activity models for conceptual understanding of probability.*

How Learners were engaged	Findings
• Physical manipulation of models • Verbal reasoning • Visual presentations	▯ Learners in school A were engaged in both physical manipulation of models and verbal reasoning for conceptual understanding of probability. ▯ Learners in school B were not engaged in physical manipulation of models but only in verbal reasoning for conceptual understanding of probability. ▯ Some learners in school B presented work without any verbal comment

4.7 Summary of findings for research question 3

Table 6: *How teachers assisted learners in enhancing clarity of concepts in probability from the constructive social activity models perspective.*

Teacher's assistance / practices	**Findings**
Setting concepts with the appropriate physical tools. Going round marking learners' work and nonstrictness to learners' solutions. Allowing learners to attempt questions on the board. Paying attention to individual differences.	TA provided appropriate physical tools for learners to enhance the clarity of concepts in probability while TA did not do so. TA did not promptly and consistently mark learners' exercise books while TB did that regularly and promptly. Both TA and TB allowed learners to attempt questions on the board but they did not pay attention to learners' individual differences

CHAPTER 5

DISCUSSION OF FINDINGS

5.1 Overview

This chapter discusses the findings of the study on use of constructive social activity models when learning probability in selected secondary schools in Mbala district. The discussion of the research findings is according to the research questions. Section 5.2 discusses what constructive social activity models learners' use when learning probability. Sections 5.3 discusses how learners engage in the use of constructive social activity models for conceptual understanding of probability. Section 5.4 discusses how teachers assist learners' in enhancing the clarity of concepts in Probability from the constructive social activity models perspective and any figures stated are with reference to chapter 4, and Section 5.5 summarises the chapter.

5.2 Research question 1: What constructive social activity models do learners use when learning probability concepts?

5.2.1 Use of concrete models

This theme emerged from three categories, the use of dice, coins and marbles of different colours. Through lesson observation, these three categories showed that learners were able to use the concrete tools to exemplify the concepts of probability. Results showed that one of the teachers had the ability to introduce and develop probability concepts using the appropriate tools such as dice, coin and marbles as shown in Figures 2, 3, and 4. Learners in this study generally expressed high levels of interest in using tangible objects as they were called to do the experiments before any definitions or formulas about probability were written on the board. This finding is supported by the MESVTEE, (2013) who argue that the use of tools, is an essential element in the classroom, to aid students in making sense of mathematics concepts and in developing their ability to reason and communicate. The researcher also believes that sense making is connected to what (Skemp 1979) called relational understanding as earlier on alluded in chapter 2. The researcher believes that meaningful use of tools could help learners develop relational understanding, hence the need to integrate various mediating tools in the content.

The curriculum also emphasizes thinking processes such as communication, interconnections, reasoning, representations, exploration, investigation and problem solving that can transform traditional teacher centered mathematics classroom situation, into learner centered environment where learners are allowed to construct new knowledge.

Furthermore, these results are congruent to what other scholars say that any formal learning which a learner comes across, a teacher should not begin a lesson by giving definitions but rather set a constructive social activity model which is a real representation of those concepts which are in abstract and intended, (Freudental, 1991; Ernest, 1991). If their prior knowledge in the introduction of scientific concept is congruent to the subject content matter and the model, then new representational models must be introduced to reconstruct new knowledge, (Lerman 1989; Stansfield & Carlton, 2011) as models are tentative schemes or structures that correspond to real objects, events, or classes of events and that have explanatory power. Models help scientists and engineers understand how things work. Models take many forms, including physical objects, plans, mental constructs or cognitive modeling, mathematical equations, and computer simulations. Scientific explanations incorporate existing scientific knowledge and new evidence from observations, experiments, of models into internally consistent, logical statements, (Stansfield & Carlton, 2011). Thus, the concrete models that were used to initiate the learning process by learners in school A were relevant and meaningful.

However, a check in the learners textbooks (Reference books) such as Progress grade 9 and 11 showed that a variety of tools have been used in framing word problems and picture model tools as shown in Figure 17 for teaching and learning purposes. Most of the picture models in Figure 17 were not used but only a limited number of tools (dice, coin, marbles) by TA and were only used to illustrate and develop less sophisticated concepts about experimental and theoretical probability as compared to Basu and Debnath (2015) findings in relation to Pascal. These results reveal that teachers and learner participants were limited on the choice of modeling tools embedded in text books and during FDGs/Semistructured interviews learners and teachers said that they were not familiar with such things which they did not use. They considered them irrelevant, inaccessible in school and could be confusing thus, only the following were used;

5.2.1.1 Dice

The dice were unbiased with doted numbers {1, 2, 3, 4, 5, 6} on each side. The teacher participant described the dice with the intention of helping learners see that rolling a die and having a certain number which is intended happens by chance as there are 6 chances. By describing the dice to the learners the teacher meant to relate abstract concepts of chance to authentic problems in real life using dice features as accurately as possible just like Dowd (2013) who presented a circle model to show why probabilities are multiplied in compound events. These findings are also in line with Basu and Debnath (2015), Efron (2013) and Kolmogorov (1956), who say that using a dice when introducing theoretical and experimental probability, gives a logical treatment of probability theory since it also deals with games of chance. It can help explicate the concepts such as equally likely occurrences but not fifty-fifty chance, random events, independent events to mention but a few. During the fifteenth century, the pragmatic approach to problems of games of chance with dice began in Italy, (Basu & Debnath 2015). In this case, the die helped learners in School A to understand probability easily as compared to learners in School B who were not exposed to one.

5.2.1.2 Coin

Learners in school A had to use coins to develop the concept of probability. The concepts of chance, favorable outcome, event, possible outcome and sample space were introduced. TA described coins as having the features on one side a coat of arm and on the other side a bird (eagle) to the learners. Describing general features of coins by the teacher to his learners and what each of the two sides would represent for Head and Tail, was vital for learners to easily identify and accurately represent concepts of probability. This is because if the teacher did not do so, it was going to be more confusing to the learners to interpret and note the outcome events as other learners were still getting confused of the features in the process of the experiment. These observation results show that even if the old denomination coins have disappeared in circulation from which the features Head and Tail outcomes originated, but the outcomes have remained static in probability content.

Meaning that even in the new books the problems framed in words still use a head and a tail as features of a coin which is not in circulation in real life. So such inaccurate representation of a coin pauses challenges as evidenced in the findings. In addition, for self-paced learning, learners would face challenges when reference books are not updated and revised because learners will not find meaning and relevance with reality. These findings are in contradiction with the Social cultural theory which posits that the physical characteristics of a model which are observable, should accurately illustrate the abstract concepts so that learners take probability as a physical property rather than abstract (Vygotsky 1979). If tool use posses challenges during the activity, it may need checking and refinement or transformations or improvisations (Schlottmann, 2001).

Furthermore, tools such as coins are so useful in the development of the theory of equally likely and fifty- fifty occurrences, and a degree of confidence that something will definitely happen. In a case of tossing an ideal coin, a priori probability either of the two possible outcomes, is 1/ 2 which Basu and Debnath (2015) elaborated further on how using a coin enabled Pascal (1623-1662) and Fermat (1601- 1665) to develop further concepts in probability clearly linking it to Binomial. The famous Pascal's triangle as seen in chapter 2 is vital to acknowledge in the teaching of probability from a model perspective in the sense that when a coin is tossed twice, there are four possible outcomes: HH, HT, TH and TT where H according to Pascal, stands for heads and T for tails. A solution of which was closely related to the binomial coefficients of $(H + T)^2 = H^2 + HT + TH + T^2 = H^2 + 2HT + T^2$ was developed.

Similarly, assuming an ideal coin is tossed three times, there are eight possible outcomes: HHH, HHT, HTH, THH, HTT, THT, TT H, and TTT. It provides further link to solution of the binomial coefficients (collecting all terms of H and T together) of $(H + T)^3 = H^3 + 3H^2T + 3HT^2 + T^3$. The binomial coefficients in this example led to the famous Pascal's triangle. In connection with the Pascal triangle, Pascal observed the famous formula for the binomial coefficients as follows; ${}^nC_r = ({}^n{}_r)$ involved in the coefficient of $a^r b^{n-r}$ in the binomial expansion of $(a + b)^n$ for any integer $n \geq 0$ and $0 \leq r \leq n$. He also linked binomial coefficients with the combinatorial coefficients that arose in probability, and used these coefficients to solve the problem of points in the case of one player requires r points and the other n points to win a game (Debnath, 2009; Debnath, 2010; Kolmogorov 1956; Efron, 2013).

Therefore, Pascal's example in the use of a coin in the development of probability concepts has significant implications in the knowledge acquisition process domain by domain. Just as a coin helped learners in school A to understand equally likely events easily. It is very important that learners are provided with opportunities to manipulate these constructive social activity models when learning probability, as they help in conceptual understanding of the topic.

5.2.1.3 Marbles

Learners in school A used marbles of different colours (green, blue, purple, red and white) to introduce the concept of picking randomly with or without replacement, mutually exclusive events, compound events and theoretical probability. This resonates with De Jager (1966) whose lesson did not start with a definition but used learners' prior experiences to explore the formal theoretical concept of Probability through observation of a pack of playing cards, dice, coin and marbles of different colours. Thus, the pedagogical approach of using physical objects to introduce the study of theoretical and experimental probability was influenced by the suggestions made by Shaughnessy (1981) and De Jager (1992) that theoretical reasoning involves predicting outcomes and then an experiment is used as a form of checking. This practice impedes effective knowledge acquisition of learners in the topic as earlier stated in Section 5.2.1 of this chapter.

5.2.2 Use of psychological models (Cognitive modelling)

Research results from lesson observations and document analysis also showed that TA and TB tasked the learners to use computational representational understanding of mind (CRUM) during the lessons without use of tools. Lesson observation results showed that there were situations framed in word problems which could not require any physical representational tools in real word. This was in order for learners to establish individual meaning making (internalization) either from a model perspective or from a given situation. Other situations demanded that learners make mental connections to real life situational models and make logical judgments. The following themes emerged; learners tried to use cognition concept from a model perspective, making logical judgments, copying, images or diagrams, and memorisation to present probability concepts. Teacher participants also asked learners to work out word problems on the board and in their exercise books. Psychological modeling is important in that it helps to interpret the intuitive reasoning of the learners, their actions and establish their misconceptions. This is in line with Jarworsky (1996), who states that cognition is the mental action or process of acquiring knowledge and understanding through thought, experience, and the senses. It encompasses processes of using computational representational understanding of mind (CRUM) such as knowledge, attention, memory and working memory, judgment and evaluation, reasoning and computation, problem solving and decision-making, comprehension and production of language, just to mention but a few. Cognitive processes use existing knowledge and generate new knowledge. This enabled learners to learn probability concepts in abstract too, especially learners in school B who had no opportunity to manipulate physical models and in that way their understanding and nonunderstanding was noted.

5.2.2.1 Cognition concept from a model perspective

In an instance where learners in school A used a concrete model the teacher assumed that the mind of the learners had then mental representations analogous to data structures and would be able to compute a given task following the procedures or algorithms demonstrated previously from a model perspective. Similary, Temperly (2014) tasked the participants to rate the expectedness of the continuation note given the two-note context on a scale of 1 to 7 after which they were require to compute the probabilities of the other probability melodic intervals. This act is in line with Jarworsky (1996) who states that cognitive processes can also use existing knowledge and generate new knowledge. For example, after the learners participated in the chance of picking marbles of different colours at random from a bag that the teacher had come with, , L_2A had to use cognitive modelling or psychological modelling to work out task 2 from the board, as there were no concrete pens. The other example was to do with letters of the English alphabet that did not have concrete models too as shown in Figure 5 A and B.

The given examples which teacher A tasked the learners with an assumption that the mind of the learners had developed mental representations analogous to these questions and would be able to compute the given tasks following the procedures demonstrated previously from a model perspective were;

1. *A box contains the following English alphabet letters; a, b, d, e, f, g,h, i, j, k. if one letter is selected at random what is the probability that it is a vowel*?
2. *.A small bag contains 6 black and 9red pens of the same type. Calculate the probability that when a pen is picked at random it will be red.*

This is a situation where a teacher wanted to test learners' retention if they were able to perform the task after a tool is withdrawn. (Gravemeijer, 1994) has described this level of modeling as formal level of mathematics and it is important in the process of learning mathematics. It enables one to work with conventional procedures and notations as shown from learners working in Figure 5.

5.2.2.2 Making judgement

Learners in school B used cognitive modeling or psychological models from the introduction of Probability to the development and conclusion. They did not use any concrete constructive social activity models but made judgments and conclusions based on their mental processes as evidenced from their excerpt conversations from the previous chapter. This is in contrast with the social cultural theory of knowledge construction as well as Hatano's (1996) Charecterisations of knowledge acquisition. Firstly, the social cultural theory posits that knowledge construction emanates as a result of learners' personal life experiences and social interaction with physical phenomenon or tools in their daily lives as members of a culture (Vygostky 1979). It views mathematics learning as a social activity of human creation of which in all its structures, rules, postulations, laws, formulas, theories and symbols have reference to the physical world of reality from which they are created (Wertsch 1984; Cobb 1994; Freudental 1991). Thus, teacher B was supposed to use real objects to represent the abstract concepts in reality in order to develop the probability concepts.

The introduction to Probability and how teacher B proceeded with the lesson development at school B was problematic, as it was in abstract where learners were required to recall the definition of probability from what they learnt in grade 9. Sometimes it may be difficult to remember concepts learnt in earlier grades. Therefore, knowledge acquisition involves restructuring as a characterisation that gives a guiding principal to an educator that, before going into details on Probability, use some basic ideas or situations. This is because constituent pieces of knowledge or concepts can change; thus, pieces of knowledge are amalgamated into a unitary during restructuring (Hatano, 1996). In addition, teacher B quickly moved to experimental probability before calling the learners to order as his lecture method was so systematic that learners were listening passively.

Furthermore, where a teacher asks learners to say '*what comes into your mind whenever you hear the word probability*?', later on gives a definition and making a judgment in abstract, is against the constructivists' epistemology principles of knowledge construction process and RME theory. The process of knowledge construction is also critical to constructivist scholars such as Bruning, et.al (2004) who argue about a set of cognitive themes that resonate with integration of indigenous knowledge in the learning process among which one of them is that: Learning is constructive, not a receptive, process. Implying that a learning situation should not aim at transmitting all abstract concepts as fixed as they are, well defined independent entities in reference books. This is because such exemplification cannot provide the important insights into either the culture or the authentic activities of that culture where learners' daily needs are met (Baise 2013; Carraher and Schliemann 1985; Dowd 2013; Lave 1988; Saxe 1991, Scribner 1984). In addition, Fredental (1991)'s use of contexts in phenomenological exploration states that the starting point of every instruction should be experientially real to the learners mind by allowing them to become immediately engaged in the contextual situation. This means that instruction should not start with the formal systems of definitions. The phenomena by which mathematics concepts appear in reality should be the source of concept formation.

Furthermore, the lesson was then developed with a word problem as a worked example on the board by TB, which learners copied in their exercise books as shown in the snapshot Figure 6. After copying the examples which were all worked by TB, learners wrote an exercise immediately. While learners from school A actively and accurately used the cognitive modeling by judging a situation in most instances more than the learners in school B. For instance, when introducing the concept of impossible probability, they were guided systematically and therefore they learnt the concepts of probability on sample space easily. The teacher even used an idea of the pizza to ensure that learners understood the concepts of impossible occurrences in probability.

The illustration about the possibility of any learner expecting to eat pizza in relation to impossible occurrences on that particular day fits in the recommendation made by Gravemeijer (1994) who commended that single-case probabilities assumed in models may well be mere conveniences that do not correspond directly to anything physical in the world such as expectations, anxieties, hopes or predictions but those can still be modeled from a fantasy world of fairy tales of cognitive modeling. Cognitive models should exemplify probability problems from real world, from the fantasy world of fairy tales as long as the problems are experientially real in the learners' mind. For example, when one is stroke with an illness, there are two possible outcomes, recovery or death and this cannot be exemplified or represented by a physical activity but cognitive modeling. From this exemplification TA proceeded to write summery notes on the board on impossible probabilities followed by example as shown in Figure 7.

5.2.2.3 Memorisations

Learners in school B used cognitive modeling by memorising the tasks procedures which the teacher demonstrated in Figure 8 without using any concrete constructive social activity models. They tried to memorise the procedures involved in order to construct the correct tree diagrams in Figure 11 and how to assign correct probabilities like the teacher did in Figure 8 and 9 in the previous chapter. TB presented the construction procedure of the tree diagram and the examples which were given were done by TB on the board and asked learners to answer the questions that followed. This is contrary to scholars who support the theory of social construction of knowledge such as, Voigt (1992, p.10), Rogoff (1990) and Saxe (1991) who argue that the prominent objective of mathematics education is not that learners produce correct answers but that they do it insightfully and by reasonable thinking. Thus, insight and reasonable thinking occurs in the zone of proximal development (ZPD). This is the difference between what children can do on their own and what they can do with assistance from others. Interactions with adults and peers in the ZPD is what promotes cognitive development (Schunk, 2012). However, TB introduced tree diagrams by giving it verbal definitions while learners were passive. The notes were written on the board and then gave an example (Figure 8 and 9) and expected learners to reproduce the procedure which he used in his example when introducing tree diagrams to the examples and exercise (Figure 11 A and B).

The researcher noticed that learners were trying to recall a certain procedure on how to construct tree diagrams and assign probabilities. In Figure 11 A, the learner inaccurately assigned the first branch representing the results or outcomes of the first toss and was assigned with probability as two thirds and one third. She was still thinking and wondering if it was the correct procedure as she could be seen from the way she was twisting her head in trying to recall the procedure. The other learner in B kept on rubbing the working with her hands as a sign that she was not sure of the correct procedure. The first branch representing the first toss had outcomes two out two, one out of two chances. These learners worked from the board at the same time such that there was no communication to confer meaning in the process of showing their working on the board (Figure 11 A and B). It is important for the learners to share their experiences through verbal interpretations of a phenomenon or situation by verbal communication in order to confer meaning. They should be seen to interact by conjecturing, agreements, disagreements, counteractions, reaction, counter examples and contradictions before they reach a common understanding of a situation (Konold 1991). Through the process of verbal communication, the teacher notes their individual actual level of development and through collaboration with peers; ideas are tested, refined and developed producing a common interpretation of a phenomena (Wertsch 1984). When teacher B gave the following class exercise which reads;

Exercise

A bag contains 6 black and 9 red pens of the same type two pens are taken at random one after another without replacement.

i. *draw a tree diagram to show all the possible outcomes*
ii. *calculate the probability of picking pens of different colors.*

During the exercise, some learners took time trying to recall the teacher's working from the previous examples, and it was difficult for them to answer the questions in the exercise as they had easily forgotten what the teacher said in his explanation. This is why the use of constructive social activity models to explicate understanding in Probability cannot be over emphasised.

The snapshot in Figure 12 shows learners as they were trying to consult each other in school B during the class exercise, without a formal discuss. The situation of a learner asking anther learner that, "*how did he do his example?"* is as result of transmitted knowledge because learners did not participate in knowledge construction, as they were passive and could not schematise. TB did not use learners' prior experiences when creating their learning environment, contrary to Hatano (1996) who argues that transmitted knowledge becomes usable in a variety of problem solving situations only after it has been reconstructed, interpreted, enriched and connected to the prior knowledge of the learner.

5.2.2.4 Copying

During lesson observations, some learners in school B could leave their own desks in search of solutions and copy from those whose work had been marked or from those whom they trusted as shown in Figure 13. Instead of the teacher being seen to organise learners in small groups stimulate their ability to compare their solutions in a class discussion (Broddie, 1996) in a logical manner, the opposite was the case, learners were busy copying from one another throughout during the exercise. In Figure 13, learners were seen seeking answers from their friends in all the parts A, B and C. They just wanted to have the working in their exercise books by copying so that their books could also be marked. Their conversations showed non understanding of the concepts and could only copy without reasoning process as established in their conversations such as; "*is this the answer?"* while pointing his finger on the solution in the friend's book. Another one said, *"hurry up I want to take it for marking*." While the other one said, "*hey don't take your book first let me check your answer."* Freudental (1973, 1991), states that a teacher should set meaningful discussions so that the interpretation of the situation presented in the contextual problem focuses on the adequacy and the efficiency of various solution procedures. The teacher should ask the learners to communicate, argue and justify their solutions as they manipulate the model and the task at hand (De Lange, 1996; Gravemeijer, 1994; Broddie 1996; konold 1991; Freudental 1991).

Furthermore, one of the applications reflecting Vygotsky's theory is reciprocal teaching. It involves interactive dialogues between teacher and small group of learners (Broddie, 1996) not just by copying. Firstly, the teacher models the activities, thereafter, the teacher and learners take turns being the teacher or group leaders leading the discussions. During the manipulation of the activity framed in word problems, learners should be encouraged to ask questions, then to determine their level of understanding, teacher can include a question-asking strategy in the instructional sequence. Thus, learners gradually develop skills, since reciprocal teaching comprises the principle of social interaction and ZPD of the Vygotskian perspective (Schunk, 2012). Peer collaboration is vital rather than learners going from one desk to another, (Vygotsky, 1979). Bruner (1984), Ratner et al., (2002) states that it reflects the notion of collective activity (Schunk, 2012). The shared social interactions when peers work on tasks cooperatively serve as an instructional function mainly in learning mathematics, science, and language arts which attests to the recognized impact of the social environment during learning (Verenikina, 2010).

5.2.2.5 Images or diagrams

Through document analysis the researcher noted learners' different computational representations of the mind on the images and diagrams as they worked without any concrete model. Various procedural imagery and diagrams which learners wrote in their various worksheets showed that they used cognitive modeling to answer questions. This is similar to self-regulation, (Schunk, 2012) which is developed through internalisation (developing an internal representation) of actions and mental operations that occur in social interactions (Miller, 1989; Wertsch, 1985). However, this self –regulation was not mainly as a result of peer to peer social interaction, but was after paying attention to lecture method by the teacher. The snapshots were taken in variations, learners who worked independently as in Figure 14, those who worked in collaboration by merely copying, Figure 15; or inquiring and Figure 16 who had never taken the book for marking.

For example, Figure 14, is the image sample of one of the learner's independent computational procedures in school B. That is a learner who worked independently without leaving her desk to go and search for solutions nor ask for help from anywhere. This learner could recall that when there are 6 black and 9 red pens and if two pens are taken at random without replacement, the first chance of picking either black or red as presented in Figure 14 is 6/15, or 9/15. The rest of the solutions were incorrectly modeled in Figure 14, indicating her level of actual development which was supposed to be nurtured by capable peers. Figure 15 also is an imagery illustration of cognitive modeling processes of a snapshot taken among the learners who worked in collaboration with each other and demonstrated some conceptual understanding. The ones in Figure 15, could be the more capable peers to interact formerly in small groups with less capable like in Figure 14. Figure 16 also is an image computational procedure by another learner in school B. The snapshot shows this learner's working from the previous exercise to the next exercise and the homework without having his work checked at any time.

The three different snapshots Figure 14, 15 and 16 analysis of learners' documents images and diagrams, show procedural competencies of leaners' cognitive skills of which if these skills were exhibited on a social plane with a constructive social activity task setting, they would demonstrate a conceptual shift from actual developmental level to their level of potential development. The more capable peers in Figure 15 would have helped the less capable peers in figure 14 and 16 at a social setting in the Zone of Proximal Development. The social cultural theory posits that knowledge construction emanates as a result of learners' personal life experiences and social interaction with physical phenomenon or tools in their daily lives as members of a culture (Vygostky 1979). The situation in figure 16 is where a learner just sits in the far back corner in the classroom and only copies the work after work without interacting with anyone until the lesson ends. While for those who worked in collaboration in figure 15, their cognitive skills were more accurate as they showed some understanding of the concepts and could have been the ones to help the less capable in a task setting. In the classroom context of learning, a teacher should be aware that learners are not completely blank as they have prior conceptions about probability from their cultural events in their usual life experiences which they have to share as peers on a constructive social activity model set as a representation of reality as they engage in collaboration with one another (Vygostky 1979; Wertsch 1984).

Furthermore, in order to develop more insight into constructive social activity model use, the researcher also used focus group discussion to establish if there were any constraints as regards the choice of what constructive social activity models to use. Knowledge on how or if any of the concrete modeling tool could affect knowledge construction was vital for the researcher because there are some important characteristics of a modeling tool to take note of as outlined by Freudental (1991, p.524) and other constructivist that, "A model is a model of a situation that is familiar to the learners." The physical characteristics of a model which are observable, should illustrate the abstract concepts so that learners take probability as a physical property than abstract (Schlottmann, 2001). Moreover, the concept of situation definition also provided the researcher with a lens through which a tool may constrain knowledge construction in a task setting as it is a way in which a setting or context is represented or defined by those operating on it (Brodie 1996). Thus, that setting or context represented, is a 'constructive social activity model.' The constructive social activity model/models being operated on by learners, may be defined differently by learners and even different from the teachers own interpretation (Wertsch 1984; Brodie 1996). However, through negotiation of learners' different situation definition by verbal communication, a common situation definition is shared which Wertsch (1984) calls intersubjectivity.

In FGDs learners demonstrated that they were not familiar with some of the modeling tools used in the textbooks which in turn could hinder their active participation and constrain knowledge construction process. The researcher asked: *'Can we identify any of* *these pictures? Any comment on how we know them?* The researcher's question was meant to elicit their knowledge of the objects in their textbooks apart from having used them in the classroom learning process. However, most of them were unfamiliar with the models shown in the pictures, except a few of them who could name some models such as playing cards and chess.

From their responses, the researcher could tell that learners had little knowledge of the tools. Knowledge of a tool or an object or model is important and must imply the following; firstly, the instance it appears learners must identify (verbal interpretations by use of familiar language, imagery, symbolic, signs etc.) (Wertsh, 1985; Miller, 1989). Secondly, is appropriate action on it (Miller, 1989), which implies physical manipulations in the very context in which it is designed. The more an individual engages in physically actions on the tools, the more they gain experience and the more they learn.

It was also assumed by the researcher that through conceptual mathematisation during the learning process, learners could have developed a deeper understanding of Probability and link of further constructive social activity models resulting in real life (Freudental 1991). It was expected that learners would subsequently apply probability concepts to other aspects of their daily lives and as such, the concepts would be reinforced and strengthened. This was sought due to the fact that learners in grade 11 could have more progressive knowledge strands on Probability concepts since they learned in grade 9. However, in the FDGs learners were also asked to identify those models from their environment that could be used to explicate probability concepts apart from those they used during the lessons or from the textbook of which they had limited relations as they could only name a few.

During the semi-structured interview with the teachers, the researcher wanted to know why in order to develop concepts of probability during the classroom learning process learners did not use some of the concrete models in the learners' reference (textbooks) or any other. Their perspective was similar to that of the learners that they were not familiar with some of the modeling tools embedded in the subject matter. Teacher A indicated that some of the models were not familiar to some learners and that he had never used such models before and could not be found anywhere in school. Hence, he only used a coin, a die, marbles and playing cards which learners were familiar with. In the case of playing cards, some learners expressed ignorance about them by stating that cards were only played by boys so they would not know anything about them.

From the semi-structured interviews, it was noted that some of the modeling tools that have been planned in the content for learners as models or aids to learning Probability and included in the text books are not culturally rooted. The tools that man uses to transform culture are situated and progressively developed, refined, validated and shared among generations and learners from that culture are supposed to be familiar (Vygotsky 1979). If tool uses possess challenges during the activity, it may need checking and refinement or transformations or improvisations. Implying that learners' textbooks should be revised from time to time and the models embed should be those that are culturally rooted and familiar to learners and teachers should improvise. For instance, if in the process of cutting vegetables you experience the knife is blunt then it requires either sharpening by rubbing against its edge back and forward on a rough solid rock or metal. Doing so, may increase the likelihood of its meaningful application in problem solving during the activity and in the future problem solving.

Considering the important implications of using constructive social activity models in a learning process, the researcher also wanted to find out from TB why either learners or the teacher could illustrate no concrete model from probability concepts. According to Freudental (1991) the constructive social activity models are developed by teachers or the pupils themselves. Freudental's (1991) theory encourages that learners should develop mathematical tools or models from which they develop insights by themselves. This implies that even learners themselves are capable of creating models of reference in their own situation and own understanding. Self-developed (or emergent) models of the learners serve to bridge the gap between informal and formal knowledge (Gravemeijer, 1994).

In the semi-structured interview, the teacher acknowledged the importance of giving chance to learners to come up with modeling tools which they are familiar with for contextual relevance although doubts about learners' capabilities to make own models due to the technical language of probability. Thus a teacher should always play a role in guiding learners in the process of knowledge construction either in the production of own concrete models or actual classroom learning process (Cobb 1991). When the researcher wanted to find out why TB did not use any concrete models in learning probability, the teacher indicated that the topic was a continuation as he assumed that learners had learnt the concepts of probability introduction in grade 9. He further indicated that he needed to speed up the teaching in order to catch up with other classes which were ahead.

In order to allow learners, feel the ownership of knowledge, there is need to engage them in all activities at all times in various contexts and experiments. Temperly (2014) demonstrates how participants were engaged in rating the expectedness of the continuation note given the two-note context on a scale of 1 to 7 with excitement. This increased the rate of cognition through active participation and not by interpretation of what is written in text books.

For example, a teacher telling learners that they already know a concept from Probability which was learnt in grade 9 is discouraging. Rather a teacher should start with a situation and establish their actual level of development in that particular context and not by assumption that they know. Knowledge acquisition is ‘situated’ in contexts; Human knowledge acquisition is conceptualised as a process of representing experience with an object, event, or conceptual entity, so that the resultant representation (i.e knowledge) can readily be used in future problem solving and understanding. Even when two or more learners know a mathematical formula which is common, how it is represented may vary among them (Hatano, 1996). Thus gaining insight of learners’ subjective conceptions during social plane as a teacher is a fundamental principle of constructivism in knowledge construction because learners are indeed not empty slats (Ernest 1991).

Moreover, Bereiter (1991) also states that all conceptualisation takes place on the social plane and learners’ constructions are vital for devising appropriate teaching. Therefore, when learners look at the relationships from many perspectives, they find the computational side of probability easier to understand (Carraher, 1996). Ernest (1991) also argues that objective and subjective mathematical knowledge recycles through social interaction. Using their subjective knowledge, learners are exposed to carry out empirical observation of probability trials from an objective phenomenon, reconstruct in new knowledge through social negotiation process and mediation for which brings about individual internalization process and new knowledge as subject. They may be well aware of a particular concept or formula but may present it differently and the teacher should not overlook that fact (Hatano 1996).

5.3 Research question 2: How do learners engage in the use of constructive social activity models for conceptual understanding of Probability?

Research question two focused on observing how learners engage in the use of constructive social activity models for conceptual understanding of Probability. Being engaged in the use of constructive social activity models for conceptual understanding of Probability implied to the researcher, observing appropriate action on it (model tool) which involves learners' physical manipulations in the very context in which it is designed. It was assumed that the more an individual engages in physical actions on the tools, the more they gain experience on the object and the more they learn. Knowledge or situation definition of a tool or an object or model implies the following; firstly, the instance it appears learners must identify (verbal interpretations by use of familiar language, imagery, symbolic, signs and so on. In order to establish how learners, engage in the use of constructive social activity model, the researcher observed grade 11 learners learning probability alongside document analysis of their individual worksheets in order to see if they were able to retain the concepts. Thereafter, conducted focus group discussion to have an in-depth understanding on what may or may not have challenged their participation or retention of concepts from a model perspective. The following themes emerged; verbal reasoning, physical manipulation, visual representation.

5.3.1 Verbal reasoning

Verbal reasoning emerged as a result of what was observed where learners tried to show understanding and reasoning using concepts framed in words without use of any concrete objects. It was one of the means that was used to evaluate the ability to think constructively, rather than through simple fluency or vocabulary recognition. This was noted either through the questions which learners asked or through responses which learners gave in the process of learning. In this way the researcher was able to note understanding and non-understanding among learners through verbal reasoning.

According to Terrell, et. al (1989), verbal reasoning tests of intelligence provide an assessment of an individual's ability to think, reason and solve problems in different ways. Research results from lesson observations were congruent to Terrell, et.al analysis where learners explained verbally in words, the models framed in Probability word problems without physical manipulation. Learners in school A were more engaged and consistent in interpreting the models and situations of fancy world of fairy tales in order to draw concepts of probability than learners in school B. In their various explanations, some learners who could not do well their teacher could do it for them without redirecting the situation to others. In school A, when the teacher noted some errors he asked capable learners to explain for those who could not explain properly but did not ask for justification as to elicit thought process. The snapshots in (Figure 18) highlight examples where the learners (L_7 B), (L_5 A) and (L_4 A) were asked to describe the following given situations through verbal reasoning. The learners in TB's class were asked to all stand and only sit down if the correct answer was given.

However, TB did not ask learners to justify their verbal responses though it is an important aspect of a social cultural theory classroom learning environment and the notion of the ZPD. Learners are supposed to question be questioned, to justify for either their correct or incorrect responses. For instance, if L_8 B was asked to justify the misconception to his response it could have been used to develop further concepts. It is therefore encoureaged to always ask learners to justify , question, argue their responses in the learning process. The notion of ZPD is one of the guiding principles in the classroom learning situation whereby, Learners are supposed to interact by conjecturing, agreements, disagreements, counteractions, reaction, counter examples and contradictions before they reach a common understanding of a situation (Konold 1991) and consequently develop individual understanding.

5.3.2 Physical Manipulation

Class observations also showed that learners physically manipulated the constructive social activity model in school A. The models they used and manipulated were coin, die and picking marbles from a bag. For example, TA put up a challenge for his learners by putting green, blue and red marbles in a bag and promised to give a coin to anybody who could pick a green marble from the bag.

Firstly, this was a positive critique for the learners to single out a chance of picking a green marble as a guided action because every human action is goal directed whether involving games of chance or any choices, even just choosing where to wait for a bus (Glynn 2010). In particular, a learners' action is goal directed at picking a green marble on condition that a coin will be given. Secondly, is appropriate action on it which implies physical manipulations in the very context in which it is designed. The more an individual engages in physical actions on the tools, the more they gain experience and the more they learn. Therefore, tools that man uses to transform culture are situated and progressively developed, refined, validated and shared among generations (Vygotsky, 1979). This is a typical example of how probability concepts can be modeled successfully and applied, by using aspects of the concrete model corresponding to, or approximating to aspects of the real world. Thus, in the process of trying to pick a green marble, the modeled situation was used to justify that probabilities refers to chances. Secondly, learners' active participation was so encouraging and overwhelming as they started rushing in front with excitement in order to be the first to win a coin if they picked a green marble. First participant attempted and picked a red marble, the teacher shook the marbles and tried the second time and a learner again picked a blue marble. The second and third participant could not pick any green marble in their first and second trials. They asked the teacher if really there was a green marble in the bag. Figure 19 shows how learners engaged in physical manipulation of the constructive social activity models of trying to pick a green marble without looking into the bag. When applied to the process of knowledge construction in probability concepts, this characterisation is linked to testing a prediction. A prediction of an outcome is made through observation of a phenomena and carrying out an experiment to test whether the prediction is accurate or not and this is what is referred to as the notion of falsifiability (De Jager 1966).

In the same way TA then called on anyone to roll a fair die which he brought to the classroom such that a 3 is obtained. Five (5) learners tried rolling a die one after another and only one of them obtained a 3. Learners found it so interesting that they were rushing and anxiously waiting to role a fair die. The snapshot in Figure 20 shows learners engaging in rolling the die one after another. TA asked learners to list the outcomes of the experiment, and L3A listed as {1,2, 3, 4,5, 6}. Asking learners to list the outcomes of an experiment is important as the outcomes must be noted in order to verify the prediction. In an experiment, learners should list all possible outcomes and their associated probabilities as Probability is associated with games of chance which imply that the participants in the games are always actively involved and not passive (Lighter 1991). Unfortunately, learners did not list the outcomes of this experiment. They were supposed to be listing by taking note of each outcome because these models exemplify experiments or activities, which can be carried out with observable results.

Learners in school A also engaged themselves in trying to toss a coin in order to have a head land up as shown in Figure 21. TA asked if anyone would toss a coin and have a head up and three learners were given chance to try. Their outcomes were as follows; L_3A had a head, L_1A a tail, and L_4A a tail. Thus, the outcomes noted by the researcher from the learners experiment were {H, T, T}.

During focus group discussion the researcher wanted to find out from the learners about what they think as regards the use of tools in the learning process. Learners were asked their views about using models in learning, their responses during FDGs were generally that the use of tools helped them in understanding the concepts of Probability and also that they made the lessons interesting.

These responses confirm that learners' use of learning constructive social activity models to represent possible outcomes for probabilities enhances their conceptual understanding. Ernest (1991) argues that objective and subjective mathematical knowledge recycles through social interaction. Using their subjective knowledge, learners are exposed to carry out empirical observation of probability trials from an objective phenomenon, reconstruct in new knowledge through social negotiation process and mediation for which brings about individual internalisation process and new knowledge as subject.

5.3.3 Visual representation

Classroom observations and document analysis showed that participants were engaged in answering a variety of questions which were seen from the documents and their presentations on the board. The working did not have follow up questions to establish the thinking process of the learners, hence only what they had written could show their process of reasoning. Observations showed that when a teacher participant called a learner or even two learners at once to work out a problem on the board, they worked silently and so the researcher could only observe the visual representation of what the learner had in mind as there were no follow up questions asked even when a learner wrote an answer whether correct or wrong on the board. Lakatos (1976) stated that a pupil who makes a wrong conjecture may have had strong grounds for making such a conjecture. Even though, learners were engaged in making visual presentations of their understanding through formulas, equations and tree diagrams as evidenced from sources such as learners' writings on the board, exercise books and worksheets, their thinking process could have been determined by probing them further. In an instance where a correct representation was written by a learner the teacher praised the learner with a 'tick' or 'Okay, good' while incorrect responses were followed by just a cross or the teacher not even saying anything in regard to the learner's representations, or the teacher orally says 'no, go and sit down!' Even if an answer is incorrect, or is not the one which the teacher was expecting or hoping to receive, it should not be ignored, exploration of a pupil's incorrect or unexpected response can lead to worthwhile discussion and increased awareness for both teacher and pupil of specific misunderstandings or misinterpretations (Jaworski, 1994).

Whether a learner is engaged in verbal explanations or just makes a visual representation of the cognitive processes, whether correct or wrong a teacher must find out what the representation means and why. Skemp (1971) states that solving a particular problem correctly does not necessarily indicate that understanding of the relevant concepts was present. Asking guided questions on the other hand could help teachers to understand the process of thinking of a learner, whether the answer was correct or wrong. The change in emphasis from product to process has found its ultimate justifications in the philosophical underpinnings of the constructivist's movement with its stress on the importance of children actively constructing mathematics (Ernest, 1991). Whatever a learner says or writes down in answer to a question is what makes sense to the learner at that moment. Asking learners how they got to the answer they gave is a good way of discovering something about their thinking and opens the way to explaining why a particular answer may not be useful under different circumstances. Successful thinking should be praised even if it does not look correct in the teacher's eyes (Mudenda, 2019).

Furthermore, in order to understand and appreciate learners' thinking, the teacher must have an utmost infinitely flexible mind, because learners sometimes start from grounds that seem inconceivable to teachers. Teachers are encouraged to provoke learners' ideas and experiences in relation to the concept being explored rather than leaving them alone to make such visual representations as evidenced in Figures 14, 16, 22, 30 and 31 inclusive. The results in this study are in view of placing emphasis on the product and not the process. This view according to Ernest (1991) presents the body of Mathematical knowledge as complete, static, convergent, platonic and not revisable. The use of language in form of questions can help learners to move into and through their ZPD. Teachers should have directed learners in such a way that would have lead them in finding the answers on their own hence ownership of mathematical knowledge. The teacher participants in the study did not challenge the learners to a degree which was appropriate to offer. The lack of challenge may have resulted in making learners not to make progress which was possible with the teacher's help. Questions can serve to extend learners' thinking further and provide opportunities for them to articulate and reflect on their thoughts (Vygotsky, 1978). They can also serve as "scaffolds" by guiding the student through a logical thinking process or by prompting the learner to think about a problem in a new way. This would help learners regardless of their ability to move to the next region, instead of remaining in the region of their comfort. Drawing from the fore going it can be concluded that learners who learn in such a way remain on the same level, since there is no scaffolding of the learning process and they find mathematics really hard and boring. Sensitivity in handling learners who give perceived wrong answers to questions, and those that are not ready to say something because they are not comfortable is very important.

Teachers should learn to implicitly engage their learners that the answer is wrong so that, it does not seem to embarrass those who have nothing to offer or who do not wish to contribute, by asking questions so that learners could discover on their own that the first attempt was wrong. Jaworski (1994) suggests that such an approach encourages the trust of the learners, and diminishes the threat of feeling foolish because their contributions might be regarded as silly, or wrong. Such an approach encourages learners to take responsibility for their learning not to despise the teacher's activities by choosing not to participate (Jaworski, 1994).

5.4 Research question 3: How do teachers assist learners in enhancing the clarity of concepts in Probability from the constructive social activity models perspective?

Research question three focused on observing what teachers did to ensure that the tension and gaps between the daily life experiences of a child and the knowledge of the experts which is in the learners' reference books is bridged using the notion of ZPD as a researcher's mirror. Wertsch (1985) contends that the essence of the ZPD is to deal with two practical problems in the learning situation: the assessment of learners' intellectual abilities and the evaluation of instructional practices.

Jaworski, (1994) argues that to think that learners would construct their own knowledge without a teacher‘s assistance is absurdity. It was therefore necessary to observe teachers settle in a natural classroom setting and see how they practiced their knowledge and model tools together with their learners so that they gradually and consciously develop scientific concepts of probability from a model perspective through scaffolding. The constructivist epistemologists also encourage teachers to note individual differences, learner capabilities from their own articulation of beliefs and gradually create conceptual shifts in individual learners' minds by mediating those concepts that learners have not yet fully developed as they interact with them, (Bryan, 2008; Flick, 2007; Wertsch, 1985). Thus, data in this regard was collected through observation, document analysis to check what learners were writing, focus group discussion; semi-structured interviews were conducted with a particular teacher. The themes which emerged were; setting the tasks or concepts with the appropriate tools, going round marking learners work and non- strictness to learners solutions, allowing learners to attempt questions on the board and lack of attention to individual differences.

5.4.1 Setting concepts with the appropriate physical tools

TA came to the classroom carrying a yellow bag in which there were marbles, coins and a dice to introduce Probability concepts that made learners curious. TA did not want to disclose what was in the bag which made learners anxious and curious to know what was happening and in that way learners were stimulated to learn. Teachers' understanding of the subject content matter includes using various forms of representations of physical tools so that learners' ideas, discussions, illustrations/ examples, explanations or clarifications make the content subject matter more comprehensible and practical to real life experiences (Cobb, 1994). The conceptions, perceptions and preconceptions of learners' experiences from different ages, backgrounds require a careful selection of an appropriate tool that would easily be manipulated and accurately bridge the gap between spontaneous and scientific concepts. The following themes emerged under this research question

In line with Constructive Theory – Vygotsky's approach which states that the "human mind is constructed through a subject's interactions with the world and is an attribute of the relationship between subject and objects" (Verenikina, 2010). Vygotsky advocates that humans do not act directly on the physical world without the intermediary of tools and these tools are what the study referred to as constructive social activity models. Therefore, in order to develop cognition of Probability concepts, teacher A set the concepts in the introduction and lesson development by use of appropriate tools such as die, coin, and marbles. This helped in assisting learners meaning making, and clarity of concepts from the model tool perspective as seen from the excerpts of the lesson conversations. The teacher used his pedagogical skill well when he said whoever rolled a dice and obtained a three, or whoever tosses a coin and obtains head, and whoever picks a green marble from the bag would walk away with a one-kwacha coin. The teacher asked learners if they remembered what they did in the experiments, and they said yes, referring to their attempts shown earlier in Figures 20 and 21; and in line with Brodie (2000), teaching in the ZPD requires teachers to teach through questioning and prompting of learners' ideas. In so doing, learners will be able to accurately use and present tools through the way they respond and write when solving mathematical problems (Wertsch, 1990).

These research findings are in line with Schlottmann's (2001) experiment which reviewed the children's intuitive reasoning competence whereby when the teacher said that whoever scored a success learners were very excited to score and be rewarded a coin. This intuition also contributes to the biases evident in adults' judgement that people act to fulfill their desires (Wellman, 1990). Their action also depends on how likely the desired outcome is.

However, the researcher noted that in trying to develop the lesson, from a constructive social activity model perspective or conceptual mathematisation according to (De Lange, 1999) TA did not relate well the outcomes of the experiment in Figure 20 and 21 to introduce experimental probability due to lack of clarity. Although the verbal explanations were quite clearer than written notes on the board, learners connected the concepts with the teacher's assistance.

In order to clarify the concepts further and link the previous constructive social activity model to Experimental probability concept, TA could not differentiate between experimental and theoretical probability. As he developed, a lesson with reference to the learners' attempts made in rolling a die in Figure 20 and tossing coin in Figure 21, he wrote summary notes on the board as shown in the Figure 23 with experiment and its corresponding outcome. For example, the experiment of rolling a die should have outcome list observed in the experiment and so is for tossing a coin. The teacher's summery notes which were written as regards the experiment were contrary to what conceptual mathematisation implies according to de Lange and the social constructivist theory who value concrete modelling situations in the learning process (Daniels 2001). The process of extracting the appropriate mathematical concept from a concrete situation is described by De Lange (1987) as conceptual mathematisation. In addition, failure to draw an accurate conclusion by the teacher confirms what the Baseline Survey Report (2018) indicated as well as Liu and Thomson (2009) findings which showed that teachers have content limitation. An impression in these results is that lack of understanding of probability concepts by the teachers is not only due to intuitive thinking but also as a result of its own representative models. This could be so because models are likely to cause difficulty in understanding probability concepts especially when taken to formal instructions according to (Konold 1991; Dowd 2013).

Such conclusions made by the teacher A in Figure 19 about experimental probability are contrary to what scholars say; for instance, the set of all possible outcomes of an experiment is the sample space of the experiment, (Debnath, 2009; Konold, 1991). An event is any subset of a sample space that is of the observers' interest. The likelihood or chance, possibility of an event to occur is probability and it is expressed as a ratio. The objective view of probability is essentially based on the statistical concept of relative frequencies of events from observations and experiments. Based on games of chance with finite number of equally likely outcomes, the objective probability is measured by the ratio of the number of favourable outcomes to the total number of outcomes, (Shaughnessy, 1993; Michael 2003; Dowd, 2013; Konold 1991, 1994). The teacher further connected the failures and success of the trials which learners were making in the experiments.

The teacher assisted learners to connect the constructive social activity model of picking marbles of different colours from the bag from Figure 19 stated earlier in this chapter, to theoretical probability as shown in the example given in Figure 25. **Independent events** and use of a **tree diagram** were developed through questioning and probing. Therefore, the teachers' activities in the classroom setting are interventions portrayed in promoting thought process and reflection on the part of a learner with request for argument in support of their assertions, (Drijvers et al 2014) but this was not done in the case of TB. TB gave learners less chance to interact and did not intervene in provoking thought process among learners. Learners worked on their own without being put in small groups and without any constructive social activity model. Figure 25 also shows the example on the independent events in succession which was set by the TA in order to assist learners' conceptual understanding of probability from a model perspective.

During focus group discussion with learners as shown in Figure 26 the researcher wanted to find out what learners experiences were as regards to teachers' assistance in seeking clarity of probability concepts from a model perspective. Learners referred to their notes and tried to reflect on through each of the sub topics in which some of them performed well while others did not. The ones who had no challenges explained to their colleagues why they did not have challenges and they shared experiences. When the researcher asked what guidance they thought they needed to get in order to answer the questions correctly, they said that the teacher needed to give them all the clear rules to follow in order to answer the questions on probability.

These responses reflect what learners felt the (TB) should have done to assist them answer all questions of probability accurately. However, the main concern of mathematics education is not that learners give correct answers but the process of arriving at the answer by use of concrete models. Moreover, one of the characterisations of knowledge acquisition as stated by Hatano (1996), is that knowledge is 'situated' in contexts; implying that human knowledge acquisition is conceptualised as a process of representing experience with an object, event, or conceptual entity, so that the resultant representation (i.e knowledge) can readily be used in future problem solving and understanding. Even when two or more students know a mathematical formula which is common, how it is represented may vary among them and so the teacher should be there to assist learners understanding (Hatano, 1996). For instance, if a triangular totaling pattern demonstrated by De Jager (1966) raises challenges on how scores are being paired, alternatively, a square tabula method would be better understood than the former. Thus gaining insight of learners' subjective conceptions during social plane as a teacher is a fundamental principle of social constructivism in knowledge construction because learners are indeed not empty slates.

5.4.2 Going round marking learners' work and non-strictness to learners' solutions

Checking and marking learners working is a way of enhancing confidence in the learners' mental processes of knowledge construction as they learn probability from a model perspective (Konold, 1991). TA gave tasks and examples relating from a model perspective but did not at any single time mark or check learners' individual working during and after class lessons except TB. The Figure 27 is a snapshot of learners working which is evident that the TA lacked consistency in helping learners get assistance and clarity of concepts when the tool is withdrawn. There is an unchecked solution for the previous class work, the homework, and the next sub heading indicating Law of probability-mutually exclusive events. No tasks were checked and during semi structured interviews, the researcher asked TA why learners working or solutions were not checked. The response was that he was usually very busy as he was the acting senior teacher and head of mathematics department and had five classes with big number of students to teach. When learners were asked the same question by the researcher during focus group discussion pertaining to how they think about the situation, they said he taught them Additional mathematics as well and so they agreed that he had work overload.

Teacher B as stated earlier did not use any of the concrete modeling tools but was consistent with marking learners work. However, he was not strict with assisting in correcting learners' misconceptions and work accurately as shown in Figure 28. The practice of not caring about learners' process skills may not help in enhancing knowledge acquisition in the topic as it is not a good practice. This resonates with the findings of Koji, Mulenga, and Mukuka (2016) who established that inappropriate approaches and methods used in the teaching was one of the reasons why learners faced challenges in the topic. Hence there is need for teachers to ensure that learners accurately correct their work in order to overcome any challenges they might be facing. The marked solution starting with a mathematical equation p in Figure 28 should have been corrected as P (Not going for remedial), but it shows non-strictness in assisting learners' solutions. Even the second question the learner started with P (die) P (coin) instead of stating a particular event of interest of rolling a die and tossing a coin but the opposite was the case. The understanding of tree diagrams construction especially when it involved picking without replacement also showed that none of the teachers indicated any follow up commentary to guide the learners where they needed help for example in Figure14 and 16. Matura and Makonye (2016) recommend that in order to reduce the intuitive misconceptions, leaners should engage in activities while the teacher scaffolds their limited constructs through checking their work. They argue that giving tasks to learners is a way of assessing their prior knowledge, establishing understanding and non-understanding, correcting their errors as learners extract concepts from a given model. Thus going through and checking learners' working, would reveal learners' behavioural patterns related to the different types of representations used in solving probability tasks that would act as the basis for new knowledge construction. Learners gain confidence in the efficiency of their work with the teacher's help such as marking, checking their work and correcting their errors (Matura & Makonye, 2016).

5.4.3 Allowing learners to attempt questions on the board

Both teachers A and B gave opportunities to learners to attempt tasks while showing their working on the board but did not remind learners to verbalize or to justify their workings as shown in Figure 29. Therefore, teachers should ensure that learners explain loudly their thinking in order to examine both oral and written responses to tasks (Mubanga, 2020).

Moreover, in the social construction of knowledge, teachers are encouraged to give learners chance to show their reasoning either by writing or through explaining. This leads to learners bringing out their spontaneous concepts to the learning. Discussing their attempts by giving explanations enables learners to refine their spontaneous concepts in probability word problems and that helps learners to develop conceptual understanding through collaborating with other learners and the teacher (Vygotsky, 1978). Vygotsky's ZPD, which explains how to advance learners' learning process is reinforced by Wertsch (1985, p. 60-61) who asserted that: Any function in the learner's cultural development appears twice, or on two planes. Teachers should be aware of the fact that learners learn even by imitations from what they see them doing and also from what they observe other people do in society.

5.4.4 Lack of attention to Individual differences

Document analysis reviewed that not much assistance was given to learners who were struggling such as the snapshot in Figure 30. Taking into account ZPD (Vygotsky 1978, p.86), none of the schools applied it in the process of learning as learners were passive, worked independently without social interactions (no small group discussions). The actual development level among learners of different capabilities was not taken care of by the teachers. Due to frustrations some learners stayed without writing the tasks because there was no attention given to their areas of need looking at Figure 31, if the learner was working in collaboration with the significant others the working from that document would not have been in the manner it is. Even though the work was canceled but it still shows non understanding of probability concepts as seen in Figure 31B where one of the solutions to the questions was 6, of which Probability value cannot be greater than 1. It was observed that these learners were ever in the corner and no attention was given to them or even finding out whether they submitted their work for marking or not. It is important for the teacher to note learners' difficulties, level of participation, their actions and how they interoperate concepts or situation definition. Learners' situation definition as described by Wertsch (1984) represents the individual actual level of development and through collaboration with peers; ideas are tested, refined and developed producing a common interpretation of a constructive social activity model.

Teachers should be able to give guided questions in form of a dialogue from a model perspective in order to encourage mathematical thinking in the learners. Learners should be encouraged to ask questions in groups between them and the knowledge gained to be shared to the rest of the members in class so that the other groups can also learn from each other. According to Cobb (1988), when a small group explains its "solution", irrespective of whether it happens to be viable or not," to the whole class, this provides a wonderful opportunity for learning. Furthermore, explanations given by different groups are opportunities for learners and their teachers to learn from the different strategies and methods which other members of the class used (Cockcroft, 1982). Presentations from different groupings can also help teachers to present work in different ways hence helping learners to have depth and breadth of Mathematical concepts**.**

Failure to note individual differences by TB was confirmed during FGDs where the researcher wanted to find out why there was so much fidgeting, copying and movements of learners from one desk to another to look for solutions in school B without any order.

The learners indicated that the problem was that most of them were behind due to missing classes and that the teacher was also too fast in explaining concepts.

The experience stated by L_7B of the teacher being too fast leaving learners far behind, according to (Verenikina, 2010), is contrary to the term ‘proximal’ in the ZPD as it implies ‘nearby,’ indicating that the assistance provided goes slightly beyond learners’ current competence complementing and building on their existing abilities. It is largely a test of a student’s developmental readiness or intellectual level in a specific domain, and it shows how learning and development are related (Schunk, 2012). So, a teacher needs to start with the learners prior conceptions as described in the ZPD notion that a teacher needs to note individual differences, learner capabilities from their own articulation of beliefs and gradually create conceptual shifts in individual learners by mediating those concepts which learners have not yet fully developed as they interact. This process brings about clarity and conceptual understanding of scientific concepts and long life skills acquisition and not rushing through the concepts without learner full participation as observed with T B (Wertsch, 1984; Schunk, 2012).

Figures 30, 31 also are snapshots of some learners working who needed assistance from the significant other as seen from their documents in school B in the process of knowledge construction. Mason (1987) states that learner’s experience is fragmentary –pupils come to the mathematics classroom with a diversity of experiences to which they attempt to relate what they do and hear, constructing stories which form the basis of their understanding. Mathematics meaning varies from individual to another and communication depends upon the ability to share each other’s meanings. Wertsch (1984) as earlier alluded to, also argues that during learner interaction with the set constructive social activity model/ models, some learners are likely to face challenges such as their inability to manipulate, interpret appropriately which in turn limit their constructs and result in making errors. They are likely to raise hands to seek clarity from the teacher or peers. Their limited interpretation of the given situation is dependent on each learner’s prior knowledge and may be able to construct that which is limited to their experience. As a result, they may need assistance from their peers or the teacher in order to be assisted in the construction of meaning to reach the potential level of development. Basing the study on Wertsch's (1984) elaborations on what 'problem solving under adult guidance or in collaboration with more capable peers,' means in the ZPD, and by using his concepts of situation definition and intersubjectivity in application, both teachers in school A and school B did the contrary.

5.5 Summary of the chapter

This chapter discussed the findings of the study on the use of constructive social activity models when learning probability. The findings revealed that only the learners in one of the two sampled schools used concrete tools in line with the social cultural theory. Although the use of concrete tools was applied in the learning process and learners' full participation in the physical manipulation of the concrete tools, when it came to conceptual mathematisation in the ZPD, some concepts were mis-interpreted. Moreover, learners as well as teachers were unfamiliar with most of the concrete representation modelling tools framed in word problems of probability thereby limiting various representations for cognition. Teacher B did not involve the grade 11 learners in their own learning, did not adequately probe learners' in order to determine their actual level of development and correct errors or even to justify their thinking on written tasks, they did not learn as inter subjective on a social plane in small groups. Problem solving under adult guidance or in collaboration with more capable peers was not applied in the learning process of probability especially in school B. In the next chapter, conclusions and recommendations are presented.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Introduction

This chapter concludes the study and highlights the major contribution the study has made to research (Section 6.3). Thereafter, the recommendations (Section 6.4) are given in order to improve the process of knowledge construction in the learning process with insight and reasonable thinking rather than focus on the product of correct answers. The increased awareness of how logical relations of scientific concepts written in textbooks are linked to tools (constructive social activity models) during the classroom learning process in the real world with regard to learners' indigenous knowledge and experiences, would foster review of certain tools embedded in textbooks for conceptual relevance. It is also vital for the curriculum planners to be aware of how the use of tools mediate learning and constrain knowledge construction process not only on the part of the teacher but also the learners. The tools can be framed coherently in word problems in the textbooks, even provided to learners but research on how they are presented in a concrete situation and interpreted in the learning process to draw conceptual understanding of scientific concepts cannot be ignored. The cognising subjects in the process of knowledge construction during learning should be checked because knowledge is not acquired by transmission alone but by active participation of all subjects. In order to produce robust knowledge and contribute to higher achievement performance in mathematics, reference to concrete real-life situations must be embraced in teaching and learning. This is because the research revealed non understanding of real representation of abstract concepts hence leading to rote learning and poor performance results as reviewed by the Examinations Council of Zambia 2017 and 2018. The chapter ends by suggesting further research in mathematics education.

6.2.Conclusions

Since this study's main caption was, "Use of constructive social activity models when learning Probability," it therefore focused on observable activities attributed to real life practices that call for learner participation and influence learner's own knowledge construction leading to understanding. This involved interpreting the meanings, relevance and appropriateness of reality of those models in application and how they foster or constrain knowledge construction process. It was meant to seek insight into how the use of appropriate tools (models) on a social setting address the relationship between the mathematical concepts of probability in relation to their different cultural context in which learners exist which was refered to as conceptual mathematisation. Thus, the learners' reference books such as Progress in Mathematics for grade 9 & 11 on pages 154 & 147-151, Zambia Senior Secondary School Mathematics grade 11 & 12 have various models to mediate the cognition of probability concepts which were explored on how learners engage in drawing conceptual understanding of probability and their conceptual relevance to the particular dynamic cultural setting.

The findings revealed that only the learners in one of the two sampled schools used a dice, coin and marbles as concrete tools in line with the social cultural theory. Although the use of concrete tools was applied in the learning process and learners' full participation in the physical manipulation of the concrete tools in this school, when it came to conceptual mathematisation in the ZPD, some concepts were mis-interpreted and misrepresented. Moreover, learners as well as teachers were unfamiliar with most of the concrete representation modelling tools framed in word problems of probability thereby limiting various representations for cognition. In another sampled school, Teacher B did not involve the grade 11 learners in their own learning, did not adequately probe learners' in order to determine their actual level of development and did not correct errors or even to justify their thinking on written tasks, they did not learn as intersubjective on a social plane in small groups. They learned the concepts in abstract without use of any physical objects and problem solving under adult guidance or in collaboration with more capable peers was not applied in the learning process. Learners were not given chance to come up with their own modelling tools as it was presented that there were insufficient reference materials to distribute to learners for self-guidance or self-paced learning. Generally, it was concluded that there was no appropriate use of concrete constructive social activity models in the process of learning and how they link to abstract concepts in probability was misinterpreted. Even though learners were engaged in the use of models with excitement in school A their conceptions were misguided from the model perspective by their teacher.

Those who did not use any models, were passive and challenged such that during class exercises they were only copying, memorising and others just waited for the lesson to come to an end without any concern from the teacher.

6.3 Contribution to the field

This study has made an academic contribution to the body of knowledge regarding research in mathematics education in the use of constructive social activity models in the learning of scientific concepts particularly probability and how teachers assist learners draw conceptual understanding of probability.

Vygotsky's (1978) Sociocultural theory of development, the ZPD notion, Wertsch (1984)'s elaborations, the post Vygotskian constructivists with the evolution of Realistic Mathematics Education (RME) scholars and related literature helped the researcher apply emphasise appropriately to the analysis of data on the use of constructive social activity models' learners use when learning probability. These assumptions enabled the researcher to establish an insight into how learner's physical manipulation of concrete tools set as a representation of abstract concept in the classroom learning imply. They gave a deep understanding on how objective concepts which are written in textbooks affect learners' interest due to the teacher's interpretations and presentations in relation to learners' experiences. Teachers did not fully engage grade eleven learners in knowledge construction from a model perspective; they did not accurately probe learners' understanding (situation definition), and no small group discussions (intersubjectivity) were observed. The learning process was largely enhanced by cognitive modelling of probability concepts and resulted in non-understanding of concepts as seen from their documents. Some concepts were applied in some studies before to determine people's inability to teach probability from a model perspective, but this study focused on the use of constructive social activity models when learning probability in a contextualised learning. Some models framed in word problems, pictured and used to illustrate abstract concepts and their contextual nature was questioned and found to be unfamiliar to learners and teachers, hence constrained the viability of knowledge constructs from the model perspective. In view of the findings of this study, the following recommendations were made:

6.4 Recommendations

The following recommendations were made in the light of policy, practice, and further research.

i. The findings show that the use of constructive social activity models in the process of learning probability is generally limited to only a few specific and less sophisticated concepts and poorly linked to experimental and theoretical probability concepts. Lesson studies and CPDs should be intensified in schools by teachers of mathematics so that there is in depth exploration of modelling tools not just by knowing how the tools are used locally but also how to integrate them in the learning process for robust knowledge development of various scientific concepts.

ii. The results also show that the suggested interpretive and elaborative framework through which effective learning takes as posited by constructivist epistemologies and the ZPD notion, was less applied in the classroom learning situations. Learners were not engaged in their own learning, in most situations, they learnt concepts in abstract, there was no verbal interpretations of concepts as individuals and as groups, teachers did not pay attention to learners' individual differences and there was no small group discussion implementation. Therefore, the researcher recommends that the Curriculum Development Centre should incorporate and emphasise Vygotsky's (1978) Sociocultural theory of development and the post Vygotskian constructivists, Hatano (1996)'s five characterisations of knowledge acquisition, with the evolution of Realistic Mathematics Education (RME conceptual mathematization) at all levels of education.

iii. In addition, teachers of mathematics to ensure designing and developing cost effective teaching/learning aids and materials/even for in-service training which are rooted in the learner's cultural practices for conceptual understanding of probability concepts.

iv. A coin is important in the development of probability theory, it has significant implications in the knowledge acquisition processes in other domains as demonstrated by Pascal. However, research results show that even if the old denomination coins have disappeared in circulation from which the features Head and Tail outcomes originated, the outcomes have remained static in probability content which contradicts with the coins in circulation in real life which have; for instance a coat of arms and an eagle on each side. MOGE through the CDCs should review books, while integrating learners' indigenous knowledge which is continuous and subjective so that the constructive social activity models embedded in the content, the activities, culture and concepts are interrelated and timely even for self-paced learning.

v. Teachers should ensure that learners take a leading role in their own learning by giving them learning materials such as textbooks so that they have enough time to come up with their own constructive social activity models in relation to particular concepts. This will enable teachers to establish what learners can do unassisted and what they can only do with assistance, so that they can intervene by providing the necessary mediation.

vi. Teachers should be strict with learners' processes of knowledge acquisition and attention to individual differences must be paid. As the process of knowledge acquisition is constrained by various factors such as background experiences, biological processes, tool use (constructive social activity model), the task set at hand and the instructions given to carry out a task, just mention but a few

vii. Results also showed that teachers had content limitations as evidenced in the misinterpretations of the concepts and non-strictness to learners' solutions whereby they did not guide learners in the learning process so as to even construct correct tree diagrams. Therefore, teachers should be engaged in more lesson studies, since Senior Standard Education Officers (SESO) are responsible for schools in a province, and Heads of Department should monitor the teachers from various schools more often to ensure that topics such as probability which have proved to be more challenging could be re enforced. Planning on how they intend to engage learners in learning by integrating various strategies, giving prompt feedback to learners and how they give meaningful illustrations that accurately challenge their prior conceptions should be planned and checked by management consistently.

6.4.1 Further research

i. There is need to conduct an exploratory research on the relationship of the interplay of culture, activities, concept and learners' diverse experiences in order to have insight on the integration of models and their various means of representation in the learning process of probability.

ii. There is need to carry out a quantitative study to examine learners' performance if the use of constructive social activity models would increase concept acquisition and improve performance of the learners in relation to their cultural set up.

iii. There is need to establish an empirical evidence to determine the causes of teacher content limitation and learner learning difficulty in probability.

iv. There is need to investigate learners' views on their limited participation in mathematics lessons and in the manipulation of constructive social activity models in probability.

v. Research into culture, activities and how concepts develop is needed.

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APPENDICES

Appendix 1

Projected Research Budget

S/N	DESCRIPTION	QUANTITY	UNITY PRICE (ZMW)	TOTAL COST (ZMW)
1	**Stationery**			
	- Reams of paper	02	60	120
	- Box of pens	01	30	30
	- Flash disk 32GB	01	175	175
2	**Research equipment**			
	-MP 3 Recorder	01	800	800
	-Typing proposal and report -	08	200	1600
	Printing: copies of proposal and Report.	08	200	1600
	- Binding: copies of proposal and report	08	100	800
	- Communication processes	-	600	600
4	**Research cost**			
	- Transport to and from	4	360	1440
	Mbala.	14	180	2520
	- Lodging per night	14	80	1120
	- Feeding per day			
5	- Ethical clearance	01	500	500
	- Poster	01	300	300
	- Editing	01	1800	1800
5	**Miscellaneous**			800
6	**GRAND TOTAL**			**K14,105**

Appendix 2

Proposed Work Plan and Time Table

S/N	ACTIVITY DETAILS	DATES	DURATION
1	Preparation of research proposal	September - October, 2019	Eight weeks
2	Development piloting recasting of data collection instruments	January, 2020	Two weeks
3	Data collection processing and analysis	February - March 2020	6 weeks
4	Report writing, typing and editing	September / October, 2020	Seven weeks
5	Proof reading, production & submission of first draft	October, 2020	Three weeks
6	Refining and submission of second draft report	November, 2020	Three weeks
7	Refining and presentation of final draft	December, 2020	Four weeks

Appendix 3

Letter of Permission from the District Education Board

All correspondences
Should be addressed to:
District Education Board secretary
Mbala District Education Board
MBALA

In reply please quote

REPUBLIC OF ZAMBIA

MINISTRY OF GENERAL EDUCATION

MBALA DISTRICT EDUCATION BOARD
P.O. BOX 420240
MBALA

20th February, 2020

Ms. Doye Chikola

u.f.s: The Headteacher
Lucheche Day Secondary School
MBALA

Dear Sir/Madam,

***RE:* PERMISSION TO CONDUCT RESEARCH IN SCHOOL: YOURSELF**

Reference is made to the stated subject matter above.

Permission has been granted to Chikola Doye to conduct research at Lucheche Day Secondary and Mbala Secondary School.

Kindly give her the support she may need.

REPUBLIC OF ZAMBIA
MINISTRY OF GENERAL EDUCATION
[illegible] 2020

Wilfred Chilala
District Education Standards Officer
For/District Education Board Secretary
MBALA

Appendix 4

Lesson observation guide

Dear Participant,

I am a postgraduate student at the University of Zambia (UNZA) pursuing a Master's degree in Mathematics Education. I am conducting a research on the topic, **"***Insight into constructive social activity models learners' use when learning probability"*. Kindly feel free as I observe record and capture the lesson. This study is purely for academic purposes and not meant to cause you any mental or physical harm. I therefore, encourage you to participate freely.

School code:……………Teacher's code: …………… Grade: ……………Class ………

Time: ………………………Number of Learners: Boys… Girls: ………………………

Observation sheet.

Topic:……………. S/Topic………………. Date………………………..

Models		**Lesson Introduction**	**Lesson development**	**Learner activity/tool use /participation**	**Comments**
From text books	**others/improvised**				
dice					
coin					
likelihood scale					
Standard deck of cards					
Jackpot bank					
Spinning wheel					
lottery					
marbles					
chess					
Tree diagrams					
Tabular presentations					
Other models					

General

observations……………………………………………………………………………..

………………………………………………………………………………………………..

………………………………………………………………………………………………..

………………………………………………………………………………………………..

Appendix 5

Document Analysis Sheet

Dear Participants,

I am a postgraduate student at the University of Zambia (UNZA) pursuing a Master of Education in Mathematics Education degree. I am conducting a research on the topic, "insight into the constructive social activity models engaged by learners in learning probability concepts**."** You are kindly requested to expose your working sheets in this research by providing me with your working for the tasks, exercises during or after the learning process. All observations on your working recorded will be taken as strictly classified. This study is purely for academic purposes and not meant to cause you any mental or physical harm. I therefore, encourage you to be free as I conduct document analysis.

School code:……………………Class code:Learner code:……………………..

Grade:……………..

Time:……………………Number of Learners: Boys……………. Girls……………….

Date:………………………Document type:………………………………………………

Document analysis schedule sample

VISUAL PRESENTATIONS	**TYPE/THOUGHT PROCESS**
Tree diagrams	correctness
Outcome listing (i.e set brackets, tables)	accuracy
Formulas/equations	Conceptions and correctness
Symbols and signs	Interpretation and meanings

Appendix 6

Letter of Consent for Parent/ Guardian of Learners

Dear Parent/Guardian

I am a second year Master of Education in Mathematics Education student at the University of Zambia (UNZA) under the supervision of Dr. Nalube, P.P. I wish to conduct a research study as a final year student, which is, "*use of constructive social activity models learners' use when learning probability*." This will help suggest to curriculum designers, to consider reviewing models for contextual and conceptual relevance as this could help in planning lessons while focusing on learners' cultural background. I wish to observe grade 11 learners first, then check their worksheets and finally wish to have focused group discussions with the learners. This is aimed at having in-depth understanding of learners' thinking process pertaining to how they relate models to concepts of probability. The focused group discussion may take about 30 minutes. Therefore, participation of your child in this study is voluntary issue and any information collected will be kept anonymous.

Please indicate by filling in the form below whether you permit your child to take part in this study. Your cooperation will be highly appreciated.

For further enquires please contact cell phone(s) 0973474520 or email 2018248723@student.unza.zm

Thanks for your cooperation
Yours Sincerely,

Chikola Doye,
I………………………………………………………….. Agree / disagree to allow my child …………………………………………………………to participate in,
PARENT/GUARGIAN
SIGNATURE…………………………………………………………...
DATE
……………………………………………………………………………………………

Thank you

Appendix 7

Focus group discussion guide

Dear participant,

I am a second year Master of Education in Mathematics Education student at the University of Zambia (UNZA) under the supervision of Dr. Nalube, P.P. I wish to conduct a research study as a final year student, which is to, "*use of constructive social activity models learners' use when learning probability*." The purpose of this study is to provide insight into constructive social activity models earners engaged in learning probability concepts." This will help suggest to curriculum designers, to consider reviewing models for contextual and conceptual relevance as this could help in planning lessons while focusing on learners' cultural background. I wish to observe grade 11 learners first, then check their worksheets and finally wish to have focused group discussions with the learners. This is aimed at having in-depth understanding of learners' thinking process pertaining to how they relate models to concepts of probability. The focus group discussion may take about 30 minutes with participants who are purposefully selected. Therefore, your participation in this study is voluntary issue and any information collected will be kept anonymous.

Sign:…………………………………………………………………………………….

Questions to provoke the discussions may be based on the challenges or successes with respect to class observation notes obtained in the classroom. The text books will be given to learners so that the discussion starts with a flow. These may be as follows;

1. What challenges did you encounter while learning probability and why?
2. How does the use of tools/objects help or challenge your understanding of probability concepts?
3. How familiar are you with these objects/things cited in textbooks for probability concepts?
4. What guidance do you think the teacher should give or should have given you in the learning process in order that you answer the questions correctly?

5. Identify any games of chance in your textbooks.
6. Explain why you think these games are played.
7. How do they relate to the concepts you have learned.
8. From what you have learned in the classroom about probability concept e.g.(likelihood), what things or tools in life can demonstrate a chance? (either from world fairy tales or physical models)
9. Discuss any games of chance that you have played before and the rules?
10. What else in your environment can demonstrate any concepts you have learned today?
11. How can the concepts you have just learned be applied to solve problems in life?

Appendix 8

Semi-Structured Interview Guide for Teachers

Dear Participant,
I am a postgraduate student at the University of Zambia (UNZA) pursuing a Master's degree in Mathematics Education. I am conducting a research on the topic, ‘' use of constructive social activity models engaged by learners in learning probability concepts".You are kindly requested to participate in this research by responding to the questions below, through filling in your answers in the spaces provided below each question. This study is purely for academic purposes and not meant to cause you any mental or physical harm. I therefore, encourage you to participate freely.

Sign…………………………………………………………………………………….

TEACHER'S CHOICE OF THE MODEL TOOLS TO USE IN ASSISTING LEARNERS' CONCEPTUAL UNDERSTANDING OF PROBABILITY.

1. What other things do you use apart from those stated in the examples/exercises in learners textbook when teaching of probability to your learners?
2. How do the use of those models you have mentioned act as examples in your probability lessons?
3. How familiar are you with the things or objects/ tools/ cited in the textbooks to use when teaching probability concepts to your learners?
4. How can you demonstrate the use of the models/or the things stated from the textbooks in order to assist learners learn probability concepts from the model perspective? For example the pictures on page 147 in the grade 9 textbooks and those cited from grade 11 text book, how can you help the learners through use of these in order to clarify the probability concepts?
5. Why didn't you use the activity models/tools/ things cited in the text book, e.g a die to demonstrate to learners how they can use it to draw conceptual understanding of probability?
6. What do you do to ensure that all the learners actively participate in the designed activity when learning probability concepts from the use of the things cited in the text book?
7. How do you help learners make real life connections with probability concepts?
8. What things in your view do you think would be appropriate to include in the learners' reference materials for conceptual relevance?

Appendix 9

Ethical clearance letter from the University of Zambia

THE UNIVERSITY OF ZAMBIA

DIRECTORATE OF RESEARCH AND GRADUATE STUDIES

RESEARCH DEPARTMENT

APPROVAL OF STUDY

4th November, 2020.

REF NO.HSSREC-2020-FEB-001

Chikola Doye
LUSAKA

Dear Ms. Chikola,

RE: "INSIGHTS INTO CONSTRUCTIVE SOCIAL ACTIVITY MODELS LEARNERS USE WHEN LEARNING PROBABILITY IN SELECTED SECONDARY SCHOOLS IN MBALA DISTRICT"

Reference is made to your protocol dated 1st February, 2020. HSSREC resolved to approve this study and your participation as Principal Investigator for a period of one year.

REVIEW TYPE	ORDINARY REVIEW	APPROVAL NO. HSSREC-2020-FEB-001
Approval and Expiry Date	Approval Date: 4th November, 2020	Expiry Date: 3rd November, 2021
Protocol Version and Date	Version - Nil.	3rd November, 2021
Information Sheet, Consent Forms and Dates	• English.	To be provided
Consent form ID and Date	Version - Nil	To be provided
Recruitment Materials	Nil	Nil
Other Study Documents	Questionnaire.	
Number of Participants Approved for Study		

Specific conditions will apply to this approval. As Principal Investigator it is your responsibility to ensure that the contents of this letter are adhered to. If these are not adhered to, the approval may be suspended. Should the study be suspended, study sponsors and other regulatory authorities will be informed.

Conditions of Approval

- No participant may be involved in any study procedure prior to the study approval or after the expiration date.
- All unanticipated or Serious Adverse Events (SAEs) must be reported to HSSREC within 5 days.
- All protocol modifications must be approved by HSSRECprior to implementation unless they are intended to reduce risk (but must still be reported for approval). Modifications will include any change of investigator/s or site address.
- All protocol deviations must be reported to HSSRECwithin 5 working days.
- All recruitment materials must be approved by HSSRECprior to being used.
- Principal investigators are responsible for initiating Continuing Review proceedings. HSSRECwill only approve a study for a period of 12 months.
- It is the responsibility of the PI to renew his/her ethics approval through a renewal application to HSSREC.
- Where the PI desires to extend the study after expiry of the study period, documents for study extension must be received by HSSRECat least 30 days before the expiry date. This is for the purpose of facilitating the review process. Documents received within 30 days after expiry will be labelled "late submissions" and will incur a penaltyfee of K500.00. No study shall be renewed whose documents are submitted for renewal 30 days after expiry of the certificate.
- Every 6 (six) months a progress report form supplied by The University of Zambia Humanities and Social Sciences Research Ethics Committee as an IRB must be filled in and submitted to us. There is a penalty of K500.00 for failure to submit the report.
- When closing a project, the PI is responsible for notifying, in writing or using the Research Ethics and Management Online (REMO),both HSSRECand the National Health Research Authority (NHRA) when ethics certification is no longer required for a project.
- In order to close an approved study, a Closing Report must be submitted in writing or through the REMO system. A Closing Report should be filed when data collection has ended and the study team will no longer be using human participants or animals or secondary data or have any direct or indirect contact with the research participants or animals for the study.

- Filing a closing report (rather than just letting your approval lapse) is important as it assists HSSRECin efficiently tracking and reporting on projects. Note that some funding agencies and sponsors require a notice of closure from the IRB which had approved the study and can only be generated after the Closing Report has been filed.

- A reprint of this letter shall be done at a fee.

- All protocol modifications must be approved by HSSREC by way of an application for an amendment prior to implementation unless they are intended to reduce risk (but must still be reported for approval). Modifications will include any change of investigator/s or site address or methodology and methods. Many modifications entail minimal risk adjustments to a protocol and/or consent form and can be made on an Expedited basis (via the IRB Chair). Some examples are: format changes, correcting spelling errors, adding key personnel, minor changes to questionnaires, recruiting and changes, and so forth. Other, more substantive changes, especially those that may alter the risk-benefit ratio, may require Full Board review. In all cases, except where noted above regarding subject safety, any changes to any protocol document or procedure must first be approved by HSSREC before they can be implemented.

Should you have any questions regarding anything indicated in this letter, please do not hesitate to get in touch with us at the above indicated address.

On behalf of HSSREC, we would like to wish you all the success as you carry out your study.

Yours faithfully,

Dr. J. Mwanza

DR. JASON MWANZA
Dip. Clin. Med. Sc., BA.M.Soc., PhD

CHAIRPERSON
THE UNIVERSITY OF ZAMBIA HUMANITIES AND SOCIAL SCIENCES RESEARCH ETHICS COMMITTEE - IRB

cc: Director, Directorate of Research and Graduate Studies
Assistant Director (Research), Directorate of Research and Graduate Studies
Assistant Registrar (Research), Directorate of Research and Graduate Studies