

**The University of Zambia**  
**University Examinations – Semester I 2001/02**

C311

Biochemistry I

**Instructions:**

- 1) Time: **Three (3)** Hours
- 2) There are two Sections (7 printed pages) in this examination paper, Section A and Section B; Answer All questions in Section A and any 6 questions in Section B

**Section A (40 marks)**

Answer **ALL** Questions in this section.

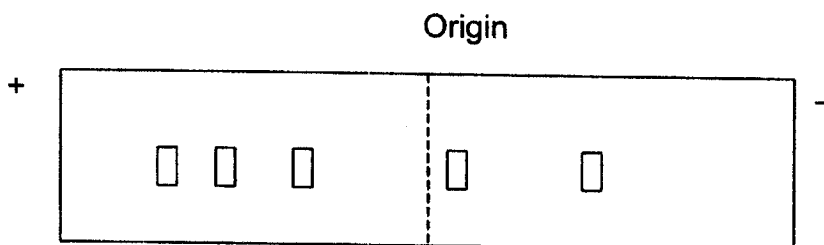
1. a) Which amino acids are the major determinants of the charge of a polypeptide chain containing all 20 amino acids?
- b) Why would a sequence of lysl residues in a polypeptide chain destabilize an  $\alpha$ -helix conformation at pH 7 (pka1 = 2.2, pka2 = 9.0, pka3 = 10.5)?

**[4 marks]**

2. Briefly explain how proteins, which form the delicate molecules of enzymes, etc., can also form tendons with tremendous tensile strength.

**[4 marks]**

3. A mixture of five peptides (P1, P2, P3, P4, and P5) is subjected to paper electrophoresis at pH 8.5. After electrophoresis, staining of the paper revealed the pattern of migration shown in the figure below. Given the following pI values for each peptide-9.0 for P1, 5.5 for P2, 10.2 for P3, 8.2 for P4, and 7.2 for P5- identify which zone corresponds to each peptide. Assume that each peptide has a mol. Weight close to 1200. If another peptide P6 (pI = 10.2 and a molecular weight about 1800) is added to the original mixture, where will you predict it to move



[6 marks]

4. The equilibrium constant  $K$  for the binding of oxygen to myoglobin is  $10^6 M$ , where  $K$  is defined as

$$K = \frac{[Mb][O_2]}{[MbO_2]}$$

The rate constant for the combination of  $O_2$  with myoglobin is  $2 \times 10^7 M^{-1}S^{-1}$ .

- What is the rate constant for the dissociation of  $O_2$  from oxymyoglobin?
- What is the mean duration of the oxymyoglobin complex?

[4 marks]

5. Irreversible inactivation of an enzyme may be confused with noncompetitive inhibition. Why? How could you distinguish between a reversible noncompetitive inhibition and an irreversible inactivator? Enzyme supply is not limiting? (**Very brief explanation!**)

**[4 marks]**

6. Write short notes on ATP synthase.

**[2 marks]**

7. Briefly describe the role of biotin in the catalytic mechanism of pyruvate decarboxylase.

**[2 marks]**

8. Glucose labeled with  $^{14}\text{C}$  at C-2 is added to a solution containing the enzymes and cofactors of the TCA and glycolysis. What is the fate of the radioactive label?

**[6 marks]**

9. How do the toxins cyanide, rotenone, antimycin A inhibit the Electron Transport Chain?

**[6 marks]**

10. Briefly explain why glycolysis is a more efficient way of metabolizing glucose than fermentation.

**[2 marks]**

**Section B (60 marks)**

Answer **ANY 6** questions in this section

All questions this section carry equal marks

11. a) After a pre-incubation with  $p$ -chloromercuribenzoate, the binding of an enzyme with its substrate was no different from that of untreated enzyme, but the catalytic activity of the treated enzyme was found to be 40% less. What conclusion can be drawn from this type of observation? **(be very brief and to the point)**

b) You have measured the initial velocity of an enzyme in the absence of inhibitor and with inhibitor A and inhibitor B. In each case, the inhibitor is present at  $10\mu\text{M}$ . The data are shown in the table below.

[S] mM	Velocity ( $\text{Msec}^{-1}$ ) $\times 10^7$ Uninhibited	Velocity ( $\text{Msec}^{-1}$ ) $\times 10^7$ inhibitor A	Velocity ( $\text{Msec}^{-1}$ ) $\times 10^7$ Inhibitor B
0.333	1.65	1.05	0.794
0.40	1.86	1.21	0.893
0.50	2.13	1.43	1.02
0.666	2.49	1.74	1.19
1.0	2.99	2.22	1.43
	3.72	3.08	1.79

- Determine  $K_m$  and  $V_{max}$  of the enzyme
- Determine the type of inhibition imposed by inhibitor A and calculate  $K_i$  (s)
- Determine the type of inhibition imposed by inhibitor B and calculate  $K_i$  (s)

**[10 marks]**

12. a) A pentapeptide obtained from treatment of a protein with trypsin was shown to contain arginine, aspartic acid, leucine, serine, and tyrosine. For determination of amino acid sequence, the peptide was cycled through the Edman degradation procedure three times. The composition of the peptide remaining after each cycle was as follows:

After cycle 1: arginine, aspartic acid, leucine, and serine

After cycle 2: arginine, aspartic acid, and serine.

After cycle 3: arginine, serine.

Determine the probable sequence of the peptide (**show your working**).

b) The absorbance of a protein solution containing 1 mg/ml albumin was measured using a 1 cm light path and was found to be 0.550 and 0.199 absorbance units at 279 nm and 290 nm respectively. Assuming that tyrosine and tryptophan are the only amino acids that absorb appreciably at these wavelengths, calculate the number of these two amino acids in a molecule of the protein given that:

(a) MW of albumin is 65 000 Dalton.

(b) The molar extinction coefficient of tyrosine is  $1.1 \times 10^3$  l/mol.cm at 279 nm and  $0.28 \times 10^3$  l/mol.cm at 290 nm.

(c) The molar extinction coefficient of tryptophan is  $5.2 \times 10^3$  l/mol.cm at 279 nm and  $4.3 \times 10^3$  l/mol.cm at 290 nm.

[10 marks]

13. Compare and contrast the Pentose Phosphate Pathway and Gluconeogenesis, paying particular attention to their roles in the body, the source of precursors and final products.

**[10 marks]**

14. Using a detailed diagram, outline the Chemiosmotic theory and then answer the following:

- (a) What are the effects of uncoupling agents on ATP production?
- (b) What happens if the inner mitochondrial membrane is made more permeable to protons?
- (c) How do the precursors for ATP production enter the inner mitochondrial membrane?

**[10 marks]**

15. Using a table, briefly describe the fate of pyruvate under aerobic and anaerobic conditions.

**[10 marks]**

16. What is the role of hormones in the coordination of glycogen synthesis and degradation?

**[10 marks]**

17. Using a detailed diagram, outline the Chemiosmotic theory and then answer the following:

- (a) What are the effects of uncoupling agents on ATP production?
- (b) What happens if the inner mitochondrial membrane is made more

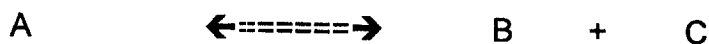
permeable to protons?

(c) How do the precursors for ATP production enter the inner mitochondrial membrane?

**[10 marks]**

18. a) Explain why thermodynamics and kinetics are each important in life processes.

b) Consider the hypothetical reactions  
(pH 7, 25<sup>0</sup>C and 1 atmosphere)



The initial concentration of A is 0.19M; at equilibrium, only 1% of A remains. Calculate:

- i)  $K_{eq}$ , and
- ii) the  $\Delta G^{01}$  for the synthesis of A from B and C { $R = 1.987$  cal/mol/K}

**[10 marks]**

**End of examination!**



THE UNIVERSITY OF ZAMBIA

UNIVERSITY EXAMINATIONS – APRIL 2002

C321

TIME: THREE (3) HOURS

ANSWER: ANY FOUR QUESTIONS

1.a Complete the following table:

	$\lambda(\text{cm})$	$\nu(\text{sec}^{-1})$	$\nu(\text{cm}^{-1})$
i.	$4.5 \times 10^{-5}$		
ii.		$3 \times 10^{13}$	
iii.			$2 \times 10^5$

b. Complete the following table:

	X[M]	A	T%	E(l/mol-cm)	b(cm)
i.			34	1540	200
ii.		0.613		250	10.00
iii.	$2.5 \times 10^{-2}$	0.167			1.00

- c. i. Calculate the energy in KJ at 200 nm, 400 nm and 750 nm per mole respectively.  
ii. What are the wave numbers at 0.8  $\mu\text{m}$ , 2.5  $\mu\text{m}$  and 15.4  $\mu\text{m}$  respectively. [ $h = 6.62 \times 10^{-34}$ ,  $N_A = 6.02 \times 10^{23}$ ,  $C = 3.0 \times 10^8$  m/s]
- d. A  $1.28 \times 10^{-4}$ M solution of potassium permanganate has a transmittance 70% at 525 nm in 2.00 cm cell. [M.M.KMnO<sub>4</sub> = 158.04].
- i. What is the absorbance and molar absorptivity of this solution?  
ii. If the concentration were doubled, what would be transmittance and absorbance respectively?  
iii. What concentration would have a transmittance of 0.5 in this cell?

- e. i. A 25 ml aliquot of Kafue River water was treated with a mild reducing agent and then excess 2, 9-dimethyl-1, 10-phenantroline. This reagent reacts with copper but not iron(II). After diluting to 50.0 ml, the solution has an absorbance of 0.388 at 455 nm. From the calibration curve, this absorbance correspond to a concentration of 3.1 ppm Cu. Calculate the copper concentration in parts per million in the original sample. [Cu = 63.546].
- ii. If the density of the water sample was 1.0g/ml, calculate the percent copper in the water.
- f. A mixture of  $\text{Ni}^{2+}$  and  $\text{Co}^{2+}$  is analyzed by forming the salicylaldotime complexes in a chloroform solvent. The molar absorptivities are  $\epsilon_{\text{Ni}} = 10$  and  $\epsilon_{\text{Co}} = 1000$ , while at 400 nm both molar absorptivities are  $5 \times 10^3$ . If a solution in a 1-cm cell exhibits an absorbance of 0.091 at 500 nm and 0.615 at 400 nm, calculate the  $\text{Ni}^{2+}$  and  $\text{Co}^{2+}$  concentrations ( in M). [Ni = 58.69 and Co = 58.932].
2. a. Explain principle of IR- spectroscopy
- b. What type of stretching and bending vibrations do you know? Sketch them.
- c. Explain quantum treatment of vibrations.
- d. Explain the principle of Raman spectroscopy.
- e. What are the differences between sources, monochromators, cells and detectors in UV + VIS and IR spectroscopy respectively.
- f. Explain different sample handling techniques in IR- spectroscopy.
- g. Calculate the wavenumber and wavelength of the fundamental absorption peak due to stretching vibration of  $\text{C}\equiv\text{N}$  group.  
 [C=12.0, N=14.0,  $k=1.5 \times 10^3$  N/m (or  $17.5 \times 10^5$  dyn  $\text{cm}^{-1}$ ),  $C=3.0 \times 10^8$  m/s,  $N_A = 6.02 \times 10^{23}$ ].
3. a. Explain the principle of atomic absorption spectroscopy (AAS) and compare it with flame photometry. Sketch them.
- b. Explain the principle of emission spectroscopy
- c. Describe type of sources for AAS and emission spectroscopy
- d. Explain Inductively coupled plasma source. Sketch a diagram.

- e. Outline briefly fluorescence method.
- f. Explain standard addition method for AAS.
- g. A 2.00 ml sample of Cow's milk was diluted to 50 ml and analyzed for Zn, using AAS. The absorbance was 0.106. A second 2.00 ml aliquot was spiked with 12  $\mu\text{g}$  of Zn before dilution to 50 ml. The absorbance was 0.245. Calculate Zn concentration in the milk as parts per million (micrograms per milliliter). [Zn = 65.38]
- h. Magnesium in blood serum can be determined by AAS. A 1.0 ml serum sample is diluted to 100 ml, and its absorbance is 0.125. A standard containing  $2.0 \times 10^{-5}\text{M}$   $\text{Mg}^{2+}$  gives an absorbance of 0.187. Calculate Mg concentration in the blood as milligram percent (milligrams per 100 ml). [Mg = 24.305].
4. a. Explain principle of  $^1\text{H}$  NMR spectroscopy.
- b. Explain  $^{13}\text{C}$  NMR spectroscopy and their advantages comparing to  $^1\text{H}$  NMR.
- c. Explain chemical shift in  $^1\text{H}$  NMR spectroscopy.
- d. Explain spin-spin splitting in  $^1\text{H}$  NMR spectroscopy.
- e. Sketch the basic components for  $^1\text{H}$  NMR instrument.
- f. Calculate the chemical shift of a particular nucleus in a 60-MHZ instrument if the reference nucleus absorbs at a magnetic flux density that is 0.063 G greater than that at which sample nucleus absorbs. [For hydrogen nucleus a magnetic flux density is  $14 \times 10^3$  G].
- g. Calculate the energy and the frequency of radiation that is required to excite a proton from the lower to the upper energetic level if the applied field has a magnetic flux density of 14,092 G. For a proton magnetogyric ratio is  $2.68 \times 10^8$  radian  $\text{T}^{-1}\text{S}^{-1}$ .
5. a. Explain principle of mass spectrometry.
- b. Sketch the basic part of mass spectrometer.
- c. Calculate the resolution in a mass spectrum if a peak that is centered at 245 amu has a peak width at 5 percent of the peak height that is 0.52 amu.

d. What accelerating potential will be required to direct a simply charged water molecule through the exit slit of magnetic mass spectrometer if the magnet has a field strength of 0.240 tesla and the radius of curvature of the ion through the magnetic field is 15.6 cm?  
[charge per ion  $eZ = 1.60 = 1.60 \times 10^{-19} \text{ C}$ ,  $N_A = 6.02 \times 10^{23}$ ].

e. Write the fragmentation of mass spectra from (i)  $\text{R-CH}_2\text{-CH}_2\text{-OH}$

(ii) and from methylphenyl - $\text{O-CH}_3$ , and from

(iii) nitro benzen, - $\text{NO}_2$  respectively.

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THE UNIVERSITY OF ZAMBIA  
UNIVERSITY SEMESTER I EXAMINATIONS , APRIL 2002

C 341

INORGANIC CHEMISTRY II

TIME : THREE HOURS

INSTRUCTIONS:

1. This examination has Six (6) questions
  2. Answer Any Four (4) questions.
  3. Each question carries FIFTEEN(15) marks.
  4. Periodic table and T-S diagrams are attached.
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- 1(a) The magnetic moment of an octahedral cobalt (II) complex is 3.90 B M .  
Determine the electron configuration of the ion in terms of  $e_g$  and  $t_{2g}$  group of orbital.
- (b) Define the term *ambidentate ligand* and give a suitable example.
- (c)✓ Write balanced chemical reactions when,
- (i) Potassium ferrocyanide is heated with concentrated sulphuric acid in aqueous medium.
  - (ii) Malonic acid is heated with phosphorous penta chloride at 150°C.
  - (iii) Calcium phosphate reacts with a mixture of silica and coke.
  - (iv) Barium xenate is heated with concentrated sulphuric acid.
- ✓ (d) Write a brief account on "Interhalogen compounds"
- 2(a) Determine the ground state R-S terms for  $d^7$  ion and calculate
- (i) Crystal field stabilisation energy (CFSE) in terms of  $D_q$  in a weak octahedral field.
  - (ii) Magnetic moment ( $\mu_s$ ).
- (b) Name the following co-ordination compounds
- (i)  $[\text{Cu}(\text{NH}_3)_4]^{2+}$
  - (ii)  $[\text{Cr}(\text{NH}_3)_2(\text{H}_2\text{O})_2 \text{Br}_2]^+$

(c) Describe contact process for the manufacture of sulphuric acid. Write all the chemical reactions involved in the process and mention the necessary conditions to obtain the maximum yield of sulphuric acid.

(d) Write balanced chemical reactions between sodium hydroxide and

(i) Dichloro hexaoxide (ii) Xenon trioxide

3(a) Name the following co-ordination compounds

(i) Hexaammine cobalt (II) hexacyanochromate (III)

(ii) Hexacyanopalladate (iv) ion.

(b) Draw all possible isomers of the following complex compounds.

(i) Square Planar  $[\text{Pt}(\text{H}_2\text{O})_2\text{Br}_2]$

(ii)  $[\text{Co}(\text{NH}_3)_3(\text{NO}_2)_3]$

(c) Using valence bond theory account for the following

(i)  $[\text{Cu}(\text{NH}_3)_4]\text{SO}_4$  is coloured and is square planar complex.

(ii)  $\text{K}_4[\text{Fe}(\text{CN})_6]$  is a low spin and diamagnetic complex.

(d) Write the structures of the following species.

$\text{N}_2\text{O}_5$  ;  $\text{XeO}_2\text{F}_2$

4 A transition metal complex believed to have less than half filled d- orbital has a high spin ground state ligand term of  $^5\text{E}_g$ .

(i) State the number of d electrons in the complex

(ii) What is the ground state term for the free ion?

(iii) If the ligand field is changed to low spin state, what will be ground state term of this new arrangement?

(iv) In changing from high spin to low spin state, is the electronic spectra also being affected?

(v) Compute the change in the ( $\mu_s$ ) magnetic moment observed in (d).

5 Comment on the following

(a) Carbon monoxide,  $\text{CO}$ , is a notoriously poor  $\sigma$ -donor ligand yet it is found on the stronger side of the spectro-chemical series.

(b) Potassium permanganate,  $\text{KMnO}_4$ , is a highly coloured (purple) complex, yet the Mn has no d-electrons.

- (c) d-d transition seriously violate the Laporte Selection rules  
i.e  $\Delta l = \pm 1$  and  $g \leftrightarrow u$ , yet they occur and are being used in analysis.
- 6 (a) Configurations  $d^4 - d^7$  have T-S diagrams that show effects of both strong and weak ligands fields. Using T-S diagrams state the value of  $Dq/B$  corresponding to the change of ground term for each  $d^4$ ,  $d^5$ ,  $d^6$  and  $d^7$  configurations.
- (b) Derive the R-S terms for a  $d^8$  configuration.
- (c) An aqueous solution of  $[\text{Cr}(\text{H}_2\text{O})_6]^{2+}$  is pale blue in colour. It shows a wide weak band at  $14,100 \text{ cm}^{-1}$  ( $f = 4.2, f = 1.0 \times 10^{-4}$ )
- (i) What spin allowed transition is attributed to this band?
- (ii) Determine  $10 Dq$ .

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END OF C341 EXAMINATION

# THE PERIODIC TABLE

Group 1 2 3 4 5 6 7 0

Period

1	H Hydrogen 1
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2	He Helium 4
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Key

Atomic Number
Symbol
Name
Relative atomic mass

3	Li Lithium 7	4	Be Beryllium 9	5	B Boron 11	6	C Carbon 12	7	N Nitrogen 14	8	O Oxygen 16	9	F Fluorine 19	10	Ne Neon 20
11	Na Sodium 23	12	Mg Magnesium 24	13	Al Aluminium 27	14	Si Silicon 28	15	P Phosphorus 31	16	S Sulphur 32	17	Cl Chlorine 35.5	18	Ar Argon 40
19	K Potassium 39	20	Ca Calcium 40	21	Sc Scandium 45	22	Ti Titanium 48	23	V Vanadium 51	24	Cr Chromium 52	25	Mn Manganese 55	26	Fe Iron 56
37	Rb Rubidium 85	38	Sr Strontium 88	39	Y Yttrium 89	40	Zr Zirconium 91	41	Nb Niobium 93	42	Mo Molybdenum 96	43	Tc Technetium (98)	44	Ru Ruthenium 101
55	Cs Caesium 133	56	Ba Barium 137	57	La Lanthanum 139	72	Hf Hafnium 178	73	Ta Tantalum 181	74	W Tungsten 184	75	Re Rhenium 186	76	Os Osmium 190
87	Fr Francium (223)	88	Ra Radium (226)	89	Ac Actinium (227)	104	Unq Unil- quadium (261)	105	Unp Unil- penium (262)	106	Unh Unil- hassium (263)	107	Uup Unil- pentium (264)	108	Uuh Unil- hexium (265)

58	Ce Cerium 140	59	Pr Praseo- dymium 141	60	Nd Neodymium 144	61	Pm Promethium (147)	62	Sm Samarium 150	63	Eu Europium 152	64	Gd Gadolinium 157	65	Tb Terbium 159	66	Dy Dysprosium 163	67	Ho Holmium 165	68	Er Erbium 167	69	Tm Thulium 169	70	Yb Ytterbium 173	71	Lu Lutetium 175
90	Th Thorium 232	91	Pa Protactinium (231)	92	U Uranium 238	93	Np Neptunium (237)	94	Pu Plutonium (242)	95	Am Americium (243)	96	Cm Curium (247)	97	Bk Berkelium (245)	98	Cf Californium (251)	99	Es Einsteinium (254)	100	Fm Fermium (253)	101	Md Mendelevium (258)	102	No Nobelium (254)	103	Lr Lawrencium (257)

▶ Lanthanoid elements

▶▶ Actinoid elements



**THE UNIVERSITY OF ZAMBIA**  
**UNIVERSITY SEMESTER I EXAMINATIONS, APRIL 2002**

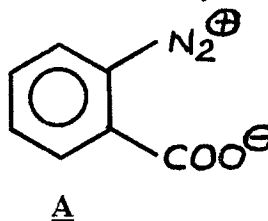
**ORGANIC CHEMISTRY III - C351**

**INSTRUCTIONS**

1. Time allowed for this paper is **THREE (03) hours**.
  2. Answer **FOUR (04) questions Only**.
  3. Each question carries **25 marks**.
  4. Marks distribution for each question is shown.
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**QUESTION – ONE**

- (a) The dry diazonium salt A, structure shown below, was subjected to a flash discharge and the reaction was monitored by a mass spectrometer. After about 50 microseconds, three peaks at  $m/z$  : 28, 44, and 76 appeared simultaneously. As the time passed, the peak at  $m/z$  76 disappeared and a peak at  $m/z$  152 approached maximum intensity.



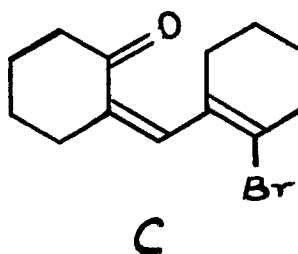
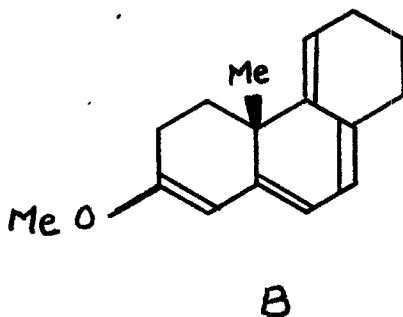
- (i) Identify the peaks at  $m/z$  : 28, 44, and 76.

(4.5 marks)

- (ii) What is the substance of mass 152 ? Propose a plausible mechanism for its formation from compound A.

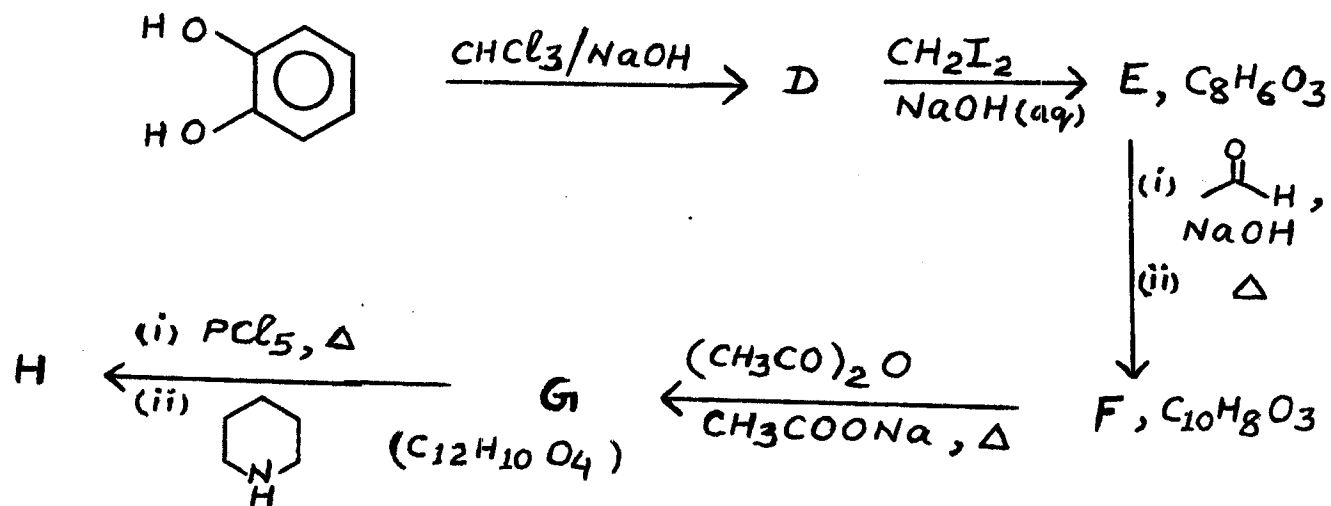
(5.5 marks)

- (b) Predict the uv – absorption maximum values for the compounds B and C , structures shown below.



(5 marks)

(c) An alkaloid H,  $C_{17}H_{19}O_3N$ , was synthesized as shown below.



(i) Give structures of compounds D – H in the above synthesis.

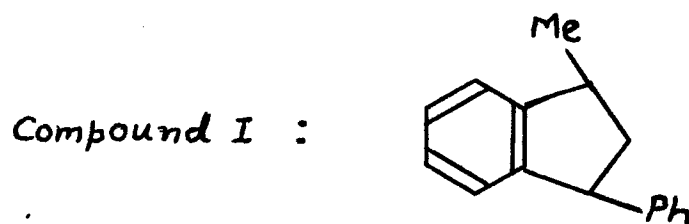
(6 marks)

(ii) Suggest a mechanism for the reaction involved in the transformation of compound F into compound G.

(4 marks)

## QUESTION - TWO

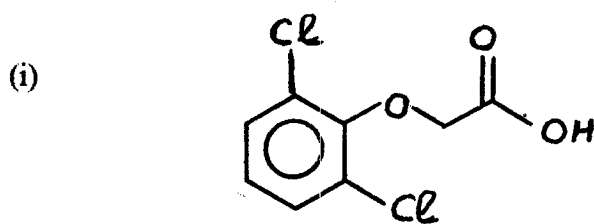
(a) Styrene ( $C_6H_5CH=CH_2$ ) dimerises in presence of phosphoric acid at  $40^\circ C$  to give a compound I, structure shown below.



Suggest a mechanism to explain the formation of compound I.

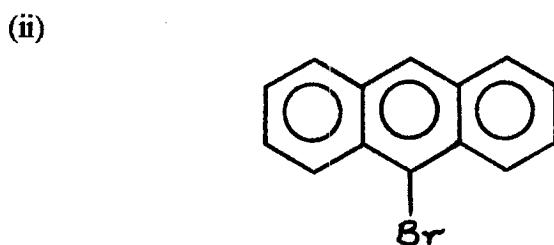
(9 marks)

- (b) Give a synthesis of each of the following compounds starting from the indicated starting materials and any other readily available reagents / materials. Show clearly the reagents needed and the reaction conditions necessary for each step of the synthesis. Reaction mechanisms are NOT required.



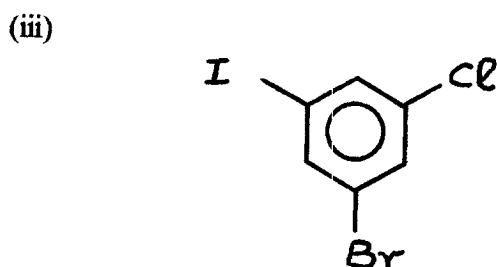
from phenol

(5 marks)



from benzene

(6 marks)

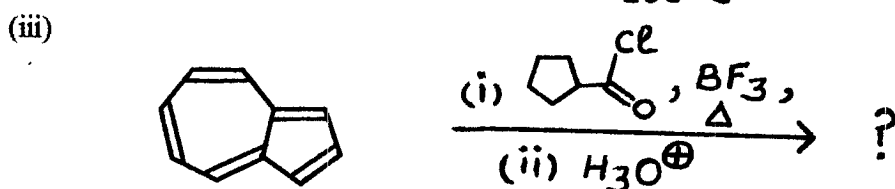
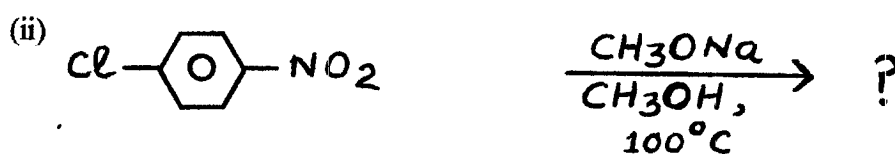
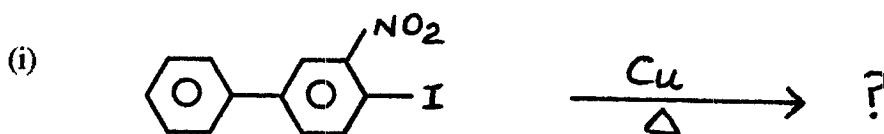


from any mono-substituted benzene

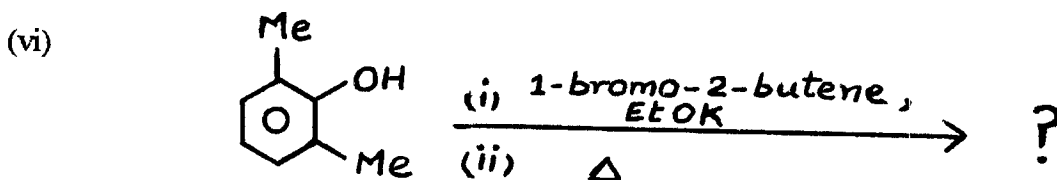
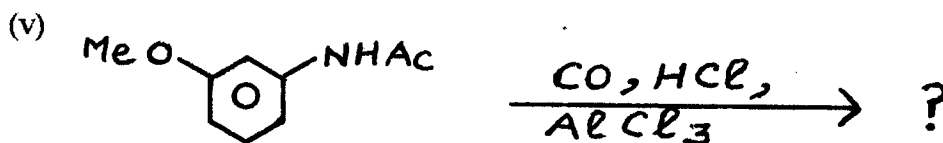
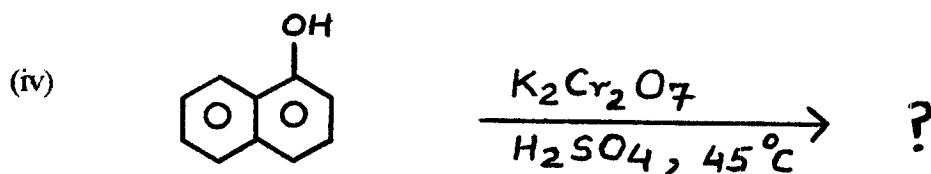
(5 marks)

### QUESTION - THREE

- (a) Predict the major organic products of the following reactions.



Question 3 (a) continued from page 3.



(9 marks)

(b) One of the two mass spectra given below corresponds to 2,2 – dimethylpentane and the other corresponds to 2,3- dimethylpentane.

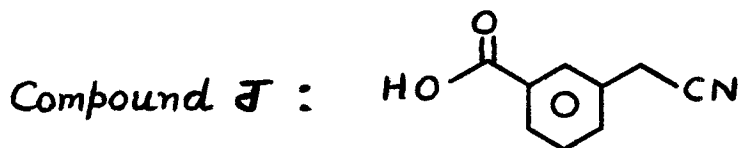
Spectrum 1 :  $m/z$  : 100 , 85 , 71 , 57 , 43 , 41 (base peak)

Spectrum 2 :  $m/z$  : 85 , 57 , 43 , 41 (base peak)

Determine which spectrum corresponds to 2,2 – dimethylpentane and which spectrum corresponds to 2,3 – dimethylpentane. Show your reasoning.

(8 marks)

(c) When o- chlorobenzoic acid is treated with potassium amide ( $\text{KNH}_2$ ) and liquid ammonia in presence of acetonitrile ( $\text{CH}_3\text{CN}$ ), there is obtained a 70 % yield of compound J, structure shown below, and 10 – 20 % yield of a 1 : 2 mixture of o- and m- aminobenzoic acids.

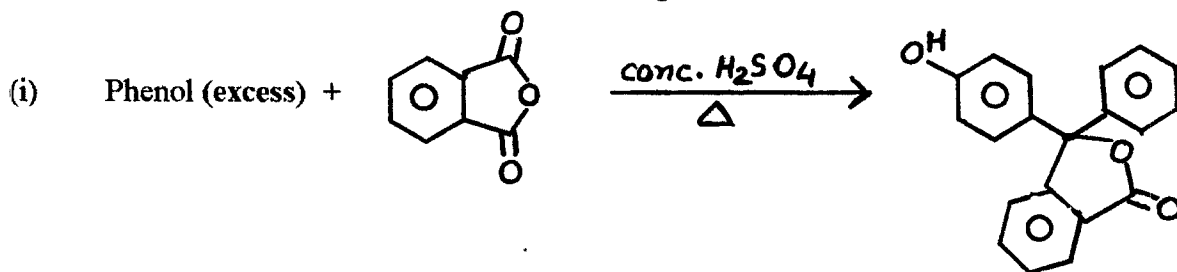


Provide a mechanistic explanation for the formation of compound J .

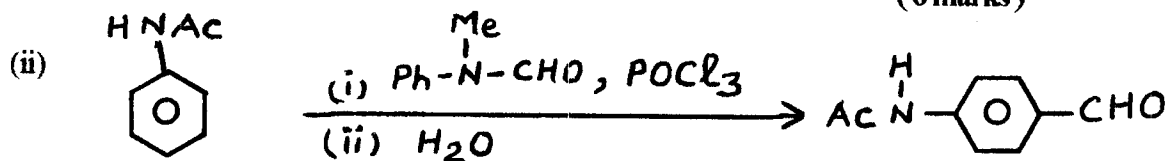
(8 marks)

QUESTION – FOUR

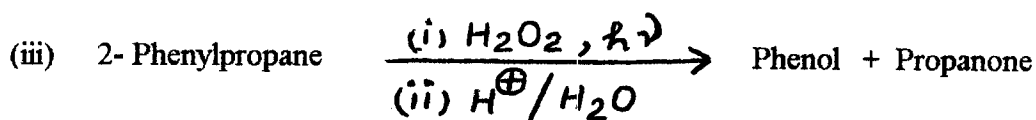
(a) Suggest plausible mechanisms for each of the following reactions.



(6 marks)

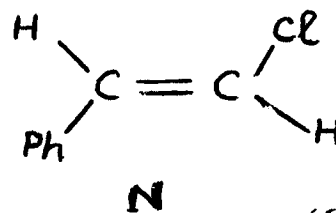
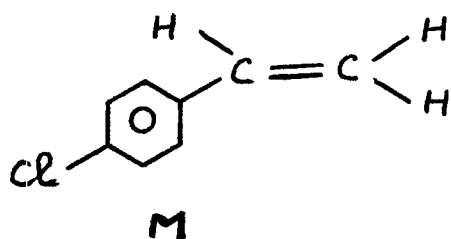


(5 marks)



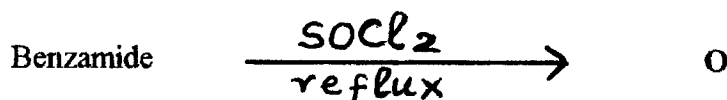
(5 marks)

(b) Show clearly how proton nmr spectroscopy can be used to distinguish compound M from compound N, structures shown below.



(6 marks)

(c) Identify the product of the following reaction.



Compound O

IR (neat),  $\bar{\nu}$   $\text{cm}^{-1}$  : selected bands at 3050, 2240, 1600, 1500

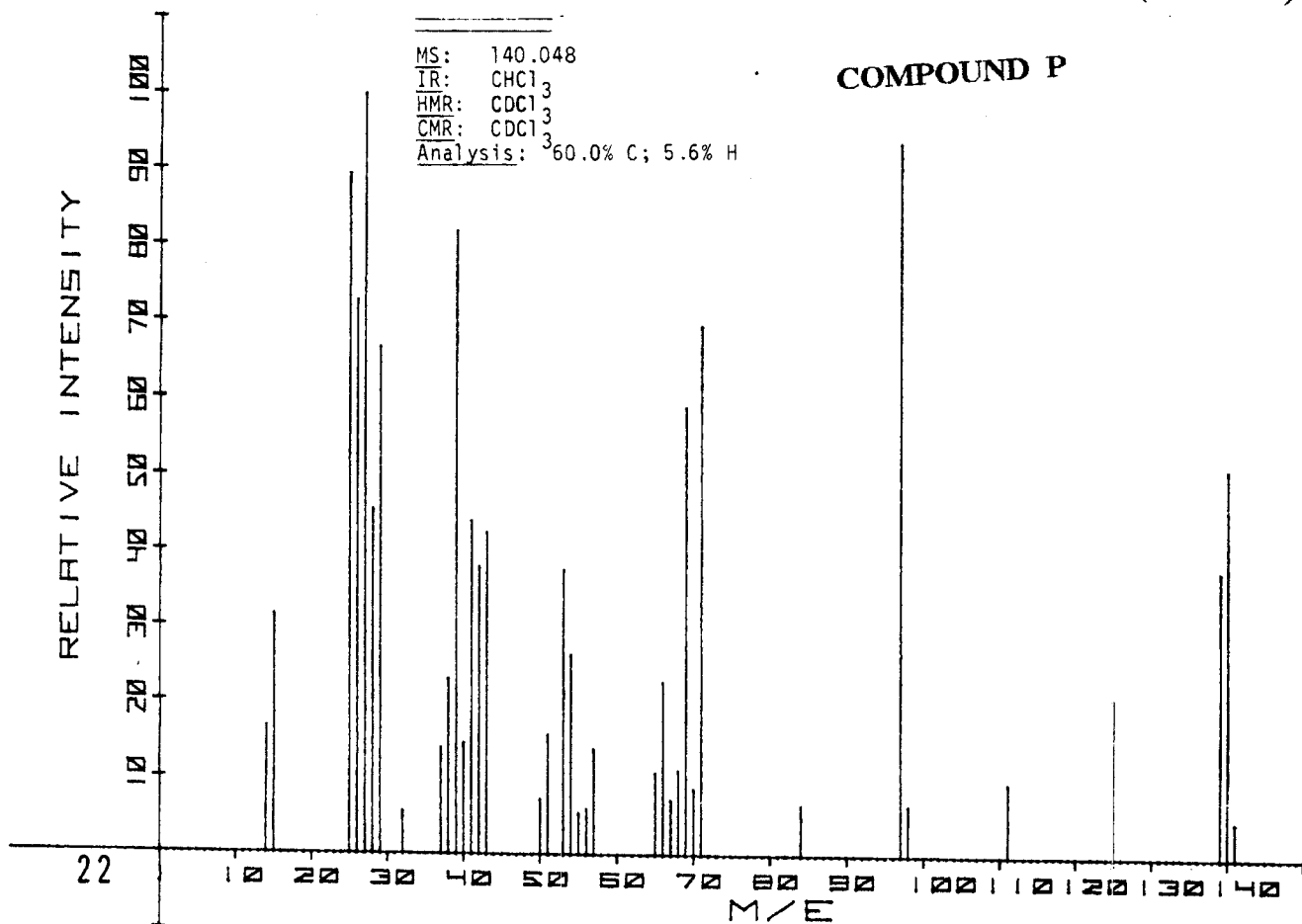
$^1\text{HNMR}$  ( $\text{CDCl}_3$ ),  $\delta$  (ppm) : 7.2 (m, 5H)

(3 marks)

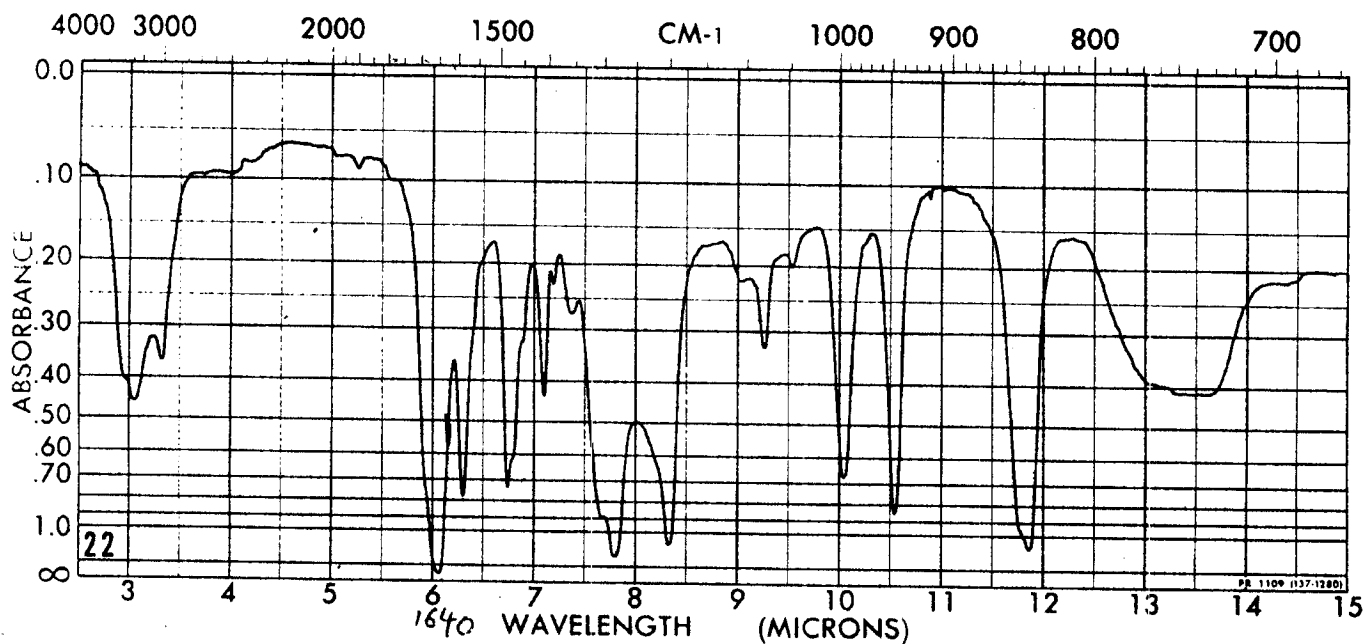
**QUESTION - FIVE**

(a) Deduce the structure of compound P,  $C_7H_8O_3$ , from the following Ms, IR,  $^1H$ NMR and CMR spectra.

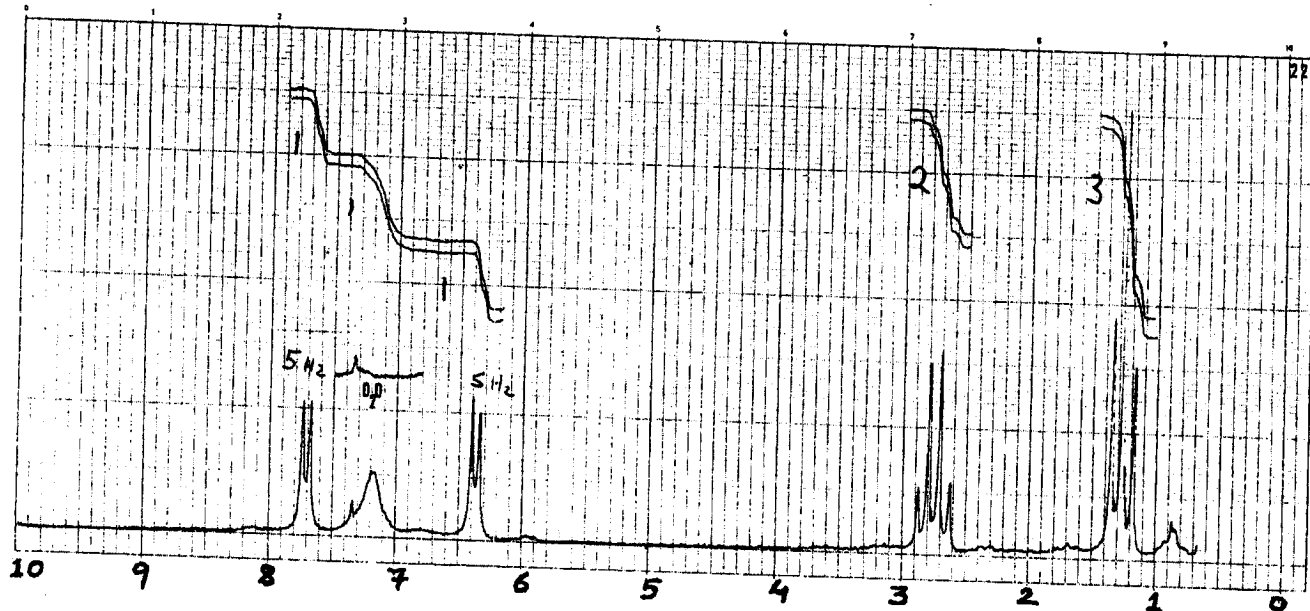
( 12 marks )



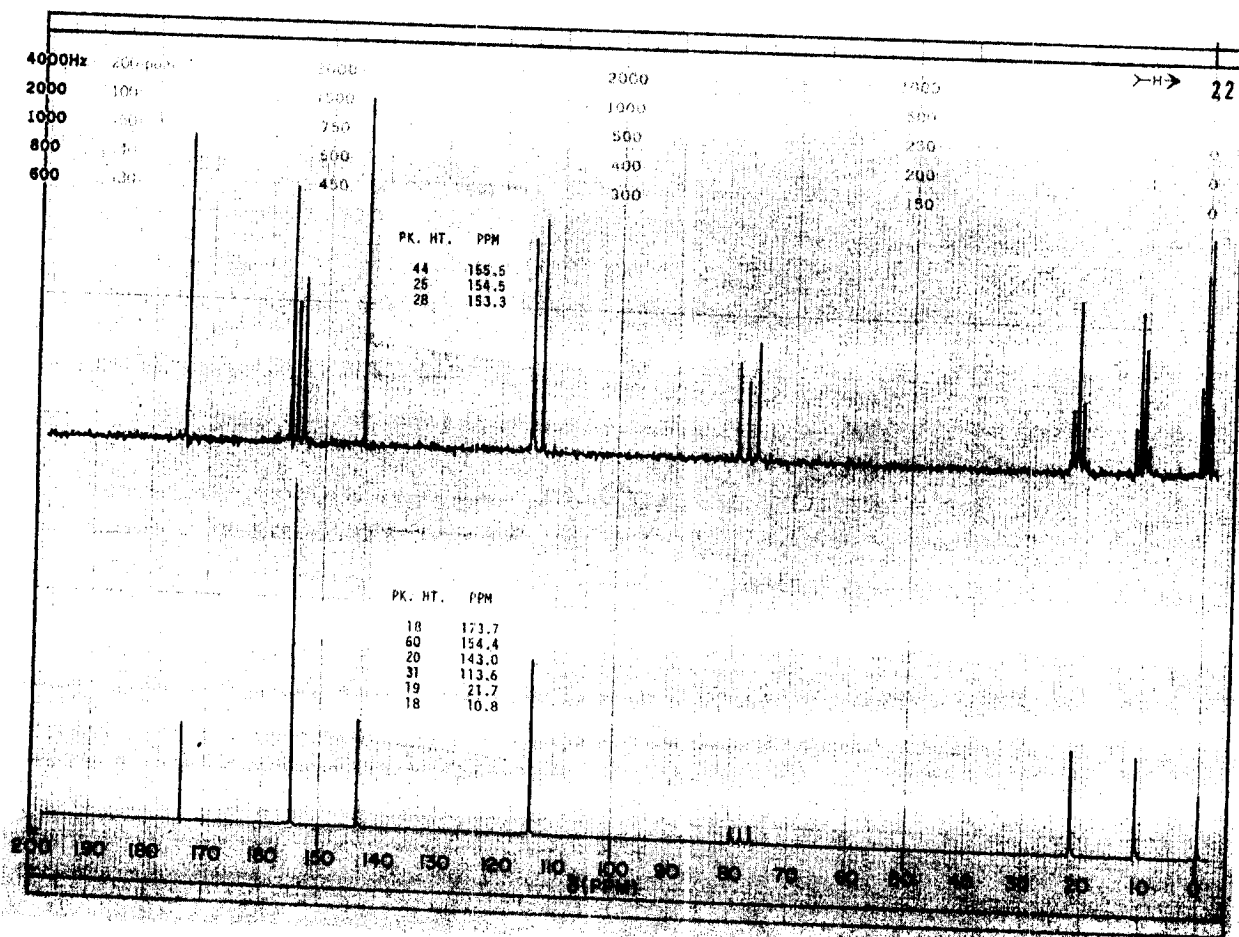
**COMPOUND P**



# COMPOUND P

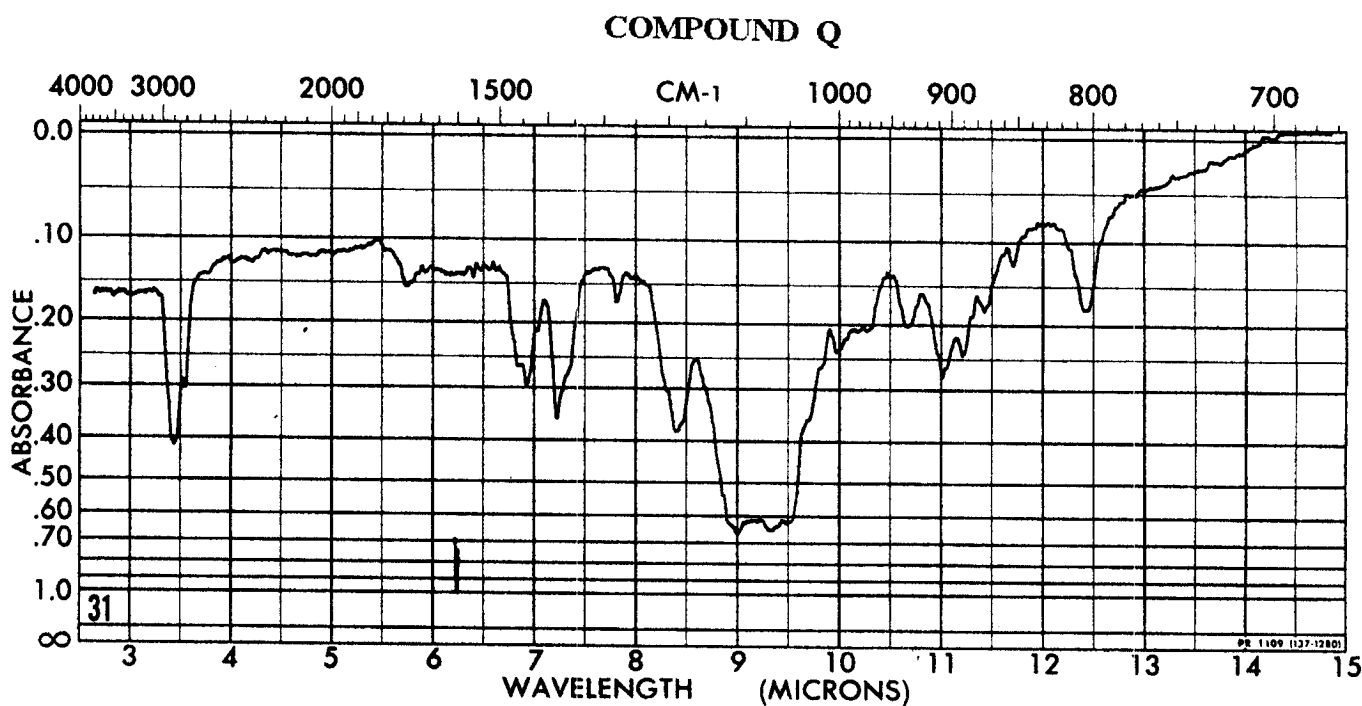
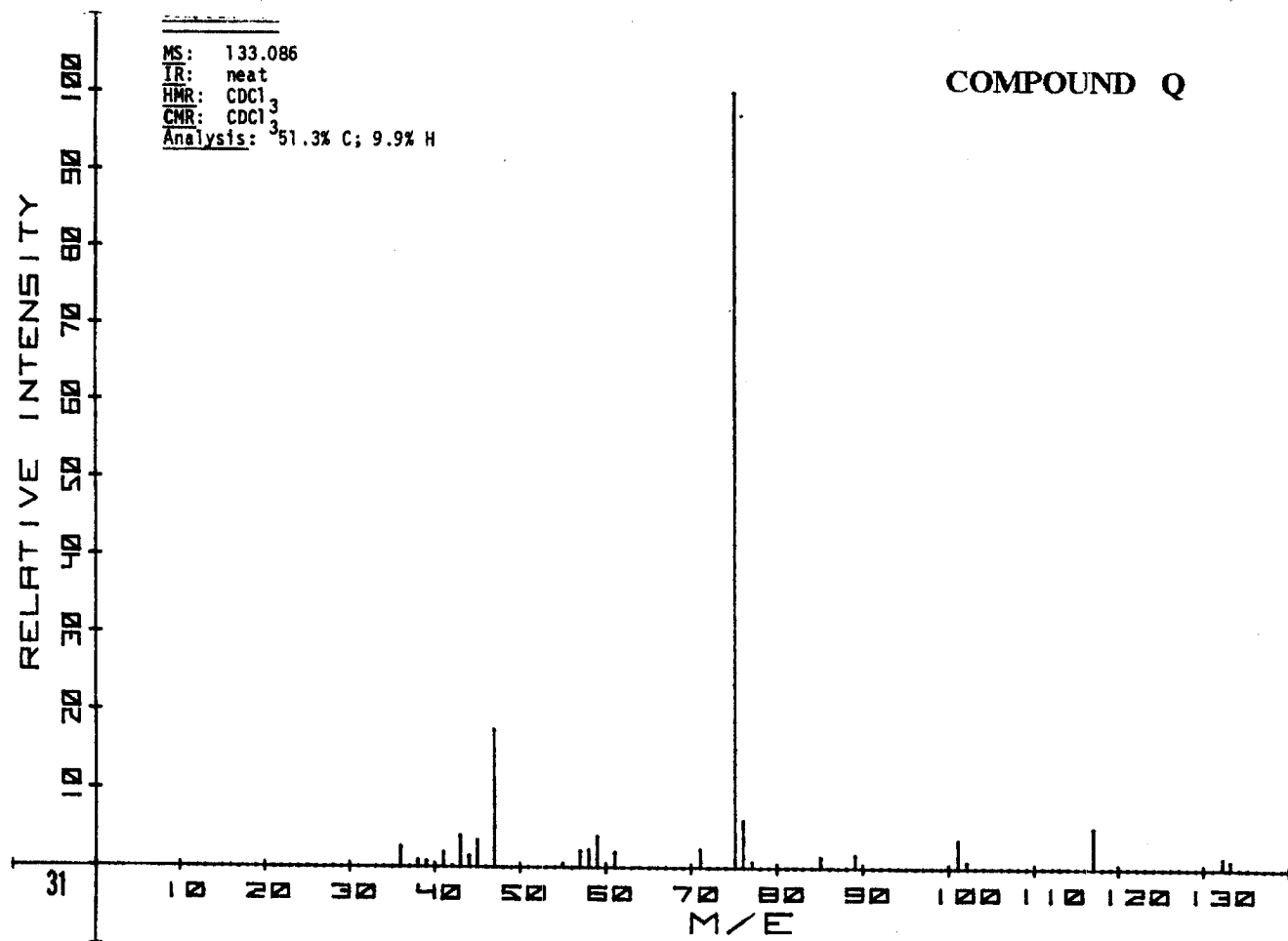


# COMPOUND P

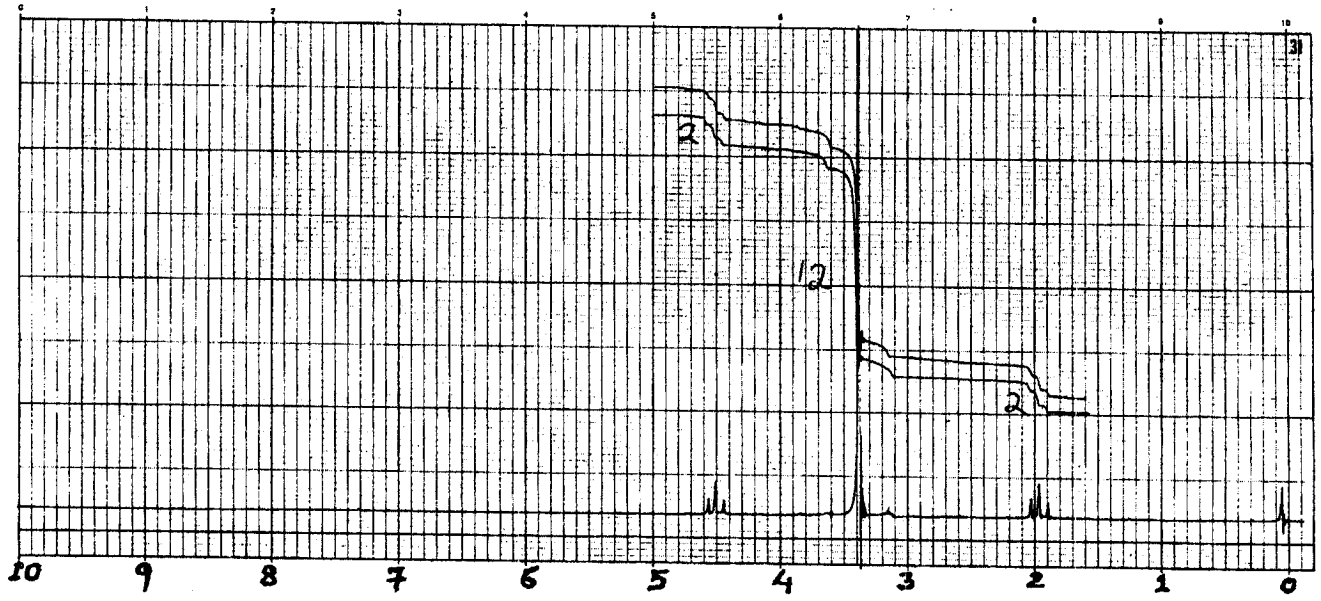


(b) Deduce the structure of compound Q,  $C_7H_{16}O_4$ , from the following MS, IR,  $^1H$ NMR and  $^{13}C$ NMR spectra.

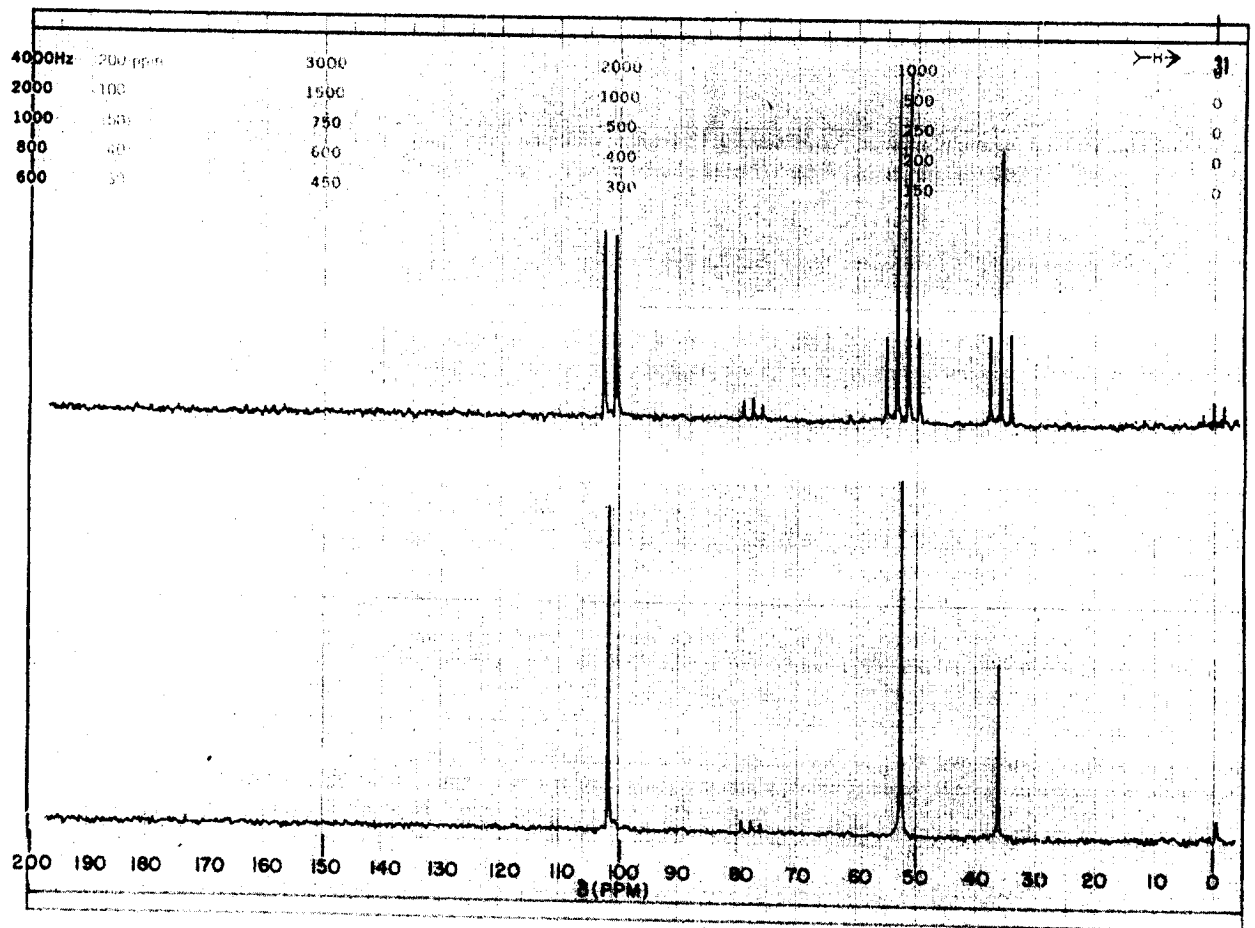
(13 marks)



COMPOUND Q



COMPOUND Q

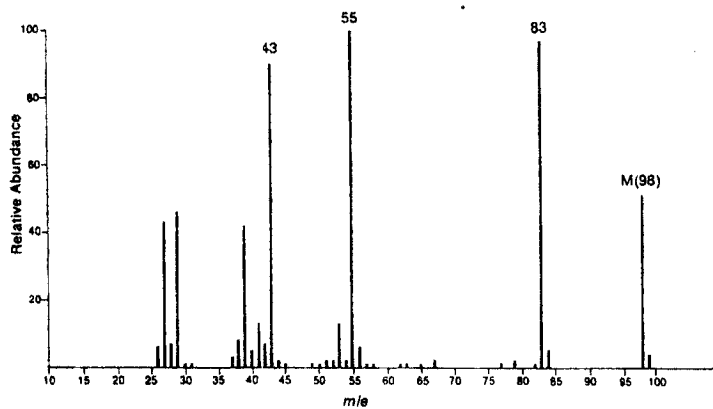


### QUESTION - SIX

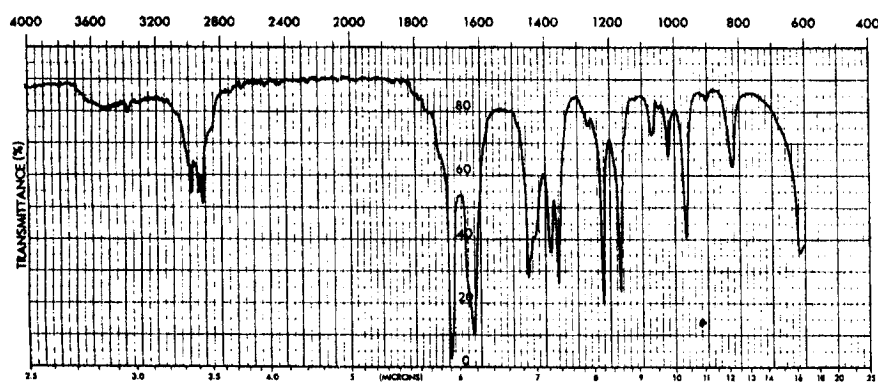
- (a) Deduce the structure of a non-nitrogenous compound R from the following MS, IR and  $^1\text{H}$ NMR spectra.

( 13 marks )

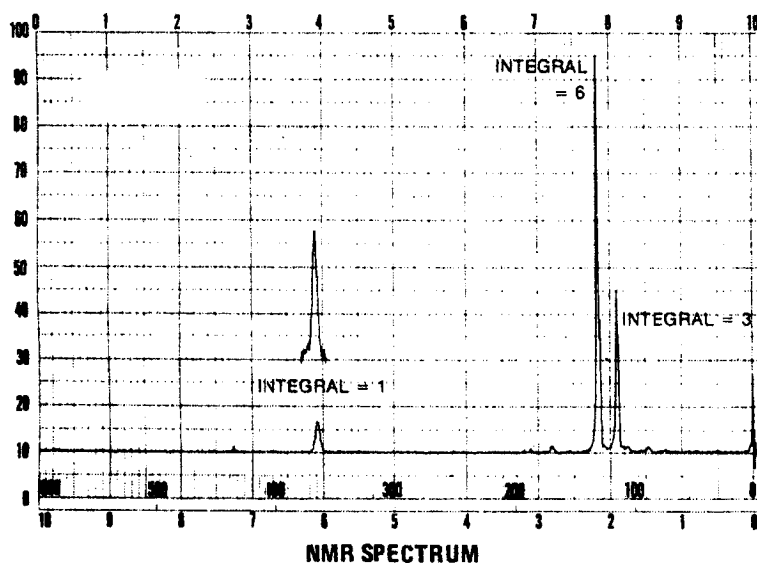
#### COMPOUND R



MASS SPECTRUM



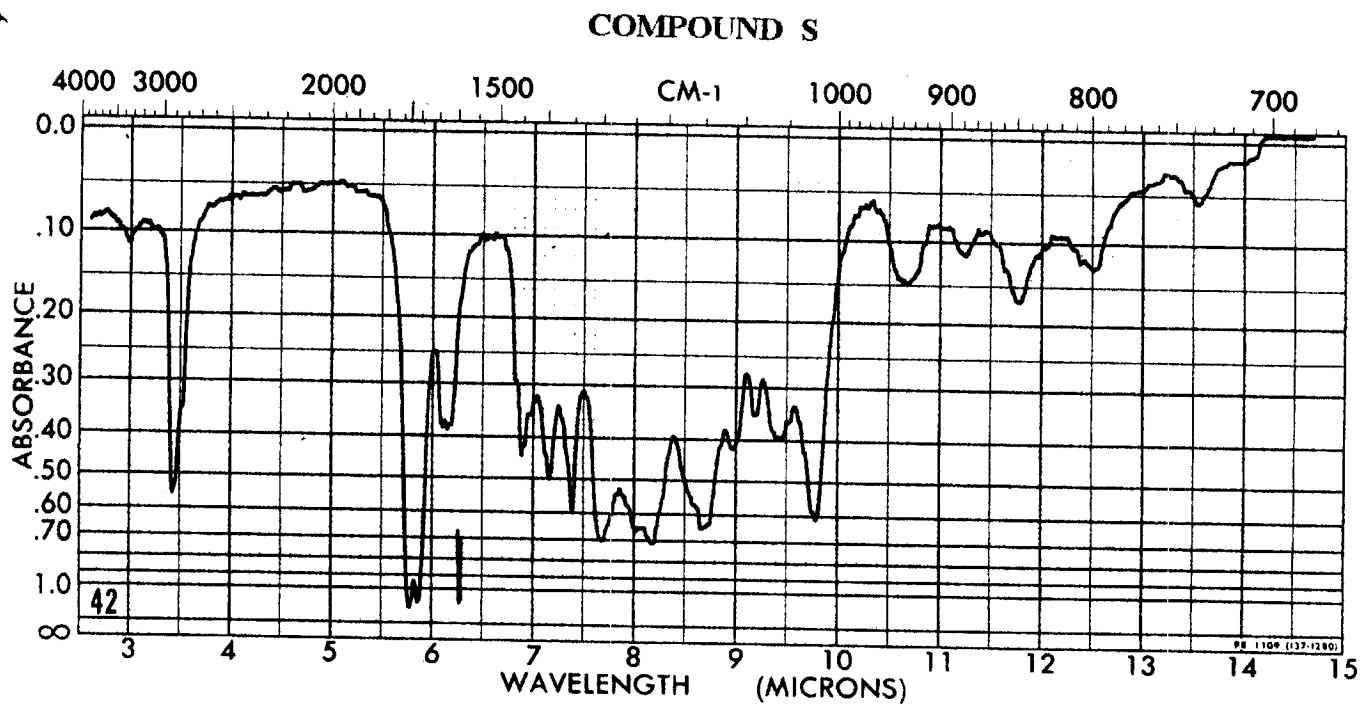
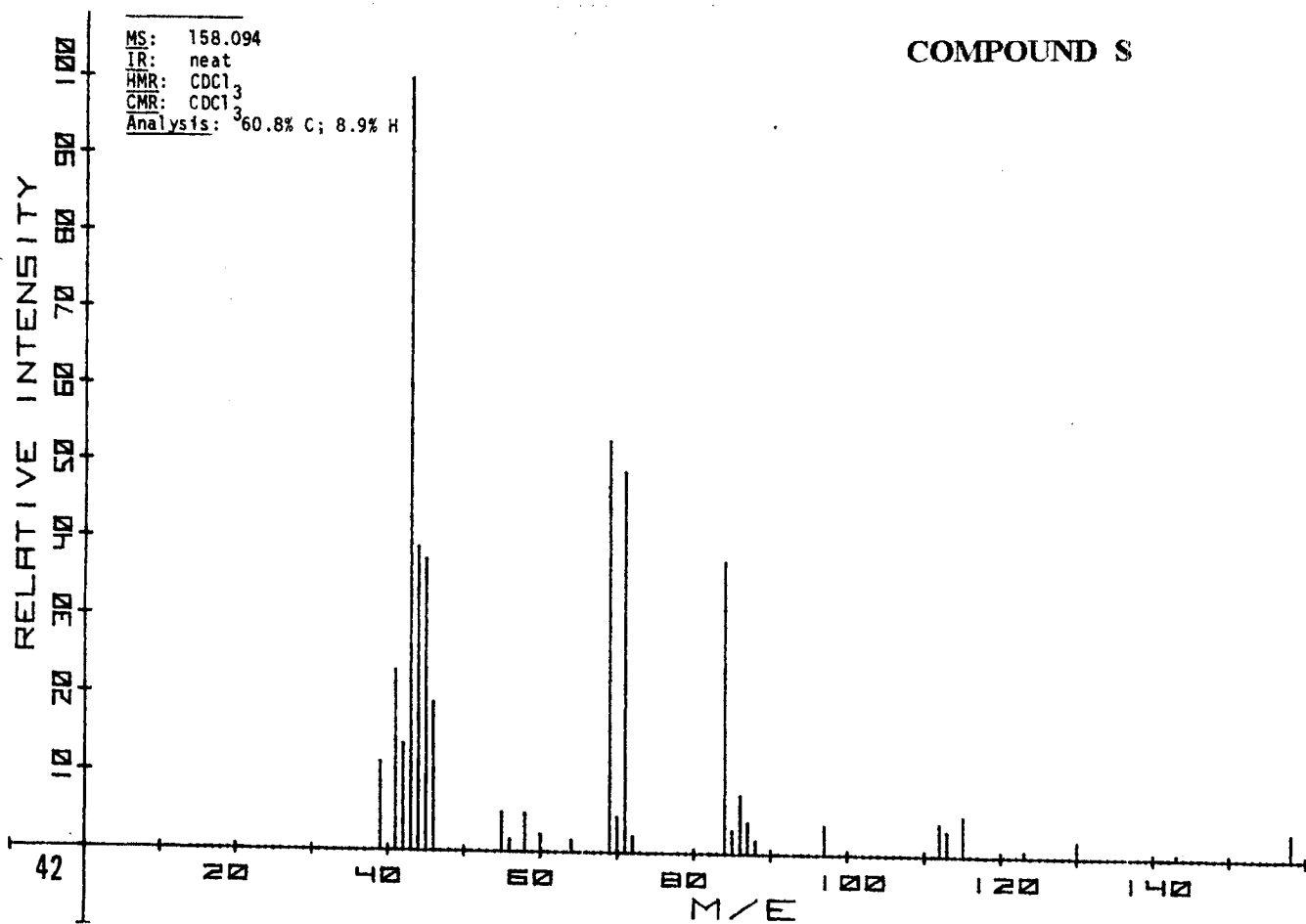
INFRARED SPECTRUM



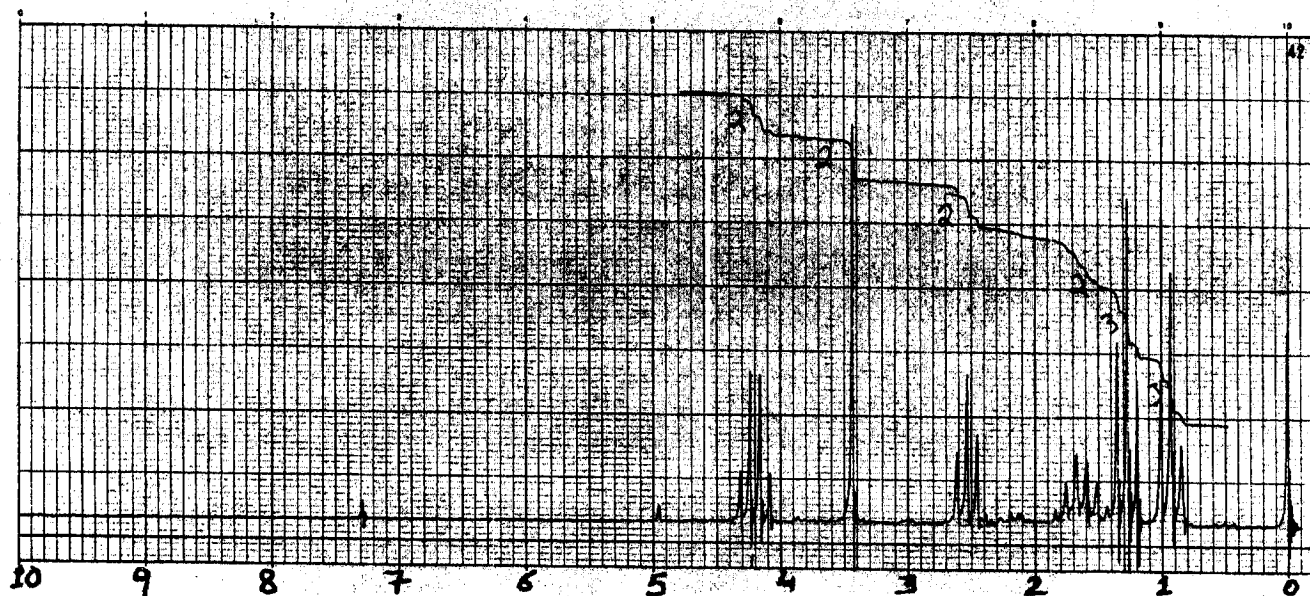
NMR SPECTRUM

(b) Deduce the structure of compound S,  $C_8H_{14}O_3$ , from the following MS, IR,  $^1H$ NMR and CMR spectra.

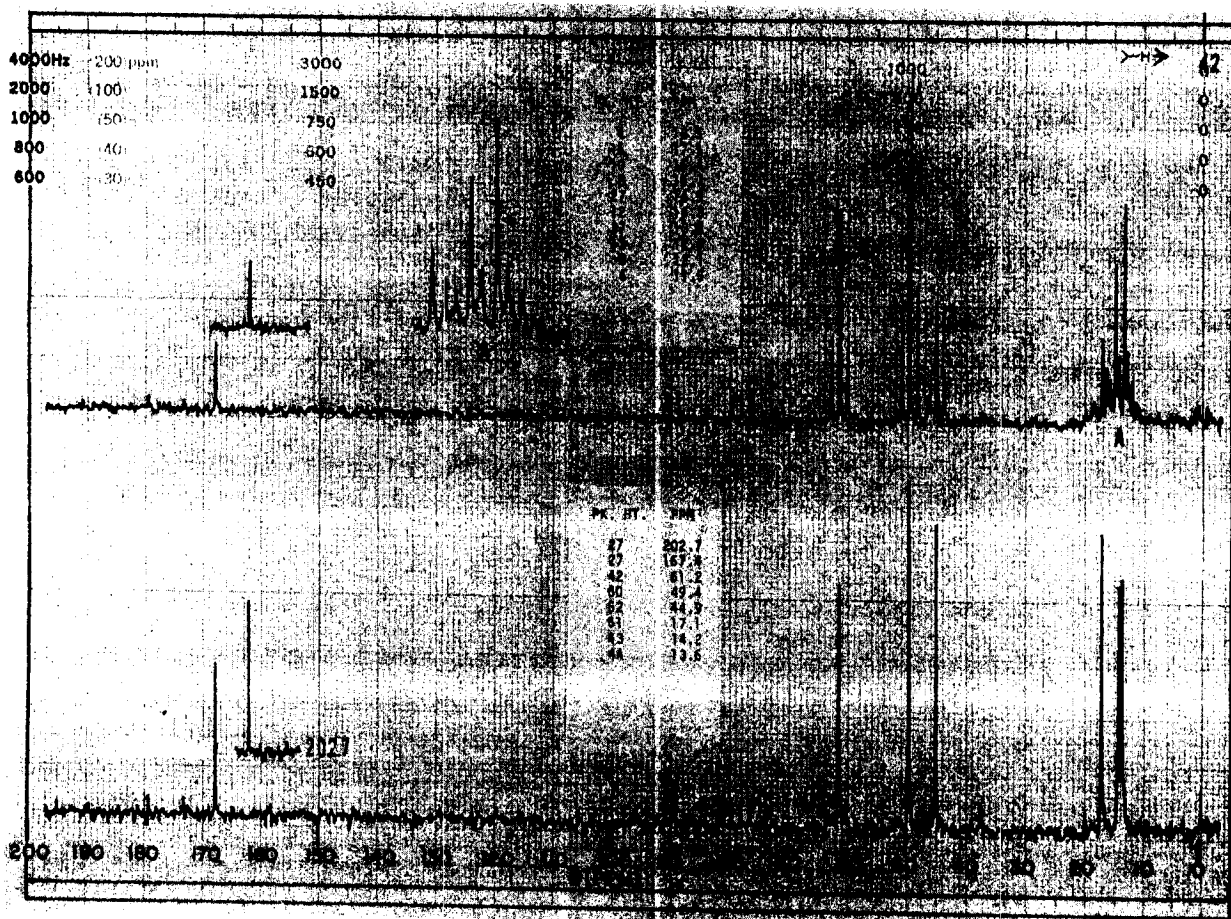
( 12 marks )



COMPOUND S



COMPOUND S



END OF EXAMINATION

**THE UNIVERSITY OF ZAMBIA**  
**UNIVERSITY EXAMINATIONS - APRIL 2002**  
**SEMESTER I**  
**CHEMICAL KINETICS AND NUCLEAR CHEMISTRY - C 361**

INSTRUCTIONS : ANSWER ALL FIVE ( 5 ) QUESTIONS

TIME ALLOWED : THREE ( 3 ) HOURS

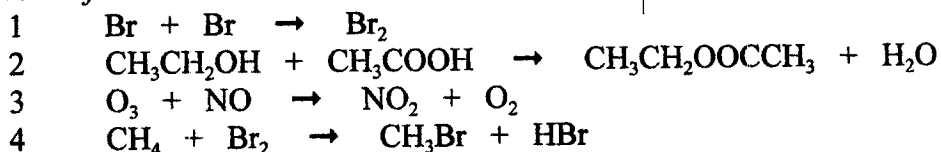
DATA  
 $R = 8.314 \text{ J. K}^{-1} \cdot \text{mol}^{-1}$

1. ( 40 Marks )

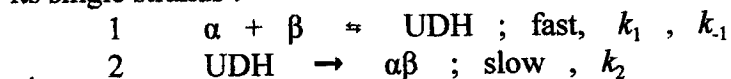
- a) The following equation is important in the kinetics study of a certain class of reactions:

$$\frac{I_L^o}{I_L} = 1 + k_Q \tau^o [Q]$$

- ( i ) To what class of reactions is the equation applicable ?
- ( ii ) Identify the equation and all the terms in it.
- b) ( i ) What is the *steric factor* ?
- ( ii ) Arrange the following reactions in increasing order of magnitude of the *steric factor* :



- c) In its stable form deoxyribonucleic acid (DNA), the fundamental carrier of genetic information, consists of two single strands  $\alpha$  and  $\beta$  which form the double helix. Consider the following mechanism for renaturation of the stable helix from its single strands :



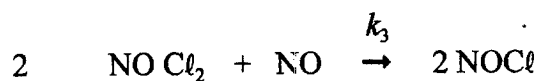
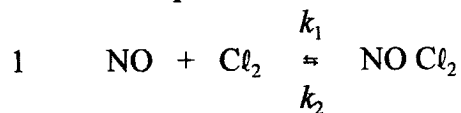
where UDH is unstable double helix intermediate.

- ( i ) Which is the rate determining step ?
- ( ii ) Derive the rate equation for the formation of the stable double helix  $\alpha\beta$ .

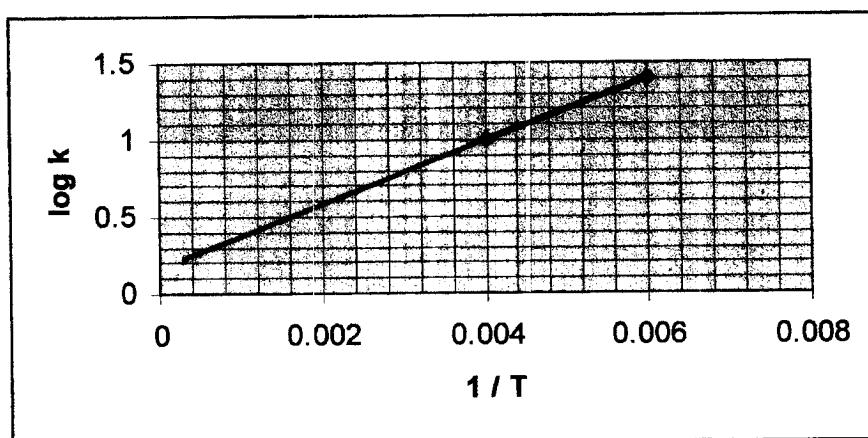


3. ( 15 Marks )

Nitrosyl chloride is formed from a reaction of nitric oxide having unpaired electron and chlorine in a two step mechanism:



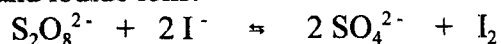
The rate constants  $k_1$ ,  $k_2$  and  $k_3$  are comparable ( i.e nearly the same or of the same order of magnitude). Trautz who investigated this reaction found that the experimental rate constant varies with temperature as shown in the graph below:



- Derive the rate law for the overall reaction.
- What is the overall order of the reaction.
- On the basis of your result in part 3 a) above, comment and explain the “apparent” anomalous temperature dependency of the rate constant of this reaction as shown in the graph above.

4. (15 Marks)

In the last laboratory experiment, you determined the activation energy of the reaction between persulphate and iodide ions:



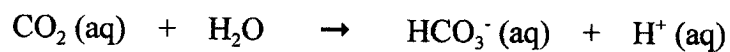
- a) Briefly discuss this experiment. In your answer
- discuss the theoretical basis of the experiment,
  - indicate or give the most plausible mechanism of the reaction
  - criticize the experiment.
- Avoid trivial details and repetition for which marks will be deducted!
- b) The following are the results of twelve of the sixteen results obtained by the class:

<u>Experimenter</u>	<u><math>E_a</math> (kJ.mol<sup>-1</sup>)</u>
96500069	40.81
96056631	50.10
96143584	50.49
96152265	31.18
97188964	45.61
95081356	45.66
94823464	40.94
96097515	48.61
96270993	50.10
98070126	47.87
96021012	41.57
97085405	46.90

- Should the result of experimenter 96152265 be discarded? Explain your answer.
  - On the basis of your answer to part 4 b) ( i ), calculate the average activation energy of the reaction.
  - Compare the standard deviation with the average deviation of these results.
  - On the basis of your answers to parts 4 b) (i) and 4 b) (iii), give the **precision** of the average activation energy of the class results.
- c) The lecturer wishes to mark the laboratory reports of this experiment using the class average value of the activation energy on the basis of a 95 % **confidence limit** of the individual values. Given that the *t* factor for a 95 % **confidence limit** for twelve measurements is 2.20, should the results of experimenter 96152265 be included in the calculation of the class average value of the activation energy? Explain your answer.

5 ( 10 Marks )

The enzyme carbonic anhydrase catalyzes the reaction



H. Devoe and G. B. Kistiakowsky, *Journal of the American Chemical Society*, **83**, 274 (1961) obtained the following data:

<u>Initial CO<sub>2</sub> concentration</u> <u>moles dm<sup>-3</sup> x 10<sup>3</sup></u>	<u>Initial rate</u> <u>moles dm<sup>-3</sup> s<sup>-1</sup> x 10<sup>3</sup></u>
1.0	0.02
2.5	0.05
5.0	0.08
7.5	0.10
10.0	0.12
25.0	0.17
50.0	0.19
100.0	0.21

Determine the values of the Michaelis-Menten constant and of  $r_{\text{max}}$ , the maximum rate.

**The University of Zambia**  
**University Examinations – Semester I 2001/02**

C411

Advanced Biochemistry I

**Instructions:**

Time: Three (3) Hours

Answer **ANY FIVE (5)** questions

Please check that you have **THREE (3)** printed pages of this paper

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1. The Nernst equation relates the electric potential  $\Delta\psi$  resulting from an unequal distribution of a charged solute across a membrane permeable to the solute to the ratio between the concentration of solute on one side and on the other:

$$m \Delta\psi = -2.3RT/F \log [S_0]_1/[S_0]_2$$

where  $m$  is the charge on the solute,  $2.3RT/F$  has a value of about 60 mV at 37°C, and  $[S_0]_1$  and  $[S_0]_2$  refer to the concentrations of solute on either side of the membrane. Consider a planar phospholipid bilayer separating two compartments of equal volume. Side 1 contains 50-mM KCl and 50-mM NaCl, while side 2 contains 25 mM KCl.

- a) If the membrane is made permeable only to  $K^+$ , e.g., by addition of valinomycin, what will be the magnitude of  $\Delta\psi$ ?
- b) If the membrane is made permeable to  $H^+$  and  $K^+$ , in which direction will  $H^+$  initially flow?

**[20 marks]**

2.
  - a) Explain in detail how the myosin head converts the energy of ATP hydrolysis into mechanical energy.
  - b) Briefly explain how smooth muscle contraction is controlled.
  - c) Actin is found in non-muscle cells. What is its role there?

**[20 marks]**

3. How is cAMP production controlled? In particular, describe the role of GTP in the process. What is the relevance of the latter to cholera?

**[20 marks]**

4. Tryptophan biosynthesis is subject to control at the genetic level.
  - a) What is the significance of tandem tryptophan codons in the *trpL* transcript? Does this phenomenon exist elsewhere?
  - b) What effect would there be on tryptophan biosynthesis if a mutation was introduced into region 3 of the *trpL* mRNA that disrupted base-pairing between region 2 and 3?
  - c) What do each of the genes in the *trp* operon encode?
  - d) Using a diagram, show how tryptophan biosynthesis is regulated by attenuation

**[20 marks]**

5. Write short notes on how the following are involved in regulation of metabolism:
  - a) allostery
  - b) adrenal gland, and

c) pancreas covalent modifications

**[20 marks]**

6. What chromatographic method would you use to separate the following pairs of substances? Justify your choice of technique by explaining which characteristics each technique you have chosen depends on, i.e. the basis of separation.

a) ala-phe-lys and ala-lys-lys

b) lysozyme and ribonuclease A

c) haemoglobin and myoglobin

**[20 marks]**

***End of Examination***

THE UNIVERSITY OF ZAMBIA  
DEPARTMENT OF CHEMISTRY  
APRIL EXAMINATIONS 2002

C421  
APPLIED ANALYTICAL CHEMISTRY

TIME: 3 HOURS

ANSWER ANY 4 FROM THE 6 QUESTIONS IN THIS PAPER

QUESTION 1

- (a) Potassium dichromate in basic solution exhibits an absorption maximum at 372 nm. A basic solution containing  $3.00 \times 10^{-5}$  M potassium dichromate transmits 71.6% of the incident light radiation at 372 nm when placed in a 1cm cell. (1) what is the absorbance and the molar absorptivity of the solution? (2) Compound X exhibits molar absorptivity of  $2.45 \times 10^3$  L/mol-cm at 450nm. What is the concentration of X in a solution which will cause a 25% decrease in radiant power for 450nm radiation when the solution is placed in a 2cm cell? (4)
- (b) Describe 3 schemes used in the analysis of silicate rocks. (3)
- (c) A soda ash sample is analysed by titration with std HCl. The analysis was done in triplicate giving the following results: 93.50, 93.58 and 93.43%. Within what ranges are you confident that the value lies? (tabulated value is 4.303) (2).
- (d) B can be determined in fertilizers in 2 ways, describe them. (3)
- (e) Physical ways can be used to try and identify types of soils in any field, discuss this statement. (3)

QUESTION 2

- (a) For the analysis of cement samples, series of stds were made and emission intensity for Na and K measured. Each std contained  $6300 \mu\text{g/ml}$  Ca as CaO to compensate for the influence of Ca upon the readings. The results are as follows:

Concentration	Emission reading	
	Na <sub>2</sub> O	K <sub>2</sub> O
$\mu\text{g/ml}$		
100	100	100
75	87	80
50	69	58
25	46	33
10	22	15
0	3	0
Cement A	28	69
B	58	51
C	42	63

For each cement sample 1.00g was dissolved in acid and diluted to 100ml. Calculate % Na<sub>2</sub>O and K<sub>2</sub>O. (4)

- (b) Describe how to determine K in the presence of sulfates and phosphates in a fertilizer using wet chemistry. (2)
- (c) Discuss the 4 methods used to collect atmospheric gases for analysis and include the determination of one of the collected gases. (3)
- (d) What weight of pyrite (impure FeS<sub>2</sub>) must be used in analysis so that BaSO<sub>4</sub> precipitate formed will be equal to half that of %S in the sample? (3)
- (e) Compare the routes by which high molecular compounds and toxic metals may disperse and re-concentrate in the environment and organisms. (3)

### QUESTION 3

- (a) Al can be analysed by titration with EDTA ( $Al^{3+} + H_2Y^{2-} \rightarrow AlY^- + 2H^+$ ). A 1.00g sample required 20.5ml EDTA for titration. EDTA was standardised by titrating 25ml of a 0.1M CaCl<sub>2</sub> solution, requiring 30ml. Calculate %Al<sub>2</sub>O<sub>3</sub> in the sample. (3)
- (b) An analyst notes that a 1ppm solution of Na gives flame emission signal of 110, while the same solution containing also 20ppm K gives a reading of 125. It was determined that 20ppm solution of K exhibited no blank reading. Explain the results. (3)
- (c) You are to analyse water that is known to have been contaminated with Hg and Cd, what instruments and approaches would you apply to try and determine the levels of these 2 toxic elements. (3)
- (d) A farmer would like to establish how fertile his farm soil is, describe the important factors that will show this and how they are determined. (3)
- (e) How would you determine B and silica in glass? (3)

### QUESTION 4

- (a) A water sample is analysed for Li by FES using std addition method. Three 0.5ml aliquots of sample are added to 5ml portions of solvent. To these are added (a) 0μl (b) 10μl and (c) 20μl of std 0.05M LiCl solution. The emission signals are 23.0, 45.3 and 68.0 for solutions a,b, and c respectively. What is the concentration of Li in the sample in ppm (w/v) (3)
- (b) Most soils contain some exchangeable cations, what are they and describe how any 2 of them can be determined. (3)
- (c) What are the advantages and disadvantages of the following techniques (a) ICP-MS, (b) GAAS, (c) AFS and (d) ICP-AES? (3)
- (d) In monitoring SO<sub>2</sub> in the environment absorption train using H<sub>2</sub>O<sub>2</sub> as an absorbent is used rather than West and Gaeke reagent. Discuss the advantages and disadvantages of this for large scale monitoring. (3)
- (e) What do you understand by the terms (i) pollution (ii) bio-concentration factor (iii) TWA (iv) eutrophication and what do they signify? (3)

### QUESTION 5

- (a) For determining glucose in blood serum, 2 methods are used, a std method (Folin-Wu) and another are used. Is there a difference in the 2 methods at 95% confidence level. Tabulated value is 4.95. (3)

New method mg/dl	std method mg/dl
127	130
125	128
123	131
131	129
126	127
129	

- (b) Discuss the important parameters you would consider to ensure that water for domestic use is acceptable for human consumption and include description of 2 determinations of these parameters. (3)
- (c) In flame spectrophotometry, discuss the differences between emission, absorption and fluorescence spectrometry, giving examples to illustrate them. (3)
- (d) The purity of a 0.287g sample of  $Zn(OH)_2$  is determined by titrating with std HCl solution requiring 37.6ml. The HCl was standardised by precipitating AgCl in a 25ml aliquot and weighing (0.46g AgCl was obtained). What is the purity of  $Zn(OH)_2$ ? (3)
- (e) Describe 2 methods you may use to separate Pb from rocks and one technique you can use to determine it from a solution of the rock. (3)

### QUESTION 6

- (a) An  $H_2O_2$  solution is analysed by adding slight excess of std  $KMnO_4$  solution and back-titrating the unreacted  $KMnO_4$  with  $Fe^{2+}$  solution. A 0.587g sample of  $H_2O_2$  solution was taken, 25ml of 0.0215M  $KMnO_4$  added. The back-titration required 5.10ml of 0.112M  $Fe^{2+}$  solution. What is the % $H_2O_2$  in the sample? ( $5H_2O_2 + 2MnO_4^- + 6H^+ \rightarrow 5O_2 + 2Mn^{2+} + 4H_2O$ ;  $5Fe^{2+} + MnO_4^- + 8H^+ \rightarrow 5Fe^{3+} + Mn^{2+} + 4H_2O$ ) (4).
- (b) TOC gives a better estimate for organic matter in water than either BOD or COD. Discuss the statement. (2)
- (c) The following absorbance values were obtained in preparing a calibration curve: blank, 0.03A; 1.00nM std, 0.11A; 2.00nM std, 0.19A; 4.00nM std, 0.35A. Plot the calibration curve. What is the most probable reason for non-compliance of the data to Beer's Law? (3)
- (d) Describe how to estimate for chemically available P and urea in fertilizers. (3)
- (e) 0.2g of copper ore is analysed iodometrically. Cu(II) was reduced to Cu(I) by iodide. ( $2Cu^{2+} + 4I^- \rightarrow 2CuI + I_2$ ). What is the %Cu in the ore if 20ml of 0.2M  $Na_2S_2O_3$  is used for titrating liberated iodine.  $I_2 + S_2O_3^{2-} \rightarrow 2I^- + S_4O_6^{2-}$ . (3).

(Use Ba = 137.3; Cu = 63.5; I = 126.9; S = 32.1; Fe = 55.8; Al = 27.0; O = 16.0; Cl = 35.5; Ca = 40.1; Zn = 65.4; Ag = 107.9)



# THE UNIVERSITY OF ZAMBIA

UNIVERSITY SEMESTER I EXAMINATION – APRIL 2002

## ADVANCED INORGANIC CHEMISTRY I – C441

### INSTRUCTIONS:

1. Attempt Any Four(4) Questions.
2. You are reminded of the need for orderly presentation of your answers.
3. Indicate the Questions attempted on the Answer Booklet cover.
4. Duration: THREE (3) hours

### USEFUL INFORMATION:

#### a. PHYSICAL CONSTANTS

Quantity	Symbol	SI Units	egs Units
Avogadro's Constant	$N_A$	$6.02 \times 10^{23} \text{ mol}^{-1}$	$6.02 \times 10^{23} \text{ mol}^{-1}$
Boltzmann Constant	$k$	$1.38 \times 10^{-23} \text{ J/atom.K}$	$1.38 \times 10^{-16} \text{ erg/atom.K}$ $8.62 \times 10^{-5} \text{ eV/atom.K}$
Bohr Magneton	$\mu_B$	$9.27 \times 10^{-24} \text{ A.m}^2$	$9.27 \times 10^{-21} \text{ erg/gauss}$
Permeability in Vacuum	$\mu_0$	$1.257 \times 10^{-6} \text{ H/m}$ $4\pi \times 10^{-7} \text{ H/m}$	Unity = 1 emu
Planck's Constant	$h$	$6.63 \times 10^{-34} \text{ J.s}$	$6.63 \times 10^{-27} \text{ erg.s}$ $4.13 \times 10^{-15} \text{ eV.s}$
Velocity of light in a vacuum	$c$	$3 \times 10^8 \text{ m/s}$	$3 \times 10^{10} \text{ cm/s}$

- b. Periodic Table and Character Tables are provided.
- c.  $^{79}\text{Br}$  is 50% abundant and  $^{81}\text{Br}$  is 50% abundant

1. (a) Determine the shapes and hence, the molecular point groups of the following species:



- (b) State **ALL** the symmetry operations of a  $C_{3v}$  point group.  
(c) Construct a multiplication table for  $C_{3v}$  point group and show, using **three** different symmetry operations, whether or not the associative property holds.
- 

2. (a) What is the value of the thermal energy,  $kT$ , in  $\text{cm}^{-1}$  at room temperature ( $25^\circ\text{C}$ )?

- (b)(i) Determine the ground state term for  $\text{Eu}^{3+}$ ,  $[\text{Xe}]4f^6$ , ion.

- (ii) The  $J_n \rightarrow J_{n+1}$  multiplet interval for a  $\text{Eu}^{3+}$  ion at room temperature ( $25^\circ\text{C}$ ) is  $204 \text{ cm}^{-1}$ . Determine  $\mu_{\text{eff}}$  for the ion and comment on its magnitude compared to the experimental value of  $3.6 \mu_B$ .

- (c) The energy separation between adjacent  $J_n$  states is given by

$$\Delta E_{J_n \rightarrow J_{n+1}} = \lambda(J+1)$$

where  $\lambda$  is of the magnitude of  $200 \text{ cm}^{-1}$ .

Determine the energy separations from the Ground State terms of  $\text{Pr}^{3+}$ ,  $[\text{Xe}]4f^2$  and  $\text{Sm}^{3+}$ ,  $[\text{Xe}]4f^5$ .

Hence or otherwise comment on the population distribution between these states in the two ions.

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3. (a) Describe the importance of Mass Spectroscopy to an Inorganic Chemist.

- (b) Describe each of the following and explain how they arise:

- (i) meta-stable peaks  
(ii) doubly charged ion peaks

- (c) Discuss the isotopic distribution pattern for  $\text{CH}_2\text{Br}_2^+$ .

- (d) State the Stevenson's Rule.
-

4. (a) By considering the Ammonia,  $\text{NH}_3$ , molecule
- (i) State the normal modes of vibration for this molecule
  - (ii) How many of these represent N-H stretching frequencies and how many represent the HNH bond angle deformation?
  - (iii) Which vibrations are Infrared Active and which are Raman Active?
- (b) State the "*Rule of Mutual Exclusion*"
- (c) Distinguish between Rayleigh scattering, Stoke and Anti-stoke scattering in Raman Spectroscopy.
- 

5. (a) The magnetic flux density within a bar of some material is 0.50T at an applied field strength of  $5.64 \times 10^5 \text{ A/m}$

Compute:

- (i) the magnetic permeability
  - (ii) the magnetic susceptibility
  - (iii) state the type of magnetism being displayed
- (b) Describe the effect of raising temperature on the magnetic susceptibility of the material in (a).
- (a) Define "*saturation magnetization*" of a paramagnetic substance.
- 

6. (a) Consider the organometallic compound  $(\text{C}_4\text{H}_6)\text{Fe}(\text{CO})_3$ . By considering the interaction between  $\text{C}_4\text{H}_6$  and Fe only.
- (i) Determine the shape of the  $\text{C}_4\text{H}_6$  SALCs and those of the frontier orbitals.
  - (ii) Determine the energies of these SALCs in terms of  $\alpha$  and  $\beta$ .
  - (iii) Deduce the orbitals on Fe available for bonding and state that nature of bonding.
- (b) By applying group theory concepts determine the  $\sigma$  - hybridisation scheme for  $\text{PF}_5$ .
- 

**END OF EXAMINATION**

A. Character Tables for Chemically Important Symmetry Groups

$C_4$	E	$C_4$	$C_2$	$C_4^3$		
A	1	1	1	1	$z, R_z$	$x^2 + y^2, z^2$
B	1	-1	1	-1		$x^2 - y^2, xy$
E	$\begin{Bmatrix} 1 & i \\ 1 & -i \end{Bmatrix}$	$\begin{Bmatrix} -i & 1 \\ 1 & -i \end{Bmatrix}$	$\begin{Bmatrix} -1 & i \\ 1 & -i \end{Bmatrix}$	$\begin{Bmatrix} i & 1 \\ 1 & -i \end{Bmatrix}$	$(x, y)(R_x, R_y)$	$(yz, xz)$

$\epsilon = \exp(2\pi i/5)$

1. The Nonaxial Groups

$C_3$	E	$C_3$	$C_3^2$		
A	1	1	1	$z, R_z$	$x^2 + y^2, z^2$
A'	1	$\epsilon$	$\epsilon^2$		$(x, y)(R_x, R_y)$
A''	1	$\epsilon^2$	$\epsilon$		$(yz, xz)$
E	$\begin{Bmatrix} 1 & \epsilon \\ 1 & \epsilon^2 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon & 1 \\ \epsilon^2 & \epsilon \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^2 & \epsilon \\ \epsilon & 1 \end{Bmatrix}$	$(x, y)(R_x, R_y)$	$(x^2 - y^2, xy)$

2. The  $C_2$  Groups

$C_2$	E	$C_2$	
A	1	1	$z, R_z$
B	1	-1	$x, y, R_x, R_y$

$C_3$	E	$C_3$	$C_3^2$	
A	1	1	1	$z, R_z$
E	$\begin{Bmatrix} 1 & \epsilon \\ 1 & \epsilon^2 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon & 1 \\ \epsilon^2 & \epsilon \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^2 & \epsilon \\ \epsilon & 1 \end{Bmatrix}$	$(x, y)(R_x, R_y)$

\* Appendix IIIA is presented in two places: (1) here, in its proper location in the sequence of appendices, and (2) as a separate section in a pocket inside the back cover.

$C_6$	E	$C_6$	$C_3$	$C_2$	$C_3^2$	$C_6^5$	
A	1	1	1	1	1	1	$z, R_z$
B	1	-1	1	-1	1	-1	
E <sub>1</sub>	$\begin{Bmatrix} 1 & \epsilon \\ 1 & \epsilon^2 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon & 1 \\ \epsilon^2 & \epsilon \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^2 & 1 \\ \epsilon & \epsilon \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^3 & 1 \\ \epsilon & \epsilon \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^4 & 1 \\ \epsilon^2 & \epsilon \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^5 & 1 \\ \epsilon^3 & \epsilon \end{Bmatrix}$	$(x, y)(R_x, R_y)$
E <sub>2</sub>	$\begin{Bmatrix} 1 & \epsilon^2 \\ 1 & \epsilon^4 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^2 & 1 \\ \epsilon^4 & \epsilon \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^4 & 1 \\ \epsilon & \epsilon \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^5 & 1 \\ \epsilon^2 & \epsilon \end{Bmatrix}$	$\begin{Bmatrix} \epsilon & 1 \\ \epsilon^3 & \epsilon \end{Bmatrix}$	$\begin{Bmatrix} \epsilon & 1 \\ \epsilon^3 & \epsilon \end{Bmatrix}$	$(yz, xz)$

$\epsilon = \exp(2\pi i/6)$

$C_6$	E	$C_6$	$C_3$	$C_2$	$C_3^2$	$C_6^5$	
A	1	1	1	1	1	1	$z, R_z$
B	1	-1	1	-1	1	-1	
E <sub>1</sub>	$\begin{Bmatrix} 1 & \epsilon \\ 1 & \epsilon^2 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon & 1 \\ \epsilon^2 & \epsilon \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^2 & 1 \\ \epsilon & \epsilon \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^3 & 1 \\ \epsilon & \epsilon \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^4 & 1 \\ \epsilon^2 & \epsilon \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^5 & 1 \\ \epsilon^3 & \epsilon \end{Bmatrix}$	$(x, y)$
E <sub>2</sub>	$\begin{Bmatrix} 1 & \epsilon^2 \\ 1 & \epsilon^4 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^2 & 1 \\ \epsilon^4 & \epsilon \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^4 & 1 \\ \epsilon & \epsilon \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^5 & 1 \\ \epsilon^2 & \epsilon \end{Bmatrix}$	$\begin{Bmatrix} \epsilon & 1 \\ \epsilon^3 & \epsilon \end{Bmatrix}$	$\begin{Bmatrix} \epsilon & 1 \\ \epsilon^3 & \epsilon \end{Bmatrix}$	$(xz, yz)$

$\epsilon = \exp(2\pi i/7)$

$C_7$	E	$C_7$	$C_7^2$	$C_7^3$	$C_7^4$	$C_7^5$	$C_7^6$	
A	1	1	1	1	1	1	1	$z, R_z$
E <sub>1</sub>	$\begin{Bmatrix} 1 & \epsilon \\ 1 & \epsilon^2 \\ 1 & \epsilon^3 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon & 1 \\ \epsilon^2 & \epsilon \\ \epsilon^3 & \epsilon^2 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^2 & 1 \\ \epsilon^3 & \epsilon \\ \epsilon^4 & \epsilon^2 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^3 & 1 \\ \epsilon^4 & \epsilon \\ \epsilon^5 & \epsilon^2 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^4 & 1 \\ \epsilon^5 & \epsilon \\ \epsilon^6 & \epsilon^2 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^5 & 1 \\ \epsilon^6 & \epsilon \\ \epsilon & \epsilon^2 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^6 & 1 \\ \epsilon & \epsilon \\ \epsilon^2 & \epsilon^2 \end{Bmatrix}$	$(x, y)$
E <sub>2</sub>	$\begin{Bmatrix} 1 & \epsilon^2 \\ 1 & \epsilon^4 \\ 1 & \epsilon^6 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^2 & 1 \\ \epsilon^4 & \epsilon \\ \epsilon^6 & \epsilon^2 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^4 & 1 \\ \epsilon^6 & \epsilon \\ \epsilon & \epsilon^2 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^6 & 1 \\ \epsilon & \epsilon \\ \epsilon^2 & \epsilon^2 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon & 1 \\ \epsilon^2 & \epsilon \\ \epsilon^4 & \epsilon^2 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon & 1 \\ \epsilon^2 & \epsilon \\ \epsilon^4 & \epsilon^2 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon & 1 \\ \epsilon^2 & \epsilon \\ \epsilon^4 & \epsilon^2 \end{Bmatrix}$	$(xz, yz)$

$\epsilon = \exp(2\pi i/8)$

$C_8$	E	$C_8$	$C_4$	$C_2$	$C_4^3$	$C_8^5$	$C_8^7$	
A	1	1	1	1	1	1	1	$z, R_z$
B	1	-1	1	1	-1	-1	-1	
E <sub>1</sub>	$\begin{Bmatrix} 1 & \epsilon \\ 1 & \epsilon^2 \\ 1 & \epsilon^3 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon & 1 \\ \epsilon^2 & \epsilon \\ \epsilon^3 & \epsilon^2 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^2 & 1 \\ \epsilon^4 & \epsilon \\ \epsilon^6 & \epsilon^2 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^4 & 1 \\ \epsilon^6 & \epsilon \\ \epsilon & \epsilon^2 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^6 & 1 \\ \epsilon & \epsilon \\ \epsilon^2 & \epsilon^2 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^7 & 1 \\ \epsilon^5 & \epsilon \\ \epsilon^3 & \epsilon^2 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^7 & 1 \\ \epsilon^5 & \epsilon \\ \epsilon^3 & \epsilon^2 \end{Bmatrix}$	$(x, y)$
E <sub>2</sub>	$\begin{Bmatrix} 1 & \epsilon^2 \\ 1 & \epsilon^4 \\ 1 & \epsilon^6 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^2 & 1 \\ \epsilon^4 & \epsilon \\ \epsilon^6 & \epsilon^2 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^4 & 1 \\ \epsilon^6 & \epsilon \\ \epsilon & \epsilon^2 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon^6 & 1 \\ \epsilon & \epsilon \\ \epsilon^2 & \epsilon^2 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon & 1 \\ \epsilon^2 & \epsilon \\ \epsilon^4 & \epsilon^2 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon & 1 \\ \epsilon^2 & \epsilon \\ \epsilon^4 & \epsilon^2 \end{Bmatrix}$	$\begin{Bmatrix} \epsilon & 1 \\ \epsilon^2 & \epsilon \\ \epsilon^4 & \epsilon^2 \end{Bmatrix}$	$(xz, yz)$

$\epsilon = \exp(2\pi i/8)$

### 3. The $D_n$ Groups

$D_2$	E	$C_2(z)$	$C_2(y)$	$C_2(x)$	
$A_1$	1	1	1	1	$x^2, y^2, z^2$
$A_2$	1	1	-1	-1	$xy$
$B_1$	1	-1	1	-1	$z, R_x$
$B_2$	1	-1	-1	1	$y, R_y$
$B_3$	1	-1	-1	1	$x, R_x, yz$

$D_3$	E	$2C_3$	$3C_2$	
$A_1$	1	1	1	$x^2 + y^2, z^2$
$A_2$	1	1	-1	$z, R_x$
E	2	-1	0	$(x, y)(R_x, R_y)$ $(x^2 - y^2, xy)(xz, yz)$

$D_4$	E	$2C_4$	$C_2(=C_4^2)$	$2C_2'$	$2C_2''$	
$A_1$	1	1	1	1	1	$x^2 + y^2, z^2$
$A_2$	1	1	1	-1	-1	$z, R_x$
$B_1$	1	-1	1	1	-1	$x^2 - y^2$
$B_2$	1	-1	1	-1	1	$xy$
E	2	0	-2	0	0	$(x, y)(R_x, R_y)$ $(xz, yz)$

$D_5$	E	$2C_5$	$2C_2'$	$3C_2''$	
$A_1$	1	1	1	1	$x^2 + y^2, z^2$
$A_2$	1	1	1	-1	$z, R_x$
$E_1$	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	$(x, y)(R_x, R_y)$
$E_2$	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	$(xz, yz)$ $(x^2 - y^2, xy)$

$D_6$	E	$2C_6$	$2C_3$	$C_2$	$3C_2'$	$3C_2''$	
$A_1$	1	1	1	1	1	1	$x^2 + y^2, z^2$
$A_2$	1	1	1	1	-1	-1	$z, R_x$
$B_1$	1	-1	1	1	1	-1	
$B_2$	1	-1	1	-1	-1	1	
$E_1$	2	1	-1	-2	0	0	$(x, y)(R_x, R_y)$
$E_2$	2	-1	-1	2	0	0	$(xz, yz)$ $(x^2 - y^2, xy)$

### 4. The $C_{np}$ Groups

$C_{3v}$	E	$C_2$	$\sigma_v(xz)$	$\sigma_v(yz)$	
$A_1$	1	1	1	1	$x^2, y^2, z^2$
$A_2$	1	1	-1	-1	$xy$
$B_1$	1	-1	1	-1	$z, R_x$
$B_2$	1	-1	-1	1	$x, R_x, yz$

$C_{3v}$	E	$2C_3$	$3\sigma_v$	
$A_1$	1	1	1	$x^2 + y^2, z^2$
$A_2$	1	1	-1	$z, R_x$
E	2	-1	0	$(x, y)(R_x, R_y)$ $(x^2 - y^2, xy)(xz, yz)$

$C_{4v}$	E	$2C_4$	$C_2$	$2\sigma_v$	$2\sigma_d$	
$A_1$	1	1	1	1	1	$x^2 + y^2, z^2$
$A_2$	1	1	1	-1	-1	$x^2 - y^2$
$B_1$	1	-1	1	1	-1	$xy$
$B_2$	1	-1	1	-1	1	$(xz, yz)$
E	2	0	-2	0	0	$(x, y)(R_x, R_y)$

$C_{5v}$	E	$2C_5$	$2C_2'$	$5\sigma_v$	
$A_1$	1	1	1	1	$x^2 + y^2, z^2$
$A_2$	1	1	1	-1	$z, R_x$
$E_1$	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	$(x, y)(R_x, R_y)$
$E_2$	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	$(xz, yz)$ $(x^2 - y^2, xy)$

$C_{6v}$	E	$2C_6$	$2C_3$	$C_2$	$3\sigma_v$	$3\sigma_d$	
$A_1$	1	1	1	1	1	1	$x^2 + y^2, z^2$
$A_2$	1	1	1	1	-1	-1	$z, R_x$
$B_1$	1	-1	1	1	1	-1	
$B_2$	1	-1	1	-1	-1	1	
$E_1$	2	1	-1	-2	0	0	$(x, y)(R_x, R_y)$
$E_2$	2	-1	-1	2	0	0	$(xz, yz)$ $(x^2 - y^2, xy)$



7. The  $D_{6d}$  Groups (Continued).

$D_{6d}$	E	$2S_{12}$	$2C_6$	$2S_8$	$2C_2$	$2S_{12}^3$	$C_2$	$6C_2'$	$6\sigma_6$	$R_x$	$x^2 + y^2, z^2$
$A_1$	1	1	1	1	1	1	1	1	1	1	$x^2 + y^2, z^2$
$A_2$	1	1	1	1	1	1	1	1	1	1	$x^2 + y^2, z^2$
$B_1$	1	1	1	1	1	1	1	1	1	1	$x^2 + y^2, z^2$
$B_2$	1	1	1	1	1	1	1	1	1	1	$x^2 + y^2, z^2$
$E_1$	2	$\sqrt{3}$	1	0	-1	$-\sqrt{3}$	-2	0	0	0	$(x^2 - y^2, xy)$
$E_2$	2	0	-2	0	2	0	0	0	0	0	$(x^2 - y^2, xy)$
$E_3$	2	0	-2	0	2	0	0	0	0	0	$(x^2 - y^2, xy)$
$E_4$	2	-1	1	2	-1	$\sqrt{3}$	-2	0	0	0	$(xz, yz)$
$E_5$	2	$-\sqrt{3}$	1	0	-1	$\sqrt{3}$	-2	0	0	0	$(xz, yz)$

8. The  $S_6$  Groups

$S_6$	E	$S_4$	$C_2$	$S_4^3$	$R_x$	$x^2 + y^2, z^2$
A	1	1	1	1	$R_x$	$x^2 + y^2, z^2$
B	1	-1	1	-1	z	$x^2 - y^2, xy$
E	1	i	-1	-i	$(x, y); (R_x, R_y)$	$(xz, yz)$
	1	-i	-1	i		

$S_6$	E	$C_3$	$C_3^2$	i	$S_6^5$	$S_6$	$\varepsilon = \exp(2\pi i/3)$
$A_6$	1	1	1	1	1	1	$x^2 + y^2, z^2$
$E_6$	1	$\varepsilon$	$\varepsilon^2$	$\varepsilon$	$\varepsilon^*$	$\varepsilon$	$(x^2 - y^2, xy); (xz, yz)$
$A_6$	1	1	1	1	-1	-1	$(R_x, R_y)$
$E_6$	1	$\varepsilon$	$\varepsilon^2$	-1	$-\varepsilon$	$-\varepsilon^*$	z
	1	$\varepsilon^*$	$\varepsilon$	-1	$-\varepsilon^*$	$-\varepsilon$	$(x, y)$

$S_6$	E	$S_6$	$C_4$	$S_6^3$	$C_2$	$S_6^5$	$C_4^3$	$S_6^7$	$\varepsilon = \exp(2\pi i/8)$
A	1	1	1	1	1	1	1	1	$x^2 + y^2, z^2$
B	1	-1	1	-1	1	-1	1	-1	$R_x$
$E_1$	1	$\varepsilon$	i	$-\varepsilon^*$	-1	$-\varepsilon$	-i	$\varepsilon^*$	z
	1	$\varepsilon^*$	-i	$-\varepsilon$	-1	$-\varepsilon^*$	i	$\varepsilon$	$(x, y); (R_x, R_y)$
$E_2$	1	i	-1	-i	1	i	-1	-i	$(x^2 - y^2, xy)$
	1	-i	-1	i	1	-i	1	i	$(xz, yz)$
$E_3$	1	$-\varepsilon^*$	-i	$\varepsilon$	-1	$\varepsilon^*$	-i	$-\varepsilon$	
	1	$-\varepsilon$	i	$\varepsilon^*$	-1	$\varepsilon$	i	$-\varepsilon^*$	

9. The Cubic Groups

T	E	$4C_3$	$4C_3^2$	$3C_2$	$\varepsilon = \exp(2\pi i/3)$
A	1	1	1	1	$x^2 + y^2 + z^2$
$E$	1	$\varepsilon$	$\varepsilon^*$	1	$(2x^2 - y^2 - z^2, xy)$
	1	$\varepsilon^*$	$\varepsilon$	1	$(2x^2 - y^2 - z^2, xy)$
T	3	0	0	-1	$(xy, xz, yz)$

$D_{6d}$	E	$2C_6$	$2C_3^2$	$2C_4$	$C_2$	$4C_2'$	$4C_2''$	$i$	$2S_8$	$2S_6^3$	$2S_8^3$	$\sigma_6$	$4\sigma_6$	$4\sigma_6$	$R_x$	$x^2 + y^2, z^2$
$A_{1g}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	$x^2 + y^2, z^2$
$A_{2g}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	$x^2 + y^2, z^2$
$B_{1g}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	$x^2 + y^2, z^2$
$B_{2g}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	$x^2 + y^2, z^2$
$E_{1g}$	2	$\sqrt{2}$	0	-2	0	2	$\sqrt{2}$	0	-2	0	0	0	0	0	0	$(xz, yz)$
$E_{2g}$	2	0	-2	2	0	0	0	-2	2	0	0	0	0	0	0	$(x^2 - y^2, xy)$
$E_{3g}$	2	$-\sqrt{2}$	0	2	0	0	$\sqrt{2}$	0	-2	0	0	0	0	0	0	$(x^2 - y^2, xy)$
$A_{1u}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	$(xz, yz)$
$A_{2u}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	$(xz, yz)$
$B_{1u}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	$(xz, yz)$
$B_{2u}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	$(xz, yz)$
$E_{1u}$	2	$\sqrt{2}$	0	-2	0	0	$\sqrt{2}$	0	-2	0	0	0	0	0	0	$(xz, yz)$
$E_{2u}$	2	0	-2	2	0	0	0	-2	2	0	0	0	0	0	0	$(x^2 - y^2, xy)$
$E_{3u}$	2	$-\sqrt{2}$	0	2	0	0	$-\sqrt{2}$	0	-2	0	0	0	0	0	0	$(x^2 - y^2, xy)$

7. The  $D_{2d}$  Groups

$D_{2d}$	E	$2S_4$	$C_2$	$2C_2'$	$2\sigma_d$	$R_x$	$x^2 + y^2, z^2$
$A_1$	1	1	1	1	1	$R_x$	$x^2 + y^2, z^2$
$A_2$	1	1	1	-1	-1	z	$x^2 - y^2$
$B_1$	1	-1	1	1	-1	xy	xy
$B_2$	1	-1	1	-1	1	$(x, y); (R_x, R_y)$	$(xz, yz)$
E	2	0	-2	0	0		

$D_{2d}$	E	$2C_2$	$3C_2$	i	$2S_6^3$	$3\sigma_6$	$R_x$	$x^2 + y^2, z^2$
$A_{1g}$	1	1	1	1	1	1	$R_x$	$x^2 + y^2, z^2$
$A_{2g}$	1	1	1	1	1	1	$(R_x, R_y)$	$(x^2 - y^2, xy); (xz, yz)$
$E_g$	2	-1	0	2	-1	0		
$A_{1u}$	1	1	1	1	1	1	z	$(x, y)$
$A_{2u}$	1	1	1	1	1	1	$(R_x, R_y)$	$(x^2 - y^2, xy); (xz, yz)$
$E_u$	2	-1	0	-2	1	0		

$D_{4d}$	E	$2S_8$	$2C_4$	$2S_6^3$	$C_2$	$4C_2'$	$4\sigma_4$	$R_x$	$x^2 + y^2, z^2$
$A_1$	1	1	1	1	1	1	1	1	$x^2 + y^2, z^2$
$A_2$	1	1	1	1	1	1	1	1	$x^2 + y^2, z^2$
$B_1$	1	1	1	1	1	1	1	1	$x^2 + y^2, z^2$
$B_2$	1	1	1	1	1	1	1	1	$x^2 + y^2, z^2$
$E_1$	2	$\sqrt{2}$	0	$-\sqrt{2}$	-2	0	0	0	$(x^2 - y^2, xy)$
$E_2$	2	0	-2	0	2	0	0	0	$(x^2 - y^2, xy)$
$E_3$	2	$-\sqrt{2}$	0	$\sqrt{2}$	-2	0	0	0	$(x^2 - y^2, xy)$

$D_{3d}$	E	$2C_3$	$2C_3^2$	$3C_2$	i	$2S_{10}^3$	$2S_{10}$	$5\sigma_6$	$R_x$	$x^2 + y^2, z^2$
$A_{1g}$	1	1	1	1	1	1	1	1	$R_x$	$x^2 + y^2, z^2$
$A_{2g}$	1	1	1	1	1	1	1	1	$(R_x, R_y)$	$(xz, yz)$
$E_{1g}$	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	0	$(x^2 - y^2, xy)$
$E_{2g}$	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	0	$(x^2 - y^2, xy)$
$A_{1u}$	1	1	1	1	1	1	1	1	z	$(x, y)$
$A_{2u}$	1	1	1	1	1	1	1	1	z	$(x, y)$
$E_{1u}$	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	-2	$-2 \cos 72^\circ$	$-2 \cos 144^\circ$	0	0	$(x^2 - y^2, xy)$
$E_{2u}$	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	-2	$-2 \cos 144^\circ$	$-2 \cos 72^\circ$	0	0	$(x^2 - y^2, xy)$

10. The Groups  $C_{\infty v}$  and  $D_{\infty h}$  for Linear Molecules

$C_{\infty v}$	$E$	$2C_{\infty}$	$\infty C_2$	$z$ $(x, y); (R_x, R_y)$	$x^2 + y^2, z^2$ $(xz, yz)$ $(x^2 - y^2, xy)$	$x^2 + y^2 + z^2$
$A_1 \equiv \Sigma^+$	1	1	1	1	1	1
$A_2 \equiv \Sigma^-$	1	1	1	1	1	1
$E_1 \equiv \Pi$	2	$2 \cos \Phi$	0	2	$-2 \cos \Phi$	0
$E_2 \equiv \Delta$	2	$2 \cos 2\Phi$	0	2	$2 \cos 2\Phi$	0
$E_3 \equiv \Phi$	2	$2 \cos 3\Phi$	0	0	0	0
$D_{\infty h}$	$E$	$2C_{\infty}$	$\infty C_2$	$i$	$2S_{\infty}$	$\infty C_2$
$\Sigma^+$	1	1	1	1	1	1
$\Sigma^-$	1	1	1	1	1	1
$\Pi_g^+$	2	$2 \cos \Phi$	0	2	$-2 \cos \Phi$	0
$\Pi_g^-$	2	$2 \cos \Phi$	0	2	$2 \cos 2\Phi$	0
$\Delta_g^+$	2	$2 \cos 2\Phi$	0	2	$2 \cos 2\Phi$	0
$\Delta_g^-$	2	$2 \cos 2\Phi$	0	2	$-2 \cos 2\Phi$	0
$\Phi_g^+$	2	$2 \cos 3\Phi$	0	0	0	0
$\Phi_g^-$	2	$2 \cos 3\Phi$	0	0	0	0

$C_{\infty v}$	$E$	$2C_{\infty}$	$\infty C_2$	$z$ $(x, y); (R_x, R_y)$	$x^2 + y^2, z^2$ $(xz, yz)$ $(x^2 - y^2, xy)$	$x^2 + y^2 + z^2$
$A_1$	1	1	1	1	1	1
$A_2$	1	1	1	1	1	1
$E$	2	0	2	0	0	$(2z^2 - x^2 - y^2, x^2 - y^2)$
$T_1$	3	-1	-1	0	-1	$(x^2 + y^2 - z^2, xy, xz, yz)$
$T_2$	3	-1	-1	0	-1	$(x^2 + y^2 - z^2, xy, xz, yz)$

11. The Icosahedral Groups\*

$I_h$	$E$	$12C_5$	$12C_5^2$	$20C_3$	$15C_2$	$i$	$12S_{10}$	$12S_{10}^3$	$20S_6$	$15\sigma$	
$A_g$	1	1	1	1	1	1	1	1	1	1	$x^2 + y^2 + z^2$
$T_{1g}$	3	$\frac{1}{2}(1 + \sqrt{5})$	$\frac{1}{2}(1 - \sqrt{5})$	0	-1	3	$\frac{1}{2}(1 - \sqrt{5})$	$\frac{1}{2}(1 + \sqrt{5})$	0	-1	$(R_x, R_y, R_z)$
$T_{2g}$	3	$\frac{1}{2}(1 - \sqrt{5})$	$\frac{1}{2}(1 + \sqrt{5})$	0	-1	3	$\frac{1}{2}(1 + \sqrt{5})$	$\frac{1}{2}(1 - \sqrt{5})$	0	-1	$(2z^2 - x^2 - y^2, x^2 - y^2, xy, yz, zx)$
$G_g$	4	-1	-1	1	0	4	-1	-1	1	0	
$H_g$	5	0	0	-1	1	5	0	0	-1	1	
$A_u$	1	1	1	1	1	-1	-1	-1	-1	-1	
$T_{1u}$	3	$\frac{1}{2}(1 + \sqrt{5})$	$\frac{1}{2}(1 - \sqrt{5})$	0	-1	-3	$-\frac{1}{2}(1 - \sqrt{5})$	$-\frac{1}{2}(1 + \sqrt{5})$	0	1	$(x, y, z)$
$T_{2u}$	3	$\frac{1}{2}(1 - \sqrt{5})$	$\frac{1}{2}(1 + \sqrt{5})$	0	-1	-3	$-\frac{1}{2}(1 + \sqrt{5})$	$-\frac{1}{2}(1 - \sqrt{5})$	0	1	
$G_u$	4	-1	-1	1	0	-4	1	1	-1	0	
$H_u$	5	0	0	-1	1	-5	0	0	1	-1	

\* For the pure rotation group I, the outlined section in the upper left is the character table; the g subscripts should, of course, be dropped and (x, y, z) assigned to the  $T_1$  representation.

# Periodic Table of the Elements

		PERIOD																																	
		1		2												13		14		15		16		17		18									
		1A		2A												3A		4A		5A		6A		7A		8A									
1	H	2	He											13	B	14	C	15	N	16	O	17	F	18	Ne										
2	Li	3	Be	4	B	5	C	6	N	7	O	8	F	9	Ne																				
3	Na	10	Mg	11	Al	12	Si	13	P	14	S	15	Cl	16	Ar																				
4	K	18	Ca	19	Sc	20	Ti	21	V	22	Cr	23	Mn	24	Fe	25	Co	26	Ni	27	Cu	28	Zn	29	Ga	30	Ge	31	As	32	Se	33	Br	34	Kr
5	Rb	26	Sr	27	Y	28	Zr	29	Nb	30	Mo	31	Tc	32	Ru	33	Rh	34	Pd	35	Ag	36	Cd	37	In	38	Sn	39	Sb	40	Te	41	I	42	Xe
6	Cs	34	Ba	35	La	36	Hf	37	Ta	38	W	39	Re	40	Os	41	Ir	42	Pt	43	Au	44	Hg	45	Tl	46	Pb	47	Bi	48	Po	49	At	50	Rn
7	Fr	42	Ra	43	Ac	44	Rf	45	Db	46	Sg	47	Bh	48	Hs	49	Mt	50	Ds	51	Uu	52	Cn	53	Nh	54	Fl	55	Mc	56	Lv	57	Ts	58	Og

* Lanthanide series:	
58	Ce
59	Pr
60	Nd
61	Pm
62	Sm
63	Eu
64	Gd
65	Tb
66	Dy
67	Ho
68	Er
69	Tm
70	Yb
71	Lu

+ Actinide series:	
90	Th
91	Pa
92	U
93	Np
94	Pu
95	Am
96	Cm
97	Bk
98	Cf
99	Es
100	Fm
101	Md
102	No
103	Lr



**THE UNIVERSITY OF ZAMBIA**  
**UNIVERSITY SEMESTER I EXAMINATION**

**APRIL 2002**

**C481 – INORGANIC INDUSTRIAL CHEMISTRY**

**TIME: THREE (3) HOURS**

**INSTRUCTIONS: ANSWER ANY FIVE QUESTIONS**

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1. State the reactions and production processes of the following sodium compounds:  
 $\text{NaNO}_2$ ,  $\text{Na}_2\text{O}_2 \cdot 7\text{H}_2\text{O}$ ,  $\text{Na}_2\text{S}_2\text{O}_3 \cdot 5\text{H}_2\text{O}$ ,  $\text{NaNH}_2$ ,  $\text{NaCN}$  and  $\text{Na}_4\text{Fe}(\text{CN})_6$
2. Describe the raw materials for ceramic and glass industries.
3. Discuss the composition and uses of Magnesium binding materials, Acid resistant cement, Building and Hydraulic limes.
4. Explain the manufacture of Portland cement (flow-Sheet is attached).
5. Discuss the Hydrogen production processes.
6. Explain the apparatus used for gas purification and absorption processes.

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# THE UNIVERSITY OF ZAMBIA

UNIVERSITY SEMESTER I EXAMINATION APRIL 18, 2002

## C491 Organic Industry Chemistry

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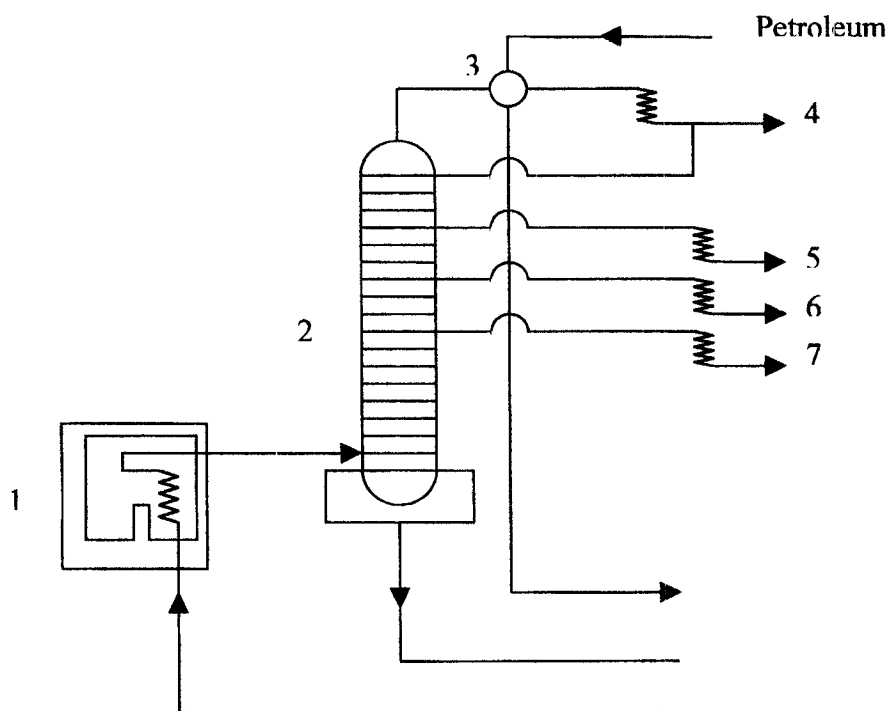
**Time** : 3 hours

**Instructions** : Answer question 1 and any other four (4) questions

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1.
  - (a) Explain in the following terms: (i) continuous process (ii) countercurrent
  - (b) In both petroleum refining and gasification of coal the efficiency of catalyst is affected by a common by product. What is this product? A process stage is often included in the process to alleviate this problem. What does this stage involve?
  - (c) Water, which is commonly used for cleaning, has a property called surface tension. Explain in brief surface tension and what effect it has on cleansing.
  - (d) Water provided in mechanical processing of logs to obtain pulp has two roles. Explain these two roles.
  - (e) What problems are associated with storage and transportation of colloidal sulfur preparations that are used as fungicides?
  
2.
  - (a) In coal gasifiers with liquid heat carriers, the coal and gasifying medium are blown onto a bath of molten heat transfer agent. What advantages does the use of liquid heat carriers have?
  - (b) In the Winkler process particle size is not critical (up to 8 mm may be used). However, the process is still affected when particles are either too small or large. Explain in a concise manner the consequence of having (i) small particles (ii) large particles.
  
3. Frictional resistance can affect the flow of fluids and result in either streamline or turbulent flow. With the aid diagrams distinguish qualitatively between the two types of flow. Indicate in brief how each comes about.
  
4.
  - (a) Distinguish between a soap and a detergent
  - (b) How does soap clean?
  - (c) Explain the function of phosphate in soaps and detergents.
  - (d) Glycerine is a by product of the manufacture of soap. Explain in brief what is involved in its recovery?

6. Pulp for manufacturing paper is conditioned before entering the papermaking machine.
- Explain what is involved in conditioning the pulp.
  - What are sizers and what function do they play in paper manufacturing?
  - Distinguish between stock and tub sizing
7. The diagram below shows part of an atmospheric-vacuum crude fractionation plant.
- Identify the unit label 2 and explain in brief the process that occurs in the unit.
  - What are the products 4, 5, 6 and 7?
  - The bottom product can be processed further to obtain more products. Explain how this done.





**THE UNIVERSITY OF ZAMBIA  
UNIVERSITY SEMESTER 1 EXAMINATIONS**

**APRIL 2002**

**CAV 251**

**ANALYTICAL/PHYSICAL/ORAGANIC CHEMISTRY**

**TIME: THREE HOURS**

**INSTRUCTIONS**

1. There are **THREE** sections in this examination paper: Section A, B and C.
2. Both Section A and B have two questions each. You are to answer **ONE** question from each section.
3. Section C has four questions. Answer **THREE** questions from this section.
4. All questions carry equal marks.
5. Show all your working and reasoning clearly.

**IMPORTANT INFORMATION**

**R = 8.314 J/mol.K**

## SECTION A. ANALYTICAL CHEMISTRY

Answer only ONE question in this section.

### QUESTION ONE

A. (i) What is the most important requirement for a sample? Explain one method used to reduce a solid sample to a manageable size. (3 marks)

(ii) What are determinate errors? Name three sources of these errors. Define the standard deviation of a set of data. (3 marks)

B. A study is being performed to see if there is a correlation between the concentration of chromium in the blood and a suspected disease. The results of analysis of blood samples from a control group and a diseased group were as follows:

Control group (ppbCr): 15, 23, 12, 18, 09, 28, 11, 10

Diseased group (ppbCr): 25, 20, 35, 32, 15, 40, 16, 10, 22, 18

Determine, at 95% confidence level, whether the differences between the two groups can be ascribed to chance or whether they are real. (6 marks)

### QUESTION TWO

A. The solubility product of the compound  $P_2X$  was found equal to  $3.58 \times 10^{-13}$  on the basis of solubility measurements and the assumption of quantitative ionisation to  $P^+$  and  $X^{2-}$  ions. Later it was found that the compound is quantitatively ionised to  $P_2^{2+}$  ions. Calculate the real solubility product of  $P_2X$ . (6 marks)

B. A 100 ml solution of 0.100M sodium acetate is titrated against 0.100M hydrochloric acid. Calculate the pH of the solution, and hence plot the titration curve, at the following points of the titration:

0%, 25%, 50%, 75%, 100% and 125%.

$K_a(\text{CH}_3\text{COOH}) = 1.75 \times 10^{-5}$ . (6 marks)

## SECTION B PHYSICAL CHEMISTRY

Answer ONE question from this section.

### QUESTION ONE

- A. Define an adiabatic process. (2 marks)
- B. One mole of an ideal gas at  $20^{\circ}\text{C}$  is compressed adiabatically so that the temperature rises to  $50^{\circ}\text{C}$ . Calculate the  $\Delta U$ ,  $q$ ,  $w$  and  $\Delta H$  for the process.  $C_v$  for the gas is  $1.5R$ . (8 marks)
- C. Distinguish between standard free energy and entropy. (2 marks)

### QUESTION TWO

- A. Three moles of an ideal gas at  $300\text{K}$  expands isothermally and reversibly from  $20\text{dm}^3$  to  $60\text{dm}^3$ . Compute  $w$ ,  $q$ ,  $\Delta U$  and  $\Delta H$ . (8 marks)
- B. Derive equations for entropy change for;
- (i) Isothermal expansion of an ideal gas from volume,  $V_1$  to  $V_2$ .
- (ii) Isochoric temperature change of a gas from  $T_1$  to  $T_2$ . (4 marks)

## SECTION C ORGANIC CHEMISTRY

### QUESTION ONE

- A. Distinguish between stereoisomerism and optical isomerism. (2 marks)
- B. There are three isomeric alkanes of molecular formula  $\text{C}_5\text{H}_{12}$ . Isomer A gives a mixture of four (4) monochlorination products when reacted with chlorine gas at  $300^{\circ}\text{C}$ . Under the same conditions, isomer B gives a mixture of three (3) monochlorination products while isomer C gives only one monochlorination product. From this information assign structural formulas to isomers A, B and C. (10 marks)

### QUESTION TWO

Name and draw structural formulas for the eight (8) isomeric alcohols of molecular formula  $\text{C}_5\text{H}_{12}\text{O}$ . Classify each as primary, secondary or tertiary. Which of the eight show enantiomerism? (12 marks)

### QUESTION THREE

- A. The presence of glucose  $\text{HOCH}_2(\text{CHOH})_4\text{CHO}$ , in the urine is a common sign of diabetes mellitus. What would you suggest (explaining in detail) as a possible test to check for the presence of glucose in urine samples?

What types of compounds would interfere with your test? (6 marks)

- B. What is meant by the terms  $\text{SN}_1$  and  $\text{SN}_2$ ?

Compare and contrast  $\text{SN}_1$  and  $\text{SN}_2$  reactions in terms of

- (i) rate determining step
- (ii) the stereochemistry of the product as related to that of the starting materials. (6 marks)

### QUESTION FOUR

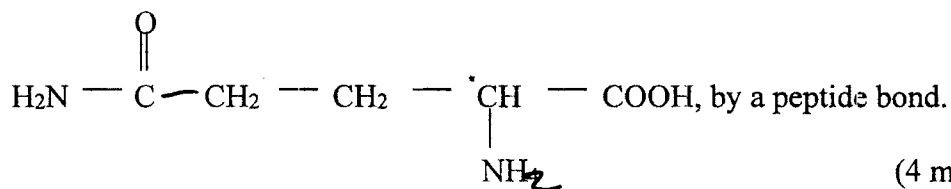
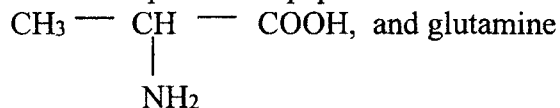
- A. (i) What is an  $\alpha$ -amino acid?

Explain what is meant by a zwitterion. (2 marks)

- (ii) Write the formula for glycine in its zwitterion form. (1 mark)

- (iii) Write the equation for the addition of potassium hydroxide to the solution in A (ii). (1 mark)

- (iv) Write the possible dipeptide structures that can be obtained by joining alanine,



(4 marks)

- C. Are  $\alpha$ -D-glucose and  $\beta$ -D-Glucose enantiomes? Explain. (3 marks)

**END OF EXAMINATION**

**Table A.1. The *t*-distribution**

Value of <i>t</i> for a confidence interval of	90%	95%	98%	99%
Critical value of $ t $ for <i>P</i> values of	0.10	0.05	0.02	0.01
Number of degrees of freedom				
1	6.31	12.71	31.82	63.66
2	2.92	4.30	6.96	9.92
3	2.35	3.18	4.54	5.84
4	2.13	2.78	3.75	4.60
5	2.02	2.57	3.36	4.03
6	1.94	2.45	3.14	3.71
7	1.89	2.36	3.00	3.50
8	1.86	2.31	2.90	3.36
9	1.83	2.26	2.82	3.25
10	1.81	2.23	2.76	3.17
12	1.78	2.18	2.68	3.05
14	1.76	2.14	2.62	2.98
16	1.75	2.12	2.58	2.92
18	1.73	2.10	2.55	2.88
20	1.72	2.09	2.53	2.85
30	1.70	2.04	2.46	2.75
50	1.68	2.01	2.40	2.68
∞	1.64	1.96	2.33	2.58

**Table A.2. Critical values of *F* for a one-tailed test (*P* = 0.05)**

$\nu_1 \backslash \nu_2$	1	2	3	4	5	6	7	8	9	10	12	15	20
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45
3	10.13	9.552	9.277	9.117	9.013	8.941	8.887	8.845	8.812	8.786	8.745	8.703	8.660
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.999	5.964	5.912	5.858	5.803
5	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818	4.772	4.735	4.678	4.619	4.558
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099	4.060	4.000	3.938	3.874
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.677	3.637	3.575	3.511	3.445
8	5.318	4.459	4.066	3.838	3.687	3.581	3.500	3.438	3.388	3.347	3.284	3.218	3.150
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230	3.179	3.137	3.073	3.006	2.936
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	3.020	2.978	2.913	2.845	2.774
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.896	2.854	2.788	2.719	2.646
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.796	2.753	2.687	2.617	2.544
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714	2.671	2.604	2.533	2.459
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646	2.602	2.534	2.463	2.388
15	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641	2.588	2.544	2.475	2.403	2.328
16	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.538	2.494	2.425	2.352	2.276
17	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548	2.494	2.450	2.381	2.308	2.230
18	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510	2.456	2.412	2.342	2.269	2.191
19	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477	2.423	2.378	2.308	2.234	2.155
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.393	2.348	2.278	2.203	2.124

$\nu_1$  = number of degrees of freedom of the numerator and  $\nu_2$  = number of degrees of freedom of the denominator.

TABLE A3  
REJECTION QUOTIENT, Q, AT DIFFERENT CONFIDENCE LIMITS

Number of observation	Confidence Level		
	Q90	Q95	Q99
3	0.94	0.970	0.994
4	0.76	0.829	0.926
5	0.64	0.710	0.821
6	0.56	0.625	0.740
7	0.51	0.568	0.680
8	0.47	0.526	0.634
9	0.44	0.493	0.598
10	0.41	0.466	0.568
15	0.338	0.384	0.475
20	0.300	0.342	0.425
25	0.277	0.317	0.393
30	0.260	0.298	0.372



# THE UNIVERSITY OF ZAMBIA

## SCHOOL OF NATURAL SCIENCES

### Department of Computer Studies

FIRST SEMESTER EXAMINATIONS FOR THE 2001/2002 ACADEMIC YEAR

CS 3021 – INTRODUCTION TO DATABASE AND FILE SYSTEMS – 17 APRIL 2002 PM

#### QUESTION 1: NORMALISATION THEORY

In some university, a student/courses/grades database is maintained. Among other information, the database contains information about the grades for "continuous assessment" and for "final examinations" obtained by each student for each individual course. The (very) junior database manager has decided to record that information in a relational table names **Grades** and having the following attributes:

StudNO	StudName	IDCardNO	YearCode	CourseNO	CourseTitle	Lecturer	GradeAsses	GradeExam
--------	----------	----------	----------	----------	-------------	----------	------------	-----------

The following suppositions hold:

- **StudNO** is the unique computer number identifying a particular student;
- **StudName** is the student's full name, and not necessarily unique;
- **IDCardNO** is the student's unique National Registration Card number;
- **YearCode** indicates the student's department and study year (e.g. CST-03 or MAT-04);
- **CourseNO** is the unique course code identifying a particular course;
- **CourseTitle** is the official descriptive title for the course;
- **Lecturer** is the full name of the lecturer for the course;
- **GradeAsses** is the grade for "continuous assessment" obtained by the student for the course;
- **GradeExam** is the grade for "final examination" obtained by the student for the course.

1.1 Draw the diagram for the functional dependencies between the attributes in this relational table.

1.2 This table is not in BCNF. Explain why, and indicate which is the normal form for this table.

1.3 Reduce the table into a set of new relational tables, all of which should be in BCNF.

(continues overleaf)

## QUESTION 2: EER MODEL AND MAPPING TO RELATIONAL TABLES

### A very simplified vehicle registration system.

A registered vehicle is identified by its registration number. For each registered vehicle the following information is to be maintained: make and type, construction year, chassis number, engine number, colour, date of registration, weight, type (either "car" or "truck" or "bus"), and owner. For trucks the following additional data are registered: number of wheels and maximum load. For busses the maximum number of passengers is registered.

The owner of a vehicle is either an individual or a company. An individual is identified by its National Registration Card Number (or Passport Number), and the following additional information is recorded: up to two first names, surname, date of birth, physical address (composed of street and number, borough, city, province), P.O.Box number and P.O.Box town. A company is identified by its VAT Number and the following information is registered: company name, company address (composed as above for individuals), P.O.Box number and P.O.Box town, phone number.

**2.1** Draw the extended entity relationship diagram for this very simplified registration system.

**2.2** Map the EER-diagram into a set of relational database tables. Indicate primary keys and foreign keys.

---

**Time for this examination: 3 hours.**

**Written exam, no books, no lecture notes, no solved exercises.**

---

**THE UNIVERSITY OF ZAMBIA**

**UNIVERSITY FIRST SEMESTER EXAMINATIONS - APRIL 2002**

**GEO 111**

**INTRODUCTION TO HUMAN GEOGRAPHY - I**

**TIME: THREE HOURS**

**ANSWER: QUESTION 1 AND ANY OTHER THREE QUESTIONS. QUESTION 1 CARRIES 40% AND THE OTHER THREE CARRY 20% EACH.**

**NOTE: Credit will be given for use of relevant illustrations. Use of a calculator and an approved atlas is allowed.**

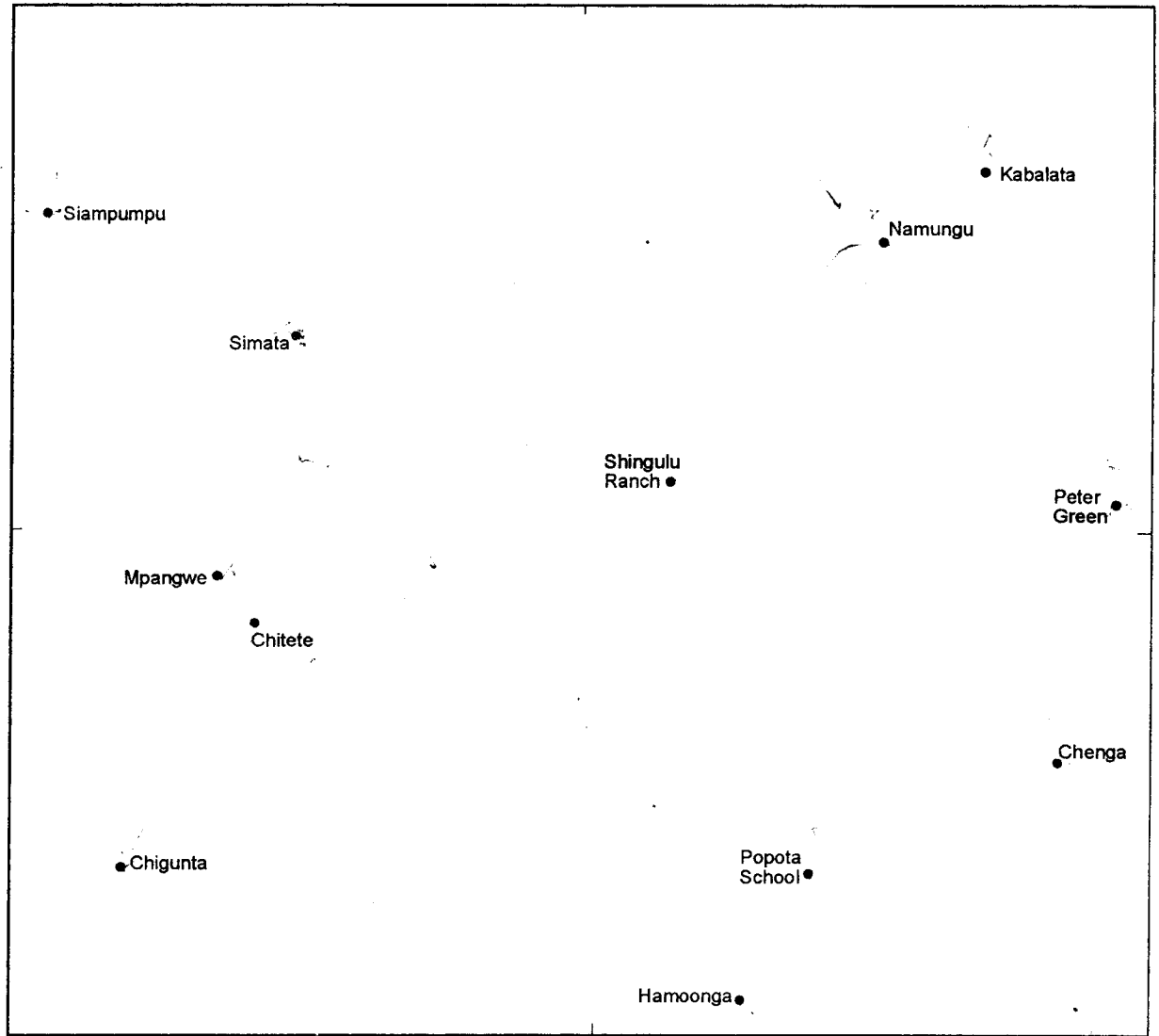
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- Q.1 With reference to Fig. 1.**
- (a) Calculate the nearest neighbour measure (RN) for the area.
  - (b) Explain the distribution pattern of settlements.
- Q.2 Outline the three major approaches in Geography and explain the trends in the development of Human Geography.**
- Q.3 'Zambia is among the highly urbanized countries in the developing world'. Discuss.**
- Q.4 Give a critical account of Weber's Location Theory.**
- Q.5 Discuss the applicability of the assumptions of Von Thunen's Model with reference to the Zambian situation.**
- Q.6 Write short explanatory notes on each of the following:**
- (a) Mean Information Field
  - (b) Jacob's Model of Cultural Development
  - (c) Site of Rural Settlements
  - (d) Rank - Size Rule
  - (e) Sector Model in Urban Landuse.

---

**END OF EXAMINATION**

Fig. 1: Distribution of Settlements.



0 2 4 km

**KEY**

- Village location

**THE UNIVERSITY OF ZAMBIA**  
**FIRST SEMESTER DEFERRED/SUPPLEMENTARY EXAMINATIONS**

JUNE 2002

GEO 175

**INTRODUCTION TO MAPPING TECHNIQUES IN GEOGRAPHY**

**PAPER I I**

**MAPS AND AERIAL PHOTOGRAPHS**

**TIME: THREE HOURS**

**ANSWER: FOUR QUESTIONS**

**NOTE: Use of calculators and approved atlas is allowed. Candidates are encouraged to use relevant illustrations where necessary**

---

Q1. ✓ The class boundaries of the first five classes of a frequency distribution are given as 28.6 - 33.1, 33.1-37.6, 37.6 - 42.1, 42.1 - 46.6 and 46.6 - 46.6 - 46.4.

- (a) Calculate the class marks of all the given five classes ✓
- (b) Calculate the class limits if, the observations are given as correct to the nearest tenth of a unit? (1)
- (c) What would the class boundaries and limits for the sixth class be?

Q2. ✓ Write short explanatory notes on all of the following

- (a) Classification of data
- (b) - Dot maps (2)
- (c) Divergent line graphs
- (d) - Choropleth maps
- (e) Scatter diagrams

Q3. This year's copperbelt agricultural show attracted a number of local exhibitors from different parts of the country. Table 1 below shows the origins of the exhibitors who traveled to Ndola for the event. Using the outline map of Zambia

provided (fig 1), display the data using a method that shows comparative volumes of exhibitors who travelled to Ndola.

Table 1: Number of Local exhibitors who travelled to the Copperbelt show

<u>Origin</u>	<u>No of exhibitors</u>
Livingstone	29
Lusaka	62
Mansa	48
Mbala	20
Mwinilunga	16
Mongu	6
Solwezi	11
Chipata	8

Source: Imaginary data

Q4. ✓ With the aid of diagrams, briefly describe any two methods used to present geographical data showing the whole divided into component parts. ✓

Q5. (a) On the outline map of Zambia provided (fig 2) show the relative populations of the towns listed in Table 2 using the most appropriate method.

Table 2: Population of Zambian Towns, 1990.

<u>Town</u>	<u>Population 1990</u>
Kabwe	166,629
Livingstone	96,116
Ndola	388,376
Solwezi	130,142
Mongu	153,221
Mansa	153,621
Chipata	314,204
Mbala	201,046
Lusaka	1,137,361

(b) Briefly comment on the merits and limitations of the method you have used.

Q6. ✓ You have been asked to make a presentation on the weather conditions of Bikavu in Congo DR present the data on monthly rainfall and temperatures for Bikavu on one diagram using the most appropriate method.

Table 3: ✓ Monthly Rainfall and temperature in Bikavu.

	J	F	M	A	M	J	J	A	S	O	N	D
Rainfall	38	70	96	113	103	28	233	288	141	51	20	42
Temp °C	9.9	10.4	13.8	17.7	21.6	23.4	24.9	24.9	23.7	22	17.5	12.1

Source: Imaginary data

**END OF EXAMINATION**

Fig. 1: GEO 175.

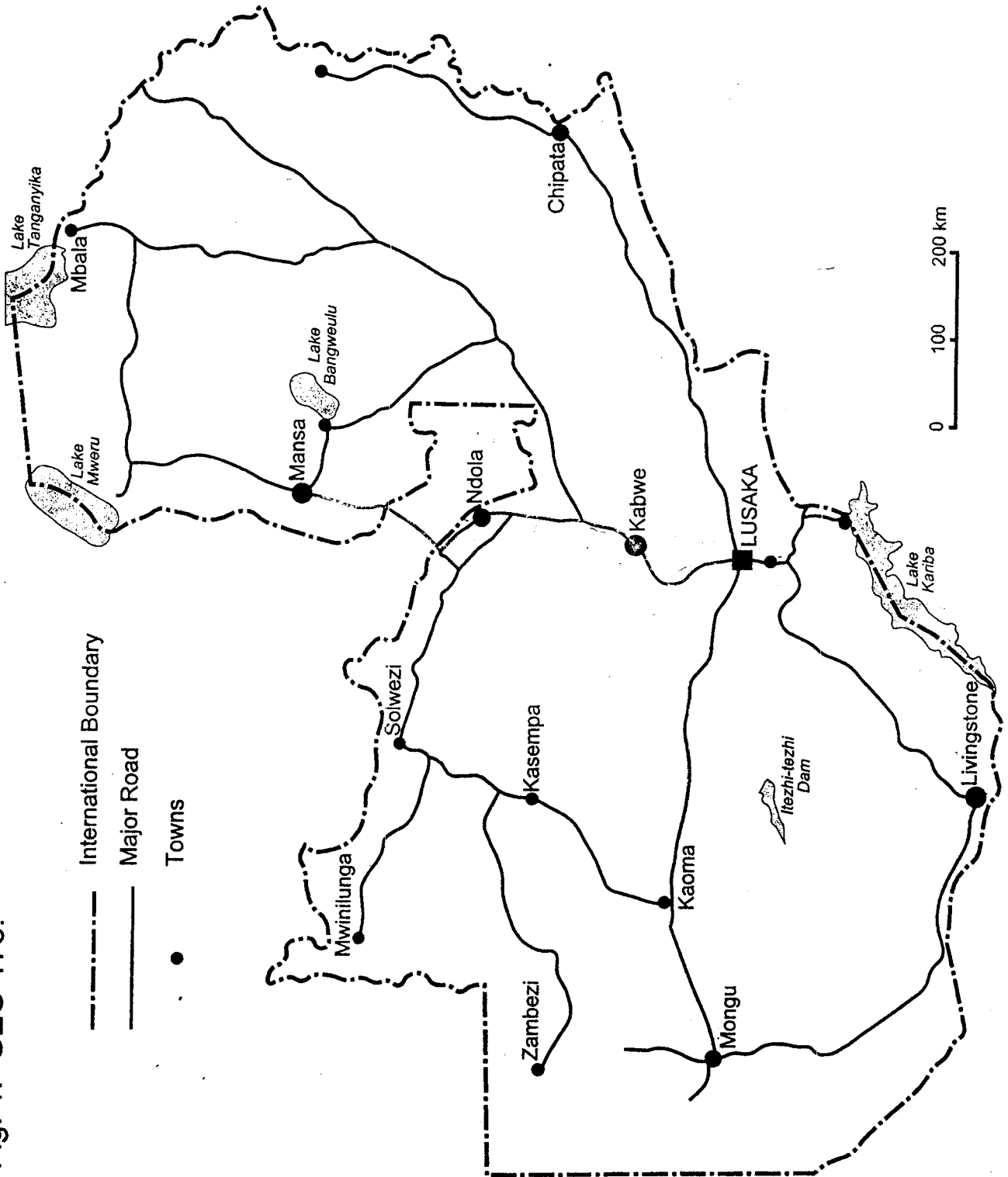
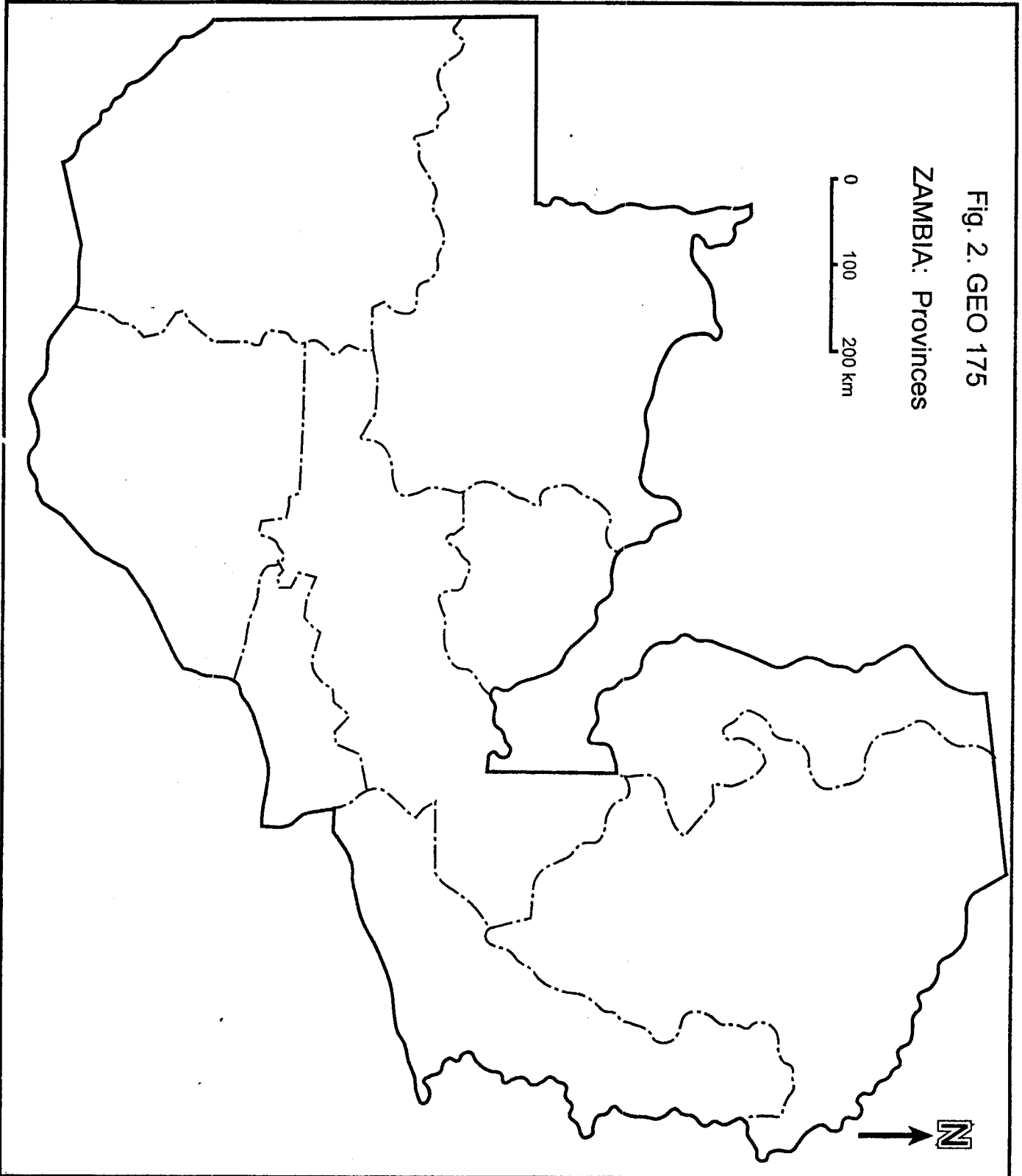
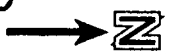


Fig. 2. GEO 175

ZAMBIA: Provinces



**THE UNIVERSITY OF ZAMBIA**

**UNIVERSITY FIRST SEMESTER EXAMINATIONS - APRIL 2002**

**GEO 211**

**THE GEOGRAPHY OF AFRICA**

TIME: THREE HOURS

ANSWER: ANY FOUR QUESTIONS

NOTE: Candidates are advised to make use of illustrations wherever appropriate.  
Use of an approved atlas is allowed.

---

- Q1. In what ways do rainfall and soils influence economic development in Africa?
- Q2. Show how racial, ethnic and religious diversity can contribute to socio-economic development in Africa.
- Q3. What is the explanation for and possible solutions to Africa's socio-economic crisis?
- Q4. Compare and contrast the strategies for socio-economic development that were implemented in Kenya and Tanzania after independence.
- Q5. Explain the causes of regional inequality in the space-economies of Ghana and Cote d'Ivoire (Ivory Cost), and suggest ways of overcoming it.
- Q6. In what ways did the Negroes of West Africa contribute to the Neolithic Revolution?
- 

**END OF EXAMINATION**

**THE UNIVERSITY OF ZAMBIA**

**UNIVERSITY FIRST SEMESTER EXAMINATIONS - APRIL 2002**

**GEO 271**

**QUANTITATIVE TECHNIQUES IN GEOGRAPHY I**

**TIME:** Three hours  
**ANSWER:** Any four questions  
**NOTE:** All questions carry equal marks

---

- Q1. Dr. Mugwagwa was requested by a water and sanitation company to assess whether or not the following statement of the research problem reached the acceptable standard. At the end of the day, he concluded that the research problem was flawed. The statement of the problem read as follows. "The existence of problems of water supply and sanitation in rural areas is compounded by several factors. These problems manifest themselves in terms of low daily per capital water consumption which even falls further in the dry season, prevalence of water related diseases, poor quality water and long distances to the water services from households".
- (a) Explain the flows which you think Dr. Mugwagwa identified in this statement.
- (b) Rewrite this statement so that the research problem becomes researchable.
- (c) Formulate three hypotheses with reference to the corrected statement (b)
- Q2. Distinguish an interview schedule from an unstructured questionnaire.
- Q3. Suppose you have been asked to come up with a sample of 300 female farmers in Zambia and you are given guidelines that the sample should be highly random with a reduced cost of sample preparation:  
Explain categorically how you would arrive at such a sample and justify the selected approach.
4. Different parts of a research proposal should be linked to one another. Discuss.
5. Assume Ms Chonoona is to under take a study to evaluate the impact of improved Fallow Technology on soil fertility in Kagoro area of Katete District. Which evaluation methodology would you recommend to her? Justify your decision.
6. Mikiwe intends to combine both the structured questionnaire and the 'Focused

Group Discussion' as instruments for collecting data in her research. Advise her on what she should expect when coding her data after using these methods.

---

**END OF EXAMINATION**

**THE UNIVERSITY OF ZAMBIA**

**UNIVERSITY FIRST SEMESTER EXAMINATIONS - APRIL 2002**

**GEO 271**

**QUANTITATIVE TECHNIQUES IN GEOGRAPHY 1**

**TIME:** Three hours  
**ANSWER:** Any four questions  
**NOTE:** All questions carry equal marks

---

- Q1. Dr. Mugwagwa was requested by a water and sanitation company to assess whether or not the following statement of the research problem reached the acceptable standard. At the end of the day, he concluded that the research problem was flawed. The statement of the problem read as follows. "The existence of problems of water supply and sanitation in rural areas is compounded by several factors. These problems manifest themselves in terms of low daily per capital water consumption which even falls further in the dry season, prevalence of water related diseases, poor quality water and long distances to the water services from households".
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Explain categorically how you would arrive at such a sample and justify the selected approach.
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**END OF EXAMINATION**

**THE UNIVERSITY OF ZAMBIA**

**UNIVERSITY FIRST SEMESTER EXAMINATIONS - APRIL 2002**

**GEO 381**

**ENVIRONMENT AND DEVELOPMENT I**

TIME: THREE HOURS

ANSWER: ANY FOUR QUESTIONS

NOTE: Illustration should be used whenever appropriate.

---

- Q1. Write short explanatory notes on all of the following:
- (a) Environment concept
  - (b) Property rights
  - (c) Externalities
  - (d) Conservation of biodiversity
  - (e) Global warming
- Q2. In what ways and to what extent is development linked to environmental systems?
- Q3. "Although the goal of sustainable development is now universally accepted, people know little about what it entails or how to achieve it" (Guilt, et al, 2001: p1). Discuss.
- Q4. Environmental degradation is a leading environmental issue and has social perspectives in the world. It affects both rural and urban areas.
- (a) What are the major causes of environmental degradation?
  - (b) What are the physical indices of environmental deterioration in both rural and urban areas?
  - (c) What are social impacts of the environmental crisis as a result of environmental degradations?
  - (d) What are the remedial measures to address the social and environmental effects of land degradation.
- Q5. Describe and explain how the limits to the productive capacity of land resources are set by climate, soil and landforms conditions and by the use and the management applied to land.

- Q6. **"An 'ecological ethic' is thus an ethic that claims that the welfare of at least some non-humans are intrinsically valuable and that because of this intrinsic value, we humans have a duty to respect and preserve them "(Velasquez, 1992, p233)**
- (a) Describe and explain principles of environmental ethics.
  - (b) What are the moral duties of human being to protect the welfare of human being and other non-human parts of the environment?
  - (c) What are the implications of environmental ethics on the industrial technology and business activities?

---

**END OF EXAMINATION**

**THE UNIVERSITY OF ZAMBIA**

**UNIVERSITY FIRST SEMESTER EXAMINATIONS - APRIL 2002**

**GEO 481**

**ENVIRONMENT AND DEVELOPMENT II**

TIME: THREE HOURS

ANSWER: ANY FOUR QUESTIONS

NOTE: Illustrations should be used whenever appropriate.

- 
- Q1. You have been hired as a team leader for a consultant team to develop an environmental management programme for Barotse flood plain ecosystem in Western province of Zambia. Explain how you would go about to develop an environmental management programme for conservation and utilization of the Barotse floodplain, using the IUCN sustainability assessment method.
- Q2. "Forest resources are not simply natural resources to be exploited by every sector without due regard to the long - term impacts of unsustainable use and mismanagement" (FAO, 2001, p26) Discuss.
- Q3. / In what ways and to what extent are the patterns and levels of urbanisation in developing countries a significant determinant of urban environmental problems and impacts.
- Q4. / Although the world is endowed with abundant water resources, it is not readily available to the majority of people in the world. Discuss.
- Q5. Energy is an essential component for economic development of a country like Zambia.
- (a) Describe and explain the major sources of energy in Zambia
  - (b) What are the environmental problems associated with energy use
  - (c) How is the country trying to address the environmental problems associated with energy consumption.
- Q6. / Many agricultural problems and constraints are usually a complex mix of natural phenomena and human activities. Discuss

---

**END OF EXAMINATION**

**THE UNIVERSITY OF ZAMBIA**

**UNIVERSITY FIRST SEMESTER EXAMINATIONS - APRIL 2002**

**GEO 911**

**POPULATION GEOGRAPHY**

**TIME:** Three hours

**ANSWER:** Any four questions

**NOTE:** All questions carry equal marks

Use of an approved Atlas and calculator is allowed

---

- Q1. 'Catholics are both pro-natalists and anti - natalists'. Discuss.
- Q2. 'The capitalist theory regarding the relationship between the rate of population growth and socio-economic development is more realistic than the Marxist one'. Discuss.
- Q3. Examine the statement that 'Industrialisation is the only recipe for Demographic Transition in any country.'
- Q4. Compare and contrast non - economic values and economic values of children in any Zambian traditional society of your choice.
- Q5. Examine Trewartha's (1953:97) argument that "Population is a pivotal element in Geography, and the one around which all others are oriented and the one from which they all derive their meaning".
- Q6. Use any two theories of migration to explain why there has been rural to urban, urban to rural and rural to rural migration in Zambia since independence.

---

**END OF EXAMINATION**

**THE UNIVERSITY OF ZAMBIA**

**UNIVERSITY FIRST SEMESTER EXAMINATIONS - APRIL 2002**

**GEO 921**

**ECONOMIC GEOGRAPHY**

TIME: THREE HOURS

ANSWER: ANY FOUR QUESTIONS

NOTE: All questions carry equal marks. Candidates are encouraged to use illustrations wherever appropriate.

---

- Q1. 'Transportation development is a pre-requisite to economic development'. Discuss this statement with reference to Zambia.
- Q2. In what ways and to what extent have the recent economic transformations contributed to the performance of the manufacturing sector in Zambia?
- Q3. 'Trade and location are the two sides of the same coin'. Discuss.
- Q4. Compare and contrast the Weberian Framework and the Variable Cost Model by Smith (1981).
- Q5. Outline and explain the variables that may be associated with industrial plant closures.
- Q6. 'Zambia is reported to have comparative advantage in the tourism sector in the sub - region and yet the competitiveness is still low.' Discuss.
- 

**END OF EXAMINATION**

THE UNIVERSITY OF ZAMBIA

UNIVERSITY FIRST SEMESTER DEFERRED/SUPPLEMENTARY  
EXAMINATIONS - JUNE 2002

GEO 951

CLIMATOLOGY

TIME: THREE HOURS.

ANSWER: QUESTION 1 AND THREE OTHERS.

---

1. Attached is a synoptic weather map covering Africa and Western Europe, valid for 12:00 Hours GMT on 26 March 2002 as supplied by the Zambia Meteorology Department. Proceed with the following tasks:
  - a) What is the highest wind speed that is depicted on the attached weather map?
  - b) Identify and label an active extratropical cyclone on the attached weather map.
  - c) On the extratropical cyclone you identified in (b), identify and label the following:
    - i. the warm front,
    - ii. the cold front,
    - iii. the warm sector,
    - iv. the cold sector, and
    - v. the occluded front.
  - d) Describe the weather you would expect as a result of the extratropical cyclone.
  - e) Why is an extratropical cyclone unlikely to hit Zambia?
  - f) Why is a tropical cyclone unlikely to hit Zambia?
2. Give an account of the factors affecting the intensity of solar radiation received by the earth.
3. Explain the mechanisms by which cooling of the air in the atmosphere, and consequently condensation, can be accomplished.

4. Write short explanatory notes on all of the following:
- a) Vertical structure of the atmosphere.
  - b) Theories on rainfall formation.
  - c) Measurement of evaporation.
  - d) The definition of drought.
  - e) The anticyclone.
5. Explain the forces governing wind motion on the surface of the earth and, using these forces, illustrate the likely wind circulation patterns around pressure cells in both hemispheres of the earth.
6. Describe the likely consequences and implications of climate change.

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END OF EXAMINATION

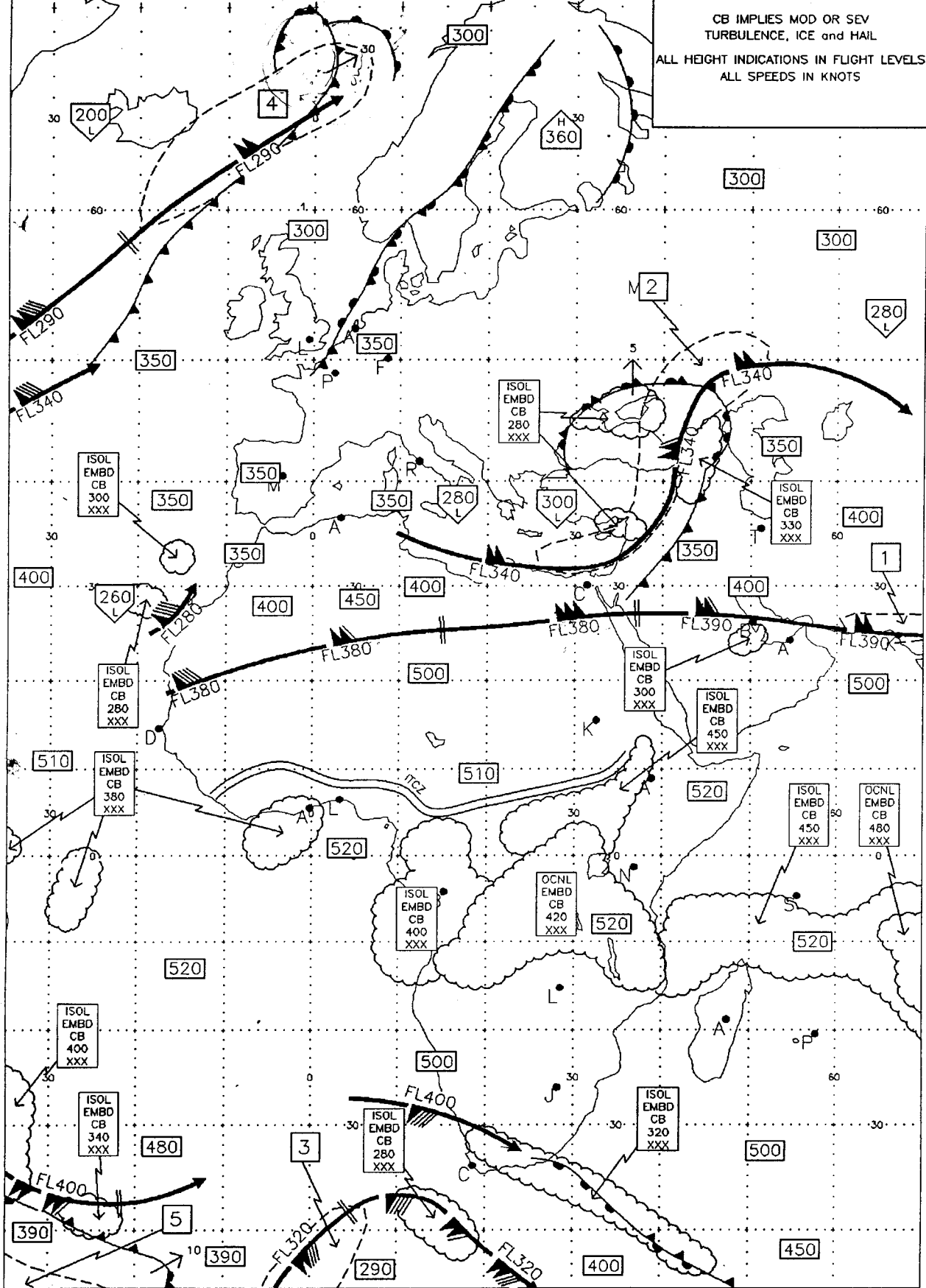
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CAT AREAS

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2	390	5	370
3	370		XXX
	280		XXX
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WORLD AREA FORECAST CENTRE  
LONDON  
FIXED TIME FORECAST CHART  
ICAO AREA C SIG WX  
FL 250-630  
VALID 12 U.T.C. ON 26 MAR 2002

CB IMPLIES MOD OR SEV  
TURBULENCE, ICE and HAIL  
ALL HEIGHT INDICATIONS IN FLIGHT LEVELS  
ALL SPEEDS IN KNOTS



# THE UNIVERSITY OF ZAMBIA

UNIVERSITY FIRST SEMESTER EXAMINATIONS - APRIL 2002

GEO 955

## GEOMORPHOLOGY

TIME: THREE HOURS

ANSWER: ANY FOUR QUESTIONS

NOTE: THE USE OF AN APPROVED ATLAS IS ALLOWED  
THE USE OF ILLUSTRATIONS IS ENCOURAGED WHEREVER  
APPROPRIATE

---

- Q1. Write short explanatory notes on all of the following:
- (a) the importance of weathering
  - (b) the structure of the earth.
  - (c) stream flow and its importance
  - (d) the types and characteristics of plate boundaries
  - (e) anthropogeomorphology and the associated major processes.
- Q2. 'Wearing down' or 'wearing back'? Outline and discuss the differing views on the development of slopes.
- Q3. Discuss the concept of the 'graded stream' and explain how a stream's progress towards a 'graded profile' may be interrupted.
- Q4. Explain the importance of rivers as agents of erosion and describe any three ways in which they may carry out this process.
- Q5. Describe the measurements that a fluvial geomorphologist would take of a stream channel in order to make a detailed study of it.
- Q6. How valid is the cycle concept in geomorphology today?

---

END OF EXAMINATION

**THE UNIVERSITY OF ZAMBIA**

**UNIVERSITY FIRST SEMESTER EXAMINATIONS - APRIL 2002**

**GEO 971**

**AERIAL PHOTOGRAPHY AND PHOTO INTERPRETATION**

**PAPER 1**

**TIME:** Three hours  
**ANSWER:** Any four questions  
**NOTE:** All questions carry equal marks

---

- Q1. Write short explanatory notes on all of the following:
- (a) Particle theory
  - (b) Spectral reflectance
  - (c) Photographic filters
  - (d) Silver halide crystals
  - (e) Distortions in aerial photographs
- Q2. Explain why photography before 1903 may have had limited use for topographic mapping.
- Q3. What are the major differences between aerial photography and satellite imaging?
- Q4. Outline and explain the major innovations that have contributed to the current status of aerial photography.
- Q5. (a) Draw the spectral reflectance curve of healthy green vegetation and explain the plant factors/properties that may be responsible for the pattern of the reflectance curve.
- (b) List three characteristics of panchromatic film necessary to produce high quality aerial photographs.
  - (c) Explain the circumstances that may necessitate the use of black and white infra-red film than panchromatic one.
  - (d) Contrast a fast film and a slow film.
- Q6. 'Maximum utilization of aerial photography for any purpose is not feasible at the moment in Zambia'. Discuss.
- 

**END OF EXAMINATION**

**THE UNIVERSITY OF ZAMBIA**  
**UNIVERSITY FIRST SEMESTER EXAMINATIONS**

**APRIL 2002**

**M 111 - MATHEMATICAL METHODS 1**

---

**INSTRUCTIONS:**

- (i) Answer *any five (5)* questions.
- (ii) Show *all essential* working clearly.
- (iii) Indicate the number of *each question* attempted in the *first column* on your *main answer book*.
- (iv) Use of calculators and tables *is not* allowed.

**TIME ALLOWED:** Three (3) hours.

---

1. a) (i) For any two non-empty sets  $A$  and  $B$ , show that

$$[(A \cup B) \cup (A \cup B^c)]^c = \phi$$

- (ii) In a single venn diagram, represent the following sets:

$R = \{\text{real numbers}\}$   
 $Q = \{\text{rational numbers}\}$   
 $C = \{\text{complex numbers}\}$   
 $I = \{\text{Irrational numbers}\}$   
 $Z = \{\text{Integers}\}$

- b) Given the functions

$$f(x) = x + 6 \text{ and } g(x) = |2x + 3|,$$

- (i) On the same axes, sketch the graphs of  $f(x)$  and  $g(x)$  indicating the points of intersection.

- (ii) Hence find the set of values of  $x$  for which

$$f(x) > g(x).$$

- c) Given the sets  $A = [-2, 1)$ ,  $B = (0, 4)$  and  $C = \{-1, 3\}$

Find: (i)  $A \cap B$  (ii)  $A' \cap C$  (iii)  $A' \cap B'$

2. a) (i) Find the real values  $a$  and  $b$  given that

$$\frac{a}{1+i} + \frac{b}{1+2i} = 1$$

- (ii) Find the conjugate of

$$z = \frac{1-i}{(3-i)^2}$$

- (iii) Find the integers  $p$  and  $q$  with  $q \neq 0$  such that

$$\frac{p}{q} = 1.1222.....$$

- \* b) Find the derivative of  $f(x) = \frac{1}{x}$  from first principles.

- c) For the functions  $f(x) = x + 1$  and  $g(x) = \frac{3}{x}$

- (i) Show that  $g(x)$  is its own inverse.  
(ii) Show that  $f(x)$  is one-to-one.  
(iii) Find  $(f \circ g)^{-1}(x)$  and its domain.  
(iv) Show whether  $f(x)$  is even, odd or neither.

3. a) (i) Rationalize the denominator:

$$\frac{3 - \sqrt{7}}{-\sqrt{7} + 1}$$

- (ii) Find the range of values of  $x$  for which

$$(1-x)(x-3)(x+4) < 0$$

- \* (b) Solve the following equations for values of  $x$  between  $0^\circ$  and  $360^\circ$  inclusive:

- i)  $\cos 2x + \sin x = 0$   
ii)  $\sec x + \tan x = \cos x$

- c) Find the range of values of  $x$  for which:

- i)  $|x-1| < 0$       (ii)  $\frac{x+4}{x+1} \leq \frac{x-2}{x-4}$

4. (a) Given  $y = \frac{3x}{(x+2)(x-1)}$

(i) Express  $y$  in partial fractions

(ii) Find  $\frac{dy}{dx}$

(iii) Find  $\frac{dy}{dx}$  when  $x = 2$ .

b) For  $g(x) = 2x^3 - x^2 - 23x - 20$ .

(i) Show that  $x + 1$  is a factor of  $g(x)$ .

(ii) Solve the equation  $g(x) = 0$ .

c) Given the function  $f(x) = \sqrt{x}$

(i) Find the domain and range of  $f(x)$ .

(ii) Sketch the graph of  $f(x)$ .

(iii) Find  $\lim_{x \rightarrow 0} f(x)$ .

(iv) Determine if  $f(x)$  is continuous at  $x = 0$ .

5. (a) Find the following limits:

(i)  $\lim_{x \rightarrow 0} \frac{(1+x)^2 - x^2}{x}$

(ii)  $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$

b) Find  $\frac{dy}{dx}$  given that:

(i)  $y = (1 + x^3)^{10}$

(ii)  $y = x^2 \sin 3x$

c) Given  $f(x) = 3\sin(x - 90^\circ)$

(i) Find the period of  $f(x)$ .

(ii) Sketch the graph of  $f(x)$  for  $-270^\circ \leq x \leq 270^\circ$

(iii) Find from your graph the solution of  $f(x) = 0$

6. a) If  $\alpha$  and  $\beta$  are roots of the equation

$$2x^2 + 7x + 1 = 0,$$

find the value of:

(i)  $\frac{1}{\alpha} + \frac{1}{\beta}$                       (ii)  $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

- b) For the graph of

$$f(x) = -4x^2 - 12x - 5$$

- i) Find the x intercepts  
(ii) Find the turning point of  $f(x)$ .  
(iii) Sketch the graph of  $f(x)$ .  
(iv) Using the sketch in (iii), state the range of values of x for which  $f(x) < 0$ .  
(v) On the same plot, sketch the graph of  $y = |f(x)|$ .
- c) Find the derivative of the following functions:

(i)  $f(x) = x \tan x$

(ii)  $g(x) = \sqrt{x} \left( 2 - \frac{1}{x} \right)$

(iii)  $h(t) = \frac{\cos t}{t}$

---

**END OF EXAMINATIONS**

# UNIVERSITY OF ZAMBIA

## UNIVERSITY FIRST SEMESTER EXAMINATIONS

APRIL 2002

### M161 – INTRODUCTION TO PROBABILITY AND STATISTICS I.

#### INSTRUCTIONS

1. Attempt any **five (5)** questions.
2. Show *all your work* to earn full credit.
3. All the questions carry equal marks.
4. You may use calculators to confirm your calculation.
5. This examination paper has 4 pages and each page is printed.

**TIME ALLOWED: Three (3) hours**

- [1] (a) A certain family is composed of a father, mother, and children. The routine family chores are as follows:

- |   |                  |    |                           |
|---|------------------|----|---------------------------|
| 1 | Cooking          | 7  | Taking children to school |
| 2 | Eating           | 8  | Making the bed            |
| 3 | Laundry          | 9  | Watering the lawn         |
| 4 | House repairs    | 10 | Washing the car           |
| 5 | House cleaning   | 11 | Reading children's books  |
| 6 | Buying groceries | 12 | Watching TV               |

- Chores done by mother are given by the set  $M = \{1, 2, 3, 5, 6, 7, 8, 9, 11, 12\}$
  - Chores done by father are given by the set  $F = \{2, 4, 5, 6, 7, 9, 10, 11, 12\}$
  - Chores done by children are given by the set  $C = \{2, 5, 8, 9, 10, 11, 12\}$
- (i) Display the set of common chores in the family.
  - (ii) What percentage of chores are done by mother?
  - (iii) Ignoring *eating* and *watching* TV, what percentage of time does mother get help from father?

- (b) A small restaurant has been hired to serve meals to participants in a workshop running for a certain number of days. The meals available are as follows:

- |                   |           |   |
|-------------------|-----------|---|
| Vegetarian meal   | – call it | V |
| Meal with chicken | – call it | C |
| Meal with beef    | – call it | B |
| Meal with fish    | – call it | F |
| Meal with tubers  | – call it | T |
| Cereal meal       | – call it | R |

The manager is concerned with the combination of meals to be served in any given day. Let  $(X, Y)$  represent meal  $X$  and meal  $Y$  are served in a particular day at lunch and supper in any order.

- (i) Display a set consisting of a pair of meals  $(X, Y)$  to be served for five days, in which a vegetable meal is always served but each of the other meal is served once in the 5 days.
- (ii) Display 2 sets of possible meal combinations to be served for six days in which each meal is served *twice* on two separate days.
- (iii) Display 3 sets of possible meal combinations to be served for 3 days in which each meal is served once in the 3 days.
- (iv) How many pairs  $(X, Y)$  are possible?

[2] (a) The selling price  $S$  of a commodity is related to the cost price  $C$  via a linear relationship:  $S = (m + 1)C$ , where  $m$  is a percentage markup.

- (i) If a commodity costs K10,000 and sells at K15,000, find the percentage markup.
- (ii) A commodity sells at K30,000 after a 30% markup, find the cost.
- (iii) At what price should a commodity be sold if it costs K5,000 and we wish to achieve a 20% markup?

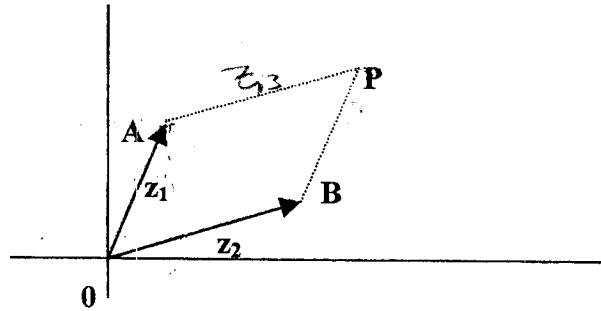
(b) Suppose that demand for shirts is related to price linearly as follows:

- When the price is \$3 per shirt, 200 shirts are sold
- When the price is \$5 per shirt, 100 shirts are sold

Let  $D$  stand for demand and  $P$  stand for price.

- (i) Write down a relationship between  $D$  and  $P$ , making use of the data above.
- (ii) By how many shirts does the demand fall when price increases by \$1?
- (iii) Beyond what price is there no demand for the shirts?
- (iv) From the relationship, determine the maximum number of shirts that can be sold.
- (v) What is the price that gives the maximum number of shirts?
- (vi) How realistic is the price in (v)? Briefly explain.
- (vii) How ideal is this relationship between demand and price? Briefly explain.

[3] (a)



In the diagram, Peter and John are walking toward P from point 0. Peter uses the route through A along  $z_1$  and John uses the route through B along  $z_2$  and both arrive at point P. Their paths form a parallelogram.

You are given that  $z_1 = 4 + 6i$  and  $z_2 = 8 + yi$

- (i) Find the value of  $y$  if P is 10 units from A
- (ii) If P is 10 units from A how far is it from 0?
- (iii) How far is A from B?

(b) Given that  $z_1 = 1 - i\sqrt{3}$  and  $z_2 = -1 + i\sqrt{3}$ , find:

(i)  $z_1 z_2 - (z_1)^2$ .

(ii)  $(z_2)^2 + z_1 \overline{z_2}$

(iii)  $\frac{z_1}{z_2} - \frac{2}{z_1 - z_2}$

(c) Given the equation  $z^2 + 2z + c = 0$

- (i) For what values of  $c$  does the equation give *real* roots?
- (ii) Solve the equation given that  $c = -24$ , express your answer in the form  $a + bi$

[4] (a) A business man has 20 rooms for rental. He charges \$10 per room per month and incurs costs on vacant rooms. He says that from his knowledge of business mathematics, if  $X$  is the number of vacant rooms and  $Y$  is the gross profit, then  $Y$  and  $X$  are related via the quadratic function below:

$$Y(x) = 160 + 12x - x^2$$

- (i) How much does he earn in rentals if all the rooms are occupied?
- (ii) How many rooms should be vacant for him to get maximum gross profit?
- (iii) *Sketch* the graph of  $Y$  versus  $X$  for  $X > 0$

(b) Given  $P(x) = x^3 - x^2 - 11x + 12$ .

- (i) Determine linear factors of  $P(x)$
- (ii) Solve  $x^3 - x^2 - 11x + 12 < 0$

- [5] (a) Resolve the following fractions into partial fractions

(i)  $\frac{5x+2}{3x^2+4x-4}$

(ii)  $\frac{2x+7}{x^3-1}$

- (b) Resolve  $\frac{1}{n(n+1)}$  into partial fractions and hence show that

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

- [6] An investor sells three products I, II and III to the three classes of customers: consumers, wholesalers, and retailers. The number of units sold (per box) and amounts of sales in dollars for the last year appear as follows:

SOLD TO	PRODUCT			TOTAL SALES EARNED(\$)
	I	II	III	
Consumers	3	4	4	6600
Retailers	6	2	6	6200
Wholesalers	7	1	5	4900
PRICE/Box	X	Y	Z	

Product I is sold at X dollars per box, Product II is sold at Y dollars per box and Product III is sold at Z dollars per box.

- (a) Using the information in the Table, form and equation in X, Y and Z for:
- total sales from consumers.
  - total sales from retailers
  - total sales from wholesalers
- (b) Solve the equations in (a) simultaneously
- (c) Determine which product contributed the
- most to the total sales
  - least to the total sales.

**END OF EXAMINATION**

UNIVERSITY OF ZAMBIA

UNIVERSITY FIRST SEMESTER EXAMINATIONS, 2001/02

M211 MATHEMATICAL METHODS III

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**INSTRUCTIONS:**

1. Write your **computer number** on all the answer books used.
2. Attempt any **five (5)** questions only.
3. Indicate the **number** of each question attempted on the main answer book.

**TIME ALLOWED:** Three (3) hours.

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1. Express the given equation of a conic section in standard form and identify the curve:

$$4x^2 - 12xy + 9y^2 - 52x + 26y + 81 = 0$$

Find its

- (i) vertex (vertices)
- (ii) focus (foci), and
- (iii) directrix (directrices).

2. (a) Discuss the graph of the curve

$$r = \frac{3}{3 + 5\cos\theta}.$$

Hence sketch the curve.

- (b) The orbit of Halley's comet is an ellipse with the sun at one focus. In terms of astronomical units (AU), the major and minor semi axes of this elliptical orbit are 18.09 AU 4.56 AU, respectively.

- (i) What are the maximum and minimum distances from the sun to Halley's comet?
- (ii) State the eccentricity of the orbit of Halley's comet.

6. Evaluate the integrals:

(a)  $\int x \tan^{-1} x dx.$

(b)  $\int \frac{dx}{\sqrt{4+x^2}}$

(c)  $\int_0^{\pi/2} \frac{1}{3+5\cos x} dx.$

7. (a) Given that  $I_n = \int_0^{\pi/2} x^n \cos x dx$ , where  $n \geq 2$ , show that

$$I_n = \left(\frac{\pi}{2}\right)^2 - n(n-1)I_{n-2}.$$

Hence find  $I_4$ .

(b) Find the distance travelled by the particle  $P(x,y)$  between  $t = 0$  and  $t = 4$  if the position at time  $t$  is given by

$$x = \frac{t^2}{2}, \quad y = \frac{1}{3}(2t+1)^{3/2}.$$

(c) The circle

$$x^2 + y^2 = a^2$$

is rotated about the  $x$  - axis to generate a sphere. Find its volume.

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**END OF EXAMINATION**

# UNIVERSITY OF ZAMBIA

## UNIVERSITY FIRST SEMESTER EXAMINATIONS APRIL 2002

### MATHEMATICS M221 - LINEAR ALGEBRA I / M225 - INTRODUCTION TO MATHEMATICS LOGIC

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**INSTRUCTIONS:** Attempt ANY Five (5) Questions  
**TIME ALLOWED:** Three (3) Hours

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1. (a) What is meant by the following terms:

(i) the matrices  $A$  and  $B$  are row equivalent? [2 marks]

(ii) a consistent system of linear equations? [2 marks]

(b) (i) Let  $A = \begin{pmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{pmatrix}$  and  $B = \begin{pmatrix} 1 & 2 & -1 & 4 \\ 0 & 0 & 5 & -3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

be matrices in the vector space  $M_{3 \times 4}(\mathbf{R})$ . Show that the matrix  $B$  is row equivalent to the matrix  $A$ . [3 marks]

(ii) Express the matrix  $C$  as a product of elementary matrices, where

$$C = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}. \quad [5 \text{ marks}]$$

(c) Given the system of equations

$$\begin{aligned} x + 2y - z &= 4 \\ 2x + 4y + 3z &= 5 \\ -x - 2y + 6z &= -7, \end{aligned}$$

determine whether or not, it is consistent. [4 marks]

If it is consistent, find a general solution for the system. [4 marks]

---

2. (a) Define the following terms:
- (i) The rank  $r$  of a non-zero matrix  $A$ . [2 marks]
  - (ii) The normal form of a matrix  $A$ . [2 marks]
- (b) Let  $A$  be the  $3 \times 3$  matrix given by

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{pmatrix}. \text{ Then}$$

- (i) determine the rank of  $A$ . [3 marks]
- (ii) reduce the matrix  $A$  to its normal form. [5 marks]

(c) Reduce the matrix  $B = \begin{pmatrix} 1 & 2 & 3 & -2 \\ 2 & -2 & 1 & 3 \\ 3 & 0 & 4 & 1 \end{pmatrix}$

to normal form  $N$  and compute the invertible matrices  $P$  and  $Q$  such that  $PBQ = N$ . [8 marks]

---

3. (a) What is meant by the following:
- (i) an invertible  $n \times n$  matrix  $A$ ? [2 marks]
  - (ii) an orthogonal matrix  $B$ ? [2 marks]
- (b) (i) Determine the values of  $x$  for which the matrix

$$A = \begin{pmatrix} x & x \\ 4 & 2x \end{pmatrix}$$

is not invertible. [4 marks]

3 (b)/cont.....

(ii) Show that the matrix

$$B = \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

is an orthogonal matrix.

[4 marks]

© Let  $A$  be the matrix given by

$$\begin{pmatrix} y+z & x & x^3 \\ z+x & y & y^3 \\ x+y & z & z^3 \end{pmatrix}$$

Then, show that  $\det A = (x+y+z)^2(y-x)(z-x)(z-y)$ , where  $\det A$  denotes the determinant of the matrix  $A$ .

[8 marks]

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4. (a) Define each of the following terms:

(i) a subspace  $W$  of a  $K$ -space  $V$ . [2 marks]

(ii) a  $K$ -linear combination of a subset  $S$  of a  $K$ -space  $V$ . [2 marks]

(b) (i) Let  $W = \{(0, a_2, a_3, \dots, a_n) \mid a_i \in K \ (i = 2, \dots, n)\}$  be a subset of the vector space  $V_n(K)$  over the field  $K$ . Then, show that  $W$  as defined is a subspace of  $V_n(K)$ . [4 marks]

(ii) Show that  $U = \{(1+a, a, b) \mid a, b \in \mathbf{R}\}$ , a subset of the vector space  $\mathbf{R}^3$ , is not a subspace of  $\mathbf{R}^3$ . [4 marks]

(c) (i) Let  $W_1, W_2, \dots, W_k$  be subspaces of a vector space  $V$  over  $K$ . Prove that the intersection,  $W_1 \cap W_2 \cap \dots \cap W_k$ , is also a subspace of  $V$  over  $K$ . [4 marks]

4 ©/cont ....

- (ii) Let  $S = \{v_1, v_2, \dots, v_k\}$  be a finite subset of a vector space  $V$  over  $K$ . Define

$$U = \left\{ \sum_{i=1}^k a_i v_i \mid a_i \in K \right\}.$$

Then, prove that  $U$  is a subspace of  $V$ .

[4 marks]

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5. (a) Define the following:

- (i) A linearly dependent subset of a vector space  $V$  over  $K$ .

[2 marks]

- (ii) A  $K$ -basis for a vector space  $V$ .

[2 marks]

- (b) (i) Let  $C = \{(a, b) \mid a, b \in \mathbf{R}\}$  be the  $\mathbf{R}$ -space  $C$ , and let

$$u = 1 + i, v = i$$

be vectors in the vector space  $C$ . Then, show that  $u$  and  $v$  are linearly dependent over  $C$ , the field of complex numbers, but linearly independent over  $\mathbf{R}$ , the field of real numbers.

[5 marks]

- (ii) Let  $V$  be the vector space of polynomials of degree  $\leq 3$  over  $\mathbf{R}$ . Determine whether  $u, v, w \in V$  are independent or dependent, where;

$$u = t^3 - 3t^2 + 5t + 1, v = t^3 - t^2 + 8t + 2, w = 2t^3 - 4t^2 + 9t + 5.$$

[5 marks]

- (c) Let  $W$  be a subspace of  $V_4(\mathbf{R})$  generated by the vectors  $(1, -2, 5, -3)$ ,  $(2, 3, 1, -4)$  and  $(3, 8, -3, -5)$ . Then, find a basis and the dimension of  $W$ .

[6 marks]

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6. (a) What is meant by the following terms:

- (i) the coordinates of a vector  $v$  in a  $K$ -space  $V$ ?

[2 marks]

- (ii) the sum of subspaces of a  $K$ -space  $V$ ?

[2 marks]

6/cont....

- (b) (i) Let  $S = \{v_1, v_2, \dots, v_m\}$  be a subset of a vector space  $V$  over  $K$  and suppose that  $S$  contains a dependent subset, say  $\{v_l, \dots, v_r\}$ . Then, show that  $S$  is also dependent. **[4 marks]**

- (ii) Find the coordinate vector of  $v = (4, -3, 2)$  relative to the basis  $\{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$  of  $V_3(\mathbf{R})$ . **[5 marks]**

- (c) Let  $U$  be the subspace of  $\mathbf{R}^5$  spanned by

$$\{(1, 3, -2, 2, 3), (1, 4, -3, 4, 2), (2, 3, -1, -2, 9)\}$$

and let  $W$  be the subspace spanned by

$$\{(1, 3, 0, 2, 1), (1, 5, -6, 6, 3), (2, 5, 3, 2, 1)\}.$$

Then find a basis and the dimension of  $U + W$ . **[7 marks]**

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**END OF EXAMINATION**

**AND GOOD LUCK!**

# THE UNIVERSITY OF ZAMBIA

UNIVERSITY FIRST SEMESTER EXAMINATIONS - 2001/02

## M231 - REAL ANALYSIS I

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- INSTRUCTIONS:**
1. Write down your **computer number** on all the answer booklets used.
  2. Attempt **five(5)** questions only.
  3. Indicate the **number** of each question attempted on the main answer book.

**TIME ALLOWED:** Three(3) hours.

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1. (a) Let  $f: X \rightarrow Y$  be a function and let  $A \subset X$  and  $C \subset Y$ . Give a definition of
  - (i)  $f(A)$ , the direct image of  $A$  under  $f$
  - (ii)  $f^{-1}(C)$ , the inverse image of  $C$  under  $f$ .
- (b) Let a function  $f: \mathcal{R} \rightarrow \mathcal{R}$  be defined by  $f(x) = x^2 + 1$ . Given that  $A = \{x: -1 \leq x \leq 2\}$  and  $C = \{y = f(x): 0 \leq y \leq 2\}$ . Find
  - (i)  $f(A)$
  - (ii)  $f^{-1}(C)$
- (c) If  $f: X \rightarrow Y$  is a function and  $A$  and  $B$  are subsets of  $X$ , prove that
$$f(A \cup B) = f(A) \cup f(B).$$

- 2 (a) Let  $S$  be a subset of  $\mathcal{R}$ , the set of real numbers. Define
- (i) an upper bound for  $S$
  - (ii) a lower bound for  $S$
  - (iii) a least member for  $S$
  - (iv) a greatest member for  $S$
  - (v) a greatest lower bound for  $S$
  - (vi) a least upper bound for  $S$ .
- (b) Given that  $S = \left\{ \frac{n-5}{2n} : n \in \mathcal{N} \right\}$ . Find
- (i) the least upper bound for  $S$
  - (ii) the greatest lower bound for  $S$
  - (iii) the greatest member for  $S$
  - (iv) the least member for  $S$ .
- (c) Let  $S$  be a non-empty set of real numbers. Prove that  $\alpha \in \mathcal{R}$  is a supremum if and only if
- (i)  $x \leq \alpha \quad \forall x \in S$  and
  - (ii) for every  $\varepsilon > 0$ , there is some  $x' \in S$  such that  $\alpha - \varepsilon < x' < \alpha$
3. (a) State (i) the completeness axiom for sets of real numbers.  
(ii) the Archimedian property for real numbers.
- (b) State and prove the Dedekind's property for the sets of real numbers  $A$  and  $B$ .
- (c) Prove that the Dedekind's property is equivalent to the completeness axiom in  $\mathcal{R}$ .
4. (a) Define (i) a sequence  $(x_n)$  of real numbers which tends to a finite limit  $l$  as  $n$  approaches  $\infty$ .  
(ii) a sequence  $(x_n)$  of real numbers which tends to infinity as  $n$  approaches  $\infty$ .

- (b) Use your definition in (a) (i) to show that the sequence  $\left( x_n = \frac{3n-1}{n+2} \right)$  tends to 3 as  $n$  approaches  $\infty$ .

- (c) Determine whether the sequence defined by

$$x_n = \frac{n^3 - n^2 \cos n + 2}{4n^3 + n^2 - 4 \sin n}$$

has a limit. If the limit exists then find it.

5. (a) (i) Define a decreasing sequence  $(x_n)$  of real numbers.  
(ii) Define a bounded sequence  $(x_n)$  of real numbers.  
(iii) State Monotone convergence theorem, without proof.

(b) The sequence  $(x_n)$  is defined

$$x_1 = \frac{5}{2}, \quad x_{n+1} = \frac{x_n^2 + 6}{5}, \quad n \geq 1.$$

(i) Using mathematical induction prove that

$$2 < x_n < 3$$

for all  $n$ .

(ii) Show that  $x_n$  is a decreasing sequence.

(iii) Deduce that  $(x_n)$  is convergent and find its limit.

6. (a) (i) Define a subsequence of a sequence  $(x_n)$  of real numbers.

(ii) State the Bolzano-Weierstrass theorem for sequences of real numbers, without proof.

(b) Prove that if a sequence  $(x_n)$  of real numbers converges to  $l$  then any subsequence of  $(x_n)$  also converges to  $l$ .

(c) Assuming that a sequence  $\left[ x_n = \left( 1 + \frac{1}{n} \right)^n \right]$  tends to  $e$  as  $n \rightarrow \infty$ , find

the limit of the sequence  $\left( \left[ 1 + \frac{1}{3n} \right]^n \right)$ .

7. (a) Let  $(x_n)$  be a sequence of real numbers. Define

(i) the limit superior of  $(x_n)$

(ii) the limit inferior of  $(x_n)$

(ii) a Cauchy sequence  $(x_n)$ .

(b) Find the limit superior and the limit inferior of the sequence defined by

$$\left( 1 + \frac{1}{n} \right) \cos n\pi.$$

(c) Prove that every convergent sequence of real numbers is a Cauchy sequence.

**END OF EXAMINATION**

# UNIVERSITY OF ZAMBIA

## UNIVERSITY FIRST SEMESTER DEFERRED EXAMINATIONS - JUNE 2002

### MATHEMATICS M261 - INTRODUCTION TO STATISTICS

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- INSTRUCTIONS:**
- a) Answer any four Questions. All the necessary work and assumptions where necessary must be given.
  - b) Statistical tables and some formulae will be provided. ~~Use of calculators is allowed.~~
  - c) Use of calculators is allowed.

**TIME ALLOWED:** Three (3) Hours

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1. (a) In December 1994, Business week magazine published some forecasts of economists and econometric models. One of the items included in the forecasts was the percentage by which prices would increase. The predictions were as follows:
- |    |    |    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|----|----|----|
| 73 | 58 | 61 | 53 | 63 | 55 | 52 | 53 | 54 | 52 | 63 | 60 |
| 56 | 55 | 53 | 51 | 54 | 49 | 60 | 55 | 55 | 51 | 51 | 45 |
| 53 | 62 | 58 | 57 | 53 | 54 | 64 | 66 | 52 | 55 | 55 |    |
- (i) Display the stem-and- leaf.
  - (ii) Find the mode, median and inrequartile range from the stem- and- leaf display.
  - (iii) Comment on the skewness of the data.
- (b) Group the data in (a) above into six intervals of equal width with the lower interval being 45 - 49.
- (i) Display the histogram and the frequency polygon on the same graph.
  - (ii) Using the grouped data compute, mean, median, mode and the variance.
- (c) BP Zambia limited wants to take a sample to determine average number of customers served at a BP filling station in an hour.

- (i) Define the population of interest.
- (ii) State the main characteristic in the population of interest.
- (iii) What is the parameter of interest.
- (iv) Describe briefly how you would take a sample of size 60, so that it is a "good" representative of the population.

2. (a) (i) A personnel officer at Barclays Bank which has 55 branch offices in Zambia, would like to be 99% sure of having the true mean number of days absent per year per employee correctly estimated within 0.5 days. A preliminary sampling indicates that the standard deviation of the number of absences is 2.3 days. How many employees' attendance records are needed as a random sample in the personnel officer's study?

(ii) In order to accurately forecast demand for its services, Mr. Sausandi Quit-smoking health programme needs to estimate the proportion of smokers who sincerely want to break the habit. In particular, Mr. Sausandi wants to be 92.5% sure of having that proportion correctly estimated to within 0.05. How many smokers are needed for the survey.

(b) (i) A careful auditing is essential to all businesses, large and small. Suppose a firm wants to compare the performances of two auditors it employs. One measure of auditing performance is error rate, so the firm decides to sample 200 pages at random from the work of each auditor and carefully examine each page for errors. Suppose the number of pages on which at least one error is found is 17 for auditor A and 25 for auditor B. Test to see whether these data indicate a difference in the true error rates for the two auditors at level of significance.

(ii) The government is interested in determining whether salary discrimination exists between men and women in private sector. Suppose random samples of 15 women and 22 men are drawn from the population of first-level managers in the private sector. The information is summarized below (annual salaries in US dollars).

	Men	Women
Mean	19,700	18,400
Standard deviation	3,100	2,300

Do these data provide sufficient evidence to indicate that the mean salary of male first-level managers exceeds the mean salary of females in that position at 0.01 level of significance.

- (a) A physiologist wished to study the effect of birth-control pills on exercise capacity. Five female subjects who had never taken the pill had their maximal oxygen up take measured (in millimeters per kilogram of their body weight) during a treadmill session. The five subjects then took the pill for a specified length of time and their uptakes were measured again, as given in the table below. Do the data provide sufficient evidence to indicate that the pill causes a decrease in maximal oxygen uptake?

Subject	Maximal	Oxygen uptake
	Before	After
1	35.0	29.5
2	36.5	33.5
3	36.0	32.0
4	39.0	36.5
5	37.5	35.0

3. The EPA sets a limit of 5 parts per million on PCB (a dangerous substance) in water. A major manufacturing firm producing PCB for electrical insulation discharges small amounts from the plant (factory). The company management, attempting to control the PCB in its discharge, has given instructions to halt production if the mean amount of PCB in the affluent exceeds 3 parts per million. A random sampling of fifty water specimens produced a mean of 3.3 and standard deviation of 0.5 parts per million.
- (i) Do these statistics provide sufficient evidence to halt the production process, use  $\alpha = 0.01$ .
- (ii) If you were the plant manager, would you want to use a large or a small value of  $\alpha$  for the test in (i)? Justify your answer.
- (b) (i) A machine used to fill beer bottles must operate so that the amount of beer actually dispensed varies very little. If too much beer is released, the bottles will over flow causing waste. If too little beer released, the bottles will not contain enough beer, causing complaints from customers. A random sample of the fills of twenty bottles yielded a standard deviation of 0.2 ml. Estimate the true variance of the fills using a 95% confidence interval. Interpret the result.
- (iii) Ophthalmologists require an instrument that can rapidly measure interocular pressure for glaucoma patients. The

device now in general use is known to yield reading of this pressure with a variance of 10.3. The variance of five pressure readings by a newly developed instrument is equal to 9.8. Does this data provide sufficient evidence to indicate that the new instrument is more reliable than the instrument currently in use.

- (a) A political candidate's team conducted a survey to determine whether television advertising campaign was worth while. Both before and after the advertising campaign random samples were taken from the candidate's constituency and each person was asked his or her voter preference. The results of the survey are shown below. Estimate the difference in the proportion of voters who favor the candidate before and after the advertising campaign using a 97% confidence interval. Interpret the confidence interval.

	Sample size	Number who prefer The candidate
Before advertising Campaign	200	85
After advertising Campaign	300	139

4. A firm wants to know the correlation between the size of its sales force and its yearly sales revenue. The records for the past 10 years are examined, and the results listed below are obtained.

Year	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995
Number of sales people (x)	15	18	24	22	25	29	30	32	35	38
Yearly sales revenue (y) in billions of kwacha	1.35	1.63	2.33	2.41	2.63	2.93	3.41	3.26	3.63	4.15

$$\sum_{j=1}^{10} x_j = 268, \sum_{j=1}^{10} y_j = 27.73, \sum_{j=1}^{10} x_j y_j = 800.62, \sum_{j=1}^{10} x_j^2 = 7,668 \text{ and } \sum_{j=1}^{10} y_j^2 = 83.8733$$

- (i) Calculate the sample correlation coefficient for sales force and yearly sales revenue.
- (ii) Test at 0.01 level of significance  $H_0 : \rho = 0.6$  versus  $H_a : \rho \neq 0.6$

- (b) The pulse rate of the adult male population between 18 and 25 years of age in Zambia is known to have a normal distribution with a mean of 72 beats per minute and variance of 95. The requirements for Zambia air force state that anyone with a pulse rate over 90 beats per minute is medically unsuitable for service in ZAF. If there are 1000 Zambia male applicants between 18

and 25 years of age how many will be found medically unsuitable on the basis of pulse rate.

(a) (i) A machine used to regulate the amount of dye dispensed for mixing shades of paint can be set so that it discharges an average of  $\mu$  ml of dye per can of paint. The amount of dye discharged is known to have a normal distribution with variance equal to 0.160. If more than 6 ml. of dye are discharged when making a particular shade of blue paint, the shade is unacceptable. Determine the setting for  $\mu$  so that only 1% of the cans for the particular shade of blue paint will be unacceptable.

(iii) A recent study involving attrition rates at UNZA has shown the 43% of all incoming freshers do not graduate within the required duration of their degree programme. If 200 freshers are randomly sampled this year, and their progress is followed, what is the probability that less than half will graduate within the required duration?

5. An experiment was conducted to compare three methods of packaging a certain frozen food. The criterion was the ascorbic acid content (mg/100gm) after a specified period of time. Seven observations were made for each packaging method. Below is an incomplete ANOVA table for the said experiment.

Source	DF	SS	MS	F
Treatment		147.35		
Error				
Total		202.31		

- (i) What are the treatments in above experiment.
- (ii) Complete the above ANOVA table.
- (iii) Do the data provide sufficient evidence to indicate difference among treatments at 0.01 level of significance.

(b) An experiment was conducted to complete the yields of pineapple juice for six different juice extractors. Because of a possibility of a variation in the amount of juice per pineapple from one truck load of pineapples to another, equal weights of pineapples from a single truck load were assigned to each extractor and this process was repeated for fifteen loads. The amount of juice recorded for each extractor for each truck-load produced the following incomplete ANOVA table.

Source	Df	Ss	Ms	F
Extractor				
Truck-load		159.29		
Error		95.33		
Total		339.33		

- (i) What are the treatments in the experiment.
  - (ii) What are the blocks in the experiment.
  - (iii) Complete the ANOVA table.
  - (iv) Do the data provide sufficient evidence to indicate difference among treatment means at 0.05 level of significance.
- (c) For the experiment in (b) above analyse the data as a completely randomized design.

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*END OF EXAMINATION*

THE UNIVERSITY OF ZAMBIA

UNIVERSITY FIRST SEMESTER EXAMINATIONS - 2001/02

M325 - GROUPS & RING THEORY

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**INSTRUCTIONS:** There are two sections in this paper. Section A and Section B. Attempt any three(3) questions from section (A) and any two questions from section (B). All questions carry equal marks.

**TIME ALLOWED:** Three(3) hours.

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SECTION A

Answer any three(3) questions from this section

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Q1. Define the following terms:-

- (i) a permutation  $\pi$  on  $A$  set  $a$  onto itself.
- (ii) the order of an element  $a \in G$ .

a) Given the permutations

$$\sigma_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix} \quad \text{and} \quad \sigma_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix}$$

evaluate

(i)  $\sigma_1^{-1}\sigma_2$       (ii)  $(\sigma_2\sigma_1)^2$       (iii)  $\frac{1}{(\sigma_1\sigma_2)}$

Which of these are even or odd?

- b) State and prove lagrange's theorem as applied to a finite group  $G$ .
- c) quote lagrange's theorem in showing that a group of prime order has no proper subgroups and is necessarily cyclic.

Q.2 What is meant by the following terms:-

- (i) a syLOW p-subgroup of the group  $G$ ?
- (ii)  $\phi$  is a homomorphism from a group  $G$  onto a group  $G'$ ?

(a) Prove that syLOW p-subgroups of  $G$  belonging to the same prime are always conjugate with one another-in  $G$ .

- (b) Show that a finite group  $G$  has a unique sylow subgroup  $P$  corresponding to a given prime  $p$  if and only if  $P$  is normal in a group  $G$ .
- (c) Let  $G$  be the group of positive real numbers under multiplication and let  $G'$  be the group of all real numbers under addition. Define  $\phi : G \rightarrow G'$  by  $\phi(x) = \log_{10}x$  for all  $x, y \in G$ . Show that  $\phi$  so defined is a homomorphism.

Q3. Define the following terms:-

- (i) a normal subgroup of a group  $G$ .
- (ii) a left or right coset of  $N$  in  $G$ .
- (a) Show that  $N$  is a normal subgroup of  $G$  if and only if  $gNg^{-1} = N$  for every  $g \in G$ .
- (b) Prove that the subgroup  $N$  of  $G$  is normal in  $G$  if and only if every left coset of  $N$  in  $G$  is a right coset of  $N$  in  $G$ .
- (c) Verify that if a subgroup  $N$  of  $G$  is normal in  $G$  then the product of two right cosets of  $N$  in  $G$  is again a right coset of  $N$  in  $G$ .

Q4. (a) Show that if  $G$  is a finite group of order ' $g$ ', then all the elements of  $G$  satisfy the equation

$$x^{1+g} = x$$

- (b) Show that there can be no simple group of order 200.
- (c) Prove that if  $H$  is of index 2 in  $G$ ; then  $H$  is normal in  $G$ .
- (d) Prove that when  $G$  is a finite group of order ' $g$ ' and if  $h_a$  is the number of elements of  $[a]$ , then  $ha \mid g$ .

## SECTION B

Answer any two(2) questions from this section.

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Q5. Define the following:-

- (i) a field  $F$  as applied in ring theory.
- (ii) an ideal  $I$  in a ring  $R$ .
- (iii) a zero divisor in a ring  $R$ .
- (a) Prove that if  $p$  is a prime number, then  $Jp$ , the ring of integers mod  $p$ , is a field.
- (b) Let  $R$  be a commutative ring with a unit element whose ideals are  $\{0\}$  and  $R$  itself. Prove that  $R$  is a field.
- (c) Give atleast two elaborated examples of a commutative ring with or without a unit element.

Q6. What do you understand by the following terms as applied to rings?

- (i) A maximal ideal  $M$  of a ring  $R$ .
- (ii) an integral domain  $D$ .

Prove that

- (a) a finite integral domain is a field.
- (b) If  $R$  is a commutative ring with a unit element and  $M$  is an ideal of  $R$ , then  $M$  is a maximal ideal of  $R$  if and only if  $R/M$  is a field.
- (c) By factorizing the polynomial  $x^2 + 1$ , show that

$\frac{Q}{\langle x^2 + 1 \rangle}$  is a field. Furthermore,

$\frac{C}{\langle x^2 + 1 \rangle}$  is not a field

Where  $Q$  is a field of rationals and  $C$  is a complex one.

Q7. Prove the following assertions:

- (a) A Euclidean ring possesses a unit element.
  - (b) If  $f(x), g(x)$  are two no-zero elements of  $F[x]$ , then  $\deg f(x) + \deg g(x) = \deg[f(x)g(x)]$
  - (c) If  $R$  is an integral domain, then so is  $R[x]$ , a ring of polynomials in  $x$ .
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**END OF EXAMINATION**

THE UNIVERSITY OF ZAMBIA

UNIVERSITY FIRST SEMESTER EXAMINATIONS - 2001/02

M331 - REAL ANALYSIS III

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**INSTRUCTIONS:** Answer any five questions.

**TIME ALLOWED:** Three(3) hours.

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- Q.1 (a) Find the interior and boundary of each of the following subsets of the real line:  
(i) the set of all integers (ii) the set of all rational numbers  
(ii)  $(0,1)$  (iv)  $[0,1]$ .
- (b) Let  $S$  be a subset of the real line. Define limit point of  $S$ . Find the limit point of each of the following subsets of the real line:  
(i)  $S = \{1,2,3,4,\dots\}$  (ii)  $S = [-1,1]$  (iii)  $\left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\right\}$ .
- (c) State and prove Bolzano-Weierstrass Theorem.
- Q.2 (a) Define open set and closed set in the real line  $\mathbb{R}$ . Prove the following:  
(i) The sets  $\mathbb{R}$  and  $\emptyset$  are open.  
(ii) The intersection of a finite number of open sets in  $\mathbb{R}$  is open.  
Show by an example that the limitation to finite intersections is essential.
- (b) Define compact set and connected set. Give two examples for both kind of sets. Prove the following:  
(i) The set  $E = (a,b)$  is not compact.  
(i) The set  $E = [a,b]$  is connected.
- (c) State Heine-Borel Theorem.
3. (a) Define uniform continuity.  
Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a function defined by  $f(x) = x^2$   
Prove that  $f$  is not uniformly continuous on  $\mathbb{R}$ .
- (b) Let  $f: [a,b] \rightarrow \mathbb{R}$  be a function which is continuous on  $[a,b]$ . Prove that  $f$  is uniformly continuous on  $[a,b]$ .

- (c) Prove that  $f(x) = \cos x$  is uniformly continuous on  $(-\infty, \infty)$ .
- Q4 (a) Define pointwise and uniform convergence for the sequence of functions  $(f_n)$ .
- (b) Let  $f_n(x) = \frac{x}{2 + nx}$  for  $0 < x < \infty$ .  
Prove that  $f_n(x)$  converges uniformly in  $[0, \infty)$ .
- (c) State Weierstrass M-Test.  
By using this test prove that  $\sum_{n=0}^{\infty} \frac{x^n}{n!}$  is uniformly convergent in  $[-1, 1]$ .
- Q.5 (a) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a continuous function. Let  $E \subset \mathbb{R}$  be a compact set. Prove that  $f(E)$  is compact.
- (b) Let  $f: [a, b] \rightarrow \mathbb{R}$  be continuous. Prove that  $f$  is bounded and takes a maximum and minimum value at points on  $[a, b]$ .
- (c) Prove that  $f(x) = \frac{1}{x}$  is not uniformly continuous on  $(0, 1)$ .
6. (a) Define function of bounded variation. Give two examples of functions which are of bounded variation.
- (b) If  $f$  is of bounded variation on  $[a, b]$ , then prove that  $f$  is bounded on  $[a, b]$ .
- (c) Give one example of a function which is bounded on  $[a, b]$  but is not of bounded variation on  $[a, b]$ .
- Q7. (a) Define step function.  
Give two examples of functions which are step functions.
- (b) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be continuous function. Let  $E \subset \mathbb{R}$  be a connected set. Then prove that  $f(E)$  is connected set in  $\mathbb{R}$ .
- (c) Let  $f: [a, b] \rightarrow \mathbb{R}$  be a continuous function. Prove that  $f$  takes every value between  $f(a)$  and  $f(b)$ .

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**END OF EXAMINATION**

# UNIVERSITY OF ZAMBIA

## UNIVERSITY FIRST SEMESTER EXAMINATIONS APRIL 2002

### MATHEMATICS M421 - STRUCTURE AND REPRESENTATION OF GROUPS

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**INSTRUCTIONS:** Attempt ANY Five(5) questions

**TIME ALLOWED:** Three(3) hours

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1. (a) Define each of the following:
- (i) the *kernel* of a group  $G$ . [2 marks]
  - (ii)  $G$  is an *internal direct product* of its subgroups  $A$  and  $B$ . [2 marks]
- (b) (i) Let  $H$  be a subgroup of  $G$ ,  $K$  a normal subgroup of  $G$ . Then, prove that  $H \cap K$  is a normal subgroup of  $H$  and that  $H/H \cap K$  is isomorphic to  $KH/K$ . [6 marks]
- (ii) Show that there is an isomorphism between the quotient group  $S_4/V_4$  and the group  $S_3$ , where  $S_4$  is the symmetric group of degree 4,  $S_3$  is the symmetric group of degree 3 and  $V_4$  is the Klein four group. [4 marks]
- (c) If  $n$  is a positive odd integer, let  $D_{4n}$  be the group of symmetries of a regular  $2n$ -gon and let  $D_{2n}$  be the group of symmetries of a regular  $n$ -gon inscribed in the  $2n$ -gon by joining vertex  $2i$  to vertex  $2i + 2$ , for all  $i \pmod{2n}$ . Then, show that  $D_{4n}$  is an internal direct sum of  $D_{2n}$  by  $Z_2$ . [6 marks]
-

2. (a) Define
- (i) a composition series of a group  $G$ . [2 marks]
  - (ii) a solvable group  $G$ . [2 marks]
- (b) (i) Let  $G$  be a solvable group and let  $H$  be a subgroup of  $G$ . Then, prove that  $H$  is also solvable. [5 marks]
- (ii) Suppose that  $H$  is a normal subgroup of  $G$  and that  $H$  and  $G/H$  are solvable. Then, prove that  $G$  is also solvable. [5 marks]
- (c) Let  $S_4$  be the symmetric group on 4 symbols; and  $S_5$  be the symmetric group on 5 symbols. Then, show that  $S_4$  is a solvable group, while  $S_5$  is not solvable. [6 marks]
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3. (a) Define
- (i) the derived group  $G'$  of a group  $G$ . [2 marks]
  - (ii) a nilpotent group  $G$ . [2 marks]
- (b) Let  $G$  be a group and  $G'$  denote its derived group. Then show that
- (i)  $G'$  is a normal subgroup of  $G$ . [4 marks]
  - (ii)  $G/G'$  is an abelian group. [3 marks]
- (c) (i) Show that every nilpotent group  $G$  has a non-trivial centre. [3 marks]
- (ii) Let  $G$  be the group defined by

$$\langle x, y \mid x^4 = y^2 = 1, yx = x^3y \rangle.$$

Then find the centre,  $Z(G)$ , of  $G$  and determine whether or not,  $G$  is a nilpotent group. [6 marks]

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4. (a) Define each of the following terms:
- (i) a *G-orbit* of an element  $b$  in  $G$ . [2 marks]
  - (ii) the *stabilizer* of an element  $b$  in  $G$ . [2 marks]
- (b) (i) Let  $G_b$  be the stabilizer of  $b$  in  $G$ . Then, prove that  $G_b$  is a subgroup of the group  $G$ . [3 marks]
- (ii) Let  $G = D_3 = \langle a, b \mid a^3 = b^2 = 1, ba = a^2b \rangle$  and  $H = \{1, b\}$  be a subgroup of  $G$ . By using a suitable group action on the left cosets of  $H$  in  $G$ , show that  $D_3$  is isomorphic to  $S_3$ , the symmetric group of order 6. [5 marks]
- (c) Let  $G$  and  $H$  be as in  $b(ii)$  above. Then,
- (i) find the *G-orbits* of 1, 2 and 3. [4 marks]
  - (ii) determine the stabilizers of 1, 2 and 3, and deduce that the stabilizers are actually subgroups of  $G$ . [4 marks]
- 

5. (a) Define the following terms
- (i) a *complete group*  $G$ . [2 marks]
  - (ii) a *semi-direct product* of a group  $Q$  by a group  $K$ . [2 marks]
- (b) Show that if  $Q$  is a complete normal subgroup of a group  $G$ , then there exists a normal subgroup  $K$  of  $G$  such that
- $$G = Q \otimes K,$$
- a direct product. [5 marks]
- (c) (i) Let  $\text{Inn}(G)$  denote the group of inner automorphisms of a group  $G$ , and  $Z(G)$  be the centre of  $G$ . Then prove that
- $$G/Z(G) \cong \text{Inn}(G). \quad [4 \text{ marks}]$$
- (ii) By using the fact that the symmetric group  $S_n$  is complete if  $n \neq 2, 6$ , or otherwise, determine the automorphism group,  $\text{Aut}(S_n)$ , of  $S_n$  for  $n \neq 2, 6$ . [2 marks]
- Show that  $S_n$  is a semi-direct product of  $A_n$ , the alternating group on  $n$  symbols, and  $C_2$ , the cyclic group of order 2. [3 marks]
-

6. (a) Define

(i) a matrix representation of a group  $G$ . [2 marks]

(ii) a completely reducible matrix representation of a group  $G$ . [2 marks]

(b) (i) Show that the map defined by

$$T(x) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

is a matrix representation of the cyclic group  $G = \langle x \mid x^4 = 1 \rangle$ .

Prove that  $T$  is irreducible over the field of real numbers  $\mathbf{R}$  but completely reducible over  $\mathbf{C}$ , the field of complex numbers. [6 marks]

(ii) Let  $T_1$  and  $T_2$  be other matrix representations of  $G = \langle x \mid x^4 = 1 \rangle$  such that  $T_1(x) = i$  and  $T_2(x) = -i$ , where  $i = \sqrt{-1}$ . Determine the characters of the representations  $T, T_1, T_2$  of  $G$ , and hence, deduce that  $T = T_1 \oplus T_2$ . [5 marks]

(c) Find the left regular representation of the group

$$G = \langle x, y \mid x^2 = y^2 = 1, xyx^{-1} = y \rangle,$$

and determine its character. [5 marks]

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**END OF EXAMINATION**

**GOOD LUCK!!**

**THE UNIVERSITY OF ZAMBIA**

**UNIVERSITY FIRST SEMESTER EXAMINATIONS - 2001/02**

**M431 - REAL ANALYSIS V**

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**INSTRUCTIONS:** Answer any five questions.

**TIME ALLOWED:** Three(3) hours.

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- Q1. (a) Prove that any infinite subset of a countable set is countable.
- (b) Define Cantor set.  
Prove that the sum of the lengths of the intervals removed from  $[0,1]$  in the construction of cantor set is 1.
- (c) Define maximal element in a partially ordered set. Explain this notion by examples. State Zorn's Lemma.
- Q2. (a) Define bounded set in a metric space  $(X,d)$ . Prove that every cauchy sequence in a metric space  $(X,d)$  is bounded.
- (b) If  $A$  and  $B$  are subsets of a metric space, then prove the following:  
(i) If  $A \subset B$ , then  $d(A) \leq d(B)$   
(ii)  $d(A \cup B) \leq d(A) + d(A,B) + d(B)$ .  
Note that  $d(A)$  denote the diameter of  $A$ .
- (c) Let  $(X_1, d_1)$  and  $(X_2, d_2)$  be metric spaces and let  $f: X_1 \rightarrow X_2$  be a function. Prove that  $f$  is continuous on  $X_1$  if and only if  $f^{-1}(G)$  is open in  $X_1$  whenever  $G$  is open in  $X_2$ .
- Q3. (a) (i) Define dense set in a metric space  $(X,d)$ . Give an example of a dense set.
- (ii) Define separable metric space  
Give an example of a metric space which is not separable.
- (b) Prove that a metric space  $(X,d)$  is separable if and only if  $X$  has a countable subset  $Y$  with the following property:  
For every  $\varepsilon > 0$  and  $x \in X$ , there is a  $y \in Y$  such that  $d(x,y) < \varepsilon$ .
- (c) Prove that the space  $c_0$  is separable.

Q4. (a) State and prove Holder's inequality for the space  $l_p^n$ .

(b) Prove that  $l_p^n$  is a metric space with the metric defined by

$$d(x, y) = \left[ \sum_{i=1}^n |x_i - y_i|^p \right]^{1/p}.$$

(c) Define norm on a vector space X.

Prove that every normed vector space X is a metric space relative to the natural metric d defined by

$$d(x, y) = \|x - y\|.$$

5. (a) Define Totally bounded set in a metric space (X,d).  
Show by an example that a bounded subset of  $l_\infty$  is not totally bounded.

(b) Prove that the subset E of the metric space (X,d) is totally bounded if and only if for every  $\varepsilon > 0$ , the set E contains a finite subset

$$\{x_1, x_2, \dots, x_n\} \text{ which is } \varepsilon\text{-dense in E.}$$

(c) Prove that every finite subset of a metric space is totally bounded.

Q6. (a) Let (X,d) be a metric space. Prove that if X has the Heine-Borel property, then X is compact.

(b) Let A be a subset of a metric space (X,d). If (A,d) is compact, then prove that A is closed and bounded.

(c) Find an open covering of  $E = (0,1)$  which does not contain a finite subcovering.

Q7. (a) Prove that if f is a one-to-one continuous function from the compact metric space X onto a metric space Y, then  $f^{-1}$  is continuous on Y.

(b) Let  $(X,d_1)$  and  $(Y,d_2)$  be metric spaces. Prove that if  $f: X \rightarrow Y$  is uniformly continuous and  $(x_n)$  is a Cauchy sequence in X, then  $(f(x_n))$  is a Cauchy sequence in Y.

(c) Let f be a continuous function from the compact metric space X into a metric space Y. Then prove that the image  $f(X)$  is a bounded and closed subset of Y.

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**END OF EXAMINATION**

THE UNIVERSITY OF ZAMBIA

UNIVERSITY FIRST SEMESTER EXAMINATIONS - 2001/02

M461 - MULTIVARIATE ANALYSIS

**INSTRUCTIONS:** Answer any four(4) questions. All essential working must be shown. Calculators are allowed. Statistical Tables will be provided.

**TIME ALLOWED:** Three(3) hours.

1. a) Define the following:
- (i) sample correlation coefficient.
  - (i) quadratic form.
- b) Prove the following:
- (i) Show that variance covariance matrix  $\sum_v$  of a random vector  $V$  with mean vector  $\mu_v$  is expressed as

$$\sum_v = E(VV^T) - \mu_v \mu_v^T.$$

- (ii)  $\bar{X}$  is an unbiased estimator for  $\mu$  and has covariance matrix  $\frac{1}{4} \sum$ , where  $X_1, X_2, \dots, X_n$  is a random sample such that each  $X_i$  has mean and covariance matrix  $\sum$ .

- c) Given the five measurements on variables  $X_1, X_2$  and  $X_3$ :

$X_1$	9	2	6	5	8
$X_2$	12	8	6	4	10
$X_3$	3	4	0	2	1

Find:

- (i) the mean vector  $\bar{X}$ .
- (ii) covariance matrix  $S$ .
- (iii) correlation matrix  $R$ .

2. a) Define the following:
- (i) random vector.
  - (ii) independence of two random vectors
- b) (i) State and prove the cauchy-shawarz inequality.  
(ii) Show that if

$$\Sigma = V^{1/2} P V^{1/2} \quad \text{then}$$

$$\rho = (V^{1/2})^{-1} \Sigma (V^{1/2})^{-1}$$

where  $V = \text{diag}(\sigma_{11}, \dots, \sigma_{22}, \sigma_{11})$ .  
Hence or otherwise, find  $\rho$  when

$$\Sigma = \begin{pmatrix} 4 & 1 & 2 \\ 1 & 9 & -3 \\ 2 & -3 & 25 \end{pmatrix}$$

- c) Let  $X_1, X_2, X_3$  and  $X_4$  be independent  $N_p(\mu, \Sigma)$  random vectors. Find
- i) the marginal distributions of the random vectors

$$V_1 = \frac{1}{4}(X_1 - X_2 + X_3 - X_4) \text{ and}$$

$$V_2 = \frac{1}{4}(X_1 + X_2 - X_3 - X_4)$$

- ii) the joint distribution of the random vectors  $V_1$  and  $V_2$  defined in (i).

- d) Let  $\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$  be  $N_3((\mu, \Sigma))$  with  $\mu^T = (-3, 1, 4)$  and

$$\Sigma = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Determine if the following are independent:

- (i)  $X_1$  and  $X_2$

(ii)  $(X_1, X_2)$  and  $X_3$

3. a) Define the following terms:

(i) trace of a matrix.

(ii) spectral decomposition of a matrix.

b) Prove the following:

(i) If  $X$  is a  $p$ -dimensional random vector and  $c$  a  $p$ -dimensional vector of constants, then  $Y = C^T X$  has mean  $C^T \mu_X$  and covariance  $C^T \Sigma C$ .

(ii) If  $\Sigma$  is a positive definite matrix and  $(\lambda, e)$  is an eigenvalue-eigenvector pair of  $\Sigma$ , then  $(\lambda^{-1}, e)$  is an eigenvalue-eigenvector pair of  $\Sigma^{-1}$ .

c) Let  $A = \begin{pmatrix} 9 & -2 \\ -2 & 6 \end{pmatrix}$

(i) Determine the eigenvalues and eigenvectors of  $A$ .

(ii) Write the spectral decomposition of  $A$ .

4. a) Define the following terms:

(i) Hotelling's  $T^2$  statistic for

$$H_0: \mu = \mu_0 \text{ versus}$$

$$H_1: \mu \neq \mu_0$$

(ii) a  $100(1 - \alpha)\%$  confidence region for  $\theta$ .

b) Prove the following:

The  $T^2$  statistic is invariant under transformations of the form

$$Y = CX + d$$

With  $C$  non singular

c) Let the data matrix for a random sample of size  $n = 4$  from a bivariate normal population be

$$X = \begin{pmatrix} 2 & 12 \\ 8 & 9 \\ 6 & 9 \\ 8 & 10 \end{pmatrix}$$

- (i) Evaluate  $T^2$  for testing  $H_0: \mu^T = (7, 11)$ .
- (ii) Specify the distribution of  $T^2$ .
- (iii) Using (i) and (ii), test  $H_0: \mu^T = (7, 11)$  versus  $H_1: \mu^T \neq (7, 11)$  at the  $\alpha = 0.05$  level.
- (iv) Evaluate the Wilk's Lambda

5. a) Find the maximum likelihood estimates of the  $2 \times 1$  mean vector  $\mu$  and the  $2 \times 1$  covariance matrix  $\Sigma$  based on the random sample,

$$X = \begin{pmatrix} 3 & 6 \\ 4 & 4 \\ 5 & 7 \\ 4 & 7 \end{pmatrix}$$

From a bivariate normal population.

- b) The various costs associated with transporting milk from farms to dairy plants were studied for 25 gasoline trucks. The variables  $X_1 =$  fuel,  $X_2 =$  repair and  $X_3 =$  capital, were measured with the following results.

$$\bar{X} = \begin{pmatrix} 12.56 \\ 8.1612 \\ 10.5444 \end{pmatrix} \text{ and}$$

$$S = \begin{pmatrix} 28.96593 & 17.21536 & 2.694567 \\ 17.21536 & 21.45285 & 6.044528 \\ 2.694567 & 6.044528 & 13.59904 \end{pmatrix}$$

- (i) Construct the 95% Bonferroni intervals for the individual cost means.
- (ii) Construct the 95%  $T^2$  - intervals for the individual cost means.
- (iii) Compare the two sets of intervals.

- c) Samples of sizes  $n_1 = 45$  and  $n_2 = 55$  were taken of home owners with and without air conditioning, respectively. Two measurements of electrical usage were considered. The first is a measure of total off-peak consumption ( $X_1$ ) during July. The resulting summary statistics are:

$$\bar{x}_1 = \begin{pmatrix} 204.4 \\ 556.6 \end{pmatrix} \quad S_1 = \begin{pmatrix} 13825.3 & 23823.4 \\ 23823.4 & 73107.4 \end{pmatrix}$$

$$\bar{x}_2 = \begin{pmatrix} 130 \\ 355 \end{pmatrix} \quad S_2 = \begin{pmatrix} 8632 & 19616.7 \\ 23823.4 & 55964.5 \end{pmatrix}$$

Find the 95% simultaneous confidence intervals for the differences in the mean components.

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**END OF EXAMINATIONS**

**THE UNIVERSITY OF ZAMBIA  
UNIVERSITY SESSIONAL EXAMINATIONS**

**MATHEMATICAL METHODS(V) - M911**

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**INSTRUCTIONS:** There are six (6) questions in this examination paper.  
Candidates are required to answer ANY five (5) questions of their choice. All questions carry equal marks.

**TIME ALLOWED:** Three (3) hours.

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Q1. Evaluate the following:-

(a) (i)  $\frac{1}{(D+3)}\{e^{-2x}\}$       (ii)  $L^{-1}\left\{\frac{5}{s+2}\right\}$

(b) Define a vector quantity?

Given that  $\phi = x^2yz^3$  and  $A = xzi - y^2j + 2x^2yk$

(i) find  $\nabla\phi$

(ii) find  $\text{curl}(A)$

(iii) prove that  $\nabla \cdot (\nabla A) \cdot A + \phi (\nabla \cdot A)$  *what is this?*

(c) Solve:

$$(3x^2 - y^2)dy - 2xydx = 0$$

Q.2 (a) Find the maximum, minimum or saddle points of the function  $f(x) = 12 + 8x^2 - x^4$ . Sketch its graph; clearly showing its features.

(b) Write down the Taylor's series for the function of  $f(x)$  at the base point  $a_0$ .

Hence evaluate Taylor's series of the function  $f(x) = e^x$  at the base point  $a = 1$ .

(c) Use the method of langrange multipliers to find the maximum value of  $w = xyz$  of all points  $(x,y,z)$  lying on the line of intersection of planes.

$$x + y + z = 30 \text{ and } x + y - z = 0$$

- Q.3 (a) Find a unit normal to the surface  $2x^2 + 4z - 5z^2 = -10$  at the point  $p(3, -1, 2)$ .
- (b) Give a physical interpretation of the directional derivative of  $f$  in the direction of the vector  $u$ ?

Consider  $z = f(x, y) = xy^2$ . Calculate the directional derivative of  $f$  in the direction of the vector  $v = 2i + 3j$  at the point  $(4, -1)$ .

- (c) Find the equations of the
- tangent plane
  - normal line

to the surface  $x^2 + y^2 = 4z$  at the point  $(2, -4, 5)$ .

- (d) Find the solutions of the following differential equation:

$$y'' + 3y' - 10y = 0$$

Show that these solutions are linearly independent by using Wronskian determinant.

Q4. (a) Solve:  $4 \frac{d^2y}{dx^2} - 12 \frac{dy}{dx} + 9y = 0$

- (b) use series method to solve the following:

(i)  $(D - 1)y = 0$

(ii)  $y' = 2xy$

(c) Find  $L^{-1} \left\{ \frac{1}{s^2 + 5s - 12} \right\}$

- Q.5 (a) Find the maximum and minimum values of  $f(x, y, z) = 4xyz$  subject to the constraint  $\phi(x, y, z) = x^2 + y^2 + z^2 - a^2$  using the method of langrage multipliers.

- (b) Find the solutions of the following initial values problem using laplace transforms:

$$y'' + 2y' + 5y = 0$$

under conditions  $y(0) = 2$  and  $y'(0) = -4$

Q.6 The distribution of voltage on a metal plate is given by

$$V = 100 - x^2 - 4y^2$$

- (a) (i) At the point (1,-2), in what direction does the voltage increase most rapidly?
- (ii) In what direction does it decrease most rapidly?
- (iii) Find the magnitude of this increase or decrease?
- (iv) In what direction does it change least?
- (b) In part (a) above, describe the path of a particle that starts at the point (1,-2) and moves in the direction of greatest voltage increase.
- (c) Given that  $\mathbf{a} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$  and  $\mathbf{B} = -4\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ . Compute
- (i)  $\mathbf{A} \times \mathbf{B}$       (ii)  $\left( |\mathbf{A} \times \mathbf{B}|^2 + 4 \right)^{1/2}$

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**END OF EXAMINATION**

**The University of Zambia**  
**Physics Department**  
**University Examinations 2001/2002**  
**P-191 : Introductory Physics - I**

All questions carry equal marks. The marks are shown in brackets. Question 1 is compulsory. Attempt four more questions. Clearly indicate on the answer script which questions you have attempted.

Time : Three Hours.

Maximum Marks : 100.

**Do not forget to write your computer number clearly on the answer script !!**

Wherever necessary use :

$G = 9.8\text{m/s}^2$  :  $P_A = 1.01 \times 10^5 \text{ N/m}^2$  :  $1 \text{ cal.} = 4.2 \text{ J}$  :  $\rho_{\text{water}} = 1000\text{kg/m}^3$   
 Specific heat capacity of water =  $4200\text{J/kg.K}$  :  $1 \text{ pascal} = 1 \text{ N/m}^2$

Some equations you may find useful :

$v_f = v_o + at$  :  $v_f^2 = v_o^2 + 2ax$  :  $x = v_o t + (1/2)at^2$  :  $f = \mu F_N$  : work =  $F.s.\cos \theta$

$Ft = m(v_f - v_o)$  : kin. energy =  $(1/2)mv^2$  : grav. pot. energy =  $mgh$  :  $W = mg$

$x = v_{\text{avg.}} t$  :  $Ft = \Delta p$  :  $\Delta PE + \Delta KE + \Delta TE = 0$  : power = work/time :  $P = Fv$

$v_{\text{avg.}} = (1/2)(v_o + v_f)$  :  $\omega_f = \omega_o + \alpha t$  :  $\omega_f^2 = \omega_o^2 + 2\alpha\theta$  :  $\theta = \omega_o t + (1/2)\alpha t^2$

$p = mv$  :  $a_T = \alpha r$  :  $L = I\omega$  :  $\tau = I\alpha = Fr$  :  $[\text{Kin. Energy}]_{\text{total}} = (1/2)mv^2 + (1/2)I\omega^2$

$1 \text{ rev} = 360^\circ = 2\pi \text{ radians}$  :  $F_c = (mv^2)/r$  :  $I = \sum mr^2$  :  $F = (Gm_1m_2)/r^2$  :  $F = -kx$

$Y = (F/A)/(\Delta L/L_o)$  :  $B = -\Delta P/(\Delta V/V_o)$  :  $W_{\text{app.}} = mg - B.F.$  :  $P = \rho gh$  :  $v_T = \omega r$

$W_{\text{app.}} = W [1 - (\rho_f/\rho)]$  :  $[(1/2)mv^2]_{\text{avg.}} = (3/2)kT$  :  $\Delta Q = cm\Delta T = nC\Delta T$

$\Delta L = \alpha L\Delta T$  :  $\Delta V = \gamma V\Delta T$  :  $\Delta W = P\Delta V$  :  $P_1V_1^\gamma = P_2V_2^\gamma$  :  $(\Delta Q/\Delta t) = (kA\Delta T)/\Delta L$

$\Delta Q = \Delta U + \Delta W$  :  $\Delta W = nRT.\ln(V_f/V_i)$  :  $R = (2u^2 \sin \theta \cos \theta)/g$  :  $t = (2u \sin \theta)/g$

$(1/2)kx^2 + (1/2)mv^2 = (1/2)kx_o^2$  :  $\omega = \sqrt{(k/m)}$  :  $v = \pm \sqrt{[(k/m)(x_o^2 - x^2)]}$

$v = \sqrt{(Y/\rho)}$  :  $f = (1/2\pi)\sqrt{(k/m)}$  :  $f = (1/2\pi)\sqrt{(g/L)}$  :  $v = \sqrt{[T/(m/L)]}$  :  $v = \sqrt{(B/\rho)}$

$PV = nRT$  :  $v = \sqrt{(\gamma RT/M)}$  :  $f = 1/\tau$  :  $\omega = 2\pi f$  :  $I_1\omega_1 = I_2\omega_2$  :  $\Delta T.E. = f.s$

area of a sphere =  $4\pi r^2$  : area of a right cylinder =  $2\pi rL$  :  $0K = 273^\circ C$  :  $a = -kx/m$

$a_{\text{max.}} = kx_o/m$  :  $a_c = \omega^2 x_o$  : P.E. =  $(1/2)kx^2$  :

**Question 1** : Sample answers : F(a), G(d)... etc. DO NOT guess the answer. For each correct answer, 2 marks. For each wrong answer, (2/3) will be deducted. No answer, zero mark. No deduction of marks for not attempting. Minimum total marks for Question 1 is zero. [  $10 \times 2 = 20$  ]

(A) Identify the vector quantity :

- (a) heat (b) work  
(c) angular momentum ✓ (d) time.

(B) In throwing a javelin, a person has greater probability of winning if he throws :

- (a) at an angle of  $45^\circ$  to the ground ✓  
(b) at an angle slightly less than  $45^\circ$  to the ground  
(c) at an angle slightly more than  $45^\circ$  to the ground  
(d) in a horizontal direction.

(C) An athlete runs some distance before taking a long jump because :

- (a) she gains energy to take her through a long distance  
(b) it helps to apply a large force  
(c) by running action and reaction force increases  
(d) by running she gives herself a larger inertia of motion.

(D) An object A hits another object B which is initially at rest. Object B will acquire the most speed when the mass of object A is :

- (a) equal to mass of object B  
(b) less than mass of object B  
(c) greater than mass of object B  
(d) greater or less than mass of object B depending on the speed of object A.

(E) A particle is moving in a circle with uniform speed. It has constant :

- (a) velocity (b) momentum  
(c) kinetic energy (d) displacement.

(F) Young's modulus for a perfectly plastic body is :

- (a) zero (b) infinite  
(c) 1 (d) finite.

(G) Two pieces of metal when immersed in a liquid have equal upthrust acting on them. Then :

- (a) both pieces have equal weights in air  
(b) both pieces have equal volumes  
(c) both pieces have equal density  
(d) both are floated to the same depth.

**Q8. (a)** Explain in short why good thermal conductors are also good electrical conductors. [2]

**(b)** A copper pipe that is 15cm in diameter and has a wall thickness of 0.35cm carries steam at 125°C through a vat of circulating water at 20°C. How much heat is lost per meter of the pipe in one second? Given,  $k_{\text{Cu}} = 400\text{W/K.m}$  [8]

**(c)** A running man has a kinetic energy which is half the kinetic energy of a running boy. The mass of the boy is half that of the man. The man increases speed by 1m/s and then his kinetic energy is the same as that of the boy; the speed of the boy remains constant. Find the original speeds of the man and the boy. [10]

**--End of P-191 Examination--**

THE UNIVERSITY OF ZAMBIA  
PHYSICS DEPARTMENT

UNIVERSITY EXAMINATION 2001/2002

P231 (PROPERTIES OF MATTER AND THERMAL PHYSICS)

TIME: THREE HOURS MAX. MARKS 100

ANSWER: FIVE QUESTIONS

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QUESTION 1

- (a) Explain what is meant by the term coefficient of viscosity and derive its dimensions in terms of length, mass, and time. (5 Marks)
- (b) Derive Poiseuille's formula for the rate of flow of a viscous liquid through a narrow tube. (10 Marks)
- (c) A horizontal tube 0.8 mm in diameter and 16 cm long is connected at one end to a constant level water tank whose height is 200 cm. If  $600 \text{ cm}^3$  of water flows through the tube in 8 minutes, find the coefficient of viscosity. (Density of water =  $1000 \text{ kg/m}^3$ , acceleration due to gravity  $g = 9.81 \text{ m/s}^2$ ). (5 Marks)

QUESTION 2

- (a) Define surface tension of a liquid and explain it in terms of intermolecular forces in a liquid. (5 Marks)
- (b) Explain in terms of the cohesive and the adhesive forces between the molecules of a liquid and molecules of a solid why surface of a liquid curves downwards at the walls of the container of the liquid. (5 Marks)
- (c) Obtain an expression for the excess pressure inside a soap bubble. (10 Marks)

### QUESTION 3

- (a) Show that the depression of a beam of length,  $\ell$ , and uniform cross-section supported on two knife-edges at its two ends and loaded in the middle with a weight  $W$  is

$$\frac{W \ell^3}{48Y I_g} \quad (10 \text{ Marks})$$

where  $Y$  is the Young's modulus of the material of the beam,  
 $I_g$  is the geometrical moment of inertia of the beam.

- (b) A light rectangular bar of length  $1 \text{ m}$  and a square cross-sectional area  $5 \times 10^{-3} \text{ m}^2$  is supported at the two ends and loaded in the middle with a load of  $0.1 \text{ kg}$ . The depression observed is  $1.96 \times 10^{-3}$  metres. Calculate the Young's modulus of the material of the bar. (5 Marks)
- (c) A long cylindrical rod of diameter  $0.4 \text{ cm}$  and length  $1 \text{ m}$  is bent into a semi-circle form by a string tied to its ends. Find the Young's modulus of the material of the rod if the tension in the string is  $50 \text{ N}$ . (5 Marks)

### QUESTION 4

- (a) A wire of original length  $\ell$  stretches by a length  $d\ell$  when a force  $F$  is applied at one end. Show that the energy stored in the wire is given  
by  $E = \frac{1}{2} F \cdot d\ell$ . (5 Marks)
- (b) A uniform steel wire of length  $4 \text{ m}$  and cross-sectional area  $3 \times 10^{-6} \text{ m}^2$  is extended by  $1 \text{ mm}$ . Calculate the energy stored in the wire if the elastic limit is not exceeded. (Young's modulus of the material of the wire is  $2.0 \times 10^{11} \text{ N/m}^2$ ). (5 Marks)
- (c) Two vertical wires  $X$  and  $Y$  suspended at the same horizontal level, are connected by a light rod at their lower ends. The wires have the same length  $\ell$  and cross-sectional area  $a$ . Weight  $30 \text{ N}$  is placed at point,  $P$ , on the rod where  $XP:PY = 1:2$ . Both wires are stretched and the rod then remains horizontal. If the wire  $X$  has a Young's modulus of  $1.0 \times 10^{11} \text{ N/m}^2$ , calculate the Young's modulus of the wire  $Y$  assuming that the elastic limit is not exceeded for both wires. (10 Marks)

### QUESTION 5

- (a) Using the kinetic theory of gases and clearly stating the assumptions made, show that for a perfect gas  $PV = \frac{NmC^2}{3}$ . (10 Marks)
- (b) The molecular weight of argon is 39.9. Calculate the root mean square translational speed of its molecules at a temperature of 273 K. (Avogadro number  $N_A = 6.02 \times 10^{23}$  particles per mole). (2 Marks)
- (c) (i) Derive Charles's law, Dalton's law and the standard gas law,  $PV = RT$ , on the basis of the kinetic theory of gases.
- (ii) Write a brief note on absolute zero temperature from the standpoint of the theory. (8 Marks)

### QUESTION 6

- (a) Show that the efficiency,  $E$ , of a Carnot engine using an ideal gas as a working substance is  $E = \frac{Q_1 - Q_2}{Q_1}$ . (10 Marks)
- (b) A steam engine takes steam from the boiler at  $200^\circ\text{C}$  ( $1.54 \times 10^6$  Pa pressure) and exhausts directly into the air ( $4.58 \times 10^4$  Pa pressure) at  $100^\circ\text{C}$ . What is its maximum possible efficiency? (3 Marks)
- (c) A dish of hot food is placed in a refrigerator maintained at  $50^\circ\text{C}$ . To cool to this temperature, the food loses 220,000 joules of energy. How much electric energy is needed to operate the compressor if we assume that room temperature is  $23^\circ\text{C}$  ( $3.60 \times 10^6$  joules = 1 kWh). (7 Marks)

### QUESTION 7

- (a) One end of a copper rod is in thermal contact with a reservoir at temperature 500 K and the other end is in thermal contact with a reservoir at temperature 300K. If 8000 joules of energy are conducted from one end to the other, with no change in temperature distribution along the rod, find
- (i) The entropy change of the reservoir
- (ii) The entropy change of the rod and
- (iii) The entropy change of the universe. (5 Marks)
- (b) A hot stone of mass,  $M_2$ , specific heat  $C_2$ , and temperature  $T_2$  is dropped into a container of cool water of mass,  $M_1$ , specific heat,  $C_1$ , and temperature  $T_1$  (where  $T_1 < T_2$ ). The final temperature of the combination is  $T_f$ . What entropy change occurs in this process? (5 Marks)

- (c) Suppose a refrigerator has a coefficient of performance of 5.5
- (i) How much energy does it consume in removing 1850 calories (0.239 calories = 1 Joule) from its interior? (5 Marks)
  - (ii) What power rating will this refrigerator have if it can remove 1850 calories from its interior each minute (0.239 calories = one joule). (5 Marks)

**UNIVERSITY OF ZAMBIA**  
**DEPARTMENT OF PHYSICS**  
**UNIVERSITY EXAMINATIONS 2001/2002**

**P251**  
**CLASSICAL MECHANICS I**

**DURATION:** Three (3) hours

**INSTRUCTIONS:** Answer only four (4) questions in total. *All questions carry equal marks.*

**MAXIMUM MARKS:** 100 %

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**Where necessary, you may need to use the following:**

$$g = 9.8 \text{ ms}^{-2}$$

$$\mathbf{r}_{cm} = \frac{\sum_i m_i \mathbf{r}_i}{m}$$

Q1. (a). (i) We can order events in time. For example, event  $b$  may precede event  $c$  but follow event  $a$ , giving us a time order of events  $a, b, c$ . Hence there is a sense of time, distinguishing past, present, and future. Is time a vector therefore? If not, why not?

(5 marks)

(b). A particle describes a path with position vector

$$\mathbf{r} = a \cos \omega t \mathbf{i} + b \sin \omega t \mathbf{j}$$

Show that

(i) the path is the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

(5 marks)

(ii) the acceleration is directed towards the origin,

(5 marks)

(iii)  $\int_1^{t+1} \mathbf{r} \times d\mathbf{r} = \omega ab t \mathbf{k}$ , and interpret this result.

(10 marks)

Q2. (a). (i) Explain why the earth is able to retain its atmosphere, unlike the moon which has none.

(3 marks)

(ii) Why does the earth's atmosphere contain no significant amounts of  $\text{H}_2$  despite the fact that  $\text{H}_2$  is the most abundant element in the universe?

(3 marks)

(b). The drag  $D$  on a sky diver of mass  $m$  is modelled by the expression

$$D = cv^2$$

where  $c$  is a constant and  $v$  is the velocity of the sky diver.

(i) Find an expression for the limiting velocity  $v_l$  of the sky diver, and

(4 marks)

(ii) Show that after falling for a time  $t$  the velocity of the sky diver is given by

$$v = v_l \frac{1 - \exp(2gt/v_l)}{1 + \exp(2gt/v_l)}$$

(10 marks)

(iii) A typical limiting velocity is  $-54 \text{ms}^{-1}$ . Show that the sky diver attains 50 per cent and 90 per cent of the limiting speed after 3.3 s and 8.1 s of fall respectively.

(5 marks)

- Q3. (a). (i) There are situations where Newton's third law fails. Give an example where this may occur. **(3 marks)**
- (ii) A coin is put on a phonograph turntable. The motor is started, but before the final speed of rotation is reached, the coin flies off. Explain. **(3 marks)**

- (b). (i) Show that the variation of gravity with height can be accounted for approximately by the following potential energy function

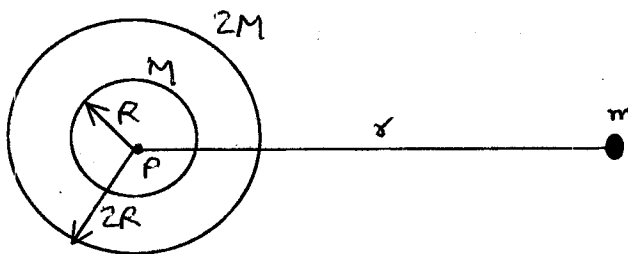
$$V = mgz \left( 1 - \frac{z}{r_e} \right)$$

in which  $r_e$  is the radius of the earth. **(8 marks)**

- (ii) Find the force given by the above potential function. From this, find the component differential equations of motion of a projectile under such a force. **(4 marks)**
- (iii) If the vertical component of the initial velocity is  $v_{0z}$ , how high does the projectile go? **(7 marks)**

- Q4. (a). State the three Kepler's laws. **(6 marks)**

- (b). The figure below shows a system of two uniform, thin-walled, concentric spherical shells. The smaller shell has radius  $R$  and mass  $M$ . The larger has radius  $2R$  and mass  $2M$ , and the point  $P$  is their common centre. A point mass  $m$  is situated at a distance  $r$  from  $P$ .



- (i) What is the gravitational force  $F(r)$  these shells exert on  $m$  in each of the three ranges of  $r$ :  $0 < r < R$ ,  $R < r < 2R$ ,  $r > 2R$ ? **(9 marks)**
- (ii) What is the gravitational potential energy of the point mass  $m$  when it is at  $P$ ? (Take the potential energy to be zero when  $m$  is infinitely far from  $P$ .) **(4 marks)**
- (iii) If the particle is released from rest very far away from the spheres, and assuming that the particle can pass freely through the walls of the shells, what is its speed when it reaches  $P$ ? **(6 marks)**

Q5. (a). Consider a one-dimensional elastic collision between a given incoming body  $A$  and a body  $B$  initially at rest. How would you choose the mass of  $B$ , in comparison to the mass of  $A$ , in order that  $B$  should recoil with

- (i) the greatest speed, (3 marks)
- (ii) the greatest momentum, and (4 marks)
- (iii) the greatest kinetic energy (5 marks)

(b). (i) Show that in a one-dimensional elastic collision the speed of the centre of mass of two particles,  $m_1$  moving with initial speed  $v_{1i}$  and  $m_2$  moving with initial speed  $v_{2i}$  is

$$v_{cm} = \left( \frac{m_1}{m_1 + m_2} \right) v_{1i} + \left( \frac{m_2}{m_1 + m_2} \right) v_{2i} \quad (5 \text{ marks})$$

(ii) Using the expressions that you obtain for the particles' speeds after collision,  $v_{1f}$  and  $v_{2f}$ , derive the corresponding result for  $v_{cm}$  after the collision. (8 marks)

Q6. (a). A named famous physicist who was fond of practical jokes, mounted a rapidly spinning flywheel in a suitcase which he gave to a porter with instructions to follow him. What happens when the porter is led quickly around a corner? Explain in terms of  $\tau = dL/dt$ .

(5 marks)

(b). Find the centre of mass of a solid hemisphere of uniform density  $\rho$  and radius  $a$ .

(10 marks)

(c). A rigid body moves in a uniform gravitational field. Show that the equation of motion of its centre of mass is

$$\ddot{\mathbf{r}}_{cm} = -g\mathbf{k}$$

where  $-\mathbf{k}$  is a unit vector in the direction of the force.

(10 marks)

@@@@@@@@ @ **END OF EXAMINATION** @@@@@@@@@@

THE UNIVERSITY OF ZAMBIA

*DEPARTMENT OF PHYSICS*

2001/2002 ACADEMIC YEAR FIRST SEMESTER EXAMINATIONS

P 261

ELECTRICITY AND MAGNETISM

**Instructions:** Answer FIVE(5) questions only. Time allowed: Three (3) hours.

Q1 (a) (i) If the electric field at a point is zero, would it be true to say that the electric potential at the same point is also zero? Explain your answer [ 2 marks ]

(ii) Three charged particles,  $Q_1$ ,  $Q_2$  and  $Q_3$  lie on a straight line and are separated by a distance  $d$  as shown in figure 1

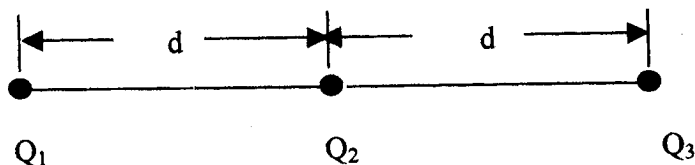


Figure 1

If charges  $Q_1$  and  $Q_2$  are held fixed, but  $Q_3$  is free to move but in fact remains stationary, how are  $Q_1$  and  $Q_2$  related? [ 3 marks ]

(b) Find the magnitude of the electric field at a distance  $a$  along a perpendicular bisector of a uniformly charged rod of length  $100a$  and total charge  $q$ , figure 2 [ 7 marks ]

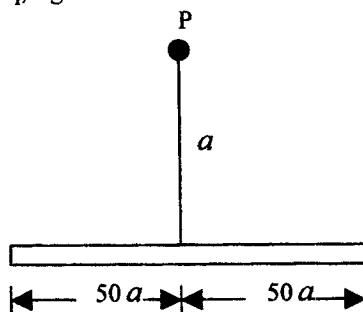


Figure 2

(c) Charge is uniformly distributed throughout an infinitely long cylinder of radius  $R$ .

(i) Show that the electric field,  $E$ , at a distance  $r$  from the cylinder axis (i.e.  $r < R$ ) is given by

$$E = \frac{\rho r}{2\epsilon_0}, \text{ where } \rho \text{ is the volume charge density} \quad [ 4 \text{ marks } ]$$

(ii) Repeat (i) for  $r > R$ ? [ 4 marks ]

Q2 (a) (i) Discuss, on a molecular level, the mechanism responsible for the modification of capacitance when a dielectric slab is inserted between the plates of a capacitor. [ 5 marks ]

(ii) A capacitor of capacitance  $C_1$  is charged to a charge  $Q$  and then isolated to avoid charge leakage. The capacitor is then connected in parallel with another capacitor of capacitance  $C_2$ . It is found that when equilibrium is established,  $C_1$  remains with only  $2/3$  of its initial charge. How are  $C_1$  and  $C_2$  related? [ 4 marks ]

- (b) A parallel plate capacitor is filled with two dielectrics of dielectric constants  $K_1$  and  $K_2$  as in figure 3. Show that the capacitance of this configuration is given by  $C = \frac{\epsilon_0 A (K_1 + K_2)}{d}$ , figure 3 [ 5 marks ]

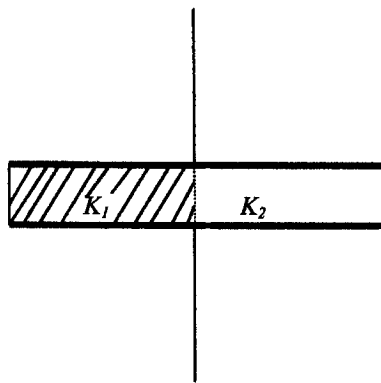


Figure 3

- (d) A parallel-plate capacitor is filled with two dielectrics of equal thicknesses as in figure 4

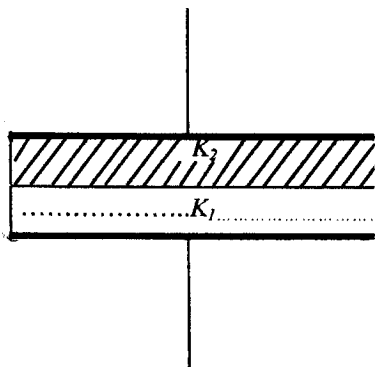


Figure 4

Show that the capacitance of this configuration is given by  $C = \frac{2\epsilon_0 A (K_1 K_2)}{d (K_1 + K_2)}$  [ 6marks ]

- Q3 (a) (i) A current of 10 amp flows in a wire of  $1\text{mm}^2$  cross section. If the density of charge carriers in the wire is  $10^{27}/\text{m}^3$ , find the average drift velocity of the electrons. [ 2marks ]
- (ii) A square aluminium rod is 1.0 m long and 5mm on edge. What is the resistance between its ends? What must be the diameter of a circular 1.0m copper rod if its resistance is to be the same [ 3 marks ]  
Take resistivity of aluminium,  $\rho_{Al} = 2.8 \times 10^{-8} \Omega\text{m}$  that of copper as  $\rho_{Cu} = 1.7 \times 10^{-8} \Omega\text{m}$   $20^\circ\text{C}$
- (b) (i) A point charge  $q$  is rotating in a circular orbit of radius  $r$ , with an angular velocity  $\omega$ . What is the current produced by this moving charge? [ 4marks ]

- (ii) An insulating disk of radius  $R$  as shown in figure 5, has a uniform charge density  $\sigma$  on its surface. It rotates with an angular velocity  $\omega$ . Find the total circulating current. (Hint: first write the expression for the current from the moving charge on a thin ring of radius  $r$  and width  $dr$ ). [ 6marks ]

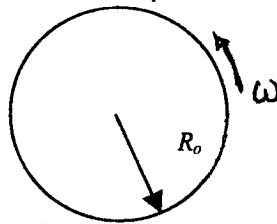


Figure 5

- (c) A piece of wire of length  $L$  has a radius  $r$  and a resistance  $R$ . What would the radius of a second wire made of the same material as the first but twice as long have to be if the second wire is to have the same resistance? [ 5 marks ]

- Q4 (a) The magnetic force,  $F$ , on an electron moving with a velocity  $v$  in a region containing a magnetic field of induction  $B$  is defined by the relation  $F = evB \sin \theta$  where  $e$  is the electronic charge and  $\theta$  is the angle between the velocity vector and the magnetic field. By carrying the argument further namely that a current in a conductor is nothing but a flow of electrons, show how this relation leads to the magnetic force on a current carrying wire. [ 4 marks ]

- (b) The wire shown in figure 6 carries a current  $i$ . What is the magnetic field  $B$  at the center C of the semi-circle arising from
- (i) the semi-circular segment of radius  $R$ ;
  - (ii) each straight segment of length  $l$ ;
  - (iii) the entire wire.
- [ 6marks ]

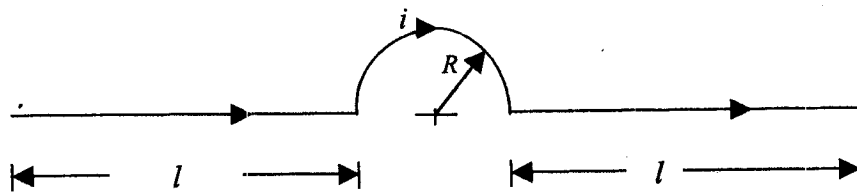


Figure 6

Question continued next page

- (c) (i) Figure 7 shows a long wire carrying a current of 30A. The rectangular loop carries a current of 20A. Calculate the resultant force acting on the loop given that  $a = 1.0\text{cm}$ ,  $b = 8.0\text{cm}$  and  $l = 30\text{cm}$  [ 4 marks ]

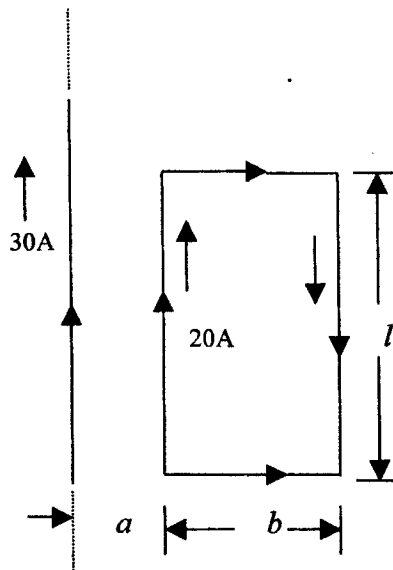


Figure 7

- (ii) Figure 7 shows a hollow cylindrical conductor of radii  $a$  and  $b$  which carries a current  $i$  uniformly spread over its cross-section. Show that the magnetic field  $B$  for points inside the body of the conductor

(i.e.  $a < r < b$ ) is given by 
$$B = \frac{\mu_0 i}{2\pi(b^2 - a^2)} \frac{(r^2 - a^2)}{r}$$

[ 6 marks ]

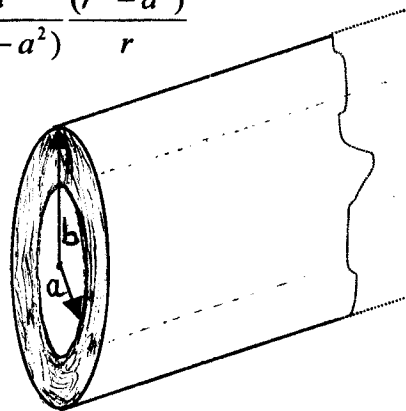
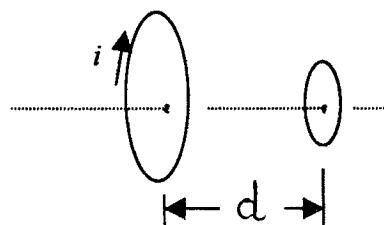


Figure 7

- Q5 (a) Two conducting loops face each other a distance  $d$  apart, figure 8. An observer sights along their common axis from left to right. If a clockwise current  $i$  is suddenly established in the larger loop,

Figure 8



- (i) What is the direction of the induced current in the smaller loop? [ 2 marks ]
- (ii) What is the direction of the force (if any) that acts on the smaller loop? [ 2 marks ]

- (b) A metal rod of length  $L$  moves on a stationary conducting loop with a velocity  $v$  in a direction perpendicular to its axis and to a constant magnetic field  $B$ , figure 9. Derive an expression for the induced *emf* in the rod
- (i) using the magnetic force on charges by virtue of the motion of the rod [ 5marks ]
- (ii) by means of Faraday's law of induction and compare with (i) [ 5marks ]

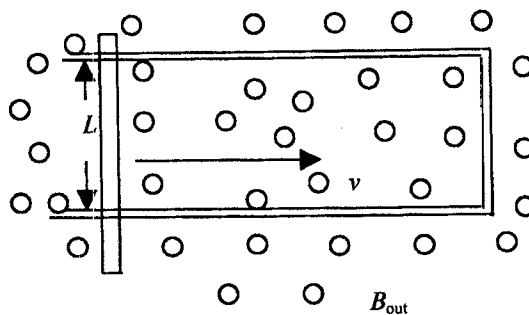


Figure 9

- (c) A current in a solenoid of 100 turns/cm changes according to the equation  $i = 3.0t + 1.0t^2$ , where  $i$  is in amperes and  $t$  is in seconds.
- (i) Plot quantitatively the induced *emf* in the coil for  $t = 0$  to  $t = 4.0s$
- (ii) If the resistance of the coil is  $0.15 \Omega$ , what is the instantaneous current in the coil at  $t = 2.0s$ ? [ 6marks ]

- Q6 (a) What is meant by the r.m.s. value of an alternating voltage or current? [ 2 marks ]
- (b) A 230V, 50Hz alternating p.d. supplies an inductor coil that has an inductance of  $0.06 H$  in series with a capacitance of  $6.8 \mu F$ . The effective resistance of the circuit is  $2.5 \Omega$ . Find the
- (i) total impedance,  $Z$ , of the series circuit; [ 6marks ]
- (ii) overall current flowing in the circuit; [ 2 marks ]
- (iii) phase angle between the current and the voltage [ 3 marks ]

- (iv) the resonance frequency for the series circuit [ 3 marks ]
- (v) the current at resonance [ 2 marks ]
- (c) State one important application of resonance that occurs in circuits such as the one cited in (b) [ 2 marks ]

Q7 (a)(i) Describe the construction of a simple form of transformer. [ 4marks ]

(ii) If the secondary coil is on an open circuit, explain, without calculation, the effect on the current flowing in the primary of [ 2marks ]

- a fall in the supply frequency and
- a reduction in the number of primary turns [ 2 marks ]

(b) Calculate the current which flows in a resistance of  $3\ \Omega$  connected to a secondary coil of 60 turns if the primary has 1200 turns and is connected to a 240-V a.c. supply, assuming that all the magnetic flux in the primary passes through the secondary and that there are no other losses. [ 5 marks ]

(c) Figure 10 shows an inductance,  $L$  connected in series with a resistance,  $R$ . If the switch,  $S$ , is suddenly connected from terminal  $A$  to  $B$ , show that the current in the circuit does not go-off instantaneously but decays according to the equation

$$I = I_0 e^{-t/\tau} \text{ where } I_0 \text{ is the maximum current in the circuit and } \tau = L/R, \text{ is the time constant of the circuit.}$$

[ 5 marks ]

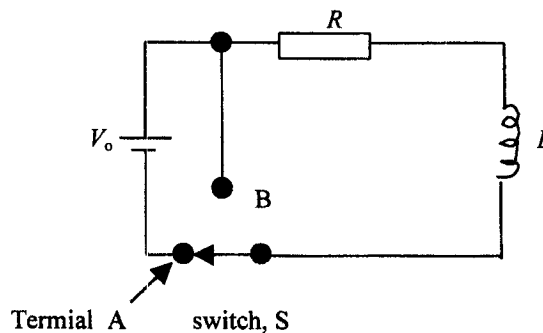


Figure 10

**END OF EXAMINATION**

THE UNIVERSITY OF ZAMBIA  
*DEPARTMENT OF PHYSICS*  
2001/2002 ACADEMIC YEAR FIRST SEMESTER EXAMINATIONS  
P 341  
INTRODUCTORY ELECTRONICS

**Instructions:** Answer four(4)questions only. Time allowed: Three (3) hours.

Q1 (a) Find the unknown currents in the circuit shown, figure 1. [ 5 marks ]

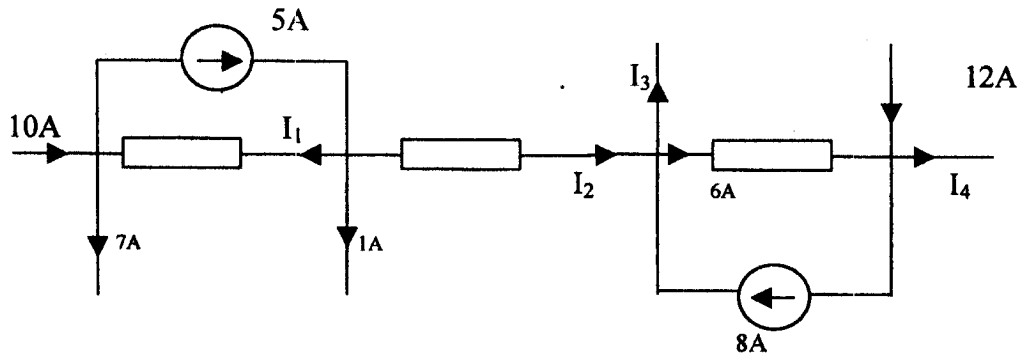


Figure 1

(b) Use Thevenin's theorem to find the current in a resistance load connected between the terminals A and B of figure 2 if the load is  $2\Omega$ . [ 7 marks ]

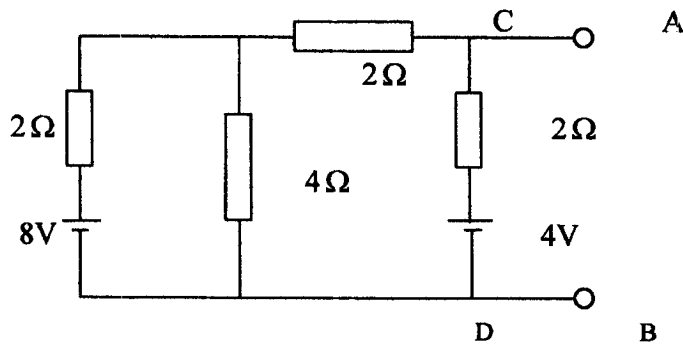
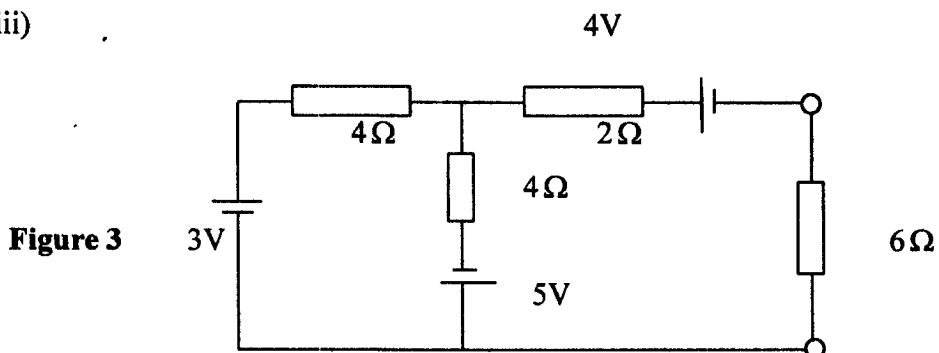


Figure 2

(c) (i) State Norton's theorem as applied to d.c. circuits clearly defining the important terms you use. [ 3 marks ]

(ii) For the circuit shown in figure 3, calculate the current in the  $6\Omega$  resistance using Norton's theorem [ 10 marks ]

(iii)



(ii) Suppose a voltage of  $10\text{ V}$  at a frequency of  $1500\text{ Hz}$  is fed into the filter cited in Q3a(i), calculate the output voltage. [ 3 marks ]

(b) (i) Two impedances  $Z_1 = 2 + j6$  and  $Z_2 = 6 - j12\ \Omega$  are connected so that they are additive. Find the resultant impedance in polar form. [ 3 marks ]

(iii) Given two currents,  $i_1 = 10\sin(\omega t + \frac{\pi}{4})$  and  $i_2 = 5\cos(\omega t - \frac{\pi}{2})$ . Find the r.m.s value of  $i_1 + i_2$  using complex number representation. [ 5 marks ]

(c) In figure 4, calculate

- (i) the current in the circuit
- (ii) the voltage drops across the following impedance combinations:  
 $R_1$  and  $L$  (i.e  $Z_1$ ),  $R_2$  and  $C$  (i.e.  $Z_2$ )
- (iii) the real power consumed by the circuit. ( Hint: use complex representation)
- (iv) the phase angle between current and voltage [ 8 marks ]

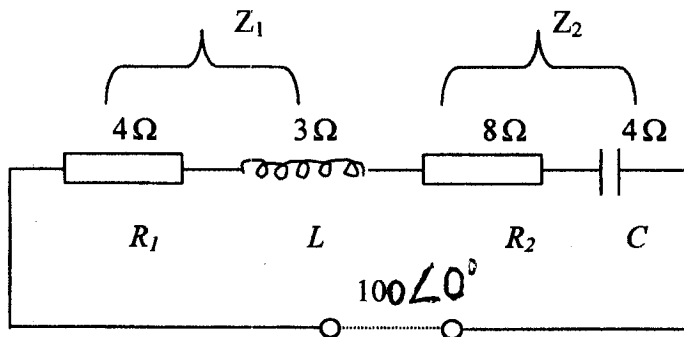


Figure 4

Q4 (a) Distinguish between the following:

- (i) Extrinsic and intrinsic semi-conductors [ 2 marks ]
- (ii) Donors and acceptors [ 2 marks ]
- (iii) n-type and p-type semi-conductors [ 2 marks ]
- (iv) Majority and minority carriers [ 2 marks ]

Q2 (a) The input and output sinusoidal wave-forms from an electronic device are described as follows:

**Input wave-form:** distance between two consecutive peaks = 2cm

**Output wave-form:** distance between two consecutive peaks = 1 cm

If the distance between two consecutive peaks of the two wave forms at one instant is 5 mm.

Find

(i) the frequency of each of the two wave-forms; [ 3 marks ]

(ii) the phase difference between the two wave-forms at the instant mentioned above

Take **time base setting** to be at **10mSec/cm** [ 3 marks ]

(b) The standard form of a sinusoidal alternating current is  $i = I_m \sin \omega t = I_m \sin \theta$ .

(i) Show that the average value of this quantity over a complete cycle is zero and that

(ii) The average value of the same quantity over half a cycle is **not** zero and is given by

$$I_{AV} = \frac{2I_m}{\pi} \text{ where } I_m \text{ is the peak value of the current. [ 8 marks ]}$$

(iii) By integrating the squares of the instantaneous values of the current over a complete cycle and averaging them, one obtains the r.m.s of the current. Show that this quantity is

$$\text{given by } I_{r.m.s} = \frac{I_m}{\sqrt{2}} \text{ where } I_m \text{ is as defined above [ 5 marks ]}$$

(c) An alternating current varying sinusoidally with a frequency of 50Hz has an rms value of 20A .

(i) Write down the equation for the instantaneous value of the current [ 2 marks ]

(ii) Write down the instantaneous value of the current 0.0125 second after passing through a positive maximum value. [ 2 marks ]

(iii) At what time, measured from a positive maximum value, will the instantaneous current be 14.14 Amps? [ 2marks ]

Q3 (a) (i) For a load resistance  $R = 10k\Omega$ , design a simple  $RL$  low-pass filter with a cutoff frequency at  $f = 1000Hz$ . Draw its circuit and sketch the transmission function (i.e.  $\frac{V_o}{V_i}$ ) versus signal frequency. [ 6 marks ]

(b) (i) What is meant by the term “doping” as applied in the manufacture of semiconductors?

[ 2marks ]

(ii) Estimate the relative concentration of silicon atoms (atoms/m<sup>3</sup>) and **electron-hole pairs** at room temperature (300K) and calculate the intrinsic resistivity,  $\rho_i$ .

[ 7 marks ]

Necessary data: **Avogadro's number** =  $6.022 \times 10^{23}$  atoms/mole

Molar mass of silicon = 28.09 g/mole

Density of silicon  $\rho_{si} = 2.33 \times 10^6$  g/cm<sup>3</sup>

Intrinsic concentration of electron-hole pairs at 300K for silicon =  $1.5 \times 10^{16}$  per cubic meter

(c) (i) How does the operation of a **Zener diode** differ from the operation of the **ordinary diodes**?

[ 2 marks ]

(ii) In the circuit shown below, figure 5, the supply voltage has to vary from a minimum of 80 volts to a maximum of 120 volts. Find the maximum and minimum values of Zener diode current.

[ 6 marks ]

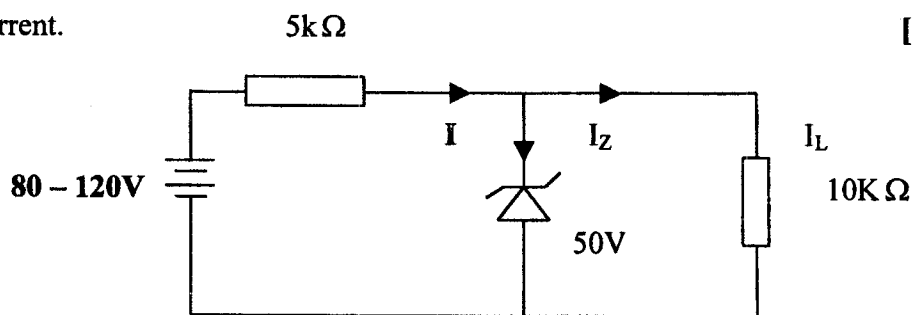


Figure 5

Q5 (a) (i) Distinguish between **input characteristics** and **output characteristics** of a bipolar junction transistor. (Hint, use graphical sketches)

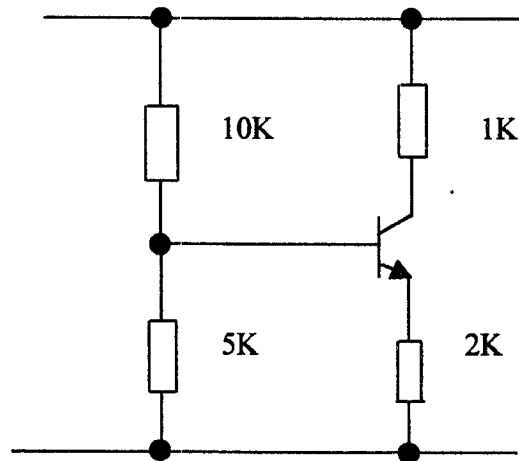
[ 3marks ]

(ii) How must a transistor be biased for faithful signal amplification?

[ 2marks ]

(b) (i) Using the voltage divider bias method, draw the d.c. load line for the transistor amplifier shown in figure 6. Assume the transistor to be of silicon with  $\beta = 100$

[ 10 marks ]



**Figure 6**

Q5(b) (ii) Distinguish between the bipolar junction transistor (BJT) and the Field Effect Transistor (FET) in relation to

- charge carriers
- transistor control
- input impedance and
- noise levels

**[ 5 marks ]**

(c) A Junction Field Effect Transistor (JFET) circuit is shown below, figure 7. Find the values of

- (i)  $V_{DS}$  and
- (ii)  $V_{GS}$

What type of channel is implemented in it? Explain your answer **[ 5 marks ]**

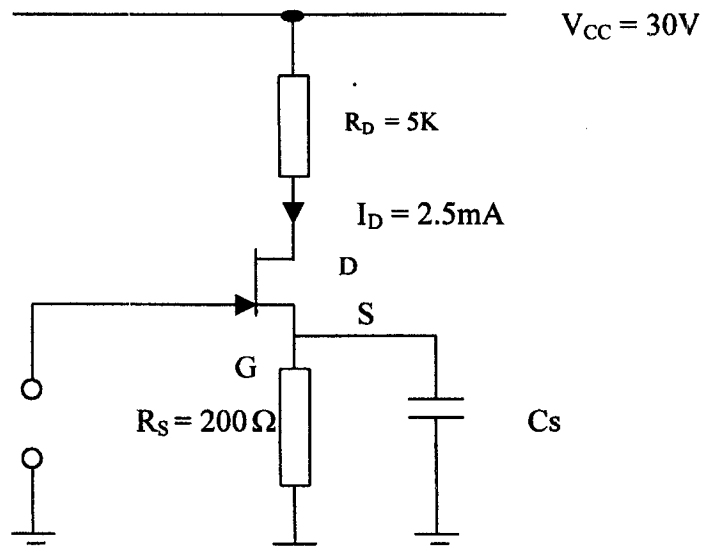


Figure 7

- Q6 (a) (i) By means of well labeled diagrams including definitions, distinguish between **positive and negative voltage feed back** and state what is meant by the terms **open loop and closed loop gains**. [ 6 marks ]
- (ii) Discuss five (5) **advantages** of negative feedback [ 5 marks ]
- (b) Using the negative voltage feedback amplifier drawn in part (a), derive an expression giving the **closed loop gain** of the amplifier and show that to a good approximation, this this quantity does not depend on transistor parameters but on the values of passive elements such as resistors. [ 8 marks ]
- (c) When negative feedback is applied to an amplifier of open loop gain 100, the overall gain drops to 50.
- (i) Calculate the fraction,  $\beta$ , of the voltage fed back to the input ;
- (ii) If this fraction is maintained, calculate the value of the amplifier gain required if the overall gain of the amplifier stage is to be 75 [ 6 marks ]

**END OF EXAMINATION**

UNIVERSITY OF ZAMBIA  
PHYSICS DEPARTMENT  
UNIVERSITY EXAMINATIONS  
FIRST SEMESTER 2001/2002

**P351**

INTRODUCTION TO QUANTUM MECHANICS

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**Time: Three hours**

**Answer: Any four questions**

**All questions carry equal marks**

**Total marks: 100**

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1. (a) Two spaceships, each measuring 100 m in its own rest frame, pass by each other travelling in opposite directions. Instruments on spaceship *A* determine that the front end of spaceship *B* requires  $5 \times 10^{-6}$  s to traverse the full length of *A*.

(i) What is the relative velocity of the two spaceships? [4]

(ii) A clock in the front end of *B* reads exactly one o'clock as it passes by the front end of *A*. What will the clock read as it passes by the rear end of *A*? [5]

(b) A particle of energy  $E$  is incident on the potential barrier  $V(x) = 0$  for  $x < 0$ ,  $V(x) = V_0$  for  $0 \leq x \leq a$  and  $V(x) = 0$  for  $x > a$ . If  $E < V_0$ ,

(i) show that for this case, the time-dependent Schrodinger equation separates into two ordinary differential equations and deduce these equations. [6]

(ii) Show that there is a finite probability of finding the particle inside the barrier. [4]

(iii) Show that the particle can be found on the other side of the barrier. [4]

(iv) If this had been a classical situation, what would have been the probability of the particle being found on the far side of the barrier? [2]

2. (a) A particle is in the potential well  $V(x) = V_0$  for  $0 \leq x \leq L$  and  $V(x) = \infty$  for  $x < 0$ ,  $x > L$ .

(i) Obtain the normalised eigenfunctions and the corresponding eigenvalues for this particle. [11]

(ii) Show that the eigenfunctions are orthogonal to one another. [4]

(b) The unnormalised wave function of the particle in (a) above is known to be

$$\psi(x) = \phi_1(x) + 3\phi_2(x)$$

where  $\phi_1(x)$  is the ground state eigenfunction and  $\phi_2(x)$  is the first-excited state eigenfunction of the potential well.

(i) Obtain the probability that a measurement of the energy of the particle yields

(1) the ground state energy, (2) the first-excited state energy (3) the second excited state energy. [6]

(ii) If the particle has a mass of  $9.1 \times 10^{-31}$  kg, what is the expectation value of the energy in eV? Assume that  $L = 10^{-10}$  m. and  $V_0 = 12$  eV. Note that  $e = 1.6 \times 10^{-19}$  C and  $\hbar = 1.054 \times 10^{-34}$  Js. [4]

3. (a) The time-independent Schroedinger equation for a one-dimensional harmonic oscillator is

$$-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \psi(x) + \frac{1}{2} m \omega^2 x^2 \psi(x) = E \psi(x)$$

(i) Prove that the eigenfunctions  $\psi(x)$  are of definite parity. [8]

(ii) Hence or otherwise, show that the expectation value of the position always vanishes. [2]

(b) The potential energy of a three-dimensional harmonic oscillator is

$$V(x, y, z) = \frac{1}{2} m \omega_1^2 x^2 + \frac{1}{2} m \omega_2^2 y^2 + \frac{1}{2} m \omega_3^2 z^2$$

(i) Deduce the energy eigenvalues of this oscillator. [11]

(ii) If  $\omega_1 = \omega_2$  and  $\omega_3 > \omega_1$ , deduce the degeneracies of the first three energy levels. [4]

4. (a) (i) Define the Hermitian conjugate of an operator. [2]

(ii) Define a Hermitian operator and explain the importance of such operators in quantum mechanics. [3]

(iii) Prove explicitly that the operator

$$H = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{1}{2} k x^2$$

is Hermitian. Assume that the operator acts on functions which vanish at  $x = \pm\infty$ . [4]

(b) (i) Prove that if two observables commute, they have simultaneous eigenfunctions. [4]

(ii) Prove that if the eigenvalue spectrum of a Hermitian operator is non-degenerate, eigenfunctions belonging to different eigenvalues are orthogonal. [4]

(c) (i) What is the interpretation of the wave function? [2]

(ii) Why are the wave properties of matter not manifest at macroscopic (classical) level? [2]

(iii) Justify the boundary conditions which apply to a wave function and its derivative at the interface between two regions. [2]

(iv) Explain what is meant by a stationary state. [2]

5. (a) (i) Obtain the operator for the angular momentum in Cartesian coordinates. [3]

(ii) The components of the angular momentum obey the commutation relations

$$[L_i, L_j] = i\hbar L_k \text{ with } i, j, k \text{ taken in cyclic order.}$$

Prove this relation for any pair of components. [5]

(c) For spin 1/2, the spin operators

$$[S_x] = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, [S_y] = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \text{ and } [S_z] = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

represent the spin components in the  $x$ ,  $y$  and  $z$  directions respectively. Thus, the spin operator is

$$[\mathbf{S}] = \hat{\mathbf{i}}[S_x] + \hat{\mathbf{j}}[S_y] + \hat{\mathbf{k}}[S_z]$$

(i) Show that the spin component in the general direction defined by the unit vector  $\hat{\mathbf{n}} = \hat{\mathbf{i}} \sin \theta \cos \varphi + \hat{\mathbf{j}} \sin \theta \sin \varphi + \hat{\mathbf{k}} \cos \theta$  is given by

$$[S_{\hat{\mathbf{n}}}] = [\mathbf{S}] \cdot \hat{\mathbf{n}} = \frac{\hbar}{2} \begin{pmatrix} \cos \theta & \sin \theta e^{-i\varphi} \\ \sin \theta e^{i\varphi} & -\cos \theta \end{pmatrix} \quad [2]$$

(ii) Prove that the eigenvalues of this operator are  $\pm \hbar/2$ . [5]

(iii) Prove that the normalised eigenvectors of this operator are

$$[\chi_{+1/2}] = \begin{pmatrix} \cos \theta/2 \\ \sin \theta/2 e^{i\varphi} \end{pmatrix} \text{ and } [\chi_{-1/2}] = \begin{pmatrix} \sin \theta/2 \\ -\cos \theta/2 e^{i\varphi} \end{pmatrix}$$

for the eigenvalues  $\pm \hbar/2$  respectively. [8]

(iv) Prove that the eigenvectors are orthogonal. [2]

6. (a) The Schroedinger equation for two particles can ultimately be reduced to the equation for the internal motion of the particles

$$-\frac{\hbar^2}{2\mu} \nabla^2 \Psi(\mathbf{r}) + V(\mathbf{r})\Psi(\mathbf{r}) = \varepsilon\Psi(\mathbf{r})$$

where  $\mu$  is the reduced mass and  $\mathbf{r}$  is the position vector of one particle relative to the other. Given that in spherical polar coordinates the Laplacian is

$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \left[ \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} \right]$$

and that

$$L^2 = -\hbar^2 \left[ \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} \right]$$

show that for the central potential  $V(\mathbf{r}) = V(r)$ , the Schroedinger equation can be separated into the radial equation

$$\left(\frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr}\right)\psi_{nlm}(r) - \frac{2\mu}{\hbar^2}[V(r) + \frac{l(l+1)\hbar^2}{2\mu r^2}]\psi_{nlm}(r) + \frac{2\mu\epsilon}{\hbar^2}\psi_{nlm}(r) = 0$$

and Legendre's equation

$$L^2 Y_{lm}(\theta, \varphi) = l(l+1)\hbar^2 Y_{lm}(\theta, \varphi) \quad [11]$$

(b) A particle of mass  $m$  and charge  $-q$  moves in the Coulomb potential  $V(r) = -q^2/4\pi\epsilon_0 r$  of another charge  $q$  fixed at the origin.

(i) Find the value of  $\alpha$  for which the function

$$\psi(r) = Ae^{-\alpha r}$$

is an eigenfunction of the system. [8]

(ii) Deduce the energy eigenvalue corresponding to this state. [3]

(iii) Calculate the expectation value of the radial coordinate  $r$  and hence give an estimate of the size of the hydrogen atom. [3]

Assume the following parameters:  $e = 1.6 \times 10^{-19} \text{C}$ ,  $m_e = 9.1 \times 10^{-31} \text{kg}$ ,  $m_p = 1836m_e$ ,  $\epsilon_0 = 8.85 \times 10^{-12} \text{C}^2/\text{N.m}^2$ ,  $\hbar = 1.054 \times 10^{-34} \text{Js}$ .

Note that  $\int_0^\infty x^n e^{-x} dx = n!$ .

\*\*\*\*\*END OF EXAMINATION\*\*\*\*\*

THE UNIVERSITY OF ZAMBIA  
PHYSICS DEPARTMENT  
UNIVERSITY EXAMINATIONS  
FIRST SEMESTER – 2001/2002  
P361 - ELECTROMAGNETISM

TIME: 3 HOURS

MAX MARKS: 100

ATTEMPT ANY FOUR QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

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You may use the following information:

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ farad/meter}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$$

$$\int \sec \theta d\theta = \ln(\sec \theta + \tan \theta)$$

The vector identities

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = -\vec{\nabla}^2 \vec{A} + \vec{\nabla}(\vec{\nabla} \cdot \vec{A})$$

$$\vec{\nabla} \cdot (f\vec{A}) = f\vec{\nabla} \cdot \vec{A} + \vec{A} \cdot \vec{\nabla} f$$

$$\vec{\nabla} \times (f\vec{A}) = \vec{\nabla} f \times \vec{A} + f(\vec{\nabla} \times \vec{A})$$

$$\vec{\nabla} \cdot (\vec{\nabla} V) = (\vec{\nabla} V)^2 + V \vec{\nabla}^2 V$$

The vector  $\vec{r}$  is directed from  $P'(x', y', z')$  to  $P(x, y, z)$ . If  $P'$  is fixed and  $P$  is allowed to move, then the gradient under this condition is given by

$$\vec{\nabla} \left( \frac{1}{r} \right) = -\frac{\hat{r}}{r^2}$$

If  $P$  is fixed and  $P'$  is allowed to move, then the gradient is

$$\vec{\nabla} \left( \frac{1}{r} \right) = \frac{\hat{r}}{r^2}$$

In spherical coordinates  $(r, \theta, \phi)$

$$\bar{\nabla}f = \frac{\partial f}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} \hat{\phi}$$

$$\bar{\nabla}^2 \equiv \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2}$$

$$\bar{\nabla} \cdot \bar{A} \equiv \frac{2}{r} A_r + \frac{\partial A_r}{\partial r} + \frac{A_\theta}{r} \cot \theta + \frac{1}{r} \frac{\partial A_\theta}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\bar{\nabla} \times \bar{A} = \frac{\hat{r}}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (\sin \theta A_\phi) - \frac{\partial A_\theta}{\partial \phi} \right] + \frac{\hat{\theta}}{r} \left[ \frac{1}{\sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r} (r A_\phi) \right] + \frac{\hat{\phi}}{r} \left[ \frac{\partial}{\partial r} (r A_\theta) - \frac{\partial A_r}{\partial \theta} \right]$$

In cylindrical coordinates  $(\rho, \phi, z)$

$$\bar{\nabla}f = \hat{\rho} \frac{\partial f}{\partial \rho} + \hat{\phi} \frac{1}{\rho} \frac{\partial f}{\partial \phi} + \hat{z} \frac{\partial f}{\partial z}$$

$$\bar{\nabla} \cdot \bar{A} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\rho) + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\bar{\nabla} \times \bar{A} = \hat{\rho} \left( \frac{1}{\rho} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) + \hat{\phi} \left( \frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \right) + \hat{z} \left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\phi) - \frac{1}{\rho} \frac{\partial A_\rho}{\partial \phi} \right]$$

$$\nabla^2 f = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial f}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

For any arbitrary vector  $\bar{A}$  and a surface S bounding a volume  $\tau$

$$\int_{\tau} (\bar{\nabla} \times \bar{A}) d\tau = - \int_S \bar{A} \times d\bar{a}$$

Poisson's Equation

$$\bar{\nabla}^2 V = -\frac{\rho}{\epsilon_0}$$

For a long solenoid of length  $L$ , the magnetic induction  $B = \frac{\mu_0 NI}{L}$  inside  
 $= 0$  outside

The vector potential at a point due to a current carrying conductor

$$\vec{A} = \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{l}}{r}$$

The magnetic induction

$$\vec{B} = \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{l} \times \hat{r}}{r^2}$$

$$\vec{B} = \bar{\nabla} \times \vec{A}$$

Vector potential due to a magnetic dipole

$$\vec{A} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

The magnetic induction at a point on the axis of a circular current carrying loop is

$$B = \frac{\mu_0 I a^2}{2(a^2 + z^2)^{3/2}}$$

The Maxwell's Equations are

$$\bar{\nabla} \cdot \vec{D} = \rho_f$$

$$\bar{\nabla} \cdot \vec{B} = 0$$

$$\bar{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

$$\bar{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

Q.1 (a) Use the expression

$$W = \frac{1}{2} \int_{\tau} V \rho d\tau$$

for the potential energy of a charge distribution and Poisson's equation to show that the energy density associated with an electric field in vacuum is given by

$$W = \frac{\epsilon_0}{2} \int_{\tau} E^2 d\tau$$

Give adequate reasoning, wherever necessary. (12 marks)

(b) Use the expression for the energy density in (a) above to show that the electric potential energy of a sphere of radius  $R$  carrying a total charge  $Q$  uniformly distributed over its volume is given by

$$W = \frac{3Q^2}{20\pi\epsilon_0 R}$$

(13 marks)

Q.2 A long cylindrical shell has inner and outer radii of  $a$  and  $b$ , respectively. The space between the two radii is filled with a class A dielectric of relative permittivity  $\epsilon_r$ . A charge of uniform linear density  $\lambda$  Coulomb/meter is distributed along its axis.

- (a) Use the generalized form of Gauss' law to calculate  $\vec{D}$  and  $\vec{E}$  at a point
- within the dielectric
  - outside the cylindrical shell

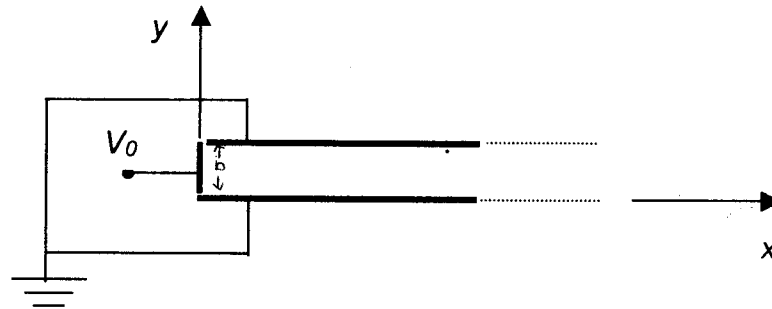
(8 marks)

(b) Calculate the bound surface charge density on the inner as well as the outer surface of the cylindrical shell. (8 marks)

(c) Calculate the bound volume charge density at a point within the dielectric. (4 marks)

(d) Show that the net induced charge on the cylindrical shell is zero. (5 marks)

- Q.3. As shown in the figure, two semi-infinite grounded parallel electrodes extend to infinity in the  $+x$  and  $\pm z$  directions. At  $x = 0$  they terminate at a potential  $V_0$ . Write down the boundary conditions for the problem and use the method of separation of variables to solve the Laplace equation. Hence obtain the general solution satisfying the boundary conditions



(25 marks)

- Q. 4 (a) Define magnetization and express it in terms of the average dipole moment per atom. (2 marks)
- (b) Obtain an expression for the vector potential at an exterior point due to a magnetized material. Hence show that a magnetized material is equivalent to a surface current density and a volume current density. (10 marks)
- (c) The magnetization of iron can contribute a maximum of 2 teslas to the magnetic induction in iron. If each electron can contribute a magnetic moment of  $9.27 \times 10^{-24}$  A-m<sup>2</sup>, how many electrons per atom on the average contribute to the magnetization? (Density of iron is 7870 Kg/m<sup>3</sup> and atomic weight 55.8). (5 marks)
- (d) A long solid cylinder is uniformly magnetized with magnetization  $\vec{M}$  in the direction parallel to its axis.
- Calculate the values of the volume and the surface currents.
  - What is the value of  $\vec{B}$  and  $\vec{H}$  inside the cylinder? (4+4 marks)

- Q.5. A conducting sphere of radius R is charged to a potential V and spun about a diameter at an angular velocity  $\omega$ . If  $\theta$  is the spherical polar angle of a point on the surface when the axis of rotation is taken as the z-axis and the centre of the sphere as the origin,

- (a) Show that the surface current density is

$$\lambda = \epsilon_0 \omega V \sin \theta$$

(9 marks)

(b) Show that the dipole moment is

$$\vec{m} = \frac{4}{3} \pi R^3 M \hat{k}$$

(8 marks)

(c) Calculate the numerical value for the dipole moment for a sphere of 10 cm radius charged to a potential of 10 kilovolts and spinning at  $1 \times 10^4$  turns/minute.

(4 marks)

(d) What current flowing through a loop 10 cm in diameter would give the same dipole moment?

(4 marks)

Q.6 (a) Use Maxwell's equations for the electromagnetic field to show that in an isotropic, linear and homogeneous medium, the  $\vec{E}$  and  $\vec{H}$  vectors satisfy attenuated wave equations.

$$\nabla^2 E - \epsilon\mu \frac{\partial^2 E}{\partial t^2} - \mu\sigma \frac{\partial E}{\partial t} = \nabla \left( \frac{\rho_f}{\epsilon} \right)$$

$$\nabla^2 H - \epsilon\mu \frac{\partial^2 H}{\partial t^2} - \mu\sigma \frac{\partial H}{\partial t} = 0$$

(8 marks)

(b) Assuming the plane wave representation  $E = E_0 \exp j(\omega t - kz)$  show that for a good conducting medium, the wave vector is given by

$$k = \left( \frac{\mu\sigma\omega}{2} \right)^{\frac{1}{2}} (1 - j)$$

(7 marks)

(c) Hence show that the  $\vec{E}$  vector is attenuated as the wave travels inside the medium with characteristic distance, the "skin depth"  $\delta$ , given by

$$\delta = \left( \frac{2}{\mu\sigma\omega} \right)^{\frac{1}{2}}$$

(4 marks)

(d) For copper,  $\sigma = 5.8 \times 10^7$  mho/m,  $\mu_r = 1$ . At what depth will the intensity of electromagnetic radio waves of frequency 3 Gigahertz incident on a copper plate reduce to 1% of its initial intensity?

(6 marks)

..... **END OF THE EXAMINATION** .....

**THE UNIVERSITY OF ZAMBIA**  
**PHYSICS DEPARTMENT**  
**UNIVERSITY EXAMINATIONS-2001/2**  
**P-401**  
**(Computational Physics-II)**

**Time:** Three Hours

**Maximum Marks:** 100

**Answer:**(i) Question 1 is compulsory.  
(ii) Any Three questions from 2,3,4,5 and 6.

*All questions carry equal marks.*  
(Marks are shown in brackets).

---

**Q.1: (a) State the errors, if any, in the following: [5]**

- (i) `a = b*sin(2x)`
- (ii) `open(2,file ='exam.data')`
- (iii) `character set*4`  
`set = 'start'`  
`tp = set`
- (iv) `character name*4`  
`name = 'phys'`  
`write(*,10) name`  
`10 format(2x, a6)`
- (v) `z = 1/3`  
`y = log(z)`

**(b) (i) Explain what you understand by non-linear least squares fit of data. [1]**

**(ii) Explain the difference between an interpolation and fitting the data by the least squares method. [2]**

**(c) An input file named 'atom.dat' contains the atomic number and the corresponding chemical symbol of an element in each line in the following form:**

1 , H  
2 , He  
....etc.

The file covers all the elements of the periodic table.

Write a program such that when the user enters the atomic number of a

chemical element, the program produces the appropriate chemical symbol. [7]

(d) You are given a subroutine  $QUAD(a,b,c,x1,x2,i)$  for solving a quadratic equation

$$ax^2 + bx + c = 0$$

where  $x1, x2$  and  $i$  are returning values, and  $a, b, c$  are constant coefficients.

(i) if  $i = 1$ , then  $x1$  and  $x2$  are real roots,

(ii) if  $i = -1$ , then the roots are imaginary with  $x1 =$  real part and  $x2 =$  imaginary part.

Write a program which inputs the parameters  $a, b, c$  and uses the given subroutine  $QUAD(a,b,c,x1,x2,i)$  and prints the real or imaginary roots with appropriate titles distinguishing the two roots. [5]

(e) The input file 'STUDENT.DAT' contains the following data [5]

column 1-12 = Name of the student.

column 14 = Gender of the student as character M or F .

column 16-17 = marks obtained in the course P401

column 19-20 = marks obtained in the course P421

column 21-22 = marks obtained in the course P441

Write a program to read the data from the input file and to create an output file 'STUDENT.OUT' with the following columns along with the titles.

Name of Student, Gender, Marks: P401, Marks:P421, Marks:P441

Assume that the total number of candidates is 20.

**Q.2:** The gravitational force  $F$  between two bodies of masses (in units of kg)  $m_1$  and  $m_2$  is given by the formula

$$F = \frac{Gm_1m_2}{r^2}$$

where  $G = 6.672 \text{ E-11 Nm}^2\text{kg}^{-2}$  and

$r =$  distances between bodies in meters.

The kinetic energy given to a body under a constant acceleration due to some external force  $F(r)$  is given by the integral

$$KE = \int_{r_{\min}}^{r_{\max}} F(r) dr$$

Write

(i) a subroutine program with the name  $EF(m1,m2,r)$  to calculate  $F(r)$ , [5]

(ii) a subroutine program with the name  $DINTEGRAL(rmin,rmax,value)$  to calculate the above integral using  $EF(m1,m2,r)$ . Here  $rmin$  and  $rmax$  are lower and upper limits of integration and 'value' is the returning value of the integral. [10]

Use the trapezoidal rule for integration:

$$\int_a^b f(x)dx = \frac{h}{2} [f(a) + f(b) + 2 \sum_{i=1}^{n-2} f(x_i)]$$

where  $h = (b-a)/n$ ,  $n =$  number of points in the interval  $(a,b)$ .

- (iii) a main program to calculate the kinetic energy gained by a falling body of mass  $m$  to earth from  $rmax$  using the DINTEGRAL subroutine. The values for  $m$  and  $rmax$  are to be input from the default device. Include in your program an output file with the name KEGAIN.OUT where the output will contain  $mass m$ ,  $rmax$  and  $KE$ , the kinetic energy gained with appropriate titles. [10]

Given: Radius of the earth = 6378.5E+03 m.  
Mass of the earth = 5.976E+24 kg.

**Q.3.** The sample data for ambient room light obtained by an optical multi-channel analyser are given in the following table

$\lambda$	Intensity $I_{exp}$	$\lambda$	Intensity $I_{exp}$
4357.84	40.0	4358.39	168.0
4357.94	41.0	4358.49	111.0
4358.04	7.0	4358.59	64.0
4358.14	66.0	4358.69	51.0
4358.24	129.0	4358.79	46.0
4358.34	165.0	4358.84	41.0

It is proposed to fit the data to a theoretical Lorentzian line shape function given by

$$I(\lambda_j) = \frac{I_0}{1 + 4(\lambda_j - \lambda_0)^2 / \Gamma^2}$$

The merit function is given by

$$\chi^2 = \sum_{j=1}^N [I_{exp} - B - I(\lambda_j)]^2$$

where  $I_{exp}$  = measured intensities for corresponding  $\lambda_j$ ,

$$B = 30.0 \quad \text{and} \quad I_0 = 168.5.$$

By minimizing  $\chi^2$  the best values of  $\lambda_0$  and  $\Gamma$  are obtained.

Write one program to do the following:

- (i) to create a data file for the data given above which are to be read from the default device (keyboard) with the file name 'INTEN.DAT' [5]
- (ii) to calculate  $\chi^2$  for  $\lambda_0$  in the range,  $4358.3 \leq \lambda_0 \leq 4358.4$  with increment  $\Delta\lambda=0.01$  and for  $\Gamma$  in the range,  $0.1 \leq \Gamma \leq 0.6$  with increment  $\Delta\Gamma=0.05$  and [12]
- (iii) to tabulate, for each  $\lambda_0$ , values of  $\Gamma$  and corresponding  $\chi^2$  on an output file 'INTEN.OUT' [8]

**Q. 4:** Consider random walk problem in two dimensions on a square lattice. The walk starts at the origin which is at a lattice point approximately at the centre of the lattice and the length of each step one unit. There is a trap at the 100<sup>th</sup> step in any direction. Once a walker reaches the trap, the walk cannot continue.

Take number of steps  $N= 10000$  and using 500 trials for finding the average,

- (i) write pseudo code to find the average number of steps taken to reach a trap. [10]
- (ii) convert the pseudo code into a program code in FORTRAN. [15]

Assume that you have a function sub routine program  $RAN0(idum)$  for calculating random numbers, where  $idum$  is the seed. This means that  $r = RAN0(10)$  gives  $r$  as a random number between 0 to 1 for seed number 10 which is an integer. Embedding such a calculation within a do.. loop will give as many random numbers between 0 to 1 as one needs for the given seed. Note that for each trial, the seed for generating random numbers should be different so as to ensure that for each trial the path taken is different.

**Q.5:** For the central potential

$$V(r) = \begin{cases} -V_0 & \text{for } r < r_0 \\ 0 & \text{otherwise} \end{cases}$$

the Schrodinger equation for angular momentum  $l=0$  with Hartree atomic units  $\eta = m = e = 1$  is given as

$$\frac{d^2 u}{dr^2} + 2(E + V_0)u = 0$$

For large enough  $r$ ,  $u(r) \rightarrow 0$ , and at  $r=0$ ,  $u(0)$  = finite..

Use the Numerov method for solving such differential equations to answer the following:

- (i) Write an algorithm in pseudo code to find the solution  $u(r)$  on the interval  $0 \leq r \leq 25$  at 500 points for a trial bound eigenenergy  $E$ . The value of trial  $E$  is to be input from the default device for an assumed  $r_0$ . [10]

Assume that  $u(0) = 0.5$  and  $u(h) = u(0)$  where  $h = r_{i+1} - r_i$

- (ii) Convert the pseudo-code into a Fortran program. [10]

- (iii) Knowing that the Numerov method has accuracy of the order  $h^6$  explain how you would proceed to determine the bound eigenenergy. [5]

The Numerov method: The second-order equation (with boundary conditions)

$$\frac{d^2 y}{dx^2} + k(x)^2 y = S(x)$$

can be solved using the algorithm,

$$\left(1 + \frac{h^2}{12} k_{n+1}^2\right) y_{n+1} - \left(1 - \frac{5h^2}{12} k_n^2\right) y_n + \left(1 + \frac{h^2}{12} k_{n-1}^2\right) y_{n-1} = \frac{h^2}{12} (s_{n+1} + 10s_n + s_{n-1})$$

The notation used is such that  $k(x)$  means that  $k$  is a function of  $x$  and  $S(x)$  means that  $S$  is a function of  $x$ .

**Q.6. (A) (i)** Explain the terms chaos, deterministic chaos, fixed point, periodic point, and fractal. [5]

(ii) A simple pendulum in the presence of a damping and driving force can be described by the equation of motion

$$\frac{d^2x}{dt^2} = -c \frac{dx}{dt} - (1 + F \cos \omega t) \sin(x)$$

where  $\omega = 2\pi f$ ,  $f$  = frequency of the driving force,  
 $F$  = amplitude of the driving force and  $c$  = damping factor.

Answer the following:

(a) What is the phase plot in this case? [1]

(b) Which is the parameter in the differential equation that will lead to chaotic behavior of the pendulum? [5]

Define the Poincare section in this case.

Which are the parameters that define the Poincare section?

Explain its importance.

(c) If  $F=0$ , will the pendulum attain chaotic behavior? Explain. [2]

**(B) (i)** Define auto correlation and show that from a signal with random noise of uniform distribution, the true signal can be recovered. [4]

(ii) We define the discrete Fourier transform  $g$  of  $f$  as

$$g(n\Delta\omega) = \sum_{m=0}^{N-1} f(m\Delta t) \exp(-in\Delta\omega m\Delta t)$$

where  $m$  and  $n$  take the values  $0, 1, \dots, N-1$  with  $N$  = the total number of points,  $\omega = 2\pi/T$ ,  $T$  = period and  $\Delta\omega \Delta t = 2\pi/N$ .

Show that

(a)  $g(n\Delta\omega) = g_{\text{even}}(n\Delta\omega) + g_{\text{odd}}(n\Delta\omega) \exp(-i2\pi n/N)$  [4]

(b) Explain how the identity in (a) leads to the technique of fast Fourier transform. Illustrate this by assuming a data set of 16 points. [4]

**@@@END OF EXAMINATION@@@**

**UNIVERSITY OF ZAMBIA**  
**DEPARTMENT OF PHYSICS**  
**UNIVERSITY EXAMINATIONS 2001/2002**

**P421**  
**SOLID STATE PHYSICS I**

**DURATION:** Three (3) hours

**INSTRUCTIONS:** Answer only four (4) questions in total. *All questions carry equal marks.*

**MAXIMUM MARKS:** 100 %

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**Where necessary, you may use the following:**

Electron mass,  $m = 9.1 \times 10^{-31}$  kg

Electron charge,  $e = 1.6 \times 10^{-19}$  C

Planck's constant,  $h = 6.63 \times 10^{-34}$  J.s

Speed of light in vacuum,  $c = 3.0 \times 10^8$  m/s

Permittivity of free space,  $\epsilon_0 = 8.85 \times 10^{-12}$  C<sup>2</sup>.J<sup>-1</sup>.m<sup>-1</sup>

Primitive translational vectors and Reciprocal lattice vectors:

$$\mathbf{A} = 2\pi \frac{\mathbf{b} \times \mathbf{c}}{\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}} \quad \mathbf{B} = 2\pi \frac{\mathbf{c} \times \mathbf{a}}{\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}} \quad \mathbf{C} = 2\pi \frac{\mathbf{a} \times \mathbf{b}}{\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}}$$

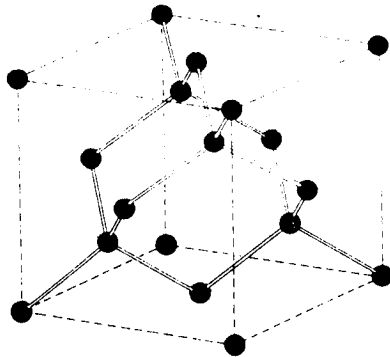
Interaction energy between two ions,  $U = \lambda \exp(-R/\rho) - \frac{q^2}{R}$

Monatomic linear dispersion relation,  $\omega(k) = \sqrt{\frac{2K(1 - \cos ka)}{M}} = 2\sqrt{\frac{K}{M}} \left| \sin \frac{1}{2} ka \right|$

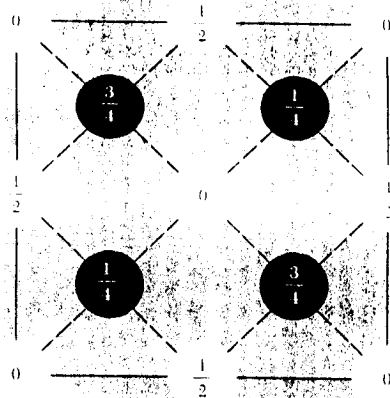
Density of states for a square lattice of length  $L$ ,  $D(\omega) = \frac{L^2}{2\pi} \frac{k}{d\omega/dk}$

Thermal energy,  $U = \int D(\omega) \langle n(\omega) \rangle \hbar \omega d\omega$

- Q1. (a) (i) Define what we mean by a *solid*. (3 marks)  
 (ii) What do you understand by a *primitive* and *non-primitive* cell? (4 marks)
- (b) (i) Explain why the diamond structure may be considered as being made up of two interpenetrating face centred cubic (fcc) lattices. See figure below. (6 marks)  
 (ii) Given that the cube edge for diamond is  $3.56 \text{ \AA}$ , calculate the distance between nearest neighbours, and show that there are  $1.77 \times 10^{23}$  atoms per  $\text{cm}^3$ . (9 marks)  
 (iii) From this, calculate the density of diamond and compare the results with the observed density. (Experimental value of the density of diamond is  $\sim 2.7 \text{ g/cm}^3$ .) (3 marks)



(a)



(b)

Crystal structure of diamond. (b) also shows the atomic positions in the cubic cell of the diamond structure projected on a cube face, with fractional heights above the base given in units of the cube edge.

- Q2. (a) The practical process of obtaining information of a crystal structure from a study of its diffraction pattern roughly falls into two stages. State these two stages. (6 marks)

- (b) The condition for Bragg diffraction of a particle with wavelength  $\lambda$  is stated as

$$\lambda = 2d \sin \theta$$

where  $d$  is the separation of the crystal planes involved and  $\theta$  is the angle between the incident wavevector of the particle and the crystal planes.

- (i) Show that this is equivalent to

$$G^2 = 2\mathbf{k} \cdot \mathbf{G}$$

(9 marks)

where  $|\mathbf{k}| = 2\pi / \lambda$ .

- (ii) Choose a set of planes with Miller indices  $(hkl)$ , and let  $\mathbf{G}$  have the same indices. Show that the spacing  $d_{hkl}$  of the lattice planes with Miller indices  $(hkl)$  is given by

$$d_{hkl} = \frac{2\pi}{|\mathbf{G}(hkl)|}$$

(10 marks)

where  $\mathbf{G}(hkl) = h\mathbf{A} + k\mathbf{B} + l\mathbf{C}$ .

- Q3. (a). (i) Distinguish between the *cohesive* energy and the *lattice* energy. (4 marks)  
 (ii) Explain why only two electrons can form a covalent bond. (4 marks)

- (b). For a system that executes simple harmonic motion about a position of stable equilibrium, the force,  $F$ , is given by

$$F = -\left(\frac{\partial^2 V}{\partial R^2}\right)_R (R - R_0)$$

where  $V$  is the potential energy and  $R - R_0$  is the deviation from equilibrium. Show that the zero-point vibration of a molecule is given by

$$\frac{1}{2} h \nu_0 = \frac{h}{4\pi\mu^{1/2}} \left[ \left(\frac{\partial^2 V}{\partial R^2}\right)_R \right]^{1/2} \quad (5 \text{ marks})$$

- (c). From the following data, find the energy required to dissociate a KCl molecule into a K atom and Cl atom. The first ionisation potential of K is 4.34 eV; the electron affinity of Cl is 3.82 eV; the equilibrium separation of KCl is 2.79 Å. [Hint: Show that the mutual potential energy of  $K^+$  and  $Cl^-$  is  $-(14.40/R)$  eV if  $R$  is given in Angstroms.] (12 marks)

- Q4. (a). Using a qualitative description, distinguish between the acoustic and optical branches of the phonon gas. (8 marks)

- (b). Consider a linear chain in which alternate ions have mass  $M_1$  and  $M_2$ , and only nearest neighbours interact.

- (i) Show that the dispersion relation for the normal modes is

$$\omega^2 = \frac{K}{M_1 M_2} \left( M_1 + M_2 \pm \sqrt{M_1^2 + M_2^2 + 2M_1 M_2 \cos ka} \right) \quad (8 \text{ marks})$$

where  $K$  is the spring constant between the ions.

- (ii) Discuss the form of the dispersion relation and the nature of the normal modes when  $M_1 \gg M_2$ . (6 marks)  
 (iii) Compare the dispersion relation with that of the monatomic linear chain when  $M_1 \approx M_2$ . (3 marks)

- Q5. (a). There are more than six consequences that arise because of neglecting anharmonic effects in solids. Give four of these. (8 marks)

- (b). (i) Discuss the heat capacity of a two-dimensional (2-D) square lattice with nearest neighbour separation  $a$  on the basis of the Debye approximation. (13 marks)  
 (ii) Show that at low temperatures the heat capacity is proportional to  $T^2$ . (4 marks)

Q6. (a). Explain briefly why the low-temperature heat capacity of a metal is often plotted in the form  $C/T$  versus  $T^2$ . (3 marks)

(b). A sample contains  $N$  electrons, which behave as free particles with energy

$$E = \frac{\hbar^2 k^2}{m^*}$$

where  $m^*$  is an effective mass. Derive formulae for the Fermi energy ( $E_F$ ) in the following cases.

- (i) The sample is one-dimensional (1-D), and is of length  $d$ . (3 marks)
- (ii) The sample is 2-D, and is of area  $A$ . (4 marks)
- (iii) The sample is three-dimensional (3-D), and is of volume  $V$ . (5 marks)
- (iv) In each case, show that the density of states at  $E_F$  is of the form

$$g(E_F) = \xi \frac{n}{E_F}$$

with  $\xi$  a number  $\approx 1$  and

$$n = \begin{cases} N/d & (1-D) \\ N/A & (2-D) \\ N/V & (3-D) \end{cases} \quad (8 \text{ marks})$$

- (v) Hence show that, irrespective of the number of dimensions,  $g(E_F)$  is proportional to  $m^*$ . (2 marks)

@@@@@@@@@@@@@@@@@END OF EXAMINATION@@@@@@@@@@@@@@@@@