TEACHERS' STRATEGIES IN THE TEACHING OF LINEAR PROGRAMMING IN SELECTED SECONDARY SCHOOLS IN MONZE DISTRICT - ZAMBIA

by

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A dissertation submitted to the University of Zambia in partial fulfillment of the requirement for the award of the Degree of Masters of Education in Mathematics Education.

THE UNIVERSITY OF ZAMBIA

LUSAKA

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DECLARATION

I, **Kaabo Emmanuel**, do hereby declare that, this dissertation entitled; "Teachers' Strategies in the teaching of linear programming in selected secondary schools in Monze District - Zambia" is my original work and has not been submitted to any institution before. All sources used have been thoroughly acknowledged.

Signed (Candidate)............................. Date…………………. 2019

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APPROVAL

This dissertation by **Kaabo Emmanuel** is hereby approved as partial fulfillment of the requirements for the award of the degree of Masters of Education in Mathematics Education by the University of Zambia.

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ABSTRACT

Mathematics plays a vital role in personal, national and global development. Despite its great importance to national development, the Zambia Grade 12 National Examination results reports low percentage passes in mathematics. The topics that continuously posed a challenge in Grade 12 Ordinary level Mathematics are linear programming, earth geometry, trigonometry and calculus. Therefore, this study interrogated teachers' strategies in the teaching of linear programming in secondary schools; the constraints and the suggested measures to overcome the constraints in the teaching and learning of linear programming. Social constructivism approaches advocate for more interactive learning environment as the theoretical framework. The descriptive research design was used to collect qualitative data from three teachers and 15 pupils. Purposive sampling technique was applied to select the participants. Data were collected using the lesson observations, interviews and focus group discussions. The data were analyzed thematically by categorizing of major and sub-themes that emerged from the study. The findings of the study indicated that to a large extent, teachers used grade nine linear inequations when introducing linear programming. This strategy was used in order to bring the interaction of inequality signs/symbols that give the basis for formation of inequations from situational statements, plotting and shading to unveil the feasible region. In the formation of inequations, terms and their associated symbols were key. The table containing terms with their associated symbols were used to help the interaction of the two. The coordinates of the vertices of the feasible region were also used for finding both the maximum and minimum costs. The constraints included: the instruction for shading in grade nine and 12 were not consistent; the learners were not exposed to variety of situational problems under which the terms and their associated symbols would be used; and the use of trial and error method for finding the maximum and minimum cost as learners were overwhelmed with coordinates resulting from missing out on correct ones needed. The suggested measures to overcome the constraints were: shading of the unwanted region should be the instruction even in grade 9 and using the coordinates of the vertices of the feasible region in finding the maximum and minimum costs/profits. The study therefore, recommends that, there should be consistence in the instruction of the region to be shaded in both linear inequation and linear programming. Use of tables containing terms with their associated symbols can be a more precise strategy in providing a better understanding of the two. Teachers should consider giving a variety of situational problems to enhance learner interaction with the terms and their associated symbols used in linear programming. Teachers of mathematics should always include linear programming in their in-house continuous professional development (CPD) and cluster meetings to address the challenges in the effective strategies in the teaching and learning of it. Moreover, further research on strategies for effective teaching and learning of linear programming in secondary schools in Zambia could be explored and see how they compare with the ones established by this study.

Keywords: *Teacher strategy, Linear programming*.

DEDICATION

This work is dedicated to my lovely wife Elector Hakanene, my mother Lemitah Kaabo, my children Blessings, Praise and Hope, my brothers and sisters and my niece Catharine for their diligent support during the period of writing this dissertation.

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ACRONYMS AND ABBREVIATIONS

FGD	Focus Group Discussion
LP	Linear Programing
MoGE	Ministry of General Education
RME	Realistic Mathematics Education
T/L	Teaching and Learning
ZPD	Zone of Proximal Development

CHAPTER ONE

INTRODUCTION

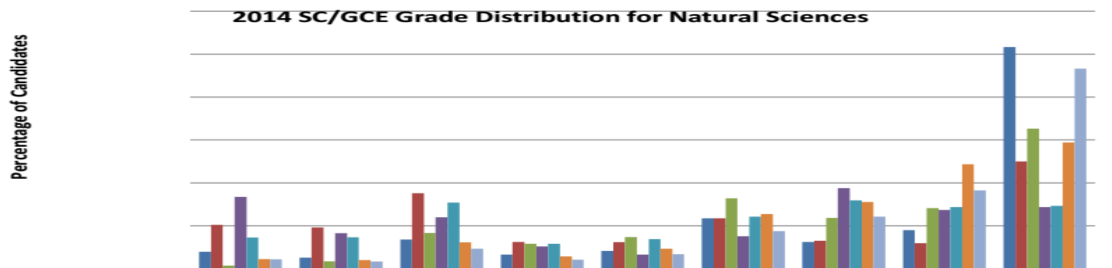
1.1 Overview

This chapter introduces the study by first presenting the background, statement of the problem, purpose of the study and objectives with their corresponding research questions. It further provides the significance of the study, scope (delimitation and limitation), operational definitions and then the chapter summary.

1.2 Background

Mathematics plays a vital role in personal, national and global development. Its fundamental role lies in its application in most social sciences like geography, government, business transactions and in house hold-chores. In addition, mathematics has been applied within various studies such as engineering, biology, medicine, economics, and in military advancement (Cockcroft, 1982). Therefore, it is against this background that most countries including Zambia have taken mathematics to be a compulsory subject in both primary and secondary school. However, despite its great importance, the learning of mathematics is considered to be challenging by majority of learners. This is evident in the Zambia national examination results, which reports that low percentage passes are most recorded in mathematics (ECZ reports, 2013, 2014, 2015, 2016). The graphs below show the summary analysis for natural science for 2014 and 2015 of the Examination Council of Zambia.

Figure 1.2 below is the 2014 Grade Distribution of natural science grade 12 .



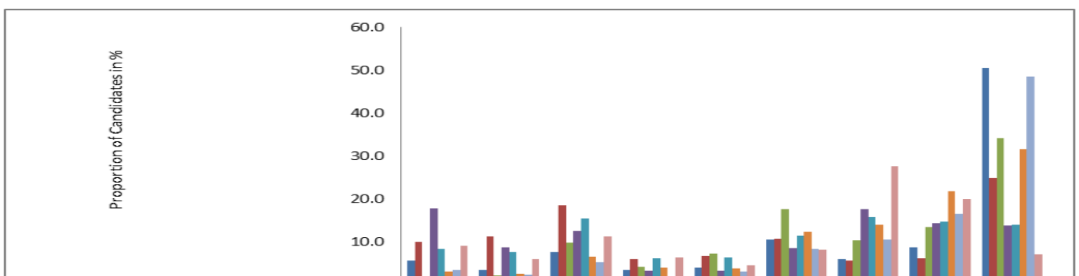
	one	two	three	four	five	six	seven	eight	nine
Maths	3.97	2.58	6.79	3.27	4.12	11.75	6.23	9.01	51.68
Add Maths	10.25	9.63	17.57	6.26	6.17	11.77	6.54	5.97	25.02
Agric Science	0.75	1.72	8.30	5.80	7.38	16.43	11.83	14.11	32.66
Physics	16.73	8.30	12.00	5.18	3.28	7.56	18.80	13.72	14.37
Chemistry	7.28	7.33	15.39	5.83	6.92	12.13	15.93	14.38	14.65
Biology	2.23	2.01	6.12	2.86	4.67	12.72	15.53	24.33	29.41
Science	2.21	1.69	4.69	2.10	3.38	8.76	12.14	18.25	46.64

Figure 1.2 SC/GCE (Grade 12) Subject Grade Distribution for Natural Science Subjects

Figure 1.1: 2014 grade 12 grade distributions for Natural sciences

Figure 1.1 above shows the grade 12 grade distributions for Natural Sciences for 2014 (ECZ, 2014). The bars in different colors represent the natural sciences subject passes following the grading system in Zambia that a grade nine is a fail. Therefore, in 2014 51.68 percent of the candidates who sat for the Grade 12 GCE examinations failed mathematics, indicating a higher percentage fail as compared to the pass which can be calculated as 48.32 percent.

Figure 1.2 below is the 2015 Grade Distribution.



	One	Two	Three	Four	Five	Six	Seven	Eight	Nine
Mathematics	5.7	3.4	7.6	3.4	4.0	10.5	6.0	8.8	50.6
Add. Mathematics	10.0	11.2	18.5	6.0	6.8	10.8	5.7	6.1	24.9
Agric. Science	0.8	2.2	9.8	4.2	7.3	17.7	10.3	13.5	34.1
Physics	17.8	8.7	12.5	3.3	3.3	8.6	17.6	14.3	13.9
Chemistry	8.3	7.6	15.5	6.2	6.3	11.5	15.9	14.8	13.9
Biology	3.1	2.6	6.6	4.0	3.8	12.3	14.0	21.9	31.7
Science	3.5	2.3	5.3	2.0	3.0	8.3	10.5	16.5	48.6
Comp. Studies	9.1	6.0	11.2	6.4	4.5	8.1	27.6	20.1	7.1

Figure 1.2: 2015 (Grade 12) Subject Grade Distribution for Natural Science Subject

Figure 1.2 above shows the subject grade distribution for Natural Science Subject for 2015 (ECZ, 2015). Therefore, it can be evidently seen that the pattern for the results in mathematics is the same with high records in failure. In 2015 the percent failure in mathematics was 50.6 percent of the candidates who sat for the Grade 12 GCE examination (ECZ, 2015) and giving a percentage pass of 49.4 percent. This largely shows that attention to some of the causes of the low percentage passes in ordinary level mathematics be examined and then the way forward be sought.

Following the reports above, I took interest to learn of the topics that attribute to low passing percentages especially for Grade 12 ordinary level mathematics. The examination council of Zambia performance reports showed that candidates have challenges in relation to mastery of the concepts and the skills taught. The topics that have continuously posed a challenge in Grade 12 Ordinary level Mathematics are linear programming, earth geometry, trigonometry and calculus (ECZ Report, 2013, 2014, 2015, 2016).

From the reports of 2013, 2014, 2015 and 2016 the Examinations Council of Zambia gives the following conclusive reports:

2013 Conclusive report (ECZ, 2013 p. 28, 2014 p.23, 2015, p. 27and 2016 p.28)

The analysis of performance at Grade 9 and 12 in the 2013 examinations in mathematics and additional mathematics showed that some candidates have serious challenges. The sample scripts show that most candidates have challenges that are related to ***mastery of the concepts and the skills in the syllabus rather than making errors in their calculations. Trigonometry, earth geometry and linear programming have been reported to be challenging topics to the candidates even in the previous years.*** *Therefore, teachers of mathematics should always include these topics in their in-house continuous professional development (CPD) and cluster meetings to address the challenges faced by candidates.*

1.1.1 The importance of strategies in teaching

The educational schemes of today are seeking to develop teachers through making them fully aware of the different teaching strategies, since the efficiency of a teacher is determined largely by his/ her choice of the teaching and learning strategies through which the objectives of the lesson are achieved and its content is fully covered; and by which students are given the ability to acquire the formerly set lesson objectives which precisely meet their needs (Ball & Perry, 2009). However, choosing the appropriate teaching and learning strategy is a complicated process. It demands deep reflection on the part of the teacher and the ability to strike a balance of the available strategies in the light of the many interrelated variables.

A teaching strategy is in essence a general plan which includes all the parts of the teaching situation; namely: the objectives, teaching methods, teaching aids and evaluation strategies. The aforementioned parts are actually the activities a teacher does in the class for the purpose of achieving the lesson's objectives. Teaching strategies are basically based on kinds of patterns and theories that are called "Learning Theories". Such theories are classified into three categories. These are: behavioral, cognitive and affective (Pongsuwan, 2011).

Further, studies have indicated that teachers need to equip themselves with more flexible approaches in the teaching of mathematics and science (Galton & Eggleston, 1979; Nelson, 1996). These alternative teaching methods strive to cater for the needs of these varied learners by establishing a conducive learning environment for students. The teaching and learning of science and mathematics is no longer regarded as only a simple and technical procedure involving teaching objectives and learning outcomes. Teachers are encouraged to adopt progressive teaching styles to accommodate the varied abilities of students, so as to enable these students excel in their learning.

Mathematics teaching and learning, students are not only required to have knowledge of numerical facts, but also understanding in problem solving abilities that are adjustable in accordance with their individual strength and weaknesses (Jones & Tanner, 2002). Therefore, helping students learn effectively is a challenge to teachers. Teachers must realize that one type of teaching method is not sufficient (Suriati Sulaiman & Tajularipin Sulaiman, 2010). Effective teaching strategies are essential in ensuring that successful teaching and learning process takes place. Individual qualities within each teacher have been recognized as a major influence on their capacity and ability in creating opportunities for their students to learn effectively (Ball & Perry, 2009). Identifying effective teaching strategies whereby teachers are comfortable in their teaching and students learn in a joyful and relaxed environment is beneficial for education. It is vital that all learning materials should include opportunities for students to develop their strengths and strengthen their weaknesses in each particular intelligence area (Lash, 2004). Therefore, it is necessary to assess the effectiveness of our current teaching style of linear programming and consider innovative ways to improve our teaching to enhance the teaching and learning effect (Delaney & Shafer, 2007).

1.1.2 Why focus on Linear Programming

The purpose of education is preparing learners for life after school. The teaching and learning of mathematics activities should be very realistic in nature. It should be providing the transformation of word problems of everyday life into mathematical problems whose intellectual power is motivation of further learning resulting into individual development and intellectual improvement (MoGE, 2013). Linear programming has an ingredient in the teaching/learning of mathematics whose usefulness is feasible in life after school. The establishment of proper connections between mathematics as a discipline and the application of mathematics in the real-world contexts is highly promoted (MoGE, 2013). Linear programming provides opportunities for an approach in the mathematics classroom that bring about real-life situational mathematics learning (Dantzig, Thapa 2006).

The history of linear programming can be traced back to George Dantzig who developed it in 1947. He was studying proposed training and logistics for the United States military training program, as a mathematical adviser to the US Air Force Controller in the Pentagon. Dantzig was also an expert on planning methods using desk calculators. His colleagues at The Pentagon, Hitchcock and Wood, asked him to find a method that would rapidly compute a time-staged operation, training and logistical supply program (Dantzig, 2002). Dantzig was influenced by the work of Leontief, who proposed a large but simple matrix structure called the "inter-industry input-output model‟ (Leontief, 1933) of the American Economy.

In the model, there was a one-to-one correspondence between the production processes and the items being produced by these processes. Dantzig (1937) also used ideas from a paper by von Neumann (1928) on game theory together with his ideas on steady economic growth to formulate a highly dynamic model that could change over time. He realized that The Air Force needed a model with alternate activities and which had to be computable; hence, he invented the simplex method. The simplex method can be described as a dynamic linear program with astair case matrix structure. The method generates a sequence of feasible iterates by repeatedly moving from one vertex of the feasible region set to an adjacent vertex with a lower value of the objective function. When it is not possible to find an adjoining vertex with a lower value, the current vertex must be optimal and termination occurs. The simplex method is a tool for practical planning of large complex system (Dantzig, 2002)

Linear programming is a mathematical technique for finding optimal solutions to problems that can be expressed using linear equations and inequalities. In a real-world problem can be represented accurately by the mathematical equations of linear programming; the method can help to find the best solution to the problem. Of course, few complex real-world problems can be expressed perfectly in terms of a set of linear functions. Furthermore, linear programs can provide reasonably realistic representations of many real-world problems — especially if a little creativity is applied in the mathematical formulation of the problem.

Linear programming can further be defined as a mathematical modeling technique useful for guiding quantitative decisions in business planning, industrial engineering and, to a lesser extent, in the social and physical sciences. It is the maximization or minimization of a specific performance index, usually of an economic nature like profit, subject to a set of linear constraints. For this exercise to qualify as linear programming the performance index should also be linear (Nkambule, 2009).

Boaler (1998:41) in her project report, stated that;

> *There is a growing concern among mathematics educators that many students are able to learn mathematics for 11 years or more but are then completely unable to use this mathematics in situations outside the classroom context. In*

various research projects adults and students have been presented with tasks in which they are required to make use of mathematics they have learnt in school. These projects have shown that in real-world mathematical situations, adults and students do not use school-learned mathematical methods or procedures.

In Boaler's quotation, it is evident that there is a disjoint in the use of real-life mathematical situation problems and the school-learnt mathematical methods which largely demonstrate lack of relational understanding in the learning of mathematics in most cases.

Well, considering its power in bringing real life problems of economics, business and decision-making skills into the classroom environment; and providing skills for determining best outcomes in a given mathematical model involving some linear relationship. Hence their application in business, economics as well as various engineering fields as determined by the aims of mathematics (Doug, 2006).

It is against this background that, linear Programming is included as the compulsory Zambian secondary school mathematics topic in the school curriculum. It helps to provide skills for determining best outcomes in a given mathematical model involving some linear relationship. This technique has found application in business, economics as well as various engineering fields. Hence a number of researches have been conducted in trying to improve the teaching and learning of linear programming. For instance, Nkambule (2009), did a study on the teaching and learning of linear programming in a grade 11 multilingual mathematics class of English language learners: exploring the deliberate use of learners' home language and Nakhanu, Shikuku and Wasike (2015) did a study in Kenya entitled Application of Linear Programming Knowledge and Skills to Real Life Contexts by Secondary School Students in Kenya which is a technique of Problem-based learning (PBL). However both had different focuses as compared to this study. Further, little is known about research that has been done in Zambia with the focus on linear programming. Therefore, this research provides the bases for understanding the strategies teachers use in the teaching of linear programming from a Zambian context and suggesting measures for effective teaching and learning of linear programming.

Table 1.1 below shows extract of what should be covered in linear programming from ordinary level mathematics syllabus.

Table 1.1: linear programming from the syllabus

TOPIC	SUB TOPIC	SPECIFIC OUTCOME	KNOWLEDGE	SKILLS	VALUES
12.2 LINEAR PROGRAMMING	12.2.1 Linear programming	12.2.1.2 Draw graphs of linear equations and inequations in one and two variables (as a recap) 12.2.1.3Shade the wanted and unwanted regions 12.2.1.3Describe the wanted or unwanted regions. 12.2.1.3Determine maximum and minimum values 12.2.1.4Use the search line to determine the maximum and minimum values 12.2.1.5Apply knowledge of linear programming in real life	•Drawing graphs of linear equations and inequations in one and two variables (as a recap) •Shading the wanted and unwanted regions •Describing the wanted or unwanted region •Finding Values in the feasible region •Using the Search line to determine the maximum and minimum values •Applying knowledge of linear programming in real life	•Interpretation of the wanted or unwanted regions. •Shading of the unwanted region. •Determination of maximum and minimum values. •Application of linear programming in real life situation.	•Logical thinking in finding the wanted region. •Planning when using graph paper

From what the syllabus demands for coverage, it is saddening to learn that detailed analysis in linear programming from the examinations council of Zambia performance review report showed that poor work was recorded: 2013 in the use of two symbols (= and <), 2014 failure to come up with correct interpretation of inequalities, and graphing to a given scale, 2015 in use of = sign in the inequations and 2016 reversing of inequality signs was common (ECZ 2013, 2014, 2015 and 2016). Why these difficulties contributing to poor results?

It is against this background therefore, that the study examined the teachers' strategies in the teaching of linear programming. Taking into account interactions that take place in the teaching and learning discourse of linear programming thus; teacher-learner interaction, learner-peer interaction and learner interaction with the learning materials. The section below highlights the statement of the problem of this study.

1.3 Statement of the Problem

Mathematics is a compulsory subject in the Zambian secondary education curriculum. Students are expected not only to study mathematics, but more importantly to achieve high grades in mathematics examinations (MGE 2013). Yet many learners find mathematics learning quite challenging. Why recording high failure rates in ordinary level mathematics resulting from challenges in the same topics (linear programming, Earth geometry and trigonometry) over the years? Why not visiting the strategies that are used in the teaching and learning of these topics, and when teachers are encouraged to adopt progressive teaching strategies to accommodate the varied abilities of students, so as to enable these students excel in their learning? It is necessary to assess the effectiveness of our current teaching strategies of linear programming and consider innovative ways to improve our teaching to enhance the teaching and learning effect (Delaney & Shafer, 2007). It is for this reason that the research examined the teachers' strategies that are employed during the teaching and learning discourse of linear programming.

1.4 Purpose of the Study

The purpose of this study was to examine teachers' strategies in the teaching of linear programming and determine the factors affecting the teaching of linear programming.

1.5 Research Objectives

The study was guided by the following objectives:

1. To examine teachers' strategies in the teaching of linear programming in secondary school mathematics.
2. To establish the factors affecting the teaching and learning of linear programming.

3. To determine the intervention measures that teachers would suggest to overcome the constraints.

1.6 Research Questions

The following were the research questions the guided the study:

1. What strategies do teachers use in the teaching and learning of linear programming?
2. What are the factors affecting the teaching and learning of linear programming?
3. What strategies would teachers suggest to enhance the effective teaching and learning of linear programming in order to overcome the constraints?

1.7 Significance of the Study

The study is important in providing teachers with the necessary insight into effective teaching strategies in the teaching and learning of linear programming. It might bring to light the kind of teacher-learner interaction that exists in most mathematics learning discourses which can be a greater drive for improvement. Determination of the constraining factors could inform teachers to adjust their instructional practices to include more motivating aspects as postulated by constructivists. The study also hopes to inform the ministry of General Education, on the measures to enhance the best teaching strategies in the teaching and learning of linear programming.

1.8 Scope of the Study

This section sets boundaries for the study (delimitation) and perceived constraining factors (limitation) which might affect the research process. It starts with the delimitations to justify the choice of the population.

1.8.1 Delimitation

The study targeted teachers of mathematics, grade 11/12 pupils in which linear programming was being taught. The study focused on teacher strategies in the teaching of linear programming in an interactive learning environment.

1.8.2 Limitation of the study

According to Yin (2011) limitations relate to elements of the study like constraints that are not under the control of the researcher. This section highlights factors that are liable to affect the outcome of the study to some extent. The study did not access learners' working to confirm with the constraining factors, because it would have meant carrying out document analysis of learners' books and doing so would have had an effect on the focus of the study especially that I was concerned with teachers' strategies in the teaching and learning of linear programming. The small sample size used also restricted the generalization of findings.

1.9 Theoretical and Conceptual framework

This study was underpinned by the social constructivism theory of Vygotsky (Vygotsky, 1978) with it major tenet the Zone of Proximal Development (ZPD). Social constructivism is a theoretical perspective that posits that knowledge and learning are products of social interactions. It contends that, knowledge and reality are actively created by social relationships and interactions and the ZPD brought in the aspect of mediation in the learning discourse of linear programming. Further under mediation, the aspect of the language of linear programming was key in the understanding of concepts. Therefore, Social interaction plays a fundamental role in the development of higher order mental function (Vygotsky, 1978). In this study, this theory guided the study in understanding the strategies teachers use as they interact with learners in socially created learning environments in the teaching and learning of linear programming.

Further, our conception of mathematical knowledge mirrors and governs the way we present the subject to our learners. From the philosophical points of view, the philosophy of the nature of the mathematical knowledge to the teacher is very important, thus the absolute and fallible notion of mathematical knowledge, which Ernest (1991) refers to as 'dualistic epistemology' (p.151, 287).

Absolute mathematical knowledge emanates from the traditional philosophical position of absolutism which advances the universal and objective nature of mathematics. It contends that mathematics is a body of infallible, objective truth, far removed from the affairs and values of humanity (Ernest, 1991).

Fallibilists view mathematics as an incomplete and everlasting work-in progress. Ernest (1991) submits that the fallibilist view of mathematical knowledge is subtler and accept that social forces do partly mound mathematics. He argued that mathematical truths are invented and not discovered and showed why 1 + 1 did not always add up to 2. Mathematics is contextual, subjective and therefore prone to modification. While retaining meaning, teachers should modify mathematics to make it more comprehensible to learners by taking advantage of available resources like language as a cultural and mediation tool. I have dwelt more on this in due course as I review literature on mathematical knowledge for teaching and linear programming knowledge for teaching.

I believe that mathematical knowledge is fallible, corrigible and revisable. This is in line with, Lakatos' masterpiece, proofs and refutations on the philosophy of dubitaility which stipulates that, "mathematics is fallible, not indubitable, changing, and like any other body of knowledge, it too grows by the criticism and correction of theories which are never entirely free of ambiguity or possibility of error and oversight and is the product of human inventiveness" (Lakatos, 1976).

Lakatos' conception of mathematical knowledge is in resonance with Paul Ernest who sides the social constructivist's view (Vygotsky, 1978) of mathematical knowledge as fallible, open to revision and that it is publicly available for scrutiny (Ernest, 1991). Notwithstanding this, in Zambia mathematics is a difficult subject. Teachers need to relate mathematics and linear programming to the practical significance.

In view of the social constructivist perspective my assumption was that secondary school teachers would be able to construct and reconstruct linear programming knowledge. Arguing from social-cultural theory of Vygotsky (1978) of instruction, I propose that linear programming be taught constructively to enhance active participation of learners and profound understanding of concepts (Ma, 1999). Therefore, social constructivism approaches would largely encourage a participative atmosphere characterized by freedom of interaction and questioning of linear programming knowledge.

Figure 1.4 below is the conceptual frame which gives the understanding of the main assumptions in the teaching and learning of linear programming that a teacher should consider. Therefore, feeding into the teacher strategies in the teaching and learning of linear programming include: the philosophy of the nature of mathematical knowledge (Ernest, 1991), the mathematical knowledge for teaching (Ball, et al, 2008), concepts of linear programming and an instructional theory social constructivism's account for knowledge, learning and teaching (Vygostky, 1978).

Figure 1.3 below shows the conceptual framework of the study.

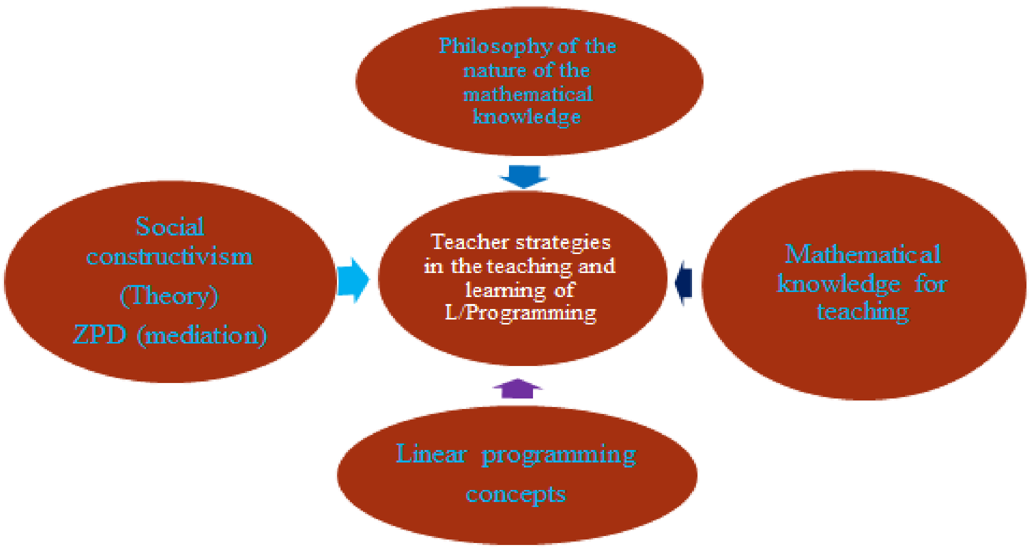


Figure 1.3: *Conceptual framework*

1.10 Operational Definitions

According to Hilton (1986: 48), "a definition is a concise statement of the basic properties of an object or concept which ambiguously identify that object." In this study, concepts are defined according to the contexts in which they were used.

Teaching strategies: The structure, system, methods, techniques, procedures and processes that a teacher uses during instruction. These are strategies the teacher employs to assist students' learning (Delaney, 2008).

Social constructivism: An established educational theory based on the principle that learners and teachers co-construct knowledge of linear programming through social processes, thus teacher-learner, learner-peer and learner interaction with the learning materials (Vygotsky, 1978).

Linear programming: A mathematical technique for finding optimal solutions to problems that can be expressed using linear equations and inequalities in the light of maximization and minimization of profits and decision making (Dantzig, 2002).

Zone of Proximal development: It is a theoretical space of understanding which is just above the level of an individual's current understanding (Vygotsky, 1978). **Knowledgeable knower**: This is a peer or teacher who provides assistance to the learner in a task beyond the learner's capability (Vygotsky, 1978).

Mediation: It is closer teacher direction and guidance in the learning discourse (Brodie, 2000)

1.11 Organization of the study

Chapter One

This chapter provided some detailed accounts of the background of the study based mostly on the reports from Examinations Council of Zambia annual performance reports. It also outlined some key items such as the statement of the problem, purpose of the study, objectives with their research questions, and the significance of the study, scope (delimitation and limitation), the theoretical and conceptual framework and finally the operational definitions.

Chapter Two

This chapter gives the main concepts/principles under which literature was reviewed. it provides the discussion of these concepts in seven (7) broad categories which include: the rationale to teaching and learning of linear programming, the importance of strategies in teaching, the philosophical nature of the mathematical knowledge, mathematical knowledge for teaching, factors affecting the teaching and learning of linear programming, mathematical knowledge for teaching linear programming and the theoretical foundations that mirror the study.

Chapter Three

This chapter discusses the research methodology that we applied to examine teachers' strategies in the teaching of linear programming. The chapter presents the type of research design that was used, study area, study population, sampling techniques, instruments for data collection, procedure for Data collection and data analysis that was employed in the study. I also took into account the ethical issues to ascertain the consents.

Chapter Four

This chapter presents the findings of the study in the light of the research questions. These findings are presented in form of themes supported by verbatim and vignettes which were captured during lesson observations.

Chapter Five

This chapter provides the discussion of the findings presented in chapter four in the light of the research objectives. The findings are further discussed in view of the literature reviewed and the theoretical foundations that mirrored the study.

Chapter Six

This chapter presents the conclusion and the recommendations made for this study in light of the findings. The recommendations unveil the teachers' practices in the teaching and learning of linear programming, it further gives suggestions on policy and then opens guide for further research in the same area.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter reviews the literature related to the strategies in the teaching of linear programming. The review is categorized into four (4) broad areas namely; mathematical knowledge for teaching, factors affecting the teaching and learning of linear programming, mathematical knowledge for teaching linear programming and related studies in linear programming. At the end of the chapter, a summary of the reviewed literature is given.

2.2 Mathematical knowledge for teaching

Teaching is a complicated practice that requires an interweaving of many kinds of specialized knowledge. In this way, teaching is an example of an ill-structured discipline, requiring teachers to apply complex knowledge structures across different cases and contexts (Mishra, Spiro, & Feltovich, 1996; Spiro & Jehng, 1990). It is therefore believed that teachers practice their craft in highly complex, dynamic classroom contexts that require them constantly to shift and evolve their understanding (Leinhardt & Greeno, 1986). Thus, effective teaching depends on flexible access to rich, well-organized and integrated knowledge from different domains (Glaser, 1984; Putnam & Borko, 2000; Shulman, 1986, 1987), this may include knowledge of student thinking and learning, knowledge of subject matter, and increasingly, pedagogical knowledge (knowing of how to teach).

The field of mathematics has recorded substantial research to investigate the kind of knowledge required for teaching mathematics. Shulman (1986) built his foundation for teaching reform on an idea of teaching that emphasizes comprehension and reasoning, transformation and reflection. He justified this emphasis by responding to four questions: "what are the sources of the knowledge base for teaching? In what terms can these sources be conceptualized? What are the processes of pedagogical reasoning and action? And what are the implications for teaching policy and educational reform?" However, this study will not engage in the discussion of the answers to these questions but they formed the bases for conceptualization of the mathematical knowledge for teaching linear programming which included subject matter content knowledge and Pedagogical content knowledge (PCK).

Shulman (1986) stressed the need for subject matter content knowledge and argued that content knowledge should go beyond knowledge of facts of the subject and that it requires understanding the structures of the subject matter. Central to this conceptualization of Pedagogical Content Knowledge according to Shulman is the notion of the transformation of the subject matter for teaching. This transformation occurs as the teacher interprets the subject matter, finds multiple ways to represent it, and adapts and tailors the instructional materials to alternative conceptions and students' prior knowledge. In this context, it is generally knowing the how and the why of teaching the subject specific. Therefore, in line with Shulman's (1987; 1986) proposition of the framework for knowledge bases, many other researchers indicate that teaching mathematics is a complex and multifaceted task. Therefore, picking a strategy in teaching linear programming demands a multi-dimensional knowledge base as advocated by Shulman.

In exploring the teacher knowledge for teaching, this study section is discussed in the light of Shulman's framework for knowledge base. Shulman (1986) claimed that Pedagogical Content Knowledge (PCK) was not accorded its real position by teachers and for this reason he called it, the blind spot, missing paradigm, missing gap or missing programme (Ernest, 1986) which enables teachers to 'get it across' (Ball and Hill,2009, p.86) to their learners. Shulman (1986, p. 8) argued that 'mere content knowledge is likely to be as useless pedagogically as content- free skill'. Instead of paying much attention to class organization, the teacher should take as first priority, the content taught, questions asked, explanations given and illustrations demonstrated in the teaching discourse which are key in the teaching of linear programming. Shulman (1987, p.8; 1986, p.227) further described PCK as 'a special amalgam of content and pedagogy that is uniquely the province of the teachers, their own special form of professional understanding (Even, Elen&Depaepe, 2015; lannin, Webb, Chval, Arbaugh, Hicks, Taylor, &Bruton 2013).

Researchers seem to agree that teaching mathematics depends, among other factors, on a specialized body of knowledge specifically for teaching mathematics (Adler, 2005;, Ball et al., 2008; Kwon, Thames, & Pang, 2012).While the link between ordinary subject content that a teacher has and students' achievement remains blurred (Ball, Lubienski, &Mewborn, 2001; Monk, 1994), there is strong evidence that links teachers' mathematical knowledge for teaching to classroom instruction and students' achievement (Ball et al., 2005; Baumert et al., 2010; Hill, Rowan & Ball, 2005; Mewborn, 2003). Shulman (1986) first studied this body of knowledge. His initial classification of the knowledge for teaching included seven categories, namely: knowledge of content, knowledge for curriculum, pedagogical content knowledge, knowledge of pedagogy, knowledge of learners and learning, knowledge of context, and knowledge of educational philosophies and objectives. Before and immediately after Shulman's initial work, different terminologies were used to describe the body of knowledge required for teaching. For instance, the terms content knowledge and pedagogical content knowledge were widely used and as such they are key in the teaching of linear programming.

Ball, Thames, and Phelps (2008) and Manson (2008) argue that differences in the understanding of and use of the terms to describe this body of knowledge caused a lot of ambiguities and limitation in its use. Grossman (1990), a member of Shulman's research team improved Shulman's categorization of the body of knowledge for teaching. She reduced Shulman's seven categories into four main categories. These categories were subject-matter knowledge, general pedagogical knowledge, knowledge of context and pedagogical content knowledge. The major contribution through Grossman's categorization and definition of the knowledge for teaching was the inclusion of beliefs as part of the pedagogical knowledge. This inclusion of beliefs as part of the pedagogical knowledge is very important in understanding how teachers would pick and use a particular strategy in a particular section of linear programming.

Further research (Fennema & Franke, 1992; Marks, 1990) followed these initial efforts to define and understand knowledge for teaching mathematics. However, these efforts were mainly disjointed and used Shulman's work to defined subject matter knowledge for teaching from a qualitative focus (Ma, 1999). Ball, Thames and Phelps (2008) developed a practice-focused framework for teachers' mathematical knowledge for teaching whose aspects have been experimentally verified. In their work, Ball, Thames and Phelps analyzed literature that was available on mathematical knowledge for teaching and isolated from the literature important components of mathematical knowledge for teaching. These components have been subjected to empirical studies by studying teaching of mathematics and not teachers (Ball et al., 2008; Hill et al., 2005). Ball and her research team derived the term Mathematical Knowledge for Teaching (MKT) to represent the special type of knowledge required only for teaching mathematics which is the same knowledge base for teaching linear programming.

Ball et al. (2008) argue that mathematical knowledge for teaching framework has two major domains namely: subject matter knowledge (SMK) and pedagogical content knowledge (PCK). For them, subject matter knowledge comprises common content knowledge (CCK), specialized content knowledge (SCK), and horizon content knowledge. Horizon content knowledge represents knowledge about mathematics that is outside the curriculum being taught, and how such knowledge is important in orienting and navigating in the classroom (Jakobsen, Thames, Ribeiro, 2013). Hill, Ball, and Schilling (2008) refer to this sub-domain as knowledge at the mathematical horizon. Pedagogical knowledge is made up of knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC) of which those play a pivotal role in focusing on the teaching of linear programming.

The common content knowledge sub-domain of subject matter knowledge is knowledge that is not unique to teachers. Rather, it consists of mathematical knowledge and skills that are required in, for instance, solving mathematics problems or performing a mathematical procedure and defining a mathematical concept. This therefore encompasses all the knowledge and skills necessary to use mathematics in general as required by both teaching and non-teaching professionals. The specialized content knowledge is "mathematical knowledge not typically needed for purposes other than teaching" (Ball et al., 2008, p. 400). For Hill et al. (2008, p. 378), this is the "knowledge that allows teachers to engage in a particular teaching task, including how to accurately represent mathematical ideas, provide explanations for common rules and procedures, and examine and understand unusual solution methods to problems". They argue that this knowledge differs significantly from Shulman's (1986) original conceptualization of subject matter knowledge which in their view is common content knowledge. Teachers require this specialized knowledge in the teaching of linear programming among other things in order to identify error patterns in students and assess whether a nonstandard approach is generalizable.

In the debate on the type of mathematical knowledge for teaching, Ball et al.'s (2008) position declared that it was unclear what exactly makes up the extra knowledge of mathematics for teaching. They referred to the question of what effective teaching of mathematics is required in terms of content understanding. Meanwhile, in their quest to elaborate Shulman's categories of knowledge bases, Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK), they suggested the domains of SMK to include common content knowledge (CCK), mathematical horizon content Knowledge (HCK) and specialized content knowledge (SCK) while classifying pedagogical content knowledge as comprising knowledge of content and students (KCS), knowledge of content and teaching (KCT) knowledge of content and curriculum (KCC) which is a demand for teachers in the light of the teaching of linear programming. These knowledge categories are illustrated in the figure below.

Figure 2.1 below shows the knowledge domains as suggested by Ball and her research team.

Subject matter knowledge **pedagogical content knowledge**

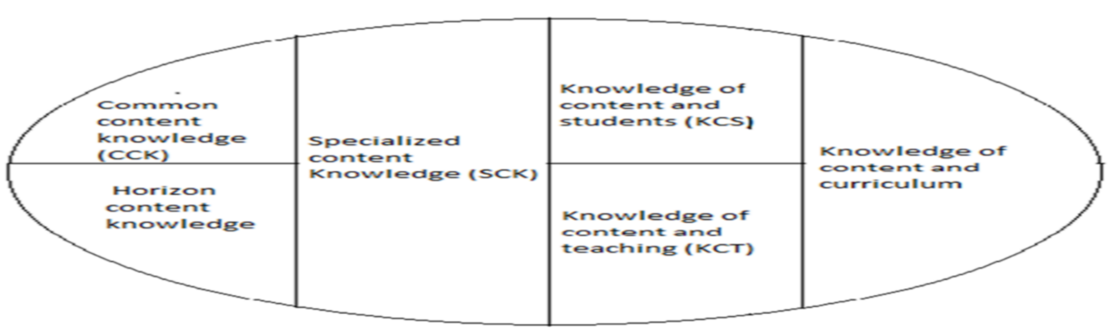


Figure 2.1: Categories of MKT (Ball et al, 2008, p. 403)

However, the above model has received some critiques/reservation. Let me state some of the concerns that have been raised regarding Ball et al.'s model. The first reservation which was raised is that, the 'line' between the Common Content Knowledge (CCK) and the Specialized Content knowledge (SCK) is blurred. This blurring is a concern shared by Thanhesier, Browning, Moss, Watanabe and Garza-Kling (2010). Yet while this is problematic in an analytical and academic way, in a practical sense it seems to make little difference where one domain finishes (CCK) and the other one begins (SCK). This issue is acknowledged by Ball et al. (2008) who refer to this as being a 'boundary' problem. They state that it is not always easy to discern where one domain divides from the next.

The second issue raised is simply in the representation of the domains in the diagram and the possibility of using the visual cue of region size to determine the importance of one domain of knowledge over another. In other words, because SCK appears to employ a larger area (Figure 2.1), the question that has been raised is: "is it therefore, perhaps more important than KCC?" however regarding this question there was no evidence in any reading suggesting levels of importance, but in any other specific circumstance, any one domain was necessarily more important than any other.

The third reservation is in using the term Pedagogical Content Knowledge (PCK) to describe the domains regarding pedagogical concerns (Hurrell, 2013). Hurrell (2013, p 59) urges that, "*perhaps Pedagogical Knowledge (PK) may have been a better term to employ as there is a strong argument to be stated that PCK actually only occurs at the overlap between the SMK and PK…PCK is actually only occurs between the knowledge bases of content, pedagogy and context.*" The fourth and final reservation is that the model does not display the possibilities of all of the interactions between the domains (Hurrell, 2013).

Therefore, due to these four arguments outlined above, Hurrell (2013) offers some consideration for refinements to Ball et al.'s (2008) Mathematical Knowledge for Teaching model (Figure 2.1). In this revised model for Mathematical Knowledge for Teaching (MKT), each of the domains of MKT has been illustrated by regions which are the same size. This is indicative that until a specific circumstance has been determined no domain is fundamentally more important than any other domain (Hurrell, 2013). Indeed it is the circumstance that determines which domain or domains have priority for the teaching of linear programming. This implies that, Mathematical Knowledge for Teaching linear programming on the part of the teacher fits in with this understanding and seeing the demonstration of any domain knowledge in my study as need arose was one of my interests. Figure 2.2 below is the suggested revised model in addressing the concerns above.

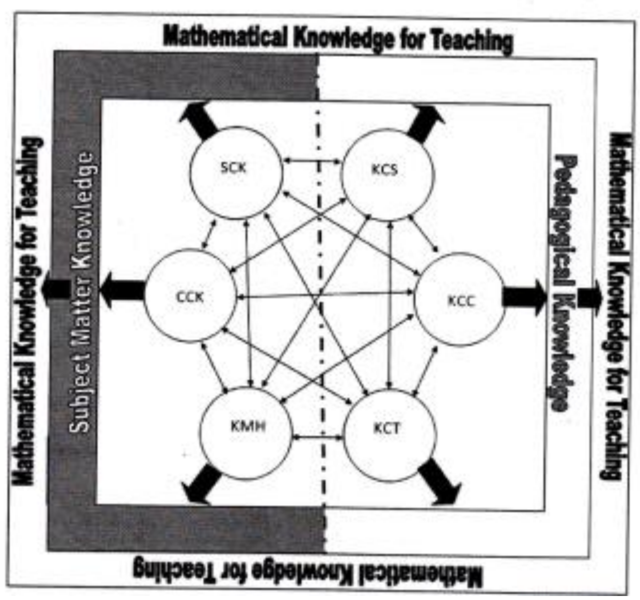


Figure 2.2: Revised model for Mathematical Knowledge for Teaching (Hurrell, 2013)

In the revised Mathematical Knowledge for Teaching model (Figure 2.2) the pedagogical domains are shown 'feeding' into an area designated as Pedagogical Knowledge' in the Ball et al. (2008) model (Figure 2.1) this field is treated as a heading and given the label of Pedagogical Content Knowledge (PCK). As previously stated, research would assert that PCK is actually the overlap of Subject Matter Knowledge (SMK) and Pedagogical Knowledge (PK) and that Figure 2.1 does not show this overlap. Rather it displays SMK and PCK sitting as headings without reference to the required interconnectedness. The final difference between the two models is in the use of arrows in the revised MKT model to illustrate the various interactions between the domains. The Ball et al. (2008) model (Figure 2.1) suggests that each of the domains is discrete and entire of itself. It could be argued that exercising any of the domains brings into play other domains, for example, it is not feasible in the context of teaching and learning of linear programming to talk about content devoid of pedagogy or pedagogy devoid of content. Even those who teach by didactic chalk and talk illustrate a pedagogy which is so important to observe too.

Teachers are quite capable of engaging in reflective practices regarding their teaching (Galae, 20l2). To facilitate this reflection occurring, it may be beneficial for some to have a model from which to work and scaffold their understanding. PCK which was devised by Shulman (1986) as a heuristic is certainly a consideration, but the Ball et al. (2008) construct with the modifications suggested may be a better place to start from due to the development of Shulman's ideas. Ball and Charalambous's research (2012) supports that Mathematical Knowledge for Teaching (MKT) contributed to instructional quality in the teaching and learning of linear programming in my study. Hurrell (2013, p 62) further argues that, *"it therefore would not seem unreasonable to suggest that if we want to improve teacher effectiveness the development of MKT is an important factor. At the very least, familiarity with this construct would allow teachers to reflect on the various domains that require development to foster PCK, and allow them the opportunity to strengthen any areas in which they may feel they are deficient."*

Research has demonstrated that teaching mathematics requires mathematical understanding beyond content knowledge necessary for practicing mathematics in general sense (Ball et al., 2005; Hill et al., 2005; Ball & Schilling 2008; Shechtman, Roschelle, Haertel, & Knudsen, 2012). This understanding helped me to check through in the teaching of linear programming on how teachers practiced their teaching in the demonstration of their knowledge base. Studies have shown that having strong subject matter content knowledge is inadequate for the mathematical knowledge necessary for teaching. However, the fact remains that we do not have an all-inclusive understanding of what mathematical knowledge is necessary for teaching across contexts and the extent to which other factors impact mathematical knowledge for teaching in teachers (Silverman & Thompson, 2008). Further research is therefore required to expand the conceptualization of mathematical knowledge for teaching by considering different contexts and how mathematical knowledge for teaching develops in these differing contexts as suggested by Silverman and Thompson in which exploring teachers' strategies in the teaching of linear programming is a part.

While there has been different mathematical knowledge for teaching models due to researchers' differing conceptions and beliefs about what mathematical knowledge is required for effective teaching, there appears to be a common understanding that teachers' mathematical knowledge for teaching and practice are interactive and dynamic. Another important and common understanding among the different mathematical knowledge for teaching models available is context which is an important consideration in the teaching of linear programming. All the models seem to suggest that teachers' mathematical knowledge for teaching is not context free as it will be affected by locality, resources, background, and other factors (Thompson, 1992; Silverman, 2008; Andrews, 2011; Delaney, 2012).

Further, according to research on teaching, strong evidence exists that many beginning teachers have limited knowledge about their own discipline (Anderson, 1988, Shulman, 1986, 1987). A teacher's professional knowledge affects all phases of instruction-lesson content and planning, implementation, assessment, and reflection. Teaching how to teach and learning how to teach is difficult. Aristotle believed that ultimate understanding rested on one's ability to transfer knowledge into teaching (cited in Shulman, 1986). Through research, several models of teacher knowledge have been developed (e.g., Elbas, 1983; Leinhardt and Smith, 1985; Shulman, 1986, 1987, Wilson, Shulman &Richert, 1987). Though differences exist between the number of sub-components identified in each of the models, there is a general agreement that four main categories of professional knowledge for teaching are: (1) general pedagogical knowledge, (2) subject matter knowledge, (3) pedagogical content knowledge, and (4) knowledge of context. All these give a greater base for a much broader understanding for a teacher in the teaching of linear programming.

Finally, according to Ball et al (2005), higher general pedagogical/psychological knowledge implies higher quality of instruction according to student perception, thus higher cognitive activation, better instructional pacing, better student-teacher relationship. Therefore, the teaching of linear programming in the light of different strategies that teachers used demanded highly the understanding of the mathematical knowledge for teaching as advocated for by different scholars above.

2.3 Factors affecting the teaching and learning of linear programming

2.3.1 Language and the learning of mathematics

Pimm (1981, 1991) argues that learning mathematics is like learning a language. Pimm analyzed the spoken and written language of mathematics and described the relationship between mathematics and language in terms of register. According to Pimm, part of learning mathematics is acquiring control over the mathematics register. Halliday (1975 cited in Pimm, 1991, p.17) defines a mathematics register as “A set of meanings that is appropriate to a particular function of language, together with the words and structures which expresses these meanings”. The mathematics register includes words, phrases, symbols, abbreviations and ways of speaking, reading and writing that are specific to mathematics.

Pimm (1991) further argued that acquiring fluency in the mathematics register requires that learners need to learn to work within the mathematics register and understand its particularity. Pimm said, learning mathematics in school comprises both informal and formal components. Informal language is the kind of language learners use in their everyday experiences to express their understanding of mathematical concepts. Formal mathematical language refers to the accepted use of terminology developed in schools.

Educators have acknowledged language as a tool for teaching and learning. Mercer (1995) argues that people use language to get things done and to engage in their interests. Using language to get things done depends on how well one is able to communicate his ideas to other people and how those people understand his ideas. In most cases, instructions are passed on through talking or writing using a language that both parties understand. In the same way, a teacher negotiates meanings with learners in a language that they both understand. Learners use language to communicate amongst themselves and with the teachers. They use language to think (Orton, 1992) and Mercer (1995) argues that language is both an individual and a social mode of thinking. Thinking is a process that assists learning and thus language structures that are not fully formed may set hurdles in learning.

According to Thurston (1995) in order to achieve the necessary comprehensively mathematical understanding, successful communication of mathematical ideas is the key. Language forms an integral part of this communication. Bohlmann (2001) contends, “Language is the medium by which teachers introduce and convey concepts and procedures through which texts are read and tasks are solved” (P.6).

Students have to understand the mathematics concept as well as communicate their understanding of these concepts verbally and in writing. The teacher presents concepts written or verbally using mathematics language, and everyday language, students have to be proficient in both languages. However, competency in everyday language does not imply competency in the mathematical language (Lemke, 1990). Therefore, the teaching of linear programming attracted the understanding of mathematical language in the light of linear programming as this was key in building the conceptual understanding of linear programming.

2.3.2 Categories of mathematics register

According to Halliday (1978), the register is characterized by field, tenor and mode. Field refers to the social activity in which participants are given tasks to do and are allowed to verbalize; tenor refers to the relationships among the participants which include group leader during the social activity; and mode refers to the way the social activity is organized and how the participants interact with each other. Green (1988) observed that acquiring a register does not only involve learning the appropriate words but also being able to predict the kind of language appropriate to the field, tenor and mode for a particular context of situation.

Discussing the register of the mathematics classroom, Green (1988) identified two kinds of registers which come into play in different situational contexts. The first is the register of formal or ‘technical’ mathematics. Monroe and Panchyshyn (1995) sub-divided the register of formal mathematics into technical terms and sub-technical terms. On one hand, they pointed out that each technical term has only one meaning which is specific to mathematics. An example of such a technical term is ‘inequation’, a terminology which conveys the mathematical concept. On the other hand, they described sub technical terms as words whose meanings vary from one subject to another or from a subject to everyday experience. For example, the word 'root' which is part of a tree has a different meaning when used in the mathematical context where it represents values of a variable in an equation, as the same understanding is employed in linear programming concepts.

The second kind of register is the language of instruction used by the teacher in different social activities in the classroom (Chapman, 1997). This register varies according to the nature of the activities pupils are engaged in the classroom. For example, if on one hand the activity is about finding the answers to a given problem, the language of instruction may include words such as calculate, solve or work out. On the other hand, if the activity involves graph work as in the case of linear programming, then the language may include words like 'shade', 'label' or 'plot'. The language of instruction also includes a range of imperative forms such as 'let,' 'suppose,' 'define,' 'given' and 'consider' as opening words in sentences. Commenting on the characteristics of the mathematics register, Pimm (1987) observed that: The most striking characteristic of the mathematics register is the number of terms it contains which have been borrowed from everyday English. Examples of such words include: face, degree, relation, power, radical, complete, integrate, legs, product, mean, real, rational and natural. The extent to which this happens is great such that it is not just certain nouns and verbs like a 'ring' or to 'differentiate' to which this borrowing applies, but it also involves a wide range of grammatical constructions. Despite all clocks becoming digital, the word 'clockwise' will still remain (Pimm, 1987, p.78).

Therefore, the mathematics register is composed of different forms of mathematical language, which also includes symbols and terminologies from everyday language which was adapted by Bubb (1994) as cited in Ballard and Moore (1987). The mathematics register is an important attribute in the teaching and learning of linear programming. It spells out the need to paying attention to symbols and phrases that are specific for linear programming.

2.3.3 Teacher's philosophical beliefs

Beliefs have been described differently by many scholars. Ernest, (1989) submitted that beliefs consist of teachers' system of mental models, conceptions, values and ideologies. Subsequent works of Shulman and colleagues also accounted for the beliefs about mathematics. Ernest, (1991) held that teachers' beliefs about the nature of mathematics affect their subject matter knowledge and learning. He contended that, one's conception of what mathematics is affects their conception of how it should be taught and what they believe to be most essential about mathematics.

Ernest (1989) described three categories of teacher beliefs about the nature of mathematics that have been widely adopted. These include the instrumentalist view, the Platonist view and the problem- solving view. Instrumentalists see mathematics as, "an accumulation of facts, skills and rules to be used in the pursuance of some external end" (Ernest, 1989, p. 250). They view mathematics as a discipline with various unrelated topics. Because of this, learning is regarded as telling. The Platonist view sees mathematics as a static body of unified, pre-existing knowledge awaiting discovery. In this category, the structure of mathematical knowledge and the interconnections between various topics are fundamentally important. However, the view restricts creativity and inventiveness. The third category is the problem- solving view in which according to Ernest (1989), mathematics is regarded as a dynamic and creative human invention; a process rather than a product.

My study related belief categories to the nature of mathematical knowledge (absolute and fallible). I did not subscribe to the instrumental and Platonist view and emphasize the problem-solving belief category. I desired to see linear programming being treated as a dynamic body of knowledge that can be created from within the classroom. I wanted to see positive teacher-learner interaction in the learning discourse, and the teacher engaging the learner in tasks that promote inter-subjectivity among learners, thus bring about shared meaning in learning (Ernest 1991).

I acknowledge that beliefs explaining a teacher's philosophy of mathematics teaching affect their attitude and further influence their knowledge structures. In view of this, I addressed attitude as a factor containing teachers' knowledge in the teaching of linear programming.

2.3.4 Teacher’s knowledge and attitude

Teacher’s attitude towards mathematics includes liking, enjoyment, interest and confidence in mathematics or the opposites, which according to Ernest, (1989) may include ‘mathephobia’. Attitudes may also include such feeling as teacher’s confidence and valuing of mathematics. Much has been reported about the significant correlation between teacher attitude and achievement in mathematics (Bishop &Nickson, 1983). These attributes were measured during the lesson observations and interviews that were conducted in the study.

Having established the effect of misconceptions, beliefs and attitudes of teachers on linear programming, I will now highlight the learner attributes in learning linear programming.

2.3.4 Learner attributes

I propose that background knowledge of learners is also an important factor in advancing their conceptual understanding. This included the knowledge acquired from preceding grade levels on the topic and other related topics as pre-requisite knowledge for linear programming to the learner. Besides, home background and experience are important facets in learning and as such, the teachers should understand learners as individuals.

Having discussed the philosophical belief of the teacher, teacher knowledge and attributes and learner attributes, I wish also to highlight what can be considered in teaching and learning linear programming

2.4 What can be considered in teaching and learning linear programming

School mathematics comprises different topics or areas of study which include probability, geometry and trigonometry. These topics may have their own register which could be classified as sub-registers. This is true in the sense that the terminologies that are used in one particular topic may not be the same as the ones used in another topic. Therefore, since there are different branches of mathematics, then each branch may have its own register.

Davidenko (2006) suggested a framework for analyzing the algebra mathematics register which has four categories. He defined each category of the register as follows.

Instrumental register: This register uses only verbs to denote actions and sequence of actions. For example, add 2, divide by 4, and plug in the value.

Procedural register: In this register, verbs are used to denote action or sequence of actions and logical connectors such as if/then and this/because are also used. For example, divide by 2 on both sides of the equation because we are applying the inverse operation of multiplication.

Conceptual register: This register uses nouns to name the concepts. Adjectives and adverbs may also be used to describe the properties of a concept or procedure. For example, a quadrilateral (noun/concept) is a four-sided (adjective/property) polygon (noun/concept).

Formal and symbolic register: In this register symbols are used for concepts (x = variable, m = slop); symbols for procedures or operations (+, -, x, ÷); symbols for relationships (<, >, =) and expressions to denote logical statements (and, or, for all).

The author concluded that the framework could be used to explore the teachers' use of language in the classroom. This framework was applied in the analysis of the classroom practices and experiences of both the teachers and pupils in the use of mathematical terminologies in the teaching and learning process of which I see it to be also useful in the classroom interaction.

Further, Bell (1991) and Sweller and Low (1992), suggested a set of specific heuristics (Steps 1-8) that provide a model for solutions of linear programming problems. It was hoped that these heuristics would help break the problem down into manageable steps and avoid or alleviate cognitive obstacles to linear programming.

Summary of Steps in Solving Graphically a Linear Programming Problem comprises the following:

Step 1: Locate the "decision variables".

Step 2: Name the decision variables, representing each by a different letter (usually x or y)

Step 3: Name the variable which must be maximized or minimized (e.g., profit, or cost) and express it in terms of x and y, the decision variables.

Step 4: What constraints (restrictions) are imposed on each of the decision variables? State these in words using "The number of ...".

Step 5: Express "The number of ..." constraints in mathematical language, using inequality symbols.

Step 6: Using x and y axes, sketch the areas defined by the inequality statements. Hence find the "feasible region".

Step 7: Find the co-ordinates of the vertices of the feasible region.

Step 8: Find the solution to the problem by calculating the profit (or cost, etc.) for each of the vertices of the feasible region.

These steps by Bell (1991) helped me compare some of the strategies teachers used in the teaching of linear programming. They brought to light some steps which teachers used during their lesson discourses in the teaching and learning of linear programming as a yard stick.

2.4.1 Application of linear programming in real – life situations

The application of linear programming knowledge and skills to real life context is further supported by Freudenthal (1973) in the realistic mathematics education advocacy. In RME, the starting point of instructional experience should be real to the children, so that they can immediately engage in personally meaningful mathematical activity (Streefland, 1991 cited in De Lange, 1996). The learners' informal mathematical activity should correspond to a basis from which they can abstract and construct more and more complicated mathematical concepts. While engaging in these mathematical activities, learners will be using horizontal and vertical mathematization and in the process learners will be using their home languages, mathematical symbols and the language of learning and teaching interchangeably. Hence, move gradually to the importance of the shift from "ordinary" language to specialized language and mathematical, symbolic representations.

In horizontal mathematization, learners are mathematising contextual tasks. In the process, learners use their informal strategies to describe and solve a contextual task. While vertical mathematization occurs when the informal strategies lead learners to solve the task using mathematical language or to find a suitable algorithm (Treffers, 1987; Gravemeijer, 1994). Horizontal and vertical mathematization happens through learners' actions as well as when they reflect on their actions.

Freudenthal (1991) contends that:

> *Horizontal mathematization leads from the world of life to the world of symbols. In the world of life, one lives, acts (and suffers); in the other one symbols are shaped, reshaped, and manipulated, mechanically, comprehendingly, reflectingly: this is vertical mathematization. The world of life is what is experienced as reality, as is symbol world with regard to abstraction. (p. 41)*

For my study, in a linear programming task, the process of extracting the important information required using an informal strategy by teachers would be called horizontal mathematization. Translating the task into mathematical language through the use of symbols and later progressing to selecting an algorithm could be considered to be vertical mathematising as it involves working with the problem at different levels.

2.5 Related studies in linear programming

Hosein, Anesa; Aczel, James, Clow and Doug (2006), did a study on the teaching of linear programming in different disciplines and in different countries. The study was an online survey of linear programming (LP) lecturers in four countries in various disciplines. The study used Biglan's classification of disciplines to show that courses in hard-pure and hard-applied subjects were more likely to teach theoretical aspects of linear programming whilst the hard-applied and soft-applied subjects looked more at the application. The disciplines were grouped as 'hard' or 'soft' and 'pure' or 'applied'. For example, a course in engineering is classified as hard-applied as it is rooted in a clearly delineated paradigm and has an application nature. Only two other classifications were used: hard-pure (e.g. mathematics) and soft-applied (e.g. business studies) as no soft-pure (e.g. history) courses were found. Further, the soft-applied disciplines were more likely to utilize software during the teaching of the topic. Also, US lecturers were more likely to teach theoretical aspects of LP whilst the UK lecturers were more likely to use common software such as spreadsheets rather than any other mathematics software. This study was done on college lecturers and its perspective was on the different discipline through which linear programming can be taught. I argue that the study did not give an African context because the countries which were involved were; Australia, New Zealand, USA and the UK. Therefore, my study focuses on the teachers' strategies in the teaching and learning of linear programming in Zambia, thus giving an African perspective.

Nkambule (2009), did a study on the teaching and learning of linear programming in a grade 11 multilingual mathematics class of English language learners: exploring the deliberate use of learners' home language. Data was collected through lesson observations for five consecutive days, reflective interview with teacher and clinical interview with two learners. This was a qualitative case study focusing on one teacher and his Grade 11 multilingual class in a township school in the East Rand. Analysis of data revealed that the teacher used learners‟ home languages to probe learners" understanding of specific terms frequently used in linear programming concepts, for example terms such as, „at least‟ and „at most. Learners' responses suggest that they drew on their home languages for the meaning of these words. The focus of this study was on use of home language in the learning of linear programming.

Nkambule (2013), did another study on immigrant learners learning linear programming in multilingual classrooms in South Africa. This was an empirical case study which used qualitative methods of data collection and analysis. The main conclusion in this study was that immigrant learners were successful in linear programming when teachers created learning opportunities by using code switching to support them. The focus of the study was an investigation of learning in a multilingual classroom. I also hoped to see in my study, teachers code switching in the teaching and learning of linear programming in providing more interaction of symbols with their associated phrases/meanings.

Nakhanu, Shikuku and Wasike (2015) did a study in Kenya entitled Application of Linear Programming Knowledge and Skills to Real Life Contexts by Secondary School Students in Kenya. The purpose of this study for Nakhanu, Shikuku and Wasike (2015) was to investigate whether using the origin test and extreme points technique could encourage and improve students' learning of linear programming. This technique is a version of Problem-based learning (PBL). Problem based learning and its effect on Kenyan Secondary School students learning outcomes in linear programming. Problem-based learning is a student-centered pedagogy in which students learn about a subject through the experience of problem solving. They learn both thinking strategies and domain knowledge. The goals of PBL are to help students develop flexible knowledge, effective problem-solving skills, self-directed learning, effective collaboration skills and intrinsic motivation (Hmelo-Silver, 2004, p. 2). This study adopted the pre-test, post-test nonequivalent group experimental design. The design involved two groups of subjects, with one group being the control and the other being the experimental group. Results of this study show that learners taught using the origin test and extreme points technique achieved better results than those taught using conventional methods. This study was conducted as an intervention measure using problem-based learning to improve students' learning of linear programming. However, my study focuses on teachers' strategies in the teaching of linear programming which scored the interest of seeing if teachers used problem-based learning approach in their teaching using the origin test and extreme point technique.

2.6 The Research Gap

Linear programming has been recorded as one of the topics that have continuously been attributed to lower passing percentage in the Zambian grade 12 national final examination results. However, the desire to achieve better grades in mathematics examinations in our society cannot be over emphasized. Research in specific topics that cause low passing percentage in mathematics in Zambia are few, and little is known on linear programming from the Zambian context. However, a few studies that have been done outside Zambia have different focus compared to my study. This is evident from the literature reviewed in Section 2.5.1 on related studies in linear programming.

Below is the summary table of related studies in linear programming and showing the gaps that have been identified.

Table 2.1: Summary on related studies and the gap

RELATED STUDIES IN LINEAR PROGRAMMING			
Author/research	Methodology	Findings	Gaps
Hosein, et al (2006). The teaching of linear programming in different disciplines and in different countries	Online survey	US lecturers were more likely to teach theoretical aspects of LP whilst the UK lecturers were more likely to use common software such as spreadsheets or mathematics software. This study was done on college lecturers.	Focused on college lecturers general approach in the T/L of LP as to whether they use some mathematics software or just theoretical
Nkambule (2009). The teaching and learning LP in a grade 11 multilingual mathematics class of English language learners: exploring the deliberate use of learners' home language.	Qualitative	Learners" responses suggest that they drew on their home languages for the meaning of these words; terms used in LP (atleast, at most,…)	The focus of the study was an investigation of learning in a multilingual classroom in the use of code switching
Nkambule (2013). Immigrant learners learning LP in multilingual classrooms in south Africa	Qualitative Case study	Immigrant learners were successful in linear programming when teachers created learning opportunities by using code switching to support them.	The focus of the study was an investigation of learning in a multilingual classroom of the immigrant learners.
Nakhanu et al(2015) Application of LP Knowledge and Skills to Real Life Contexts by Secondary School Students in Kenya (PBL)	pre-test, post-test nonequivalent group experimental design.	learners taught using the origin test and extreme points technique achieved better results than those taught using conventional methods	Its focus was on Problem based learning and its effect on Kenyan Secondary School students learning outcomes in linear programming and the use of origin test and extreme points technique.
Therefore, my study focused on teachers' strategies in the teaching and learning of linear programming in secondary school mathematics.			

2.7 Summary of the literature review

Table 2.2 below gives the summary of the literature reviewed the light of the teacher strategy in the teaching of linear programming.

Table 2.2: Summary of literature review

Section	Areas of focus	Key principles/concepts
2.1	Mathematical knowledge for teaching (MKT)	➢ Knowledge domains specific for teachers in teaching linear programming
2.2	Factors constraining the T/L of linear programming.	➢ Language of mathematics playing a pivotal role in the teaching of L/P ➢ Teacher's philosophical beliefs of the nature of L/P knowledge ➢ Teacher's attitude may include such feeling as teacher's confidence and valuing of mathematics. Much has been reported about the significant correlation between teacher attitude and achievement in mathematics. ➢ Learners' background knowledge is an important factor in advancing their conceptual understanding, taking into considerations their experiences from their social environments.
2.3	What can be considered in T/L of LP	➢ Studies about the T/L of linear programming with their proposed approaches. ➢ Application of LP in real – life situations as the most advocated for in bringing the much needed meaningful learning for personal, national and globe development.
2.4	Related studies	➢ None had a focus on teachers' strategies in teaching of LP

Therefore, in the next chapter I present the methodology of the study.

CHAPTER THREE

METHODOLOGY

3.1 Introduction

This Chapter discusses the research methodology that I applied to examine teachers' strategies in the teaching of linear programming in selected secondary schools in Monze District of Southern Province. The chapter presents the type of research design that was used, study area, study population, sampling techniques, instruments for data collection, procedure for Data collection and data analysis that was employed in the study. The chapter also addresses ethical issues to ascertain the consents.

3.2 Research design

A research design can be defined as, "the arrangement of conditions for collection and analysis of data in a manner that aims to combine relevance to the research purpose with economy in procedure" (Kothari and Garg 2014, p. 29). It is a blue print for collecting and analyzing data. Similarly, Babbie and Mouton (2008) describe a research design as a plan or a blue print that guides and informs how the study will be conducted. This entails all the procedures by which the research problem is tackled to arrive at the findings. The research design is important as it directs and guides the researcher during the research process.

This study used a descriptive research design. According to Paggarwal (2012, p. 231), "descriptive or normal survey is that method of investigation which attempt to describe and interpret what exists at present in the form of conditions, practices, processes, trends, effects, attitudes, beliefs, etc… it is an organized attempt to analyze, interpret, and report the present status of social institutions, groups or area." It is concerned with the phenomena that are typical of the normal conditions. It explores into the conditions or relationships that exist, practices that prevail, beliefs, points of view or attitudes that are held, processes that are going on, influences that are being felt and trends that are developing (Paggarwal, 2012). Therefore, for my study to be descriptive I sought to describe and interpret what exists at present in the form of teacher's practices in terms of strategies in the teaching of linear programming. This method helped me to interpret and report the present strategies that teachers are using in teaching linear programming.

Collis and Hussey (2003) describe descriptive research as research which describes phenomena as they exist, that it is used to identify and obtain information on the characteristics of a particular problem or issue. Since I sought to understand the phenomena in the light of the strategies teachers used in the teaching and of linear programming in its natural setting, hence this design. The qualitative approach was used in understanding of the phenomena in context-specific settings, such as "real world setting where the researcher does not attempt to manipulate the phenomenon of interest (Patton, 2001, p.39). The approach was employed so as to precisely capture the respondents' views and giving a deeper understanding of lessons observed.

3.3 Study Site

To examine the teachers' strategies in the teaching and learning of linear programming, it is important to describe the site settings (Patton, 2002; Ball, 1990). The study was undertaken in three (3) secondary schools of Monze District in the Southern Province of Zambia. Monze is located about 180km from Lusaka, the country's capital city. The three schools were purposively picked, that is, a boys' school, a girls' school and a co-educational school. Monze is inhabited by the Tonga speaking people. Below is the map of Zambia showing the location of Monze District.

Figure 3.1: Study site (map of Zambia showing the location Monze)

3.4 Study Population

A population is defined as a group of individuals, objects or items from which samples are taken. It also refers to the large group from which the sample is taken (Berge, 2007). The target population for this study comprised three (3) secondary schools (for boys only, for girls only and co-education schools), and teachers of mathematics in Monze District.

3.5 Study Sample

A sample is a smaller group obtained from the accessible population. Each member or case in the sample is usually known as participants. Sometimes subjects may be termed as 'respondents' or 'interviewees' (Mugenda, 2003). The sample size for this study included 3 mathematics teachers from 3 different schools and 15 pupils. In total the study sample size comprised of 18 respondents. Merriam (1998) points out that the size of sample should neither be excessively large, nor too small. However, it should be optimum meaning that it should fulfill the requirements of efficiently, representativeness, reliability and flexibility (Merriam, 1998). In fact, a small sample for observations, post – lesson interviews and FGD ensured a high level of reliability and helped to concentrate resources on obtaining reliable information or what the researcher thinks to be the representative or average person (Cohen et al, 2007). Deliberate sampling enables the researcher to select the desirable sample convenient enough to collect (Merriam, 1998). Sandelowski (1995) also points out that determining adequate sample size in qualitative research is ultimately a matter of judgment and experience on the part of the researcher, and researchers need to evaluate the quality of the information collected in light of the uses to which it will be put. Therefore, in this study sample (n=18) was suitable for the qualitative nature of this study, as it was giving a good representation of the schools in my study.

Table 3.1 below shows the table on distribution of the participants**.**

Table 3.1: Distributions of participants

School/location	***Teacher psuedo name***	***Pupils***	***Total***
Boys school (Peri-urban)	***Tr- B***	***5***	***6***
Girl school (Peri-urban)	***Tr- G***	***5***	***6***
Co-school (Urban)	***Tr- M***	***5***	***6***
Total	***3***	***15***	***18***

The study comprised of 3 teachers and 15 pupils.

The choice of the three schools was based on the understanding of the type of schools in the Zambian society, thus the boys, girls and co-education school. I desired to see the teaching and learning of linear programming and the interaction thereof in three different learning environments, for boys only, girls only and co-education.

3.6 Sampling techniques

The study being qualitative used a non-probability sampling techniques to come up with the sample size. The selection of the schools was purposive thus the boys, girls and co-education school. In view of this, I desired to see the teaching and learning of linear programming and the interaction thereof in three different learning environments, for boys only, girls only and co-education. Purposive sampling was employed to select three (3) teachers thus, the grade 12 mathematics teachers where linear programming is taught. Lastly 15 pupils were selected from three (3) secondary schools using purposive sampling of five pupils out of the 45 of average class from each school with the help of their teacher in order to get rich information on the teaching and learning of linear programming. This was a reasonable sample size for a qualitative study because it provided the data needed as it fulfilled the requirements of efficiency, representativeness, reliability and flexibility (Merriam, 1998).

3.7 Data collection methods

Data of qualitative nature were collected using lesson observations, post-lesson interviews and focus group discussions. The procedure, purpose and merits of each instrument in this section is explained below.

3.7.1 Lesson observations

The study used lesson observations to collect data as participant's present lessons using lesson observation guide (Appendix 3). Observation method is known for eliminating subjective bias (Kothari & Garg, 2014) and, "the information obtained under this method relates to what is currently happening; it is not complicated by either the past behavior or future intention" (ibid, 2014, p. 9). In fact, it is a good method for collecting information on respondents' feelings such as attitudes (Kothari & Garg, 2014) toward teaching and learning of linear programming. Sidhu (2006, p.165) said that for observations to be valid, "it is essential that the actual observations be made of the job activity and the product of such activity". Moreover, direct observations are a more natural way of collecting data (Burgess, 2009, 2007; Sidhu, 2006). Therefore, I observed the lessons on linear programming so as to understand what exactly happens in a teaching and learning classroom environment. During observations, I sat in the participants' class of their regular mathematics time and used the video recorder to record the lesson during the teaching session (McMillan & Schumacher, 1993).

3.7.2 Post-lesson interview

To examine teacher's knowledge of strategies in the teaching of linear programming, data was also collected using semi-structured interviews (Appendix A). Direct post-lesson interviews were conducted to enable the researcher control the sample and minimize missing returns from participations (Kothari & Garg, 2014). Data collected by this method is fairly reliable with 'completeness and accuracy' (Francis, 2004, p. 14). Scholars contend that interviews are a useful means of exploring someone else's ideas or thinking about something (Edwards, 1996). In fact, interviews are said to be the best way of collecting data because, "through the respondent's incidental comments, facial and bodily expressions, tone of voice, gestures, reactions, feelings, attitudes, evasions and non-cooperation, an interview can acquire information that would not be conveyed by any other way' (Sidhu, 2006, p, 146). Thus, by conducting interviews helped me to have a better understanding of teachers' strategies of teaching of linear programming. Talking to participants helped me to establish their beliefs about and attitude towards linear programming better. In order for me to capture data in its totality, the interviews were audio-recorded after the lesson and the transcribed see Appendix A for the transcribed interviews.

3.7.3 Focus Group Discussions

Focus group discussions are also advantageous in qualitative research because varied opinions on the topic can be obtained from the respondents. McMillan and Schumacher (1993) recommend that focus group discussion is used in a qualitative research as a data collection tool because it simultaneously solicits for opinions and experiences of participants. Focus Group Discussions were used with pupils as key informants, the discussions were audio-recorded and then transcribed (see appendix B). The focus group discussions were used in order to get the learners views of their experiences in the teaching and learning of linear programming. It helped me in knowing some of the challenges they faced in the teaching and learning of linear programming. Having discussed the data collection instrument, I now discuss data collection procedure and the time line

3.8 Data collection procedure and time line

The study was conducted in term 3 of the school calendar from September to November 2017 and term 1 of 2018 from January to February. Data were collected firstly through lesson observations and followed by post-observation interviews for teachers thereafter; pupils were engaged in Focus Group Discussion. The lessons in linear programming were all observed, thus observing each teacher in all the linear programming lessons in order to allow a more natural setting on the part of the participants and to observe how the topical lesson progressed. Video-recording of the lessons and audio-recording of post-observation interviews enable me to capture reality in its totality. I followed this order in collecting data because, after observing the lessons it was easier for me to ask the teachers views/reasons in line with the strategies they used in different stages of the topic; and then the focus group discussion followed to try to understand the learners experiences during the lesson. Having presented the instrumentation and procedure for data collection, I now provide an outline of instruments and procedures for data analysis.

3.9 Data analysis and procedures

Qualitative data mainly from lesson observations, post-lesson interviews and focus group discussion (Ary et al, 2010) were analyzed thematically in their specified units rather episodes (introduction, development and conclusion). According to Bryman (2008), the thematic analysis is an independent qualitative descriptive approach meant for identification, categorization of the main themes that emerge in the responses provided by the research participants. A theme is basically a group of linked categories describing similar meanings and usually emerges through inductive analytic process. This inductive analytical process allow for themes to emerge from the data rather than searching for pre-determined themes (Strauss and Corbin, 1998). The thematic analysis enables identification of key ideas within the data collected; and for its usefulness in analyzing open-ended data from the interview transcripts (Wilson, 2010). In this study episodes where generated as introduction, development and the conclusion of the lessons. From these episodes, themes were identified in line with the research objectives in order to address the research questions. During analysis, data were recorded from lesson observations, post-lesson interviews and focus group discussions. These were then transcribed, edited, coded, categorized, tabulated (Kothari & Garg, 2014).

The interpretation of the video lessons was using social constructivism tenet, thus examine teacher-learner interaction, learner interaction with peer/learning materials. Further, also taking into account, the teacher mediations as advocated in the ZPD, teaching techniques and seeking alternative to learners' misconception in the learning discourses in the light of knowledge construction. Yin (2009) argues that the researcher should study the output so as to determine whether there is any meaningful pattern coming out. Creswell (2009) also adds that a reflection should be made on the general meaning of the overall information when analyzing data. Lastly, conclusions were drawn based on the findings

3.10 Trustworthiness

In order to ensure trustworthiness in my study, participant own words were used in the presentation of findings. Further, the themes after data analysis were subjected to expert review to see whether they are in line with recordings and recognizable (Merriam, 1998). Thus, my supervisor cross examined them to ensure their credibility in the study.

3.11 Ethical considerations

According to Cohen et al (2007, p. 51) "Ethics concern right and wrong, good and bad…" this is the question of norms and values. Research ethics seek to protect human participants, serving the interest of participants and examine specific activities for the ethical soundness and informed consent (Patton, 2002). Before commencement of the study, I obtained clearance from the Ethics Committee of the University of Zambia. Informed consent was sought from all participants. All the participants were made aware of the nature and purpose of the study and informed that their participation would not affect their status in the school in any way. The participants' right to privacy and confidentiality was respected. To guarantee the anonymity of participants, pseudo names were used instead of actual ones. The study findings were also shared with the participants.

CHAPTER FOUR

PRESENTATION OF RESEARCH FINDINGS

4.1 Overview

This chapter presents the findings of the study on the teachers' strategies in the teaching of linear programming in secondary school mathematics in Monze District, Southern Zambia. The purpose of the study was to examine the strategies that teachers used in teaching of linear programming and determining factors affecting its teaching. Therefore, in this presentation of findings, the following codes have been used for identification of the participants. Tr-B (Teacher at the boys' school), Tr-M (Teacher at the co-education school), Tr-G (Teacher at the girls' school), FGD-B (Focus Group Discussion Boys school), FGD-M (Focus Group Discussion Co-education school) and FGD-G (Focus Group Discussion Girls' school). While in the lesson dialogues pupils will be coded as Pupil $_{\mathbf{B1..}}$(Pupil at boy school), Pupil $_{\mathbf{G1..}}$(Pupil at girl school) and Pupil $_{\mathbf{M1..}}$(Pupil at a co-education school). The study being qualitative, the data are presented in various categories of themes in line with the objectives. The following are the research Objectives:

I. To Examine teachers' strategies in the teaching and learning of linear programming in secondary school mathematics
II. To established factors affecting the teaching and learning of linear programming
III. To determine the intervention measures that teachers would suggest to overcome the constraints.

In order to authenticate the interpretations drawn from the data set, the research questions guided the presentation of findings and these are:

1. What strategies do teachers use in the teaching and learning of linear programming?
2. What factors affecting the teaching and learning of linear programming?
3. What intervening measures do teachers suggest to overcome the constraints?

The findings are presented in the following order: teachers' qualification and years of experience in the teaching of mathematics as the information was captured in the interview in order to broaden the understanding of the participants, then the themes shall be discussed in the light of the research questions. The themes under question one include: the use of grade 9 linear inequations and coordinate geometry as pre-requisite knowledge, use of a table with symbols and their associated terms during the interpretation of problem statements in forming inequations/inequalities, enlisting of terms and the associated symbols when forming inequations from situational problems, use of trial and error method and boundary point (vertices of the feasible region) when finding the coordinate points for the maximum and minimum cost/profit. Themes for question two are: instruction for the region to be shaded, translating situational statements into inequalities and use of trial and error method. Question three had the following themes: consistence of the region to be shaded for linear inequations and linear programming, more learner interaction with situation problems, using vertices and boundary points of the feasible region and engage with real life application of linear programming.

4.2 Demographic characteristics of the teachers

The data about the teachers who participated in the study was obtained during the interview (Appendix A). There were two diploma holders and one was a degree holder who taught at senior secondary school level and participated in the study. All these who participated had taught for more than 5 years. The following section gives light to the strategies which were used in the teaching and learning of linear programming.

4.3 Teachers' strategies in the teaching and learning of linear programming in secondary school mathematics.

Research question number one was on the strategies teachers use in the teaching and learning of linear programming. In order to establish the strategies teachers used in the teaching and learning of linear programming in the secondary school mathematics, lessons were observed, semi- structured interviews on teachers and focus group discussions were conducted. Therefore, the main strategies which the study identified were: use of inequations and coordinate geometry, tables involving terms and their associated symbols, enlisting the terms to associate with the symbols, use of trial and errors method and use of boundary points.

4.3.1 Use of linear inequations and coordinate geometry

From the lesson observations, the teacher had different strategies in introducing the topic. The use of grade nine Inequations/Inequalities and coordinate geometry were the strategies in the introduction of linear programming. Therefore, the study estalished that two out of three teachers used linear inequations from junior secondary school (Tr – B and Tr – G), and then one used coordinate geometry of the straight lines (Tr – M).

4.3.1.1 Linear Inequation/inequality

Linear inequation/inequality was use by Tr – B and Tr – G in introducing linear programming. However, each one used the strategy differently. I therefore present the ways in which each one used it. The following are examples on how these teachers introduced linear programming are provided as follows:

Tr – B had the following lesson flow;

> Tr - B: *Today we are going to be looking at Linear Programming. However we are going to start by looking at graphing of linear equation/ inequalities in one variable.*
>
> Tr – B*: Write (graphing of linear inequation in one variable)on the chalkboard.*
>
> Tr - B: *In grade 9 you remember; we were shading the wanted region*
>
> Pupil $_{B\,1}$: *Agree*
>
> Tr – B: *But now we are going to be shading the unwanted (false) region.*
>
> *Tr – B: Let us look at the following examples.*
>
> *Figure 4.1 shows one of the graphs which was done.*

Figure 4.1 below shows some of the graphs which were plotted for linear inequation/inequalities in one variable. The following was the dialogue which was engaged:

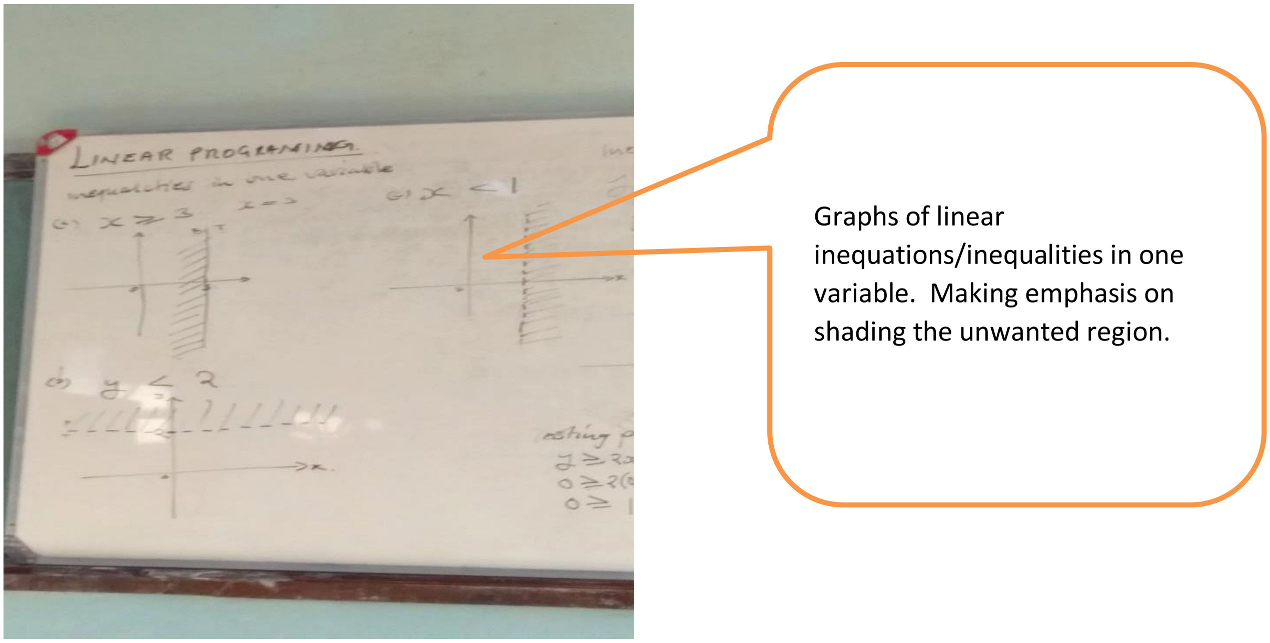


Figure 4.1: *linear inequations/inequalities in one variable (Tr – B, October 2017)*

Tr – B: *We are now going to look at linear inequation/inequality in two variable.*

Tr – B: *Write (graphing of linear inequation/inequality in two variable)*

Tr – B: *We are going to start with an inequation y greater or equal to x.*

Tr – B*: When plotting graphs of inequation and inequality in two variables we use a table, say*

X	*1*	*3*
Y		

When x=1, y=1

When x=3, y= 3

Figure 4.2 below shows the graph of linear inequation in two variables and the test point which was used to determine the region to be shaded.

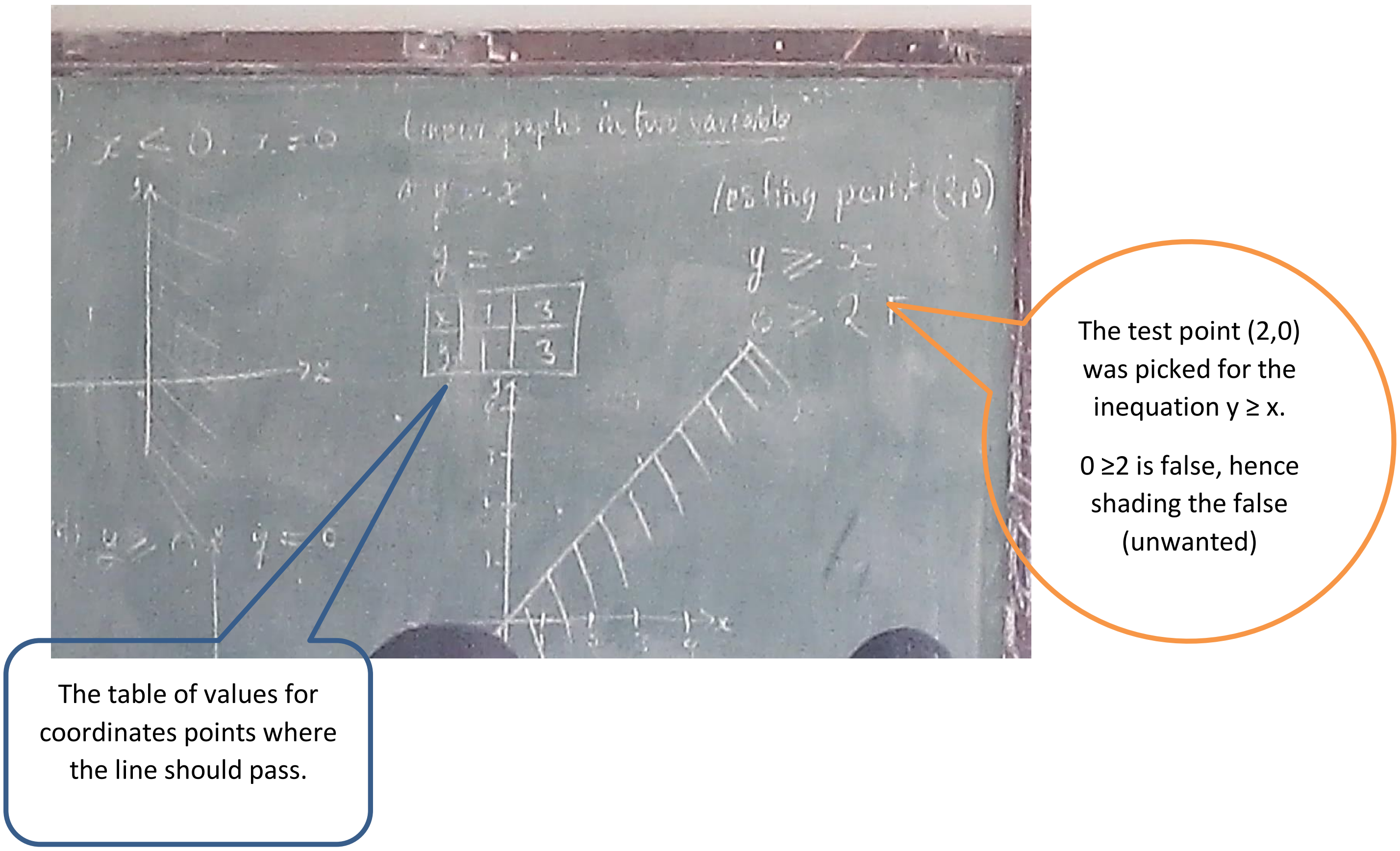


Figure 4.2: *Linear inequations in two variables (Tr – B, October 2017)*

During plotting, the teacher did explain as follows;

Tr - B: *Since it is greater or equal to (≥), then the line has to be bond.*

Tr - B: *Then to determine where to shade we use a test point. Anyone to choose a point?*

Pupil $_{B\ 2}$: *(2, 0)*

Tr - B: *Then we substitute in our inequation y greater or equal to x, thus y = 0 and x =2, we have 0 greater or equal to 2 which is false and the point (2,0) is below the line, then we shade*.

However, when Tr – B was asked during the interview as to why he opted to use this strategy in introducing the topic, he had this to say:

"Aah the method that I used to introduce the topic Linear Programming, first of all I had to look at the issues ***of inequalities (< or >) in one variable*** *and we moved on to* ***inequations in two variabl****e and also believe in what we call starting from the known to the unknown the preliminaries are covered first of all, and I had to go back as far as grade nine to re-kindle the interest of Linear Programming to where it starts from." (Tr* **-** *B: September 2017)*

Tr – G also did introduce the topic using linear inequation/inequality of junior secondary school. She started with showing the learners the inequality symbols with their corresponding meaning (phrases associated to them), and then went on giving examples on how to solve linear inequations in one variable as shown in Figure 4.3 below.

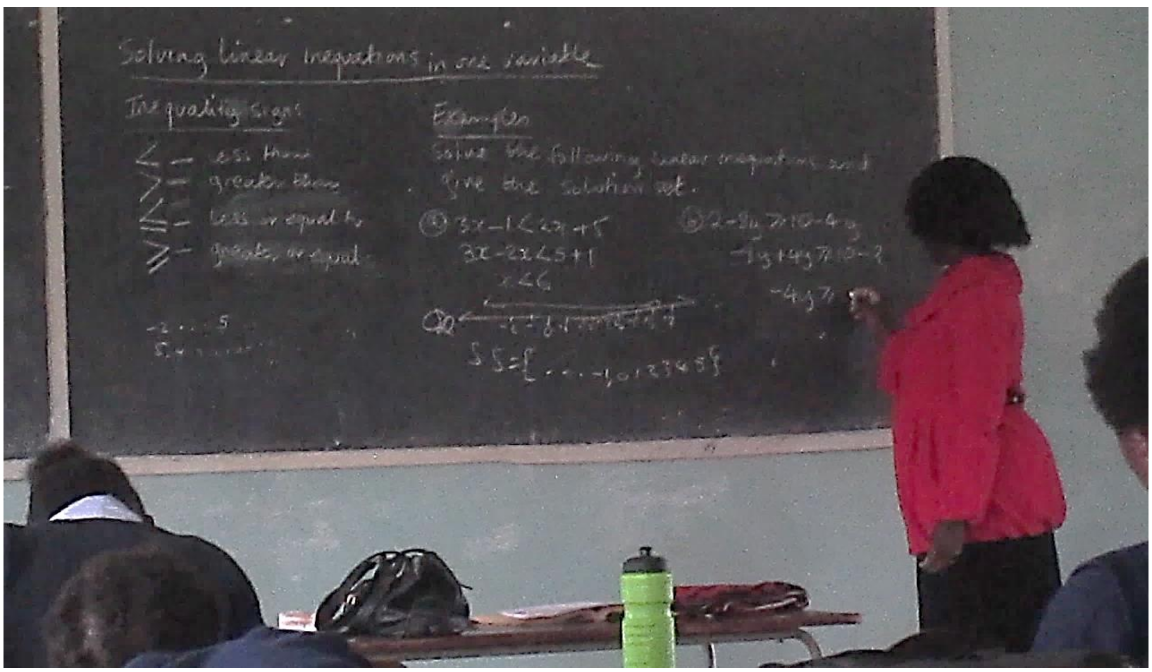


Figure 4.3*: Solving linear inequations/inequalities (Tr – G, February 2018)*

However, the following is the discussion on how Tr – G used linear inequations/inequalities.

Tr – G: *Why do people do business?*

Pupil $_{G1}$: *to earn money*

Pupil $_{G2}$: *agree*

Tr – G: *Anyone with a different answer?*

Pupil $_{G3}$*: to get some profit from their money*

Tr – G*: In mathematics, there is a topic called linear programming which deals with maximizing and minimizing profits and costs respectively.*

Tr – G*: But before we get to look at it in details, we are going to look at linear inequations/inequalities.*

Tr – G*: Writes on the chalkboard (solving linear inequations in one variable) as shown in figure 4.3 above.*

However, when she was asked in the interview as to the reason for using this strategy, she had the following to say:

> *" Aal to me I was like thinking like, since linear programming is more of the business set up. And when we look at the questions which come at the end, they talk more of profits and then minimizing the cost. So which means mostly linear programming is to do with that and this is the topic most of the time, with the background of business administration that I have, I saw that it is used to plan, to plan activities and so forth. Linear programming is used to know whether you are making a loss or a profit in business, as you have seen most of the questions that comes, they will be first be talking about maximizing the profit and then minimizing the cost. So that's why I say, ok let me first come up with this introduction in this way and then also the pupils they need something in real life, a tangible thing because most of the time they ask why we learn mathematics it is very difficult how am I going to use this. So, it is better if I approach it in that way. Telling the pupils that we can use that topic in this, they could also be interested in.." I also started with* ***linear equations in one variable and two variable****,*

and I don't even dwell much on the equations, coz I was just reminding them because they needed the idea of solving the equations and inequations like for me personally I prefer to go for inequations than for me to go for equations. Since linear programming will be dealing much with aah inequation signs." (Tr - G: February 2018).

4.3.1.2 Coordinate geometry

Tr - M did introduce using coordinate geometry. He started by writing some notes on chalkboard about deriving inequations that defines a region. The Figure 4.4 below shows some of the main areas under which coordinate geometry was used in the introduction.

Figure 4.4: *Forming inequation that defines a region (Tr – M, September 2017)*

Deriving inequations that defines a region.

From the notes TR-M explained the notes.

Tr – M: *Recall that the inequation that defines a particular region are presented by a straight line. Thus we need to know the number of lines that bond the required region.*

NOTE: Each straight line is represented by the standard equation of a straight line.

Y = mx + C: where x and m are any points on the line

m: is the gradient

c: is the Y intercept

If the line is parallel to either the X or Y axis such a line will have the gradient of zero (0).

Tr - M*: All the points along the line will be presented by x and y. So we are to use the standard equations to get to know the equation of the lines shown in the vignette.*

Figure 4.5 below shows the teacher demonstrating how to find the inequation/inequalities that defines the feasible region.

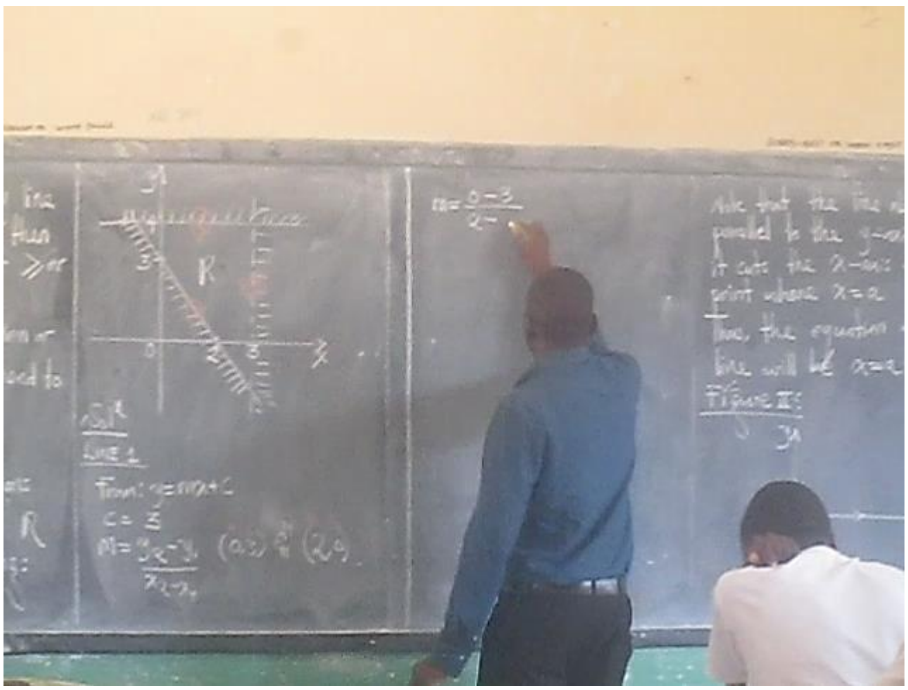


Figure 4.5: *Defining the inequation* (Tr – M, September 2018)

Tr – M: *How many lines defines the region R.*

Pupil $_{\mathbf{M\,1}}$: *3*

Tr - M: *we label the lines 1, 2 and 3.*

Pupil $_{\mathbf{M\,2}}$*: label the line.*

Tr - M: *the point is that, is this line parallel to y - axis or is it cutting the y - axis?*

When asked during the interview, as to why the teacher opted for this strategy, the response was:

> *"Aaah coz you would note that eey the boundary of the lines that form the solution set to the inequalities when you ought to represent an inequality on the XOY-plan, it should be represented as a straight line.*
>
> *-Basically, it follows that the equation or that **represent a straight line is an equation of a straight line**, so thus pupils ought to know how to derive the equation of the straight line. So, I wanted to try and bring out the pre-requisite knowledge they have on coming up with equation of the straight lines. Then from there trying replacing where there is an equal sign on the equation of the **straight line bringing up the inequality sign or symbols** rather." (Tr - M: October 2017).*

4.3.2 Use of tables involving symbols and their associated meaning

One out of the three teachers used a table of the symbols against the likely phrases/words that would accompany the meaning of the symbol. This was during the formation of inequations from the situational word problem/statements. The teacher was discussing how to identify the phrase that could give the objective condition in the situational statement and check from the table to see the symbol/sign that can be associated with the symbols. Figure 4.6 below shows the symbols and the associated key words/phrases that Tr -M used.

The example showing how Tr – M used the table is as follows:

> Tr - M: *when formulating inequations we ought to look out for some key words representing a symbol.*
>
> Tr - M: *so we are going to have a table to show the symbol and their associated symbols.*
>
> Tr – M: *Write the table on the chalkboard.*

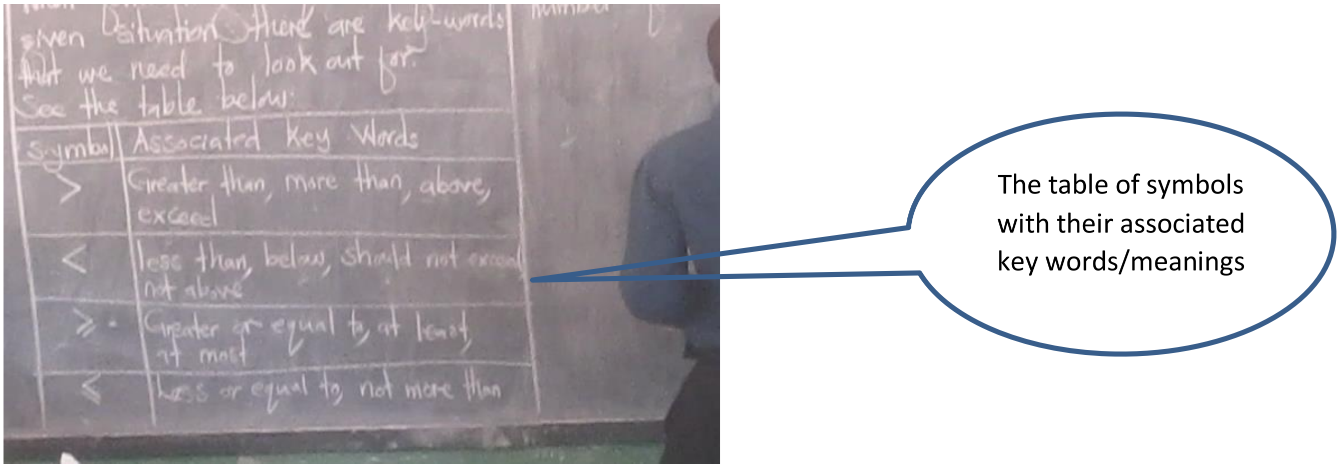


Figure 4.6: Table of symbols with their associated meaning *(Tr – M, September 2017)*

Tr – M: *Let us now look at this example of a situation where we can form some inequation.* □

A businessman selling electronic items wishes to buy two kinds of cell phones namely: LG and Hisense. An LG phone cost K2, 800 and a Hisense phone cost K4, 200. He intends to buy at least 6 phones in all and has a budget of K42, 000. In additional he wants to have at least 3 but not more than 5 Hisense phones.

Let x represents the number of LG phones and y to represent the number of Hiense phones. Formulate four (4) inequations satisfying the above information.

Tr - M: solution.

X = # of LG phones

Y = # of Hiense phones

Tr - M: what are some of the key phrase which can be presented as an inequation?

Pupil $_{M3}$: at least 6, at least 3 and not more than 5.

Tr - M: now at least 6, what inequality sign can we use?

Pupil $_{M4}$*: greater or equal*

Tr - M: what about not more than 5?

Pupil $_{M5}$*: its less or equal to*

TR: now the inequation would be?

$2x + 3y \leq 30,\ x + y \geq 6,\ y \geq 3$ *and* $y \leq 5$

Figure 4.7 below is showing the inequation that have been formed from the situational statement above.

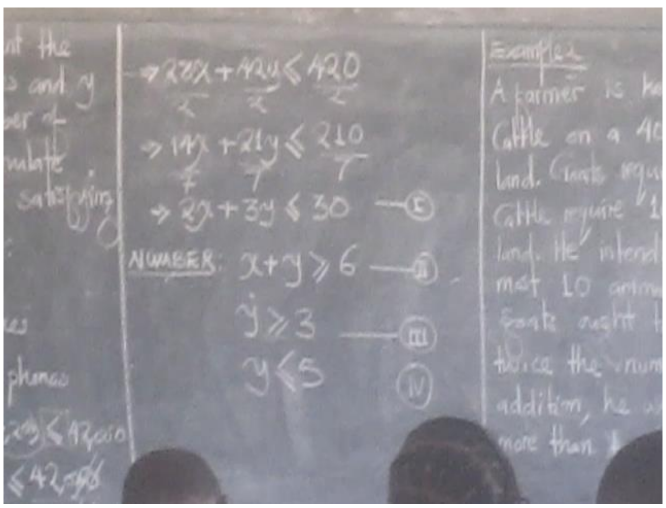


Figure 4.7: Inequations from situational statement *(Tr – M, 2017)*

4.3.3 Enlisting the phrase to associate with the symbols

The strategy of enlisting the phrases with their associated symbols was use by two out of the three teachers. This strategy was mainly used by Tr – B and Tr – G in a similar way. However, a typical example of one of the questions which was discussed by Tr – B is as follows:

Tr – B: Reads the situational statement from one of the exam past papers;

Makwebo prepares Hungarian and beef sausages, she prepares at least 40 Hungarian and at least 10 Beef sausages. She prepares not more than 160 sausages. The number of Beef sausages is not more than the number of Hungarian sausages. Given that x *represent Hungarian sausage and y Beef sausages. Write four inequalities which represent these conditions.*

Tr – B: *The key is the phrase at least which represent* $\geq$

Tr - B: *The word at least represent* $\geq$ *and that should be known.*

Tr – B: *Therefore, at least 40 means…*

Pupil $_{B\,5}$**:** *it means that x greater or equal to 40, y greater or equal to10*

Tr - B: *Then, not more than means that number or less.*

Tr – B: *The statement, number of beef sausage is not more than the number of Hungarian sausages means?*

Pupil $_{B\,6}$: *y less or equal to x*

Tr – B: *What about the statement, "she prepares not more than 160 sausages"?*

Pupil $_{B\,7}$: *it can mean* $x + y \leq 160$.

Tr – B: *Correct..!*

Tr - B in figure 4.8 below is not using any table but just straight from the situational statement given, he is helping the learners enlisting the terms to be associated with the symbols.

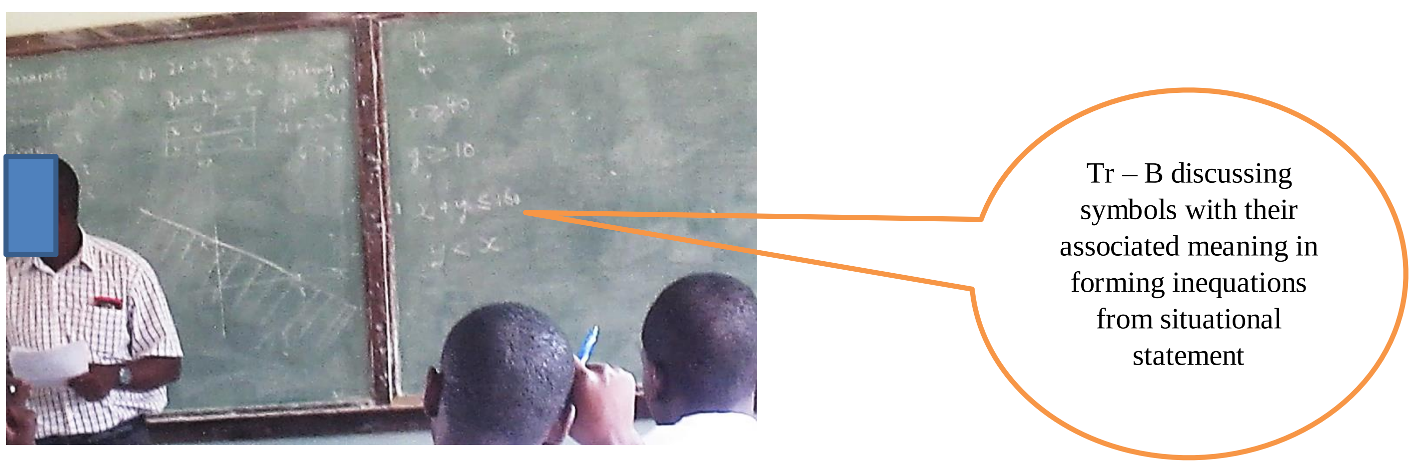


Figure 4.8*: Forming inequations from situational statement* (Tr – B, October 2017)

When he was asked during the post – lesson interview, he had this to say:

These symbols are very important because you find learners sometimes they make mistakes on less than or greater than so it is up to the teacher to have strategies on how best he can introduce those to the learners especially the issue to do with less or equal to, sometimes they don't understand what it means when we say less or equal to. The emphasis should be done before much is done in the teaching of the signs (>, <, ≤ and ≥) in Linear Programming. So those symbols are very important because they are the ones that we use through and through ***and words like at least, at most they also accompany those symbols, so must marry the symbols and the words like less than, should be key when discussing how to form inequations from situational statement*** *(Tr - B: October 2017)*

4.3.4 Use of the test points

I also took interest in considering some of the strategies teachers used in the discussion of the Shading of the unwanted region. Through lesson observations, all the three teachers used the test point as a more ideal approach in determining the side of the line to be shaded.

Figure 4.9 shows Tr -G explaining the best way of determining the region to be shaded using the test point.

Tr – G: *When deciding the side of the line to shade, we can use a test point.*

Tr – G: *Take for instance the line* $y - x > -6$

Tr – G: *Anyone to choose a point we can use to test the false part to shade?*

Pupil $_{G4}$: *(4, -3)*

Tr – G: *Is it up or down?*

Pupil $_{G5}$: *down*

Tr – G*: Let us substitute (teacher does the solving on the board and final result -7 greater than -6*

Tr – G: *So -7 greater than -6 is false, where do we shade?*

Pupil $_{G6}$: *down*…!

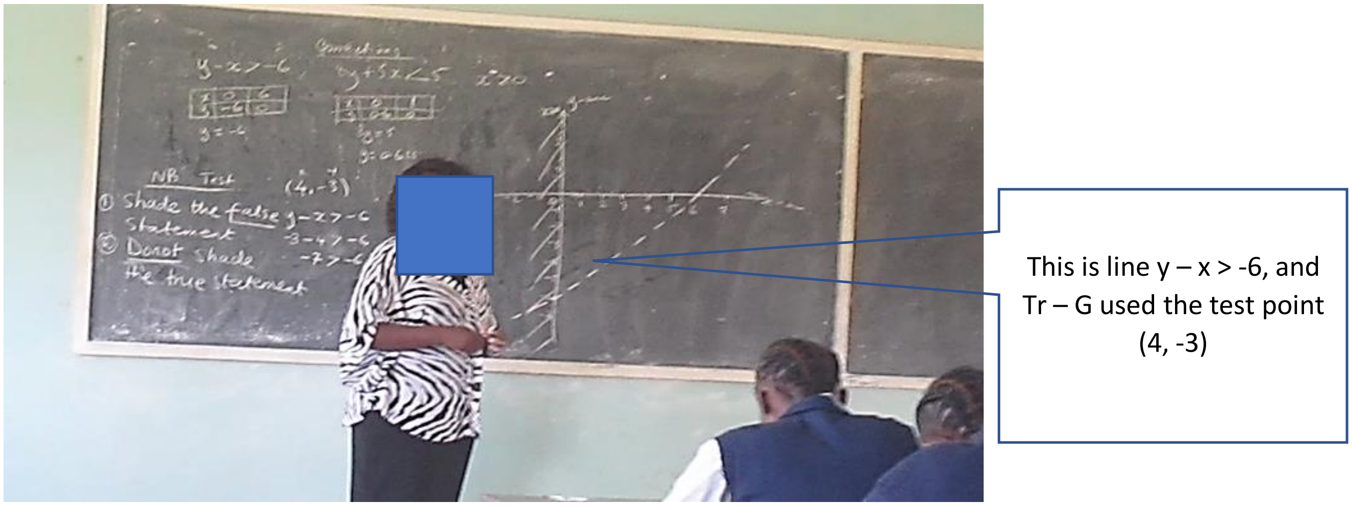


Figure 4.9: *Use of the test point for the region to be shaded (Tr – G, February 2018)*

Figure 4.10 below shows Tr – G using another test point to try and test the other side of the line

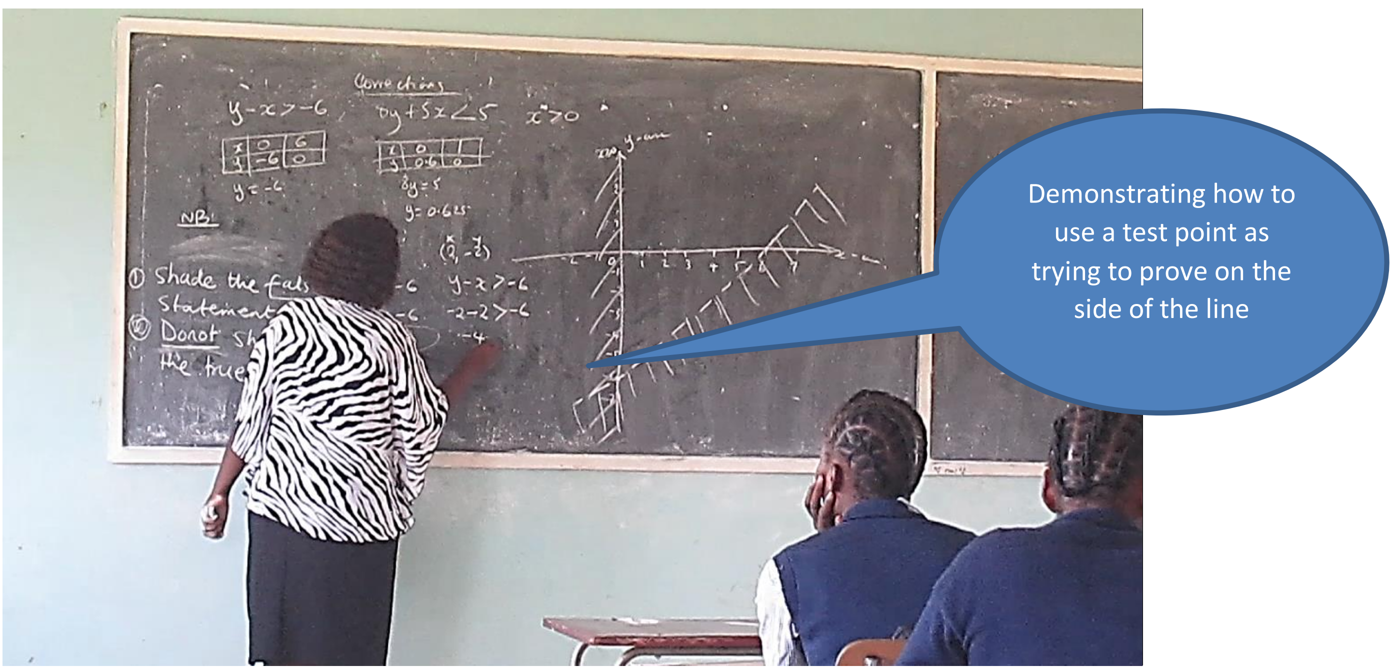


Figure 4.10: *Demonstrating the test point (Tr – G, February 2018)*

Tr – G: If we want to prove on the other side of the line, we pick another coordinate point on this side (pointing to other side of the line where a point wasn't picked)

Tr – G: what point can we use?

Pupil $_{G7}$: (2, -2)

Tr – G: OK, we substitute in the inequality y – x greater than - 6

Tr – G: (Shows the substitution on the board) -2 – 2 greater than -6

Tr – G: we now have – 4 greater than - 6

Tr-G: it is a true statement, then no shading!!

The test point strategy was also used by Tr – M when defining the lines that bounded some region. Thus, after finding the equation of the line then replacing the equal sign with an inequality sign as shown in Figure 4.11 Below.

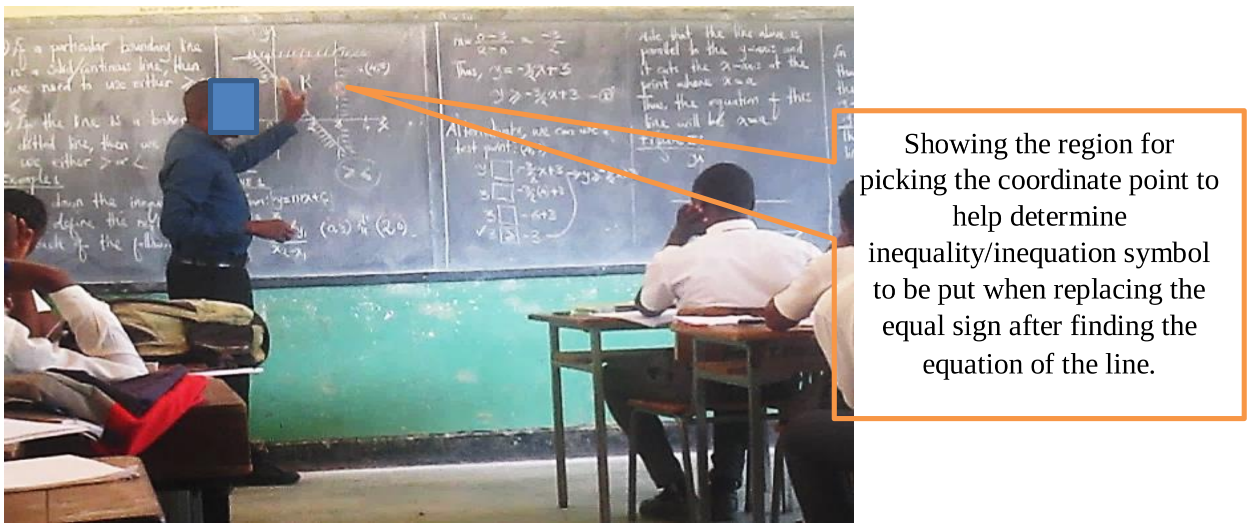


Figure 4.11: *showing where the point should come from (Tr – M, September 2017)*

Figure 4.11 shows Tr – M explaining that you have to pick a coordinate point from the un shaded part of the line, when you want to find the symbol to use to replace the equal sign of the equation of the line in question.

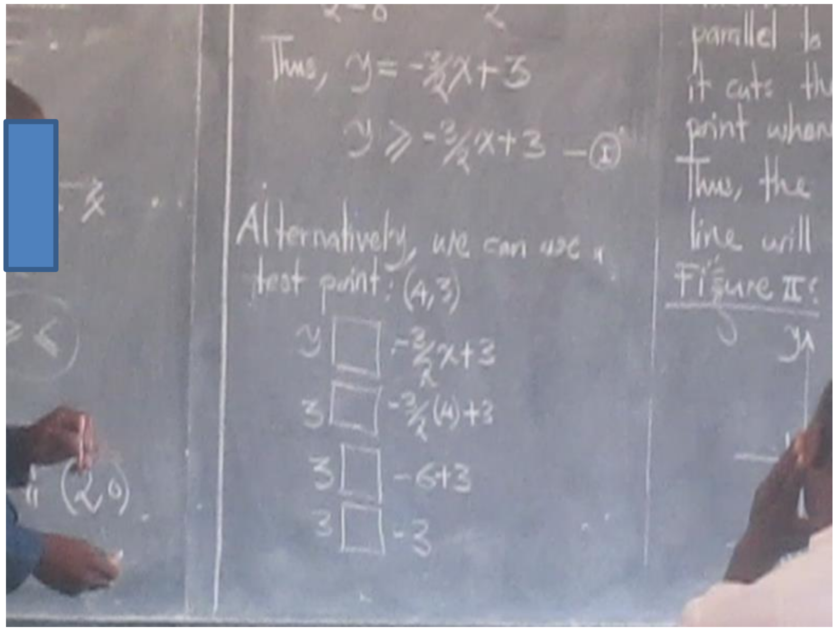


Figure 4.12: *Using a test point (Tr – M, September2017)*

The figure 4.12 above shows how teacher Tr – M was using the test point in determining the symbol to be put in changing the equation to an inequation.

Tr- M: *We can decide the symbol by using a test point.*

Tr – M: *let us pick a point from the feasible region, you can suggest the point*

Pupil $_{M\,6}$: *(4, 3)*

Tr – M: *Good!!*

Tr- M: *Now we can write, y* □ $\frac{-3}{2}x + 3$ *(see Figure 4.11 above)*

Tr – M: We have 3 □ *- 3, what symbol can we put to make a true statement?*

Pupil $_{M\,8}$*: greater than*

Tr- M: Now since the line in bond we will put?

Pupil $_{M\,9}$*: greater or equal to*

Tr – M: And the inequation is y≥ $\frac{-3}{2}\,x + 3$

4.3.5. Use of Trial and Error

The location of the coordinate point for the maximum profit/cost was one area of concern for me. There were two strategies that were identified in the teaching and learning of how to locate the coordinate points for the maximum and minimum profit/cost. These include the use of the coordinate that make up the vertices and both the boundary and the inner points of the feasible region.

4.3.5.1 Vertices of the feasible region

One of the strategies for finding the coordinates that can give the maximum/minimum cost/profit which was used was the use vertices of the feasible region. Two out of the three teachers did use this strategy, and discussion was as follows:

> Tr - M: *These are the vertices of the feasible region (shows on the grid board in figure 4…)*
>
> Tr – M: *What are these points?*
>
> Pupil $_{\mathbf{M10}}$: *(40, 10)*
>
> Tr – M: *What about the next vertex?*
>
> Pupil $_{\mathbf{M\ 11}}$: *(40, 40)*
>
> Tr – M: *The next one?*
>
> Pupil $_{\mathbf{M12}}$: *(80, 80)*
>
> Tr – M: *Ok, and the last one?*
>
> Pupil $_{\mathbf{M\ 13}}$: *(150, 10)*
>
> Tr – M*: Now we can calculate the profits using the profit function* $p = 3x + 2y$
>
> Tr - M*: We are going to substitute these coordinates points in the profit function to see which one would give us the maximum profit.*
>
> Tr – M*: (40, 10) = 140, (80, 80) = 400 and (150, 10) = 470*
>
> Tr - M*: We have seen that (150, 10) gives us the max profit, hence we need 150 Hungarian and 10 beef to give the maximum profit.*

Figure 4.13: *Teacher showing the vertex (Tr – M, September, 2017)*

However, figure 4.14 below shows the coordinate point Tr – M was emphasizing in figure 4.13 at one of the vertices of the feasible region.

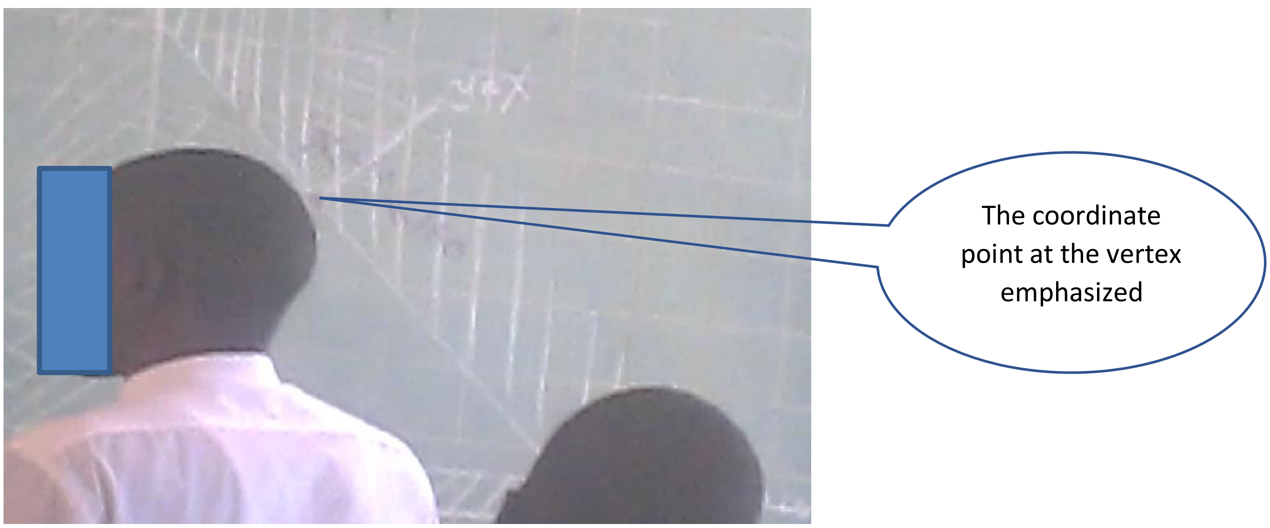


Figure 4.14: *Tr – M September 2017.*

In support of this strategy during the post – lesson interview, Tr – M had the following response to make with regards to this strategy;

> *Yes especially on finding the maximum profit or minimum cost or minimum profit or cost thereof. I would suggest… other than using the search line, which could be time consuming as moving across axes like from the Y and X-axis, using that search line which moves upwards aah after one has clearly stated the region and defining the inequalities. I feel the easier way would be using the points which make up the solution*

set but then it would be difficult if one picks all the points because sometimes you would find that there are quite a number of points which will be in a solution set following the scale that is given, ***so the most ideal thing on how to get the maximum / minimum profit are the points which are on the edge of the solution set or region that makes up the solution set, just identify those coordinate point which are on the edges then someone will be able to check of course if there four inequalities expect just to have about four edges****. So with the four points, one can quite try and see which points gives him or her the maximum profits or cost, other than the points which are just on the edges will work out----yes." (Tr - M: September, 2017)*

Further teacher Tr - G during post – lesson interview in support of the use of vertices in finding the maximum/ minimum profits/costs said*, "The best is to go for the points where the lines intersect. Those are the main points mostly that give us the profit. The emphasis should be on the whole numbers since the questions will be talking about chickens and so on." (Tr – G: February 2018).*

4.3.5.2 Use of both the boundary and inner points

The use of the boundary and inner points in finding the coordinate for the maximum/minimum profit/cost was used. One out of the three teachers used this strategy. This strategy demanded picking a number of points from both the boundaries and the inner part of the feasible region. These points are then tested against the profit function in order to determine the one which gives the maximum/minimum cost/profit.

Figure 4.15 below shows how Tr – B was using trial and error method in finding the coordinates for the maximum/minimum cost/profits.

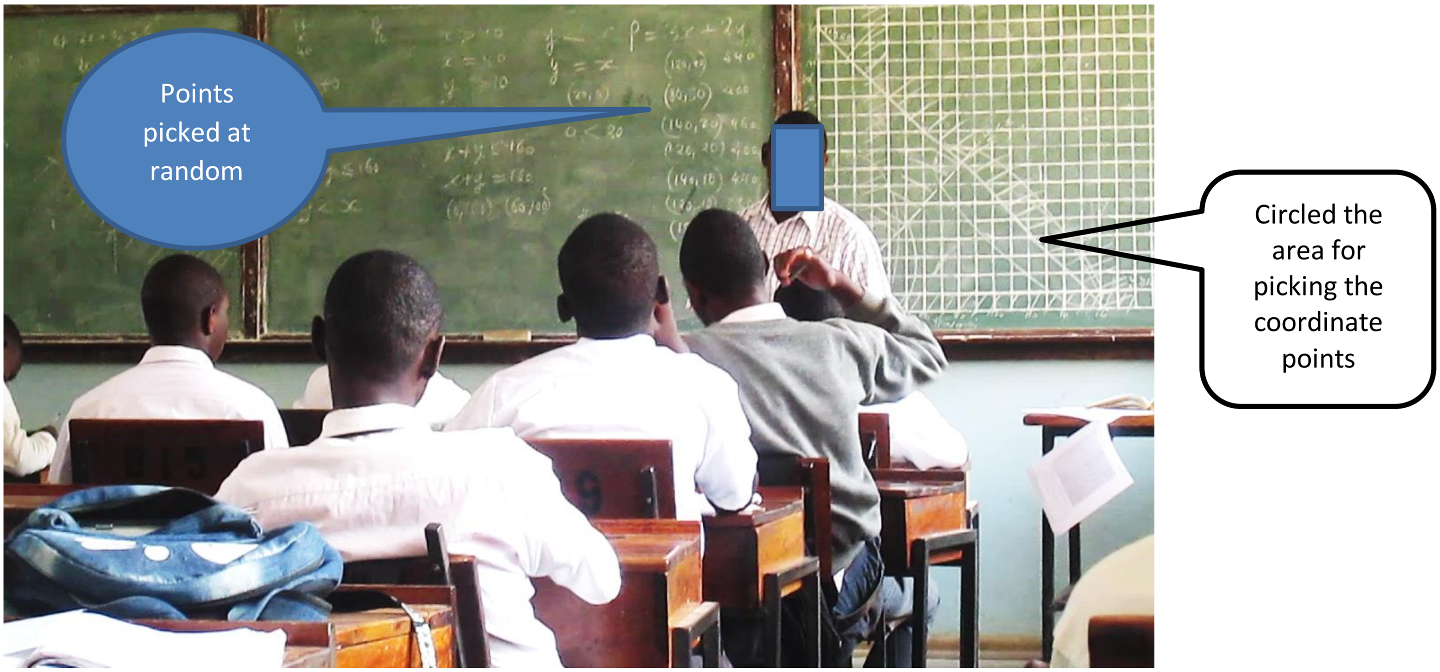


Figure 4.15: *(Tr - B October, 2017)*

Tr – M*: The profit function in this statement is P= 3x + 2y*

Tr – M*: The profit on the sale 3x for Hungarian and 2y for beef sausage. how many of each is required to make maximum profit?*

Tr - M*: We consider the points found in the feasible region and those on the bounders.*

Tr - M*: Which points can we use and since we are concerned with maximum profit, we are to use points on far right.*

Tr – M: Circles the region to pick the points from.

Tr - M*: We list the points*

Pupil $_{\mathbf{B\,8}}$*: Mention the points.*

Tr - M*: Writes on chalkboard (120, 20) = 400, (80, 30) =300, (140, 20) =460, (140, 10) = 450, (100, 10) =320, and (150, 10) =470*

Tr – M: Now which coordinate gives maximum?

Pupil $_{\mathbf{B\,9}}$*: (150, 10)*

Tr – M*: We can conclude that, for maximum profit 150 Hungarians and 10 beef sausages are needed.*

Further, when Tr - B was asked about the method of finding the coordinate for the maximum and minimum profit/cost, his response was:

> *"Yeah there is a----methods that we normally use, where you draw what we call a searching line- and the searching the search line, once you draw the searching line, then you move it---you use a ruler where you have to move it from the searching line parallel going up wards, the last point that will give you the maximum, all those it is not very clear method especially to pupils, they take long to understand it ok. Mostly the method I would prefer is the trial and error, where you pick points here and there, where you think that probably these are the points that will give me the maximum and then pushing in the profit function, then the one that gives in the maximum, that's the one they will pick on. But off course that is time consuming in an exam that method probably will chew a bit of time for the learner." (Tr - B: October, 2017).*

Table 4.1 below is a summary of objective 1 on strategies teachers used.

Table 4.1: Teachers' strategies

Name of Teacher	Introduction	Formulating inequations from problem situations	Shading	Finding max/min profit/cost
Tr – M	Used coordinate Geometry	Used a table of symbols with their associated phrases	Used a test point	Used vertices of the feasible region
Tr – B	Used linear inequation/inequality	Enlisting phrases to associate with symbols	Used a test point	Used trial and error method (picking points at random) from both the feasible region/boundary
Tr – G	Used linear inequation/inequality	Enlisting phrases to associate with symbols	Used a test point	Used vertices of the feasible region

4.4 Factors affecting the teaching and learning of linear programming

The second research question was on factors affecting the teaching and learning of linear programming in secondary school mathematics. With regards to the second research question, the following were the views obtained from participants;

4.4.1 Instruction for the region to be shaded

During the interview, teacher Tr - G did observe the inconsistence of the instruction for the region to be shaded for the inequation/inequalities taught in grade 9 and in linear programming in grade 12. For instance, in grade 9 the instruction is about shading of the wanted region whereas in linear programming it is shading the unwanted region.

Therefore, she had this to say:

> *The schemes say "they are following the provincial scheme called the common scheme for the province" that we have to teach linear equation in one variable and two variables. Then aah----after doing that, then we shade the wanted region according to the schemes because it is indicated that, we shade the wanted aah region, even in grade 9 we were shading the wanted region. So like for me I would prefer and I would love, these people who set these schemes to go for what – inequations and avoid the issue of equation. They just go straight and to say, lets solve inequations and then we shade the unwanted region because, we will be shading the unwanted region continuously than for us to start saying* ***we shade the wanted region and then again change to the unwanted region****. So, I would rather, we go straight for these; from the word go we shade the unwanted region than for us to be changing, and if we go with this strategy, like for me with the experience that I got, I have found that pupils have gotten used and get confused and at the end of the day I have not achieved my objective. I would have confused my pupils more when I say, or like shading the wanted region like in grade nine, then again shading the unwanted region in grade 12. So that it really confuses the pupils. (Tr - G: February 2018)*

4.4.2 Plotting the inequations/inequalities

Plotting of inequation/inequalities on the XOY-plain was one of the difficult the learners faced in the teaching and learning of linear programming. Further during the interview, Tr – M also pointed out that,

"I note one challenge where a few pupils had difficulties like in --- some could quite alright come up with the expected inequality but when it comes to plotting it on the XOY- plane, aah some could make minor mistakes quite alright they could get one point correctly and make a mistake on the other point, so they would draw the correct line but fail to give the correct shading" (TM: September, 2017)

It was also echoed during the focus group discussion, FGD-PM and FGD - PB learners lamented having a challenge on plotting of lines on the XOY-plain. One participant in FGD- PB said,

> *" -When you are given an inequation and told to put it on the graph. The part that I found challenging was where to decide which part is true and which one is false, when you are given in an equation form, but when it is straight forward it was very easy." (FGD-PB: October, 2017)*

4.4.3 Translating situational statements into inequations/inequalities

The problems in understanding and interpreting the terms that are largely associated with the correct inequality symbols was noted. This generally resulted in failing to form inequations/inequalities from the situational statements. One teacher lamented saying*:*

> *The challenges that I have noted are---especially with our pupils that we have here, most of our pupils are coming from background where, there are not so used to using this foreign… the queen's language. So---they would, maybe I would say think in their mother tongue languages, then trying to bring that or **integrate it with English language**, most of them fail to do that---aah, so basically it could **maybe the language barrier I think**—there is need to maybe inculcate in our learners this reading culture, it is not there so they **fail to connect the phrases which are used as words and try to understand what is really expected of them do.*** (Tr - M: September, 2017).

Further, during the FGD-PM, it was mostly stressed that finding inequalities given a statement was really a challenge. One learner pointed out that*," A situation where you are given a statement and how to find inequalities is a challenge. But -It is simple if you understand the languages that, so if you understand those languages it will be simple." (FGD-PM: September, 2017)*

Tr- B also had this to say,

> "Aaah *one of the most challenging things that I have seen in the teaching and learning of Linear Programming is that most learners,* ***they fail to come up with the inequalities from the words.*** *Because Linear Programming has to do with the description of words then from those words you have to come up with your---with the inequality you are going to use-you find that, if* ***a learner does not understand the words like at least****, he will put a less than instead of a greater than, then everything become distorted, so the instruction of the inequalities from the words given, is one of the most challenging task that the learners normally face. If you go through, you do a lot of examples and a lot of exercises helping the learners to understand exactly which symbol to use at what time." (Tr - B: October, 2017)*

4.4.4 Use of trial and error method

The use of trial and error method in finding the coordinate point for the maximum/minimum points was the main strategy used. However, this strategy was used differently in that, Tr-M and Tr – G used vertices of the feasible region with a focus on coordinates with whole numbers and Tr – B used both boundary and inner points of the feasible region.

The use of trial and error method for both boundary and inner points when finding the coordinate point for maximum/minimum cost/profit was a challenge. Learners were overwhelmed with coordinates resulting in missing out on correct ones needed was a challenge recorded for the strategy.

It was evident from the focus group discussions when one learner pointed out that,

> "*As I would like to say like what my friend has said, when finding maximum point, I don't know if it's only our teacher who taught us or using some formula other than* ***letting pupils just guess, it more like probability****, like you pick any two point that you*

think will give maximum profit of the region. I think they should find some ***way of giving them a formula so that it is straight forward.*** *And also, after doing the lesson on Linear Programming, you should refer to other books on the same topic." (FGD-PB: October, 2017).*

Another learner further said, "*the part for find maximum profits, our teacher didn't very explain properly on how to find points. -Finding maximum profits, it's like you chose the points by guessing."(FDG - PB: October, 2017).*

However, Tr – M pointed out saying,

"Others prefer using trial and error method, but for me I see ***it as time consuming and probably causing learners to miss out on the most needed*** *coordinates. Hence I advise my pupils to go for the boundary points precisely on the edges of the feasible region." (Tr – M: September 2017).*

Table 4.2 below is the summary of question 2 on factors affecting LP

Table 4.2: factors affecting the teaching and learning of linear programming

Topic Area	**Constraining factor**
Formulating inequations from problem situation	➢ Translating situational statements into inequations/inequalities.
Shading	➢ The instruction of shading the true region in linear inequations and false in linear programming
Finding the maximum/minimum costs/profit	➢ Use of trial and error method of both the boundary and inner points of the feasible region. Learners were overwhelmed with coordinates points resulting in missing out on correct ones needed

4.5 Suggested measures to overcome the constraints

The third research question was to establish the measures that teachers would suggest to overcome the identified constraints. Therefore, this section shall highlight the suggested measures to overcome the constraints.

4.5.1 Consistence of the region to be shaded for linear inequations and linear programming

The pre- requisite knowledge for linear programming is mostly linear inequation. The pre-requisite knowledge of plotting inequations and inequalities from junior secondary mathematics is key. However, the inconsistence of the region to be shaded for linear inequation and in linear programming is causing a great challenge where shading is concerned. Therefore, from this finding the introduction is anchored on linear inequation as pointed out by Tr - B that:

> *The first thing I think when you are tackling Linear Programming, it is also important to look at—the pre-requisites of what is needed to accompany Linear Programming. Like I said at the beginning is* ***that you need to go back as far as grade 9 work, you start from inequalities ok, how to shade inequalities in one variable, inequalities in two variables****. (Tr - B: October,2017).*

However, the instruction for shading in grade 9 and grade12 were not consistent, in grade 9, the shading is for the wanted region while in grade 12 it is for the unwanted.

Tr – G had this to say,

> *"When it comes to shading, we should just go straight and to say, lets solve inequations and then we shade the unwanted region because, we will be shading the unwanted region continuously than for us to start saying we shade the wanted region and then again change to the unwanted region following what the provincial schemes are stating which is a build up from grade nine linear inequations.* ***So I would rather, from the word go we shade the unwanted region than for us to be changing, and if we go with this strategy, like for me with the experience that I got, I have found that pupils have gotten used, and get confused and at the end of the day I have not achieved my objective****." (Tr – G: February 2018)*

It was suggested further that the shading of unwanted region should be after testing the point for the side of the line which is false. This was further encored by the learners in the FGD- G that the use of the test points was easy and making it less difficulty to determine the false region.

4.5.2 More learner interaction with situational problems

Linear programming is mostly about maximizing and minimizing costs/profits from situational problems. Therefore, more learner interaction with phrases/words and their associated symbols has to be fostered. However, from the observations, learners did not interact fully with the situational problems in order to build more understanding of the terms and their associated symbols. This was evident when a question on how best can learners be engaged in order to provide more basis for understanding the terms and symbols used in linear programming.

Teacher Tr – B had this to say:

> *"There **must be a lot of interaction between the teacher and the learners**. You know its not to like offloading everything on the learner eey the teacher should give chance to the learners sometimes learners to come and demonstrate on the board, what they have understood from the sentence and even ask the learners to plot let say if you are plotting the inequality. First of all you start with plotting the line then shading the inequality can be given to the learners so that they rely participate in the learning, then the teacher always demonstrating and the learner become like on lookers which is not a good approach – ok-yes sir." (Tr - B: October,2017)*

4.5.3 Using vertices and boundary points of the feasible region

In order to overcome the challenge of finding the coordinates for the maximum profits/costs, the following were some of the suggestions that Tr – M and Tr – G gave;

> *"Yes especially on finding the maximum profit or minimum cost or minimum profit or cost thereof. I would suggest… other than using the search line, which could be time consuming as moving across axes like from the Y and X-axis, using that search line which moves upwards aah after one has clearly stated the region and defining the inequalities. I feel the easier way would be using the points which make up the solution*

set but then it would be difficult if one picks all the points because sometimes you would found that there are quite a number if points which will be in a solution set following the scale that is given, so the most ideal thing on how to get the maximum / minimum profit are the ***points which are on the edge of the solution set or region that makes up the solution set,*** *just identify those coordinate point which are on the edges then someone will be able to check of course if there four inequalities expect just to have about four edges. So with the four points, one can quite try and see which points gives him or her the maximum profits or cost, other than the points which are just on the edges will work out----yes."(Tr - M: September,2017)*

Further teacher Tr - G also suggested that,

> *"The pupils have a problem in getting the points on the graphs. Emphasis should be that when you are getting the points, you should be getting the points from the wanted region. After getting the profit equation, you substitute the coordinate value in the profit equation then you can find the maximum profit among those points which are going to be inside.* ***The best is to go for the points where the lines intersect****. Those are the main points mostly that's where they give us the profit or where the profit will come where the lines intersect. The emphasis was on whole numbers since the questions will be talking about chickens and so on which cannot have half chicken coz they can give numbers with points. Then we use the profit equation to substitute the one with greater value will be the maximum. And emphasis that the answer will come where the lines are intersecting. For the minimum, it is to get from the left side coz that where you found smaller numbers like zeros and so on. Then the question should be first to calculate to the maximum profit." (Tr-G: February 2018)*

Teachers explained that the use of vertices of the feasible region was more ideal for finding the maximum/minimum points for the costs/ profits.

4.5.4 Engage with real life application of linear programming

On this one, one out of the three teachers gave this view;

> *"Aaah other than just bringing out like question in class, I think it is ideal that pupils were able to see the actual situations given to them. Take for instance, Linear Programming is so ideal like in the business circles, agricultural, trading, marketing*

and purchasing and supply those areas, so ***I feel pupils were able to or taken out of a classroom situation and take for instance if you are talking of someone who is say… buying two types of chickens maybe broilers and layers****. You take them out of the classroom situation and they go and see the actual situation and then they are able to see the intentions limitation and identify the constrains which are there, other than just being taught, it is more less like a theoretical thing, I think it could be better if they are like brought out of the classroom and be able to see the actual situation on how they can interpret and note constrains which are there." (Tr - M: September,2017).*

Table 4.3 below is the summary of objective question 3 on suggested measures to overcome the constraints.

Table 4.3: suggested measures

Constraining factors	**Suggested measures**
Demand for shading the wanted region in linear inequation and then the unwanted in linear programming	➢ Being consistent in the region to be shaded thus shading the unwanted or vice visa in linear inequation and linear programming
Less learner interaction with situation problem of linear programming	➢ More learner interaction with situational problems ➢ Engage with real life application of linear programming
Trial and error method in finding the coordinate for the maximum/ minimum cost/profit	➢ Using vertices and boundary points of the feasible region for coordinates giving the maximum/minimum cost/profit

4.5 Summary of the chapter

This chapter has presented the research findings from the participants on the teachers' strategies in the teaching and learning of linear programming in selected secondary schools in Monze District of southern province. The table below shows the summary of findings from each research question.

Table 4.4: Summary Table of Chapter

OBJECTIVE 1: ***TEACHER STRATEGIES USED***				**OBJECTIVE 2:** ***CONSTRAINING FACTORS***		**OBJECTIVE 3*****: SUGGESTED MEASURES TO OVERCOME THE CONSTRAINTS***	
	Tr – M	**Tr - G**	**Tr – B**				
Introduction	Used coordinate Geometry equation of a straight line	Used grade 9 linear inequation/ inequalities	Used grade 9 linear inequation/inequalities	**Topic area**	***Constraining factors***	**Constraining factors**	***Suggested measures***
Formation of inequations/ inequalities from situational problems	Used a table of symbols with their associated phrases/words	Enlisting phrases to associate with symbols without a table	Enlisting phrases to associate with symbols without a table	Inequality/ inequation formation	*Less learner interaction with situational problems*	Shading the wanted vs the unwanted	*Consistence in both linear inequation and programming*
Shading	Used a test point	Used a test point	Used a test point	shading	*Instruction (wanted vs unwanted)*	Less learner interaction with situational problems	*More interaction Engage with real – life application of LP*
Finding Coordinate of the for max/min profit/cost	Used vertices of the feasible region	Used vertices of the feasible region	Used both the inner and boundary points of the feasible region	Finding the coordinate for max/min cost/profit	*Trial and error method of both the inner and boundary point of the f region*	Trial and error of both the inner and boundary points	*Use the vertices of the feasible region*

CHAPTER FIVE

DISCUSSION OF THE FINDINGS

5.1 An overview

The previous chapter presented the findings of the study on the teachers' strategies in the teaching of linear programming in senior secondary schools in Monze District of Southern Province. The purpose of the study was to examine the strategies that teachers used in teaching of linear programming and determining factors affecting its teaching. This chapter therefore, focuses on the discussion of findings under the sub-themes that emerged in line with the objectives of the study. Reference is also made to the literature reviewed and the theoretical framework so as to authenticate the findings. Further a meta-analysis of these findings is also made.

The following were the research questions of the study:

1. What strategies do teachers use in the teaching of linear programming?
2. What factors affecting the teaching and learning of linear programming?
3. What intervening measures do teachers suggest to overcome the constraints?

5.2 Strategies teachers use in the teaching of linear programming

Objective number one was to examine teachers' strategies in the teaching and learning of linear programming in secondary school mathematics. In order to establish the strategies, the study focused on how linear programming was taught in the classroom. The strategies that the study identified included: use of inequations from the junior secondary mathematics and coordinate geometry for introduction, use of tables for terms and their associated symbols interaction, enlisting terms and their associated symbols from situational problems, use of test point to determine the area to be shaded, use of trial and error method for finding the coordinate for maximum/minimum cost/ profit and use of the vertices/boundary points of the feasible region for finding the coordinate for maximum/minimum cost/profit.

5.2.1. Linear inequation and coordinate geometry

Linear inequation was used by two out of three teachers as a strategy in the introduction of the topic linear programming. Two out of the three teachers used linear inequation as the pre – requisite in the introduction. One out of the three teachers introduced the topic using coordinate geometry as pre-requisite knowledge into the topic.

Teacher Tr – B and Tr - G introduced the topic by having a recap on the presentation of linear equations in one and two variables from junior secondary school mathematics (grade 9). They gave out a number of inequations in both one and two variables, and showed how they were represented on the XOY plain. During lesson observations, it was observed that teacher Tr – G started by talking about issues of business in relation to linear programming, and then proceeded in solving inequation as a pre-requisite knowledge needed for linear programming and she did support during post-lesson interviews that it was more ideal for one to introduce it like that as linear programming is about business.

Introducing linear programming using coordinate geometry, teacher Tr – M started from the standard equation of a straight line, finding the equation of a given line. He gave examples of lines drawn on the same graph bounding some region and found the equations of those lines with appropriate inequality symbols that defines the shaded region. In this strategy, the teacher during post – lesson interview argued that since linear programming is about plotting lines on the Cartesian plane, this was somewhat better for him as pre-requisite knowledge.

The use of different strategies in the introduction demonstrates the teachers' integrated knowledge in trying to provide the much-needed base for linear programming. This observation is also supported by Leinhardt and Greeno (1986) who did observe that, teachers practice their craft in highly complex, dynamic classroom contexts that require them constantly to shift and evolve their understanding. Whereas Shulman (1986 & 1987) observed that, effective teaching depends on flexible access to rich, well-organized and integrated knowledge from different domains.

Further, the use of different strategies by the teachers, demonstrated some required knowledge for teaching linear programming. This is supported by Ball (2009) in the discussion on the Mathematical knowledge for teaching. The main knowledge domain of pedagogical content knowledge which constitutes the knowledge of content and students (KCS), knowledge of content and curriculum (KCC) and the knowledge of content and teaching (KCT) was demonstrated. It was evident that all the three teachers tried to link with previous knowledge where the curriculum knowledge was very important which was key in building into the knowledge of content and student (KCS) and knowledge of content and teaching (KCT). For Hill et al. (2008, p. 378), this is the "knowledge that allows teachers to engage in a particular teaching task, including how to accurately represent mathematical ideas, provide explanations for common rules and procedures, and examine and understand unusual solution methods to problems."

The above observation further is in line with Vygotsky (1978) who suggested that teaching geared to developmental levels that have already been achieved will be ineffective, and that the only 'good learning' is that in advance of development. Therefore, the teaching of linear inequations and coordinate geometry was in line with social constructivism tenet about learning in the Zone of Proximal development. All the three teachers tried to mediate learners from their zone of knowing to the zone of new knowledge, thus building from what they can do as solving linear inequations and coordinate geometry of straight lines to how the same can be used in the further realm of knowledge which is linear programming which falls in with what they cannot do at this stage.

5.2.2 Forming inequations/inequalities from situational problems

The development of the topic was mainly focused on two sub-themes; formations of inequations from situational statements and plotting of lines to create a feasible region. The strategies in the teaching and learning of phrases/words and symbols are at the center of linear programming. It was observed that during lesson observation that attention was paid in the teaching and learning of phrases/words and symbols. Further during post-lesson interviews they all gave support that linear programming is about understanding terms with the associated symbols. According to Davidenko (2006) the phrases and symbols used in linear programming fall in the formal and symbolic register of the language of mathematics. In this register, symbols are used for concepts (x = variable, m = slop); symbols for procedures or operations (+, -, x, ÷); symbols for relationships (<, >, =) and expressions to denote logical statements (and, or, for all). However, the main symbols that are used in linear programming are; the inequation and inequality sign, which include: ≤ (less or equal to), ≥ (greater or equal to), < (less than), > (greater than) and = (equal to). The analysis of data from lesson observations showed that, one out of the three teachers used a table of phrases/words with their associated symbol while the other two enlisted the phrases/words and their associated symbols from situational problems in forming inequations.

5.2.3 Use of a table and enlisting the phrases with their associated symbols

The use of a table of terms and their associated symbols in the formation of inequations from situational problems was observed to have provided the most needed help and interaction of the two. Tr – M used a table and it was precise in terms of relating the phrases with their corresponding symbols. In the use of table during post-lesson interview, the teacher argued that it was easy for the pupils to relate the terms with their associated symbols from the table. The use of a table in giving the interaction of the phrases and their associated symbols resonate well with Gredler (1997) in his four general perspectives that inform how one could mediate the learning within a framework of social constructivism using cognitive tools perspective. Cognitive tools perspective focuses on the learning of cognitive skills and strategies of which the table was more precise and ideal. Further the use of the table was like the meditational tool in the teaching and learning of linear programming.

The enlisting phrases to associate with the symbols direct from situation problem without a table provided less interaction as learners mostly depended on the teacher to identify the phrases/words to be associated with the symbol. The common approach in the formation of inequations/inequalities was the identification of words that would be expressed as inequality symbol. These are words that give the constraints/conditions in the statement. These are words like; at least, at most, not more than, more than and equal to. What is key was the interaction which the teacher engaged the learners with the situational statement in identifying phrases that can be associated to inequation/inequality symbols in order to familiarize them with the usage of these words.

The above observation is in line with Jaworski (1992) in her argument about three elements inherent in constructivist mathematics teaching which are: the provision of a supportive learning environment; offering appropriate mathematical challenge; and nurturing processes and strategies that foster learning. Therefore, the choice of the strategy is of great importance for effective teaching of linear programming in the light of formation of inequations/inequalities. However, this calls for more interaction of the learners with the language of mathematics.

5.2.2.1 Language of mathematics

The teacher who used the table was able to relate the phrases in the question with the symbols in the table. For instance, the phrase like; ‘at least’ and ‘not more than’ were associated with symbols and making it much easier to formulate the inequation ($2x + 3y \leq 30$, $x + y \geq 6$, $y \geq 3$ and $y \leq 5$) as required. The other two teachers discussed phrases that were to be associated with the symbols direct from the statements without a table, thus identifying the phrase for the constraints with the appropriate symbols to present the phrase.

In view of the above, the teaching of the mathematical language was very important. This is in line with Newman (1983) who pointed out that although teachers have often assumed that incorrect solutions to problems have arisen from a lack of understanding of mathematical concepts or a deficiency in computing skills, but Newman argued that, the errors have been caused by inadequate understanding of the language of mathematics. Newman further argued that, while curriculum planners typically comment on the importance of language factors in mathematics learning, they subsequently paid little attention to how language factors influence mathematics learning. Then the major source of difficulty with mathematical word problems is the fact that the language of mathematics and the language of common English usage often differ radically. This is in agreement with the understanding that linear programming involves word problems that result in maximizing and minimizing costs. Rosental and Resnick (1974, p. 817) cited by Ellerton (1996), described word problems in arithmetic as; tasks which require the integration of linguistic and arithmetic processing skills. In word problems, a situation is described in which there is some modification, exchange, or combination of quantities.

Therefore, the teaching and learning of inequality terms and symbols is like learning a language. I strongly observed that, the teaching and learning of these terms and symbols should have been handled as in the teaching of language. This is supported by Psychologists and educators in many parts of the world who have acknowledged the issue of language as a tool for learning (Mercer, 1995; Orton, 1992; Vygotsky, 1986). Mercer (1995) argues that people use language to get things done and to engage in their activities. Therefore, Language is seen as situated, jointly produced and organized by content to construct meaning through interaction in the mathematics classroom. The importance of this understanding on the part of the teacher in the teaching and learning of linear programming is very key.

It is further understood that, the meaning of concepts in terms of linear programming phrases and symbols in a mathematics classroom is negotiated by teachers to learners in a language that they both understand. Therefore, the teaching and learning of terms like; at most, at least, more than, not more than and equal to, as conceptual terms for inequality symbols that are used in linear programming situational statements may fall in the informal language. Informal language is the kind of language that learners use in their everyday experiences to express their understanding of mathematical concepts. Formal mathematical language refers to the accepted use of terminology developed in schools (Pimm, 1991). Therefore, in this study the negotiation of meaning in terms of phrases was very important. The social interaction to engage learners in building conceptual understanding was the main task in the formation of inequations from situational problems. This is highly supported by the social constructivism theory tenet of mediation which mirrored this study.

5.2.4 Graphing the inequations/inequalities (plotting) using the test point

The plotting of inequations was taken to be difficult. However, the graphing involved drawing a line and shading the unwanted region. The common strategy was where two coordinate points were identified by taking an assumption of one value of the variable, for linear inequation/inequality in two variables and for the one in one variable, to identify a point/s where the line could pass.

When it came to shading, all the three teachers used the testing point method, where you get a point from each side of the line and substitute for the value of the variables to the point which would satisfy the inequation/inequality in question. The point which does not satisfy the inequation/inequality would be considered to fall in the unwanted region hence it was to be shaded. This was where the instruction for shading was key. It was at this stage that the interference of shading the wanted region from linear inequation was very common. Besides, the errors were mostly on not paying attention to the inequality symbols when shading the false/unwanted region that gave the representation of inequation/inequality. This strategy was supported by all the three teachers during post-lesson interview that it was more ideal and very easy for pupils in identifying the region to shade.

5.2.5 Use of trial and error method

The location of maximum or minimum costs/profits was one area that I paid attention to. Two different strategies were used in the discussion of the location of the coordinate for the maximum/minimum cost/profit. These are; the use of the vertices of the feasible region and using both the boundary and the inner points of the feasible region. Tr-M and Tr-G both used the coordinates of the edge (vertices) of the feasible region.

Further, of the two strategies used, the one for finding coordinates of the vertices of the feasible region was more effective as it seemed much easier to locate the coordinates and then substitute the coordinates of these vertices in the profit function. This strategy gives much light in lessening the challenge of locating the maximum or minimum profits. To the contrary, the one for both the inner and boundary points of the feasible posed some challenges as learners were overwhelmed with coordinates resulting to missing out on the correct one needed for the maximum/minimum cost/profit. This challenge was echoed out by learners during the focus group discussions that the strategy was more like guessing when finding the coordinate and that, they hoped for a more precise approach preferably a formula.

The methods of using a search line and origin test and extreme point technique were very unpopular. None of the three teachers used any of the two methods. However, during the post-lesson interview, the teachers did mention in passing that in books, it is recommended that the search line can be used in finding the coordinate point for the maximum/minimum cost/profit, but they strongly lamented that this method was time consuming. Hence they could not use it. For the origin test and extreme point technique, nothing was said about it therefore, it was very far from being used.

This sub-section of the topic gives the greatest application of linear programming. Dantzig, (2002) defined linear programming as a mathematical method for determining a way to achieve the best outcome (such as maximum profit or lowest cost) in a given problem (mathematical model) for some list of requirements which can be represented as linear relationships. Linear programming arose as a mathematical model developed during the Second World War to plan expenditures and returns in order to reduce costs to the army and increase losses to the enemy.

The coordinates for the maximum/minimum cost/profit is at the helm of linear programming. All is done in order to arrive at the coordinate for maximization/minimization of costs/profits. It is at this point where the application of linear programming is appreciated in various fields of economics, business and engineering. Therefore, using vertices to find points for maximum/minimum costs/profits aligns with Bell (1991) who also suggested a set of specific heuristics (steps 1-8) that can provide a model for solutions of linear programming problems, thus Step 7: Finding the co-ordinates of the vertices of the feasible region and Step 8: Finding the solution to the problem by calculating the profit (or cost, etc.) for each of the vertices of the feasible region.

5.3 Factors affecting the teaching and learning of linear programming

With regards to the second objective of the study, I sought to establish the constraining factors in the teaching linear programming in secondary school mathematics in Monze District, Southern Province of Zambia. The constraining factors in the teaching and learning of linear programming were unearthed from the lesson observations and interactions with participants. The following were the most prominent ones; the region to be shaded in linear inequation and linear programming, translating situational statements into inequations/inequalities, the trial and error method of using both the boundary and inner points when finding the coordinates that can give the maximum/minimum costs.

5.3.1 The region to be shaded in linear inequation and linear programming

The pre- requisite knowledge with regard to linear equation learnt in the previous grades, was very important in building the much-desired understanding for the topic. For instance, the topic linear equation has been taught in grade 9 and the instruction on the region to be shaded is significantly different from the instruction given in grade 12 under linear programming. In grade 9, the instruction is about shading the wanted region contrary to the instruction for grade 12 where the shading is for the unwanted region. One teacher pointed out during post-lesson interview that, from the onset in grade 9, learners should be instructed to shade the wanted region so as to lessen the challenges in grade 12 when it comes to linear programming.

When it came to plotting the inequations, the greater challenge was on shading the correct region for the correct solutions to be realized. However, it was observed that this challenge was as a result of the pre-requisite knowledge learnt in the previous grades, where the shading was for the wanted region. Therefore, with the change of the instruction, thus shading the unwanted region, learners found it challenging and, in most cases, they are required to use the testing point. Then after using the testing, shading the false region was influenced by shading the true region.

It was therefore observed that learners did not fully interact with each other to familiarize themselves with the use of the testing point and also getting to remember that it is about shading the unwanted region. However, during the focus group discussion learners pointed out that, the use of the testing point was quite easy in determining the region that was to be shaded.

5.3.2 Translating situational statements into inequalities

The formation of inequation/inequalities from situational statements was one area learners had difficulty with. The challenge was identifying the inequality symbols to associate with the conditional phrases in the statement. This challenge was as the result of less learner interaction with a variety of situational problem statement of linear programming.

From the lessons observed, less was done by the teachers in mediating the construction of knowledge through learner to learner interactions. This was evidenced by the fact that out of three teachers who took part in the study, only one tried to encourage learner interaction by grouping learners on certain activities. The other teachers only used the learners in solving the inequations on the board, thus the teacher-learner interaction which was well demonstrated while the learner to learner interaction was less attended to.

The in-depth discussion of the mathematical terms/ phrases is at the center of conceptual understanding of the teaching and learning of linear programming. This is supported by Pressley (1998), and Monroe and Orme (2002) who argued that, most researchers agree that discussing mathematical words and phrases can help pupils learn mathematical vocabulary accurately and overcome misunderstanding of mathematical concepts. This means that mathematics has its own vocabulary which must be learnt. Therefore learners ought to be engaged within themselves so as to help them interact with the mathematical vocabulary.

MacDonald (1990) also found it necessary to develop pupils' language of learning in terms of both the meaning of words in relation to learning and their use, before engaging them in meaningful dialogue during the learning process. Therefore, in this connection, Orton (1992) also argued that language is for both communication and thinking, and that we mostly think in our first language. This implies that communication of the mathematical language need to be translated into usual language to allow thinking.

Further, the interest in relationship between language and learning in general is not new. Some theories (Vygotsky, 1962; Piaget, 1952) have suggested that language determines and defines thought. Vygotsky (1962) had the opinion that language and thinking are inescapably linked, and even though they first appear independently in infants, they quickly merge into a single function as humans develop into fully social beings (Vygotsky, 1962). Von Glasersfeld (1995) observed that language is a tool which enables people to construct meanings when they talk to each other in the group to which they belong through social interaction.

In support of the above, it is argued that, the role of the mathematics teacher is thus to help learners overcome the challenges posed by mathematical language. Chard (2003) observed that the teacher needs to plan instruction that would engage learners in using appropriate mathematical vocabulary. Murray (2004) encourages the teacher to use classroom discussion when introducing mathematical terminologies and connecting them with more familiar words which learners know.

5.3.3 The trial and error method of using both the boundary and inner points

Two different methods of finding the coordinates were used, and these are using the coordinates of the vertices and both the boundary and the inner points of the feasible region. However, the use of both the boundary and the inner points was challenging as learners were overwhelmed by the coordinates resulting in missing out on the correct one needed. Finding the coordinates for the maximum profits/costs is a product of having the correct inequations/inequalities, plotting and shading. It was to the pupils concern during the focus group discussion that using this strategy was like choosing the points by guessing. Therefore, if the question process was not well handled, the challenge of the coordinates for the maximum also arises.

5.4 Suggested measures to overcome the constraints

The third research objective was to establish the measures that teachers would suggest to overcome the constraints. This section discusses the suggested measures to overcome the constraints.

5.4.1 Consistence instruction on the region to be shaded

The study found that the use of the grade 9 inequations/inequalities in both one and two variables can, to a larger extent help in rekindling the interest and ultimately motivate learners in the teaching and learning of linear programming. Two of the three teachers introduced the topic by having a recap of the linear inequations from grade 9.

However, it was noted that the instruction on the region to be shaded for grade 9 and also the common schemes of work prepared by the provincial team demanded that, the shading should be for the wanted region which provides the solution set to the inequation/inequalities in question. This contradicts with the instruction for grade 12 linear inequations which demands shading the unwanted region. According to Thurston (1995), in order to achieve the necessary comprehensively mathematical understanding, successful communication of mathematical ideas is the key.

5.4.2 Providing a variety of situational problems and creating a more interactive learning environment

The challenge on symbols in the formation of inequations in the teaching and learning of linear programming was mostly pronounced. However, it was observed that, providing a variety of situational problems and creating an interactive learning environment can help learners practice the use of symbols efficiently. This observation is in affirmative with Nakhanu, Shikuku and Wasike (2015) who did a study in Kenya entitled Application of Linear Programming Knowledge and Skills to Real Life Contexts by Secondary School Students in Kenya. They used a Problem-based learning (PBL) approach which advocated the use of real – life problems in the teaching and learning of linear programming. And used the origin test and extreme points technique in the teaching and learning of linear programming.

This measure corresponds with the learning policy under the Ministry of General Education of learner - centered instructional approach (MOE, 2013), which also aligns with Gredler (1997) in his perspective of Idea-based social constructivism. Idea-based social constructivism sets education's priority on important concepts in the various disciplines. These "big ideas" expand learner vision and become important foundations for learners' thinking and on construction of social meaning (see section 2.7) for more on the art of creating an interactive learning environment by Vygotsky in his construct of learning in the ZPD.)

The study further observed that the role of the teacher is very paramount in creating this environment. This involves helping the learners to do the tasks they could not have done on their own as advocated by Vygotsky (1978) in his construct of the Zone of proximal development.

The teacher's role in teaching Mathematics in the ZPD becomes one of purposeful instruction, a mediator of activities and substantial experiences allowing the learners to attain their Zone of Proximal Development (Blanton, 1998; Rueda et al., 1992). The implications are that the teacher's task is to push the student's ZPD toward higher and higher levels of competence and complexity.

5.4.3 Use of the vertices of the feasible region.

The study further found that the challenge of determining the points that would give the maximum profit can easily be mitigated by using the vertices of the feasible region. This was observed and two of the three teachers stressed the importance of using this method unlike the search line method and the one for trial and error.

Apart from the use of coordinates at the vertices of the feasible region, two teachers (Tr – M and Tr – B) suggested the use of the profit margin equation of which the different coordinate points are substituted in the profit function to determine the maximum profit. Then for the minimum cost/profit, the same process was suggested. However, this time it is about picking the points of the vertices at the far left of the visible region.

This revelation is in line with Bell (1991) who suggested a set of specific heuristics (Steps 1-8) that provide a model for solutions of linear programming problems. The following are some of the steps that were suggested;

Step 6: Using x and y axes, sketch the areas defined by the inequality statements. Hence find the "feasible region".

Step 7: Find the co-ordinates of the vertices of the feasible region.

Step 8: Find the solution to the problem by calculating the profit (or cost, etc.) for each of the vertices of the feasible region.

The study revealed how these steps were addressed by teachers during their lesson discourses.

5.4.4 Engage with real life application of linear programming

Learner motivation is very important in teaching. Therefore, the use of real life situations is an important ingredient in the teaching and learning of linear programming.

Below is a simple example of an entrepreneur

Figure 5.1: *Application of Linear Programming*

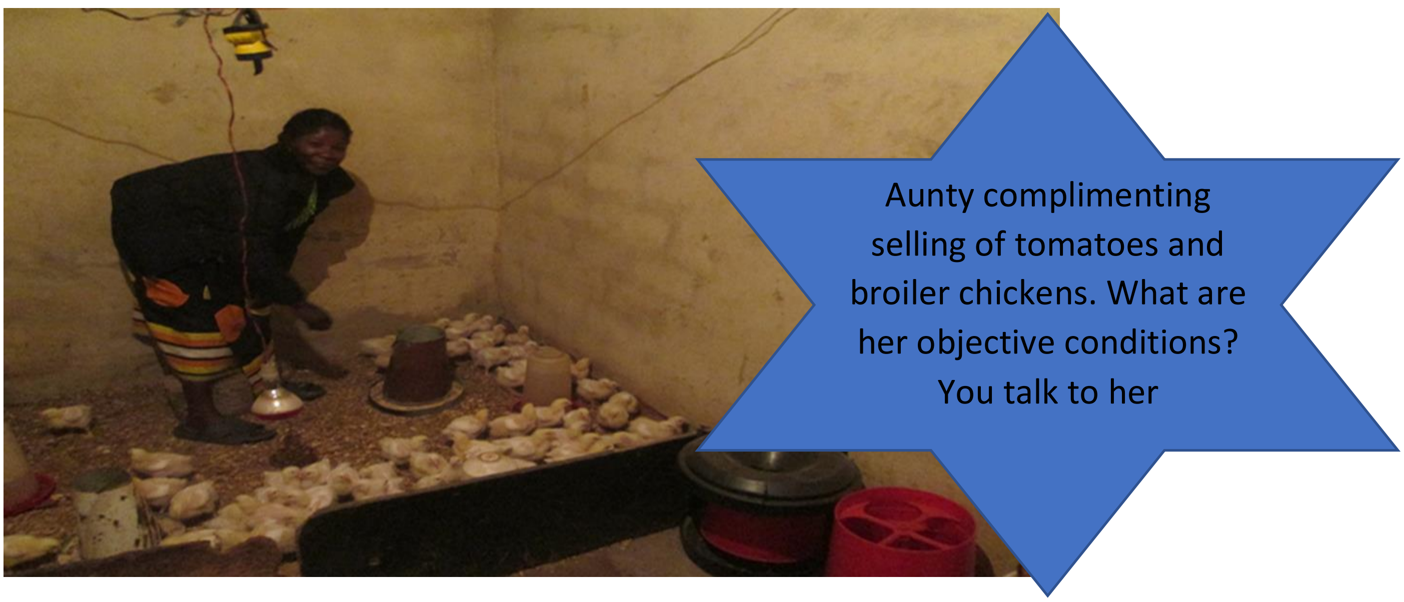


Figure 5.2: *Application of Linear Programming*

The **figures 5.1** and **5.2** above shows a woman selling tomatoes and layering broiler chickens respectively. The two figures are typical examples of an entrepreneur person who can be interacted to learn about some of the objective conditions in her business. If learners can visit such people to interact with them and ask questions like; how many boxes of tomatoes do you normally order for your business, how many broiler chickens do you normally keep, how much is a box of tomatoes and how much do you order these chickens at, what are your estimates on the feed and drugs for the chicken up to the time they are ready, are there any other things you don’t do in order not to make a loss or things you do to maximize your profit? These are some of the questions that can be asked when you make a visit (field trip) and then when they go back to class, from the answers they would have gotten, in groups for increased interaction they can formulate inequations/inequalities and then make class presentations which can be quite motivating and educative in the light of linear programming. This was raised by teacher Tr-M during the post-lesson interview as one way in motivating learners to appreciate linear programming.

The above sentiments are in agreement with Bredo (1994) in situated cognition. He argued that, situated cognition makes the distinction between descriptions and the ‘reality out there’. Knowledge can be represented, but "knowledge is never in hand" (Newell, 1984). "The map is not the territory" (Korzybski, 1941). The situated cognition perspective as advocated does not deal primarily with the relationship between entities as distinct and separate. Instead, it considers the system—context, persons, culture, language, inter-subjectivity—as a whole coexisting and jointly defining the construction of meanings (Bredo, 1994). It is against this background that if learners are engaged in real life application of linear programming can be a more effective strategy.

This is further supported by Freudenthal (1973) in the realistic mathematics education advocacy. In RME, the starting point of instructional experience should be real to the children, so that they can immediately engage in personally meaningful mathematical activity (Streefland, 1991 cited in De Lange, 1996). The learners’, “informal mathematical activity should correspond to a basis from which they can abstract and construct more and more complicated mathematical concepts. While engaging in these mathematical activities, learners could be using horizontal and vertical mathematization and in the process learners would use their home languages, mathematical symbols and the language of learning and teaching interchangeably. Hence, move gradually to the importance of the shift from “ordinary” language to specialized language and mathematical, symbolic representations.

In a linear programming task, the process of extracting the important information required using an informal strategy such as trial and error to solve the task, would be horizontal mathematization. Translating the task into mathematical language through the use of symbols and later progressing to selecting an algorithm like: $2x + 3y \leq 60$; $x + y \geq 6$, $y \geq 3$ and $y \leq 5$ could be considered vertical mathematising as it involves working with the problem on different levels which fit in well with the construct of realistic mathematics education. Therefore, this understanding is supported by the constructivists who believe that learners are the makers of meaning and knowledge. At the heart of constructivist philosophy is the belief that knowledge is not given but gained through real experiences that have purpose and meaning to the learner, and the exchange of perspectives about the experience with others (Piaget & Inhelder, 1969; Vygotsky,1978).

This is further affirmed by Gredler (1997) in his perspective of pragmatic or emergent approach, where he argues that Social constructivists with this perspective assert that the implementation of social constructivism in class should be emergent as the need arises. Its proponents hold that knowledge, meaning, and understanding of the world can be addressed in the classroom from both the view of individual learner and the collective view of the entire class (Cobb, 1995; Gredler, 1997)

5.5 Summary of the chapter

This chapter has presented the discussion of the findings on teachers' strategies in the teaching of linear programming in Monze District of Southern Province. The discussion looked at strategies used in the introduction of the topic, development and the conclusive part of finding the coordinates for maximizing and minimizing the cost/profit. The discussion further covered the areas of challenge/constraints in the teaching and learning of linear programming and the measures which were suggested by the teachers to overcome the challenges.

The study established that teachers used different strategies in the teaching and learning of linear programming. These strategies are in line with the call for teachers' adoptions for more progressive teaching styles to accommodate the varied abilities of students, so as to enable these students excel in their learning. The domains of the mathematical knowledge for teaching as advocated by Ball and her team were demonstrated by the teachers in the teaching of linear programming. The study further established that the phrases and the symbols that are specific for linear programming are in line with different researchers who cited a great importance of the language of mathematics in conceptual understanding of linear programming. The literature that was reviewed is in line with the social constructivist theory where knowledge and learning are products of social interaction. The theory mirrored the effective strategies that teachers used to cultivate an interactive learning environment. Therefore, the mathematical knowledge for teaching and the language of mathematics in the light of linear programming are effective in the domains of social constructivism. The next chapter presents the conclusion and recommendations of the study.

CHAPTER SIX

CONCLUSION AND RECOMMENDATIONS

6.1 Overview

The foregoing chapter discussed the findings of the study on the teachers' strategies in the teaching of linear programming in selected secondary schools in Monze District of southern Province. This chapter presents the conclusions and recommendations of the study based on the findings.

6.2 Conclusion

The study revealed six main strategies that teachers used in the teaching of linear programming in selected secondary schools in Monze District of Southern Province. These strategies are: the use of linear inequation and coordinate geometry of the straight line for introduction, use of the tables of terms and their associated symbols/signs in the formation of inequation from situational statements, enlisting terms and associated symbols when forming inequation from situational problems, use of trial and error method when finding the coordinate for the maximum/ minimum costs/profit and use of the boundary points for finding the coordinates for the maximum/ minimum cost/profit.

The study further established the constraining factors in the effective teaching of linear programming and the intervening measures that were suggested to overcome the constraints. The constraining factors were; the region to be shaded for linear inequation and linear programming were not consistent, less learner interaction with variety of linear programming questions/situational statements and getting inequations to form linear programming questions to enhance the understanding of the use of the terms and associated inequality symbols was inadequate. The trial and error method was not very effective as learners were overwhelmed by the coordinate points resulting in missing out on the correct ones needed. The intervention measures that were brought to light include; the instruction for the region to be shaded for linear inequation and linear programming to be consistent, more learner interaction with a variety of linear programming questions to enhance understanding of terms and symbols used in linear programming and use coordinates of the vertices of the feasible region for the maximum/minimum costs/profits would be more effective.

The study has established that the teaching and learning of linear programming has much to do with understanding of linear programming conceptual terms and symbols. Therefore, attention to the teaching strategies, thus building from introduction to its conclusion should provide a much needed interaction with the terms and symbols of linear programming and providing real – life situational contexts to enhance its application in an interactive learning environment.

This study therefore has contributed to the body of knowledge that, the more precise strategies in the teaching of linear programming linking from introduction to conclusion can be realized. In introducing the topic, building from inequations/inequalities and coordinate geometry could be considered. While in the development, the use of the table with symbols and their associated terms/meaning could be precise and in conclusion which is more to do with maximum/minimum cost/profit the use of vertices is more precise. I hope these strategies can largely help to advance the understanding of the effective ways in the teaching of linear programming.

6.3 Recommendations

The following recommendations were made based on the findings of the study: The study established that teaching and learning of linear programming was key in bridging the real life contextual learning environment into a classroom situation as its application is highly appreciated in the world of business, economics and engineering fields. It is therefore being recommended that all pupils at senior secondary school get the much-needed knowledge of linear programming for life after school.

The study further, recommends that, the region to be shaded either wanted or unwanted (false) should be consistent rather the same in linear inequations and linear programming in order to provide a much deeper connection of linear inequations and linear programming. Use of tables containing terms with their associated symbols can be a more precise technique to help the interaction of the two. Teachers should consider giving a variety of situational problems to enhance learner interaction with the terms and their associated symbols used in linear programming. Teachers should consider using coordinates of the vertices of the feasible region for finding the maximum/minimum cost/ profits as a more precise technique. Inviting or visiting an entrepreneur person to learn about the objective conditions in their business can be quite motivating and can provide the much-needed focus for linear programming in practice. Therefore, teachers of mathematics should always include linear programming in their in-house continuous professional development (CPD) and cluster meetings to address the challenges in the effective strategies in the teaching and learning of it.

Further research on strategies for effective teaching and learning of linear programming in secondary schools in Zambia could be explored and see how they compare with the ones established by this study. A research with the focus on the learners could be welcome.

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APPENDICES

APPENDIX A: FACE TO FACE INTERVIEW GUIDE FOR MATHEMATICS TEACHERS

Interviewer: ______________________________

Interviewee:______________________ Sex ( ) School:_________________

.Date:___________

Place:___________________________ Start Time:______________

Please note that this is a purely academic study which seeks to investigate the effective use of social constructivism methods in the teaching and learning of linear programming in selected secondary schools in Monze district.

1. Why did you choose to introduce linear programming the way you did?

2. How did you plan to teach sign/symbols with their associated meaning?

3. Why did you use those strategies in each sub-topic?

4. What are some of the constraints/challenging learners face in learning linear programming?

5. What challenges you as teacher face when teaching linear programming?

6. Anything you would suggest to better the teaching and the learning of linear programming.

End Time:______________ Thank you for your time and participation in this study.

INTERVIEW TRANSCRIPTIONS FOR TEACHERS

Tr - M

Q1 Aaah coz you would note that eey the boundary lines that form the solution set to the inequalities when you ought to represent an inequality on the XOY-plan, it should be represented as a straight line.

-Basically, it follows that the equation or that represent a straight line is an equation of a straight line, so thus pupils ought to know how to derive the equation of the straight line. So, I wanted to try and bring out the pre-requisite knowledge they have on coming up with equation of the straight lines. Then from there trying replace where there is an equal sign on the equation of the straight line bringing up the inequality sign or symbols rather.

Q2 Yes – There after aah, I thought it would be ideal if we are going to look at each of the symbols individually like the less than symbol, and we look at the appropriate phrases or words that can be associated with say the symbol of less than, greater than, the appropriate words, which can be used, other than greater than, you can use like more than if you are telling of less or equal to, maybe you can get to use not more than and other phrases which could be associated with the---some symbols; inequalities.

Q4. Challenges

-I noted the few challenges---mm I note one challenge where a few pupils had difficulties like in --- some could quite alright come up with the expected inequality but when it comes to plotting it on the XOY- plane, aah some could make minor mistakes quite alright they could get one point correctly and make a mistake on the other point, so they would fail to draw the correct line, anyway they were able to shade the expected regions quite alright.

Q4 Aah with respect to the Teaching of linear P, the challenges that I have noted are---especially with our pupils that we have here, most of our pupils are coming from background where, there are not so used to using this foreign… the queen's language.

So---they would, maybe I would say think in their mother tongue languages, then trying to bring that or integrate it with English language, most of them fail to do that---aah, so basically it could maybe the language barrier I think—there is need to maybe inculcate in our learners this reading culture, it is not there so they fail to connect the phrases which are there as way words and try and understand what is like really expected of them. Yes, and aah on the learning part the learners aah-the challenges some pupils still have problems with computational skills.

yes, there are things which we would take that by the time someone is in grade 12 and teaching this topic, they would be able to subtract or multiply or evaluate an equation but you note that you still have to revise those small-small things from their junior secondary Education levels, so we find ourselves to start from the foundation when ought to just try and build on, but we have to revise the foundation like a lot of time.

Q5. Aaah other than just bringing out like question in class, I think it is ideal that pupils were able to see the actual situations given to them. Take for instance, L/P is so ideal like in the business circles, agricultural, trading, marketing and purchasing and supply those areas, so I feel pupils were able to or taken out of a classroom situation and take for instance if you are talking of someone who is say… buying two types of chickens maybe broilers and layers. You take them out of the classroom situation and they go and see the actual situation and then they are able to see the intentions limitation and identify the constrains which are there, other than just being taught, it is more less like a theoretical thing, I think it could be better if they are like brought out of the classroom and be able to see the actual situation on how they can interpret and note constrains which are there.

Q6. Aah---Yes especially on finding the maximum profit or minimum cost or minimum profit or cost thereof. I would suggest… other than using the search line, which could be time consuming as moving across axes like from the Y and X-axis, using that search line which moves upwards aah after one has clearly stated the region and defining the inequalities. I feel the easier way would be using the points which make up the solution set but then it would be difficult of one picks all the points because sometimes you would found that there are quite a number of points which will be in a solution set following the scale that is given, so the most ideal thing on how to get the maximum / minimum profit are the points which are on the edge of the solution set or region that makes up the solution set, just identify those coordinate point which are on the edges then someone will be able to check of course if there four inequalities expect just to have about four edges. So with the four points, one can quite try and see which points gives him or her the maximum profits or cost, other than the points which are just on the edges will work out----yes.

Tr - B

Q1. Aah the method that I used to introduce the topic L / P, first of all I had look at the issues of < inequalities in one variable and we moved on to < in two variable and also believe in what we call starting from the known to the unknown the preliminaries are (covered) first of all, i had to go back as far as grade nine to re-candle the interest of L / P to where it start from.

Q2. These symbols are very important becoz you find learners sometimes they make mistakes on less than or greater than so it is up to the teacher to have strategies on how best he can introduce those to the learners especially the issue to do with less or equal to, sometimes they don't understand what it means when we say less or equal to. The emphasis should be done before much is done in the teaching / < of L / P. So those symbols are very important because they are the symbols that we use through and through and words like at least, at most they also accompany those symbols, so must marry the symbols and the words like less than, "This is what it means" then also the issue of shading the unwanted region, that you come up with what we call the feasible region yeah.

Q4. there must be a lot of interaction between the teacher and the learners. You know its not to like offloading everything on the learner eey the teacher should give chance to the learners sometimes learners to come and demonstrate on the board, what they have understood from the sentence and even ask the learners to plot let say if you are plotting the inequality. First of all you start with plotting the line then shading the inequality can be given to the learners so that they rely participate in the learning, then the teacher always demonstrating and the learner become like on lookers which is not a good approach – ok-yes sir.

Q5. Aaah one of the most challenging thing that I have seen in the teaching and learning of L / P is that most learners, they fail to come up with the inequalities from the words. Becoz L/ P has to do with the description of words then from those words you have to come up with your---with the inequality you going to use-you find that, if a learner does not understand the words like at least, he will put a less than instead of a greater than, then everything become distorted, so the instruction of the inequalities from the words given, is one of the most challenging task that the learners normally face. If you go through, you do a lot of examples and a lot of exercises helping the learners to understand exactly which symbol to use at what time.

Q6. Yeah from what I gave and what I marked and----thee---assessment I made was that, I would say am comfortable 85% of my learners would be able to answer the question on L/P with confidence ok, because at least there able to shade and come up with a feasible area, and from the feasible area, they are able to deduce the maximum or the main points as required by the question.

Yeah please --!! The first thing I think when you are tackling L/P, it also important to look at—the pre-requisites of what is needed to accompany L/P. like I said at the beginning is that you need to go back as far as grade 9 work, you start from inequalities ok, how to shade inequalities in one variable, inequalities in two variables. You develop your lesson from the known to the unknown and you will find that, you will kindle the interest of learners by so doing, but if you first start from somewhere above, you will find that most of the learners will be lost and it will be very difficult. And also the emphasis on those symbols that we use like greater than, less or equal – those should be married with certain words like; at least, at most, or maximum, minimum. Once you do that, then you will find that you are going to move with the learners and no one will be left behind. So that what I would say on the strategies on how to improve T/L of L/P

Yeah there is a----methods that we normally use, where you draw what we call a searching line- and the searching the search line, once you draw the searching line, then you move it---you use a ruler where you have to move it from the searching line parallel going up wards, the last point that will give you the maximum, all those it is not every clear method especial to pupils, they take long to understand it ok. Mostly the method I would prefer is the trail and error, where you pick points here and there, where you think that probably these are the points that will give me the maximum and then pushing in the profit function, then the one that gives in the maximum, that's the one they will pick on. But off course that is time consuming in an exam that method probably will chew a bit of time for the learner.

INTRODUCTION .Tr – G

Aal to me I was like thinking like, since linear if programming is more of the business set up. And when we look at the questions which come at the end, they talk more of profits and then minimizing the cost. So which means mostly linear programming is to do with that and this is the to topic most of the time with the background of business administration that I have, I saw that linear programming is used to plan, to plan activities and so fourth and linear programming is used to know whether you are making a loss or a profit in business, as you have seen most of the questions that comes, they will be first be talking about maximizing the profit and then minimizing the cost. So that's why I say,ok let me first come up with this introduction in this way and then also the pupils they need something in real life, a tangible thing because most of the time they ask why do we learn mathematics it is very difficult how am I going to use this. So it is better if I approach it in that way. Telling the pupils that we can use that topic in this, they could also be interested in.-I also started with linear equations in one variable and two variable, and i don't even dwell much on the equations, coz I was just reminding them because they needed the idea of solving the equations and inequations like for me personally I prefer to go for inequations than for me to go for equations. since linear programming will be dealing much with aah inequation signs. So I said if a dwell much again on the -e- equations, pupils will be getting confused, they be putting an equal sign where we need an inequality sign, they be putting and as you already know that even pupils when they are solving inequations, they like putting an equal sign there as inequation. That's why I just looked at it that, I just make it as ----am not going to dwell much according to even to my schemes. The schemes say that we have to teacher linear equation in one variable and two variables.

Then aah----after doing that, then we shade that, what the wanted region according to the schemes because it is indicated that, we shade the wanted aah region.Now if we are doing to shade, the of course we wanted to shade the wanted region in grade nine. Now linear equation, you find that aah equation at the end, we will be again shading the unwanted region, coz we emphases that can we shade the unwanted region.

So like that if we dwell much on the first part of…..equations and showing them the wanted region, and then at the end of the day again we change or I change to say … now we are shading the unwanted region. It becomes a confusion on the part of the pupils because they will be questioning that at first you were saying wanted region, aah we were shading the wanted region, again we are changing to the unwanted region. So like for me I would prefer and I would love, these people who set these schemes to go for what – inequations and avoid the issue of equation. They just go straight and to say, lets solve inequations and then we shade the unwanted region because, we will be shading the unwanted region continuously than for us to start saying you the wanted region and then again change to the unwanted region. So I would rather, we go straight for these; from the word go the unwanted region. Than for us to be changing and that one, if we go with strategy, like for me with the experience that I got, I have found that pupils have gotten used and get confused and at the end of the day I have not achieved my objective. I have confused my pupils more where I say, or like shading the wanted region like in grade nine, then again shading the unwanted region in grade 12. So that it really confuses the pupils. If we can just start to say,let us go with the unwanted region, we solve the inequation then we go for the unwanted region and I have seen that, that one have not given much problems to the pupils, we have at least done the……..

Q2 -Language/symbolism.

For like for me symbols, when you look at linear equation, basically we talking about the symbols greater than or less than and so fourth. The moment the pupils do not understand or do not know we one greater and which one is less, that becomes a problem through out they will have that problem of which part to shade or which part not to shade. So it's from the word go that they should understand the inequalities, that's why when am teaching I have found a system, because we know that left, we say left handside, when you fold the left handside like that, it is less whatever position you are sitted and then when you fold your right hand side it is greater and then whatever position you are sitted, I found that this system, it works-

Then again left starts with less, so it is easy for the pupils to know that left is for less and right is for greater. So I advice this system that whatever position first do that and it has worked for the right parts which an looking for, if it is the unshaded or whatever.

-Atmost /Atleast

Ok, the least I have seen that in linear equation there are certain words we use so again it also goes with the words are there. Pupils have to be aware of the words and they should understand for them to give the correct thing. So I have discovered that the words that we use in linera programming we are going to use the same words like at least, and at mostly, at least is the more prominent and it is the most used in most of the questions. So now like for me on my part first of all, I have to make my pupils understand these words.coz if they won't understand these words again they won't bring out what I want. They won't give me those inequalities which I want eeche or the sign especially, they give me a wrong things. So we have to make them understand less or equal tomus when dowe use it that's why they are those words like available, atmost at leas.

Then at least is a word also the word which confuse these people and it confuses pupils. Because when we look at least the way it is used, am sure this word is misused , as you so when pupils when ask them, they think at least means, it just means less, because them when you say at least means less. They take it like in the language or maybe in our language when you say at least what-what it will mean less when it means, the minimum should be that number that is at least the minimum meaning we can have more-or expect more. We have to cleave them from the—point of view of the lay man way of understanding,they way they take it. So you clear that by trying to explain those explainations and just involve them and the answer will definitely going to come out. As you saw whem they were discussing themselves, the answer could come out the way they understand , the could argue then at the end of the day the answer will come out and at least they understand.

-Maximum/Minimum profit.

The pupils have a problem in getting the points on the graphs. Emphasis should that when you are getting the points, you should be getting the points from the wanted region. After getting the profit equation, you substitute the coordinate value in the profit equation then you can find the maximum profit among those points which are going to be inside. The best is to go for the points where the lines intersect. Those are the main points mostly that's where they give us the profit or where the profit will come where the lines intersect. The emphasis was on whole numbers since the questions will be talking about chickens and so on which where were cannot have half chicken coz they can give numbers with points. Then we use the profit equation to substitute the one with greater value will be the maximum. And emphasis that the answer will come where the lines are intersecting. For the minimum it to get from the left side coz that where you found smaller numbers like zeros and so on. Then the question should be first to calculate to the maximum profit.

Q5 –Challenges -Shading the wanted region when later you want them to shade unwanted region.

Material- Like graph papers

APPENDIX B: FOCUS GROUP GUIDE FOR PUPILS

Interviewer: ___________________________

School:__________________.   Date:____________

Place:__________________________   Start Time:_____________

1. Do you see mathematics to be a very important subject in the school curriculum? For your answer give two reasons

2. Did find learning linear programming easy or challenging? Give reasons

3. Is there any challenge you can sight in the way your teacher introduced the topic?

4. Anything which was interesting in the introduction?

5. Which area of linear programming did you find it interesting? Give reason.

6. Which area of linear programming did you find it challenging? Explain

7. Any suggestion you want to make in you learning of linear programming?

End Time:________________________

Thank you for your time and participation in this study.

FOCUS GROUP TRANSCRIPTIONS

FGD-PM

Q1. Yes – it helps to express the numbers of items which can be bought or sold using inequalities, it enables to come up with the solution set where you would account for the items used maybe in business

-it helps us to explain a number of things

-it helps us to approximate, it is not having accurate – it helps know the range in which things should be.

Q2. Yes-mathematics is part of the academic, when you go to the senior level, mathematics is something which is needed and—mathematics is also important for knowing the calculation in others like calculate simple interests in purchases which actually used in business, it is important in academics.

Q3. Sometimes I do it personally coz in groups sometimes it maybe that you are the only one who knows, so it comes boring. If I fail; a question I go to my teacher for help.

-we study—some we use past papers of past exams- we

Q4. There some challenges, like finding profits

-A situation where you are given a statement, and how to find inequalities.

-It is simple if you understand the languages that, so if you understand those languages it will be simple.

-But you are supposed to know them becoz there are only about 4-5 somewhere there.

-Where you apply it that's a challenge.

-The only challenge is on how to find profits, on finding inequalities I have no problem.

Q5. I had diffecults when plotting the lines and shading

-The challenge on how to plot a line

-Finding the inequality that's where the challenge was

-Plotting- But I managed to get the inequality

-Finding the coordinates points of maximum

Q6. No suggestions,

FGD - PB

Q1. Yes, aah it is more like statistics it helps to collect data, say for instance, we tell you to buy a certain number of books, thus English and mathematics, so you have to decide which ones will be less and which one would be more. But in actual sense both has to be equal. Coz you are taking it as one subject is not more important than the other.

-It is more about business when determining maximum and minimum profits

-I would like to say that once fully understood, this topic, usually it comes in paper 2, which carry twelve marks it is easy to answer and pass the exam.

-I feel L/P helps like expressing solutions of graphical, it is like expanding from statistics as Kaumba has said.

Q2. Yes sir, when you build interest in mathematics you also improve and become good at it. It improves your thinking capacity, let say you able to calculate and keeping you awake. If you know how to solve equations and the like. It is possible for you to pass other subjects cause you will be active.

Q3 Yes – just for reference

Q4 Yes – the part for find maximum profits, our teacher didn't very explain properly on how to find points.

-Finding maximum profits, its like you chose the points by guessing.

-Other challenge was, you find you are given lines already drawn and then you are told to name those lines or make an equation for the lines – making equations from statements.

-When you are given an equation and told to put it on the graph. The part that I found challenging was where to decide which part is true and which one is false, when you are given in an equation form, but when it is straight forward it was very easy.

Q4. Aaah maybe you don't know something, you can refer to someone, maybe to Kaumba.

-Maths require attention, it is not like other subjects like commerce.

Q5 – Yes – when plotting the points on the graph.

-Determining the testing points of the lines as passing through the origin that was very simple.

Q6. As I would like to say like Inambao said, when finding maximum point, I don't know if its only our teacher who taught us or using some formula other than letting pupils just guess, it more like probability, like you pick any two point that you think will give maximum profit of the region. I think they should find some way of giving them a formular so that it is straight forward. And also after doing the lesson on L/P, you should refer to other books on the same topic

-should have a number of refers.

-it should also be taught in different perspective coz sometimes question may not come from statements sometime question comes with the graph already graphed.

APPENDIX C: LESSON OBSERVATION GUIDE/CHECKLIST (teacher strategies in a social constructivist approach)

Observer: ___________________________________

School:______________________ Teacher sex( )

Class_____________________

Lesson:____

Date:____________________

Time_______________________

No-	OBSERVATION	YES	NO
1	Assessment for pre-requisite knowledge of learner		
2	Discuss inequalities symbols (semiotic mediation)		
3	Learner engagement in class discussion of inequality symbols		
4	Positive teacher- learner interaction		
5	Teacher asking higher order questions for learner engagement in critical thinking		
6	Learner- peer interaction		
7	Teacher creation of a collaborative learning environment		
8	Teacher mediating learner misconceptions		
9	Teaching seeking explanation of learner point of view for clarification		
10	Learner interaction with other learning materials		
11	Work with pupils to translate a problem into the		

	appropriate mathematical terms		
12	Ask pupils to express a mathematical idea in another way		
13	Ask pupils to check their own answers		
14	Ask pupils to work independently		
15	Ask pupils to work in collaboratively on tasks worth of group effort		
16	Ask pupils to work in collaboratively although the tasks are not worth of group effort		
17	Explains a procedure for solving a problem		
18	Encourage pupils to reflect on a problem before solving it		
19	Encourages pupils to think about what strategy might be appropriate for a given task		
20	Asks pupils to read mathematical text and explains the text		
21	Asks pupils to read mathematical text and asks pupils to explain what they have read		
22	Asks convergent question (question that requires a specific answer)		
23	Asks a divergent question (open-ended, for which there are many possible answers)		
24	Asks pupil to explain how they found the answer		
25	Follows up on a pupil’s incorrect answer		

26	Follows up on a pupil’s correct answer		
27	Engages in discussion with single pupil		
28	Engages in discussion with a group of pupils		
29	Asks pupils to discuss whether the results obtained is reasonable		
30	Asks pupil(s) to check if their answer is correct		
31	Provide effective support to all pupils who need it		
	LESSON PLAN		
32	Do lesson plans have clear outcome that portrays social constructivism tenets		
33	Do the lesson outcomes build on previous T/L		
	TEACHER KNOWLEDGE		
34	Do they show a thorough knowledge of the subject content covered in the lesson?		

APPENDIX D: INFORMED CONSENT TO PARTICIPATE IN RESEARCH FOR TEACHERS

My name is Mr. KAABO EMMANUEL. I am from The University of Zambia, Great East Road Campus. From the Department of mathematics and science Education (mathematics Education).

Your participation in this study is entirely voluntary. Please read the information below and ask questions about anything you do not understand, before deciding whether or not to participate. We are asking you to take part in the research study because we are trying to learn about the extent to which teachers effectively use social constructivism methods in the teaching and learning linear programming in selected secondary schools of Monze district.

1. Participation in this study is voluntary and there are no monetary or rewards in any form.

2. There are no risks in taking part in this study.

3. If you feel like not taking part in this study, no one will be upset with you or even if you accept and decide to change your mind later and want to stop.

4. You can ask any questions that you have about the study. If you have a question later that you didn't think of now, you can call me on

+260977- 190127 or ask me next time.

5. Signing your name at the bottom means that you agree to be in this study.

____

Name and signature of participant

Date:___________________________________

APPENDIX E: INFORMED CONSENT TO PARTICIPATE IN RESEARCH-PUPILS

My name is Mr. KAABO EMMANUEL. I am from The University of Zambia, Great East Road Campus. From the Department of mathematics and science education (mathematics Education).

Your participation in this study is entirely voluntary. Please read the information below and ask questions about anything you do not understand, before deciding whether or not to participate. We are asking you to take part in the research study because we are trying to learn about the extent to which teachers effectively use social constructivism methods in the teaching and learning linear programming in selected secondary schools of Monze district.

1. If you agree to be in this study we shall ask you questions about the practice of teaching and learning of linear programming.

2. There are no risks in taking part in this study except in some cases the research my take a few photos and record the discussion.

3. If you feel like not taking part in this study, no one will be upset with you or even if you accept and decide to change your mind later and want to stop you still do so.

4. You can ask any questions that you have about the study. If you have a question later that you didn't think of now, you can call me on

+260977-190127or ask me next time.

5. Signing your name at the bottom means that you agree to be in this study.

____Date:________________________

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