THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

INSTITUTE OF DISTANCE EDUCATION

2019/2020 ACADEMIC YEAR

CHE 2415 BASIC INORGANIC CHEMISTRY

M 4100 COMPLEX ANALYSIS

MAT 1100 FOUNDATION MATHEMATICS

MAT 2100 ANALYTIC GEOMETRY AND CALCULUS

MAT 2602 INTRODUCTION TO STATISTICS

PHY 3621 ELECTROMAGNETIC THEORY

UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

Department of Chemistry

2019 Academic Year Final Exam

CHE2415

Basic Inorganic Chemistry

Answer any FOUR Questions **Duration: 3 Hours**
Each question carries 15 marks
Start each question on a *fresh right hand page* of your answer booklet

Question 1

(a) Electromagnetic radiation of wavelength 242 nm is just sufficient to ionise the sodium atom. Calculate the ionisation energy of sodium in $kJmol^{-1}$.
(b) The electronic energy in hydrogen atom is given by E_n $(-2.18 \times 10^{-18}\ s) / n^2 J$. Calculate the energy required to remove an electron completely from the n = 2 orbit. What is the longest wavelength of light in cm that can be used to cause this transition?
(c) In Rutherford experiment, a thin foil of gold, a heavy atom, was bombarded by α-particles. If a thin foil of light atoms e.g. aluminium were used, what difference would be observed in terms of deflections and backbouncing?
(d) (i) How many electrons will be present in subshells having m_s value of -½ for n = 4?
(ii) If n is equal to 3, what are the values of quantum numbers l and m?
(iii) When is the energy of an electron equal to zero? Explain your answer.

Question 2

(a) Explain how a chemical bond is formed.
(b) Although geometries of NH_3 and H_2O molecules are distorted tetrahedral, bond angle in water is less than that of ammonia. Discuss.
(c) Arrange the bonds in order of increasing ionic character in the molecules: LiF, K_2O, N_2, SO_2 and ClF_3.
(d) What is meant by the term bond order? Calculate the bond order of O_2, O_2^+,O_2^-

Question 3

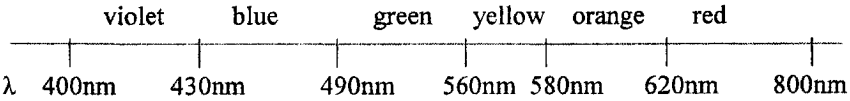
(a) Why do elements of the same group have similar chemical and physical properties?
(b) Why are cations smaller and anions larger in radii than their parent atoms?
(c) Why is 1st Ionisation energy (IE) of sodium lower than 1st IE of magnesium but 2nd IE is much higher than 2nd IE of magnesium?
(d) Increasing order of reactivity among group 1 is Li< Na<K<Rb<Ce. For group 17 it id F>Cl>Br>I. Explain

Question 4

(a) (i) Why are solids rigid?
(ii) Why do solids have a definite volume?
(iii)Solid A is a very hard electrical insulator in solid as well as in molten state and melts at extremely high temperature. What type of solid is it?
(b) (i) Ionic solids conduct electricity in molten state but not in solid state. Explain
(ii) What type of solids are electrical conductors, malleable and ductile?
(c) Distinguish among (i) simple (ii) Face-centred and (iii) body-centred cubic unit cells.
(d) Which of the following lattices has the highest packing efficiency (i) simple cubic (ii) body- centred cubic and (iii) hexagonal close-packed lattice?

Question 5

(a) How many ions are produced from the complex $Co(NH_3)_6Cl_2$ in solution?
(i) 6 (ii) 4 (iii)3 (iv)2. Explain and illustrate your choice.
(b) Deduce which one amongst the following ions has the highest magnetic moment value:
(i) $[Cr(H_2O)_6]^{3+}$ (ii) $[Fe(H_2O)_6]^{2+}$ (iii) $[Zn(H_2O)_6]^{2+}$
(c) Given high spin $[Fe(H_2O)_6]^{3+}$ and low spin $[Fe(CN)_6]^{3-}$ with crystal field splitting energy of 171 kJ/mol and 392 kJ/mol respectively and the electromagnetic spectrum below,



(d) Draw and label a figure to show the splitting of d-orbitals in an octahedral crystal field.

Question 6

(a) By using the Molecular Orbital (MO) approach for each of the following ions, N_2, N_2^{2-} and N_2^+, determine (i) the bond order of N_2 (ii) magnetism of N_2 and (iii) their order of increasing bond strength.
(b) VBT is premised on two assumptions. State the assumptions.
(c) What condition must be fulfilled by participating orbitals for maximum overlap between them to occur?
(d) Differentiate between a d-block element and a transition metal.

END OF EXAMINATION

PERIODIC TABLE OF ELEMENTS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

hydrogen 1 **H** 1.00794																	helium 2 **He** 4.002602
lithium 3 **Li** 6.941	beryllium 4 **Be** 9.012182											boron 5 **B** 10.811	carbon 6 **C** 12.0107	nitrogen 7 **N** 14.00674	oxygen 8 **O** 15.9994	fluorine 9 **F** 18.9984	neon 10 **Ne** 20.1797
sodium 11 **Na** 22.98977	magnesium 12 **Mg** 24.3050											aluminium 13 **Al** 26.981538	silicon 14 **Si** 28.0855	phosphorus 15 **P** 30.97376	sulphur 16 **S** 32.065	chlorine 17 **Cl** 35.453	argon 18 **Ar** 39.984
potassium 19 **K** 39.0983	calcium 20 **Ca** 40.078	scandium 21 **Sc** 44.95591	titanium 22 **Ti** 47.867	vanadium 23 **V** 50.9415	chromium 24 **Cr** 51.9961	manganese 25 **Mn** 54.93805	iron 26 **Fe** 55.845	cobalt 27 **Co** 58.9332	nickel 28 **Ni** 58.6934	copper 29 **Cu** 63.546	zinc 30 **Zn** 65.409	gallium 31 **Ga** 69.723	germanium 32 **Ge** 72.64	arsenic 33 **As** 74.9216	selenium 34 **Se** 78.96	bromine 35 **Br** 79.904	krypton 36 **Kr** 83.798
rubidium 37 **Rb** 85.4678	strontium 38 **Sr** 87.62	yttrium 39 **Y** 88.90585	zirconium 40 **Zr** 91.225	niobium 41 **Nb** 92.90638	molybdenum 42 **Mo** 95.94	technetium 43 **Tc** [98]	ruthenium 44 **Ru** 101.07	rhodium 45 **Rh** 102.9055	palladium 46 **Pd** 106.42	silver 47 **Ag** 107.8682	cadmium 48 **Cd** 112.411	indium 49 **In** 114.818	tin 50 **Sn** 118.710	antimony 51 **Sb** 121.760	tellurium 52 **Te** 127.60	iodine 53 **I** 126.9045	xenon 54 **Xe** 131.293
caesium 55 **Cs** 132.90545	barium 56 **Ba** 137.327	lutetium 71 **Lu** 174.967	hafnium 72 **Hf** 178.49	tantalum 73 **Ta** 180.9479	tungsten 74 **W** 183.84	rhenium 75 **Re** 186.207	osmium 76 **Os** 190.23	iridium 77 **Ir** 192.217	platinum 78 **Pt** 195.078	gold 79 **Au** 196.96655	mercury 80 **Hg** 200.59	thallium 81 **Tl** 204.3833	lead 82 **Pb** 207.2	bismuth 83 **Bi** 208.980	polonium 84 **Po** [209]	astatine 85 **At** [210]	radon 86 **Rn** [222]
francium 87 **Fr** [223]	radium 88 **Ra** [226]	lawrencium 103 **Lr** [262]	rutherfordium 104 **Rf** [261]	dubnium 105 **Db** [262]	seaborgium 106 **Sg** [266]	bohrium 107 **Bh** [264]	hassium 108 **Hs** [269]	meitnerium 109 **Mt** [268]	darmstadtium 110 **Ds** [271]	roentgenium 111 **Rg** [272]	ununbium 112 **Uub** [285]		ununquadium 114 **Uuq** [289]				

key

element name
atomic number
symbol
atomic mass

lanthanum 57 **La** 138.9055	cerium 58 **Ce** 140.116	praseodymium 59 **Pr** 140.90765	neodymium 60 **Nd** 144.24	promethium 61 **Pm** [145]	samarium 62 **Sm** 150.36	europium 63 **Eu** 151.964	gadolinium 64 **Gd** 157.25	terbium 65 **Tb** 158.9253	dysprosium 66 **Dy** 162.50	holmium 67 **Ho** 164.930	erbium 68 **Er** 167.259	thulium 69 **Tm** 168.934	ytterbium 70 **Yb** 173.04
actinium 89 **Ac** [227]	thorium 90 **Th** 232.038	protactinium 91 **Pa** 231.0359	uranium 92 **U** 238.0289	neptunium 93 **Np** [237]	plutonium 94 **Pu** [244]	americium 95 **Am** [243]	curium 96 **Cm** [247]	berkelium 97 **Bk** [247]	californium 98 **Cf** [251]	einsteinium 99 **Es** [252]	fermium 100 **Fm** [257]	mendelevium 101 **Md** [258]	nobelium 102 **No** [259]

1. Planck's Const. $h = 6.626 \times 10^{-34}$ J.s
2. Avogadro's Const. $N_A = 6.022 \times 10^{23}$ 1/mol
3. Speed of light $c = 2.9979 \times 10^{8}$ m/s

15

4. $1\text{J} = 1\text{kg}\cdot\text{m}^2\text{s}^{-2}$
5. Mass of electron $m_e = 9.11 \times 10^{-31}$ kg
6. 1 electron volt $= 1.602 \times 10^{-19}$ J
7. R_H (Rydberg Const) $= 1.097 \times 10^{7}\,\text{m}^{-1} = 2.179 \times 10^{-18}$ J

The University of Zambia

Institute of Distance Education

Department of Mathematics & Statistics

2019/2020 Academic year Final Examinations

M4100 - Complex Analysis

Time allowed : Three (3) hrs Full marks : 100

Instructions:
- Attempt **any five (5)** questions. All questions carry equal marks.
- **Full credit** will only be given when **necessary work** is shown.
- Indicate your **computer number** on all answer booklets.

This paper consists of 4 pages of questions.

---

1. a) i) Let $z = x + iy$. If $z + \frac{1}{z}$ is real, show that either z is real or $|z| = 1$.

   ii) Express $z = -1 + i$ into polar form.

   b) Let $U(x, y) = 2x^2 + 2x + 1 - 2y^2$.

   i) Show that U is harmonic.

   ii) Hence, compute the harmonic conjugate for U.

   c) (i) If z lies on the circle $|z| = 2$, show that

$$\frac{1}{|z^4 - 4z^2 + 3|} \leq \frac{1}{3}.$$

   (ii) Find all the roots of $(-1 - \sqrt{3}i)^{\frac{1}{2}}$ and display them on the appropriate circle.

2. a) i) State the Cauchy-Riemann equations.

   ii) Prove that if $z = x + iy$, $f(z) = xy + iy$ is nowhere analytic.

b) (i) Shade the domain defined by

$$|\text{Arg}z| \leq \frac{\pi}{6}, \ \text{Re}z < 1.$$

(ii) If n is a positive integer, show that

$$(1+i)^{4n} - (1-i)^{4n} = 0.$$

c) Compute the following complex limits:

(i) $\lim_{z\to\infty} \frac{2z^4+1}{z^4+1}$ (ii) $\lim_{z\to -2i} \frac{z^3-8i}{z+2i}$.

3. a) i) Is the function

$$f(z) = \frac{z^2 + (2-i)z - 2i}{z-i}$$

continuous at $z = i$? If not, can it be made continuous by redefining at $z = i$?

ii) Show that the given function $f(z) = (xy^2 + 2) + iy$ is not analytic at any point.

b) (i) Solve the following equation $e^{4iz} = i$ for z.

(ii) Find the principal value of $(1-i)^i$.

c) i) Find all complex values of the given logarithm: $\log(i^4)$.

ii) Using the definitions in exponential forms, show that $\cos iy = \cosh y$.

4. (a) Evaluate

$$\int_C f(z)dz$$

where $f(z) = y - x - 3x^2 i$, $z = x + iy$ and C is the contour comprising one line segment from $z = 0$ to $z = 1 + i$.

(b) Let C be the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$ that lies in the first quadrant. Without evaluating the integral, show that

$$\left| \int_C \frac{dz}{z^2+1} \right| \leq \frac{\pi}{3}.$$

(c) Find the singular points (that is where $f(z)$ is not defined) of the following functions and determine if they
are isolated or not.

(i)

$$f(z) = \frac{e^{\frac{1}{z}}}{z^2+1}.$$

(ii)

$$\frac{3z^3+2}{\sin 2z}.$$

5. (a) (i) Cauchy-Goursat theorem for a simple region states that : Suppose that f is analytic on a domain D. Let Γ be a piecewise smooth simple closed curve in D. Then

$$\int_\Gamma f(z)dz = 0.$$

Using the above theorem, evaluate

$$\int_C \frac{1}{\cos z}dz$$

when C is the circle $|z| = 1$ which is positively oriented.

(ii) Evaluate

$$\int_\Gamma \frac{z}{(z-2)^2}dz$$

where Γ is the circle $|z| = 1$.

(c) (i) Express $\frac{z}{(z+3)(z+i)}$ into partial fractions.

(ii) Hence, evaluate

$$\int_C \frac{zdz}{(z+3))(z+i)},$$

where C is the circle $|z| = 2$ taken in the positive sense.

6. (a) State, without proof, the Cauchy integral formula.

(b) Evaluate

$$\int_C \frac{\sin z}{(z-\frac{\pi}{2})}dz,$$

where C is the circle $|z| = 4$ traversed in the positive sense.

(c) Evaluate

$$\int_C \frac{e^{\pi z}}{z(z+2)}dz,$$

when $C = \{z : \ z = 3e^{i\theta}, \ 0 \leq \theta \leq 2\pi\}$.

7. (a) (i) Find the zeros of $f(z) = \frac{(z^2+i)(z^2-2z+3)}{(z-3)}$.

(ii) Show that the function $f(z) = \overline{z}$ is not analytic at any point.

(b) The Argument Principle states that

$$\int_C \frac{f'(z)}{f(z)}dz = 2\pi i[N - P]$$

where N is the number of zeros of $f(z)$ that lie inside C and P is the number of singularities of $f(z)$ that lie inside C, and counted according to their multiplicity.

Use the Argument Principle to evaluate

$$\int_C \frac{f'(z)}{f(z)}dz$$

if C is the circle $|z| = 3$ for

$$f(z) = \frac{2\sin z - 1}{(z-1)^2(z+\frac{1}{2})}.$$

(c) Find the Laurent series expansion of

$$f(z) = \frac{z}{(z+1)(z-3)}.$$

The University of Zambia-Institute of Distance Education

Department of Mathematics & Statistics

2019/2020 **Academic Year, Final Examinations**

MAT1100: Foundation Mathematics

Monday 17*th* **AUGUST** 2020

Time Allowed: **3 hours**

Instructions:

1. There are **Six (6) questions** in this examination. Attempt **any four (4)**.
2. Indicate your **computer number** on all your answer booklets.
3. **Full credit** will only be given when necessary working is shown.
4. **Calculators** are not allowed.

This examination has (4) **pages** of questions.

---

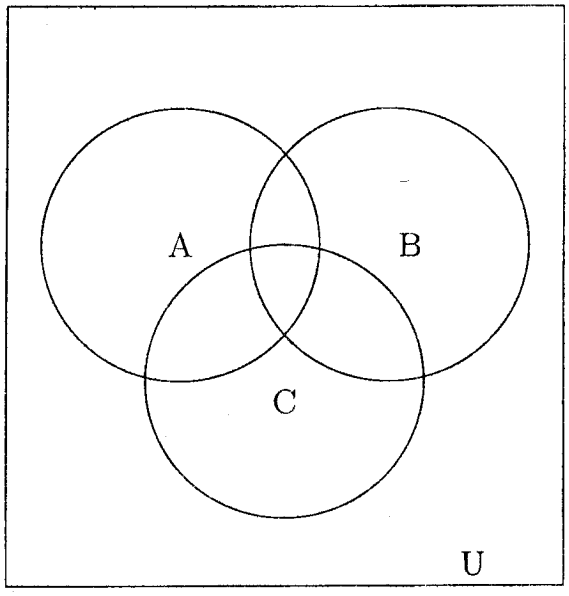
1. (a) i. Let $A = [-2, 3]$, $B = (-1, 10]$, $C = \{x \in \mathbb{Z} : -2 \leq x \leq 15\}$ and the universal set be $\mathbb{R}$. Find the set $(A^c \cup B) \cap C$ and display your answer on the number line.

   ii. Let U be the universal set. Given that $X \cap Y = \emptyset$, simplify the following as much as possible:

   $$(X \cup Y) \cap (X^c \cup Y^c) .$$

   iii. On the figure below, shade the region corresponding to

   $$(A \cap B) \cup (C - (A \cup B)).$$



(b) Find $|z|$, given that

$$z = \frac{1}{2+i} + \frac{1}{1-2i}.$$

(c) Let $*$ be a binary operation on $\mathbb{R}$, the set of real numbers, defined by

$$a * b = a^2 + 2b.$$

i. Find $(-1) * 3$ and $3 * (-1)$.

ii. Is $*$ commutative on $\mathbb{R}$? Justify your answer.

[14, 6,

2. (a) Let

$$f(x) = -2x^2 + 4x + 9.$$

i. By completing the square, express $f(x)$ in the form

$$f(x) = a(x-h)^2 + k.$$

ii. Find the turning point, and x and y intercepts.

iii. Sketch the graph of $f(x)$.

(b) Given that α and β are roots of the quadratic equation

$$2x^2 + 6x - 3 = 0,$$

i. find the value of $\alpha + \beta$ and of $\alpha\beta$.

ii. find the value of $2\alpha^3 + 2\beta^3$.

(c) Let $p(x) = 6x^3 - 25x^2 + 23x - 6$.

i. Find the remainder when $p(x)$ is divided by $(2x+1)$.

ii. Given that $x - \frac{1}{2}$ is a factor of $p(x)$, factorize $p(x)$ completely.

iii. hence, or otherwise, solve the eqaution $p(x) = 0$.

[8, 8, 9]

3. (a) Evaluate each of the following, leaving your answer in the form

$$\frac{1}{4}\left(\sqrt{a} + \sqrt{b}\right).$$

i. $\sin(105°)$.

ii. $\cos\left(\frac{\pi}{12}\right)$.

(b) Solve the following equations; for $0° \leq x \leq 360°$.

i.

$$\sqrt{2}\sin x \cos x - \cos x = 0.$$

ii.

$$2\sin^2 x + 5\cos x - 4 = 0.$$

(c) Prove the following identities:

i.

$$\frac{\cos x}{1 - \tan x} + \frac{\sin x}{1 - \cot x} \equiv \sin x + \cos x.$$

ii.

$$\frac{\cos x \csc x}{\tan x + \cot x} \equiv \cos^2 x.$$

[8, 9, 8]

4. (a) Solve each of the following equations;

i.

$$9^{2x+3} = \left(\frac{1}{27}\right)^{3x+1}.$$

ii.

$$\log_5 6 + \log_5 2x^2 = \log_5 48.$$

(b) On the same set of axes, sketch the graphs of

$$f(x) = 1 - \left(\frac{1}{4}\right)^x$$

and

$$g(x) = \log_{\frac{1}{4}}\left(\frac{1}{x}\right) + 1.$$

(c) Given that

$$f(x) = 3\sin 2(x + 30°) \quad \text{for } 0° \leq x \leq 360°,$$

i. Find the period, amplitude and phase shift.

ii. Hence, or otherwise, sketch **two revolutions (cycles)** of the graph of $f(x) = 3\sin 2(x + 30°)$ for $x \geq -30°$. Clearly, label the intercepts.

[8, 8, 9]

5. (a) Differentiate the following function using the first principle. $f(x) = \frac{1}{2-\sqrt{x}}$.

(b) Differentiate each of the following:

i.
$$y = \tan^{-1}(e^{-3x}).$$

ii.
$$x\cos y + \ln\left(\frac{y}{x}\right) = 5.$$

(c) Given that
$$f(x) = x(x^2 - 4),$$

i. Find the turning point(s) and determine whether maximum or minimum.

ii. Sketch the graph of $f(x)$.

[6, 9, 10]

6. (a) i. Integrate
$$\int \frac{(x-2)(x+3)}{\sqrt{x}}\ dx.$$

ii. Show that
$$\int_0^{\frac{\pi}{2}} e^{(\cos x+1)}\sin x\ \ dx = e^2 - e.$$

(b) Given that
$$B = \begin{pmatrix} -1 & 1 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & k \end{pmatrix},$$
find the value of k if B is a singular matrix.

(c) i. Find the value of a, b, c, d and e given that
$$(2 + 3x)^4 = a + bx + cx^2 + dx^3 + ex^4.$$

ii. Sketch the graph of the circle given by
$$x^2 + y^2 - 6x - 4y = -4.$$

[9, 4, 12]

The University of Zambia

Institute of Distance Education

Department of Mathematics & Statistics

2019/20 Academic Year Examinations

MAT2100 – Analytic Geometry and Calculus

INSTRUCTIONS: (1) There are 7 questions in this paper and all questions carry equal marks. **Answer any FIVE (5).**
(2) Show necessary working to avoid loss of marks

TIME ALLOWED: **Three (3)** hours.

DATE: **18th August, 2020 AM**

---

1. (a) (i) By translation of axes, express the equation of the conic section

$$9x^2 - 36x + 4y^2 + 24y + 36 = 0$$

in standard form and hence, identify the conic. *(6)*

(ii) Sketch the graph of the conic in (i) indicating its line(s) of symmetry, vertex or vertices, and focus (or foci). *(4)*

(b) The equation of the conic section is given by

$$xy = 2.$$

(i) Use a suitable rotation of axes to transform the equation into its standard form and hence, identify the conic. *(6)*

(ii) For this conic in (b) find its focus (or foci). *(4)*

(20 marks)

2. (a) (a) Evaluate each of the following limits:

(i) $\lim_{x\to 4} \frac{\sqrt{x}-2}{x^2-16}$ *(4)*

(ii) $\lim_{x\to 0} (\tan x)^x$. *(5)*

(b) (i) State the Rolle's theorem (without proof). *(2)*

(ii) Given the function $f(x) = x^2 - 3x + 2$, find the interval (a, b) and $c \in (a, b)$ for which the conclusion of the Rolle's theorem is satisfied. *(5)*

(iii) Find the first four terms of Maclaurin's series expansion of $\ln(1 + x)$. *(4)*

(20 marks)

3. (a) Evaluate each of the following integrals:

(i) $\int \frac{1}{(4-x^2)^{3/2}} dx$ *(4)*

(ii) $\int \frac{1}{x^2+2x+10} dx$ *(4)*

(iii) $\int \frac{x^2}{x^2-1} dx.$ *(4)*

(b) (i) Find the integral

$$\int xe^{3x} dx.$$ *(4)*

(ii) Find the volume generated by revolving the first quadrant area bounded by the parabola $y^2 = 8x$ and the line $x = 2$ about the $x-$axis. *(4)*

(20 marks)

4. (a) (i) Find the distance from the point $P(1,,1,7)$ to the plane through the points $A(3,-1,6)$, $B(1,5,5)$, $C(4,-6,4)$. *(5)*

(ii) Find the equation of the plane through the point (5,-2,3) which is parallel to the plane $7x + 4y - 3z = 12$. *(5)*

(b) (i) Find the unit normal vector $\boldsymbol{N}$, for the plane curve

$\bar{R}(t) = (\cos 2t)\boldsymbol{i} + (2 \sin t)\boldsymbol{j}$ at $t = \frac{\pi}{4}$. *(5)*

(ii) The equation of a space curve is given by

$$\bar{R}(\mathrm{t}) = \sqrt{2}(\mathrm{t})\boldsymbol{i} + e^t\boldsymbol{j} + e^{-t}\boldsymbol{k}.$$

Find the vector equation for the line tangent to the curve at the point $P(0,1,1)$ i.e. at $t = 0$. *(5)*

(20 marks)

5. (a) Given that $z = 3xe^{xy}$, find

(i) $\frac{\partial^2 z}{\partial x^2}$ (ii) $\frac{\partial^2 z}{\partial y \partial x}$ (iii)) $\frac{\partial^2 z}{\partial y^2}$. *(6)*

(b) (i) Use the chain rule for partial derivatives to find $\frac{\partial w}{\partial u}$ for

$w = x^2 + yz, x = uv, y = u - v, z = 2u^2v,$

when $u = -1$ and $v = 2$. *(7)*

(ii) Find the maximum or minimum or saddle point for the surface defined by

$z = 5xy - 7x^2 + 3x - 6y + 2.$ *(7)*

(20 marks)

6. (a) Find the general solution of each of the following differential equations:

(i) $\frac{dy}{dx} = x - y + 1.$ *(5)*

(ii) $\frac{dy}{dx} + \frac{1}{x}y = x.$ *(5)*

(b) (i) Find the particular solution of the differential equation

$$2y\frac{dy}{dx} = x + 3,$$

when $y = -1$ at $x = 0$. *(5)*

(ii) Verify that each of the given equations is exact and find its general solution:

$$(x^2 + y^2)dx + 2xydy = 0.$$ *(5)*

(20 marks)

7. (a) Find the general equation of the differential equation

(i) $\frac{d^2y}{dx^2} = \sin x - 3x^2$ *(5)*

(ii) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = xe^x$ *(8)*

(b) Find the solution of the differential equation satisfying the given conditions:

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0,\ y(0) = 1 \text{ and } y'(0) = -1.$$ *(7)*

(20 marks)

END OF EXAMINATION

THE UNIVERSITY OFZAMBIA

UNIVERSITY EXAMINATIONS

MAT 2602 – INTRODUCTION TO STATISTICS

AUGUST EXAMINATIONS - 2020

INSTRUCTIONS:

1. Answer any **FIVE (5)** questions.
2. All questions carry equal marks.
3. Show all the necessary work to earn full marks.
4. Write down the questions attempted on the front page of the main booklet.
5. Use of calculators and mathematical tables is allowed.

TIME ALLOWED: Three **(3)** hours.

---

1 [a] A random sample of size 2 will be selected, with replacement, from the set of numbers {1, 3, 5}.

[i] List all possible samples and calculate $\bar{x}$ for each.

[ii] Determine the sampling distribution of $\overline{X}$

[iii] Calculate the $E[\overline{X}]$ and $Var[\overline{X}]$

[b] A random sample of size 100 is taken from a population having a mean of 20 and standard deviation of 5. The shape of the distribution is unknown.

[i] What can you say about the sampling distribution of the sample mean $\overline{X}$.

[ii] Evaluate $P(20.75 < \overline{X} < 32 \cdot 25)$

2 Measurements of the left-hand and right-hand gripping strengths of 10 left-handed writers are recorded:

	1	2	3	4	5	6	7	8	9	10
Left hand	140	90	125	130	95	121	85	97	131	110
Right hand	138	87	110	132	96	120	86	90	129	100

[i] Do the data provide evidence that people who write with the left-hand have a greater gripping strength in the left hand than they do with the right hand?

[ii] Construct a 90% confidence interval for the mean difference.

3. **[a] [i]** If X and Y are independent variables,

Prove that $Var[X+Y] = Var[X] + Var[Y]$

[ii] Let $X_1, X_2, X_3 ....., X_n$ be mutually independent and identically distributed random variables with mean μ .

Show that $\sum_{i}^{n}(X_i - \overline{X})^2 = \sum_{i}^{n}(X_i - \mu)^2 - n(\overline{X} - \mu)^2$

[b] A company operates four machines three-shifts each day. From production records, the following data on the number of breakdowns are collected.

	MACHINES			
SHIFT	A	B	C	D
1	41	20	12	16
2	31	11	9	14
3	15	17	16	10

The company wishes to know whether machine breakdown is associated with the shift.

[i] State the null and alternative hypothesis

[ii] State the statistic.

[iii] At 0.05 level of significance, test whether breakdowns are independent of shifts.

4 As part of an investigation of employee turnover, an industry-wide survey of people in sales management positions gave the number of years of experience in sales-related positions. These are data are summarized in the following table:

	Males	Females
Sample size	80	70
Mean years	21.7	18.5
Standard Deviation of years	9.3	4.8

[i] Construct a 95% confidence interval for the difference of means $\mu_1 - \mu_2$

[ii] Do the data on experience in sales provide strong evidence that the mean years of sales experience for males for males is different from that of females. [Test with means $\alpha = 0.02$]

5 [a] Given the following ANOVA table:

Source	Sum of squares	Degree of freedom(d.f.)
Treatment	24	5
Error	57	35
Total	81	40

Carry out the F test for equality of means taking to $\alpha = 0.05$

[b] You are given the following summary statistics of data:

$n_1 = 15 \qquad \bar{x} = 72 \qquad \sum(x_i - \bar{x})^2 = 28$

$n_2 = 12 \qquad \bar{y} = 72 \qquad \sum(y_i - \bar{y})^2 = 22$

[i] Obtain $s^2{}_{pooled}$

[ii] Test $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 > \mu_2$ with $\alpha = .05$
Determine the P – value and comment on its size.

[iii] Construct a 95% confidence interval for $\mu_1 - \mu_2$.

6 In a study to determine the influence of training on the time required to do a complete assembly job, 15 new employees were given amounts of training ranging from 3 to 12 hours. After training, their times to complete the job were recorded. Let *x* denote the duration of training (in hours), and ley *y* denote the time to do the job (in minutes). The following summary statistics were recorded:

$\bar{x} = 7.2 \quad S_{xx} = 33.6 \quad S_{xy} = -57.2 \qquad \bar{y} = 45.6 \quad S_{yy} = 160.2$

[i] Calculate the least squares estimates $\hat{\beta}_0$ and $\hat{\beta}_1$.

[ii] Determine the equation of the best fitting straight line for these data.

[iii] Calculate the residual sum of squares SSE.

[iv] Do the data substantiate the claim that the job time decreases with more hours of training? [Determine by testing $H_0 = \hat{\beta}_0$]

[v] Estimate the mean job for 9 hours of training and construct a 95% confidence interval.

[vi] Predict the time to do a job for $y^* = 35$ hours and comment on your result

---

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
School of Natural Sciences
Department of Physics
2020 Academic Year
Final Examinations

PHY3621 – ELECTROMAGNETIC THEORY

Duration: 3 hours Maximum Marks: 100

Instructions

- This examination paper contains 7 questions. Attempt any 5 questions.
- Each question carries 20 marks. Marks allocated for each question are indicated in brackets [ ].
- Show all your working clearly. Omission of essential work will result in loss of marks.
- Follow carefully instructions written on the answer booklets.

CONSTANTS THAT MAY BE USEFUL

$e = 1.602 \times 10^{-19}$ C
$m_p = 1.67 \times 10^{-27}$ kg
$m_e = 9.11 \times 10^{-31}$ kg
$\varepsilon_o = 8.85 \times 10^{-12}$ $C^2N^{-1}m^{-2}$
$h = 6.63 \times 10^{-34}$ Js
$\mu_o = 4\pi \times 10^{-7}$ Ns^2C^{-2}
$c \cong 3.0 \times 10^8$ m/s

1. (a) There are currently just four known basic forces three of which are: strong, weak and gravitational forces. Name the remaining basic force. [2]

(b) Three electric fields are given as follows: $\vec{E}_1$ is 10 N/C and points at 30.0° above the positive x axis, $\vec{E}_2$ is 30 N/C and points at 10.0° below the positive x axis, and $\vec{E}_3$ is 10 N/C and points at 80.0° below the negative x axis. Determine the resultant in terms of

i. components [4]

ii. unit-vector notation [2]

iii. magnitude [2]

iv. direction [2]

(c) Given that $\mathbf{A} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$ and $\mathbf{B} = B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}} + B_z\hat{\mathbf{z}}$, use properties of unit vectors $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ to show that

i. the dot product can be written as $\mathbf{A} \cdot \mathbf{B} = A_xB_x + A_yB_y + A_zB_z$, [4]

ii. the cross product can be written as

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}.$$ [4]

---

2. (a) By using the definition of ∇, show that the divergence of the gradient of a function f results in the Laplacian of the function f:

$$\nabla \cdot (\nabla f) = \nabla^2 f.$$

[4]

(b) A rod of length l, has a uniform positive charge per unit length λ and a total charge Q. Calculate the electric field at a point P that is located along the long axis of the rod and a distance a from one end. [8]

(c) A rod of length l, located along the x axis (with one end at the origin) has a total charge Q and a uniform linear charge density λ. Find the electric potential at a point P located on the y axis a distance a from the origin. [8]

---

3. (a) By using the equality of cross derivatives:

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right),$$

show that the curl of the gradient vanishes:

$$\nabla \times (\nabla f) = 0.$$ [4]

(b) A ring of radius a carries a uniformly distributed positive total charge Q. Calculate the electric field due to the ring at a point P lying a distance x from its center along the central axis perpendicular to the plane of the ring. [8]

(c) Write down the Maxwell's equations and give the names given to each one of them. [8]

---

4. (a) By using the vector identity

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C},$$

show that the curl of the gradient is always zero:

$$\nabla \cdot (\nabla \times \mathbf{v}) = 0.$$ [4]

(b) A disk of radius R has a uniform surface charge density σ. Calculate the electric field at a point P that lies along the central perpendicular axis of the disk and a distance x from the center of the disk. [8]

(c) Find

i. an expression for the electric potential at a point P located on the perpendicular central axis of a uniformly charged ring of radius a and total charge Q. [4]

ii. an expression for the magnitude of the electric field at point P. [4]

---

5. (a) By using the definition of ∇, show that the curl of the curl of a vector function $\mathbf{v}$ is:

$$\nabla \times (\nabla \times \mathbf{v}) = \nabla(\nabla \cdot \mathbf{v}) - \nabla^2 \mathbf{v}.$$ [5]

(b) A uniform electric field $\mathbf{E}$ is directed along the x axis between parallel plates of charge separated by a distance d. A positive point charge q of mass m is released from rest at a point next to the positive plate and accelerates to a point next to the negative plate. Find the speed of the particle at the negative plate. [5]

(c) A uniformly charged disk has radius R and surface charge density σ.

i. Find the electric potential at a point P along the perpendicular central axis of the disk. [5]

ii. Find the x component of the electric field at a point P along the perpendicular central axis of the disk. [5]