LEARNERS' PROBLEM-SOLVING PROCESSES IN CALCULUS AT GRADE 12 LEVEL: A CASE STUDY OF SELECTED SECONDARY SCHOOLS IN LUSAKA DISTRICT, ZAMBIA

By

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(M. Ed-Mathematics Education)

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DECLARATION

I **Julius Zulu,** hereby solely declare that the work contained in this dissertation has been composed and written by me and that this work is as the result of my own individual effort. I further sincerely declare that this research has not been previously for any academic award at any other higher education institution, and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

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APPROVAL

This Dissertation of **Julius Zulu** has been approved as fulfilling the requirements for the award of the Degree of Master of Education in Mathematics Education (M. Ed-Mathematics) by the University of Zambia.

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ABSTRACT

Poor essential workings in Mathematics is an attribute of poor problem solving processes. The study explored learners' problem solving processes in Calculus at Grade 12 level. Calculus was introduced when the curriculum was revised in 2013 and involves basic differentiation and integration at this level. The study sought to establish Grade 12 learners' problem solving processes in Calculus, identify the challenges Grade 12 learners' encounter in solving Calculus problems, and determine strategies teachers and learners would suggest to improve problem solving-skills in Calculus. Twenty learners and two teachers at two secondary schools in the Lusaka district of Lusaka province, Zambia, participated. A qualitative study approach, which followed a descriptive case study design, was used. Data was collected using lesson observations, focus group discussions, and semi-structured interviews. Video and audio recordings were used to capture observations and interviews, respectively, in their totality. Thematic analysis was used to analyse data. The four principles of problem solving by Polya namely, understanding the problem, devising a plan, executing the plan and looking back guided the analysis. Although learners' read, re-read and wrote Calculus functions before solving, they experienced difficulties in underlining key important words; writing calculus formulas; simplifying Calculus problems; applying appropriate Calculus formulas; and had no reflective skills during and after solving Calculus problems. The challenges included failure to: substitute $f(x + h)$ and $f(x)$ when working from first principles, cite Calculus notations, cite the correct formula when working from first principles, and apply appropriate basic Mathematical concepts. Moreover, learners had challenges with understanding the language of Calculus, and teachers' teaching approaches. In view of these findings, it was recommended that teachers should use problem solving approaches which assist learners in identifying key words in the problem, devising Calculus formulas, monitoring each step during solving and looking back after solving. Applications of basic concepts in earlier grades should also be consolidated and revised on an on-going basis. It was further recommended that teachers should focus on the development of the formulas and introduce Calculus symbols in early grades while learners should practise basic concepts to enhance understanding of Calculus.

Key words: Calculus, Problem solving processes, Integration, Derivative

DEDICATION

To

Magret Nyirongo and Chrissy C. Zulu

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ABBREVIATIONS AND ACRONYMS

CDC	Curriculum Development Centre
CPD	Continuing Professional Development
DEBS	District Education Board Secretary
ECZ	Examinations Council of Zambia
FGDs	Focus Group Discussions
MESVTEE	Ministry of Education, Science, Vocational Training and Early Education
MOE	Ministry of Education
MoGE	Ministry of General Education
NCTM	National Council of Teachers of Mathematics
NRC	National Research Council
PCK	Pedagogical Content Knowledge
PSP	Problem Solving Process
STEM	Science, Technology, Engineering, and Mathematics
UNZA	University of Zambia

CHAPTER ONE

INTRODUCTION

1.1. Overview

This chapter introduces the study by firstly presenting the background, the statement of the problem, purpose of the study and objectives. It also provides the significance of the study, scope of the study (delimitations & limitations), theoretical and conceptual framework, operational definitions and concludes with the layout of the study.

1.2. Background to the study

The Zambian Education Curriculum was recently reviewed and restructured in 2013, a process that started way back in 1999 (MESVTEE, 2013c). The Secondary School Mathematics Curriculum was among the affected areas; topics were aligned according to content and process domains (strands). The content domain was defined under six thematic areas namely; Numbers and Calculation, Algebra, Geometry, Measurement and Estimation, Relations (Functions), and Statistics and Probability (MESVTEE, 2013). Following the overhaul review of the Zambian Education Curriculum, the O-level Mathematics Syllabus was also revised to incorporate Calculus and linked to the Secondary School Curriculum (MESVTEE, 2013) (See appendix 1 for detail, p.145). Other topics that were added in the O-level Mathematics Syllabus include "Computer and Calculator, Arithmetic and Geometric Progressions, Composite Functions, Inverse Functions, Graphs of Cubic Functions, Standard Deviation in Statistics, and Trigonometric Equations (ECZ, 2016, p. 2).

Calculus was newly introduced at Secondary level to ordinary learners who were not allowed to study such advanced Mathematics before. The initiative was anchored on the usefulness of Calculus in all sectors of life. It was also the Ministry of General Education (MOGE) policy to introduce Calculus at this level to create a bridge for further education, where it is required in many tertiary courses (MESVTEE, 2013). In light of this, it was for this reason that Calculus was introduced at Secondary school level in Zambia, in order to help ordinary learners develop an analytical mind at Grade 12 level.

At the time of the study, the topic ran from Grade 12 up to tertiary level. Calculus was then a new term in the Secondary School sector, although some of the concepts implicitly existed at Secondary School level under "Coordinate Geometry" in Grade 11 which covered "Coordinate and Mid-point, Length of a straight line between points, Gradient, Equation of straight lines, and Parallel and Perpendicular lines" (MESVTEE, 2013, p.18) (See appendix 2 for detail, p.146). Thus, the revised Zambian Education Curriculum Framework introduced Calculus at Grade 12 level for the first time in O-level Mathematics Syllabus. However, Calculus concepts covered at this level include "differentiating functions from first principles, using the formula for differentiation, calculating equations of tangents and normals, explaining integration, finding Indefinite integrals, evaluating simple definite integrals, and finding the area under the curve" (MESVTEE, 2013, p. 13).

Nevertheless, since the introduction of Calculus at Grade 12 in 2013, the 2013 revised O-level Mathematics Syllabus and examinations have undergone two cycles in high schools in 2016 and 2017 (ECZ, 2016; ECZ, 2017). This means that the 2016 and 2017 examinations at Grade 12 included questions from the topic Calculus (ECZ, 2016, p. 12; ECZ, p. 25 & p. 120). In the final Mathematics examination for 2016, Calculus was awarded 2 marks out of 80 in paper 1 (See appendix 3 for detail, p. 147) which is 2.5% for the whole examination and 5 marks out of 100 in paper 2 (See appendix 4 for detail, p. 148) which is 5% for the whole examination. In the final Mathematics examination for 2017, Calculus was awarded 2 marks out of 80 in paper 1 (See appendix 5 for detail, p.151) which is 2.5% for the whole examination and 3 marks out of 100 in paper 2 (See appendix 6 for detail, p.150) which is 3% for the whole examination. Hence, the Calculus theme is quite significant in the Zambian Mathematics curriculum as it is international.

Despite the integration of Calculus at Grade 12 level, learners' "performance in Calculus has not improved significantly and "some candidates scored zero" (ECZ, 2016, p. 12 &116; ECZ, 2017, p. 118) (See appendix 7 for detail, p. 151). The report by ECZ 2016 and 2017 examinations indicates that poor performance in Calculus was as the result of poor essential workings when solving Calculus problems (ECZ, 2016, p.28; ECZ, 2017, p.25).

In Zambia, poor essential workings has been a problem that has contributed to poor performance in Mathematics for quite some time. I studied the Mathematics examiners' reports of the Examination Council of Zambia for Senior Secondary Certificate from 2013 to 2017. The objective was to find areas of poor learners' performance with the ultimate goal of identifying a possible area of research which would enhance problem solving skills in Calculus. The purpose of the examiners' reports is to provide feedback to teachers, learners, policy makers and other stakeholders on learners' performance in the examination, with recommendations on how any issues identified may be addressed (ECZ, 2014; ECZ, 2015). A repeating theme in the examiners' reports was the learners' poor essential workings when solving mathematical problems in general and Calculus problems in particular. The following are some of the notable extracts from the reports:

i. *Failure to show essential workings by the candidates contributed to poor performance in Mathematics (ECZ, 2013, p.21).*

ii. *Some challenges that candidates faced were as a result of lack of mastery of concepts and skills, failure to understand mathematical terms like express, simplify, evaluate, to mention but a few and failure to show essential working (ECZ, 2014, p. 16).*

iii. *Low performance in Mathematics was as a result of candidates' poor presentation of work. The report recommended that essential working that leads to the final answer should be critically analysed by teachers when marking class work in order for them to provide appropriate feedback to the learners would help to improve performance (ECZ, 2015, p. 28).*

iv. *Some candidates omitted necessary steps in their solutions (poor essential workings (ECZ, 2016, p.28). Therefore, it is important for teachers to emphasize showing of essential working, as this will also help to teach learners to use the calculator to evaluate explicit expressions.*

v. *Lack of mastery skills and poor essential workings (ECZ, 2017, p.25). Therefore, the report even recommended that teachers should ensure that showing essential workings during instruction in the classroom is emphasized if candidates have to earn all the marks in any given assessment as well as examination (ECZ, 2017, p.25).*

In all the reports, poor essential workings was cited as a major deficiency in learners' work, in particular, when solving mathematical problems. Detail analysis also shows that even after introducing Calculus, poor essential workings still persisted. Globally, scholars have argued that poor essential workings, is as a result of poor problem solving processes. The processes include failure to understand problems, devise plans, carry out plans and look back (NCTM, 2014; Kilpatrick et al., 2001). The fact that poor essential workings has continued to cause learners underperform in Calculus since 2016, calls for serious investigation. Thus, poor essential workings in Mathematics in general and Calculus in particular motivated me to explore learners' problem solving processes in Calculus at Grade 12 level. In the next section, I now elaborate on my personal experience for researching on learners' problem solving processes in Calculus at Grade 12 level.

1.3. Personal experience

The 2013 school year was a year of great change for me. In that year, I was transferred to a new school and within months moved up a grade level from eleventh to twelfth grade. With the change of grade, came a change of Curriculum and Mathematics Syllabus. The Ministry of General Education had recently revised the Curriculum Framework and the O-level Mathematics Syllabus (MESVTEE, 2013). However, the 2013 revised O-level Mathematics Syllabus and the 2013 revised Curriculum Framework came with the integration of problem solving and Calculus. This was a new challenge for me, but at the same time, it was a good chance to expand my repertoire of mathematical problem solving techniques and teaching techniques, not just in mathematics but as well as Calculus in particular. I wanted to continue researching and working with my learners to help them become better and effective problem solvers. In order to do this, I reflected upon my teaching practices and my problem-solving skills in Calculus and how they related to influence problem-solving processes in my classroom(s).

During that time, I noticed that my learners struggled with problem-solving processes in Calculus topic towards routine and non-routine problems. Learners would complain, either that Calculus was too hard or they would give up solving Calculus related problems. Some of the reasons they gave where not understanding the problem, failure to devise a plan, failure to solve the problem and in many instances failure to evaluate the answer. I also observed that some learners were failing to show essential workings when given Calculus problems that resulted to many incorrect answers. Poor essential workings also led to loss of marks allocated to Calculus problems. Despite my continued efforts to conduct teacher instructed lessons, on how to solve problems and improve their solving skills, there were many instances where I found my learners struggling. By the end of the year, I was tired of reminding learners on issues of poor problem solving skills and lack of showing essential workings in Calculus.

Furthermore, as a teacher of Mathematics, I experienced instances during the classroom where learners understanding of a particular concept in Calculus was not well developed, and hence their problem-solving skills flawed. This meant that learners were likely to hold these misconceptions themselves, which would lead to poor problem solving skills in Calculus. Indeed, in my discussions with the learners, as a teacher, I did realize that they were having misconceptions. When it came to assessing learners' work, the misconceptions went unnoticed and the struggle for poor essential workings by the learners in Calculus continued. To some extent, I never bothered to find out how my learners solved Calculus problems and I would say I overlooked this important concept because all what I cared for, was just to teach Calculus in the classroom. Therefore, when I was aware of this; the above experiences described plus the need to carry a research by exploring learners' problem solving processes in Calculus at Grade 12 level.

It was against this background that I decided to explore learners' problem solving processes in Calculus at Grade 12 level; taking into account to understand what are these poor essential workings with specific focus on Calculus. However, in the context of this study, poor essential workings meant failure to understand the problem, devise the plans, carry out plans, and look back. Which meant that, for the learners who got the Calculus problem correct, it was not a problem because they excelled through the process, whether explicitly or implicitly, they were able to manage the process. But for those learners who had difficulties, it was a sign that they did not exhibited good essential workings. Moreover, you can only see poor essential workings by focussing on those learners who failed to solve Calculus problems or those learners who had difficulties in solving Calculus problems. Hence, the need to understand what are these poor essential workings by focusing on learners' problem solving processes in Calculus at Grade 12 level. Therefore, the section below highlights the statement of the problem in view of the concerns identified above.

1.4. Statement of the problem

Although Calculus has been introduced at Grade 12 level for the first time following the overhaul of the curriculum framework in Zambia, the reports by the ECZ for 2016 and 2017 indicates that learners' poor performance in Calculus was as the result of poor essential workings (ECZ, 2016, p.12 & 118; ECZ, 2017, p.25 & 120), and available literature show that poor essential workings in Mathematics in general and Calculus in particular is as the result of poor problem-solving processes (NCTM, 2010; Polya, 1957, Polya, 1988). These processes include failure to; understand the problem, devising plans, carrying out plans, and looking back. The fact that poor essential workings has continued to cause learners underperform in Calculus, calls for serious interventions in order for performance to improve. Therefore, it is against this reason that I decided to understand what these poor essential workings are by focusing on learners' problem solving processes in Calculus at Grade 12 level, particularly to find out; if learners understand Calculus problems before solving? If learners devise Calculus plans before solving? How learners execute Calculus plans? If learners evaluate Calculus problems after solving? What challenges do learners encounter during problem solving processes? What problem solving strategies can improve learners' problem-solving skills in Calculus? The need for answers to these questions created a gap which necessitated this empirically-informed investigation.

1.5. Purpose of the Study

The study sought to explore learners' problem-solving processes in Calculus at Grade 12 level.

1.6. Specific objectives of the study

This study was based on the following objectives:

1. To establish Grade 12 learners' problem solving processes in Calculus.
2. To identify the challenges Grade 12 learners' encounter in solving Calculus problems.
3. To determine the strategies teachers and learners would suggest to improve problem-solving skills in Calculus.

1.7. Research questions

This study was guided by the following research questions:

1. What are Grade 12 learners' problem solving processes in Calculus?
2. What challenges do Grade 12 learners encounter in solving Calculus problems?
3. What strategies would teachers and learners suggest to improve problem solving skills in Calculus?

1.8. Significance of the Study

Successful undertaking of the study was envisaged to bring to the fore an understanding of learners' problem solving processes in Calculus at Grade 12 level. This study could contribute to the body of knowledge in Mathematics Education in four ways: Firstly, teachers might be aware of learners' problem solving processes in solving Calculus problems. Secondly, determination of the challenges was also meant to inform teachers about the challenges learners encounter, and problem solving strategies that can enhance performance in Calculus. Thirdly, the study could also inform the Ministry of General Education on the strategies that can enhance performance in Calculus. Such information could be used in the planning of continuing professional development (CPD) programs for teachers of Mathematics. Fourthly, findings could contribute significantly to research in Mathematics Education on problem solving processes in Calculus at Grade 12 level, locally and globally and further contribute to promotion of STEM Education.

1.9. Delimitation of the study

Delimitations are used to indicate how the study is narrowed in scope (Creswell, 1994). This study was restricted to two senior secondary school of Lusaka District of Central Zambia. I used two schools in order to have an in-depth understanding of learners' problem solving skills in Calculus. In addition, this was restricted to the Grade twelves only. According to MESVTEE (2013), Calculus is only taken at Grade 12 level.

1.10. Limitations of the Study

The section highlights factors that could have affected the outcomes of the study to some extent. Limitations are those conditions which are beyond the control of the researcher and may also place restrictions on the conclusions of the study (Best and Kahn, 2009). Firstly, being a teacher educator, my visitation initially caused anxiety and panic in the teachers, but I assured them that the study was part of my academic progression and not for monitoring purposes.

Calculus being among the last topics in O-level Mathematics Syllabus, was likely to be taught in the third term. In fact, Calculus is schemed in term three at national level in secondary schools in Lusaka District. Considering that this study was done in the first term and that some schools might not have planned to teach the topic, I sought permission from school authorities to arrange for lessons. To some extent this could have affected teachers' classroom practices. Above all, focus on learners was foregrounded and focus on teachers was backgrounded, therefore foregrounding teachers would have brought about different effects to the way I designed my study. In the next section, I now discuss the theoretical and conceptual framework that guided this study.

1.11. Theoretical framework

My study was informed by Gestalt theory of problem solving (Wertheimer, 1959). Gestalt theory of problem solving holds that problem solving occurs within a flash of insight. By insight, Wertheimer (1959) the guru of this theory proposed to mean *getting a problem,* *understanding the problem, devising a plan, solve the problem*, and after getting a solution, *looking back* to see if the answer one gave, seems reasonable.

Gestalt theory also provides a phenomenological description of problem-solving as a problematic situation in which the problem-solvers desires a solution to a problem but has no immediate solution to the problem (Wertheimer, 2006; Mayer, 1995). In view of this, Gestalts identified two types of thinking that take place during problem-solving namely productive thinking and reproductive thinking. According to Gestalt theory, productive thinking involves deep understanding of the phenomenal structure of a problem in order to create the solution. In view of productive thinking, Gestalt theory further holds that problem solving emphasize the connection between reasoning and various cognitive abilities such as *intelligence, intellect*, *attention* and *working memory* (Wertheimer, 1959). From the cognitive point of view, Wertheimer (1959) submitted that working memory encompasses meta-level processes such as *planning, monitoring* and *evaluating* during problem solving.

On the other hand, the Gestalt theory holds that reproductive thinking involves the mechanical application of previously learned concepts and experiences in order to solve new problem situations. Reproductive thinking leads to good performance on retention problems but poor on transfer problems. The guru of this theory argued that when dealing with problem solving, one thing that is important is what prior knowledge problem solvers have and how efficiently they access it (Wertheimer, 1959). He also contended that successful problem involves linking new knowledge to what is already known. These links can take different forms, such as adding to, modifying, or reorganizing knowledge or skills (Wertheimer, 1959). Due to the failure of Gestalt theory to explain on the indicators that can be used to divulge if learners have *understood Calculus problems, devised Calculus plans, executed Calculus plans*, and *evaluated Calculus plans*, Polya's (1959) indicators of problem solving discussed in chapter on sub-section 2.5 helped to achieve this to bridge the gap.

In my study of exploring Grade 12 learners' problems solving processes, I found Gestalt theory of problem solving to be the most appropriate. The theory succinctly spells out how problems in Mathematics are solved; *getting a problem, understanding the problem*, *devising a plan, solve the problem,* and after getting a solution, *looking back to see if the answer one gave, seems reasonable.* Therefore, Gestalt theory helped me establish how Grade 12 learners *understand Calculus problems*, *devise Calculus plans, execute Calculus plans,* and *how* *learners evaluate Calculus answers* to see if the answer one gave, *seems reasonable* or *not* in a natural setting. This was adhered to by observing Calculus lessons using video recordings and taking vignettes of learners' problem solving processes for the qualitative phase. Moreover, I found the notion of *productive thinking* and *reproductive thinking* more appropriate for the process of understanding mathematical problems and the application of previously learned concepts and experiences in order to solve new Calculus problems. This helped to establish if Grade 12 learners apply or link previous learnt mathematical knowledge to new learnt Calculus concepts. It is for these reasons that this Gestalt theory guided my study by exploring learners' problem solving processes in Calculus at Grade 12 level. Figure 1.1. below summarises Gestalt theory of problem solving:

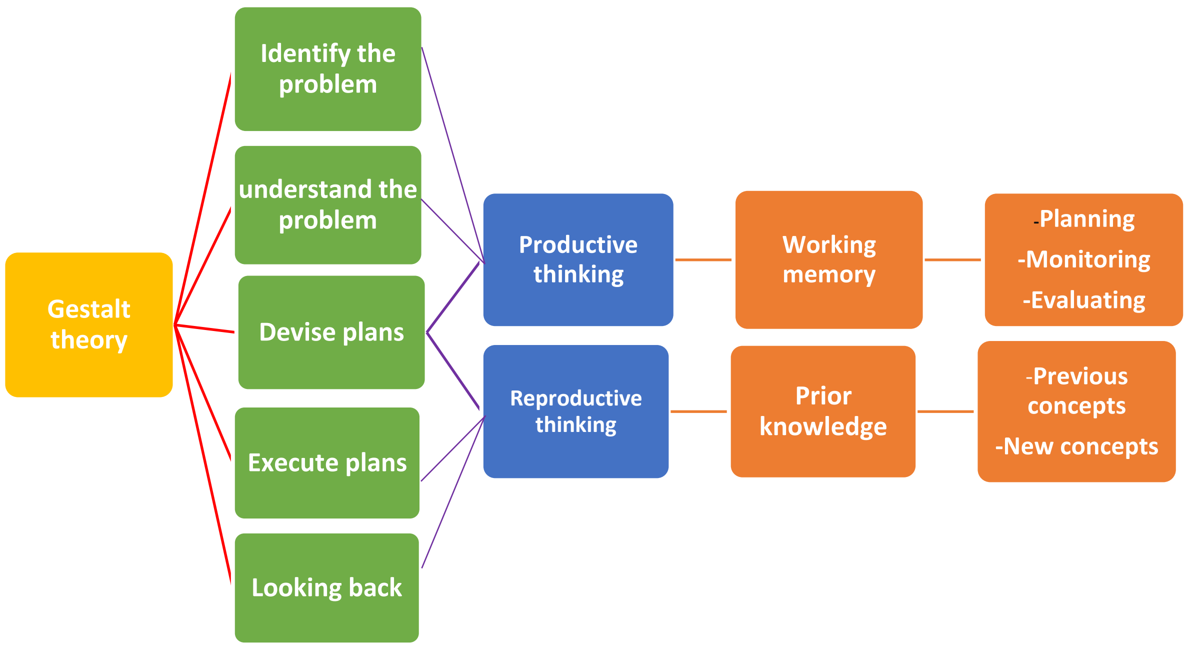

Figure 1.1: Summary of Gestalt theory of problem solving

1.12. Conceptual framework

The conceptual framework below on Figure 1.2. gives the understanding of the main assumptions on learners' problem solving processes in Calculus at Grade 12 level. To come up with the above conceptual framework, literature was incorporated into the conceptual framework. Literature review explained the theoretical framework broadly in terms of problem solving processes required in solving mathematical problems in general and Calculus problems in particular. The theoretical framework informed the conceptual framework mainly by enhancing problem solving processes of the learners.

Problem-solving process begins when a learner engages in thoughts by understanding a Calculus problem with indicators read, re-read, identify key words, and draw graphs. When a learner has understood a Calculus problem, he/she then move into the devising stage plan. Devising a Calculus plan is the second stage in the problem-solving process. In order to solve the Calculus problem, the learner needs to create external representations of their models or a plan based on devising formulas, functions, simplifying, and rough workings.

The third stage, which is carry out the plan during problem solving process, is crucially important if a learner expects to find the correct Calculus solution. At this stage, the learner needs to solve the Calculus problem by monitoring, applying appropriate formulas, and keeping and saving results. After carrying out a Calculus plan, a learner then arrives at the fourth stage which is evaluating or looking back after the learner has solved the Calculus problem. Look back or evaluating the Calculus solution is not the final answer, but rather the outcome from carrying out a set of procedures on a mathematical representation. The result obtained is important yet it needs to be interpreted within the problem's context.

The conceptual framework emphasizes the dynamic and cyclical nature of the problem-solving activity. This means that this framework starts at understanding the problem and proceeds clockwise. The solid black arrows represent "backtracking" between each step, and the square shapes behind each stage of problem-solving processes represents the experiences with solving Calculus problems that are embedded throughout the whole problem solving process. In a nutshell, the solid black arrows however represent that when the problem solver fails to devise a plan, he/she can go back to the first stage which is understanding the problem and vice versa.

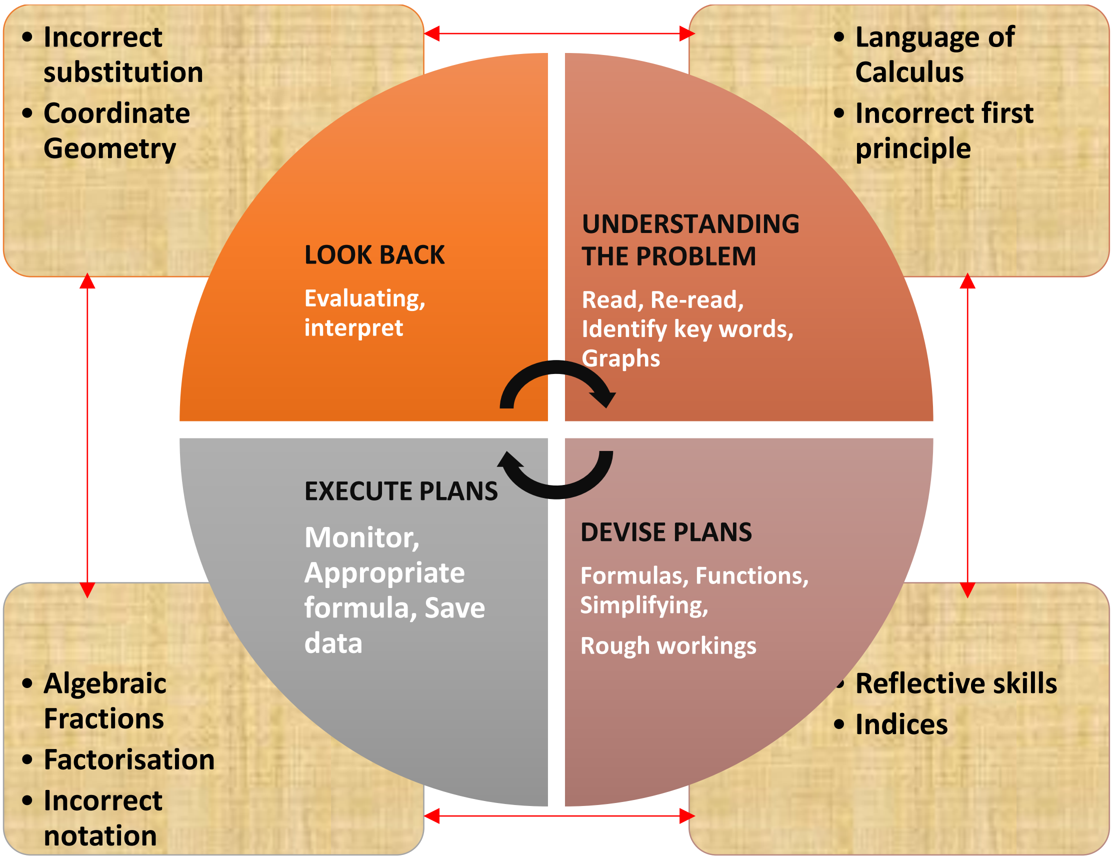

Figure 1.2.: Conceptual Framework of the Study

Having provided the theoretical and conceptual framework, I now present the operational definitions of terms used in this study.

1.13. Operational Definitions of terms

"A definition is a concise statement of the basic properties of an object or concept which unambiguously identify that object or concept" (Hilton, 1986, p. 48). In this study concepts were defined operationally; according to the contexts in which they were used.

Calculus: Calculus refers to a topic in Mathematics that deals with limits, functions, derivatives, integrals, and infinite series (NRC, 1989). In this study, the term Calculus referred to a "topic in Mathematics at Grade 12 that deals with derivatives and integrals".

Problem: A mathematical problem is a task (a) which the student is interested and engaged and for which he wishes to obtain the resolution, and (b) for which the student does not have a readily accessible mathematical means by which to achieve that resolution" (Schoenfeld, 1993,

p. 71). In this study, the term problem solving referred to when learners encounter a Calculus problem in the classroom, it is a problem to them because they have no immediate solution but when they go through the process of how problems are solved the goal is realized.

Problem Solving: Problem solving means engaging in a task for which the solution method is not known in advance (Schoenfeld, 1988, p. 52). In the context of this study, the term problem solving means engaging in a Calculus task for which the Calculus solution is not known in advance but when engaged in solving the Calculus solution is realized.

Problem solving processes: "Problem-solving processes are a process of understanding the problem, devise a plan, carry out the plan and evaluate the solution to see if the answer found is reasonable" (Polya, 1945, p. 222). In this study, problem solving processes means a step by step procedure by which learners put concepts, knowledge and skills by identifying the Calculus problem, understand the Calculus problem, devise Calculus plans, carry out Calculus plans, and evaluate the solution to check if the Calculus answer found is correct or not.

Problem solving strategies: Problem solving strategies means method(s) by which a problem can be solved (Biddlecomb & Carr, 2010, p. 2). In this study, problem solving strategies means a method by which Calculus problems can be solved.

Poor essential workings: Poor essential workings meant to mean failure to understand the problem, devise the plans, carry out plans, and look back by focusing on learners who had difficulties solving Calculus problems (Polya, 1957; NTCM, 2010).

1.14. Organization of the study

Chapter 1 introduces the study by giving the background. The motivation for this study is fourfold: The rationale of integrating Calculus in the Zambian O-level Mathematics Syllabus, curriculum change, why focus on Calculus, and personal experience. It also outlined some key items such as the statement of the problem, purpose of the study, objectives with their research questions, and the significance of the study, scope (delimitation and limitation), the theoretical and conceptual framework and finally the operational definitions.

Chapter 2 presents a review of literature related to the problem under investigation. Literature is presented under the following sub-headings: a brief history of problem solving, research in problem solving (2010-2017), defining of problems and problem solving, definition of problem-solving processes, Polya's problem solving model, metacognition training in Calculus, and procedural and conceptual knowledge in Calculus. The chapter also takes care of experiences with solving Calculus and strategies for problem solving in Calculus. The chapter concludes with different studies involving problem solving processes in Calculus and gaps identified.

Chapter 3 gives the methodology which includes the research design, approaches, methods, techniques, instruments and procedures for collecting and analysing data.

Chapter 4 presents an analysis of qualitative findings. Polya's (1957) problem solving processes namely understanding the problem, devising the plan, carrying out the plan and looking back guided the process, first vertically (within-case analysis) then horizontally (across-case analysis). Nevertheless, only the horizontal analysis has been reflected in this document. Lesson observation schedule was accompanied by video recordings. FGDs and interviews were used to complement the lesson observations. The chapter ends with a summary just after analysis of findings regarding constraints and measures.

Chapter 5 provides the discussion of the findings presented in chapter four in the light of the research objectives. The findings are further discussed in view of the literature reviewed and the theoretical foundations that mirrored the study.

Chapter 6 provides the conclusion and recommendations based on the findings. The conclusion summarises the study while recommendations provide more suggestions to inform policy, practice and future research in Mathematics and Calculus Education.

CHAPTER TWO

LITERATURE REVIEW

2.1. Introduction

This chapter explores and acknowledge the works of other researchers on learners' problem solving processes in Mathematics in general and Calculus in particular. This chapter begins with an overview of a brief historical perspective on problem solving. It is followed by research from 2010 to 2017 on problem solving processes in Mathematics in general and Calculus in particular. It was important to note the origin of problem solving and how it has evolved into today's world of solving problems. The origin of problem solving is later followed by the definitions of problem and problem-solving, and problem solving as a process respectively. The rationale of this is that the themes suggested provide better understanding of various concepts of what a problem is and what problem solving processes are in the context of this study. This helped me to operationalise what a problem is and what problem solving processes are.

In the second section of this chapter, literature on Polya's (1957) four-staged problem solving processes are reviewed which informs my study. Specifically, I have used the four-staged problem solving processes to connect elements of successful problem solving to practices for learning how to solve problems within Polya's (1957) indicators. The coverage of this provides the conceptual framework on problem solving processes for solving. I have ended the section by reviewing literature on procedural and conceptual knowledge of Calculus problems and metacognition training in Calculus**.** The rationale of reviewing on procedural and conceptual knowledge is that these two types of knowledge are a pillar to my study in the sense that for any problem in Mathematics to be solved in general and Calculus in particular, procedural and conceptual knowledge are always at play. Besides, reviewing literature on metacognitive training in Calculus was meant to establish if learners think, reason, conjecture, ask questions before, during, and after solving Calculus problems.

The third section of this chapter contains a description of research on experiences with solving Calculus problems. I have ended the section with inventory measures to surmount challenges to solving Calculus problems. The final section in this chapter reviews literature on related studies on problems solving and problem solving processes in Calculus. However, it should be noted beforehand, that due to the scarcity of literature around problem-solving processes in Calculus in Zambia and my study being topic-specific, my literature review makes reference to both local and international studies. These related studies on problem solving and problem solving processes helped me establish the gap knowledge and understand how these related studies informed my study.

2.2. History of Problem Solving

An emphasis in Mathematics education on problem solving has a long history. Stanic and Kilpatrick (1988) traced the history of problem solving in Mathematics Education, giving examples of problems from as far back as ancient Egypt and China. They noted that whereas *"problems have occupied a central place in the school Mathematics curriculum since antiquity, ... problem solving has not"* (p. 1). It was not until the second half of the 20th century that problem solving came to the forefront of research in Mathematics Education.

In the 1980s in particular, problem solving had the attention of many of those seeking reform in Mathematics Education. The recommendation of the NCTM (1980) is that *"problem solving must be the focus of school Mathematics in the 1980s"* (p. 1) began a decade of research into various aspects of problem solving, from how experts solve problems to effective ways to teach problem solving.

The importance of problem solving in doing, learning, and teaching Mathematics was recognized well before the NCTM and others made it a primary focus in the 1980s. Certainly humankind has always faced problems, mathematical and otherwise, and has devised ways to solve them. To understand a modern view of problem solving in the context of teaching and learning Mathematics, one must look to the mid 20th century. Most notably, Polya (1945, 1957, 1962) emphasized problem solving in school Mathematics and published several books on the topic including the seminal work *"How to Solve It"* and two volumes of Mathematical Discovery.

Many years before research on problem solving became popular, Polya described the nature of problems, problem solving, and the teaching of problem solving. This is the reason why today it is difficulty to discuss the history of problem solving without referring to George Polya. Many consider Polya to be the father of modern thought on problem solving. In his note book *"How to Solve It"*, he provides a wealth of information and includes a list of four problem-solving steps, which are; *Understanding the problem*, *Devise a plan*, *Carry out the plan* and *Look back at the solution.* The four-stage problem solving processes opined by George Polya improves problem solving skills if used appropriately not just in Calculus but as well as in Mathematics.

In the section that follow, I now elaborate on some of the specific areas of research on the history of problem-solving in Calculus from 2010-2017. I wish to argue that enthusiasm in the Mathematics Education community for problem solving and problem-solving instruction was particularly high in the 2010s. According to Shoenfeld (2013), this enthusiasm began to decline in the late 2000s. At the very least, the late 2010 saw a change in the kinds of questions Mathematics Education researchers were asking about problem solving. For example, in 2010 many researchers shifted their focus from the role of the teacher to the role of curriculum in helping students become better problem solvers.

2.3. Research in Problem-Solving: 2010-2017

In 2010, there was a lull in research in problem solving. Mugisha (2012) offered a few explanations:

1. Other issues have drawn attention away from problem solving"—issues such as "beliefs about the nature of Mathematics, sociocultural influences on Mathematics learning, applications of Mathematics, and assessment.
2. We think we already know all about problem solving.
3. Constructivism has replaced problem solving as the dominant 'ideology' driving Mathematics Education research.
4. Problem solving is even more complex than we once thought.

Beginning in the 2012s, many Mathematics Education researchers (Swagnagan, 2012; Schoenfeld, 2013) have focused their efforts on investigating the effectiveness of standards-based Mathematics curriculums—that is, curriculums that were written in response to the

NCTM (2010) Curriculum and Evaluation Standards for School Mathematics and, more recently, Principles and Standards for School Mathematics (NCTM, 2010). It is clear that before the 21st century, the focus of many research has been on teaching using problem solving, much of which is teacher-centred, rather than specifically on problem-solving as a process which is learner-centred. This study contributed to this existing gap by specifically looking at learners' problem-solving processes in Calculus at Grade 12 level, although at learner educational level in particular, in Zambia.

Several years in 2014 to 2017, some Mathematics Education researchers still turned their attention to research on teaching Mathematics through problem solving (Kabila, 2016; Loci & Hoc, 2014; Razal & Mansyure, 2017). Recently, teaching learners how to solve problems as a component of problem solving processes in relation to Calculus is a relatively new area of interest, so research is scarce (Flores & Garcia, 2017**).** In as much as research is scarce on problem-solving processes in Calculus, little is known on how learners solve Calculus problems and what problem solving strategies can improve learners' problem solving skills in Calculus. Hence my study and its location in the Zambian context.

In light of this, it is therefore important to note that the history of problem solving has a great profound influence on mathematical curriculum in today's classroom although a lot of research has been done on teaching through problem solving than on problem-solving as a process (problem-solving process). I elaborate that my concern in this study was on problem-solving processes particularly in Calculus. I can also argue that the history of problem solving has provided a backbone on which my study rets in two ways: (1) my study has been informed on how the history of problem solving evolved and (2) the history has led to the identification of the knowledge gap in the context of this study. In this discourse, the history on problem solving has led to intellectually undergird the next section, "definitions of problem and problem solving".

2.4. Definitions of Problems and Problem Solving

There is a huge amount of literature about problems and problem solving (Lester et al., 1994; Schoenfeld, 1993) and in this study, I take a broad perspective by distinguishing between two views of mathematical problem solving including problems and problem solving respectively.

Before going into the different perspectives on mathematical problem solving I have specified the use of the concept of problems and problem solving in this study.

I follow Schoenfeld (1993) definition of what a mathematical problem is. Schoenfeld states:

> *"For any student, a mathematical problem is a task (a) which the student is interested and engaged and for which he wishes to obtain the resolution, and (b) for which the student does not have a readily accessible mathematical means by which to achieve that resolution" (p. 71).*

However, in order to have a substantial definition on problem solving**,** I also followed Polya's (1962) definition of what problem solving is. Polya states:

> *"finding a way out of a difficulty, a way around an obstacle, attaining an aim which was not immediately attainable" (p. v).*

Elsewhere, Polya (1962) specified this broad conception of problems and problem solving in terms of Mathematics:

> *"our knowledge about any subject consists of information and know-how......What is know-how in Mathematics? The ability to solve problems—not merely routine problems but problems requiring some degree of independence, judgment, originality, creativity" (p. vii–viii).*

In line with Polya's (1962) argument, Lambdin (2003) defined a problem as a situation in which disequilibrium and perplexity occur. However, Schoenfeld (1988) distinguished between mathematical tasks that are problems and those that are exercises. He claimed that both are important but that students in many high school Mathematics classrooms engage primarily in completing exercises and rarely, if ever, are challenged to solve problems. A problem, in this sense, is a task for which the method of solution is not immediately obvious, and which is likely to take more than just a minute or two.

On the other hand, Mayer (1985) described problems and problem solving as follows:

> *"A problem occurs when you are confronted with a given situation—let's call that the given state—and you want another situation—let's call that the goal state—but*

there is no obvious way of accomplishing your goal. … Problem solving refers to the process of moving from the given state to the goal state of a problem" (pp. 123–124, italics in original).

Like Polya's description above, Mayer's definition applies to problem solving in general and is not unique to Mathematics.

Schoenfeld (1992), in his review of the literature on problem solving, noted the broad range of definitions used in discussions of mathematical problem solving. The term problem solving *"has been used with multiple meanings that range from 'working rote exercises' to 'doing mathematics as a professional"* (p. 334).

The NCTM (2000) offered a definition of problem solving similar to those above but applied it specifically to Mathematics. In the Standards, the NCTM defined problem solving using different phrasing at different points, but the following is representative:

"Problem solving means engaging in a task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understandings" (p. 52).

The following summary of the descriptions of problem solving proposed by Polya, Schoenfeld, the NCTM, and others highlights the ideas most relevant to the present study. In the context of this study, problem solving was taken to mean *"when learners encounter Calculus problems in the classroom, it is a problem for them because they have no immediate solution but when they go through the process of how problems are solved the goal will be realized".* This means that when learners are presented with the Calculus problem to solve, the goal is not known but when they engage into the process of solving the Calculus problem they eventually realize the goal.

Henceforth, it is imperative to note that something could be a problem for one person and not for another, and that once a problem is solved it is no longer a problem for the person who solved it. Kilpatrick (1985) noted:

“*to be a problem, it has to be a problem for someone”* (p. 2, emphasis added) and also, *“researchers in mathematics education have long accepted the truth that a problem for you today may not be one for me today or for you tomorrow”* (p. 3).

Therefore, in this study, whether a problem for one person might not be a problem for another, this was not the concern in this study. My concern, however, in this study was to explore learners’ problem-solving processes in Calculus at Grade 12 level. In another perspective, what this means to this current study is that whether it is a problem for someone or not, problem solving is a broad frame in my study.

However, a problem must present a challenge to the solver. Recall Polya’s (1962) comment: “Where there is no difficulty, there is no problem” (p. 117). According to Hiebert and Wearne (2003), a problem should be difficult, but not too difficult: “Allowing Mathematics to be problematic does not mean making Mathematics unnecessarily difficult, but it does mean allowing students to wrestle with what is mathematically challenging” (p. 6). While problem solving in Calculus is my broad frame, I am well informed by Polya (1945) and Tall (2006) that a mathematical problem causes a student to become uncomfortable when they cannot find a solution. In light of this, it is important to note that when individuals encounter a Calculus problem and find themselves in an uncomfortable situation and if they do not have the means to change it, they do not overcome the situation. In short learners fail to solve a Calculus problem. Therefore, it is in this instance that learners realize what they have encountered is not an exercise in Calculus, but rather a Calculus problem that requires affirmative processes to record correct answers.

The assumption I made was that focusing on learners’ problem solving processes at Grade 12 level was of much benefit to understand what are these poor essential workings by the learners? This was one way of answering the problem. This was of help because it’s really worrying to find out that because of poor essential workings some candidates “scored a zero” on Calculus problems (MESVTEE, 2016, p.12). Hence, exploring on learners’ problem solving process in Calculus at Grade 12 level was to bridge up the gap between teachers’ problem solving skills and learners’ problem solving skills on Calculus problems. Just like George Polya in 1945 said, researching on how students solve problems is key to uplift problem solving challenges in the education sector, hence for the study specifically in Zambia. In my next section, I now proceed to look at what is meant by problem-solving processes.

2.5. Definition of Problem-Solving Processes

In accounting for the process of problem solving, many writers have attempted to clarify what is meant by problem-solving processes in Mathematics (Lester, Masingila, Mau, Lambdin, dos Santos & Raymond, 1994). Similar to the definition of problem solving there are many different conceptions of what problem-solving processes entails. However, problem solving process has been described by researchers in different ways and range from Polya's (1957) rather simplistic description.

Of many definition of what is meant by problem-solving processes, I follow Polya's (1945) definition of what problem-solving processes is. Polya coined:

> *"Problem-solving processes as a process of understanding the problem, devise a plan, carry out the plan and evaluate the solution to see if the answer found is reasonable"* (p. 222).

However, in order to have a substantial definition on problem solving processes, I also follow Wertheimer's (1959) definition of what problem solving is. The guru (Wertheimer, 1959) of Gestalt theory of problem solving states:

> *"Problem-solving processes as a process of identifying the problem, understanding the problem, devise a plan, carry out the plan and look back".*

Building on Polya's (1957) and Wertheimer's (1959) ideas on problem-solving processes, NTCM (2010) also asserts that, problem-solving processes as an integral part of Mathematics learning is what is often called learning how to problem solve or learning how to solve problems. Additionally, problem solving processes is a step-by-step procedure or is a journey to find a solution, not a destination (NCTM, 2010; NCTM, 2014). Also, Kilpatrick et al., (2001) states that in general terms, problem-solving processes are means by which pupils put concepts, knowledge and skills to work during problem solving.

Consequently, it would appear that there may be many definitions of problem-solving processes which make a contribution to an investigation or to the solving of a problem, yet the summation of these problem-solving processes is itself considered to be a process. Therefore, from the four definitions of what problem solving processes are (Polya, 1957; Wertheimer, 1959; NCTM, 2010; Kilpatrick et al., 2001), problem solving processes in the context of this study particularly in Calculus means:

> *"A step-by-step procedure or process by which Grade 12 learners put concepts, knowledge and skills by identifying the Calculus problem, understanding a Calculus problem, devising a Calculus plan, carrying out a Calculus plan, and finally, evaluating the solution to check if the answer found is correct or not".*

Nevertheless, I can therefore argue that problem-solving process is merely a general guide of how to proceed in solving problems. However, it can also be said that problem solving processes are just acts of *defining a problem; determining the cause of the problem; identifying, prioritizing and selecting alternatives for a solution; and implementing a solution*. My argument is supported by Polya (1957) who argues that it is through these four processes that determines how problem(s) will be solved. Therefore, having looked at what problem solving processes really are by scholars stated above and having defined what problem solving processes really are in the context of this study, in the sub-section I now dwell much on Polya's four-stage problem-solving processes and their indicators.

2.6. Polya's Problem-Solving Model

As mentioned above in chapter 1, several models of problem-solving frameworks have been developed, many of them born out of research in Mathematics. Although there are various problem solving models, I wish to state that Polya's four-stage problem-solving model is central to this study. I have chosen Polya's problem-solving model because I find Polya's problem solving model very useful to my study as it informs this study on how problems are solved particularly not only in Calculus as a course or topic but as well as in general Mathematics courses or topics.

It is sufficient to also state here that several, if not all, problem solving models have been based on Polya's (1945/1957) four-stage description of problem solving as a process. Polya (1957) described four phases of problem solving that is *understanding the problem, devising a plan, carrying out the plan,* and *looking back* as mentioned earlier on in chapter 1 when the study was unfolding. This model in this study is used in two ways; *to examine as to whether the problem-solving process is discussed explicitly* and *to structure the analysis of the students' presentations on the chalkboard*. Thus, many people have interpreted the use of these processes as a sequential process. Also, there are numerous Mathematics textbooks that reduce problem solving to a four-step procedure but as Goldin (2004) observed, one can better interpret Polya's work by considering problem solving as a dynamic and cyclic process in which problem solvers move among the phases as they work through a problem. For example, one may find that devising a Calculus plan helps in understanding the Calculus problem or that looking back at the solution leads to better ways to solve the Calculus problem.

Even though these phases are non-sequential and a problem solver does not necessarily leave one phase before entering another, it is helpful to describe each phase individually (Kilpatrick et al., 2001). Before I discuss each stage individually, I want to set forth that in this study I am using problem-solving processes as discussed above on section 2.4 to refer to:

> *"A step-by-step procedure or process by which Grade 12 learners put concepts, knowledge and skills by identifying the Calculus problem, understanding a Calculus problem, devising a Calculus plan, carrying out a Calculus plan, and finally, evaluating the solution to check if the answer been found is correct or not".*

In that line, all along I have been saying is that my concern in this study is to find out how learners solve Calculus problems and by seeking to explore learners' problem solving processes in Calculus at Grade 12 level, my research instruments hinges on the following problem solving processes and their indicators that are discussed in the next sub-section.

2.6.1. Understand the problem

Understanding the problem requires identifying the unknown, the data, and the conditions of the problem. As Polya (1957) stated, *"It is foolish to answer a question that you do not understand"* (p. 6). He also noted that a problem solver must be motivated to solve the problem, and that teachers can motivate students by choosing good and interesting problems that are at the right difficulty level. Actively reading a problem supports learners to make sense of it. However, the depth and quality of learners decoding and subsequent understanding of the text impacts their success (Polya, 1957). I go with Wilburne (2006) who argued that at this stage students should be encouraged to come to terms with the problem by restating the mathematical problems in their own words and picking out the relevant information necessary to the solving of the problem. I have acknowledged Wilburn argument because his argument has informed this study that not all problems consist of useful information. Wilburn's (2006) assertion is also supported by Polya (1957) who argued that not all mathematical problems consist of useful information. Therefore, before Calculus problems are solved, it is very important to identify the most important information that can help problem solvers to understand the problem. This also means that certain problems consist of very unimportant information. However, this study tried to explore if learners read Calculus problems and identify important or underline or circle or highlight key and unimportant information on Calculus tasks before solving. With reference to the ECZ (2016) Performance review reporting that candidates *"scored zero on Calculus problems"* (p. 12), it was very interesting to find out if leaners' understand Calculus problems.

Besides, Polya (1957) acknowledged that reading the problem before solving helps to understand the problem and helps to identify key important terms. Correspondingly, researchers (Pape, 2004; Verschaffel et al., 2000; Staulters, 2006) affirmed that reading and understanding a text influences which schemata are activated to solve the problem; hence this initial step in the problem-solving process is important. Polya (1957) further argued that at times a problem cannot be understood for the first time. Hence the problem will require rereading it (Polya, 1957). Polya (1957) therefore recommended that students before solving a mathematical problem should be rereading the problem. In this view, this study tried to explore if learners reread Calculus problems before solving.

Therefore, it is worthwhile to state that this first step by Polya (1957) (understanding the problem) is key to knowing how to go about the problem. As simple as that sounds, this is often the most overlooked step in the problem-solving process. This may seem like an obvious step that does not need mentioning, but I might say in order for a problem-solver to find a Calculus solution, he/she must first understand what is been asked to find out. Just like Polya as stated above, *"it is foolish to answer a question that you do not understand",* I totally agree with him because who solves a problem without understanding the problem? Even in real life situations we all ought to understand our daily life problems before solving them. Thus, after learners have understood the problem, Polya (1957) stated that the next stage should be devise a plan.

2.6.2. Devise a plan

One must have at least a cursory understanding of the problem in order to come up with a plan of attack (Polya, 1957). Devising a plan is often the crux of the solution process. In fact, according to Polya (1957), *"the main achievement in the solution of a problem is to conceive the idea of a plan"* (p. 8). The plan may develop slowly or dawn on the problem solver rather suddenly. One may devise a plan by comparing the problem to a previously solved problem or by solving a simpler—or similar—problem. During planning stage: "a connection between the data and the unknown is investigated and whether the operations to be made are known, giving the students a plan. If students are not sure of a connection between the data and the unknown, they should simplify the problem or solve part of the problem" (Polya, 1957, p.9).

During planning stage, Polya (1957) highlighted the following strategies that are suitable for solving mathematical problems: drawing pictures, acting the problem out, using models, searching for pattern, making tables or charts, breaking the problem into smaller more manageable parts, formulas, rough workings, writing equations, using logical reasoning, guessing and checking, using mathematical equipment, working backwards from a solution, making lists and solving similar simpler problems. I can proclaim that this list is not quite exhaustive as it contains the suggested strategies appropriate for use with school students. Since this study aimed at exploring learners' problem-solving processes in Calculus at Grade 12 level, I therefore explored if learners'; devise Calculus formulas before solving; write Calculus functions/equations before solving; show rough workings; and simplify Calculus problems before solving. This helped teachers to have a glimpse if their learners'; devise Calculus formulas before solving; write Calculus functions/equations before solving; show rough workings; and simplify Calculus problems before solving. In addition, this however helped me to establish if learners' devise Calculus plans or not. Referring to Polya (1957), if problem solvers effectively devise a plan, then they are more likely to move to third stage of the problem-solving framework which involves carrying out a plan or creating an effective situation model.

2.6.3. Carrying out the plan

Carrying out the plan is the third phase Polya (1957) described. He compared devising a plan to carrying it out: "To devise a plan, to conceive the idea of the solution is not easy. It takes so much to succeed; formerly acquired knowledge, good mental habits, concentration upon the purpose, and one more thing: good luck. To carry out the plan is much easier; what we need is mainly patience" (p. 12). Patience is required not only for carrying out the plan, but also for making necessary adjustments to the plan or even abandoning the plan altogether and devising a new plan. In view of this, Bostic and Pape (2010) have argued that students who know more than one way to solve a problem are more likely to give the correct answer to a problem. However, Polya (1957) contended that during this stage, it is important to monitor each and every step when working on a mathematical problem. He said that when working on a mathematical problem, making mistakes is an obvious thing hence the need to check each step made. In his note book *"How to Solve It",* Polya (1957) advised that certain mathematical problems require the appropriate formula. Unfortunately, it is very rare for most problem solvers to stick to the appropriate formula required by the questions but instead end up using a different formula (Polya, 1957).

Therefore, since the principal objective of this study was to explore learners' problem solving processes in Calculus at Grade 12 level, I was also concerned with finding out if learners' monitor each and every step when solving Calculus problems and to explore if learners use the appropriate Calculus formula before solving as required by the question. Exploring if learners double check each step when solving the Calculus problem helped me to establish if learners were able to identify mistakes when solving Calculus problems. This study also aimed at finding out if learners' keep and save data/results/answers when solving Calculus problems. As good problem solvers work on a problem, they pay attention to their solution process both during and after they have solved the problem (Polya, 1945, 1957).

2.6.4. Looking Back

Looking back includes checking the answer, but it is much more than that. It involves reviewing both the problem and the solution; looking for other solution methods; considering extensions, connections, and related problems; and reflecting on one's solution process. Polya (1962) claimed that *"the best time to think about methods may be when the reader has finished solving a problem"* (p. xii). It is particularly difficult to motivate students to look back after solving a problem, but according to Polya (1957), *"a good teacher should understand and impress on his students the view that no problem whatever is completely exhausted"* (p. 15). What Polya's argument means to this study is that teachers of Calculus for instance should be emphasizing in their Calculus classes on the importance of looking back after solving the Calculus problem. I have tried to justify Polya's arguments how it informs my study because his argument is very valid and fits very well in this study.

I therefore argue that during reflection, the solution is checked in terms of the original problem. My argument resonates with Schoenfeld (2013) who argued that reflecting back on the original problem helps students to check if the problem has been answered or not. However, for this study, it was of importance to find out if learners evaluate their solution(s) during and after the process of solving Calculus problems. However, Figure 2.1 shows Polya's problem-solving model.

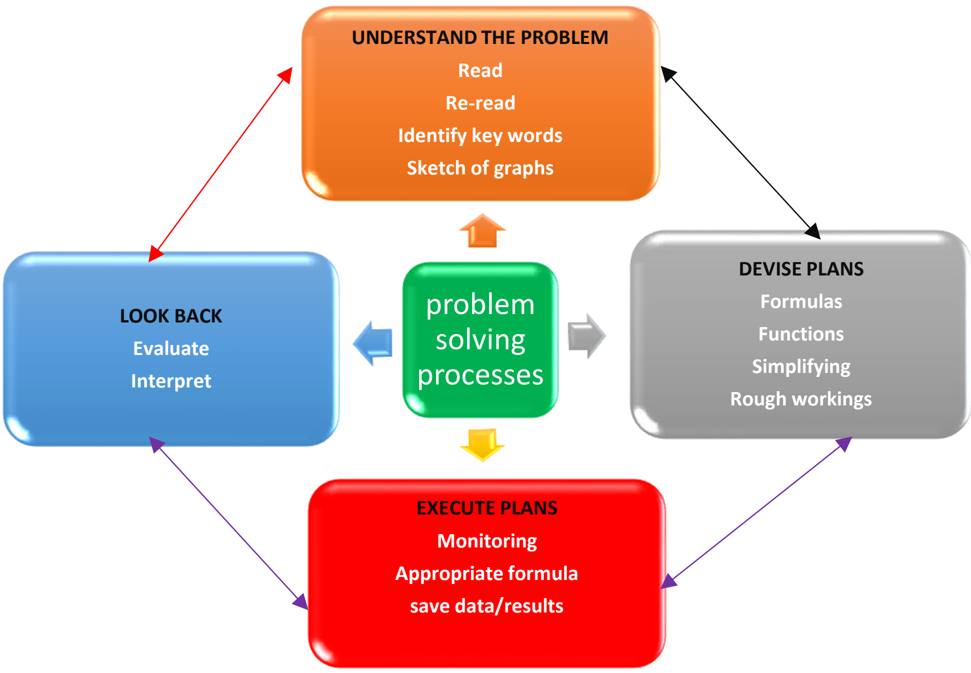

Figure 2.1: Polya's (1945) four-stage problem solving model

Figure 2.1 above describes the stages of the problem-solving processes and they play a vital role when solving problems not just on Calculus problems but also any other mathematical problems. Nonetheless, Polya's (1945) problem solving model is a problem solving-strategy that can be used to improve learner's problem solving skills (NCTM, 2014; NRC, 1989). However, in this study, the focus was not to establish if Polya's (1957) problem solving model can improve learners' problem solving skills in Calculus but this study used Polya's (1957) to explore learners' problem solving processes in Calculus at Grade 12 level. Trying to establish if Polya's (1957) problem solving framework can improve learners' problem solving skills in Calculus would have meant changing focus of the study and the design of the study. Having looked at Polya's (1957) problem solving model which informed this study, I now proceed to review literature on procedural and conceptual knowledge in solving Calculus problems.

2.7. Procedural and Conceptual knowledge in solving Calculus problems

In Mathematics Education, there are elaboration of different mathematical understandings that learners might hold. Hiebert and Lefevre (1986) proposed that the mathematical knowledge that a learner could have can either be procedural or conceptual. Researchers (Byrnes & Wasik, 1991) have all acknowledged the importance of procedural and conceptual knowledge for Mathematics learners and the roles the two knowledge sets play in the development of mathematical proficiency. Hiebert and Lefevre′s (1986) further elaborates that the framework of conceptual and procedural knowledge, characterizes the mathematical or Calculus knowledge that a learner can have. Since it is the mind that acquires and constructs knowledge (Hatano, 1996; von Glasserfeld, 1989), it is crucial to ascertain the prior knowledge the mind has already to help with the construction of new mathematical knowledge.

According to Hiebert & Lefevre (1986), conceptual knowledge is "knowledge that is rich in relationships" (p.6). Conceptual knowledge can also be regarded as knowledge that is rich in relationships and relates to the principles that refine understanding of Mathematics and also refers to the interconnections between ideas that explain and give meaning to mathematical procedures. Hiebert & Lefevre (1986) further described conceptual knowledge as that knowledge which is part of a network comprised of individual pieces of information and the relationships between these pieces of information. Perhaps it can be argued that conceptual knowledge then concerns thinking on the nature of mathematical objects. On the other hand, procedural knowledge is regarded as "rules or procedures for solving mathematical problems" (Hiebert & Lefevre, 1986, p.7). In similar view, Evan and Lappan (1994) opined that procedural knowledge includes both a familiarity with the symbolic representation of

Mathematics and knowledge of using them. They further argued that it encompasses knowledge of rules and procedures for carrying out mathematical calculations and how to solve mathematical problems.

Arguing in line with the above scholars, it can be argued that since procedural and conceptual knowledge are key to solving Calculus problems, in the context of this study, it was assumed that Grade 12 learners comprehend concepts of differentiation and integration, knew how to differentiate Calculus problems using first principles; $\frac{dy}{dx} or\ f'(x) or\ y' = \frac{\lim_{h\to0} f(x+h)-f(x)}{h}$, rules of differentiation $\frac{dy}{dx}\ or\ f'(x) or\ y' = nx^{n-1}$, and knew how to integrate Calculus problems using the integral formula; $\int ax^n dx = \frac{ax^{n+1}}{n+1} + c$. In addition, it was also assumed that Grade 12 learners cite correct Calculus formulas such as $\frac{dy}{dx} or\ f'(x) or\ y' = \frac{\lim_{h\to0} f(x+h)-f(x)}{h}$, $\frac{dy}{dx}\ or\ f'(x)\ or\ y' = nx^{n-1}$, $\int ax^n dx = \frac{ax^{n+1}}{n+1} + c$ when solving Calculus problems, and note the difference between $f(x)$ and $f'(x)$ or y and y'. However, establishing whether Grade 12 learners lack procedural knowledge was achieved by how learners carried out Calculus problems, how learners cited Calculus formulas, and how learners noted the difference between difference between $f(x)$ and $f'(x)$ or y and y' after solving Calculus problems.

Literature across the globe has shown that procedural knowledge is built on conceptual understanding (NCTM, 2014; Schoenfeld, 1992; Kilpatrick et al., 2001). In relation to NCTM (2014), Schoenfeld (1992), and Kilpatrick et al., (2001) statement that procedural knowledge is built of conceptual knowledge, I can therefore argue that the two strands should work hand in hand. What this meant to my study is that I assumed that the Grade 12 learners should be in a position to know the procedures of finding the derivative of functions using first principles, explain the concepts of differentiation, rules of differentiation, explain integration, evaluate simple integrals, find indefinite integrals, and calculate equations of tangents and normals.

In Figure 2.2., for instance, when a Mathematics teacher teaches a Calculus concept to learners effectively, the learners understand the Calculus concept resulting in them forming appropriate concept images which adequately resemble the target concept. In such cases, learners properly align incoming mathematical knowledge with their old mathematical knowledge (through assimilation or accommodation) (Piaget, 1968) thereby forming a more comprehensive concept image. Conversely, if the teacher has weak conceptual knowledge, he/she deploys distorted knowledge to learners. This distorted knowledge may influence learners to form invalid concept images which by definition are misconceptions which influence their mathematical thinking. Similarly, if a learner has a very defective concept image, he/she may not receive incoming knowledge properly, no matter how well articulated by any medium. Rather, the learner might form a distortion/misconception of the new correct knowledge. If the learners are only taught procedures, they only have learnt half-truths because procedures are only useful in specific contexts.

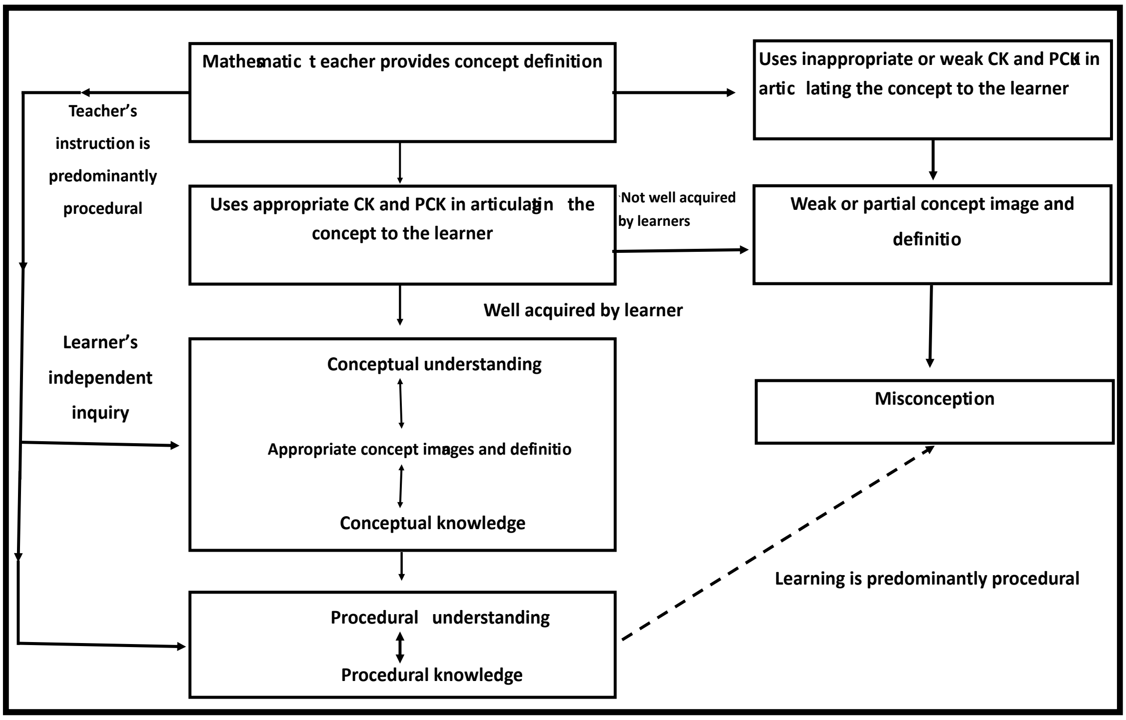

Figure 2.2.: The dissemination of knowledge from teachers to learners (adopted from Luneta, 2008)

Loosely speaking, one can say that conceptual knowledge is about thinking whereas procedural knowledge is about doing. In Calculus, procedural knowledge incorporates strategies of finding limits, sketching graphs of functions, optimizing and minimizing functions, rules for differentiation and integration, symbolism and so on. In particular, it should be mentioned that procedural knowledge does not explain why for instance the gradient function; $\frac{dy}{dx}$ vanishes at the turning points of a function. Procedural knowledge is also not concerned with the definition of the derivative, or arguments on why a function tends to the limit learners have calculated (Makonye, 2011). In the next section, I review literature on metacognitive training in Calculus.

2.8. Metacognition training in Calculus

Metacognition is loosely defined as thinking about your own thinking (Flavell, 1977). More formally metacognition refers to "higher order thinking which involves active control over the cognitive processes engaged in learning. Activities such as planning how to approach a given learning task, monitoring comprehension, and evaluating progress toward the completion of a task are metacognitive in nature" (Livingston, 1997, p. 1**).** In this study, it was presumed that leaners think about their own thinking when solving Calculus problems and immediately after solving. I therefore agree in line with Flavell (1977) that metacognition is loosely defined as thinking about your own thinking. What this meant to my study is that solving Calculus problem requires thinking about thinking. This also implies that understanding Calculus problems, devising Calculus problems, solving Calculus problems, and evaluating Calculus problems requires thinking about thinking. It is also very imperative to comprehend that both teachers and learners require metacognitive skills such as posing and asking questions, self-talking explaining, guessing/conjecturing etc. that could improve their problem-solving skills in Calculus.

However, my argument resonates with Montague (2007) who argued that students should be taught to effectively use metacognitive techniques such as self-questioning and self-monitoring while solving problems. Therefore, I can assert that students can effectively learn metacognitive skills through instruction that can come best from their teachers. I say this because it is through teachers' instructions that influence learners thinking about thinking in Calculus. Also, my thoughts once more resonate with Livingston (1997) who argued that teachers are the best instrument who could help learners improve their metacognitive skills in Calculus. Following the argument by Montague (2007) who has stated that students should be taught to effectively use metacognitive techniques such as self-questioning and self-monitoring while solving problems, my concern in this study was also to find out what metacognitive techniques learners' employ when solving Calculus problems. This helped me to establish if learners' think about thinking before solving Calculus problems.

Moreover, Schoenfeld (1988) has emphasized the value of metacognition when solving Calculus problems. According to him, there is evidence that when students get coaching in solving Calculus problems that includes attention to such things—when they are encouraged to think about issues like "What are you doing? Why are you doing it? How will it help you solve the problem?"—their problem-solving performance can improve dramatically. (p. 290).

By appreciating the great works teachers can do to improve leaners' metacognitive skills for instance in Calculus, leaners can literally on their own improve their metacognitive skills. For instance, learners can engage in metacognitive skill during problem solving when they engage in self-talk to check their understanding of the Calculus problem, acknowledge and organize existing data concerning the Calculus problem, weigh alternative choices of plans or heuristics, change their choice of plans during their working of the Calculus problem, or when they check or test their Calculus solutions for being reasonable or correct.

Additionally, Brown *et al.,* (1983, p. 107) as cited in Schoenfeld (1985), argues that metacognitive control can be characterized as follows: "These processes include *planning* activities prior to understanding a problem (predicting outcomes, scheduling strategies, various forms of vicarious trial and error, etc.), *monitoring* activities during learning (testing, revising, rescheduling one's strategies for learning), and *checking* outcomes (evaluating the outcome of any strategic actions against criteria of efficiency and effectiveness)". On the same line as Brown *et al.* (1983, p.107) as cited in Schoenfeld (1985), I can argue that I have found metacognitive control components such as planning, monitoring and checking to be very important to my study in that, since my study aimed at exploring learners' problem solving processes in Calculus at Grade 12 level, metacognitive control components helped me find out if learners plan before solving Calculus problems, if learners monitor their steps when solving Calculus problems, and if learners check their Calculus answers immediately after solving. These are among other issues this study tried to address which are synonymous to Polya's (1957) problem solving process. Having looked at metacognitive training in Calculus, in the next section, I now review literature on experiences with solving Calculus problems.

2.9. Experiences with Solving Calculus problems

Reviewed literature has shown that understanding the limit concept require the understanding of first principles (Orton, 19983a; Eisenberg, 1993). For instance, Makonye (2011) did a study on Exploring Learners' Difficulties in Solving Grade 12 Differential Calculus. His study established that only 30% of the students correctly recalled the definition of the derivative at the point $(x, f(x))$ as $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$. He also found out that 40% of the learners had some inaccuracies in recalling this definition where learners for example had put a plus instead of a minus required to denote the infinitesimal change in the mantissa, thereby committing a structural error. In the context of this study, it was presumed that Grade 12 learners cite the correct first principles $\frac{dy}{dx}$ $or\ f'(x)\ or\ y' = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ when solving differential Calculus problems. The study by Makonye (2011) was vital to the current study in terms of slight similarities in research approach which was qualitative. Besides, the study by Makonye (2011) also informed the current study on how learners find it a challenge to cite the correct first principles when solving Calculus problems. However, his study was different to the current study as it did not clearly bring out why learners find it a challenge to recall or cite the correct first principles formula when solving Calculus problems. My current study achieved this by asking the learners during FGDs. This was one of the issues this study tried to establish.

A study done by Makgakga and Makwakwa (2016) entitled "*Exploring Learners' Difficulties in Solving Grade 12 Differential Calculus"* revealed that learners encountered a number of challenges substituting between $f(x+h)$ and $f(x)$ in the first principles formula $\frac{dy}{dx}$ $or\ f'(x)\ or\ y' = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$, in order to get the result. Their study further revealed that when learners were given to work out the Calculus function; $f(x) = 2x^2 - x$ using first principles, the substitution made by the learner between the terms $f(x+h)$ and $f(x)$ was $f'(x) = \lim_{h\to 0} \frac{x^2-2(x+h)-(x^2-2x)}{h}$. This substitution was wrong substitution in which the learner did not consider squaring the bracket in the first term. Makgakga and Makwakwa (2016) further found that the second term should have been $(x+h)$ because of the negative sign between the first and second terms, and lastly the last term should have been $(2x^2 - 2)$ instead of $-2x^2 - x$.

In a similar study, Makonye (2011) researched on "*Learner Mathematical Errors in Introductory Differential Calculus Tasks: A Study of Misconceptions in the Senior School Certificate Examinations in South Africa*. The research focused on the nature of errors and misconceptions learners have on introductory differentiation as exhibited in their 2008 examination scripts. Research findings showed that students wrongly substituted between $f(x+h)\ and\ f(x)$ when they were given to find the derivative function of $y = -3x^2$. In his study, one student wrote $\frac{\lim_{h\to 0}((-x+h)-3^2}{h}$ to represent the formula $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$, that he/she had presented.

Despite having not explained why learners encountered difficulties in substituting the terms between $f(x+h)\ and\ f(x)$ when working from first principles, the study by Makgakga and Makwakwa provided the background to the current study. The current study was different because it tried to bring out reasons why learners encounter difficulties in substituting between $f(x+h)\ and\ f(x)$ when working from first principles. This however was achieved by asking the learners during FGDs. In addition, Makgakga and Makwakwa (2016) and Makonye (2011) studies did not look at strategies that can improve learners' problem solving skills in Calculus but this current study established problem solving strategies that can improve learners' problem solving skills in Calculus. Hence contributing to the knowledge gap. However, the assumption I made in the current study was that learners were able to substitute between $(x+h)\ and\ f(x)$ when working from first principles.

Besides, literature across the globe has shown that there is a difference between $dy\ and\ dx$ or $y\ and\ y'$ in the field of Mathematics. For instance, Orton (1983a) in his study entitled "*Students' understanding of differentiation"* observed that students had difficulties in dealing with differentiation symbols and could not interpret the meaning of $\frac{\delta y}{dx}\ or\ \frac{dy}{dx}$ notations. Consequently, the study by Orton (1983a) informed this study about the challenges learners encounter difficulties when solving Calculus problems for instance noting the difference between dy and dx when solving Calculus problems. However, Orton's (1983a) left a gap for the current study as he did not clearly explain why learners encounter difficulties with noting the difference between dy and dx when solving Calculus problems. This was one of the issues this study addressed. Besides, the other knowledge gap is that Orton's (1983a) did not look at strategies that can improve learners' problem solving skills in Calculus in relation to Calculus notations. This was also one of the issues this study tried to address. However, in the context of this study, the assumption was that Grade 12 learners know the difference between dy and dx know Calculus symbols such as $\frac{\delta y}{dx}\ or\ \frac{dy}{dx}$, $\lim_{x\to\infty} f(x)$ and $\int f(x)\,dx$.

In addition, scholars have argued that in order to understand Calculus concepts and procedures of tackling Calculus problems, competence with algebraic skills play an important role (Makonye, 2011; Orton, 1983a). In a similar view, Dlamini (2017) in his study entitled "*Exploring the causes of the poor performance by Grade 12 learners in Calculus-based task"* points out that in order to be good in Calculus and solve Calculus problems without difficulties, learners should also know how to apply the laws of exponents, simplify coefficients and variables or expressions, competence in algebraic skills, solve linear and quadratic equations and inequalities, factorize, and find the roots of a function. I wish to argue in line with Dlamini's (2017) statement because algebraic skills and the knowledge required in Calculus include the manipulation of algebraic expressions by adding, subtracting, multiplying and dividing expressions.

Furthermore, a study done by Makonye (2011) found that 85% of the students sample simplified $-\frac{1}{6x^3}\ to - 6x^3\ or - 6x^{-3}$; simply transferring 6 to the numerator. This was one of the most common errors shared by different learners in the context of applying the laws of indices to solve Calculus problems. In addition, students failed to detach 6 and $x^3\ in\ 6x^3$ and the first group had an arbitrary error of regarding the fraction $\frac{1}{6x^3}$ as equal to $6x^3$. He further observed that these students disregarded the fraction concept and changed the question to a form that was accessible and sensible to them thereby committing an arbitrary error. This is because mathematical arguments in Calculus are often expressed and communicated in Indices terms. This therefore meant that with no competence in Indices, epistemic access to Calculus is doubly difficult. This is why I believe Indices is such an important pre-calculus topic.

Arguably, reviewed literature across the globe has shown that lack of factorization skill affect learners' problem solving skills in Calculus. For instance, in a report by the Department of Basic Education (2012) in South Africa, learners were given $g(x) = \frac{x^2+x-2}{x-1}$ to simply to test their understanding of differentiation. The report however revealed that the question $g(x) = \frac{x^2+x-2}{x-1}$ was poorly answered because many learners did not first simplify $g(x)$ to $x + 2$ by factorising the numerator and then dividing by the denominator. Moreover, the report also revealed that many of those that did this were unable to determine that $g'(x) = 1$ (DBE, 2012). Thus, it can be argued that factorization is another mathematical concept used in some topics of Mathematics and Calculus is no exception. If a candidate has not mastered it his/her performance in Mathematics is at risk.

Despite these studies (Dhlamini, 2017; Makonye, 2011; Makonye & Luneta, 2010; DBE, 2012; Orton, 1983a) informed the current study that learners experience difficulties in applying basic mathematical concepts i.e. Algebraic skills, Indices, Fractions, and Factorization, the knowledge gap is that these studies did not establish reasons why learners experience difficulties in applying basic mathematical concepts when solving Calculus problems. This is another area this study tried to address hence contributing to the knowledge gap. In addition, the other knowledge gap is that these studies (Orton, 1983a; Dhlamini, 2017; Makonye, 2011; Makonye & Luneta, 2010; DBE, 2012; Orton, 1983a) did not look at problem solving strategies that can improve learners' problem solving skills in Calculus regarding application of basic Mathematical concepts. However, in the context of this study, it was presumed that Grade 12 learners had competence in solving Calculus problems that require basic application of algebraic skills, laws of indices, fractions, and factorization. Having discussed experiences with solving Calculus problems, I wish to highlight the strategies for problem solving in Calculus.

2.10. Strategies for Problem Solving in Calculus

"Strategies are groupings of actions, mental or physical, designed to solve a problem" (Biddlecomb & Carr, 2010, p. 2). There are many strategies that learners can use to find the correct Calculus answer. However, learners need to know these strategies and utilize them. "In learning Mathematics in general and Calculus in particular, the learners should have the opportunity to discover by themselves a way to reach the solution to the problem" (Cotic & Zuljan, 2009, p. 300). Arguing in line with Cotic and Zuljan, learners should learn how to properly use strategies for solving Calculus problems. With the background knowledge of mathematical problem solving strategies, learners are better able to solve any Calculus problem that may arise. Bridging on Biddlecomb and Carr (2010) ideas of what problem solving strategies is, in the context of this study, problem solving strategies mean a method by which Calculus problems can be solved.

"John Dewey pointed out that the best way to gain deeper understandings of a subject is to search for better methods to solve problems" (Stanic & Kilpatrick, 1988, p. 43). These better methods or strategies help students to complete the Calculus problem solving process. By focusing on problem solving strategies, learners can learn various ways of overcoming any Calculus obstacle. Hence, it can be argued that teachers instructing learners on the various strategies of problem solving can promote learning on a higher level of thinking and those strategies should foster proficient problem-solving skills in Calculus for instance. However, in the context of this study, it was of importance to establish what problem solving strategies would teachers suggest to improve learners' problem solving skills in Calculus.

"Research indicates that when teachers explicitly teach strategies, this instruction appears to have a positive impact on children's numerical knowledge" (Biddlecomb & Carr, 2010, p. 6). The keyword here is explicitly. When teaching instructional strategies to solve Calculus problems the results are positive. Therefore, it can be argued that teachers need to teach the strategies and suggest effective problem solving strategies in order for the learners to internalize them and make them become background knowledge in order to improve their problem-solving skills. This was one of the issues this study tried to establish. In this view, learners may not know the value of the new strategies until they apply them to a Calculus problem and have success in finding the Calculus solution.

Literature across the globe has shown that after teaching various strategies to the learners, teachers should be promoting mathematical discourse when presenting a Calculus problem to the class (Kilpatrick et al., 2001). Arguing in line with Kilpatrick and his colleagues, it was of importance to know what problem solving strategies teachers would suggest to improve learners' problem solving-skills in Calculus. Besides, "it is important for the teachers to challenge and encourage the students towards independent search of various paths to the solution by discussing and comparing these in class" (Cotic & Zuljan, 2009, p. 300). By having mathematical discourse, I believe learners can have more knowledge not only of how to use the strategies but also to suggest problem solving strategies that can enhance their problem-solving skills in Calculus. In fact, having in-class discussions of the different strategies can illustrates to the learners that there is not just one way to solve a Calculus problem. "This could help the learners to develop intuition and creativity, convergent and divergent thought, as well as to acquire the ability to plan and evaluate" (Cotic & Zuljan, 2009, p. 300). Moreover, learners need these skills to further their mathematical problem solving abilities in Calculus. This type of learning is important for the 21st century student to know how to think differently about a given Calculus problem. Hence the need to understand problem solving skills that can improve learners' problem solving skills in Calculus.

According to literature, there are many strategies and tools to help learners improve problem solving in Calculus (Schoenfeld, 1992). One strategy for visual learners is the use of manipulatives. "Manipulatives-physical materials to support learning such as blocks or tilesare ubiquitous in early years educational settings across cultures" (Manches, O'Malley, & Benford, 2010, p. 622). Manipulatives are helpful to learners in visualizing what they are reading in the word problem. They are able to concretely look at the problem and physically manipulate the materials into finding a solution. "The use of physical materials to support young children's education can be traced back to education pioneers such as Fröbel and Montessori" (Manches, O'Malley, & Benford, 2010, p. 623).

Other strategies that are used for problem solving are drawing pictures, making charts, working backwards, and guess and check (Rickard, 2005). Perhaps learners who are visual learners can benefit from the strategy of Drawing Pictures. However, this makes the problem more concrete and real for the student. Rickard (2005) further contented that making charts is a method that is good for organizing data to find a solution while working backwards is sometimes a good strategy when the problem presented does not offer a forward solution. Lastly, guess and check is always an excellent strategy to use even after you have already used a previously mentioned strategy. It never hurts to go back and check your work when solving Calculus problems. Therefore, these problem-solving strategies highlighted by Rickard (2005) are key to solving mathematical problems.

Mathematical problem solving strategies are taught with the instructional purpose to produce positive results in particular Calculus results. A study was conducted in a seventh-grade middle school mathematics class in Maryland on the influence of using problem solving strategies. This study found very positive results with the students' motivation towards problem solving. "Learning problem-solving strategies has positively influenced the students' attitudes towards solving more challenging problems in my classroom" (Shears, 2005, p. 9). This study showed that teaching students strategies that would aid in the process of problem solving had a positive influence on the students' motivation. Shears (2005) also indicated that problem-solving strategies would be valuable for all students to learn. This is a good study which shows how positive teaching and strategies helped the students become successful problem solvers.

These strategies are not meant to overload the learners' learning or problem solving capacity. There are meaningful skills that every learner should receive from their teacher. Quality Mathematics instruction "should equip the students with declarative and procedural knowledge and skills and allow them to gradually grow independent" as discussed on section 2.6 of this chapter (Cotic & Zuljan, 2009, p. 307). However, it can be argued that learners equipped with these skills will have more knowledge of how to independently solve Calculus problems. The following section highlights some of the related studies and gaps identified.

2.11. Related studies and gaps identified

A study within Zambia sought to analyze the perceptions of heads of department and Mathematics teachers towards the teaching and learning of Mathematics using problem solving method in a lesson study cycle. This was done by Kabila (2016) from the University of Zambia in conjunction with the Zimbabwean Open University. The aim was to find out the perceptions of heads of department and Mathematics teachers towards teaching and learning Mathematics using problem solving method in a lesson study cycle. The study was guided by Vygotsky's Zone of Proximal Development theory. The findings of the study indicated that teachers and Heads of Department were familiar with the problem-solving method. Another important finding of the study was that there were no opportunities given to the learners to share different or similar ideas even in the group work. The findings of Kabila's study is vital because it has informed this study that teachers were familiar with problem-solving method. This helped me to establish what problem solving strategies teachers would suggest that can improve learners' problem solving skills in Calculus. Moreover, Kabila's (2016) study was centered on teaching and learning Mathematics using problem-solving method. This research left a gap since it bordered on teaching and learning mathematics using problem solving. There was need for another study to specifically look at problem solving as a process that is to explore problem solving processes in Calculus at Grade 12 level that is to investigate how problems are solved involving Calculus in particular.

Mugisha (2012) did an exploratory study by investigating problem solving skills in Calculus which was the case of UNISA first year students, utilizing a retrospective study design Calculus using past examination scripts between 2006 and 2009 in South Africa. A sample of 200 scripts was selected and the performance in the four questions on the paper was analyzed based on different concepts in Calculus. Mugisha's (2012) study revealed that when an assessment using rubric and Polya's problem solving strategies was implemented, it demonstrated that the majority of students were capable of understanding and recognizing the nature of Calculus problems presented to them and students were also able to display good solution skills but many of them could not successfully implement those skills. Mugisha's study acts as a basis where this current study was grounded as I tried to explore Grade 12 learners' problem solving processes in Calculus. Mugisha's (2012) informed this study about the proposed problem solving processes in Calculus. However, unlike the study by Mugisha (2012) that used rubric and Polya's problem solving strategies to assess learners' performance, this current study was centered on Polya's problem solving model and Gestalt theory to interrogate Grade 12 learners' problem solving processes in Calculus at Grade 12 level. Trying to establish if Polya's (1957) problem solving processes can improve learners' problem solving processes in Calculus would have meant changing the focus of the study and the design of the study. Hence contributing to the knowledge gap. Besides, the study findings revealed that learners lacked the ability to evaluate Calculus answers after solving. This also left a gap for the current study to establish reasons why learners do not evaluate Calculus answers immediately after solving.

In a similarly study, Mugisha, Doungno and Mogari (2014) did an exploratory study into the assessment of students' performance in the first-year Calculus using past examination scripts between 2006 and 2009 in South Africa. The aim of study was confined to the investigation of the assessment of Calculus students at the University first year level, using the module Calculus A (code: MAT 112) students of the University of South Africa (UNISA) as a case study. This study further revealed that the use of the rubric demonstrated that the majority of students were capable of (1) understanding of a problem; (2) recognizing what is required; (3) developing a solution strategy; (4) implement the solution strategy, (5) ''hesitating''; (6)''honesty'', and (7) display ingenuity in the solution process. The study by Mugisha, Doungno and Mogari (2014) provided a background to the current study because of slight similarities in terms of the research approach (qualitative research approach). Unlike Mugisha, Doungno and Mogari (2014) study that used rubric assessment and Polya's (1957) problem solving model to assess the performance of the students in Calculus, the current study used Polya's (1957) model of problem solving to explore Grade 12 learners' problem solving processes in Calculus at Grade 12 level. Trying to establish if Polya's (1957) problem solving processes can improve learners' problem solving processes in Calculus would have meant changing the focus of the study and the design of the study. This was among the gaps this current study sought to offer some contributions though from the Zambian context.

In the case of South Africa again, Paul (2016) explored on Grade 12 learners' productive thinking processes when solving optimization problems using Calculus. The purpose of this study was to explore learners' productive thinking processes when solving optimization problems using a Calculus approach. The sample for the study consisted of 50 Grade 12 learners who were randomly selected from mathematics and science supplementary instruction Centre. The results of the study indicated that learners struggled to decode contexts of problems and to represent it symbolically, formulate equations and functions and apply Calculus methods to solve the problem and interpreting the solutions. Nevertheless, the current study considers Paul's (2016) study to be very vital because it informs the current study that learners' struggle to decode contexts of Calculus problems thereby failing to represent Calculus problems symbolically, formulate Calculus equation, apply methods to solve the Calculus problem, and interpret the Calculus solutions. Paul's (2016) study and the current study have similarities in some variable like Calculus as a topic, learners at senior secondary (Grade 12 learners) and both studies have titles that are topic specific which is Calculus. However, the study by Paul (2016) study however did not look at factors that affected learners' struggle to decode contexts of Calculus thereby failing to represent Calculus problems symbolically, formulate Calculus equations, apply methods to solve Calculus problems, and interpret the Calculus solutions. This was also the focus of this study intended to explore. The current study established problem solving strategies that can improve Grade 12 learners' problem solving skills in Calculus which Paul's (2014) did not hence contributing to the knowledge gap.

In another study, Tarmizi (2010) undertook a study on visualizing students' difficulties in learning Calculus in Malaysia. The purpose of this study was to investigate student's performance in solving Calculus problems and further analyzed student's difficulties in solving the problems. His study found that students needed to monitor their steps in problem solving or deriving problem solution. In addition, the study by Tarmizi (2010) revealed that much attention should be directed to fostering student's ability to plan for problem solution. Tarmizi's (2010) study is similar to the current study in the sense that both studies have interest in analyzing student's difficulties in solving Calculus problems. Although Tarmizi study looked at student's difficulties in solving Calculus problems, his study did not clearly explain the factors that causes students difficulties in solving Calculus problems. The current study tried to identify the challenges learners encounter when solving Calculus problems particularly in the Zambian context. This however entails that Tarmizi's (2010) study plays a significance role to the current study on which my study rest. Additionally, this current study unlike the study by Tarmizi's did not look at problem solving strategies that can improve learners' problem solving skills in Calculus hence contributing to the knowledge gap.

Makgakga and Makwakwa (2016) undertook a study by exploring learners' difficulties in solving Grade 12 differential Calculus in the republic of South Africa. A convenient sample of thirty-seven (n=37) learners of a secondary school in the Polokwane District of the Limpopo province, South Africa, participated in this study. An interpretive paradigm was used for this study, which followed an explanatory sequential mixed-methods design. The results revealed that 52% of the learners performed better in finding the derivative of functions by using the rules of differentiation, compared with 23% of the learners who performed better on the questions by using first principles. The interviews showed that learners experienced difficulties when substituting $(x + h)$ in the formula if the function comprised more than one term, and also when dividing by in the formula. Using the rules of differentiation, learners experienced difficulties when given functions involving fractions. Their study is similar to this current study as it employed Grade 12 learners' which was the case in this current study. Despite their study having not looked at and clearly not explaining on what factors caused leaners to experience difficulties when substituting $(x + h)$ in the formula by using the first principle, this current tried to achieve this in problem solving context. In addition, their study did not look at strategies that can improve learners' problem solving skills in Calculus which this current study looked at.

More recently, Rizal and Mansyur (2017) researched on Calculus Problem Solving Behaviour of Mathematic Education Students in Indonesia. The purpose of this study was to obtain a description of the problem-solving behaviour of Mathematics Education students. The study showed that the behavioural of problem-solving from Mathematics Education students were as follows: (1) In understanding the problems by solving Calculus problems together, they tend to put their first attention to an image, reading the texts slowly and repeatedly, then in a whole and more focused on the sentence with equations, numbers or symbols improved their problem-solving skills, (2) In making problem-solving plans, they tend to use a similar formula to the previous problems they met, and (3) Resolving the problems as planned. (4) Examining the truth of problems solving by re-reading and paying attention to the algorithms and logic from the previously completed work.

Their study is significant to this study as it informed this current study that when understanding Calculus problems, students needs to put their attention to an image, read the Calculus texts slowly and repeatedly. Although their study informed this current study that when understanding Calculus problems, students needs to put their attention to an image, read the

Calculus texts slowly and repeatedly, their study did not look at how students interpreted Calculus solutions after solving which the current study. The current study also aimed at finding out how often learners monitor their solving processes which their study did not look at. These are two key issues this current study sought to address. Having looked related studies and gaps identified, below is the summary Table (2.1.) of literature review illustrating themes and key issues.

2.12. Summary of the chapter

This chapter presented a number of issues risen from literature review on the history problem solving, research into problem solving, definitions problems and problem solving, procedural and conceptual knowledge, metacognitive training in Calculus, experiences with solving Calculus problems, and strategies for problem solving in Calculus as summarized on Table 2.1.

Themes	**Key issues**	**Relating to the current study**
History of problem solving	▪ Problem solving is traced as far back as ancient Egypt and China. ▪ In the 20th century problem solving came to the forefront of research in Mathematics Education. ▪ The guru of problem solving is George Polya (1945) ▪ History of problem solving has profound influence mathematical curriculum in today's classroom.	▪ My study has been informed on how the history of problem solving evolved. ▪ The history led to the identification of the knowledge gap in the context of this study.
Research into problem solving	▪ Beginning of 2010 a lot of research was on teaching through problem solving ▪ Recently research on problem solving process is new area of interest though research is scarce	▪ My study has been informed on how the history of problem solving evolved from 2010 to 2017. ▪ The history led to the identification of the knowledge gap in the context of this study.

Definitions of problems and problem solving	Problem	Problem solving	▪ The definition provided better understanding of various concepts of what a problem is and what problem solving processes are in the context of this study. ▪ This helped my study to operationalize what a problem is and what problem solving is.
	▪ Task ▪ Exercise ▪ A problem for one person but not for another	▪ Engage in a task ▪ Solution method is unknown	
Polya’s problem solving model	▪ Understanding (read, re-red, identify terms & sketch graphs) ▪ Devise plans (functions, formula, rough work, & simplify) ▪ Carry out plans (Monitoring, appropriate formula, keep results) ▪ Look back (evaluating answers/prove & interpret)		▪ Polya’s problem solving model informed my study about how problems are solved particularly not only in Calculus as a course or topic but as well as in general Mathematics courses or topics.
Procedural and conceptual knowledge	Procedural	Conceptual	▪ These two types of knowledge informed my study that in the sense that for any problem to be solved in Mathematics in general and Calculus in particular, they are always at play
	▪ Rules ▪ Procedure ▪ Symbols ▪ Formulas ▪ Doing	▪ Definitions ▪ Concepts ▪ Principles and relations ▪ Thinking	

Metacognitive training in Calculus	▪ Control of one's own thinking, self–regulation, thinking skills, posing and asking questions, self-talking, guessing/conjecturing ▪ Metacognitive control is characterized as planning, monitoring and checking	▪ Metacognitive training helped the current study to establish if learners think, reason, conjecture, ask questions before, during, and after solving Calculus problems.
Experiences with solving Calculus problems	▪ Incorrect first principles ▪ Incorrect substitution ▪ Incorrect notations ▪ Application of basic Mathematical concepts, and Language of Calculus.	▪ These experiences to solving Calculus challenges informed my study about some of the experiences leaners encounter in solving Calculus. ▪ Reviewing on these experiences helped me to identify the knowledge gap.
Strategies for problem solving	▪ Manipulatives ▪ Drawing pictures ▪ Making charts ▪ Drawing graphs ▪ Working backwards ▪ Guess and check	▪ These strategies informed my study on about some of the problem-solving strategies that enhances learners' problem solving skills in Calculus. ▪ Reviewing on these strategies helped me identify the knowledge gap.

Having provided the literature review, I now present the methodology of this study

CHAPTER THREE

RESEARCH METHODOLOGY

3.1. Introduction

This chapter discusses the general methodology utilized in the study. Methodology is about describing the strategies that was useful in carrying out the study. It endeavors to highlight the research design, population, sampling, data, collection methods and instruments, data analysis, rigor and the ethical considerations. Orodho (2003) defined methodology as an outline, scheme or plan that is utilized to generate answers to the research questions.

3.2. Research design

Kilpatrick (1978) noted that researchers should formulate research questions before considering specific research methods. When it comes to investigating learners' problem solving processes in Calculus, I believe that, since learning is complex and nuanced, more meaning can be found in a close examination of a few learners than in a broad look at a large sample of learners. For that reason, I chose a qualitative method to address the research questions in the present study. Berg (2007) described this kind of research:

> *"Qualitative research properly seeks answers to questions by examining various social settings and the individuals who inhabit these settings……. also, qualitative procedures provide a means of accessing unquantifiable facts about the actual people researchers observe and talk to. Qualitative research methods are useful for addressing different kinds of questions than those that quantitative methods can address" (p.8).*

Lester (1985) advocated qualitative methods for conducting research in problem solving instruction. He stated, "adopting a holistic view of problem solving and problem solving instruction necessitates the use of naturalistic [inquiry] rather than traditional scientific research paradigms" (p. 52). By naturalistic inquiry he was referring to qualitative research done in a natural setting such as a classroom. I employed a qualitative research paradigm because I needed to develop deeper insights and an in-depth understanding of the learners' problem solving processes, challenges they encounter in solving Calculus problems, and bring out problem solving strategies that can improve their problem-solving skills in Calculus in a natural setting such as a classroom (Merriam, 1998).

Of the many qualitative designs, *a descriptive case study* (Berg, 2007) was the most appropriate for addressing the research questions and to conclusively answer the research questions in the current study. According to Berg (2007), a descriptive case study is a method of collecting information by interviewing or observing a sample of individuals. It can be used to collect information about people's attitudes, tone of voice, gestures, opinions or habits on any educational or social issue. I employed descriptive case study because I wanted to bring out issues as they exist in natural setting. This enabled the me to report on the findings regarding the topic, learners' problem solving processes in Calculus at Grade 12 level. This was through the use of the lesson observations, FGDs, and the interviews with the respondents.

Of the subjects Berg listed, a *group* and *institutions* best describes the focus of the case study. That is, I targeted only two secondary schools in Lusaka district as institutions. In terms of a *group,* I only targeted Grade 12 learners who were doing Calculus. Some have criticized case study research for not producing findings that are generalizable, but Berg (2007) argued, "when case studies are properly undertaken, they should not only fit the specific individuals, groups, institutions or event studied but also generally provide understanding about similar individuals, groups, institutions and events…. the logic behind this has to do with the fact that few human behaviours are unique, idiosyncratic, and spontaneous" (pp. 295–296). Having provided the research design for my current study, the next section describes the site where this study was carried out from and the reasons for the site chosen forth.

3.3. Location of the Study

The study site was undertaken in Lusaka district (Figure 3.1) of Lusaka province, Zambia. According to Msabila and Nalaila (2013) there are many motivating factors that could influence the researcher's choice of the study site, such as; the nature and incidence of the problem, research time frame, and data accessibility, clients' interest and instructions, resource availability, performance in a particular field, goals and objectives of the study. Therefore, I selected Lusaka province as the study site for my study because the performance in Mathematics from 2014 to 2017 has been below 40%. The reason for the choice of the study site is backed by the Examination Council of Zambia, 2016 general performance analysis that reported that, "the average performance of mathematics in Lusaka province from 2014 to 2016 has been below 40 percent (36.53%)" (ECZ, 2016, p. 24, ECZ, p.2).

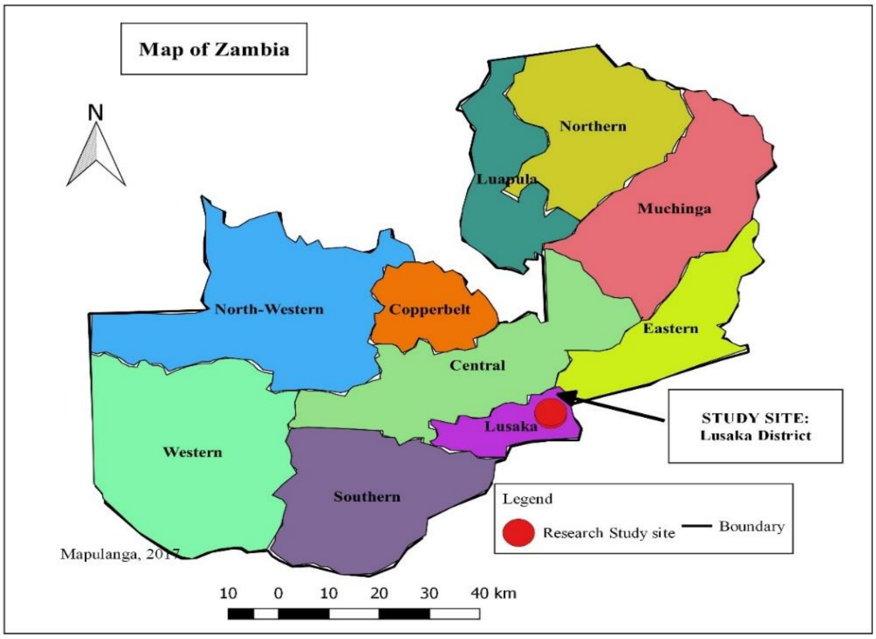

Figure 3.1: Map of Zambia showing location of Lusaka district in Lusaka Province

3.3. Target population

The target population were Grade 12 teachers of Mathematics and Grade 12 learners from two secondary schools in Lusaka District of Zambia. The choice of teachers was based on the reason that teachers are the ones who teach Calculus at Grade 12 level and the choice of the learners was based on the reason that they are the ones that learn Calculus. In addition, I included this cohort of Grade 12 teachers of Mathematics and Grade 12 learners due to its unique feature of being the third product under the revised curriculum and O-level Mathematics Syllabus.

3.4. Study sample

In this study, the sample size I employed comprised of two (1 from school A & 1 from school B) Grade 12 teachers of Mathematics and twenty (10 from school A & 10 from school B) Grade 12 learners of Calculus from two (2) secondary schools (school A & school B) making a total sample of twenty-two respectively in Lusaka district of Zambia. This was a reasonable sample size for a qualitative study because it provided the data needed as it fulfilled the requirements of efficiency, representativeness, reliability and flexibility (Merriam, 1998) as it included Grade 12 teachers (who teaches Calculus) and learners' (who learns Calculus). In fact, a small sample for observations, FGDs, and interviews ensured a high level of reliability and helped to concentrate resources on obtaining reliable information (Hannagan, 1997).

3.5. Sampling procedure

In this study, I employed purposive sampling technique because the study was a naturalistic inquiry which has design strategies, flexibility and purposeful sampling (Best & Kahn, 2009). The power of purposeful sampling lies in selecting information rich in cases for in-depth analysis (Merriam, 1998). To ensure that Grade 12 teachers who teach Calculus take part in this study, purposively sampling technique was employed (Maxwell, 2005). I selected Grade 12 teachers of Mathematics who teach Calculus at Grade 12 because not all the Grade 12 teachers of Mathematics at the time of this study teach Calculus. In light of this, 2 classes for the Grade twelves (Grade 12 X from school A and Grade 12 Y from school B) where purposively selected and the teacher of that class automatically become the participant.

In order to select Grade 12 learners to participate in this study, ten (10) Grade 12 learners from each class (Grade 12 X and Grade 12 Y) were purposively selected. The Grade 12 learners were purposively selected with the help of the teachers. They were purposively selected because Calculus is taught at Grade 12 hence the need for Grade 12 learners. I selected the Grade 12 learners with the help of the teachers because I aimed at involving three (3) categories of Grade 12 learners (high ability solvers, middle ability solvers, and low ability solvers) in Mathematics in general and Calculus in particular. The criteria of choosing the high ability solvers (2 from school A & 2 from school B), middle ability solvers (2 from school A & 2 from school B) and low ability solvers (1 from school A & 1 from school B) was based on their general performance in Mathematics**.** I included variety of abilities of the learners in order to establish how different abilities solve Calculus problems. In fact, not considering the abilities would have distorted the establishment of the learners' problem solving processes in Calculus and this would have resulted into picking learners of the same abilities because of choosing at random. Moreover, not considering high ability solvers, middle ability solvers, and low ability solvers would have meant focusing on one ability for instance, just high leaving out middle and low or middle leaving out high and low or low leaving out high and middle hence the need of employing three (3) abilities. In addition, not considering a mixed would have overshadowed the challenges they encounter in solving Calculus problems and problem-solving strategies that can improve their problem-solving skills in Calculus of the lower ability or middle ability or high ability because the higher ability would have taken advantage of the situation.

Of the many purposive sampling techniques, I proposed to use homogeneous purposive sampling procedure (Merriam, 1998). Homogeneous purposive sampling procedure was used because I wanted to seek answers to questions by examining various social settings with the Grade 12 learners who inhabit these settings and share same characteristics. In addition, I took advantage of the revised curriculum change as an additional motivating factor for an opportunistic sample. Berg (2007) points out that opportunistic sampling is believed to minimise self-selection bias. Having provided the sampling procedure for this study, I now discuss data collection methods in the next section.

3.6. Data collection methods

This study is qualitative in nature and only qualitative data collection methods where used (Merriam, 1998; Burgess, 1995). For this study, three research methods where used to collect the required information from the respondents and these included: Lesson observations, FGDs, and Semi-structured interview. Therefore, the following subsection provide more details about the data collection methods and procedures regarding the same.

3.6.1. Lesson observations

I used lesson observation schedules to collect data as Grade 12 learners were solving Calculus problems (See appendix 8 for detail, p. 152). In fact, lesson observations is a tool that provides information about the actual behaviour (McMillan & Schumacher, 1993). Moreover, “direct observation is useful because some behaviour involves habitual routines of which people are hardly aware” (Kasonde, 2013, p. 44). Merriam (1998) said that lesson observations is a special instrument of collecting data in a qualitative research. Observation is a data collection procedure whereby researchers try to understand what is happening in a given setting by paying attention, video recording, watching and listening carefully (Patton, 2002).

Video recording was very helpful to collect data through lesson observation (Patton, 2002; McMillan & Schumacher, 1993). One of the advantages of using a video camera to record field observations is that it allows the researcher not only to capture the physical environment but also to revisit the images later and relive the experiences during analysis (Merriam, 1998). Indeed, as McMillan and Schumacher (1993) observe, “audio-visual technology makes it possible……….to analyse people and events with a degree of particularity that would have been impossible just a decade ago….” (p. 144). In fact, the purpose of video recording on how Grade 12 learners were solving Calculus problems was to enhance trustworthiness of these qualitative results. This helped me capture data in its totality.

3.6.2. Focus group discussions

In this study, I used FGDs in order to collect data on the challenges Grade 12 learners encounter in solving Calculus problems and to collect data on strategies learners would suggest to improve their problem-solving skills in Calculus (See appendix 9 for detail, p. 153). According to Bryman (2008), FGD is a type of group interview, which consists of a number of participants with the inclusion of the interview moderator. In addition, FGDs was used to collect data pertaining to Grade 12 learners’ problem solving processes in Calculus. The idea of using FGDs to collect data pertaining to Grade 12 learners’ problem solving processes in Calculus was to comprehend with the findings from lesson observations. However, FGDs with the learners where held after the whole topic Calculus (differential Calculus & integral Calculus) was taught. To enhance trustworthiness of these qualitative results, I audio-recorded interviews after the lessons. This helped me capture data in its totality.

3.6.3. Semi-Structured Interviews

Semi-structured interviews were used with the teachers of Calculus. The rationale of the interviews with the teachers was to have an in depth understanding, opinions, and views pertaining to their learners' problem solving processes in Calculus. In addition, interviews with the teachers were used because I wanted to understand on some of the challenges their learners encounter in solving Calculus problems, and determine strategies they (teachers) would suggest to improve their learners' problem-solving skills in Calculus (See appendix 10 for more detail, p. 154). McCracken (1988) and Merriam (1998) argue that data collected by this method is fairly reliable and allows respondents to freely respond to an issue, and allows the researcher to gather in-depth information. In addition, I used direct personal interviews to enable me control the sample and minimize missing returns from participants (Kothari, 2004; McMillan & Schumacher, 1993). Scholars across the globe contended that interviews are the best way to collect data because it helps the researcher to have feelings, opinions, attitudes, views, and are useful in gathering in-depth data (Kvale, 1996). To enhance trustworthiness of these qualitative results, I audio-recorded interviews after the lessons. This helped me capture data in its totality. In the next section, I discuss data collection procedure and time line.

3.7. Data collection procedure and time line

This study utilized the qualitative method to collect the required data and analyse and interpret it. The actual data was collected in term 1 of the school calendar for school A, between 2nd March, 2018 and 23rd March, 2018; and of the school calendar for school B, between 2ndApril, 2018 and 29th April, 2018. This means that the period of data collection was 6 weeks (3 weeks for school A & 3 weeks for school B). In order to solicit views from the teachers and learners, I collected data as follows:

Qualitative data was first collected through lesson observations. I observed and documented a total of 18 lessons; nine for each class (e.g. Class A and Class B). This means that for each class (e.g. Grade 12 X for school A and Grade 12 Y for school B) I observed and documented nine (9) lessons (3 lessons per week for Class A & 3 lessons per week for Class B) because Mathematics appeared thrice each week on the school time-table. From the nine (9) lessons observed and documented per each school, class A had 6 single lessons (2 single lessons per week) and 3 double lessons (1 single lesson per week) as appeared on the school time-table. Class B had 3 single lessons (1 single lesson per week) and 6 double lessons (2 single lessons per week) as appeared on the school time-table. However, in the context of this study, double lessons were counted as one. In line with the New 2013 Curriculum Framework and the New 2013 O-level Mathematics Syllabus D on the time allocated to Mathematics lesson, I observed the whole lesson on Calculus. In order to observe a full lesson, I therefore sat in the participants' class during their regular Mathematics time and I used a video camera to capture data in its totality during a teaching and learning session (Berg, 2009; Berg, 2007; Creswell, 2014). In each class: Grade 12 X and Grade12 Y, I first observed the teacher on how they teach Calculus and solve Calculus problems.

However, I video-recorded and documented a total of 20 videos; one for each of the learners (A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, and A_{10} at school A) and learners (B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8, B_9, and B_{10} at school B). This enabled me to take vignettes when learners were solving Calculus problems both on the chalkboard and in their Mathematics exercise books. In addition, this provided a platform for me to establish Grade 12 learners' problem solving processes in Calculus. It also enhanced determination of factors liable to constrain learners in solving Calculus problems. Thus, I used video recording for retrospective analysis (Bryman, & Burgess, 1994). The rationale of video recording was to enhance trustworthiness of these qualitative results. This helped me capture data in its totality.

In order to have a deep understanding, probe questions and have an in-depth knowledge on Grade 12 learners' problem solving processes in Calculus, deep understanding of the challenges Grade 12 learners encounter in solving Calculus, and establish inventory measures to surmount the challenges learners' encounter in solving Calculus problems, FGDs with the purposively selected Grade 12 learners where conducted after the whole topic Calculus was taught and observed. In this case, learners were subjected to structured interview questions during FGDs (Jorgenson, 1989). To do this, I grouped the learners' in groups of 5 (translating to two focus groups per school). Cohen, Manion and Morrison (2004) advises that a focus group usually comprises five to eight individuals who share certain characteristics, which are relevant for the study. In this case with the sample size (n=20) of the Grade 12 learners, I had 4 FGDs in total with the learners for the 2 classes (Grade 12 X for school A and Grade 12Y for school B), meaning from each class I had 2 FGDs with 5 setts of the learners. Each FGDs comprised of 2 high ability learners, 2 middle ability leaners, and 1 low ability learner. The reason for putting learners in mixed ability groups was to get a collection of challenges they encounter in solving Calculus problems across abilities and a collection of strategies that can enhance problem solving skills in Calculus. In addition, the interviews with the mixed ability started after allowing them to watch the video of their solving process of Calculus problems. The points, views and opinions I got from the participants during FGDs were audio taped using an electronic recorder or audio recorder. In fact, this method of collecting data allowed me to ask semi-structured questions, probe and ask follow-up questions in order to solicit for in-depth information from the participants (Jorgenson, 1989 & Creswell, 2009).

Then after each FGDs with the learners, I used semi-structured interviews that comprised of open-ended interview and closed-ended questions to collect data from the Grade 12 teachers of Mathematics concerning to Grade 12 problem solving processes in Calculus and establish problem solving strategies that can improve their learners' problem solving skills in Calculus. This was done after the whole topic Calculus was taught. Rubin and Rubin (1995) advise that during research, it is also important to look at participants who administer teaching than the participants who are learning. However, this justifies why Grade 12 teachers of Mathematics were partially involved in this research study. Audio-recording of post-lesson interviews enabled me to capture reality in its totality. However, the interviews with each participant started after allowing them to watch the video of their lesson. Having provided data collection and procedure for this present study, in the next sub-section I now provide how qualitative data was analysed.

3.8. Data analysis

In order to analyse qualitative data, data that was collected from interviews and FGDs was analysed thematically (McMillan & Schumacher, 1993). During analysis, data recorded from lesson observations, interviews and FGDs was transcribed, edited, coded, categorised and tabulated (Kothari & Garg, 2014). However, I interpreted the video lessons using Polya's (1957) problem solving model to explore learners' problem solving processes in the form of understanding the problem, devising plans, carrying out plans, and looking back. Understanding the problem was gauged by learners' knowledge of reading problems, rereading problems, identifying key important words, and sketching graphs. Devising plans was characterised by learners' proficiency of writing formulas, writing equations/functions, rough workings, and simplifying certain parts of a problem. Carrying out plans was gauged by learners' competence in monitoring each step when solving a problem, appropriate formula, and keeping track and save results/answers/data. Looking back was characterised by learners' reflective skill in checking an answer after solving and interpreting answers after solving. However, triangulation of data was also achieved by comparing what learners said during FGDs with the vignettes I took during video recordings when learners were solving Calculus problems. The following section addresses the trustworthiness regarding this study.

3.9. Trustworthiness of the study

To ensure trustworthiness in my study, participants own words and vignettes (pictures) of pupils' exercises showing problem solving process were used in the presentation of findings. Further, the themes after data analysis were subjected to expert review to see whether they were in line with recordings and recognizable (Merriam, 1998; Adler, 1996). Thus, my supervisor cross examined them to ensure their credibility in the study. The following section addresses the ethical issues regarding this study.

3.10. Ethical considerations

Ethical considerations are important to research and I ensured that all key components of ethics were addressed by using agreed standards (Morrow, 2009). Cohen, Manion and Morrison (2004) asserts that ethics concern right or wrong, good and bad. Ethics are moral principles which guide conduct and are held by individuals, a group, or a profession. Just like many scholars have placed importance on ethical considerations in any research study, Creswell (2009) argues that research ethics requires that researchers engage in ethical practices and anticipate ethical issues prior to the study. Before commencement of the study, I obtained clearance from the Ethics Committee of the University of Zambia and the Department of Mathematics and Science Education (See appendix 11 for more detail, p. 155). This served to promote the integrity of my research and that of my institution (University of Zambia). I also observed originality, quality and ownership of data as well as honesty (Creswell, 2014). Having been allowed by the University of Zambia to go out for research, I sought the authority of the District Education Board Secretary (DEBS), Lusaka District, to gain entry into schools (See appendix 12 for more detail, p. 156). Informed consent of respondents was sought (See appendix 13 for more detail, p. 157) before proceeding with the research. All the participants were made aware of the nature and purpose of the study. To guarantee the anonymity of participants, I disassociated names from responses during coding and used pseudonyms where learners from school A were codded A_1 to A_{10} and learners from school B were codded B_1 to B_{10}. Teachers were also codded by (teacher A from school A and teacher B from school B) so as to protect their identities (Creswell, 2014). Their faces were not captured in the final document. After all, the information they provided was more important than their personal details. I never shared the information with anyone else apart from individual participants and my supervisor. In addition, the learners were informed that their participation in the study would not affect their Grades in school and for the teachers, it would not affect their teaching performance as the data collected was strictly meant for research purposes.

3.11. Summary of the chapter

In summary, this chapter presented the description of the methodology used in the study. The next chapter present the study findings according to the research questions.

CHAPTER FOUR: RESULTS

4.1. Overview

In the previous chapter, I described the research methodology, which was employed in the study to come up with the results which are presented in this chapter. The themes that are presented in this chapter emerged from the data collected from lesson observations, FGDs, and interviews. In this study, data analysis for research question one was aided by Polya's (1957) problem solving processes and their indicators whilst research question two and three were analysed based on the themes that emerged from FGDs and interviews. However, triangulation of data was also achieved by comparing what learners said during FGDs with the vignettes I took during lesson observations when learners were solving Calculus problems.

Vignettes are rich pockets representing data (Miles & Huberman, 2004). These are summaries that can be pulled together in a focussed way for interim understanding. Vignettes are focused description of a series of events taken to be representative, typical or symbolic. They are vivid and they are representative captures. The following codes have been used for identification of the participants; Teacher A (teacher at school A), Teacher B (teacher at school B); learners (A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, and A_{10} at school A) and learners (B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8, B_9, and B_{10} at school B). However, the data analysis procedure in this study was specifically guided by the following research questions:

1. What are Grade 12 learners' problem solving processes in Calculus?
2. What challenges do Grade 12 learners encounter in solving Calculus problems?
3. What strategies would teachers and learners suggest to improve problem solving skills in Calculus?

4.2. Grade 12 learners' problem solving process in Calculus

Research question number one was on Grade 12 learners' problem solving processes in Calculus. In order to establish Grade 12 learners' problem solving processes in Calculus, lessons were observed, and focus group discussions were conducted.

4.2.1. Understanding Calculus problems

In this section, the analysis of Grade 12 understanding of Calculus problems was gauged by learners' knowledge of reading Calculus problems, rereading Calculus problems, identifying key important words, and sketching graphs.

4.2.1.1. Reading Calculus questions

Research results shows that learners read Calculus problems before solving. All the twenty learners during FGDs stated that they read Calculus problems in order to understand Calculus problems before they solve. For instance, the following excerpts for learners (A_5, B_1, and B_4) are the typical examples as shown below:

> *"Yes, Sir I do, they say read to understand, so when you read you are going to understand, so I always make sure that I read before I solve"* **(A_5).** *"Yeah! It is very important to read through Calculus questions so that you grasp what the question is asking you to do"* **(B_1).** *"Always Sir because by reading it will give you a picture on how you are you are going to tackle the questions"* **(B4).**

Responses also shows that although learners read Calculus, it did not guarantee them success in tackling Calculus problems and getting the correct answer. For instance, when I had asked them if reading Calculus problems guaranteed them solving and getting correct Calculus answers, the following excerpts by learners (A_1, A_3, & B_6) act as typical examples:

> *"Even if I read Calculus questions before I start solving, it does not help me get correct answers"* **(A_1).** *"No Sir, it does not help me find correct answers"* **(A_3).** *"Hmmmm Sir, sometimes reading helps me to find correct answers and sometimes not"* **(B_6).**

4.2.1.2. Rereading Calculus questions

It was found out that learners (15 out of 20) representing (75%) reread Calculus problems. In nearly of all cases, when learners read a Calculus problem for the first time and did not understand, learners during FGDs expressed that they continued to reread Calculus problems until they either understood the problem or decided to skip it as evidenced by the following verbatim expressed by learners (B_4 & B_8):

> *"…. I make sure I reread Calculus problems four times to be sure that I am answering the right question"* **(B_4).** *"Calculus problems when solving requires patience in order to understand…… So, I always reread the Calculus problem so that I am certain of the question I am being asked to either differentiate or integrate. It's good to be sure of what I am been asked to do starting to solve because you may just end up making mistakes when you start solving Sir"* **(B_8).**

4.2.1.3. Identifying key information/words

Results from lesson observations shows that learners (20 out of 20) representing (100%) do not underline key important words such as integrate, differentiate, limit, tangents, normals, gradient, coordinates etc. From the 20 learners observed during lesson observations, 7 learners representing (35%) experienced difficulties underlining key important words but, showed competence of understanding derivative concepts and got correct Calculus answers. The following vignettes (Figure 4.1a. & Figure 4.1b.) by learners (A_3 and B_1) act as typical examples:

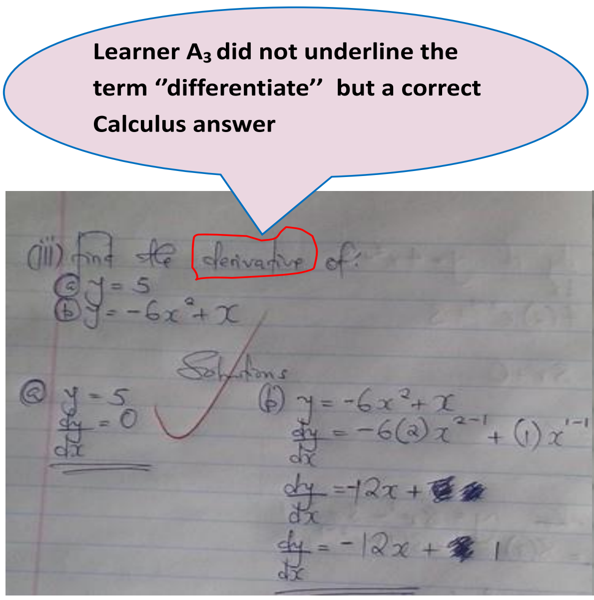

Figure 4.1a: Learner A_3 problem solving process

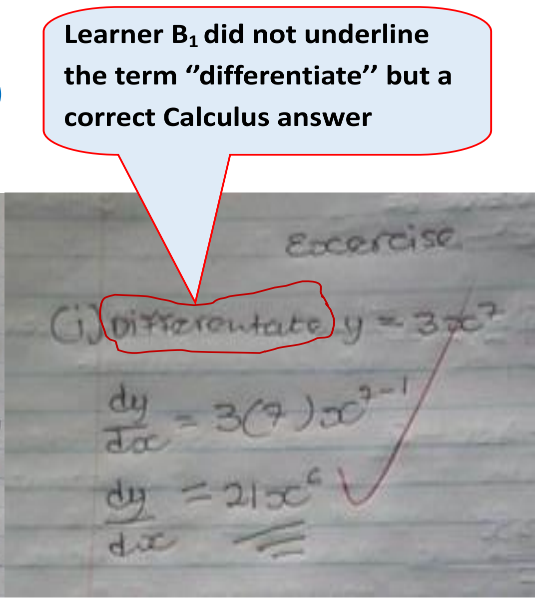

Figure 4.1b.: Learner B_1 problem solving process

In addition, results show that 13 from 20 representing (65%) learners did not underline key important words and got incorrect Calculus answers as evidenced by the following vignettes (Figure 4.1c. & Figure 4.1d.) for learners (A_8 and B_8):

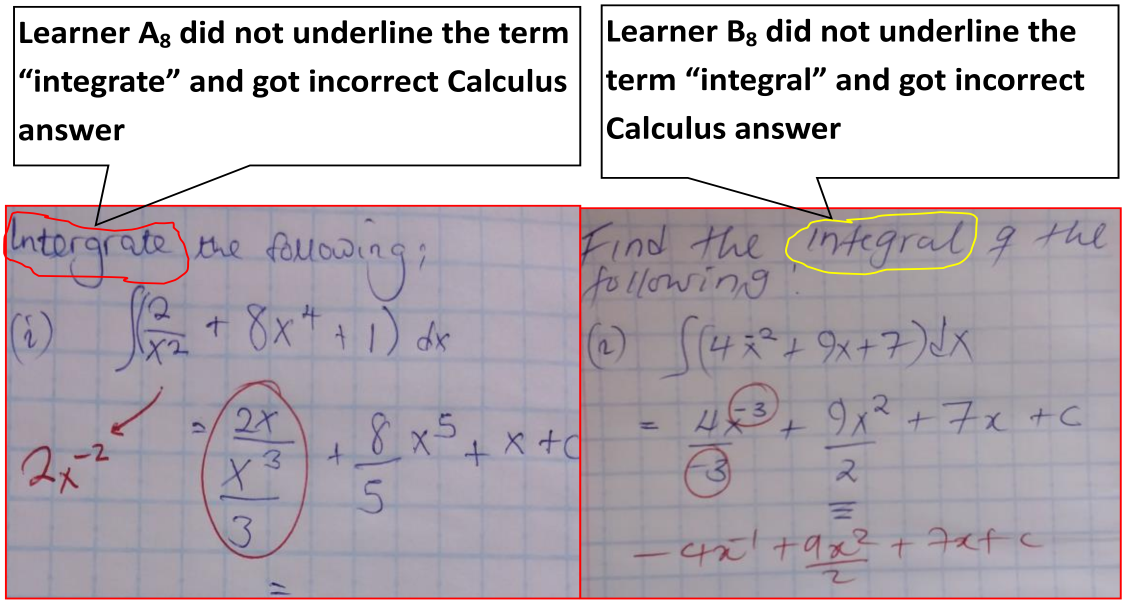

Figure 4.1c: Learner A_8 problem solving process **Figure 4.1d: Learner B_5 problem solving process**

Besides, results from lesson observations comprehended with the results from FGDs where learners (all the twenty) said they do not underlining key important words before solving. The following extracts by learners (A_1, A_3, B_5, and B_7) act as typical examples:

> *"……Hmmmm its rare for me to do that"* **(A_1).** *"……. I don't underline Sir I just start solving* **(A_3).** *"No I don't* **(B_5).** *"……I can't remember doing that Sir"* **(B_7).**

When learners expressed that they do not underline key words before solving, I took interest to know why and the following excerpts in form of verbatim by learners (A_{10} and B_4) are typical examples:

> *"We don't underline because our teacher when teaching doesn't emphasize that it is important to underline Sir"* **(A_{10}).** *"Hmmmmm we don't know that underlining key words before starting to solve is important Sir…..We are never told by our teacher that before solving questions we have to underline key words. Us we just start solving"* **(B_4).**

4.2.1.4. Sketching graphs

Results of the study show that learners (20 out of 20) representing (100%) experienced difficulties in sketching simple graphs in order to help them understand Calculus concepts and understand Calculus problems. During FGDs, learners elaborated that sketching simple graphs in order to aid their understanding of Calculus concepts and Calculus problems was really a challenge. The following excerpts by learners (A_9 and B_2) act as typical examples:

> *"I don't sketch Sir…. Even if on some questions like finding the gradient functions requires drawing graphs monga ka (like a) graph to help you understand how maybe a point is touching the tangent Sir"* **(A_9).** *"It's rare for me to do that, actually I don't because I find it a challenge to sketch graphs"* **(B_2).**

Similarly, results from lesson observations show that learners (20 out of 20) representing (100%) experienced difficulties in sketching graphs, although they showed competence of understanding derivative concepts by finding the $x - coordinates$ as evidenced by the following vignette (4.1e.) for learner (B_{10}):

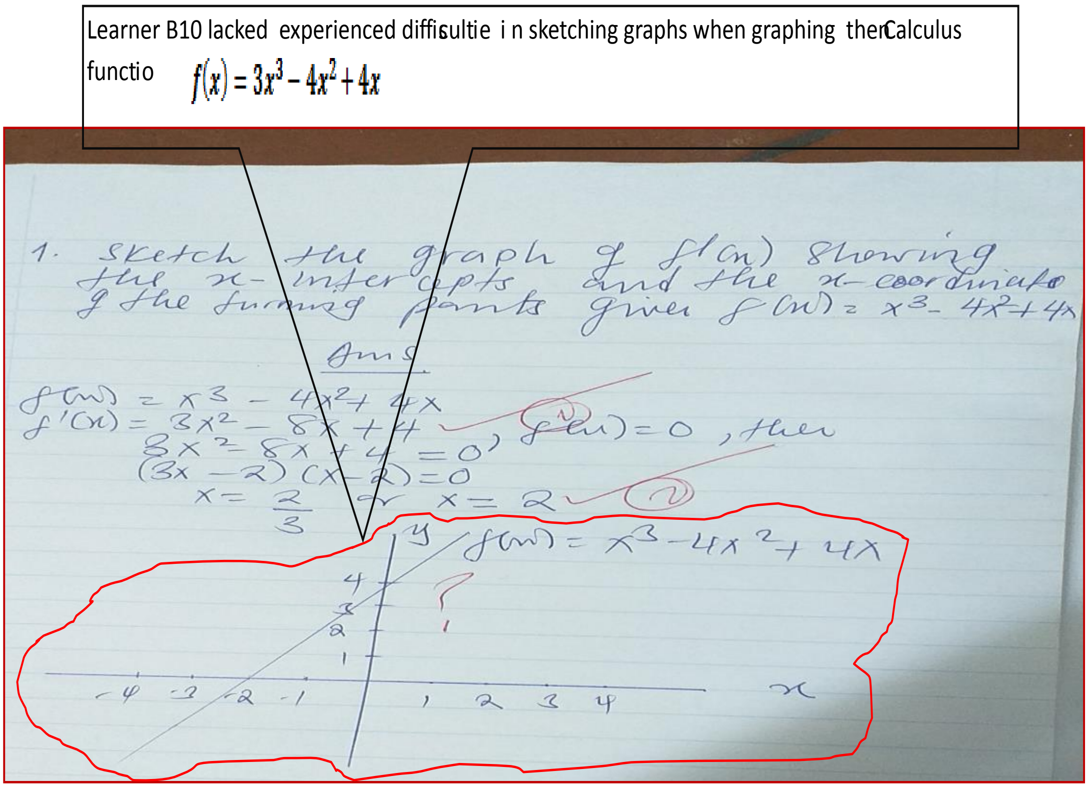

Figure 4.1.1.4a: learner B_{10} problem solving process

4.2.2. Devising Calculus plans

In this sub-section of section 4.2.1., the analysis of devising Calculus plans was characterised by learners' proficiency of writing formulas, writing equations/functions, rough workings, and simplifying certain parts of a problem within Polya's (1957) indicators.

4.2.2.1. Writing Calculus formulas

It was observed from lesson observations that learners (18 out of 20) representing (90%) had difficulties in writing Calculus formulas before solving. Thus, 2 out of 18 of the learners (A_2 and A_6), did not devise Calculus formulas but got correct Calculus answers as evidenced by the following vignettes (Figure 4.2a. & Figure 4.2b.) and 16 out of 18 of the learners (B_5 and B_7) did not devise Calculus formulas and got incorrect Calculus answers as evidenced by the following vignettes (Figure 4.2c. & Figure 4.2d.):

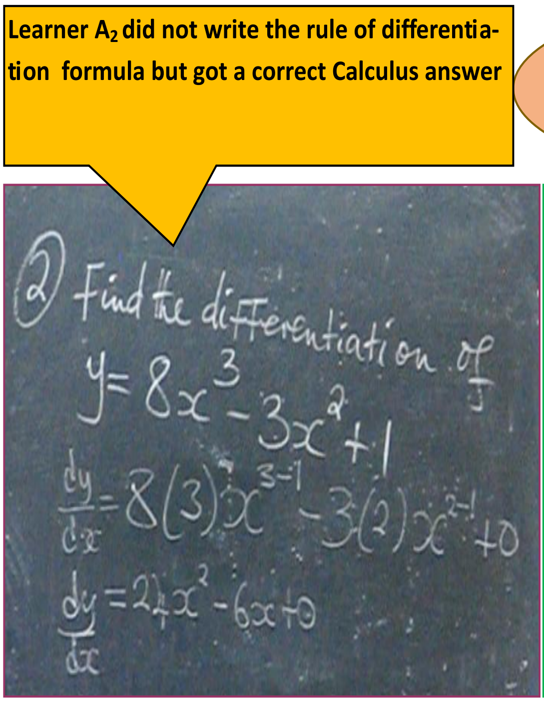

Figure 4.2a: Learner A_2 Problem solving process

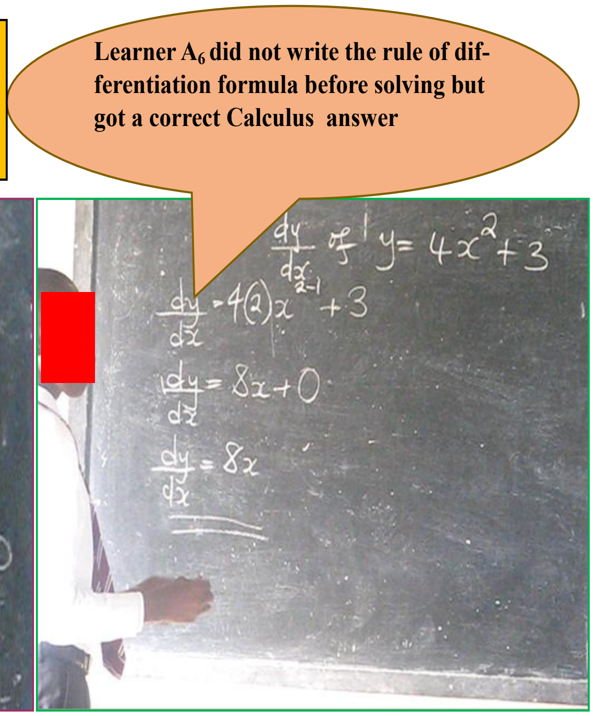

Figure 4.2b: Learner A_6 problem Solving process

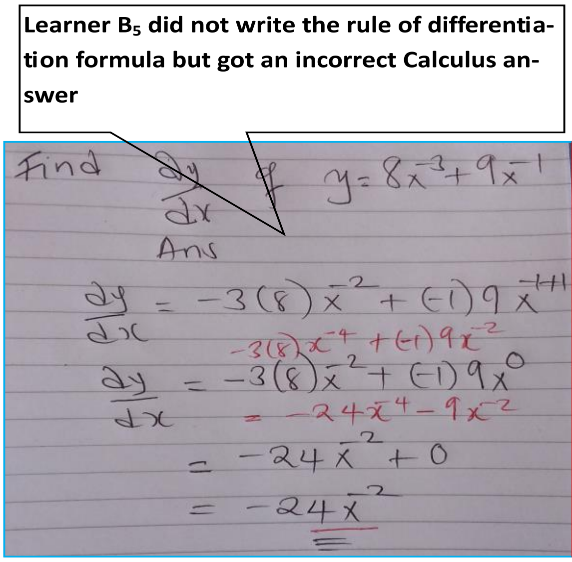

Figure 4.2c: Learner B_5 problem solving process

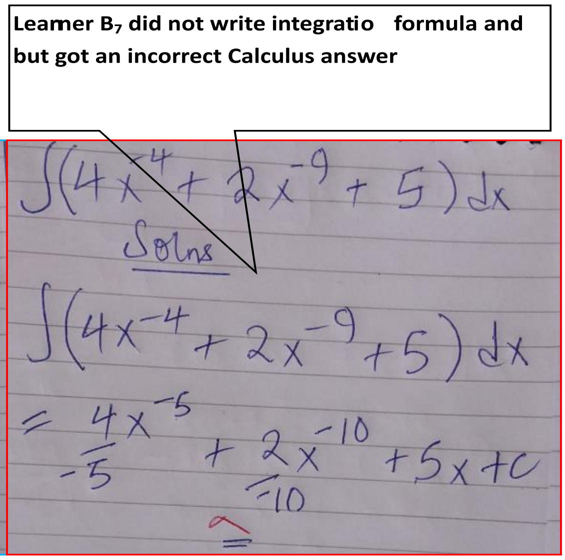

Figure 4.2d: Learner B_7 problem solving process

Of all the learners, results from lesson observations shows that only 2 learners (B_3 and B_8) out of 20 representing 20% showed competence of writing Calculus formulas before solving but got incorrect Calculus answers as evidenced by the following vignettes (Figure 4.2e. & Figure 4.2f.):

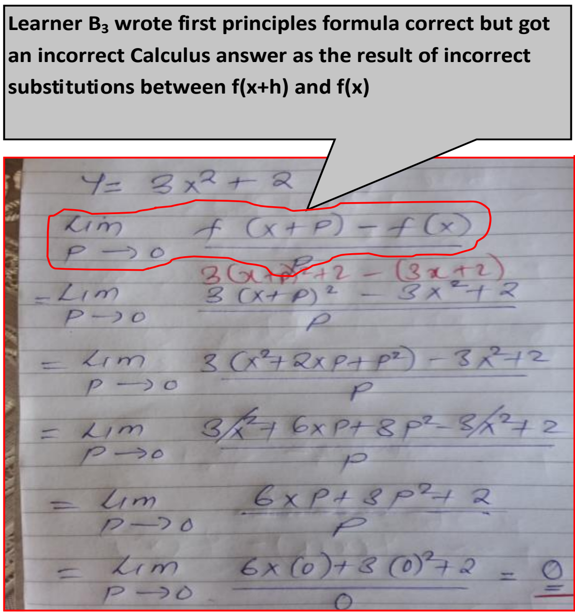

Figure 4.2e: Learner B_3 problem solving process

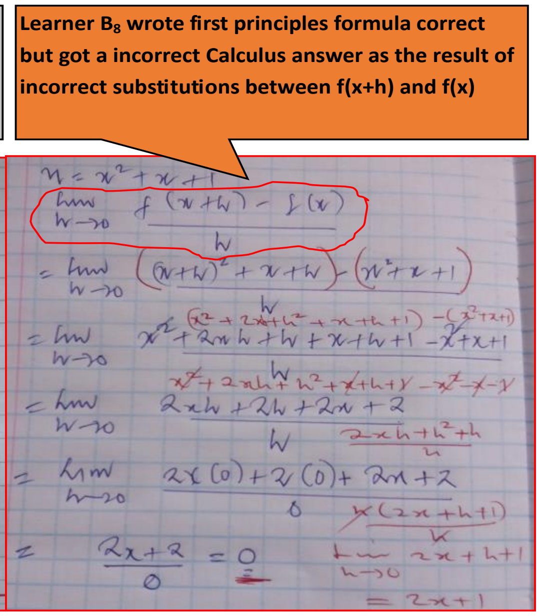

Figure 4.2f: Learner B_8 problem solving process

After noticing that majority of learners experienced difficulties in writing Calculus formulas before solving during lesson observations, I asked them during FGDs why in nearly all the Calculus problems they solved, very few of them wrote Calculus formulas before solving. The following excerpts in form of verbatim act as typical example:

"We were not writing the formulas because there was no emphasis from the teacher that formulas are that important in Calculus…. Actually Sir, all the Calculus questions in class were all solved without writing the formula first. We just started just writing dy/dx then we continue solving" **(B1).**

Moreover, even the teachers (A and B) when solving Calculus problems were not writing Calculus formulas before solving as evidenced by the following vignettes (Figure 4.2g. & Figure 4.2h.):

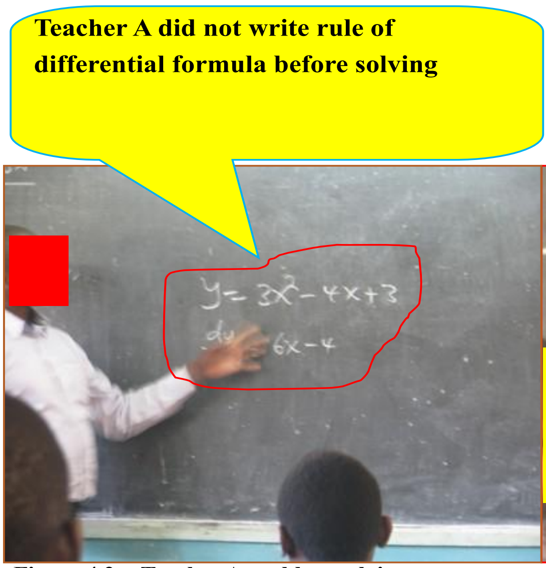

Figure 4.2g: Teacher A problem solving process

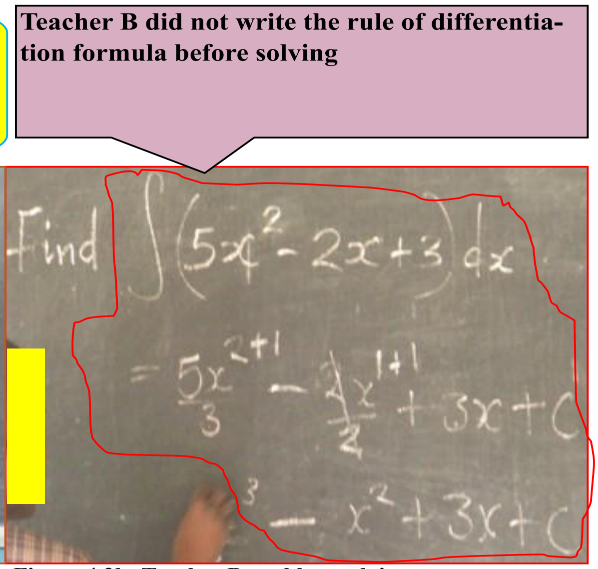

Figure 4.2h: Teacher B problem solving process

With regards to teachers views if learners devise Calculus formulas before solving, teacher A and teacher B during an interview expressed that: *"Yes they write formulas before solving"* **(Teacher A).** *"Yes they always do that before they start solving"* **(Teacher B).**

4.2.2.2. Writing Calculus equations/functions

Results from lesson observations show that (16 out of 20) of the learners representing (80%) displayed Calculus functions before solving and got correct Calculus answers. They also showed competence understanding differentiating each term. The following vignettes for learners (A_2 and B_5) act as typical examples (Figure 4.2i. & Figure 4.2j.):

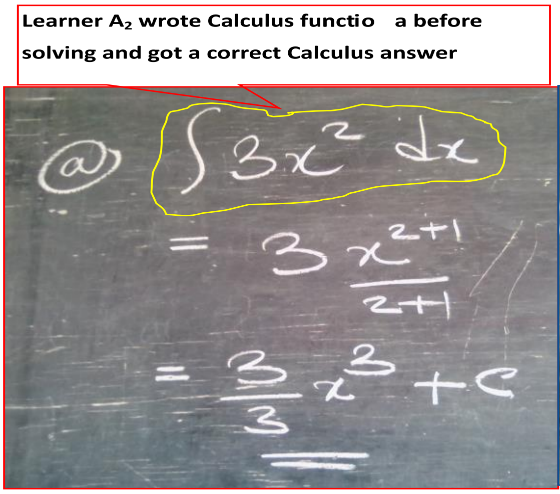

Figure 4.1.2.2a: Learner A_2 problem solving process

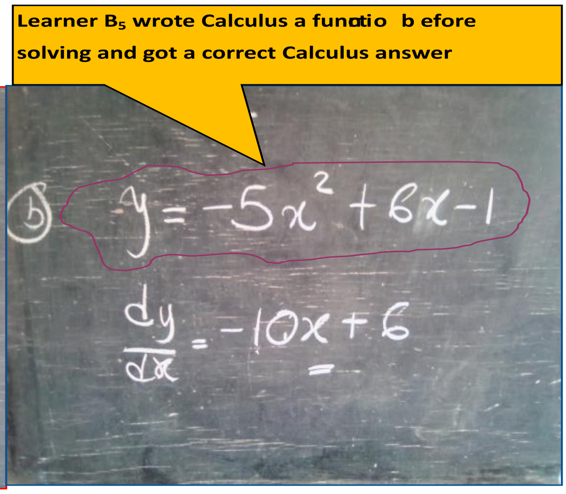

Figure 4.1.2.2b: Learner B_5 problem solving process

Likewise, responses from FGDs with the learners showed that learners displayed Calculus functions before solving as evidenced by the following excerpts for learners (A_2, B_1 and B_4) as typical examples:

> *"I extract the equation from the question because it is the same question that you have to solve"* **(A_2).** *"I always write down the equation before starting to solve Sir, it gives confidence"* **(B_1).** *"It's impossible to start solving a question minus writing down the equation which you will have to solve"* **(B_4).**

With regards to teachers views if their learners display Calculus functions before solving, teacher A and teacher B shared these views as revealed below in form of verbatim:

> "…. *Yeah learners write down equations before starting to solve and that is what I always emphasize when I am teaching because any mathematical problem the first thing is you write down the equation first. Writing a problem shows that the problem has been identified from the main question and this give confidence"* **(Teacher A).** "*Yes, they do that before they start solving the question*" **(Teacher B).**

Of all the learners observed during lesson observations and interviewed during interviews, results of this study have shown that only (4 out of 20) representing (20%) wrote Calculus formulas, showed competence of understanding derivative and integral concepts but got incorrect Calculus answers. The following vignettes (Figure 4.2k. & Figure 4.2l.) for learners (A_6 and A_9) act as typical examples:

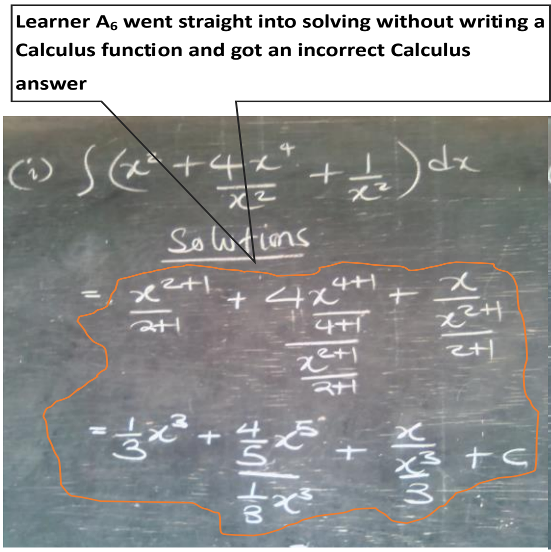

Figure 4.2k: Learner A_6 problem solving process

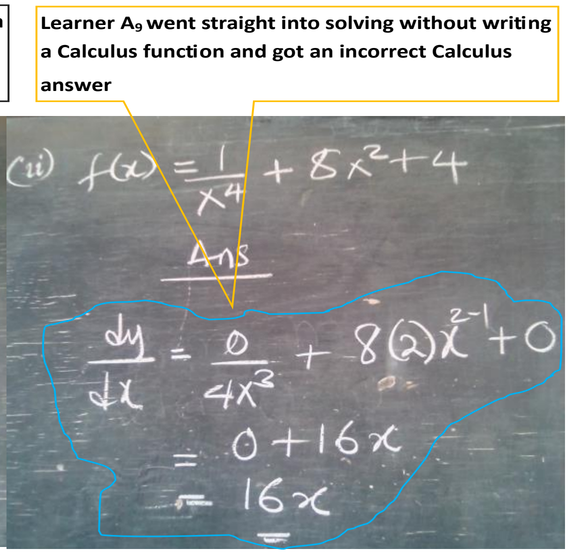

Figure 4.2l: Learner A_9 problem solving process

4.2.2.3. Rough workings

Results from lesson observations reveal that learners (18 out of 20) representing (90%) show rough workings before solving Calculus problems. In nearly all cases, learners extensively relied much on rough workings before tackling Calculus problems and showed competence of integrating each term. The following vignettes for learner B_2 to integrate; $\int(2x-3)^2dx$ and $\frac{\int x^4-3x+1}{2x^3}\,dx$ and learner B_{10} to integrate; $\int\frac{x^4-3x+1}{2x^3}dx$ act as typical examples (Figure 4.2m. & Figure 4.2n.):

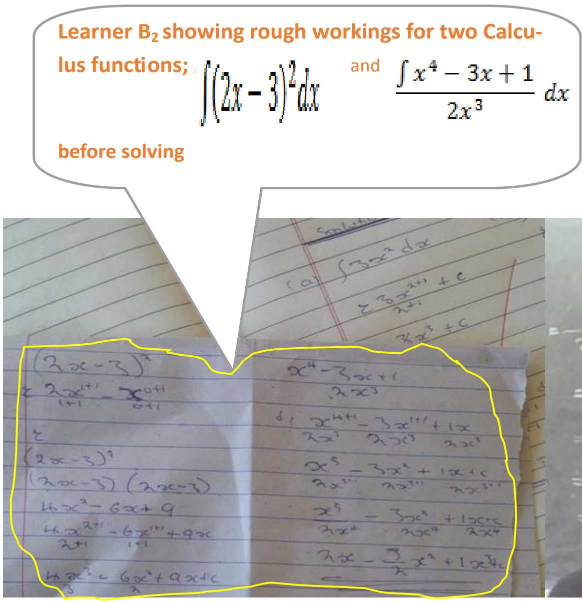

Figure 4.2m: Learner B_2 problem solving process

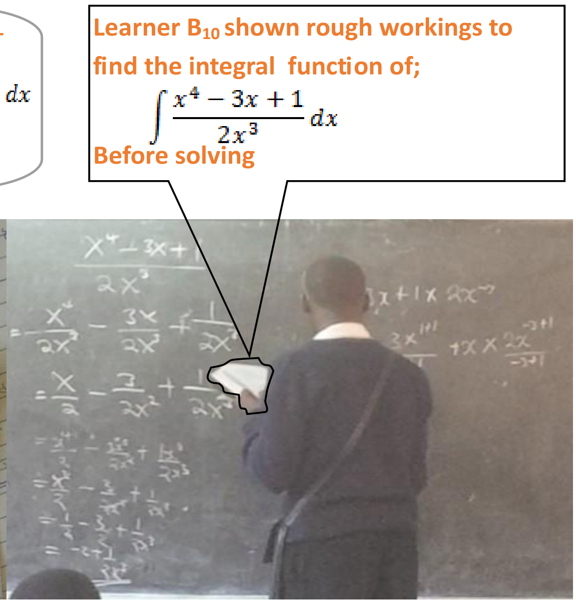

Figure 4.2n: Learner B_{10} problem solving process

Similarly, results from lesson observations is substantiated by learners' responses from (FGDs). Learners during (FGDs) expressed that they show rough workings before solving Calculus problems as evidenced by the following excerpts in form of verbatim by learners (A_2, B_6, and B_9) as typical examples:

> *"I show some rough work before I start solving Calculus problems"* **(A_2).** *"When I am about to solve Calculus problems, I make sure I write down on a small piece of paper before starting to solve in my exercise book"* **(B_6).** *"I solve somewhere before I seriously start solving"* **(B_9).**

Teachers views were in accordance with learner's views. The following verbatim explains teacher A and teacher B views about rough workings.

> *"……Yes, I have seen in most instances when I give them work to do they will first solve somewhere before they solve in their books. So, I can say they do"* **(Teacher A).** *"Some they just start solving there and then and some they don't* **(Teacher B).**

4.2.2.4. Simplifying Calculus problems

In terms of simplifying Calculus problems before solving, (17 out of 20) representing (85%) of the learners observed during lesson observations experienced difficulties in simplifying the Calculus problems before solving. In many instances, learners went into applying the rule of differentiation both on the numerator and denominator without simplifying Calculus problems before solving. Although this was the case, learners showed competence of differentiating and integrating each term. For instance, the following vignettes for learners (A_7, B_9, B_{10}, and A_5) act as typical examples (Figure 4.2o., Figure 4.2p., Figure 4.2q., & Figure 4.2r.):

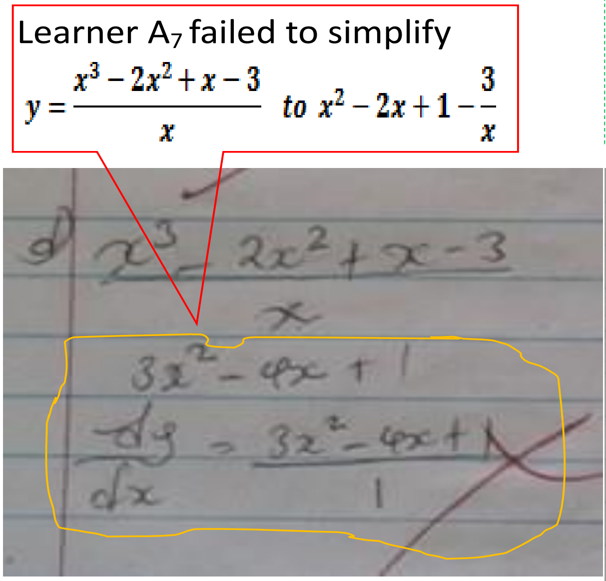

Figure 4.2o: Learner A_7 problem solving process

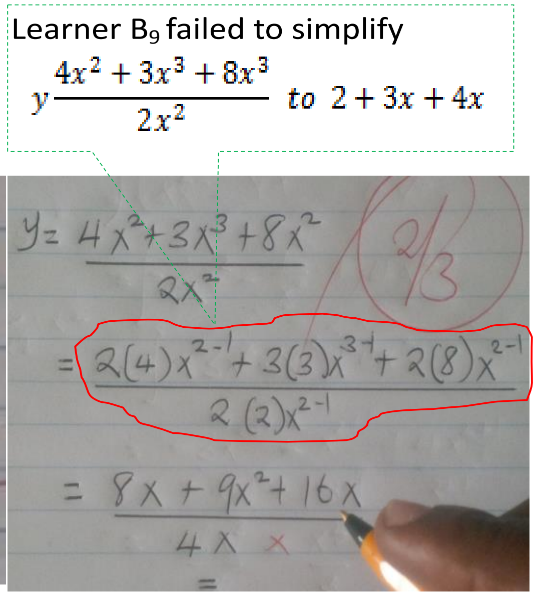

Figure 4.2p: Learner B_9 problem solving process

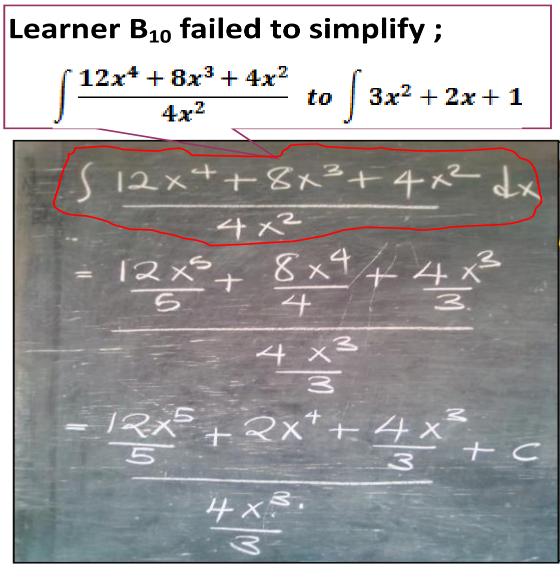

Figure 4.2q: Learner B_{10} problem solving process

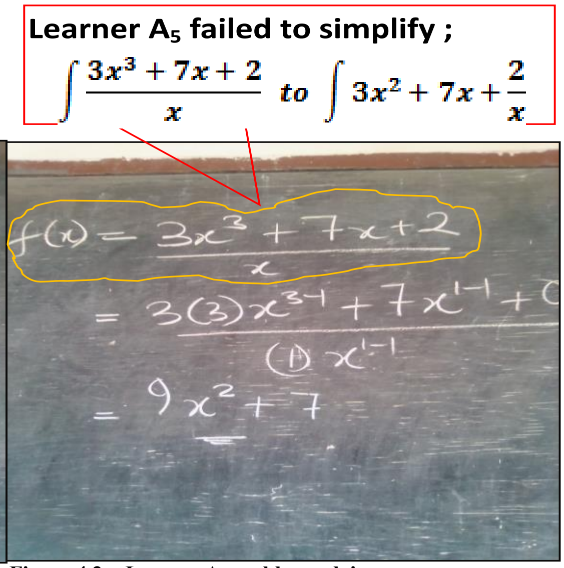

Figure 4.2r: Learner A_5 problem solving process

Also, results from (FGDs) show that learners experienced difficulties in simplifying problems before solving as evidenced by the following excerpts:

> *"….I rarely simplify Calculus problems before I start solving"* **(A_{10}).** *"Sometimes Sir"* **(B_7).** *"I don't simplify Sir because it's the challenge I am facing. There those questions that comes in the form of fractions and those that you have to expand first I have some difficulties"* **(B_8).**

Of all the observed and interviewed learners during FGDs, only (2 out of 20) representing 10% simplified Calculus problems before solving and showed competence of integrating each term. For instance, the following vignettes (Figure 4.2s. & Figure 4.2t.) for learners (A_2 and B_2) act as typical examples:

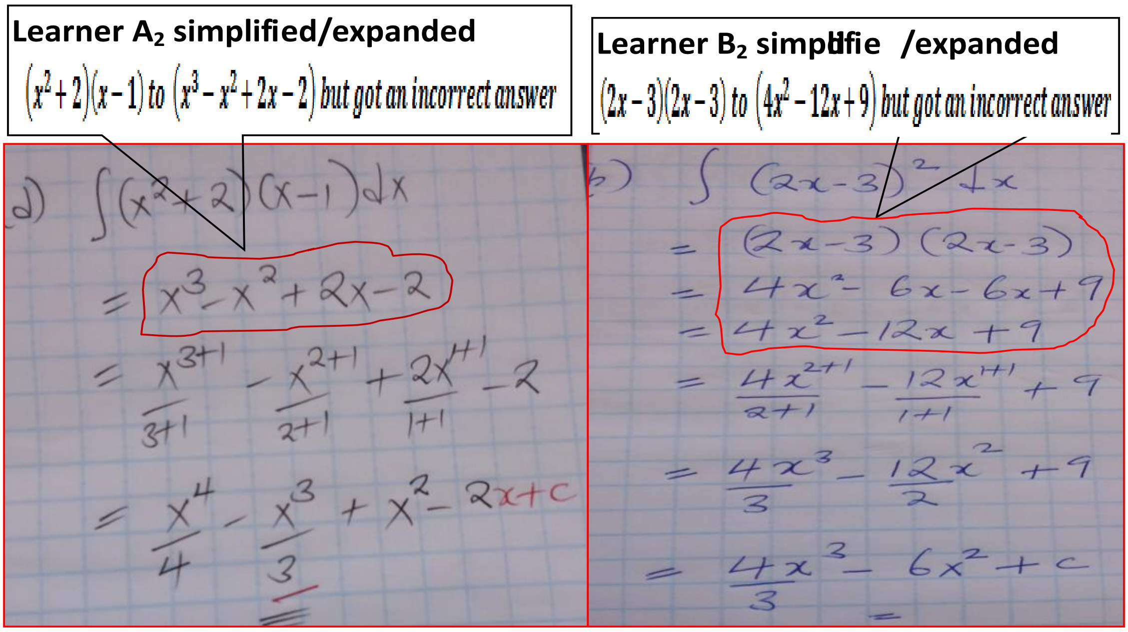

Figure 4.2s: Learner A_2 Problem solving process

Figure 4.2t: Learner B_2 problem solving process

After noticing that majority of the learners experienced difficulties in simplifying Calculus problems before solving, I asked them during (FGDs) why they experienced difficulties in simplifying Calculus problems before solving. The following excerpts for learners (A_4, B_7, and B_9) acts as typical examples:

> *"……. We have forgotten the concepts of simplifying problems"* **(A_4).** "*We were not told by our teacher that it is important to simplify before solving"* **(B_7).** *"Like I said the other day Sir, Calculus its like a combination of topics like Algebra, so the reason why we failed to simplify before solving is that we have forgotten the concepts of Algebra"* **(B_9).**

Consequently, when I interviewed the teachers to find out if learners simplify Calculus problems before solving, the following excerpts acts as typical examples for teacher (Teacher A & Teacher B):

"……Simplifying Calculus problems is another challenge I have noticed from our learners" **(Teacher A).** *"Depending on the nature of the question some they do simplify and some they don't"* **(Teacher B).**

4.2.3. Carrying out a plan

In this sub-section of section 4.2.1., the analysis of carrying out plans was gauged by learners' competence in monitoring each step when solving a problem, appropriate formula, and keeping track and save results/answers/data within Polya's (1957) indicators.

4.2.3.1. Monitoring each step when solving Calculus problems

Results from lesson observations show that learners (20 out of 20) representing (100%) experienced difficulties in monitoring each step when solving Calculus problems. It was observed that during the process of solving, learners did very little checking of their work to see if they were on the right track or not. This led to incorrect derivative results or integral results despite showing competence of differentiating and integrating each term. For instance, the following vignettes (Figure 4.3a., Figure 4.3b., & Figure 4.3c.) for learners (A_1, A_{10} and B_2) act as representative example:

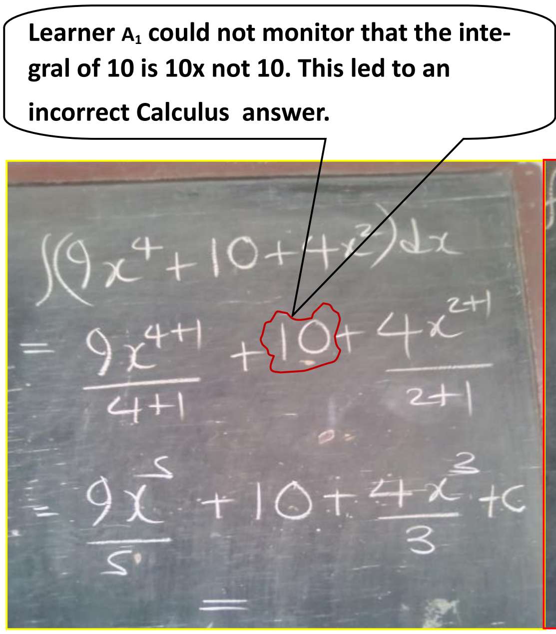

Figure 4.3a: Learner A_1 problem solving pro-

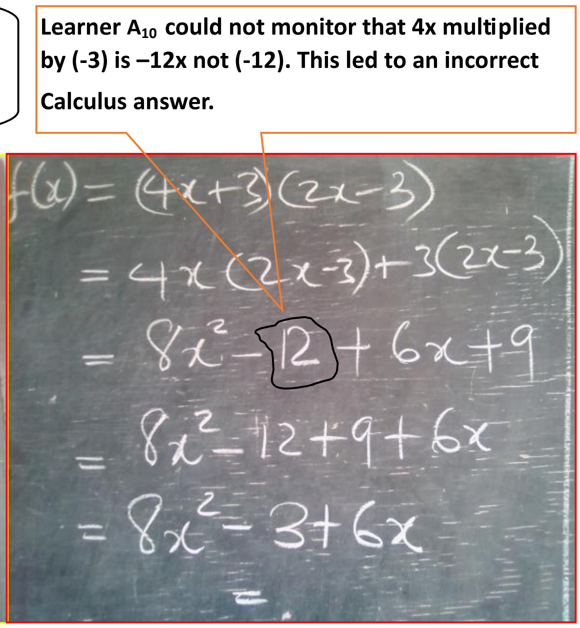

Figure 4.3b: Learner A_{10} problem solving pro-

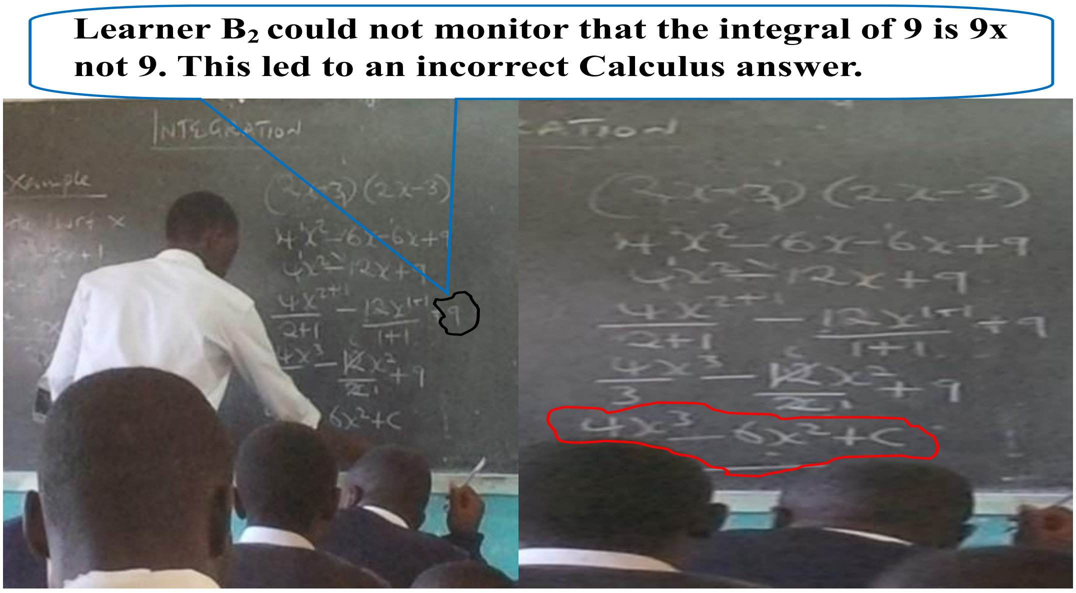

Figure 4.3c: Learner B_2 problem solving process

Also, results from the (FGDs) show that learners experienced difficulties in monitoring each and every step when solving Calculus problems. For example, when I had asked if they monitor their steps during the process of solving Calculus problems, the following excerpts explains learners (A_1 & B_2) stances about monitoring their steps when solving Calculus problems as typical examples:

> *"…….I rarely check my work when I am solving Calculus questions"* **(A_1).** *"Once I start solving the question it means I have to find the solution to the problem. There is no need to check my solution process"* **(B_2).**

In some instances, in line with the same question, learners thought by monitoring the steps whilst working on the problem, they will not be able to finish solving on time. For instance, *"I don't monitor myself Sir due to time and time in Mathematics will always be not enough"* **(A_7).** *"Due to time Sir, I don't"* **(B_5).**

Correspondingly, both teacher A and teacher B seem to have observed that learners do not monitor their steps when solving Calculus problems as evidenced by the following excerpts:

> *"…. I don't think our children monitor their steps when solving Calculus questions"* **(Teacher A).** *That is the challenge these children have, when they are solving they don't even check for simple multiplications of signs. You will find that where it is*

supposed to be a negative sign they will put a positive sign or vice versa" **(Teacher B).**

Sometimes, some learners got stuck when solving Calculus problems as I was observing them. In this case, three (3) of them got stuck as they were working on Calculus problems. When asked during FGDs what are some of the things they do when they get stuck, learners said: *"Asking my friends"* **(A_3).** *"Asking my fellow pupils or my teacher"* **(A_4).** *"Looking back in the book or checking if the formula used is the right one"* **(B_4).**

4.2.3.2. Appropriate Calculus formula

In view of using the required formula or method to solve Calculus problems, results from lesson observations show that learners (18 out of 20) representing (90%) used the rule of differentiation instead of using first principles as required by the question. Although this was the case, learners obtained correct Calculus answers by differentiating each term correctly. The following vignettes for learners (B_6 and B_8) (Figure 4.3d. & Figure 4.3e.) act as typical examples:

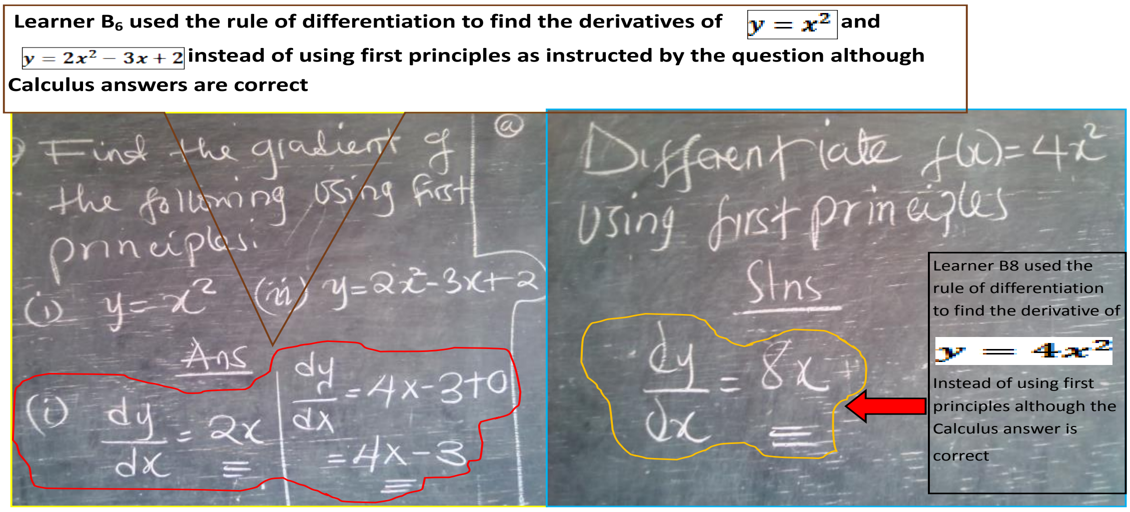

Figure 4.3d: Learner B_6 problem solving processes

Figure 4.3e: Learner B_8 problem solving process

When I had asked why learners opted to use rule of differentiation to Calculus problems instead of using first principles as guided by the question, the following excerpts act as typical evidence:

"I find it a challenge to use the formula for first principle than the rule of differentiation formula" **(A_5).** Also, learner **A_9** explained that: *"The formula for the first principles is too confusing to be used".*

Similarly, both teacher A and teacher B during interviews said that they observed that instead of using first principles as instructed from the main Calculus question, learners opted to use the rule of differentiation formula to solve Calculus problems as evidenced by the following excerpts:

"I have observed that our children do not follow instructions. Instead of using first principles as instructed by the question they end up using the rule of differentiation. Following instructions in Mathematics is important and failure to follow instructions sometimes prompt teachers and even examiners to mark our children's answers wrong even though their answers are correct" **(Teacher A).** *"Our learners are fond of using first principles to solve Calculus problems instead of using the rule of differentiation as instructed by the questions"* **(Teacher B).**

4.2.3.3. Keeping Calculus answers

Results from FGDs show that learners (20 out of 20) representing (100%) kept and saved answers whilst working on Calculus problems as evidenced by the following passages by learner (A_6 and A_8):

"Yes Sir I do keep data because you may use the answer from the first stage to apply it to the second stage of the question" **(A_6).** *"I do save as well Sir and it is important to keep data because mathematics is step by step procedure, so the answer you may find in the first stage can be used in the next stage"* **(A_8).**

Similarly, results from FGDs comprehends with lesson observations I took about learners saving data or results for next part of the questions as evidenced by the following vignettes (Figure 4.3f. & Figure 4.3g.) by learners (A_9 and B_5):

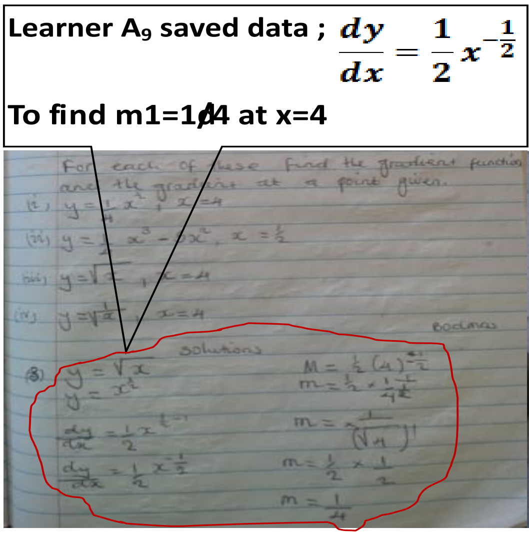

Figure 4.3f: Learner A_9 problem solving process

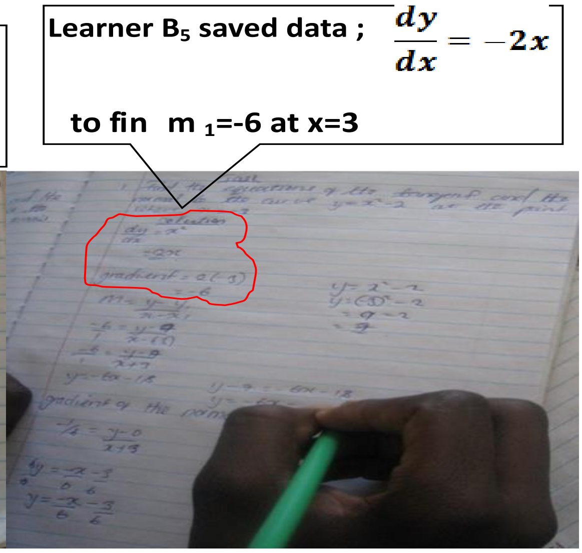

Figure 4.3g: Learner B_5 problem solving process

Keeping and saving data was one thing that was highly mentioned by both teacher A and teacher B more especially when they were teaching on equation of the tangents and normals, gradient function at a particular point (along the $x - axis$) and when finding the coordinates of the points. For instance, in one of the lessons I observed, teacher A and B said:

> "*Finding the gradient function at a point is something that is easy, so what you have to do is, you first find the derivative of a function you have been given, then after you have differentiated whether using the rule of differentiation or power rule formula or by the first principle formula, keep the derivative function (always remember to keep). After you have kept the derivative function, substitute x with the value given (along the $x - axis$)*" **(Teacher A).** "*You have to always remember that after you have found the gradient you have to keep the answer and after you have found the equation of the tangent, remember to use the gradient you had found and substitute it in this formula; $m = \frac{1}{-a}$ or M_1M_2=-1 then use the new gradient value now to find the equation of the normal*" **(Teacher B).**

4.2.4. Evaluating Calculus answers/looking back

In this sub-section of section 4.2.1., the analysis of looking back was characterised by learners' reflective skill in checking an answer after solving and interpreting answers after solving within Polya's (1957) indicators.

4.2.4.1. Checking Calculus answers after solving

Results from lesson observations indicate that all the twenty learners (20 out of 20) that participated in this study experienced difficulties in evaluating derivative results or integral results after solving. In all cases, learners had difficulties to look back to check if the answer they had found was correct or not but showed competence by integrating each term. For instance, the following vignettes (Figure 4.4a. & Figure 4.4b.) by learners (B_2 and B_4) act as typical examples:

Learner B_2 incorrect answer and failed to look back to check if the answer was correct or not. The correct answer could have been; $\frac{4x^3}{3} - 6x^2 + 9x + c$ not $\frac{4x^3}{3} - 6x^2 + c$

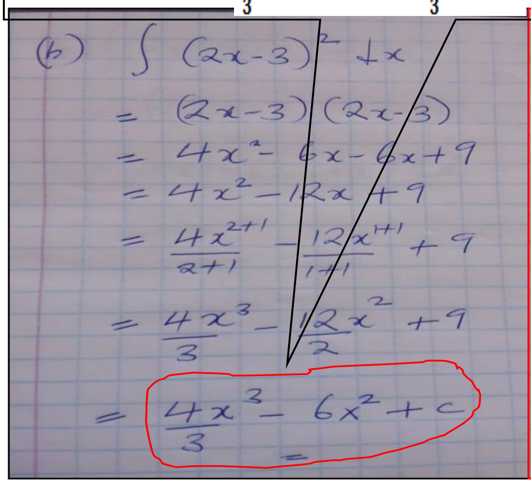

Figure 4.4a: Learner B_2 problem solving process

Learner B_4 incorrect answer and failed to look back to check if the answer was correct or not. The correct answer could have been; $\frac{x^4}{2} - \frac{3x^2}{2} + x$ not $\frac{3x^4}{4} - \frac{3x^2}{2} + x$

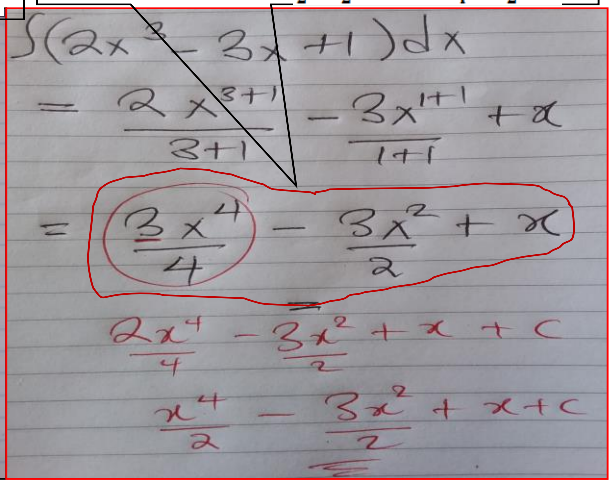

Figure 4.4b: Learner B_4 problem solving process

Correspondingly, results from (FGDs) revealed that learners experienced difficulties in evaluating Calculus answers after solving as evidenced by the following excerpts by learners (B_2, B_4, and B_6) as typical evidence:

> *As for me Sir I have faith in my pen and calculator so I don't evaluate"* **(B_2).** *"No I hope for the best"* **(B_4).** *"I don't evaluate for fear of if I find the right answer I might exchange it with a wrong answer"* **(B_6).**

During (FGDs), I asked them why they were not looking back or evaluating their Calculus answers after solving. However, learners mentioned that they were not taught how to evaluate or prove if their answers were correct or wrong immediately after solving Calculus problems. For instance, the following excerpts by learners (A_3 and B_1) act as typical examples:

"…. We were not told to check if our answers are wrong or correct" **(A_3).** *"Proving hmmmmmm we were not taught"* **(B_1).**

Consequently, when I interviewed the teachers to find out if learners look back after solving Calculus problems, the following excerpts acts as typical examples for teacher (Teacher A and Teacher B):

"Our children do not evaluate answers after solving and this has costed them marks. You will find that as soon as they finish solving, they rarely go back to the question and the answer being found. In fact, what I have observed is that they fail to relate that the inverse of differentiation id integration. So, it is important for them to start evaluating their answers to check if there any mistakes made" **(Teacher A).** *"Hmmmmm!!! Most of them do not. Once they have solved I have realized that they do not check or go back to see if they have made any mistake or not"* **(Teacher B).**

4.2.4.2. Interpreting Calculus answers

Results of the study indicate that very few learners (4 out of 20) representing (20%) interpreted Calculus answers after solving. For instance, during (FGDs), learners cited that they experienced difficulties to interpret Calculus answers after solving as evidenced by the following by learners (A_8 and B_1):

"……I don't interpret Calculus answers after solving" **(A_8).** *"I never thought it's really important to interpret answers after solving. what I know is if it is to calculate I will calculate, if it is evaluate I will evaluate just like that. Interpreting is something I was not doing*" **(B_1).**

This however also conforms with teacher A during interviews who expressed that:

"………. It's very rare to see our children interpret the answers after finding. Maybe it could be as result of chizungu (English)".

4.2. Summary of findings for research question 1

Table (4.1.) for research question 1 on Grade 12 learners' problem solving processes

Processes	Description of the process	Findings
Understanding the problem	1. Reading 2. Reading 3. identifying key words 4. Sketching graphs	1. Learners read Calculus problems. 2. Learners reread Calculus problems. 3. Leaners were not underlining key words despite showing competence of understanding derivative and integral concepts. 4. Learners experienced difficulties in sketching simple graphs despite showing competence of understanding derivative and integral concepts.
Devising a plan	1. Calculus formulas 2. Calculus functions 3. Rough workings 4. Simplifying Calculus problems	1. Learners experienced difficulties in applying Calculus formulas despite showing competence of understanding derivative and integral concepts. 2. Learners devise Calculus equations and showed competence of understanding derivative and integral concepts. 3. Learners devise rough workings. 4. Learners experienced difficulties in simplifying Calculus problems but showed competence of understanding derivative and integral concepts.

Carrying out a plan	1. Monitoring each step when solving Calculus problems 2. Appropriate Calculus formula 3. Keeping track and saving answers	1. Learners were not monitoring each step when solving Calculus problems despite showing competence of understanding derivative and integral concepts. 2. Learners opted to use rule of differentiation instead of first principles despite showing competence understanding derivative and integral concepts. 3. Learners kept track and saved answers and showed competence of understanding derivative and integral concepts.
Looking back	1. Checking Calculus answers after solving 2. Interpreting Calculus answers	3. Learners experienced difficulties in evaluating Calculus answers but showed competence of understanding derivative and integral concepts. 4. Learners experienced difficulties to interpret Calculus answers after solving but showed competence of understanding derivative and integral concepts.

4.3. Challenges encountered by Grade 12 learners in solving Calculus problems

The previous section provided findings on Grade 12 learners' problem solving processes in Calculus as indicated by the first research question. Research question two sought to identify challenges Grade 12 learners encounter in solving Calculus problems. The current section (4.3.) considers five (5) challenges learners encountered in solving Calculus problems. It provides answers to the second research question and all came from respondents. This question was answered through (FGDs) with the learners and semi-structured interviews with the teachers. Vignettes were used to supplement the findings.

4.3.1. Incorrect first principles formula; $\frac{dy}{dx}$ *or* $f'(x)$ *or* $y' = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

Failure to cite the correct formula when working from first principles was one of the mentioned constraint learners encountered in solving Calculus problems. Results from FGDs show that (18 out of 20) representing (90%) of learners had difficulties in citing the correct formula when working from first principles. Instead of writing the correct first principle formula with a negative sign between $f(x+h)$ and (x) ; $\frac{dy}{dx}$ *or* $f'(x)$ *or* $y' = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$, the findings reveal that majority of the learners cited the incorrect first principles with a positive sign between $f(x+h)$ and $f(x)$; ($\frac{dy}{dx}$ *or* $f'(x)$ *or* $y' = \lim_{h\to 0}\frac{f(x+h)+f(x)}{h}$). For instance, the following excerpts give concerns raised by learners (A_1 and A_2) act as typical examples:

> *" ......I have a challenge using the first principles formula. Sometimes I swap the signs Sir. With the power rule formula, I have no problem"* **(A_1).** *"……. I find it a challenge to use the formula for the first principles. I have no problem using the rule of differentiation formula. The rule of differentiation formula is very simple to use"* **(A_2).**

However, learners' responses are also comprehended with the following vignettes during lesson observations (Figure 4.5a., Figure 4.5b., & Figure 4.5c.) for learners (A_1, B_6 and B_7):

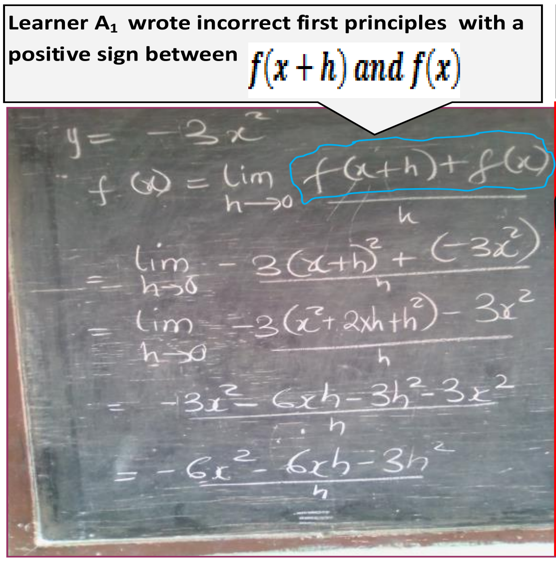

Figure 4.5a: Learner A_1 problem solving process

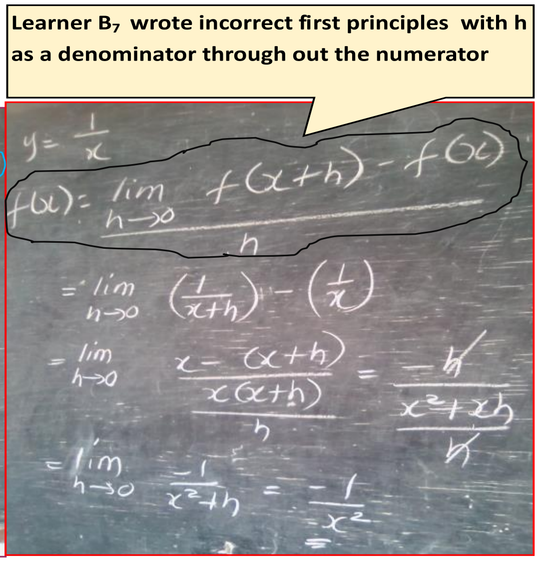

Figure 4.5b: Learner B_7 problem solving process

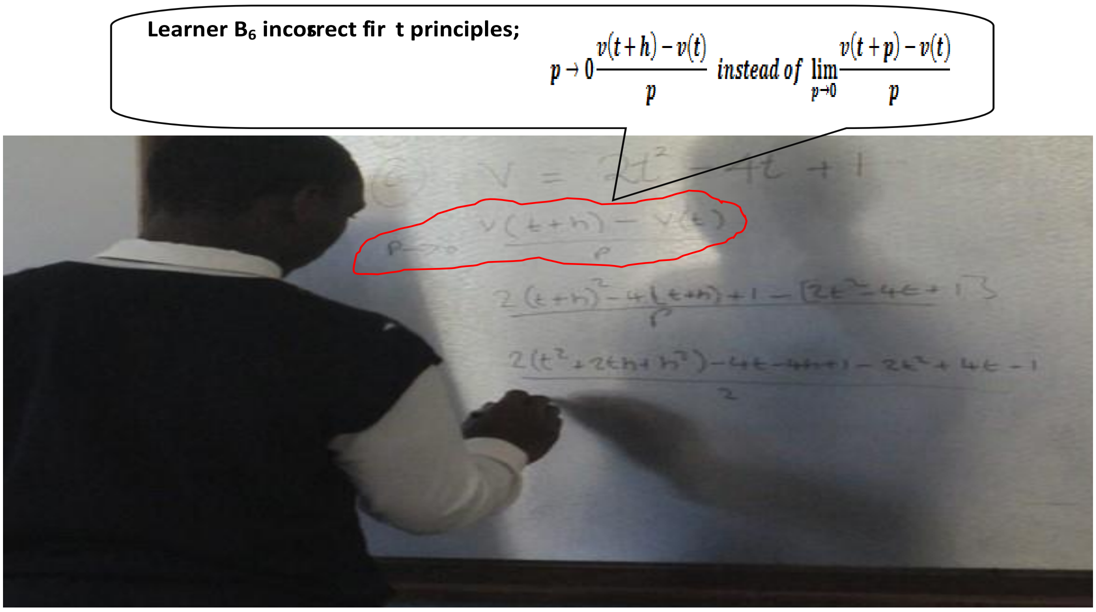

Figure 4.5c: Leaner B_6 problem solving process

Despite learners cited the incorrect first principles, they showed competence of understanding derivative concepts. Furthermore, (FGDs) with the learners revealed reasons why majority of them cited incorrect first principles formula when solving Calculus problems. The following responses by learners (A_5 and B_{10}) acts as typical examples:

> "*We just memorise these formulas Sir*" **(A_5).** *"Our teacher was too fast when teaching us how to solve Calculus problems using first principles"* **(B_{10}).**

4.3.2. Incorrect substitutions between $\mathbf{f(x+h)}$ and $\mathbf{f(x)}$

Results showed that learners experienced difficulties in substituting the variables and coefficients between $f(x+h)$ and $f(x)$ when working from first principles. For instance, learners (17 out of 20) representing (85%) during FGDs mentioned that they had difficulties substituting between $f(x+h)$ and $f(x)$ when working from first principles when solving Calculus problems as evidenced by the following excerpts for learners (A_4 and B_7) as typical examples:

> *"……. The challenge we the learners are facing sir is with substituting between $f(x+h)$ and $f(x)$. I think teachers should be explaining more on how to use the formula Sir"* **(A_4).** *"The main problem is the formula. We are not able to substitute even when given a straight function without a fraction"* **(B_7).**

Similarly, results from lesson observations showed that learners encountered difficulties in substituting the terms between $f(x+h)$ and $f(x)$ as evidenced by the following vignettes (Figure 4.6a., Figure 4.6b., & Figure 4.6c) for learners (A_3, B_1, and B_9) as typical examples:

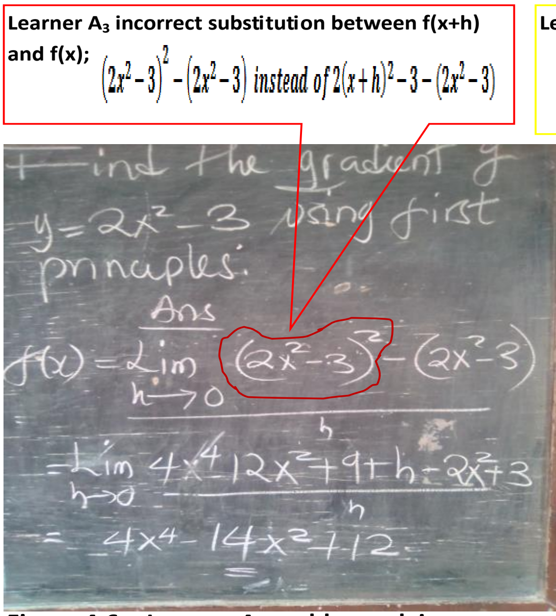

Figure 4.6a: Learner A_3 problem solving process

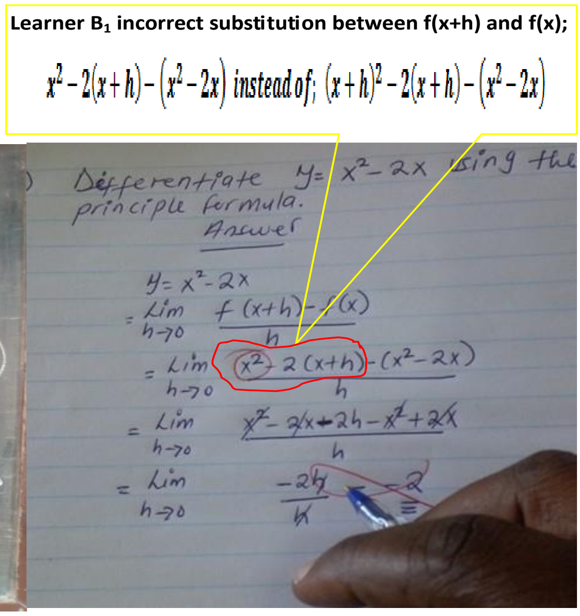

Figure 4.6b: Learner B_1 problem solving pro-

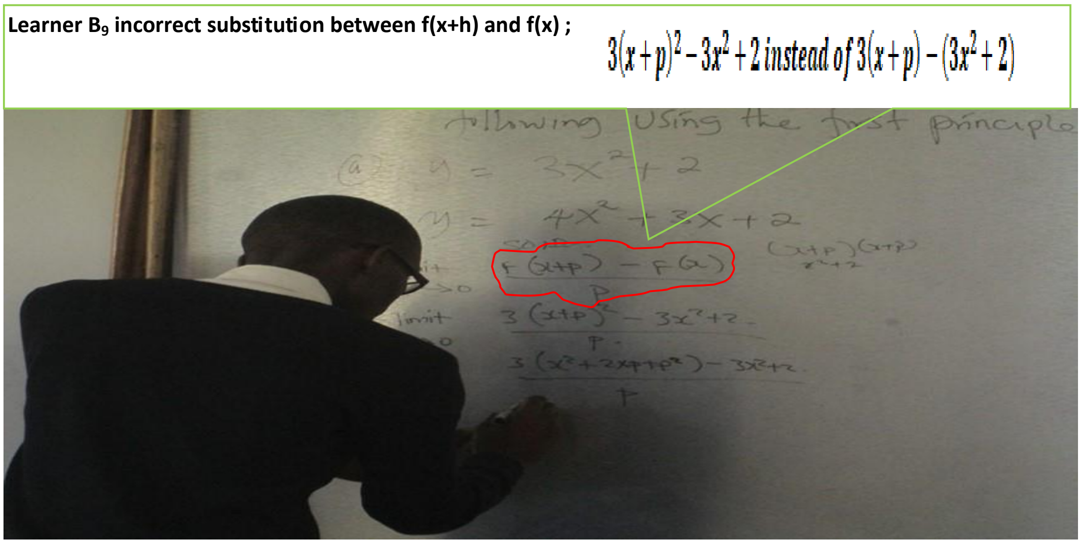

Figure 4.6c: Leaner B_9 problem solving process

In order to understand why learners experienced difficulties in substituting $f(x+h)$ and $f(x)$ when working from first principles, during FGDs, I asked them to explain why they faced such a challenge. The following responses acts as typical examples:

> *"………Due to poor foundation in Mathematics"* **(A6).**"*I didn't understand how to substitute between $f(x+h)$ and $f(x)$ when the teacher was teaching. I didn't understand because the teachers not explain properly and was too fast and too much short-cuts"* **(A5).** *"First principles formula is difficulty to use. We just memorise this formula. This has affected my substitution between $f(x+h)$ and $f(x)$. Its difficulty to find correct answers like this Sir"* **(B4).**

Similarly, teacher A during interviews expressed that:

> *"Some of these children do wrong computation mostly when substituting using the first principle formula……. They can't substitute properly between $f(x+h)$ and $f(x)$.*

4.3.3. Incorrect notation between $f(x)$ and $f'(x)$

Results from FGDs showed that about 14 out of 20 representing 60% of the learners cited having difficulties in noting the difference between $f(x)$ and $f'(x)$ or y *and* y'. For example, learners (A_5 and B_2) during FGDs expressed that:

"……It is very confusing to note the difference between $f(x)$ from $f'(x)$ or y from y'. I think our teacher should be able to emphasise when we have to use $f(x)$ and $f'(x)$. All what we know is Calculus is just $\frac{dy}{dx}$ and when we are solving Calculus problems we go straight to differentiate or integrate" **(A5).** *"……I don't know the difference between $f(x)$ and $f'(x)$. When solving Calculus problems, I write either of the two"* **(B_2).**

Correspondingly, responses from (FGDs) by the learners were also substantiated by the following vignettes during lesson observation for learners (A_{10} and B_3) who treated Calculus symbols to be the same (Figure 4.7a. & Figure 4.7b.):

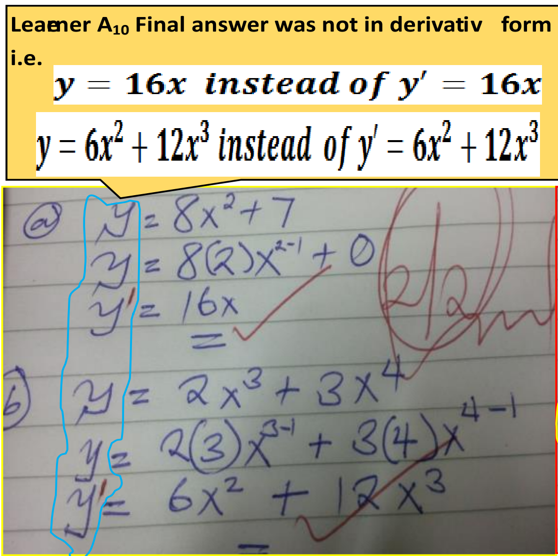

Figure 4.7a: Learner A_{10} problem solving process

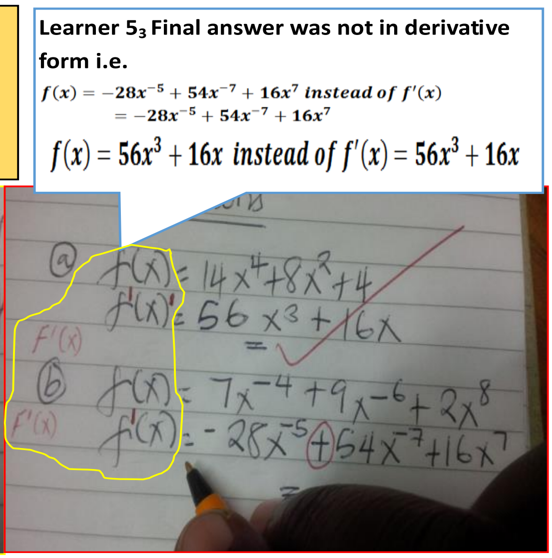

Figure 4.7b: Learner B_3 problem solving process

However, during a class session, both teacher A and teacher B had to emphasise on notation after noting that learners' final answers where not in derivative form but in functional function. This led both teacher A and B to put a prime on (y and $f(x)$) in "red" when marking to show that the Calculus answer was supposed to be in derivative form not in function form as evidenced on vignettes (Figure 4.7a. & Figure 4.7b.).

However, during FGDs, I took interest to find out why most of the learners Calculus answers after solving were not in derivative form but in functional form despite showing competence of differentiating each term correctly (Figure 4.7a. & Figure 4.7b.)? The following responses acts as typical examples:

"Most of these Calculus symbols we are just learning them now" **(A_1).** *"We have never come across these symbols in other topics in Mathematics, this is for the first-time Sir"* **(B_2).** *"Symbols like $\frac{dy}{dx}$ we are just learning and knowing them now, in other topics there are no symbols like these"* **(B_8).**

In a similar view, teacher A during interviews expressed that:

Our learners have a problem with the lettering of the symbols that we use in Calculus. For example, you will just hear learners talking about $\frac{dy}{dx}$ instead of them focusing on the meaning of differentiating with respect to y and x.

4.3.4. Application of Basic Mathematical Concepts

4.3.4.1. Algebraic skills

About (12 out of 20) learners representing 60% cited that applying algebraic concepts was one of a constraint they encountered in solving Calculus problems. During (FGDs) learners cited having difficulties with adding the like terms together or grouping the terms, expanding and simplifying expressions when solving Calculus problems. For instance, the following excerpts by learners (A_3 and B_7) act as typical examples:

".....I have a serious problem with writing the correct expansion, adding unlike terms and multiplying of the signs Sir" **(A_3).** *"……..The challenge I face when solving Calculus questions is when I am given a problem that requires first to expand"* **(B_7).**

However, the following vignettes (Figure 4.8a. & Figure 4.8b.) for learners (A_{10} and B_5) corresponds with the findings from (FGDs) concerning having challenges in applying algebraic skills:

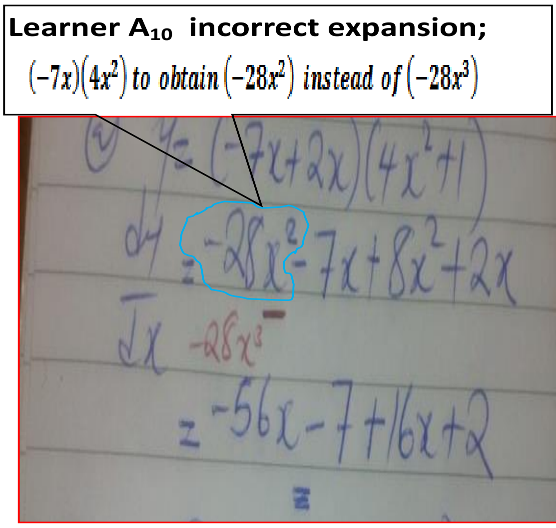

Figure 4.8a: Learner A_{10} problem solving pro-

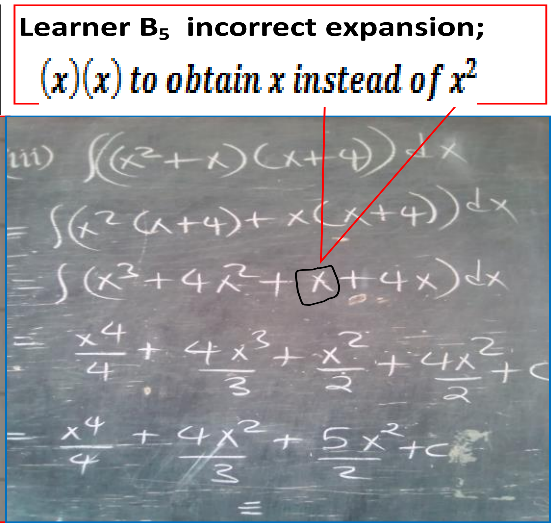

Figure 4.8b: Learner B_5 problem solving pro-

Looking at how learners wrestled with applying algebraic concepts when solving Calculus problems, I asked them during FGDs why they experienced difficulties in applying algebraic concepts when solving Calculus problems, despite showing competence of differentiating and integrating each term (Figure 4.8a. & Figure 4.8b.). Learners expressed that poor foundation in topics like Algebra affected them to apply algebraic concepts as evidenced by the following passages for learners (A_6 and B_2):

> *"…….I have a poor foundation in Mathematics more especially in topics like Algebra"* **(A6).** *"It is due to a poor foundation I have in Algebra. I didn't understand it very well when my former teacher in Grade 10 taught me"* **(B_2).**

4.3.4.2. Laws of indices

Responses from the learners during FGDs revealed that they experienced difficulties in applying the concepts of laws of indices when solving Calculus problems. For instance, 19 out of 20 representing 95% of the learners cited having difficulties with converting fractional indices into power rule and converting surds into power rule as evidenced by the following verbatim:

> *"……For me Sir I have the problem when changing the square root in an index form and the all a lot of it become sour. I think we learnt that in Grade Ten (10) but Hmmmm I couldn't remember how we were changing into an index form when we were learning in class"* **(B6).** *"……The majority of us to my best of my knowledge we have a poor background from primary schools were mathematics concepts were*

not fully understood. This has been a challenge for a lot of us pupils and this makes us fail to understand Calculus problems and solve Calculus problems because you find that for some questions in Calculus we have to apply the laws of indices" **(B_8).**

Consequently, learners' responses during FGDs corresponded with the following vignettes I got during lesson observations where learners had difficulties in tackling Calculus problems involving surds and tackling Calculus problems involving fractional indices as evidenced by the following vignettes (Figure 4.9a. & Figure 4.9b.) by learners (A_8 and B_3) acting as typical examples:

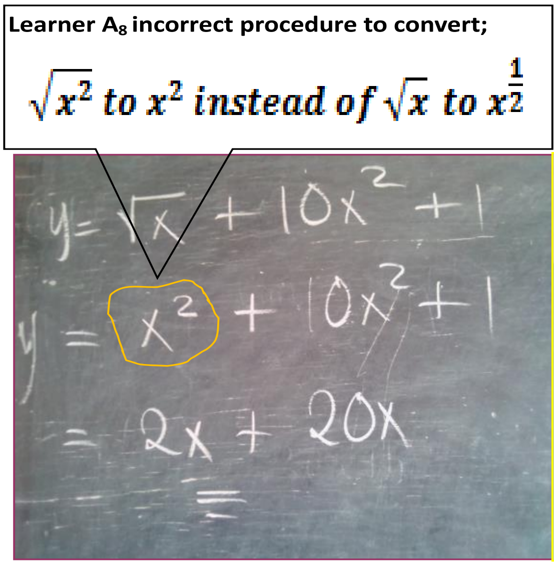

Figure 4.9a: Learner A_8 problems solving process

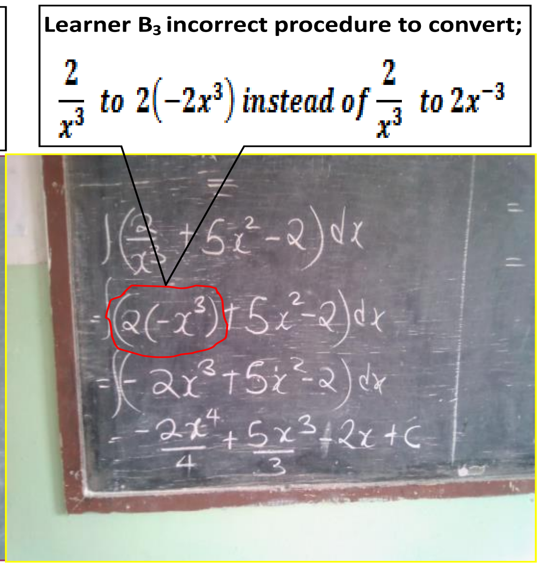

Figure 4.9b: Learner B_3 problems solving process

Similarly, the following vignettes (Figure 4.9c. & Figure 4.9d.) by learners (A_4 and B_{10}) act as typical examples:

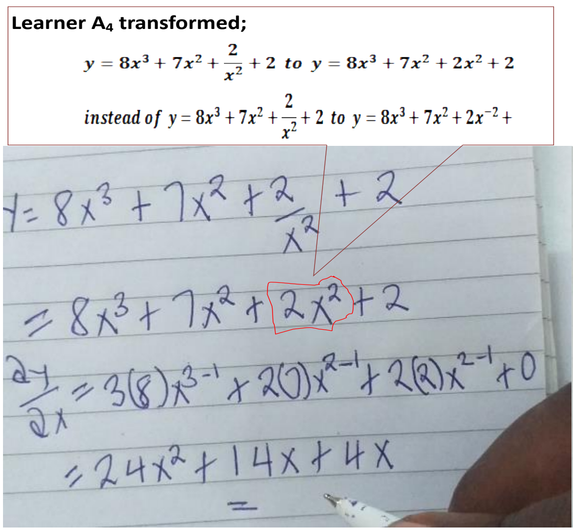

Figure 4.9c: Learner A_4 problem solving process

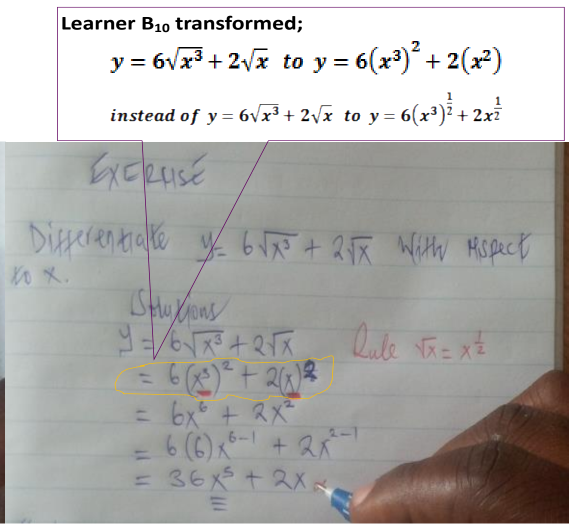

Figure 4.9d: Learner B_{10} problem solving process

During FGDs, I took interest to find out why learners had difficulties with converting fractional indices into power rule and converting surds into power rule despite showing competence of differentiating and integrating each term (Figure 4.9a., Figure 4.9b, Figure 4.9c. & Figure 4.9d.). However, learners brought out several reasons as evidenced below by the following passages:

> *"It's all because of poor foundation in topics like Indices. I remember we learnt in Grade 10"* **(A_1).** *"I did not understand the topic Indices properly in Grade 10. Our teacher when teaching was too fast. This affected my understanding Sir"* **(B_7).**

Consequently, teacher A response during interviews resonated with learners' responses on the applications of the laws of indices affecting their problem-solving skills in Calculus. For instance, during interviews, teacher A explained that:

> *" …. One biggest challenge I have found with our children is relating Calculus to prerequisite knowledge to topics like Indices. This is another area of a topic learners have completely forgotten and they have found Calculus problems more challenging because of lack of simple basics they learnt in Grade 10".*

However, teacher B never expressed any views about applying the laws of indices being a hindrance to learners' problem solving skills in Calculus.

4.3.4.3. Factorization

Learners also cited having difficulties in applying the concepts of factorisation when solving Calculus problems. Majority of the learners (14 out of 20) representing 70% during FGDs complained of having forgotten the concepts of finding the factors (Product and the Sum). For instance, the following verbatim by learners (A_8 and A_9) act as typical examples:

> *Factorising especially when you reach at a certain stage where you need to factorise and simplify it"* **(A_8).** *"I have a problem with factorising first before I differentiate or integrate Sir"* **(A_9).**

Similarly, the vignettes I got during lesson observations also showed that learners encountered difficulties when solving Calculus problems that required simplifying by factorisation as evidenced by the following vignettes (Figure 4.10a. & Figure 4.10b.) for learners (A_7 and A_{10}):

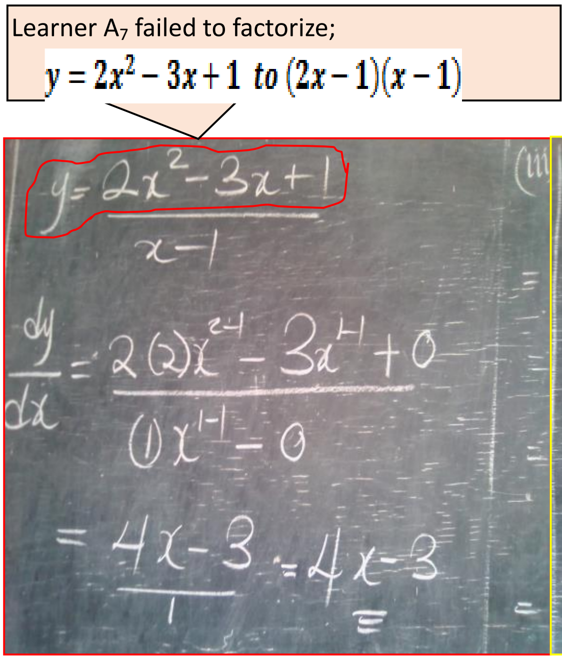

Figure 4.10a: Learner A_7 problem solving process

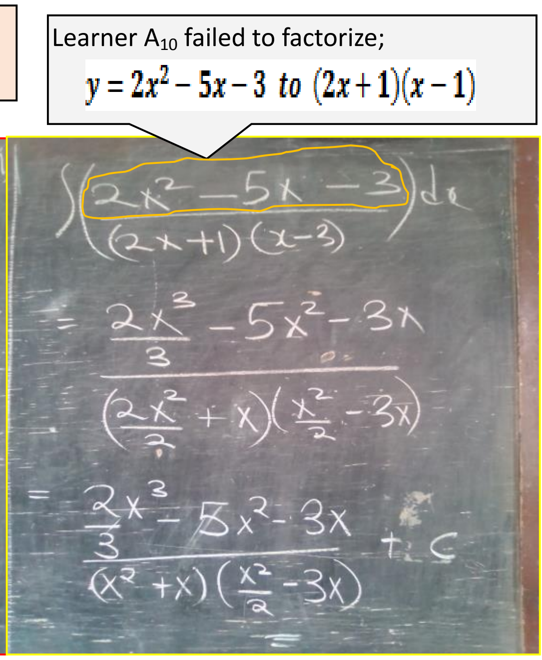

Figure 4.10b: Learner A_{10} problem solving process

However, during FGDs with the learners, I asked them why they experienced difficulties in solving Calculus problems that required simplifying first by factorization despite showing competence of differentiating and integrating each term correctly (Figure 4.10a. & Figure 4.10b.). For instance, learners (A_4 and B_7) expressed that:

> *"Sir a majority of us lack the knowledge of applying what we learnt in other Mathematics topics. This has contributed to wrong answers because we are failing to factorise"* **(A_4).** *"What I can say is that we have poor background knowledge in topics like Factorisation and Algebra"* **(B_7).**

The comments made by the learners regarding having difficulties to factorise when solving Calculus problems corresponded with teacher A during interviews. For instance, teacher A explained that:

> *"Majority of these children have poor background knowledge or pre-requisite knowledge Sir……. Our children they fail to see the interlink between previous topics, they fail to connect the previous topic to the topic at hand, for example when*

you come to differentiation it uses much of factorisation and much more on Algebra. So because of poor background on factorisation and Algebra you will find that learners find it very hard to solve Calculus problems".

However, teacher B never mentioned anything about lack of applying the concepts of factorisations been one of a challenge learners encounter in solving Calculus problems during interviews but during lesson observations, teacher B cautiously urged learners to go back to Grade 10 work and grasp the concept on how to find two factors: *"I see a majority of you have difficulties to factorise and finding the factors. It's like you have completely forgotten the concepts and it shows you don't regularly practice mathematics topics"* **(Teacher B).**

4.3.4.4. Algebraic fractions

Failure to apply concepts of algebraic fractions was also one of the difficulties learners encountered in solving Calculus problems. For instance, 16 out of 20 representing 80% of the learners during FGDs whined having difficulties with fractional problems in solving Calculus problems as supported by learners (A_7, A_9, and B_3) as typical examples:

> "*……. I have a challenge with fractional questions Sir. It is very easy for me to find the derivative or integrate Calculus questions that are not in fractions"* **(A_7)**
> *"……Factional problems are a problem Sir, I fail to differentiate like that"* **(A_9).**
> *"My problem is with fraction questions"* **(B_3).**

Equally, findings from lesson observations show that learners omitted the part of simplifying Calculus problems involving fractions and applied rule of differentiation or rule of integration as evidenced by the following vignettes (Figure 4.11a. & Figure 4.11b.) for learners (A_3 and B_6):

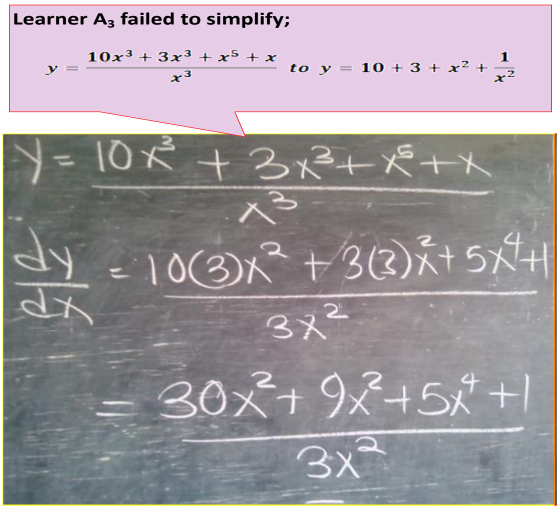

Figure 4.11a: Learner A_3 problem solving process

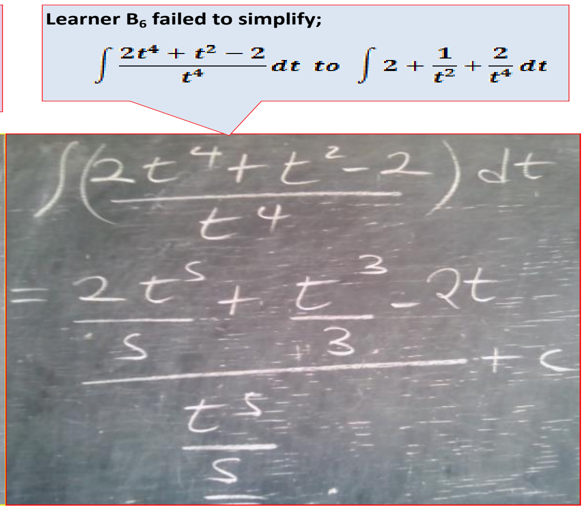

Figure 4.11b: Learner B_6 problem solving process

In order to have an in-depth understanding as to why learners omitted the part of simplifying Calculus problems involving fractions despite showing competence of differentiating and integrating each term (Figure 4.11a. & Figure 4.11b.), learners (A_{10} and B_2) during FGDs expressed that:

> *"I did not understand the topic Algebra very well in Grade 10 when our teacher was teaching. This has contributed to this Sir"* **(A_{10}).** *"It's because of poor background knowledge in topics like Fractions, Algebra, Functions, and Factorisation. These topics we learnt in Grade 10. I think we need to pay attention to these topics Sir"* **(B_2).**

Additionally, teacher A and teacher B expressed that:

> *"……. Most of the learners cannot find derivatives and integrals of functions involving fractions"* **(Teacher A).** *"These learners have challenges with Calculus problems that are in the form of fractions. They are finding it very hard to solve simple algebraic fractions. Also, just adding, subtractions and divisions is really a challenge for them"* **(Teacher B).**

4.3.4.5. Coordinate Geometry

Majority of the learners (16 out of 20) representing 80% during FGDs spoke passionately about having difficulties in applying the concepts of Coordinate Geometry when solving Calculus problems. When asked about the difficulties they encounter when solving Calculus problems, the following excerpts for learners (A_7 and B_7) are the typical examples.

> "*For me I have the problem with applying different concepts to one question like concepts of Coordinate Geometry that I have forgotten"* **(A_7).** *"Failure to remember the concepts of Coordinate Geometry we learnt in Grade 11 Sir"* **(B_7).**

Teacher B during interviews also expressed that:

> *"One such challenges that is coming out so strongly is relating Calculus to prerequisite knowledge more especially connecting to Calculus to topics like Coordinate Geometry you find that some children have forgotten how to handle Coordinate Geometry, they find it very hard even on how to find and the meaning of the gradient…. So, as we try to explain it under differentiation that you can find the gradient using the rules of differentiation or using the first principle formula they will find it very hard to understand. They will think it's completely a new concept other than connecting it to what they already know just the gradient except now it's not the gradient of a straight line but it should be the gradient function because it for a curve".*

Teacher A never expressed any concern about applying the concepts of Coordinate Geometry being a hindrance to learners when solving Calculus problems.

4.3.4.6. Arithmetic skills

About 15 out of 20 representing 75% of the learners during FGDs cited having difficulties in applying simple basic arithmetic signs such as $-X-= +, +X+= +, -X+= -, and + X-= -$ when solving Calculus problems. The following excerpts gives the concern by learners (A_8 and B_6):

> *"……. Sir the issue of multiplying the signs like negative and positive is the challenge I face when solving Calculus questions"* **(A_8).** *"……. I have the with multiplying the signs"* **(B_6).**

Similarly, vignettes I got from lesson observations also show that learners experienced difficulties in multiplying simple arithmetic skills when solving Calculus problems. Learners also showed competence of differentiating and integrating each term. For example, the following vignettes (Figure 4.12a. & Figure 4.12b.) for learners (A_1 and B_3) act as typical examples:

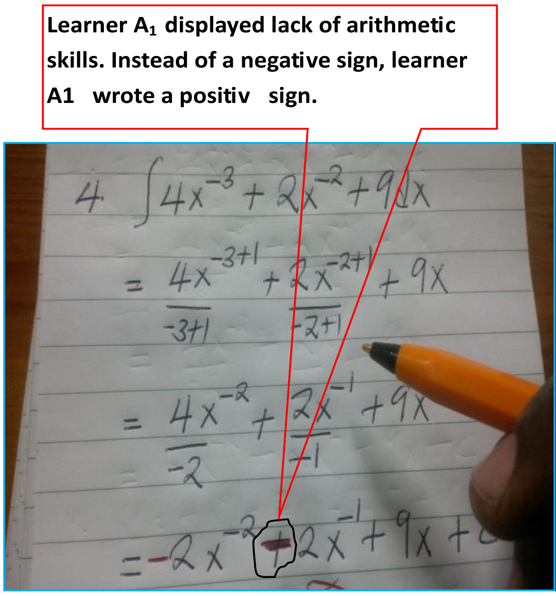

Figure 4.12a: Learner A_1 problem solving process

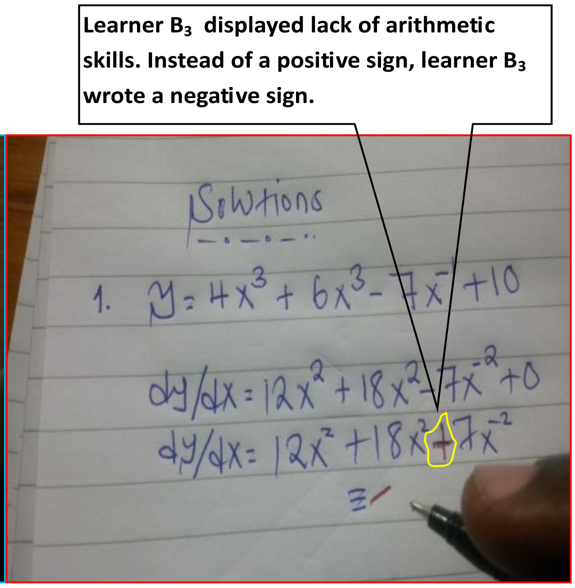

Figure 4.12b: Learner B_3 problem solving process

4.3.5. Teachers teaching approaches

Findings of the study indicate that teachers teaching and solving approaches affected learners' problem solving skills. Learners (18 out of 20) representing 90% expressed that teachers and solving approaches contributed to failure to understand Calculus concepts and procedures to solve Calculus problems as evidenced by the following verbatim by learners (A_4 and A_7):

> *"……Our teacher is very fast when teaching and not willing to answer questions when we haven't understood. This somehow affect our understanding of Calculus concepts. Sometimes when we ask sometimes, he says these are simple problems or that question is simple use your brain or ask your friends. This somehow affect how I find answers"* **(A_4).** *"……. I don't understand the approaches our teacher uses when teaching. When teaching the teacher is fast"* **(A_7).**

4.3.6. Reflective skills

Majority of the learners (17 out of 20) representing 85% expressed having difficulties with evaluating their answers immediately after solving as evidenced the following verbatim for learners A_9 and B_4:

> *"……. I don't evaluate my answers after solving. Once I finish solving that is the end no need to go through. In short Sir, how to prove if my answer is correct is where my problem is"* (A_5). *"The problem I face after solving is how to evaluate afterwards"* **(A_9).** *"Sir how to prove if my answer is correct"* **(B_4).**

4.3.7. Language of Calculus

Study findings indicate that learners deemed language and its grammar as one of a constraint affecting their problem-solving processes in Calculus. Majority of the learners (18 out of 20) representing 90% expressed that they had difficulties in understanding the language of Calculus as substantiated by the following opinions by learners (A_3, B_3, and B_8) which corresponded with the views of learner A_1, A_8, B_5, and learner B_{10}'s views. Arguably, learner A_3, B_3 and B_8 said that:

> *"……The problem I face is lack of grasping what the question is asking me to do"* **(A3).** *"......The way some of the sentences are constructed Sir its really difficulty to understand Calculus questions"* **(B3).** *"……It's quite cumbersome to understand the type of grammar that they use in some questions"* **(B_8).**

Lack of understanding the language of Calculus was also expressed by teacher A. According to teacher A:

> *"…. Our children have the problem of understanding Calculus questions this somehow is affecting them how they solve Calculus. Understanding questions is key in mathematics so if children are failing to read, it becomes a challenge".*

4.3. Summary of findings for research question 2

Table 4.2.: Challenges Grade 12 learners encounter in solving Calculus problems

Challenge	**Description of the challenge**	**Findings**
1. In correct first principle formula	$(\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)+f(x)}{h}),(p \to 0\,\frac{v(t+h)-v(t)}{p}),\ and\ \frac{\lim_{h\to 0} f(x+h)-f(x)}{h}$	-Learners experienced difficulties in citing the correct first principle formula although they showed competence in understanding derivative concepts.
2. Incorrect substitutions	$between\ f(x+h)$ and $f(x)$ when working from $(\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h})$	-Learners experienced difficulties in substituting between $f(x+h)$ and $f(x)$ when working from first principles.
3. Incorrect notations	$f(x)$ and $f'(x)$, $y\ and\ y'$	-Learners experienced difficulties in noting the difference between $f(x)$ and $f'(x)$, $y\ and\ y'$ but showed competence in understanding derivative concepts.
4. Applications of basic mathematical concepts		
i. Algebraic skills	Grouping the terms, expanding and simplifying expressions	Learners experienced difficulties in applying algebraic skills but showed competence in understanding derivative and integral concepts.

ii.	Indices	Negative index, Surds/Radicals, Fractional index, and Power rule	Learners experienced difficulties in applying the laws of indices but showed competence in understanding derivative and integral concepts.
iii.	Factorization	Product and Sum, like terms	Learners experienced difficulties in applying the factorizing skills but showed competence in understanding derivative and integral concepts.
iv.	Fractions	Simplifying, like terms	Learners experienced difficulties in solving Calculus problems in form of fractions but showed competence in understanding derivative and integral concepts.
v.	Coordinate Geometry	Gradient, tangent, normals	Learners experienced difficulties in applying coordinate geometry skills but showed competence in understanding derivative and integral concepts.
vi.	Teachers teaching approaches	Less explanations, Fast, short-cuts	Learners experienced difficulties in understanding teachers teaching approaches because they explained less and were too fast.
vii.	Reflective skills	Checking, interpreting	Learners experienced difficulties in evaluating Calculus problems but showed competence in understanding derivative and integral concepts.
viii.	Language of Calculus	Terminology terms like limits, derivatives, integrate etc.	Lack of understanding Calculus terms affected learners’ problem solving skills in Calculus.

4.4. Suggested Strategies to improve learners' problem solving skills in Calculus

While the preceding section presented findings on challenges, this section (4.4.) gives the way forward to address the challenges identified in line with the third research question. Research question number three tried to establish inventory measures to surmount the challenges Grade 12 learners encounter in solving Calculus problems. This question was answered by both teachers' through interviews and learners through FGDs. The seven measures were reported by the participants during FGDs and interview item to suggest ways in which learners' problem solving skills in solving Calculus problems can be improved. This was also supplemented by the findings from the Mathematics lessons I observed using the video camera.

4.4.1. Teachers teaching approaches

Majority of the learners (18 out of 20) representing 90% during FGDs suggested that giving enough explanations by their teachers can improve problem-solving skills in Calculus. For instance, in the following quotes, learners (B_1 and B_4) of which their views coincided with learners (A_1, A_2, A_3, B_6, and B_9) expressed that:

> *"…. When teaching, teachers should be explaining so that we the pupils can understand Calculus concepts and our teacher should be avoiding shortcuts when solving Calculus questions"* **(B_1).** *"On my point of view Sir, we need a more simplified and a more comprehensive teaching method that will help us understand Calculus concepts at the beginning of the topic"* (**B_4).**

Consequently, teacher A during interviews suggested that in order to improve learners' problem solving skills in solving Calculus, teachers should be patient when teaching Calculus so that learners can understand Calculus concepts. The following is teacher A excerpts in verbatim form:

> *"….. A teacher surely must take time systematically in an organised manner and bring about concepts that depend on each other in the processes of teaching Calculus. So, the teacher must be mentally prepared to give more explanations*

when teaching and off course internalise all the concepts that he or she is about to teach so that the learners can understand".

Additionally, teacher B during face to face interview said:

"Teachers when teaching Calculus not only to rely much on teacher centred method and not to be fast when teaching".

4.4.2. Supplying Calculus text books

Despite learners did not cite that lack of learning materials such Calculus textbooks was a hindrance in solving Calculus problems, learners (16 out of 20) representing 80% during FGDs cited that, having more Calculus texts books in their school library can improve their problem-solving skills in Calculus. For instance, in the following excerpts, learners (A_4 and B_2) expressed that:

"I think our skills in Calculus can be improved if our school library can be stocked with Calculus books where we can be referring to and practice on our own at times" **(A_4).** *"....Our library should be stocked with more books on Calculus. We lack books on Calculus, I think if more books on Calculus are available, we can be researching, practicing on our own and solve more Calculus problems"* (**B_2**).

Correspondingly, teacher A and teacher B during interviews expressed that:

"Government should start supplying more Calculus textbooks for the purpose of reference for us teachers and for the children to study" (**Teacher A**). *"Government through the MOGE has to start stocking quality and quantity number of Calculus textbooks for reference and studying"* (**Teacher B**).

4.4.3. Pre-requisite knowledge in pre-calculus topics

Majority of the learners (19 out of 20) representing 95% recommended that having a strong background knowledge in topics like Indices, Functions, Coordinate Geometry, Algebra, fractions, factorisation can improve their problem-solving skills in Calculus. The following quotations for learners (A_3 and B_6) provided below act as typical examples:

> *"…I think Sir one needs to understand topics like Factorisation and Indices"* **(A_3).**
> *"By studying it different composition such as Coordinate Geometry, Factorisations and others"* **(B_6).).**

The same understanding was held by both teacher A and teacher B during interviews who expressed great concern on pre-requisite knowledge. However, teacher A spoke highly about learners' having pre-requisite knowledge. The following excerpts are provided as typical examples:

> *"……I think from my vast experience, one of the challenge one would be lack of pre-requisites because a topic like Calculus I feel there are prerequisites which should be looked briefly before teaching the topic. I know these kids at Grade 10, even at Grade 9 they learn Indices and by the time they reach Grade 12 a lot of them may have forgotten the concepts. So, it is very important before introducing or teaching Calculus that such prerequisites topics such Indices and algebra should be tackled first briefly off course for maybe a period or even the full period then the next period you begin teaching Calculus. Prerequisites knowledge in Indices we are talking about are powers or exponents, surds and so on because they are involved in Calculus so if we briefly go over these topics then we can make their understanding faster and easier.* **(Teacher A).** *".... One good strategy is that learners should have a strong background in algebra. Teachers should be emphasizing on the importance of algebraic skills and how important are. I have noticed that the majority of the pupils have serious problems with applying the rules of indices and the concepts of factorisations and in order to improve their skills in Calculus, I think topics like Indices and functions also are key because algebraic skills is what drives mathematics"* **(Teacher B).**

4.4.4. Understanding the language of Calculus

Learners (17 out of 20) representing 85% during FGDs spoke passionately about the difficulties posed by lack of understanding the language of Calculus when solving Calculus problems. Learners during FGDs spoke zealously about understanding the language of Calculus in order to improve their problem-solving skills in Calculus. For instance, the following verbatim by learners (A_2 and B_9) act as typical examples:

> *"……. Since the first step when solving in any mathematical question, you have to read and understand the question, then you develop a plan, so as for me I think reading and understanding the question and also coming up with a plan in that way you will have a clear picture of the kind of the question you are solving"* **(A_2).** "*I think by understand Calculus questions before starting to solve Calculus problems our solving skills can be improved. Once we understand the grammar been used and have a culture of reading Calculus problems before starting to solve Calculus problems, we can improve in Calculus Sir"* **(B_9).**

Similarly, teacher A during interviews expressed that:

> *"I feel one of a strategy that can improve our children's problem-solving skills in Calculus can be through understanding the Calculus questions. It's like most of us our children just start solving Calculus problems without understanding what the problem is saying this becomes very hard for them to solve most the questions".*

Teacher B however, never expressed any concern towards understanding Calculus problems been one of a strategy that can improve learners' problem solving skills in Calculus.

4.4.5. Homework

Majority of the learners (14 out of 20) representing 70% during FGDs recommended that in order for their problem-solving skills in Calculus to be improved, teachers on daily basis should be giving them homework. For instance, the following excerpts are typical examples by learner (A_1 and B_7):

> *"I feel one strategy in order to be in Calculus is when teachers start giving us homework daily so that when we are at home, we can be going through as part of revisions"* **(A_1).** *"By giving us more homework because the majority of us pupils take homework as for revisions"* **(B_7).**

Similarly, teachers (A and B) during interviews regarded giving homework as a good strategy that can improve learners' problem solving skills in Calculus as evidenced on the following verbatim:

> "*Teachers should have a practice of giving homework at least once in a week or even twice in a week the reason for this is that the very questions that will be for homework will be a continuation of what the teacher has been doing in class. This will help our children learn new Calculus solving techniques So, you know these children once they learn at school they close books and when they reach at home they don't do anything at all so the teacher really must be on them so that you know...... when certain concepts in mathematics are practiced continuously these children tend to internalise the concepts and there after they don't have challenges. But one problem I have seen with our pupils is that they do not have time to practice at home they are so playful and get involved in other vices which may not be good for them"* **(Teacher A).** "*Strategy can be giving them more homework's so that their minds are preoccupied and their mind will be preoccupied with work to do work to do, the more they do the topic becomes the language in them because they will do more"* **(Teacher B).**

4.4.6. Collaborative work

Learners (17 out of 20) representing 85% during FGDs spoke highly about solving Calculus problems with capable peers in order to improve their problem-solving skills in Calculus. The following verbatim are typical examples:

By solving different Calculus questions with our friends Sir" **(B_8).** *"Asking our friends and teachers to help us understand Calculus concepts where we did not understand can be of he3p*" **(B_{10}).**

Teacher B during interviews also recommended group work as one of a good strategy that can improve learners' problem solving skills in Calculus as evidenced by the following verbatim:

".... One of them should be introducing more of group work so that they do more interactions among themselves in that, if they learn from each other in a more simplified language when a friend is explaining maybe even in vernacular it can make them understand easier what really is the concept here. So, maybe we need to do more of group work where we encourage more learner centred lessons where a teacher just facilitates and we encourage them to research on their own before the lesson".

Teacher A never suggested group work as a strategy that can improve learners' problem solving skills in Calculus.

4.4.7. Introducing Calculus symbols in early grades

In view of experiencing difficulties with Calculus symbols, learners (19 out of 20) representing 95% during FGDs cited that Calculus symbols such as $\frac{dy}{dx}, f'(x) or\ y', \lim h \to 0$ should be introduced in early Grades in order to improve their problem-solving skills in Calculus as evidenced by the following excepts for learners (A_2 and B_5) acts as typical examples:

"Hmmmmmm Sir, I think symbols like $\frac{dy}{dx}$ should be introduced in previous grade. In that way we will be good in Calculus **(A_2).** *"…….I think Calculus symbols should be introduced in early grades system so that lets say for example you are in Grade 8 or 10, you begin from there knowing these symbols by the time you get there in Grade 12, you have knowledge"***(B_5).**

4.4. Summary of findings for research question 3

Table 4.3.: Suggested measures to overcome the challenges

Strategies	**Description of the strategy**	**Findings**
Teachers teaching approach	Explaining, asking questions, avoiding short-cuts,	Comprehensive teaching approach can improve learners' problem solving skills in Calculus.
Calculus textbooks	Availability, quantity, quality	Supply of Calculus textbooks for reference purposes can improve learners' problem solving skills in Calculus.
Pre-requisite knowledge in pre-calculus topics	Indices, Algebra, Functions and Relations, Coordinate Geometry, Factorisation etc.	Pre-requisite knowledge in pre-calculus topics can improve learners' problem solving skills in Calculus.
Understanding the language of Calculus	Limits, derivative, integrate, preliminary	Understanding of the language of Calculus can improve learners' problem solving skills in Calculus
Homework	Assignments	Giving learners homework on daily basis can improve learners' problem solving skills in Calculus.
Collaborative work	Ideas, knowledge, working together	Solving Calculus problems with capable peers can improve learners' problem solving skills in Calculus.
Introducing Calculus symbols in early grades	$\frac{dy}{dx}, f'(x) or\ y', \lim h \to 0$	Introducing Calculus symbols in early grades can improve learners' problem solving skills in Calculus.

4.5. Summary of the chapter

This chapter presented findings from the participants on the learners' problem solving processes in Calculus at Grade 12 level, challenges they encounter, and suggested measures to overcome the challenges as presented on Table 4.4 below:

OBJECTIVE 1: GRADE 12 LEARNERS' PROBLEM SOLVING PROCESSES	
Process	**Description of the process**
Understanding the problem	-Reading, Rereading, Identifying key words, and Sketching graphs
Devising a plan	-Calculus formulas, Calculus functions, Rough workings, Simplifying Calculus problems
Carrying out a plan	-Monitoring, Appropriate Calculus formula, and Keeping track and saving answers
Looking back	-Checking and Interpreting Calculus answers
OBJECTIVE 2: CHALLENGES GRADE 12 LEARNERS ENCOUNTER IN SOLVING CALCULUS PROBLEMS	
Challenge	**Description of the challenge**
Incorrect first principle formula	$(\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h)+f(x)}{h}), (p \to 0 \frac{v(t+h)-v(t)}{p}), and \frac{\lim_{h \to 0} f(x+h)-f(x)}{h}$
Incorrect substitutions	$between\ f(x+h)$ and $f(x)$ when working from $(\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h})$
Incorrect notations	$f(x)$ and $f'(x)$, $y\ and\ y'$
Algebraic skills	Grouping the terms, expanding and simplifying expressions
Indices	Negative index, Surds/Radicals, Fractional index, and Power rule
Factorization	Product and Sum, like terms

Fractions	Simplifying, like terms
Coordinate Geometry	Gradient, tangent, normals
Arithmetic skills	Multiplying signs ($-X + OR + X -$)
Teachers teaching approaches	Less explanations, Fast, short-cuts
Language of Calculus questions	Terminology
OBJECTIVE 3: SUGGESTED MEASURES TO IMPROVE PROBLEM SOLVING SKILLS IN CALCULUS	
Strategies	**Description of the strategy**
Teachers teaching method	Explaining, asking questions, avoiding short-cuts
Calculus textbooks	Availability, quantity, quality
Pre-requisite knowledge in pre-calculus topics	Indices, Algebra, Functions and Relations, Coordinate Geometry, and Factorisation
Understanding the language of Calculus	Terminologies such as Limits, derivative, integrate
Homework	Assignments
Collaborative work	Sharing ideas, knowledge, skills, strategies, and working together
Introducing Calculus symbols in early grades	$\frac{dy}{dx}, f'(x) or\ y', \lim h \to 0$

Having looked at presentations of findings, the next chapter discusses these findings.

CHAPTER 5

DISCUSSION OF FINDINGS

5.1. Introduction

This chapter discusses the findings as presented in chapter 4. Just like in the previous chapter, the discussion will be done in subsections. Section 5.2. discusses learners' problem solving processes in Calculus within Polya's four problem solving processes and their indicators. Sections 5.3. and 5.4. provide discussions on the constraints learners' encounter in solving Calculus problems and measures respectively. Section 5.5. summarises the chapter.

Research question 1: What are Grade 12 learners' problem solving processes in problems?

5.2. Grade 12 learners' problem solving processes

5.2.1. Understanding Calculus problems

This sub-section provides the discussions of understanding of Calculus problems within Polya's (1957) first stage of problem solving with their indicators. These included reading Calculus problems, re-reading Calculus problems, underlining key words, and sketching graphs.

5.2.1.1. Reading Calculus problems

The study revealed that learners read Calculus problems before solving. Learners during FGDs cited that they read Calculus problems before solving in order to understand the problem (Section 4.2.1.1.). Learners responses are in harmony with Gestalt theory of problem solving by Wertheimer (1959) who asserted that reading a mathematical problem before solving helps problem solvers to understand the problem at hand (See section 1.11. for detail.). In view of this, it can be argued that actively reading a Calculus problem supports learners to make sense of the problem at hand and subsequent understanding of the terms such as differentiate, integrate, limit etc. Moreover, Polya (1957) has argued that it is foolish to answer a question that you have not read and you do not understand (See section 2.6.1. for detail). He further argued that it is important to read mathematical questions before solving for "understanding purposes" (Polya, 1957, p.23).

Although learners read Calculus problems before solving, results have also shown that reading Calculus problems did not guarantee success in solving Calculus problems (Section 4.2.1.1). This finding is in accordance with the guru of problem solving process Polya (1957) who has argued that reading a mathematical problem does not guarantee success in tackling the question. In a similar view, Hiebert and Carpenter (1992) stated that "any individual task can be performed correctly without understanding" (p. 89). In view of this, Polya (1957) argued that it is important to read mathematical questions before solving for "understanding purposes" and get the solution correct" (p.25). Consequently, Verschaffel and his colleagues (2000) argued that students who take the necessary time and energy to understand the text and develop a situation model are likely to solve the problem but that does not guarantee success. In fact, learners' vignettes (4.2k., 4.2l., 4.2o., 4.2p. etc.) provide adequate evidence that reading does not guarantee success, because despite learners saying they read Calculus problems before solving, their Calculus answers were wrong.

5.2.1.2. Re-reading Calculus problems

The study revealed that learners reread Calculus problems before solving. During FGDs, learners expressed that the idea of rereading Calculus problems was to locate important pieces of information and to understand the main goal of the questions (Section 4.2.1.2.). Learners responses are in harmony with Polya (1962) who contended that when students reread a mathematical problem, they consolidate their knowledge and develop their ability to solve the problem (See section 2.6.1. for detail). Available literature has also shown that when tasks contain unfamiliar terminology or more words than typically seen in translating problems, students need to adequately reread the problem in order to understand and make sense of the problem (Polya, 1957; Schoenfeld, 1992). Moreover, there is some evidence that students' who reread problems before solving have higher chances of solving the problem and find a sensible or reasonable answer (Verschaffel et al., 1999), and success on problem-solving also depends on rereading the problem before solving (De Corte et al., 2000).

Additionally, learners during FGDs expressed that they reread Calculus problems many times whenever they did not understand Calculus problems the first time they reread (Section 4.2.1.2.). Besides, learners during FGDs also expressed that the purpose of rereading Calculus problems many times was to make sure that they fully understand Calculus problems before solving as they feared making silly mistakes when working. In nearly all cases, when learners read Calculus problems for the first time and did not understand, they continued to reread until they either understood the problem or decided to skip it (Rose, 1991).

5.2.1.3. Underlining key Calculus words

The study found out that learners experienced difficulties in underlining key important words such as integrate, differentiate, tangents, coordinates, normals, and limits before solving (Figure 4.1a. & Figure 4.1b.). Rose (1991) does affirm to this observation in her study that students do not underline key important words to help them understand mathematical problems. While the finding of this study resonates with the study done by Rose (1991), the current study established three (3) reasons why learners experienced difficulties in underlining key important words before solving, and these were: (1) their teachers were not underlining as well (Figure 4.1g. and Figure 4.1h.), (2) they did not know that underlining key terms before solving is important (Section 4.2.1.3.), and (3) they were not told by their teachers that it is important to underline key words before solving.

Although learners experienced difficulties in underlining key words before solving, results have shown that some learners (35%) found correct derivative and integral results and applied correct derivative and integral concepts (Figure 4.1a. & Figure 4.1b.). Similarly, learners (65%) who experienced difficulties in underlining key words showed competence of understanding derivative and integral concepts despite getting incorrect Calculus answers (Figure 4.1c. & 4.1d.). Montague (2005) argued that it is possible to fail to identify key information from a mathematical question and yet get a correct answer. In fact, Polya (1957) argued that some mathematical problems contain unnecessary information, therefore it is important that problem solvers take keen interest in identifying important information before solving. Despite that being the case, Montague (2005) further advised that even though it is possible to fail to highlight key information from a mathematical question and yet get a correct answer, problem solvers in Mathematics are encouraged to first identify what is important and what is not important from a mathematical question as that would enhance their understanding of the problem and identify the formula to use. However, literature has also shown that underlining or highlighting key words is designed to help students restate the mathematical problems in their own words, therefore strengthening their comprehension of the problem (Cunningham & MacGregor, 2014; Polya, 1957). As Polya (1988) notes, highlighting key important words on mathematical questions is key to finding correct solutions and identifying the right formula to use (See section 2.6.1. for detail). As simple as that sounds and easily stated by the guru of problem solving, highlighting key words is often the most overlooked step in the problem solving process, yet it is fundamental for learners to first highlight key important words before solving in order to understand what they are being asked to find out or solve and for easy identification of Calculus formulas.

5.2.1.4. Sketching graphs

The study established that learners experienced difficulties in sketching simple graphs, in order to help them understand Calculus concepts before solving when finding the minimum and maximum points (Section 4.2.1.4.). Abd (2004) does affirm that learners find it a challenge to sketch graphs when finding the minimum and maximum points. Besides, Newman (1983) argued that "Graphic material is often considered to be an aid in Mathematics, but for some children it is an extremely difficult task to interpret the implied meanings. Moreover, even findings from lesson observations shows that learners experienced difficulties in sketching simple graphs, in order to help them understand Calculus concepts (Figure 4.1e.). Looking at how learners struggled to sketch simple graphs to aid their understanding of Calculus problems, there is need for learners to understand basic concepts of Coordinate Geometry when finding the gradient, stationary points, equations of tangents and normals which is normally learnt in Grade 11. Although this was the case, learners showed competence of understanding derivative and integral concepts during problem solving (Figure 4.1e.). As noted by Orton (1983a), an elementary grasp of what is meant by stationary points, turning points, minimum points, maximum points and points of inflexion, and the nature of the gradient or rate of change at such points, may all be obtained by graphing.

5.2.2. Devising Calculus plans

This sub-section provides the discussions of devising Calculus plans within Polya's (1957) second stage of problem solving with the indicators. These included devising Calculus formulas, devising Calculus functions, showing rough workings, and simplifying Calculus problems.

5.2.2.1. Writing Calculus Formulas

Results of this study show that learners experienced difficulties in writing Calculus formulas before solving whether working from the rule of differentiation or working from the rule of integration (4.2a., 4.2b., Figure 4.2c. & Figure 4.2d.). Learners' during FGDs cited that they were not writing Calculus formulas because their teachers too were not writing Calculus formulas before solving (Figure 4.2g. & Figure 4.2h.). In all the video recordings which I took, I observed that teachers too were not citing Calculus formulas before solving. This somehow influenced learners not to devise Calculus formulas before solving. Although results have shown that learners had difficulties in citing Calculus formulas before solving, some learners' (2 out of 18) Calculus answers were correct (4.2a. & 4.2b.). This showed that learners understood derivative and integral concepts. In addition, even the learners who had difficulties in citing Calculus formulas before solving (16 out of 20) and got incorrect Calculus answers showed competence of applying the concepts of derivative and integral concepts correctly (Figure 4.2c. & Figure 4.2d.). In view of this finding, it can be argued that citing a mathematical formula in general and Calculus formula in particular before solving can give problem solvers an advantage to obtain correct solutions. This claim is supported by Polya (1957) who said devising a mathematical formula to use guides how the problem will be solved and how the final answer will be found (See section 2.6.2. for detail). Hence the argument that failure by the learners to devise Calculus formulas before solving is a clear indication that they lack procedural knowledge (See section 2.7. for detail).

5.2.2.2. Writing Calculus Functions

Findings from observations showed that learners who devised Calculus functions before solving and got correct Calculus answers (Figure 4.2i. & Figure 4.2j.). Devising Calculus functions before solving resonate well with Polya (1957) who said that extracting the function from the main mathematical question not only will it help you to identify the formula to use, but also gives you confidence in tackling the problem of any situation (See section 2.6.2. for detail). In fact, learners' concerns towards the importance of devising Calculus functions before solving conforms with the findings from observations and learners' responses during FGDs (Section 4.2.2.2.). Pedagogically, this means that devising Calculus functions before solving shows how systematics a problem can be solved because it is the starting point of actual problem solving process.

Besides, findings also showed that some learners were not writing Calculus functions before solving and got incorrect Calculus answers (4.2k. & 4.2l.). Therefore, despite learners were not displaying Calculus functions before solving, learners showed competence of understanding derivative and integral concepts because they were able to apply these concepts correctly despite their answers being wrong (4.2k. & 4.2l.). However, literature across the globe has shown that failure to extract an equation from the main question before solving sometimes led to incorrect solutions (Krulik & Rudnick, 19987). In view of this, it can be submitted that the equation can then be developed from the translation of the problem through different designs e.g. translating a problem into picture form and from that the equation can then be developed (Florida Department of Education, 2010).

5.2.2.3. Showing rough workings

The findings revealed that learners extensively relied much on rough workings before solving Calculus problems (Figure 4.2m. & 4.2n.). In view of this finding, Polya (1957) emphasized the need to show some rough workings as one of a strategy before solving (See section 2.6.2. for detail). Besides, Schoenfeld (1992) pointed out that rough workings provide evidence of thinking process used to solve mathematical problems. Correspondingly, through rough workings, problem solvers can manage to identify the correct formula to use, the method or strategy to use, and how the problem will be tackled (Pape, 2004). In fact, it can be argued that it is beneficial to maintain rough workings because it gives the problem solver the direction on how a Calculus problem will be solved. In addition, although not all Calculus problems will need some rough workings, albeit to also argue that rough workings are a key to obtain the correct derivative or integral result and they might help problem solvers find mistakes made while checking on the rough sheet or paper before, during and after solving.

5.2.2.4. Simplifying Calculus problems

Research findings have shown that learners experienced difficulties in simplifying Calculus problems before solving them (Section 4.2.2.4.). Although the learners experienced difficulties in simplifying Calculus problems before solving, learners showed competence of applying derivative and integral concepts even if their Calculus answers were wrong (Figure 4.2o., Figure 4.2p., Figure 4.2q., & 4.2r.). However, failure by the learners to simplify Calculus problems before solving led to incorrect derivative and integral results as evidenced by the vignettes which I captured during lesson observations (Figure 4.2o., Figure 4.2p., Figure 4.2q., & 4.2r.). I can infer that failure by the learners to simplify Calculus problems before solving shows that they could not apply their prior knowledge of simplifying algebraic expressions. My assertion is supported by Nalube (2014) who argued that if learners are unable to simplify expressions and solving equations; constructing equations from given problems; and interpreting from context into algebraic language, it is a clear indication that they lack prior knowledge in Algebraic concepts and this could be due to errors, difficulties or misconceptions they might have. In addition, Polya (1957) said omitting the process of simplifying is a common thing by most problem solvers. However, if problem solvers are not sure about the connection between the known data and the unknown data, it is important to break the problem into smaller pieces (See section 2.6.2. for detail).

The current study further revealed several reasons why learners experienced difficulties in simplifying Calculus problems before solving them that include: (1) forgetting how to simplify, (2) not been taught how to simplify by their teachers, and (3) forgetting concepts of Algebra (Section 4.2.2.4.). Moreover, the study further found out that some learners showed competence of simplifying Calculus problems before solving but got incorrect Calculus answers (Figure 4.2s. & Figure 4.2t.). In view of these findings, Schoenfeld (1992) stated that it is possible to break the problem into simple parts and yet fail to obtain the correct solution. Therefore, I can argue that systematic problem solving process in my opinion often involves step-wisdom meaning knowing that the best way to solve a particular Calculus problem if it is too complex, it has to be broken up into a series of logical steps, rather than to try to solve it all at once because solving a complex Calculus problem at once can lead to an incorrect Calculus result or solution.

5.2.3. Carrying out Calculus plans

This sub-section provides the discussions of carrying out of Calculus plans within Polya's (1957) third stage of problem solving with the indicators. These included monitoring each step when solving Calculus problems, applying the appropriate Calculus formulas, and keeping and saving Calculus answers.

5.2.3.1. Monitoring

Research findings indicate that learners experienced difficulties in monitoring their steps when solving Calculus problems (Figure 4.3a., Figure 4.3b., & Figure 4.3c.). This led to incorrect Calculus answers. This observation aligns with Mugisha (2012) who observed that, learners experienced difficulties when monitoring their steps during the process of solving. However, during FGDs, learners cited that, they were not monitoring their steps when solving Calculus problems because; they believed that monitoring each step would lead to failure to finish solving on time (Section 4.2.3.1.). In light of this, Kilpatrick et al., (2001) stated that sometimes the mistakes students make when solving mathematical problems is possibly due to time factor. He further contended that students fear that going through the mathematical problem whilst solving would result into not finishing solving the problem. Although this was the case by the learners, Polya (1988) advised that it is important to check each step during the process of solving, than worrying much on time (See section 2.6.3. for detail). Despite the results showing that learners experienced difficulties in monitoring their steps during the process of solving, some learners showed competence of applying the correct procedure to solve Calculus problems and understood derivative and integral concepts (Figure 4.3a., Figure 4.3b., & Figure 4.3c). Based on these findings, it can be argued that failure to monitor each step when solving Calculus problems could be as a result of lack of metacognitive skills (See section 2.8. for detail). The study further established that when learners got stuck whilst working on Calculus problems, they asked their fellow pupils, teachers and sometimes looked back in their Mathematics books (Section 4.2.3.1.).

5.2.3.2. Use of Appropriate Calculus formula

In view of using the required formula or method to solve differential problems, results have shown that learners opted to use the rule of differentiation than first principles (Figure 4.3d. & Figure 4.3e.). In line with this finding, the guru of problem solving Polya (1957) argued that it is very rare for most problem solvers to stick to the appropriate formula required by the questions, but instead they end up using a different formula (See section 2.6.3. for detail). During FGDs, learners submitted that working from the rule of differentiation was easy than working from first principles (Section 4.3.3.2.). During interviews, teachers also expressed a concern that some learners opted to use the rule of differentiation instead of first principles (Section 4.3.3.2.). Learners also showed competence of understanding derivative and integral concepts because they differentiated each term correctly (Figure 4.3d. & Figure 4.3e.). This finding is not consistent with the study done by Rose (1991) who found that students followed the instructions of the questions. Although their Calculus answers were correct, learners needed to use the suggested method required by the question because Mathematics is about following instructions (NCTM, 2010; NRC, 1989). In view of this, it can be argued that failure by the learners to use the suggested method as required by the question can lead to loss of marks despite the answer being correct. In fact, teachers during interviews also expressed a concern that failure to follow instructions by the learners when solving Calculus problems could lead to loss of marks and could prompt examiners during examinations to mark the answers wrong (Section 4.2.3.2.).

5.2.3.3. Keeping Calculus answers

The study showed that the majority of learners kept and saved answers whilst working on Calculus problems. When finding the equations of the tangents, normals, and gradient functions, during lesson observations, it was observed that learners saved Calculus answers for the next part of the question (Figure 4.3f & Figure 4.3g.). In fact, during FGDs with the learners, they said they kept Calculus answers during the process of solving (Section 4.2.3.3.). Similarly, keeping and saving Calculus answers was highly emphasized by their teachers during interviews especially when they were teaching on equation of the tangents and normals, gradient function at a particular point (along the $x - axis$) and when finding the coordinates of the points (Section 4.2.3.3.). Thus Polya (1957) adds that during the process of carrying a mathematical problem, problem solvers should be keeping track of what they have solved because you can use what has been kept for the next part of the problem (See section 2.6.3. for detail). As advocated by Cockcroft (1982), keeping and saving data whilst solving problems is useful to show other people what they have or had done and it is also helpful in finding errors, should the right answer not be found. Although learners kept Calculus answers whilst solving, it has been established that most of the kept Calculus answers were incorrect (Figure 4.3g.). However, it can be argued that the fundamental cause of incorrect solutions by learners could be as a result of lack of arithmetic skills such as multiplying four basic mathematical signs i.e. $-X-$ which is equal to a positive sign, $-X+$ which is equal to a negative sign, $+X+$ which is equal to a positive sign, and $+X-$ which is equal to a negative sign.

5.2.4. Evaluating Calculus answers/Looking back

This sub-section provides discussions of looking back within Polya's (1957) fourth stage of problem solving with the indicators. These included checking Calculus answers and interpreting Calculus answers.

5.2.4.1. Checking Calculus answers after solving

The study has established that learners experienced difficulties in checking if their Calculus answers were correct or not after solving (Section 4.2.4.1.). The challenge was that they were not aware that they should reflect on their solutions for mistakes immediately after solving. This contributed to incorrect Calculus answers (Figure 4.4a. & Figure 4.4b.). Besides, during interviews, teachers also expressed a concern that their learners were not evaluating or looking back immediately after solving (Section 4.2.4.1.). This finding aligns with Gestalt theory by Wertheimer (1959) who asserted that the step of looking back after executing the plan is challenging to problem solvers as it requires insight. Although learners experienced difficulties in checking if their Calculus answers were correct or not, results also showed that learners were able to expand Calculus problems and applied derivative and integral concepts correctly (Figure 4.4a. & Figure 4.4b.).

Based on this finding, I can therefore infer that failure by the learners to look back or evaluate Calculus answers could be as a result of lack of metacognitive skills (Kilpatrick et al., 2001)

(See section 2.8. for detail). Hence the argument that learners lack this type of skill. However, during FGDs, learners expressed that they experienced difficulties to look back because of the following factors: (1) they were not told by their teachers, that looking back after solving is important, (2) they had a misunderstanding that looking back would take much of their time and they would be unable to attend to other Calculus questions on time, (3) they did not see the need to recheck their Calculus answers after solving, (4) they expressed that they have faith in their ball point and calculator, and (5) they said they hoped for the best (Section 4.2.4.1.). In view of this, Polya (1945) argued that a good teacher should understand and impress on his/her students with the view that no problem whatsoever is completely exhausted. There remains always something to do; with sufficient study and penetration, we could improve any solution, and, in any case, could always improve our understanding of the solution" (p. 15) (See section 2.6.4. for detail).

5.2.4.2. Interpreting and reporting Calculus answers

Results indicated that very few of the learners interpreted Calculus answers after solving. During FGDs, learners expressed that immediately after solving Calculus problems, they were not attaching the meaning to most of the Calculus solutions such as on stationary points, inflexion points, maximum points and minimum points (Section 4.2.3.2.). This finding aligns with Verschaffel and his colleagues (2000) who found that learners do not attach meanings especially on mathematical word problems. Correspondingly, research has shown that students find it difficult to attach the meaning of the problem after solving and often perform poorly (Pape, 2004). Nevertheless, interpreting results implies examining the outcome from an implemented strategy, thinking about what it means, and reflecting on the result's reasonableness given the problem's context (Verschaffel et al., 2000), yet many ineffective problem solvers offer the outcome from a completed strategy as the final solution without interpreting it or reflecting on its appropriateness (Gordon, 2004). In view of this finding, it can be argued that interpreting Calculus problems require re-examining the Calculus answers obtained and determining the solution's reasonableness given the problem's context.

Research question 2: What challenges do Grade 12 learners encounter in solving Calculus problems?

5.3. Challenges encountered by Grade 12 learners in solving Calculus problems

5.3.1. Incorrect first principle formula; $\frac{dy}{dx}$ *or* $f'(x)$ *or* $y' = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

Regarding citing the correct formula when working from the first principles, learners had serious challenges in citing the correct formula when working from first principles (Section 4.3.1.). Literature across the globe as shown that the correct first principle formula is $\frac{dy}{dx}\, or\, f'(x)\, or\, y' = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ (Makonye, 2011; Orton, 1983b). However, when working from first principles, it was observed that some learners wrongly cited first principles with a plus sign; ($\frac{dy}{dx}\, or\, f'(x)\, or\, y' = \lim_{h \to 0} \frac{f(x+h)+f(x)}{h}$) instead of a minus sign; $\frac{dy}{dx}\, or\, f'(x)\, or\, y' = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ between $f(x+h)$ and $f(x)$ (Figure 4.5a.). Other incorrect first principles which learners cited during lesson observations included; $\frac{\lim_{h \to 0} f(x+h)-f(x)}{h}$ instead of; $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ (Figure 4.5b) and $p \to 0 \frac{v(t+h)-v(t)}{p}$ instead of; $\lim_{p \to 0} \frac{v(t+p)-v(t)}{p}$ (Figure 4.5c.). This observation was in consonant with the study done by Makonye (2011) as well as the study done by Makgakga and Makwakwa (2016) who found out that some of the learners used the positive sign instead of the negative sign in $y' = \lim_{h \to 0} \frac{f(x+h)+f(x)}{h}$ when working from first principles (See section 2.11. for detail). Although some learners experienced difficulties in citing the correct first principles formula, results showed that they had understood derivative concepts (Figure 4.5a, Figure 4.5b., & Figure 4.5c.). Teachers also during interviews expressed concerns that some learners experienced difficulties in citing the correct first principles (Section 4.3.1.).

Although the study findings were in coherence with the studies done by Makonye (2011) and Makgakga and Makwakwa (2016), the studies done by Makonye (2011) and Makgakga and Makwakwa (2016) did not establish incorrect first principles formulas such as; $p \to 0 \frac{v(t+h)-v(t)}{p}$ *, and* $\frac{\lim_{h \to 0} f(x+h)-f(x)}{h}$ learners cited when solving Calculus problems which the current study established (Figure 4.5a, Figure 4.5b., & Figure 4.5c.). In addition, the current study further established the reason why some learners cited the wrong formula when working from first principles. During FGDs, learners expressed that citing the wrong first principles was due to memorizing the formula (Section 4.3.1.). Based on the learners' concerns, I can therefore argue that memorizing first principles could be as a result of how first principles was introduced by teachers in the classrooms. When introducing first principles, it was observed that teachers could not show how first principles is derived. Hence, learners failed to link the concept of first principles and how first principles is used to solve derivative functions. Thus, they could not carry out differentiation successfully. Besides, learners also said that, failure to cite the correct formula when working from first principles was as a result of their teachers' teaching approaches. In addition, some learners during FGDs expressed that they did not understand because their teachers were too fast when teaching them how to solve derivative functions using first principles (Section 4.3.1.). Learners concerns conforms with Herbert (2012) who contended that some learners lacked knowledge of using the correct formula suggesting a lack of advanced mathematical thinking which could have been as the result of the method of teaching and learning of Calculus.

Arguably, literature has shown that failure to cite the correct formula when working from first principles for some learners for example, putting a plus instead of a minus required to denote the *infinitesimal* change in the mantissa showed that Grade 12 learners were fixed to the addition operation because that was the first sign they learnt at school (Davies, 1984, p.23). Therefore, failure to recall or cite the correct formula could be as a result of lack of procedural knowledge (Kilpatrick et al., 2001; Herbert, 2011) (See section 2.7 for detail).

5.3.2. Incorrect substitutions between $\boldsymbol{f(x+h)}$ and $\boldsymbol{f(x)}$

The results of this study had established that learners experienced difficulties in substituting the terms between $f(x+h)$ and $f(x)$ when working from first principles; $\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ (Section 4.3.2.). As a result, incorrect substitution between $f(x+h)$ and $f(x)$ by the learners led to incorrect derivatives (Figure 4.6a., Figure 4.6b. & Figure 4.6c.). With regards to incorrect substitutions, teachers also during interviews expressed concern that some learners experienced difficulties in substituting between $f(x+h)$ and $f(x)$ when working from the first principles formula (Section 4.3.2.). Literature reviewed has also shown that students encounter some difficulties in substituting between $f(x+h)$ and $f(x)$ when working from the first principles formula (Makgakga & Makwakwa, 2016) (See section 2.9. for detail). Although some learners experienced difficulties in substituting, results have also shown that learners showed competence of understanding derivative and integral concepts (Figure 4.6a., Figure 4.6b. & Figure 4.6c.).

Despite the findings of this study are in coherence with reviewed literature (Makgakga & Makwakwa, 2016), the current study further established three reasons why learners experienced difficulties in substituting between $f(x+h)$ and $f(x)$ when working from first principles: (1) teachers when teaching how to substitute between $f(x+h)$ and $f(x)$ were too fast and taught in short-cut ways, (2) poor foundation in pre-calculus topics, and (3) memorising first principles instead of understanding how the formula is developed and operates (Section 4.3.2.). In light of this, it is fair to argue that these three (3) reasons greatly affected learners' problem-solving skills in Calculus which contributed to incorrect derivatives. I can also assert that the competences of learners in Calculus hinge upon the mathematical constructs of substitutions and variable, more especially when working from first principles. The ability to substitute is key competence in Calculus as it enables learners to manipulate and handle variables by substituting them with others whenever a Calculus function links them.

In addition, I can postulate that incorrect substitutions by some Grade 12 learners such as; $\lim_{h\to 0}\frac{\left(2x^2-3\right)^2-(2x^2-3)}{h}$ instead of $\lim_{h\to 0}\frac{2(x+h)^2-3-(2x^2-3)}{h}$ (Figure 4.6a.; $\lim_{h\to 0}\frac{x^2-2(x+h)-(x^2-2x)}{h}$ instead of $\lim_{h\to 0}\frac{(x+h)^2-2(x+h)-(x^2-2x)}{h}$ (Figure 4.6b.), and $\lim_{h\to 0}\frac{3(x+h)^2+2-3x^2+2}{h}$ instead of $\lim_{h\to 0}\frac{3(x+h)^2+2-(3x^2+2)}{h}$ (Figure 4.6c.) could be as a result of weak procedural knowledge and conceptual knowledge (See section 2.7. for detail). This could mean that some Grade 12 learners lacks both procedural knowledge and conceptual knowledge. However, if Grade 12 learners had these types of knowledge, probably they could have substituted the terms between $f(x+h)$ and $f(x)$ correctly when working from first principles without any challenge and apply the concepts of the formula to solve derivative functions and find reasonable answers. This proclamation was drawn from the incorrect substitutions displayed by the learners.

5.3.3. Incorrect notations between $f(x)$ and $f'(x)$

The study has showed that some learners had serious difficulties in citing the correct notations between $f(x)$ and $f'(x)$ when solving Calculus problems (Section 4.3.3.). During lesson observations, most of the learners' Calculus answers were not in derivative form after solving (Figure 4.7a. & Figure 4.7b.). Failure to note the difference between (x) and $f'(x)$ can lead to led to incorrect Calculus answers. As a result, failure by the learners to note the difference between (x) and $f'(x)$ prompted teachers to add prime in red ink on learners Calculus answers (Section 4.3.3.), after acknowledging that learners had issues with taking note the difference between $f(x)$ and $f'(x)$. This result is not in agreement with a study done by (Orton, 1983a) who noted that students had difficulty in dealing with differentiation symbols and could not interpret the meaning of $\frac{\delta y}{dx}$ or $\frac{dy}{dx}$ notations (See section 2.8. for detail*).* Orton's (1983a) study is not in agreement with the current study because his students had difficulties with interpreting the meaning of $\frac{\delta y}{dx}$ or $\frac{dy}{dx}$ whilst in the current study learners had difficulties noting the difference between $f(x)$ and $f'(x)$. Moreover, □$f(x)$ in Calculus implies the function is in functional form whilst $f'(x)$ in Calculus implies the function is in derivative form. Although learners lacked the ability to note the difference between $f(x)$ and $f'(x)$, results have shown that some learners understood the concepts of derivatives and obtained correct Calculus answers because they differentiated each term correctly (Figure 4.7a. & Figure 4.7b.).

Failure to take note of the difference between $f(x)$ from $f'(x)$ need to be taken very seriously by Grade 12 learners because much of Calculus is communicated through symbols, and one cannot understand and do Calculus without understanding its symbolism because mathematical knowledge is abstractly represented via written symbols. In Vygostskian terms, symbols are also tools that signify concepts as well as the oil that lubricates mathematical discourse with and across persons (Vygotskian, 1987). In Vygotskian terms, symbols are both signs and tools because they communicate mathematical concepts and help learners to think about mathematical ideas. In addition, it can be also argued that failure by some Grade 12 learners to note the difference between $f(x)$ and $f'(x)$ could be as the result of lack of procedural knowledge (Evan & Lappan, 1994**)** (See section 2.7 for detail). As argued on Sub-section 2.7 in chapter 2, these symbols are representations of concepts and failure to write symbols correctly shows lack of procedural knowledge.

The current study further established three reasons why learners failed to note the difference, $f\ (x)\ and\ f'(x)$, and understanding Calculus symbols such as $\frac{dy}{dx}$. The first reason was that learners argued that Calculus symbols such as $\frac{dy}{dx}$, y' $f'(x)$ and $\int f(x)\ dx$ are never learnt in topics of Mathematics. Secondly, learners cited that never at one point in their previous grades did they came across these symbols. Thirdly, learners cited that their teachers were not emphasizing about the difference between $f\ (x)\ and\ f'(x)$ (Section 4.3.3.).

5.3.4. Application of Basic Mathematical Concepts

5.3.4.1. Algebraic skills

The study has established that learners encountered difficulties in applying algebraic concepts such as adding the like terms together or grouping the terms, expanding and simplifying expressions when solving Calculus problems (Figure 4.8a. & Figure 4.8b.). This finding is aligned to Dlamini (2017) who argued that in order to be good in Calculus, students should be good in algebraic skills (See 2.9. for detail). However, during FGDs, learners submitted that they experienced difficulties in applying algebraic skills when solving Calculus problems due to poor foundation in topics like Algebra (Section 4.3.4.1.). Learners concerns are supported by Nalube (2014) who argued that you cannot perform well in Mathematics if you have poor foundation in key topics like Algebra. She further went on to say that Algebra is the anchor to all Mathematics topics. Arguing in line with Nalube’s (2014) assertion, it can be therefore argued that incorrect adding of the like terms together or grouping the terms, expanding and simplifying expressions when solving Calculus problems revealed that some learners had poor pre-requisite knowledge in Algebra. In other words, this meant that learners could not link their previous mathematical knowledge which they learnt in Algebra to the already existing knowledge. As a result, majority of the learners found incorrect derivative results and incorrect integral results despite understanding derivative and integral concepts despite showing competence of differentiating and integrating each term correctly (Figure 4.8a. & Figure 4.8b.). My argument is supported by Gestalt theory of problem by Wertheimer (1959) under reproductive thinking which advocates for problem solvers to have prior knowledge before beginning to learn a new topic in Mathematics (See section 1.11. for detail).

5.3.4.2. Indices

The study established that learners encountered difficulties in applying the laws of indices when solving Calculus problems (Section 4.3.4.2.). It was observed that learners had great difficulties in converting fractional indices into the power rule and surds into the power rule (Figure 4.9a., Figure 4.9b., Figure 4.9c., & Figure 4.9d.). By far this was one of the most prominent challenge of the learners on this item and learners' deficiencies in mathematical operations hampered their learning of Calculus that led to incorrect Calculus answers (Figure 4.9a., Figure 4.9b., Figure 4.9c., & Figure 4.9d.). Despite the results showing that learners had difficulties in applying basic laws of Indices, some learners showed competence of understanding the derivative and integral concepts because they were able to differentiate and integrate each term (Figure 4.2.4.2a., Figure 4.2.4.2b., Figure 4.2.4.2c., & Figure 4.2.4.2d.). In this view, Makonye (2011) in his study also observed that students were found to have numerous errors in Algebra, such as failing to apply laws of indices (See section 2.9. for detail). On the contrary, the study done by Makonye (2011) did not establish that learners also experience difficulties when converting surds into natural indices i.e. $\sqrt{x}$ to $x^{\frac{1}{2}}$ which the current study established (Figure 4.9a. & Figure 4.9d.).

In addition, the current study had established some reasons as to why learners experienced difficulties in converting fractional indices into the power rule and surds into the power rule. During FGDs, learners expressed that they had difficulties in converting fractional indices into the power rule and surds into the power rule because of (1) poor background knowledge in a topic like Indices and (2) they did not understand Indices when they learnt it in Grade 10 (Section 4.3.4.2.). Looking at how learners wrestled with applying the laws of Indices to solve Calculus problems, it meant that some learners had poor prior knowledge and could not hope to perform well in problem situation if by themselves could not diligently remember the laws of indices which they learnt as far back as Grade 8 (Index Notation) and in Grade 10 (Indices). In fact, the need to apply prior knowledge to solve mathematical problems has been highly emphasized by Gestalt theory of problem solving under reproductive thinking (See section 1.11. for detail).

5.3.4.3. Factorization

The results of the study indicated that learners had difficulties in applying factorization concepts when solving Calculus problems (Figure 4.10a. & Figure 4.10b.). Despite learners showing difficulties in simplifying Calculus problems by factorization, results had also shown that learner shad competence of applying derivative and integral concepts (Figure 4.10a. & Figure 4.10b.). However, teachers also during interviews expressed that their learners experienced difficulties in applying concepts of factorization when solving Calculus problems (Section 4.3.4.3.). However, learners showed competence of differentiating and integrating each term despite their Calculus were incorrect (Figure 4.10a. & Figure 4.10b.). Based on the findings from lesson observations, FGDs and interviews, it can be argued that failure by the learners to apply the concepts of factorisations clearly enlightens that they completely burry off their mathematical concepts learnt in previous topics. In Zambia for instance, a topic like Factorization in the O-level Mathematics Syllabus is normally taught and learnt in Grades 8-11 and Grade 12 learners are expected to apply Factorization concepts without any challenge. Consequently, the DBE (2012) in South Africa reported that learners experienced difficulties in solving Calculus problems that required to be simplified first by factorization (See section 2.9. for detail). However, the current study established reasons why learners experienced difficulties in solving Calculus problems that required applying factorisation skills or concepts. Learners during FGDs expressed that they failed to apply factorisation skills or concepts when solving Calculus problems due to poor background knowledge in previous topics like Factorisation (Section 4.3.4.3.). Moreover, teachers also expressed concern that Grade 12 learners failed to see the interlink between previous topics to the topic at hand (Calculus) when differentiating Calculus that requires the concepts of Factorization (Section 4.3.4.3.).

5.3.4.4. Fractions

Research findings indicate that learners experienced difficulties in solving Calculus functions involving fractions (Section 4.3.4.4.). In nearly all cases, during lesson observations, learners applied the rules of differentiation or rules of integration to both the numerator and denominator without simplifying Calculus functions involving fractions. This led to incorrect Calculus answers (Figure 4.11a. & Figure 4.11b.). This finding suggest that failure by the learners to simplify algebraic fractions led to inappropriate differentiation and integration.

Although learners seemed to have experienced difficulties with Calculus problems that were in the form of fraction, they showed competence of differentiating and integrating each term correctly (Figure 4.11a. & Figure 4.11b.). This observation was aligned to the study done by Makgakga and Makwakwa (2016) who found that learners experienced difficulties when given Calculus functions involving fractions of the variable (See section 2.11. for detail).

During FGDs, learners brought out two important reasons why they encountered difficulties in solving Calculus problems that were in the form of fractions. One of the reason was due to poor background knowledge in topics like Algebra, Fractions, and Functions and the second reason was that they hardly understood these topics when they learnt them in their previous Grades (Section 4.3.4.4.). Based on these findings, it can be fairly argued that lack of algebraic manipulation and poor prior knowledge on algebraic fractions are the reasons why learners encountered difficulties in solving Calculus problems involving fractions. In other words, it shows that Grade 12 learners have a weak link relationship with algebraic fractions. In Zambia, Algebra is learnt in Grades 8-12. My argument is also supported by Gestalt theory that advocates for prior knowledge when solving mathematical problems. Moreover, teachers during interviews also said, one of the reasons why learners experienced difficulties in solving Calculus problems that were in the form of fractions was as a result of poor background knowledge in previous topics of Mathematics (Section 4.3.4.4.).

5.3.4.5. Coordinate Geometry

Findings established that learners exhibited inadequacies in applying Coordinate Geometry concepts when solving Calculus problems (Figure 4.1e., 4.3f, & 4.3g.). Failure to apply Coordinate Geometry concepts contributed to incorrect derivative results. Similarly, teachers during interviews mentioned that learners had difficulties of relating Calculus to pre-requisite knowledge more especially connecting of Calculus to topics like Coordinate Geometry (Section 4.3.4.5.). This however meant that learners had poor prior knowledge on topics like Coordinate Geometry. In addition, it can be also be argued that a topic like Calculus hinges heavily on Coordinate Geometry and it is very difficult to understand Calculus concepts copiously without the knowledge of Coordinate Geometry. Hence the argument that failure by the learners to link previous concepts to the one at hand could be as a result of poor reproductive thinking as argued by Wertheimer (1959) the guru of Gestalt theory of problem solving (See section 1.11. for detail). For instance, learners could not link the concept of maxima/minima, finding the gradient, finding equations of tangents and normals, and finding gradient functions at a particular point they learnt in Grade 11 in order to solve Calculus problems in Grade 12.

5.3.5. Teachers teaching approaches

The study established that teachers teaching approaches affected how learners solved Calculus problems. For instance, learners during FGDs expressed concern that their teachers were too fast when solving Calculus problems (Section 4.3.5.). Learners further expressed that this led to less understanding of Calculus procedure of solving problems, derivative and integral concepts. Moreover, learners also cited that teachers' teaching approaches contributed to memorizing of Calculus formulas such as the rule of differentiation formula, first principles formula, and the rule of integration formula without understanding how those formulas were derived (Section 4.3.5.). Similarly, Makgakga and Makwakwa (2016) in their study found that learners' poor understanding of Calculus concepts was as a result of teachers' teaching approaches (See section 2.11. for detail). In addition, the current study further established that learners complained of teachers solving Calculus problems in a short-cut-way (Section 4.3.5.). Learners during FGDs further cited that each time their teachers were solving Calculus problems, they solved Calculus problems in short-cut ways without explain procedures. This however affected how learners understood the procedure of solving Calculus problems.

In addition, learners during FGDs also indicated that their teachers focused only on calculations and did not explain what the derivatives were, how formulas worked, how concepts were applied in solving Calculus problems (Coe, 2000; Ubuz, 2007). In my own view, it can be argued that teachers' teaching strategies used to solve Calculus problems specifically were determined by two factors: (1) the skill and the sophistication level of the teacher, and (2) the range of mathematical tools that the teacher had previously mastered. I have argued like this because Polya (1957) explained that teachers' skills and resource tools they use when teaching problems has some serious effect on students' problem solving skills. Hence the need for teachers to understand the above-mentioned factors.

5.3.6. Reflective skills

The study findings revealed that learners experienced difficulties in applying reflective skills during problem solving process in Calculus (Figure 4.4a. & 4.4b.). Leaners during FGDs expressed that they had challenges on how to prove if derivative results or integral results were correct or not (4.3.6.). Although learners experienced difficulties at looking back after solving, some learners were able to expand Calculus problems and applied derivative and integral concepts correctly (Figure 4.4a. & 4.4b.). This observation was in affirmative with Tarmizi (2010) who found out that students had difficulties when proving if the answer that was found was correct or not (See section 2.11. for detail). Ideally, reflective skill is aligned to fourth step of problem solving by Polya (1957) and Gestalt theory of problem solving by Wertheimer (1959) which is looking back, requiring one to demonstrate the skill or ability to recheck his/her work for mistakes in every action taken as discussed in chapter 1 on sub-section 1.11. and chapter 2 on sub-section 2.6. Several authors (Kilpatrick et al., 2001) have argued that lack of checking the computation of the problem when solving lead to incorrect solutions. Hence the comment that failure by some Grade 12 learners to look back after solving could be as a result of lack of metacognitive skills (See section 2.8. for detail). I therefore infer that it is important that learners are given opportunities and time to experiment and conjecture results so that they can be proving if their Calculus answers found make sense or not.

5.3.7. Language of Calculus

Although learners were able to read and re-read Calculus problem before solving, results showed that learners had serious difficulties in understanding the language of Calculus. Learners during FGDs cited that terms like differentiate, integral, limit were a challenge to them because they failed to understand what the terms meant (Section 4.3.7.). The expressions by the learners critically puts English Language competency into perspective. For instance, learners during FGDs could not express themselves in English because they could code switch (Bemba and English) which is tied to language of learning and teaching (LoLt), and which can be argued that there was no positive transfer leading to many mistakes.

In addition, teachers also during interviews expressed concern that learners experienced difficulties in understanding the language of Calculus (Section 4.3.7.). This finding meant that understanding the language of Calculus problems seems to be a key predictor of difficulties which learners encountered in solving Calculus problems. Besides, since language is a key mediating artefact in acquiring knowledge (Vygotsky, 1978), students who do not understand the language find that language acts as a translucent instead of a transparent resource that illuminates new mathematical knowledge (Lave & Wenger, 1991). In fact, if language structures are similar there is positive transfer and if there is no similarity there is negative transfer (Makonye, 2011). As the majority of learners used Bemba and Nyanja languages which are not similar to English, the language of learning and teaching (LoLT) is argued that there was not much positive transfer leading to many errors and misconceptions in Calculus due to lack of comprehending the language. This affected learners Calculus solutions.

Research question 3: What strategies could teachers and learners suggest to improve problem solving skills in Calculus?

5.4. Suggested Strategies to improve learners' problem solving skills in Calculus

In this section the researcher discussed findings on measures to address challenges.

5.4.1. Comprehensive teaching approaches

With regards to teachers' role in helping learners understand Calculus concepts, knowledge acquisition, and competency development, learners during FGDs expressed that a comprehensive teaching methodology could be essential in order to improve their problem-solving skills in Calculus (Section 4.4.1.). In addition, learners during FGDs expressed concern that when teachers are teaching and solving Calculus, they should not only rely on how to solve Calculus problems but also explain Calculus concepts (Section 4.4.1.). Scholars have argued that an effective teaching methodology is one that puts learners at the centre of learning and helps them understand concepts (Kilpatrick et al., 2001; Schoenfeld, 1985). Schoenfeld (2013) also stated that teacher centred approaches constrain students to understand mathematical concepts and this affects their performance in class. He further emphasised that teachers of Mathematics should not only rely on teacher cantered approaches but should also blend in learner centred approaches.

Arguing in line with Schoenfeld, teachers play a key role in knowledge acquisition and without employing effective teaching methods like learner centred approaches it can be difficult for learners to understand Calculus concepts, write correct notations, carry out Calculus problems correctly, factorise correctly, and substitute correctly i.e. between $f(x+h)\ and\ f(x)$. In fact, it can be also suggested that when teaching Calculus, teachers should not only rely only teacher centred approaches but instead also employ learner centred approaches. Learner centred approaches are tenet of the theory of constructivism where learning is seen as an active process, and not a passive one (Von Glasersfeld, 1995a).

5.4.2. Supplying Calculus textbooks

Although the findings of this study never highlighted the problem of inadequate or inappropriate Calculus textbooks as one of the challenge affecting learners' problem solving processes skills in Calculus. However, in trying to improve learners' problem solving skills in Calculus, learners during FGDs and teachers during interviews suggested that having Calculus text books in their school library would improve problem-solving skills in Calculus. In this view, learners advocated that a topic like Calculus requires more Calculus books so that at their free time they could be solving more Calculus problems with their peers (Section 4.4.2.). Scholars across the globe have argued that we learn from existing literature (Reys, Reys & Chavez, 2004; MOE, 1996). Textbooks are important in education and that their shortage contributes to poor quality of education provided in schools. Therefore, respondents' concerns for the need for more and quality textbooks was in line with existing educational standards.

5.4.3. Pre-requisite knowledge in pre-calculus topics

The current study established a need for learners to have pre-requisite knowledge in appropriate topics of Mathematics in order to improve their problems solving skills in Calculus. Among the topics learners suggested during FGDs that could improve their problem-solving skills in Calculus were Functions and Relations, Coordinate Geometry, Algebra, Indices, Fractions, and Factorization (Section 4.4.3.). However, the concerns expressed by the respondents were in harmony with reviewed literature by different scholars in Mathematics (Lam, 2009; Herbert, 2009) who argued that Calculus is a combination of different topics in Mathematics such as Algebra, Indices, and Functions and Relations.

Moreover, learners concern also comprehends with the teachers' responses during interviews who also suggested that understanding pre-calculus topics could improve problem solving skills in Calculus (Section 4.4.3.). This calls for learners to know and understand that in order to have good problem solving skills in Calculus, appropriate pre-calculus topics such as Functions and Relations, Coordinate Geometry, Algebra, Indices, and Factorisation are cardinal, and teachers of Mathematics should always ensure that these pre-calculus topics are emphasised and revised on an ongoing basis. Apparently, having pre-requisite knowledge in these pre-calculus topics could help learners understand how these topics and a topic like Calculus interlink because prior knowledge is another aspect that plays a significant role in Calculus solving. In fact, the importance of prior knowledge is highly emphasized by Gestalt theory of problem solving (See section 1.11. for detail).

5.4.4. Understanding the language of Calculus

Results showed that understanding the language of Calculus could improve learners' problem solving skills in Calculus. The majority of the learners during FGDs recommended that, understanding the language of Calculus could be one of the good strategy that could improve their problem-solving skills in Calculus (Section 4.4.4.). In line with this finding, literature has shown that familiarity with mathematical terminology improves problem-solving ability contexts (Verschaffel et al., 1999). In view of this, I therefore argue that Mathematics is a language that relies on symbols but it also includes graphs, charts, and texts to decode. In fact, "reading completely depends on being able to understand the structures of texts and nuances of language; to interpret authors' ideas; and to visualize, evaluate, and infer meanings" (Ball & Bass, 2003; p. 29). Ideally speaking, understanding the language of Calculus and Mathematics knowledge are woven together, but further explorations with Calculus problems that are not translation tasks are necessary.

5.4.5. Giving Homework

Owing to the dares and dilemmas encountered by the learners in solving Calculus problems, learners proposed that teachers should be giving them homework on daily basis or thrice in a week, that could improve learners' problem-solving skills in Calculus (Section 4.4.5.). This concern by learners to be given homework by their teachers on regular basis conforms with the Ministry of Education (1996) National Policy on Education: “Educating Our Future” which stipulates that each school will be required to have a clear schedule of performance-monitoring activity that checks pupils’ progress and that prominent among these will be homework given to pupils on a regular basis. However, this study found out that each school had domesticated the policy to suit its own local programmes and environment. Arguing in line with the learners’ concerns, giving them homework on daily basis would provide teachers of Mathematics with feedback from their learners on how well they had grasped the daily lessons and skills involved in tackling homework assignments in particular Calculus assignments.

During FGDs, learners also expressed that if their teachers can be giving them homework on regular basis, they could be taking homework as part of revisions (Section 4.4.5.). In relation to learners’ responses, Dennis (2007) contended that pupils take homework as a tool for revision, it gives the pupils opportunity to learn new things. Denis (2007) argument is also supported by teachers who said that giving homework helps students learn new things (Section 4.4.5.). Additionally, scholars have, however argued that giving homework to students improves their performance in Mathematics (Kohn, 2006; Christopher, 2007). Besides, learners expressed that by giving them homework on regular basis, it could be an opportunity for them to practice what was learnt in class (Section 4.4.5.). Moreover, teachers during interviews also expressed that giving learners homework could be a good strategy so that their minds are preoccupied with work pertaining to Calculus assignments (Section 4.4.5.).

5.4.6. Collaborative work

In an effort to improve learners’ problem solving skills in Calculus, learners during FGDs suggested that solving Calculus problems with their friends could help them share ideas, encourage one another and understand Calculus problems together (Section 4.4.6.). Learners’ responses during FGDs comprehends with teachers’ responses during interviews who also suggested that learners solving Calculus problems with their fellow pupils could improve their problem-solving skills in Calculus (Section 4.4.6.). Respondents responses conforms with reviewed literature which said that collaborative work has been found to promote the use of higher-order thought processes, foster the use of academic problem-solving skills and perspective-taking skills, increase opportunities for oral rehearsal of information, and encourage and involve peers in learning which can increase friendship, acceptance, and cognitive processing skills (Schoenfeld, 1992; Lester, 1985).

Arguing in line with the learners' concerns, Schoenfeld (1985) argued that when students talk to others, they have the opportunity to reflect on their ideas and to develop the language skills needed to express those ideas. Based on Schoenfeld's ideas, it can be argued that when learners engage in solving Calculus problems with their peers, the opportunity for interactions to occur among the learners is much more likely than in a large group setting. Thus, in order to have an effective collaborative work, teachers of Mathematics should be at centre of the process as facilitators. In fact, the notion of teachers been at center of the process as facilitators was highly emphasized by themselves (Section 4.4.6.). Consequently, the challenges learners encountered in solving Calculus problems occurred because they did not have "More Knowledgeable Others" (Vygotsky, 1978) to help them learn Calculus and solve Calculus problems. As Vygotsky has argued, learning begins with an interaction in the social plan where learners are passed on society's valued knowledge from previous generations to create a Zone of Proximal Development (Vygotsky, 1978).

5.4.7. Introducing Calculus symbols in early Grades

In an effort to address the challenges posed by failure to differentiate or note the difference between $f(x)\ and\ f'(x),\ y\ and\ y'$, learners suggested that Calculus symbols such as $f'(x), f(x), y, y', \frac{dy}{dx}, \lim_{h\to 0}\ and\ \int f(x)\,dx$ should be introduced in early Grades (Section 4.4.7.). Learners further expressed that understanding of these Calculus symbols in early Grade could improve their problem-solving skills in Calculus as they will be more familiar with the symbols. In fact, reviewed literature as shown that understanding of mathematical symbols can help learners appreciates Mathematics (DBE, 2015). Similarly, Makonye (2011) has stated that understanding of Calculus symbols such as $\frac{dy}{dx}\ and\ \lim_{h\to 0}$ can improve performance in Calculus.

5.5. Summary of the chapter

This chapter discussed the findings of the study on learners' problem solving processes at Grade 12 level. Findings showed although majority of the learners' read Calculus problems, reread Calculus problems, wrote down Calculus functions, display rough workings, and keep track and save data before solving, it was found out that learners lacked the ability to underline key important words; writing Calculus formulas; simplifying Calculus problems; sketch graphs; use appropriate Calculus formula before solving; and lacked reflective skills when solving and after solving Calculus problems.

It was found that the challenges included failure by learners to: cite the correct formula when working from first principles; substitute the terms between $f(x+h)$ and $f(x)$ when working from first principles; cite the notation between $f(x)$ and $f'(x)$; and applying appropriate basic mathematical concepts i.e. Algebraic skills, laws of indices, Factorization, Algebraic fractions, Coordinate Geometry, and Arithmetic skills. Moreover, learners had challenges with: teachers teaching approaches, reflective skills, and understanding the language of Calculus.

To address the constraints established from the study, participants suggested the need for an adequate supply of Calculus textbooks, comprehensive teachers teaching approaches, pre-requisite knowledge in pre-calculus topics such as (Algebra, Coordinate Geometry, Fractions, Indices, Functions and Relations, Factorization), understanding the language of Calculus, simplifying Calculus problems, learners to be given homework, learners solving Calculus problems with their peers, and introducing Calculus symbols in early Grades.

CHAPTER 6

CONCLUSION AND RECOMMENDATIONS

6.1. Introduction

This chapter concluded the study and highlights the major contribution it has made to research (Section 6.1). Thereafter, the recommendations (Section 6.4.) are passed in order to improve poor performance in Calculus as reported by the Examination Council of Zambia for 2016 and 2017, and further research in Mathematics Education.

6.2. Conclusion

The study explored learners' problem solving processes in Calculus at Grade 12 level in the context of poor essential workings. In terms of understanding Calculus problems, it has been established that learners' read and re-read Calculus functions before solving in order to understand the question and to locate important pieces of information before solving. The study has also showed that learners experienced difficulties in underlining key important words and sketching simple graphs before solving. In terms of devising Calculus plans, it has been found that learners showed competence of devising Calculus functions and showing rough workings but experienced difficulties in citing Calculus formulas and simplifying Calculus problems before solving. In terms of carrying out Calculus plans, results showed that learners kept Calculus answers during the process of solving but experienced difficulties in monitoring their steps during problem-solving processes. The study further established that learners opted to use the rule of differentiation instead of first principles because it easy working from the rule of differentiation than working from first principles. In terms of looking back, results have shown that learners experienced difficulties in looking back to check if the answer found is correct or not and very few learners interpreted Calculus problems immediately after solving. Based on learners' problem solving processes, poor essential workings in Calculus as reported by Examinations Council of Zambia could be as a of result of failure to underline key Calculus words, devise Calculus functions, Calculus formulas, simplify Calculus problems, apply the appropriate Calculus formula, monitor each step during problem solving, and looking back to see if the Calculus answer found is reasonable or not.

Learners' problem solving processes were constrained by failure to: cite the correct formula when working from first principles; substitute the terms between $f(x+h)$ and $f(x)$ when working from first principles; cite the notation of derivative between $f(x)$ and $f'(x)$; and Applying Basic Mathematical Concepts (Algebraic skills, laws of indices, Factorization, Coordinate geometry, and simplifying Algebraic fractions). Additionally, results have also shown that learners had challenges with: teachers teaching approaches, reflective skills, and understanding the language of Calculus. According to these findings, results show that learners' difficulties were enhanced by poor prior knowledge (in topics like Algebra, Functions, Coordinate Geometry, and Functions), lack of metacognitive skills, memorizing, lack of procedural knowledge and conceptual knowledge, lack of commitment, and teachers' teaching approaches.

In response to research objective three that sought to address the challenges established from the study, participants suggested the need for an adequate supply of Calculus textbooks and related materials, comprehensive teachers teaching approaches, pre-requisite knowledge in pre-calculus topics such as (Algebra, Coordinate Geometry, Fractions, Indices, Functions and Relations, Factorization), understanding the language of Calculus, learners to be given homework, learners solving Calculus problems with their peers, and introducing Calculus symbols in early Grades. In view of the findings, the next section highlights the contribution of this study to the field of Mathematics Education in general and Calculus Education in particular.

6.3. Contribution to the field

Globally, this study has also made a theoretical contribution to the body of knowledge regarding research in Mathematics and Calculus Education. Polya's (1957) problem solving processes and Gestalt theory of problem solving (Wertheimer, 1959) has helped to emphasise appropriate analysis involving data collection of Understanding the problem (read, reading, underlining key words, and sketching graphs), Devising plans (equations, formulas, simplifying, and rough workings), Carrying out plans (monitoring each step during problem solving, appropriate formulas, and keeping and saving results), and looking back (proving or checking and interpreting and reporting answers) which are scarcely applied in many studies. In view of the findings I made the following recommendations:

6.4. Recommendations

The foregoing recommendations were made in the light of policy, practice and further research in Mathematics and Calculus education based on the findings of the study:

6.4.1. Policy

1. Need for the MoGE to devise strategies to incorporate all teachers teaching Calculus. One good strategy that can improve learners' problem solving skills in Calculus could be orienting teachers to problem solving processes, which helps learners to understand Calculus problems, devise Calculus problems, execute Calculus plans, and look back after solving.

2. MOGE need to supply quality and quantity of Calculus textbooks in line with the revised Mathematics curriculum in all secondary schools.

3. MoGE should ensure that there is efficient CPD in secondary schools so that teachers should continue sharing and acquiring new knowledge and skills based on best practices of teaching and learning of Calculus. Most importantly solving Calculus problems.

4. MoGE should be conducting orientation workshops to help teachers develop interest in Calculus, share and acquire new knowledge and skills based on practices of teaching and learning of Calculus. I received concerns that some teachers shun to teach Calculus as it has been perceived to be a difficulty topic.

5. In order for the learners to be more familiar with Calculus symbols, Government through CDC need to introduce Calculus symbols such as $\frac{dy}{dx}, y' or\ f'(x)\ and\ \lim_{h\to 0}$ in early Grades I.e. Grade 10 or 11.

6.4.2. Practice

1. Both In-service and Pre-service teachers need to undergo staff development-related activities; continuing professional development (CPD) whilst in training. This could help teachers share and acquire new knowledge and skills based on best practices of teaching and learning of Calculus.

2. Applications of basic mathematical concepts in earlier grades should also be consolidated and revised on an on-going basis.

3. Teachers should be carrying out a diagnostic assessment to determine what learners know about pre-calculus topics such as Algebra, Functions, Indices, Fractions, Coordinate Geometry, and Factorization.

4. There is need for learners to be identifying key words, devise Calculus functions, devise Calculus formulas, simplifying Calculus problems, applying appropriate Calculus formulas, monitoring each step during problem solving and looking back after solving.

5. Learners should be solving Calculus problems with capable peers. This could improve learners' problem solving skills in Calculus.

6. Teachers should be using problem solving approaches which assist learners in identifying key words in the problem, devising calculus formulas, monitoring each step during solving and looking back after solving.

7. Teachers should be explaining how working from first principles is derived. This could illuminate the idea of learners memorizing Calculus formulas instead of understanding how first principles formula operates.

8. Teachers when teaching Calculus should be employing learner centred approaches whilst they act as facilitators.

9. There is need for teachers of Mathematics to teach leaners that there is a difference between $f\ (x)\ and\ f'(x)$ or $y\ and\ y'$.

6.4.3. Further research

1. Need to explore learners' problem solving processes in basic application of differential Calculus and basic application of integral Calculus.

2. Need to interrogate learners' attitudes toward learning Calculus in relation to problem solving processes.

3. Need for an independent research to investigate if Polya's (1957) problem solving processes can improve Grade 12 learners' problem solving skills in Calculus.

4. Need to interrogate teachers' problem solving processes in Calculus. This could help establish if teachers' problem solving processes improves learners' problem solving skills in Calculus.

5. Need to compare learners' performance in other Mathematics topics that may influence learners' performance in Calculus.

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APPENDICES

APPENDIX 2

Extract showing the current of Coordinate Geometry in Secondary School

Topic	Subtopic	Specific outcome	knowledge	Skills	Values
INTRODUCTION TO CALCULUS	12.7.1Differentiation 12.7.2 Integration	12.7.1.1Explain concept of differentiation 12.7.1.3Differentiate functions from first principles. 12.7.1.4 Use the formula for differentiation 12.7.1.8 Calculate equations of tangents and normals 12.7.2.1 Explain integration 12.7.2.3 Find Indefinite integrals 12.7.2.2 Evaluate simple definite integrals 12.7.2.3 Find the area under the curve	- Explaining the concept of differentiation -Differentiating functions from first principles -Product rule; chain rule and quotient rule Indefinite integrals, -Arbitrary constant, Definite -integrals, Stationary points, Secant, Tangents -Normal -Explain integration as the reverse of differentiation -Rule of integration $\frac{dy}{dx} = ax^n; \int ax^n dx = \frac{ax^{n+1}}{n+1}$ +c □ Area under the curve	Interpretation of differentiation and integration -*Application* of definite integrals. *Estimation* of area under the curve.	*Appreciation* of calculus. *Curiosity* in differentiating and integrating. *Critical thinking* in using rules for differentiation and integration.

Source: MESVTEE (2013) - "O" Level Mathematics Syllabus Grades 10 to 12.

APPENDIX 2

Extract showing the current of Coordinate Geometry in Secondary School

Topic	Subtopic	Specific outcome	knowledge	Skills	Values
COORDINATE GEOMETRY	11.6.1 Coordinate and the mid point 11.6.2 Length of a straight line between two points 11.6.3 Gradient 11.6.4 Equation of a straight line Parallel and perpendicular lines	11.6.1.1 Calculate the mid-point of two points 11.6.2.1 Calculate the length of a straight line 11.6.3.1 Calculate the gradient of a line segment 11.6.4.1 Find the equation of a straight line 11.6.4.1 Find the gradients of parallel and perpendicular lines Use gradients of parallel and perpendicular lines to find equations	• - Gradient • Mid point • Length (distance formula) • Gradient point form • Gradient Intercept form • Double intercept form • Parallel lines • Perpendicular lines	• Calculation • Drawing • Sketching • Substitution • Labelling • Deduction	• Problem solving • Application • Appreciation • Reasoning • Recognition • Interpretation • Relation

Source: MESVTEE (2013) - "O" Level Mathematics Syllabus Grades 10 to 12.

APPENDIX 3

Extract showing the 2016 paper 1 final examination Calculus question

For Examiner's use

17 **(a)** Evaluate $5^{\circ} + 5^{1}$.

(b) In the answer space below, is an incomplete simple program pseudocode for calculating and outputting the volume of a cylinder V, given the base radius r and the height h. complete the program by filling in the blank spaces with appropriate statements.

Answer: **(a)** .... [2]

(b) Begin

Enter... ;

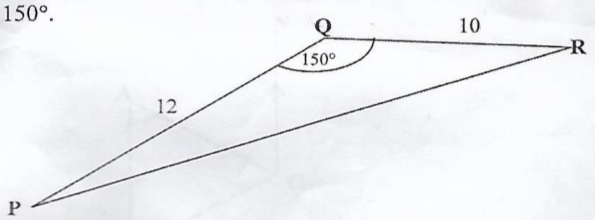
V = ... ;

Output V;

End. [2]

18 **(a)** Find $\int (6x^2 - 5)\,dx$.

(b) The diagram below shows triangle PQR in which PQ = 12cm, QR = 10cm and $P\hat{Q}R = 150°$.



Calculate the area of triangle PQR.

Answer: **(a)** .... [2]

(b) .... [2]

APPENDIX 4

Extract showing the 2016 paper 2 final examination Calculus question

4 **(a)** **(i)** Construct a triangle ABC where AB = BC = CA = 7cm. [1]

(ii) Measure and write the size of $\angle CAB$. [1]

(b) Within the triangle ABC, construct the locus of points

(i) equidistant from AB and BC, [1]

(ii) 4cm from B, [1]

(iii) 3cm from AB. [2]

(c) A point R, within triangle ABC, is such that it is nearer to BC that AB, less than 3cm from AB and less than 4cm from B. Shade the region in which R must lie. [2]

---

5 **(a)** Simplify $\frac{x-1}{x^2-1}$. [2]

(b) The first three terms of a geometric progression are $x + 1$, $x - 3$ and $x - 1$. Find

(i) the value of x, [3]

(ii) the first term, [1]

(iii) the sum to infinity. [3]

---

6 The equation of a curve is $y = x^3 - \frac{3}{2}x^2$. Find

(a) the equation of the normal where $x = 2$, [3]

(b) the coordinates of the stationary points. [3]

APPENDIX 5

Extract showing the 2017 paper 1 final examination Calculus question

7 **(a)** For the sequence 11, 13, 15, 17, …, find the 13^{th} term.

(b) If the arithmetic mean of 5 and c is 11, what is the value of c?

Answer: **(a)** …………………………… [1]

(b) …………………………… [2]

---

8 **(a)** If $A^T = (1 \quad -2 \quad 3 \quad -4 \quad 5)$, write the matrix A.

(b) Find the derivative of $y = 2x^3 - 2x^2 - 3x + 1$, with respect to x.

Answer: **(a)** …………………………… [1]

(b) …………………………… [2]

---

9 **(a)** Given that E = {1, 2, 3, 4, 5, 6, 7, 8}, A = {1, 8} and B = {2, 3, 4, 5, 6, 7}, list $(A \cup B)'$.

(b) Solve the equation $25^x = 5$.

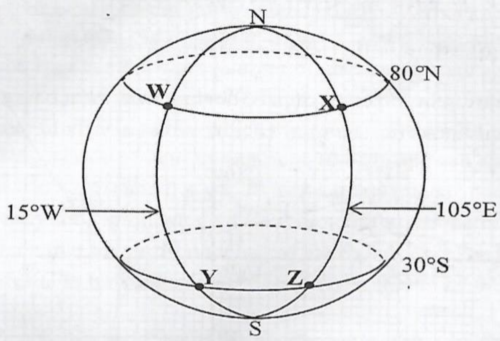
Answer: **(a)** …………………………… [1]

(b) …………………………… [2]

APPENDIX 6

Extract showing the 2017 paper 2 final examination Calculus question

9 (a) W, X, Y and Z are four points on the surface of the earth as shown in the diagram below. (Take π as 3.142 and R = 3437nm)



(i) Calculate the difference in latitude between W and Y. [2]

(ii) Calculate the distance in nautical miles between

(a) X and Z along the longitude 105°E, [2]

(b) Y and Z along the circle of latitude 30°S, [2]

(b) Find the coordinates of the points on the curve $y = 2x^3 - 3x^2 - 36x - 3$ where the gradient is zero. [4]

(c) Evaluate $\int_{-1}^{3} \left(3x^2 - 2x\right) dx$. [2]

APPENDIX 7

Extract showing an analysis performance in Calculus

On	*Topic*	*General performance*	*Comments*
6	(a) Differentiation	Poor	Most candidates lacked knowledge of the use of differentiation to find the equation of the normal.
	(b) Application of differentiation	Poor	Some just equated the original equation to zero and solved for x leading to wrong solutions.

Source: ECZ (2016)-Examination Council of Zambia, 2016 Performance Review Report

APPENDIX 8

Classroom Observation Guide/schedule for learners

LESSON ELEMENTS TO CHECK/OBSERVE			
Item #	Elements to check (learner activity)	Observation(s)	Comments and Suggestion
a.	Understanding the problem		
1.	Reading a Calculus text/problem		
2.	Rereading a Calculus text/problem		
3.	Underlining or Highlighting key terms etc. on the problem/question		
4.	Sketch simple graphs		
b.	Devising a plan		
1.	Writing Calculus formulas before solving		
2.	Writing Calculus functions before solving		
3.	Simplifying certain parts of Calculus problems		
4.	Showing rough solving skills		
c.	Carry out the plan		
1.	Checking or monitor each step of the plan		
2.	Apply/use appropriate Calculus formula		
3.	Showing essential workings when solving a Calculus problem		
4.	Keeping track and save results/data		
d.	Looking back		
1.	Check if the answer is correct		
2.	Check if the answer is incorrect		
3.	Interpret Calculus answers		

APPENDIX 9

Focus Group Discussion for Pupils

Part 1: Understanding Calculus problems

1. Do you read a Calculus text before solving Calculus problems? Why?
2. Do you reread a Calculus text before solving Calculus problems? Why? How many times would you say you reread the Calculus text?
3. Do you identify key words/information before solving Calculus problems?
4. Do you draw a sketch(s) resenting a Calculus problem before solving?

Part 2: Devising Calculus plans

1. Do you write Calculus formulas before solving Calculus problems?
2. Do you write Calculus functions before solving?
3. Do you show rough solving skills before solving Calculus problems? why?
4. Do you simplify Calculus problems before solving? why?

Part 3: Implementing a Solution Plan/Carrying out a plan

1. Do you check each step when solving Calculus problems?
2. Do use appropriate Calculus formulas when solving Calculus problems?
3. Did you keeping track and save results/data when solving Calculus problems?
4. Did you solve the Calculus problem independently?
5. Did you solve the Calculus problem collaboratively?

Part 4: Reflection/evaluating/re-examining

1. Did you check if the answer is correct after solving a Calculus problem?
2. Did you check if the answer is incorrect after solving a Calculus problem?
3. Did you interpret the answer of a Calculus problem?

Part 5: Challenges to solving Calculus problems

1. What are some of the challenges do you encounter when solving Calculus problems?

Part 6: Strategies to improve problem solving skills in Calculus

1. What are some of the problem-solving strategies that can improve problem solving skills in Calculus?

Part 7: promoting metacognition

a. Does the teacher ask questions, explain or make comments that encourage students to be reflective about problem solving in Calculus? If so, how?

End of Interview

Thank you very much for your participation!

APPENDIX 10

Interview for teachers

Part 1: Understanding Calculus problems

1. Do learners read Calculus problems before solving?
2. Do learners reread Calculus problems before solving?
3. Do learners identify key words/information before solving Calculus problems?
4. Do learners draw a sketch(s) resenting Calculus problems before solving?

Part 2: Devising Calculus plans

1. Do learners write Calculus formulas before solving?
2. Do learners write Calculus functions before solving?
3. Do learners show rough solving skills before solving?
4. Do learners simplify Calculus problems before solving?

Part 3: Implementing a Solution Plan/Carrying out a plan

1. Do learners check/monitor each step when solving Calculus problems?
2. Do learners use appropriate Calculus formulas when solving Calculus problems?
3. Do learners keep track and save results/data when solving Calculus problems?
4. Do learners solve the Calculus problem independently?
5. Do learners solve the Calculus problem collaboratively?

Part 4: Reflection/evaluating/re-examining

1. Do learners check if the answer is correct after solving a Calculus problem?
2. Do learners check if the answer is incorrect after solving a Calculus problem?
3. Do learners interpret answer(s) after solving Calculus problems?

Part 5: Challenges to solving Calculus problems

1. What are some of the challenges do learners encounter when solving Calculus problems?

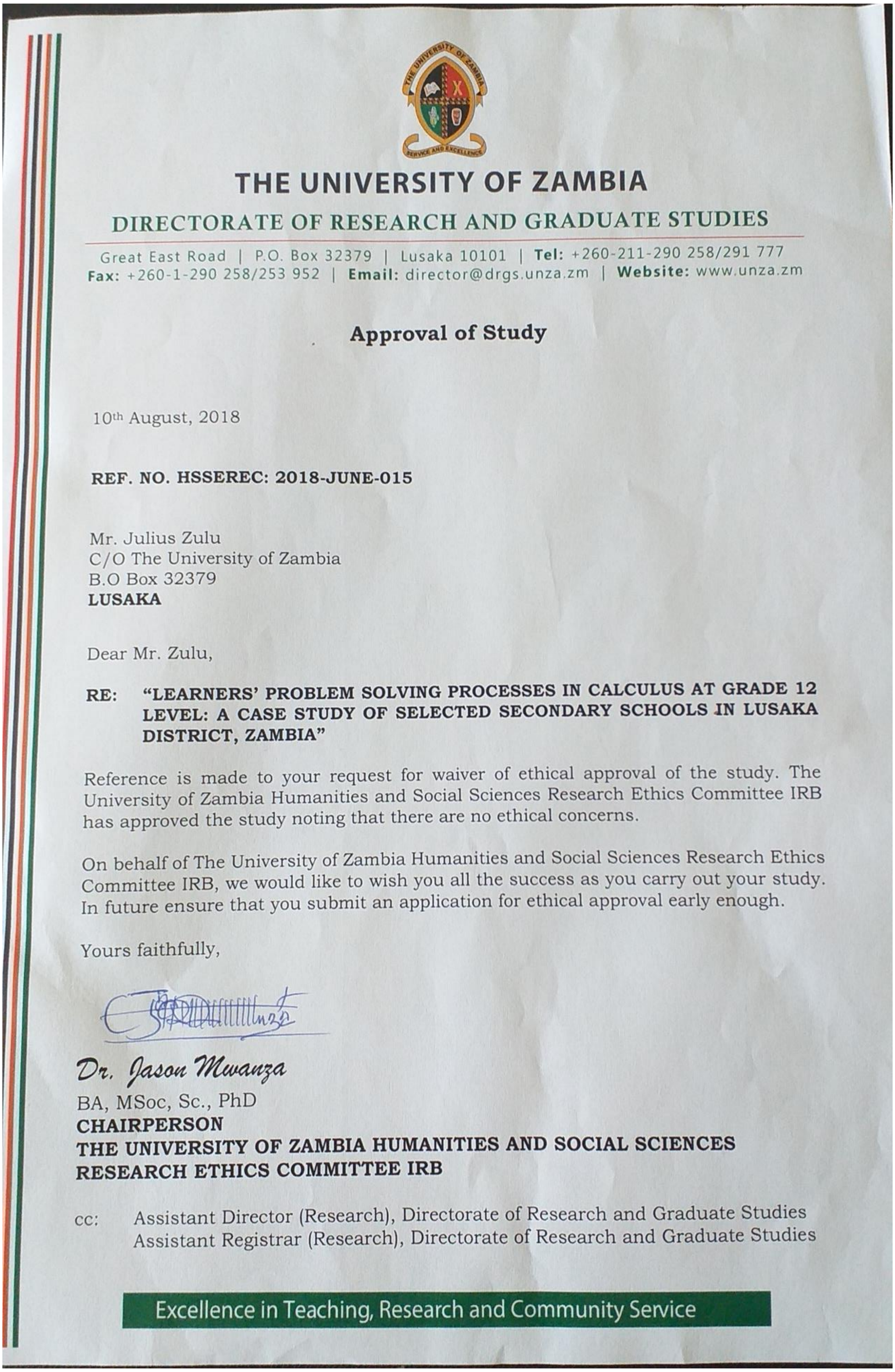
Part 6: Strategies to improve problem solving skills in Calculus

1. What are some of the problem-solving strategies that can improve learners' problem solving skills in Calculus?

Thank you for your time and cooperation

APPENDIX 11

Clearance letter for data collecting from the ethical committee, The University of Zambia



THE UNIVERSITY OF ZAMBIA

DIRECTORATE OF RESEARCH AND GRADUATE STUDIES

Great East Road | P.O. Box 32379 | Lusaka 10101 | Tel: +260-211-290 258/291 777
Fax: +260-1-290 258/253 952 | Email: director@drgs.unza.zm | Website: www.unza.zm

Approval of Study

10th August, 2018

REF. NO. HSSEREC: 2018-JUNE-015

Mr. Julius Zulu
C/O The University of Zambia
B.O Box 32379
LUSAKA

Dear Mr. Zulu,

RE: "LEARNERS' PROBLEM SOLVING PROCESSES IN CALCULUS AT GRADE 12 LEVEL: A CASE STUDY OF SELECTED SECONDARY SCHOOLS IN LUSAKA DISTRICT, ZAMBIA"

Reference is made to your request for waiver of ethical approval of the study. The University of Zambia Humanities and Social Sciences Research Ethics Committee IRB has approved the study noting that there are no ethical concerns.

On behalf of The University of Zambia Humanities and Social Sciences Research Ethics Committee IRB, we would like to wish you all the success as you carry out your study. In future ensure that you submit an application for ethical approval early enough.

Yours faithfully,

Dr. Jason Mwanza
BA, MSoc, Sc., PhD
CHAIRPERSON
THE UNIVERSITY OF ZAMBIA HUMANITIES AND SOCIAL SCIENCES RESEARCH ETHICS COMMITTEE IRB

cc: Assistant Director (Research), Directorate of Research and Graduate Studies
Assistant Registrar (Research), Directorate of Research and Graduate Studies

Excellence in Teaching, Research and Community Service

APPENDIX 12

Permission letter for final data collection-Provincial Education office, Lusaka

All correspondence should addressed to the District Education Board Secretary

Telephone: 0211-240250/240249 0955 623749
E-mail: desbsisk@yahoo.co.uk

DEBS/101/1/19
In reply please quote

REPUBLIC OF ZAMBIA

MINISTRY OF GENERAL EDUCATION

DISTRICT EDUCATION BOARD SECRETARY
P.O. BOX 50297
LUSAKA

Mr Smikala

REPUBLIC OF ZAMBIA
MINISTRY OF GENERAL EDUCATION
13 MAR 2018
DEPUTY HEADTEACHER
OLYMPIA PARK SECONDARY SCHOOL

15th February, 2018

Mr. Julius Zulu
Computer No.: 2017012021

RE: REQUEST TO UNDERTAKE FIELD WORK FOR MASTERS STUDENT: MR. JULIUS ZULU

I hereby write to introduce the above named dully registered student at the University of Zambia, School of Education. Mr. Zulu would like to conduct an academic field work for his Masters programme in your school which will also involve data collection to which this office has no objection. However, ensure that this programme does not interfere with the learning schedule.

Kindly render him support where possible.

B. Mwanza (Mr.)
DISTRICT EDUCATION BOARD SECRETARY
LUSAKA DISTRICT

sw*

APPENDIX 13

Participant's consent form

1. I have discussed the research project to be undertaken by **Julius Zulu** regarding "Learners' Problem Solving Processes in Calculus at Grade 12 level: A Case Study of Selected Secondary Schools in Lusaka District, Zambia" whose purpose is to explore learners' problem solving processes in Calculus at Grade 12 level.
2. I fully understand the purpose and give approval to participate by:
   - Being observed and video-recorded
   - Being interviewed and audio-recorded
3. My participation, however, is on condition that:
   - Authority has been given for the research to be done in this school
   - I can ask the researcher or his supervisor(s) any questions about the study
   - The only people who will see the information will be the researcher and his supervisor(s)
   - There will be no written reports in which the school and I could be identified
   - I can withdraw from being involved at any stage without having to give any reasons

Thank you

Participant's signature:…………………………………

Date:…………………………………..………………….