SECONDARY SCHOOL TEACHERS' MATHEMATICAL KNOWLEDGE FOR TEACHING QUADRATIC EQUATIONS: A CASE OF THREE SECONDARY SCHOOLS IN KATETE DISTRICT

BY

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LUSAKA

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DECLARATION

I, **Titus Luka Mbewe**, do hereby declare that this thesis represents my own work and that it has not been previously submitted for a degree at this or any other university.

Signature: ………………………………   Date: ……………………………………

APPROVAL

This Thesis of Titus Luka Mbewe is approved as fulfilling the requirements for the award of the Degree of the Doctor of Philosophy in Mathematics Education of the University of Zambia.

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Chairperson/ Board of Examiners	Signature	Date
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Supervisor	Signature	Date
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DEDICATION

To my wife Esnala for the support during my studies. My children, Faith, Talandila, and Titus Junior for giving me strength which gave focus on my studies. My late parents, Luka Phillip Mbewe and Agness Joseph Banda, for the insights that are still shinning in me. My brothers and sisters for constantly encouraging me to complete the studies.

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ABSTRACT

In the last three decades, there has been a growing interest on how teachers utilise the specialised knowledge for mathematical knowledge in classroom instruction. This study was an inquiry focusing on mathematical knowledge for teaching quadratic equations.

The study was a qualitative and sought to answer the following three (3) research questions: (1) What subject matter knowledge did the three mathematicsteachers possess when teaching quadratic equations? (2) What strategies did the three mathematics teacher use in teaching quadratic equations? and (3) How did the three mathematics teachers' address pupils' errors and misconceptions when teaching quadratic equations..?.Data were collected using questionnaires, pre-lesson interviews, lesson observations and post-lesson interviews. Thematic analysis was used to analyse the data according to the components identified in the study and these were were subject matter knowledge, knowledge of pedagogy, and knowledge of learner conceptions.The findings of the study revealed that the three teachers had adequate knowledge of the subject matter but was limited to procedural knowledge. Secondary school mathematics teachers' ability to make appropriate connections among mathematical concepts to generate different solutions and representations for problems, to address learner difficulties and misconceptions effectively and choose appropriate teaching strategies were heavily dependent on the depth and breadth of their subject matter knowledge. All three participants used a similar teaching approach to overcome learners' difficulties and misconceptions such as going through the examples given.

The study recommends that teaching instruction should address both procedural and conceptual knowledge as they complement each other for effective lesson delivery. The study contributes to an emerging study of MKT by consolidating the importance of the three components identified in the study namely: subject matter knowledge, pedagogical knowledge and the knowledge of learner conceptions in the teaching of quadratic equations.

Keywords: *Mathematical knowledge for teaching, subject matter knowledge, pedagogy, procedural knowledge, conceptual knowledge*

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LIST OF ACRONYMS AND ABBREVIATIONS

CCK	Common Content Knowledge
CK	Content Knowledge
COACTIV	Cognitive Activation
KAT	Knowledge for Algebra Teaching
KCT	Knowledge of Content and Teaching
MCK	Mathematical Content Knowledge
MKT	Mathematical Knowledge for Teaching
MoE	Ministry of Education
MPCK	Mathematical Pedagogical Content Knowledge
PCK	Pedagogical Content Knowledge
PK	Pedagogical Knowledge
QUANTUM	Quality Mathematical Education for Teachers in South Africa
SCK	Specialised Content Knowledge
SMK	Subject Matter Knowledge
TEDS-M	Teacher Education Development Study in Mathematics

CHAPTER 1

BACKGROUND OF THE STUDY

1.0 Introduction

About 100 years ago, knowing content of the subject matter meant that teachers could teach the subject efficiently (Shulman, 1986). The content of the subject matter outweighed the pedagogical skills which were considered as the second important component for the teacher to be qualified.

In the second half of the twentieth century, there was a concept of the change from the behaviourist theorists, which consisted classroom management, teaching methodologies and questioning techniques to the pedagogical skills, which teachers needed in order to effectively have ideal instruction in the delivery of lessons.

Shulman (1986) wanted to find the solution of the question:"What is the missing paradigm in the teaching profession?" It was found that the missing paradigm was Pedagogical Content Knowledge (PCK), which systematically combined Content Knowledge (CK) and Pedagogical Knowledge (PK). This as outlined by Shulman (1986) meant that teachers need to utilise analogies, illustrations and examples to make the subjects more comprehensible to the learners. This study considers the fact that knowledge for teaching combines both understanding of the content and pedagogy. Grossman et al. (2005), looked at the centrality of PCK as necessary for the teacher education curriculum and teachers need to have PK about the subject matter, so that they can reach a great variety of learners to explain the content. Bransford et al. (1999), emphasised how it was important for teachers to have a considerable skill in teaching.This was to concentrate on comprehension rather than memorisation and prepare learners to enhance learning and perception. The studies conducted by Shulman prompted an emergence of a specialised knowledge, which mathematics teachers needed to acquire special knowledge for teaching mathematis which was later referred to as mathematical knowledge for teaching (Ball, 1990).

Ball and Bass (2005) pointed out that the iterative process for developing mathematical knowledge for teaching attempted to understand how teachers became knowledgeable as they practice for the same topics in their teaching and how learners interpret and use representations of particular topics.

1.1 Mathematical Knowledge for Teaching

Most of the studies carried out on mathematical knowledge, up to now, have highlighted its nature and development instead of focusing on how it is implemented in the classroom environment concerning specific subjects (De Jong et al., 2002).

When MKT is used for a specific topic, it shows teachers' comprehension both for the difficulties learners face while studying the topic and the most effective instructional strategies, analogies demonstrations and other details that are essential for the learners to understand the topic (Veal & Makinster, 1999). Although every mathematics topic has its own concepts, terms, they all have various instructional strategies and methods to achieve the objectives. This entails that teachers need to have divergent methods for a specific topic. Teachers' being knowledgeable about the topic influences their teaching effectively (Kapyala et al., 2008). This brings us to the importance of mathematics in the Zambian curriculum.

1.2 Importance of Mathematics in the Zambian Curriculum

The Zambian Government through the Ministry of Education attaches great importance to the teaching and learning of mathematics. This is an undisputable aspect in that mathematics is taught as a compulsory subject from primary to secondary school. With regard to the importance of mathematics, the Ministry of Education (1996, p. 59) states:

> The Ministry of Education will give priority to the improvement of mathematics and science in high schools of all types. To this end, it will ensure that all the schools will offer general academic programmes and will devote more of their resources and time to the teaching of mathematics and science.

From the above quote, it can be noted that the policy of the Zambian system of education places particular importance on the improvement of mathematics and science.Further, MOE (2013), states that "... Mathematics prepares and enhances the learners' prospect of employment and further education".

Mathematics has become critical in terms of entry into tertiary institutions. For example, a prospective student is to enter a college of education to train as a primary school teacher, he or she should possess five-ordinary level passes of which mathematics, any science subject and English should be compulsory (MOE, 2015).

Zambia is in need of suitably qualified doctors, engineers and many other scientifically-oriented professionals.This means that for the prospective students to enroll in any of these fields, there are supposed to have a minimum of a credit (Grade 6) pass in mathematics. Therefore, there is need for Zambia to have appropriately qualified personnel in order to address the professional demands of the country and to improve the levels of scientific and technological advancement.This will enable growth and expansion of the economy, which is needed. Zambia is far from this ideal situation. Therefore, improvement in learner performance is extremely vital because the lack of expertise influences the general outlook of the country.

Despite the importance attached to mathematics in the national curriculum documents, pupil performance at the national examinations at grades 9 and 12 has persistently been a source of great concern.

1.3 Performance of Pupils at Grades 12

With regarding the performance of mathematics at Grade 12, the Zambia Education Framework (2013, p.4) records:

> Regarding to performance, there is some improvement in the achievement levels but practical subjects, mathematics and sciences continue to record unsatisfactory results. On average, less than two thirds of the candidates who sit for either Junior Secondary Leaving Examinations or the School Certificate Examinations, pass mathematics each year.

The underperformance of pupils at school certificate level is something which gives the researcher an academic impetus to conduct this study. Table 1.1 below shows the performance of candidates according to grades in mathematics from 2017 to 2020.

Table 1.1: Mathematics Grade 12 Results for 2017, 2018, 2019 and 2020 According to the Grades

YEAR	GRADES (%)								
	ONE	**TWO**	**THREE**	**FOUR**	**FIVE**	**SIX**	**SEVEN**	**EIGHT**	**NINE**
2020	6.6	4.2	9.4	4.2	5.0	12.9	6.8	8.9	42.0
2019	6.2	4.2	9.3	4.4	5.1	11.5	7.4	8.6	43.3
2018	6.2	4.2	9.6	3.9	5.4	12.9	7.5	8.2	41.9
2017	6.3	4.3	9.6	4.9	5.3	14.0	6.8	7.5	41.3

Source: (ECZ, 2017, 2018, 2019 and 2020 School Certificate Examination Results Highlights)

Table 1.1 above shows that about 41 per cent of the candidates who sat for Mathematics from 2017 to 2020 failed mathematics. Slightly above 6 per cent of the candidates got division one in mathematics. The underperformance by candidates in Mathematics at Grade 12 national examinations is something which is not healthy in the education sector (MOE, 2013). Jones, Hopper and Franz (2008, p.307) made an observation on underperformance in mathematics and recorded that "No matter the psychological or socio-economic reasons, poor mathematical ability has serious consequences, and as educators we must address the question of why so many students are failing".

In the Zambian curriculum, algebra comprises more than half of what is taught in the Zambian mathematics syllabus. The topics under algebra include the following: equations (linear, simultaneous and quadratic equations), and inequations, factorisation and simplification of algebraic expressions, graphs, variables, sequences and series as shown in Figure 1.1 below:

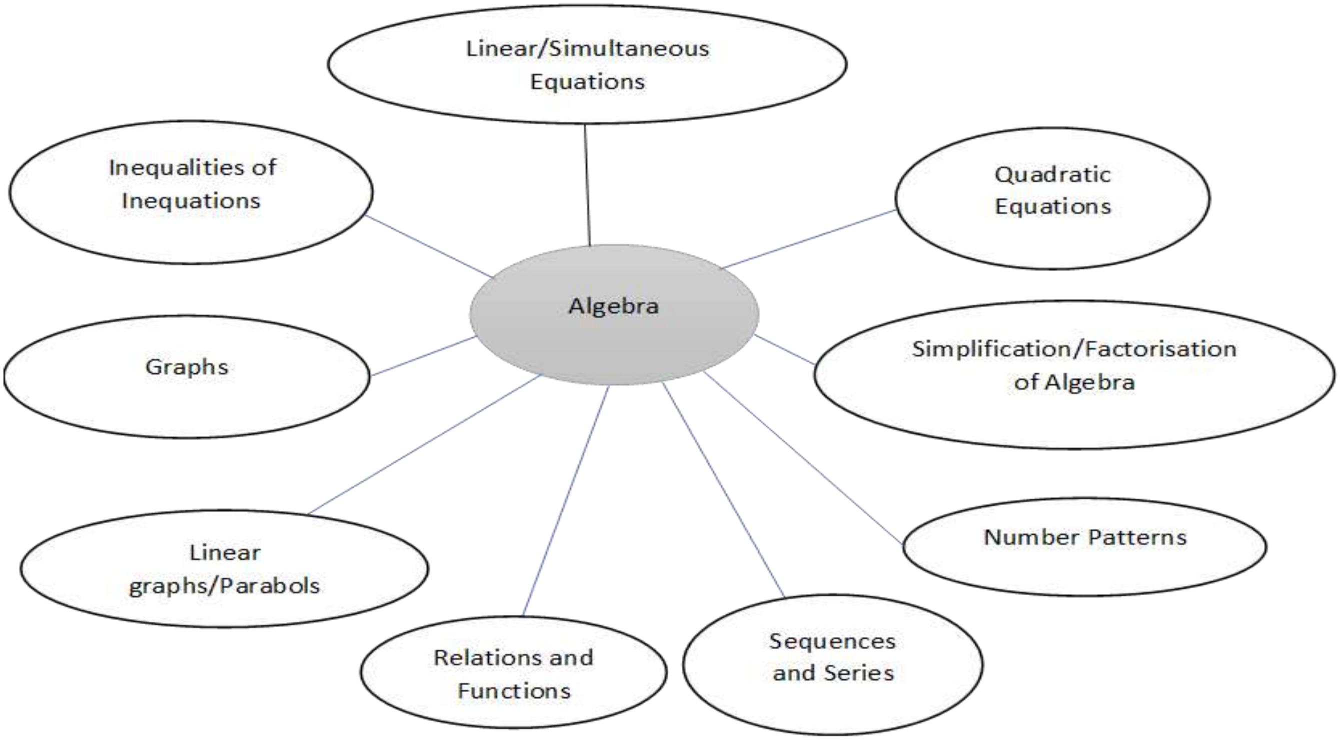


Figure 1.1: Secondary School Algebra Topics in the Zambian Syllabus

1.4 Why Focus on Quadratic Equations

Quadratic equations presented itself as a mathematical concept, which the researcher could focus on in this study for various reasons.

Firstly, the researcher wanted to have a deeper understanding of mathematical knowledge for teaching which is rather complex in terms of teachers' time and energy. The researcher thought of conducting this study with a limited timeframe on one mathematical concept on quadratic equations.

Further, the concepts learnt from the topic of quadratic equations are used later in higher mathematics classes at colleges and universities, especially when dealing with polynomial functions (Curran, 1995). In addition, quadratic equations were selected because it has many uses in career-related professions such as business, engineering and science where the concept is used for modeling ideal situations. In business, the concept learnt from quadratic equations may be used to help in forecasting profit and loss by looking at the maximum and minimum, respectively.

The U-shape of a parabola is incorporated in science in the construction of structures such as it appears on: Suspension bridges, parabolic reflectors of satellite dishes, radio telescope, and cross-section of automobile headlights. Therefore, the quadratic equations lie at the heart of modern communication (Brown et al., 2007). Good insight into quadratic equations will enable learners to deal with different types of equations such as linear, exponential and logarithmic equations and this will lead to an understanding of real-life uses of this concept (Curran, 1995). The idea of quadratic equations is also used in the military when predicting where the artillery shells will hit the earth (Center, 2012).

Understanding quadratic equations is critical for pupils' understanding of further mathematics such as polynomials and calculus. There also appears to be an agreement in literature that many secondary school pupils find the concepts of quadratic equations as one of the most conceptually challenging topics in the curriculum (Kotsopoulos, 2007; Didis, 2011).

Finally, quadratic equations are one of the oldest branches of mathematics with the Babylonians having used algebraic methods about 2000BC. However, symbolic algebra only emerged about 1500AD. The French mathematician Viete (1540-1603) is credited to the early development of symbolic algebra (Van Reeuwijk, 1995). It was Viete who used algebraic symbols to come up with the quadratic formula (Olteanu, 2006). The researcher was, therefore, motivated to carry out a study on one of the oldest and most challenging branches of mathematics.

In the Zambian system of education, learners experience difficulties when answering quadratic equations as reported by consecutive chief markers' reports for Grade 12 and General Certificate of Education (GCE). The candidates at this level experience difficulty when answering questions involving solving quadratic equations of the form: $1-5p-2p^2=0$. Those who attempted to solve the quadratic equation: $1-5p-2p^2=0$ using the formula wrote $a=1$, $b=-5$ *and* $c=2$ there by getting the answers wrong (ECZ, Examiners Report, 2005).

According to the chief examiners, other candidates failed to expand the given expression on the left hand side correctly and there by obtaining expressions such as:

$6x^2 - 7x + 2 = 3$ or $6x^2 - 7 - 2 = 3$(ECZ Examiners Report, 2012).

If a good number of candidates' experience difficulties in solving quadratic equations in the manner suggested above, it is worth investigating the teaching and learning processes.

1.5 Teaching and Learning of Mathematics in Zambia

In Zambia, the overall teaching and learning process occurs in a highly examination-oriented context. What followsis that a secondary school will be rated highly if it produces very good results at both grades 9 and 12. This trend seems to affect not only pupils learning methods, but also teachers' teaching approaches, in the sense that learning to answer examination questions accurately is seen as more important than actual meaningful learning. The teacher, in most cases, would take a lot of time drilling pupils on how to answer examination questions than how to understand a topic.

There are many factors which affect pupils' performance in mathematics and that may inform failure rates in mathematics at grades 9 and 12. Among these are how pupils are taught mathematics and teachers' mathematical knowledge.

1.6 Statement of the Problem

Secondary school teachers need to have multiple representationsand an understanding of the way learners perceive and learn to solve quadratic equationsif they are to convey a deeper understanding of their content. Various, research has been conducted at the primary school level (Ball, Hill, and Bass, 2005) with fewer more recent efforts to examine mathematical knowledge for teaching at the secondary level (Ferrini-Mundy et al., 2005). It is against this background that research needs to be conducted at secondary school level.

In the past three decades, various studies have focused on learners' understanding of different aspects of equation solving (Kieran, 1988), Linchevski, and Herscovics (1996), andLi (2007). The findings from these studies partially demonstrate the complexity of learning and teaching of equations.This could be a valuable and important resource for teachers of algebra to enrich their knowledge. The literature on learners' understanding of equations also motivates parallel studies on algebra teachers' mathematical knowledge to ensure that teachers have conceptual understanding and processes in which learners have difficulties.

From the mid-eighties, several mathematics education researchers have devised perspectives on teachers' content for teaching in general (Shulman, 1986, 1987), knowledge for teaching mathematics in general (Ball and Bass, 2005) and mathematical knowledge for teaching a particular topic such as algebra (Fellini-Mundy, 2003, and Li, 2013).

Different scholars such as Ball, Bass, Hill, and Schilling (2005) developed instruments for assessing knowledge needed for teaching mathematics.These have been validated or tested for trustworthiness by research groups (Ferrini-Mundy, 2005). However, most of the instruments which have been developed are mostly for primary school level mathematics concepts such as number and numeration, and multiplication and division. It is in this regard that systematic research and applicable instruments for secondary level mathematics are needed or required.

In another review on frameworks developed for understanding MKT, Scheiner (2015) found that the current generic frameworks for mathematics teachers' knowledge portray differences of opinion and lack of clarity about the nature of teachers' knowledge. Scheiner (2015), therefore, suggests that research on knowledge for teaching should focus on specific mathematical concepts.

In a literature review on studies that have been conducted on MKT, Hoover, Mosvold, Ball and Lai (2016) found that although there is a rising interest on the mathematical knowledge that is specific to teaching, there is lack of theoretically grounded, well defined and shared conception of MKT. They, therefore, argue that, "mathematical knowledge for teaching needs to be elaborated for specific mathematical topics and tasks of teaching, across educational levels," (p. 17).

There is inadequate research on teachers' mathematical knowledge at secondary school specifically in key topics such as quadratic equations. Existing studies on mathematics teachers' knowledge of mathematics topics such as quadratic equations are neither abundant nor systematic.

1.7 Purpose of the Study

The purpose of the study was to evaluate teachers' mathematical knowledge for teaching quadratic equations. The focus was on subject matter knowledge, knowledge of pedagogy, and knowledge of learners.

1.8 Research Questions

This study aimed at answering the following research questions:

1 What subject matter knowledge for teaching quadratic equations at Grade 11level did the three secondary school mathematics teachers possess?
2 What strategies did the three secondary school mathematics teachers use in teaching quadratic equations at Grade 11 level?
3 How did the three secondary school mathematics teachers address pupils' errors and misconceptions related to quadratic equations at Grade 11 level?

1.9 Significance of the Study

Reviewing the background of the study to the research problem as presented earlier, this section points out the most important features informing the significance of the study. The term mathematical knowledge for teaching (MKT) was first used by Ball (1990).Since then, academicians have examined how MKT flourishes, what components it has and how they relate to each other. Previous studies claimed that the nature of MKT is specific (Berry, 2004; Veal and Makinster, 1999; Ball, 1990). Nonetheless, there was not enough to show how MKT was specific and how teachers convey their knowledge of different subjects to the learners. Consequently, it has been described in the literature that more topic specific MKT studies need to be done in the classroom to find out how teachers utilise their MKT whilst transferring knowledge to the learners (Ball, 1990; Shannon, 2006).

Having been was conducted in the real classroom environment, the study may provide valuable information on MKT literature about the nature of the construct and effective knowledge that teachers utilise while teaching. Contrary to other studies, this study was carried out by taking into account the teaching experiences of teachers in a real classroom.The study intends to provide mathematics teachers with valuable knowledge on practical teaching of quadratic equations besides giving theoretical knowledge. Experienced teachers of mathematics are liable to have many instructional methods and strategies, which might help other teachers teaching the same topic. Sharing MKT can improve both teachers' practical and theoretical knowledge (Ball, 1990; Longhran et al., 2004). Consequently, giving detailed information of experienced teachers about their practices and sharing them may act as a source for teachers' education programmes and in-service teacher training.

This study attempts to respond to calls for better understanding of mathematics knowledge for teaching. In the past, education research has dwelt much on the aspects of mathematical knowledge for teaching and learning mathematics in general.However, more studies need to be done on specific topics of mathematics so that generalised conclusions could be made.

It can be argued that studies on specific topics such as quadratic equations can make teachers understand the errors or misconceptions, which pupils have in understanding that topic. Teachers may develop the conceptual understanding of the cognitive processes of the pupils as they learn specific topics. This is important as teachers need to put much emphasis on conceptual understanding as opposed to procedural understanding.

The study on mathematics teachers' knowledge of quadratic equations is important because it may reveal the necessary condition for teaching methodologies and approaches that could help pupils have conceptual understanding of algebra in general and quadratic equations in particular. Shulman (1986) stated that the research on what teachers should know should also be supplemented with research on the teaching of particular subjects.Studying the kinds of MKT used in teaching specific topics such as quadratic equations is important. It may help researchers and mathematics teacher educators to evaluate the depth and breadth of individual mathematics teacher' own conceptions of the approaches and best ways of teaching and learning them.It can also help to make sense of teachers' instructional preferences, reasoning and decision-making.

Finally, such studies could contribute to identifying the most crucial features of MKT (Subject matter knowledge and pedagogical content knowledge) and support the effective teaching. They also facilitate substantial growth in pupils' algebraic proficiencies, including the recent emphasis on developing reasons and sense making skills (Graham, Cuoco, and Zimmerman, 2010).

1.10 Theoretical and Conceptual Framework

The theoretical framework that informs the study is drawn from the works of Scheiner (2015). This is what he called on the broad perspective of mathematical knowledge for teaching (MKT) to include an epistemological, cognitive, and didactical dimension. Thus, the theoretical framework for this study shall be referred to as (Mathematics) teachers' knowledge: epistemological, didactical and cognitive perspective.The three dimensions in the framework were considered as useful lenses in investigating the mathematical knowledge. Epistemological dimension of mathematical knowledge for teaching is a thorough understanding of a particular discipline by the teacher and how knowledge is structured within the subject.

In teaching quadratic equations, the mathematics teachers' subject matter knowledge may include understanding of quadratic expressions, solving linear equations, solving quadratic equations using various methods such as factorisation method, completing the square method, and the formula method. A more in-depth understanding would include the relationship which exists between the concepts and the methods. For example, the relationship which exists between completing the square method and the formula method. The subject matter method would help in answering research question 1, which reads: What state of teachers' subject matter knowledge for teaching quadratic equations did the three secondary school mathematics possess?

Didactical dimension has to do with teachers' knowledge of the pedagogical form as the content is represented in the instructional media such as visual aids, textbooks and other teaching strategies (Ball et al., 2008).

Teachers' knowledge of didactical representations exists at two levels. These are macro level which in the Zambian context is the Curriculum Development Centre (CDC), and the micro level, which in this case is the classroom level which is the focus of this study. At micro level, it is important that the teacher presents the work in such a way that it is comprehended by the learner. The teacher must go beyond the subject matter knowledge in order to provide the didactical representations. The teacher must help the learners to understand that one quadratic equation can be solved using a variety of methodologies. This helps the learners to develop the strengths and limitations of each method. This helped the researcher answer research question 2 on thestrategiesthe three secondary school mathematics teachers use in teaching quadratic equations.

Cognitive dimension represents teachers' knowledge of the cognitive form of the content in learners' mental representations. Addressing learner difficulties by the teacher is the core of the cognitive dimension of mathematical knowledge for teaching (Ball, 1990).

Teachers' thorough understanding of learner's conceptions in the solving of quadratic equations would involve not only recognising the errors or misconceptions but also anticipating the likely causes of such. This helped the researcher to answer research question 3, which investigated how mathematics teachers in the three secondary school addressed pupils' errors and misconceptions related to quadratic equations.

1.11 Limitation of the Study

The study had a number of limitations. One of the limitations was that the presence of the observer in the classroom could have been an obstacle to accessing information. In order to combat this, the researcher started observing the lessons a week earlier before the actual teaching of quadratic equations.

The other limitation was that the categories of teacher knowledge proposed by this study might not be assumed to be comprehensive, but they can be regarded as a basis for understanding knowledge for teaching quadratic equations.One way that the researcher intended to combat this limitation was to clearly describe thedata gathering techniques and analysis. This would help other researchers to observe mathematics teachers teaching other topics.

The other limitation is that this study was conducted over a short period of time compared to the studies that aimed at building theory for mathematical teacher knowledge, for example, studies by Adler and Davis (2006), Ball et al. (2008), and Herbst et al. (2009). These studies observed mathematics lessons taught by several teachers over a long period of time and on different mathematical topics. However, this was not possible due to limitedtime and resources. Although the data generated in this study covered a narrow range in terms of quadratic equations content, it might be sufficient for theory proposition and descriptions. Secondly, data was generated from real classroom settings (Yin, 2009). Thirdly, because all lessons were observed under each topic and each teacher was teaching the same topic in several streams at the same level, this made me to collect large volumes of data over a short period of time. Furthermore, the lessons that were observed towards the end of each lesson did not yield new results when analysed.This meant that a saturation point had been reached and data collection could be stopped (Cohen et al., 2007). Additionally, the triangulation of the lesson observation with data from the interview and questionnaire, afforded the researcher an opportunity to complement and confirm the findings.

Another limitation was the choice of the participants in the study, which may not represent the views of other teachers. To combat this limitation, the researcher employed a number of data collection methods such as interviews, observations and questionnaire to triangulate data.

The study did not also take into consideration students' views about quadratic equations. This means that the categories of knowledge revealed by the study might not have captured enough information regarding what learners went through when developing quadratic equations concepts. Through the analysis of the learners' views, and the classroom interactions between and among learners, the researcher would have captured more insights regarding the complexities of mathematical knowledge for teaching quadratic equations.

Since video recorders were not allowed in classrooms by some Headteachers, observations were not conducted with the video recorder as planned. Therefore, a voice recorder and field notes were employed to compensate the video recording.

1.12 Thesis Outline

The thesis is presented in five chapters. Chapter 1 is the background to the study with special attention to given to defining mathematical knowledge for teaching. The significance of the study and the statement of the problem are also given in this chapter. Chapter 2 presents a review of some relevant literature related to MKT. The chapter also presents a review of some theoretical frameworks related to teacher knowledge and MKT. Chapter 3 presents the research methodology that was used in this study. This includes the philosophical orientation of the study, the research design, and the details of the participants. Data collection methods, data analysis procedures, trustworthiness and ethical issues are also discussed in this chapter.

Chapter 4 presents the findings of the study in relation to each research question. Chapter 5 presents a discussion of the findings which are classified into different categories of teacher knowledge for teaching quadratic equations. The summary of the major findings and recommendations of the study are also discussed. The chapter also presents the limitations of the study and concluding remarks.

CHAPTER 2

REVIEW OF RELATED LITERATURE

2.0 Introduction

This chapter reviews the necessary literature regarding the mathematical knowledge for teaching which teachers need to possess to effectively teach quadratic equations. The literature is reviewed under the following themes: development of mathematical knowledge for teaching, subject matter knowledge, knowledge of teaching strategies, knowledge of learner conceptions and previous research carried on the topic of quadratic equations.

2.1 Studies Conducted on Mathematical Knowledge for Teaching

Issues of what knowledge a teacher should possess in order to effectively teach mathematics is very critical in modern trends of education (Ball, Hill, & Bass, 2005). Traditionally, research has been devoted to issues of preparation (aspects of teaching) but less attention has been given to how much teachers need to know the subjects they teach (Ball, Thames, & Phelps, 2008).

Before the 1980s, education research comprised two major types of research studies of which both were behaviourist in nature. The first one was "process-product" oriented and this has a large set of studies which described the relationship between teacher conduct or behaviour and learner achievement (Hill, Rowan, and Ball, 2005). The typical behaviour of teachers, which some studies examined included group work of learners. Although there were a lot of studies done by the behaviourist movement, they were not particularly useful in producing data, which we would use to improve pedagogy. This study tried to pick it up from what existed over the years between the behaviourist and the emerging studies which look at addressing pedagogy issues.

The motivation for studies that lookat subject matter teacher's knowledge started in the mid-1980s, when Lee Shulman in 1985 made the presidential address at the American Educational Research Association (AERA) annual meeting. In his address, Shulman argued that there was a lack of focus on knowledge of subject matter in the process-product research on teaching which placed much emphasis on issues to do with classroom management and neglecting specific content knowledge that teachers needed to have to teach effectively. In his research, dubbed *"growth for teaching"*, Shulman emphasised on the importance of subject matter knowledge and pedagogical content knowledge, which combined theory and practice. In an attempt to justify the importance of subject matter knowledge, Shulman (1986, p. 9) stated that, "the teacher need not only understand *that* something is so; they must further understand *why* it is so". This is in tandem with Even and Tirosh (1995, p. 6) stated, "insufficient subject matter knowledge, on the part of the teachers, does not seem to be sporadic, infrequent phenomenon, but rather a wide spread one whose consequences for the actual teaching should be investigated". The argument justifies the importance of subject matter knowledge as a component of mathematical knowledge for teaching and hence, the need to carry out a study.

Shulman's seminal presentations on knowledge of teaching are closely related to the studies done by Grossman (1990) who looked at subject matter knowledge in form of substantive and syntactic knowledge. Substantive knowledge determines how teachers facilitate learners' inquiry as well as their own learning about the subject. On the other hand, syntactic knowledge deals with knowing how new concepts and knowledge are brought into the field. The researcher raised the question 'Do teachers possess both substantive and syntactic knowledge? If they do which one is dominant as they engage pupils in meaningful discussion?

In response to Shulman's call on the importance of PCK, Ball et al. (2001) responded by pointing out that research using traditional measures of teacher content knowledge (e.g., degrees obtained or mathematical courses taken) "…leaves obscured the nature of teachers' knowledge" (p. 443). The researcher agrees with Ball and others (2001) in that, this study focuses on what goes on in the classroom rather than using traditional approaches of assessing MKT of academic credentials such as degrees or mathematical courses taken. The MKT is knowledge of mathematics used in doing the work of teaching and it includes and goes beyond the PCK that Shulman had originally proposed.

It is from the studies done by Ball and others that a model to describe mathematical knowledge for teaching was developed (Ball, Thames, and Phelps, 2008; Hill and Ball, 2004; Rowan, and Ball, 2005). Ball's (2008) framework breaks Shulman's categories of subject matter knowledge and pedagogical content knowledge (*PCK*) into two distinct pieces as shown in Figure 2 below:

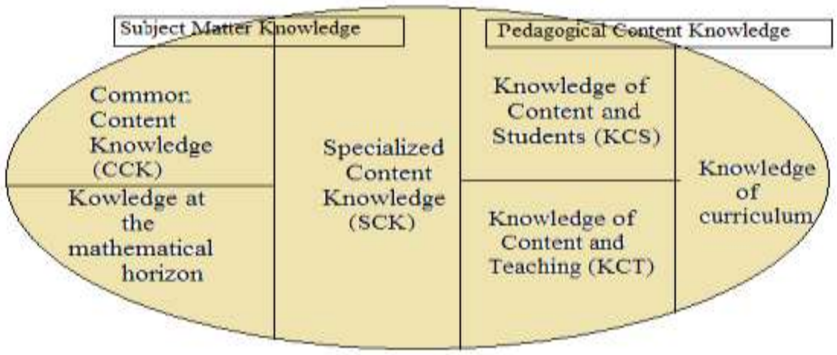


Figure 2.1: Mathematical Knowledge for Teachers

(Source: Ball et al., (2008). Mathematical Knowledge for Teaching

The MKT framework by Ball et al. (2008), for defining the knowledge needed by teachers for effective mathematics teaching, leaves many questions unanswered. For example, which specific knowledge is needed and mostly used? What type of teacher knowledge results in learner learning? It is against this background that the current study was conducted to provide the various interactive processes through which the teacher's subject matter, knowledge of student learning and knowledge of pedagogy are blended.

Ball, Bass, and Hill (2005) investigated whether and how teachers' mathematical knowledge for teaching added to learners' gain in mathematics achievement during the early stages of primary school level. The results suggested that teachers' mathematical knowledge for teaching positively affected learners' gain in mathematical achievement during the early stages of primary school. The results further indicated that teachers' mathematical knowledge was important even when teaching at a primary school. The idea of mathematical knowledge for teaching is essential to this study.

Two major features emerged from the studies conducted by Ball et al., (2008). Such knowledge must be simplified to the level of the learner and a practice-based theory be developed. The practice-based theory views mathematics teaching as involving substantial mathematical work, which focuses on mathematics that emerges within the core domains of teaching tasks such as posing mathematically sound questions and problems to probe or promote learner understanding.

The practice-based theory by Ball and Bass (2005) were important in addressing the pedagogy component of the mathematical knowledge for teaching quadratic equations. In this study, it wasthe researcher's view that such knowledge can only be seen when the teacher is in class. The researcher takes cognisant of the fact that such knowledge cannot be assessed by looking at teacher credentials such as degrees.

MKT framework has provided insights into the teaching of mathematics by emphasising that it involves some mathematical work. The framework also acknowledges the usefulness of classroom interactions between the teacher and the students. This means that the framework acknowledges that mathematics is socially constructed. However, McCrory et al. (2012), argued that Ball's MKT framework does not take into account the complexities of secondary school mathematics teaching because it was developed based on elementary and middle school mathematics. This study, argues that despite being developed based on elementary and middle school mathematics, the MKT framework enhances understanding of teacher knowledge in relation to the subject. The further division of SMK and PCK into sub-domains sheds light onto the complexities and demands of the teaching of mathematics.

The study further concurs with Herbst and Kosko (2012) that the framework is too general because it assumes that knowledge for teaching of different mathematical fields is the same. However, Mathematics has several fields that require different content, pedagogic and reasoning skills. Hence, researchers might mainly use the framework as a guiding tool for exploring and developing MKT frameworks for specific fields of mathematics. The results of the studies done by Ball and others reviewed that mathematical knowledge for teaching was important and needed more attention. Having looked at the previous studies and career as a teacher, the researcher thought of carrying out the study in mathematical knowledge for teaching quadratic equations in Katete District of Zambia.

Other studies on MKT aimed at establishing evidence on whether teachers' mathematical knowledge influences students' achievement. Hill, Rowan and Ball (2005) conducted a large-scale survey in which they followed teachers' and learners' achievements over a period of time from first to third grade. Their studies found that teachers' mathematical knowledge is directly related to students' mathematical achievements. This means that teachers with high MKT score provide better instructional experiences for their students. They, therefore, suggested that improvements on teachers' mathematical knowledge can result into improvements in students' mathematical achievements. Hill et al. (2008), observed that early studies on MKT took two approaches; the deficit approach and the affordance approach. The studies, which used the deficit approach aimed at establishing links between a teacher's lack of mathematical understanding and patterns in his/her mathematics instruction. The studies which used the affordance approach aimed at highlighting MKT that was useful during instruction. Hill et al. (2008), explained that although these studies have generated more knowledge, they have several shortcomings. The studies are fine grained in the sense that they are qualitative studies, which mainly focused on one topic in the context of only one lesson. The studies also analysed the relationships within one teacher, and they have not linked teacher knowledge to students' achievement. As such, the aim of their studies was to continue establishing links between MKT and instruction.

Hill et al. (2008), used mixed methods of data collection in USA to examine the link between teacher's MKT and quality of instructional practice. Their studies found that there is a strong, significant and positive association between MKT and teachers' mathematical quality of instruction. They explained that strong MKT helps teachers to play different roles in their teaching of the subject appropriately. These include providing explanations for thinking and general conceptual discussion of procedures, appropriate selection and sequencing of mathematical tasks, use of multiple representations, making connections between informal and formal mathematical ways, good use of mathematical language, and several others. Another MKT study concerning teaching of secondary school mathematics was conducted using COACTIV model (Krauss et al., 2008). The components of COACTIV were Shulman's (1986) CK and PCK teacher knowledge categories. The CK test was developed using the secondary school mathematics curriculum, and the PCK test was developed from its subcategories (Krauss et al., 2008). After analysing the tests that were administered to secondary school teachers, they found that there was a high correlation between CK and PCK. Krauss et al. (2008), concluded that CK supports the development of PCK. This review shows that the studies on MKT took different approaches. The common aspect of these studies was that they studied MKT with respect to a whole mathematics curriculum. This implied that the researchers assumed that it is possible to generalise MKT for all primary and secondary school mathematical fields.

Chick, Pham, and Baker (2006) conducted a study on the teachers' mathematical knowledge for teaching subtraction. It was a qualitative study and data was collected through questionnaires, lesson observation, and interviews. The framework consisting of subject matter knowledge, instructional strategies, and knowledge of learner conceptions were used to identify mathematical knowledge for teaching.

The framework was based on the fact that teachers needed to possess adequate subject matter or content before they can deliver the lesson. Secondly, the teacher needed to possess proper strategies to make content accessible to the learner (Shulman, 1986). Finally, the teacher needed to have knowledge on learner conceptions, which was evidenced if the teacher showed interest in the learners' prior knowledge, dealt with learners' difficulties during the lesson, and took care of possible learner misconceptions.

This study benefitted from the study conducted by Chick, Pham and Baker by adopting mathematical knowledge for teaching from the three perspectives thus: knowledge of the subject matter, pedagogical knowledge, and knowledge of learner conceptions.

Lin and Rowland (2016) were commissioned to review the psychology of mathematics education (PME) studies related to pre-service and in-service teachers' knowledge and teaching development. The aim of the review was to find out how much research had been conducted in the field from 2006 to 2016. The review focused on several mathematics teacher education issues including teacher knowledge, teacher beliefs, teacher education, educator education, professional development, and professional growth. Lin and Rowland (2016) found that there were two hundred and twenty papers submitted to PME with a focus on mathematics teacher knowledge.

They also found that most of the research on mathematics teacher knowledge was conducted in USA and Europe and there was only one paper that originated from Africa. The authors argued from their findings that Shulman's (1986) powerful categories of teacher knowledge have not yet been conceptualised in African countries.

The study also found that the largest research on mathematical work for teaching in Africa were conducted in South Africa. The studies were conducted by Adler and her colleagues under the project called quality mathematical education for teachers (QUANTUM). One of the goals of the

QUANTUM project were to elaborate on MKT. As part of the project, Adler (2005) studied how knowledge is being assessed in mathematics teacher education programmes in South Africa. Her justification for such a study on mathematics education was that there were fewer people taking up advanced study of mathematics (Adler, 2005). She viewed this as a threat to the development of the mathematics discipline as well as to the provision of scientists, engineers and mathematically well-qualified teachers for the South African schools. The study focused on the kind and quality of mathematics being taught in South African secondary schools.

Adler’s study aimed at establishing if the notion of unpacking is being valued as part of mathematical competence needed by teachers. She identified all teacher education courses and analysed their formal assessment tasks. The results indicated that the mathematics in the teacher education course was highly compressed. This implied that teachers did not have an opportunity to learn to unpack mathematics. Analysis of assessment tasks in quadratic equations showed that most tasks demanded unpacking of mathematics. They did not reflect the concept of teaching. Adler argued that the absence of unpacking in the courses is a clear indication that it is a challenging issue in mathematics. She further suggested that there is need for a detailed study of the actual teaching practice that can help to improve understanding of unpacking. Although the study depicts unpacking of mathematics as general mathematical work, the later studies in relation to the QUANTUM project were conducted on specific topics.

As part of the QUANTUM project, Adler and Davis (2006) studied mathematical practices revealed in formal assessments across a range of mathematics teacher education courses in South Africa. The researchers assumed that there is specificity to the mathematics that teachers need to know and know how to use. They view unpacking of mathematical ideas as the most important element of knowledge for the work of teaching mathematics. Their analysis was based on assessment tasks. One of their aims was to find out the mathematical knowledge as well as the teaching practices embedded in the assessment tasks. Thus, the questions that guided the analysis of assessment tasks focused on the primary and secondary objects of the content (whether mathematics and/or teaching) and the mathematical knowledge revealed by the tasks. For the tasks that revealed both mathematics and teaching objects, Adler and Davis (2006) were interested in determining the object which was prioritised in a particular assessment task. The study findings showed that despite being specifically designed for teachers, mathematics education courses were dominated by mathematical knowledge, which involved the ability to demonstrate mastery of mathematical concepts and their procedures. Adler and Davis (2006) characterised the courses as compressed and unelaborated Mathematics which does not demand any display of understanding. Thefindings of the study also showed that most of the tasks that were presented in a compressed form required some unpacking. They argued that the prevalence of compressed mathematical tasks in the assessment is a sign of lack of knowledge of mathematics for teaching. Further, the researchers recommended that studies be conducted to elaborate the idea of mathematics for teaching.

In further exploration of the idea of mathematics for teaching under QUANTUM project, Kazima, Pillay and Adler (2008) investigated mathematical work demanded by teachers during teaching of probability and functions. They used the eight aspects of problem solving suggested by Ball et al. (2004), as a theoretical framework for their study. Kazima et al. (2008), condensed the eight aspects into the following six categories of problem-solving; definitions, explanations, representations, working with learners' ideas, restructuring tasks and questioning. Their study mainly focused on the mathematical work of teaching that is performed by teachers when introducing and explaining probability and function concepts, and the resources used. They observed and analysed lessons by two teachers, one was teaching probability and the other was teaching functions in South African secondary schools. The study findings showed that the teachers used different mathematical approaches in introducing concepts, ideas or procedures to the learners. This implied that the teachers dealt with different mathematical work. The results also indicated that the teachers differed in their areas of focus and emphasis during the lessons. The teacher who was teaching probability captured all categories of mathematical work in different degrees, while the one who was teaching functions captured only some of the categories. Kazima et al. (2008), argued that the differences in approaches used by the teachers implied that different topics demand different mathematical work. In conclusion, they argued that the tasks of problem solving suggested by Ball et al. (2004), have specific meanings across mathematical topics and teaching approaches. Therefore, they suggested, that research on mathematics for teaching should be investigated further and be specific to mathematical topics.

Another study, which was part of the QUANTUM project was conducted by Adler (2010). She used Shulman's (1986) theoretical framework of teacher knowledge to further explore the notion of mathematics for teaching and its importance in teaching and learning of mathematics, and in teacher education.

Adler argued that "strengthening our understanding of the mathematical work of teaching, what some refer to as mathematics for teaching, is a critical dimension of enhancing its teaching and learning," (Adler, 2010, p. 1). She justified this argument using two examples of geometry lessons from school mathematics classrooms under the QUANTUM project. In the first example, a teacher gave students a task with an aim of developing angle properties of a triangle. In the second example, a teacher asked students to work in groups to find the number of diagonals in a 700-sided polygon. In both examples, Adler was interested in illustrating and illuminating four components of mathematical work: designing the task, mediating learner progress, valuing and evaluating learner responses, and managing the integration of mathematical content and mathematical processes as foci in the lesson. The results of the study indicated that the tasks constructed by the two teachers were not similar to the usual tasks found in South African mathematics textbooks. This meant that there was a mathematical work that was involved during development of the tasks. Adler (2010) noticed that mathematical reasoning was in focus in both examples and that the learners developed multiple solution paths for each task. This implied that the teachers were required to analyse the learners' responses and do mathematical judgments. In conclusion, Adler argued that mathematics for teaching matters, especially that of designing and mediating a task. Therefore, she suggested that educators embrace deeper understanding of the complexities of teaching, and know their work in teacher education (Adler, 2010).

The studies by Adler and her colleagues under the QUANTUM project agree with Ball et al. (2004) that teaching mathematics involves carrying out some mathematical tasks. The studies have shed more light on what is involved in several tasks of teaching mathematics in relation to different topics. Regarding teaching, the study by Adler (2010) has clarified two tasks; designing tasks and evaluating learner responses. However, they have not clarified on mathematical knowledge required for carrying out the tasks. Ball et al. (2004), explained that tasks of teaching mathematics depend on mathematical knowledge.

Malambo (2015) conducted a qualitative study that explored Zambian mathematics student teachers' content knowledge of functions and trigonometry. The students were final students who had completed mathematics education courses. In that study, content study was investigated as CCK and SCK. Data was collected through paper test and semi-structured interviews. Paper test data was analysed through descriptive statistics and qualitative techniques while interviews were analysed through content analysis.

The findings showed that CCK for both functions and trigonometry were not proficient and students showed a shallow understanding. The findings, therefore, suggested that the study of advanced mathematics does not automatically result in students' comprehensive understanding of secondary school mathematics. The data collection methods from the study conducted by Malambo were beneficial to the current study.

Jakobsen and Mosvold (2015) conducted a review of an empirical research on MKT in Africa. The review was done with an aim of illuminating what had already been investigated on MKT, and how MKT had been approached in African countries. There were several issues highlighted in the review in relation to the number of studies done, the methodology used and the target groups. The review pointed out that there were very few studies were conducted on MKT in Africa and most of the studies were qualitative, hence, they did not use any measures of MKT. The researchers, therefore, suggest that more studies should be done in different African countries using existing measures of MKT. The review by Jakobsen and Mosvold (2015) was in agreement with Lin and Rowland's (2016) observation that there was limited research on MKT that has been conducted in Africa. This suggests that the work that was involved in the teaching of mathematics was not clearly understood in most of the African countries. Although Jakobsen and Mosvold (2015) have emphasised that research on MKT should focus on measuring MKT using existing measures, it was also necessary to focus on understanding MKT for African countries. This is because the contexts in which the MKT frameworks and its measures were developed are different from the African context. As such, there might be in this study some issues and interests concerning African countries to which the frameworks were developed could not be adopted. So, to successfully adapt the existing MKT measures to suit a particular African country, researchers might need to begin from a framework that is sensitive to some contextual issues.

The next subheading looks at the knowledge of the subject matter as the first component of the mathematical knowledge of teaching.

2.2 Knowledge of the Subject Matter

One of the elements of mathematical knowledge for teaching identified in this study is subject matter knowledge. It must be noted that insufficient subject matter knowledge is a concern as it could hinder the teaching and learning process.

Ball (1990) conducted a study on 252 prospective primary and secondary school teachers on fractions, focusing on the division of zero. Data was collected through interviews and questionnaires. The findings reviewed that prospective teachers had problems in explaining the meaning of division in relation to fractions. Ball's study also noted that the prospective teachers' subject matter knowledge was based on memorisation, and that they were unable to create a representation for the problem or explain the cognition beyond their calculation even though they could operate them correctly. In other words, prospective teachers lacked knowledge of connections between mathematical concepts or ideas. She noted that most of the mathematics teachers perceived mathematics as a set of rules and facts, and following procedures to arrive at a desired solution. The study further noted that knowing mathematics for prospective teachers is knowing how to do it. Therefore, most of the prospective teachers would rather their learners know how to carry out algorithms than teach the underlying reasoning that makes algorithms work out as they do. The data collection methods of interviews and questionnaires were of interest to the current study.

Ball, Bass, and Hill (2005) investigated whether and how teachers' subject matter knowledge for teaching added to learners gain in mathematics achievement during the early stages of primary school level. It was a qualitative study and data were collected through lesson observation and interviews. The results were that teachers' mathematical knowledge for teaching positively affected learners' gains in mathematical achievement during the early stages of primary school. The results further indicated that teachers' mathematical knowledge was important even when teaching at a primary school.

Although the studies conducted by Ball, Bass and Hill (2005) were on fractions, the aspects of data collection methods and data analysis procedures were still of interest in the current study. In this study, the issue of subject matter knowledge was a critical entity of mathematical knowledge for teaching quadratic equations. Further, although the subject matter studied by Ball, Bass and Hill

(2005) was on a primary school mathematics topic, the study made inferences about knowledge for teaching in a general manner with the hope of shading light on how teacher knowledge can be used for effective teaching.

Grossman (1990) carried out a study on the subject matter knowledge of teachers of English. It was a qualitative case study, which involved two teachers of literature in English. The purpose of the study was to find out how subject matter knowledge of teachers of Literature in English affected the interpretation of the texts. Data was collected through interviews and observation and was analysed through categorisation into themes. The study by Grossman (1990) showed that teachers' knowledge and beliefs about the purpose for teaching a subject had an impact on the instructional beliefs. For instance, one of the participants viewed teaching literature as explaining the given text in depth, whereas the other participant put emphasis on helping students to relate the text to their own lives. The two participants used different instructional materials and activities to pursue their goals.

Grossman (1990) further indicated that teachers' lack of content knowledge affected the style of instruction. In teaching the material, they were uncertain of the fact that teachers may choose to lecture rather than involve the learners in meaningful learning activities. Data was analysed using coding system and coming up with themes. Although the study by Grossman was based on English language, her study was of interest to this study.The influence of subject matter on quality of teaching, data collection methods of observation and interviews which she used, and thematic analysis of data were beneficial to this study.

Grossman (1990) identified four knowledge domains for teachers in general and these included, pedagogical knowledge, subject matter knowledge, pedagogical content knowledge (PCK) and knowledge of content. She proposed that PCK was composed of four central components. The first component was knowledge of beliefs about the purposes for teaching a subject at different grade levels. She noted that knowledge of learners' understanding, conceptions and misconceptions of particular topics in a subject is a component of PCK. Moreover, she stated that knowledge of curriculum materials available for teaching a particular subject matter and knowledge of instructional strategies and representations for teaching particular topics are components of PCK.

Grossman (1990) stated that the development of teacher knowledge is influenced by those four knowledge components and that the preparation of teaching should include sufficient training in each of these knowledge components.This should be done so that teachers have the capacity to transform their subject matter into a form that learners can access. This study looked at the four teacher knowledge domains and how the teacher would actually use them when teaching quadratic equations. Table 2.1 below shows Grossman's model.

Table 2. 1: Grossman Teacher Knowledge Model (Grossman, 1990. p. 5)

SUBJECT MATTER KNOWLEDGE		
Syntactic structures	Content	Substantive structures

GENERAL PEDAGOGICAL KNOWLEDGE		
Learners and learning	Classroom management	Curriculum and instruction

PCK		
Conceptions of Purposes for Teaching Subject Matter		
Knowledge of understanding	Curricular knowledge	Knowledge of instructional strategies

KNOWLEDGE OF CONTEXT		
STUDENTS		
Community	District	School

Yusof and Zakaria (2010) did a study that explored and described the level of MKT of three teachers focusing on the topic of functions at secondary school level mathematics. The study found that the teachers who participated lacked conceptual knowledge on the topic of functions, hence, their lessons were not accurate and lacked substantial comprehension for the learners. As for this study, the researcher focused on how the lessons were conveyed to the learners on the topic of quadratic equations. The elements of the subject matter knowledge that were observed in the study included the use of analogy, representation, use of symbols, examples and explanations that were suitable for providing conceptual and procedural explanations. Specifically, this particular study sought to explore and describe the elements of teacher's subject knowledge in the topic of quadratic equations. Table 2.2 below shows the MKT levels as identified by Yusof and Zakaria (2010).

Table 2.2: Mathematical Knowledge for Teaching Levels Identified by Yusof and Zakaria (2010)

LEVEL	INDICATOR
1	• Teacher explains the wrong concept or a concept which is not clear to the learners. • The teacher asks low level questions • The lesson is teacher-centred. • The teacher is unable to detect learners' difficulties with the topic. • The teacher does not relate the lesson being presented to learners' existing knowledge.
2	• The teacher explains the correct mathematical concept but gives the same type of examples to back his or her explanations that is, no variety of examples are given. • Teaching is teacher-centred. • The teacher has an awareness of learners' difficulties but does not probe further through asking questions that allow learners to speak out their ideas about the topic.
3	• Teacher's explanations of concepts are more accurate and clear along with the incorporation of suitable examples. • Learners' participation is seen to be positive through the provision of relevant activities which provide both conceptual and procedural understanding of mathematical principles being studied.

Source: Yusof and Zakaria, 2010, p. 34

In the current study, the researcher observed the levels of mathematical knowledge that the three participants used as they presented their lessons.

Bukova- Guzel (2010) investigated pre-service mathematics teachers' MKT by using solid objects. He found that teachers participating in the study did not pay attention to possible learner misconceptions. The study was qualitative and data was collected through semi-structured interviews, analysis of lesson plans and video recordings of instructional applications. The framework of the analysis of mathematical knowledge used in that study in Table 2.3 used knowledge of teaching strategies, knowledge of learners and knowledge of curriculum. Two of these components; knowledge of teaching strategies and knowledge of learners, were incorporated in this study. Table 2.3 below shows components of the MKT as outlined by Bukova- Guzel (2010).

Table 2.3: Mathematical Knowledge for Teaching Outlined by Bukova- Guzel (2010)

KNOWLEDGE OF TEACHING STRATEGIES AND MULTIPLE REPRESENTATIONS	**KNOWLEDGE OF LEARNER CONCEPTIONS**	**KNOWLEDGE OF CURRICULUM**
• Using appropriate activities in instruction • Using real life examples and analogies in instruction • Utilising different instructional strategies in presentations • Making use of different representations i.e. in graphics, tables, formulas etc.	• Having knowledge of learners' prior knowledge • Using real life examples and analogies in instruction • Having knowledge of difficulties learners will face during learning • Having knowledge of possible learner misconceptions • Having knowledge of learner differences	• Being aware of the elements of the mathematics curriculum (Conceptions and purposes) • Being aware of the varieties of instructional tools in the mathematics curriculum and how to use them • Being aware of the instruments to measure learner learning and how to use them

Source: Bukova- Guzel's, 2010

2.3 Procedural and Conceptual Knowledge

Another form of subject matter knowledge is conceptual and procedural knowledge. Although, the two orientations of *conceptual* and *procedural* teacher knowledge have been studied for a number of years before the 1980s, the major works in describing these two types of knowledge have been done by Hiebert (1986).In their edited book titled *Conceptual and Procedural Knowledge*. Hiebert and Lefebvre (1986, p. 3) defined *conceptual knowledge* as a "connected web of knowledge" while *procedural knowledge* is divided into two parts namely; the recognition of proper 'forms' and "knowledge of rule, algorithms, and procedure". In an effort to give a clear understanding of procedural and conceptual knowledge, Hiebert and Lefebvre (1986), describe the difference between the two as a procedural knowledge associated with rote learning while conceptual knowledgeis associated with meaningful learning.

Hiebert (1986) made a major contribution to mathematics education by discussing the importance of both conceptual and procedural knowledge in which they emphasise that meaningful learning can only take place if both types of knowledge exist. The involvement of both conceptual and procedural knowledge as outlined by Hiebert was beneficial in this study because the concepts are critical in understanding subject matter knowledge.

Star (2007) pointed out that some key notions such as procedural and conceptual understanding still required more careful characterisations in terms of knowledge and knowledge of quality. In understanding this fact, he worked with more than 600 prospective teachers at the University of Texas between 2001 and 2005. During this period, Star noted that procedural and conceptual understanding of basic mathematical routines such as basic operations of integers and whole numbers must complement each other and must not be treated in isolation. He further claims that conceptual knowledge should include not only what is known (procedures) but also why the procedures can be known. Star (2007), suggests that there is such a thing as "deep procedural knowledge", which can exist independently from conceptual knowledge. This type of knowledge is associated with understanding, flexibility, and above all critical judgment (Star, 2005). What this means is that a teacher must be flexible when choosing a procedure that is efficient for a given mathematical problem. The study by Star was a major insight of this study in the sense that the researcher focused on both the procedural and conceptual knowledge as part on an integral component of mathematical knowledge for teaching. Whereas Star carried a study on prospective teachers, this study focuses on serving teachers.

Kotsopoulos (2007) reported that learners needed to develop procedural and conceptual knowledge through various learning experiences in an integrated manner. The Australian Academy of Science (AAS) also recognised the intertwined relationship between procedural fluency, conceptual understanding, and problem-solving and reasoning due to the hierarchical nature of mathematics (AAS, 2015). The cognitive work done by Kirschner, Sweller, and Clark (2006) gave an explanation for the necessity of fluency with prerequisite knowledge. They note that without prerequisite fluency, short-term memory becomes overloaded and unable to effectively process the new concepts being learned. Hattie (2009) noted that procedural fluency with prerequisite knowledge, even at a very early stage, was highly predictive of latter success. The key prerequisite concepts and processes necessary to engage meaningfully with quadratic equations include the following: basic whole number understanding, fraction computation, linear algebraic procedures, and or coordinated geometry.

The key process in working with quadratic equations involves solving or finding the *x* intercepts should there be any. In most curricula, this has involved: factorisation, completing the square method, and the use of the quadratic formula. Each of these methods has its own advantages and disadvantages when it comes to teaching, learning and applications. Research has shown that learners and teachers stay away from some techniques in preference of the factorisation method, generally, using coefficients that are easy to factorise. Since learners' ability to perform fractional and radical arithmetic had been reported low, Bosse and Nandakumar (2005), sought to conduct a study whereover emphasis on relatively simple factorisation was a concern as many quadratic equations cannot be solved by factorisation method. Further, other methods such as completing the square and using the formula that are more efficient or develop conceptualisation may be ignored in teaching (Bosse and Nandakumar, 2005). For instance, factorisation method using algebraic tiles links quadratic equations with basic multiplication division concepts through the area model of rectangles and squares (Howden, 2001). Geometric models are useful in adding to the comprehension in developing the quadratic formula through completing the square method (Norton, 2015).

Barnes (1991), suggested using graphical calculators to plot quadratic equations with no roots, one root, or two roots and linking this to the discriminate values. Research suggests that teachers tend to avoid teaching alternative methods due to high instances of process skill errors with techniques such as the quadratic formula and completing the square method (Zakaria et al., 2010).

In addressing the issue of procedural and conceptual knowledge, Even and Tirosh (1995), examined teachers' knowledge of subject matter in the presentations on the concept of fractions. The study was based on the idea of generating appropriate representations and explanations for a concept should not be based on the facts, rules and procedures but also know why they are true. He referred to such knowledge as 'knowing that' and 'knowing why'. In their study, one participant knew that 4 divided by 0 is undefined but did not explain why. Therefore, the participant would tell students that it was one of the mathematical proposition or axiom that should be memorised. According to Tirosh and Even, the 'knowing that' is the procedural knowledge and the 'knowing why' the conceptual knowledge. Thus, Even and Tirosh concluded that teachers' knowledge of the subject matter of students thinking had a strong influence on their pedagogical decisions. The current study also focused on the way teachers respond to certain questions, which learners pose to them as opposed to mere memorisation of facts and algorithms.

In studying the subject matter knowledge and the lesson study method, Leinhardt and Smith (1985) carried out a case study on the organisation and content of arithmetic teachers' subject matter knowledge of fractions. Their main reason for the choice of fractions was that it was one of the more difficult topics at primary level. Although the subject matter knowledge was a primary mathematics topic, the authors made analyses and inferences about knowledge in a general manner with the "hope that a detailed analysis of expert teachers' fraction will shed light on how knowledge in effective teaching is" (Leinhardt and Smith, 1985, p. 249). The findings suggested that the lesson structure and the subject matter knowledge were related but not completely equivalent. The study supported the assertion of Shulman and others' that knowledge for teaching is made up of general content knowledge and more specific knowledge, which they referred to as pedagogical content knowledge. Further, the findings of the study revealed that experienced teachers had adequately developed the necessary knowledge for teaching, while the novice teachers were lacking in certain areas. Experienced teachers were determined by the performance of their learners and the lesson structure knowledge. Theseteachers also demonstrated well-organised scripts and routines, which they had developed over time. The novice teachers, on the other hand, did not properly organise their time and spent more time on assignments than did the experienced teachers (Leinhardt, 1991).

The study carried out by Leinhardt and Smith (1985), is relevant to this study as it looked at both lesson structure knowledge and subject matter knowledge. With regard to lesson structure knowledge, this study paid much attention to the pedagogical processes the teacher portrayed including the way the work was structured. On the other hand, the researcher of the currents study focused on the content knowledge of the quadratic equations.

Ma (1999) conducted a comparative study on the teachers' understanding of mathematics in China and the United States of America (USA). He used written assessments and interviews on both experienced and novice teachers in both countries. Questions on subtraction with regrouping, multi-digit multiplication, division of fractions and area and perimeter of a rectangle were the focus of the comparative study. The purpose of the study was to explore teachers' understanding of fundamental or basic mathematics in China and the USA. Besides studying students' knowledge of mathematical procedures and algorithms, the researcher also examined mathematical teachers' knowledge in this respect. The findings of the study revealed that the experienced teachers who had more than 10 years teaching experience had more profound mathematical knowledge for teaching than their novice counterparts. Further, Ma (1999) noted that profound mathematical knowledge consisted of both procedural and conceptual knowledge, and it was the base for successful mathematics teaching.

The results also showed that the novice teachers that were interviewed were able to display "algorithmic competency" in solving mathematical problems. Ma (1999) notes that many of the novice teachers were fond of emphasising on the understanding of procedures and getting the correct answer. This is what Shulman (1987) referred to as knowledge limited to "*that is so*" Experienced teachers were interested in making sense of conceptual knowledge of understanding; what Shulman called "*understanding why something is so".* However, Ma noted that the majority of mathematics teachers were very brief when it came to explanations involving conceptual understanding. This current study benefitted from that of Ma (1999) in terms of data collection methods of interviews, which were used in this study.

Tumuklu and Yesildere (2007) conducted a study on pre-service primary school teachers' subject matter knowledge on the topics of fractions, decimal fractions and integers. The study was qualitative and data was collected through lesson observation. Findings of the study were that pre-service primary school teachers' subject matter knowledge was inadequate as they could not identify and assist learners who had difficulties and misconceptions. The findings of the study suggested that, for teachers to present a topic in such a way that was understood by the learners, they needed to possess adequate subject matter knowledge. This finding is similar to the objective of the current study also looked at the subject matter knowledge in relation to identification and assistance of learners with difficulties and misconceptions. The data collection methods and analysis by Tumuklu and Yesildere (2007) were, therefore, beneficial to the current study.

Stein, Baxter, and Leinhardt (1990) investigated the effects of limited subject matter knowledge on the topic of functions. It was a case study involving a highly recommended primary school teacher with 18 years of teaching experience. Data was collected through observations of video recordings. The findings of the study revealed that there was a direct connection between subject matter knowledge and his teaching. The study further revealed that the teacher, though highly experienced, had inadequate subject matter knowledge on the topic of functions. The current study benefitted from this study by using similar data collection methods.

As shown above, studies by several authors, indicate that subject matter knowledge is an important component of mathematical knowledge for teaching. In this study, subject matter knowledge is one of the components researched on.Lesson observations aided this by way of display of accurate mathematical information, sequencing of facts and flexibility of presenting explanations by the teacher.

Shulman (1986, 1987) pointed out the importance of both subject matter knowledge and pedagogical knowledge of which the latter emphasises on knowledge of instructional strategies.

2.4 Knowledge of Instructional Strategies

Modern trends in education demand that the teacher should involve learners in the teaching and learning process. It is against this background that the choice of an instructional strategy is important. Although different lessons may require different methods, the choice of an instructional strategy does not only depend on the subject matter knowledge but also on the learners' level of understanding (Shulman, 1987).

One of the earliest studies to be conducted on the instructional strategies was done by Carpenter et al. (1988). The study investigated primary teachers' pedagogical skills on addition and subtraction problems. It was found that teachers were good at predicting learners' performance on problem-solving tasks but were unable to predict what strategies the learners would use to solve the problem. They noted that this was evidenced when teachers failed to make their instructional decisions based on the strategies that learners use to solve problems, but they paid attention to the level of difficulty of the question. This study tried to make an observation whether the level of learner understanding was considered in the choice of instructional strategy.

In trying to understand the pedagogical knowledge of pre-service primary school teachers, Ball and Wilson (1990) carried out a study on the topics of division, place value, fractions, area and perimeter. The main aim of the study was to find out how subject matter knowledge affected the instructional strategies in the delivery of the lessons. It was a mixed study involving written assessment and lesson observation. The findings of the study showed that a good number of pre service teachers lacked conceptual understanding of basic mathematical topics and were unable to represent them in a meaningful way to teach for understanding. The current study benefitted from that of Ball and Wilson through data collection methods and analysis.

Kinach (2002) conducted a study on pre-service teachers on the instructional practices on the topics of addition and subtraction of integers. It was a qualitative study in which the participants were observed in lesson delivery. The findings of the study revealed that pre-service teachers' mathematical conceptions were likely to influence their decisions about how to teach mathematics. Furthermore, the findings showed that most of the pre-service teachers perceived that knowing mathematics was to get the right answer. This study benefitted from the study conducted by Ball and Wilson (1990) in that it gave an insight for the researcher to conduct a topic study on quadratic equations at secondary school level.

Lim (2007) conducted a study on the characteristics of mathematics teaching in Shanghai, China. The aim of the study was to find out instructional strategies teachers use when teaching mathematics. The study was qualitative and involved lesson observations. The findings of the study pointed to the fact that success in teaching a specific mathematics topic depended on the depth and breadth of the teachers' pedagogical knowledge. Furthermore, the findings revealed that teachers with strong pedagogical knowledge selected teaching strategies that were appropriate for the level of development of their learners.

Baumert etal. (2009), in their study, which involved teachers' mathematical knowledge, cognitive activation in the classroom and learner progression, mention three components of instructional strategies that are critical in ensuring and sustaining insightful learning processes in a mathematics lesson. The first being that the lesson should be cognitively challenging and well-structured. Secondly, the teacher should ensure that there is proper monitoring of the learning processes and prompt individual feedback from the learners. Finally, there should be efficient class and time management. From the above discussion, teachers need to select teaching strategies that encourage class discourse and support learners through guided practice. They also need to address learners' misconceptions.

The current study, was to investigate the kind of instructional strategies that the teachers used in the teaching of quadratic equations. The intention was to observe whether the strategies used suited the topic that was being taught during the specific lesson and if the examples used provided the best learning opportunities for the learners. The next section discusses another component of mathematical knowledge for teaching, which is knowledge of learners' conceptions.

2.5 Knowledge of Learner Conceptions

Shulman (1986) proposed that knowledge of learner thinking particularly knowledge of common conceptions, misconceptions, and difficulties that learners encounter when learning particular content are essential parts of the knowledge base required for successful teaching.

Ball (1990) emphasised on the need for teachers not to only depend on the "right or wrong analysis", but also use student errors as a mirror into student conceptual understanding of their errors. Ball (1990) is trying to make the point that errors are a normal part of the learning process. The question remains "do teachers address error analysis as they teach quadratic equations?"

Although there is a rich and extensive body of literature on learner understanding, many have focused on the concept of a function held by practicing teachers (Hitt, 1994), and prospective teachers (Bishop and Stump, 2000; Stump, 2001; Wilson, 1994).

Earlier studies such as those conducted by Tirosh, Even, and Robinson (1998) show that experienced teachers understood and found ways to engage the learners with common error analysis of conjoining algebraic expressions such as: $2a+4a=6ab$.On the other hand, less experienced teachers noticed the error but did not understand why it occurred and did not engage the learners in resolving it. The current study benefitted from that of Tirosh, Even, and Robinson by way of putting emphasis on how teachers engage learners on identification of error analysis.

Hitt (1994) conducted a study on teachers' concept of functions. The main purpose of the study was to investigate whether teachers had misconceptions about the construction of functions. The study was quantitative and data was collected using a questionnaire from 117 mathematics teachers. The findings of the study were that teachers had a tendency of thinking in terms of continuous functions with little proficiency in constructing such functions. In other words, teachers rarely considered the alternative discontinuous functions. Unlike the study conducted by Hitt (1994), which focused on the misconception of the teachers on functions, this study looked at the misconceptions, which the learners faced in the learning of quadratic equations and how the teachers identified and assisted such learners.

Bishop and Stump (2000) conducted a study on 32 pre-service primary school teachers. It was a qualitative study in which data was collected through class observation, interviews, and written assignments. The main aim of the study was to explore teachers' conceptions of algebra. The study found that the majority of the teachers gave either a non-algebraic or procedural definition of algebra. Moreover, the pre-service teachers expressed limited understanding and appreciation of the generalisation perspective of algebra and that they could not differentiate arithmetic from algebra. The current study benefitted from that of Bishop and Stump through data collection methods and analysis of data.

Stump (2000) conducted a studyon pre-service teachers who participated in a secondary school methods course. It was a qualitative case study whose aim was to identify knowledge of learners' difficulties with the slope and their representations for teaching slope. Data was collected from written assignments, interview transcripts, and transcripts from algebra lessons. The study found that the knowledge of representations for teaching the concept of slope provided meaningful problem-solving activities for their learners. Stump concluded that the pre-service teachers' limited exposure to non-traditional curriculum materials might be the reason for developing a weak knowledge of slope in real world setting. This study tends to benefit from the study conducted by Stump (2000) in the sense that it will also look at learner difficulties and misconceptions in the teaching and learning of quadratic equations.

Kilic (2011) investigated pre-service secondary school teachers' knowledge of their students. The purpose of the study was to learn about how a methods course and its associated field experience, support the development of pedagogical content knowledge for pre-service secondary school teachers. The results of this study revealed that having strong subject matter knowledge is essential but not sufficient for an effective teacher. The results also revealed that pre-service secondary school teachers had insufficient knowledge of learner conceptions and that they were unable to address learners' errors and address them. The study further revealed that once the teachers noticed the learners' errors, they resorted to repeating how to carry out the procedure. The current study benefitted from that of Kilic (2011) through data collection methods and analysis.

Sibuyi (2012) conducted a study on effective teachers' pedagogical content knowledge in the teaching of quadratic functions. The purpose of the study was to investigate the pedagogical content knowledge held by the two teachers who were classified as effective, since their learners had consistently achieved good results at Grade 12 final examinations. It was a qualitative case study involving two teachers. The results of the study suggested that the two teachers had adequate subject matter knowledge but limited knowledge on the aspects of teaching strategies and knowledge of learners' pre-conceptions and misconceptions on the topic of quadratic equations. The current study benefitted from that of Sibuyi through data collection methods and analysis.

Falkner, Levi, and Carpenter (1999) conducted a study on the Grade 6 primary class. The main purpose of the study was to find out the understanding of the equal sign. The Grade 6 learners were presented with the question "$8 + 4 = x + 5$. It was a quantitative study. The question was first given to 24 learners in one grade 6 class and surprisingly, all of them gave the solution 12. Later, the question was given to all 145 Grade 6 learners in the school and all of them gave the solution as either 12 or 17. The findings of the study indicated that the teachers had limited knowledge of and about learner thinking in algebra. Furthermore, the results revealed that teachers rarely gave attention to learners' conceptions and ways of thinking. Finally, the findings revealed that teachers' reactions to learner thinking, and conception were based on the judgement of the answers in terms of right or wrong and provided the learners with teachers' own explanation for the right answer. This study benefited from that of Falkner, Levi, and Carpenter (1999) by taking a keen interest in understanding the learners' conception by observing the teaching and learning process.

Another study conducted by Schleppenbach et al. (2007), investigated the use of errors in the classroom situation by comparing teachers from China and the United States of America (USA). The findings reported that USA teachers tended to avoid and hide student errors whereas Chinese teachers tended to direct students to think about the original question in conceptual ways. On the other hand, in theUSA teachers used a typical strategy of repeating the procedure until the students recognised their errors.This study like that of Schleppenbach et al. (2007), looked at identification of error by the learners.

Li (2007) carried out a study in which he investigated secondary school algebra teachers' mathematical knowledge for teaching algebraic equations. It was a quantitative study and 72 teachers participated in the study by completing a questionnaire and a written response instrument. Eight participants were then invited to a follow up semi-structured interview. The findings from the studies revealed that participants provided a wide range of instances of student misconceptions and difficulties in learning how to solve linear and quadratic equations. Further, the results showed that teachers' subject matter knowledge played a central role in the reasoning and decision-making specific contexts. This study, like that of Li (2007) emphasised on learner difficulties and misconceptions.

The literature on teachers work with errors, suggests a number of analytic processes that teachers need to follow as they engage students in error analysis. The teacher needs to identify an error (Peng and Luo, 2009), then attend to the error (Jacobs et al., 2010), and finally, decide how to engage the learners (Peng and Luo, 2009). The researcher adopted the above error analysis as a way of assessing how teachers deal with the errors, which learners encounter as they learn quadratic equations.

The studies conducted by other scholars as outlined above acknowledged the importance of mathematics knowledge for teaching and in some instances, teachers showing insufficient subject matter knowledge, emphasising on procedural knowledge when teaching and unable to identify and predict learner misconceptions, which teachers possess when teaching specific mathematics topics. Therefore, this study looked at how teachers addressed possible errors and misconceptions as they taught quadratic equations.

The next section discusses the studies conducted by various researchers on quadratic equations.

2.6 Previous Research Carried on the Topic Quadratic Equations

For a topic as specific as quadratic equations, there is not a significant research base. There are, however, several studies, which have been carried out on learners and teachers on a unit of quadratic equations that can provide insights into this study. The resounding evidence in mathematics educational research is that the learners' performance in quadratic equations is exceptionally below average and does not significantly improve even after instruction (Chaysuwan, 1996; Vaiyavutjamai, et al., 2005).

Learners have been found struggling when solving for x in the form $x^2 = k > 0$ and $(x-r)(x-s)=0$ where r and s are any real numbers (Vaiyavutjamai et al., 2005). The most concerning of all the data was that, out of a sample of 29 second year university students in the United States of America (USA) who were pre-service middle school students' mathematics specialist teachers, 37 per cent could answer the two questions correctly (Vaiyavutjamai et al., 2005). Other than the studies done by the researchers noted above, there is a deficit in research and lack of empirical evidence on learners' performance and teachers' mathematical knowledge with respect to quadratic equations.

Bosse and Nandakumar (2005) conducted a study of college courses.Twenty-seven college algebra textbooks, established that the employment of factorisation method was not as efficient as utilising the quadratic formula and completing the square. They further criticised the traditional way of teaching of solving quadratic equations using factorisation method compared to other methods such as the quadratic formula and completing the square. The argument advanced by Bosse and Nandakumar (2006) is that the strength of using techniques of 'the completing square method' and the ‘quadratic formula’ for solving quadratic equations, is that they are used for all forms of quadratic equations and more effective than the factorisation method, which is appropriate only for quadratic equations with rational roots.

Vaiyavutjamai and Clements (2006) conducted a study in Thailand on 231 students from two government schools. The study was meant to investigate how traditional lessons on quadratic equations using factorisation, completing the square and the formula methods were presented to Grade 9 pupils. The study was both qualitative and quantitative and was composed of 18 lessons of 50 minutes each, which were observed and tape recorded. Pre-tests and post-tests were conducted as well as interviews for exploring students’ in-depth cognitive understanding of quadratic equations. The findings from this study revealed that pupils had misconceptions in variables which act as hindrance for them to understand quadratic equations. The pupils did not realise that the two equations $x^2 +3x + 2 =0$ and the $(x + 3) (x – 1) = 0$ are equivalent and that the factored form is but another form of a quadratic equation. The current study benefitted from that of Vaiyavutjamai and Clements (2006) by carrying out a similar study in Zambia.

The study carried out by Vaiyavutjamai and Clements (2006) further exposed that traditional methods of teaching quadratic equations promote rote learning at the expense of relational understanding or meaningful learning. Skemp (1987) defined two types of mathematical reasoning as relational and instrumental. Instrumental understanding is the knowledge of rules and how to carry out a mathematical procedure without necessarily thinking. In short, Skemp (1987), calls instrumental understanding as ‘rules without reasons’. On the other hand, relational understanding is knowledge of what to do and why. Relational understanding has to do with both rules and reasons. Skemp (1987) emphasises that ‘reasons without rules’ are meaningless, dangerous and appropriated as ‘rules without reasons’. The issues of relational and instrumental understanding outlined by Skemp (1987) were beneficial to the current study.

Leong et al. (2010), stated that factorisation method can be meaningful if it is taught through the use of algebra tiles or geometrical representations. To concretise the situation, they carried out a lesson study including pre-tests and post-tests. The result of the study showed an improvement of students learning through the use of algebra tiles in teaching. The study conducted by Leong et al. (2010), paved way for the current study by looking at the other dimension of quadratic equations.

Li (2011) conducted a study on an individual teacher during a three-lesson sequence on solving a quadratic equation using the method of applying the quadratic formula.He focused on the teacher's knowledge of the subject matter, pedagogical dimensions, and learner's conceptions. The research suggested that the teacher utilised more of the subject matter in making pedagogical decisions with less dependence on representations. The teacher also drew on her knowledge of her own learners' level of mathematical proficiency in making instructional decisions on representations and decisions on trimming learning objectives during classroom instruction. Li concluded that it was difficult to gauge the level of conceptual understanding developed because the teacher put emphasis on procedure, and many learners could have remained at this level of understanding. This study was beneficial to the current study because it also looked at the procedural and conceptual knowledge of the teachers as they teach quadratic equations. The current study looked at the three components of mathematical knowledge for teaching as outlined by Li (2011).

Didis and Erbas (2015) carried out a study on performance and difficulties faced by learners in formulating and solving quadratic equations with one root. The main purpose of the study was to investigate the performance difficulties of Grade 10 learners in solving quadratic equations with one root using symbolic equation and word problem representations. The study was qualitative and convenient sampling was used to come up with the intended sample. Data was collected through an open-ended questionnaire comprising eight symbolic equations and four-word problems. Semi-structured interviews were also conducted with sixteen learners.

The findings revealed that although the learners have difficulties in solving both symbolic quadratic equations and quadratic word problems, they performed better in the content of symbolic equations than in quadratic word equations. Learner difficulties in solving symbolic problems were mainly associated with arithmetic and algebraic manipulation errors whereas in quadratic word equations learners had difficulties in understanding the context.They, therefore, were unable to formulate the equation to be solved. Didis and Erbas (2015 concluded that the difference in the structural property of the symbolic equations and word problems representations affected learners' performance in the formulating and solving of quadratic equations with one root. The study by Didis and Erbas (2011) was significant to the current study in terms of data collection tools and analysis. It also enabled the researcher to observe how the teachers identify learner difficulties and consequently address them.

O’Connor and Norton (2016) conducted a qualitative case study on learning difficulties in quadratic equations. The purpose of the study was to investigate learners’ difficulties in quadratic equations. The study was qualitative and involved 25 participants. A written test was used to collect data associated with the topic. Descriptive analysis was used to analyse data. The study revealed that learners’ success was inhibited by errors that arose from lack of procedural understanding regarding fractions, algebraic processes, and conceptual understanding regarding algebraic conventions. The researcher found that learners’ difficulties are categorised into four patterns namely; difficulties in fractional reasoning, difficulties in conceptual understanding, regarding algebraic misconceptions and lack of procedural understanding and inefficient techniques without this pre-requisite knowledge, working without understanding the nature of quadratic equations was hindered. The study by O’Connor and Norton was beneficial to the current study as it focused on the intertwined relationship on both procedural and conceptual knowledge of the teachers regarding quadratic equations.

Lima (2008) carried out a study which was aimed at finding the connection or the relationships between the procedure of solving quadratic equations with the formula and the general principles for solution of the equations. Mixed methods approach was used and 77 students participated in the study. A written test assessment was used to collect data followed by interviews. The findings revealed that only 7 out of 77 students succeeded in solving quadratic equations, which were written in a non-standard form. The study also revealed that learners were dependent on the procedure represented by the quadratic formula to get the correct solution simply by following the set rules.Regardless of the problem structure, they demonstrated no comprehension of the underlying concept. The study concluded that learners demonstrated a complete lack of understanding of the connection between the procedure of solving quadratic equations with the quadratic formula and algebraic principle for solution of equations. This study was beneficial to the current study as it showed the gap in research in terms of the methods, which teachers used in order to ensure that learners would not experience such difficulties.

Zakaria and Maat (2010) conducted a case study on the analysis of learners’ errors in learning of quadratic equations. The purpose of the study was to determine the learners’ errors in learning of quadratic equations. The survey included 30 learners from a secondary school in Jambi, Indonesia. Diagnostic test was used as the instrument of the study that included three methods of solving quadratic equations namely; factorisation, completing the square and the formula. Interviews were also used to identify at which level learners' errors occurred when solving quadratic equations. The type of error was based on Newman's Error Hierarchy Model that includes reading type error, comprehension, transformation, process, skill and encoding error. Data was analysed using descriptive statistics, percentage and frequency.

The findings revealed that most learners made errors in transformation and process skills when solving quadratic equations and no error was recorded in reading. The number of learners who made encoding errors and carelessness was negligible. The learners' error in solving quadratic equations was due to their weaknesses in mastery of concepts in algebra, fractions, integers and algebraic expressions. This study was beneficial to the current study as it sought to find out if the teachers had the knowledge of learners' errors prior to teaching the topic of quadratic equations.

Didis, Bas and Erbas (2011) conducted a study on the difficulties learners faced on solving quadratic equations using factorisation method. It was a quantitative study in which a written test paper assessment was used. The findings of the study revealed that learners found work challenging for a variety of reasons. One reason was that learners, primarily used instrumental or procedural approaches without considering underlying concepts or structures in which "they tried to apply these rules thinking neither about what why they did so, nor whether what they were doing was mathematically correct" (p. 7). This study was beneficial to the current study as it aims at finding out more about the method teachers used.Where they at the level of procedural knowledge only or was conceptual knowledge also employed. The next subheading looks at theoretical frameworks for teacher knowledge.

2.7 Theoretical Frameworks for Teacher Knowledge

Teachers' content knowledge has been an important area of study in response to Shulman's (1986) notion of teacher knowledge. There are various conceptualisations of mathematics teacher knowledge, which build on Shulman's (1986) framework of teacher knowledge. As such, several frameworks of mathematics teacher knowledge have been developed by different scholars based on their conceptualisation of teacher knowledge. The following sections present Shulman's theoretical framework of teacher knowledge and some of the frameworks that build on it in relation to mathematics education.

2.7.1 Shulman's Framework for Teacher Knowledge

Shulman (1986) proposed a framework for understanding teacher knowledge. He argued that there is a specialised form of knowledge that is unique to teachers and teaching. This special form of knowledge distinguishes teachers from subject matter specialists. The categories of teacher knowledge suggested by Shulman (1986) include subject matter content knowledge (SMK), pedagogical knowledge (PK), pedagogical content knowledge (PCK) and curricular knowledge (CK). SMK, "refers to the amount and organisation of knowledge per se in the mind of the teacher," (Shulman, 1986, p. 9). Shulman emphasised that apart from knowing how an algorithm works, a teacher also needs to know why the algorithm works, and on what grounds its warrant can be either accepted or denied. This agrees with suggestions by Herbst et al. (2009), and Jones et al. (2009), that the teachers' responsibility is not only to show students how to prove a geometric statement but also to help them to understand why the proof works. Shulman (1986) described PCK as a blend between content knowledge and pedagogical knowledge. He pointed out that this type of knowledge is necessary for making content understandable to students. PCK includes knowledge of powerful representations, analogies, illustrations, examples, explanations, demonstrations, and knowledge of students' misconceptions in relation to a particular topic. He proposed that more studies should be conducted on PCK to better understand it in relation to a particular subject.

Curricular knowledge is the knowledge of the full range of programmes designed for the teaching of particular subjects and topics at a given level. It also includes knowledge of different kinds of instructional materials available in relation to those programmes and their use. In continuing with studying teacher knowledge, Shulman (1987) further categorised teacher knowledge as follows:

i. Content knowledge. This referred to the subject matter knowledge which teachers had.
ii. General pedagogical knowledge, with special reference to those broad principles and strategiesof classroom management and organisation that appear to transcend subject matter.
iii. Curriculum knowledge, with particular grasp of the materials and programs that serve as"tools of the trade" for teachers.

iv. Pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding.
v. Knowledge of learners and their characteristics.
vi. Knowledge of educational contexts, ranging from workings of the group or classroom, the governance and financing of school districts, to the character of communities and cultures.
vii. Knowledge of educational ends, purposes, and values, and their philosophical and historical grounds (Shulman, 1987, p. 8).

Shulman (1987) argued that PCK is of special interest among the seven categories of teacher knowledge. This is because it represents a blending of content and pedagogy, into an understanding of how particular topics or issues are organised, represented, and adapted to the diverse needs and abilities of learners. Shulman's work offered useful characteristics of content knowledge for effective teaching in general. The work also shifted researchers' attention from pedagogy only to both CK and PCK. Ball et al. (2001), justifies PCK as an essential idea in teacher knowledge for three reasons. Firstly, it fills the gap left when the focus is only on teacher's credentials. Secondly, it improves understanding of the knowledge required for teaching to a larger extent. It is through Shulman's framework that educators and researchers realised that teachers' mathematical knowledge is different from that of a mathematician. Thirdly, it has produced new methods of teacher study that have resulted in generation of rich knowledge (Ball et al., 2001).

Ball et al. (2008), acknowledged two contributions of Shulman's framework. Firstly, it shifted researchers' attention from general aspects of teaching to the role of content in teaching. Secondly, it represents content understanding as a special component to the profession of teaching. However, Jones, Mooney and Hurries (2002) argued that Shulman's model may be too simplistic because it does not distinguish between the nature of different school subjects. They noted that the teaching of mathematics is complex and different from the teaching of other subjects, hence, need for specialised framework. Ball et al. (2001), also observed that the framework might be very difficult to use because it does not provide specific guides for teaching mathematics well. Furthermore, Ball et al. (2005), argued that Shulman's framework might be difficult to operationalise because it does not clarify the difference between CK and PCK. The researcher's view is that Shulman (1986) does not present his framework as a final tool for understanding knowledge for teaching any subject but as a lens for guiding further research on knowledge for teaching a particular subject.

The framework acts as a revelation for researchers to realise that there are different dimensions of teacher knowledge for every subject. By suggesting that more research be conducted to ensure deep understanding of PCK for specific subjects, Shulman was acknowledging that the demands for teaching of different subjects are unique and distinct in nature. This could be the reason why despite the criticisms, several studies have used Shulman's framework as a lens for understanding and developing categories of teacher knowledge for different subjects including mathematics.

2.7.2 Teacher Education Development Study in Mathematics (TEDS-M) Framework

Tatto et al. (2008), developed the TEDS-M framework, which was used to develop instruments for investigating and evaluating both primary and lower secondary teacher education preparation around the world. The authors aimed at developing test items for enabling different countries to analyse their education systems and policies and compare their expectations about future mathematics teachers. The TEDS-M theoretical framework of teachers' professional competence regard MKT as comprising Shulman's (1986) two teacher knowledge categories; mathematical content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK) (Tatto et al., 2008). The domains of MCK include the mathematical content that the teachers are required to teach, and mathematics beyond the level that the teachers are required to teach. The domains of MPCK included mathematics curricular knowledge, knowledge of planning, and knowledge of enacting Mathematics (Tatto et al., 2008). The assumptions of the TEDS-M framework are as follows:

i. Teacher education is understood and implemented differently across national settings and evenbetween institutions within the same country.
ii. Teacher education and teacher learning are complex, contested processes.
iii. Knowledge of the content to be taught is a crucial factor influencing the quality of teaching.
iv. Teacher education requires understanding of and addressing how teachers should think about mathematics, teaching, and learning.
v. Teacher education embodies a developmental logic of how teachers acquire professional knowledge for the teaching of mathematics and other subjects.
vi. Knowledge for teaching involves consideration of the situational contexts where teachers will teach.

vii. Teacher education is assumed to be linked to student achievement, but this relationship is poorly understood.

The assumptions of the TEDS-M project agree with those of different scholars. For instance, the assumption that teachers' content knowledge influences the quality of teaching, agrees with several authors who argue that the successful teaching of mathematical concepts depends crucially on the subject knowledge of mathematics teachers (Hill et al., 2008). The assumption also supports Hill et al. (2005), who argued that teacher practices influence students' achievements in mathematics.

The TEDS-M framework was developed based on assumptions that agree with many scholars. This implies that the framework was developed by considering issues that concern teacher knowledge. This study agrees with the assumptions of the TEDS-M project because they hinge on the importance and complexity of teacher education. Lin and Rowland (2016) argue that the TEDS-M framework ignores the power of mathematics classrooms in advancing teachers' professional development by assuming that teacher knowledge can be evaluated only through pencil and paper tests taken outside mathematics classroom. This means that the framework ignores the usefulness of teacher-students' interaction in mathematics education. Besides, we argue in this study that the framework also ignores the importance of assessing students' thinking in mathematics education. Like some mathematical teacher knowledge frameworks, the PCK categories of TEDS-M framework describe general knowledge for both primary and secondary school pre-service teachers. This implies that the framework does not recognise that PCK is determined by specific subject matter knowledge.

2.7.3 Knowledge for Algebra Teaching (KAT) Framework

Building on Ball's framework (2008), McCrory et al. (2012), developed a framework of knowledge for algebra teaching (KAT). The framework was designed for primary and secondary school algebra and it was developed after conducting extensive studies on two main topics in algebra; algebraic equations and expressions, and linear relationships. McCrory et al. (2012), sought to investigate teacher knowledge demands for these topics because they are foundational to school algebra and posed challenges to students in tertiary institutions. Their argument was that, despite knowing that advanced mathematical knowledge and mathematical knowledge closer to the practices of teaching is necessary for secondary teacher effectiveness, there is little evidence to inform decisions about what to emphasise during the preparation of algebra teachers. This is because there are no measures of teacher knowledge that would capture advanced mathematics and mathematics closer to the practices of teaching. As such, their work attempted to develop a theory that would test teachers' MKT of Algebra. The KAT framework comprises three categories of knowledge: knowledge of school algebra, knowledge of advanced mathematics, and mathematics for teaching knowledge. These categories reflect three perspectives of knowledge teachers needed to be taught in pre-service for effective teaching of algebra. Knowledge of school algebra includes proficiency in what the teachers are going to teach (McCrory et al., 2012). Ball et al. (2008), called this CCK while Shulman (1986) called it SMK. Knowledge of advanced mathematics includes other mathematical knowledge like college mathematics. This agrees with Shulman who suggested that the level of content of secondary school mathematics teachers should be the same as that of mathematics majors. McCrory et al. (2012), argued that this category of knowledge gives a teacher broader and deeper understanding of school algebra. Mathematics for teaching knowledge is that which is useful in teaching, but not included in the teachers' formal mathematical education (McCrory et al., 2012). Examples of this knowledge include knowledge of different definitions of a particular mathematical concept, and how those definitions influence understanding of the mathematics that follows. They explained that this type of knowledge is related to SCK suggested by Ball et al., (2008).

Apart from the three categories of teacher knowledge, the KAT framework also contains categories of work that teachers do when teaching algebra. These include decompressing, trimming and bridging. McCrory et al. (2012), argued that a teacher could only accomplish the three activities of the work of teaching mathematics (decompressing, trimming and bridging) if they have enough knowledge of school mathematics, knowledge of advanced mathematics and MKT. Decompressing means working backwards to unpack content from mature and complex understanding to simple and understandable form. They suggested that decompressing could be relevant in analysing students' mathematical work and thinking, as well as designing, modifying and selecting mathematical tasks. Ball et al. (2004), also suggested that decompressing or unpacking should be one of the tasks of a mathematics teacher.

Trimming involves reducing the complexity of mathematics content into ways that make the content accessible to students while maintaining its integrity (McCrory et al., 2012). It includes interpreting and transforming mathematics into a form that is understandable to students while preserving rigor and originality of the concept. This task was also suggested by Shulman's (1986) framework of teacher knowledge under PCK category. Bridging involves making various kinds of links and connections among mathematical concepts between students' thinking and the lesson objectives. Bridging activity is also one of the tasks of teaching suggested by Ball et al., (2004). There are several strengths of the KAT framework. Firstly, it was developed based on analysis of secondary school teachers' subject matter knowledge of mathematics. This means that the framework was developed based on in-depth analysis of the issues that concern teaching of secondary school mathematics. The other strength is that the KAT framework agrees with both Shulman (1986) and Ball et al. (2008), that teaching of mathematics involves doing some work, which requires special kind of knowledge. However, the researcher argue that the KAT framework does not strengthen insights into understanding of the work of teaching Mathematics. This is because several scholars like Shulman (1986), Ball et al. (2004; 2008) already suggested the categories of teacher knowledge and the activities of teaching suggested by the framework. Furthermore, McCrory et al. (2012), assumed that despite being designed specifically for teaching of algebra, the KAT framework could still be used in any other field of mathematics because it is widespread and general. As already argued, MKT is specific to mathematical fields, hence, an algebraic MKT framework might not be applicable to quadratic equations.

2.7.4 Theoretical Framework that Guided the Study

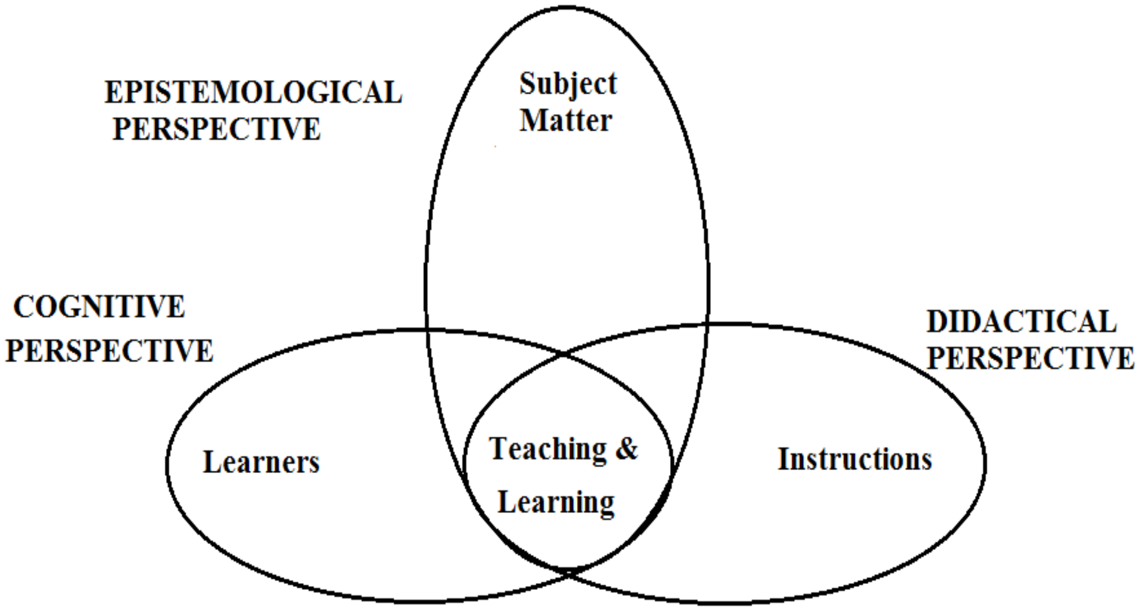
Many of the theoretical frameworks that inform debate in education and mathematics education, in particular, are theories of learning whereas the focus of this study lies in the practice of teaching. This is a field of study which is not fully developed. Researchers have proposed several ways of distinguishing among the many facets of knowledge needed by a teacher to teach effectively.

The theoretical framework that guided this study has several sources. It is based on the assumption that effective teaching requires a teacher to attend and endeavour to understand the mathematics concepts and the cognition of their learners (Maher, 1998). In agreement with Shulman (1998) and Ball et al. (2005), our perception recognises that to teach mathematics effectively, requires knowledge of mathematics that "goes beyond the knowledge of the subject matter per se to the dimension of subject matter knowledge for teaching" (Shulman, 1986, p. 9). However, our view of mathematics transcends an epistemological stance that it entails "ways of representing and formulating the subject that makes it comprehensive to the learners" (Shulman, 1986, p. 9).

In contrast, we embrace the idea that teachers cannot convey knowledge to the learners. Instead, they can invite learners to engage with a mathematical task and rationally or logically connect with the learners to comprehend their emergent mathematical concepts and reasoning, as they build on their knowledge. Researchers can infer teachers' mathematical knowledge for teaching by analysing their practice in action, including interaction with learners, questions they ask and issues they make salient or noticeable to the learners.

Although the discipline has specific frameworks mentioned in the literature they differ in detail. Many of them meet in efforts to further extend and refine the construct of the mathematical knowledge for teaching. Drawing on the work of Shulman (1986) and Ball et al. (2005), the teacher should have the capacity to transform the subject matter and deconstruct one's knowledge into a less polished final form when critical components are accessible and visible.

To theorise the mathematical knowledge for teaching, the theoretical framework that informs the study is drawn from the works of Scheiner (2015), of which he called on a broader perspective of mathematical knowledge of teaching to include an epistemological, cognitive and didactical perspective. Hence, for the purpose of this study, the mathematical knowledge for teaching shall be referred to as (mathematics) teachers' knowledge focusing on epistemological, cognitive, and didactical perspective.



(Mathematics) Teachers'knowledge: Epistemological Didaction & Cognitive Perspective.
Adapted from Scheiner (2015)

The researcher assumes that the epistemological, cognitive and pedagogical dimensions are essential to mathematical knowledge for teaching. Many studies revealed that teachers have their own beliefs about teaching and learning of mathematics, shaped by their own school experience and are hard to change through teacher education courses (Ball, 1988). In this study, the researcher is aware of secondary teachers' beliefs about teaching and learning of mathematics but will not attempt to assess these beliefs. Instead, the researcher tried to learn about mathematical knowledge for teaching quadratic equations by looking at the epistemological, cognitive, and didactical dimension through interviews, observation, and questionnaire.

2.8 Definitions of Terms for Mathematical Knowledge for Teaching

In trying to define each of the three dimensions of mathematical knowledge for teaching, the researcher used mathematics in general and quadratic equations in particular as examples.

2.8.1 Epistemological Dimension

In education research, the subject matter of mathematics is organised into branches such as arithmetic, algebra, number theory, analysis, geometry, applied mathematics, and probability and statistics. For example, algebra has three sub-branches, which include elementary algebra, abstract algebra and linear algebra. Abstract algebra has specialised theories in groups, rings, fields, and other structures.

Epistemological dimension does not exist in an abstract form, but it appears in textbooks, journals, monographs and other publications in terms of algorithms, reasoning methods and problem-solving strategies.

Teachers' subject matter knowledge is divided into distinct categories by different scholars. For instance, Shulman (1968a) looks at two types of structures of the subject matter content knowledge and these are substantive and syntactic structures (Ryan and Mc Crae, 2005). Substantive structure refers to the ways in which basic concepts and principles of the discipline are organised in order to relate them to the facts, while syntactic structures refers to a set of ways in which truth and validity of a discipline can be established (Ryan and Mc Crae, 2005). On the other hand, Ball (2005) and Bass et al.'s (2006), give a clear distinction between common content knowledge and teachers' specialised content knowledge, and Ferrini-Mundy et al. (2003, 2004), look at the division between school mathematics knowledge and advanced mathematics.

The previous few results have indicated that there is a positive connection between teachers' subject matter preparations and pupil achievement. Nonetheless, there was also evidence of a "threshold effect" on pupil achievement. This refers to the minimal additional effect on pupil achievement which refers to the minimal additional effect of teachers' mathematics studies beyond five undergraduate courses (Wilson and Floden, 2002).

What this simply means is that although strong mathematics background is important, it is not in itself a sufficient condition for effective teaching. What would constitute effective teaching may also depend on their knowledge of other forms of mathematics that is comprehended by the pupils and is presented in a professional way.In other words, pedagogically presented.

2.8.2 Cognitive Dimension

Knowledge of learner conceptions domains represents teachers' knowledge of the thinking or cognitive form of the content in pupils' mental representations. The content being discussed here is that which results from pupils' interacting with the teacher, the subject matter, other pupils, instructional technologies, and other factors in the existing environment. This particular knowledge could partially be observed from and assessed by the questions that pupils pose and the answers provided to questions raised by the teacher or other pupils. This is during classroom interactions and those given in form of continuous assessment, tests and home works. It is important to note that in order for a teacher to teach effectively, he or she needs to be interested in and also very knowledgeable about the level of pupils to current trends.The teacher must also understandthe nature of their pre-conceptions, misconceptions, difficulties, non-typical ideas, ways of thinking and their learning trajectories across topics, units and school curriculum.

Knowledge of learner conceptions helps us to understand the classes of pupil errors (Leinhardt and Smith, 1985). The classes of pupil errors further help to understand what makes specific topics easy or hard for a certain groups of pupils (Shulman, 1986a).

Studying teacher knowledge of how secondary school pupils understand specific topics in mathematics could lay a powerful foundation for continuous professional development (CPD) meetings that are similar to the lesson study cycle.

2.8.3 Knowledge of Didactic Representations

Knowledge of didactic representations domain has to do with the teachers' knowledge of the pedagogical form. This happens as the content is represented in the instructional media such as visual aids, textbooks and other teaching forms. The content of the knowledge of didactic representations is that which the immediate product is when the subject matter is being linked and tailored by individual teachers for specific group of learners that is through particular ways of sequencing content and also by using examples, explanations and questioning techniques.

As the discipline of mathematics does not exist in isolation from other disciplines, its knowledge also exists as systems of perceptions, conceptions and beliefs that may affect an individuals' mind or that which is shared by a group of learners. Whenever a learner wants to communicate a mathematical idea to fellow learners, he or she would use a media such as symbols, graphs, diagrams and charts. The outcome of a communication process would be highly dependent on the selection, use and above all, interpretation of the media by each participant.

It must be made clear that mathematics teaching and learning offer a special form of communication between teachers and learners and among the learners themselves. In the Zambian education system at national or macro level, the Curriculum Development Centre (CDC) specifies the coverage and sequence of the topics for a specific course of study.This is done in consultation with other stakeholders such as parents and teachers (MOE, 1996). At micro or implementation stage, the teachers and learners interact. The teachers use definitions, explanations and exercises given in the textbooks to explain the concepts to the learners. The teachers pose questions to the learners and the way they respond, influences the relationships between the mathematics content and the learners.

Knowledge of didactic representation as revealed by other eminent scholars such as Ball et al. (2006) (knowledge of content and teaching), Artique et al. (2001)(the didactic dimension), Leinhardt and Smith, (1985) (lesson structure knowledge) and in Shulman (1986a) and Ferrini-Mundy et al. (2004),(pedagogical content knowledge - PCK) and teaching knowledge category, respectively. Also, Shulman's (1986a) "curricular knowledge" could be looked at as a unique type of knowledge of didactic representations.

Mathematics teachers like any other teachers of other disciplines may acquire knowledge of didactic representations through various ways. These include continuous professional development (CPD) meetings, capacity-building workshops and teaching practices. It must be born in mind that a teacher must not only teach mathematics using a general pedagogical approach, but also be content specific. This is important in the sense that each topic may have a different approach from the other.

The next section focuses on the three domains of content knowledge for teaching namely; epistemology, cognitive and knowledge of didactic representations can be applied to the teaching of quadratic equations.

2.8.4 Epistemological Dimension in Teaching Quadratic Equations

In teaching quadratic equations, secondary school mathematics teachers need to have the knowledge of the subject matter, which may include but not limited to the following: multiplication of two binomials, distributive property, solving linear equations of the form $bx + c = 0$, and the zero-product property knowledge on various strategies of solving quadratic equations such as:

graphical method, completing the square, quadratic formula and using geometrical tiles should also be possessed.

A more depth understanding of the teachers' knowledge of subject matter would include the connections of the concepts and the type of method, which would apply to a particular or specific quadratic equation and the strength and limitations of each method. As the case is in quadratic equations, the factorising method has limitations, in the sense that, it cannot apply to all forms or types of quadratic equations, while completing the square and the quadratic formula are generic.

The knowledge of learner conceptions involves an interaction between teachers and learners and among learners themselves. During this kind of interaction, the teacher is able to identify errors, misconceptions and difficulties, which learners have in understanding the concepts regarding the solving of quadratic equations. It is during this interactive session between teachers and learners and among the learners themselves that the teacher should find the root cause of certain misconceptions and difficulties which learners face. The teacher with a strong epistemological background is one who has knowledge of how mathematical concepts are related and why mathematical procedures work (Ball, and Bass, 2000; Skemp, 1987). Therefore, a teacher should be able to relate a particular mathematical concept with others and justify the reasons behind the mathematical procedures to promote pupils' understanding. In this study, the teacher should be able to explain the relationship between 'completing the square' and the 'quadratic formula' methods of solving quadratic equations.

2.8.5 Knowledge of Didactic Representations in Solving Quadratic Equations

Knowledge of didactic representations in teaching quadratic equations is two-fold. The first one is at macro or curriculum level. It involves the sequencing of the quadratic equations from the simplest to the most complicated and the reasoning behind the particular sequencing. At the implementation stage, it is important that learners are exposed to hands-on (practical approach) and visual tools such as algebra or geometrical tiles and scientific calculators. It is important to note that as it has already been explained earlier, the teacher should be able to explain the strengths and limitations of each method.

The basic entities and utilities that a mathematical procedure involves include basic algorithms, alternative algorithm and strategies, and related concepts and procedures. Specific to this research, the working definitions were given for the key terms and specific examples for each case.

A mathematical procedure is an operation that acts on an initial input and produces expected results. The inputs and outputs can be numerical, graphical or geometrical. In school mathematics, typical examples of procedures would include counting, simplifying fractions and algebraic expressions, performing the four basic operations on whole numbers, integers, fractions and other forms of rational and real numbers.

In college or university mathematics, advanced mathematics procedures may include finding derivatives and integrals of given functions, computing limits of sequences or functions and finding solutions to different equations.

A strategy, on the other hand, is basically a sequence of actions and reasoning for carrying out a procedure, which may or may not be well-defined or generalisable to a variety of instances. Different strategies may exist for carrying out a particular procedure. Forexample, a learner may use different methods of solving quadratic equations such as factorisation method, completing the square, using the quadratic formula or using the graphical method.

In general, when a strategy is well-adopted and able to produce accurate results, it is then called an algorithm or a formula. A good example is the quadratic formula, which has widely been accepted as the formula for solving quadratic equations. In another or simplified way, an algorithm is a well-defined, finite and generalisable set of actions for carrying out a mathematical procedure.

There are other possible strategies for carrying out a procedure, which are not formally introduced in secondary school mathematics. These are referred to as alternative algorithms and strategies. Alternative algorithms and strategies are different from basic algorithms in the sense that the former have lower levels of generic efficiency and precision than the latter. More often basic algorithms are less transparent than the alternative strategies.

Related concepts, procedures and properties serve as direct foundations for a given procedure. The researcher illustrates below one example of how the three conceptual aspects of a mathematical procedure- can be used in solving quadratic equations.

The basic algorithms for solving a quadratic equation are completing the square and the quadratic formula. It must be noted that the factorisation method is not regarded as a basic algorithm, because it is not generic. Alternative strategies such as graphical, geometrical tiles to solve quadratic equations have been put across by Hoffman (1976),

The standard algorithm for solving quadratic equations such as completing the square and the quadratic formula can be used in solving every type of quadratic equation.In this case, they are the most efficient among all the other types of strategies.

Finally, teachers with a strong didactic dimension have rich abilities of teaching activities and are able to choose, tasks, examples, representations and teaching strategies that are appropriate for the pupils. Further, teachers need to know how to facilitate classroom discourse and manage time for classroom activities well.

2.9 Applicability of the Theory to the Study

The mathematical knowledge for teaching, the theoreticalframework developed for this study which encompasses 'epistemological, didactic and cognitive dimension' is appropriate for this study as shown in Table 2.4 below. The table shows how the concepts derived from the theory can be used to answer the research questions.

Table 2.4: Concepts Drawn from the Theoretical Framework

Dimension of the theory	**Concepts**	**How am using the concepts in the study**
Epistemology	Knowledge of subject matter	(a) How the teacher presents facts, and their relationships among them.
Didactic Dimension	Knowledge of Instruction	(a) How the teacher plans and organises the work. (b) The type of the teaching strategies the teacher uses.
Cognitive Dimension	Knowledge of learner conceptions.	(a) How the teacher takes care ofpupils' common difficulties and anticipated misconceptions.

2.10 Operationalisation of Mathematical Content Knowledge for Teaching as Enshrined by Shulman's Work

Shulman (1986) introduced a new way of thinking about mathematical knowledge for teaching (MKT) and popularised the PCK, which he called the missing paradigm. He categorised the knowledge needed for teaching into two categories namely; content knowledge for teaching and PCK. According to Shulman (1986, p. 9), content knowledge is defined as "the amount and organisation of knowledge per se in the mind of the teacher". Pedagogical content knowledge is defined as "the knowledge of generic principles of classroom organisation and management and the like" (ibid, p. 14).

Shulman then subdivided content knowledge for teaching into three subcategories: subject matter content knowledge (SMCK), pedagogical content knowledge (PCK) and curricular knowledge. SMCK includes facts and concepts in a discipline and an understanding of how knowledge is generated and structured within the subject.PCK, on the other hand, is "the particular form of CK that embodies the aspects of content most germane to its teachability" (Shulman, 1986, p. 9). It combines pedagogy (art of teaching) and content (aspect of knowledge). Additionally, PCK also includes an understanding of the conceptions and preconceptions of learners, and the factors affecting the difficulty of learning specific topics of study. Thus, drawing on Shulman's explanation, the two key issues of PCK are knowledge of teaching (KoT) and knowledge of students thinking (KoS).

Curricular knowledge, according to Shulman (1986, p. 10) is defined as a:

"... full range of programmes designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programmes, and the set of characteristics that serve as both the indicators and contraindications for the use of particular curriculum or programme materials in particular circumstances".

Curricular knowledge also includes among other things, knowledge of curriculum materials for a given topic in a particular grade and the familiarity with the topics and issues that have been and will be taught in the same discipline during the preceding and later years in the school (Shulman, 1986).

2.10.1 Operationalisation of Mathematical Knowledge for Teaching as Enshrined by Ball et. al (2008)

Ball et al. (2008), built on their work from Shulman's seminal presentation of 1986, which pioneered the aspects of PCK as a special type of knowledge needed by the teacher in order to successfully perform his duty. Further, Ball and her colleagues looked at two distinct types of mathematical knowledge for teaching namely: specialised content knowledge (SCK) and common content knowledge (CCK).

SCK is that type of mathematical knowledge, which is uniquely possessed by the teacher (Ball et al., 2008). This type of knowledge may include among other things the ability to explain why procedures work and what they essentially mean, this is in addition to appraising the methods learners use when solving computational problems and determining whether such methods would be generalisable to other problems (Hill and Ball, 2004; Ball et al., 2008).CCK is a type of knowledge, which is not unique to a teacher as other users of mathematics such as nurses, bankers and engineers are to hold such knowledge (Ball et al., 2008).

From the two types of knowledge; SCK and CCK explained above, a question that may immediately arise is whether relationships exist between SCK and CCK or whether SCK exists independently from CCK? In response to the above questions, Ball et al. (2008), found that the results for the questions representing SCK of teaching mathematics were statistically separable from the results on the CCK items. The results, therefore, show that there is a place in professional preparation for concentrating on teachers' specialised knowledge.

According to Ball et al. (2008), CCK and SCK of mathematical knowledge comprise one domain of teachers' mathematical knowledge for teaching and that is content knowledge (CK). The mathematical knowledge for teaching is divided into two categories namely; subject matter knowledge (SMK) and pedagogical content knowledge (PCK). SMK is subcategorised into CCK, SCK, and the knowledge at the mathematical horizon (knowledge of students' future mathematics curriculum). PCK is broken into knowledge of content and students (KCS) and knowledge of content and teaching (KCT). KCS refers to knowledge of students' and their cognition for solving problems (Ball et al., 2008; Hall et al., 2004). KCT includes appropriate curriculum design and choice of representations, materials, and explanations for learners. Finally, knowledge of curriculum is the knowledge regarding materials in the discipline one is studying (Shulman, 1986).

The works by Shulman and Ball cited above have been synthesized into the following theoretical framework of knowledge for teaching mathematics, which will serve as a guide for this study. Table 2.5 shows the theoretical framework of knowledge for teaching mathematics.

Table 2.5: Theoretical Framework forKnowledge for Teaching Mathematics

Variable	**Mathematical knowledge for teaching**	
Construct	Subject matter knowledge (SMK)	Pedagogical content knowledge (PCK
Dimension	Cognitive dimension	Pedagogical dimension
	Knowledge, reasoning	Knowledge of teaching (KoT);knowledge of student thinking (KoS)
	Knowledge	

Variable	Instructional practice	
Aspect	PCK	
Dimension	Knowledge of teaching (KoT)	Knowledge of student thinking (KoS)

2.11 Chapter Conclusion

In this chapter, the concept of mathematical knowledge for teaching was discussed with special reference to the work done by other scholars. The three elements identified in this study namely: knowledge of the subject matter, knowledge of teaching strategies and knowledge of learner conceptions were also discussed. The implication of each element was discussed in relation to the teaching of quadratic equations. The chapter ends with the presentation of the theoretical framework and the conceptual framework that were used for the collection and analysis of data in this study.

CHAPTER 3

RESEARCH METHODOLOGY

3.0 Introduction

The purpose of this study was to evaluate teachers' mathematical knowledge for teaching quadratic equations. The under listed research questions guided the study:

1. What subject matter knowledge for teaching quadratic equations at Grade 11level did the three secondary school mathematics teachers possess?
2. What strategies did the three secondary school mathematics teachers use In teaching quadratic equations at Grade 11level?
3. How did the three secondary school mathematics teachers address pupils' errors and misconceptions related to quadratic equations at Grade 11 level?

This section discusses the philosophical orientation, research design, participants, methods of data collection, data analysis and trustworthiness of the study, which will assist in answering the research questions outlined above.

3.1 Philosophical Orientation

In this study, the researcher does not hold extremist view that suggest that either reality only exists in one's mind or is purely objective but rather to acquire an understanding of how mathematics teachers make sense of the whole phenomena as they teach quadratic equations. The assumption of the study is that teachers' mathematical knowledge for teaching, and its use in teaching are significant to the context of teaching and learning. The variables in the study include the topics and subject being taught on one hand and the learning characteristics on the other. The researcher holds the view that this knowledge can only be identified through observing the actual teaching process.

The study was conducted from an epistemological position that propagates the principle that what we know is always negotiated in cultural and social context. This implies that people construct meanings of reality according to their social settings. According to Cohen, Manion and Morrison (2007), the interpretivist paradigm attempts to understand the subjective world of human experience and to preserve the integrity of the phenomena under investigation. The interpretivist approach seeks to interpret an understanding the world in terms of its actors (Cohen et al., 2007). As such, interpretivist researchers make efforts to get inside the context and to understand from within. This approach's meaningis constructed by the researcher as the original meanings from the peoples' action. As such, researchers guided by interpretivist are required to employ naturalistic methodologies like observation and interviewing. These suggestions imply that the study of teaching and learning of mathematics is better understood through observation of classroom practices.

3.2 Research Approach

This study aimed at developing a deep understanding of MKT regarding the topic on quadratic equations. Most of the studies on MKT have been conducted by adopting a qualitative approach (Ball, 1990; Ball, 1995; Shulman, 1983). It is, therefore, against this background that the methodology of the present study is based on qualitative approach.

The study followed a qualitative research design and this assisted in generating data relevant to teaching and learning of quadratic equations in relation to mathematical knowledge for teaching. The characteristics of qualitative research are frequently presented as contrasts to the more traditional quantitative approaches (Lincoln and Guba, 2000).

Qualitative research stresses the socially constructed reality studies of phenomena in their actual settings to make sense of, or interpret, the meanings people bring to them (Denzin and Lincoln, 2000). Further, qualitative research encourages emergent as opposed to predetermined designs in which researchers focus on process rather than simply outcomes.

Researchers also hold the view that generalised concepts are built from data that have been gathered and grouped together (Bogdan and Biklem, 1998). More importantly, qualitative researchers hold the view that one needs to get close to the people in order to understand how they think, learn and know the world around them (Bogdan and Biklem, 1998). In using this design, the researcher intends to get closer to the people and collect data through observing people in natural settings. The qualitative researchers set up strategies and procedures to enable\\them collect data from the informants (Bogdan and Biklem 1998).

Bryman (2008) explained that there are three major criticisms against qualitative research. The first criticism is that qualitative research is too subjective. He explained that qualitative research findings are usually criticised of relying too much on the researcher's views about what is significant, and upon the close personal relationship that the researcher establishes with the respondent. The second criticism is that qualitative research is difficult to replicate because there are no standard procedures to be followed and data is generated from unstructured interviews and observations (Bryman, 2008). According to Bryman (2008), this criticism arises from the claim that the investigator himself is the main instrument of data collection, as such, they choose what to concentrate on and what to observe. Thirdly, he noted that findings from qualitative research are criticised for being generated from a very small sample. Consequently, it is impossible to generalise the findings to other settings because they are not a representative of the population being studied. However, Bryman (2008) reacted to these criticisms by arguing that the aim of qualitative research is not to generalise to a population but to generate theory through understanding of behaviour in the context in which the study is conducted. This implies that qualitative research is mainly concerned with the quality of the theoretical inferences that are made out of the data and not the representativeness of the sample to its population.

3.3 Research Design

The present study is based on qualitative approach and the case study was adopted as one of the qualitative designs. Case study is defined by Meriam (1998, p. 21) as a "holistic description and analysis of a single instance, phenomenon or social unit." MKT is influenced by context, learners as well as teachers' characteristics such as experience and content knowledge.

Thus, the nature of MKT, requires a holistic description and the teacher has to be analysed separately. Moreover, in a case study, the case should be a single entity and a unit that has boundaries and cases are generally chosen because they represent an instance of the issue or hypothesis being studied (Merian, 1998). Additionally, the boundaries and context of a case study have to be informative about the research design (Yin, 2003).

In this study, the 'case' is each mathematics teacher in a public secondary school in Katete District of Zambia. Further information about teaching context of each case such as classroom condition, size, teaching experience, gender, etceterawas given in detailed descriptions. Figure 3.1 highlights the procedure of the study.

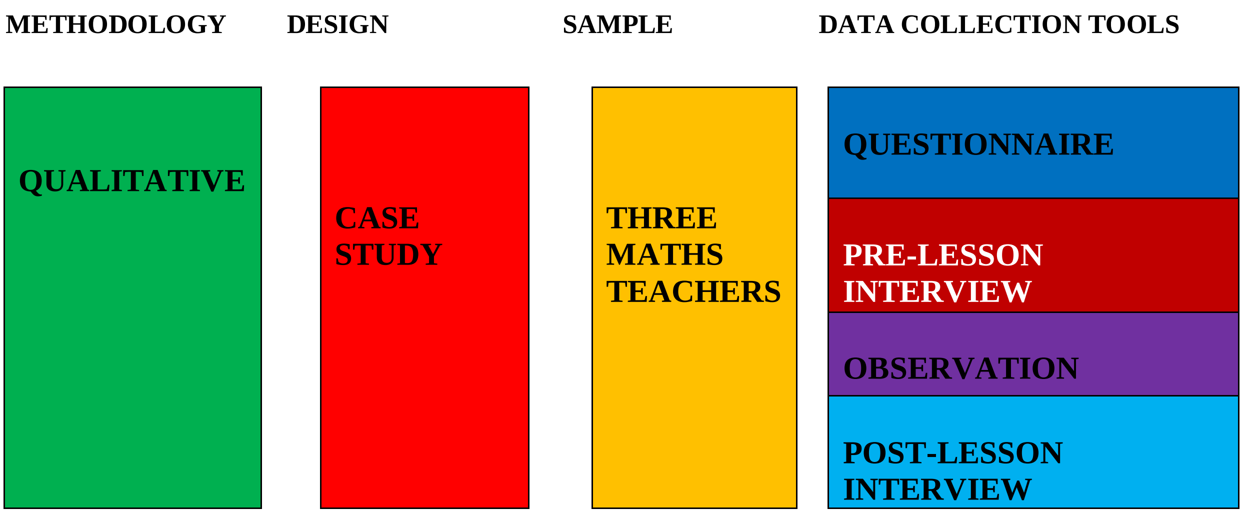


Figure 3.1: The Procedure of the Study

3.4 Participants of the Study

Qualitative studies do not aim at generalising the findings of a study as quantitative studies do. Instead, they seek to have an in-depth understanding of a phenomenon. Therefore, proper sampling procedures are key in order to come up with a sample.

In this study, purposive sampling method was employed to gather deeper and richer information about mathematics teachers' mathematical knowledge for teaching. Purposive sampling entails that the researcher desires to discover, understand and gain insight. The powerful aspect of purposive sampling is selecting information-rich cases in order to get in-depth information. To carry out purposive sampling, the most important task is to determine the selection criteria for the interest of the study (Patton, 1990).

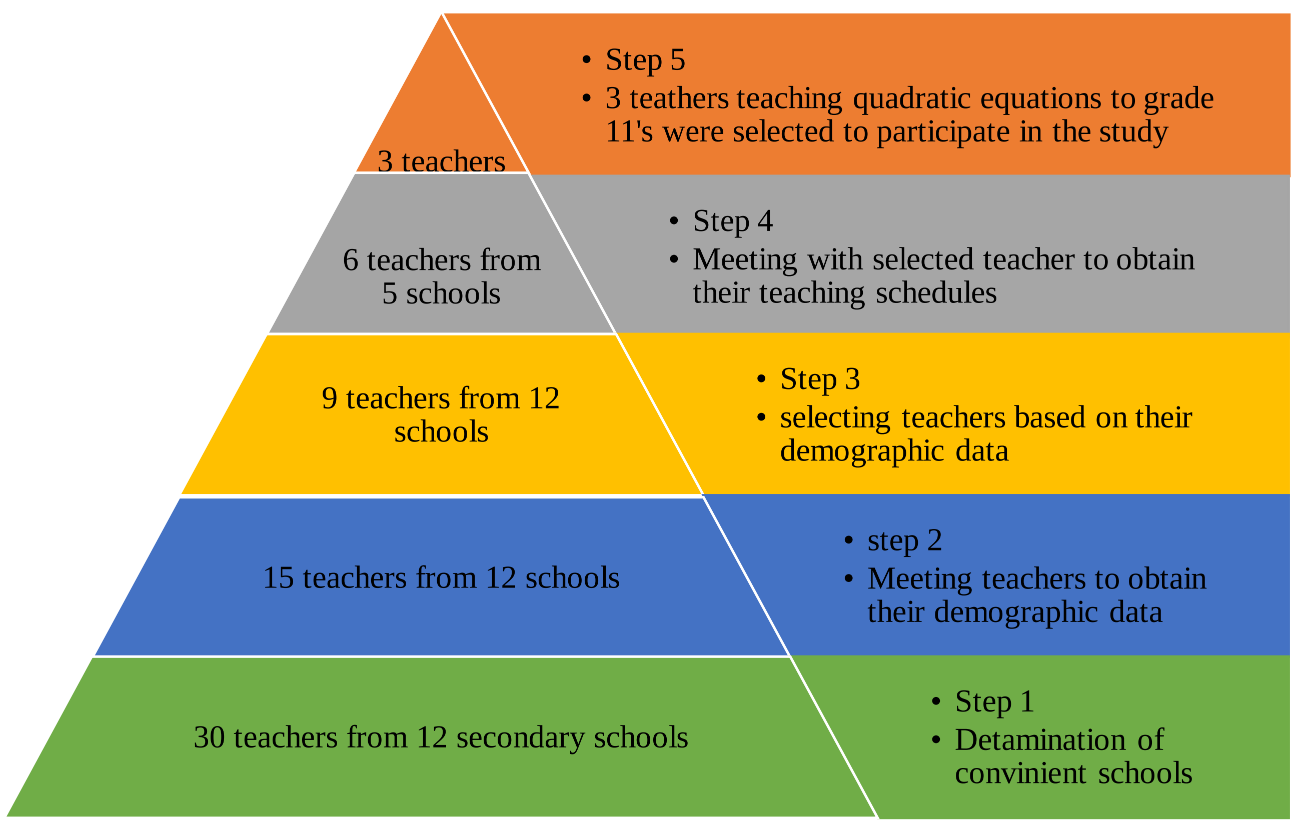
The criteria of selectinginformation-rich cases are based on the purpose of the study and the literature on MKT studies. The study was conducted with three secondary school mathematics teachers who were purposively selected from three schools in one of the small towns in the Eastern Province of Zambia. Like in Kazima et al. (2008), study, the identification of appropriate participants in the current study began with finding a reputably successful, qualified and experienced secondary school mathematics teacher who was willing to take part in the study.

Therefore, the first requirement was that the participants should be sought in a public secondary school. The second requirement was that the teachers to participate in the study must have at least four years of teaching secondary school mathematics.

There were two assumptions for selecting teachers with long teaching experience. The first assumption was that by the end of four years teaching experience, the teachers would be familiar with the whole secondary school mathematics curriculum. Secondly, it was assumed that teachers with long experience had rich information about knowledge for teaching quadratic equations, which they had accumulated over the years. Content knowledge, pedagogical knowledge and the teaching experience were utilised as the criteria for selecting the participants in the study. This is based on the argument by Grossman (1999), in which he highlighted that teaching experience was a major factor in the development of MKT (Grossman, 1999).

The participant teachers' selection procedure through purposive sampling was conducted in five steps as shown in Figure 3.2 below.

Figure 3.2: Selection Procedure through Purposive Sampling



Firstly, the selection of the location of the secondary school as the study area was made and conveniently chosen. The convenient school was selected to enable the researcher increase the accessibility of the data and to spend time with the teacher. This activity was to be completed by January 2020.

In the second step, schools were visited by the researcher in the second week of February 2020. That week was selected to reach teachers at their free time, especially in the afternoon when they did not have classes. In this meeting, the researcher tried to obtain the demographic data in order to select appropriate participants. The teachers were requested to tell a brief story about their teaching. After the meeting, they were 15 teachers from 12 schools who became volunteers to participate in the study.

In the third step, the analysis of teachers' demographic data was employed according to the selection criteria including subject matter knowledge, pedagogical knowledge and teaching experience. This step resulted in selecting 6 teachers from 7 schools as the candidates of the study.

In the fourth step, the candidate teachers were visited again in the second week of March 2020. This was to obtain information about their weekly teaching schedule, which was to be later employed to arrange the teacher observations. In this step, schedules of 6 teachers from 5 schools for Grade 11 class on quadratic equations were obtained.

In the fifth and last step, weekly teaching schedules of teachers were taken into consideration because each of the teachers were to be observed for 80 minutes per week. When the weekly teaching schedules were evaluated, the number of teachers were reduced to 3 and these were collected. To increase as much variation in the sample as possible, the limitation of sample selection was based on the observation schedule.

At the end, the teachers who were most information-rich cases, based on the selection criteria, and who had an appropriate schedule for observation were selected as the participants of the study. All the evaluation processes of the participant selection resulted in 3 secondary school mathematics teachers (one female and two males) being selected. All the teachers had different characteristics which increased the opportunity to identify patterns and contrasts for the cases (Miles and Huberman, 1994).

3.5 Research Sites

The study was conducted in three public secondary schools in Katete District. The schools were purposively selected because they were the three public secondary schools in the district. Additionally, theyhad the largest number of mathematics teachers out of the nine secondary schools at the time of the study. Secondly, because it was easy for the researcher to have a wide range of experiences needed for this study. All the schools and the teachers were easily accessible by the researcher.

3.6 Study Population

The study population comprised all mathematics teachers in the three schools of Katete District who had a Grade 11 class in the year of study.

3.7 Data Collection Methods

In order to gather in-depth information on MKT, questionnaire, pre-lesson interview, lesson observations, and post-lesson interview were utilised in the study. Creswell (1998) referred to this type of data collection as multiple source of information. Each data collection tool aimed at investigating one or more components of MKT. Table 3.2 indicates the data collection tools and investigated MKT components.

Table 3.1: Data Collection Tools

DATA COLLECTION TOOLS	INVESTIGATED MKT COMPONENTS
Questionnaire	Subject Matter Knowledge Pedagogy Knowledge
Pre-Lesson Interview	Pedagogical Knowledge Knowledge of Learners
Lesson Observation	Subject Matter Knowledge Pedagogical Knowledge Knowledge of Learners
Post-Lesson Interview	Pedagogical Knowledge Knowledge of Learners

3.7.1 Questionnaire

The questionnaire was designed to generate responses about teachers' mathematical knowledge for teaching quadratic equations, which was essential for answering research questions (1) and (3). It followed these two formats: multiple choice questions and open-ended questions.The multiple-choice questions in this assessment were written to measure the first domain of the theoretical framework on the subject matter knowledge. Johnson and Christensen (2015) acknowledge that a questionnaire can be used to collect qualitative data if its questions are open-ended. The questionnaire was composed of two parts. Part 'A' comprised background information on the participants while part 'B' assessed mathematical knowledge for teaching. The two further define a questionnaire as "a self-report data collection tool that each research participant fills out as part of a research study". They argue that questionnaires can be used to elicit information about thoughts, feelings, and perceptions and to collect qualitative data.

A questionnaire is one of the data collection instruments, which can help the researcher collect multidimensional data within a short period. The open-ended questions have an advantage of assisting the researcher to get comprehensive information on teachers' cognition and reasoning to the specific questions, which may be unique to this study. The content of the assessment questions was specific to the solving of quadratic equations as it assumed that teachers of mathematics at a secondary school were familiar with the topic.

In this study, a questionnaire was a valid tool for answering research questions 1 and 3, which assessed subject matter knowledge and knowledge of learner conceptions, respectively. The questionnaire was reviewed by two mathematics teacher education specialists to determine if the terms were consistent with the stated curriculum objectives and the literature. This was employed in constructing content validity of the instruments. Prior to the actual study, a pilot study was conducted on 2 secondary school mathematics teachers. The criteria for selecting the participants under the pilot study was similar to the selection of the participants under the main study. During the pilot study, space was created for the respondents to make criticisms and recommendations for improving the questionnaire. Such enlightenment of the results of the pilot study, helped to revise basic issues such as change in the question statement to increase the comprehension of the participants before the administration of the actual study. During the administration of the questionnaire, teachers were informed verbally that their identities would remain confidential by use of pseudonyms. The questionnaire was administered to three secondary mathematics teachers at their available times.The participants were given 90 minutes to complete the questionnaire.Table 3.2 below is the summary description of the items and their purpose in relation to the questionnaire.

Table 3.2: Summary Descriptions of the Items on Assessment Questionnaire in Relation to the Research Question

Item	**Summary /description of item**	**Purpose of item (in relation to research question**
4	How teachers would present a lesson on the teaching of the quadratic equations using the formula method	Research question 1 (Subject matter knowledge)- To find out the procedural and conceptual knowledge Research question 2 (Teaching strategy)
5	Typical errors expected when teaching solving quadratic equations using the formula	Research question 3- Anticipated or expected errors, difficulties, or misconceptions
6	Pre-requisite knowledge learners need to possess before teaching solving quadratic equations using factorisation method	Research question 3- Importance of prerequisite knowledge to address anticipated errors and difficulties
7	Is the quadratic equation $x^2 \quad +.4x+6=0$ Factorable or not. Give reasons	Research question- Subject matter knowledge
8	Mistake committed by the candidate when solving the quadratic equation $(2x-1)(3x-2)=3$	Research question 3- Identification of errors and misconceptions- Knowledge of learner conceptions.

3.7.2 Lesson Observation

While there have been a number of frameworks developed around the topic of mathematical knowledge for teaching, the majority of the researchers agree that the knowledge needed by the teachers can only be seen by looking at the actual process of teaching (Ball and Bass, 2002; Thames and Phelps, 2008; Davis and Simmt, 2006). Another justification for observing teachers is that teachers "...are not always able to articulate their practical knowledge" (Berliner, 2004, p. 206). Simply asking mathematics teachers what they know would not necessarily get at the heart of the mathematical knowledge for teaching. Therefore, to evaluate mathematical knowledge for teaching quadratic equations that teachers possess, the researcher needed to examine the teacher in the process of teaching.

Lesson observation was also forvalidating the information given during pre-lesson semi-structured interviews. It also gave the opportunity to seek explanation for the nature of interaction that took place within the classroom. Above all, observation method is important in that it reduces bias (Ghosh, 2013). It provides useful information and opportunity to obtain content-rich data about pedagogical application, subject matter knowledge and knowledge of learners. During lesson observation, teachers' concrete pedagogical action that is employed for a particular reason in response to MKT, was also examined (Longtan et al., 2004).

The main focus of the lesson observations was the teachers' interpretation of the concepts, questions to pupils and dealing with pupils' correct and incorrect answers. This was to answer research question 3. Furthermore, lesson observations helped to answer research question 2 by observing teachers' instructional practices when teaching quadratic equations. More specifically, lesson observations were used to resolve research question 1 and tried to observe the context of the teachers' delivery of the subject matter, which includes teacher approach to the subject matter, questioning the pupils and the nature of feedback provided to the pupils.

During the observation, the researcher was positioned as a non-participant observer. By "being there", there was no intention to participate in anything, but just watch and make sense of peoples' verbal and non-verbal behaviour (Marshall and Rossman, 1999). All the teachers were observed three times and audio recorded all of the observations, which allowed reviewing the same events for any number of times to get a more detailed account of the discourse and what went on in the classroom.

Lesson observations lasted for 10 weeks; from the first week of September 2020 to the second week of November 2020. The observations started a week earlier before the teaching of quadratic equations topic in the classroom to make sure that both the teachers and learners were familiar with the researcher. There was no observation data collected before the teaching of quadratic equations. Field notes and a voice recorder were used for obtaining data. Participants were informed about the observation and their permissions were obtained before collecting the data. The voice recordings were transcribed and every kind of teachers' and learners' talks, if they were related to teaching quadratic equations. The transcription was completed with field notes. The analysis of the data observed was utilised according to the aim of the study to investigate MKT components.

The study adopted the observation schedule from Chick, Bakar, and Pham (2006), in which mathematical knowledge for teaching mathematics was understood from three perspectives.These were subject matter knowledge, didactic knowledge and knowledge of learner conceptions. The observation model was also used by Kilic (2009); Sibuyi (2012). The classroom observation schedule is shown in Table 3.2 as shown below:

Table 3.3: Classroom Observation Protocol

MKT ELEMENT TO BE OBSERVED	**EVIDENT WHEN THE TEACHER…**	**OBSERVED PRACTICE DISPLAYED**
(a)Knowledge of the subject matter	1. Exhibits deep and thorough conceptual understanding of identified aspects of quadratic equations 2. Identifies critical mathematical components within the concept of quadratic equations that are fundamental for understanding and applying that concept 3. Displays skill for solving quadratic equations	

(b) Knowledge of teaching strategies	1. Uses appropriate activities in instruction 2. Uses real-life examples and analogies in instruction 3. Utilises different instructional strategies	
(c) Knowledge of learners' conceptions	1. Addresses learners' misconception 2. Displays expectations of possible difficulties learners may face during learning and addresses such 3. Discusses learners' ways of thinking about a concept 4. Shows an awareness of the instruments to measure learner thinking and how to use them	

Source: Adapted from Chick, Baker and Pham (2006).

To answer research question1 which sought to find out what subject matter knowledge the teachers possess in the teaching of quadratic equations, a lesson observation schedule assessed the following elements:

Knowledge of the subject matter of the topic in which the teacher is engaged. That is procedural and conceptual knowledge. This is where the following were used to assess this element in terms of accuracy of mathematical facts presented; flexibility of presentation; sequential representation of facts; flow of ideas and hierarchical presentation of facts.

Knowledge of teaching strategies. These are the elements that enabled the teacher to present the lesson in a way that was comprehensible to the learners involving the following guidelines: organisation of the lesson; choice of examples; representations, and appropriate teaching strategies.

Knowledge of learner conceptions (misconceptions and pre-conceptions) about quadratic equations. The following were used to check the teacher's knowledge of learner conceptions: assessing learners' understanding; identifying errors and misconceptions made by the learners, addressing learners' difficulties, and determining their sources.

3.7.3 Interviews

Interviews are generally accepted as the main source of qualitative data needed for the understanding of the study (Meriam, 1998; Paton, 2001). In fact, interviews were crucial data collection tools for MKT because ideas of participants, which were not observable to the researcher could be gotten through interviews as they enabled them to reflect their ideas related to teaching of quadratic equations.

In this study, two separate interview protocols were prepared; pre-lesson and post-lesson interviews. Data was collected using semi-structured and unstructured interviews, which were done prior to lesson observations and after lesson observations respectively. The interviews were designed to generate data for research questions 1, 2 and 3, which looked at subject matter knowledge, teachers' teaching strategies and identification of learners' errors and misconceptions. According to Kvale (1996, p. 6), an interview is "a conversation that has structure and a purpose". This means that for every interview, there must be at least, two people who are engaged in a conversation which is intended to elicit some information. There are three types of qualitative interviews: informal, semi- structured, and standardised open-ended interviews (Patton, 1990). The informal and standardised interviews are also called unstructured and structured interviews, respectively.

The structured interviews involve the use of predetermined set of questions and follows a rigid laid down procedure. On the other hand, unstructured interviews are characterised by a flexibility of approach to questioning. Further, in unstructured interview, the interviewer is given the greater freedom to find out specific information pertaining to the study. Semi-structured interviews also contain a set of guided questions, which the interviewer does not follow strictly but allows for probing depending on the type of information needed for the study (Patton, 1990; Kothari, 2011).

Semi-structured interviews were used to explore teachers' knowledge on quadratic equations. According to Cohen and Manion (1994), interviews are used to, for instance, follow up unexpected results, validate other methods or go deeper into motivating the respondents and their reasons for responding as they did. It is through the use of the semi-structured interviews that in-depth information about teachers' knowledge, reasoning and cognition which cannot be revealed if other research methods such as questionnaires are used was collected.

The set of interview questions have been organised in such a way that it was able to elicit general information about the teachers practice and experiences regarding the teaching of quadratic equations. Further, the interview questions also probed further, the teachers' conceptual knowledge. The interviews addressed and consequently, aligned the responses to the theoretical framework with an aim to answer the research questions. The set of questions also addressed the aspects of domains as addressed in the theoretical framework namely; knowledge of the teachers' subject matter, knowledge of the learners' conceptions and knowledge of didactic representations. The researcher conducted pre-lesson and follow-up interviews after the lesson observation. The teachers were interviewed before the lesson to find out more about their mathematical knowledge of teaching.

The purpose of this interview was to find out how teachers organised the lessons, the teacher's knowledge of the key concepts to be taught, and teaching strategies to be used. The interview was also used to examine how the teacher planned to assist learners with difficulties, assessment tasks, and any expectations of learners' misconceptions that the teacher might come across and also to triangulate with lesson observations. The interviews were audio recoded for the researcher not to lose track of the data and to enable him refer to it when need arose. The researcher adopted the interview model from Chick, Bakar, and Pham (2006). Just like in the observation model, the interview guide was composed of subject matter knowledge, didactic knowledge, and knowledge of learner conceptions.

3.7.3.1 Pre-lesson Teachers Interview Questions

The teachers were interviewed before each lesson in order to find out more about their mathematical knowledge for teaching quadratic equations. The purpose of the interviews was to find out how the teacher organised the lessons, teachers' subject matter knowledge, teaching strategies to be used, how the teacher planned to assist learners with difficulties, and any anticipations of learners' misconceptions.It was also to triangulate lesson observation data.

In this study, pre-lesson interviews were conducted to obtain a more complete picture of mathematics teachers' MKT on quadratic equations. In the pre-lesson interviews, MKT components: subject matter knowledge, knowledge of learners and knowledge of teaching strategies were investigated based on content covered on quadratic equations. Pre-lesson interviews were semi-structured in nature and conducted after the completion of the questionnaire and before lesson observation.

The semi-structured interview enabled the researcher to ask further questions to deepen the understanding of teachers' teaching and obtain data related to the purpose of the study. Two mathematics educators examined the interview questions based on the research questions and the purpose of the study.

The pre-lesson interview was piloted before being employed in the study. This was conducted ontwo mathematics teachers at a public secondary school.The criteria for selecting pilot cases were similar to actual selection process of the participants of the study. During the pilot study, the two participants were asked to make mention of the questions, which were not clear in order to help modify the pre-lesson interview. All necessary revisions were made to construct the final version of the pre-lesson interview protocol. During the administration of the main study, the entire interview with the participants was voice recorded with the permission of each participant of the study and transcribed. The pre-lesson interviews were conducted with the participants at their available and agreed time and took about 80 minutes.

The set of semi-structured interview questions are shown in Table 3.2 below:

Table 3.4: Semi-structured Interview Guide

Elements of MKT	**Question related to MKT**	**Responses**
Knowledge of the subject matter	1. What are the key concepts in the lesson that you are about to teach? 2. Illustrate the sequence you will follow in order to teach these key concepts. 3. Does the lesson involve any procedural knowledge that the learner must know? If so what does the procedure involve?	
Knowledge of the instructional strategies	1. Which instructional strategy will you employ in order to successfully deliver the lesson? 2. Why did you choose this strategy? 3. In your selection of examples, have you chosen real life examples?	
Knowledge of learner conceptions	1. What learners' prior knowledge is regarded as important before the key concepts can be successfully taught to the learners?	

	2. What possible learner misconceptions do you anticipate in this lesson? 3. How will you help learners who experience difficulties in the lesson?	

Adapted from Chick, Baker, and Pham (2006)

3.8 Pilot Study

A pilot study is a "small study conducted prior to a larger piece of research to determine whether the methodology, sampling, the instruments and analysis are adequate and appropriate" (Bless and Higson-Smith, 2000, p. 155). This mini-research is intended to test measuring instruments or the procedure to be followed in the actual project. The participants came from the selected schools with at least 5 years of teaching experience.

In this study, the pilot testing helped the researcher to see if the participants were able to interpret each question correctly and regulate the time needed to complete the interview. It was also used to refine the instruments and help identify unforeseen problems that might emerge during the main investigation. The researcher conducted the pilot study on the questionnaire to two selected mathematics teachers. The same teachers were asked to participate in a pilot interview. The final pilot study helped the researcher to provide clues for the final revision for the questionnaire and to restructure the interview questions.

3.9 Post- Lesson Interview

Although data were collected from pre-lesson interview and classroom observation, teachers were interviewed after observation of the quadratic equations lesson. This was to obtain ideas of teachers on teaching quadratic equations and to deepen the observation data in line with the purpose of the study.

The post-lesson interview questions were prepared based on the collected data of each participant. Each one of the teachers' teaching was different and exemplified different MKT.Accordingly, each teacher's post-lesson interview questions were different from the other. The post-lesson interview ranged from 30 minutes to 60 minutes and was audio recorded with the permission of the participant of the study and transcribed.

3.10 Data Analysis

Data analysis in qualitative research is both an iterative and an on-going process. In this regard, data and interpretations are continuously checked with respondents who acted as sources and differences in opinions are negotiated and outcomes are agreed upon, understood and reflected. Glesne (1999, p. 130), defined qualitative data analysis as “Working with the data, you describe, create explanations, pose hypotheses, develop theories, and link your story to other stories”.

Although there are various ways of analysing data in qualitative research, it becomes a common sense in qualitative study, that data analysis is a continuous task from the beginning of research to the end. In this study, data analysis was performed simultaneously with the collection of data to enable the researcher to focus and shape the study as it proceeded (Glesne, 1999). Patton (2002) advised that the art of developing some manageable classification and coding scheme is probably the first step towards analysing interview data.

In this study, the researcher started analysing data by managing which was collected through questionnaire, interview and class observation. Interpretive data analysis was data according to the conceptual framework.The researcher transcribed the interviews recorded in audiotapes and transformed them into text files that were stored and reviewed. In order to analyse the interviews well, the researcher took notes and other artifacts from the interviewee and read them through in order to make potential themes. Later, the interviews were transcribed verbatim and entered into the interviewees’ rating matrix database.

The researcher followed a similar approach with the audio tapes recorded during the classroom observations. During class observations, the researcher outlined the flow of the lesson, basically to see what was happening in the lesson with the comments that guided the analysis.

The next step in the analysis of data, was description. Using the data from multiple sources such as interviews, questionnaire and class observations, the researcher articulated ‘thick’ and rich description of the persons and context they were in, in terms of school and the curriculum. To make connections among stories, analysis began with identification of the themes emerging from the raw data (Strauss and Corbin, 1990). During the analytical coding, the researcher identified the conceptual categories into which the phenomena observed would be grouped.

Data collected was analysed in line with the three categories identified in the theoretical framework namely; subject matter knowledge, knowledge of the teaching strategies, and knowledge of the learners' conceptions. Knowledge of the subject matter was analysed by checking the teacher's conceptual understanding of the topic and the procedural knowledge demonstrated by problem-solving. Knowledge of teaching strategies was analysed through the checking for use of appropriate activities such as the use of real-life examples and use of different teaching strategies. Finally, knowledge of learners' conceptions was analysed through the teacher's ability to address the learners' misconceptions and anticipation of possible learners' difficulties and discussions of learners' ways of thinking. The above stated categories and themes were investigated through lesson observations, interviews, and questionnaire. The next section looks at the trustworthiness of the study.

3.11 Quality and Rigour of the Study

After completing the construction of the research tools, it is necessary to maintain quality standard of the research. To maintain quality standard, Guba and Lincoln (1998) suggest the following criteria, which were followed to maintain the quality standard in this study.

3.12 Trustworthiness of the Study

Many qualitative researchers have been criticised for being too subjective, or the number of cases too small and above all, that mere talking is not a scientific method (Merriam, 1995). Patton (2002) stated that researchers should take validity and reliability issues into account while designing a study, analysing results, and judging the quality of the study. However, understanding the concepts of validity and reliability of the study in qualitative study are different from those of the quantitative study (Patton, 2002). It is for this reason that qualitative research is based on different assumptions about reliability and consider validity and reliability from a perspective congruent with the philosophical assumptions underlying the paradigm of qualitative research (Merriam, 1998). While quantitative studies answer questions of validity and reliability of the study, qualitative studies address the issue of trustworthiness of a study. Based on this perspective, Lincoln and Guba (1986) defined credibility (as an analog to internal validity, transferability (as an analog to external validity), dependability (as an analog to reliability) and conformability (as an analog to objectivity) as signs of trustworthiness in qualitative studies (p. 76 - 77). The amalgam of credibility, transferability, dependability, and conformability formed the trustworthiness of this study. The evidence in support of the trustworthiness of this study was described under the following titles:

3.12.1 Credibility

Lincoln and Guba (1985) claimed that the most important factor in establishing trustworthiness is confirming credibility. Credibility in qualitative research seeks to answer the following questions: 'How do the findings march with reality'. Credibility is the degree to establish the confidence of the study. To increase the aspect of credibility in qualitative study, Merriam (1998) offered six strategies thus: triangulation, member checks, long-term observation, peer examinations, participatory modes of research and minimising biases of research. In addition to the aforementioned, strategies, Shenton (2004) offered frequent debriefing sessions, peer scrutiny of research and examination of previous research findings as strategies to increase credibility of the study.

Triangulation was the first method adopted to establish credibility. Triangulation is a procedure where researchers search for convergence among multiple sources of information to form themes or categories in the study (Creswell and Miller, 2000). Yin (2009) stated that when you triangulate, the data facts of the case study are supported by more than a single source. There are four types of triangulations and these are: data triangulation (use of a variety of data sources in a study), theory triangulation (use of perspectives to interpret a single set of data), methodological triangulation (use of multiple methods to study a single problem) and investigator triangulation (using a number of researchers to obtain data) (Patton, 2002).

In this study, data triangulation was achieved by using multiple sources including questionnaire, pre-lesson, observations and post-lesson interviews. The investigator triangulation was achieved by inviting another colleague to observe the teachers teaching quadratic equations. This co-observer was purposively selected for observation, because she was knowledgeable on the construct of MKT and how to conduct observations. The co-observer observed each of the participants for 60 minutes. These observations allowed her to gain experience about teachers' teaching. She was instructed on how to observe, take notes, use voice recorder and to transcribe the observation according to the data collected. After observation, the researcher and the co-observer came together to discuss the observations with the help of the data collected. When inconsistences between the researcher and the co-observer emerged, they focused on these parts and tried to reach the consensus.

Long-term observation also helped to ensure credibility.The researcher spent five weeks in the classroom observing and talking to the teachers. Peer debriefing meant requesting another researcher to review and comment on the findings (Merriam, 1998). In this study, another researcher who had experience in MKT and qualitative research was asked to comment on the findings of the study. This was with a view of increasing credibility of the study by means of peer debriefing. Consequently, the researcher based the findings of the study on her comments.

3.12.2 Dependability

In qualitative research, dependability, as an analog to reliability means that readers of the study should acknowledge the consistency and dependability of the findings (Lincoln and Guba, 1985). Dependability shows whether the research findings are congruent with the data collected.

The aim of dependability in a qualitative study is to decrease the errors and bias in the study (Yin, 2009). Dependability in qualitative study means that if another researcher pursues the same study, s/he should get similar findings. Merriam (1998) stated that in order to increase dependability of the study, the researcher shall give a detailed description of how data was collected, how categories were derived and how decisionswere made throughout the investigation. To increase dependability in this study, intense description and audit trail were used. Under intense description, all the necessary details in terms of methodology were given. Audit trail was given special attention in the study and other doctoral students were given time to go through the instruments and provided timely advice.

3.12.3 Transferability

The term transferability is defined as the generalisation of the findings beyond the given study (Yin, 2009). Transferability in qualitative studies refers to what extent the findings of the studies can be applied to other cases. The main aim of the investigation in qualitative study is to have an analytical generalisation from the particular set of findings to broaden theories rather than statistical generalisation like the case is in quantitative research.

To increase transferability in this study, intense description on the cases was applied. In this case, three mathematics teachers had their physical and cultural environment described in detail. The study had more thanone participant and this was also a strategy to increase transferability.

Although the aim of the study was to investigate mathematics teachers' MKT on quadratic equations, generalisation of the findings was not a concern of the study. However, the findings of the study could be shared with other mathematics studies having similar characteristics to further understand mathematics teachers' MKT on other topics.

3.12.4 Conformability

Quantitative studies emphasise that the research should be value-free and should be objective. These studies are based on interpretations and are largely regarded as subjective. The term conformability refers to the degree of neutrality of the researcher while interpreting the data. In order to maintain conformability in qualitative research, the researcher must ensure that the findings of the study are based on the result of the experiences and ideas of participants of the study, rather than characteristics and preferences of the researcher. Triangulation, admission of researchers' bias and detailed methodological description are some of the strategies to promote the conformability of an investigation (Shenton, 2004). In this study, the applied strategies to increase the conformability were triangulation, admission of researcher' bias and peer debriefing.

3.13 The Role of the Researcher

The role of the researcher in qualitative studies is different and more complex than in quantitative studies, as the nature of qualitative studies are open ended and less structured. Conducting qualitative study is mainly based on the researcher, in terms of primary data gathering and analysing data in qualitative studies (Merriam, 1998).

The concern for the researcher's existence in the context of qualitative study was held by Patton (2002). This was by means of the researcher's explanation of the degree of his participation, revelation and existence. The participation in the study ranges from full participation to complete observer. In this study, the researcher took the role a complete observer, which means he did not participate in any teaching activity of the classroom but merely observed from the back of the classroom.

With regard to this study, the researcher informed the participants about observation and their permission was obtained before collecting data. The role of the researcher was made clear and explained to the participants at the beginning of the study (Marshall and Roseman, 2006).

3.14 Ethical Considerations

At the most elementary level, ethics in research have to do with not doing wrong to others but being respectful and fair to them. Ethics has been defined as:

> ...a matter of principled sensitivity to the rights of others, being ethical limits the choices we can make in pursuit of truth. Ethics say that while truth is good, respect for humanity and dignity is better, even if, in the extreme case, the respect of human nature leaves one ignorant of human nature (Caven, 1977 as cited in Cohen et al., 2002, p. 56).

The above quote illustrates the importance of ethical considerations when conducting a research project involving humans as participants, hence qualitative study is not an exception. Human rights and cultural respects and beliefs are important aspects to consider when conducting research. Merriam (1998) posits that ethical issues in qualitative studies are likely to emerge with regard to the collection of data and in the reporting of the findings. It is in relation to these two areas (collection of data and reporting of the findings) of the research process that ethical issues were considered when conducting this study.

Christensen and Johnson (2015) highlighted some ethical considerations, which were applied in this study. The participants were required to give their informed consent. This entails that participants were at liberty to agree or disagree to participate in the study after being informed of its purpose, procedures, benefits and risks among others. In this study, informed consent were sent to the parents of pupils whom the participants teach. The parents were required to give consent to their children for them to take part in the study. The participants were also given the freedom to withdraw from the study. They wereinformedthat the study was voluntary and that they were free to leave the study project whenever they chose to do so and would face no penalty.

Confidentiality and anonymity were addressed as part of ethical issues to protect privacy in research. Confidentiality entails that the researcher must not reveal the identity of the participant to any other apart from the researcher himself and his staff. On the other hand, anonymity means keeping the identity of the participant from everyone, including the researcher. For this study, the three participants were allocated pseudonyms as Faith, Talandila and Titus to protect the participants' identities.

Ethical considerations of this nature reflect that even in the teaching of mathematics, the teachers participating in this study were treated with respect and dignity. Consent was sought from all participants including the pupils' parents. All participants were free to choose to participate or withdraw. Interview transcripts and all observational notes were kept under lock and key so that no one other than the researcher could access the information. Finally, the researcher sought authority from the Ethics Committee of the University of Zambia (UNZA).

3.15 Summary

This chapter describes the research methodology used. It focuses on the research design of the study, which is qualitative. The research method was discussed in detail with regard to sampling procedures, data collection instruments and analysis. Trustworthiness of the study was alsoestablished. Special attention was drawn to the elimination of bias and ethical issues.

CHAPTER 4

PRESENTATION OF FINDINGS

4.0 Introduction

This chapter, is a presentation of the findings of the study, conducted on three secondary school mathematics teachers on the teaching and learning of quadratic equations. The purpose of the study was to investigate teachers' mathematical knowledge for teaching quadratic equations. The MKT was investigated with respect to subject matter knowledge, knowledge of teaching strategies, and knowledge of learners. The study sought to answer the following three questions:

1) What state of teachers' subject matter knowledge for teaching quadratic equations at Grade 11level did the three secondary school mathematics teachers possess?
2) What strategies did the three secondary school mathematics teachers use in teaching quadratic equations at Grade level 11?
3) How did the three secondary school mathematics teachers address pupils' errors and misconceptions related to quadratic equations at Grade 11 level?

To answer the research questions, qualitative design was used.Data were gathered from a variety of sources namely; observations of mathematical lessons, interviews, and questionnaire. Table 4.1 below shows how the data collection methods were used to answer the research questions.

Table 4.1: Data Collection Methods and the Research Questions Answered

RESEARCH QUESTION	DATA COLLECTION METHOD		
	QUESTIONAIRE	INTERVIEW	LESSON OBSERVATION
1. What is the state of teachers' subject matter knowledge for teaching quadratic equations?	✓		✓
2. What strategies do teachers employ to teach quadratic equations?	✓	✓	✓
3. How do teachers' address pupils' errors and misconceptions related to quadratic equations?	✓	✓	✓

To analyse the data collected, cross analyses were done.This was to enhance triangulation and look at the data from all the instruments in a holistic manner. The analysis of data was meant to yield overarching findings related to the research questions by comparing and connecting the themes from all the instruments.

4.1 Biography of the Participants

Of the three participants hereby referred to as Faith, Talandila and Titus all pseudonyms, two were males and one was female. The lowest qualification of the participants was a Secondary Teachers' Diploma and the highest was a Bachelor of Arts Degree with Education at the time of the study.

4.1.1 Description of the Participants

Faith was a female, aged 35.Her highest qualification wasa Diploma in Secondary Teaching specialised in mathematics and history.She was pursuing a degree in mathematics at a public university under distance education mode at the time of study. Although she was trained to teach history, she was not teaching history at the time of the study because the school had adequate history teachers. Additionally, although Faith only had a diploma at the time of the study, she was seconded to teach senior classes because of lack of graduate teachers in the mathematics department. In the Zambian system of education, a diploma holder is trained to teach junior secondary school classes; grades 8 and 9 and not senior secondary classes. Therefore, this was an anomaly. During the time of the study, she had been teaching mathematics for five years and had taught quadratic equations for three years. ***Talandila:*** was a male teacher aged 29. His was a holder of the Bachelor of Science Degree in Mathematics. At the time of the study, he had been teaching for five years and had taught quadratic equations for three years. The participant had other responsibilities such as patron of the mathematics component of the Junior Engineers Technicians Scientists (JETS) and also a member of the Guidance and Careers Department, at the school.

Titus was a male teacher aged 44. His highest professional qualification was a Bachelor of Arts Degree in Mathematics Education. Besides, he had a Diploma in Secondary Education specialising in mathematics. He had been teaching for twenty-one years at the time of the study with eleven of these years teaching mathematics at secondary school level. He was Acting Head of Department for Mathematics comprising 11 members of staff.

4.2 Mathematical Knowledge for Teaching

As earlier discussed in chapters 1 and 2, mathematical knowledge for teaching was defined in relation to the subject matter knowledge, knowledge of pedagogy and knowledge of learner conceptions. The ability to solve quadratic equations, give justification for the answer, provide valid explanations of how mathematical concepts are related and give reasons why particular solutions were incorrect, were perceived as indicators of subject matter knowledge. The choice of teaching activities and examples, representations, promoting classroom discourse, pupil motivation and creativity, were perceived as indicators of knowledge of pedagogy. Identification of possible learner difficulties, misconceptions and errors were perceived as indicators of knowledge of learner conceptions.

4.3 Presentation of the Findings

This section presents the findings and analysis of the study It is organised according to the categories that were already identified in the conceptual framework. The section provides an analysis based on the teachers' ability to explain and justify reasoning in quadratic equations. Since multiple data collection methods such as class observation, interviews and questionnaire were used, cross analyses of data were used in order to ensure triangulation. In a bid to answer the research questions precisely, the analyses of data were based on the three key components of the mathematical knowledge for teaching as identified in the literature review and elaborated in the theoretical framework.These were the subject matter knowledge, didactical knowledge and knowledge of learner conceptions.

4.3.1 Subject Matter Knowledge of Quadratic Equations

Subject matter knowledge in the teaching of quadratic equations was investigated in this section. The first research question "What is mathematics teachers' knowledge of the subject matter knowledge regarding the topic of quadratic equations?" was evaluated with the help of the questionnaire and lesson observation. Teachers' ability of knowledge of facts and procedures of solving quadratic equations, teachers' ability to justify their reasoning and explanation of how concepts are related were perceived as competences of their level of the subject matter. The next section discussesthe subject matter knowledge which include teachers' knowledge of facts and procedures in solving quadratic equations.

4.3.2 Knowledge of Facts and Procedures of Solving Quadratic Equations

This section looks at the facts and procedures of solving quadratic equations as a theme, which emerged under subject matter knowledge of mathematics knowledge for teaching. The three levels identified include the following: Level 1 is one in which the teacher's explanations are not clear while Level 2 is a level in which the teacher's explanations are clear, but the teacher gives the same type of examples to back his or her explanations.Level 3 is the one in which the teacher's explanations of concepts are more accurate, clear, incorporate suitable examples given and are of many variety including situational and application concepts. The levels identified are according to Yusof and Zakaria (2010) as outlined in the related literature. Table 4.2 below shows the knowledge of facts and procedures of solving quadratic equations according to the levels outlined above.

Table 4.2: Talandila's Knowledge of Facts and Procedures of Solving Quadratic Equations

PHASE OF THE LESSON	TEACHER/PUPIL ACTIVITY	COMMENT
Introduction of the Discriminant	The teacher introduced the discriminant, D = $b^2 - 4ac = 0$. Using the example earlier given that is, $x^2 + 5x + 6 = 0$, he proceeded as follows: $D = b^2 - 4ac$ $D = 5^2 - 4(1)(6)$ $D = 25 - 24$ $D = 1$ He concluded by saying that since theDiscriminant was a perfect square.In case 1 then, the quadraticequation can be solved by using factorisation method. The teacher did not explain the genesis of the Discriminant.	Since the teacher did not explain what the genesis of the discriminant was, the teacher's explanation was unclear and uncoordinated hence, it was categorised as level 1
Giving of examples leading as introduction to the completion of the Square Method	The teacher wrote the following questions on the board and solved on the chalk board. Solve for x, $x^2 = 9$	The teacher did not explain the reason of introducing the square
	$x^2 = 9$ $\sqrt{x^2} = \sqrt{9}$ $x = \pm 3$ *or* $x^2 - 9 = 0$ Teacher ***Talandila*** did not explain the reasons as to why the square root signs were introduced.	
Giving of examples	The teacher gave the examples as a way of introducing solving quadratic equations using factorisation method. The examples are as shown below: Example 1 $x\,(x + 2) = 0$ $x\,(x + 2) + 0 = 0$ $x^2 + 2x + 0 = 0$ Product = 0 Sum = 2 Factors =2 and 0, Product = 2 x 0 = 2 and sum = 2 + 0 = 2 $x^2 + 2x + 0x + 0 = 0$	Since the teacher was able to explain the mathematical concepts in a correct way but gave the same type of examples to back the explanations (no variety of examples were given), it was categorised as Level 2

	$x\ (x + 2) + 0(x + 0) = 0$ Therefore, $x = 0$ or $x + 2 = 0$, $x = 0 – 2$, $x = -2$ Example 2 Solve $m\ (m- 3) = 40$, using the factorisation. The teacher proceeded as follows: Teacher- What is the first step to do? Pupils in chorus- remove the brackets Then the teacher proceeded and expanded the equation by removing the brackets $m^2 -3m- 40 = 0$ *product=1x -40 = -40* *Sum = -3* The teacher went further and said that we look for two numbers if when multiplied we get -40 and when added we get -3. The teacher listed all the possible factors as shown (5, -8), (-5, 8), (-2, 20), (-20, 2), (4, -10), (-4, 10). Through inspection with the pupils, the teacher started eliminating the possible factors and finally remained with the desirable factors of (5, -8) as 5 x -8 = -40, and 5 + (-8) = -3. The teacher further explained that we write the sum as in this case -3 as indicated below: $m^2 - 8m + 5m - 40 = 0$ $m\ (m – 8) +5\ (m – 8) = 0$ $(m + 5)\ (m- 8) = 0$ $m + 5 = 0$ or $m – 8 = 0$ $m = 0 – 5$ or $m = 0 + 8$ $m = -5$ or $m = 8$	

It can be observed from Table 4.2 above that ***Talandila's*** knowledge of facts and procedures varied as he operated both at level 1 and 2 during the introduction and development of the lesson.

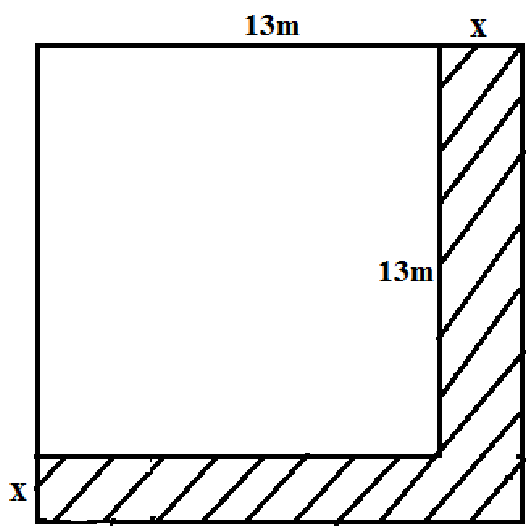
Table 4.3: Faith's Knowledge of Facts and Procedures of Solving Quadratic Equations

PHASE OF THE LESSON	TEACHER/PUPIL ACTIVITY	COMMENT
Giving Examples in the teaching of solving quadratic equations using completing the square method.	The teacher introduced the lesson by stating that if the quadratic equation has no perfect factors it could not be solved using factorisation method but it could be solved using other methods such as completing the square method. At this point, Solve $3x^2 + 4x - 5 = 0$, using completing the square method ***Teacher:*** *Take the constant to the other side,* $3x^2 - 4x = 5$. *Check the coefficient of* x^2, *which is 3, and divide each term by 3* $\frac{3x^2}{3} - \frac{4x}{3} = \frac{5}{3}$ $x^2 - \frac{4x}{3} = \frac{5}{3}$, *Teacher explains that the coefficient, of x which is* $\frac{-4}{3}$ *should be divided by* $\frac{1}{2}$ *and squared.*	The teacher used the traditional approach of giving the same type of examples without paying attention to giving other varieties or forms of examples hence this was categorised as Level 2
	$x^2 - \frac{4x}{3} + (\frac{-2}{3})^2 = \frac{5}{3}\ (\frac{-2}{3})^2$ $(x - \frac{2}{3})^2 = \frac{5}{3} + \frac{4}{9}$ $(x - \frac{2}{3})^2 = \frac{19}{9}$ ***Teacher:*** *What is the opposite of square?* ***Pupil:*** *Square root* Teacher proceeded as shown below: $x - \frac{2}{3} = \pm\sqrt{\frac{19}{9}}$ $x - \frac{2}{3} = \pm 1.45296635$ $x = \frac{2}{3} \pm 1.45296635$ $x = \frac{2}{3} + 1.45296635$ *or* $\frac{2}{3} - 1.452966315$ $x = 2.12$ *or* -0.79 Another example was given: Solve the quadratic equation. $2x^2 + 7x + 3 = 0$ using completing the square method and the teacher solved using the steps shown in the example above. ***Faith*** **Lesson Observation 2**	

Table 4.3 shows that ***Faith*** used the same type of examples without paying attention to other forms of examples. It is against this background that her lessons were categorised as level 2.

Table 4.4: Titus's Knowledge of Facts and Procedures of Solving Quadratic Equations

PHASE OF THE LESSON	TEACHER/PUPIL ACTIVITY	COMMENT
Presentation of the lesson on solving quadratic equations using factorisationmethod	**Lesson observation – *Titus*** A man is 37 years old and his son is 8 years old. How many years ago was their product 96? The pupil was asked to solve the question on the chalkboard. ***Pupil***: *let the years of the man be x and that of the son be y* $x \times y = 96$, $xy = 96$ equation.1, $x - y = 29$, $x = 29 + y$... equation 2 $(29 + y)(y) = 96$ $29y + y^2 = 96$ $29y + y^2 - 96 = 0$ $y^2 + 29y - 96 = 0$ Sum = 29 Product = - 96 Factors = 32 and – 3 $y^2 + 32y - 3y - 96 = 0$ $y(y + 32) - 3(y + 32) = 0$ $(y - 3)(y + 32) = 0$ $y - 3 = 0$ or $y + 32 =$ $y = 3$ or $y = -32$. We pick $y = 3$ because age cannot be in negative. We substitute the value of y in the equation $x = 29 + y$ as shown below: $x = 29 + 3$, $x = 29+3$, $x = 32$ Since $x = 32$ and the current age is 37, then 37 – 32 = 5. Hence it was 5years ago when the product of their ages was 96. The teacher introduced another method as shown below: *Let x* stand for the number of years ago for the father and the son: $37 - x$ and $8 - x$ *and* their product be equal to 96 $(37 - x)(8 - x) = 96$ $296 - 37x - 8x + x^2 = 96$ $296 - 45x + x^2 - 96 = 0$ ***Teacher;*** *we write the quadratic equation in standard form of* $x^2 + bx + c = o$ as shown below	The teacher gave a variety of examples including situational and application concepts in order to make the learners understand the practical implications of quadratic equations. Above all the explanations were more accurate and clear. Hence, it was categorised as Level 3

	$x^2 – 45x + 200 = 0$ Sum = $-45x$, $x^2 – 45x + 200 = 0$ Sum = $-45x$ Product = $200x^2$ factors = $-40x$ and $-5x$ $x^2 – 40x – 5x + 200 = 0$ $x (x – 40x) -5(x – 40) = 0$ $(x – 40) (x – 5) = 0$ $x – 40 = 0$ or $x – 5 = 0$ $x = 40$ or $x = 5$ The teacher explained that since it was five years because we cannot pick 40 because the man is 37years now.	
Teaching of solving quadratic equations using completing the square method	The builder is changing the floor plan of a house to meet thedesignerof the new customer. The present plan calls for a square dining area that measures 13m by 13m. The customer would also like the dining hall to be a square but with an area of 250m^2. How much will be the dimensions of the room? ***Teacher:*** *What should be done because the new customer wants it to 250m^2?* ***Pupil***: Wrote on the chalk board. $(13 + x) (13 + x) = 250m^2$ The teacher drew the diagram on the chalk board as shown below:  Teacher: $(13 + x) (13 + x) = 250$ ***Teacher:*** *What is the other way of writing* $(13 + x) (13 + x)$? ***Pupil:*** $(13 + x) 2$ ***Teacher:*** *This can be written as: $(13 + x) 2 = 250$*	***Titus*** gave a variety of examples and were situational in nature.So, the lesson was categorised as Level 3

	Teacher: *How can we complete the square?* P*upil:* $(13+x)^2 = \pm 250$ ***Teacher****: Yes... What is the next step?* ***Pupil:*** *mmm.... let me try*. She wrote: *13 + x = ±15. 8113883* *x = - 13 + 15.8113883 or -13 – 15. 8113883* *x = 2.81 or -28.8* Teacher concluded that the dimensions of the room will be 2.81+ 13 = 15.81m by 18.81m (***Titus*** lesson observation # 2).	

It can be observed from Table 4.4 above that ***Titus*** used a variety of examples, which also included real life ones. It is because of this that ***Titus***' lessons were categorised at level 3 according to Yusof and Zakaria (2010).

4.3.3 Explanation of How Concepts AreRelated through Pre-RequisiteKnowledge

Three indicators were identified as outlined in the literature according to Yusof and Zakaria (2010). In this category, the first level is where the teacher does not relate the lesson being presented to learners' existing knowledge whereas the second level is where the teacher relates the lesson being presented to the existing knowledge but does not take time to explain the linkage. Level 3 is where the teacher relates the lesson being presented to the learners' existing knowledge and takes time to explain the linkage through procedural and conceptual knowledge. Table 4.5 below outlines this competence and indicators as earlier explained.

Table 4. 5: Explanation of How Concepts AreRelated through Pre-Requisite Knowledge

Explanation of how concepts are related through pre-requisite knowledge	**LEVEL 1: Teacher does not relate the lesson being presented to learners existing knowledge** **The Case of *Faith*- First Lesson Observation** ***Introduction*** The teacher began by defining the quadratic equation as follows ***Faith:*** '*A quadratic equation is an equation of the form ax^2 + bx + c = 0, where a, b and c are constant numbers'*. ***Faith:*** *We are going to look at solvingquadratic equations using factorisation method.* ***Faith:*** *What is to factorise*? ***Pupil****: 'To factorise is to reduce an equation'.*

	Pupil: *'To factorise is to simplify an equation'.* The teacher then gave the example. ***Example*** Solve the quadratic equation, *'$2x^2 + 7x – 15 = 0$'* using factorisation method. The teacher introduced the topic but did not have a direct link to the existing topic because no specific examples of factorisation were given. **Level 2: The teacher relates the lesson being presented to learners' existing knowledge but gives the same type of examples to explain the concept.**

The next subsection takes a look at the observation schedule of the subject matter knowledge as shown in Table 4.6 below.

Table 4.6: Subject Matter Knowledge Observation Schedule

Name of Participant	Ability to solve quadratic equations	Justification for the answer	Valid explanation	Relationship of mathematical concepts
Talandila	✓			✓
Faith	✓			✓
Titus	✓	✓	✓	✓

It is seen from Table 4.6 above that all the three participants had the ability to solve all forms of quadratic equations. However, ***Faith*** and ***Talandila*** had problems to give valid explanations and justification for the answer. It was also observed that when pupils made a mistake, Teacher ***Faith*** and ***Talandila*** would just ask another pupil to solve the question in the correct way.They would not take time to explain why the particular solution was incorrect.

All three participants were able to explain the relationships among the concepts of the quadratic equations. For instance, all the participants were able to explain the fact that the quadratic formula was derived using completing the square method. ***Titus***, on the other hand, gave valid explanations in order to help pupils understand that "the square root of a negative number is a complex number and it is taught in further mathematics". This was in response to a pupil who said that a negative number had no square root. Table 4.7 below shows thematic representations for subject matter knowledge:

Table 4.7: Subject Matter Knowledge Theme, Code and Category

Code	Category	Theme
Ability to solve problems	Common content knowledge	Subject matter knowledge
Justification for the answer Valid explanations of the relationship of mathematical concepts	Specialised content knowledge	

4.4 Teaching Strategies

The second research question of the study was, 'What strategies do teachers use when teaching quadratic equations?' This section discusses responses to this question. Teaching strategies which the three secondary mathematics teachers used when teaching quadratic equations were evaluated with the help of pre-lesson and post-lesson interviews and lesson observations.

4.4.1 Faith'sKnowledge of Teaching Strategies

Teacher knowledge about teaching strategies were investigated in this section. The knowledge of teaching strategies of the participant teachers was presented with topic specific teaching strategies of quadratic equations.

4.4.1.1 Topic Specific Teaching Strategies

Teacher knowledge of the topic specific strategies about quadratic equations were investigated in this section. The knowledge of teaching strategies of the participant teacher was presented with two subheadings: representations and activities used when teaching quadratic equations. Teacher's knowledge of teaching strategies was evaluated with the help of the pre-lesson interview, lesson observation and the post-lesson interview. The data analysis for representation of teaching strategies leads to three categories namely: activities, use of examples and instructional strategies utilised during the teaching of quadratic equations.

4.4.1.2 Strategies Used by *Faith* to Overcome Learners' Difficulties While Teaching Quadratic Equations

The participants were asked if they assisted learners who experienced difficulties while teaching quadratic equations. Data on teaching strategies to overcome difficulties while teaching quadratic equations were obtained through pre-lesson interviews, post-lesson interviews and lesson observations.

The classification of learning difficulties and their respective teaching strategies employed by ***Faith***aregiven in Table 4.8.

Table 4.8: Teaching Strategies Employed by Faith to Overcome Learners' Difficulties Quadratic Equations

DIFFICULTIES	**TEACHING STRATEGIES USED TO OVERCOME LEARNERS' DIFFICULTIES WHILE TEACHING QUADRATIC EQUATIONS**
CONCEPTS	Quadratic equations involving multi-mathematical operations e.g. $\pm$, square root sign, and division sign. STRATEGY USED: Learners are allowed to do more practice on a variety forms of quadratic equations such as including those which were in standard form and those which were not in standard form.They also solve them using the formula method (Lesson observation and pre-lesson interview).
RELATIONSHIPS	Learners have difficulties in understanding the relationship between completing the square method and the formula method. STRATEGY USED: ***Faith*** started teaching the completing the square method before the formula method. She encouraged learners to use completing the square method, so that they would appreciate the formula (Lesson observation, pre-lesson interview).

UNDERSTANDING	Learners have problems in finding the factors when using the factorisation method, especially when the quadratic equations had big numbers as coefficients. STRATEGY USED: The learners were given a variety of quadratic equations which they were asked to solve using the factorisation method with an emphasis of those which had big numbers as coefficients (Lesson observation, pre-lesson interview)

4.4.2 Talandila's Knowledge of Teaching Strategies

Teacher knowledge about teaching strategies were investigated in this section. The knowledge of teaching strategies of the participant teachers was presented with topic specific teaching strategies of quadratic equations.

4.4.2.1 Topic Specific Teaching Strategies

Teacher knowledge of the topic specific strategies about quadratic equations were investigated in this section. The knowledge of teaching strategies of the participant teacher was presented with two subheadings: representations and activities used when teaching quadratic equations. Teacher's knowledge of teaching strategies was evaluated with the help of the pre-lesson interview, lesson observation and the post-lesson interview. The findings for representation of teaching strategies lead us to three categories namely: activities, use of examples and instructional strategies utilised during the teaching of quadratic equations.

Teacher knowledge of the topic specific strategies about quadratic equations were investigated in this section. The knowledge of teaching strategies of the participant teacher was presented with two subheadings: representations and activities used when teaching quadratic equations. Teacher's knowledge of teaching strategies was evaluated with the help of the pre-lesson interview, lesson observation and the post-lesson interview. The data analysis for representation of teaching strategies leads to three categories namely: activities, use of examples and instructional strategies utilised during the teaching of quadratic equations.

4.4.2.2 Strategies Used by *Talandila* to Overcome Learners' Difficulties While Teaching Quadratic Equations

The participants were asked if they assisted learners who experienced difficulties while teaching quadratic equations. Data on teaching strategies to overcome difficulties while teaching quadratic equations were obtained through pre-lesson interviews, post-lesson interviews and lesson observations.

The classification of learning difficulties and their respective teaching strategies employed by ***Talandila*** are given in Table 4.9.

Table 4.9: Teaching Strategies Employed by Talandila to Overcome Learners' Difficulties Quadratic Equations

DIFFICULTIES	TEACHING STRATEGIES USED TO OVERCOME LEARNERS' DIFFICULTIES WHILE TEACHING QUADRATIC EQUATIONS
CONCEPTS	Quadratic equations involving multi-mathematical operations e.g. $\pm$, square root sign, and division sign. STRATEGY USED: Learners are allowed to do more practice onavariety of forms of quadratic equations and ensure that they practice solving the quadratic equations using the quadratic formula (Lesson observation and pre-lesson interview)
RELATIONSHIPS	Learners have difficulties in understanding the relationship between completing the square method and the formula method. STRATEGY USED: emphasise on deriving the quadratic formula using completing the square method (Lesson observation, pre-lesson interview).
UNDERSTANDING	Learners have problems in solving quadratic equations, which have a negative value of "b". Most of the learners did not consider the negative which is already in the formula. STRATEGY USED: More practice given to the learners. the teacher gave topic tests to the learners. He also usually gave more quadratic equations of all sorts ranging from those in standard formof $ax^2 + bx + c = 0$ and those not in standard form (Lesson observation, pre-lesson interview).

4.4.3 Titus'sKnowledge of Teaching Strategies

Teacher knowledge about teaching strategies were investigated in this section. The knowledge of teaching strategies of the participant teachers was presented with topic specific teaching strategies of quadratic equations.

4.4.3.1 Topic Specific Teaching Strategies

Teacher knowledge of the topic specific strategies about quadratic equationswere investigated in this section. The knowledge of teaching strategies of the participant teacher was presented with two subheadings: representations and activities used when teaching quadratic equations. Teacher's knowledge of teaching strategies was evaluated with the help of the pre-lesson interview, lesson observation and the post-lesson interview. The data analysis for presentation of teaching strategies leads to three categories namely: activities, use of examples and instructional strategies utilised during the teaching of quadratic equations.

4.4.3.2 Strategies Used by *Titus* to Overcome Learners' Difficulties While Teaching Quadratic Equations

Teachers were askedif they assisted learners who experienced difficulties while teaching quadratic equations. Data on teaching strategies to overcome difficulties while teaching quadratic equations were obtained through pre-lesson interviews, post-lesson interviews and lesson observations.

The classification of learning difficulties and their respective teaching strategies employed by ***Faith*** are given in Table 4.10 below.

Table 4.10: Teaching Strategies Employed by Faith to Overcome Learners' Difficulties Quadratic Equations

DIFFICULTIES	TEACHING STRATEGIES USED TO OVERCOME LEARNERS' DIFFICULTIES WHILE TEACHING QUADRATIC EQUATIONS
CONCEPTS	Learners have difficulties on the signs, especially when using the formula method.They usually forget the sign which is on 'b' STRATEGY USED: Learners are allowed to do more practice. Learners were given a number of quadratic equations and asked to solve using different methods of solving quadratic equations such as factorisation

	method, completing the square method and formula method (Lesson observation and pre-lesson interview)
RELATIONSHIPS	Learners had difficulties in understanding the relationship between completing the square method and the formula method. STRATEGY USED: ***Titus*** started teaching the completing the square method before the formula method as indicated in the following quotation (Lesson observation, pre-lesson interview). "I also like to emphasise to my learners to practice more on the solving of the quadratic equations using the completing the square method because it is useful in further mathematics at college and university" (Pre-lesson interview)
UNDERSTANDING	Learners have difficulties when the quadratic equations have recurring digits, learners tend to round off on the way, thereby getting wrong answers. STRATEGY USED: The teacher always encouraged the learners not to round off on the way but do so when they on the last stage of solving the question (Lesson observation, pre-lesson interview) The teacher also gave them more questions of this matter so that they could have more practice (Pre-lesson interview)

4.5 Teaching Strategies Used When Teaching Quadratic Equations Using the Formula Method

One of the methods used to solve quadratic equations was through the formula. When the questionnaire was administered, the three participants responded that they would first teach the pupils how the formula was derived and expected them to understand and remember each step. This was also true as this was seen during class observation. The extract below shows the extract from ***Titus*** on the introduction of the formula method of solving quadratic equations. When asked why they would teach the learners how the formula was derived, the following responses as outlined in Table 4.11 below were given:

Table 4.11: The Reasons Given Over the Choice of Approach

S/N	**NAME OF PARTICIPNT**	**REASON(S) GIVEN**
1	***Faith***	I would use the approach because the pupils will understand better
2	***Talandila***	For pupils to get the connection between the completing square method and the formula method
3	***Titus***	Derived formula helps pupils understand and remember

During lesson observations, it was noted that all the participants introduced the lesson on solving quadratic equations using the formula by first deriving the formula using completing the square method. Below is an extract of a lesson as presented by ***Titus***:

The teacher led the class into deriving the formula and he wrote the standard form of quadratic equations: $ax^2 + bx + c = 0$

Teacher: *Take the constant c to the other side and it will become*: $ax^2 + bx = -c$

Teacher: *Divide the equation throughout by 'a' in order to make the coefficient of x^2 unit or 1* as follows: $\frac{ax^2}{a} + \frac{bx}{a} = \frac{-c}{a}$ and this becomes: $x^2 + \frac{bx}{a} = \frac{-c}{a}$.

Teacher: We half the coefficient of x which in this case is $\frac{b}{a}$ and square it and this becomes:

$x^2 + \frac{bx}{a} + (\frac{b}{2a})^2 = \frac{-c}{a} + (\frac{b}{2a})^2$. Adding the component of the right-hand side the equation becomes:

$x^2 + \frac{bx}{a} + (\frac{b}{2a})^2 = \frac{-c}{a} + \frac{b^2}{4a^2}$, and this is equal to

$x^2 + \frac{bx}{a} + (\frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}$. At this point, the teacher explained that the left-hand side of the equation can be factored as shown below:

$$(x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}.$$

He then introduced a square root to both sides as shown below:

$$\sqrt{(x + \frac{b}{2a})^2} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Table 4.12 below shows the observation schedule in relationship with the knowledge of pedagogical representations.

Table 4.12: Observation Schedule for Knowledge for Pedagogical Representations

Name of participant	Promoting classroom discourse	Pupil motivation	Ordinary examples	Real life examples	Creativity
Talandila			✓	✓	
Faith			✓		
Titus	✓	✓	✓	✓	✓

It is evident from Table 4.12 that all the participants gave examples to the pupils as a way of presenting concepts to the pupils. However, only ***Titus*** and ***Talandila*** gave real life examples. With regard to pupil motivation, only ***Titus*** was able to motivate pupils so that they could appreciate the importance of learning quadratic equations. Hedid this by telling the pupils that quadratic equations were important in demarcating of plots by the municipalities.

Titus also motivated pupils at the end of the lesson by asking them what they had learnt. All the participants used questioning techniques as a form of presenting information to the pupils. However, only ***Titus*** was able to promote real classroom discourse by allowing both pupil-pupil and teacher-pupil interaction. In terms of creativity, ***Titus*** was outstanding in this category in the sense that he was able to allow pupils to express themselves by solving problems on the chalkboard and use a variety of methods to solve a single question. Table 4.13 below shows thematic representation of the pedagogical knowledge.

Table 4.13: Theme: Pedagogical Knowledge, Code and Category

Code	Category	Theme
Promoting classroom discourse Creativity Pupil motivation Real-life examples	Questioning Motivation Pedagogy	Knowledge of pedagogical representations

4.6 Knowledge ofTeachers on LearnerConceptions about Quadratic Equations

In this section, teachers' knowledge about learners in learning quadratic equations was investigated and evaluated through the pre-lesson interview, lesson observation, and the questionnaire. The findings from the questionnaire have already been analysed. This section looks at the findings of teacher knowledge about learners evaluated through pre-lesson interview and lesson observation and was analysed under two titles: pre-requisite knowledge required for learning quadratic equations and knowledge of learners' difficulties while learning quadratic equations.

4.6.1 Faith'sPre-Requisite Knowledge Required for Quadratic Equations

In this section, pre-requisite knowledge required for learning quadratic equations was evaluated with the help of pre-lesson interview and lesson observation. When asked the pre-requisite knowledge, which was required for teaching quadratic equations, the respondent indicated that learners needed to have the knowledge of quadratic functions, solving linear equations, combining like terms, graphing using the XOY-Plane and the zero factor theorem.

Faith emphasised that there was need for learners to know about the quadratic functions becausethere were similar in nature.The only difference between the two was that the quadratic equation was equated to zero. She further indicated that learners are taught quadratic functions before being taught quadratic equations. She also mentioned that "learners required to know about graphing on the XOY-Plane because one of the methods for solving quadratic equations was graphical method".

However, during lesson observation, ***Faith*** did not take pre-requisite knowledge required by the learners into consideration. Below is the extract to justify the point from ***Faith's*** lesson observation.

Introduction

The teacher began by defining the quadratic equation as an '*an equation of the form* $ax^2 + bx + c = 0$*, where a, b and c are constant numbers'.*

She further indicated that for that particular lesson, they would look at solving quadratic equations using factorisation method. She asked the pupils to define what factorisation was and the pupils gave the following answers:

Pupil: 'To factorise is to reduce an equation'.

Pupil: 'To factorise is to simplify an equation'.

The results of data analysis of pre-requisite knowledge required by ***Faith*** are shown in Table 4.14 below.

Table 4.14: Faith's Pre-RequisiteKnowledge Needed for Teaching Quadratic Equations

SKILLS REQUIRED FOR LEARNING QUADRATIC EQUATIONS	**RESPONSE**
PROBLEM-SOLVING	Learners need to have an "idea of problem solving of linear equations because at some point the quadratic equations will be reduced to linear equations before a solution is found (Pre-lesson interview)
COMBINING LIKE TERMS	Learners should be able to know how to combine like terms together for them to solve quadratic equations (Pre-lesson interview)
COORDINATE GEOMETRY	Learners should be able to know how to plot on the XOY-Plane as this will be important when they are taught how to solve quadratic equations using graphical method (Pre-lesson interview)

4.6.2 Faith'sKnowledge of Learners' Difficulties in Learning Quadratic Equations

The difficulties of learners are presented in this section, with examples of responses. Teachers' knowledge of learners' difficulties while learning quadratic equations was evaluated with the help of pre-lesson interview, lesson observation, and questionnaire. The findings on ***Faith's*** knowledge of learners' difficulties in learning quadratic equations, werefrom the pre-lesson interviewsand lesson observation.

When asked during pre-lesson interview about the difficulties, which learners face in learning quadratic equations, ***Faith*** responded as follows:

> *"learners faced a number of difficulties in learning quadratic equations. Learners had difficulties in finding roots if the discriminant* $b^2 - 4ac$ *is not equal to zero. The leaners have difficulties in understanding the quadratic formula properly, especially because the formula has a negative b.If the quadratic equation has a negative value of the coefficient of b, learners tend to forget or ignore the other negative"(Lesson observation).*

Faith showed that she had knowledge of learner difficulties when she emphasised on the proper writing of the quadratic formula. She explained that once the quadratic formula is not properly written, the learners were bound to get an incorrect solution.

During the post-lesson interview, she explained that *"lack of practice on the integers was the major reason for the difficulties which learners faced in the learning of quadratic equations"*.

She further indicated that the source of difficulties, which learners face while learning Quadratic Equations was*: "the topic of integers, which is not well-handled during the early years of secondary school education"*.

The findings of ***Talandila's*** knowledge of learners' difficulties in learning quadratic equations are shown in Table 4.15 below.

Table 4.15: Source of Difficulties and Responses

DIFFICULTIES	**RESPONSE OF *TALANDILA***
RELATIONSHIP	Learners have difficulties in understanding the relationship between completing the square method and the formula method (Pre-lesson interview). The interview except as shown below: ***Researcher****: How about when teaching completing the square method, what difficulties or misconceptions do you anticipate when teaching this method?* ***Talandila:*** *When the coefficient is not 1, then the pupils find difficulties, especially those who are faster they overlook this*
UNDERSTANDING	Learners have problems in finding the factors when using the factorisation method, especially when the quadratic equations had big numbers as coefficients (Lesson observation). ***Researcher****: What other misconceptions or difficulties do you anticipate?* ***Talandila:*** *They also have difficulties in finding the factors when teaching factorisation method (Pre-lesson interview).*
CONCEPTS	Quadratic equations involving multi-mathematical operations e.g. $\pm$, square root sign, and division sign. ***Researcher:*** *Any difficulties or misconceptions you anticipate your learners to have, before you teach quadratic equations?* ***Talandila:*** *There is also an issue of understanding the quadratic formula properly. For example, in the formula, there is negative (b) and if the quadratic equation there is a negative, the pupils tend to miss it.* *They also have difficulties in finding the factors when teaching factorisation method.*
INTEGERS	The topic of integers is not properly taught in junior secondary school mathematics. This is because integers are cardinal for understanding future mathematics (Pre-lesson interview)
PROBLEM-SOLVING USING COMPLETING THE SQUARE METHOD	Lack of understanding of squares and square roots makes learners to find it difficult to understand the operations of completing the square method (Pre-lesson interview)

4.6.3 Talandila'sTeachers' Knowledge of Learners about Quadratic Equations

Teachers' knowledge of learners in learning quadratic equations was investigated and was evaluated through the pre-lesson interview, lesson observation and the questionnaire. This section, looks at the findings which were analysed under two headings: pre-requisite knowledge required for learning quadratic equations and knowledge of learners' difficulties while learning quadratic equations.

4.6.4 Talandila'sPre-RequisiteKnowledge required forTeaching Quadratic Equations

In this section, pre-requisite knowledge required for learning quadratic equations was evaluated with the help of pre-lesson interview and lesson observation.

When ***Talandila*** was asked what pre-requisite knowledge he would consider to be important before teaching quadratic equations, his response was as follows:

> "*learners needed to be knowledgeable in algebra, integers, especially in the use of positive and negative signs. Learners were also supposed to be conversant with squares, square roots and difference of two squares, especially when dealing with the quadratic formula. Learners needed to have pre-knowledge of solving linear equations in one variable.*"

However, during lesson observation, ***Talandila*** took cognisant of learners' pre-requisite knowledge into consideration. The lesson observation extract below shows this assertion as shown below:

The teacher introduced the lesson by giving two questions to the class and these were: (a) $x^2 + 5x + 6 = 0$ and (b) $x^2 + 4x - 3 = 0$. The teacherasked the pupils to solve the equation using the factorisation method. The pupils experienced difficulties in solving question (b) using factorisation method.

Talandila: *It is difficult to solve a quadratic question like* $x^2 + 4x - 3 = 0$ *because it had no simple factors and hence, the factorisation method cannot do.*

Presentation

The teacher introduced the discriminant, D = $b^2 - 4ac = 0$. Using the example earlier given:

$x^2 + 5x + 6 = 0$, he proceeded as follows:

$D = b^2 - 4ac$

$D = 5^2 - 4(1)(6)$

$D = 25 - 24$

$D = 1$

Talandila: *Since the discriminant is a perfect square, in this case, 1, the quadratic equation can be solved using factorisation method.*

Using the discriminant, the teacher considered the quadratic equation thus:

$x^2 + 4x - 3 = 0$

$D = b^2 - 4ac = 0, a = 1, b = 4, c = -3$

$D = 4^2 - 4(1)(-3)$

$D = 16 + 12$

$D = 28.$

Talandila**: "***Since the discriminant is 28 and is not a perfect square, the quadratic equation: $x^2 + 4x + 3 = 0$ has no simple factors. Therefore, itcan be solved using other methods such as completing the square method which we are going to look at in this lesson".*

4.6.5 Talandila's Knowledge of Learners' Difficulties in Learning Quadratic Equations

In this section, the difficulties of learners were presented with examples of responses. Teachers' knowledge of learners' difficulties while learning quadratic equations was evaluated with the help of pre-lesson interview, lesson observation, and questionnaire.

When asked about the learner difficulties and misconceptions, which he anticipated, ***Talandila*** respondedas follows:

> *"they have difficulties in solving quadratic equations using the formula because it has a lot of basic operations such as addition, subtraction, division, and square root. The learners have problems in understanding the computations of solving quadratic equations using completing the square method, especially when the leading coefficient is not equal to one.Learners do not know the relationship between completing the square method and the formula method because teachers just introduce the formula method as an entity without deriving the formula using completing the square method."*

During lesson observation, ***Talandila*** showed that he anticipated learner difficulty when he asked the learner to solve the equation; $(x-2)^2 = 9$ on the chalkboard. Below is an excerpt from second lesson observation of ***Talandila***.

Solve $(x - 2)^2 = 9$,

The teacher asked one of the pupils to solve the problem on the chalk board. She proceeded as follows:

$(x - 2)^2 = 9$

$x^2 - 4 = 9$

$x^2 = 9 + 4$

$x^2 = 13$

$x = \pm\sqrt{13}$

The teacher asked another pupil to solve the problem and correct the misconception. She progressed as shown below:

$(x - 2)^2 = 9$

$\sqrt{(x-2)^2} = \sqrt{9}$

x – 2 = $\pm$ *3*

x = 2 $\pm$ *3*

x = 2+ 3 or 2-3

x = 5 or -1

(Source: ***Talandila's*** second class observation, date??)

The findings of ***Talandila's*** knowledge of learners' difficulties in learning quadratic equations is shown in Table 4.16 below.

Table 4.16: Talandila's Knowledge of Learners' Difficulties in Learning Quadratic Equations

DIFFICULTIES	**RESPONSE OF *TALANDILA***
RELATIONSHIP	Learners have difficulties in understanding the relationship between completing the square method and the formula method.
UNDERSTANDING	Learners have problems in solving quadratic equations, which have a negative value of b. Most of the learners do not consider the negative, which is already in the formula. The interview except shows the assertion ***Researcher****:* What learner misconceptions and difficulties do you anticipate before you teach the topic of quadratic equations? ***Teacher****:* In most cases, the pupils have difficulties in the signs where you say plus or minus. When you come to the quadratic equation you say *x* is equal to negative *b*. If the quadratic equation has a negative value of *b*. most of the pupils do not capture the negative value of *b* which is there in the formula.
CONCEPTS	Quadratic equations involving multi-mathematical operations e.g. $\pm$, square root sign, and division sign (Pre-lesson interview).
INTEGERS	The topic of integers is not properly presented at the junior level and learners carry on with the misconceptions up to further mathematics (Post-lesson interview)
PROBLEM-SOLVING USING COMPLETING THE SQUARE METHOD	Lack of proper presentation of the concept and concentration of learners getting the right answer only (Pre-lesson interview)

4.6.6Titus' Knowledge of Learners' Difficulties in Learning Quadratic Equations

In this section, the difficulties of learners were presented with examples of responses. Teachers' knowledge of learners' difficulties while learning quadratic equations was evaluated with the help of pre-lesson interview, lesson observation, and questionnaire. The findings of the items on the questionnaire was done earlier. The findings in this section on ***Titus'*** knowledge of learners' difficulties in learning Quadratic Equations are frompre-lesson interview and lesson observation.During the pre-lesson interview, ***Titus*** explained that:

> *"learners had difficulties in finding the solutions of quadratic equations,which have recurring digits as the learners like rounding off on the way thereby getting a wrong answer. He further explained that learners had difficulties in solving quadratic equations using completing the square method especially when the coefficient of the leading power is not a unit. Learners had problems with the operations when using the formula method such as, if the value of b has a negative, the learners usually forget to multiply with the other negative value of b".*

It was observed during the lesson that ***Titus*** took time to explain the correct presentation of the quadratic formula. He anticipated that many learners had difficulties in writing the correct form of the formula.

The findings of ***Titus'*** knowledge of learners' difficulties in learning quadratic equations is shown in Table 4.17 below.

Table 4.17: Titus' Knowledge of Learners' Difficulties in Learning Quadratic Equations

DIFFICULTIES	TITUS' RESPONSE
RELATIONSHIP	Learners have difficulties in understanding the relationship between completing the square method and the formula method.
UNDERSTANDING	Learners have difficulties when the quadratic equations have recurring digits, learners tend to round off on the way, thereby getting wrong answers.
CONCEPTS	Learners have difficulties on the signs, especially when using the formula, they usually forget the signs which is on 'b'
SOURCES INTEGERS	The topic of integers is not properlypresented at the junior level. Therefore, learners carry on with the misconceptions

	even further in other topics in mathematics (Post-lesson interview)
PROBLEM-SOLVING USING COMPLETING THE SQUARE METHOD	Lack of proper presentation of the concept and concentration of learners getting the right answer only (Pre-lesson interview)

Question 7 on the questionnaire allowed the participantsto identify typical errors that pupils make in the process of learning and using the quadratic formula.It also looked at what strategies they would use to help the pupils improve their comprehension. Table 4.18 below shows responses concerning the typical errors, which learners commit as they solve quadratic equations using the formula method.

Table 4.18: Errors Committed by Pupils as they Use the Quadratic Formula

Error category	**Name of participant**		
	Faith	***Talandila***	***Titus***
Computational Errors	✓	✓	✓
Not writing the equation in standard form	✓		✓
Dropping of the negative *b* sign in the formula	✓	✓	✓
Dealing with the $\pm$ symbol		✓	
Dividing by *2a*			✓

It is evident from Table 4.18, that all the three participants responded that learners experienced computational errors. This could be as a result of multiple operations in the quadratic formula such as the use of square root and new symbol ($\pm$), integrated in one formula.This is not surprising that the quadratic formula could cause all sorts of errors when it is being used by the learners.

One of the computational mistakes and difficulties arise when learners drop the negative sign, which is on (*b*), especially when the value of *b* in the quadratic formula has a negative itself. The other difficulty, which learners experience is on the division of *2a* when they just consider numerator to be $b^2 - 4ac$. Another common mistake, which the learners experience is when the quadratic equation is not written in standard form as they may find no matching digits in the quadratic equation and the letters in the quadratic formula.

Question 10 on the questionnaire was an extract from the Examinations Council of Zambia (Examiners Report, 2011) in which a candidate solved the quadratic equation:

$(2x-)(3x-2)=3$ as shown below:

$(2x-1)(3x-2=3$

$$2x-1=3 or 3x-2=3 \quad x=2 or x=1\frac{2}{3}$$
$$2x=4 or 3x=5$$
$$x=\frac{4}{2} or x=\frac{5}{3}$$

The participants were asked to explain what error the candidatemade when s/he solved the equation that way.Secondly, they were asked to explain the possible strategies that they would use to overcome the identified misconceptions or errors. The following responses were provided as shown in Table 4.19 below to show what error the candidate was committing when solving the quadratic equation.

Table 4.19: Error Which Candidate Committed

NAME OF PARTICIPANT	WHAT ERROR WAS THE CANDIDATE COMMITING WHEN SHE SOLVED THE EQUATION THAT WAY
Faith	The candidate was moving the constant to the right hand side so that it would become easier to factor the terms on the right hand side
Talandila	The candidate forgot to equate the equation to zero or s/he thought that the equation was already factorised
Titus	The candidate was trying to overgeneralise the zero product theorem as though the property was applicable even when the right hand side was not equal to zero or that s/he was misapplying the strategy for solving the quadratic equation $ax^2 + bx = 0$

PARTICIPANT	MISCONCEPTIONS IN SOLVING QUADRATIC EQUATION $(2X-1)(3X-2)=0$	STRATEGIES TO OVERCOME THE MISCONCEPTIONS
Faith	(1) That quadratic equations are difficuly (2) Solving quadratic equations without rearranging	(1) Solve quadratic equations using appropriate methods (2) The methods used should be clearly understood so that application is not a problem
Talandila	(1) Not expressing the quadratic equation in the form: $ax^2+bx+c=0$ (2) Not expanding the equation	(1) Teach on how to express any quadratic (2) equation in the form of: $ax^2+bx+c=0$ (3) Teach on how to expand and group like terms together
Titus	(1) Lack of understanding of the product rule that *ab=0* either *a=0* or *b=0*	(1) Emphasise on the zero product rule that *ab=0* only if *a=0* and *b=0* and that the quadratic equation has to be in the form of *f(x) =0*

Table 4.20 shows the observation schedule for knowledge of learner conceptions.

Table 4.20: Observation Schedule for Knowledge ofLearner Conceptions

Name of participant	Addresses learners' misconceptions and errors	Displays expectations of learners' possible difficulties	Awareness of instruments to measure pupils' learning	Allowing learners to use variety of methods	Ability to assist learners with difficulties
Faith			✓		✓
Talandila	✓	✓	✓		✓
Titus	✓	✓	✓	✓	✓

It can be noted from Table 4.20 above that all the participants were aware of the instruments used to measure pupils learning outcomes such as oral questions, written in-class exercises and home work. Furthermore, all the participants showed the ability to assist learners with difficulties. This was evident when the participants were going round marking the work. In some instances, the teachers themselves would actually help by way of solving the perceived difficulty questions with the learners on the chalkboard. Teacher ***Talandila*** and ***Titus*** also took some time to address learners' misconceptions and errors by solving problems themselves on the chalkboard. ***Talandila,*** for example, reminded the learners that the question: Solve $100x^2 = 16$ was a quadratic equation, which could best be solved using the difference of two squares. Only ***Titus*** was able to allow learners to solve one quadratic equation using a variety of methods. Table 4.21 shows the thematic representations for the knowledge of learner conceptions.

Table 4.21: Knowledge of Learner Conceptions Theme, Codes and Category

<table>
<tr><th>Code</th><th>Category</th><th>Theme</th></tr>
<tr><td>Addressing learners' misconceptions and errors</td><td rowspan="3">Learner difficulties and misconceptions</td><td rowspan="5">Knowledge of learner conceptions</td></tr>
<tr><td>Displays expectations of learners' possible difficulties</td></tr>
<tr><td>Ability to assist learners with difficulties</td></tr>
<tr><td>Awareness of instruments to measure pupils' learning</td><td rowspan="2">Assessment</td></tr>
<tr><td>Allowing learners to use a variety of methods</td></tr>
</table>

4.7 Chapter Summary

The chapter presented the findings of the study. The findings on mathematical knowledge for teaching quadratic equations were discussed in relation to the three categories: subject matter knowledge, pedagogical knowledge, and knowledge of learner conceptions. Further, cross analyses of data were done.

CHAPTER 5

DISCUSSION OF RESULTS

5.0 Overview of the Chapter

The previous chapter was an analysis of the data. This chapter is the summary of the study, discussion of the findings, recommendations, and suggestions for future research.

5.1 Summary of the Study

The study was designed to address and contribute to an emerging component of teacher knowledge in mathematics education. The main purpose of the study was to investigate the teaching and learning of quadratic equations in relation to mathematical knowledge for teaching.

A qualitative study design was used in which three participants were purposively chosen according to their experiences. These were those with less than three years, between five and ten years, and over ten years. Questionnaire, pre-lesson interview, lesson observation and post-lesson interviews were conducted to carry out the study. The researcher made three observations on each of the three participants. Finally, the questionnaire was also administered to the three participants.

The following research questions guided the study:

(1) What state of teachers' subject matter knowledge for teaching quadratic equations at Grade 11 level,did the three secondary school mathematics teachers possess?

(2) What strategies did the three secondary school mathematics teachers use in teaching quadratic equations at Grade 11level?

(3) How did the three secondary school mathematics teachers address pupils' errors and misconceptions related to quadratic equations at Grade 11 level?

For the purpose of this study, the theoretical assumptions have been shaped by the works of Scheiner (2015).This is where he called for a broader perspective of mathematical knowledge for teaching to include epistemology, cognitive, and didactical perspective. Epistemological dimension of mathematical knowledge for teaching represents subject matter knowledge while cognitive dimension represents knowledge of learner conceptions.Didactical dimension represents knowledge of the pedagogical or instructional form. In this study, mathematical knowledge for teaching is defined as consisting of knowledge of subject matter, knowledge of pedagogy and knowledge of learner conceptions.

5.2 Discussion of the Findings

5.2.1 Presentation of Findings

Data was collected through lesson observations, interviews, and the questionnaire. As discussed in the previous chapter, this was important as mathematical knowledge would easily be observed when teachers were in action. The researcher managed to observe a total of nine lessons. Each participant was observed three times. As explained in the previous chapter, the three themes: subject matter knowledge, knowledge of pedagogy and knowledge of learner conceptions were regarded to be critical in understanding mathematical knowledge for teaching quadratic equations. The following is a discussion on the findings according to the research questions.

Research Question 1: What state of teachers' subject matter knowledge for teaching quadratic equations at grade 11did the three secondary school mathematics teachers possess?

Knowledge of subject matter is an important component of mathematical knowledge for teaching. A deep understanding of mathematical concepts, facts and algorithms is important for effective teaching to occur (Borko and Putnam, 1996). In trying to answer research question 1, cross analyses of data from questionnaires, interviews and class observations were considered. The following themes of the subject matter knowledge were identified: knowledge of facts and procedures, prerequisite knowledge and skills necessary for teaching quadratic equations, and applicability of a particular method of solving quadratic equations.

5.2.1.1 Knowledge of facts and procedures of solving quadratic equations

Based on the findings from the study, all the three participants had adequate knowledge of quadratic equations. As noted in the previous chapter, all the participants were able to solve the quadratic equations using all the methods. The findings revealed that the three secondary school mathematics teachers had theirsubject matter knowledge consisting of memorised facts and procedures. This was visible when the participants were teaching solving quadratic equations using the formula method. Although all the participants taught the derivation process of the formula using completing the square method, only ***Titus*** emphasised the need for the learners to understand the derivation well. Both ***Faith*** and ***Talandila*** emphasised the desire for the learners to master the formula. This confirms the views of the other two scholars Even and Tirosh (1995) who referred to such knowledge as “knowing that’ and not knowing why”.

The participants also showed inadequacies in the subject matter knowledge when they were asked to explain why the quadratic equation: $x^2 + 4x + 6 = 0$ was not factorable. All the participants responded that it was not factorable. It is not wrong to say that the quadratic equation is not factorable because it has no simple factors. However, a teacher with strong mathematical knowledge should have emphasised that the quadratic equation is not factorable when all the coefficients are required to be integers. Nonetheless, in later algebra, quadratic equation will be factorable when the coefficients are not limited to the set of integers. This simply showed that teachers’ subject matter knowledge was procedural and the teachers were not able to explain the reasoning behind the rules or basic procedures. The procedural nature of their subject knowledge was echoed in their ways of teaching mathematics as they perceived the teaching of mathematics as telling of procedures.

The participants’ mathematical knowledge was similar to what Skemp (1978) referred to as instrumental understanding or rules without reasons as opposed to relational understanding, which has to do with rules and reasons. The findings of this study are consistent with the previous studies conducted by Ball (1990a) on preservice elementary teachers on division with fractions, which found that teachers lack conceptual understanding of mathematical ideas and procedures and that their mathematical knowledge was primarily limited to mathematical rules. She further noted that teachers were unable to make connections between mathematical ideas. The only difference between Ball’s (1990a) and the current study was that Ball’s study was a large scale study while this was primarily a small-scale qualitative study. The findings of this study support that of Vaiyavutjamai and Clements (2006), which posed that traditional methods of teaching quadratic equations promote rote learning. For teachers to be efficient in the teaching of quadratic equations, they must combine Skemp’s instrumental and relational understanding of the subject matter. To justify the relationship between procedural and conceptual understanding, this study is in strong agreement with Star's (2007) contention that the learning of procedures must be connected with conceptual understanding in order tofosterthe comprehension of concepts.

The next section discusses the findings of the second theme, which was identified under the subject matter knowledge.

5.2.1.2 Prerequisite knowledge and skills necessary for teaching quadratic equations.

Prerequisite knowledge is an important aspect in teaching mathematics. A teacher should possess such knowledge to make relationships between mathematical concepts. The teachers' abilities to make sound connections between mathematical concepts and explaining the reasoning on mathematical concepts was highly determined on the conceptual knowledge of mathematics. For instance, when the participants were asked which method of solving quadratic equations between the formula and completing the square they would introduce first, all the participants indicated that they would first introduce completing the square method before introducing the formula method.They gave a similar explanation that the formula method was derived by completing the square. Question 5 of the questionnaire also tried to find out how they would introduce the formula method of solving quadratic equations. They all pointed to the fact that they would use the completing the square method to derive the formula.

Question 8 from the questionnaire asked the participants about the pre-requisite knowledge, which pupils needed before being taught solving quadratic equations using factorisation method. The participants' responses were varied. All the options given were necessary pre-requisite knowledge needed for learners before teaching solving quadratic equations using factorisation method. ***Faith,*** for instance, did not realise that solving the linear equation of the form: $ax+b=0$ was a pre-requisite knowledge for teaching factorisation method. This simply meant that she could have thought that solving linear equations and solving quadratic equations were not directly related or that the two linear equations emerged and each needed to be solved separately when the quadratic equation wasfactored into the form : $(ax+b)(cx+d)=0$.

The findings of this study validate the prior research conducted by Ball (1990a) who posit that teachers' mathematical knowledge had a tendency to rely on their previous knowledge about the content. Although Ball (1990a) contended that mathematics teachers were unable to make connections between mathematical ideas, this study found that teachers were able to make sound connections with other mathematical ideas as all the participants were able to show the relationship that existed between the quadratic formula and the completing the square method. Furthermore, the participants always referred to the previous knowledge in the introduction before presenting the lesson of the day.

Below is adiscussion on the applicability of a particular method of solving quadratic equations as another theme under the subject matter knowledge.

5.2.1.3 Applicability of using a particular method of solving quadratic equation

The participants were asked to show proficiency in the subject matter area of quadratic equations when they were required to show the applicability of using a particular method of solving a quadratic equation. It was also meant to find out if the participants were able to make meaningful connections between mathematical concepts. This question was meant to find out if the participants were aware of when exactly to use a particular method knowing that there are multiple methods of solving quadratic equations. ***Titus*** and ***Talandila*** gave the right response during the interview by saying that all the methods were applicable to solving all forms of quadratic equations except factorisation method. On the other hand, ***Faith*** responded that only the formula method was applicable to solving all forms of quadratic equations while the rest were not. This may be attributed to the professional qualifications of the participants.Because both ***Titus*** and ***Talandila*** were holders of the first degree, they could have sound and adequate subject matter knowledge pertaining applicability of using a particular method of solving quadratic equations. This can also, be attributed to the teaching experience. For instance, ***Titus*** had more than twenty years of teaching experience. The study found that both experience and professional experience were necessary to the formulation of meaningful mathematical connections. The findings of this study were consistence with Leinhardt (1991) who found that experienced teachers demonstrated well-organised scripts and routines while novice teachers on the other hand did not.

In summary, the subject matter knowledge for teaching quadratic equations was mostly procedural and incoherent among the three respondents. The validity of the participants' explanations on how certain procedures or rules worked were highly dependent on their subject matter knowledge.

The next subheading looks at the findings for research question 2.

Research Question 2: What strategies did the three secondary school mathematics teachers use in teaching quadratic equations at Grade 11level?

This research question assessed teachers' knowledge of pedagogy, which included planning instructions, teaching strategies, and assessment (Borko and Putnam, 1996). In trying to answer the question, the researcher used interview schedule and observation. The researcher, as shown in the previous chapter, identified two indicators namely: choice of examples and teaching strategies and representation of work.

5.2.1.4 Choice of examples

The inclusion of examples is a common feature, which teachers of mathematics use as they teach mathematics. The study attempted to establish how well the teachers used the inclusion of examples in their lessons. It was found that, out of the three participants, only ***Titus*** used the real-life examples as he taught quadratic equations.***Faith*** and ***Talandila*** used ordinary examples. The decision of which example to use in order to teach a particular lesson revealed that the participants did not pay attention to how the examples would facilitate pupils' understanding, quadratic equation. They looked at their surface features such as the number of steps involved in solving the quadratic equation.

The findings of this study are generally consistent with the findings of other studies which stressed the fact that teachers lack knowledge of instructional strategies and of mathematical representations (Ball, 1990a; Grossman, 1990). It is with this background that the study found the need for pupils be given opportunities to make sense of the subject matter through use of real-life examples rather than merely memorising rules and procedures.

The next section looks at the teaching strategies and representation of work.

5.2.1.5 Teaching strategies and representations of work

This particular component of mathematical knowledge for teaching focused on how teachers were able to plan and teach lessons using a variety of teaching strategies. It also looked at how they engaged learners through questions and other forms of assessment such as homework and class exercises. The study found that mathematics teacher's knowledge of teaching strategies was highly dependent on their subject matter knowledge. Teachers, in most cases, explained the concepts on how to carry out procedures or apply rules according to the algorithms in order to solve the quadratic equations. Furthermore, the teachers were interested in the accuracy of the answer and the rules leading to the answer.

However, ***Titus*** seemed to have a different approach in that he emphasised on the need for learners to have conceptual understanding of the lesson.

This was evident when Titus said: "pupils need to be knowledgeable about the importance of the learners knowing how to derive the quadratic formula".

This as earlier explained in the previous chapter could be as a result of the fact that ***Titus*** was moresenior in terms of service and at the same time, he was the head of department.

The results of this study isin consistentwith that of Star (2007) who argued that conceptual knowledge should not only comprise known (procedures) but also why the procedures should be known. The study is also consistent with what Shulman (1986) indicated that teachers need not only understand that something is so, but also understand why it is so. The findings of the study confirm the views of other scholars who posit that mathematics teachers lack knowledge of instructional strategies and representations (Ball, 1990; Grossman, 1990).

Finally, the findings complemented the results of prior research by Ball (1991) who pointed out that when mathematics teachers' conceptual knowledge was strong; they put more emphasis on explaining why procedures work and how mathematics concepts are related. The next subheading discusses research question 3, which looked at the strategies that teachers used in identifying possible learner difficulties and misconceptions.

Research Question 3: How did the three secondary school mathematics teachers address pupils' errors and misconceptions related to quadratic equations at Grade 11 level?

Knowledge of learner conceptions has to do with knowing the characteristics of a group of learners and planning instruction according to the needs of learners (Fennema and Franke, 1992). The researcher assessed the teachers' knowledge of learner conceptions by looking at the ability of teachers' to anticipate learners' difficulties and misconceptions about the subject matter and also the ability to explain how to address them effectively. The knowledge of learner conceptions is two-fold.These are knowledge of the subject and the effectiveness of the pedagogical or teaching strategies.

Knowledge of learner conceptions demands that the teacher knows the subject matter of the subject area as well as imparting effective teaching strategies. The key indicators of the knowledge of learner conceptions as presented in the previous chapter include identification of possible learner difficulties and misconceptions, assistance given to learners who experience difficulties and allowing learners to use a variety of methods as the teacher teaches quadratic equations. The following were the indicators of teachers' knowledge of learner conceptions.

5.2.1.6 Teachers' ability in identification of possible learner difficulties and misconceptions

Teachers need not only to be able to help learnerswhen they experience difficulties and misconceptions but should also be able to plan effectively before they teach to either avoid or deliberately bring into light common learner misconceptions and errors. In this case, teachers need to be able to identify the source of learner difficulties and errors in order to be able to correct them with ease.

Through lesson observations, the researcher identified a variety of possible learner difficulties and misconceptions in solving quadratic equations, which includedcomputational problems, problems with the square root, the negative sign and the $\pm$ sign. The frequency of the possible difficulties and misconceptions identified by the teachers for the learners could be used as substitute or proxy for the actual frequency of the difficulties and misconceptions made by the learners when learning.

The researcher observed that such possible misconceptions committed by the learners were related to the learners' insufficient knowledge and skills in arithmetic. The teachers teaching strategies revealed that although the teachers were able to identify learner possible difficulties and misconceptions, theywere unable to recognise the gap in solving quadratic equations as they focused on the procedural steps. In other words, they attributed learners' misconceptions to their inability to remember and correctly perform procedures. Teachers emphasised on improving learners' procedural skills. The study further found that the teachers' perception of mathematics as well as their subject matter knowledge had an impact on how they would present a particular lesson on quadratic equation. It was also found that when the learners gave an incorrect answer, the teachers tended to explain the correct answer rather than explaining how they arrived at that answer. This means that although the teachers were able to identify misconceptions and errors, they did not do much identify the source of misconceptions and errors.

The findings of the study are consistent with other scholars such as Even and Tirosh (1995) who found that teachers were reluctant to make an attempt to understand the source of learners' responses. Thus, the teachers missed the opportunity to detect the gaps in learners' mathematical understanding and were unable to help them construct their mathematical knowledge. The teachers were unable to notice the conceptual knowledge that learners were lacking that made them fail to solve a given quadratic equation. They thought that they would eliminate learners' possible difficulties and misconceptions by explaining how to solve the problem procedurally. The next subheading looks at teachers'ability to assist learners who experienced difficulties in the learning of quadratic equations.

5.2.1.7 Assistance rendered to learners who experienced difficulties

As indicated above, it was observed that teachers identified that learners experienced a number of difficulties as they were learning quadratic equations and so, were able to assist. It was observed that the teachers assisted the learners during the introduction as a way of recap from what they learnt in the previous lesson. However, the assistance was restricted to explaining the correct answer rather than how they found the answer. In other words, the teacher tried to labour to explain the correct way of solving the question rather than trying to know the source of the misconception thereby missed an opportunity to detect the gap in the learners' mathematical understanding to construct their mathematical knowledge. This was evident for both ***Faith*** and ***Talandila*** who merely went through the difficulty question during the introduction of the next lesson. What mattered the most for both of them was the procedural knowledge and not conceptual knowledge. It was also observed that from the responses of the questionnaire there werealot of shortcomings on the part of the participants in addressing the concept, and on assisting the learners who had misconceptions and difficulties in the learning of quadratic equations.

When the participants were asked to explain how they would assist the student who solved the quadratic equation:$(2x-1)(3x-2)=3$ wrongly, both ***Faith*** and ***Talandila*** explained that the student did not identify that the question was not a quadratic equation, and that they would assist them by revisiting the factorisation method of solving quadratic equations. The responses were not comprehensive and did not address how this would be done. However, it was only ***Titus*** who was able to explain how he would be able to handle it. He indicated that this would be by explaining the concept in relation to zero product property and that the quadratic equation was to be in the form $f(x) = 0$. ***Titus'*** response was more comprehensive and complete than that of ***Faith*** and ***Talandila***. Partly this could be attributed to the experience he had compared to the other two participants.

The findings of this study revealed that the power to detect and eliminate learners' misconceptions is not dependent on the subject matter knowledge. The study findings are not consistent with prior research conducted by Even and Tirosh (1995) who noted that the teachers' ability to detect and eliminate learners' misconceptions in a particular topic is related to their knowledge about those topics. The findings of this study are in consistence with what other scholars such as Ball, Thames, and Phelps (2008) who noted that specialised content knowledge (SPK) of teachers should also include knowledge of content and students (KCS), which should include the anticipation of learners' difficulties, common errors and misconceptions that learners may face in a specific topic. This study attests that, if a teacher does not possess KCS, then he or she may not be able to address or eliminate the possible errors, which may hinder the proper understanding of certain concepts.

The next subheading discusses another important aspect of knowledge of learner conceptions and that is whether teachers allow the learners to use a variety of methods as they teach quadratic equations.

5.2.1.8 Teachers' ability to allow learners use a variety of methods for solving quadratic equations

Another theme which emerged under the knowledge for learner conceptions was the teachers' ability to allow learners to use a variety of methods in solving quadratic equations. This was arrived at because there are a number of methods for solving quadratic equations such as factorisation method, formula method, completing the square method and graphical method. When the participants were asked if they allowed learners to use a variety of methods when teaching quadratic equations, they all responded that they did. However, from the classroom observation, it was notedthat Titus and Talandila allowed learners to use other methods of solving quadratic equations but restricted the solving of quadratic equations to what they were presenting in that particular lesson. For instance, if they were teaching on solving quadratic equations using factorisation method, the learners were compelled to solve the quadratic equation using that method only at that time. It was also observed that the questioning was limited to closed questions as opposed to open questions. The advantage of open type of questions over closed type of questions is that the former requires the learners to express the ideas unlike the latter, which requires the learners to give one-word answer or solve a question using a prescribed algorithm or formula. The findings of this study support the views of other scholars Ball and Bass (2003) who noted that teachers of mathematics must ask or pose mathematically sound questions and problems to probe, assess and promote student learning with a view of establishing and managing mathematical discourse. The next subheading discusses the major findings of the study.

5.2.1.9 Major findings of the study

Based on the data collected from the study, there were four major findings related the mathematical knowledge for teaching (MKT) quadratic equations. The major findings are summarised as follows:

(1) The subject matter knowledge of the participants was adequate although limited to procedural knowledge. The participants lacked conceptual understanding as they emphasised more on the masterly of the rules and procedures with a view of finding the correct answer.

(2) It was also noted from the study, that the teachers' procedural nature of the subject matter influenced the teaching strategies in which they perceived teaching as knowing to do mathematics or merely following the set of rules and procedures (Ball, 1990).

The repertoire or the range of teaching skills was limited to procedural undertaking as opposed to conceptual understanding. It was observed that only ***Titus*** was able to use real-life examples, while the other two were not able to use real-life examples.Therefore,the teachers failed to help pupils develop a conceptual understanding of mathematical concepts that they were trying to exemplify.

(3) It was further noted that the lack of subject matter knowledge in some aspects of quadratic equations hindered the three participants'ability to generate appropriate strategies and necessary representations to eliminate pupils' difficulties and misconceptions as they emphasised on the rules and procedures to assist pupils solve problems correctly.

(4) The teachers of mathematics were unaware of the sources of difficulties in pupils' mathematical reasoning and assumed that pupils had weaknesses in their procedural knowledge and not conceptual knowledge.

5.3 Contribution to the Body of Knowledge

The study contributes to an emerging study of MKT by consolidating the importance of the three components identified in the study namely: subject matter knowledge, pedagogical knowledge and the knowledge of learner conceptions in the teaching of quadratic equations.The ability of the three participants to make appropriate connections among mathematical concepts to generate different solutions and representations for problems, to address learner difficulties and misconceptions effectively, and choose appropriate teaching strategies, were heavily dependent on their subject matter knowledge.

In observing the teachers teach in the classroom, all three participants followed the same pattern of teaching the quadratic equation.This was by beginning with solving using the quadratic formula followed by completing the square method and, finally, the formula method. It is clear that both completing the square method and the formula method were generic in nature as they could be used to solve all forms of quadratic equations, while the factorisation method was not generic in nature as it was limited to solving quadratic equations which had factors. It was observed that the completing the square method and the formula method were less appreciated as the learners appeared to be more comfortable with factorisation method which usually is the first method to be taught to the learners. This study recommends that mathematics textbook writers should introduce the concept of solving quadratic equations using the completing the square followed by formula method and then factorisation method. This will help learners to appreciate the completing the square and the formula method, which are generic in nature.

5.4 Recommendations

The recommendations in this study are two-fold, for practice and for further research.

5.4.1 Practice

The findings revealed that teachers of mathematics under study, had sufficient subject matter, which was limited to procedural knowledge. The study strongly recommends that the teachers should incorporate both procedural and conceptual knowledge as they teach, so that learners have a better understanding of the subject matter.

It was observed that teachers had difficulties in identifying the sources of learner difficulties and misconceptions. The study, therefore, recommends that in order to improve teachers' knowledge of learner conceptions on the teaching of quadratic equations, the teachers should use the learners' responses as a source of help of revealing the learners' thoughts on the concepts of quadratic equations. The teachers need to pose questions that require learners to explain their ideas about quadratic equations.

Furthermore, teachers' preparation of the lesson should include or reflect anticipated or possible learner difficulties and misconceptions. This is with a view to addressing them as they teach. Finally, teachers of mathematics should be exposed to workshops and continuous professional development (CPD) meetings not only to deal with subject matter knowledge but also to focus on knowledge of learner conceptions and pedagogy that could be used to effectively teach quadratic equations.

5.4.2 Further Research

In this study, it was not easy to draw a conclusion about the level of mathematical knowledge for teaching quadratic equations because it involved limited number of teachers. It would be prudent that another research be conducted on a similar study involving a bigger sample of participants to include teachers' knowledge in other topics such as integers, functions, algebraic expressions and trigonometry.

Although the study was undertaken with experienced teachers, there seems to be challenges that the teachers are facing in terms of both CK and PCK. The blame is put on the pre-service education for not concentrating on these aspects of knowledge during teacher preparation. Additionally, experience was also regarded as a rich source of the teachers' knowledge base (Shulman, 1986). This shows that there is need to explore why the teachers are unable to acquire enough CK and PCK in teaching quadratic equations.

The study can be repeated with pre-service teachers to find out if there are some aspects of knowledge that were not revealed by the in-service teachers. Undertaking the study with pre-service teachers might also reveal categories of teacher knowledge, which are acquired from pre-service education and those which are acquired through experience.

The findings, which came out prominently in this study were that the subject matter knowledge of mathematics teachers determined their teaching strategies. Future research could also be conducted on the relationship of mathematical knowledge for teaching and learner achievement.

Further, the findings of the study also revealed that teachers of mathematics subject matter knowledge were more procedural than conceptual. This was observed through an emphasis of allowing learners to follow the procedures of solving a problem leading to a correct answer. This shows that teachers of mathematics, teach it based on their understanding and belief of what mathematics is all about and how it should be taught. It, therefore, recommendedthat a study to look at the correlation between mathematical knowledge for teaching and the teachers' mathematical beliefs be conducted.

5.5 Conclusion

The major aim of teaching is to enhance learners' understanding of the subject matter. It is against this background that teachers need to be equipped with various knowledge and skills including mathematical knowledge for teaching in order for them to establish and maintain that goal. It is, therefore, prudent that teacher education programmes such as continuous professional programmes (CPD) should provide opportunities for mathematics teachers to develop their knowledge and skills for effective teaching.

Teachers of mathematics also need to be familiar with the conceptual foundations of a particular subject-matter and also be able to know how learners learn it. They should also know the strategies which facilitate pupils' learning and comprehension of the subject matter, and also be able to prepare instructional materials that enables them to effectively delivery of the lessons. The teachers of mathematics should also have opportunities to spend much time in a classroom setup as observers as well as practitioners to learn more about learners and teaching.

The results of the study revealed that the subject matter knowledge, which teachers possess consists of memorised rules, facts, and procedures. They lacked conceptual understandings of quadratic equations as observed in their arguments about the connections between mathematical ideas. They were unsure about the relationship between the given concepts.

Pedagogical knowledge can be understood as a mixture of various knowledge and skills that enable teachers to make subject knowledge understandable to the learners. To achieve a desirable learning outcome, teachers must create a conducive learning environment for the benefit of the learners. The findings of the study revealed that the teaching of mathematics was based on the teachers' decision about the examples that they would use when teaching a particular lesson. Teachers tended to tell learners how to carry out a particular procedure without justifying their meaning.

Teachers' knowledge of learner conceptions was closely linked with knowledge of subject matter and knowledge of pedagogy. The findings of the study revealed that teachers were unable to identify what conceptual knowledge learners were lacking insolving quadratic equations. The assumption was that learners did not know the rule that they needed to solve a particular problem. Therefore, teachers failed to eliminate learners' misconceptions and difficulties by explaining how to solve the mathematical problem procedurally.

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APPENDICES

Appendix 1: Questionnaire

I am Titus Luka Mbewe a student from the University of Zambia, pursuing a Doctor of Philosophy Degree in Mathematics Education in the Department of Mathematics and Science Education. The topic of my study is "Secondary School Teachers Mathematical Knowledge for Teaching Quadratic Equations". The purpose of the study is to describe the teaching and learning process of quadratic equations in relation to mathematical knowledge for teaching. The questionnaire is strictly for academic purposes and the responses will be treated as confidential. Thank you in advance for accepting to be part of this study.

1 Sex

2 How long have you taught mathematics at secondary school?

Less than five years

Between 5-10 years

Between 10-20 years

More than 20 years

3 How long have you taught mathematics at senior secondary (Grades 10 to 12)

level?

Less than five years

Between 5-10 years

Between 10-20 years

More than 20 years

4 (a) If you were to use the quadratic formula as a method of solving quadratic equations, which of the following approach would you use?Demonstrate how the formula is derived and expect pupils to understand and remember each step.

Demonstrate how the formula is derived and expect pupils to remember the main

ideas only.

Explain the main ideas behind the formula and expect pupils to remember and use

the formula only.

Introduce and use the formula directly and explain how the formula is derived in

later chapters.

Introduce and use the formula directly without giving any instruction.

Other explain

4(b) Explain why you would use the approach you selected above?

....
....

5Explain the typical errors the pupils make in the process of learning and using

the quadratic formula. For each, explain the error and the typical strategy of

helping the pupils improve their comprehension.

Possible error or misconceptions strategy for improving comprehension

(a)

(b)

(c)

6 In order to carry out the factorisation method of solving quadratic equation, what knowledge of algebra and skills are essential for pupils to fully understand the factorisation method? Tick all that apply

Combining like terms

The zero-factor theorem

Solving linear equations of the form $ax + b = 0$

The distributive law

Expressing the equation in standard form

Multiplying two binomials

7 A teacher teaching quadratic equation says that $x^2 + 4x + 6 = 0$ cannot be solved using factorisation method because the trinomial $x^2 + 4x + 6$ is “not factorable” Do you agree?

If you do how would you explain to the pupils what it exactly means that the trinomial is “not factorable” If no, why?

8 A Grade 12 pupil in a certain school solved the quadratic $(2x- 1) (3x – 2) = 3$ asillustrated below:

$(2x – 1) (3x – 2) = 3$

$(2x – 1) (3x – 2) = 3$

$2x – 1= 3$ or $3x – 2 = 3$

$2x = 4$ or $3x = 5$

$x = \frac{4}{2}$ or $x = \frac{5}{3}$

$x = 2$ or $1\frac{2}{3}$

Source: Examinations Council of Zambia (Examiners report, 2011).

What error was the pupil thinking when s/he decided to solve the equation that way?

How do you assist the student to overcome the misconception?

Appendix 2: Pre-Lesson Interview Schedule

I am Titus Luka Mbewe a student from the University of Zambia (UNZA), pursuing a Doctor of Philosophy Degree in Mathematics Education in the Department of Mathematics and Science Education. The topic of my study is "Secondary School Teachers Mathematical Knowledge for Teaching Quadratic Equations". The purpose of the study is to describe the teaching and learning of quadratic equations in relation to mathematical knowledge for teaching. I will ask a series of questions about solving quadratic equations.

Elements of MKT for the Study	**Questions related to the MKT**	**Response**
(a) Knowledge of the Subject matter	(1) What methods of solving quadratic equations do you know? (2) Are all the methods of solving quadratic equations applicable to all forms of solving quadratic equations? (3) If not, how do you guide the pupils about which method to use for a particular question? (4) Have you ever used geometrical tiles to solve quadratic equations?	

(b) Knowledge of pedagogical representations	(1) How do you introduce the topic of solving quadratic equations using-factorisation method, quadratic formula, completing the square method (2) Which method of solving quadratic equations do you start teaching between quadratic formula and completing the square method (2) Which of the two methods of solving quadratic equations; quadratic formula and completing the square, do you like when teaching quadratic equations. Give reasons. (3) In your selection of examples to be used in this lesson, how you included real-life examples.Give reasons for your answer.	
(c) Knowledge of learner conceptions	(1) Which learners' prior knowledge is regarded to be important before the above key concepts can successfully be taught to the learners?	

	(2) What possible learner misconceptions/ difficulties do you anticipate regarding the lesson? (3) How will you assist learners who experience difficulties with the lesson? (4) Have you prepared an assessment instrument to evaluate whether the purpose of the lesson has been achieved? (5) Do you allow learners to use a variety of methods when solving quadratic equations. (6) Have you learnt any new method of solving quadratic equations from the learners?	

Interview Question: Source Adapted from Chick, Baker and Pham (2006).

Appendix 3: Observation Schedule

Element to be observed	Evident when the teacher	Observed practice displaced
(a) Knowledge of the subject matter	(1) Exhibits deep and thorough understanding of identified aspects of quadratic equations. (2) Identifies critical mathematical components within the concept of quadratic equations. (3) Displays skills for solving problems in the area of quadratic equations.	
(b) Knowledge of pedagogical representations	(1) Uses appropriate activities in instruction. (2) Uses real-life examples and analysis in instruction (3) Utilises different instructional strategies in presentation.	

(c) Knowledge of learners conceptions.	(1) Addresses learners' misconceptions. (2) Displays expectations of possible difficulties learners may face during learning and address such. (3) Discusses learners' ways of thinking about a concept. (4) Shows an awareness of the instruments to measure pupils learning and how to use this.	

Source: Adapted from Chick, Baker, and Pham (2006).

Appendix 4: Parental Consent Form

I agree to allow my child........................ to take part in a study entitled "Mathematical Teacher Knowledge and Instructional Practices: A Case of Quadratic Equations.", which is being conducted by Mr Titus Luka Mbewe from the University of Zambia (UNZA) under the Department of Mathematics and Science Education. The aim of the study is to find out the kinds of knowledge and experiences, which secondary school mathematics teachers possess. I do understand that I may not allow my child to take part in the study. My child can withdraw from the study at any time and at no penalty. The results of my child's participation will be confidential and will not be released to anyone without my prior consent, unless otherwise required by law. The researcher hopes to learn something that may help educate mathematics teachers so that they can help other children learn mathematics better in the future.

I understand the procedures under which the study will be conducted. I, therefore, agree to allow my child take part in the study.

....

Signature of the Parent/ Guardian Date Signature of the Researcher Date

Appendix 5: Participant Teacher Consent Form

I ... agree to participate in the study entitled "Mathematical Knowledge and Instructional Practices: A Case of Quadratic Equations", which is being conducted by Mr. Titus Luka Mbewe from the University of Zambia (UNZA) under the department of Mathematics and Science Education. The aim of the study is to find out the kinds of knowledge and experiences which secondary school teacherspossess as they teach quadratic equations. I understand that participation is entirely voluntary and I can withdraw my consent at any time without penalty. Further, no risk of psychological, social or physical harm is foreseen except for some rare stress when the researcher observes the lesson or during the interview session and that no future risks are foreseen.

The results of the participation will be confidential and will not be released in any form without prior consent, unless otherwise required by the law. My name and any details that identify me as a participant will be changed in any written reports to protect confidentiality.

I understand the procedures under which the study will be conducted. I, therefore, accept to participate in this study. I have been given a copy of this form.

....

Signature of the Researcher Date Signature of the participant Date

Appendix 6: Interview with Teacher Talandila Date

19th September, 2017

The except below shows the interview with teacher Talandila regarding the subject matter knowledge of quadratic equations.

1. *Researcher:* In which grade is the topic of quadratic equations taught?
2. *Teacher*: In Grade 8 just a bit, an introduction, then in Grade 11.
3. *Researcher:* What methods of solving quadratic equations do you know?
4. *Teacher:* I know of four methods
5. *Researcher:* Which are these?
6. *Teacher:*Factorisation method, completing the square method, formula
7. method and the graphical method
8. *Researcher:* Are all the methods applicable to solving all forms of quadratic equations?
9. *Teacher:* mmm... No, there are some equations that you may find difficulties when you use factorisation method when the quadratic equation has no Simple factors.
10. *Researcher:* Which methods in your view are applicable to solving all forms of quadratic equations?
11. *Teacher:* The formula method, it's the simplest actually.
12. *Researcher:* Any other methods which you think are applicable to all forms of quadratic equations?
13. *Teacher:*Completing the square method, even graphical method although, it is very involving and the accuracy may be compromised.
14. *Researcher:* How do you guide the pupils about which particular method to use?
15. *Teacher:* There is a way of testing whether the quadratic equation has simplefactors. I use the discriminant test : $b^2 - 4ac$. If the discriminant is not a perfect square it means that equation has no simple factors and you cannotuse factorisation method. You can either use completing the square methodor formula method. If the discriminant method is giving

negatives, then theequation has no real roots and secondary school we do not deal withcomplex numbers or with square roots of negative numbers.

16. *Researcher:* Have you ever used geometrical tiles to solve quadratic equations?
17. *Teacher:*mmm.... Geometricaltiles?
18. *Researcher:* Yes, geometrical tiles.
19. *Teacher:* Aaaa...No
20. *Researcher:* How would you introduce the lesson of solving quadraticequations using factorisation method?
21. *Teacher:* I would first remind the pupils of factorisation of quadratic expressions and linear equations, Then I will also remind them of the product rule $a \times b = 0$. Either a or b =0, implying that a or b represents the factors of the quadratic equations.
22. *Researcher:* How would you introduce the lesson of solving quadraticequations sing the quadratic formula?
23. *Teacher:* This is an order which you are supposed to follow as you teach quadratic equations. We begin with factorisation method, followed bycompleting the square which will help the pupils understand the formulamethod. So the pupils will find it easy to follow the formula method if they have done completing the square method.
24. *Researcher:* How would you really introduce the formula method? Would you just tell the pupils that this is the formula method or there is way you can do it?
25. *Teacher:* I will first start by deriving it using completing the square method and then tell the pupils that this is the formula method which we are going to be using.
26. *Researcher:* Which of the two methods completing the square method or formula method, do you prefer to teach your pupils?
27. *Teacher:* I would favour the quadratic formula. reason being that most of thepupils have difficulties in understanding completing the square method and sometimes they interchange the signs when using completing the square method thereby ending up getting wrong answers. Quadratic formula does not consume much time and it is faster and more accurate than completing the square method.
28. *Researcher:* When giving examples, do you include real life examples?
29. *Teacher:* Yes, I have.
30. *Researcher:* Can you give these examples?

31. *Teacher:* There are some questions which involve area of a piece of land andbecause area is squared, it can be used to find side of the piece of land.
32. *Researcher:* What learners pre knowledge do you regard to be important before teaching quadratic equations?
33. *Teacher:* Algebra, integers that is signs, squares and square roots and thedifference of two squares. Pupils should also understand linear equations that is finding the missing number. They can use the idea of linear equations to understand the quadratic equations except that in quadratics you are looking for two solutions. even factorisation and indices help pupils to understand quadratic equations.
34. *Researcher:* What learner misconceptions and difficulties do you anticipate before you teach the topic of quadratic equations?
35. *Teacher:* In most cases, the pupils have difficulties in the signs where you sayplus, or minus. When you come to the quadratic equation you say x is equal to negative$b$. If the quadratic equation has a negative value of b. most of the pupils do not capture the negative value of b which is there in the formula.
36. *Researcher:* How do you assist learners who experience difficulties in thetopic of quadratic equations?
37. *Teacher:* After giving pupils some work, I identify difficulties which theyhave and thereafter, have a makeup class.
38. *Researcher:* What assessment instruments do you use to assess if the purposeor objective of the lesson has been achieved?
39. *Teacher:* Normally, after teaching, I ask questions randomly; for example, *what have we learnt today?* Sometimes, I give tests or home works.Although home works are not good because pupils can copy from one other. But when you are asking oral tests you can learn whether pupils have understood or not.
40. *Researcher:* Do you allow learners to use a variety of methods when solvingquadratic equations?
41. *Teacher:* Yes, I do if the question does not specify which method to use, butif I specify, which method to use then the pupils must follow.
42. *Researcher:* Have you ever learnt any method of solving quadratic equationsfrom the pupils?

43. *Teacher:* Yes, I, have although the method which I learnt from a pupil could not work for other types of quadratic equations. Even that learner could notexplain nicely how that method of solving quadratic equations was working.
44. *Researcher:* Any other issues of interest you may wish to share with me pertaining the teaching and learning of quadratic equations?
45. *Teacher:* Most of the pupils find it difficult to understand quadraticequations because of the introduction itself. Teachers normally come in to class and say, "*today we are going to learn look at quadratic equations".* Without giving them the connection of why they should learn quadratic equations. Because if you learn why, you can learn toknow how to do it. So, there is need to prepare them. It is alsoimportant for them to know the prerequisite knowledge.
46. *Researcher:* Thank you so much.

Appendix 7: Interview with Teacher Faith

Date 21st September, 2017.

1. *Res:*Have you taught the topic of quadratic equations?
2. *Faith*: Yes, I have.
3. *Res:*In which grade do you teach the topic of quadratic equations?
4. *Faith*: We usually teach it in Grade 11 but if we are fast with workcoverage sometimes in Grade 10.
5. *Res:*What methods of solving quadratic equations do you know?
6. *Faith:* I know of,factorisation method, graphicalmethod, completing thesquare method, and in turn, we also use the formula method.
7. *Res*: Are all methods of solving quadratic equations applicable to all forms ofquadratic equations?
8. *Faith:* No....Not all methods of solving quadratic equations are applicable to all forms of quadratic equations. Factorisation, completing the square and graphical are not except for the formula method which is applicable to all.
9. *Res:* Do you know of any other method of solving quadratic equations apart from the ones you have listed above?
10. *Faith*: These are the four major methods I normally use. I have not yet discovered another method.
11. *Res:* How do you guide the pupils about which method to use, since not allmethods are applicable to all forms of quadratic equations?
12. *Faith*: We usually guide them, first to see the type of quadratic equation. Since by this time, they already have an idea of factorisation of quadraticexpressions, factorisation by grouping. Pupils will be able to know which quadratic equation can best be solved using factorisation method. When theyfail to find factors under factorisation method they will be encouraged to use quadratic formula since it is applicable to all forms of quadratic equations.
13. *Res*: In your view is completing the square method of solving quadraticequations applicable to solving all forms of quadratic equations?

14. *Faith*:Yes, it is because the same quadratic formula is derived from completing the square method. What is important is that pupils should understand all the stages and in that way they will not miss out.
15. *Res*: Have you ever geometrical tiles to solve quadratic equations?
16. *Faith*: Mmmm.... No
17. *Res*: How would you introduce the lesson of solving quadratic equations using factorisation method?
18. *Faith:* You explain in such a way that you have two products which are equal to zero. It is as good as finding the factors which should be equated to zero. Then you solve for the variables.
19. *Res:* How would you introduce the lesson of solving quadratic equations using the formula?
20. *Faith*: It is best explained when pupils know where the formula is coming from. So, it is important to teach completing the square method so that the pupils can understand. Then you can derive the formula from the standard quadratic formula, which is: $ax^2 + bx + c = 0$. You emphasise the fact that *a*should not be equal to zero. Encourage the pupils not to memorise the formula and in this way they will understand.
21. *Res*: How would you introduce the lesson of completing the square method?
22. *Faith*: You first see the type of the quadratic equation which you have. You emphasise the point that the constant (*a*) should always be equal to one. Ifthey have another number rather than one in the constant (*a*) theyshould divide that number throughout the terms by that number. Thenthey will be able to do the other steps that follow.
23. *Res*: Which method between quadratic formula and completing the square do you like when teaching?
24. *Faith*: I like using the quadratic equation.
25. *Res*: Do you have any reasons as to why you prefer teaching quadraticformula to completing the square method?
26. *Faith*: Yes, because I will not be stranded per se as long as I understand what to do
27. *Res*: When teaching quadratic equations, do you include real life examples?
28. *Faith:*Yes, we do. As we conclude the lesson, after teaching all the fourmethods that is: factorisation, completing the square, formula and graphical.We usually include real life examples.

29. *Res*: Give an example of a real life example which you may give under quadratic equations?
30. *Faith*: Mmm....not really
31. *Res*: What pre-knowledge of the pupils do you regard to be important before
32. teaching quadratic equations?
33. *Faith:* They should have the knowledge of coordinate geometry, inequationsand inequalities. This will help them plot the coordinates and find the necessary scale.
34. *Res*: Any other pre-knowledge which the learners need to know before they learn quadratic equations?
35. *Faith*: They should also have knowledge of ratio and proportion and scale drawing.
36. *Res*: What possible difficulties do you anticipate that learners may face as you teach quadratic equations?
37. *Faith*: Where they need to find the roots of $b^2 - 4ac$. If it is not a perfect square root, they will still need a calculator to find the square root.
38. *Res*: Any other difficulties or misconceptions you anticipate your learners tohave before you teach quadratic equations?
39. *Faith:* There is also an issue of understanding the quadratic formula properly. For example, in the formula there is negative (b) and if in the quadratic equation there is a negative, the pupils tend to miss it. They also have difficulties in finding the factors when teaching factorisation method.
40. *Res*: How about when teaching completing the square method, whatdifficulties or misconceptions do you anticipate when teaching this method?
41. *Faith*: When the coefficient is not 1 then the pupils find difficulties, especially those who are faster they overlook this.
42. *Res*: How do you assist learners who experience difficulties in quadraticequations?
43. *Faith*: I divide the pupils in small groups and give them questions so thatthey can assist each other.
44. *Res*: How do you assess if the objective of the lesson has been achieved?
45. *Faith*: Through oral questions. because if they are given homework they mayjust help one another and I may not get a genuine feedback.
46. *Res*: Do you allow learners to use a variety of methods when solving quadratic equations?
47. *Faith*:Yes, I do allow them especially if the question is open, but if comes

48. with a condition I don't.

49. *Faith*: Have you learnt any new method of solving quadratic equations form the learners?

50. *Faith*: I have learnt though you find that the same may not work forother quadratic equations.

51. *Res*: Any other information which you may wish to share with me regarding the teaching and learning of quadratic equations?

52. *Faith*: I have discovered that pupils usually have difficulties inquadratic equations if they did not learn well linear equations and inequalities. If they did not understand integers and algebra they mayalso find the topic of quadratic equations difficulty. They may also have difficulties in understanding completing the square method if they do not have adequate knowledge on squares and square roots.

53. *Res*: Thank you so much for your time.

Appendix 8: Pre-Lesson Interview with Teacher Titus

DATE 26th SEPTEMBER, 2017.

1. *Res*: How long have you taught mathematics at secondary school?

2. *Titus*: I have been teaching mathematics at secondary school for ten yearsnow.

3. *Res*: What grades have you taught at secondary school level mathematics?

4. *Titus:* All the grades.

5. *Titus*: You mean grades 8 to 12?

6. *Res*: Yes

7. *Titus*: Since you have taught mathematics at all the grade levels atsecondary school. Are you familiar with the topic quadratic equations?

8. *Res:* Yes, I am.

9. *Titus*: In which grade is the topic of quadratic equations taught?

10. *Res*: It is taught in Grade 11

11. *Titus*: What methods of solving quadratic equations do you know?

12. *Res*: Factorisation, completing the square method, graphical and theformula method

13. *Res*: Are all these methods applicable to solving all forms of solving quadratic equations?

14. *Titus*: Yes, they are applicable except for factorisation method, which cannot be used to solve quadratic equations which have solutions in decimal form.

15. *Res:* How best do you guide the pupils about which method to use?

16. *Titus*: There are two forms of quadratic equations. Those which arefactorisable and those which are not. We usually advise the pupils to first try to solve the quadratic equation using the factorisation method and if it cannot work, then they can use the formula or completing the square method. We usually advise the pupils to use graphical method if we have a graph paper.

17. *Res*: How about the graphical method. Is it applicable to all forms of quadratic equations?

18. *Titus*: Yes, although the solutions of such quadratic equations are estimated ones.

19. *Res*: Have you ever used geometrical tiles to solve quadratic equations?

20. *Titus*: Tiles....No I have not.

21. *Res*: How do you introduce the lesson of solving quadratic equations using factorisation method?

22. *Titus*: It is a build up from what they learnt at junior level under factorisation of linear equations and quadratic expressions. At senior level, when we teach quadratic equations we tell them that we equate to zero.

23. *Res*: How do you introduce the topic of solving quadratic equations using the formula?

24. *Titus*: The formula is usually taught as the third method. The first one beingfactorisation, then completing the square and the formula method. We firstderive the formula from completing the square method and when that is doneit becomes simpler to the learners. This is necessary as pupils will be able toknow where the formula is coming from.

25. *Res*: When deriving the formula, is it a must that pupils should master all the steps regarding deriving the formula?

26. *Titus*: Yes, they have to know that, because do not know some pupils may ask where the formula is coming from because they may feel that it has been imposed on them.

27. *Res*: Which of the two methods completing the square or formula method would you prefer to teach to your pupils?

28. *Titus*: I would prefer teaching completing the square method to the formula method.

29. *Res*: Any reasons for choosing completing the square?

30. *Titus*: Yes, because if they use completing the square method and pupils know the steps they will be able to get all the marks, unlike the formula method because there are other things we look, for example, if the over sign is not written in complete way the pupils may lose marks.

31. *Res*: In your selection of examples, do you include real life examples?

32. *Titus*: Yes, I do. For example, the roads usually have two faces, these are the smooth surface and the rough surface. I would ask the pupils to calculate the time taken to travel on a smooth surface and the rough surface without giving the distances.

33. *Res:* What prior knowledge or topics do you regard to be important before you teach the topic of quadratic equations?

34. *Titus*: The pupils should be conversant with integers (adding and subtraction of integers), solving of linear equations as well as factorisation and be able to locate points on the XOY-plane.

35. *Res*: What learner difficulties or misconceptions do you anticipate before you teach the topic of quadratic equations?

36. *Titus*; Yes, for example, if the solution of the quadratic equation is recurring, pupils like rounding off on the way thereby getting wrong solutions. Failure by the pupils to follow the right steps as they solve quadratic equations.

Appendix 9: Appoval of Study by Ethics Committee (Univerity of Zambia)

THE UNIVERSITY OF ZAMBIA

DIRECTORATE OF RESEARCH AND GRADUATE STUDIES

Great East Road Campus | P.O. Box 32379 | Lusaka 10101 | **Tel:** +260-290 258/291 777
Fax: (+260) 211 290 258/253 952 | **Email:** director.drgs@unza.zm | **Website:** www.unza.zm /directorates/drgs

Approval of Study

15th February, 2019

REF. NO. HSSREC: 2018-AUG- 051

Mr. Titus Luka Mbewe
University of Zambia
School of Education
P.O. Box 32379
Lusaka

Dear Mr. T. L. Mbewe,

RE: "MATHEMATICAL KNOWLEDGE FOR TEACHING QUADRATICAL EQUATIONS"

The University of Zambia Humanities and Social Sciences Research Ethics Committee IRB has approved the study noting that there are no ethical concerns.

On behalf of The University of Zambia Humanities and Social Sciences Research Ethics Committee IRB, we would like to wish you all the success as you carry out your study.

In future ensure that you submit an application for ethical approval early enough.

Yours faithfully,

Dr. J. Mwanza
CHAIRPERSON
THE UNIVERSITY OF ZAMBIA HUMANITIES AND SOCIAL SCIENCES RESEARCH ETHICS COMMITTEE IRB

CC: Director Directorate of Research and Graduate Studies
Assistant Director (Research), Directorate of Research and Graduate Studies
Assistant Registrar (Research), Directorate of Research and Graduate Studies
Senior Administrative Officer (Research), Directorate of Research and Graduate Studies

Towards Improving Service and Excellence in High Education Beyond Fifty Years