USING MENTAL CALCULATION TO PERFORM THE FOUR MATHEMATICAL OPERATIONS: EXPERIENCES OF GRADE 2 TEACHERS WITH ADDITION AND SUBTRACTION IN SELECTED SCHOOLS IN LUSAKA, ZAMBIA

By

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DECLARATION

I, MUHAU TABAKAMULAMU, declare that this Thesis

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APPROVAL

This Thesis of MUHAU TABAKAMULAMU has been approved as fulfilling the requirements for the award of the Degree of Doctor of Philosophy in Mathematics Education by the University of Zambia

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ABSTRACT

Since the 1980s, mathematics educators around the world have accepted the view that to develop numeracy effectively, young children need initially to use strategies for mental calculation rather than to memorise textbook procedures. The term *mental calculation* refers to informal, often untaught calculation methods used by children to solve a variety of arithmetic problems. This primarily qualitative study used a quasi-experimental design to assess the extent to which teachers in early primary mathematics in Zambia could adopt the use of strategies for mental calculation for double-digit addition and subtraction. The study also sought to determine the impact of teachers' use of such strategies on pupils' performance in mathematics.

Participants were 10 Grade 2 teachers (5 from control schools and 5 from experimental schools) and 311 Grade 2 pupils. The Grade 2 pupils, 167 of whom were from experimental schools and 144 from control schools, were evenly distributed with regard to sex, age, intellectual ability and socioeconomic background. In experimental schools the study had two phases: Grade 2 teachers attended a staff development workshop to prepare them to implement the study; followed by the main study when the teachers implemented in the classroom the ideas discussed during the workshop. In control schools the same set of mathematics topics were covered, but Grade 2 teachers there used the usual textbook procedures. Data were collected through lesson observations, interviews with teachers, teachers' journal entries, and document analysis; while pupils' performance was assessed by means of two numeracy tests. One numeracy test was administered at the beginning of the main study, and the other at the end of the study.

Results of the study suggested that over the ten-week implementation period teachers in experimental schools changed their existing beliefs about mathematics teaching and learning and to some extent their classroom practices as well, to support the use of strategies for mental calculation for double-digit addition and subtraction. Their pupils performed significantly better in the post-test compared to pupils in control schools, and developed more positive attitudes towards learning mathematics. Based on these findings, it was recommended among other things that, to show pupils that their informal solution strategies were valued, teaching in early primary mathematics should build on them. At the same time teachers should demonstrate that standard textbook procedures resulted from pupils' informal strategies and were more efficient.

DEDICATION

This Thesis is dedicated to my parents, my late father Pastor Moses Tabakamulamu Singulwani and my mother Namatama Muyunda Tabakamulamu, who together struggled with little resources to ensure that I had a good education, although my father did not live long enough to see the final results of his efforts; and to my wife Kubecte and my children Namatama and Sepo, who now understand why I had to work so hard to complete this task.

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LIST OF ABBREVIATIONS AND ACRONYMS

ADB	African Development Bank
ADF	African Development Fund
AFRODAD	African Network on Debt and Development
CDC	Curriculum Development Centre
CSO	Central Statistical Office
DfEE	Department for Education and Employment
ECZ	Examinations Council of Zambia
ENL	Empty Number Line
FBE	Free Basic Education
GNI `	Gross National Income
GRZ	Government of the Republic of Zambia
JCTR	Jesuit Centre for Theological Reflection
MARK	Mathematics Rainbow Kit
MMD	Movement for Multiparty Democracy
MoFNP	Ministry of Finance and National Planning
MoE	Ministry of Education
MoH	Ministry of Health
NBTL	New Breakthrough to Literacy
RME	Realistic Mathematics Education
SAP	Structural Adjustment Programme
SITE	Step In To English
TDRC	Tropical Diseases Research Centre
UNESCO	United Nations Educational and Scientific Organisation
UNICEF	United Nations Children's Fund
UNIP	United National Independence Party
UNZA	University of Zambia
ZATEC	Zambia Teacher Education Course

CHAPTER 1

INTRODUCTION

1.0 Introduction

The main aim of this primarily qualitative study was to assess the extent to which teachers in Zambia could learn to foster the use of strategies for mental calculation relating to double-digit addition and subtraction in the early primary grades. The study also sought to determine the impact of teachers' use of these strategies on pupils' performance in mathematics at this level of education. The term *mental calculation* is currently used in mathematics education to describe informal solution methods usually developed and used by children to solve a variety of arithmetic problems, including those involving double-digit addition and subtraction. The methods are said to be *strategic* in that they involve an element of choice, that is, in using them children have to determine *strategically* and *flexibly* the most appropriate solution method for a given situation (Threlfall, 2000; 2002).

1.1 Background to the Study

To understand why there was a need to conduct this study in Zambia, it is necessary to know something about the country and its education system. In view of this, Section 1.1.1 gives brief facts and figures about Zambia, and Section 1.1.2 discusses the education system in the country, describing its organisation and structure. Section 1.1.2 also attempts to show in what ways the education system in Zambia was affected by changes in the country's governance system since independence from Britain on 24 October 1964. Section 1.1.3 focuses on the issue of medium of instruction in education in Zambia, an issue that has continued to be debated since European missionaries first introduced formal education in Northern Rhodesia (as Zambia was called until 1964) in the 1880s. Finally, Section 1.1.4 describes the performance of pupils in mathematics since the 1970s, which made it necessary to conduct this study in Zambia.

1.1.1 Zambia: Brief Facts and Figures

This study was conducted in Lusaka, the Capital City of Zambia, a land-locked, Sub-Saharan African country lying between latitudes 8^0 and 18^0 south, and longitudes 22^0 and

34^0 east. Zambia shares its borders with eight other African countries. To the north is the Democratic Republic of Congo; Tanzania is in the north-east; Mozambique and Malawi are on the east; in the south are Zimbabwe, Botswana, and Namibia; and Angola is on west (see Figure 1).

The last population census conducted in 2000 revealed that Zambia, which covers an area of 752, 612 square kilometres, had a total population of 9.9 million people, which was increasing at an annual rate of 2.7 percent (CSO, 2003a; 2003b). By 2006 the population of Zambia was estimated to be 11.7 million and increasing at an annual rate of 2.9 percent (MoFNP, 2006a).

Figure 1: Map of Zambia Showing Neighbouring Countries (Maps of the World, 2007).

The figures for 2000 represent an average population density of 13.1 persons per square kilometre, indicating that Zambia is a sparsely populated country (CSO, 2003a). However, the population density varied according to area of residence. For example, Lusaka Province had the highest population density at 64 persons per square kilometre, while North-Western Province, one of the least developed provinces in Zambia at the time of the study, had a population density of only 4.6 persons per square kilometre (CSO, 2003a; 2003b; MoFNP, 2007).

Zambia is one of the most highly urbanised countries in Sub-Saharan Africa. In 2000 about 35 per cent of the Zambian population lived in the cities, mainly in the two most urbanised provinces of Lusaka and Copperbelt (CSO, 2003a; 2003b). Consequently, the major urban centres such as Lusaka were overcrowded, and most of the urban poor lived in unplanned settlements on the fringes, without adequate access to social services such as clean drinking water (MoFNP, 2007).

Since the mid-1980s mortality statistics in Zambia have been compounded by high incidences of HIV and AIDS, estimated in 2007 to be around 14 percent nation-wide among people aged between 15 and 49 (CSO, MoH, TDRC, UNZA, and Macro International Inc., 2009). This made it necessary to include the factor of HIV and AIDS in population projections. Thus, for example, the CSO (2003b) estimated that with HIV and AIDS the population of Zambia in 2025 will be 21.3 million, whereas without HIV and AIDS it would be 23.6 million. Kelly (2000), in his book *Planning for Education in the Context of HIV/AIDS,* argues that policy-makers and educational planners in countries such as Zambia with high incidences of HIV and AIDS, should adopt a similar way of thinking in planning the management of their education systems. That is, to ensure survival of their education systems, they should "[mainstream] the AIDS perspective in all aspects of policy formulation, planning and action" (p. 45), to take into account the loss in human resources likely to result from the impact of HIV and AIDS.

Since independence in 1964, Zambia has passed through three identifiable governance systems: First Republic (1964-1973), Second Republic (1973-1991), and Third Republic (1991 to date). Briefly, the First Republic was characterised by continuation of the multiparty politics of the colonial era, and an accelerated expansion of educational facilities at all levels, to counteract the policies of the colonial administration that left Zambia at independence in an exposed position, as it lacked trained human resources to take over from the Colonial Government. The Second Republic was a period during which Zambia was under One-Party rule, guided by the socialist oriented philosophy of Zambian humanism (see e.g. Kaunda, 1974). Following a cabinet decision in December 1990 to reintroduce multiparty politics, the United National Independence Party (UNIP) Government led by President Kenneth Kaunda, which had ruled Zambia since independence, was defeated in the general elections held at the end of 1991, and was replaced by the Movement for Multiparty Democracy (MMD) led by President Frederick Chiluba (Carmody, 2004). This marked the beginning of the Third Republic.

As one would expect, Zambia's economic policies changed in accordance with the kind of governance system in place. The First Republic had a market economy designed before independence to play the role of financier to the colonial power Britain (AFRODAD, 2007). The economy was nevertheless buoyant, driven by profits from copper mining (Kelly, 1991). In fact, it has been estimated that in current US Dollar terms Zambia's Gross National Income (GNI) per capita in 1964 was US $200, which was higher than that of Botswana and Thailand at the time (MoFNP, 2006a). This made it possible for the Zambian Government to restructure and rebuild the country's education system in the years immediately after independence (Mwanakatwe, 1974).

During the Second Republic, a period when the political leadership in the country espoused socialist-oriented policies, Zambia's economy was controlled by the State, many heavy industries having been nationalised in the 1970s (see Carmody, 2004). This period coincided with a sharp decline in the country's economy resulting from a fall in the prices of copper on the international market, coupled with a sharp rise in the price of oil triggered by the oil wars of the early 1970s (Kelly, 1991). It is, however, generally believed that the dire economic situation at this time of the country's history was worsened by the fact that One-Party rule created a huge party machinery at different levels of society, entirely supported by the country's treasury (Carmody, 2004). It is also true that Zambia's involvement in the liberation wars in Southern Africa during the period contributed to the country's financial woes. For example, during the wars Zambia increased its defence budget to be better able to protect itself from incursions by forces from Southern Rhodesia; Zambia also experienced, during the same period, limited access to sea ports in Southern Africa, forcing it to rely on expensive air freighting to survive (GRZ/UNESCO, 1979).

Convinced that Zambia's financial problems were due to the socialist policies of the Second Republic, the political leadership in the Third Republic opted for a free market economy, which included implementation of a Structural Adjustment Programme (SAP) supervised by the World Bank (Carmody, 2004; Kelly, 1994; 1998/99). The World Bank prescribed structural adjustment programmes to 'correct' ailing economies in many developing countries, which it believed got that way either because their governments played too prominent a role in matters relating to industrial production, had over-valued exchange rates and large budget deficits, and/or implemented protectionist trade policies (The World Bank, 1994). However, rather than improving Zambia's economy, SAP turned out to be even more economically debilitating. This was particularly the case in the education sector, which suffered reduced funding, resulting in further dilapidation of educational facilities and services, and failure of many children from poor families to access education (Carmody, 2004, Kelly, 1994; 1998/99).

In 2002, the second MMD Government that came into power in 2001 announced a Free Basic Education (FBE) policy for pupils in Grades 1 to 7. All user fees were removed, and it was no longer necessary for pupils who could not afford to purchase school uniforms to wear them in order to attend lessons (Sampa, 2003; de Kemp et al., 2008). The FBE policy significantly raised enrolment figures in primary school, particularly among orphans and children from poor families (de Kemp et al., 2008). The same policy was credited with an increased net enrolment ratio in basic education, which reached 95 percent in 2005 (MoE, 2005). However, according to Henriot (2006), the FBE policy had its negative side. Citing a study conducted in Lusaka by the Jesuit Centre for Theological Reflection (JCTR), Henriot argues that whereas the FBE policy indeed removed *direct* costs of schooling, such as user fees, Parent Teachers' Association charges, and so on, it left room for school authorities to continue administering *indirect* costs of schooling, such as those relating to the cost of books and supplies. These continued to prevent many children from poor families from accessing school education. Furthermore, the FBE policy created a financing gap between the grants given to schools and what schools were able to make through various levies before the policy was declared (ADB/ADF, 2006). Anecdotal reports suggested that many schools tried to cover this gap by asking new pupils to bring with them on first reporting for lessons, items such as a ream of paper for use during examinations, and implements such as hoes for cleaning school surroundings.

1.1.2 The Education System in Zambia

For purposes of educational administration, Zambia is currently divided into ten main units: the Ministry of Education (MoE) Headquarters in Lusaka, and nine provincial education offices. Provincial education offices are further sub-divided into 72 district education offices, which deal with matters at the local school level. The Ministry of Education Headquarters is responsible for provision of education in the country and for overall policy formulation. A Permanent Secretary, who is answerable to the Minister of Education, heads the Ministry, and is assisted by five directorates, each of which performs particular functions relating to the organisation and management of education in the country. (See Figure 2).

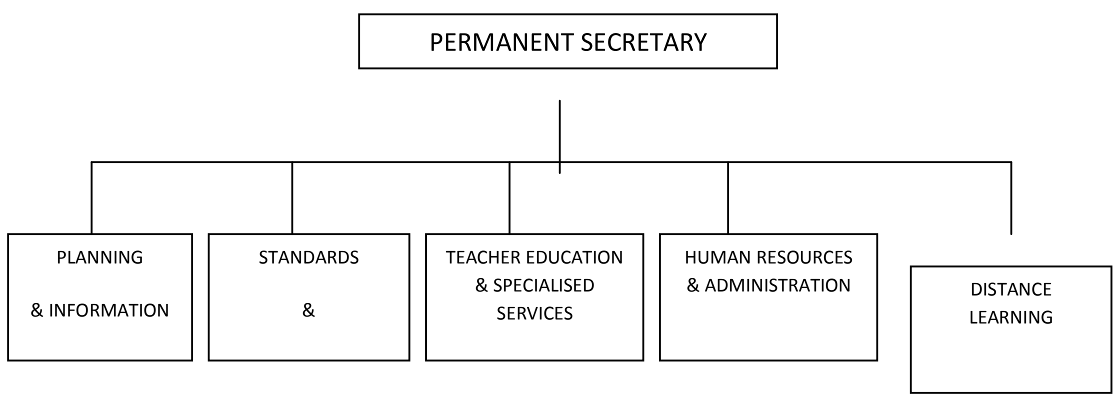


Figure 2: Organisational Chart of the Ministry of Education Headquarters (Adapted from MoE, 2001c).

The Directorate of Planning and Information is responsible for the overall planning and development of education, while that of Standards and Curriculum deals with curriculum development and maintenance of educational standards throughout the country. The function of the Directorate of Teacher Education and Specialised Services is to develop and deploy teachers in the country; and has responsibility for special education, guidance and counselling, and for provision of (school) library services. The same Directorate also facilitates the work of other Government ministries and stakeholders, who are more directly involved in the provision of early childhood education. The Directorate of Human Resources and Administration ensures effective and efficient educational administration in the country; while that of Distance Education is responsible for provision of distance education and adult literacy classes. [For more details see *Ministry of Education Strategic Plan 2003 - 2007* (MoE, 2002)]

Before the Ministry of Education began devolving some of its functions to districts and local schools around 2000, the work of provincial education offices was more or less that of a conduit for problems, issues, reports, and so on, from districts and local schools to the Ministry of Education Headquarters; and of decisions from Ministry Headquarters to districts and local schools (MoE, 1997; 2001d)). Following decentralisation, the work of provincial education offices changed. They became facilitators/participants in decisions made at district and school level, which now had some jurisdiction over matters relating, for example, to teacher discipline and sponsorship of teachers on continuing professional development (CPD). Thus decentralisation enabled provincial and district/school level educational administrators to exercise both administrative and management functions.

At the time of the study, the formal education system in Zambia had a 7 - 5 - 4 structure. That is, seven years of primary education (four years of *lower* primary and three years of *upper* primary); five years of secondary education (two years of *junior* and three years of *senior* secondary education, now referred to as high school education); followed by *tertiary* education, whose duration in the case of many bachelor's degrees was four years (MoE, 1996). This 7 - 5 - 4 structure however represented only the 'usual' study programmes. For example, the duration of a number of study programmes in tertiary institutions varied from one year to seven years, depending upon the kind of qualification sought. Also, since the early 1980s *basic schools* have emerged, which provide education for children in Grades 1 to 9 (MoE, 1992). Such schools were expected to increase in number as the education system continued to evolve. In fact, it was a long-standing goal of education policy-makers and planners in Zambia from the 1970s, to provide every child with nine years of basic education (MoE, 1977b; 1992). Therefore, educational planners and policy makers hoped that the development of basic schools would eventually change the structure of the school system in Zambia to 9 - 3 – 4. That is, nine years of basic education; three years of high school education; and four years of university education, terminating with a bachelor's degree (MoE, 1996).

In the meantime, competitive national examinations at Grade 7, Grade 9, and Grade 12 continued to be held, since there was a nationwide shortage of school places (MoE, 1996). This meant that for the foreseeable future significant numbers of pupils would be eliminated from the education system at the end of Grade 7, 9 and 12, because they failed to perform sufficiently well in examinations at these grade levels to reach the 'cut-off points' that would enable them to proceed to the next levels of education. Thus, for example, in 2007 only 54.54 percent of pupils who took the Grade 7 terminal examination entered secondary education; while only 38.71 percent managed to move from Grade 9 to Grade 10 (MoE, 2007).

1.1.3 Medium of instruction in Education

A key issue that has dogged the education system in Zambia since pre-colonial times relates to which language to use as the medium of instruction in schools, particularly from Grade 1 to 4. When European missionaries first arrived in Northern Rhodesia in the early 1880s and introduced *formal* education (Carmody, 2004; Gadsden, 1992; Mwanakatwe, 1974, Snelson, 1970), they found the Africans living in the territory speaking a variety of Bantu languages (Posner, 2001). Faced with this situation, the missionaries decided that the language of instruction in schools would be the local language of a given area (Carmody, 2004; Prah, 2000; Snelson, 1970).

By 1890, Northern Rhodesia had come under the control of the British South African (BSA) (Marten and Kula, 2008). However, educational provision in the territory continued to depend almost entirely on the initiative of the missionaries, since the BSA Company was reluctant to make any financial contributions to education (Carmody, 2004; Gadsden, 1992; Snelson, 1970). Therefore, the position taken by the missionaries concerning language of instruction in schools was maintained.

When Northern Rhodesia came under the direct control of the British Crown in 1924, colonial administrators adopted a similar position concerning language of instruction in schools. They proposed that four dominant African languages (Bemba, Nyanja, Tonga and Lozi) should be used as media of instruction in schools for Africans (Kelly, 1995;

Shana, 1980). In 1930 the Advisory Board of Native Education recommended that the "mother-tongue…would be used to teach the mechanics of reading and writing and that English…would replace [it]…thereafter" (Shana, 1980, p.5). UNESCO endorsed this position in the early 1950s, adding that English could also be introduced early in African schools but only as a school subject (Shana, 1980).

The medium of instruction issue came up again towards independence in 1964. Following a recommendation of the Primary Education Committee to introduce English as a subject of study from Grade 1, the then Ministry of African Education effected the recommendation in 1962 (Shana, 1980). In 1963, a UNESCO team comprising Australian education experts visited Northern Rhodesia to assist the Government plan the future development of its education system. They recommended that English should be the universal medium of instruction from Grade 1, and that a policy decision should be made to effect the change (Linehan, 2004; Kelly, 1995; Shana, 1980). In view of this, the Zambian Government in 1965 adopted English as the medium of instruction in schools from Grade 1, and made it into law as part of the Education Act of 1966 (Mwanakatwe, 1974; Shana, 1980).

The decision by the Zambian Government to adopt English as a universal medium of instruction from Grade1 was based on a number of considerations. According to Mwanakatwe (1974), English was considered a neutral language in that it was not allied to any of the Zambian languages. Adopting it as a medium of instruction therefore prevented divisions based on tribal lines, thereby promoting national unity. It was also argued that producing teachers who were fluent in the vernacular languages and developing the necessary educational materials in them, would be costly. Furthermore, it was pointed out that learning in a multiplicity of languages would retard academic progress; and that, in fact, learning English opened the way to the world's accumulated stock of knowledge.

Many educationists in Zambia believe that the decision taken in 1965 to adopt English as a medium of instruction from Grade 1 was a mistake (CDC, 1983; Kelly, 1995; Kashoki, 1990). Kashoki in particular argues that the decision was questionable *politically*, *culturally*, *socially*, *psychologically* and *pedagogically*:

> Politically, it is wondered whether it is in the interest of an independent African country for a foreign language to feature so prominently and so pervasively in the life of its citizens. Socially and culturally, there are disturbing questions whether the pre-eminence of a foreign language may not in the long run have irreparable, deleterious effects on the cultural fibre of the nation. Psychologically and pedagogically, it is questioned whether in fact instruction in a foreign language promotes effective learning (p. 81).

Furthermore, according to Kashoki (1990), the argument that English was a neutral language was flawed, since Zambia at the time had school children who were either native or near-native speakers of English, and who would be favoured by the English medium policy.

Zambia had the chance to reverse this English medium policy in the 1970s, when it reviewed its education system, so that any necessary reforms could be implemented. By that time it was clear that the adoption of the English medium policy from Grade 1 was ill-advised, as it had impaired children's educational development particularly in the early primary grades (CDC, 1983; Kelly, 1995). In view of this, the draft education policy, *Education for Development*, which the Ministry of Education published in 1976, stated that school children would be introduced to formal education through a language in which they could communicate. The document proceeded to identify seven Zambian languages (Bemba, Nyanja, Tonga, Lozi, Kaonde, Lunda, and Luvale) as media of instruction from Grade 1 to 4, in the areas where they were dominant. It further proposed that:

> English will be introduced into the curriculum from Grade 1, in a gradual manner appropriate to the age of the children and the subject matter to be taught, so that students may be prepared for the use of English as medium of instruction from Grade 5 onwards (p. 11).

However, following nation-wide debates focusing on its contents, *Education for Development* was rejected; so also was its proposal on medium of instruction in schools (Carmody, 2004). It has been suggested that the document was rejected because it assumed that Zambia was going socialist, an assumption that was opposed by many educated Zambians, who owed their privileged positions to the existing education system (Carmody, 2004; Clarke, 1979; Coombe, 1979; Hoppers, 1985; Lungu, 1985). This group agreed that educational reform was indeed needed but not the type that altered significantly the basic distribution of power in society (Clarke, 1979).

Educational Reforms (1977), a document that was eventually accepted and adopted in 1978 as the official Government policy on education, retained the English medium policy. In the document, the Ministry of Education justified this decision on the basis of the following among others:

- it was not practicable in a multi-lingual society like Zambia to adopt the mother tongue as medium of instruction;
- the cost of developing learning materials in different Zambian languages could be prohibitive;
- if English was going to be the medium of instruction in higher grades, it should be introduced as early as possible;
- designating Zambian languages as media of instruction in schools would hamper the mobility of both teachers and children within the country; and that,
- certain concepts in mathematics, science and technology could not be expressed precisely in Zambian languages, and it would take some time before this could be achieved.

However, given mounting evidence that the English medium policy had impaired the education of many Zambian children, and following the World Conference on Education for All held in Thailand in 1990, educationists and stakeholders finally agreed that if education should benefit all children, it should be delivered through the medium of a Zambian language (Kelly, 1995). This decision was reflected in *Focus on Learning* (1992), the official Government policy on education since *Educational Reforms* (1977). *Focus on Learning* argues that although the English medium policy had advantages, such as those listed above, they were more than offset by the disadvantages. As the document states:

> Too early an emphasis on learning through English means that the majority of children form hazy and indistinct concepts in language, mathematics, science and social studies….The use of English, to the exclusion of local languages, as medium of instruction in schools leads to a downgrading of these languages….[It sends] the implicit message…that schooling should direct one away from traditional society towards a life, areas of employment, and living circumstances that require the use of English (p. 28).

Kelly (1995) is even blunter in his criticism of the English medium policy. He describes the period during which this policy was in place as Zambia's blighted years, partly because it produced a generation of Zambians who were ill-prepared to tackle the country's economic problems, its droughts, and the pandemic of HIV and AIDS. Furthermore Siluyele (1996), addressing an annual conference of the Zambia Association for Mathematics Education (ZAME), an organisation that comprised mainly mathematics teachers and teacher educators, argued that it was unfair to expect young children who did not understand English to struggle to make sense of the language and, at the same time, to learn mathematics in the language. He therefore urged his audience to put pressure on policy-makers and politicians, to allow children to learn mathematics in their mother tongue at least during the first four years of formal education.

The new policy on language of instruction resulting from the debates above was included in *Educating Our Future* (1996), Zambia's current education policy. Concerning medium of instruction in education, *Educating Our Future* states that, "all pupils will be given an opportunity to learn initial basic skills of reading and writing in a local language; whereas English will remain as the official medium of instruction" (p. 39). In *The Basic School Curriculum Framework* (2000a), the Zambian Government went even further, stating that:

> ...at all levels [of basic school education] and in all subjects teachers are encouraged, whenever necessary and relevant, to use the familiar language for explanations, clarifications, questions and answers. Likewise, pupils are at liberty to use the language in which they are able to express themselves confidently and competently, even when the medium of instruction is English (p. 23).

To implement the new language policy on education, the Zambian Government with the help of local and foreign language experts, established the Primary Reading Programme (PRP). The PRP developed targeted literacy development interventions at all levels of primary education; also producing syllabuses and other learning materials, and training teachers to use them (Linehan, 2004; Sampa, 2003). The PRP begins at Grade 1 with *New Breakthrough to Literacy* (NBTL), a course intended to enable Grade 1 children to master the basics of reading through seven official Zambian languages (Bemba, Nyanja, Tonga, Lozi, Kaonde, Lunda, Luvale), and to develop some oral ability in English (MoE, 2002). NBTL is followed at Grade 2 level by *Step In To English* (SITE), which takes "pupils, who have mastered the basics of reading in their local language and who have some oral English vocabulary, and introduce[s] them to reading in English with comprehension" (MoE, 2001b, p. 33). From Grade 3 to 7, pupils encounter the *Read On* series, whose aim is to "ensure that whatever level of literacy skills pupils possess when they start Grade 3, they will have developed these skills and become fluent readers and writers in both English and their Zambian Language by grade 7 (MoE, 2001b, p. 34).

Has the medium of instruction issue in education in Zambia finally been resolved? It seems unlikely. According to Hawes (1979), in Africa issues relating to language of instruction tend to be highly political, and their resolution is rarely, if ever, based on educational grounds alone. He adds: "When they are made, they are almost inevitably subject to mistrust and misunderstanding by some sections of the community" (p. 76). This may already be happening. For example, in 2000 the Ministry of Education commissioned a study in selected provinces, which investigated what stakeholders outside the education system wanted added to the basic school curriculum. The study found, among other things, that 77 percent of parents and 73 percent of pupils in the sample opposed the current language policy in education in Zambia, because they believed that English was the language that distinguished educated from uneducated Zambians. If such sentiments increased in future, it is possible that the current language policy in education, arguably the most democratic since independence in 1964, could be due for revision. Indeed, Linehan (2004), who participated in the development of PRP materials, suggests that the present medium of instruction policy in Zambia should be viewed as a temporary measure, which will need to be reviewed later.

1.1.4 Learner Performance in Primary Mathematics

Since the 1970s nation-wide surveys have consistently indicated unsatisfactory learning results in numeracy among primary school pupils in Zambia. For example, in 1974 Sharma and Henderson conducted a study that investigated numeracy achievement at Grade 3 level in Zambia. This study found that Grade 3 pupils generally did not master the numeracy skills expected at their grade level. In 1980 Nkwanga, after meeting with primary school teachers and primary teacher educators, concluded that primary school pupils indeed performed poorly in mathematics, and that the main cause was the pupils' failure to understand the English mathematics vocabulary expected at their level of education. Furthermore Kelly (1991), in his book *Education in a Declining Economy*, which discusses how Zambia's economic decline from 1975 to 1985 affected educational performance in the country, also refers to "grave weaknesses in the learning (and teaching) of mathematics and science at primary and secondary levels" (p. 114). But Kelly also noted that the low performance in mathematics and science existed before 1975, when the economy was booming, adding that "Unless the fundamental problems are addressed, any improvements will be merely cosmetic and transient" (p. 115). In view of this, Kelly (1991) suggested the following lines of inquiry, which he felt might be effective in addressing the problem of unsatisfactory performance in mathematics at primary school level in Zambia:

> First, how has learning mathematical and scientific concepts in a foreign language affected children's grasp, development, and use of these subjects? Second, how effective is the training primary school teachers receive to teach mathematics and scientific concepts, techniques, and procedures in a language that is not their own to a pupil who has difficulty in handling the language of communication (p. 125)?

What was surprising, however, was that despite the change in the policy regarding medium of instruction in schools described in Section 1.1.3, the pattern of unsatisfactory performance in numeracy at primary school level continued. This is indicated by results of national assessments conducted by the Ministry of Education through the Examinations Council of Zambia (ECZ) (Kelly and Kanyika, 2000; MoE, 2001a; 2003; 2006; 2008). For example, the performance of Grade 5 pupils in the national assessment conducted in 1999 indicated very low levels of *actual* learning taking place in mathematics, with only one-quarter of the pupils included in the survey reaching the minimum expected level of performance (Kelly and Kanyika, 2000).

The Ministry of Education in the midi-1990s recognised the severity of this problem and began the groundwork to address it. In *Educating Our Future* (1996), the Ministry of Education identified development of *basic numeracy* and *problem-solving skills* as the priority target for primary mathematics education. The aim was to ensure that "those who leave school are able to *function* effectively in society, while those who continue in school have an adequate basis for further education" (p. 14). Later, in *The Basic School Curriculum Framework* (2000a), the Ministry of Education settled for the term *essential numeracy*, and identified its components. That is, on completion of primary education pupils would understand the meaning of the numbers from zero to one million; and use this range of numbers to perform the four fundamental arithmetic operations of addition, subtraction, multiplication, and division. They would also be able to carry out calculations involving fractions and percentages, and length, area and volume; and to apply these skills in typical everyday situations.

While the goals to be achieved were so clearly outlined, it seemed that little or nothing was said about *how* to achieve them. Indeed, it was unclear what teachers would do *differently* in the classroom to help pupils achieve essential numeracy. Furthermore, in *Teachers' Curriculum Manual* (MoE, 2001a), a document published specifically to help primary school teachers put into practice the contents of *The Basic School Curriculum Framework*, no guidance was given concerning implementation of the new policy on numeracy development in early primary mathematics. In contrast, *Teachers' Curriculum Manual* contained detailed guidelines for the teaching of other primary school subjects.

In *Mathematics Rainbow Kit, MARK*: *Teachers' Guide* (2004), a book which focuses on numeracy development at Grade 1 level, the Ministry of Education seems to acknowledge the omission. It states that since publication of *Educating our Future* in 1996, very little was done *practically* to improve teaching and learning of numeracy in primary school. The *MARK,* as this document came to be known, adopted in early primary mathematics the teaching methodologies used in *New Breakthrough to Literacy* (NBTL), which significantly improved reading levels among primary school pupils in the first two grades (Linehan, 2004; Sampa, 2003).

But the *MARK* continued a feature of traditional early primary mathematics in Zambia, emphasis on learning of *formal* place value concepts and use of standard procedures, which relevant research around the world in the previous three decades identified as *impeding* rather than *aiding* young children's mathematical progress (Beishuizen and Anghileri, 1998; Blote et al., 2000; Kamii and Dominick, 1997; Plunkett, 1979; Romberg, 1995). According to this research, introducing instruction in *formal* place value concepts and use of standard procedures early in children's learning of mathematics forces them to abandon their own mathematical thinking, to adopt 'adult' methods whose logic they often do not understand. As Hart (1981) states, teaching a standard procedure to children who do not understand the logic behind it is harmful to their mathematics progress, in that what they see their teacher doing is magic rather than problem solving. In contrast, children's own informal solution methods, "are often based on the individual's understanding of the situation rather than on the memorization of a set of procedures" (Thompson, 1994, pp. 342–343).

Indeed, it has been observed that "Most countries, and in particular those most successful at teaching number, avoid the premature teaching of standard written methods in order not to jeopardise the development of mental strategies" (UK Department for Education and Employment [DfEE], 1999, p. 7). In Japan, mathematics teachers routinely encourage the use of students' methods (Grows and Cebulla, 2000), which might explain why Japanese school children are well known for their excellent performances in international mathematics competitions. Consequently, many mathematics educators around the world have accepted the view that, rather than ignoring children's informal calculation strategies, numeracy teaching in the early primary grades should build on them. Furthermore, instruction in *formal* place value concepts and discussion of standard written algorithms should be postponed to later primary grades (Beishuizen and Anghileri, 1998; Blote et al., 2000; Thompson, 1999; Menne, 2001).

One wonders if there was a link between primary school children's unsatisfactory development of numeracy in the past in Zambia and the dominant teaching methods used at that level of education. A number of authors, acquainted with the education system in Zambia, think that such a link existed (see Jeremy, 1980; Joseph, 1982; Kelly, 1991). In particular Kelly (1991), whose studies on Zambian education significantly influenced education policy development in the country particularly in the 1980s and 1990s, described prevailing teaching approaches in primary school in Zambia as being *inflexible* and *unimaginative,* emphasising "factual knowledge, memorization, and convergent thinking" (p. 133). Kelly believed that the same teaching approaches were responsible for the difficulty many Zambian children experienced in learning mathematical and scientific concepts beyond primary education.

Thus if no steps were taken to teach mathematics *differently* in the early primary grades in Zambia, the new policy on numeracy learning would most likely not yield the desired results. This study, therefore, piloted in early primary mathematics an alternative approach to numeracy development, which used strategies for mental calculation for double-digit addition and subtraction. The aim was not necessarily to replace existing numeracy development approaches, but to increase the choices teachers had in this regard. The study was conducted at Grade 2 level, the grade level at which double-digit addition and subtraction is normally first introduced in Zambia.

1.2 Statement of the Problem

Since the 1970s nation-wide surveys conducted in Zambia indicated unsatisfactory learning results in numeracy at primary school level (Sharma and Henderson, 1974; Kelly, 1991; Kelly and Kanyika, 2000; MoE, 2001a; 2003; 2006; 2008). During that period, early primary mathematics instruction emphasised instruction in formal place value concepts and use of standard procedures. Accumulating research evidence around the world suggested that, rather than helping children to develop numerical skill, this approach hindered their progress in mathematics (Kamii and Dominick, 1997). Could the use of strategies for mental calculation, which helped improve children's numerical skill in countries such as the Netherlands, produce better learning results in numeracy among primary school pupils in Zambia? To what extent would teachers in Zambia learn to foster the use of this numeracy development approach in early primary mathematics? Questions such as these indicated a significant gap in knowledge which called for systematic investigation.

1.3 Purpose of the Study

The purpose of this study was to assess the extent to which teachers in Zambia could learn to foster the use of strategies for mental calculation relating to double-digit whole number addition and subtraction in early primary mathematics; and to determine the corresponding impact of this on pupils' performance in numeracy.

1.4 Objectives of the Study

Specific objectives of the study were:

- To determine the concerns of teachers related to adopting and piloting a teaching and learning approach based on the use of strategies for mental calculation for double-digit whole number addition and subtraction at Grade 2 level;

- To study the appropriateness of this teaching and learning approach for implementation by primary school teachers in Zambia in relation to the beliefs they held about the teaching and learning of primary mathematics.
- To determine the ability of the teachers, given their mathematics content knowledge, for teaching the use of strategic methods for double-digit addition and subtraction at Grade 2 level;
- To determine the impact of implementing this teaching and learning approach on Grade 2 pupils' learning and performance in numeracy.
- To establish what needed to be done further, if the pilot had to be extended to the country as a whole, to foster the use of strategies for mental calculation in early primary mathematics.

1.5 Research Questions

The study was guided by the following research questions:

1. How do teacher concerns related to adopting a teaching and learning approach based on the use of strategic methods for double-digit addition and subtraction, such as the splitting/partitioning methods, interact with the implementation process?
2. In what ways do teachers' existing beliefs about mathematics teaching and learning interact with the use of strategies for mental calculation in primary mathematics?
3. How sufficient is the mathematics content knowledge of primary school teachers in Zambia, for teaching the use of strategies for mental calculation?
4. Is there a difference in performance on numeracy tasks between primary school children in Zambia who use strategic methods for double-digit addition and subtraction, such as the splitting/partitioning methods, and their colleagues in the country who use procedures as currently taught? If so, in what ways?
5. In view of questions 1 to 4 above, what needs to be done further if the pilot is to be extended to the country as a whole, to foster the use of strategies for mental calculation in early primary mathematics in Zambia?

1.6 Significance of the Study

As indicated earlier, for several decades primary school children in Zambia did not attain levels of numeracy development expected by society, it would seem partly because of the numeracy development approach traditionally used. This study, the first of its kind in Zambia, piloted an alternative numeracy development approach in the early primary grades, based on the use of strategies for mental calculation. Therefore, its results made an original contribution to research in the area of numeracy teaching and learning at this level of education, and could be useful to all groups and organisations involved in the mathematics education of young children, such as class teachers, textbook writers, curriculum specialists, and primary teacher educators.

Specifically, the results of this study could:

- Increase the choices class teachers and primary teacher educators have with regard to thinking about young children's learning in numeracy.
- Provide recommendations of practical value to curriculum developers, textbook writers and policy makers in the Ministry of Education for the design of new curricula, which aim to maximise opportunities for development of numeracy in young children.
- Suggest to the groups named above and to other researchers in mathematics education in general the extent to which teaching methods might contribute to young children's inadequate development of numeracy in Zambia.

1.7 Definition of Terms

The following terms have been used in the study as follows:

Formal Education: A system of education that is arranged hierarchically and which runs from primary school to university and other forms of higher education. This kind of education is said to promote individual creativity, uniqueness and competitiveness (Marah, 2006). It differs from ***traditional*** African education which taught children not to think in terms of themselves as individuals but as members of the community (Marah, 2006), by inducting them into the rituals, laws and cultural symbols of their culture, which acted as some kind of glue that kept members of the tribe together (Carmody, 2004).

Mental Calculation: This refers to informal, often untaught solution methods used by young children to solve a variety of arithmetic problems, such as those involving adding and subtracting double-digit whole numbers. Such methods are called ***strategic*** in that children who use them have to decide which method will work best in a given situation.

Numeracy: The term *numeracy* was used in this study to indicate the extent to which one is *comfortable* in dealing with numerical information encountered in everyday life, such as when reading a newspaper containing graphs and charts, or when constructing a family budget (see Askew et al., 1997; Cockcroft, 1982; Jeffery, 2002; Nunes and Bryant, 1996).

Splitting or Partitioning Methods: These are methods which involve separating either one or both numbers being added or subtracted ideally into multiples of ten, fives and ones. The use of such methods allows young children to avoid an immature encounter with the process of regrouping, which many young children find difficult to understand.

Teaching by Telling: This refers to a kind of mathematics teaching which treats mathematics as a 'finished' product, and which is characterised by teachers attempting to transfer mathematical knowledge from themselves to pupils, and expecting pupils to reproduce in exercises, tests, or examinations, exactly what was demonstrated in worked examples in the classroom.

1.8 Limitations and delimitations of the Study

Although this study was intended to address the extent to which teachers in lower primary school in general (i.e. Grade 1-4) could learn to foster the use of strategies for mental calculation, it was confined to Grade 2 teachers and classes only. This was mainly because in Zambia, Grade 2 was the grade level at which double-digit addition and subtraction was first introduced. The study was also conducted in four schools only, whereas there were many more schools in Lusaka that had Grade 2 classes. This was mainly because it was impossible to study more schools, given the limited financial and material resources I had. Furthermore, the study was primarily qualitative, which necessarily required *understanding* well a relatively small sample of schools. However, I took care to choose participating schools that shared similar characteristics with other schools in Lusaka, and in the nations as a whole, so that it was possible to generalise the findings of the study beyond the immediate research settings.

Ideally, a study of this kind should have included interviewing both teachers and pupils. In this case only teachers were interviewed. This was mainly because the study focused on the ability of teachers to bring about the type of learning conditions being promoted in the study. Pupils only came in to help me evaluate teachers' ability to foster the use of strategies for mental calculation in early primary mathematics. This required knowing something about pupils' performance in mathematics associated with their teachers' use of such strategies. Finally, this study would have been more complete if it had included all the four mathematical operations (i.e. addition, subtraction, multiplication, and division). However, the limited resources I referred to above meant that it was simply not possible to accomplish this.

1.9 Layout of the Thesis

This thesis is divided into six chapters arranged as follows:

Chapter 1 introduces the study. It gives the background to the study, including an overview of the education system in Zambia; the statement of the problem; the purpose and objectives of the study; research questions; and the significance of the study. The chapter also gives operational definitions of some terms used in this study, which could be interpreted differently by different people.

Chapter 2 gives a review of the literature related to the problem under investigation. It is grouped around the following main themes related to the problem addressed: influences for change in numeracy teaching/learning practices; nature of children's mental calculation strategies for double-digit addition and subtraction; and the teaching of mental calculation. Other themes included the theoretical basis for current changes in numeracy teaching and learning practices, which hopefully will help the reader to understand the philosophical assumptions behind the study; teacher change and current developments in numeracy teaching practices; and evaluation of school mathematics innovations. The last part of the review is a summary of the literature discussed, and attempts to show in what ways this literature was limited as far as the problem under investigation in this study was concerned.

Chapter 3 discusses the research design and methodology used in the study. The chapter begins with a statement of the main aims of the study, which is followed by a description and justification of the broad methodological approach adopted; and the mixed-methods strategy used. The chapter includes a description of the sites where the study was conducted, the aim being to enable readers to have a feel of the areas in which experimental and control schools were located and to make an independent judgement regarding their comparability, particularly with regard to factors such as parents' socio-economic background that can affect the academic performance of children.

Chapter 4 presents the findings of the study relating to both teachers and pupils. It begins with demographic characteristics of participating teachers, and then attempts to answer research questions relating to them. The data presented is mainly qualitative. The chapter goes on to present the findings of the study relating to pupils. In this regard it begins with an assessment of participating pupils' achievement of numeracy before the study, as measured by their performance in a numeracy test administered at the beginning of the study, before discussing their learning experiences and performance in numeracy during the study. The data presented in this regard is mainly quantitative.

Chapter 5 discusses results of the study relating to both teachers and pupils. It starts with results relating to teacher learning, followed by results relating to pupil learning.

Chapter 6 presents the conclusions and recommendations arising from the study. In this chapter, I outline the conclusions and recommendations that appear to me to arise from the study's findings, and also suggest areas for further research.

CHAPTER 2

REVIEW OF RELATED LITERATURE

2.0 Introduction

This chapter presents a review of the literature related to recent changes in numeracy teaching and learning practices in early primary mathematics. The literature reviewed is drawn from three main areas: (a) curriculum reform in numeracy teaching and learning practices in early primary mathematics and its theoretical basis; (b) educational change and staff development; and (c) monitoring and evaluation of mathematics innovations.

The review has seven main sections. Section 2.1 discusses developments around the world in the last three decades that have influenced recent changes in early primary numeracy teaching and learning practices; Section 2.2 looks at resulting change in the content of early primary mathematics, mainly the kinds of strategies for mental calculation children use to solve problems involving double-digit addition and subtraction; Section 2.3 considers ways in which these strategies might be taught; and Section 2.4 outlines the theory of learning reflected in current changes in numeracy teaching and learning practices in early primary mathematics. Section 2.5 focuses on the process of introducing curricular innovations at school level and how stakeholders, particularly teachers, fit into the process; and Section 2.6 examines the process of monitoring and evaluating mathematics interventions. Finally, Section 2.7 is a summary of the literature reviewed and was intended to show how this literature was limited as far as the problem under investigation was concerned.

2.1 Influences for Change in Numeracy Teaching and Learning Practices

In the last three decades there has been growing support within the mathematics education community for the view that numeracy teaching in the early primary grades should de-emphasise learning of standard rules and procedures and instead encourage the use of mental calculation strategies (Beishuizen et al., 1997; Thompson, 1994). This change in thinking was influenced by a number of developments around the world, particularly results of related research in the Netherlands (Beishuizen, 1997; Beishuizen and Anghileri, 1998); renewed interest in mental calculation in the UK (DfEE, 1999); and introduction in the US in the 1980s of standards-based mathematics curricula, which promoted use of children's invented strategies in the mathematics classroom (Carroll, 2000; Heirdsfield and Cooper, 2002). I discuss each of these three examples in turn.

2.1.1 Realistic Mathematics Education in the Netherlands

Since the 1970s Dutch educators experimented with various methods of teaching numeracy in early primary education, in order to identify more realistic approaches with reference to children's life experiences and abilities (Treffers, 1993; van den Heuvel-Panhuizen, 2001). These experiments were encouraged by the work of the renowned Dutch educator Hans Freudenthal (1905–1990), who believed that developing children's competence with number did not require an early introduction of *formal* place value concepts and the use of standard written algorithms (Beishuizen and Anghileri, 1998).

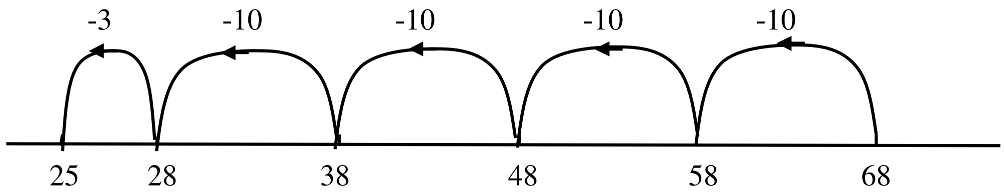
Following these experiments, Dutch educators adopted a new approach to fostering numeracy in early primary mathematics, called Realistic Mathematics Education (RME) (Treffers, 1993; van den Heuvel-Panhuizen, 2001). The term *realistic* was used in this context not only to refer to mathematics that is imbedded in real life contexts, but also to any other mathematics activities that may be real in the child's mind, including fairy tales and formal mathematics (van den Heuvel-Panhuizen, 2001). In other words, RME attempted to incorporate children's everyday reality into their mathematics education and served as a source for learning mathematics (Treffers, 1993).

RME departed from traditional early primary numeracy teaching, which emphasised the use of standard vertical algorithms, by promoting the view that mathematics teaching should build on children's mental strategies; and postponing formal place value instruction and discussion of standard written algorithms to later grades (Beishuizen and Anghileri, 1998; Menne, 2001; Thompson, 1999; Treffers, 1993). RME did this, for example, by encouraging and teaching children when adding two whole numbers first to partition or split them into multiples of ten and ones, before adding the separate parts to reach the final result (Klein et al., 1998).

In this regard two main methods were adopted, that is *1010* and *N10* (Beishuizen 1993; Beishuizen et al, 1997). \as these authors show, to add (or subtract) two whole numbers using the *1010* method involves splitting *both* numbers into multiples of 10 and ones before adding (or subtracting) the different parts, for example 25 + 8 = 10 + 10 + 5 + 5 + 3 = 20 + 10 + 3 = 33. Using the *N10* method required splitting only the second number: 25 + 8 = 25 + 5 + 3 = 30 + 3 = 33. Since in this case only the second number was split or partitioned, the calculation would involve less work if the larger number was always written down first, so that the smaller number was partitioned. That is, the calculation is easier for 25 + 8 than for 8 + 25.

Although Dutch children have shown preference for the 1010 method, wanting to use it in all situations, it is the less efficient of the two methods particularly in cases where vertical subtraction would require regrouping, that is 'carrying' and 'borrowing'(Beishuizen, 1993). For example, in a study which involved Grade 3 children who were competent users of both methods, Beishuizen et al. (1997) found that users of the 1010 method took longer to compute answers than did users of the N10 method. This is because as problem complexity increases the 1010 method is more difficult to use than the N10 method, since it involves "a greater number of steps and a heavier load on working memory" (Beishuizen et al., 1997, p. 89). Consequently, Dutch mathematics educators taught formally the use of the N10 method and left the 1010 method for children to use informally if and when they thought necessary (Beishuizen, 1993).

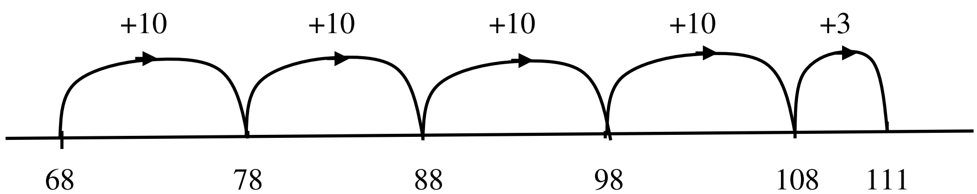
The use of an *empty number line* (ENL) as a model for performing addition and subtraction (Beishuizen, 1997), was another innovation of RME thinking which gained acceptance in other countries for example the UK (DfEE, 1999). The ENL is a 'number line' with no markings on it and pupils themselves decide which numbers to include and where to position them on the line (Beishuizen, 1997). For example, using the ENL to perform the subtraction 68 - 43 requires first drawing a straight line, marking the number 68 on it towards the right hand end, and then performing backward 'jumps' on the line in multiples of tens and ones equivalent to the absolute value of 43. Figure 3 illustrates this process.



i.e. 68 – 43 = 25

Figure 3: Subtraction by 'Jumping' Backwards on the Empty Number Line

The addition 68 + 43 can be worked out in a similar way but writing 68 at the left hand end of the line and 'jumping' in the forward direction as shown in Figure 4:



i.e. 68 + 43 = 111

Figure 4: Addition by 'Jumping' Forward on the Empty Number Line

Whether one 'jumps' towards the right or towards the left of a given number (i.e. whether one counts *on* or *backwards*) is a decision that children themselves have to make, depending on the question asked. When used together, the N10 method and the ENL comprised a powerful device for adding and subtracting whole numbers, which improved Dutch children's efficiency and accuracy in performing whole number addition and subtraction (Beishuizen and Anghileri, 1998).

The examples above demonstrate that children using the partitioning/splitting methods could obtain accurate answers to multi-digit addition and subtraction problems without having a mature understanding of place value concepts. If similar addition/subtraction problems were performed using standard vertical methods, regrouping would be involved which, as we saw in Chapter 1, many young children tend not to understand. Thus a big advantage of the ENL is that its use only requires pencil and paper; and can easily be adopted in countries such as Zambia with resource-starved primary mathematics classrooms.

2.1.2 The National Numeracy Strategy in the United Kingdom

In the UK implementation of the National Numeracy Strategy (NNS) in the 1990s involved rethinking of the way mathematics education was structured in the early primary grades (SCAA, 1997; Thompson, 2001a). According to these authors, before the 1950s mental arithmetic, which mainly consisted in pupils giving quick answers to 'rapid-fire' questions from teachers, was a regular part of the primary mathematics curriculum in Britain. Following the mathematics education reforms of the 1960s and 1970s, which were influenced by Piaget's research and which emphasised individualised learning and understanding of mathematical structure, mental arithmetic was neglected and almost disappeared from primary mathematics classrooms. Its inclusion in the NNS and the perception by members of the public that the poor performance of British children in international mathematics competitions was due to the absence of mental arithmetic from the curriculum, gave rise to renewed interest in mental methods (SCAA, 1997; Thompson, 2001a). As suggested by contents of some textbooks used in the UK, this development included adoption of the Dutch idea of an empty number line as a model for performing addition and subtraction. [See e.g. Koshy and Murray's (1996) *Unlocking Numeracy*; and the three books by Skinner et al. (2001), Mosley et al. (2001) and Askew et al. (2001) with the common title *Teaching Mental Strategies*, which were published to help teachers at different levels of primary education to implement the National Numeracy Strategy].

Related to the developments above in the UK was the question of what phrase to use to describe the new curriculum component (Thompson, 1999a; 2001a). Finally, the term *mental calculation* was adopted in preference to the traditional term *mental arithmetic*. This was done to emphasise the fact that unlike mental arithmetic, which relied mainly on recall, mental calculation included using strategies and therefore needed to be taught more carefully (Thompson, 2001a). Interestingly, equivalent Dutch phrases for mental arithmetic and mental calculation were respectively *working in the head* and *working with the head.* According to Thompson (1999a; 1999b), *working in the head* suggested mental recall; while *working with the head* included the idea that when performing mental calculation children could write something down to help their thinking.

2.1.3 Standards-based Mathematics Curricula in the United States

In *Curriculum and Evaluation Standards for School Mathematics* (1989), the influential National Council of Teachers of Mathematics (NCTM) state that mathematics education in elementary (i.e. primary) school in the US should de-emphasise rote memorisation of rules and procedures and instead pay more attention to mental computation. A decade later, in *Principles and Standards for School Mathematics* (2000), the NCTM stated:

> Mathematics teaching in the lower grades should encourage students' [informal] strategies and build on them as ways of developing more general ideas and systematic approaches…. [Teachers] should honor individual students' thinking and reasoning and use formative assessment to plan instruction that enables students to connect new mathematics learning to what they know (p. 76).

Mainly because of this, mental computation became the defining characteristic of reform in US elementary mathematics classrooms (Carroll, Fuson and Diamond, 2000).

It would seem that increased interest in mental calculation in elementary school mathematics in the US was also influenced by the belief that doing mental calculation increased children's *number sense* (Carroll, 1996; 2000; Heirdsfield and Cooper, 2002; Kamii and Dominick, 1997; Sowder, 1992a). Number sense is a concept which mathematics educators particularly in the US and other countries (e.g. Australia) used to describe a wide range of mathematical abilities, including general understanding of number and operations; ability to develop and use appropriate solution strategies; and having an inclination and an ability to use quantitative methods to communicate, process, and interpret information (McIntosh et al., 1992). According to Heirdsfield and Cooper (2002), mental calculation promoted number sense in that it enabled children to learn about numbers, make decisions about procedures, and create solution strategies; and also promoted a greater understanding of number structure and properties, which helped children develop conceptual understanding.

Kamii and Dominick (1997) provided a more direct link between using mental calculation and developing number sense in a study in the US which involved children in grades 1 to 4 and their teachers. (I referred to the same study in Chapter 1, where I used it to describe the possible harmful effects on young children's numeracy development of an early introduction of formal place value concepts and use of standard procedures).

Kamii and Dominick (1997) arranged that for a whole year all children in Grade 1 were not taught (both in school and at home) the usual standard place value based procedures for addition and subtraction with regrouping; but were instead encouraged to develop and use their own strategies. Some Grade 2 and Grade 3 classes were taught standard algorithms while some were not; while all children in Grade 4 were taught the standard algorithms and followed the usual textbook programme. All participating classes were heterogeneous and comparable, since at each grade level children were mixed routinely at the beginning of each school year. At the end of the year the children were given a test involving multidigit addition and multiplication, some problems presented horizontally and others vertically, followed by an interview to allow them to explain their solution methods.

According to Kamii and Dominick (1997), the children in Grade 4, all of whom were taught only standard algorithms, produced correct answers less often than children in Grades 2 and 3 who did not learn to use the algorithms. Furthermore, interviews showed that even in cases where both the 'algorithm' and the 'no algorithm' children at the same grade level got correct answers, less than 25% of the 'algorithm' group could explain how they got answers compared to over 80% in the 'no algorithm' group. Kamii and Dominick also observed that, whereas the 'algorithm' classes added numbers in columns as per standard procedure, they often forgot how much they had carried and wrote down wrong answers. The ‘no algorithm’ classes on the other hand usually performed addition by splitting the numbers in multiples of tens and ones and adding the tens and ones separately, often getting correct answers.

More strikingly, for purposes of this study, was the finding that among children who were still unsure about place value, learning to use standard algorithms tended to make them lose even the little knowledge of place value they had. Kamii and Dominick (1997), agreeing with Plunkett (1979), concluded that teaching children to use standard algorithms redirected “their attention from trying to make sense of numbers to remembering procedures" (p. 59). Kami and Dominic (1997) therefore concluded that teaching algorithms was harmful to children's mathematical development because it 'untaught’ place value, discouraged children from developing number sense, thereby forcing them to give up their own thinking about number.

I felt that if the results obtained by Kamii and Dominick (1997) were true, it was time to reconsider the long held practice in primary mathematics in Zambia of teaching standard algorithms almost from Grade 1. However, I do not agree with what Kami and Dominich (1997) seem to be implying that because of the difficulties associated with young children's learning of standard procedures, such procedures should not be taught at all. The point of convergence of opinion in mathematics education in this regard seems to be that teaching of standard procedures should be delayed until children were old enough to understand their logic and to appreciate the fact that standard procedures are intended to make calculations more efficient and more accurate (DfEE, 1999).

2.2 Mental Calculation Strategies Used by Children

This section discusses the results of research concerning the kinds of strategies for mental calculation children use to solve a variety of addition and subtraction problems. Although determining the nature of mental strategies used by children in Zambia was not one of the aims of the present study, it was important to review these strategies because the study included a staff development workshop for primary school teachers, which sought to acquaint the teachers with the range of children's mental methods for double-digit whole number addition and subtraction. I begin the discussion of children's solution methods by looking at single-digit addition and subtraction since the informal methods children use for double-digit addition and subtraction are an extension of these.

2.2.1 Children's Strategies for Single-digit Addition and Subtraction

Children's fundamental understanding of addition and subtraction evolves from their experiences with early counting (Baroody and Standifer, 1993). Initially children solve addition problems using the strategy called *counting-all*. This, for example, involves grouping two sets together and using fingers or physical materials to count all items starting with those in the first set and finishing with elements of the second (Baroody and Standifer, 1993; Carpenter and Moser, 1983; Hughes, 1986; Nunes and Bryant, 1996). As children gain more experience, they begin using a more efficient strategy called *counting-on.* Beginning with the total of one set, usually the larger, they count on from there to obtain the sum of the two sets (Baroody and Standifer, 1993; Carpenter and Moser, 1983; Nunes and Bryant, 1996). According to Carpenter and Moser (1983), although children's use of the above strategies persists over a long period of time, ultimately they begin solving addition problems using number combinations rather than counting. For example, in working out the value of 5 + 7 children will first find the more familiar sum 5 + 5 = 10, and then add 2. In other words, they will rely on retrieval and use of familiar number facts to accomplish the addition.

Children use similar strategies to solve subtraction problems (Baroody, 1984; Baroody and Standifer, 1993; Carpenter and Moser, 1983). Supposing they are given the following problem: “You have five bananas, and you give three of the bananas to your brother. How many bananas do you have left?” To solve the problem, young children will rely on a number of strategies, including *separating from*, *counting down*, and *counting up* (Baroody, 1984; Baroody and Standifer, 1993; Carpenter and Moser, 1983). As Baroody (1984) points out, *separating from* involves removing from the larger set the required number of items and then counting the remainder to find the answer. *Counting down* is similar to separating from, in that it also depends on children’s understanding of subtraction as taking away. It involves “starting with the number of the initial amount, counting backward a number of times equal to the amount being taken away, and announcing the last number counted to indicate the amount left” (Baroody and Standifer, 1993, p. 79).

Because many children find counting backwards more difficult than counting forward, as their subtraction work begins to include increasingly larger numbers, they may start using the *counting up* strategy (Baroody, 1984). *Counting-up* involves starting with the smaller number (e.g. 2 in the statement 5 – 2 = ?) and “counting forward until the larger number is reached-while keeping in track the number of steps in the forward count” (Baroody and Standifer, 1993, p. 79). Eventually, children will start using number combinations rather than counting in solving subtraction problems, which number combinations are often based on addition rather than subtraction. For example, to explain why 13 – 7 = 6, children might simply say they knew that 7 + 6 = 13 (Carpenter and Moser, 1983, p. 22).

2.2.2 Children’s Strategies for Double-Digit Addition and Subtraction

Children’s informal methods for multi-digit whole number addition and subtraction are closely related to, and an extension of the counting-based methods they use to add and subtract single-digits (Fuson et al., (1997). There are, however, so many varieties of these double-digit addition and subtraction methods that a complete listing of them is impossible, particularly that the same child can use a different method to solve the same question at different times and under different circumstances (Threfall, 2000).

Nevertheless, researchers have attempted to classify these methods according to certain characteristics (Suggate, 1995; Fuson et al, 1997; Thompson, 1999c; Threlfall, 2002). I discuss below the classifications developed by Fuson et al (1997), which is one of the most comprehensive and yet easy to understand. Fuson et al (1997) identify four kinds of informal methods children use to solve problems involving double-digit whole number addition and subtraction, namely: (a) Beginning with one number and moving up or down by tens and ones; (b) Decomposing tens and ones and adding or subtracting the tens and ones separately; (c) Changing both numbers; and (d) Mixing the above strategies. (As mentioned in Section 2.1.1, Dutch educators have collapsed this same range of methods into two main categories, namely *1010* and *N10*).

According to Fuson et al (1997), using the first strategy to find the value of 27 + 36 involves beginning with 27 and counting-up in sequences of tens and ones (i.e. 27, 37, 47, 57, 58, 59, 60, 61, 62, 63) to arrive at 63. The subtraction 55 – 33 would be done in a similar way, but by counting-down from 55 (i.e. 55, 45, 35, 25, 24, 23, 22) until one gets to 22. Using the second method, one way to obtain the sum 27 + 36 would be to write 20 + 30 = 50 and 7 + 6 = 13, then adding up the totals, that is 50 + 13 = 63. Changing both numbers involves compensating. For example, instead of performing the addition 27 + 36 directly, a child could change the numbers to ones that are more convenient to work with, and then make adjustments to compensate for the changes made. That is, 30 + 35 = 65, taking away 3 and adding 1 gives 63. Using mixed methods simply means combining the above methods.

Before proceeding, it is important to pause and ask two related questions about children's use of strategies for mental calculation: How do young children develop the numerical skills mentioned above? Why are they more likely to commit errors when they use adult methods (such as those normally prescribed in school textbooks) than they do when they rely on their own informal solution methods? With regard to the first question, the answer is simply that young children develop numerical abilities through interaction with their physical and social environment (Aubrey, 1993; Ginsburg and Bacon, 1993; Guberman, 1999); and seem to do so almost as naturally as they acquire language

(Starkey and Gelman, 1982). It would seem, also, that there is virtually nowhere in the world where children could grow up without their environment proving them with opportunities to develop numerical abilities (Ginsburg and Baron, 1993). Ginsburg and Baron ask: "In what culture, however impoverished, does the child lack things to count? In what culture cannot one add to what one had before? *Mathematical phenomena appear to be universal in the physical world*"(p. 5, my emphasis). It is not surprising then that by the time children begin school around the ages of six or seven, they already know something about solving addition and subtraction problems involving small numbers (such as 2 and 3), provided the problems are imbedded in real or hypothetical contexts (Hughes, 1981; 1986). However, variations in mathematical knowledge have been observed at these ages among children belonging to different social classes and cultures. For example, before receiving formal instruction in mathematics children from poorer residential areas in western countries tend to perform less well than those from the more affluent families (Hughes, 1981; 1986); while Asian children such as those in China and Japan tend to outperform their peers in western countries, such as the United States (Guberman, 1999).

Concerning the second question, the reader will remember that in Chapter 1 I argued that while young children generally fail to follow the logic behind adult solution methods, they use their own informal solution strategies with understanding. Nunes and Bryant (1996) explain why this might be the case:

> [Children's informal methods allow] them to calculate and to think of the values that they are working with at the same time. When tens and hundreds are added…*they are spoken of as tens and hundreds*. In contrast, in [standard] written arithmetic we set the meaning of numbers aside during calculation. We operate with digits and speak about them as if they were all units, following the same rules as we move from units to tens and hundreds. This approach seems to detach the children from the meaning of what they are trying to calculate and thereby makes it easier for bugs [i.e. errors] to appear in their solutions (p. 107, emphasis in original).

Given the above statement, it seems surprising that until the present study was conducted, mathematics educators in Zambia had not considered the potential benefits of using children's methods to improve mathematics performance in the early primary grades. I now turn to the question of whether or not strategies for mental calculation can be taught.

2.3 Teaching Strategies for Mental Calculation

Two main views have emerged in mathematics education concerning the teaching of strategies for mental calculation. One view, widely shared in the US, is that children can and do *invent* mental strategies for solving a variety of arithmetic problems. They do not need to be taught such strategies *formally*, but should be *encouraged* to invent them (Carpenter et al, 1997; Carpenter et al, 1999; Carroll, 1996; 2000; Foxman and Beishuizen, 1999; 2003; Fuson et al, 1997; Heuser, 2005; Suggate, 1995; Thompson, 1994; 1999; 2001). In this connection Grouws and Cebula (2000) state:

> To increase the opportunities for invention teachers should frequently use non-routine problems, periodically introduce a lesson involving a new skill by posing it as a problem to be solved, and regularly allow students to build new knowledge based on their intuitive knowledge and informal procedures (p. 18).

This may be the reason why the Cognitively Guided Instruction (CGI) Project in the US, whose aim was to help teachers incorporate children's mathematical thinking in the classroom, did not prescribe particular techniques for teaching mental calculation strategies: As CGI researchers Franke et al. (1998) stated: "We try not to direct the ways in which the teachers choose to implement their teaching practice. There does not exist one way of implementing CGI" (p. 69).

The other view, prominent in Europe particularly in the UK and the Netherlands, is that there is a need to teach children strategies for mental calculation to help make children's methods more efficient and to allow more children to have access to a variety of strategies through classroom discussion (Askew, 1998; Beishuizen, 1993; DfEE, 1999; Mosley et al., 2001; Thompson, 2001). Furthermore, research evidence particularly from the UK suggested that teaching strategies for mental calculation made a difference in children’s development and use of such strategies (Threlfall, 2000; 2002). Indeed, it has been observed that “it is higher attaining children who employ a range of mental calculation strategies, whereas below average children often rely on inefficient counting procedures or taught formal algorithms” (Murphy, 2004, p. 4). Therefore, not to teach such strategies would be to deny below average learners the chance to learn mathematics meaningfully. Consequently, for many mathematics educators in Europe the important question is not *wheth*er strategies for mental calculation can be taught, but *how.*

Nevertheless, some mathematics educators in Europe have expressed concerns that teaching strategies for mental calculation *formally* could lead children to use them mechanically, as they do with standard procedures (Heuser, 2003; Murphy, 2004; Threlfall, 1998). In view of this, Threlfall (2000) has suggested that the teaching of strategies for mental calculation should focus on *developing* rather than *acquiring* strategies. He suggests that teachers can achieve this by promoting a culture of invention among children, which includes helping them to believe that there are no right or wrong ways to get answers to problems; that methods which work for some people may not work for others; that they should work as they see fit without worrying too much about how others get answers; and that they should be willing to experiment even if that meant sometimes being unsuccessful. This was the view adopted in the present study, whose main aim was to determine the extent to which primary school teachers in Zambia could learn to teach in this way.

2.4 Theories of learning reflected in developments in early primary mathematics teaching and learning.

As the reader will have noted, the changes in numeracy teaching and learning practices described in this chapter represent a fundamental redefinition of what counts as viable mathematical activity for young children. They reflect a *social constructivist* approach to mathematics teaching and learning (Cobb et al., 1993; Simon, 1993; Cobb, 1995; Yackel and Cobb, 1996), which states that mathematics learning and activity involves "the *individual* construction of knowledge, the *social* construction of knowledge, and the *interaction* between the two" (Simon, 1993, p. 100, my emphasis). Since the present study was guided by this same theory of learning, I will spend some time explaining what social constructivism means and try to show how it relates to my study.

Social constructivism is a result of combining aspects of *radical constructivism* and *sociocultural theories of learning* (Confrey, 1995). To be clear about its meaning, it is important to know something about how radical constructivism and sociocultural theories of learning may be distinguished from each other. In view of this, clarifying this distinction will be my focus in the next two sections (i.e. sections 2.4.1 and 2.4.2) below. This is followed in Section 2.4.3 by a discussion of the debate that has raged in the past two to three decades among educators/researchers, concerning whether it is the radical constructivists or the sociocultural theorists who provide a more valid explanation of the process of human knowledge acquisition. In this regard, I will argue that it is not possible to explain adequately the process of human knowledge acquisition using as a theoretical basis *either* radical constructivism alone *or* sociocultural theories of learning on their own, since explaining this process requires taking into consideration the views of *both* radical constructivists and sociocultural theorists. In other words, it requires adopting a social constructivist perspective. I use Section2.4.4 to highlight implications of social constructivism for the mathematics classroom.

2.4.1 Radical Constructivism

In 1987 Ernest von Glasersfeld, the leading advocate for the adoption of constructivist ideas in mathematics and science education in the last three decades (Steffe and Kieren, 1994; Matthews, 2000; McCarty and Schwandt, 2000), stated that about ten to fifteen years earlier mathematics educators believed that knowledge could be *transferred* from the teacher to the student, and that the only question was *how* best to implement the transfer. In other words, it was believed that it was possible for mathematics learners to apprehend *accurately* what they are taught.

This view of mathematics learning is an example of what is called the *representational view of mind* (also known as the *correspondence theory*), which holds that *to know* is to represent accurately in one's mind what is outside the mind (Cobb et al., 1992). In other words, "knowledge consists in having a mental representation that corresponds to reality *as it really is*" (Chapman, 1999, p. 31, emphasis in original). As Chapman (1999) points out, on the surface this view of learning appears to be plausible, as illustrated by the fact that when a person who knows a particular object is asked to describe it, he or she will first try to *imagine* how the object looks and, on seeing the object again, check how closely his/her description *matched* reality.

However, accepting the representational view of mind creates problems. For example, it has been observed that the representational view of mind "conflicts with the empirical finding that mathematical meanings are socially and culturally situated" (Cobb et al., 1992, p. 7). That is, the context and circumstances in which a person learns mathematics influences *what* is learned, as well as *how* the subject is understood. This point is illustrated for example by studies conducted in Brazil, which indicated that child candy sellers' construction of mathematical meanings was closely linked to their participation in the activities of their trade (Cobb et al., 1992).

The theory of constructivism is in direct opposition to the representational view of mind. It posits that learning is an active, individual process of knowledge construction by the cognising agent, rather than a process of absorption or copying (Jaworski, 1988; Simon and Schifter, 1991; Parcker and Goicoechea, 2000; Windschitl, 2002; Terhart, 2003; von Glasersfeld, 1993; 1995). In other words, "the learner is not *given* knowledge, but actively constructs it [him or] herself, and that learning, or coming to know, is a process of adapting one's view of the world as a result of this construction" (Jaworski, 1988, p. 292, emphasis in original).

It is important to mention that the use of the term *constructivism* in the literature can be confusing because different authors may define it differently (Burbules, 2000; Windschitl, 2002). In fact, according to Nola (1997) (cited in Windschitl, 2002) there may be at least a dozen conceptions of constructivism out there. My reference in this chapter has been to that form of constructivism where, as Phillips (2000b) puts it, the centre of interest "is the *psychological* understandings of individual learners" (p. 7, my emphasis). In the literature, this form of constructivism is distinguished from other existing varieties by using a number of expressions with similar meanings, including *psychological constructivism*, *cognitive constructivism*, *individual constructivism, Piagetian constructivism*, and *radical constructivism,* all of which are intended to indicate that knowledge construction is an *individual*, *inward* process (Rodriguez, 1998).

Although current interest in the constructivist approach to mathematics learning only emerged in the last quarter of the 20th century (Steffe and Kieren, 1994), the theory of constructivism itself has a long history (Phillips, 2000a). For example, constructivist views are expressed in the works of Immanuel Kant in the eighteenth century; and in the writings of John Dewey in the early part of the twentieth century (Jaworski, 1994; Terwel, 1999; Phillips, 2000b; Howe and Berve, 2000; Packer and Goicoechea, 2000). Bredo (2000) describes Immanuel Kant as being "undoubtedly the principal originator of constructivist thought in philosophy and psychology" (p. 128); and attributes to him the unquestionably constructivist statement that, "we only know nature as a sum total of…images or representations in our mind" (Kant, 1783, cited in Bredo, 2000, p. 129). Liebert and Liebert (1995) clarify this point:

According to Phillips (2000b) John Dewey was a constructivist because

> He stressed that learners must be active, he advocated the use of projects and inquiry methods, he attacked the acquisition by students of 'cold storage knowledge' (knowledge that was acquired passively or by rote and that students did not know how to use), he regarded learning as best proceeding in social contexts, and wanted the classroom to be seen as an interactive community" (p. 14).

While I cannot authenticate the claim that Immanuel Kant was a constructivist, because of lack of access to his writings, I can see why John Dewey is regarded as having been one. In *Democracy and Education* (1916), considered as his most comprehensive statement on education (Park, 1968; Hansen, 2002), Dewey makes statements that seem to align him with modern constructivists. He criticises 'passive' school education, describing it as mostly "[consisting] in setting up rules by which pupils are to act of such a sort that even after they have acted, they are not led to see the connection between the result-say the answer-and the method pursued", adding, "So far as… [the pupils] are concerned, the whole thing is a trick and a kind of miracle" (p. 81). In another of his works Dewey (1923) condemns the results of such an education:

> How many students…were rendered callous to ideas, and how many lost the impetus to learn because of the way in which learning was experienced by them? How many acquired special skills by means of automatic drill so that their power of judgement and capacity to act intelligently in new situations was limited? How many found what they did learn so foreign to the situation of life outside the school as to give them no power or control over the latter?' (cited in Simpson, 2001, p. 196).

Dewey makes a more direct reference to constructivist approaches to teaching and learning in *Democracy and Education*, when he describes the process of education as one involving "continual reorganizing, reconstructing, [and] transforming" (p. 78) ones' experiences. This statement sounds rather like modern constructivists' argument that knowledge is actively constructed by the learner and not passively received from outside. Indeed, as Mason (1959) observes:

> In [John Dewey's]…notion that there is always some experience present (hence, the use of the term *re*construction), we have the recognition of experience as something a person has. In the notion that this is reconstructed through social participation, we

> have the recognition of something external to the individual making an impact upon and taking effect in the growth and development of the person (p. 116).

Mason's analysis of John Dewey's statement makes it clear that he (John Dewey) was indeed a constructivist, in fact a *social* constructivist, as I will define the term in Section 3.3 below.

Most commentators however agree that the current constructivist movement in education which began in the mid-1970s has its primary roots in the research and theorising of the Swiss psychologist Jean Piaget (1896 – 1980) (Gallagher and Reid, 1981; Crain, 1992; Jarwoski, 1994; Steffe and Kieren, 1994; Cobb, 1994; von Glasersfeld, 1995; Rodriguez, 1998; Gunstone, 2000; Howe and Berv, 2000; Matthew, 2000). Piaget believed that intellectual development from infancy to adulthood is characterised by movement through a number of stages, each marked by particular ways of thinking about the world, called *cognitive schemas* (Bourne and Russo, 1998; Fontana, 1995; Goswami, 1998; Flavell, Miller and Miller, 1993; Piaget, 1962 and 1973; Piaget and Inhelder, 1969).

According to Piaget, this movement is made possible by means of two main psychological processes, *assimilation* and *accommodation* (Crain, 1992; Piaget, 1950). Assimilation refers to the child's "relating of new information to pre-existing structures of knowledge and understanding" (Meadows, 1993, p. 198), illustrated for example by the action of an infant who, assuming that all objects can be sucked, tries to assimilate a new object by sucking. Accommodation, on the other hand, refers to the child's modifying of existing cognitive schemas when these become inadequate to cope with new information (Piaget, 1950). Thus Piaget viewed knowledge acquisition as a process whereby through interacting with the environment an individual uses past experience to *construct* new knowledge (Crain, 1992; Bettencourt, 1993), which view of human knowledge acquisition is essentially the same one held by many modern constructivists.

Indeed Ernest von Glasersfeld admits that his views on constructivism are based on those of Piaget's. He states: "Although I do not continually cite Piaget, I sincerely hope one realizes that almost everything I write herein can be written only because Piaget spent some 60 years establishing the basis for a dynamic constructivist theory of knowing" (von Glasersfeld, 1995, p. 6). Nevertheless, von Glasersfeld's own version of Piaget's psychological constructivism, called *radical* constructivism, has been criticised as being extreme, particularly with regard to the claims he makes concerning what it means for an individual "to know" something (Burbules, 2000; Ernest, 1998; Howe and Berv, 2000). According to Von Glasersfeld, radical constructivism rests on two main principles, namely that:

1. Knowledge is not passively received but actively built up by the cognising agent;

2. The function of cognition is adaptive and serves the cognisation of the experiential world, not the discovery of ontological reality (von Glasersfeld, 1987a, cited in Jaworski, 1994, p. 16).

The first principle given by von Glasersfeld's, which boils down to saying "knowledge is made rather than found" (Bredo, 2000, p.131), is uncontroversial. According to Burbules (2000), it is almost a truism to say that all learning is constructed since "understanding and evaluating new ideas and skills, even those of the most apparently rote character, requires reinterpreting them in the light of one's existing understandings and abilities" (p. 327). This might explain why, as Jaworski (1994) states, von Glasersfeld refers to people who accept his first principle alone as *trivial* constructivists.

It is the statement of the second principle that is the controversial part and whose acceptance, according to Von Glasersfeld, makes one a *radical* constructivist (Jaworski, 1994). As Bredo (2000) puts it, this principle amounts to saying that "*the objects and properties that we experience and know are…in some manner products of human (mental or physical) activity* (p. 131, emphasis in original). This view is endorsed by von Glasersfeld himself:

> What is radical constructivism? It is an unconventional approach to the problems of knowledge and knowing. It starts from assumptions that knowledge, no matter how it is defined, is in the heads of persons, and that the thinking subject has no alternative but to construct what he or she knows on the basis of his or her own experience. What we can make of experiences constitutes the only worlds we consciously live in…Taken seriously, this is a profoundly shocking view" (von Glasersfeld, 1995, cited in Phillips 2000b, pp. 10 - 11).

What von Glasersfeld says above is indeed shocking because it "isolate[s] each of us in a universe of our own construction" (Phillips, 2000b, p. 12). The unpalatable implication of von Glasersfeld's statement is that "individual knowers can construct truth that needs no corroboration from outside of the knower, making possible any number of 'truths'"

(Howe and Berv, 2000, pp. 32-33). Put differently, this would mean that "no form of knowing can make any kind of a claim to have a higher value or privileged position" (Terhat, 2003, p. 27).

As Howe and Berv's (2000) observe, accepting this view of learning would create difficult pedagogical puzzles for schools: "What is the teacher trying to teach…if [students] are all busy constructing their own private worlds? What are the grounds for getting the world right? Why even care whether these worlds agree?" (p. 33). Howe and Berv conclude, therefore, that for school learning to be meaningful there should be "a shared world", that is a world made up of shared meanings. Serpel (1993) also shares this view. He states that "many of society's institutions are based on the premise that knowledge is shared. *No laws, or schools, or libraries would make any sense in the absence of this premise. No communication could take place*" (p. 357, my emphasis). As will become clear in Section 3.3, acceptance or rejection of the idea of a "shared world" is a key differentiating characteristic between the educational implications of radical constructivism and those of social constructivism. But before we can look at that in detail it is important first to discuss sociocultural theories of learning.

2.4.2 Sociocultural Theories of Learning

Sociocultural theories of learning on the other hand emphasise the role played by sociocultural factors in shaping people's knowledge construction, thereby viewing knowledge acquisition primarily as a process of enculturation (Crain, 1992; Cobb, 1994; Packer and Goicoechea, 2000, Windschitl, 2002). An important example of sociocultural theories of learning is *situated cognition* theory (Packer and Goicoechea, 2000), which has attracted much attention in education circles in recent years because of a recognition that context plays a significant role in learning (Van Oers, 1998). Situated cognition theory states that learning is closely linked to the context under which it occurs (Billet, 1996; Kirshner and Whitson, 1997; Putnam and Borko, 2000; Schliemann, 1998; van Oers, 1998). More specifically, "How a person learns a particular set of knowledge and skills, and the situation in which the person learns, become a fundamental part of what is learned" (Putnam and Borko, 2000, p. 4). Thus, like other sociocultural theories of learning, situated cognition theory emphasises the importance of the social context in which learning takes place.

Sociocultural theories of learning are associated with the name of the Russian psychologist L. S. Vygotsky (1896-1934) (Cobb, 1994; Packer and Goicoechea, 2000). Vygotsky believed that in order to understand an individual's mental functioning, it is important to examine his or her sociocultural background (Crain, 1992; Wertsch, 1985). Vygotsky's views in this regard were influenced by Marxist ideas, which were being promoted in the former Soviet Union at the time Vygotsky was conducting his researches in the 1920s and the early 1930s (Cole and Scribner, 1978; Confrey, 1995; Crain, 1992; Wertsch, 1985). Karl Marx argued that history is a dialectical process (i.e. a series of conflicts and resolutions) and that historical changes in society and in material life produced changes in human nature (Cole and Scribner, 1978). Furthermore, according to Crain (1992), Marx believed that "What people think…depends on their material life-the ways in which they work, produce, and exchange goods-at a certain point in historical development" (p. 196). Crain (1992) adds that Marx's collaborator, Friedrich Engels extended the argument by stating that, it was not only what people thought that depended on historical development but also their cognitive capacities, which for example changed in line with technological development. Indeed, Engels argued further, tool-use affected the way humans think, so that once our ancestors began to use tools their minds expanded and they began to think and view their environment differently (Crain, 1992).

Vygotsky was deeply impressed by these ideas. He concluded that, "social changes were accompanied by fundamental changes in thought processes" (Cole, 1976, p. xiii). In particular, Vygotsky argued that the appropriation of cultural tools and sign systems (e.g. language, numerical systems, writing) did not simply facilitate an individual's mental functioning, but created new behaviours and fundamentally transformed the way in which the mind functioned (Vygotsky, 1978; Wertsch, 1985; Wertsch and Tulviste, 1996). With regard to language in particular Vygotsky argued that it was the single most important psychological tool possessed by humans, which set them apart from other animals (Vygotsky, 1978; Crain, 1992). And unlike Piaget, who regarded young children's speech to be merely "egocentric", a product of their inability to understand the world (e.g. Piaget, 1962), Vygotsky attributed an important role to children's speech. He argued that children's speaking aloud to themselves, which Piaget believed indicated the egocentric nature of their speech, was actually a way of reasoning as they solved problems (Vygotsky, 1962; 1978). According to Vygotsky (1978), "The more complex the action demanded by the situation and the less direct its solution, the greater the importance played by speech in the operation as a whole" (pp. 25-26). He further argued that as children grew older, and speaking aloud to themselves appeared to fade, they had actually began to carry out such dialogues more inwardly and silently (Vygotsky, 1962; Crain, 1992). Vygotsky therefore believed that speech did not simply enable children to participate intelligently in the social life of their group; it played another role, that is facilitating their own, individual thinking (Crain, 1992).

Vygotsky further argued that in order to reach the highest levels of thinking, including learning concepts and applying them correctly, children needed instruction by adults or more able peers (Vygotsky, 1978; Crain 1992; Meadows, 1993). In relation to this, Vygotsky (1962) envisaged the process of concept formation, which began in childhood, to take a long time to complete since the psychological tools children needed to complete the process only developed in puberty.

According to Vygotsky (1962), concept formation involves three basic phases. The first, called the *syncretic heaps* phase, is characterised by children grouping objects by trial and error, because of poorly understood perceptual factors. The objects grouped together may not even be related at all but the child, in his or her own subjective thinking, assumes that they are related. However, if it is suggested to the child that his or her basis for grouping is wrong, he or she will revise the basis for grouping, but again based on poorly understood perceptual factors.

The next phase towards concept formation, called *Thinking in Complexes,* is an improvement on the child's understanding of the basis for grouping together similar objects. According to Tuomi (1998), Vygotsky believed that this phase of concept formation is an improvement on the previous one because, whereas during the syncretic heaps phase the child mistakenly believed that his or her own subjective impressions reflected actual relationships among objects, in this phase his or her basis for grouping objects is determined by connections that actually exist among the objects. Vygosky (1962) however argues that "a complex is not formed on the plane of abstract logical thinking, [implying that] the bonds that create it, as well as the bonds it helps to create, lack logical unity…" (p. 62). In other words, thinking in complexes does not yet reflect the ways in which adults form concepts.

Vygotsky (1962) states that advanced thinking in complexes results in formation of *pseudo* (or *potential*) concepts, as opposed to *genuine* (or *true*) concepts. Pseudo concepts act as a bridge between thinking in complexes and true concept formation; and the words children use to represent such concepts often tally with the meanings that adults ascribe to the same words, which facilitates communication between children and adults. However, Vygotsky (1962) also believed that although children's use of pseudo concepts might coincide with adult's use of the same concepts, it did not mean that they (children) had learned to generate abstract concepts, as the thinking behind them was different. Berger (2005) clarifies this point using mathematics as an example:

> The pseudo concept can be used to explain how the student is able to use mathematical signs (in algorithms, definitions, theorems, problem-solving, and so on) in effective ways that are commensurate with that of the mathematical community even though the student may not fully 'understand' the mathematics object" (p. 159).

According to Vygotsky (1962), the third and final phase towards formation of concepts is development of adult concepts, which he divided into two categories: *spontaneous* (or *everyday*) concepts and *scientific* concepts. He believed that children can acquire spontaneous concepts (such as the concept of *house*) through everyday experiences. As Wellings (2003) states,

> The development of spontaneous concepts can be considered an inductive process. Grounded in the concrete activities of everyday life, spontaneous concepts are formed through the aggregation and synthesis of lived experiences. Similar to the process of inductive reasoning, the development of spontaneous concepts is dependent on pattern recognition, comparisons made between multiple events, reflection on activities, and the use of analogical reasoning" (pp. 6 – 7).

According to Crain (1992), Vygotsky argued that when children acquired concepts this way, the meanings they ascribed could be restricted. For example, although children could use the concept of *Grandmother* correctly in communications with adults, their understanding of its meaning might be limited in the sense that its application was tied to a particular person, their own grandmother. In view of this, an understanding of the true concept could only be achieved when children, for example, listened to the teacher explaining family relationships using a family tree.

With regard to acquisition of scientific concepts Vygotsky believed that children needed instruction from teachers (Vygotsky, 1962; Wellings, 2003). Nevertheless, Vygotsky (1962) also believed that both spontaneous and scientific concepts played an important role in the process of concept formation; and that there was, in fact, a kind of symbiotic relationship or interdependence between the two kinds of concepts: Otero and Nathan (2008) clarify this point:

> As academic [or scientific] concepts are introduced, often in formal learning environments, a learner filters [their] interpretations…(including terminology and symbols) through his or her experience-based understanding of the world. At the same time, the learner's experience-based concepts evolve as he or she attempts to cast them in the form of academic language. Through this process, learners try out academic language presented thorough schooling, increase awareness of their own experience-based concepts, and begin to develop academic language that allows them to generalize their experience-based concepts beyond the concrete experiences to which they were tied (p. 500).

In discussing the importance of instruction to enable children achieve the highest levels of cognitive development Vygotsky introduced the notion of the *Zone of Proximal Development* (ZPD), which he argued represented the gap between a child's assisted and unassisted intellectual potential. He defined the ZPD as

> [The] distance between the [child's] actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers (Vygotsky, 1978, p. 86).

Vygotsky introduced the ZPD to deal with two practical problems in psychology: full assessment of children's intellectual abilities; and evaluation of the quality and effectiveness of instruction (Wertsch, 1985; Meadows, 1993; Wertsch, Tulviste and Hagstrom, 1993). With regard to assessment Vygotsky argued that conventional tests were inadequate since they only indicated the child's actual developmental level but said nothing about the child's ability to learn new material beyond his or her present levels of cognitive functioning (Werstch, 1985; Crain, 1992). Concerning instruction Vygotsky argued that there was little benefit to be gained by aiming teaching at the bottom of the ZPD because the child's mental functioning there was already mature. Neither was there profit in teaching aimed above the top of the ZPD, since the tasks the child may be required to undertake may then require mental abilities that he or she could not master (Meadows, 1993). Thus for Vygotsky (1978) the only good learning was that which was towards but not above the upper part of the ZPD. Vygotsky thus believed that the ZPD would give teachers a truer picture of each child’s potential (Crain, 1992).

The foregoing suggests that Vygotsky's theory of cognitive development comprises three main points. Das Gupta and Richardson (1995) have summarised these as: social interaction as the main motor of cognitive development; use of cultural tools, which shaped one’s knowledge and thought; and importance of instruction by adults or peers.

I mentioned in the previous section that psychological constructivist accounts of cognitive development have been criticised for being one-sided, in that they focused too narrowly on learners’ *individual* construction of knowledge, neglecting sociocultural influences on the constructions. Sociocultural theories of learning have also been criticised for being one-sided. For example, Vygotsky argued that understanding the mental functioning of individuals required studying the interaction between developmental and cultural forces, but his own studies and those of many of his fellow soviet theorists in this area mostly focused on cultural forces (Wertsch, 1985; Crain, 1992). Furthermore,

> [Vygotsky] studied the ways in which speech, memory aids, writing, and scientific concepts transform the child's mind, but he didn't examine the ways in which the child's inner, spontaneous development might affect cultural forces….He gave us a good picture of how children internalize their culture, but he told us little about how they might challenge or criticize their culture, as a stubborn two-year old or an idealistic adolescent might do (Crain, 1992, p. 218).

Vygotsky's emphasis on the importance of instruction in cognitive development has also been criticised. His theory allows the possibility that adults will focus too much on children's future development, pushing them forward, before they have developed fully the character of their present stage (Crain, 1992). It has also been pointed out that assisting children in the manner Vygotsky suggests could undermine children’s ability to think for themselves, as it might encourage them to depend on others to know what and how to think (Crain, 1992; Confrey, 1995). In view of this, Confrey (1995) has concluded that Vygotsky's theory of instruction may be in conflict with the need for children to be inventive and creative or to express dissent; and thus could legitimise authoritarianism on the part of adults.

Although Vygotsky's name is the one often mentioned in connection with the development of sociocultural theories of learning, he did not work alone. His colleagues A.N. Leontiev and A.R. Luria also made important and significant contributions in this regard (Meadows, 1993; Minnick, Stone and Forman, 1993). In fact Meadows (1993), reviewing the relevant literature on the subject, argues that it may be difficult to determine the extent to which what is currently believed to be Vygotsky's views concerning development of sociocultural theories of learning are actually his own. Meadows' argument in this regard is based on three main points. First, Vygotsky died prematurely with his work incomplete, so that his ideas were developed by others, who have disagreed on some aspects of his work. Second, similar ideas were made contemporaneously by other theorists and had also appeared a century earlier; so that what we understand as Vygotsky's theory in this regard may have incorporated some of these insights. Finally, Vygotsky was translated into English from the Russian only comparatively recently; and there have been severe and pervasive problems in translating him. Therefore, the understanding and interpretation of his work by those who cannot read Russian may be affected by the quality of the translation.

While it is not my intention in this chapter to defend Vygotsky's legacy, but to show in what ways social cultural theories of learning differ from radical constructivism, it may be useful to see what can be said about the three issues Meadows raises above. It seems to me that apologists for Vygotsky would not have much difficulty responding to Meadows' first two points. Vygotsky indeed died prematurely and his work was taken up by his close collaborators (such as Luria) and his students. Because of this, my speculation is that the extent of disagreement over aspects of Vygotsky's work would not be great. Yanitsky and Ferrari (2008) indicate that the studies conducted by Vygotsky's colleagues and students were based on research principles set out by Vygotsky himself, who thus in a way continued to speak through them. Indeed, "Not only was Lev Vygotsky an extremely talented and versatile psychologist, he was also a gifted teacher, *fostering a generation of young scholars who continued his wide-ranging research, even after his untimely death in 1934*" (Yanitsky and Ferrari, 2008, p. 119, my emphasis). With regard to Meadows' second point, it necessary only to point out that knowledge is not a private entity; it can be used and modified by others so long as they acknowledge the source. Nevertheless, one can claim that Vygotsky clarified issues relating to sociocultural theories of learning to such an extent that it makes sense to think of him as the main brain behind their development.

It is Meadows' third point that seems to create a problem. Gillen (2000) argues that English translations by different authors of the same original works by Vygotsky have tended to result in different versions of Vygotsky, supporting Meadows' (1993) view that educators who have not read Vygotsky's works in the original Russian may have a mistaken view of his ideas. Gillen further states that even English titles of Vygotsky's best known books are mistranslations of original Russian titles. For example, according to Gillen a more accurate translation of the Russian title of Vygotsky's book *Thought and Language* (1962) should be *Thinking and Speech*. "*Thinking and Speech* is more concrete and immediate…than the more abstractly philosophical [title] *Thought and Language*. The effect of the mistranslation [has been] to move away from the active register of pedagogy into epistemological realms" (Gillen, 2000, p. 187), thereby somewhat changing the focus of Vygotsky's ideas. Because of these and other issues related to translation of Vygotsky's works from the original Russian to English, Gillen feels that "we in education….should question what we think we know about Vygotsky's work" (p. 185).

Does this mean that Vygotsky's popularity among educators will now diminish? It is difficulty to tell. However, it is clear that "heated debates about Vygotsky's legacy [will] continue" (Yasnitsky and Ferrari, 2008, p. 130). In view of this, it isunsafe to conclude that current revelations about translation problems in Vygotsky's works constitute the final judgement on his legacy.

Having discussed salient features of both radical constructivism and sociocultural theories of learning, we are ready to look at social constructivism.

2.4.3 Social Constructivism

In the last two sections, my aim was to distinguish between radical constructivism and sociocultural theories of learning. This distinction may be summarised as follows:

> [From] one perspective [radical or cognitive constructivism], learning is viewed primarily as a process of active individual construction, whereas, from the other perspective [sociocultural theories of learning], it is viewed primarily as a process of enculturation into established…practices (Cobb, 1995, p. 364).

There has been intense debate over the years with regard to which of these two views of learning provides a truer explanation of how humans develop knowledge (von Glasersfeld, 1993; Bereiter, 1994; Cobb, 1994; 1995; Ernest, 1998; Packer and Goicoechea, 2000). However, many commentators now believe that this debate may be unnecessary since there is a sense in which learning can be viewed *both* as a process of active individual construction of knowledge and enculturation (Bredo, 2000; Burbules, 2000; Cobb, 1994; Jaworski, 1994; Schliemann, 1998; Schwandt, 2003; Tobin and Tippins, 1993). Schwandt (2003) for example argues that individual construction of knowledge always has a historical and sociocultural dimension, in that it occurs "against shared understandings, practices, language, and so on" (p. 305). Tobin and Tippins (1993) clarify this point:

> [Knowledge] is personally constructed but socially mediated. That is, knowledge only exits in the minds of cognizing beings, but cognizing beings only exist in a socio-cultural sense. From the outset, an organism constructs knowledge in the presence of others who are able to perturb the environment in such a way that a learner's experiences are constrained by the presence of others (p. 6).

Furthermore, it has been noted that when Piaget argued that learning was a process of active individual construction he did not mean that cognitive development arose exclusively from such constructions of reality (Crain, 1992; von Glasersfeld, 1995; Duveen, 1997). According to von Glasersfeld (1995), a careful reading of Piaget's original works would reveal that "in almost every book he reiterates that the most important occasions for accommodation arise in social interaction (p. 11).

Based on the above considerations and on the results of their own research, Cobb and his associates have argued convincingly that the process of mathematics learning can only be understood if we accept that mathematical knowing has a *social* as well as a *cognitive* aspect (Cobb et al., 1994). "Once we do so, we begin to understand how it is possible for students to construct for themselves the mathematical practices that, historically, took several thousand years to evolve" (Cobb et al., 1994, p. 28).

According to Simon (1993), in arguing this way Cobb and his associates have succeeded in bringing together the cognitive and social sides of learning "to develop a useful social constructivist lens for looking at mathematics activity in classrooms" (p. 99). In fact, it would seem that the very nature of mathematics and the nature of the demands of its teaching and learning render the subject amenable to investigation through social constructivist lens. As Confrey (1995) notes,

> Mathematics has the dual character of being both a language (a symbol system) and an underlying model of relationships among actions with objects. As such, it fits closely with the Vygotskian description of sign-sign relationships and decontextualised knowledge. At the same time, its development in relation to human actions on objects gives it a prominent place in Piagetian analysis. Furthermore, mathematics teaching requires the recognition of mathematics as a sociocultural achievement worthy of reproduction in new generations (p. 203).

It is this social constructivist position that I have adopted as the theoretical framework for the present study. This view of learning has in the last two to three decades guided a number of successful curriculum innovations around the world, not only in mathematics but also in science. A good example in this regard is the mathematics education project called Cognitively Guided Instruction (CGI), which started in the mid-1980s in the US. The main aim of CGI was to help mathematics teachers to understand children's thinking about mathematics and how to use this in mathematics teaching (Knapp and Peterson, 1995; Fennema et al., 1996; Franke et al., 1997; Carpenter et al., 1999). As the above authors indicate, CGI used a social constructivist approach both during workshops for teachers (to help them become acquainted with empirical findings on children's mathematical thinking and to re-examine and discuss their own thinking in this regard) and in the school mathematics classroom.

Fennema et al (1996) report that CGI research provided strong evidence that "one way to improve mathematics instruction and learning is to help teachers understand the mathematical thought processes of their students" (p. 432). Furthermore, according to Carpenter et al. (1999), CGI classes achieved significantly higher levels of problem-solving ability than did non-CGI classes even at kindergarten level, where young children in CGI classes demonstrated an ability to solve mathematical problems that their teachers previously believed was not there.

Another successful curriculum project in this regard was the Cognitive Acceleration through Science Education (CASE), which started in the late 1970s at King’s College London, United Kingdom, with the main aim of determining whether changing secondary pupils' cognitive ability would improve their performance in science (Bennett, 2003;

Mbano, 2003). When developing CASE materials the researchers considered both *cognitive* and *social* aspects of learning:

> From Piaget's work came the theory which underpinned the selection of the CASE activities, and the approach of introducing and then resolving cognitive conflict. From Vygotsky's work came the theory which underpinned the inclusion of discussion in the activities to help children think about their learning…and learn from each other (Bennet, 2003, p. 70).

According to Bennett (2003), the evidence from CASE experiments was persuasive, so much so that many schools in the United Kingdom adopted and implemented CASE ideas, "making a far greater impact on [science] classroom practice than any other research study" (p. 67). Mbano (2003) used CASE materials in a similar study in Malawi. She concluded that CASE lessons increased pupils' cognitive development levels and performance in the Malawi School Certificate Examination.

A sister Project, called Cognitive Acceleration in Mathematics Education (CAME), which was introduced at King's College London in the 1990s, also used a social constructivist approach. According to Goulding (2002), CAME registered good success with regard to pupils' mathematics achievement, although not as good as that recorded in the CASE project. These examples provide evidence that social constructivism is a viable theoretical basis for research in mathematics education and can also guide teaching and learning activities in the mathematics classroom.

2.4.4 Implications of Social Constructivism for the Mathematics Classroom

What, then, are the implications of social constructivism for the mathematics classroom? An important implication is that school mathematics is a *cultural* activity whereby, through interaction with fellow students and with the teacher, individual children *negotiate* mathematical meanings (Cobb et al., 1994; Yackel and Cobb, 1996). That is, learning mathematics involves interaction with each other and with the teacher through classroom discussion, so that individual understandings or perspectives that differ from the understanding the teacher wants students to gain become adjusted and a joint theme achieved (Voigt, 1898).

In such classrooms the ability to manage effectively whole-class discussion is a key competence that teachers need to develop to cope with the requirements of mathematics teaching (McClain and Cobb, 2001; Simon, 2000; Simon and Schifter, 1991; Voigt, 1998; Yackel and Cobb, 1996). Indeed, in social constructivist classrooms,

> Linguistic communication [is]…supremely important-teachers encouraging pupils to talk, and listening to them; providing opportunity for pupils to talk and listen to each other; encouraging open negotiation of meanings without fear of being thought foolish or wrong. Teachers need to use pupils' apparent misconceptions…to gain insight into their images and constructions" (Jaworski, 1988, p. 295).

Consequently, the mathematics being learned is no longer viewed as a finished product to be passed on to learners as was the case in traditional classrooms, but as a *fallible* body of knowledge that is open to discussion (Jaworski, 1988; 1994). I therefore agreed with mathematics educators who have argued that it is social constructivism that best describes the activity of learning mathematics. That is, learning mathematics is like participating in a culture; the individual constructs his or her own mathematical understandings but does this in the presence of others, whose views on the subject will modify those of the individual. In the end, the knowledge individual learners take away from the mathematics classroom is a matter of negotiation, negotiation because it involves a certain level of give and take. This is the theoretical framework that guided the present study.

Although social constructivism implies certain ways of going about the business of learning and teaching, it does not *prescribe* a particular method of teaching (Simon and Schifter, 1991; Simon, 2000; Howe and Berv, 2000; Windshitl, 2002; Terhat, 2003). Teachers can use different methods, including direct teaching, since what is of most importance is the *quality* of student constructions (Howe and Berv, 2000). Howe and Berv (2000) further state that the main requirements for social constructivist teaching are that teachers should actively promote a fallible view of knowledge; and that instruction must be designed to interact with children's experiences and mathematical thinking, so that from these they can construct their own mathematical understandings.

Thus constructivist pedagogy is in agreement with the stance taken by the Cockcroft Committee in the United Kingdom in the early 1980s, which investigated the teaching of mathematics in primary and secondary schools with reference to the mathematics requirements of post secondary education and places of work. When asked to recommend a *definitive* method of effective mathematics teaching, the Committee said they could not but stated that:

> Mathematics teaching at all levels [of education] should include opportunities for
>
> - Exposition by the teacher;
> - Discussion between the teacher and pupils and between pupils themselves;
> - Appropriate practical work;
> - Consolidation of fundamental skills and routines;
> - Problem solving, including the application of mathematics to everyday situations;
> - Investigational work (Cockroft, 1982, p. 71).

The view of mathematics teaching and learning described as social constructivist in this chapter is different from that prevailing in many primary classrooms in Zambia. On the contrary, teachers in Zambia tend to be viewed as all-knowing knowledge transmitters; and learning mathematics is seen as mainly involving listening and absorbing the information being given out by the teacher (Kelly, 1991a). In other words, primary mathematics education in Zambia has, until now, been guided by the traditional approach to mathematics teaching. Yet meaningful whole-class discussion, for example, with its promise of developing in learners some measure of mathematical autonomy, can occur even in large classes such as exist in many schools in Zambia.

2.5 Teacher Change and Developments in Early Primary Numeracy Teaching Practices

Having discussed current changes in early primary numeracy teaching and learning and the learning theory that has been driving them, it is important to ask: How do teachers fit into these changes? It is important to mention from the outset that these movements toward change could not have been contemplated without assuming that teachers would be able and willing to implement them. Indeed, all well-planned school change programmes or innovations are accompanied by actions deliberately aimed at enabling teachers to cope with the change (Joyce and Showers, 2002). Thus, for example, implementation of RME in the Netherlands included introduction of a new teacher education programme which “emphasized strongly the principles of realistic mathematics education both in mathematics content and in the process of the course” (Wubbels et al, 1997, p. 20). Various groups of mathematics educators in the US undertook similar actions when standards-based mathematics curricula were introduced in schools in the

1990s. For example, the CGI Project implemented staff development programmes to help teachers cope with expectations of the new curriculum (Franke et al, 1997; Knapp and Peterson, 1995). In this section, therefore, I discuss issues relating to implementation of change at school level and how teachers fit into it. This issue was particularly important in the present study, a major part of which was to work with teachers in Zambia to implement change in the way children learned numeracy.

Michael Fullan in *The New Meaning of Educational Change* (2001) provides a useful framework for discussing implementation of change at school level. He identifies nine interactive factors, grouped into three categories, which he argues can either help or hinder the successful implementation of school innovations. Fullan uses the following model (Figure 5) to illustrate these factors at play, a model which I adopted as a basis for structuring my discussion in this section. My intention, however, was not to *apply* the model in my study in Zambia, as it does not tally completely with the way educational change occurs in the country. The model nevertheless helped in clarifying issues that might otherwise have been overlooked. I discuss each set of factors in turn below, explaining its application in the Zambian context.

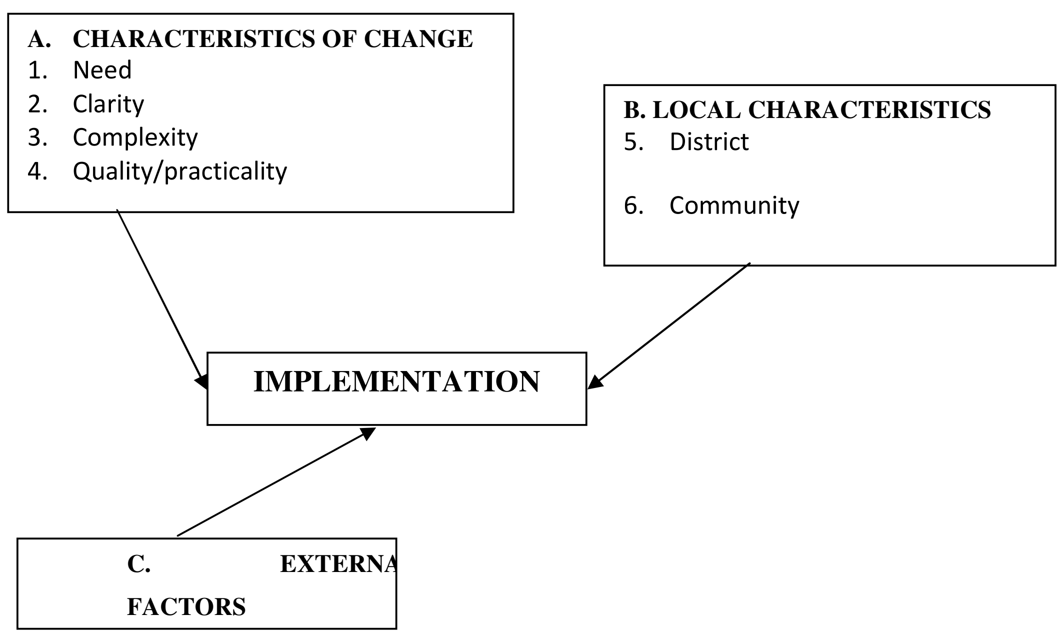


Figure 5: Factors Influencing School Change (Fullan, 2001)

2.5.1 Characteristics of Change

By characteristics of change Fullan (2001) means events at school level. Fullan argues that to have a greater chance of succeeding school innovations should address priority needs at local school level and have clear goals and clearly defined means of achieving such goals. He adds that school innovations should also not be too complex (e.g. requiring too much in terms of material usage or being radical with regard to the amount of change expected on the part of teachers) and should be well-planned and attend to matters of quality. Guskey (1986) and Joyce and Showers (2002) share these views. As Guskey points out:

> Programs that are dramatically different from teachers' current practices or that require teachers to make major revisions in the way they presently teach are unlikely to be implemented well, if at all. Therefore, if a staff development effort is to be successful, it must clearly illustrate how the new practices can be implemented incrementally, without too much disruption or extra work…If a new program does require that major changes be made, it is best to ease into its use rather than expect comprehensive implementation at once (Guskey, 1986, p. 9)

Picking up this point, Joyce and Showers (2002) argue that one of the first and most crucial things to do when planning school interventions is to arrive at decisions about suitable training content for staff development workshops, adding that selection of training content in this regard should be done by local schools in collaboration with district level educational personnel, and should reflect the needs of teachers and the school. Furthermore Smith and Freeman (2002), referring to the same issue, stress the need for "the school's planning team [coming] to significant consensus regarding what constitutes the problem at hand using existing data and information" (p. 310).

However, in making the above statements Joyce and Showers (2002), and Smith and Freeman (2002), had in mind a situation whereby the need for change is recognised by the school itself, which then takes the necessary steps to effect the change either using its own resources or with the help of outsiders. School innovations taking this route represent the ideal situation; and there is the possibility that schools' involvement in identifying the needs to be addressed by an intervention can mean teachers' commitment to implementing the intervention (Fullan, 2001).

While it is important before conducting a staff development workshop to achieve congruency between perceived needs of *individual* schools and the aims of an intervention, this has not always been possible in Zambia. For example, many of the interventions that have been implemented in schools in the country have been large scale, nationwide projects initiated and supported by the Government (usually with donor help). It was assumed that what was done at national level was in line with the needs of the whole education system, and of individual schools.

In the case of the present study the problem addressed was also a national one, which had been disseminated to schools in relevant policy documents. For example, in *The Basic School Curriculum Framework* (2000), an official Government publication distributed to schools nationwide, it is stated that "Numeracy, or mathematics, is considered the most difficult subject by both pupils and teachers. Learning results remain unsatisfactory in Zambia. This must be rectified by all possible means…." (p. 14). Indeed, my initial visits to participating schools convinced me that the teachers there were aware of this problem and wanted something done to address it. Because of this, it was perhaps not necessary to ensure that the needs addressed by the present study coincided with those of participating schools before implementing the staff development workshop. Furthermore, according to Fullan (2001), although initially teachers might be ambivalent about participating in a staff development programme, their interest may be aroused once the programme has started since "people often become clearer about their needs only when they start doing things; that is, during implementation itself" (p. 76). There was no good reason to suppose that the teachers who participated in the present study were different in this regard.

What is most important then, as Fullan (2001) points out, is to ensure that during the early part of implementing a school innovation teachers can see that both the needs being addressed are significant and that some progress is being made towards meeting them. This leads to the issue of how to design staff development programmes that have a meaningful impact. In this regard, it has been suggested that two conditions should be met: to design training that will enable teachers to learn new knowledge and skills and to transfer that knowledge and skill to the classroom; and to create conditions that ensure that teachers learn the new skills more quickly and easily (Joyce and Showers, 2002; Smith and Freeman, 2002).

Simon and Schifter (1991) share this view. They argue that staff development programmes for implementing mathematics interventions in schools should enable teachers to construct more powerful ideas that will enable them to understand better the relationship between mathematics teaching and learning. In other words,

> Teachers must be challenged at *their* levels of mathematical understanding and problem solving ability, allowing them to not only increase their mathematical knowledge, but also to experience a depth of mathematical learning that, for most of them, is unprecedented. Concurrently, teachers [should be] asked to reflect on these learning experiences. The process of making sense of new and discrepant experiences precipitates the modification of previously held ideas about learning and teaching (Simon and Schifter, 1991, p. 312).

To achieve the above, staff development programmes should include exploration of the theory or rationale guiding the new skills to be adopted, demonstration of the skills, practicing the skills, and peer coaching, that is teachers helping one another to implement the intended change (Joyce and Showers, 2002). According to Joyce and Showers exploration of the theory guiding implementation of a new skill or strategy through lectures, discussions and readings is especially important, as it can help teachers understand the concepts and principles behind it. As Richardson (1990) points out,

> Without an understanding of the theoretical framework and the opportunity to talk about how the premises in the theory agree with the teachers' own premises, teachers may accept or reject practices on the basis of whether they meet the personality needs of the teacher and other more ecologically created concerns such as classroom management…and content coverage (p.16).

Nevertheless, research has shown that exploration of theory alone would not be enough; for knowledge gained through participation in professional development activities to have a greater chance of being transferred to the classroom all the four training components should be included (Joyce and Showers, 2002)

Following the above suggestions, I tried during the staff development workshop that formed part of this study not only to expose participating teachers to relevant recent research on mental calculations but also to provide opportunities for demonstrating new skills. Teachers also had the chance to experience as learners the solution of relevant mathematics problems; to compare and discuss solutions and solution strategies; and to plan lessons together.

Although Fullan (2001) has not included it in his model, perhaps because his focus was on school change as it affects local school personnel such as teachers, another important issue to consider when thinking about change at local level is the nature of the students and the classes participating in the change programme. According to Joyce and Showers (1995), students who are used to learning in a particular way may find it difficult to understand why their teacher has suddenly changed his or her teaching style; and can force him or her to return to what they are used to doing. This was not much of a concern in the present study, however, since participating Grade 2 pupils were quite young and were not yet set in their thinking about which methods teachers should use.

2.5.2 Local Characteristics

Fullan (2001) and other authors such as Benham (1996) and Richardson (1990), argue that while the school may be the unit of change, successful implementation of school change requires the support not only of teachers and school administrators, but also of the local community and district-level education administrators. Nevertheless, it is teachers whose input matters the most (Fullan, 2001; Gill and Thompson, 1995; Gitlin and Margonis, 1995; Richardson 1990). For example, both the literature on teacher change and that on learning to teach, suggest that teachers exercise considerable control not only with regard to *whether* change will be implemented, but also *how* it will be implemented (Richardson, 1990). Therefore, "educational change depends on what teachers *do* and *think* (Fullan, 2001, p. 115, my emphasis). In view of this, my discussion in this section will focus mainly on the interaction between teachers and school change.

Discussions focusing on what teachers *do* and *think* in the context of educational change at school level revolve around three main teacher characteristics: *teacher beliefs*, *teacher knowledge*, and *teaching practice* (Borko et al., 1997; Fishman et al., 2003; Nelson, 1997; Wiske et al., 2001). Indeed Askew et al. (1997) in *Effective Teachers of Numeracy* concluded that teacher beliefs, teacher knowledge and classroom practice were the key factors to be examined in determining whether or not a particular teacher is effective. They defined effective teaching as the ability to bring about identified learning outcomes. Therefore, to understand how educational change might affect teachers and their teaching, it is important to consider these three concepts.

2.5.2.1 Changing Teacher Beliefs

It has long been accepted that teachers' beliefs, defined as "tacit, often unconsciously held assumptions about students, classrooms, and the academic material to be taught" (Kagan, 1992, p. 65), are a vital component of teachers' professional development and teaching practice (Cooney and Shealy, 1997; Hermans et al., 2008; Nespor, 1987; Pajares, 1992; Peterson et al., 1989; Richardson, 1994; Verloop et al., 2001). Nespor (1987) in particular states that "[teachers'] beliefs play a major role in defining teaching tasks and organizing the knowledge and information relevant to those tasks" (p. 324). Thus teachers' beliefs "underlie planning, decision making, and behaviour of teachers in the classroom" (Nespor, 1987, p. 328).

The association between teacher beliefs and teaching practice is however not so direct, as it is difficult to establish a simple cause and effect relationship (Correa et al, 2008; Warfield et al., 2005). According to Warfield et al. (2005), this might reflect the fact that teacher "[beliefs] do not necessarily form a cohesive unit; [so that] it is not unusual for an individual to hold contradictory beliefs [thereby] making it difficult to determine how *particular* beliefs influence instruction" (p. 442, my emphasis). Furthermore, the constraining nature of educational environments can also cause dissonance between teacher beliefs and teaching practice (Handal, 2003). As Handal illustrates, teachers who have accepted progressive ideas may still find it difficult to implement them, because parents and colleagues expect them to teach in traditional ways, they have a syllabus to follow, or have to think about incoming external examinations.

Nevertheless, successful change in teaching practice can only occur if accompanied by change in teacher beliefs. The literature on teacher change however suggests that changing teacher beliefs is not an easy task (Shechtman, 2002). According to Correa et al (2008) a possible reason for this is that teaching is a *cultural* activity, that is, it is an activity that includes culturally shared beliefs which have been unintentionally acquired in childhood, when learning to teach, and during school experience as a beginning teacher. They add that there might also be a high degree of connectedness among beliefs, "so that one [belief] about teaching cannot be changed without affecting another" (p. 141). Indeed, it would seem that teachers might not change their beliefs about teaching and learning even when they can see evidence that contradicts their usual way of looking at things (Nespor, 1987; Tillema, 1995). Furthermore, the extent to which teachers will accept new information (e.g. a new teaching approach) is determined by the extent of congruence between the new information and teachers' existing content related beliefs (Tillema, 1995). Thus when a teacher tries new ideas or activities his or her focus is on whether or not they agree with what he or she already believes about teaching and learning; those that do not may be dropped or radically altered to fit in with the teacher's usual way of doing things (Richardson, 1994).

But to say that teacher beliefs are difficult to change is not the same thing as saying that they *cannot* be changed. Teacher beliefs can be modified, altered, or even replaced with new ones (Franke et al., 1997; Lubinski and Jaberg, 1997). For example, many of the teachers who participated in the CGI Project in the US changed their initial beliefs about mathematics teaching and learning to bring them more in line with the view that mathematics teaching should build on children's thinking in ways that enhanced understanding (Franke et al., 1997). As Franke et al. point out, the CGI view of mathematics education entailed accepting for example that children come to any learning situation with some knowledge of mathematics and should be allowed to use it; that this knowledge was acquired from their interaction with problems; and that what teachers learn from listening to children should inform instructional decisions. These new beliefs were different from those that characterised traditional US mathematics classrooms, which focused mainly on encouraging children to produce correct answers by following steps demonstrated by teachers (Jacobs et al., 1997).

How individual beliefs about teaching and learning influence the extent to which teachers accept to use new teaching approaches was also an issue in Zambian education. For example, in a study that examined classroom interaction inside primary school classrooms, Maimbolwa-Sinyangwe and Chilangwa (1995) found that teachers tended to believe that girls would always perform poorer than boys in mathematics and science, and that there was not much anyone could do about it. As a result, the teachers had low expectations of girls' achievement in these subjects.

There was also a general tendency among primary school teachers in Zambia to rely for their lesson presentation on pre-planned lessons in teachers' handbooks. When this was coupled with a country-wide shortage of children's books, it meant that primary mathematics teaching was dominated by a view of teaching as telling (Kelly (1991a).

Since the present study emphasised the view that teaching was *facilitating* learning, it forced participating teachers to confront their original beliefs about teaching and learning and to consider the extent to which such beliefs might have impeded their pupils' achievement in numeracy. In doing this I followed Borko et al's (1997) argument that, "when teachers' beliefs are incompatible with the intentions of the staff development team and are not challenged, the teachers are likely to either ignore new ideas or inappropriately assimilate them into their existing practices" (p. 270).

2.5.2.2 Teacher Knowledge and Educational Change

The publication of Shulman's framework for analysing the knowledge base for teaching (Shulman, 1986; 1987) generated great interest among educational researchers in the idea of *teacher knowledge*. Since then, several studies have focused on teacher knowledge, although authors have sometimes used related terms such as *personal knowledge*, *the wisdom of practice*, *professional craft knowledge*, *action-oriented knowledge*, and *content and context related knowledge* (Verloop et al., 2001). According to Verloop et al. the use of these terms reflects the fact that

> …in the label "teacher knowledge", the concept "knowledge" is used as an overarching, inclusive concept, summarizing a large variety of [teacher] cognitions, from conscious and well-balanced opinions to unconscious and unreflected intuitions. This is related to the fact that, in the mind of the teacher, components of knowledge, beliefs, conceptions, and intuitions are inextricably intertwined (p. 446).

Although Shulman (1986; 1987) originally defined teacher knowledge as comprising seven different but related categories, most of researches in mathematics education since then have focused on two categories, that is *subject matter knowledge* and *pedagogic content knowledge* (Carpenter et al., 1988; Chinnappan and Lawson, 2005; Mewborn, 2001; Moreira and David, 2008; Nathan and Petrosino, 2003; Rowland et al., 2005; Ryan and McCrae, 2005/2006; Steele, 2005). Indeed, some authors have narrowed the meaning of teacher knowledge to these two aspects (Goulding et al., 2002; Poulson, 2001; Sherin, 2002; Verloop et al., 2001). Other authors have gone even further and grouped subject matter knowledge and knowledge about teaching the subject under a single category called *pedagogic content knowledge* (Askew et al., 1997; Wang and Paine, 2003). Thus Wang and Paine (2003) state that "[pedagogic content knowledge] includes a deeper understanding about subject matter, in particular, its content, inquiry, and dispositions…and a proper representation of these ideas for particular groups of students in instruction" (p. 76). In view of this, my discussion of teacher knowledge below will focus on subject matter knowledge and pedagogic content knowledge.

Shulman (1986) defined subject matter knowledge as knowledge not limited to the facts or concepts in a given domain but which includes an understanding of "the variety of ways in which the basic concepts and principles of the discipline are organized to incorporate facts… [and] the set of ways in which truth and falsehood, validity or invalidity, are established" (p. 9). He adds:

> Teachers must not only be capable of defining for students the accepted truths in a domain. They must also be able to explain why a particular proposition is deemed warranted, why it is worth knowing, and how it relates to other propositions, both within the discipline and without, both in theory and in practice (p. 9).

According to Edwards and Ogden (1998), while it may be possible for secondary school teachers who graduate in a single subject to achieve such a level of subject matter knowledge, it is not be reasonable to expect the same level of proficiency on the part of primary school teachers, who teach across the breadth of the whole curriculum. Bibby (1999) agrees with this view and suggests that it is unrealistic to expect generalist primary school teachers to attain such a high level of mathematics content knowledge. Indeed, it has been noted that a sizable number of serving and prospective elementary (i.e. primary) school teachers in the US did not have a solid understanding of the mathematics they were expected to teach (Ball et al., 2005; Lo et al., 2008; Mewborn, 2001; Steele, 2005). Instead, they tended to rely

> on previously learned rules to solve problems and when asked to explain why a particular rule worked…frequently replied by re-stating the steps for carrying out the rule or appealing to an external authority…rather than attending to the reasoning behind those steps" (Lo et al., 2005, pp. 6–7).

And according to Ministry of Education (1992) the majority of candidates for primary teacher education in Zambia did not pass mathematics at School Certificate level, suggesting that they were unlikely to attain levels of mathematics content knowledge consistent with Shulman's definition. My own observations in primary school classrooms and in primary colleges of education in Zambia suggested that the teachers in general developed only an *instrumental* knowledge of mathematics and would most likely teach 'rules without reason,' as Skemp (1978) has put it.

With regard to *pedagogic content knowledge* Shulman (1987) states that it is "the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to diverse interests and abilities of learners, and presented for instruction" (p. 8). In other words, as Sherin (2002) puts it, "*Pedagogic content knowledge* is the knowledge specifically for teaching the domain, including understanding how to present the facts and concepts to facilitate learning and knowledge of the typical understandings and misunderstandings of students (p. 119, emphasis in original). Thus the key idea is teachers' ability to *transform* their subject matter knowledge for classroom teaching (Deng, 2007).

The process of transforming one's subject matter knowledge into pedagogic content knowledge assumes that one has the subject matter knowledge in the first place. Given the fact that the majority of primary school teachers (for example in the US and in Zambia) did not have a secure understanding of the mathematics they were expected to teach, the likelihood of this transformation occurring at a level consistent with the definition was rather low. As Bibby (1999) states:

> To suggest that there is one pedagogical content knowledge is unrealistic and requires the removal of primary [school] teachers from their context. It might be more realistic, if rather contrived, to talk of pedagogic content *knowledges*, which would bring back the complexity that is the lot of the primary teacher and place them back in their context (p. 230).

In recent years, a more pertinent issue concerning teacher knowledge in mathematics has had to do with what teachers need to know to teach mathematics more effectively. Until the late 1980s, mathematics educators believed that in order to teach mathematics effectively teachers needed to take *more* advanced courses in the subject. Accordingly, attempts were made to find a direct link between teachers' mathematical knowledge, measured in terms of the number of advanced academic mathematics courses taken, and student achievement in the subject (Carpenter et al., 1988; Davis and Simmt, 2006; Grossman et al., 1989; Huckstep et al., 2003; Moreira and David, 2008). This belief has been difficult to substantiate. Only a weak relationship has been observed between the number of advanced mathematics courses teachers have taken and the performance of their students on standardized examinations, suggesting that an emphasis on more advanced mathematics courses for teachers was inappropriate (Davis and Simmt, 2006).

As Davis and Simmt conclude, this indicated that teachers needed a *different* kind of understanding of mathematics to be able to teach the subject effectively.

The above conclusion is not new. At the beginning of the last century Dewey (cited in Grossman et al, 1989) observed that: "Every study or subject…has two aspects: one for the scientist as scientist; the other for the teacher as teacher. These two aspects are in no sense opposed or conflicting. But neither are they immediately identical" (p. 24). Working along similar lines Ball and her colleagues in the US in the late 1990s conducted studies that eventually identified the concept of *mathematical knowledge for teaching,* an attempt to differentiate the mathematical knowledge needed for classroom teaching from that required in other mathematics-intensive careers such as engineering (Ball, 2000; Ball, 2003; Ball and Bass, 2003; Ball et al., 2004; Ball et al., 2005; Moreira and David, 2008). According to Ball et al. (2005), to identify *mathematical knowledge for teaching* does not require first examining the curriculum prospective teachers will teach. What is required is to ask oneself questions such as: "What do teachers *do* in teaching mathematics and in what ways does what they do demand mathematical reasoning, insight, understanding, and skill?" (p. 17). In other words, to understand what teachers should learn in mathematics to teach the subject effectively it is necessary to undertake a job analysis of what they actually do in the classroom (Ball, 2000). This led to the conclusion that *mathematical knowledge for teaching* comprised two main strands: the common mathematical knowledge anyone who has studied mathematics should have, and specialised mathematical knowledge which only teachers should know (Ball et al., 2005).

A related issue in many western countries and in some Asian countries has been how to measure prospective and serving primary school teachers' possession of *mathematical knowledge for teaching* (Afamasaga-Fuata'i, 2007; Ball et al., 2005; Ball and Rowan, 2004; Baker and Chick, 2006; Cavey et al., 2006; Chinnappan and Lawson, 2005; Davis and Simmt, 2006; Goulding and Suggate, 2001; Hill et al, 2004; Huckstep et al, 2003; Turnuklu and Yesildere, 2007; Zuzovsky and Libman, 2006). This led to the development of numeracy tests for example in the UK (Drake, 2002; Goulding and Suggate, 2001; McNamara et al., 2002), the aim being to ensure that only individuals who had a good grounding in numeracy became teachers (MacNamara et al., 2002).

As one would expect, some educators criticised this development. They argued that nothing worth measuring could be measured by some of the multiple-choice test questions used in the tests; that teaching and teacher learning were complex endeavours that could not be probed sufficiently by the instruments used; and that testing teachers meant 'deskilling' and 'deprofessionalising' them (Ball et al., 2005). Also, Drake (2002) has argued that numeracy tests were not a legitimate way to prevent inadequately qualified teachers from being certified, adding that the numeracy tests amounted to political interference in education, which cheapened the idea of teacher certification.

Other researchers have supported the use of tests to evaluate teachers' mathematical knowledge. In this connection Ball and her colleagues have argued that such tests helped build the teaching profession in that they identified gaps in teachers' mathematics subject matter knowledge which, when filled, resulted in improved student learning and achievement (Ball et al., 2005). Accordingly, they developed instruments for measuring teachers' understanding of mathematical knowledge for teaching (Ball and Rowan, 2004; Hill et al., 2004).

Nevertheless, teacher knowledge is an important factor to consider when implementing school innovations (Nelson, 1997; Romberg, 1997; Sherin, 2002; Verloop et al., 2001). According to Nelson (1997), when standards-based mathematics curricula were introduced in the United States a good number of mathematics teachers there held a largely *algorithmic* view of mathematics. That is, they focused on teaching mathematical skills without giving much thought to reasons why the skills worked. If the teachers were not helped to reconstruct this knowledge, Nelson argues, they would not be able to support the kind of mathematics instruction demanded by the new standards-based curriculum, whose aim was to construct a deep understanding of the subject.

A relevant example in this regard is the requirement for teachers to acquire skill in managing effectively classroom discussion, an important part of the constructivist approach to mathematics teaching. This is a key skill that enables teachers to help children construct their own mathematical understandings, and to share and justify their ideas (Ball, 1993; Forman et al., 1998; Krummheuer, 2007; Sherin, 2002; White, 2003; Whitenack and Knipping, 2002; Wood, 1999; Yackel, 2002). According to Yackel (2002), a necessary core-requisite to developing the ability to manage classroom discussion effectively is having “a thorough understanding of…students’ current mathematical conceptual possibilities and constraints, and…[of] the relevant underlying mathematical concepts, including those that might be connected across the curriculum” (p. 439). She believes that this is necessary because “teachers can only create and capitalize on opportunities to pursue mathematical ideas and connections across the curriculum to the extent that they are able to recognize such opportunities" (p. 439).

The question of how teachers’ knowledge might affect implementation of change in mathematics was even more important in Zambia than it was in the western world. In countries such as the UK and the US initial teacher education for prospective primary school teachers is a four-year degree programme, which could provide the opportunity to learn more mathematics. In Zambia, the primary teacher education programme lasted only two years, and involved only the study of basic numeracy. Because of the greater likelihood that candidates for primary teacher education had failed mathematics at School Certificate level (MoE, 1992), it was no wonder that mathematics lessons at this level of education were of poor quality.

Primary teacher educators in Zambia feared that the situation would get worse before it got better. In 2000 the Zambian Government transformed the two-year residential primary teacher education programme into one comprising one year residential training in a college of education, followed by one year of school-based training. It was reasoned that these changes made teacher education more effective and met increasing demands for trained teachers in the primary sector. But, as I learned during a period of attachment at a primary teachers' college in 2001, primary teacher educators felt that this transformation made it even more unlikely that student teachers would develop the necessary competency to teach mathematics effectively, as they would spend less time in colleges of education. Partly because of this the Ministry of Education after 2007 decided that the one-year school-based component of the training would be reduced to only three months (equivalent to the length of one school term), so that prospective teachers could spend more time in colleges of education. Nevertheless, I did not consider inadequate preparation in mathematics on the part of participating Grade 2 teachers as constituting a serious problem for my study since the staff development workshop which was part of the study would equip them with the relevant mathematical and theoretical knowledge to enable them implement the planned interventions.

2.5.2.3 Changing Teaching Practices

According to Cohen and Ball (2001), *teaching practices* refers to

> …how [teachers] frame and use academic tasks, acquaint themselves with what students know and can do, enact the instructional discourse, and mediate the [learning] environment…[which] influence how teaching and learning unfold and hence the opportunities for learning that students have and can use" (p. 75).

The above definition suggests that a number of factors influence an individual teacher's teaching practices, including personal conceptions about: the nature of the subject matter being taught; 'best' approaches to teaching and learning of the subject; and what the teacher understands by effective classroom organisation. It should therefore be expected that teachers will resist demands to change their teaching practices as it often means abandoning more comfortable procedures and routines for carrying out teaching tasks, developed over a period of time (Gitlin and Margonis, 1995). Indeed, as Msila (2008) states, "whether voluntary or imposed, all change [in one's teaching practices] involves loss, anxiety and struggle" (p. 202). Researchers should therefore do all they can to understand the internal processes individual teachers go through as they experience change, and to find ways of making the change less traumatic (Senger, 1999).

How can professional development activities be designed so that they *encourage* teachers to change their teaching practices for the better? According to DiCerbo and Duran (2006), the relevant literature identifies the following as some of the 'best practices' in this regard, best because they produce both desirable change in teaching practices and gains in student learning:

- Professional development programmes which are clearly articulated and results-driven are more likely to produce desired change in teaching practices and gains in student learning.
- Professional development that is likely to produce the desired results includes rigorous instruction in subject matter in the relevant area; provides information on how children learn in the subject area; and demonstrates teaching strategies specific to the content discussed (see also Borko, 2004). Rigorous instruction in subject content is necessary to enable teachers become experts in their field and have the confidence to use different teaching techniques.
- Professional development that allows teachers to learn in a manner similar to how they will teach in the classroom, including experiencing as learners the material

that they will teach, is more likely to produce the desired change in teaching practices. Change might even be greater in teaching practices and student achievement if professional development activities were conducted over a longer period of time, and provided opportunities for teachers to engage in active discussion; plan together; implement what they learn, and receive in-class support, for example coaching either by peers or by experts.

- Professional development activities should be designed specifically for transfer and application in the classroom. Examples include developing actual lesson plans so that teachers can see how the materials they are learning can be applied in the classroom; setting goals and how to overcome potential obstacles; sending teachers reminders so that new instructional methods are not put aside and quickly forgotten; and follow-up coaching.
- Professional development activities that were part of larger, on-going projects were more likely than stand-alone ventures to produce the desired change in teaching practice and student learning.

A related issue is the sequence in which change in teaching practices, teacher beliefs and teacher knowledge occurs. According to Guskey (1986), teachers will only change their teaching practices when they see that the new ideas they are adopting have the potential not only to expand their own personal knowledge and skills but, most importantly, to enhance their effectiveness with students:

> [When] teachers *see* that a new program or innovation enhances the learning outcomes of students in their classes; when, for example, they *see* their students attaining higher levels of achievement, becoming more involved in instruction, or expressing greater confidence in themselves or their ability to learn, then, and perhaps only then, is significant change in their beliefs and attitudes likely to occur (p. 7, my emphasis).

In other words, according to Guskey, change in teaching practices occurs *before* change in beliefs: He represents this change diagrammatically as follows:

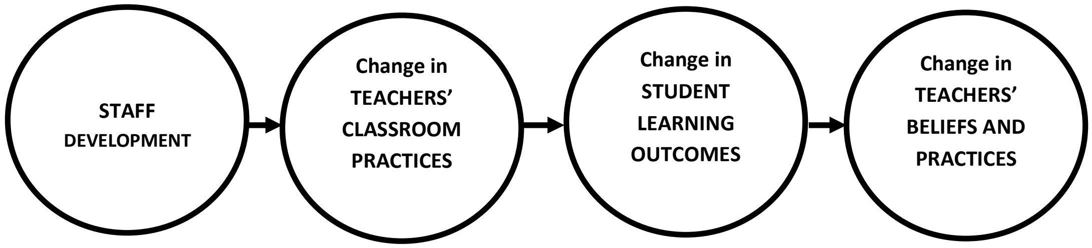


Figure 6: Guskey's model of teacher change (Guskey, 1986)

Rogers (2007) conducted a case study of a primary mathematics teacher, which investigated the relationship among professional learning, classroom practices, and teacher beliefs. She concluded that the changes exhibited by the teacher over a period of one year agreed with Guskey's view that change in teaching practices happened before change in beliefs. However, she also states that "this process appears cyclic in nature rather than linear as many changes in practice may need to be made and the professional learning ongoing before a change in student learning outcomes [is] observed" (p. 639). In line with her observations, Rogers revised as follows Guskey's (1986) model of teacher change:

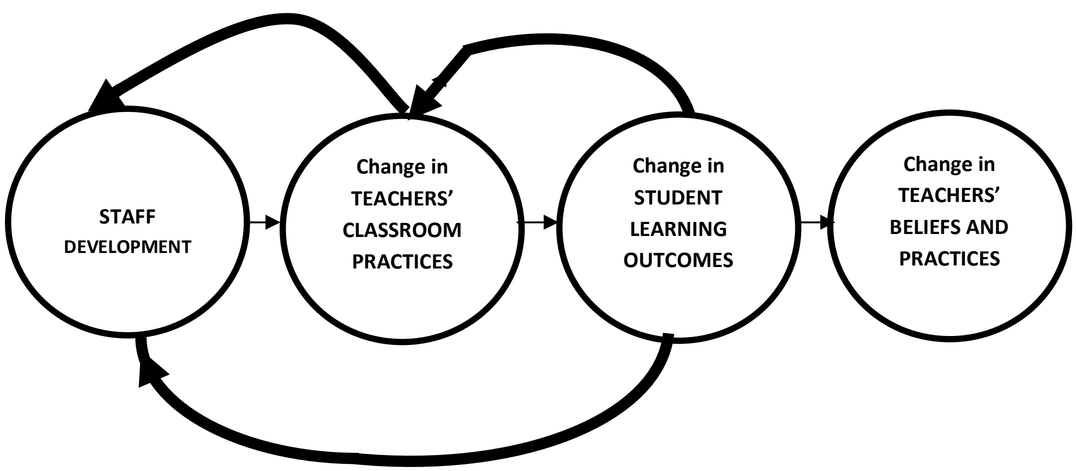


Figure. 7: Rogers' revised version of Guskey's model for teacher change (Rogers, 2007)

However, Franke et al (1997), Cooney (2001), and Parker et al (2007) insist that the question with regard to whether it is beliefs or teaching practices that change first has not been settled. For example, according Parker et al (2007) whether beliefs change first followed by teaching practices, or vice-versa, may depend on teacher type:

> Novice teachers can change attitudes first because they don't have so much built-up experience, but they are too new at teaching to change their practices as quickly as veteran teachers can. On the other hand, veterans can change their practices, but won't change their beliefs until feedback from their students over time proves to them that the changes are worth it (p. 5, my emphasis).

Furthermore, in their studies which were part of the CGI Project Franke et al (1997) found no single pattern of change among the mathematics teachers who participated in the project: one group of teachers changed their teaching practices before changing their beliefs; a second group changed their beliefs before changing their practices; and a third changed both beliefs and practices simultaneously. However, Franke et al also found that significant and lasting changes in classroom practices were not possible unless they were accompanied by changes in beliefs. They observed for example that while changing teaching practices alone was sufficient initially to help teachers engage in transforming their mathematics teaching, proceeding further in this direction was only possible if beliefs also changed. In view of this, the authors concluded that the likelihood of change in teaching practices becoming self-generating and self-sustaining was greater if such change was accompanied by change in beliefs.

Nevertheless, Guskey's (1986) argument that teachers tend to adopt only those teaching practices that in their view have the potential to enhance student learning is true. Richardson (1990) cites a number of authors who hold the view that teachers' adoption of particular teaching practices is influenced largely by their classroom experiences and that it may well be that teachers consider experience to be their only teacher. But Richardson also argues that classroom experience is only educative when it includes reflection, with research providing a theoretical framework for change:

> Research…[can] provide practitioners not just…findings in the form of activities or behaviors that work, but ways of thinking and empirical premises related to teaching and learning. These ways of thinking can be used to heighten teachers' awareness of their own beliefs, provide content for their reflections, and help them develop their justifications (p. 16).

We are now ready to consider Part C of Fullan's (2001) model, which focuses on how external factors might influence change at school level.

2.5.3 External Factors

According to Fullan (2001) the support of external factors (e.g. ministries of education) is crucial if change at school level should succeed. To have this support, school innovations should be planned in such a way that they fit in with existing government policies on education (Fullan, 2001; Parker et al., 2007).

The need for school innovations to agree with government policy on education was taken seriously in this study. As I mentioned earlier, the problem the study attempted to address (i.e. unsatisfactory learning achievement in primary mathematics) was identified by the Zambian Ministry of Education (2000a), which disseminated it to stakeholders, and invited any strategies or interventions that could raise levels of achievement in numeracy at primary school level. The alternative approach to developing numeracy in early primary mathematics which the present study piloted was therefore in line with government policy on education.

2.6 Evaluating Mathematics Innovations

Gathering relevant data to determine the degree of success of an innovation at school level, whether from the point of view of teaching or learning, is an essential part of the process of implementing school change programmes (Burden and Nichols, 2000; Joyce and Showers, 2002). Without an evaluation, it is not only difficult to interpret outcomes; it could also mean discovering at the end of the exercise that none of the planned activities had been implemented (Joyce and Showers, 2002). In view of this, evaluation of school interventions should aim at documenting the process that occurred during implementation of the intervention (Maruyama and Deno, 1992).

Burden and Nichols (2000) argue that such an evaluation can be achieved by: investigating exactly what is occurring in the efforts to bring about the proposed change; identifying the degree of match and mismatch between proposed plans and what is actually occurring, which makes it possible to comment and speculate on most likely reasons for successes and failures; and suggesting positive ways in which success might be enhanced and failures minimised, during the current implementation process and in similar efforts in future.

To accomplish this, evaluators (or researchers) need to stay in close contact with the programme and the participants, which I did in this study, so that they can use their judgement and expertise to respond to any problems that might arise during the implementation process (Lau and LeMahiue, 1997). Indeed as Joyce and Showers (2002) argue, it is only after participating teachers have received sufficient help, and have actually began implementing the new content and practices, that it makes sense to start thinking about how students respond to the changes being implemented.

The adoption of constructivist views in mathematics education beginning in the 1980s affected not only how mathematics was taught and learned, but also how mathematics education research was done, including the way data relating to school interventions was collected and analysed (Ernest, 1998). According to Ernest, before the 1980s research in mathematics education used mainly quantitative methods, but since constructivism was accepted qualitative methods began to take over. As part of this shift, the case study method (adopted in the present study) became one of the most prevalent methods for studying change in mathematics education (Franke et al., 1997). This has meant increased use of qualitative methods such as interviews with teachers; clinical interviews with children; lesson observations; and use of teachers' journal entries (see Kelly and Lesh, 2000 for a detailed discussion).

Thus in studying the effect on teachers' practices of participation in the CGI Project in the United States, Franke et al. (1998) selected as cases for intensive study only 3 teachers out of a total of 21 CGI teachers from a number of participating schools in one school district. The three teachers were selected because "[they] characterized the different patterns of change [seen] across the 21 teachers" (p. 70). Having done this the authors used interviews, informal interactions, and formal and informal observations of lessons, to understand how these teachers changed and what change meant for them. Such methods can be used in any school system in the world, provided appropriate attention is given to local conditions.

2.7 Summary of the Literature Review

The foregoing is a review of the literature on current change in early primary numeracy teaching practices. The review has included discussion of the nature of this change;

factors that influenced the change; what happens particularly at local school level when change is introduced; and how school change may be evaluated.

The studies reviewed have indicated that using mental calculation strategies as an approach to numeracy development in early primary mathematics can result in better attainment of numeracy by children than using standard (textbook) procedures, whose logic many children fail to understand. According to these studies, most of which were conducted in Europe, the US and Brazil, one explanation of the supremacy of mental calculation strategies over procedures as currently taught is that the use of mental strategies allows children to retain the *meanings* of numbers when performing calculations.

The studies also show that researchers in different countries have categorised children's mental strategies in different ways. In the Netherlands, mathematics educators have distinguished between two main methods: *1010* and *N10*. They have taught children to use these methods in conjunction with an empty number line, a model for performing addition and subtraction that has shown much promise with regard to improving young children's performance in numeracy. In the US on the other hand, four main types of children's mental methods have been identified with regard to double-digit addition and subtraction. They include: beginning with one number and moving up or down by tens and ones; splitting numbers in multiples of tens and ones and then adding or subtracting the separate parts before summing up; changing both numbers to ones more convenient to work with and then making the necessary adjustments in the final answer; or mixing these strategies.

Furthermore, the studies have shown that to learn to foster the use of mental calculation strategies in early primary the mathematics teachers need to develop, among others, the skill of managing classroom discussion effectively. Developing this skill is based on the assumption that teachers have an in-depth knowledge of the mathematics they teach; and understand children's mathematical thinking at that level of education.

This research, however, was limited in that it was based mainly on studies conducted outside Zambia. It was unclear for example whether or not the reported gains in children's learning of numeracy in the Netherlands, which were linked to the use of mental calculation strategies, could be replicated in Zambia, which has a different education system. It was also not clear to what extent the academic/professional qualifications of primary school teachers in Zambia, and the beliefs they held concerning the teaching and learning of mathematics, could help or hinder their ability to foster the use of mental calculation strategies in primary mathematics. In view of this, it was important for me to carry out the present study in Zambia not only to corroborate findings with studies conducted abroad but, most importantly, to determine whether an alternative approach to numeracy development in the early primary grades might be the remedy for poor learning results in primary mathematics in the country. In other words, I hoped that the results of the study would provide data that stakeholders in the mathematics education of young children in Zambia could use in their efforts to raise levels of numeracy in early primary mathematics.

CHAPTER 3
RESEARCH DESIGN AND METHODOLOGY

3.0 Introduction

This chapter describes the research design and methodology used in this study. There are four main sections. Section 3.1 presents the overall methodological approach adopted, providing a rationale for its choice and giving a brief account of its evolution and use as a research strategy. According to Crotty (1998), a description of the research methodology does not only comprise explaining the methodology itself but includes providing an account of the *rationale* for the choice of the methods used and how they will be employed. This is what I have done below. Section 3.2 identifies the particular research design adopted in this study; and discusses its strengths and limitations. Section 3.3 describes the sample used in the study, and how it was selected. Finally, Section 3.4 discusses the qualitative and quantitative methods used in the study and what they involve. This section also describes research sites, the process by which access to them was obtained, and the procedures adopted for data collection.

3.1 Broad Methodological Approach

The main aim of this primarily qualitative study was to assess the extent to which primary school teachers could learn to foster the use of strategies for mental calculation for double-digit addition and subtraction in early primary mathematics in Zambia; and to determine the impact of teacher learning in this regard on corresponding pupil learning and performance in numeracy. The findings of the study were intended to suggest ways in which to address the current problem of unsatisfactory learning results in numeracy among primary school children in Zambia (MoE, 2000).

To achieve the aims of the study, I devised a numeracy development approach based on the use of strategies for mental calculation relating to double-digit whole number addition and subtraction suitable for Grade 2 pupils in Zambia. I trained teachers to pilot this numeracy development approach over a period of about ten weeks in selected schools in Lusaka, during which time I examined how both the teachers and their pupils interacted with the learning approach. It was through this process that I hoped to be able to assess the extent to which primary school teachers in Zambia could learn to foster the use of strategies for mental calculation in early primary mathematics.

Examining the implementation of the numeracy development approach required doing a number of things, including making several site visits to schools to observe lessons in progress; interviewing mainly teachers to understand, for example, the decisions and motivations that influenced their actions in the classroom; and administering tests to evaluate corresponding pupil performance. As this process generated both qualitative and quantitative data, I used a *mixed-methods approach* to investigate the issues involved. According to Creswell and Garret (2008), "mixed methods is an approach to inquiry in which the researcher links, in some way (e.g. merges, integrates, connects), both quantitative and qualitative data to provide a unified understanding of a research problem" (p. 322).

Mixing qualitative and quantitative methods within the same study is a relatively new practice in social science research in general (Creswell, 2003; Creswell and Garret, 2008; Johnson and Onwuegbuzie, 2004; Snape and Spencer, 2003). To the best of my knowledge it is even more so in educational research in Zambia, where quantitative research still dominates. Because of this, I describe in some detail below what a mixed methods approach involves.

As late as the early 1990s, relatively few studies in education had attempted to use a mixed methods approach in a single study and very little had been written about this approach in the available literature (Brannen, 1992). Committed to particular research traditions, methodological purists on both the quantitative and qualitative fronts were reluctant to attempt studies that mixed the two sets of methods in the same study (Brannen, 1992; Johnson and Onwuegbuzie, 2004; Snape and Spencer, 2003). They argued that combining qualitative and quantitative methods in the same study was impracticable since these methods implied "very different theoretical perspectives and different conceptualizations of research problems, whereby different realities or different aspects of reality are observed and captured" (Brannen, 1992, p. 5). They also pointed out that not many researchers had the appropriate skills to undertake studies adopting such an approach, and that using a mixed methods approach could result in collecting incongruous sets of data (Brannen, 1992).

Since then the situation has changed significantly. As Ritchie (2003) states, in recent years several authors have provided useful frames of reference for the guidance of researchers intending to use both qualitative and quantitative methods in the same study (see e.g. Creswell, 2003; Darlington and Scott, 2002; McConney et al, 2002; Stecher and Borko, 2002; Tashakkori and Teddlie, 1998; Johnson and Onwuegbuzie, 2004). Indeed, certain kinds of research are best conducted using this approach (Patton, 2002; Creswell, 2003; Snape and Spencer, 2003; Ritchie, 2003; Johnson and Onwuegbuzie, 2004). I therefore agree with Flyvbjerg (2006) that:

> Good social science is problem driven and not methodology driven in the sense that it employs those methods that for a given problematic, best help answer the research questions at hand. More often that not, a combination of qualitative and quantitative methods will do the task best (p. 242).

Put differently, combining qualitative and quantitative methods in the same study might not only be necessary in certain situations; it could be the strategy of choice.

So far I have explained *why* a research approach combining qualitative and quantitative methods was the most appropriate strategy for attaining the aims of the present study, and have given a brief account of the evolution and use of this research strategy. In the remainder of this section, I describe in general terms how qualitative and quantitative methods might be mixed in a single study before identifying, in Section 3.2.1, the particular mixed methods approach used in this study.

As indicated earlier, a number of authors have suggested ways in which qualitative and quantitative methods might be appropriately combined in a single study. Indeed some, for example Tashakkori and Teddlie (1998), have devoted entire books to discussing the issues involved. Among these authors, Creswell's (2003) account of how qualitative and quantitative methods can be mixed in the same study seems to be one of the most useful, being brief, easy to follow, and yet comprehensive enough to include most aspects of the subject. It is the one I have adopted to structure my discussion of the subject below.

Creswell (2003) suggests six alternative strategies for mixing qualitative and quantitative methods in a single study differentiated on the bases of three main criteria. That is, the sequence in which qualitative and quantitative methods are applied, whether one set of methods plays a more dominant role in the study than the other, and whether the two sets of methods are applied simultaneously. The first three mixed methods strategies treat qualitative and quantitative methods *sequentially*, that is, they involve two distinct data collection phases: one set of methods (quantitative or qualitative) is applied before the other. The *sequential explanatory strategy* involves collecting quantitative data first and analysing the data before doing the same with qualitative data. This is primarily a quantitative approach, with qualitative data used to assist in interpreting the findings. The reverse of this strategy, a primarily qualitative approach, is called the *sequential exploratory strategy*. The third strategy in this group is called the *sequential transformative strategy,* which involves first collecting *either* quantitative data or qualitative data and analysing the data, before doing the same with the other set of data. In this case, if resources permit, priority can be given to *both* sets of methods, so that neither the quantitative side nor the qualitative side is dominant.

The last three strategies for combining qualitative and quantitative methods in a single study described by Creswell (2003) involve collecting quantitative and qualitative data *concurrently,* that is, at one and the same time. The *concurrent triangulation strategy,* as its name suggests, is intended to take advantage of the strengths of either method and to offset the weaknesses inherent in each method. It involves collecting quantitative and qualitative data concurrently, with the results of the two methods usually integrated during the data interpretation phase. The *concurrent nested strategy* also involves collecting qualitative and quantitative data simultaneously but with one set of methods (either qualitative or quantitative) being predominant and guiding the study. The data collected using the two methods are then integrated at the analysis stage. The third strategy in this group, the *concurrent transformative strategy*, is similar to the *sequential transformative strategy* in that it can take on the design features of either the triangulation approach or the nested approach. Quantitative and qualitative data are collected simultaneously and integration of different data may occur at the analysis stage.

I must mention at this point a key aspect of qualitative research which was part of this study. In the qualitative tradition researchers are encouraged to be *reflexive.* Among other things, this means that qualitative reports should include researchers' *personal* experiences during fieldwork, including problems and confusions encountered, choices made, mistakes committed, and so on, as this information is critical to understanding their reports and evaluating the findings (Ball, 1990; Ortlipp, 2008; Patton, 2002; Watt, 2007). Such introspective information also helps novice researchers, for example, to understand their biases, feelings and thoughts during fieldwork, which contributes to their development as qualitative researchers (Watt, 2007). I now discuss the particular mixed methods strategy used in the present study.

3.2 Research Design

3.2.1 The Mixed Methods Strategy Used in this Study

This study used the *concurrent nested strategy* described above, which involves simultaneous collection of qualitative and quantitative data but with either the qualitative methods or the quantitative methods being dominant and thus guiding the study. In the present case the qualitative side was dominant. In other words, my study was primarily qualitative but with embedded quantitative features to address particular questions. According to Creswell (2003), this research strategy can be represented diagrammatically by an outer rectangle and an inner rectangle as shown below (Figure. 8). The outer rectangle represents the dominant set of methods, that is, the qualitative methods indicated by the letters QUAL; and the inner rectangle the less dominant quantitative methods, denoted by the letters QUAN.

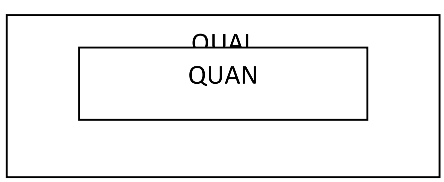


Figure 8: Concurrent Nested Mixed Methods Strategy with Qualitative Methods Dominant (Creswell, 2003).

As Creswell (2003) points out, this mixed methods strategy has many advantages. Apart from the fact that as a mixed methods approach, it makes it possible to gain perspectives from different research methods, it also allows the researcher to collect quantitative and qualitative data simultaneously in a single data collection phase. Creswell (2003) also identifies some limitations of this mixed-methods strategy. In his opinion, the available literature does not offer much advice with regard to how to integrate the data within the analysis phase of the study or how to resolve discrepancies that might occur between qualitative and quantitative data.

McConney et al. (2002), however, have addressed many of the issues relating to integration of qualitative and quantitative data in a single study, demonstrating a flexible method of integrating such data which can be tailored to suit particular situations. With regard to the question of how to resolve possible discrepancies in qualitative and quantitative data, mixed methods researchers have pointed out that such discrepancies should not necessarily be viewed as a problem (Johnson and Onwuegbuzie, 2004; Patton, 2002; Ritchie (2003). As Ritchie points out, the purpose of mixing qualitative and quantitative methods is *not* to get the same results from different angles, but "to achieve an extended understanding that neither method alone can offer. *It is then up to the researcher to explain why the data and their 'meaning' are different"* (p. 43, my emphasis). In fact, as Darlington and Scott (2002) argue, obtaining such discrepancies should be viewed as an opportunity rather than a constraint, as it may simply indicate that further work is required to understand what is going on, work that can lead to more interesting findings. They conclude, therefore, that discrepancies in data arising from mixing qualitative and quantitative methods in the same study should be expected and that what matters is how researchers respond to them.

3.2.2 Quasi-experimental Design

In the present study the above mixed-methods approach was carried out within the framework of a quasi-experimental design called the non-equivalent control group design (Cohen et al, 2000; Cook and Campbell, 1979; Creswell, 2003) illustrated in Figure 9.

Experimental:	O_1---------------X-----------------O_2

Control:	O_1---------------------------------O_2

Figure 9: Non-equivalent Control Group Design (Cook and Campbell, 1979).

In Figure 9, X represents the treatment (i.e. the special numeracy development approach based on the use of mental strategies piloted in Grade 2 classes in experimental schools), and the Os represent the pre-tests and post-tests administered. The subscripts show the time order of administration of procedures in the experiment from left to right, while the dotted line between the two rows indicates that there was no random assignment of participants to comparison groups, that is existing groups (i.e. whole classes) were used. Grade 2 pupils in experimental schools studied the special numeracy development approach for a period of about ten weeks, while at the same time their colleagues in control schools covered similar mathematics topics but using procedures as currently taught at Grade 2 level in Zambia. Pre-tests were administered at the beginning of the main study and post-tests immediately after implementation of the treatment about ten weeks later.

Quasi-experimental designs of the type described above have been criticised for not including random assignment of research participants to comparison groups. In *true* experiments randomisation serves the purpose of achieving equivalence of comparison groups from the outset, so that post-test differences between them can be attributed to intervention effects (Cook and Campbell, 1979; Reichardt and Mark, 1998; Creswell, 2003). Because the quasi-experimental design described above does not include this feature, it is considered to be weak on *internal validity*, that is, it is not sufficiently strong to rule out alternative causal explanations for the outcomes of a study (Cook and Campbell, 1979; Reichardt and Mark, 1998; Black, 1999). It is particularly vulnerable to threats due to *subject characteristics*, that is, participants' entry characteristics such as age, gender, ability, and socio-economic background, which can affect one's school performance (Muijis, 2004). These can lead to problems with interpretation of results, as post-test differences between comparison groups might be due to existing differences in subject characteristics (Wallen and Fraenkel, 2001; Neuman, 2003).

Given the above concerns, I selected comparison schools purposively to ensure some level of similarity among them with regard to pupil background factors (such as parental socio-economic status, gender, grade level and ability) and school-based factors (such as availability of educational resources and qualifications of teachers), which also have implications for children's school performance (see MoE, 2006). Finally, I administered pre-tests, so that if from the outset pupils in experimental and control schools were not equivalent with regard to mathematics performance, this could be taken into account at the data analysis stage. I did not, however, expect that these measures would rule out completely these threats to internal validity, but simply to *reduce* their effects. Indeed, even with randomisation threats to internal validity are never completely eliminated (Cook and Campbell, 1979; Wallen and Fraenkel, 2001). Furthermore, randomisation is an ideal that is often impracticable in school settings, so that in most cases researchers have to work with already formed groups (Wallen and Fraenkel, 2001). Thus despite their shortcomings quasi-experimental designs offer researchers an opportunity to carry out experimental studies in situations such as school settings where often nothing better is possible (Cook and Campbell, 1979; Maruyama and Deno, 1992). They also have one major advantage that true experiments do not have: they are conducted in *natural* rather than *artificial* settings. According to Muijis (2004), this means that if treatment effects are found, one can be confident that they work in real settings and not only in artificial laboratory settings, which "*makes quasi-experimental research a good way of evaluating new initiatives and programmes in education* (p. 29, my emphasis).

3.2.3 Case Studies

All Grade 2 teachers in experimental schools were studied in-depth as special cases to identify differences among them with regard to levels of adoption and implementation of the aims of the study, and to explore ways in which such differences might relate to students' development of strategic methods for double-digit addition and subtraction. A case study is "an intensive, holistic description and analysis of a single entity, phenomenon, or social unit" (Merriam, 1998, p.34). Case studying is deemed particularly suitable for investigating issues relating to process in interventions (such as the one implemented in this study) and in other treatment programmes (Merriam, 1998; Yin, 2003).

3.2.4 Staff Development Workshop

Before the main study began five Grade 2 teachers from experimental schools attended a staff development workshop that was scheduled to run for two weeks, to prepare them to implement the intervention in their schools. All were females, three from one school and the remaining two from the other school. In one school (the one with three Grade 2 teachers) the workshop was held for two weeks as intended. At the second school it was not possible to hold a two-week workshop because the Grade 2 teachers there had other commitments during the period, and we had only one week for the exercise. At both schools the workshop was held in a classroom, during school holidays.

The workshop had two specific objectives: to introduce the teachers to research-based information on the range of mental strategies used by children in performing double-digit whole number addition and subtraction (including the splitting/partitioning methods such as the 1010 and N10 methods and use of the empty number line), and to allow the teachers to experience these methods as *learners.* Workshop activities were based on contents of a training manual (Appendix 1) which I developed by adapting for the Zambian context at grade 2 level relevant subject matter from a three-book series on teaching mental calculation developed by a group of mathematics educators in the UK (i.e. Askew et al, 2001; Mosley et al, 2001; Skinner et al, 2001). The manual provided sufficient content for teachers themselves to learn how to use mental calculation strategies for addition and subtraction, and also included suggestions with regard to possible ways in which to teach the strategies to children. The complete workshop programme appears in Appendix 2.

In line with the social constructivist theory of learning, workshop activities used teachers' past experiences as a starting point for interactions with them. Past experiences are important in the knowledge construction process in that they influence what one finally comes to know (Reece and Walker, 1997). In view of this, one of the first things teachers did during the workshop was to solve a number of double-digit addition and subtraction problems mentally, using any method they chose, after which the merits and demerits of their different mental strategies used were discussed by the group. The final days of the workshop were devoted to working with the teachers to plan lessons using the teachers' own formats. These lessons included only as much content as the teachers felt could be covered in the time allowed for each mathematics lesson at Grade 2 level.

3.3 Sample

3.3.1 Sample and Sample Selection

The sample comprised ten Grade 2 teachers and 311 Grade 2 pupils from four state schools in Lusaka. Two of the schools were used as experimental schools and the other two as control schools. The decision to conduct the study in state schools only was based on the fact that at the time of the study the majority of schools offering primary education in Zambia were state-owned. Therefore, conducting the study in such schools increased the chance that the findings of the study would be applicable to the country as a whole.

Sample selection was done by means of a combination of *purposive* and *convenient* sampling. Purposive sampling is a sampling procedure whereby the investigator handpicks participants, research sites, documents or visual materials, that are most likely to help him or her understand the problem being addressed and, therefore, answer the research question (Creswell, 2003). Put differently, the investigator wants to discover, understand, and gain insight into a given situation and therefore selects a sample from which he or she can learn the most about the situation (Merriam, 1998). Convenience sampling on the other hand involves selecting a sample taking into account a number of factors, such as easy access to research sites, willingness of individuals to serve as research participants, and availability of time and financial resources (Cohen and Morrison, 2000). This combined sampling approach was necessary in this study because I was looking for schools that had Grade 2 classes, could be reached easily in terms of transport and cost of travel; and where gate keepers (e.g. school heads) would allow me to conduct the study.

To select the sample I first visited the offices of district educational administrators for permission to enter Lusaka schools and to obtain lists of schools in the City, from which I could choose the schools in which to conduct the study. My experience of earlier visits to this place suggested that I would spend at least an hour in a queue, as I would join other people (e.g. head teachers) who would also want to see the same educational administrator. However, it turned out that the educational administrator in question knew me and, while passing through the corridor where we were sitting, he saw me and invited me into his office. After I explained to him my mission, permission was promptly granted.

With permission obtained, I identified two residential areas in opposite sides of Lusaka which were similar with regard to socio-economic status, as indicated by the kinds of economic activities residents engaged in. I then decided which of the two residential areas would have the two experimental schools and which one the two control schools. It was important in this study to locate experimental and control schools as far apart as possible, to reduce the likelihood of treatment effects filtering to control schools during the course of the study. The four schools finally chosen were comparable with regard to a number of variables that can affect outcomes in educational settings, such as availability of teaching resources, qualifications of teachers, performance in public examinations at Grade 7, and so on.

3.3.2 Description of Research Sites

The purpose of this section is to provide a description of the areas used as research sites. This was important for two main reasons. First, the study used a quasi-experimental design with non-equivalent groups. It is therefore necessary to provide sufficient information about research sites so that readers can judge for themselves the comparability of experimental and control schools (Cohen et al, 2000; Gay, 1996). Second, and perhaps most importantly, this study was primarily *qualitative.* A detailed description of research sites is necessary to enable readers to get a feel of the setting and hopefully be able to visualise what it was like for a child to attend school in the area (Miles and Huberman, 1994; Patton, 2002; Stake, 1995; Chenail, 1995). As Chenail (1995) states:

> The readers have to have a clear picture of the data's setting so that they can begin to have a perspective from which to judge the observations being made by the researcher regarding the data. *Without the setting, without the developed characterization, there can be no context and with no context for the data, there can be no significant meaning in the analysis* (p. 4, my emphasis).

One might wonder why it is so. According to Ball (1990) and Patton (2002), the physical environment of a setting can influence how people *act* in that setting. As Ball (1990) has observed: "Social actors 'present' themselves differently in different settings….Teacher behaviour within the formalities of the classroom often contracts sharply with back-stage life in the faculty lounge or staffrooms….[Students] also show different selves in different settings" (Ball, 1990, p. 162).

According to Miles and Huberman (1994), to describe a setting adequately one needs not only to give an account of the *social* setting but also the *historical* context. Following these authors, I discuss the origins, development, and socio-economic statuses of the residential areas in which participating schools were located, before describing the schools themselves.

3.3.2.1 Control Schools

Control schools were located in a large township situated about 6 km southeast of Lusaka's main commercial centre. According to Ndhlema (2000), this township had one main (tarred) road used as the bus route, and a number of gravel roads in poor condition. It also had a clinic, a police post, a post office, a council site office, two basic schools, two secondary schools, and some private preschools. A few homes had their own water and electricity, while the majority of households drew water from communal sources.

The township was first occupied in the 1950s when employees of a European contractor built temporary homes there (Lusaka Housing Project Evaluation Team, 1977). Until the early 1970s, it was officially classified as a *squatter* settlement. As Seymour (1977) states:

> The defining characteristic of *squatter settlement* [was] illegality of land tenure: squatters [were] those who [occupied] land owned by others, without the landowner's consent. Squatter settlements [were] also illegal in the sense that they [existed] without the approval of the local authority, and [infringed] its regulations regarding land use and building standards (p. 33).

Because they were illegal, squatter settlements lacked community facilities and services such as clean drinking water and access to roads, and from time to time were demolished by city planners (Lusaka Housing Project Evaluation Team, 1977).

The development of the earliest squatter settlements in the urban areas of Northern Rhodesia has been linked to failures of the Colonial Government's housing policy for African employees. As the Lusaka Housing Project Evaluation Team (1977) points out, colonial administrators regarded African employees as village dwellers, who lived in the urban areas temporarily to provide their labour, and who would return to their villages on retirement. Accordingly, the Government provided African employees with housing deemed suitable for single men. African housing was thus "tied to employment, the loss of a job normally resulting in eviction from a house" (Seymour, 1976, p. 45). It was therefore retired African men who did not want to return to their villages, or were in self-employment, who became the first residents of squatter settlements.

Realising that squatter settlements provided low cost housing to absorb the increasing urban population, the new Zambian Government in the 1970s changed the policy towards squatter settlements from *demolition* to *upgrading* (Mulenga, 1991; Seymour, 1976; Rakodi, 1982; Todd et al., 1978). The Government also recognised that inhabitants of squatter townships were not all indolent people, but included many who "were gainfully employed either in the formal or informal sector" (Chikwanda, 1977, p. 19). The new policy of squatter upgrading, called the Lusaka Squatter Upgrading and Site and Services Project, was officially implemented in 1974 with World Bank funding. It focused on enabling residents to build themselves more decent homes, thereby improving their quality of life, and providing for the growth of employment opportunities in general. Thus attempts were made to increase residents' access to services such as clean drinking water, roads and clinics, and to provide markets where they could sell their produce (Lusaka Housing Project Evaluation Team, 1977).

Although project evaluation studies, particularly those commissioned by the World Bank or the Zambian Government, concluded that many of the aims of squatter upgrading were achieved (Keare and Parris, 1982; Mulenga, 1991), living conditions in former squatter areas did not improve much over the years. Knauder (1982) described living conditions in former squatter settlements as extremely poor, with many families living in tiny homes and drinking water from unsanitary wells located near pit latrines. About twenty years later, Ndhlema (2000) reported similar living conditions. She describes the townships as lacking adequate access to social services such as clean drinking water and proper sanitation, adding that residents lived in unhealthy, crowded conditions that exposed them to diseases such as diarrhoea, dysentery and cholera. My own observations of living conditions in former squatter townships during fieldwork, and reports I read in daily newspapers, led me to a similar conclusion.

However, the composition of residents in the township where control schools were located had changed over the years to include low income state employees such as teachers, health workers, police officers, local authority employees, and some retired civil servants. The remainder of the residents comprised mainly small scale business people: traders owning small shops, market stalls or engaged in street vending, and manufacturers of beds, chairs, lounge suites, metal buckets, and so on. Inevitably household incomes varied, ranging from as low as K50, 000 per month (Ndhlema, 2000) to as high as several millions of Kwacha a month in the case of shop owners. Nevertheless, many school children in the area could be categorised as coming from poor homes, since about 55% of residents were classified as poor (Ndhlema, 2000). I now describe the two control schools located in the township.

School A was classified by the Ministry of Education as a basic school, that is, a school providing education to pupils in Grades 1 to 9. It was located about one hundred metres from the township's main bus station and was surrounded by a high security wall. When lessons were in session, gaining access to the school required knocking on the sheet metal gate to attract the attention of the school guard, who would then open the gates to let one in. Although surrounded by poorly built houses with no regular pattern, the school itself was in a very good state of repair, having undergone rehabilitation a few months earlier. It had its own water reticulation system, and was connected to the national electricity grid. During my field work, a number of workers could be seen landscaping the area between classroom blocks and the main ablution block. I learned later that one reason why school surroundings looked clean was that school had been closed at a certain time during the year, because of a cholera outbreak. One of the conditions for its re-opening was to improve water reticulation. The administration section of the school had a separate office for the head and the school secretary, who had a working land-phone and a manual typewriter. The school also had a number of reams of paper for production of test papers, a staff room, a storeroom, and a sports field.

However, the schools' performance in public examinations at Grade 7 was poor, with less than a quarter of pupils who took the examination each year qualifying to enter Grade 8. Indeed, despite school buildings looking neat and attractive from the outside, inside the classrooms the situation was different. Several pupils crowded around desks meant for two or three children, making it difficult during administration of pre-tests and post-tests to ensure that pupils did not copy answers from each other. In accordance with requirements for literacy lessons under the NBTL strategy, all Grade 2 classrooms had a bookshelf holding a number of readers, and a mat in front of the classroom where pupils would sit during whole class lessons.

School B was similar in many ways to School A. It was also a basic school surrounded by a security wall, was undergoing rehabilitation at the time of the study, and had attractive school buildings which were in a good state of repair. The school also had piped water and was connected to the national electricity grid. However, unlike School

A, the classrooms here, particularly in the higher grades, appeared less overcrowded. I learned that school was also closed for a while during a cholera outbreak. When it re-opened, several children did not return because their parents decided to find them school places elsewhere. However, at Grade 2 level the situation was not much different from School A. Grade 2 pupils whose classrooms were undergoing rehabilitation shared accommodation with colleagues in classrooms that still had to be worked on. Because of this, it was also difficult during the administration of pre-tests and post-tests to keep pupils from seeing one another's answers. As was the case with School A, inside each Grade 2 classroom was a shelf holding some readers, and a mat where pupils sat during whole class lessons.

The school had a separate office for the head but no office for a secretary. It had a land-phone which was not working. However, there was a working duplicating machine, a storeroom, and a small library which also served as an office for senior teachers. Like at School A, only a quarter of pupils who sat for the Grade 7 final examinations each year were selected to proceed to Grade 8. Teachers in the school told me that this progression rate from Grade 7 to Grade 8 was the norm nation-wide. I also learned that the number of textbooks in the school per subject taught was equivalent to half the total number of pupils per class, which meant that during lessons two pupils shared one textbook.

3.3.2.2 Experimental Schools

Experimental schools were located in a large township situated north east of Lusaka's main commercial centre. The township was first developed in 1967 as a site and service residential area, that is, an area officially set aside for members of the public who met certain conditions to build themselves houses on a layout provided by city planners. City planners provided building loans and technical assistance, and services such as piped water, access to roads, schools, clinics, and markets (Mwenda, 1978; Knauder, 1982).

Although site and service areas were created to benefit the urban poor, city planners' insistence on high construction standards prevented many members of the target group from taking up the available plots, thereby allowing higher income families to move in (Rakodi, 1991). Because of this, site and service residential areas eventually came to include a variety of house patterns, ranging "from structures similar to squatter houses up to houses beyond the standards of formal low cost housing areas" (Knauder,1982, p. 20).

Another reason for the variety in house patterns in such areas was that over the years the authority of city planners to assign plots to new residents was sometimes challenged by local political leaders, who unofficially allocated plots to members of their own parties (Nyathi, 1978). Consequently, some buildings did not meet city planning standards, while others were located in areas previously meant for construction of schools or clinics. In the end, as Mwenda (1978) has put it, there was unplanned development on a planned layout.

The degeneration of site and service areas over the years into partly planned and partly unplanned areas meant that eventually there was little to separate them from former squatter compounds, particularly with regard to living conditions at household level. Indeed, like their counterparts in the former squatter compounds, residents of site and service areas comprised low-income civil servants (teachers, health workers, police officers, and employees of the local authority, etc.) and retired civil servants. There were also small business people such as shop owners, operators of transit and passenger vehicles, owners of market stalls, and manufacturers of beds, chairs, and so on. Therefore, school children living in former squatter townships and their counterparts in site and service areas had much in common as far as socio-economic backgrounds were concerned. I now describe the two experimental school located in the area.

School X was classified by the Ministry of education as a Basic School. It was surrounded by a security wall intended to protect school property from being vandalised. However, at the time of the study, a section of the wall opposite the main gate had fallen to the ground, so that a number of people from the nearby community passed right through the school grounds whenever the main gate was open. Just outside the school were small grocery stores which played music from time to time, but whose volume was such that the music could not always be heard from inside the classrooms.

Classrooms in School X were generally in a worse state of repair than classrooms in control schools. In the classroom block for grades 1 and 2 the planks holding the roof were badly damaged by termites and during the rainy season ants invaded classrooms, forcing teachers and children to leave the classroom for a while. Nevertheless, each classroom had a usable chalkboard, some wall charts made by the teachers in the school, desks for pupils, and a mat and a cupboard required during literacy lessons.

The administration section of the school had a separate office for the head teacher but none for a secretary. There was also no secretary. The school did not have a staff room although teachers could prepare lessons from a small room used as an office for senior teachers. However, it had a storeroom, a manual typewriter (in a state of disrepair), electricity in one classroom block, piped water, and a science kit. Science kits comprised a set of laboratory equipment used by teachers to demonstrate science experiments. Pupils watched but often did not themselves touch the equipment, which suggests that pupils in the school probably did not develop the kinds of scientific skills that require a hands-on approach.

Concerning performance in public examinations at Grade 7, School X was no different from the other schools already described: less than a quarter of pupils who sat for Grade 7 examinations each year qualified to enter Grade 8. However, teachers told me that the school had sufficient teaching/learning resources, such as textbooks, for between 25 and to 50 % of the children in a given class.

School Y was classified by the Ministry of education as a middle-basic school, which in Zambia was another way of saying a primary school. That is, it provided education for children in Grades 1 to 7. Despite this, the school had junior secondary classes (Grades 8 and 9) as a fund raining venture under what was called *the Academic Production Unit* (APU). APU referred to secondary school classes for children whose examination marks at Grade 7 or Grade 9 were not high enough for official selection to Grade 8 or Grade 10 respectively. The school was surrounded by a security wall and was located next to a large Christian church on one side, and a place where people could buy and drink beer, usually called a bar in Zambia, on the other. The bar played loud music throughout the day and only stopped doing so when the electricity company turned off power in the area.

The administration section of the school had a separate office for the head teacher and for the secretary, although at the time of the study there was no secretary. There was no telephone but the school had a duplicating machine in the office meant for the secretary, which was not working. The school had a storeroom for teaching materials, a staff room, and a science kit. There was piped water, a newly-built ablution block, and one classroom which had electricity. It was the same classroom I used as the venue for the staff development workshop.

Most classrooms needed repairs, as a number of windows were broken and walls needed repainting. Nevertheless, each classroom had a usable chalkboard, a teachers' chair (but no teacher's table), some wall-charts made by the teachers themselves, and student desks. The desks were originally meant to seat three pupils but at the time of the study at least four pupils crowded around one desk. In addition, each Grade 2 class had a mat in front for whole class activities, and a cupboard as required under the NBTL programme. With regard to availability of teaching and learning materials, particularly books, these were sufficient for less than a quarter of the total number of children in each class. As was the case in the three other participating schools described earlier, less than a quarter of all the pupils who took the Grade 7 public examination qualified to enter Grade 8.

3.4 Methods

3.4.1 Lesson Observation

Lesson observations helped me to learn how the five Grade 2 teachers in experimental schools implemented the curriculum based on mental strategies. Observation is a data collection procedure whereby researchers try to understand what is happening in a given setting by paying attention, watching and listening carefully (French et al, 2001; Neuman, 2003; Patton, 2002). "The non-participant researcher takes a detached stance to the phenomena, and aims to be 'invisible', either in fact or in effect (i.e. being ignored). The participant-observer is seen as involved in the process or activity" (Walliman, 2001). Between these two extremes is a continuum representing a variety of degrees of researcher participation in the activities of those being studied (Merriam, 1998; Patton, 2002). In the present study I located myself between the two extremes but towards the participant-observer end. For example, although I was visibly present in the classroom while teachers conducted lessons (and pupils referred to me as 'Teacher'), I did not teach any lessons or help pupils who did not understanding what they were learning. When pupils asked me to help them solve mathematics problems, I always tried to steer their attention towards the class teacher.

I recorded observations in the form of *field notes.* These comprised descriptions of what took place and in what physical setting, including the statuses, roles and activities of key players, and my own personal reflections (Jorgensen, 1989; Lofland and Lofland, 1995; Patton, 2002). (See Appendix 3 for the form used) Figure 10 illustrates the seating arrangement in Grade 2 classrooms, which was adopted by the Ministry of Education when it implemented NBTL lessons. It shows pupils' desks arranged to form four 'Learning Groups' based on ability. I usually sat in the position marked O (for observer), with the shelf holding reading materials directly behind me. The chalkboard (C) and the mat (M), used for whole class lessons, are also shown.

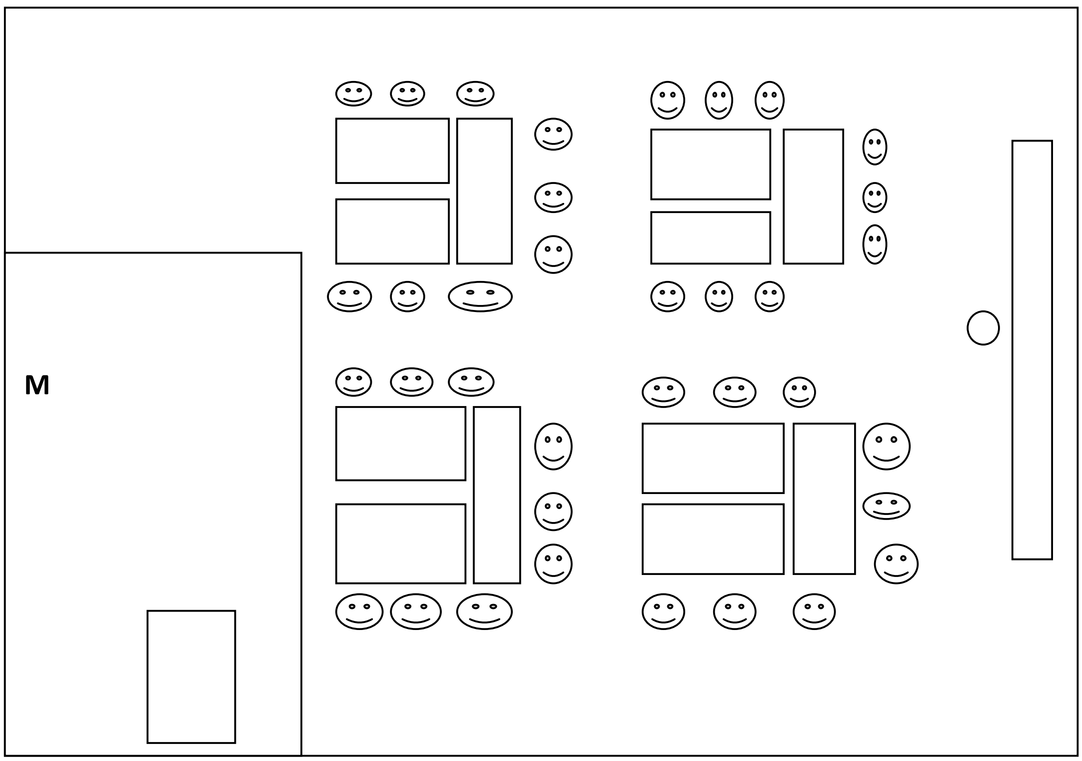


Figure 10: Seating Arrangement in Grade 2 Classes

Field notes are the main method for recording observations during fieldwork (Bryman, 2004; Lofland and Lofland, 1995; Neuman, 2003). However, in recent years a number of researchers have used a video camera either to supplement field notes or as an independent method of data collection on its own (Angrosino and Mays de Perez, 2003; Arksey and Knight, 1999; Downing, 2008). One of the advantages of using a video camera to record field observations is that it allows one not only to capture the physical environment but also to revisit the images later and relive the experiences (Downing, 2008). Indeed, as Angrosino and Mays de Perez (2003) observe, "[audio-visual] technology makes it possible…to record and analyze people and events with a degree of particularity that would have been impossible just a decade ago…" (p. 144).

However, not every research situation calls for the use of a video camera (Downing, 2008). In fact, using a video camera can create a number of problems for the researcher. For example, it is difficult for research participants not to notice its presence, which can influence them to behave in ways they otherwise would not (Arksey and Knight, 1999; French et al., 2001; Mason, 2002).

> Cameras and other forms of [audio-visual] recording equipment can make people feel self-conscious, sometimes frightened or intimidated, or as though they are under surveillance. Equally they can prompt people to want to capture the limelight and "be a star", without always fully appreciating the consequences (Mason, 2002, p. 118).

Thus research participants who find the video camera too intrusive may object to its use; while those who welcome its use might behave artificially. There is therefore the possibility that the data captured will not reflect events on the ground. Furthermore, when using a video camera one tends to want to capture everything rather than being selective, which can complicate the process of analysis (Arksey and Knight, 1999). Finally, the video camera has a restricted range, so that some important information can be missed particularly if the researcher him or herself is operating the equipment (French et al, 2001). Given these potential problems regarding the use of a video camera, and not wanting impressionable Grade 2 pupils to spend their lesson time watching the camera rather than listening to their teacher, I concluded that in my study a video camera would be more of a liability than an asset, and did not use it.

3.4.2 Interviews with Teachers

Three different types of interviews with teachers were conducted in experimental schools to obtain information relating their experience of the study. Kvale (1996) defines a research interview as a conversation between two partners about a theme of mutual interest, aimed at generating knowledge through a dialogue. All interviews with teachers were semi-structured, so that I could follow up interesting responses and/or stories and at the same time obtain specific information relating to the study. The interviews were tape-recorded.

3.4.2.1 Group Interviews

Group interviews (Appendix 4) were conducted in experimental schools at the end of the staff development workshop. I used them to determine what participating teacher had learned from the workshop regarding the use of mental strategies for adding and subtracting double-digit whole numbers, and to evaluate the effectiveness of the workshop. Group interviews are interviews whereby "several participants are interviewed at the same time and place" (French et al., 2001, p. 132). Such interviews are useful in evaluating educational innovations such as the one implemented in this study and, in this case, the groups interviewed can comprise teachers and/or children involved in the process (Cohen et al., 2000; Patton, 2002; Watts and Ebbut, 1987).

A major disadvantage of group interviews is that, if the group includes people of different statuses it is possible for the 'high status' members to dominate or inhibit the views of other members of the group (Gillham, 2000; French et al., 2001). It is therefore important to ensure either that there are no marked differences in the statuses of group members and/or that the interviews are managed skilfully to reduce the likelihood that some group members will dominate the deliberations (Gillham, 2000). In the present study all participants in group interviews were Grade 2 teachers who had worked together for a long time and were similar in social status.

3.4.2.2 Post-observation Interviews

Post-observation interviews with teachers (Appendix 5) followed lesson observations. As Wallen and Fraenkel (2001) state, "interviewing…is an important way for a researcher to check the accuracy of…the impressions he or she has gained through observation" (p. 440). My aim was to conduct these interviews immediately after observing lessons. Since all Grade 2 lessons in experimental schools took place in the afternoon, if the lesson I observed happened to be the last one and there was no time to interview the teacher at the end of the day, I would return the next day to do it.

Post-observation interviews enabled me to understand what the teachers wanted to achieve during the lesson and how their beliefs about mathematics teaching and learning aided or hindered their teaching. The interviews also gave teachers the opportunity to reflect on their experiences during the lesson and to articulate their evolving understanding of possible implications of such experiences for their role as teachers in mathematics classrooms using strategies for mental calculation. Franke et al. (1998) used a similar approach in conducting post-observation interviews with teachers who participated in the CGI Project in the US. They explored what teachers wanted to achieve during the lesson, why they chose particular courses of action, what they had learned about the students, and how the teachers planned to use what they had learned during the lesson in planning and conducting future lessons.

3.4.2.3 Post-treatment Interviews

A post-treatment interview (Appendix 6) was conducted with each participating Grade 2 teacher in experimental schools at the end of the study. The aim was to elicit the views of the teachers concerning the impact of the study on their teaching of mathematics, and on their pupils' mathematical learning. I also used the same interviews to asses the extent to which both the teachers and the pupils learned to use strategic methods for double-digit whole number addition and subtraction. Furthermore, the interviews helped me to judge if teachers' beliefs were changing to be more in line with the use of strategies for mental calculation, and whether or not the teachers would recommend extension of the pilot to the whole country.

3.4.3 Teachers' Journal Entries

On the first day of the staff development workshop I distributed notebooks to participating teachers, which they used throughout the study to record personal understandings and reflections regarding their involvement in the study. Several researchers have used teachers' journal writings in this way to complement direct observation and interviewing (Connelly et al., 1997; Lofland and Lofland, 1995; Simon and Schifter, 1991). According to Simon and Schifter (1991), teachers' journal entries "[can give] an indication of their learnings, understandings, and implementation experiences" (p. 317).

3.4.4 Numeracy Assessment Test

During the first two weeks of the implementation phase, but before experimental schools had covered much ground on the experimental curriculum in the classroom, class teachers administered a numeracy assessment test (pre-test) in both experimental and control schools. They *read* each question, while pupils listened, repeating questions as necessary to ensure that pupils could hear the question, but did not explain what the questions required to be done. Pupils wrote their answers in answer booklets which used pictures of familiar animals to indicate where answers to particular questions were to be written.

The pre-test was intended to determine if comparison groups were similar or different at the beginning of the study, and to provide baseline data with which to measure the effectiveness of the intervention. As Gay (1996) states: "The purpose of…baseline measurements is to provide a description of the target behaviour as it naturally occurs *without* the treatment. Thus, the baseline serves as the basis of comparison for assessing the effectiveness of the treatment" (p. 377, emphasis in original).

Since test questions and instructions regarding their administration were presented in English, before the teachers administered the test I met them to agree on how to conduct the exercise and to identify appropriate phrases to use in Chinyanja (the dominant Zambian language spoken in and around Lusaka). As explained in Chapter 2, it has been a policy of the Ministry of Education in Zambia since 1996 to allow lower primary school pupils initially to learn all subjects (except English Language) in a familiar (Zambian) language and begin weaning children off this towards Grade 4. Policy makers recognised that some Zambian children would attend school in areas where the dominant Zambian language spoken was different from their own. However, they argued that it was easier for Zambian children to learn another Zambian language than to learn a foreign language such as English. Because of this, no special language arrangements were made on behalf of such children.

The post-test, a *parallel* rather than the same test as the pre-test, was administered at the end of the intervention. According to Cohen et al. (2000), "The pretest may have questions that differ in form or wording from the post-test…[so long as the two] test the same content" (p. 334) and have the same level of difficulty. Differences between the pre-test and the post-test were minor, and pupils used the same answer booklet for both tests. For example, in both the pre-test and the post-test, Question 1 presented pupils with a drawing showing five bicycles. In the pre-test pupils were asked to write down the number of bicycles in the picture, while in the post-test they gave the total number of bicycle wheels. Both tests were an adaptation of a validated test used in the Leverhume Numeracy Research Programme with Year 1 children in the UK, the equivalent of Grade

1 pupils in Zambia. It was adapted and used in my study with the permission of the authors based at King's College, University of London.

In basing a Grade 2 test on the equivalent of Grade 1 mathematics subject matter content I followed the practice adopted by Sharma and Henderson (1974) in their nation-wide survey of the achievement in mathematics of Grade 3 pupils in Zambia. Sharma and Henderson used Grade 2 mathematics subject matter content to develop a test for Grade 3 pupils, reasoning that they would most likely have covered the mathematics subject matter meant for Grade 2 children.

Commentators have pointed out that administering a test twice (or giving a similar test on the second occasion), paused a threat to the internal validity of a study, in that test takers' experience from the first test will influence their performance on the second test (Gay, 1996; Wallen and Frankael, 2001). However, this was not a problem in the present study since testing effects applied equally to both experimental and control groups. Furthermore, participants were young children and about ten weeks elapsed between the two administrations of the test, which reduced the possibility that the children saw a connection between the pre-test and the post-test (Gay, 1996). Copies of the pre-test, the post-test, and the answer booklet appear in appendices 7, 8, and 9 respectively.

3.4.5 Document Analysis

Analysis of existing documents (e.g. syllabuses, lesson plans, student performance records, objects in the classroom, etc) relating to the teaching and learning of mathematics at Grade 2 level enabled me to understand what pupils at this Grade level were expected to learn and to know in mathematics. According to Merriam (1998), "[documentary] data can furnish descriptive information, verify emerging hypotheses, advance new categories and hypotheses, offer historical understanding, track change and development, and so on" (p. 126).

Nevertheless, existing documents should be used with caution. It is important to determine their authenticity and accuracy before using them, since it is possible that the information organisations release to members of the public might differ from that which they keep on their own files (Mason, 2002; Merriam, 1998; Patton, 2002). As these authors indicate, once questions of authenticity and accuracy have been considered, existing documents can be used in much the same way as data from interviews and observations.

3.4.6 Procedures for Data Collection

According to Rudestam and Newton (2001), a description of procedures for data collection should not only mention how data were collected but also how access to research sites was obtained and the exact steps taken to contact research participants and win their co-operation. Following Rudestam and Newton (2001), I describe below how I gained access to research sites, and then provide an account of the procedures I employed to collect the data needed to answer my research questions.

3.4.6.1 Negotiating Access to Research Sites

I explained in Section 3.3.1 how I obtained permission at district level to conduct my study in Lusaka schools. The letter submitted for this purpose appears in Appendix 10. After this, I approached head teachers for permission to work in their schools. I decided first to meet heads of two schools in the residential area where I hoped to have experimental schools. I did this so that Grade 2 teachers there could start preparing to attend the staff development workshop scheduled for August 2004, coinciding with school holidays at the end of the second school term. I met the school heads, explained to them what I intended to do, showed them my letter of permission from district educational administrators, and they allowed me to conduct the study in their schools.

My next task was to meet Grade 2 teachers in these schools, to persuade them to participate in the study. The teachers were initially unwilling to be involved partly because it meant attending the staff development workshop during school holidays, and they had already made plans about spending the holidays. After I explained that the proposed study would benefit their pupils' learning of mathematics and that by attending the staff development workshop and participating in implementing the experimental numeracy development approach they themselves would be advancing their professional development, they agreed to participate. I then wrote formal letters inviting each of them to participate in the study and included information about what else would be involved (Appendix 11).

Afterwards, I visited the area where I hoped to have two control schools to meet head teachers and ask their permission to include their schools in the study. After meeting them and explaining my mission they also granted me permission to work in their schools. I then talked to Grade 2 teachers, who quickly agreed to participate in the study. It would seem that the teachers' quick positive response was influenced partly by the fact that their role in the study coincided with their normal duties as teachers. They would administer pre-tests and post-tests and cover similar mathematics topics as would their colleagues in experimental schools, but using procedures as currently taught. Surprisingly, teaching schedules in control schools indicated that the mathematics topics I wanted Grade 2 teachers to cover during the study were the very ones they intended to teach at the beginning of the next school term.

3.4.6.2 Data Collection

For teachers in experimental schools, data collection effectively started on the first day of the staff development workshop, when I distributed notebooks to them and asked them to record in them their learning experiences as the study progressed. I began writing field notes at the same time.

When schools re-opened for the third term in September 2004 data collection continued with administration of pre-tests, first in experimental schools and then in control schools. It was important to start this exercise in experimental schools so that administration of pre-tests could be completed quickly to allow the teaching of the experimental numeracy development approach to begin. Since each participating teacher had two Grade 2 classes, whose lessons followed each other, it was possible to administer the test to all Grade 2 pupils in a given school on the same day. As mentioned earlier, in each participating school class teachers administered pre-tests in my presence. The day before, I met the teachers to explain the test administration process and to agree on which words to use in Chinyanja when translating test questions from English into this language.

During the administration of pre-tests in experimental schools it became clear that participating teachers were finding the process rather long and tiring, particularly given that they had to do it twice on the same day. This was partly due to the fact that at the beginning of the tests pupils tended to take rather long (in many cases about 30 minutes) to complete particulars on the cover page of their answer booklets, which included writing down their name. I was surprised at how difficult many children found the task of writing their own name on the answer booklet and how long it took them to do so.

In one classroom, after the class teacher and I had concluded that we were making no progress in this regard, I suggested to her that we use an example to help the pupils see what we wanted them to do and where to write their names. The teacher then reproduced on the chalkboard the required information on the cover page of the answer booklet and told the pupils: “If your name is Martin, for example, you should write Martin here”, pointing to the area where pupils’ names were to be written. To our amazement, when we went round to check if the children had done the correct thing we found that several of them, including girls, had written “Martin” as their name! I wondered if the NBTL programme had concentrated too much on teaching children to read, to the exclusion of helping them to learn to write even their names.

Another reason for the long test sessions was that teachers were required to ensure that pupils had written their answers to one question before introducing the next question. However, pupils took rather long to complete one question before moving to the next, and teachers had to wait for them. To reduce the amount of work involved in test administration for the teachers and for myself, we agreed that during the post-tests only the Grade 2 class each teacher considered to be the better in terms of ability between her two classes would take the examination. Consequently, I did not include in the analysis the scripts of pupils in the less-able other classes, although we allowed them to continue participating in the study.

Nevertheless, I allowed one teacher to continue with two classes because it was clear from the staff development workshop that she understood most completely the aims of the study and would implement them effectively. Since my intention in the study was to determine whether and to what extent both Grade 2 teachers and their pupils could learn to use strategies for mental calculation for double-digit whole number addition and subtraction, I felt that this teacher's classes offered me the best chance to learn about this. If the use of mental strategies could not be effective in her classes they would probably not work in any other Grade 2 teachers’ classes in experimental schools.

When it was time to administer pre-tests in control schools, only one class per teacher participated, namely each teacher's better class in terms of mathematical ability. Immediately after pupils had completed pre-tests, I started making site visits to experimental schools. My first concern was to ensure that Grade 2 teachers in the schools began implementing the experimental numeracy development approach, as without doing so there would be nothing to observe. I found that the teachers continued to need my assistance with regard to how to present certain topics in the experimental numeracy development approach throughout the study. As I mentioned earlier, I recorded lesson observations in the form of field notes. I also conducted post-observation and post-treatment interviews and used a tape recorder to record them.

3.4.7 Data Analysis

This study generated both qualitative and quantitative data. In what follows I explain how the data of each kind were analysed.

3.4.7.1 Analysis of Quantitative Data

Quantitative data were analysed by means of a combination of descriptive and inferential statistics. Descriptive statistics, mainly calculation of frequencies and percentages, were used in discussing Grade 2 pupils' mathematical knowledge and skills before the alternative numeracy development approach was implemented in experimental schools. Inferential statistics (i.e. t-tests) were used mainly to determine if there were any statistically significant differences in mathematics performance between Grade 2 pupils in experimental schools and control schools at the beginning and end of the study. I used the statistical package SPSS to calculate the t-values.

There has been some debate in recent years among researchers with regard to the adequacy of reporting treatment effects in experimental studies in terms of statistical significance tests only (Capraro, 2004; Muijs, 2004; Kellow, 1998; Thompson, 2002; Wright, 2003). It has been argued that results of statistical significance tests alone do not make it possible for readers to understand fully what a given study achieved. As Muijs (2004) states, "knowing that [a] relationship is [statistically] significant does not tell us whether the effect is strong or weak" (Muijs, 2004, p. 136). In other words, *statistical* significance is not the same thing as *practical* significance (Kellow, 1998).

It has therefore been suggested that reports of statistical significance should be supplemented by other estimates of treatment effects, such as *effect size.* According to Gay (1996),

> Effect size is a numerical way of expressing the strength, or magnitude, of a reported relationship, be it causal or not.…*[For] an experimental study…effect size expresses how much better (or worse) the experimental group performed as compared to the control group*" (p. 267, my emphasis).

Thus effect size provides a measure of the practical value of a treatment. Because of this, in addition to using significance tests (t-tests) in this study I also calculated in terms of effect size an estimate of the impact of the study on pupil learning. However, it is important to point out that currently there are a number of alternative ways to calculate effect size. Kellow (1998) discusses three different measures of effect size, and Thomson (2002) suggests that there may well be over a dozen such measures. Among these, calculation of Cohen's *d* has gained popularity among researchers (Thompson, 2002) and is the one I adopted in this study. When only two groups are being compared, as happens when t-tests are used to assess statistical significance, Cohen's *d* is calculated as follows:

***d* = (Mean of Group A - Mean of Group B) ÷ Pooled standard deviation**

where Pooled Standard Deviation = (Standard Deviation of Group A + Standard Deviation of Group B) ÷ 2.

Since Cohen's *d* gives effect size in standard deviation units, its interpretation can be facilitated by consulting the standard normal distribution table (Kellow, 1998). Researchers have given guidelines for determining whether an effect size is small, medium, or large (Muijs, 2004; Thompson, 2002; Wright, 2003). According to Muijs (2004) these are as follows: 0 - 0.20 = weak effect; 0.21 - 0.50 = modest effect; 0.51 - 1.00 = moderate effect; and > 1.00 = strong effect. However, these guidelines are suggestive rather than prescriptive, as researchers still have to reach consensus over the matter (Thompson, 2002). Furthermore the importance of an effect size, whether small or large, is determined by a number of factors. As Kellow (1998) indicates, an intervention implemented for a short time with little financial investment that generates a small effect size may still be considered important, on the assumption that a similar longer and well funded intervention would probably have had a bigger impact. On the other hand, a long, well-funded intervention that results in a moderate effect size may not be worth pursuing further.

3.4.7.2 Analysis of Qualitative Data

Qualitative data mainly from interviews and field notes were first transcribed fully (i.e. transformed into *written* form) before being analysed. I did not anticipate the difficulty involved in actually performing transcription and the sheer length of time it would take to complete even a single interview transcript. This forced me to put the task aside hoping to return to it at some opportune time in future, a future that almost never came as I kept putting off the decision to resume transcription for several weeks fearing what the task involved. It was not until I learned that despite appearing intimidating at first transcription becomes easier as one keeps *practising* it in subsequent research projects (Bird, 2005) that I gathered enough courage to go back and complete the task.

Once all transcripts were ready I analysed them manually, using the constant comparative method (Maykut and Morehouse, 1994; Neuman, 2003; Rudestam and Newton, 2001; Strauss and Corbin, 1998). As Neuman (2003) shows, this method involves mainly making three passes through the data. The first, called *open coding* requires reading through the data carefully and noting the themes or categories (also called codes) which might be there. The second pass through the data, called *axial coding*, involves trying to see how the categories already identified are related, so that major categories and sub-categories can be identified. The aim, as Rudestam and Newton (2001) put it, is to "assess how major categories relate to each other and to their subcategories" (p. 158). The final phase, called *selective coding*, is intended to bring together the themes identified in the data to determine how they hang together. The central theme is identified, creating a theoretical model of how the other themes or categories in the data relate to it.

Following the above procedure, I read through each transcript a number of times and then (literally) cut up sections of it which seemed to me to relate to different themes. I pasted these under appropriate labels using wide pieces of manila paper, and then read them again to be sure that they represented the main theme earlier identified. It is these themes which I used to structure my presentation and discussion of results, which I supported with appropriate verbatim quotes from what the teachers said.

CHAPTER 4

FINDINGS OF THE STUDY

4.0 Introduction

This chapter presents findings of the study relating to both teachers and pupils, as indicated by the research questions stated in Chapter 1. According to Rudestam and Newton (2001), most results chapters begin with a description of the sample, for example giving demographic information relating to participants, before examining research questions. This is what I have done in presenting the results of the study below.

Section 4.1 presents results relating to participating teachers. In this regard Section 4.1.1 gives the demographic characteristics of participating teachers in experimental and control schools, using the letters A, B, C, and so on, rather than names, to hide the teachers' identities. The same identifiers are used in later sections of the chapter and in subsequent chapters when referring to the same teachers. Section 4.1.2 discusses the extent to which teachers in experimental schools learned to foster the use of strategies for mental calculation relating to double-digit whole number addition and subtraction in early primary mathematics. It begins with a brief portrait of each teacher before answering relevant research questions about that teacher. Finally, Section 4.1.3 compares and contrasts the behaviour of the teachers with regard to the same set of questions.

Section 4.2 presents the results of the study relating to pupils. Section 4.2.1 examines the numeracy achievement of participating Grade 2 pupils at the beginning of the study, that is, before the alternative numeracy development approach based on the use of strategies for mental calculation was implemented in experimental schools. The results obtained in this case were based on the pupils' performance on the pre-test, and provided baseline information with which the effectiveness of the alternative numeracy development approach implemented in experimental schools was measured (Borg and Gall, 1989; Kyriakides, 2002). Section 4.2.2 considers findings relating to pupils' learning to use strategies for mental calculation for double-digit addition and subtraction. Finally, Section 4.2.3 presents findings concerning post-treatment differences between pupils in experimental and control schools. These were mainly based on quantitative data relating to the performance of the two comparison groups on the post-test, although related qualitative measures of impact assessment were also considered.

4.1 Findings Concerning Teacher Learning

4.1.1 Demographic Characteristics of Participating Teachers

Tables 1 and Table 2 show demographic characteristics of participating Grade 2 teachers in experimental and control schools respectively. The tables use the following abbreviations to represent acceptable qualifications for teaching at primary school level in Zambia: ECTC for *Early Childhood Teacher's Certificate*, PTC for *Primary Teacher's Certificate*, and PTD for *Primary Teacher's Diploma*. The PTC and the PTD were two-year teacher education programmes designed by the Ministry of Education. The PTC had been around in more or less the same form since the 1960s, while the PTD was only introduced after 2002 and served as an avenue for the continued professional development of teachers holding the PTC. Holders of both the PTC and the PTD taught at all levels of primary education, that is, from Grades 1 to 7. On the other hand, the ECTC was an initiative of privately owned colleges and its duration varied from one to two years, depending on the college offering it. It prepared teachers to handle classes from pre-school up to Grade 4. In both tables a tick [✔] identifies teaching qualifications already obtained, while the term *current study programme* is used to show qualification towards which teachers were working at the time of the study.

Table 1: *Demographic Characteristics of Participating Teachers in Experimental Schools*

Teacher	Sex	Age Range	ECTD	PTC	PTD	Current Study Programme	Teaching Experience (Years)	Teaching Experience at Grade 2 Level (Years)
A	F	40-49		√	√		16	3
B	F	30-39	√				3	2
C	F	40-49		√			18	5
D	F	50-59		√	√		16	7
E	F	40-49		√		PTD	25	<1

Table 2*: Demographic Characteristics of Participating Teachers in Control Schools*

Teacher	Sex	Age Range	ECTD	PTC	PTD	Current Study Programme	Teachin g Experie nce (Years)	Teaching Experience at Grade 2 Level (Years)
F	F	20-29		√		PTD	5	3
G	M	30-39		√			4	1
H	F	20-29		√		PTD	4	2
I	F	20-29		√		PTD	3	2
J	F	20-29		√			4	1

Table 1 and Table 2 show that teachers in experimental schools were generally older and had taught longer in primary school than their colleagues in control schools. However, the two groups of teachers were not much different with regard to teaching experience at Grade 2 level. In fact, at the time of the study Teacher E, the oldest person among the ten, had taught at Grade 2 level for less than a year. Furthermore, only one of the teachers was male. This sex distribution pattern was common among teachers in lower primary school in Zambia at the time of the study. Finally, as mentioned in Chapter 3, the Grade 2 pupils taught by these teachers were similar in several respects, including age, socio-economic background, and numbers of boys and girls in each class.

4.1.2 Learning to Teach Strategies for Mental Calculation

As mentioned in Chapter 1, the main aim of this study was to asses the extent to which teachers in lower primary school in Zambia could learn to foster the use of strategies for mental calculation in early primary mathematics. In view of this, each of the five participating Grade 2 teachers in experimental schools was studied as an individual case. When these individual cases were compared and contrasted with regard to the extent to which each of them learned to foster the use of strategies for mental calculation relating to double-digit addition and subtraction, the result was a multiple-case study.

A useful way to present the results of a multiple case study is first to consider each case individually, and then to compare and contrast the individual cases (Merriam, 1998; Miles and Huberman, 2002; Patton, 2002; Yin, 2003). That is, one first does a *within-case analysis* before doing a *cross-case analysis* (Merriam, 1998). The current section, therefore, will present results relating to each of the five individual case studies separately, and the next section (Section 4.1.3) will focus on cross-case analysis. Discussion of the individual cases will be structured according to four themes, namely *teacher concerns*, *teacher beliefs*, *teacher knowledge* and *teaching practice*, as suggested by the research questions about teachers given in Chapter 1.

4.1.2.1 Teacher A

Teacher A was one of the oldest and most experienced teachers in experimental schools. She had both the PTC and the PTD. Nevertheless, when the study began she had only taught at Grade 2 level for three years. During the study, Teacher A had a relative at home who was unwell. She told me that if the condition of that relative worsened she would discontinue her participation in the study. I expected all participating teachers to put in their best, so that I would be able to determine whether or not the use of strategies for mental calculation improved pupils' learning results in numeracy in lower primary mathematics. Therefore, Teacher A's situation worried me because it possibly meant that she might not be committed to the study, and I had no way of telling when she would discontinue her participation. However, it turned out that her relative's condition remained stable during the study, so that Teacher A was able to participate in the study until it ended.

4.1.2.1.1 Initial Concerns

Three main concerns troubled Teacher A at the beginning of the study: She was convinced that the timing of the study was wrong; she believed that Grade 2 pupils would not understand the mathematical ideas we were trying to implement; and she was sure that her Grade 2 pupils would not be able to develop the ability to explain or justify publicly their mathematical reasoning in the classroom. Teacher A expressed the first of her concerns towards the end of the group interview at her school:

Researcher: Does any of you have anything more to say concerning the workshop we have just completed?

Teacher A: Initially, I thought it was too late to start a workshop.

Researcher: What do you mean? Did you think it was too late in your professional life as a teacher?

Teacher A: No.

Researcher: Or too late before schools re-opened for the third term?

Teacher A: Too late for our students, since they are going into the third term. I thought the best [time to hold the workshop] would be in the first term.

Teacher A was right in thinking that the study should have been conducted in January, when the school year began in Zambia. It would have made it possible, if necessary, to follow-up the pupils during the year while they were still in the same grade. However, a combination of factors made this difficult to accomplish. For example, research funds only became available to me in the third quarter of the year, and I could not delay the staff development component since I was not sure whether the Grade 2 teachers I was to work with would be available the next January. However, I found out later that Teacher A's conclusion that the study was ill-timed, was partly influenced by her initial lack of understanding of what mental calculation involved. Her Grade 2 classes had done double-digit addition and subtraction in Grade 1, and she had concluded that there was therefore nothing else left for them to do that would be different. This is suggested by what she says in the following excerpt from the post-treatment interview:

> I thought it was too late for Grade 2 pupils to do this work, because this work was done in the third term of Grade 1. So I thought...nothing was going to happen. I thought we were not going to achieve anything.

Teacher A was also concerned that the mental strategies we discussed during the workshop were too advanced for Grade 2 pupils. She especially doubted whether her Grade 2 pupils had the ability to partition double-digit numbers into multiples of ten and units. During the group interview she said:

> Partitioning [double-digit] numbers? Solving word problems? There I am pretty sure I am worried. Will Grade 2 pupils understand the language and be able to find the answers?

Finally, Teacher A was not sure that her Grade 2 pupils could develop the ability to explain or justify publicly in the classroom their solution strategies. However, when I asked her during the same group interview what she would think if classroom events suggested that she was wrong in this regard, she indicated readiness to change her views:

> It is difficult to tell right now, since…we still have to teach these lessons. We shall talk…about it after we have seen how it works [in the classroom]. There are teachers in my school who, like me, are asking: "Can a Grade 2 child do this

mathematics?" That is my worry right now. I am yet to find out if a Grade 2 child can do this work.

4.1.2.1.2 Final Concerns

As is clear from her last statement above, Teacher A was *willing* to try things out in the classroom and to accept classroom evidence concerning whether or not her Grade 2 pupils could learn to use strategies for mental calculation. Indeed, by the end of the study Teacher A's initial concerns had all disappeared. I reminded her about them during the post-treatment interview:

Teacher A: I had so many worries, especially during the staff development workshop. I looked at those numbers, which we said children would be partitioning, also doing it on the empty number line. I thought that was just too much for small ones. It worried me because we are used to giving pupils an example, and saying can you do exactly what I have done...So, when we were going through this research during the workshop, I thought maybe children wouldn't be able to express their own [mathematical] ideas.

Researcher: What do you think about those worries now, since we have gone through the study?

Teacher A: Looking back? As you saw when you observed my lesson, a child was able to stand up and go to the front of the classroom, and explained to her colleagues how she worked out an answer. Previously I used to say, "I don't think a Grade 2 child could do such a thing". Now I know that they are able to go to the chalkboard and even do some writing. At times even to discuss some writing….[Laughs].

Researcher: What do you think is the reason why at that time you did not believe that [Grade 2] children could do this work?

Teacher A: [Long pause]. Because...I was used to just telling them what to do.

Teacher A's last statement above is significant. She acknowledges the role she played in the past in connection with her pupils' failure to develop mathematical autonomy, which perhaps shows the extent to which classroom evidence had convinced her about the inaccuracy of her previous assessment of Grade 2 pupils' mathematical abilities. However, Teacher A's change in this regard occurred slowly over the duration of the study. She sometimes experienced frustration as she taught the necessary strategies for mental calculation, and at times even hoped that I would call off the study. This came out when I asked her at the end of a post-observation interview whether there was anything else she wanted to say about the lesson I had observed or about what she was doing in general. She responded:

> At times when I start a lesson, [for example] a new topic, sometimes I get discouraged. I always think, "These children are not doing anything. We are not going to achieve anything". That's why…when you said, "I want to observe your lesson", I was happy as I thought that you also would be discouraged when you see that they are not doing anything. I thought this work was…so difficult. But now they are trying.

Teacher A expanded on her statement above during the post-intervention interview.

> Now it is like we have as usual fast learners who are able to do the work properly, and slow learners, especially when it comes to using the empty number line. I wanted pupils to use the number line but they would simply add for example 9 + 5 and write down the answer. I said, "Can you show the answer on the number line", but they would simply write 9 + 5 = 14. "I want the 14 to be shown on the number line. Don't add directly". As we went on they were able to do what I was telling them.

Teacher A's frustrations would have been less if she had remembered that using flexible mental strategies can also become prescriptive if pupils are forced to adopt the teacher's thinking. It is obvious to me, however, that Teacher A, who previously was used to telling pupils exactly what to do and expecting them to follow her instructions, must indeed have found this learning process frustrating at first.

4.1.2.1.3 Initial Beliefs about Mathematics Teaching and Learning

Teachers A's fears and concerns at the beginning of the study were influenced by the beliefs she held about mathematics teaching and learning in primary school. As mentioned in Chapter 2, teachers' existing beliefs can aid or hinder their ability to foster the use of mental strategies in early primary mathematics. Knowing this, I always asked teachers at the beginning of post-observation interviews to tell me what they liked or disliked about the lesson I had observed. This allowed me to find out what they thought made mathematics lessons interesting or uninteresting, which revealed something about what they believed concerning mathematics teaching and learning.

Initially Teacher A believed that a mathematics lesson was interesting if pupils could repeat what she had demonstrated on the chalkboard. This was in line with her belief that teaching was *telling*, as the following extract from a post-observation interview conducted at the beginning of the study illustrates:

Researcher: What did you like most about your mathematics lesson today?

Teacher A: At the beginning of the lesson it was interesting in the sense that [the pupils] were able to follow and [to] do what I wanted, whether it was adding or subtracting, especially when we were counting in 10s. If it came to counting on in 10s, they were able to add. When I said, “Can we count back [in 10s]”, they were able to count back; they were able to subtract. On that part, it was interesting.

The reverse was also true. Teacher A did not like mathematics lessons where pupils were unable to follow her classroom demonstrations, as the same post-observation interview suggests.

Researcher: Anything you didn't like about the lesson?

Teacher A: You know it is funny. At times they [the pupil] were able to explain things, to express themselves, but could not write down what they were saying. For example, when I said count on in 10s starting at 15…they wrote [the next number as] 20. Some however were able to see what we were doing. But others were not. I don't know if they forgot that we were counting in 10s. They started at 15, one wrote 20, others 25. But a few were able to do it, [especially when] I asked them to count up to 100, going 10, 20, 30. That was simple. But when I said, "Now can you start from 15 counting in 10s", some pupils were writing the next number as 15. I don't know. May be they were thinking about the times table for 5.

4.1.2.1.4 Final Beliefs about Mathematics Teaching and Learning

As the study progressed, and as Teacher A encountered more and more classroom evidence of her pupils’ ability to use strategies for mental calculation for double-digit addition and subtraction, her beliefs about what made a lesson interesting began to change. Rather than expecting pupils to *repeat* exactly what she had taught them, she was happy simply to see that they arrived at the correct result, regardless of the solution method they had used. The following extract from a post-observation interview held about a month after the study began illustrates this point.

Researcher: Is there anything you liked about the lesson I observed, so much that you think you should look at again in future?

Teacher A: I think the ways pupils were able to find solutions. It is like now they don't necessarily follow examples from the teacher. Each one has his [or her] own ways of finding answers.

Researcher: Why do you like that?

Teacher A: It is like children are now thinking on their own. You don't have to tell them what to do or to follow what you want so long as they come up to the same point using different ways.

Wanting to learn how much Teacher A had changed her views about mathematics teaching from *telling* to *facilitating* learning, I questioned her further. The following transcript from the same interview shows that Teacher A was beginning to espouse a constructivist approach to teaching, which suggested that she no longer saw herself as a provider of knowledge to pupils but as a facilitator of learning:

Researcher: One of your colleagues told me that when she explains something clearly she expects children to understand it. What do you think?

Teacher A: If you explain clearly they will understand it but they won't do it the way you expected. They will do it in different ways and…when they do it differently it will show that they understand.

Researcher: So, even when you explain something clearly, children may not do it the way you have explained?

Teacher A: No.

Researcher: But differently?

Teacher A: Yes, differently.
Researcher: Did you believe this before the study?
Teacher A: Before the study I would say, "You have found the answer. But was this the way I explained it to you?"

Researcher: And if it wasn't?

Teacher A: If it wasn't…[Pause]. Ah…we wanted the children to do it in the same way we had explained. But this time I have leant that, even if they do it in their own way as long as it makes sense, at least they have understood. As you saw in today's lesson, I didn't do the explaining but pupils because of [their knowledge of] partitioning …you saw how we went bit by bit. They were able to explain this and that. But when it came to writing, they just went straight to the answers. But they did the partitioning may be once then found the answer.

At the beginning of the study Teacher A believed that Grade 2 pupils were too young to answer questions meaningfully during mathematics lessons, let alone contribute their own mathematical ideas. Her views in this regard also changed. A post-observation interview conducted a few weeks into the study suggested that Teacher A was particularly impressed with her pupils' ability to find different ways of solving problems and evaluating one another's answers:

Researcher: It has been said that young children know more mathematics than teachers think they do. Did you observe anything in your lesson that confirmed this?

Teacher A: They know more [mathematics] because they were not convinced when one of them gave an answer. They wanted to know if it was correct or wrong. When I said, "He [one pupil] has tried, who else wants to try?" they raised their hands, meaning that they know more mathematics. They are able to convince another person that that was a wrong answer.

Researcher: Are you saying they can tell when an answer is wrong?

Teacher A: Yes.

Researcher: But can they suggest something better?

Teacher A: Yes, because after one of them gives an answer, you can see more hands going up, showing that they think the answer is wrong. Then they will say [in Chinyanja], "*Na peza so na so na so*". [That is, this is the way I worked it out].

4.1.2.1.5 Initial Mathematics Content Knowledge

As shown in Table 1, Teacher A was not only a seasoned educator in terms of number of years of teaching experience; she was also well-qualified as a primary school teacher, having obtained both the PTC and the PTD. Nevertheless, she was convinced that her existing mathematics content knowledge was insufficient for understanding the mathematical ideas needed to teach strategies for mental calculation comfortably. She acknowledged this during the post-treatment interview:

Researcher: Do you feel that the mathematics you already knew as a teacher was sufficient to understand the ideas on mental calculations that we discussed during the Staff Development Workshop?

Teacher A: [Long pause]. It wasn't sufficient.

Researcher: Do you mean that even the mathematics you did during the Primary Teacher's Diploma wasn't enough for this work?

Teacher A: No. There should be more mathematics in the Diploma Programme.

Researcher: Are you saying that you need to learn more mathematics itself to teach these ideas comfortably and also to understand more about mental calculation?

Teacher A: Yes.

I had assumed that the mathematics content covered as part of the PTC was adequate for comprehension of mental calculation strategies for double-digit addition and subtraction. However, according to Teacher A not even the diploma course was helpful in this regard. She also felt that while the staff development workshop was useful in increasing her knowledge of mathematics, it was limited in that two weeks was not long enough. It should also have included more learning of mathematics content, use of a textbook, and should have been followed by similar workshops aimed at improving teachers' mathematics content knowledge in general, rather than focusing only on the mathematics associated with mental calculation. The extract below, taken from the post-treatment interview illustrates these points:

Researcher: What further help do you need to be able to teach strategies for mental calculation more comfortably?

Teacher A: We need material.

Research: What kind of material?

Teacher A: We need textbooks as usual. The experimental syllabus we were following was not a detailed one, such that sometimes I got stranded.

Researcher: Do you mean that there were times when you wanted to understand something better but couldn't because the training manual wasn't clear?

Teacher A: Yes.

Teacher A's mathematics content knowledge was indeed insecure. She experienced difficulty teaching particularly the use of an empty number line as a model for performing addition and subtraction. There is a hint of this in the following extract from the post-treatment interview. We need to notice, however, that Teacher A did not say this directly but instead talked about her pupils' mathematical inadequacies.

Researcher: According to your experience, which topics or parts of topics were most uncomfortable to teach?

Teacher A: Especially that one…about the number line. I think the pupils had problems on subtraction. Addition was better.

Researcher: Do you mean the number line with numbers already shown or the one we called the empty number line?

Teacher A: The pupils were putting the numbers on it themselves. I said 5 – 2 and expected them to draw a number line and to add numbers to it on their own up to the point where they wanted the line to reach and then to subtract. That one, I think, is still a problem because they would go beyond the number that we were subtracting from. I had every now and then to insist that, if you say 10 - 5 make sure that the number line goes only up to 10, and then you come back.

Using the empty number line to perform subtraction has indeed proved challenging to many teachers around the world (see Chapter 2). It is clear, however, that Teacher A's initial subject content knowledge in mathematics was inadequate, which negatively affected her ability teach mental calculation.

4.1.2.1.6 Final Mathematics Content Knowledge

I expected that attending the Staff Development Workshop would improve Teacher A's mathematics content knowledge. It was not clear to me, however, to what extent this happened if at all. Nevertheless, whenever Teacher A did not understand the mathematics in the training manual she sought help from fellow Grade 2 teachers at her school, all of whom were participating in the study. The following extract from the post-treatment interview illustrates this:

Researcher: What did you do when you felt that you did not understand what you were supposed to teach?

Teacher A: Since there are three of us [participating in the study] at this school…I would go to my friends and ask.

Researcher: How clear would you be after such consultations?

Teacher A: Since there are three of us, I would ask one person, then another, and compare the explanations. If things were clear, I would know what to do.

Allowing participating teachers to seek help from each other was the very reason I wanted *all* Grade 2 teachers in experimental schools to participate in the study. Teacher A's behaviour in this regard then was in line with what I had hoped to see happening.

4.1.2.1.7 Initial Teaching Practice

Teacher A's classroom practice was initially characterised by the idea that teaching is telling. For example, lesson observations established that when helping pupils to solve mathematics problems Teacher A not only mentioned what materials pupils needed but also spelled out exactly what they had to do to arrive at the answer. She confirmed this during the post-treatment interview:

> [We] never used to consider children's [previous mathematical] experiences. It was always teachers' experience. You want to impart what you know to the pupils, not trying to find out what the pupils already know…I would just give an example and tell the pupils to write it down. I think that example would take only five minutes. Then I would say, "Can you begin writing the exercise".

How different was the teaching practice of Teacher A at the beginning and at the end of the study? The next section will answer this question.

4.1.2.1.8 Final Teaching Practice

As a result of participation in the study, Teacher A became convinced that *telling* was not an effective approach to developing mathematical ability in children, as the following extract from the same interview above illustrates:

Researcher: The study has come to an end. How will the experience you gained during the study affect your work as a teacher in future?

Teacher A: Although the research is coming to an end, I at least have an idea about how to start a lesson in mathematics. Whatever topic, whichever books we are using, even these old ones, I will try to use the knowledge I have learned [from this study]. Even if I want pupils to do 5 + 6, I will not simply say, find stones, count them, and do this. No. This time I will just ask a question and allow pupils to work out the answer; and then tell me how they did it….In the past I used to say: 5 + 6; can you get five stones, then get six more stones. Now count all of them together. That's what I was doing. I won't do it again.

Another indicator that teachers are allowing children to learn mathematics meaningfully is the amount of classroom discussion that takes place. Several weeks after the study had begun, it was clear from lesson observations and interviews that Teacher A's classes were not doing well in this area. The following extract from a post-observation interview conducted during the second month of the study suggests this.

Researcher: To what extent would you say there was meaningful discussion between yourself and the pupils and among the pupils during the lesson?

Teacher A: When I asked them [questions] they were able to explain. But there is still that problem. Last time you said they should also ask me questions. On that one [laughs] it is like ah…they are not doing well.

Researcher: Do you find that while you can ask the pupils questions they cannot ask you questions or ask each other?

Teacher A: They can't. I don't know whether it is lack of vocabulary….They…just don't know what to ask.

Researcher: In the past teachers said children will not talk during mathematics lessons because they cannot speak English. But now they are using Zambian languages. Do you think their understanding of the language is not sufficient for them to ask questions?

Teacher A: No. It is just that they don't have questions to ask. But they have the answers. They are able to explain to their friends. As you saw during the lesson, those small girls would come to the front of the class and write on the chalkboard. When I asked one of them, "Can you explain what you have done", she answered "I partitioned this number". She is able to explain to her colleagues. The ability to explain is there.

Teacher A hoped that her Grade 2 pupils would eventually learn to engage in classroom discussion. She said so towards the end of the same interview:

Researcher: Do you think these children will learn not only to tell others *how* they did something but also to explain *why* they think their method is best?

Teacher A: [Long pause]. Yahhh… [Laughs, apparently unsure what to say].

Researcher: Do you believe that it is possible for them in spite of their current limitations to reach a point where they can explain not only *how* they worked out answers but also *why* they think their way of doing things is the best way?

Teacher A: I think it is possible with enough time.

By the end of the study Teacher A was convinced that a semblance of classroom discussion was taking place in her mathematics lessons, something which my observations of her lessons confirmed. The following extract from the post-treatment interview also illustrates this point:

Researcher: If teachers from a school that did not take part in the study visited your classroom during a mathematics lesson, what would they see happening that might not happen in their own lessons?

Teacher A: I think the discussion part. That is, you ask pupils questions and they come [to the front of the classroom] to explain. If after they have written an answer you say, "How did you do this?" children will be able to explain how they got the answer, whereas normally we would just mark if the answer was right. But this time we are able to ask, "How did you do this?" And the child will say, "I was doing this, this, this". May be that is what they will see.

In trying to get pupils to talk about their solution strategies Teacher A often asked the class in Chinyanja "Mwai peza bwanji?" [How did you work it out?]. However, Teacher A was not looking for *justification* of solution methods but only explanations of the *procedure* followed. She seemed to be convinced that it was unnecessary to ask young children to justify their reasoning, as suggested by the following transcript from a post-observation interview:

Researcher: You were asking pupils: "How did you work it out?" What kind of answer did you expect from them?

Teacher A: How they worked it out.

Researcher: What about *why*?

Teacher A: About why?

Researcher: Yes, for example when you ask, “How did you work out the answer?” A child might say, "I was taking away". W*hy* was he or she taking away? Did you expect them to tell you only *how* they did it and not *why*?

Teacher A: Why they answered the *how* question only is because I told them: "This work you are going to count back; this work you are going to count forward". So, maybe that is why I was just interested in *Mwai peza bwanji* [How did you work it out?], since it was [simply] about counting back and counting forward. They were just expected to give me the answer, "Taipeza so, taipeza so" [We worked out the answer this way].

Thus Teacher A herself was not clear about the importance of questions that required pupils to give a justification of their solution strategy.

4.1.2.1.9 Summary of Initial and Final Characteristics of Teacher A

Table 3 provides a synopsis of the classroom behaviour and characteristics of Teacher A at the beginning and at the end of the study, measured against the four named themes.

Table 3: *Initial and Final Characteristics of Teacher A*

THEME	INITIAL	FINAL
Concerns	Study wrongly timed; should have been conducted in January, at the beginning of the school year. Experimental mathematics curriculum too advanced for Grade 2 pupils	Grade 2 pupils can understand and do mental calculation strategies.
Beliefs	Children learn mathematics by following teachers' demonstrations exactly. Only taught procedures should be used during mathematics lessons. Lessons are interesting when pupils give correct answers to teachers' questions, doing things exactly as taught.	Pupils can use own solution procedures during lessons, so long as the procedures work. Grade 2 pupils can discuss mathematics meaningfully during lessons if encouraged.
Mathematics content knowledge	Had both the PTC and the PTD, which included learning of some mathematics content Believed that the mathematics content of the PTC and the PTD was insufficient for learning to teach strategies for mental calculation. Wanted training workshop to include learning of more mathematical content to enable her to teach mental strategies more confidently. Experienced difficulty understanding use of empty number line	Still encountered difficulty figuring out how to implement some aspects of the alternative numeracy development approach. Coped by consulting fellow teachers participating in the study.
Teaching Practice	Characterised by telling. Did not consider children's mathematical thinking and expected them to use only taught procedures.	Encouraged classroom discussion. Allowed pupils to use untaught solution methods so long as they justified them.

4.1.2.2 Teacher B

Among the five participating Grade 2 teachers in experimental schools, Teacher B was the youngest and least experienced. At the time of the study, Teacher B had only taught Grade 2 classes for 2 years, which was all the teaching experience she had had. She also held what was then considered the lowest qualification for teaching in primary school, that is, the ECTC. This certificate allowed her to teach only up to Grade 4, whereas the full primary course in Zambia went up to Grade 7.

When I first invited Teacher B to participate in the study, she told me that she was once a pupil at a high school in Zambia where I had taught mathematics for several years. This gave me the hope that she would agree to work with me on my research project. However, she also told me that since school holidays were about to begin she had already made plans to travel outside Lusaka to spend the holidays with relatives elsewhere.

When I emphasised the benefits of participating in terms of her own professional development, and the possible gains for her pupils' learning of mathematics, she agreed to stay and participate in the study. I should mention, however, that Teacher B turned out to be the most forthright person among all participating Grade 2 teachers in experimental schools in speaking her mind concerning what she liked or disliked about the study, sometimes to my discomfort. Her two colleagues from the same school, who were also participating in the study, were more reserved in this regard.

4.1.2.2.1 Initial Concerns

Initially, Teacher B thought that participating in the study was a waste of her time and that of her pupils. She felt that there was nothing different her pupils would learn in mathematics at Grade 2 level that they had not yet learned. As the following excerpt from the post-treatment interview suggests, her experience during the study may have hardened these concerns:

Researcher: What was your reaction at the beginning of the study when I first invited you to take part? How did you feel?

Teacher B: [Laughs]. At first I thought it was just...a waste of time, knowing that there is already mathematics at Grade 2 level.

Researcher: When you say you thought it was going to be a waste of time, were you thinking about your time or that of your pupils?

Teacher B: Both my time and that of the learners.

Researcher: Were you worried that your pupils might not pass their tests?

Teacher B: Yes, I was worried; and I am still worried. I am worried because more able pupils have learnt a lot but slow learners have worsened.

Researcher: Are you saying that if the study had not taken place the 'slow learners' would not have gone down as much as you think they have?

Teacher B: OK. Slow learners would perhaps be [scoring] at 40 [%], and more able learners at 60 [%].

Researcher: At 60? But you think it is now different?

Teacher B: It is different. Slow learners are maybe at 25 and active learners at 55.

Researcher: So according to you this study has widened the gap in performance between the more able and the less able pupils in your classes?

Teacher B: Yes.

Later in the same interview Teacher B explained further why she believed that the study had affected negatively her pupils' mathematical progress:

> The active [i.e. more able] learners...learn fast. Whatever method you introduce, they will understand that method. But the weaker...learners...they don't seem to get what I am teaching. When I was following the official syllabus I could give the pupils homework or remedial work. But with this [new] syllabus, you just give homework to a child who doesn't know a thing. You just say, "Your mother or your brother will…help you at home". [But] even at home, they won't understand what you want the children to do.

She added:

> We can't wait for less able pupils. If it was in those days [i.e. before the study] we could give them homework. They would just copy the work; and I would say, "Go and do this at home," and they would be helped at home. Now with these new methods, who is going to help them at home?

Teacher B was correct in saying that parents and other children in the home did not understand what was meant by strategies for mental calculation and could therefore not help pupils with homework. Unfortunately, I could not help parents understand something about the new ideas their children were learning, as I had neither the time nor the resources to do so. Teacher B was also convinced that the experimental numeracy development approach required moving too quickly from one topic to another, leaving less able pupils behind. Lesson observations however did not confirm this. In fact, they confirmed the opposite: that Teacher B covered the associated mathematical ideas at much slower pace than did her two colleagues. As a result, I feared that by the end of the study she might not have taught enough topics in the experimental syllabus to allow for meaningful evaluation of the effectiveness of the intervention.

4.1.2.2.2 Final Concerns

Teacher B's initial concerns about participating in the study did not disappear during the course of the study. In fact, they increased. For example, she maintained throughout the study that the experimental numeracy development approach did not help her pupils in anyway but impeded their mathematical progress. Thus Teacher B was different from Teacher A, whose initial concerns were resolved by the end of the study.

4.1.2.2.3 Initial Beliefs about Mathematics Teaching and Learning

Like Teacher A, Teacher B's concerns about participating in the study were linked to her existing beliefs about mathematics teaching and learning. Initially, Teacher B believed that young children arrived in school without any mathematical ideas of their own; all they eventually got to know about mathematics came from the teachers. She also believed that teaching *always* resulted in learning; and that what teachers taught in mathematics was learned by pupils in the exact form it was taught. Not surprisingly, when I asked her during a post-observation interview what she believed about how children learned mathematics. She immediately responded: "They learn from the teachers. As long as I teach, I believe that they have learned".

Accordingly, during mathematics lessons Teacher B from time to time reminded pupils: "If you do not keep looking to see what I am doing, you will not understand". In other words, Teacher B believed that learning mathematics depended on how clearly the teacher explained things and how carefully and attentively the pupils followed the proceedings. Thus Teacher B's personal views about mathematics teaching and learning at the beginning of the study differed from the constructivist view, which holds that no matter how clearly teachers explain their thinking about particular mathematical ideas children will construct their *own* understandings of those ideas.

4.1.2.2.4 Final Beliefs about Mathematics Teaching and Learning

There was evidence that during the course of the study Teacher B's initial beliefs about mathematics teaching and learning had changed. Classroom events showed her that Grade 2 pupils could do things in mathematics she thought they could not. The following extract from a post-observation interview illustrates this point:

Researcher: During the staff development workshop you were not sure that Grade 2 children could understand strategies for mental calculation. What do you think now?

Teacher B: [Laughs]. They can understand them.

Researcher: But were they not able to do that before the study?

Teacher B: OK, even at that time some pupils were able. Even at this time some pupils can't.

Researcher: When you compare how you felt about your pupils' mathematical abilities at that time and how you feel about their abilities now, is there any difference?

Teacher B: OK, when we started the workshop I wasn't sure that Grade 2 children could do word problems because we had not done word problems with young children before.

Researcher: But you now think they can?

Teacher B: They can, but not all of them.

During another post-observation interview, which followed a lesson on partitioning of double-digit numbers into tens, fives and units, I asked Teacher B if she had seen anything during the lesson that convinced her that her pupils had more mathematical ability that she previously imagined. Her answer was similar to what she said in the interview above:

> Yes some...pupils are able to partition numbers...but others can't. And those who can do it on their own, they are few. Some who can do it are able to tell others how to do it.

Teacher B was thus grudging in her acceptance of the fact that her pupils could learn to use mental strategies for adding and subtracting double-digit whole numbers. She kept stressing that while some of her pupils could do so, others could not. It is possible then that Teacher B's change in beliefs about mathematics teaching and learning went deeper than she was willing to acknowledge.

4.1.2.2.5 Initial Mathematics Content Knowledge

As shown in Chapter 2, the amount and type of mathematics content knowledge teachers have can influence their ability to foster the use of strategies for mental calculation in the mathematics classroom. Teacher B qualified to teach in primary school after obtaining the ECTC. The mathematics content that was taught in this teacher education programme was less than that included in either the PTC or the PTD. I therefore expected that among the five teachers implementing the experimental numeracy development approach, Teacher B would at least initially be the most insecure in terms of understanding mathematics.

However, during interviews Teacher B stated that her mathematics content knowledge was sufficient for teaching strategies for mental calculation. However, the following extract from the post-treatment interview suggests that she did not want to acknowledge any limitations on her part with regard to knowledge of mathematics. She always assumed that any difficulties she encountered in teaching mental calculation in the classroom were due to pupils' own intellectual limitations.

Researcher: Were there times during the study when you felt that these ideas on mental strategies…were difficult to teach?

Teacher B: There were times when I found it difficult. The problem was that, whenever I wrote a word problem on the chalkboard I had to repeat it three or four times before children could understand it. That was time consuming.

Researcher: Are you saying that you did not need to repeat questions so many times before the study? You would just move on even if children had not understood the question?

Teacher B: I could just write addition [sums] on the chalkboard. My pupils know about numbers. 1 + 5. I would go through the work perhaps just once...and they would be able to see what to do.

Researcher: So before the study...what you wrote on the chalkboard were only numbers and no stories about those numbers?

Teacher B: Yes, only numbers and no stories

Since I wanted to understand the extent to which teachers' mathematical knowledge influenced their ability to foster the use of mental strategies in lower primary mathematics, in another post-observation interview I asked Teacher B a similar question but focusing on lesson preparation.

Researcher: As you know, before you teach something you have to understand it yourself. Did you find the mathematical ideas you were teaching easy to understand or were there times when you were not sure you understood well what you were going to teach?

Teacher B: Of course I used to understand. But when I think of a Grade 2 child! It was hard on the child, not on me. Me, I knew what I was doing.

Later in the same interview, I changed the question:

Researcher: To what extent would you say that the kind of mathematics you did during teacher training was sufficient to help you understand the mathematical ideas you were teaching?

Teacher B: The ideas...were just OK. Now, when I think of the children!

Researcher: So, in your case as far as understanding what you were teaching was concerned, there was no problem. It is the children who had a problem.

Teacher B: Yes.

Teacher B thus maintained that her existing mathematics content knowledge was sufficient for teaching mental calculations and that she did not experience any difficulties either in understanding or in teaching the mathematical ideas involved. There was evidence however suggesting that what she said above might not be very true. For example, during the post-treatment interview, she acknowledged experiencing problems creating word problems and numerical examples on double-digit addition/subtraction for her Grade 2 pupils.

Researcher: When I gave the staff development workshop I was hoping that it would enable us to teach mental calculation confidently. Do you think that training should have been longer than two weeks?

Teacher B: [Laughs]. The training was OK, but…there were times whereby you would go back to the …to the what?

Researcher: To the training manual accompanying the experimental numeracy development approach?

Teacher B: Yes, you go back to the workshop materials. "What do I teach?" "How will my children understand?" Those are the questions I used to ask myself. With me, I can create a few questions for myself....Now for the pupils, when I compare with the textbook [incomplete statement]….There are parents who are able to buy textbooks for their children. But for this thing [strategies for mental calculation] we never used to use those books. And some parents used to come and ask: "Why has the mathematics changed? Our children seem to be doing nothing".

Researcher: What was your answer then?

Teacher B: They will learn something. There is something we want them to know.

Researcher: That is an interesting answer. You were saying earlier that…when you would go back to the notes in the training manual…it wasn't very easy for you to see exactly what you were going to teach?

Teacher B: It wasn't exactly that. What I am trying to say is, when you refer to the textbook…sometimes things are easier. You refer to the textbook and [the] work is there. [You] just take the work from the textbook. Then for homework you [just] write the work on the chalkboard.

Researcher: So, those with a textbook would just copy from the book?

Teacher B: No, they would copy from the chalkboard and go home and tell their helpers or whoever they were [to help them do the work].

Researcher: But this time?

Teacher B: This time there was nowhere else slow learners could turn to apart from me. And it was not easy to create questions for slow learners. They haven't learnt much and I am worried about that. I feel I haven't done anything for the slow learners.

There was further evidence that Teacher B's content knowledge in mathematics in relation to teaching mental strategies was not as sure as she suggested it was. When I asked her later in the interview what other subject matter she would like included if I were to repeat the staff development workshop, her response suggested that she often ‘got stuck’ during lesson preparation while trying to understand what she was to teach.

Researcher: If I were to repeat the staff development workshop we had at the beginning of the study to make it more effective, what would you want it to include?

Teacher B: [Laughs]. OK, I think next time we as the people participating…should prepare the questions we will teach the children.

Researcher: All the questions that would be needed?

Teacher B: Yes. What we want the children to learn

Researcher: But we tried to do that during the Staff Development Workshop although we didn’t develop all the questions. I can see now that there was a need to prepare all the questions, so that you could pick questions from a given source.

Teacher B: I think so. [Without all the questions available], when you are stuck you can’t go to a fellow teacher, since she is also stuck. But [with all the questions available] you could go to a colleague and say, “What are we supposed to do here?

Teacher B was not only reluctant to say anything that might suggest that she had an inadequate knowledge of mathematics, she was also unwilling to say anything that might reflect poorly on her colleagues’ mathematical knowledge, as the following extract from the post-treatment interview suggests:

Researcher: Do you think that all primary school teachers in Zambia should learn about these ideas on mental calculation?

Teacher B: I once invited some colleagues who are not taking part in the study to come and see our work. Their reaction was: “This stuff is difficult”. [Laughs]. You see, you write 25 + 35. Then you ask a kid to partition the numbers, or you do it yourself. They said, “All this! Why don’t you just use the usual method?” I said: “No, this is a new method”. They replied: “No one can cope with this”.

Researcher: By ‘usual method’, did they mean vertical addition?

Teacher B: Yes.

Researcher: How well do children understand that ‘usual method’?

Teacher B: OK. Grade 3 and 4 children at least do better than Grade 2. They have understood ‘borrowing’ and ‘carrying’, but Grade 2 children have not.

Researcher: What did your colleagues mean when they said “This is difficult”?

Teacher B: They meant there is a lot of work involved. You partition numbers; again you bring the numbers together …just like that.

Researcher: So what they really meant was that this work would make a teachers' life difficult, and they would rather do the usual thing?

Teacher B: More or less. [Laughs].

4.1.2.2.6 Final Mathematical Content Knowledge

What can we say about Teacher B's levels of mathematical knowledge at the beginning of the study and at the end? From the foregoing it is clear that at the beginning of the study Teacher B's knowledge of mathematics in relation to teaching mental strategies was insecure. Since she was reluctant to talk about this, however, it is difficulty to say how far her mathematics content knowledge changed during the study.

4.1.2.2.7 Initial Teaching Practice

Change in one's teaching practice essentially means change in the type of activities one engages in during lessons. At the beginning of the study, probably influenced by her experiences during the Staff Development Workshop, Teacher B stated that she would allow pupils in her classes to use any solution procedures that worked, even if these differed from what was recommended in prescribed textbooks. The following extract from the group interview which marked the end of the Staff Development Workshop, illustrates this point:

Researcher: What skills do you think you have learned as a result of participating in this workshop?

Teacher B: Before the workshop we depended mainly on procedures from the textbooks, and children used in the exam only the methods taught in the classroom. But now we will teach our children to use different methods.

Later during a post-observation interview, Teacher B explained further what she meant by depending on the textbook. Incidentally, the interview extract below also suggests that before the study Teacher B taught mathematics only because she was *required* to do so but that as a result of participating in the study she began teaching mathematics to help children *learn* the subject.

Researcher: You have taught mental calculation for sometime now. How do you view your role as a teacher of mathematics compared to what you thought it was before the study?

Teacher B: Before the study I could just get the textbook, get the work, and give the children.

Researcher: Copy from the textbook?

Teacher B: Yes. But this time I have to make my own work for the children. And you can't just leave the children with their work, you just write work on the chalkboard and you go out. No, you have to do a lot of explaining.

Researcher: When you used to leave the work from the textbook on the chalkboard for pupils to do, how was their performance?

Teacher B: Since it was just maybe 3 + 2, 2 + 1, they were able to do it.

Researcher: And you marked the work without finding out why the answers were right or wrong?

Teacher B Yes, we just used to mark. But this time pupils need to understand. They also need to think.

Researcher: I am glad to hear that.

Teacher B: [Laughs]. No jokes this time, ah.

Researcher: It would seem that before the study your aim as a teacher was not really to help pupils understood what they were learning in mathematics. What then was your aim in teaching mathematics in the past?

Teacher B: Just to teach.

Researcher: And then?

Teacher B: Just to teach. [Laughs].

In short, before the study the classroom practice of Teacher B reflected the view that teaching mathematics involved telling pupils what to do and expecting them in exercises and examinations to reproduce that.

4.1.2.2.8 Final Teaching Practice

What changes occurred in Teacher B's classroom practice as a result of participating in the study? The foregoing suggest that at least three changes occurred: her thinking about her role as a teacher changed; her attitude towards mathematics teaching changed positively; and she reduced her level of reliance on mathematics textbooks. Teacher B confirms these observations in the following extract from a post-observation interview:

Researcher: As a teacher, what…were you doing in mathematics lessons before the study that you are not doing as much now, and what are you doing now that you were not doing before?

Teacher B: OK, before the study I used to give a lot of [worked] examples and I used to do most of the talking.

Researcher: What was the aim of giving a lot of examples?

Teacher B: After giving a lot of examples, I gave them work to do. But this time, I just introduce [the work] then they are able to give their own examples

Researcher: So what do you think is your role as a teacher now?

Teacher B: To supervise [i.e. to facilitate learning].

An important measure of the extent to which a teacher is promoting the use of mental calculation strategies in the classroom is the amount and quality of classroom discussion that takes place there, which includes children defending their solution methods. The extract from a post-observation interview below suggests that before the study Teacher B made little or no attempt to encourage children to talk about their mathematics during lessons. She tried to do this more often during the study, although her pupils initially had difficulty engaging in any meaningful classroom discussion mainly because they were doing it for the first time.

Researcher: When you asked pupils during the lesson to explain what they were doing, they were not speaking loud enough to be heard. Is anything the matter? Do they explain their solution methods to your satisfaction?

Teacher B: Some do. Others just write down the answers.

Researcher: Why are some pupils unable to explain things clearly when they are using a Zambian language, which they know well?

Teacher B: I can’t really tell. As long as pupils know what they are doing, it’s OK with me.

Researcher: Are you saying they know what they do, but cannot explain it?

Teacher B: They know what to do but explaining is a bit difficult for them. I don’t know why. They will learn as we go along.

Researcher: Is it because they are learning it for the first time?

Teacher B: Yes, they are getting the idea now. Yesterday, one [pupil] told me: “Ah, ni zai peza mwamene ni zai pezela” [i.e. I will work it out my own way]. And she got it right.

Researcher: But she could not explain her method?

Teacher B: OK, she used her own method and the answer was correct. I wanted her to use the partitioning method but she refused, saying: "Ni zai peza mwamene ni zai pezela”.

Researcher: That’s interesting, because she seems to be saying: “This is my method and I will stick by it”. What I would like you to do is to ask her: “Why do you think your way of doing things is better than mine?” Find out what reasons she has for defending what she has decided to do.

In another post-observation interview two weeks later, there was more evidence that Teacher B’s pupils were getting used to the idea of classroom discussion, although they still did not do enough to explain their solution methods.

Researcher: The idea of children talking about their mathematics, defending what they have done, allowing other children to ask questions, and so on, is very important. How well do you think your pupils are doing in that area?

Teacher B: They are trying, because you can give a problem on the chalkboard and tell a child to explain how he or she came up with that answer. That child will tell you what she or he did: “I counted because here you said this and this; and I counted four sweets and three bananas. I put them together. They gave me seven”.

Researcher: What about explaining their answers? To what extent have they developed the habit of asking? For example, if one of them gives an answer to a question, they ask: “How did you do it?"

Teacher B: OK, pupils haven’t been asking each other questions, but I ask them questions.

Researcher: Have you tried to persuade them to ask each other questions?

Teacher B: OK, if they notice a mistake in what their friend is doing, they will say: "Ah, simwamene; iyayi simwamene". [No, that's not the way to work out the answer. No, it is not the way to do it"].

Researcher: Do they ask each other: "Can you explain to us what you have done?"

Teacher B: No.

Researcher: Is there anything you could do to help them do that? Research suggests that, when children ask one another questions and explain their answers it helps them understand better their mathematical thinking. Maybe you should continue pressing them to do it.

Teacher B: I will try.

Researcher: Yes, see how far they will go.

During the post-treatment interview which came at the end of the study I mentioned classroom discussion again. By then Teacher B believed that given enough time her pupils would develop the ability to engage in meaningful classroom discussion.

Researcher: In what ways would you say taking part in the study has affected the pupils?

Teacher B: Before the study they never used to discuss or say anything about the way they write their answers or their work. But from the beginning of the study we learnt that we should encourage them to do it.

Researcher: How well are they doing it?

Teacher B: Not very much. They were just trying.

Researcher: How hard have you been trying to make them talk?

Teacher B: I wanted them to talk much more but they never satisfied me. I think it is because they were learning it for the first time; learning….[Laughs].

Researcher: Do you think that if you had more time, they would learn to express themselves more freely?

Teacher B: Yes.

4.1.2.2.9 Summary of Initial and Final Characteristics of Teacher B

Table 4 summarises Teacher B's initial and final characteristics as measured against the four named themes.

__Table 4:__ Summary of Initial and Final Characteristics of Teacher B

THEME	INITIAL	FINAL
Concerns	Study cancelled holiday plans. Study a waste of time for since pupils had covered similar topics. No need to learn mental strategies since Grade 2's already had a maths curriculum. Pupils will not succeed in future public exams if they waste time participating.	Study widened gap between more able and less able pupils since it did not allow time for remedial work. Parents unable to help children with homework since they do not understand this kind of maths.
Beliefs	Children have no own maths ideas of their own; they learn all from teachers. Children learn from teachers. As long as I teach I believe they have learned". To succeed in maths children must listen quietly and follow teachers' examples exactly.	Grade 2 pupils can understand the maths involved in learning to use mental strategies. Children can do word problems. Would accept children's answers even if they used untaught procedures.
Mathematics Content knowledge	Certificate in early childhood education; little maths content included. Found constructing simple word problems difficult. Wanted textbook with examples and exercises, so that teachers would use the same questions in class. Experienced difficulty understanding maths topics in experimental syllabus;	Claimed to have learned something new. Reluctant to mention any difficulties experienced.
Teaching Practice	Copied work from and taught exactly what was in textbooks. Gave several examples from textbook. Did not follow up children's answers. Taught maths because she had to. Discussed only numerical examples; word problems not done.	Started using children's thinking. Believed Grade 2 children could use mental calculation strategies. Encouraged classroom discussion. Considered her role as a teacher as that of facilitator of learning. Gave fewer worked examples

4.1.2.3 Teacher C

Teacher C was one of the most senior Grade 2 teachers in experimental schools, both in terms of age and teaching experience. Despite her seniority, Teacher C did not obtain any other teaching qualifications beyond the PTC.

Because of frequent pupil absenteeism from her mathematics lessons during the early part of the implementation period, which was the case in all other participating Grade 2 classes in experimental schools, I did not realise it until about two weeks later that her two Grade 2 classes were in fact single-sex classes: one for boys, the other for girls. The two classes were participating in an on-going nation-wide Ministry of Education experiment, which sought to determine whether or not single-sex classes offered particularly girl children a better chance to succeed academically. Teacher C's classes were therefore different from other Grade 2 classes in experimental schools, each of which had both boys and girls in roughly equal numbers. The following passage from my research diary gives an indication of how I felt when I discovered that Teacher C had two single-sex classes, and what decision I had to make to resolve the issue:

> I have just learnt that Teacher C's two classes are single sex classes. But how do I deal with choosing one of these two classes for inclusion in the analysis? Clearly, these classes are unlike the others taking part in the study, all of which have both boys and girls. Should I include both? This will not solve the problem. Maybe I should exclude both of them from the analysis.

I settled for the second option, that is, to exclude both classes from the analysis of pupil performance in the two numeracy tests. Since the study was mainly about assessing the extent to which the teachers could to learn to foster the use of strategies for mental calculation, and pupils only came in to help me evaluate this, I decided to continue working with Teacher C, that is, observing her lessons, conducting interviews with her, and so on. I was sure this was the best thing to do in the circumstances. It also meant that I did not have to ask Teacher C and her classes to withdraw from the study, with the possible negative consequences this might have had on my study.

4.1.2.3.1 Initial Concerns

Initially Teacher C had two main concerns that made her reluctant to participate in the study. First, she was not sure that I had official permission to conduct a study in her school. When she learned that I had the permission, she concluded that since she was a Grade 2 teacher she had no choice but to participate in the study. Second, Teacher C feared that the alternative numeracy development approach based on the use of strategies for mental calculation would prove too advanced for her young Grade 2 pupils. The following extract from the post-treatment interview illustrates these points:

Researcher: How did you feel when I first came to your school, explained what I intended to do in your school, and invited you to participate?

Teacher C: At first I thought you had no permission from the Ministry of Education or the head of the school. Then you showed me the letter [of permission from the Ministry of Education], and then there was nothing I could do [but to participate], since the Ministry had allowed you to go round the schools. So, since I am a Grade 2 teacher I supposed that I had to attend the [staff development] workshop.

Researcher: So at the time you couldn't see yourself having any choice at all in the matter? All you could see was that you happened to be teaching a Grade 2 class and you had to take part?

Teacher C: Yes.

Researcher: Apart from the fact that you just took part because you happened to be a Grade 2 teacher, did you have any [other] concerns or worries about participating in the study?

Teacher C: I was also unsure [about participating] because some of the topics we were going to teach were not fit for Grade 2s. I saw the topics...and I concluded that, maybe, the pupils would only be barely able to cope, since they are so young.

4.1.2.3.2 Final Concerns

By the end of the study Teachers C's initial concerns were more or less resolved. For example, during my first post-observation interview with her she seemed genuinely excited and surprised that her pupils could understood the mathematical ideas associated with learning to use strategies for mental calculation. These were the same mathematical ideas she previously thought were too advanced for them. She even suspected that my presence in the classroom during the lesson under review had caused her pupils to do better. The following transcript from the interview illustrates this.

Researcher: What did you like most about your lesson today?

Teacher C: I don't understand what happened in today's lesson. Maybe it was because they saw you.

Researcher: What do you mean?

Teacher C: The pupils were able to answer questions. They were even able to explain their answers. Somehow, splitting numbers was easy for them. I don't know how, but this work was simpler for them today. After going through their books, I think most of them got at least two out of four [correct].

Researcher: How did you feel? What ideas came into you mind?

Teacher C: They are able to do it! You saw what happened when I started the lesson. They did it on their own. I didn't do it. They did it on their own.

In another post-observation interview about a month later Teacher C was surprised to see that a boy who was normally shy and reserved responded quickly to her questions, and explained his method to other members of the class.

Researcher: Is there anything you saw or heard in today's lesson which caused you to say to yourself: "I didn't expect young children to be able to do that!"

Teacher C: The boy who was counting from 35 to get the answer 49. OK, I did not know that he could do it. Normally he is a shy boy but today he was even able to talk to the others [about] how he got the answer.

Researcher: What about the fact that when…you put the question [across] to him he answered immediately. Did that [also] surprise you?

Teacher C: Yes because for Grade 2 pupils finding the sum 35 + 14 is not easy. I thought he could not do it. But he got the answer very fast, like the others. 49. So I was very surprised.

Researcher: How did he do it?

Teacher C: How he did it? [Laughs]. Maybe as I was writing the question on the chalkboard he was already adding.

Researcher: Maybe so but there could be another reason. The method he explained in front of the class involved counting, which would take a long time to arrive at the answer. Could there be something else he may have done?

Teacher C: I don't know how he came to the right answer so quickly. [Laughs].

It was not surprising then that during the post-treatment interview Teacher C stated that she herself had started enjoying mathematics.

Teacher C: This time I have become more interested in maths because I have seen that children are able to do maths on their own. And that is because [they] are given time to do it using their own methods, not just depending on our methods.

Researcher: Do you mean you like mathematics more yourself now?

Teacher C: OK, because it is like children have become more interested as well.

Researcher: And you have become more interested in teaching them?

Teacher C: Yes.

But during the same post-observation interview Teacher C complained that her school only allowed three-thirty minute mathematics lessons per week, which made it difficult for her less able pupils to complete mathematics exercises:

> As you saw, some children, the [less able] groups, they need more time. When I tell them to go out [at the end of the mathematic lesson] they are still writing. .

Because of this, without consulting school management Teacher C increased learning time for mathematics from three thirty-minute lessons *per week* to one thirty-minute lesson *per day*. However, this does not seem to have helped much as at the end of the study, during the post-treatment interview, Teacher C again complained about insufficient time for mathematics lessons. By this time, she had also become convinced that the experimental numeracy development approach was being implemented too fast for most of her Grade 2 pupils:

Researcher: You have now gone through the study. You have taught these topics yourself. What do you think now concerning Grade 2 children's ability to understand them?

Teacher C: Some topics, students can do…well, but some topics…we are just too fast for them. We are not doing them [the topics] at a slow pace. We are going too fast just to cover [the content needed to learn to use strategies for mental calculation].

Thus while some of Teacher C's initial concerns were resolved during the study, by the end of the study new ones had emerged to replace them.

4.1.2.3.3 Initial Beliefs about Mathematics Teaching and Learning

During the Staff Development Workshop Teacher C's showed through her body language that she was not interested in the subject matter we were discussing. She had already concluded that it was not a good idea to teach Grade 2 pupils strategies for mental calculation, which she believed were too advanced for them. I also noticed during my first observations of her mathematics lessons that whenever a child successfully explained a calculation procedure he or she had used, Teacher C asked the class "to clap for him or her". This suggested that Teacher C only accepted pupils' answers if they tallied exactly with what she had in mind. In other words, she was initially like her fellow Grade 2 teachers in the school who believed that to learn mathematics effectively pupils needed to develop the ability to reproduce *exactly* in exercises and tests what their teachers had demonstrated in the classroom.

4.1.2.3.4 Final Beliefs about Mathematics Teaching and Learning

Teacher C began revising her beliefs about mathematics teaching and learning early in the study. The following extract from the group interview conducted at the end of the Staff Development Workshop seems to support this view.

Researcher: Now that you have learned that children know some mathematics before they begin school and that when you teach them you should listen to what they know, how do you plan to use this knowledge in your future lessons?

Teacher C: OK, since we now know that children know something before they even begin Grade 1, we shall…OK, I myself …will try to use this message, I mean this method, in the classroom situation and try to see if the pupils will understand.

And since this is a research, if this thing works, [and] works out well, maybe it will be adopted, so that children…can begin using their own methods starting from Grade 2. As they go on, even [up] to Grade 7, they will score better marks; [and] Zambia will become a better country in mathematics.

Researcher: But how would you feel if children continued using these…mental methods all the way to Grade 7, knowing that there are certain prescribed methods that they should learn?

Teacher C: That will depend on whether or not this research is adopted.

Teacher C's growing receptivity to the use of mental methods in primary mathematics as indicated above was partly due to her conviction that the use of mental calculations in mathematics resembled what was already happening in literacy lessons at Grade 2 level nation-wide. As suggested by the following extract from the group interview, Teacher C was convinced that the methodologies used in the Primary Reading Programme (discussed in Chapter 2) paralleled those we were using in mathematics during the study:

Teacher C: I think this experience of using [mental] maths is the same as the *Step In To* series, which we have now started [implementing at Grade 2 level].

Researcher: The English one?

Teacher C: Yes, the *Step In To English*. OK, in English the pupils are there…they are just there [and] teachers just guide [them] to do more work on their own. Children discuss in pairs or a group. It is the same with maths. If a child is very smart, [he or she can show] the other learners the method and children will have more motivation to do their own work in all subjects. A child can even have the courage and zeal to do everything on her own and not to depend mostly on the teacher. So I think this is the same as it is in *Step In To English*. It is the same in maths.

Researcher: Which means the common element there is to encourage children to talk about what they are learning?

Teacher C: Yes, to do everything on their own.

Furthermore, during her own school days in the 1980s Teacher C experienced what used to be called *Mental Arithmetic*, which involved mainly 'working in the head' and giving quick answers to the teacher when called upon to do so. Mental arithmetic was a common feature of school mathematics in Zambia from independence in 1964, until it was abandoned sometime in the 1980s. When I asked Teacher C and her two colleagues (Teacher A and Teacher B) during the same interview what they would say if officials from the Ministry of Education inquired about the desirability of extending the pilot to the whole country, Teacher C's recollection of her own experiences with mental arithmetic made it easier for her to support the idea:

Teacher C: You asked what we would say if somebody [were to come] from the Ministry of Education?

Researcher: Yes, if they said, "You see, we have heard about this thing. Now we want to try it somewhere [else], but we are not sure. What advise do you give?"

Teacher C: I would…say that sometime back in [mathematics] classrooms, as I experienced it myself…in the eighties [1980s], or what [others] learnt in the seventies [1970s], we used to do mental arithmetic….A teacher would come in the class, he [or she] asks…questions; then the pupils…[do] it mentally.

But now, this mental arithmetic was phased out. I don't know how…they removed it. [Although] mental work [was]…phased out, I think it was a better method of doing maths. Because, as we used to do it, we used to say 2 by 2 in our own heads up to 2 by 12. We used to know the [times] table for 1 up to 12 mentally. But these days, these children who are in school, they don't know the [times] tables, they…just…copy from the back of their notebooks.

Researcher: Yes.

Teacher C: And I think that if the Ministry of Education adopts this research, it will be nice.

Thus as a result of participating in the study Teacher C changed her existing beliefs about mathematics teaching and learning, and about young children's mathematical abilities. At the beginning of the study, Teacher C believed that mathematics teaching involved *telling* children who knew little or nothing about the subject what they should know. By the end of the study she believed that young children knew some mathematics even before they began formal education. Teacher C herself developed a more positive attitude towards mathematics and mathematics teaching.

4.1.2.3.5 Initial Mathematics Content Knowledge

As mentioned earlier, despite her long teaching experience Teacher C did not acquire teaching qualifications beyond the PTC. The mathematics curriculum in this teacher education programme included only basic ideas in the subject, so that many teachers exited the programme with an insecure knowledge of mathematics. Nevertheless, during the post-treatment interview Teacher C maintained that her mathematics content knowledge at the beginning of the study was adequate for learning to teach strategies for mental calculation for double-digit addition and subtraction. However, lesson observations and face to face interviews suggested that at the beginning of the study Teacher C often experienced difficulty understanding procedures for splitting double-digit numbers into tens and ones. She was also not sure how to help children apply the procedures in appropriate situations. The following extract from a post-observation interview illustrates this:

Researcher: Do you need any help with the mathematical ideas you are teaching? I mean, are there ideas in the experimental syllabus that you found children couldn't learn easily and you were wondering what to do?

Teacher C: I was wondering just what to do with these addition numbers, 10 and 100, that is, how to split them. At first I couldn't get the logic behind. I sat down at home today. As I was making a lesson plan…[pause] I saw the idea behind, as a result I was even able to do all these questions.

Researcher: What is that 'idea behind', which you saw?

Teacher C: I didn't have an idea why [for example] 17 = 10 +5 + 2. OK, maybe it has been too long since we had that [staff development] workshop. So, I tried to do it on my own; then I was able to get the examples [correct] as I have done them here. [Shows some worked examples].

Researcher: OK. One reason we are doing this is that we want pupils to learn to add double-digit numbers, for example 17 + 27. They need to learn to split the two numbers. That is, to write 17 as 10 + something, and 27 as a multiple of 10 plus something. They will then add the multiples of ten together and the ones together. That makes addition easier.

Teacher C: Hmmm. [Indicating agreement].

Another indication that Teacher C's level of mathematical knowledge was inadequate and needed enhancing, for her to teach more comfortably the use of mental strategies for adding or subtracting double-digit whole numbers, was suggested by what she said about the training I gave teachers at the beginning of the study. She complained that the training manual I had provided was inadequate for her teaching needs. She preferred a *detailed* textbook, containing all necessary worked examples and exercises, so that all participating teachers would give exactly the same work to pupils. The following information Teacher C volunteered towards the end of the post-treatment interview illustrates this point.

Researcher: Is there anything else about the study that I have not mentioned but which you feel we should discuss?

Teacher C: Maybe…we were doing this [work] with no guidance. We had no textbooks to guide us. We were just doing it from our heads….There were no guiding steps and guiding syllabuses. We were just doing it by ourselves, that is, giving children examples.

Researcher: But we had that scheme of work we developed together, which showed what you were to teach. That wasn't sufficient, was it?

Teacher C: No. It was not…because you would find that maybe I am giving my class the sum 14 + 35, and Teacher B is giving [her class] 25 + 10. So, it was a bit of a problem. It would have been better if we had the same examples in all [Grade 2] classes.

Researcher: So that you would give children exactly the same examples?

Teacher C: Yes.

Researcher: Wouldn't that take away your flexibility and independence as a teacher?

Teacher C: No.

Researcher: So if you were to undergo the staff development workshop again you would want participating teachers to use exactly the same examples and exercises?

Teacher C: Yes.

Lesson observations showed that Teacher C covered very little mathematics content in each lesson. She often repeated work already done in previous lessons and appeared reluctant to move ahead to the next topics, although I could see that her pupils wanted to learn more in the time available. I am not sure why Teacher C did this. My suspicion is that she was not sure she could understand the mathematics involved, and would rather wait for other participating Grade 2 teachers in her school to catch up so that they could move ahead together.

Worried that Teacher C's slow pace during mathematics lessons might not allow me to be in a position at the end of the study to assess meaningfully the extent to which her pupils and herself had learned to use of strategies for mental calculation for double-digit addition and subtraction, I asked her during a post-observation interview why she seemed to cover very little work in each lesson. I did this rather hesitatingly, fearing to hurt her feelings and possibly make it difficult for her to continue co-operating with me during the remainder of the study.

Researcher: By the way, it would seem that in thirty minutes you didn't give pupils much work to do?

Teacher C: OK. I just gave them one or two sums because if I gave them maybe five, they would not be able to finish. My aim is for them to understand how to do it.

Researcher: Is this something that you have arrived at as a result of your experience [in the classroom] or did somebody say you should give very little work?

Teacher C: No, this is my own doing [i.e. this is based on my own independent judgement].

Researcher: You know how much they can do in one lesson?

Teacher C: Normally, if it is addition…like here, I gave them how many? There are one; two; three; four. I give them four to five in a day because they are able to do it. If I gave them ten, they won't be able to do it.

Researcher: Can I ask you to do me a favour? When you start discussing addition of big numbers [i.e. double-digit addition], can you please give them at least four sums as an exercise? I would like to see what happens. Don't protect them from too much work. Let's see how much they can do. Let's see if that much work will break them or something.

Teacher C: OK, I will try.

Although Teacher C was in general reluctant to acknowledge shortcomings in her knowledge of mathematics, she sometimes acknowledged that she might need help to understand what she was to teach next. The extract below from a post-observation interview, which followed a lesson on the procedure for splitting double-digit numbers into tens and ones, suggests this:

Researcher: Do you need any help from me with the next lesson?

Teacher C: Maybe. I will see next Tuesday how I will deal with this…whether it will be a flop. If it will be a flop, then I will call upon you to give me extra help. But if I succeed on Tuesday, then it will be OK. I will also see if the class understands, since we did this splitting in the first lessons.

Researcher: I think that will be alright.

Teacher C: If I succeed, then it will be OK.

4.1.2.3.6 Final Mathematics Content Knowledge

It is clear from the foregoing that initially Teacher C's mathematics content knowledge was inadequate for learning to foster the use of strategies for mental calculation involving double-digit addition and subtraction. It is difficult, however, to tell how much this changed during the course of the study, since she was reluctant to talk about it. However, because of the study Teacher C learned to prepare her own mathematics exercises suitable for Grade 2 pupils, whereas previously she depended wholly on the prescribed textbook. She also learned to use children's mathematics thinking in her lesson preparation. This learning started during the Staff Development Workshop, as the following extract from the group interview at her school suggests.

Researcher: What skills do you think you have learned as a result of participating in this workshop? In other words, are there some things concerning teaching that you were not able to do but which you now think you can, or things you were not able to do well but can do better now?

Teacher C: Since we began this research…I have learnt so much in that before the study we used to depend much on textbooks from the Ministry of Education, prepared by the people there. But now, after this workshop, I think we will be able to prepare our own work, what we think is suitable for children.

Researcher: OK.

Teacher C: And I am happy to learn that young children know how to count and to do some mathematical calculations even before they begin Grade 1. So, [as teachers] we should begin from what children already know and proceed from there. So, I have learned many skills.

4.1.2.3.7 Initial Teaching Practice

What was Teacher C's usual teaching practice before the study? The following extract from the post-treatment interview supplies part of the answer:

Researcher: What did you used to do in the mathematics classroom before the study that is different from what you are doing now?

Teacher C: We used to do as we were taught at the college [during initial teacher training].

Researcher: What do you mean?

Teacher C: OK, previously I used to work out examples on the chalkboard with the help of the pupils, and then give then one or two [similar] sums to do. Then I would say, "Start writing".

This suggests that Teacher C taught mathematics the same way Teacher A and Teacher B did, that is, she expected pupils to do exactly what she had demonstrated on the chalkboard. Indeed, lesson observations at the beginning of the study indicated that even when she was teaching strategies for mental calculation, she seemed to expect her pupils to learn the strategies as presented in the classroom and to apply them in appropriate situations as she determined them herself. She did not encourage individual pupils to share mental methods with other pupils but simply to apply taught mental methods exactly as she had used them in worked examples.

4.1.2.3.8 Final Teaching Practice

Teacher C's teaching style underwent some change during the course of the study, as the following extract from a post-observation interview conducted about a month into the study suggests:

Researcher: When we started the study you held the view that children learned everything they would possibly learn in mathematics from the teacher. When you look at your job as a teacher now, what do you think you should be doing in the mathematics classroom compared to what you were doing before the study? Do you see any difference?

Teacher C: Yes, there is a difference. My role is just to guide.

Researcher: What do you mean?

Teacher C: OK…the children are finding their own answers. They are doing it on their own. You are just there to guide them, or to control the classroom situation. They will be there on their own debating. And they will be finding their own answers, using their own methods. So, as a teacher you just need to know how somebody got the answer. If the answer is right I will just mark it, unless the answer is wrong.

Thus one indication of teacher C's changing teaching style was her growing tolerance for learners' use of different solution strategies. Teacher C provided further confirmation of this later in the same interview. At one point during the interview she told me that her next lesson would focus on splitting double-digit numbers and that she would use the example 57 = 50 + 5 + 2. But almost immediately she added:

> But maybe they [the pupils] will have their own way of doing it. Maybe they can say 30 + 20. I wouldn't know their answers. These are my own examples.

This comment suggested that Teacher C accepted the fact that teaching mathematics involved a certain level of negotiation between her own views as a teacher and those of the pupils. However, it would seem that Teacher C had a limited view of what it meant to use children's thinking in teaching mathematics. She thought it consisted in allowing individual learners to come to the front of the classroom to say something about the solution strategies they used. This is illustrated by the following extract from a post-observation interview:

Researcher: How are the experiences you have had in the classroom [since the study began] shaping the way you look at things? For example, when you look at the way you are doing things now in the classroom and compare that with your previous classroom behaviour, do you see any differences?

Teacher C: OK. Previously, I could not call a pupil to come to the front [of the classroom] and tell others how he or she got an answer.

Researcher: You were not calling them to the front? Was it because if you did they wouldn't come or you yourself were not willing to do so?

Teacher C: OK. They could come, but I never used that method.

Researcher: What was the reason?

Teacher C: OK. I just used to give them sums on the chalkboard. Let's say, maybe it is 38 + 10. I would just stand there, then I would just ask the pupils while they are there at their desks. Then somebody [i.e. a pupil] would just stand up and answer the question.

Researcher: But not coming forward to the front of the class?

Teacher C: No. But this time I call upon them to come and explain to the others in front [of the class], which makes children…eager to do maths because [they think] they are being taught to be like a teacher.

Researcher: Yes, I heard you during the lesson saying, "You are the teacher". You were encouraging them to talk?

Teacher C: Yes, they become motivated because they will say they were the teacher.

Researcher: They talk about that?

Teacher C: Yes. [Laughs]. So they are happy to say, "Today, I was the teacher; I was able to teach".

I have said before that the extent to which participating teachers encouraged classroom discussion offered a better measure of how much they had internalised the ideals of the study. Evidence from interviews and lesson observations indicated that right from the beginning of the study Teacher C tried to encourage children to engage in classroom discussion. She often did this by asking pupils in Chinyanja, "Wai peza bwanji?" [How did you arrive at this result?], to which question pupils usually replied, "Nenzo pendela" [I was counting], but would often fail to explain *why* they thought counting was the most appropriate strategy to use in the given situation.

When I asked Teacher C during a post-observation interview at the beginning of the study to tell me about the quality and quantity of classroom discussion taking place among her pupils, she seemed to believe that a good deal of discussion was taking place. The following extract from the interview, which was conducted about two weeks after the beginning of the study, seems to illustrate this:

Researcher: One thing I was interested to see…as I observed your lesson, was the extent to which you encouraged pupils to discuss what they were learning; and to justify their answers. How well do you think you succeeded in doing that today?

Teacher C: I succeeded because I just asked a pupil to tell us how she got the answer and she did. In fact, I did not encourage her to do it. She just did it on her own, just as two other girls did.

Researcher: Do you think other pupils learned something from what she said?

Teacher C: Yes, I think they did, since it was a fellow pupil who was explaining to them. They did because they all got at least two out of the four answers correct. At least that is half.

Researcher: I was also expecting to see if you would allow the children to ask fellow pupils to explain their solution methods. Do you think your pupils would be able to do that?

Teacher C: Yes, they can ask. I will try to do that tomorrow.

But much of the talking that took place among Teacher C's pupils during mathematics lessons consisted of attempts to explain *how* they arrived at a given answer, rather than *why* the procedure used was valid and appropriate. Indeed, during the study Teacher C's pupils never reached the point in their mathematical development when they could defend a position they had taken. This may be because, as the following extract from the post-treatment interview suggests, Teacher C did not encourage them to do so.

Researcher: How far have your pupils developed the ability to question other pupils answers, so that those pupils can defend their answers?

Teacher C: I haven't tried that so far. I haven't tried it. [Pupils] just come to the front and explain how they got the answer, but not defending their answer. Maybe I will try that [next time].

Researcher: How do you hope to do that?

Teacher C: I will give a child [a question]; then she will come to the chalkboard and explain what she did to the others. She will have to defend her own answer, how she did it.

True to her word, the following week Teacher C tried to get children to defend their answers, but without success. At one point during the lesson, Teacher C wrote 35 + 14 on the chalkboard and invited the class to work out the answer. Almost immediately a boy shouted 49, prompting the following exchange:

Teacher C: Wai peza bwanji? [How did you work out the answer?]

Boy: Nenzo pendela. ["I was counting"; and then he proceeded to demonstrate using fingers how he had counted forward 14 steps from 35 to 49].

When asked to explain why he chose to use this method to work out the answer the boy hesitated and eventually just kept quite, seemingly unwilling to be drawn into a discussion about the validity of his chosen solution method. Despite such limitations, lesson observations showed that by the end of the study Teacher C's pupils were becoming used to classroom discussion. Teacher C herself noticed this, as the following extract from the post-treatment interview illustrates.

Researcher: Have you noticed any changes in the pupils' behaviour during lessons, in the period we have been working together on this research?

Teacher C: OK, the children have become too noisy. Maybe the noise they are making is mathematical language, because they have been free, at liberty. Before the study, I didn't used to give them freedom.

Researcher: What did you used to do? Tell them to keep quiet?

Teacher C: Yes, yes. When I am there on the floor they must keep quiet. [The term *on the floor* referred to whole class instruction. The teacher would sit on a chair and pupils would sit down around her on a mat to listen to her].

Researcher: Throughout the lesson?

Teacher C: No, they could speak. I would pose a question to them, e.g. 4 + 2. They would raise their hands up but to talk. But now since you said they must have time to discuss…it has become noisier in the class, because I said you can ask your friends, discuss with your friends. They are able to discuss because they are using Chinyanja.

Researcher: Do they also ask you to help them?

Teacher C: Yes, they do, "What is that sum? We didn't understand".

Researcher: When you look back at what you were doing yourself in the classroom, to what extent would you say you succeeded in helping them to talk about mathematics, asking questions, questioning things, etc?"

Teacher C: I was able to tell them that this time it is up to you to discuss with your friends. If you don't know anything asks your friend if she is better than you, so that at least she can help you to understand.

Thus Teacher C tried her best to encourage classroom discussion, and her pupils developed the skill to a certain degree. However, like her colleague Teacher B, Teacher C also believed that implementing the alternative numeracy development approach made it difficult for her to give remedial help to less able learners. For example, during a post-observation interview Teacher C said:

> Normally I try to assist less able pupils. But with this [mathematics], there is no time for remedial classes. So, I just leave them behind, and we just go on like that. If I had more time, I would assist them. But now, I don't have the time for remedial cases. There is no time.

Teacher C added:

> Maybe if we had ample time I would do something. You see, my first Grade 2 class comes in at 12 00 hours but normally it is almost 12 10 or 12 30 when we have settled down; and the pupils go home at 14 30. I have another class at 14 30. So, if we took time to provide remedial work by the end of the day maybe we would have taught only two subjects.

During the same interview, Teacher C told me that in the 1980s mathematics learning time at Grade 2 level was three hours per day. After the 1980s, the school age population increased significantly, forcing the Government to introduce double or even triple sessions in lower primary school (see Kelly, 1991a). This made it necessary to reduce learning time to only two hours thirty minutes per day.

4.1.2.3.9 Summary of Initial and Final characteristics of Teacher C

Table 5 summarises Teacher C's initial and final characteristics, as measured against the four themes named in the table.

Table 5: *Summary of Initial and Final Characteristics of Teacher C*

THEME	INITIAL	FINAL
Concerns	Not sure if researcher had permission to conduct the study in her school. Forced to participate since the researcher had permission from educational administrators. Topics on mental calculation too advanced for young children.	Surprised that Grade 2 pupils could do mental calculation. Enjoyed teaching and learning mathematics more. Topics on mental calculation being covered too quickly, no time for remedial lessons.
Beliefs	The mathematics of mental calculation was too advanced for Grade 2 pupils. Trying to teach mental calculation at Grade 2 level was unwise. Taught by telling, believing that teachers were the source of all that children learned in mathematics.	Willing to be guided by classroom evidence. Mental calculation reminded her of personal experiences with mental arithmetic. Accepted pupils' use of untaught solution strategies, thought study should be extended nationwide.
Mathematics Content Knowledge	Experienced difficulty understanding concepts on mental calculation.. Often needed help to teach topics on mental calculation, and covered very little content per lesson.	Still unsure how to teach some topics on mental calculation. Needed a detailed textbook in addition to training manual; had difficulty creating simple questions for pupils.
Teaching practice	Discouraged use of untaught methods, did not follow-up pupils' mathematical ideas. Taught strategies for mental calculation by telling.	Allowed pupils to use untaught methods during lessons; did not follow-up answers effectively. Viewed teaching as facilitating learning, encouraged classroom discussion.

4.1.2.4 Teacher D

Teacher D was the oldest of the five Grade 2 teachers in experimental schools. At the time of the study she had 16 years of teaching experience, seven of which were at Grade 2 level. Once Teacher D learned what the study was about, she wanted to participate. I learned later that Teacher D's willingness to join the study team was partly due to the fact that she had participated as a trainer of trainers during the introduction of the PRP (see Chapter 1), and was therefore not afraid to be part of school innovations.

Teacher D was also a well qualified primary school teacher, as she held both the PTC and the PTD. Among the five participating Grade 2 teachers in experimental schools, Teacher D understood best what the study was about and implemented it with a high degree of fidelity. She thus presented me with the best chance to determine to what extent it was possible to introduce the use of strategies for mental calculation relating to double-digit addition and subtraction in early primary mathematics in Zambia.

4.1.2.4.1 Initial Concerns

Despite Teacher D's readiness to take part in the study, she had three main concerns when the study began. First, she believed that mathematics was difficult both to understand and to teach. Because of this, she feared that her existing knowledge of mathematics would prove inadequate to understand the mathematics content that would be discussed during the Staff Development Workshop. Second, knowing from experience the amount of work and the variety of resources normally required when implementing new programmes at school level, she wasn't sure she would have the energy to cope with the tasks she would be called upon to bear during the study. Finally, Teacher D was worried that if learning to use strategies for mental calculation proved too difficult for her pupils, who were already showing signs that they disliked mathematics, it might completely kill off the little desire they still had to learn the subject. The following extract from the post-treatment interview illustrates this.

Researcher: I would like you to look back to the time when I first asked you to take part in the study. How did you feel at that time about participating?

Teacher D: I was a bit hesitant, because when you hear about a research from the higher institution [i.e. university], then you look at your own background, sometimes you are a bit hesitant. I was not very comfortable. That was my first reaction.

Researcher: You said when you looked at your own background, what do you mean by that?

Teacher D: OK I have always viewed mathematics to be a difficult subject. Even when I stand in front of the children trying to teach, I sometimes sit back and say to myself: "Maybe somebody [else] could have done this better". Because I am a teacher, when I teach I expect the learners to get whatever I have taught. But you find only two pupils in the whole class [understanding]. That gives me homework at the end of the day, and I say to myself: "I think I am not capable of teaching mathematics". [Laughs].

Researcher: So, one of your concerns…was that you were not sure if your existing knowledge of mathematics would enable you to understand the mathematics associated with mental calculation?

Teacher D: Yes, yes.

Researcher: Did you think that what we were going to discuss [during the staff development workshop] would be too difficult to teach?

Teacher D: Yes that's what I thought.

Researcher: Was there anything else you were worried about?

Teacher D: Yes. When you look at research, then you are thinking of…too many resources to use. I thought, now time had come for me to use a lot of resources. And I was already dealing with the NBTL literacy programme, which also needed a lot of resources. So, I was saying, "How was I going to divide my time?"

During the same interview, I told Teacher D that many school teachers were uncomfortable with school innovations that altered significantly the way pupils were taught, because they worried about how such innovations might affect their pupils' learning. Did Teacher D have such fears at the beginning of the study?

Teacher D: Yes, I did. You see, my pupils are already having problems understanding what is in the course books. And I thought that this [study] was going to make them escape from the mathematics classroom altogether.

Researcher: They were having problems already, and you were saying: "This new study will worsen things?"

Teacher D: Yes. I think the way we teach. We don't give pupils enough time to think about why we do certain things. It is just 30 minutes of mathematics. So I thought: "Ah…a new concept? The children won't cope".

4.1.2.4.3 Final Concerns

Teacher D's initial fears and doubts concerning whether or not to participate in the study had disappeared by the time the study was coming to an end. She confirmed this during the post-treatment interview.

Researcher: It has been over nine weeks now since the study began. How do you feel today about the study?

Teacher D: I think I am very happy about the outcome, because at the beginning I had problems with my questioning technique. I didn't know how to handle the pupils. I think both the pupils and I had much to learn [during] the first week or so. But then after that we became very comfortable. I think both of us have enjoyed [ourselves]; we have enjoyed the nine weeks. Learners are very free; they can explain anything to anybody at anytime. And it gives [me] room to rest.

Researcher: I am glad to hear that. One of the aims of the study is to see whether or not it is possible for pupils to develop the attitude of questioning things. When they see something they question it. They don't take it as if it is always true, but they ask questions about it. And yes, things have changed for you personally as well.

Teacher D: Yes, very much. And I am enthusiastic about teaching mathematics. I think I have even shelved most of the [prescribed mathematics] lessons, because I want to see that learners understand what they are learning about, and I am very happy.

Thus Teacher D had not only forgotten about her fears and concerns at the beginning of the study but had also become a more enthusiastic primary school teacher of mathematics.

4.1.2.4.3 Initial Beliefs about Mathematics Teaching and Learning

Like her colleagues in experimental schools, Teacher D initially believed that when young children first arrived in school they had nothing whatsoever in the way of mathematical knowledge and depended wholly on the teacher to develop such knowledge. The following extract from the group interview conducted immediately after the Staff Development Workshop illustrates this point. Interestingly, even at that time

Teacher D was already questioning the validity of her existing beliefs about mathematics teaching and learning.

Researcher: How has the [staff development] training affected your beliefs about mathematics teaching and learning? Anything you believed that you are now questioning?

Teacher D: I always believed that children came into school empty especially in terms of mathematics. And I was the one to tell them what to do. But I think it is no longer the case. During the training I experimented with my own children. I asked them to solve some of the problems we had discussed here. The response was overwhelming! They are using the very strategies we have been learning.

An entry in Teacher D's research diary at the time indicates just how much the Staff Development Workshop affected her views and understanding about what it meant to teach young children mathematics:

> My attitude towards numeracy has turned 180^0 just after the briefing by the researcher. I wondered throughout my teaching career why learners had difficulty grasping [the] concepts I taught. I have discovered that learners have [their] own ways of solving problems in numeracy and have different [solution] methods. I didn't even [used to] ask how they came up with an answer, or better still, what method they used to come up with their answers. The learners [themselves] did not even ask questions. I thought they were shy. Not true. I didn't train them to ask their peers [questions]. Interaction was always teacher-pupil, never pupil-teacher.

However, although Teacher D was sure that something must have been wrong with her previous beliefs about mathematics teaching and learning, she was not clear about the new beliefs with which to replace them.

4.1.2.4.4 Final Beliefs about Mathematics Teaching and Learning

Change in Teacher D's beliefs about mathematics teaching and learning began to appear during implementation of the experimental numeracy development approach. This is illustrated by what she says in the following extract from a post-observation interview, conducted four weeks into the implementation phase of the study, following a lesson on the use of the empty number line as a model for performing addition and subtraction:

> I think from the time I started working on this research…my…approach [to] mathematics [teaching] has changed. Why? I thought it was just myself and the chalkboard, the two of us, and the learners were only listeners. But it is no longer going to be like that, because I have discovered…[that] the children know that

> when you add two numbers the numbers become big. When you subtract the numbers are small….They will even tell you…[that] for adding you go to…they don't say to the right but they will say I will add from this number going to the right, though not in so many words but they will just point: "Tiyenda uku ba Teacher" [i.e. Teacher, we should go in this direction]. And then for subtraction, the bigger number starts and then it goes to the left. And they always find the [correct] answers, except perhaps [for] the jumps [on the empty number line]. On top of the jumps they don't write minus whatever number, but they know how to arrive at an answer.

Teacher D did not only change her views with regard to understanding what it meant to teach or learn mathematics in lower primary. She also wanted primary school teachers in non-participating schools nation-wide to undergo a similar experience, as this would enable them develop confidence in teaching mathematics. When I asked her and her colleague at the end of the Staff Development Workshop what advice she would give to educational administrators at national level who wanted to know if the training she had received should be extended to other primary school teachers nation-wide, she replied:

> I would tell them that I think it is very important that other primary school teachers explore activities about the use of mental calculation strategies. It will enable them to feel very confident in…teaching mathematics in lower primary.

Teacher D maintained this view during the post-treatment interview, when I repeated the question:

> I would tell them that, I think that is the right direction [to go]….This research…is very, very different because you, sir, have also taken the trouble of sitting in the classroom and seeing pupils working. That has helped me a lot, and I don't see myself going back to the traditional methods….I think I would strongly advise the Ministry of Education and yourself to continually come and see how far we shall go. And I am just praying hard that I take these [Grade 2] children up to Grade 4. I would like to...move [up] with them and see what difference [using strategies for mental calculation] makes. I am going to use the empty number line throughout. I will also use all the methodologies that I have learned, [and] see how far I will go.

Thus whereas Teacher D previously believed that teaching mathematics in the early primary grades involved giving mathematical knowledge to children who knew absolutely nothing about the subject, she now accepted that young children already know some mathematics and teaching them requires first understanding what they know and starting from there.

4.1.2.4.5 Initial Mathematics Content Knowledge

I mentioned earlier that Teacher D had both the PTC, the basic qualification for teaching in all primary grades in Zambia, and the PTD. Incidentally, during a post-observation interview Teacher D had a copy of one of the mathematics education modules for the diploma programme, which included the topic m*ental strategies*. I asked her what this topic involved:

Researcher: What was the aim of including mental mathematics in that module? What did the authors expect teachers to do in the classroom?

Teacher D: It was not explained the way you have [explained it] in this research. It was like the traditional one [i.e. mental arithmetic]….You come into the classroom, you ask a few questions, and the pupils give answers. Then you go into your lesson.

Researcher: So you were mainly interested in pupils' answers?

Teacher D: Yes. Not investigating the methods pupils used. Maybe the new version of the module includes finding out how children worked out answers. But the one we used did not.

Researcher: What about when you yourself were learning as a diploma student? How did you learn that part of the module called mental strategies?

Teacher D: I think the same way, the way I have explained. Eh…just asking questions. Not probing or finding out how one arrived at the answer, or what methods one used….That part was not covered, otherwise I should have remembered. Maybe the new version of the module covers it, I am not sure.

Thus the term *mental strategies* as it appeared in the primary diploma teacher education programme that Teacher D underwent did not mean the same thing as *mental calculation*, the subject of this study. The aim of this section was to discuss Teacher D's initial mathematics content knowledge, that is, her knowledge and understanding of mathematics before the study was implemented in her school. I now turn my attention to this.

I mentioned earlier that when I first invited Teacher D to participate in the study, she was very enthusiastic to do so. As mentioned above, this did not mean that Teacher D had no difficulty understanding how to teach strategies for mental calculation relating to double-digit addition and subtraction. She did. For example, when I asked Teacher D during the group interview what she thought should be added or removed from the Staff Development Workshop, to ensure that it met her mathematics learning needs, she responded:

> I wish you had extended [the coverage of the workshop]. You know, we have four operations [addition, subtraction, multiplication, and division]. We have tackled only two: addition and subtraction. I wish we had also done the aspect of multiplication and division.

Teacher D's statement above suggests that, while she may have been happy with how the two operations she mentions were handled during the Staff Development Workshop, she was not sure how to deal with the other two. She also experienced difficulty in the classroom when teaching how to perform double-digit subtraction using the **N10** method, which suggested that she herself did not understand how the procedure worked. The following extract from the post-treatment interview illustrates this point:

Researcher: Is there anything concerning our discussion today or concerning the study itself…you would like to say which might inform me a little more about your classroom experiences during the study?

Teacher D: Ah…in my lesson yesterday I noticed some little confusion. A few days before I used the 1010 [method] and had no problems. But when I came to discuss the N10 [method], there was that little confusion.

Researcher: Where exactly?

Teacher D: The pupils, ah…I don't know. They expanded quite well. For example 27 + 19. They understood this to be 27 + 10 + 9. But then, after that I don't know whether they were trying to split the 9 to make 5 and 4. Now, in their addition they included…27 + 9 + 5 + 4. I don't know what they were doing. That's what I will try and find out today. The 1010 [method], they did very well without any problems. And the empty number line, no problem. But [with] this N10 [method], there was a problem. I don't know why….Maybe I didn't explain it properly.

Although Teacher D states above that she did not encounter any difficulties with the 1010 method and that her pupils learned it well, there was evidence suggesting that she did, as the following extract from the same interview above indicates. (For a detailed discussion of how to use both the **1010** and **N10** methods to perform double-digit addition and subtraction, see Chapter 2).

Researcher: How much consultation has been taking place between you and your colleague [Teacher E]…who is also participating in the study?

Teacher D: Actually a lot, especially when we were using the 1010 method to do subtraction. We had to do a lot of thinking ourselves. For example, here is 36 – 19. In normal circumstances we would say 30 – 20, then add a 1. We told the children that since we had added a 1, we now needed to take away a 1 from the remaining 6, so that we get a 5. That concept took us a long time. We did a lot of consultation.

Researcher: But…I saw a little girl in your class do it correctly yesterday….She was able to subtract and add something. And I said to myself, "How did she get the idea so easily, so clearly?"

Teacher D: Yes, the 1010 method was easier for them. In fact, when I gave them the N10 [method] they still wanted to continue with 1010. So, I think I will give them more time to exhaust [1010] and then move on to N10 again.

Researcher: Would you then say, teaching these ideas was challenging?

Teacher D: Very challenging and very interesting.

4.1.2.4.6 Final Mathematics Content Knowledge

According to Teacher D, the Staff Development Workshop helped her develop the confidence she needed to teach mathematics effectively in lower primary school. For example, during the group interview she said: "I feel more knowledgeable in mathematics than I have ever been. I don't know why….I am very comfortable". Why did Teacher D reach this conclusion? Her statement below, from the same interview, answers the question:

> I have learned a lot of things [during the Staff Development Workshop]. I will give an example of the number line. I never even thought there was a number line that could be called *empty*.…So, that is one skill I think I have learnt, that from an empty number line I can add and I can subtract without any problems, and I am able to pass on the skill to the learner.

Lesson observations confirmed that while Teacher D's initial mathematics content knowledge was somewhat shaky, by the end the study she was teaching the use of strategies for mental calculation quite effectively indeed. Not surprisingly at the end of the interview Teacher D, referring to her state of mathematical knowledge before the study, said: "I think we [teachers] take some of these things for granted without even understanding them".

4.1.2.4.7 Initial Teaching Practice

At the beginning of the study, Teacher D's classroom practice during mathematics lessons was consistent with her belief that mathematics teaching comprised *telling*. That is, transferring knowledge from the teacher's head to those of the pupils, with little or no consideration being given to children's own ideas. Indeed, lesson observations indicated that in spite of having attended the Staff Development Workshop, which stressed the importance of classroom discussion to pupil learning in mathematics, Teacher D did not show much tolerance for pupils' wrong answers or answers that differed from what she had taught. Accordingly, during the group interview which, as earlier stated, took place immediately after the Staff Development Workshop, Teacher D described her previous classroom behaviour during mathematics lessons as domineering.

> I always thought that I was the only one in charge, who could pour whatever knowledge I had [into pupils' heads]….I thought I was the only one who could ask [questions]…I have discovered now that the learners themselves have questions and I was a hindrance because…each time I said to the learners, "Any questions?" there was no response. Why? Because I never trained them right from the beginning to ask their friends questions, but from today I will.

In a post-observation interview, conducted towards the end of the study, Teacher D added:

> We didn't give learners a chance to open up. It is only in this…research you have brought to our classroom that I have seen the value of us [teachers] listening [to learners' mathematical thinking]…and [of] the learners themselves exploring whatever sums they have and telling others what they have done....Otherwise before [the research] it was a closed issue.

When I asked Teacher D during the post-treatment interview to reflect on her previous teaching practice, she described herself as having been 'strict', wanting learners to do exactly as she had taught them. Teacher D explained why she did this:

Researcher:	Why do you think you were so strict in the past?
Teacher D:	I think [it is] the way I have been trained. I was trained to make sure that the children follow... the way I have taught them.
Researcher:	So you wanted the children to reproduce what you had taught them?

Teacher D: Yes, but then I have realised that that is not the best way. There are many ways of arriving at the answers.

Researcher: At the time when the aim of your teaching was to enable children to reproduce what you had done, if children actually managed to do just that, would it mean they had understood [what you were teaching]?

Teacher D: I don't think they understood anything…because they were not even able to explain. Even the ones who got the answers correct; they were not able to explain to their friends…what they had done. They would just say, "Well, I got it right!" That's all.

4.1.2.4.8 Final Teaching Practice

As is evident from the interview extracts above, during the course of the study Teacher D realised that her previous teaching style had impeded her pupils' progress in mathematics. Lesson observations showed that because of this, Teacher D began adopting a more constructivist approach to mathematics teaching and learning, which included becoming more and more tolerant of pupils' divergent views during lessons. The following extract from a post-observation interview conducted a month before the conclusion of the study illustrates this point:

Researcher: Now that you have seen what children can do [in mathematics], how do you hope to use these ideas in your future lessons?

Teacher D: I think I will be more flexible this time than I was before, because I used to be tailored [i.e. to stick] to what was laid down for me.…I will no longer insist on children doing things in a particular way only. I will accept whichever way children use, so long as it makes sense. And I may use the same methods [myself], if I am interested in one, to help the other learners.

Researcher: You have made a very important point there. When one pupil brings out an idea that you think works, it is important to allow that pupil to explain to the others and let the others question the idea. And then they [can] decide whether [or not] they want to take it up.

During the post-treatment interview Teacher D gave a similar but more detailed answer when I asked her to state in brief what she thought she had learnt from participating in the study.

Researcher: If you were to…summarise the things that you have learned during the study…what would you say?

Teacher D: OK. One, I have understood that [young] children come with enough information from home about mathematics, which I was not able to tap, but this time I am able to tap from them. And I have been observing what they do unlike before. So, both the pupils and myself are really watching each other and we are learning from each other. So, I think the children can do a lot of mathematics if I don't hinder them as a teacher.

Researcher: The study is coming to an end but your teaching continues. So, how is this experience going to affect your teaching [in future]?

Teacher D: I think this time I will be very flexible. I will not be rigid as I was in the past and I will try to learn from the pupils. I will try to find out a lot more [about] what they know than I used to. I think I have discovered [that] each pupil is unique in his or her own way. In this classroom we have put the pupils in four big groups, according to how we have assessed them in other subjects. And I took it for granted that because in other subjects they did not do well, even in mathematics I thought they wouldn't. But I was wrong because I have seen [that] the weaker pupils, some of the weaker pupils, are doing far much better than the pupils who I thought were doing very well. So, I think [from now on] I am going to give the learners a chance.

But how much classroom discussion did Teacher D allow during mathematics lessons, to give learners the opportunity to think for themselves and to construct their own mathematical understandings? Lesson observations indicated that at the beginning of the study nothing took place in Teacher D's mathematics lessons that could be called classroom discussion. Although Teacher D acknowledged pupils' correct answers by saying "Yes" or "Good", she showed little tolerance for wrong answers; and neither 'correct' nor 'wrong' answers were investigated further. She therefore denied pupils the chance to learn form each other why some answers were deemed correct and others wrong. The pupils themselves, unused to self-expression in mathematics lessons, did not seem to understand why it was suddenly necessary to explain their methods to their colleagues or to justify their reasoning particularly when they had already successfully completed a calculation.

Within a few weeks of the implementation phase of the study, lesson observations showed that Teacher D was making an effort to help children talk more about the mathematics they were learning. She often said, "Wai peza bwanji" [How did you work it out], which prompted pupils to try to explain their solution strategies. She also encouraged them to use similar expressions to demand an explanation from other pupils who had given answers to mathematics problems. As one would expect, Teacher D had more success with some pupils than with others. For example, following a lesson on the use of the empty number line to perform double-digit addition and subtraction, Teacher D wrote in the 'self-evaluation' section of her lesson plan:

> Harriet, Margaret and Fresher explained well how to partition and make jumps [on the number line] to solve addition and subtraction sums. I still have to encourage [the] Papaya, Orange and Mango [learning] groups to explain issues.

Two months later it was clear from observing Teacher D's lessons that with regard to facilitating classroom discussion she had made significant progress. This was not surprising since, as noted earlier, among all participating Grade 2 teachers in experimental schools she was the one who seemed to have understood best the aims and objectives of the study, and who implemented them with a fairly high degree of fidelity and accuracy. The following extract from a post-observation interview illustrates Teacher D's own judgement concerning the progress her pupils were making towards achieving meaningful classroom discussion.

Researcher: How far do you think your pupils have gone in developing the habit of questioning each other's work, ideas, ways of working out answers, etc.?

Teacher D: I think the one they have satisfied me on is the [aspect of] explanation. They have really come out. I am still working on the children probing others. But they are able to explain to others. So, that is one thing I am satisfied about. I think I will carry on with that kind of teaching strategy.

Researcher: You are being modest. When I was sitting in your class I overheard them also questioning a fellow pupil who was in front of the class working out something on the chalkboard. They asked: "Wai peza bwanji?" [How did you work that out?]. They are not only beginning to learn to justify their answers but also to question one another's answers.

Teacher D: I have also noticed that even shy pupils are opening up. And whether the answer is wrong or correct, they don't mind. They will just stand up and say whatever. So, that too is progress.

Researcher: I am happy to hear that. As we were saying during our training, if children can take a position in mathematics and defend it against other children's questionings; that makes them think.

During the post-intervention interview I asked Teacher D again how much discussion took place among her pupils particularly when they were left alone to do some work in mathematics. As the following extract from the interview suggests, her response indicated that she now believed that her Grade 2 pupils had attained a reasonably high degree of competence in so far as classroom discussion was concerned.

Researcher: You have already mentioned that pupils question each other's answers and so on. But how much discussion really takes place? When you give them work and leave them to do it on their own how much discussion do you see taking place among them as they work?

Teacher D: This time they talk a lot of mathematics. Especially that group [pointing to one of the learning groups in the class].

Researcher: The more able group?

Teacher D: Yes, they discuss a lot. And there are times when one of the children will stand [up] and say, "These are talking a lot and copying from each other". I don't think that that is copying; that is discussion.

Researcher: Yes, someone looking at them might think that they are copying from one another, when in fact they are engaged in a discussion.

Teacher D: I sometimes ask the less able to try and solve a problem on the chalkboard. Other pupils will say: "Aaah, that number is not supposed to be like that. Move it that way". Or sometimes a child will just stand up and show a friend how it is supposed to be done.

At the end of the interview I asked Teacher D to tell me what Grade 2 teachers from schools that did not take part in the study might notice happening in her mathematics lessons that did not happen in theirs. She replied unhesitatingly:

> I think the interaction that goes on. From Grade 1 to Grade 4 [it is] very rare to find a teacher during a mathematics lesson interacting well with pupils, where the pupils are asking each other questions, the teacher is also probing, and so on. It is more pupil-centred; the teacher is just facilitating. This is what they would see.

Thus whereas at the beginning of the study Teacher D's teaching practice was characterised by *telling*, by the end of the study she had moved towards a more constructivist approach to mathematics teaching and learning.

4.1.2.4.9 Summary of Initial and Final Characteristics of Teacher D

Table 6 summarises initial and final the characteristics of Teacher D as measured against the four named themes.

Table 6: *Summary of Initial and Final Characteristics of Teacher D*

THEME	INITIAL. BEHAVIOUR	FINAL BEHAVIOUR
Concerns	Unsure about study's possible effects on pupil learning in maths. Feared that her existing knowledge of maths might not be sufficient to understand mental strategies? Thought she might not have the energy to cope, since school innovations involved use of many resources. Feared that pupils would abandon learning maths altogether, if learning mental calculation proved too difficult.	Pleased with the outcome of her participation in the study; both pupils and herself learned a lot. Became more motivated to teach maths and eager to see that pupils understood what they learned.
Beliefs	Pupils had no mathematical ideas of their own, apart from what they learned from teachers. Teaching maths is telling pupils what to do and expecting them to do it. Pupils should not express own views during mathematics lessons; they should accept knowledge as given by teachers.	Pupils can understand and use mental strategies. Pupils' past failures in maths were possibly due to her teaching style. Mental methods made learning maths a pleasure for pupils. Supported extension of study nation-wide.
Mathematics content knowledge	Felt that the mathematics content of the PTC and the PTD was inadequate for learning to teach mental strategies. Found teaching mental strategies challenging, especially explaining how to use the N10 and 1010 methods.	Training workshop made her more knowledgeable in maths than ever before. Needed to learn more mathematics content from similar training workshops.
Teaching Practice	Characterised by attempts to transfer maths knowledge from teacher to pupil, and wanted pupils' answers to reflect exactly what she had taught. Did not take into account pupils' mathematical thinking; did not tolerate wrong answers from pupils. Her teaching style prevented learners from asking questions.	Observed pupils more and tolerated use of untaught solution methods. Expressed willingness to use children's methods if necessary, and adopted more flexible teaching approaches. Encouraged classroom discussion and asked more questions. Invited colleagues to observe her lessons.

4.1.2.5 Teacher E

Among all the teachers taking part in the study, Teacher E had taught in primary school the longest, and was due to retire from the teaching service the following year. She had spent almost her entire professional career teaching in upper primary, that is, Grade 5, 6 and 7. At the time of the study she had only taught at Grade 2 level for less than a year.

Teacher E was teaching in the same school as Teacher D, a school that had three Grade 2 teachers. The third teacher turned down my invitation to participate in the study, citing commitments elsewhere. However, when Teacher D and Teacher E started implementing the alternative numeracy development approach based on the use of strategies for mental calculation, this teacher would sometimes visit their classrooms to find out what they were teaching. She would then try the same ideas with her own Grade 2 classes.

Initially, Teacher E seemed unwilling to participate in the study. She showed this by appearing bored and uninterested particularly during the first day of the Staff Development Workshop, when I was explaining the theoretical framework behind the use of strategies for mental calculation in early primary mathematics. It would seem that her lack of enthusiasm on the first day of training was partly due to my explanation of the theoretical framework, which was perhaps abstract and failed to connect with what she had known during her many years of teaching in primary school. In fact, I learned later that at one point Teacher E had contemplated withdrawing from the study, but was persuaded by Teacher D to stay on until the end.

4.1.2.5.1 Initial Concerns

All the four other teachers in experimental schools already discussed so far admitted to having fears and concerns when first contacted to take part in the study. For example, all were unsure about whether or not their pupils would understand strategies for mental calculation, while some doubted the adequacy of their own mathematical knowledge in relation to teaching the mathematical ideas to be implemented. Teacher E was different in this regard. She stated that when first contacted to participate in the study no such fears or concerns clouded her mind. She had concluded from the beginning from the beginning of the study that her participation would benefit her pupils. The following extract from the post-treatment interview illustrates this point:

Researcher: How did you feel when I first invited you to take part in the study?

Teacher E: There was nothing that I felt. I just thought I should get involved.

Researcher: I was wondering whether in getting involved you had any concerns, since this was something new that you had not done before.

Teacher E: My concern was only for the pupils themselves.

Researcher: What do you mean?

Teacher E: I thought that what we did during the Staff Development Workshop could be a foundation for the children…to know a variety of mental strategies. But even on my part, I have learned something that is very useful to the children.

Researcher: What I was trying to find out was whether during the [Staff Development] training you had any feelings like, "Can Grade 2 children learn this?" You didn't have any such thoughts at all did you?

Teacher E: Ahhh….maybe on few things here and there but hmmm….I never had serious observations on that one, because I thought it was good for the children to learn.

4.1.2.5.2 Final Concerns

The foregoing would seem to suggest that at the beginning of the study Teacher E did not have any worries or concerns about participating, as she had concluded that her pupils would benefit from the study.

4.1.2.5.3 Initial Beliefs about Mathematics Teaching and Learning

Like Teacher D, Teacher E initially believed that children lacked mathematical knowledge of any kind when they first enrolled in school. While in school, they had nothing to offer in terms of mathematical thought, but received everything from the teacher. It was only during the Staff Development Workshop that Teacher E learned for the first time that young children knew some mathematics before starting school, and that they did not learn everything from teachers. During the group interview at her school I asked Teacher E if the training she had received had in anyway challenged her existing beliefs about mathematics teaching and learning. She replied rather tentatively:

> I think maybe…pupils sometimes found mathematics difficult because we never allowed them to explore….So, whenever they came into the classroom…ah…the children thought whatever the teacher said was the only correct thing.

This shows that before the study Teacher E did not allow children the freedom to explore mathematical ideas. She believed that they were in school precisely to learn from her, and it was her duty to tell them what they needed to know.

4.1.2.5.4 Final Beliefs about Mathematics Teaching and Learning

The Staff Development Workshop made Teacher E realise that something was wrong with her existing beliefs about mathematics teaching and learning, which might have affected negatively her pupils' progress in mathematics. The following extract from a post-observation interview further illustrates this point.

Researcher: Did anything happen during the lesson that made you think, "Ah, I didn't know that these children are capable of doing this!"

Teacher E: Yes, there are so many things that I noticed as I was teaching. Some of the pupils solved the problems and I was puzzled: "How did they get it?" I realised that these children know something. They have got something which they can express on their own.

Further evidence from the classroom presented Teacher E with similar surprises. During a post-observation interview two weeks later, which followed a lesson on single and double-digit whole number addition, I asked her again if her pupils had done anything during the lesson that surprised her. Her response:

> I think there were two children who did much better than their classmates. When I completed the lesson, after you had left, I brought them together again. I was trying to find out from them how actually they got the answers and what methods they had used. I discovered that the children knew exactly what they were doing.

Thus Teacher E was beginning to realise that young children did not need to be 'spoon-fed' as it were. They had their own ways of solving simple problems in mathematics, and all they needed was to be allowed to demonstrate this. It would seem that this realisation began forming during the Staff Development Workshop for, during the group interview at her school, she had advised me to extend the pilot nationwide:

> You should not stop here. You should extend [your study] to other [primary school] teachers…so that…we [will all] be able to use this [approach to numeracy development]; then I know children will…like mathematics. You will have a scenario where even girls will come to like mathematics.

Teacher E repeated this idea of extending the study to all primary teachers in Zambia during the post-treatment interview.

Researcher: Suppose that Ministry of Education officials asked you: "We have heard about this study and were wondering whether you think it is necessary to have it extended to as many schools as possible". What would you say and what reasons would you give for saying it?

Teacher E: OK, if they come I will…[Hesitates].

Researcher: Yes, they want to find out.

Teacher E: They want to find out what we have been doing?

Researcher: And whether you think other teachers should experience this work.

Teacher E: I would freely tell them that…everybody [every primary school teacher] should have an idea about this, because it is educative. And it must continue, really.

Researcher: And if they asked: "But what will your pupils benefit from this?"

Teacher E: I will not hesitate. I will tell them that the children actually have benefited, because they are [now] able to participate and [to] explain [things] on their own.

Evidently, realising that her previous beliefs about mathematics teaching and learning were somehow invalid, Teacher E was enthusiastic about allowing other primary school teachers in the country to undergo a similar experience.

4.1.2.5.5 Initial Mathematics Content Knowledge

At the time of the study Teacher E, who already held the PTC, was studying for the award the PTD. In fact, she wrote her final Diploma examinations the week after I first visited her school to meet school administrators and to persuade Grade 2 teachers to participate in the study. Thus apart from her long teaching experience, she was also a well-qualified primary school teacher. Nevertheless, Teacher E doubted the adequacy of the mathematics she studied in the Diploma programme for learning to foster the use of strategies for mental calculation relating to double-digit addition and subtraction in early primary mathematics, as suggested by the following extract from the post-treatment interview:

Researcher: Do you think that your present knowledge of mathematics is sufficient for teaching these ideas on mental calculation? Or would you need a workshop that would focus on learning mathematics itself?

Teacher E: Yes, I have to learn mathematics itself. I think that is when I can be more knowledgeable about mathematics.

Researcher: You did the primary diploma. How sufficient was the mathematics in it for doing this work?

Teacher E: It is not enough. Yes, there is something there but it is not enough. We still need to learn more mathematics.

Indeed, throughout the study Teacher E needed much tutorial help to implement the experimental numeracy development approach, as the following extract from a post-observation interview illustrates. Incidentally, I had already signalled the end of the interview when Teacher E indicated that she wanted us to continue talking.

Researcher: You were saying?

Teacher E: I began my lesson with an addition word problem, involving single digits. Now, here [pointing to a section in the scheme of work we had developed at her school] it says discuss a variety of mental methods….My worry is...from here are we using the numbers 6, 7, 8 and 9, in terms of '5 and a bit'?

Researcher: Yes. If you are adding 9 + 7 for example, to make the calculation easier 9 becomes 5 + 4 and 7 becomes 5 + 2. Then you add the 5s together and the ones. So, always you talk about how many 5s are there and how many ones remain.

Teacher E: I am getting it now. Even on this one [pointing to another section in the scheme of work] where it says, work as above but ensure that children understand how to use a marked number line. Now, on this one, do we have to use a number line?

Researcher: Sometimes splitting does not help much particularly when you are subtracting numbers where the second digit in the number being subtracted is bigger than the second digit of the number from which it is being subtracted. In that case a number line could make things easier.

It would seem, therefore, that at least at the beginning of the study Teacher E was unsure about what to do to implement the study, which perhaps indicated the inadequacy of her knowledge of mathematics for implementing the experimental numeracy development approach.

4.1.2.5.6 Final Mathematics Content Knowledge

Evidence from interviews and lesson observations indicated that as the study progressed, Teacher E continued to experience problems particularly with regard to teaching mental strategies for double-digit subtraction using the 1010 and N10 methods, and the empty number line. However, she believed that whatever problems she experienced in this area were due to her pupils' failure to follow her explanations. The following extract from the post-treatment interview illustrates this point:

Researcher: Did you find teaching mental calculations easy or difficult?

Teacher E: Sometimes the teaching part was not easy, but then as I continued working with them [the pupils] at least some of them could grasp or have an idea about the work I was doing. For example, I once gave [the class] some subtraction sums. I think it was double-digit numbers. Maybe we can use 24 – 19. Splitting the numbers was difficult for them.

Researcher: 24 – 19. That means you split either both of the numbers [i.e. 1010 method] or just one of the numbers [i.e. N10 method]. So, you are saying the children had difficulty splitting the numbers in terms of 10s, 5s, and 1s?

Teacher E: OK, on that one they had no problems. But they had problems [subtracting numbers] when the first number had a smaller digit in the ones position and the second number had a bigger number in the ones position.

Researcher: OK. How did you solve that one?

Teacher E: Ah….I asked my friend [Teacher D] and …she assisted me.

Researcher: It is good that you talked to your colleague about it. So you yourself didn't find the ideas themselves difficult to understand but it is the pupils who did?

Teacher E: Ah….Maybe the pupils had some difficulty with grasping the concepts. But once you explain to them fully, some of them could do the work without any problems. Only that in a class…we can have some slow learners of course.

My conclusion was that, although Teacher E did not acknowledge that her own mathematical content knowledge was inadequate, as perhaps it is difficult for teachers to do so, it actually was. I had hoped was that the Staff Development Workshop would enhance her knowledge of mathematics sufficiently to enable her teach mental calculation confidently. Apparently, this might not have been the case.

4.1.2.5.7 Initial Teaching Practice

Like her fellow Grade 2 teachers in experimental schools, Teacher E also believed that on first entry into school children had no mathematical experiences of their own, so that teachers were the source of everything they learned in mathematics. Accordingly, her classroom practice before the study was characterised by *telling*. That is, she attempted to transfer her won understanding of mathematics to her pupils. For example, in one of her lessons, which I observed, Teacher E put a word problem on the chalkboard and went around the classroom checking how children were tackling the problem. After doing this she displayed to the class the work of one child, and the following exchange ensured:

Teacher E: Do you see how this one did it? Did she do it well?

Class: Yes.

Teacher E: Yes, this is how you should do it.

Teacher E: [Picks another child's notebook and shows the child's work to the class]. Do you see how she did it? Did she do it well?

Class: No.

Teacher E: So don't do it this way. [She picks another child's notebook]. What about this one?

Class: This is done well.

This exchange between Teacher E and her class indicated that she did not accept or discuss further pupils' wrong answers. Instead, she wanted pupils' work to reflect her own classroom demonstrations. Teacher E also believed that, if a teacher explained clearly a mathematical idea, children would learn it effectively. Thus when I remarked during a post-observation interview that her pupils had impressed me with their knowledge of number partitioning, she immediately replied:

> [The exercise] was not difficult for them, because I think from my explanation they got it right. They could have found it a problem if I had not explained it well.

To what extent then did her teaching practice change, if at all, during the study? I address this question in the next section.

4.1.2.5.8 Final Teaching Practice

Right from the beginning of the study Teacher E showed that she wanted to change her teaching practice, to be more in line the social constructivist approach to mathematics teaching and learning. The following extract from the group interview at her school illustrates this point:

Researcher: What [teaching] skills do you think you have learned as a result of participating in this workshop?

Teacher E: I have discovered that the [teaching] approach I used mostly in the classroom somehow, maybe somewhere, made pupils not to behave creatively enough.

Researcher: I would like to be clear about what you mean. What kind of teaching approach were you using that you think may have caused your pupils not to do as well as they might have done in mathematics?

Teacher E: Maybe I didn't give them that freedom of …ah…I didn't give enough time for them to think.

In a post-observation interview conducted during the first two weeks of the implementation phase of the study, Teacher E clarified what she meant in the statements above. In the same statement, she also distinguishes between her classroom behaviour before the study, and during the study.

> I think we didn't used to deal much with children's mathematical experiences. What we used to do was to take…work directly from the textbook and give to children without maybe discussing their experiences with them. [But] this is what we [now] have seen. If we can…find out what…children already know, I think they would be able to enjoy mathematics, than us giving them directly from the [text]books.

By the end of the study Teacher E believed that her teaching approach in mathematics was different from that of other primary school teachers who did not experience the study. This is illustrated by what she said in the following extract from the post-treatment interview:

Researcher: If teachers from a school that did not participate in the study walked into your classroom during a mathematics lesson, what would they notice happening which does not happen in their own classrooms?

Teacher E: I think what they would notice as they come into the classroom is the free participation of the children and the way the work is going to be introduced. For example, when we look at that work…on the chalkboard, we are used to vertical [addition and subtraction]. When they see that, some of them will be wondering why I have done it like that. But that is the easiest. [That way] children can…get the work [correct]. So, I think when they come in and they see what I am doing, I will not regret [i.e. ashamed]. I will even be able to show or to explain to them why I have done this and that.

Researcher: A teacher at another school who is also involved in this study told me that some of her colleagues who are not participating sometimes come into her classroom and say, "What is this thing you are doing?" wondering whether it is of any benefit at all, since it looks so different. Have you had colleagues come in and question you like that?

Teacher E: OK, there are some who come in and they find work on the chalkboard. And I leave it [there] deliberately. I don't rub it off. So, they ask: "What kind of work are you doing now?" So I tell them: "This is the same kind of work, only that we have got so many different methods of arriving at the same answer". Then I explain to them how I did it. Some of them appreciate.

As I have pointed out in each of the four cases already discussed, the amount and quality of classroom discussion taking place during lessons indicated best how much pupils were allowed to talk about the mathematics they were learning. How did Teacher E do in this area? My first observations of Teacher E's lessons showed that there was really nothing happening in her mathematics lessons that could be called classroom discussion. However, as the study progressed I observed that Teacher E was trying more and more to get her pupils talking about mathematics during lessons. Like the other four participating teachers, to get pupils to talk about what they were learning, Teacher E often asked them the question: "Mwai peza bwanji?" [How did you work it out?]. And the children usually replied, "Nenzo pendela," [I was counting]. However, no amount of pressing persuaded them to explain *why* they counted to get the answer. In the following extract from a post-observation interview Teacher E confirms this and tries to explain why this was the case:

Teacher E: Explaining doesn't mostly…come out from them.

Researcher: What is the problem?

Teacher E: Maybe we [teachers] have not given much attention to that from the beginning. It is only now, I think, that we are trying to make sure that children explain how they arrive at an answer. But through learning about the use of strategies for mental calculation, I think we are going to make sure that children are able to explain what they have done whenever they are solving mathematics problems.

Researcher: Do you think they will actually reach a point when they will not only be able to talk about the procedure they used, but also why they think what they did was the right thing to do?

Teacher E: They will. The more we teach them; the better they will become at doing it.

By the end of the study Teacher E was sounding more upbeat about the prospects of her pupils engaging in meaningful discussion during mathematics lessons, as illustrated by the following extract from the post-treatment interview.

Researcher: One of the things we have been trying to find out in this study is whether it is possible for Grade 2 pupils to learn to talk about their mathematics not only with the teacher but also among themselves. When you consider what you have seen in the classroom over the weeks that we have been working together, to what extent would you say that your pupils are now able to do this?

Teacher E: OK, especially that group there [pointing to one of the learning groups in the class]…the best in the class. Whenever I give them some work they usually discuss it among themselves, asking one another, "How did you get it?" Sometimes you see a child moving from one desk to another trying to consult colleagues. And sometimes they come to the front of the class, where I am, asking, "Teacher, have I made the correct move here?" So I correct them or I just give them directions; then they complete it on their own.

Researcher: I remember that at the beginning of the study it was difficult to make children stand up to talk about what they had done. When you ask a child to stand up now to defend a position, how well and how readily do they do it?

Teacher E: OK, there are a few children who have become good at it. They are courageous, [for example] Sylvia. She can explain to you what she has done. Sometimes a child will even stand up and say, "I have done it this way…" If a colleague has made a mistake…they will say, "No, he has made a mistake. He was supposed to do this or that".

But in trying to get children talking about the mathematics they were learning Teacher E was limited by her inadequate knowledge of Chinyanja, a Zambian language different from her own but which she had to use to communicate with her pupils. As she said during a post-observation interview,

> I try my level best [to ask questions that encourage classroom discussion] only that language is still a problem [for] me. Sometimes I ask a question in one way, again I [change and] say it in another way because I am not very conversant with Chinyanja. Sometimes I feel that maybe I haven't asked it the right way. Maybe that is why the children haven't done well.

Although as is evident from above, Teacher E was trying to change her teaching practice to be more in line with a social constructivist approach to mathematics teaching and laerning, she still had a limited understanding of some of the terms we were using. For example, she believed that classroom discussion was something that only happened at the beginning and end of a lesson. For example, during a first post-observation interview she said:

> I think mostly [classroom discussion] is done during the beginning of the lesson and at the end of the lesson. That's why, whenever we finish our lessons we must call the children [together to sit on the mat at the front of the classroom] and…find out whether they have understood what they have done or not.

During another post-observation interview two weeks later Teacher E again said:

> What I do is I bring the [pupils] to the front here [to sit on the mat]. Before I give them any new work I would like to find out from them [what they know]…. I…use experience to find out what they know, so that as they go to the lesson they are already aware of what they are supposed to do. So, I encourage them to be together as a group.

Furthermore, although in face-to-face interviews Teacher E indicated that she wanted to move away from a view of mathematics teaching as knowledge transfer from teacher to pupil, evidence from classroom observations showed that her classroom practice did not go the same way. Right up to the end of the study, Teacher E would explain the steps required to apply a given procedure and insisted that pupils should follow the steps she had demonstrated. For example, I observed a lesson where Teacher E demonstrated the use an empty number line to perform single-digit addition and subtraction. After completing the demonstration, she told the class:

> I will give you an exercise. When I do, I want you to do it exactly the same way I have shown you on the chalkboard….When you draw an empty number line make sure to follow all the steps. OK?

While the class was doing the exercise, I tried to find out from a child sitting next to me how she was faring. It became clear to me that Teacher E's practice of wanting to see learners follow strictly the steps she had outlined limited their initiative. The child had written down in her notebook: 9 – 5 = 4.

Researcher: Wai peza bwanji answer iyi? [How did you arrive at this answer?]

Child: Ba Teacher ba kamba kuti ni kupenda 1, 2, 3, 4, 5. [The teacher said we have to count 1, 2, 3, 4, 5].

The next task she tackled was 5 – 2, which she again worked out correctly.

Researcher: Iyi inangu wai peza bwanji? [How did you work out this other one?]

Child: Ba Teacher ba kamba kuti ni kupenda 1, 2 . [The Teacher said that we count 1, 2 (i.e. starting with 5 and counting two steps back)].

Thus although the answers were correct in both cases, the fact that the child repeated the line "The teacher told me to….", indicated that she was merely following Teacher E's instructions and may not have understood the logic behind the procedures used. In short, although Teacher E wanted to change her teaching style to bring it in line with that required to foster the use of mental strategies in lower primary mathematics, until the end of the study her classroom practice continued to suggest something else.

4.2.5.5 Summary of Initial and Final Characteristics of Teacher E

Table 7 summarises the initial and final characteristics of Teacher E during the study, as measured against the four themes named in the Table.

Table 7: *Summary of Initial and Final Characteristics of Teacher E*

THEME	INITIAL BEHAVIOUR	FINAL BEHAVIOUR
Concerns	None. Happy to participate, sine it would be good for the pupils. Concerned only to see that pupils benefited from participation.	Stated that she felt the same as at the beginning of the study.
Beliefs	Pupils had no mathematical understandings of their own apart from what they learned from teachers. Believed that if pupils followed her explanations they would learn mathematics. Therefore, ability to reproduce taught procedures indicated that pupils knew mathematics.	Previous teaching approach was partly responsible for pupils' poor performance in mathematics. Surprised at pupils' ability to use their own mental strategies and believed that learning mental strategies helped pupils' progress in mathematics. Therefore, she supported extension of the study nation-wide. Still believed that teachers' role is to impart knowledge to pupils, and that 'the more we teach, the better they will learn".
Mathematics content knowledge	At time of study she had the PTC and was studying for the PTD. Thought that the mathematics content of both qualifications was insufficient for learning to teach strategies for mental calculation. Needed to learn more mathematics content to implement study more competently. Needed constant guidance from researcher to implement study; also assisted by her colleague.	Stated that she picked up new mathematical ideas during the study. . Claimed that she herself did not experience difficulty understanding the topics she had to teach, but that the pupils did. Continued to need tutorial help throughout the study to understand what she had to teach
Teaching Practice	Teaching practice characterised by telling and requiring pupils to follow worked examples exactly. Did not allow pupils to think for themselves, i.e. to use their own mathematical knowledge.	Encouraged use of mental strategies in the classroom, but used traditional teaching methods up to end of study. Encouraged classroom discussion, so that pupils could exchange mathematical ideas.

4.1.3 Cross-Case Analysis

Section 4.2 provided a portrait of each of the five participating teachers from experimental schools. It also examined the extent to which each teacher learned to foster the use of mental strategies for double-digit addition and subtraction at Grade 2 level, measured against four themes: *concerns*, *beliefs about mathematics teaching and learning*, *mathematics content knowledge*, and *teaching practice*. The present section does a cross-case analysis of the same set of themes.

4.1.3.1 Teacher Concerns

As indicated earlier, asking teachers to implement an innovation at classroom level, particularly when the innovation requires a fundamental re-alignment in the way teachers perform their teaching duties, can generate in them certain worries and concerns (Maruyama and Deno, 1992). The present section compares and contrasts these concerns, as each of the five teachers experienced them, at the beginning and at the end of the study.

There was no common pattern with regard to the way the five teachers in experimental schools reacted when I first explained to them the nature of the study and invited them to participate. This is illustrated by following statements made by the teachers during interviews:

> Initially I thought it was too late to start a workshop….I thought it was too late for the Grade 2s to do this work, because this work had been done in the third term of Grade 1. So I thought nothing [useful] was going to happen. I thought we were not going to achieve what you wanted (Teacher A).
>
> At first I thought the study was a waste of time, knowing that there is already mathematics at Grade 2 level. Mathematics is there (Teacher B).
>
> At first I thought you had no permission from the Ministry of Education or the head of the school. Then you showed me the letter [of permission]; and then there was nothing I could do [but to participate], since the Ministry had allowed you to go around the schools. So, since I am part of the Grade 2 teachers I suppose that I [had] to attend the workshop (Teacher C).
>
> I was a bit hesitant because when you hear about a research from a higher institution [such as the University of Zambia] then you look at your own [mathematical] background; sometimes you are a bit hesitant. I was not very comfortable; that was my first reaction....I have always viewed mathematics to be a

> difficult subject. Even when I stand in front of the children trying to teach, sometimes I sit back and say: "Maybe somebody [else] could have done this better". Because I am a teacher, when I teach I expect the learners to get whatever I have taught correct. But you find only two pupils in the whole class [getting it right]. So, that gives me homework at the end of the day; and I say to myself: "I think I am not capable of teaching mathematics. (Teacher D).

> [There] was nothing that I felt…but I thought I should get involved. (Teacher E).

Thus Teacher A thought that conducting a workshop in the third term (rather than in the first term) was a mistake particularly that her pupils had already done preliminary work on double-digit addition and subtraction the previous year. According to Teacher B, the whole idea of doing a study on double-digit addition and subtraction would be a waste of time, since this topic was already part of the Grade 2 mathematics curriculum and repeating it would simply not help. Teacher C on the other hand would not participate in the study unless the researcher had permission from relevant educational administrators to conduct research in her school. When she learned that the researcher had obtained the necessary permission, she concluded she had no choice but to participate. Teacher D was concerned about exposing her inadequate knowledge of mathematics, particularly to someone from a higher institution of learning, such as a university. Only Teacher E, who was nearing retirement, did not seem to be troubled by anything when invited to participate.

Despite apparently different reactions when first invited to participate in the study, four of the teachers were more or less unanimous with regard to what they thought might be the effect of the study on their pupils' learning in mathematics. They were worried that: (a) Grade 2 pupils would not understand the mathematical ideas associated with learning to use strategies for mental calculation mainly because they were young and the ideas too advanced; and that (b) the study would slow down the pupils' mathematical progress, thereby jeopardising their chances of success in future examinations, particularly the Grade 7 terminal examination which determined who proceeded to secondary education.

> [Grade 2 children] partitioning numbers? There I am pretty sure I am worried. Even with word problems. Will they understand the language and be able to find the answers? [I am worried] because, if a child fails to understand then…[he or she is] not going to achieve anything in life. (Teacher A).

> I am worried that the children will not pass the examinations. (Teacher B)
>
> [I thought] some of the topics we were to teach were not fit for Grade 2's. I checked the experimental curriculum. I saw the topics and concluded that…the children would not be able to cope, since they are so young. (Teacher C).
>
> The pupils were having problems already with what we have in the course books. And I thought that this study was even going to make them escape [i.e. run away] from the mathematics classroom altogether. (Teacher D).

Teacher E however saw things differently. She decided to participate because she believed that the mathematical experiences she would undergo during the Staff Development Workshop could provide "a foundation for children...to know a variety of mental strategies", which would be good for them.

By the end of the study Teacher A, Teacher B and Teacher D had overcome their initial concerns. When I asked them during individual interviews at the end of the study to reflect on their fears and concerns at the beginning of the study and to tell me to what extent they still felt the same way, two of them said:

> Now the children are trying [i.e. are making some progress]….It is like we now have as usual fast learners who are able to do the work properly…[and] slow learners, especially when it comes to using the empty number line. (Teacher A).
>
> Some topics students can do but some…we are just too fast for them [the students]. We are covering the topics too fast. (Teacher C).

Whereas Teacher A and Teacher C were rather guarded in expressing what happened to their initial fears and concerns as a result of participating in the study, Teacher D was more open and enthusiastic about her experiences.

> I think I am very happy about the outcome….I think both the pupils and I had so much to learn [during] the first week or so. But then after that we became very comfortable. I think both of us have enjoyed ourselves….Learners are very free, they can explain anything to anybody at anytime…. And I am [now] enthusiastic about teaching mathematics. I think I have even shelved most of the [regular mathematics] lessons, because I want to see that learners understand what they are learning about, and I am very happy. (Teacher D).

Teacher B on the other hand had replaced one set of concerns with another. As pointed out in Section 6.2, at the beginning of the study she was convinced that participating would be a waste of her time and that of her pupils. Although by the end of the study she accepted that some of her pupils had benefited from the study, she insisted that these comprised a minority few and that most of her pupils actually lost some ground in their mathematical progress, adding:

> I am still worried….My worries are that, [more able] pupils have learnt a lot but the slow learners have worsened.

Teacher B felt this way because, whereas before the study she could give pupils homework and expect parents (or brothers and sisters at home) to help them do the work, she could not do that with mental calculations, as no one at home understood the ideas involved. She also felt that the pace at which the experimental curriculum was being covered was too fast, and the amount of work she was expected to do too great, so that she could not provide remedial assistance to those who needed it.

4.1.3.2 Beliefs about Mathematics Teaching and Learning

In this subsection I consider changes in beliefs about mathematics teaching and learning experienced by the five teachers from experimental schools. Again, I consider this from two viewpoints: teacher beliefs at the beginning of the study, and what appears to have changed in this regard by the end of the study.

Evidence mainly from interviews and lesson observations suggested that before the study all five teachers held two main beliefs with regard to children's learning of mathematics. That is, young children entered school without any mathematical understandings of their own and, therefore, received all their mathematical knowledge from teachers; and, in the classroom children learned mathematics and succeeded in the subject by doing *exactly* what teachers demonstrated in worked examples. Put differently, if children attended lessons and paid attention, the amount and quality of mathematics they learned depended on the ability of their teachers to explain the procedures involved, and on the children reproducing faithfully teachers' ideas in exercises, tests and examinations. The following extracts from individual interviews with four of the teachers illustrate this point:

> Before the study, if a child used a procedure other than the one I had taught in the classroom I would say, "You have found the answer. But is this the way I explained it to you?" We wanted the children to do it in the same way we had explained. (Teacher A).

> Children learn from the teachers. As long as I teach, I believe that they have learned. (Teacher B).

> I always believed that children came into school empty especially in terms of mathematics. And I was the one to tell them what to do. (Teacher D).

> We never allowed the children by themselves to explore mathematics. So, whenever they came into the classroom they thought that whatever the teacher said was the only correct thing. (Teacher E).

How did participation in the study impact on such beliefs? There were indications right from the beginning of the study that the teachers who implemented the study were willing to change their views about mathematics teaching and learning if classroom evidence challenged their previous beliefs. For example, two of the teachers gave the following statements when asked how the staff development workshop had impacted on their existing beliefs about mathematics teaching and learning:

> It is difficult to tell [if what I believe is wrong or right], because…we still have to implement the experimental curriculum. We shall talk more after we have seen how it works in the classroom (Teacher A).

> [If] this thing [i.e. research] works and works out well, maybe it will be adopted (Teacher C).

Evidence from interviews, classroom observations, and entries in teachers' research journals showed that, with the possible exception of Teacher B, all the other four teachers had changed their previous beliefs about mathematics teaching and learning by the end of the study. They became more accommodating of children's different ways of thinking in mathematics, as the following statements indicate:

> If you explain something clearly [pupils] will understand it but they won't do it the way you expected. They will do it in different ways and …when they do it differently it will show that they understand, because they have a variety [of ways of solving mathematics problems]. (Teacher A).

> This time I have become more interested in mathematics because I have seen that children are able to [solve mathematics problems] on their own, and it is because [they] are given time to do it using their own methods, not just depending on our methods. (Teacher C).

> I was trained to make sure that the children follow …[exactly] the way I have taught them….I have realised that that is not the best way. There are many ways of arriving at the answers. (Teacher D).

> There are so many things I have noticed as I was teaching [the experimental syllabus]. Some…[pupils] were able to solve mathematics problems on their own; and I would be puzzled, wondering, “How did they get that?” Later on I concluded that these children know something; they have got something, which they can express in their own ways. (Teacher E).

Teacher B's views in this regard were different from those of her colleagues'. Although classroom evidence convinced her that her pupils were capable of learning the very mathematics she previously thought they could not, she insisted that only a few of then could do so. Accordingly, when I asked her during a post-observation interview to compare what she believed about her pupils’ mathematical ability at the beginning of the study and what she had witnessed her pupils doing during the study, her reply was:

> Yes, some pupils can partition numbers but others can’t do it. And those who are able to do it on their own, they are few.

Therefore, by the end of the study 4 of the teachers in experimental schools no longer believed that young children had no mathematical experiences of their own which they could share during lessons.

4.1.3.3 Mathematics Content Knowledge

All the five teachers who implemented the experimental curriculum were qualified primary school teachers, with some having obtained further qualifications beyond the required minimum for teaching in primary school in Zambia. Nevertheless, evidence from lesson observations and interviews showed that at the beginning of the study all five teachers had an insecure knowledge of mathematics, so that they had difficulty understanding the basic mathematical ideas associated with learning to teach mental strategies for double-digit addition and subtraction in lower primary.

For example Teacher A, one of the most well qualified of the five teachers (see Table 1), acknowledged that the mathematics content she studied as part of her teacher education programmes was insufficient for teaching comfortably strategies for mental calculation. In common with the other four teachers, Teacher A felt that she could have found implementing the study easier if the Staff Development Workshop had included learning of other types of mathematics content than only focusing on that associated with the use of strategies for mental calculation relating to double-digit addition and subtraction. She also thought that her lesson planning and classroom teaching would have been less problematic if, in addition to the training manual, the study had provided a detailed textbook, containing teachers' notes, worked examples, and pupils' exercises. Teacher B and Teacher C, from the same school, shared these views.

> When you refer to the textbook, sometimes things are easier. You refer to the textbook and the work is there. You just take the work from the textbook. Then for homework you just write the work on the chalkboard.….[This time there was no textbook and] there was nowhere else slow learners could turn to apart from me. And it was not easy [for me] to create questions for slow learners. They haven't learnt much and I am worried about that. I feel I haven't done anything for the slow learners (Teacher B).

> We had no guidance when [implementing the experimental curriculum]. We had no textbooks to guide us. We were just doing it from our heads…. We were just doing it by ourselves (Teacher C).

Teacher C did not only expect to see a detailed textbook but also wanted all teachers implementing the alternative numeracy development approach to use in the classroom exactly the same examples and exercises:

> [The training manual was also insufficient] because you would find that, for example, I am giving my class the sum 14 + 35 and Teacher B is giving her class 25 + 10. So, it was a bit of a problem. It would have been better if we had the same examples in all Grade 2 classes.

I must admit that I was unprepared for these criticisms, particularly that the Staff Development Workshop conducted at the beginning of the study did not only involve discussing the mathematics associated with learning to teach mental strategies for double-digit addition and subtraction but included discussion of other issues as well. For example, the last part of the workshop involved sitting down with the teachers to draw up a scheme of work to guide their lesson planning; to plan together the initial lessons; and to work together to create some of the questions that the teachers used in the classroom.

Thus my intention was not to give the teachers *all* the information they would need in the classroom to implement the study. I left room for teachers to be creative and flexible during lesson planning, as the study aimed to develop similar levels of creativity and flexibility in pupils. As Warfield et al. (2005) state: “In order to realize the goal of enabling students to become autonomous learners, it is necessary for the teachers of mathematics also to become autonomous learners” (p. 440). By *autonomy* the authors meant ability to make one’s own decisions in thinking about mathematical ideas and solving mathematics problems. Therefore, helping the five teachers become autonomous learners of mathematics was a legitimate goal to pursue.

The other two teachers, namely Teacher D and Teacher E, whom we have so far not mentioned in this section, also indicated their need to learn more mathematics content to be better able to foster the use of mental strategies in early primary mathematics. Teacher D admitted, for example, that at the end of the staff development workshop she felt more knowledgeable in mathematics than ever before. Teacher E on the other hand was reluctant throughout the study to say anything that might reflect poorly on her knowledge of mathematics. She maintained that any problems she experienced in the classroom during the study were due to pupils' intellectual limitations. Nevertheless, she also admitted during the post-intervention interview that:

> I [need] to learn more mathematics itself. I think that is when I can be more knowledgeable about mathematics.

The difficulties the five teachers experienced when implementing the experimental curriculum were most acute when they tried to use the N10 method to perform double-digit addition and subtraction. (Use of the N10 methods is explained fully in Chapter 4). It is important to point out that this was probably not strange as teachers in the Netherlands (see Chapter 4), where the N10 method originated, also experienced problems teaching the procedure. In short, at the beginning of the study the mathematics content knowledge of the five Grade 2 teachers who implemented the experimental curriculum was insufficient for understanding the mathematics associated with learning to teach mental calculation strategies.

Was any change apparent in the teachers' mathematics content knowledge by the end of the study? It was difficult to determine with finality the extent to which teachers increased in knowledge in this regard. Right up to the end of the study I was still getting questions from some of the teachers in experimental schools, suggesting that they still needed to understand mathematics better to implement the study meaningfully.

However, it would seem that the teachers' *pedagogic content knowledge* (see Chapter 2) increased. For example, when asked to identify the new teaching skills they had learned during the study four of the teachers responded as follows:

> Although the study is coming to an end, at least I have an idea on how to start a lesson in mathematics. Whatever the topic, whichever books we are using, even these old [officially approved] books we are using, I will try to use the knowledge I have learned. Even if I just wanted children to find 5 + 6, I will not just say 5 + 6 and then say: "What is the answer? Get stones; Count them; What?" No, I will just ask a question; then I will expect the pupils themselves to find the answer; then they will tell me how they did it. In the past…I would just say, "5 + 6. Can you get five stones, and then get six [more] stones. Now count them all". That is what I was doing. I think I won't do it again. (Teacher A, Post-treatment interview).
>
> Since we began this research …I have learned so much, in that previously we used to depend much on textbooks from the Ministry of Education, prepared by the people there. But now, after this workshop, I think we will be able to prepare our own work, what we think is suitable for the children. (Teacher C, Group interview).
>
> I have understood that young children enter school with a certain amount of mathematical knowledge learned from home, which I was not able to tap but which this time I am able to tap from them. And, unlike before, I am able to observe what they do. So, both the pupils and I are really watching each other and we are learning from each other. I think the children can do a lot of mathematics if I don't hinder them. (Teacher D, Post-intervention interview)
>
> I have discovered from this workshop that, the approach that I used in the classroom most of the time somehow… made the children not to behave creatively enough [during mathematics lessons]….Maybe I didn't give pupils that freedom of expression, I didn't give enough time for them to think. (Teacher E, Group interview).

The above statements were confirmed during lesson observations although, as mentioned in Section 6.2, in the case of Teacher E words did not always translate into actions.

4.1.3.4 Teaching Practices

To learn to foster the use of mental strategies in early primary mathematics, participating teachers in experimental schools required among other things to view themselves not as providers of knowledge, but as facilitators of learning. This section discusses the extent to which the five teachers moved in this direction.

Data from lesson observations, interviews, and teachers' journal entries indicated that at the beginning of the study all five participating Grade 2 teachers from experimental schools practised the *traditional* way of teaching mathematics (Clements and Battista, 1990; Valero, 1999). Traditional instruction is:

> …a particular kind of interaction among teacher, students and mathematical knowledge, where mathematics is mainly procedures, the teaching is…information transmission controlled by the teacher, and learning is an acquisition of information and a mechanical training" (Valero, 1999, p. 28).

In line with this view the teachers' classroom practices were initially characterised by *telling*, that is, attempting to transmit knowledge from their own minds to those of pupils. Not surprisingly, the teachers insisted that children should prove that they had understood what they were taught by reproducing in classroom exercises and tests/examinations what was demonstrated by teachers in worked examples. The statements below, which the teachers gave when describing the approaches they used in teaching mathematics before the study, illustrate this:

> We never considered children's experience; it was always teachers' experience. You want to impart what you know to the pupils, not first trying to find out what the pupils already know. (Teacher A, Post-intervention interview).

> We depended mainly on procedures from the textbooks; and in examinations children used only the methods that the teacher has given. (Teacher B, Group interview).

> I would work out examples on the chalkboard with the help of pupils; and then give them one or two [similar] sums to do. Then I would say, "Start writing". (Teacher C, Post-observation interview).

> I was trained to make sure that the children follow…and do exactly the same way I have taught them…. I don't think they understood what I taught them because they were not even able to explain anything. Even the ones that got the answers correct…were not able to explain to their friends what they had done. They would just say, "Well, I got it right!" That's all. (Teacher D, Post-intervention interview).

> We did not used to deal much with children's mathematical experiences. What we used to do was to take work directly from the textbooks and give to children without maybe discussing their experiences with them. [Teacher E, Post-observation interview).

Data from lesson observations, interviews and teachers' research diaries indicated that by the end of the study four of the five teachers (excluding Teacher E) had changed their teaching styles, so that they were more likely to support the use strategies for mental calculation in early primary mathematics. Their mathematics lessons were characterised by increased classroom interaction, as evidenced by the readiness and ability of their pupils to explain their answers when requested to do so, and the teachers made genuine attempts to encourage classroom discussion. For example, in describing what colleagues from schools that did not participate in the study might notice was different if they walked into a classroom taught by one of the five during mathematics lessons four of the five teachers gave the statements below. As the reader will notice, a theme running through all the responses is the freedom with which pupils would express themselves during mathematics lessons, something that was absent from the teachers' mathematics classrooms at the beginning of the study.

> What they might notice has changed is the discussion part, where you ask pupils questions and they come to the front of the class to explain; or where maybe after they have written an answer you say, "How did you do this?" A child is able to explain how she got the answer, whereas normally we would just mark if the answer is right. But this time we are able to ask, "How did you do this?" And the child will say, "I was doing this, this, this" (Teacher A).

> They will see that the children are free. And they would see me going round discussing with the pupils, encouraging the pupils….In the past I just used to give pupils sums on the chalkboard to do. Let's say, maybe it is 38 + 10. I would just stand there; then I would just ask the pupils while they are there at their desks. Then a pupil would stand up and answer the question….But this time, I call upon them to come to the front and explain to the others....which makes the children eager to do maths, because they think they are being taught to be like a teacher. (Teacher C).

> I think the interaction that goes on among the children....[Pupils] are asking each other questions, the teacher is also probing, and so on; and the learners working from the chalkboard from time to time. It is not always the teacher. This time it is more pupil-centred, and the teacher is just facilitating....I have invited in your absence sometimes teachers from the upper grades and the other Grade 2 teacher who is not participating in the study. She sits to see what is happening and also asks the pupils: "How did you find this? I would like to know". Then they would explain to her and, from there, she would move to her own class and present the same lesson. In a way, this is what they would see. (Teacher D).

> I think what they will be able to notice, as they come into the classroom, is the free participation of the children….[Also], the way the work was going to be introduced would be very different. For example, when you look at that work which is on the chalkboard there, we are used to vertical addition or subtraction. So, when they see that, some of them might be wondering why we have done it like that. But that is the easiest. Children will be able to get that work correct. So, I think when they come in and see what I am doing, I will not be ashamed. I will even be able to show them or to explain to them why I have done this and that. (Teacher E).

Even Teacher B, who maintained all along that the study did not benefit her pupils, agreed that some positive change had occurred in her classroom practice as a result of participating in the study:

> Yes, [they would notice some change]. My children would be able to partition numbers and find the answers quickly while theirs would not do that….I would explain my work to children very clearly and allow children to participate in the lesson.

However, as Teacher B's statement above seems to imply, she still believed that *telling,* especially when done well, was the best way to ensure that pupils learned what the teacher wanted them to learn. This reflected the thinking, contrary to the constructivist view of learning being promoted in the study, that it was possible to transfer knowledge in the exact form from the teacher's mind to that of the pupil. In fact, Teacher B was only waiting for the study to come to an end so that she could go back to her usual way of doing things in the classroom. For example, she told me at the end of the study that:

> The next few days after the study we will go back to our usual textbook, because I feel as if the pupils have not learned anything. They will go half-baked to Grade 3, because what we have done here in Grade 2 [during the study] will be different from what they will find in the first term of Grade 3….We haven't done much this term; we haven't done anything on 'borrowing'. So, when they go to Grade 3 that is the work they will find. Now, since they don't know how to apply 'carrying' and 'borrowing'…[they will experience problems in Grade 3]. We should have started [discussing 'carrying' and 'borrowing'] here in Grade 2.

Teacher B's point above was valid particularly that at the back of each participating teachers' mind was the question of how the mathematical ideas being implemented during the study would affect children's chances in the examination at the end of Grade 7, which opened the door to secondary education. However, as I stated in Chapter 1, my intention in introducing primary school teachers to the use of mental strategies during mathematics lessons was not to replace or supplant the use of vertical addition and subtraction. It was to broaden the choices available to both teachers and pupils with regard to method of solution when faced with a given mathematics problem.

The statements above given by the other four teachers (A, C, D, and E) suggested that they were encouraging classroom discussion as much as possible during lessons. However, although there was an increase in the level of interaction during lessons, it focused on children explaining *how* they worked out an answer rather than *why* a given procedure was appropriate for use in a given situation. Lesson observations confirmed this and the teachers admitted that somehow this aspect of classroom interaction could not be made to work.

4.2 Findings Concerning Pupil Learning and Performance

The main aim of this study was to assess the ability of Grade 2 teachers in experimental schools to foster the use of mental calculation strategies for two-digit addition and subtraction in early primary mathematics. To achieve this, it was also necessary to determine the impact of the teachers' implementation of the experimental curriculum on pupil performance in mathematics. The aim of this chapter is to present results of the study in this regard.

4.2.1 Pre-treatment Numeracy Achievement of Participating Grade 2 Pupils

A total of 311 Grade 2 pupils wrote both the pre-test and the post-test. Of the 311 pupils 167 were from experimental schools and 144 from control schools. All participating Grade 2 pupils were from classes with roughly equal numbers of boys and girls. Table 8 below provides a synopsis of pupil performance on the pre-test, with the percentage of correct answers being indicated by (%).

Table 8: *Percentages of Correct Answers Obtained in Experimental and Control Schools per Test Item on the Pre-test*

Question No.	Experimental Schools (n = 167)		Control Schools (n = 144)		Total (n = 311)	
	f	%	f	%	f	%
Q.1	166	99	140	97	306	98
Q.2 (a)	146	87	130	90	276	89
	133	80	118	82	251	81
Q.3 (a)	151	90	131	91	282	91
(b)	127	76	114	79	241	78
(c)	138	83	115	80	253	81
(d)	119	71	125	87	244	79
Q.4 (a_1)	147	88	115	80	262	84
(a_2)	143	86	107	74	250	80
Q.5 (a_1)	143	86	116	81	259	88
(a_2)	128	77	104	72	232	75
(b_1)	130	78	87	60	217	70
(b_2)	83	50	51	35	134	43
Q.6 (a_1)	139	83	112	78	251	81
(a_2)	106	64	85	59	191	61
(b_1)	136	81	106	74	242	78
(b_2)	97	58	83	58	180	58
Q.7	140	84	99	69	239	77
Q.8	92	55	62	43	154	50
Q.9	146	87	135	94	281	90
Q.10 (a_1)	65	39	34	24	99	32
(a_2)	26	16	11	8	37	12
Q.11 (a_1)	58	35	24	17	82	26
(a_2)	31	19	13	9	44	14
Q.12 (a_1)	116	70	77	54	193	62
(a_2)	12	07	10	07	22	07
Q.13 (a)	119	71	86	60	205	66
(b).	73	44	70	49	143	46
Q.14 (a)	121	73	118	82	239	77
(b)	100	60	79	55	179	58
Q.15 (a)	124	72	84	58	208	67
(b)	38	23	25	17	63	20
(c)	87	52	81	56	168	54
(d)	02	01	07	05	09	03
Q.16 (a)	24	14	20	14	44	14
(b)	80	48	60	41	140	45
Q.17	83	50	71	49	154	50
Q.18 (a)	115	69	97	67	212	68
(b)	134	80	110	76	244	79

*Percentages rounded off to the nearest whole number.

To understand the contents of Table 8, it is necessary to know something about what the test items assessed. In view of this, before I point out what the table shows concerning children's performance on particular aspects of numeracy, I will describe briefly the focus of each test item or groups of test items examining similar mathematical ideas.

4.2.1.1 Writing down the Number of Objects in a Small Set.

Questions 1 and 2 tested understanding of similar mathematical ideas. Question 1 assessed pupils' ability to recognise, count and write down the number of items in a small set containing 5 items. Question 2 had two parts. Part (a) presented pupils with a drawing showing a number of flowers in a vase. Pupils were required to draw more flowers in the vase, so that the total number of flowers reached a pre-specified number. In part (b), pupils were given a number and asked to draw as many flowers.

The task in Question 1 proved very easy, as practically all pupils (98%) performed it successfully. However, adding more objects of the same kind to a given set to reach a certain total, as required in Question 2 (a), was not as easy, as the percentage of pupils who performed the task successfully dropped to 89%. The percentage of successful attempts dropped even further to 81% when pupils were given a number and asked to draw an equal number of objects. Thus whereas practically all the pupils (98%) could count and write down the correct number of objects in a small set, fewer pupils (89%) could add to an existing set a given number of similar objects to reach a pre-specified total. Even fewer pupils (81%) were successful when the task involved drawing a set of objects to equal a given number.

4.2.1.2 Writing Down a Single-digit Number that is a Specified Number More Than or Less Than Another Single-digit Number

All four parts of Questions 3 assessed pupils' ability to write down a single-digit number that is a given number *more than* or *less than* another single-digit number. Table 8 shows that when the number added or subtracted in the *more than* or *less than* situation was small (i.e. 3), higher percentages of success were recorded: 91% for *more than* and 81% for *less than*. On the other hand, when the single-digit number to be added or subtracted was relatively large (e.g. 6 or more) the success percentage was lower in each case (78% and 79%). Thus the success rate in these situations was not based so much on whether the question required *addition or subtraction*, but on the *size* of the number added or subtracted and on the final result obtained.

4.2.1.3 Determining Missing Single or Double-digit Numbers on a Number Line

Questions 4 and 10 presented pupils with an incomplete number line and required them to identify the missing and to write them in appropriate positions on the number line, two missing numbers. In Question 4 the range of numbers on the number line was 0 to 10. In Question 10 the range was 0 to 50 and the spaces between consecutive numbers were narrower.

With regard to Question 4, which involved the numbers 0 to 10, 84% of the pupils succeeded in identifying the first (i.e. the lower) missing integer and 80% the second. In Question 10, where the range 0 to 50, the percentage of successful attempts at the answer was 32% for identifying and writing down the first number, and only 12% for the second number. Thus, whereas more than 80% of the pupils were successful when only the numbers 0 to 10 were involved and the spaces between consecutive numbers were wider, the task proved very difficult when the range was wider (0 to 50) and the spaces between consecutive numbers were narrower.

4.2.1.4 Translating Single or Multi-digit Numbers from the Verbal to the Symbolic Form and Writing Down Other Numbers One More or One Less Than the Given Single or Multi-digit Numbers

Questions 5, 6, 11 and 12 involved translating single or multi-digit numbers from the verbal to the symbolic form and then identifying and writing down other numbers that were either *one more than* or *one less than* the given numbers. Table 8 shows that, with regard to writing down the correct numerical form for a single or double-digit numbers (Questions 5 and 6) 70% or more of the pupils arrived at the correct answer, with the highest success rate (88%) being attained when the number involved was a single-digit one [Question 5 (a_1)]. When pupils were asked to write down a number that is *one more* or *one less* than the number they had already identified and written down, the percentage of pupils who got correct answers dropped to as low as 43% [Question 5 (b_2)].

Questions 11 and 12 required writing down the correct numerical symbol for a three-digit integer given verbally. In this case, 62% of the pupils successfully wrote down the correct symbol for *Two hundred* but only 26.4% for *One hundred nine***.** In the latter case, the most frequently cited incorrect form of the answer was 1009.

Having written down the numbers mentioned above pupils, the pupils were further required to identify and write down other numbers that were *one more than* or *one less than* the given numbers. Table 8 shows that this was a virtually impossible task for the Grade 2 children in the sample: 14% succeeded in identifying and writing down the number that is *one more than* one hundred nine; while only 7% could do so in the case of two hundred. Thus whether or not the Grade 2 pupils succeeded in identifying a number that is *one more than* or *one less than* a given number depended on the size of the given number and, it would seem, on whether or not its final digit was zero or nine.

4.2.1.5 Using Grouping to Perform Multiplication and Division by 5 or by 10

In questions 7, 8 and 13 pupils were shown pictures of a box containing 5 cakes and a bag containing 10 apples, and were then told specifically that each box contained 5 cakes and each bag 5 apples. The pupils were then shown briefly (i.e. without being allowed sufficient time to count objects physically), a number of similar boxes and bags, and were required to determine the total number of cakes in the boxes and the total number of apples in the bags. Follow-up questions required the pupils to determine how many boxes of cakes or bags of apples would be required to pack a particular number of cakes or apples.

Table 8 shows that, in the case of determining how many cakes or apples were in a given number of boxes or bags, the success percentage was high: 77% in the case of cakes and 66% for the apples. However, working out how many boxes or bags would be required to pack a given number of cakes or apples, percentages of pupils who succeeded sin arriving at the correct answers came down. That is, 50% for the question involving cakes (Question 8) and 46% for apples (Question 13). Therefore, in these cases Grade 2 pupils dealt more successfully with situations involving smaller numbers (i.e. 5) than with those involving larger numbers (i.e. 10).

4.2.1.6 Solving Addition and Subtraction Word Problems Involving Single or Double-digit Numbers

Questions 9 and 18 assessed pupils' ability to solve simple addition or subtraction word problems, involving single-digit or double-digit numbers. Table 8 shows that in the case of addition more pupils (i.e. 90%) got the correct answer when the result of adding single digits was another single digit (Question 9). When single digits were added to give a double-digit number [Question 18 (a)], the number of pupils who got the correct answer dropped to 68%. In Question 18 (b), which involved subtracting a single-digit number from a double-digit number to give a single-digit answer, the percentage of pupils succeeding in getting the correct answer was 79%.

Thus, more Grade 2 pupils in the sample solved successfully word problems involving addition of small (single-digit) numbers than could solve word problems involving one or more double-digit numbers. Similar results were obtained with regard to subtraction. The number of children who successfully solved the second type of question was smaller than the number of pupils who successfully worked out the correct answer in the case involving subtraction of a single digit- number from a double-digit number.

4.2.1.7 Performing Multiplication by 4

Question 14 measured ability to perform multiplication by 4. The question had two parts. The first was: "Four times two equals…." In this case 77% of the pupils got the correct answer. The second part was "Four times five equals….", where the percentage of pupils who worked out the answer correctly was 58%. Thus more children got the correct answer when four was multiplied by a small number (i.e. 2), than when it was multiplied by a larger number (i.e. 5).

4.2.1.8 Adding or Subtracting 10 or 100

Questions 15 and 16 assessed the ability to add ten or one hundred to a single, double or three-digit number; or to subtract ten or one hundred from a single, double or three-digit number. Table 8 shows that more than 60% of the pupils could add ten to a single number and get the correct answer [Question 15 (a)], while only 20% successfully added ten to a double-digit number [Question 15 (b)]. Furthermore, 'taking away' ten from a double-digit multiple of ten [Question 15 (c)] was accomplished by a little over 50% of the pupils. However, almost all the pupils failed to ‘take away ten from seven hundred’, a task that was accomplished by only 3% of the pupils.

With regard to adding one hundred to a single-digit number [Question 16 (a)] the percentage of correct answers was only about 14%. However, more than three times the number of pupils (i.e. 45%) were able to 'take away' one hundred from 'four hundred' [Question 16 (b)]. This relatively higher success rate was probably influenced by the fact that, without prior agreement with me class teachers used a popular street language translation for one hundred (i.e. one Zali), which simplified the question to: "Take away one *Zali* from four *Zali*".

4.2.1.9 Grouping Objects to Represent Given Proportions

Question 17 measured children's understanding of the concept of 'half,' when applied to grouping of an even number of objects. Pupils were shown a diagram, comprising eight balls. The task was to encircle half the number of balls. About 50% of the pupils did this successfully, which indicates that the task was neither easy nor difficult for them.

4.2.2 Learning to Use Strategic Methods for Double-digit Addition and Subtraction

To attain proficiency in using mental strategies for double-digit addition and subtraction pupils need to develop the following pre-requisite mathematical skills (Mosley et al, 2001; Skinner et al, 2001):

- Counting forward and counting back in 1s, 2s, 3s, 4s, 5s and 10s from a given number.
- Partitioning the single-digit numbers 6, 7, 8 and 9 into 5 and 1s (e.g. 8=5 + 3).
- Partitioning double-digit numbers into a near multiple of 10, and then in terms of 5 and 1s (e.g. writing 59 as 50 + 5 + 4).
- Adding or /subtracting double-digit numbers by first partitioning either one or both numbers and then adding the parts separately to find the final result.
- Using an empty number line (ENL) as a device for adding/subtracting numbers.

In view of this, mathematics lessons in experimental schools focused on helping pupils to develop the above skills. Accordingly, my lesson observations were also intended to determine to what extent participating pupils acquired the same skills; although I also looked out for other ways in which teachers taught mental calculations, for example word problems and encouraging classroom discussion. Related data from interviews with teachers and contents of teachers' research journals/diaries were also used.

I outline below results of the study regarding the extent to which participating Grade 2 pupils attained the skills listed above.

4.2.2.1 Counting Forward and Counting Back in 1s, 2s, 3s, 4s, 5s and 10s

Lesson observations and examination of pupils' notebooks indicated that more than half the number of participating Grade 2 pupils could count forward in 1s up to 100 fairly easily. They could also count forward in 1s from a given number (e.g. 50) up to 100. More than 80% of the pupils were also able to count forward in 5s or in 10s from a given number up to 100, particularly when only multiples of these numbers were used.

Compared to counting forward in 5s, counting forward in 4s caused the pupils some difficulty. It is perhaps important to mention that counting forward in 5s was part of the official Grade 2 mathematics curriculum at the time of the study, whereas counting forward in 4s was not. Furthermore, while the majority of participating Grade 2 pupils had no difficulty counting forward to 100 in 1s, 2s, 5s and 10s, they were unable to identify missing numbers in sequences involving 1s, 2s, 5s, and 10s, for example the sequence 5, 10, ---, 20. Both Teacher B and Teacher C seem to have noticed this, as the following the following statements written in the 'self-evaluation' section of their lesson plans suggest:

> A number of children found it hard to write the correct missing number. They know how to count in 1s, i.e. 1, 2, 3, …20 but failed to write the correct missing number. The weaker groups failed even to copy correctly from the chalkboard. The work will be taught again tomorrow (Teacher B).
>
> The lesson [on completing sequences] was taught but most of the pupils could not do the sums right (Teacher C).

After her lesson the following day, a more satisfied Teacher B was able to write:

> The lesson [on completing sequences] was taught over 2 days because pupils didn't do well at first. So after doing the same task for the second time at least a few pupils did well.

Compared with counting forward, counting back was generally difficult for the pupils, which frustrated some of the teachers. For example, referring to less able pupils' unsatisfactory performance on counting back in 5s when compared to that of the more able groups Teacher B wrote in her research diary:

> Everything is being done by the active [i.e. more able] pupils. Weaker learners [i.e. less able pupils] seem not to be part of the class. I can't give them homework-who will guide them at home? They fail to understand the idea of counting in 5s. Now, what am I going to give them? They will get weaker. If only this research was [delayed until] Grade 4.

Just as counting forward in 4s was harder than counting forward in 5s, counting back in 4s was also more difficult for many Grade 2 pupils than counting back in 5s. Also, I noticed while observing Teacher B's classes that her pupils found it much easier to guess the next number in a sequence such as 40, 36, 32, 28 than to identify missing numbers in the middle of a similar sequence such as 40, 36,----, 28. Thus the level of difficulty pupils encountered in counting back seems to have been influenced by a number of factors, such as the nature of the numbers involved and whether or not the sequence required to identify the next number or to find missing numbers in between.

An example from Teacher D's class illustrated the ease with which pupils accomplished the process of counting back. In one of her lessons Teacher D wrote the numbers 42, 32, 22, 12 on the chalkboard and asked her class: "How do you get these?" Almost immediately one of the pupils, a girl, answered: "*Ni ku chosa mo ma 10, 10, 10, 10, …*". That is, "It is a matter of taking away 10, 10, 10, 10,…". After emphasising this and telling her class that if they "took away 10s" they would get the correct result, Teacher D asked the class to count back in 10s from 92. The whole class immediately chanted in unison: "92, 82, 72, 62, 52, 42, 32, 22, 12, 2".

Interestingly, counting back from 49 to 41, for example, was achieved by more than 80% of the Grade 2 pupils in the study, while the same group of pupils utterly failed to count back from hit 40 to 39 (or 30 to 29, 0r 40 to 39). It was like the pupils could not see that when counting back in 1s the next number after 40 was 39. Therefore, in general pupils failed seemed to fail to 'cross over' from a higher decade to the one below it when counting back in 1s.

In summary, more than 80% of Grade 2 pupils in experimental schools could count forward fairly easily in 1s, 2s, 5s and 10s, but were not so confident when counting forward in 4s. Counting back was generally harder for them to accomplish particularly when done in 4s, and they found counting back in 1s from 40 to 39 (or 50 to 49, or 80 to 79, and so on) virtually impossible to do.

4.2.2.2 Splitting/Partitioning Single-digit Numbers in Terms of 5 and 1s

The ability to express a single-digit number as a sum of two other numbers is a key skill on the way to developing proficiency in using strategies for mental calculation for double two-digit addition and subtraction. This is because mental strategies for adding single or double-digit whole numbers involve first partitioning the numbers and then adding or subtracting the parts separately.

However, in teaching children in experimental schools to split/partition single or double-digit numbers, the aim was not only to show that such numbers can be expressed as a sum of two or more numbers (for example 5 = 3 + 2). The teachers and I hoped to get the children to realise that a number such as 9 can be written in terms of '5 and 1s' as 5 + 4. This simplifies working out the value of 6 + 8, for example, which then becomes 5 + 1 + 5 + 3 = 10 + 4 = 14. In this way, pupils can circumvent the need to add 6 and 8 directly, which might require using the often-misunderstood process of regrouping.

In teaching partitioning of single-digit numbers some Grade 2 teachers in experimental schools initially asked children to express a number such as 9 as a sum of any two smaller numbers, resulting in pupils writing 5 + 4, 8 + 1, 6 + 3, and so on. Almost all the pupils (i.e. more than 80%) achieved this. This is confirmed by Teacher C who, after teaching the procedure, wrote as follows in the 'self-evaluation' part of her lesson plan:

> The lesson [on expressing single-digit numbers as sums of two or more other numbers] was taught and most of the pupils [found] the numbers with little difficulty**.**

To move children from simply expressing single-digit numbers in terms of two or more smaller numbers towards using '5 and a bit' Teacher D asked her Grade 2 class to complete expressions such as 9 = 5 + …. Eventually, the majority of the pupils learned to do so, expressing for example 7 as 5 + 2, 8 as 5 + 3, and 9 as 5 + 4. Teacher D wrote in her research journal:

> Partitioning of [single-digit] numbers was easy for learners. They had little difficulty expressing 9 as 5 + 4, 7 as 5 + 2 and 8 as 5 + 3.

Even Teacher B, who maintained throughout the study that learning to use strategic methods for double-digit addition and subtraction did not do her pupils any good, was happy with the results of her teaching in this regard. A corresponding entry in her research journal reads:

> After explaining the partitioning of [single-digit] numbers in various ways we [participating teachers] asked pupils to express [single-digit] numbers as sums of two other numbers, e.g. 5 can be expressed as 5 + 0. Pupils came up with 4 + 1, 3 + 2. On this topic at least most of the pupils had come to their senses. They were able to express 7, 8 and 9 [as sums of two or more smaller numbers] with little difficulty. They were able to express these numbers even in terms of '5 and a bit'. I only hope they won't get lost along the way. Others were writing 9 = 5 + 1 + 1 + 1 + 1 or 5 + 2 + 2. At least I felt like the old days [i.e. before the study] when I could mark all the books with a smiling heart.

According to Teacher B, whenever she gave pupils an exercise in mathematics before the study differences in performance between less able and more able pupils were not big and she could mark pupils' books 'with a smiling heart'. Teacher B believed that this feeling of satisfaction disappeared when she got involved in the study. However, one can see from the teachers' statements above that learning the splitting/partitioning procedure did not give participating Grade 2 pupils difficulty.

4.2.2.3 Splitting/Partitioning Double-digit Numbers.

With regard to splitting/partitioning two-digit numbers the main aim of the lessons given during the study was to help pupils learn that a two-digit number such as 89 can be written as a sum of the highest possible multiple of 10 plus '5 and a bit'. That is 89 = 80 + 5 + 4. The purpose of doing this was again to make calculations easy, as it means that pupils can avoid a pre-mature encounter with the idea of regrouping, and instead use familiar number facts to accomplish the addition.

Lesson observations and examination of pupils' notebooks showed that more than half the number of participating Grade 2 pupils could perform the procedure. Examples of what they did in this regard included 27 = 20 + 5 + 2; 32 = 30 + 2; and 59 = 50 + 5 + 4. Other pupils, however, partitioned double-digit numbers somewhat differently. They identified correctly the highest multiple of 10 in a given number but did not use the form '5 and a bit' for the remaining part(s). Their notebooks showed 78 for example expressed as 70 + 8 or 70 + 4 + 4 or 70 + 4 + 3 + 1. Although what they did would not simply the calculations involved in adding or subtracting double-digit numbers, it showed that they at least understood the process of partitioning double-digit numbers.

.In another classroom in the same school, a child partitioned 95 as 10 + 50 + 10 + 10 + 10 + 5. A third child wrote 34 = 10 + 10 + 10 + 4. I concluded that as far as some pupils were concerned, partitioning a number simply meant writing it as a sum of other numbers. The pupils who had difficulty 'seeing' the highest multiple of 10 in a given double-digit number actually also partitioned the numbers wrongly. For example, one child incorrectly wrote 49 as 10 + 10 + 10 + 10. Seeing this, the other children in the class asked him: "*Iyo 40 waipeza bwanji?*" That is, "How did you get that 40?" In response, the child could only say "*Ya choka muli 49*", that is, "It came from 49", but could not elaborate further.

4.2.2.4 Double -digit Addition/Subtraction Using Splitting/Partitioning Methods.

The main aim of the experimental lessons in this regard was to help pupils develop the skill of adding or subtracting double-digit numbers using the 1010 and N10 methods (see Chapter 2). Both methods rely on pupils' ability to split/partition numbers before adding or subtracting the separate parts.

To perform double-digit addition and subtraction using the 1010 method involves partitioning *both* numbers preferably into a near multiple of ten plus '5 and a bit,' and then using one's knowledge of familiar number facts to add the parts separately. For example: 37 + 26 = 30 + 5 + 2 + 20 + 5 + 1 = 30 + 20 + 5 + 5 + 2 + 1 = 50 + 10 + 3 =63

The N10 method on the other hand involves leaving the first number intact and partitioning only the second number. As mentioned in chapter 2, it is most effective when used in conjunction with the empty number line (ENL). However, although the N10 method is more efficient than the 1010 method, many children find it rather difficult to use it and tend to use the 1010 method almost exclusively even though it is unsuitable in certain situations.

4.2.2.4.1 Addition Using the 1010 Method

The data collected, mainly through lesson observations, indicated that by the end of the study about half the total number of participating Grade 2 pupils could use the 1010 method to perform double-digit addition accurately. For example, in one of Teacher A's lessons, the task was to work out the sum 26 + 34. A boy volunteered to perform the addition. He wrote on the chalkboard:

$$26 + 34 = 20 + 4 + 2 + 30 + 2 + 2,$$

but did not proceed further. He had succeeded in partitioning the numbers more or less acceptably but appeared not to know what to do next. Another boy came forward and wrote:

$$26 + 34 = 20 + 2 + 1 + 2 + 1.$$

He then stopped, looking as if he did not know what to do next. At this point Teacher A stopped him from proceeding and invited a girl to attempt the calculation. She wrote:

$$26 + 34 = 20 + 5 + 1 + 30 + 3 + 1.$$

The significant thing is that all three children knew how to partition the numbers involved and did it in different ways. What they all seemed not be aware of initially was that to find the total they needed to add up the separate parts, bringing together pairs of numbers whose sums were easy to get, and then adding the sums up to get the final answer. Indeed, it would seem that Teacher A did not make it clear at first that this is what the children were expected to do. Finally, a third child (a boy) stepped forward and completed what the girl above had begun, writing:

$$26 + 34 = 20 + 5 + 1 + 30 + 3 + 1 = 50 + 10 = 60.$$

As an exercise Teacher A gave the children the sum 37 + 28. One child (a boy) worked it out as follows:

$$37 + 28 = 30 + 3 + 4 + 20 + 4 + 4 = 65.$$

Another child (a girl), indicating that different children worked in different ways, partitioned the numbers differently but also arrived at the correct answer:

$$37 + 28 = 30 + 20 + 7 + 8 = 65.$$

When Teacher A asked the children to work out the sum 44 + 59, the same girl, almost immediately, said the correct answer was 103. Asked to show how she worked it out, she wrote on the chalkboard:

$$44 + 59 = 40 + 50 + 9 + 1 + 3 = 103.$$

What I saw happening in Teacher A's Grade 2 classes, concerning pupil performance on double-digit addition using the 1010 method, was also true about pupils in the other Grade 2 classes in experimental schools. Examples of pupils' activities in Teacher B's classes illustrate this. During a lesson on double-digit addition, Teacher B wrote 7 + 18 on the chalkboard and first asked the pupils to give the meaning of the arithmetical sign (+) connecting 17 and 18. Immediately the whole class shouted back, "*Kui kila pamodzi*", that is, "Adding together".

Teacher B then invited the children to work out the answer. A girl went to the chalkboard and worked out the answer correctly by partitioning the two numbers into a multiple of ten and '5 and a bit' as follows:

$$17 + 18 = 10 + 5 + 2 + 10 + 5 + 3 = 20 + 10 + 5 = 35$$

At this point Teacher B turned the exercise into a 'boys versus girls' competition. She wrote two other addition sums (56 + 37 and 19 + 26) on the chalkboard and announced that the boys would attempt the first and the girls the second. Before the pupils could begin working out the answers, a boy, surprising me with his boldness, called out to Teacher B: "Teacher, come here. I want to ask a question". When Teacher B came over to the boy's desk, he asked: "Are we going to be given rewards for competing with the girls?" Teacher B ignored the question and asked the class to begin calculating the answers, which the children did. Although initially a group of boys at a desk near where I was sitting struggled without success to find the answer, one of them did finally succeed in arriving at the correct answer, working as follows:

$$56 + 37 = 50 + 30 + 5 + 5 + 2 + 1 = 80 + 10 + 3 = 90 + 3 = 93$$

Teacher B then went through the procedure again, even though the boy had already done so. As mentioned in Chapter 6, Teacher B believed that if the teacher gave a clear explanation of some piece of mathematics, the children would understand it. It was now the girls' turn to work out the answer to 19 + 36. One girl was able to partition the two numbers in terms of tens, fives and ones, but could not proceed further. That is,

$$19 + 26 = 10 + 5 + 4 + 20 + 5 + 1.$$

Another girl then took over and completed the calculation, writing:

$$30 + 5 + 5 + 4 + 1 = 40 + 5 = 45.$$

Again Teacher B went through the whole procedure as given by the girl, explaining every step. In short, participating Grade 2 pupils in experimental schools understood reasonably well how to use the 1010 method to perform double-digit addition, as demonstrated below by pages from two pupils' notebooks. They were trying to work out the values of 27 + 16 and of 38 + 17 respectively. Notice that the pupil tackling 27 + 16 combined horizontal and vertical addition flexibly, an indication of the choices she had to make while working out the answer. On the other hand, the pupil who was trying to find the value of 38 + 17 did not complete the calculation, although it is clear from what she had already accomplished that she was heading in the right direction.

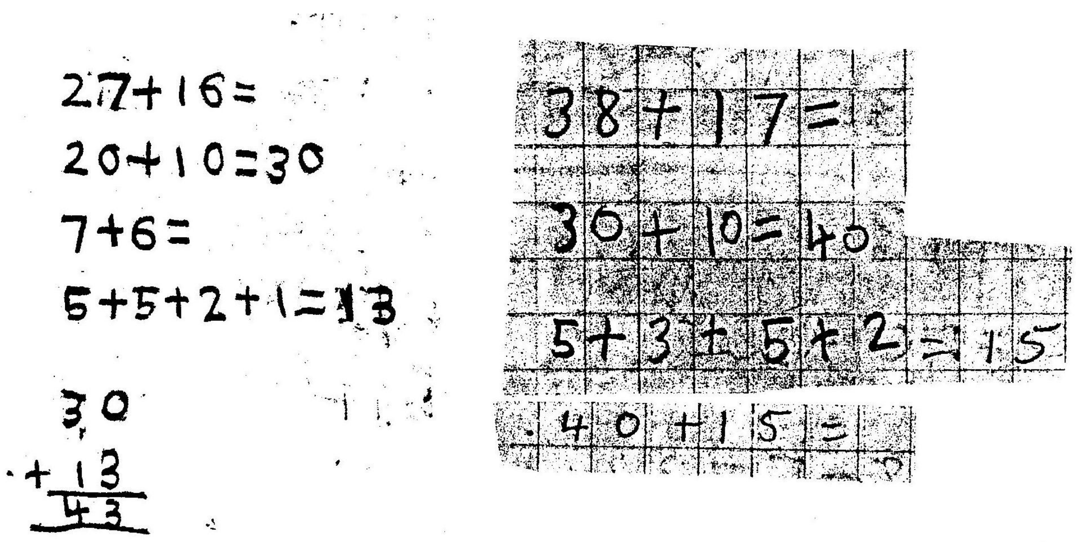


Figure 11: Examples of Pupils Use of Splitting Methods to Add Double-digit Numbers

4.2.2.4.2 Subtraction Using the 1010 Method

As mentioned in Chapter 4, the 1010 method is not very effective with double-digit subtraction. In this case the N10 method is most appropriate. For example, while using the 1010 method is straightforward when subtractions such as 39 - 17 are involved, things get rather complex when given something like 32 - 27. To attempt to calculate the answer using the 1010 method in this case would require at some point employing the 'borrowing' procedure which, as mentioned in Chapter 4, children in general find difficult to understand. Because of this, the Grade 2 teachers in experimental schools generally avoided using the 1010 to subtract two-digit numbers.

4.2.2.4.3 Addition Using the N10 Method

I have already explained above the procedure involved in using the N10 method to add or subtract two-digit numbers. Lesson observations indicated that Teacher D was the only teacher who was able to complete all the lessons in the experimental numeracy development approach, and to teach the use of the N10 method.

During one of her lessons, which I observed, Teacher D wrote 32 + 17 on the chalkboard and invited the class to work out the answer using the N10 method. One boy, using the 1010 method, attempted unsuccessfully to work out the answer. He wrote 32 + 17 = 30 + 10 + 7 = 47, leaving out the 2, perhaps forgetting that 32 = 30 + 2. Teacher D then asked the class whether or not the calculation was correct. This prompted the class to shout in unison: "No". However, none of the class members could say why they thought the calculation was wrong. I wondered if the class could tell when a mathematical calculation was incorrect but lacked the confidence to explain what they thought was wrong with it. A girl also attempted to work out the answer but did not arrive at the correct result. Teacher D, addressing the whole class, then said:

> The two [the boy and the girl] have arrived at different answers. The boy has accepted that his answer is wrong; but who can tell us what is wrong with his calculation?

Different children attempted without success to explain why. In this lesson pupils seem to have failed to use the N10 method correctly. Indeed, as mentioned in Chapter 4, the N10 method is most effective when used in conjunction with the empty number line (ENL) discussed in Section 7.2.5 below. Nevertheless, I was happy to see that even though the pupils could not explain why the calculation was wrong, they were ready and willing to question the accuracy of their colleagues' answers. This was one of the abilities I hoped Grade 2 pupils would develop as a result of participating in the study, which encouraged a way of mathematics teaching and learning that allowed interaction between teachers and pupils and among pupils.

4.2.2.4.4 Subtraction Using the N10 Method

At my next visit to the same class Teacher D set pupils two subtraction problems, that is 77 - 16 and 54 - 18. A girl attempted the first question, writing 77 - 16 = 77 - 10 - 6. However, rather than completing the calculation using the N10 method by writing

$$77 - 16 = 77 - 10 - 6 = 67 - 6 = 61,$$

she marked out tallies on the chalkboard and tried to count them to arrive at the final answer. She did not succeed. A boy also used a similar approach but failed to arrive at the correct answer.

Teacher D encouraged the class to participate. Eventually, another girl tried to tackle 54 - 18. She began in a similar way but again used tallies and arrived at the wrong result, 37. A third girl took up the task and completed the calculation successfully. All the children, including those who failed to arrive at the correct answer, were able to explain the procedures they used.

4.2.2.5 Double-digit Addition and Subtraction Using an Empty Number Line

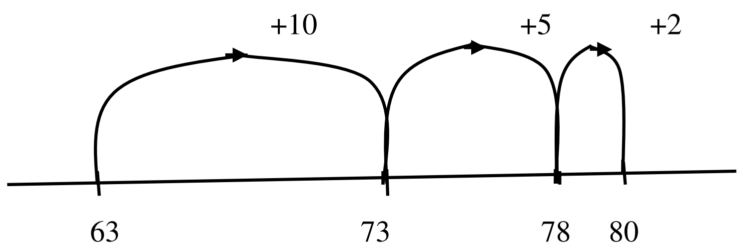
As pointed out in Chapter 4, the empty number line (ENL) is a model devised by Dutch mathematics educators for performing multi-digit addition and subtraction. It differs from the full number line, where required integers are listed consecutively, in that the only numbers shown on it are those pupils will actually work with. In general, using the ENL to add or subtract numbers involves starting with the larger of the two numbers and making 'jumps' (for example in tens) towards the *right* if one is addiing and towards the left if it is subtraction. Thus efficient use of the ENL depends on one's ability to split/partition numbers in appropriate ways.

4.2.2.5.1 Double -digit Addition Using the Empty Number Line

With regard to the ability of Grade 2 pupils in experimental schools to perform single and double-digit addition the data collected suggested mixed results, depending upon the class teacher's own level of understanding of how to use the ENL. Teacher D was the only exception. Lesson observations confirmed that she used the ENL appropriately to add and subtract double-digit numbers. Her pupils too applied the method relatively efficiently.

Teacher D introduced double-digit addition using the ENL by first asking pupils in which direction they would 'jump' on the number line when performing addition or subtraction. One pupil answered: "MINUS ti bweelela ku mbuyo; PLUS tiyenda ku sogolo". That is, "MINUS means we go backwards; PLUS means we go forward". In another lesson, a few days later, Teacher D invited the class to use the ENL to calculate the sum 63 + 17. She drew a straight line on the chalkboard for volunteers to use to illustrate the answer.

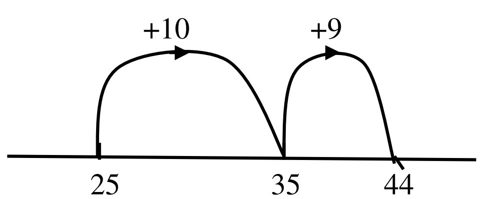
A boy stepped forward and wrote the number 63 at the right-hand end of the line, prompting the rest of the class to shout: "*Iyayi, uenela ku yambila uku*" (i.e. "No, you should start from that side"), as they pointed towards the left-hand end of the line. He obliged, wrote 63 at the left-hand end of the line, and partitioned the number 17 into 10 + 5 + 2. His final answer looked something like Figure 12:



i.e. 63 + 17 = 80

Figure 12: A Pupil's Use of an Empty Number Line to add 63 + 17

Another boy, from the other of Teacher D's two Grade 2 classes, worked out the value of 25 + 19 by first writing 19 as 10 + 9; and then used the ENL to arrive at the correct result 44 as shown in Figure 13 below.



i.e. 25 + 19 = 44

Figure 13: A Pupils' Use of an Empty Number Line to add 25 + 19

When asked to state why they partitioned numbers in the ways they did the children mentioned above would always say, "So that it becomes easier to do the sums".

Incidentally, in writing numbers on the ENL above the boy had initially written 36 where 35 is. This caused the other children in the class to shout in unison that 36 was wrong and that he should instead write 35. The examples cited above suggest that not only were the pupils in Teacher D’s classes sufficiently acquainted with the use of the ENL as a model for performing double-digit addition, they also could tell when a colleague's solution procedure was correct or incorrect. This indicated the children's growing confidence in their ability to do this kind of mathematics. The following are examples of the work of Teacher D's pupils in this regard:

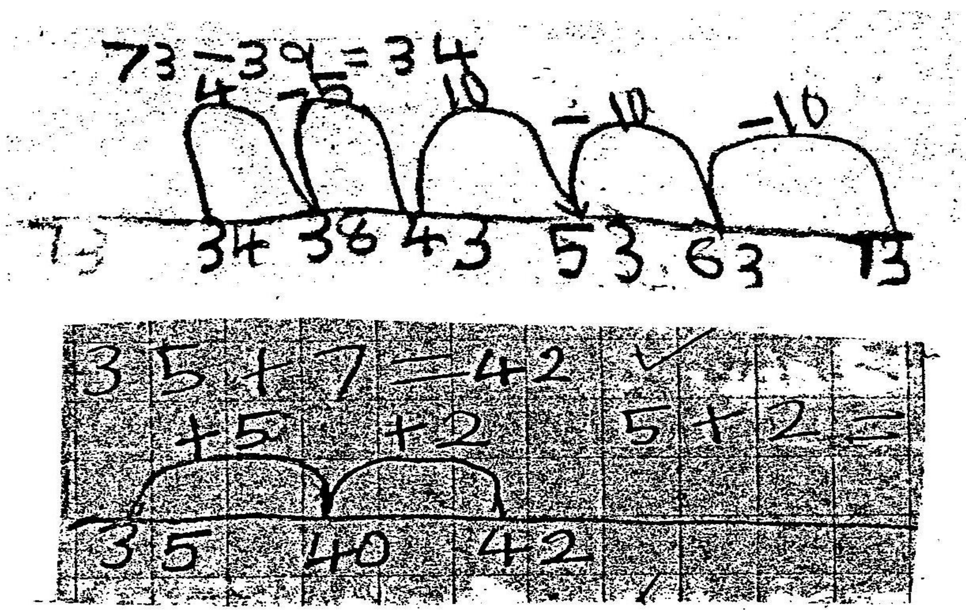


Figure 14: Examples of the Use of the Empty Number line in Teacher D’s classes

Other Grade 2 classes in experimental schools did not use the ENL to add (or subtract) double-digit numbers efficiently, although their answers were sometimes correct. For example, some pupils listed consecutive integers on a number line starting from 0, which was unnecessary. This is illustrated by the following work (Figure 15) taken from a pupils’ notebook.

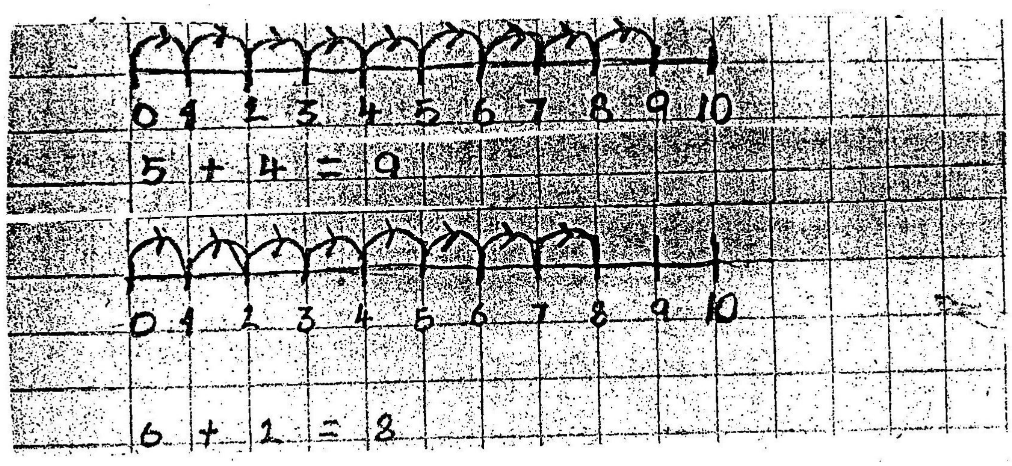


Figure 15: Pages from a Pupil's Notebook Showing Inefficient Use of an Empty Number line

As Gravemeijer (1994) points out, writing down consecutive counting numbers as shown in the diagram above means using the ENL inefficiently, since it requires counting several single steps to arrive at the answer. A related inefficient use of the ENL is highlighted in the diagram below drawn by a girl in Teacher A's class, who used a series of single-step jumps to work out the value of 5 + 9.

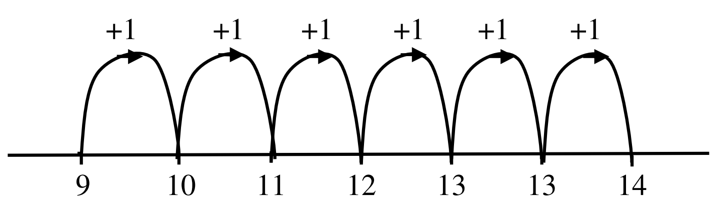


Figure 16: Inefficient Use of an Empty Number Line to Find 5 + 9

Other pupils performed double-digit addition using inefficient counting strategies, after which they drew number lines to illustrate how they had worked out the answer. Therefore, they did not use the ENL as a device for calculating the sum of two numbers, but merely as a way of diagramming the solution. I saw this in Teacher B's class. More than 80% of the pupils there worked out answers by counting with their fingers *before* drawing a number line to illustrate the process. Teacher B herself noticed this, and perceptively wrote in her research journal:

> Funny thing is that before they [the pupils] use a number line they already know the answer.

Nevertheless, almost all participating Grade 2 pupils in experimental schools understood that double-digit addition was easier if before drawing the number line one re-arranged the order in which the two numbers to be added appeared, so that the larger number was written down first. Thus in working out the value of 5 + 9 the girl from Teacher A's class referred to above first wrote 9 + 5 before drawing the number line. When asked why she did this, she replied, "I started with nine because it meant fewer jumps".

4.2.2.5.2 Double-digit Subtraction Using the Empty Number Line

By the end of the study only Teacher D and Teacher E's pupils had done this topic. Teacher E began work on the use of the ENL to subtract double-digit numbers as time allocated for implementation of the experimental numeracy development approach came to an end. Teacher E's pupils used the ENL incorrectly in this regard, perhaps indicating that she herself was still unclear about how to use the ENL to subtract double-digit numbers. For example, in working out the value of 19 - 8 the pupils relied on an awkward counting strategy (using both fingers and toes) to do the calculation and, having found the answer, proceeded to illustrate it diagrammatically.

Teacher D's classes exhibited better understanding of what is involved in using the ENL to perform double-digit subtraction. For example, in one lesson I observed Teacher D asked her pupils to use the ENL to find the value of 42 - 37. Although the first two children who volunteered to work out the answer did not arrive at the correct final result, they demonstrated that they understood the procedure to be followed. The first child, a boy, proceeded unsuccessfully as follows:

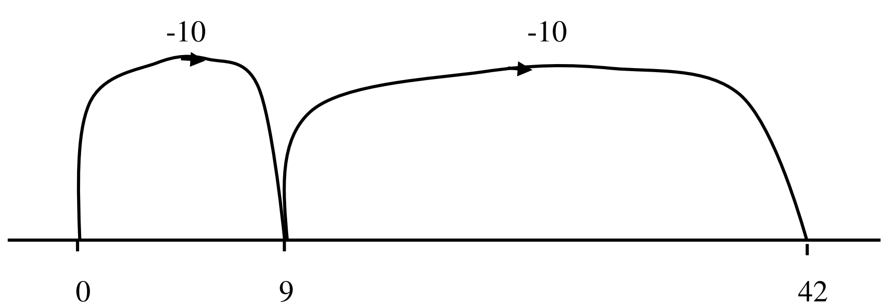


Figure 17: Unsuccessful Use of an Empty Number Line to find 42 - 37

The second child, a girl, although demonstrating that she new in which direction to go on the number line, was not only unable to arrive at the correct answer but also seemed confused with regard to the numbers to be included on the ENL. Her diagram looked something like Figure 18 below.

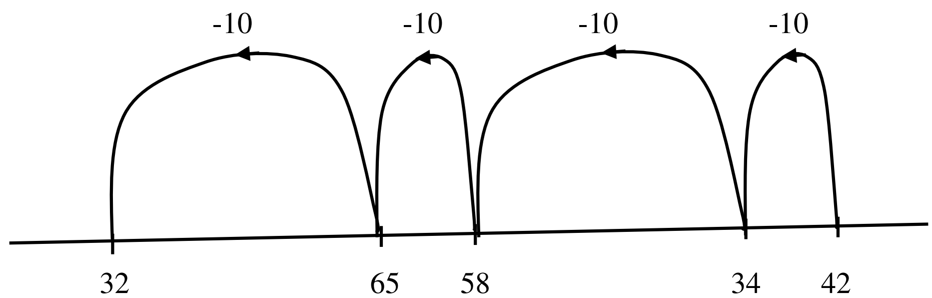
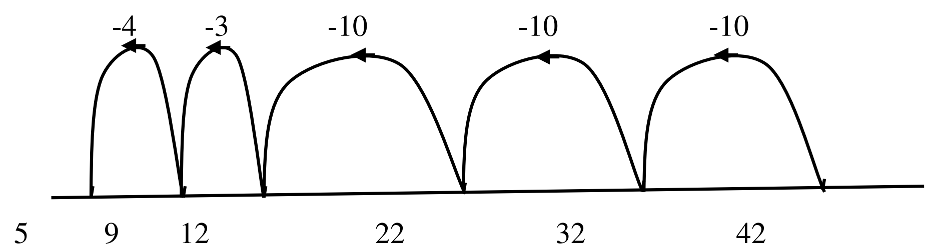


Figure 18: Second Example of Unsuccessful Use of an Empty Number Line to Work Out the value of 42 - 37

Finally another girl, from one of the more able learning groups in the classroom, stepped forward and worked out the answer accurately as shown below:



i.e. 42 – 37 = 5.

Figure 19: Correct Use of an Empty Number Line to Find the value of 42 - 37

Not only could the girl work out the correct answer; she also explained clearly the procedure she used. Her explanation, originally in Chinyanja, went like this:

> From 42 I took away 10 and got 32; from 32 I took away 10 and got 22, and from 22 I took away 10 and got 12. From 12 I took away 3 and got 9, then from 9 I took away 4 and got 5.

This pupil however did not explain *why* her method worked, but only *how* she went about working out the answer. One of the things I was trying to find out during the study was whether or not children could go beyond mere explanation to justification of the method used. In general the pupils were not able to do this. It would seem that the pupils' failure in this regard was partly due to the fact that before the study teachers generally did not ask pupils to justify their solution strategies. Thus for example in reflecting on what she had learnt during the staff development workshop Teacher D wrote in her research diary:

> I didn't used to ask…what method [learners] used to come up with an answer. And the learners did not even used to ask questions. I thought they were shy. Not true. I didn't train them to ask their peers [questions]. Interaction was always teacher-pupil, never pupil-pupil.

Although Teacher D made this statement before the main study began (i.e. before experimental schools started implementing the alternative numeracy development approach), like many of her colleagues in experimental schools when implementation began she often failed to recognise opportunities to do what she says in the statement above. For example, during a mathematics lesson I observed that one of her girl pupils used the ENL successfully to find the sum 32 – 19 (as illustrated by Figure 20), having first written: 19 = 10 + 5 + 4.

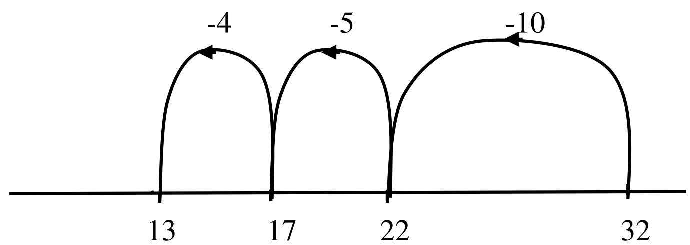


Figure 20: Correct Use of an Empty Number line to Work out the Value of 32 - 19

When the girl completed the solution Teacher D asked the class, "Where did this girl get the numbers -4, -5, -10?" One child answered: "From 19". I thought this was an excellent opportunity for Teacher D to press the child to justify her answer. She did not; and the rest of the class was denied the chance to learn why their colleague's answer was deemed correct.

4.2.3 Post-treatment Differences between Experimental and Control Schools

The data collected indicated at least two types of post-intervention differences between the experimental and control groups, which were not there at the beginning of the study. That is, quantitative differences and qualitative differences.

4.2.3.1 Quantitative Differences

As noted in Chapter 3, in selecting schools to comparison groups (i.e. control and experimental) I tried to ensure that experimental and control groups were as similar as possible with regard to factors that can affect educational outcomes, such as pupil ability and socio-economic background. I therefore assumed that there would be no statistically significant differences in performance on the pre-test between the pupils in the two comparison groups. This was confirmed by the data in Table 9.

***Table 9**: Independent t-test Results Showing Performance of Experimental and Control Schools on the Pre-test*

Group Statistics

Comparison Groups		N	Mean	Std. Deviation	Std. Error Mean
SCORE	Experimental	167	23.96	7.69	.59
	Control	144	22.41	7.67	.64

Independent Samples Test

	Levene's Test for Equality of variances		t-test for Equality of Means						
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Sig. Error Difference	95% Confidence Interval of the Mean	
								Lower	Upper
SCORE									
Equal Variances Assumed	.312	.577	1.773	309	.077	1.55	.87	-.17	3.27
Equal Variances not assumed			1.773	302.525	.077	1.55	.87	-.17	3.27

Table 9 shows that the t-test for independent samples indicated no significant difference in performance on the pre-test (t = 1.773, df = 309, two-tailed $p = 0.077$) between Grade 2 pupils in experimental schools (M = 23.96, SD = 7.69) and their colleagues in control schools (M = 22.41, SD = 7.67). (The test was administered at the beginning of the intervention and the maximum possible mark was 39).

Post-test results however showed significant differences in performance between the two comparison groups, as Table 10 indicates:

Table 10: *Independent t-test results showing performance of Experimental and Control Schools on the post-test*

Group Statistics

Comparison Groups		N	Mean	Std. Deviation	Std. Error Mean
SCORE	Experimental	167	25.47	7.35	.57
	Control	144	23.38	7.91	.66

Independent Samples Test

	Levene's Test for Equality of variances		t-test for Equality of Means						
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Sig. Error Difference	95% Confidence Interval of the Mean	
								Lower	Upper
SCORE									
Equal Variances Assumed	.142	.707	2.408	309	.017	2.09	.87	.38	3.79
Equal Variances not assumed			2.395	294.583	.017	2.09	.87	.37	3.80

Table 10 shows that the t-test for independent samples indicated a statistically significant difference (t = 2.408, df = 309, two-tailed $p = 0.017$) in performance on the post-test between the experimental group (M = 25.47, SD = 7.35) and the control group (M = 23.38, SD = 7.91). This result suggested that the treatment (i.e. the experimental curriculum on double-digit addition and subtraction) helped pupils in experimental schools to improve their performance on the numeracy test. To determine the *practical* significance of this result, I calculated its effect size using Cohen's d, where,

d = (Mean of Group A - Mean of Group B) ÷ (Pooled standard deviation, where pooled standard deviation = (SD of Group A + SD of Group B) ÷ 2.

Using the data in Table 10 we have:

$$d = (25.47 - 23.38) \div [(7.35 + 7.91) \div 2] = 0.2739 \text{ or } 0.3 \text{ (to one decimal place).}$$

Therefore, the mean in experimental schools was 0.3 standard deviations above that in control schools. In terms of Z-values, as indicated by consulting a standard normal distribution table (Kellow, 1998) the mean was 12 percent higher in experimental schools than in control schools. In other words, the treatment enabled 12 percent of the pupils in experimental schools who would otherwise have failed the test to pass the test.

I also compared the performance on the numeracy tests in experimental schools before and after the intervention (Table 11).

***Table 11**: Related t-test Results for Pupils in Experimental Schools*

Paired Samples Test

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Test 1	23.96	167	7.69	.59
	Test 2	25.47	167	7.35	.57

Paired Samples Correlation

	N	Correlation	Sig.
Pair 1 Test 1 & Test 2	167	.836	.000

Paired Samples Test

	Paired differences					T	Df	Sig. (2-tailed)
				95% Confidence Interval of the Difference				
	Mean	Std. Deviation	Std. Mean Error	Lower	Upper			
Pair 1 Test 1-Test 2	-1.51	4.32	-.33	-2.17	-0.85	-4.510	166	.000

The results of the related t-test above indicates that the performance of the experimental group on the pre-test (M = 23.96, SD = 7.69) and on the post-test (M = 25.47, 7.35) differed significantly (t = -4.51, df = 166, two-tailed p < 0.001). The experimental group performed significantly better on the post-test than on the pre-test.

In the case of performance in control schools the related t-test (Table 12) also indicated that results on the pre-test (M = 22.41, SD = 7.67) and on the post-test (M = 23.38, SD = 7.91) differed significantly (t = -2.084, df = 143, two-tailed p = 0.039).

Table 12: *Related t-test Results for Pupils in Control Schools*

Paired Samples Test

	Mean	N	Std. Deviation	Std. Error Mean
Pair 1 Test 1	22.41	144	7.67	.64
Test 2	23.38	144	7.91	.66

Paired Samples Correlation

	N	Correlation	Sig.
Pair 1 Test 1 & Test 2	144	.742	.000

Paired Samples Test

	Paired differences					T	Df	Sig. (2-tailed)
	Mean	Std. Deviation	Std. Mean Error	95% Confidence Interval of the Difference				
				Lower	Upper			
Pair 1 Test 1-Test 2	-.97	5.60	.47	-1.89	-5.0E-02	-2.084	143	.000

The results of the two related t-tests (Table 11 and Table 12) show that Grade 2 pupils in both experimental and control schools improved their performance on the post-test as compared to the pre-test. However, the gain recorded by pupils in experimental schools was much higher, as indicated by the fact that the post-test scores of the two comparison groups differed significantly (Table 10). Also, the pre-test and post-test standard deviations of the experimental group were respectively 7.69 and 7.35. This suggested that the experimental curriculum reduced variability among individual scores. On the other hand, pre-test and post-test standard deviations in control schools were respectively 7.67 and 7.91, indicating that teaching of standard procedures in the schools increased variability among individual scores. In other words, less able pupils performed more poorly and more able pupils much better.

4.2.3.2 Qualitative Differences

As mentioned above, the other post-test differences between experimental and control schools were qualitative in nature. These included differences in pupil morale, interest to learn mathematics, and levels of absenteeism. According to Mohr (1999),

> [using] qualitative research to determine the causes of intentional human behavior, such as whether or not those behaviors were induced by a program being evaluated, must involve a search for the operative reason behind the behaviours. In most cases this would undoubtedly involve obtaining information from the subjects whose behavior is at issue….It might [also] be necessary to obtain information from many other people, from documents, and from histories of relevant events (p. 75).

I am going to argue that this study produced an *operative reason* that explains the changes in pupil behaviour stated above. However, to understand this reason requires some background information. In all four nation-wide surveys of learning achievement in primary schools in Zambia published so far, pupil absenteeism was identified by more that 95% of teachers and head teachers in the samples as a problem that affected negatively pupil learning (Kelly and Kanyika, 2000a; MoE, 2001; 2003; 2006). For example, the report for 2001 states: “Teachers who thought that [pupil] absenteeism was generally a problem [in primary school] taught 96% of pupils [who participated in the study]. Only 1.9% of pupils were taught by teachers who felt that it was not a problem” (MoE, 2001, p. 31).

Maimbolwa-Sinyangwe and Chilangwa (1995) reached a similar conclusion about six years earlier. Their study conducted in Lusaka and in Chipata (Eastern Province), which focused on learning inside primary school classrooms, found that high rates of pupil absenteeism in primary school constituted a serious problem that affected negatively pupil learning, and which needed to be addressed quickly. My own observations of pupil attendance patterns in both experimental and control schools during the early part of the study would seem to support this view. For example, because of high rates of absenteeism, which made it look as though different Grade 2 pupils attended lessons on different school days, it was not until two weeks after the study had begun that I became aware of that Teacher C, from one of the two experimental schools, taught two single-sex classes.

As the following statements from two of the participating Grade 2 teachers commenting on pupil absenteeism suggest, the present study reversed the trend at least for the duration of the implementation period:

> Absenteeism has gone down. It was very high before the study. Each time I just had a few pupils coming to school. But it is no longer the case. They now come in numbers. And if I don't come to school they follow me home [laughs]…to find out why and to remind me that I have not marked their homework. So you can see they are enthusiastic about making sure that their work is …marked. And if they get anything wrong they want to do it again. (Teacher D, Post-intervention interview).

> There wasn't much absenteeism this term….maybe it is because it was my first time to handle this class….But also maybe the pupils have realised that what we they are learning is important. Most of them have attended lessons regularly throughout the term. Only…a few who were unwell did not come to school; and their parents would come here to tell me the reason for their child's absence. Otherwise, the attendance wasn't bad at all this term. (Teacher E, Post-intervention interview).

With regard to learning mathematics the teachers reported that during the implementation period pupils developed more positive attitudes towards the subject, as the following statements show:

> The pupils are no longer reluctant to participate in classroom activities. Right from the beginning of the study they have shown increasing interest in learning mathematics. Before the study, during mathematics lessons, many pupils would say, "Teacher, may I go outside", and would not come back….Also, before the study they didn't care much whether they got a sum right or wrong. It didn't bother them….They would just sit back and wait for the next lesson [in a different subject]. But this time, they will even tell me: "Teacher, I haven't finished that

> question. I need more time to complete it. And I say to myself, "I think now I am communicating well with the pupils". They have changed. (Teacher D, Post-intervention interview).

> What I have seen is that this type of work has made the pupils busier. They have become very busy. Even though there is that increased level of noise in the classroom, it is educative noise and it does not bother me. And, in fact, this time they [the pupils] will never miss any chance to have their work marked. Immediately they finish a task they want me to mark it….Before the study I used to say to them, "Leave your books, I will mark them later. I do not have enough time to do it now". But this time, they want their work marked before they leave for home. (Teacher E, Post-intervention interview).

> To my knowledge, if pupils in a class are able to ask questions it means that they like the subject; otherwise they wouldn't ask questions. If you ask pupils a question and they are just looking at you, it means they are not interested. But if they are able to answer; that means they like the subject. They like [mathematics] because they are able to answer the questions I ask. And the number of pupils whose hands go up when I ask questions has increased. So, the pupils are now more eager to learn. (Teacher C, Post-intervention interview).

An incident that occurred while I was observing one of Teacher D's mathematics lessons seems to confirm what the teachers said above, regarding increased pupil interest in learning mathematics. During the lesson a man whom both of us (Teacher D and I) did not know suddenly walked into the class and headed directly towards Teacher D, who was in front of the classroom, while I was seated at the back, and said:

> I know now that it is true you are working hard as a teacher, because I have seen it for myself. My daughter now doesn't give me much time to rest. She sometimes even wakes me up at night so that I can help her with her mathematics homework. She has really become interested in learning mathematics.

Immediately after saying this, the man introduced himself as the father of one of the girls in the class (whom he named) and then left the class.

What could explain such dramatic change in pupil attitudes towards mathematics? Part of the reason is that, during the study Grade 2 teachers in experimental schools (though not all of them) thought more about what they would teach and spent more time both on their own lesson preparation and on teaching mathematics. This seems to be supported by what Teacher E volunteered to say below, after I had announced the end of our discussion during the post-treatment interview:

> From my observations, I can say that there is something happening to the pupils. I can say that many of them are enjoying this mathematics. I think it is because we

> [both teachers and pupils] are spending more time on it, good time on it, than we used to in the past. We were giving very little time to mathematics. We were mostly concerned with literacy lessons, which came once or twice a day. We were not giving much time to mathematics because it only comes twice a week [i.e. had two thirty-minute periods per week]. But now we are doing it almost daily. So, I think now the pupils are enjoying mathematics.

The significance of what Teacher E says above becomes clear when her statement is contrasted with what Maimbolwa-Sinyangwe and Chilangwa (1995) reported as one of the main reasons primary school pupils absented themselves from lessons generally.

> Our…observation was that…many pupils stayed away from school [because]…*there was very little work done in class*, and…*teachers did not prepare teaching notes* and tended to repeat topics and to choose easy subjects that could be taught [by] reading passages from textbooks and asking pupils comprehension questions afterwards (p. 26, my emphasis).

Thus, qualitatively, the study improved attendance levels of Grade 2 pupils in experimental schools during mathematics lessons; and engendered more positive attitudes among them towards learning mathematics.

4.3 Chapter Summary

This Chapter presented the findings of the study relating to both Grade 2 teachers and their pupils, as indicated by the research questions in Chapter 1. Concerning teachers, the chapter examined the extent to which the five teachers in experimental schools learned to foster the use of strategies for mental calculation relating to adding and subtracting double-digit whole numbers. It started by considering the ability of each of the five teachers to do so, followed by an analysis of the performance of the teachers as a group. Although there were variations in the extent to which individual teachers learned to foster the use of strategies for mental calculation in lower primary mathematics, by the end of the study four of the teachers had at least moved mentally towards adopting teaching practices that supported the use of mental strategies in this regard.

With regard to pupils, the chapter first presented results on numeracy achievement at Grade 2 level before the study, as determined by the pupils' performance on a pre-test based on mathematics subject matter normally covered in Grade 1. The results of this test would be used as baseline information to measure the effectiveness of an intervention implemented in experimental schools. The Chapter then focused on findings relating to the extent to which Grade 2 pupils in experimental schools developed the necessary pre-requisite skills for attaining proficiency in using mental calculation strategies for two-digit addition and subtraction. Finally, post intervention characteristic of participating Grade 2 children in both control and experimental schools were analysed to determine whether or not implementation of the intervention in experimental schools had resulted in differences in numeracy development among pupils in the two groups of pupils. The next chapter discusses findings of the study.

CHAPTER 5

DISCUSSION OF FINDINGS

5.0 Introduction

This chapter discusses the findings of the study. It has two main sections. Section 5.1 discusses findings relating to the extent to which Grade 2 teachers in experimental schools learned to foster in the early primary grades the use of strategies for mental calculation for double-digit addition and subtraction; and Section 5.2 looks at the impact of the teachers' implementation of the experimental numeracy development approach on their pupils' performance in mathematics. Section 5.1 has four sub-sections, each of which considers a particular aspect of change in teacher behaviour resulting from participation in the study. These are: teacher concerns (Section 5.1.1), teacher beliefs (Section 5.1.2), teachers' mathematical content knowledge (Section 5.1.3), and teaching practices (Section 5.1.4). Section 5.2 is also divided into four sub-sections. Section 5.2.1 deals with pre-treatment characteristics of the numeracy achievement of participating Grade 2 pupils in both control and experimental schools; Section 5.2.2 looks at the ability of pupils in experimental schools to develop the pre-requisite skills necessary for attaining proficiency as mental calculators; Section 5.2.3 discusses quantitative post-treatment differences between pupils in control and experimental schools; and Section 5.2.4 the post-treatment qualitative differences.

5.1 Discussions of Findings Concerning Teacher Learning

As shown in Chapter 4, the changes in teacher behaviour at the beginning and at the end of the study discussed below relate to four main themes: teacher concerns, teacher beliefs, teachers' mathematics content knowledge, and teaching practices. I now proceed to discuss what the study has revealed concerning teacher behaviour in relation to each of these four themes.

5.1.1 Teacher Concerns

How did change-related concerns affect teachers' participation in the study? The study confirmed that implementing change at school level always generates a number of fears and concerns among teaching staff in particular (Holloway, 2003). Two of the five teachers in experimental schools felt that children would not benefit from the exercise since the mathematical ideas to be taught demanded cognitive abilities beyond what the pupils possessed. In other words, taking part in the study would be a waste of time both for the teachers and the pupils. Among the remaining three teachers, one was concerned about the possibility that I may not have obtained permission from relevant educational authorities to conduct a study in the schools, in which case I had no business asking her to participate; another was worried about the adequacy of her existing mathematical knowledge, lest it should prove inadequate to support her learning of the experimental numeracy development approach; while the third (Teacher E) claimed to have felt no particular worries and concerns, since her involvement in the study would benefit the learners.

Teacher E was due to retire from the teaching service the following year. Research (see Enns, 2007; Hargreaves, 2005) seems to suggest that teachers nearing retirement tend not to want to participate in school reforms since they no longer see "much purpose in making changes or setting objectives for themselves" (Enns, 2007, p. 6). The fact that Teacher E was so receptive to change was therefore rather strange. Could it be that knowing she would retire the following year, and could do nothing about it, she was no longer concerned about anything? If this was not the case, then Teacher E's behaviour could be the exception to the general view, as suggested by Hargreaves (2005), that as teachers age and begin to focus their attention on retirement, they become more resistant to school change.

Although the five teachers above reacted differently to the invitation to take part in the study, the underlying reason for their reactions seemed to have been their concern for the welfare of their pupils. They were not sure if by agreeing to participate in the study they would be furthering or harming their pupils' mathematical progress. This general concern is exemplified by Teacher B's statement that "I am worried that the children will not pass the examinations", by which she meant specifically the Grade 7 leaving examination at the end of primary education, which determined who proceeded to enter secondary education.

However, by the end of the study most of the five teachers in experimental schools had overcome their initial fears and concerns and were more concerned with what to do to help pupils advance their mathematics progress. Evidence from the classroom had convinced the teachers that young children had higher levels of mathematical understanding than the teachers earlier imagined although, as usual, some children made more progress than others. It was not surprising, then, that at the end of the study one of the teachers (Teacher D) remarked:

> I think I am very happy about the outcome [of this experiment]…. I think both of us [that is, the pupils and I] have enjoyed ourselves….Learners are free; they can explain anything to anyone at anytime.

Teacher B however felt rather differently from the other four teachers. Although like the others four teachers she had also moved from worrying about the possibility of children not being able to learn anything useful during the study, to being concerned about the fact that the children progressed through the experimental numeracy development approach at different rates, she maintained until the end of the study that the research would not benefit her pupils. In view of this she intended, as soon as the study was over, to re-teach the concepts covered using textbook rules and procedures.

The reactions displayed by the teachers in experimental schools when they first learned about what the study would involve were not strange. As Holloway (2003) states:

> Each [school] administrator, each principal, each teacher approaches a new program, any change, with a personal set of concerns....Individuals question: *Why should I do this? How long is it going to take me to work through this? I know my kids and I don't think this will work* (p.1, Emphasis in original).

Furthermore Holloway (2003), citing the Concerns-Based Adoption Model (CBAM), states that when confronted with a new school change programme teachers' concerns usually go through seven stages as follows:

- *Awareness*: Aware that an innovation is being introduced but not really interested or concerned with it.
- *Informational*: interested in some information about change.
- *Personal*: wants to know the personal impact of the change.
- *Management*: Concerned about how the change will be managed in practice.
- *Consequence*: interested in the impact on students or the school.
- *Collaboration*: Interested in working with colleagues to make the change effective.
- *Refocusing*: Begins refining the innovation to improve student learning results (pp 1–2).

The data collected in this study suggests that all five teachers in experimental schools went through at least the first six stages of concern enumerated above. For example, the teachers became aware of the intended innovation when I explained its nature and asked them to participate; and they showed their interest in the new information to be disseminated when they agreed to attend the Staff Development Workshop, during which the new ideas were discussed. The teachers' worries about how to implement the study and what it might mean for their pupils' learning of mathematics indicated that they were concerned with the management and consequence of the intended change. Finally, when the teachers experienced difficulty understanding what to do to implement the study they consulted more knowledgeable colleagues participating in the study within the same school. Thus for example, Teacher A consulted her colleagues Teacher B and Teacher C; and in the other experimental school Teacher E sought help from Teacher D before proceeding with the experimental lessons. However, it is difficult to determine to what extent the teachers reached the final stage of concern-refocusing- in that although they all *talked* about the changes they wished to see in their classroom practice only Teacher D seemed to have *actually tried* to refine the innovation, so that her pupils' mathematical knowledge could advance. Indeed, it would appear that there was not enough time before the study concluded for the other four teachers in experimental schools to begin thinking about how to refine the ideas implemented to suit their own pupils.

Finally, the school change literature (see Chapter 2) indicates that unless teachers' concerns are identified and addressed, the intended change is not likely to take place. In this connection Holloway (2003), citing relevant literature, states that individual teachers' concerns can be identified by careful questioning at the beginning or at the end of the programme or both, before attempting to address them. As shown in Chapter 4, I identified teacher concerns at the beginning of the study and at the end, and tried to address them by clarifying what the teachers wanted to be clarified and in general being there when they needed me.

5.1.2 Teacher Beliefs

This study has shown that, initially all five teachers from experimental schools held the view that when young children first begin school they did not have any mathematical ideas of their own; and that all the mathematical knowledge they eventually developed was given them by their teachers in the classroom. Therefore, pupils should not be allowed the freedom during lessons to express their own views about mathematics but should accept without questioning the mathematical knowledge that teachers *gave* them. Consequently, to succeed in learning mathematics, pupils should listen carefully to teachers' explanations and follow exactly what they heard their teachers say or do.

Not surprisingly, the teachers also believed that an interesting mathematics lessons was one where pupils answered teachers' questions correctly, following taught procedures and no others. Inversely, mathematics lessons were not interesting if pupils did not answer teachers' questions correctly because they did not listen carefully to teachers' instructions and could not do exactly what the teacher expected. Such beliefs illustrated the *traditional* approach to mathematics teaching, which is characterised by teachers attempting to transmit mathematical knowledge from their own minds to those of pupils, telling pupils exactly what procedures and solution methods to use, and looking out only for correct answers and no others (Valero, 1999; Skovsmose, 1999).

One might ask: How did the five Grade 2 teachers in experimental schools develop such beliefs about mathematics teaching and learning? As Teacher C stated in Chapter 4, one source of beliefs of this kind was the way in which the teachers were trained to think during initial teacher education. It would seem that such training gave student teachers the impression that it was their responsibility to *pass on* knowledge to their pupils, with pupils' role being mainly that of careful listeners. Another factor that might contribute to the development of the teacher beliefs outlined above, is the important role played by examinations (e.g. the Grade 7 terminal examination) in teaching and learning situations at primary school. As the Ministry of Education (1992) states: "The successful performance of their students is a powerful source of professional satisfaction and public justification for teachers...[Teachers] are often blamed, and may blame themselves, for student failures" (p. 43). Given such pressures, it is not surprising that teachers will do anything to ensure that pupils learned only that which will ensure success in examinations.

How do these results compare with previous relevant research conducted in Zambia? To the best of my knowledge no research of Zambian origin has examined teacher beliefs in relation to the teaching of particular school subjects such as mathematics. However, it is possible to piece together examples of teacher beliefs in results of studies that focused on other areas of teacher behaviour. The study by Maimbolwa-Sinyangwe and Chilangwa (1995) I referred to in Chapter 6, which investigated among other things teacher perceptions about the learning of girls and boys inside primary school classrooms in Chipata and Lusaka, is a case in point. Maimbolwa-Sinyangwe and Chilangwa found that when responding to pupils' correct answers to teachers' questions, primary school teachers in Lusaka for example either said "good", "very good", asked the class to clap in honour of the pupil who had given the correct answer, or repeated the answer. If the answer was wrong the teachers would either tell the class the answer was wrong, asked another child to answer the same question without saying the first answer was wrong, asked the child who had given the wrong answer to pay more attention next time, made no comment, or simply shout; 'no'. Thus the teachers accepted only correct answers and did not investigate further pupils' answers, whether correct or incorrect. I see here the belief, as exemplified by Teacher B (see Chapter 4), that pupils did not bring any knowledge of their own to the classroom but learned everything from teachers.

However, it was clear from the beginning of the study that some of the five teachers would change their views in this regard if classroom events produced disconfirming evidence. As the study progressed and as the teachers in experimental schools began to see that Grade 2 pupils understood mathematics at a much deeper level than previously thought possible, their beliefs began to shift towards the direction required to support the teaching of strategies for mental calculation. Some of the teachers even began to suspect that their pupils' past failures in mathematics might have been due to the teaching approaches they used, which approaches for example did not include further investigation of pupils' answers. I was not surprised, then, that by the end of the study, four of the teachers had abandoned their earlier beliefs about mathematics teaching and learning, and were trying to adopt more constructivist approaches to mathematics teaching and learning. Nevertheless, as we shall see in Section 5.1.4, the new beliefs did not always correspond to what the teachers did in the classroom. Indeed, in a review of the relevant literature Kotsopoulos and Lavigne (2008), concluded that, "Despite motivation and openness to change in order to improve teaching and, most importantly, student learning, teachers often resort to what is familiar in terms of *how* and *what* to teach" (p. 5). When teachers behave this way, it might be partly because their colleagues in the school, school administrations, parents and, perhaps, the school system itself, expect the teachers to teach in 'acceptable' ways, that is, ways which will ensure the success of their pupils in public examinations.

5.1.3 Mathematics Content Knowledge

This study has suggested that, despite all five teachers in experimental schools having the minimum qualifications for teaching in primary school in Zambia, with some having added extra qualifications to the minimum requirements, at the beginning of the study all the five teachers did not understand mathematics enough to teach the basic mathematical ideas associated with the use of strategies for mental calculation relating to double-digit addition and subtraction. Indeed, four of the teachers admitted that the mathematics content which they learned as part of their teacher education programmes was not sufficient to understand such strategies. They acknowledged that the two-week staff development workshop conducted at the beginning of the study had helped increase their understanding of mathematics, but they also felt that to develop confidence in teaching mathematics they needed more workshops of a similar kind which focused on learning of mathematics content. Given that the primary school timetable at the time of the study allowed only about three thirty-minute lessons per week at Grade 2 level, one would wonder just how much learning of mathematics actually took place nationwide at this level of education.

But the fact that the five Grade 2 teachers had an insecure knowledge of mathematics was perhaps not surprising. In 2001, I spent 25 days in a primary college of education observing lessons, with a view to learning something about what was involved in training primary school teachers. I noticed that during lessons in the social sciences (e.g. civics, language, etc) student teachers behaved like college students anywhere, actively participating in classroom discussions, analysing issues, and so on. But when it was time for mathematics lessons they would become strangely quiet and docile, apparently afraid of the subject. Since I was in a college of education well-known for producing the best primary school teachers, I had no reason to believe that the training of primary school teachers in mathematics elsewhere in the country would be different. Indeed, it would seem that the training the teachers received particularly in mathematics envisaged working conditions where lesson preparation was a matter of copying directly from teachers' guides, with little or no decision making on the part of the teacher. This maybe the reason why some of the five teachers experienced difficulty using the manuals I had provided, complaining that the manuals should have included all necessary worked examples and exercises teachers would need during the study, so that they would use exactly the same worked examples and set pupils the same exercises. However, my intention in the study was to allow the teachers to use their own initiative not only when preparing lessons but also when constructing suitable questions for pupils.

I mentioned in Chapter 2 that the ZATEC syllabus introduced after 2000 assumed that the mathematics student teachers learned in secondary school was sufficient for teaching in primary school. Accordingly, ZATEC did not stress the learning of mathematics content during teacher education. However, none of the five teachers in experimental schools had followed this course, although the youngest among them (Teacher B) underwent a teacher education programme that allowed her to teach only up to Grade 4. The other four teachers all followed a primary teacher education programme that included the study of substantial mathematics content, to the O' level standard. One would assume, therefore, that anyone who completed this programme would be well prepared to handle the demands of teaching mathematics at primary school level. This, however, was not the case. As Kelly (1991a) seems to suggest, the problem was the cognitive level at which college lecturers taught mathematics. Mathematics lessons did not treat topics in the subject sufficiently deeply to permit development of confidence among student teachers.

Nevertheless, lesson observations suggested that by the end of the study the teachers had become more confident in teaching mathematics, although it was unclear to me just how much their content knowledge in mathematics had improved. Indeed, up to the end of the study some of the teachers continued asking me questions that indicated that they still had much to learn in mathematics before they could implement the study effectively.

5.1.4 Teaching Practices

As suggested in Section 5.1.2, at the beginning of the study the teaching practice of all the five Grade 2 teachers in experimental schools reflected the traditional approach to mathematics teaching. In other words, their teaching was characterised by attempts to transmit mathematical knowledge as enshrined in official textbooks and teachers' guides into the minds of the pupils. Accordingly, the teachers did not consider pupils' own mathematical ideas and experiences during lesson preparation and presentation, and insisted that pupils should use only taught solution procedures in working out answers to mathematics problems and to present answers exactly as teachers had demonstrated during lessons. Thus the teachers were inflexible, one would even say undemocratic, in that they pushed children to follow exactly other people's mathematical knowledge and experiences, as laid down in textbooks and teachers' manuals.

I have already mentioned elsewhere in this Chapter that one of the reasons the five teachers in experimental schools used the traditional approach to mathematics teaching was that, as Teacher C stated, they were following what they were taught to do during initial teacher education. It would seem that a second reason was the busy schedule student teachers endured in college, which did not leave much room for creativity. For example, describing a typical day in the college life of a primary education student teacher in Zambia during the 1980s, Kelly (1991a) states:

> The [college curriculum is so] rigidly organized….that [it] provides little time for private, independent study and does not encourage the use of initiative or innovative teaching techniques. Trainees have a very full, carefully timetabled day, which they spend largely being taught the contents and methodology of handbooks.

> The outcome is that the trainees develop an unimaginative teaching style that emphasises factual knowledge, memorization, and convergent thinking (p. 133).

Kelly (1991a) adds that this "rigid, conservative training program in the colleges [was] closely linked to the college tutors' lack of knowledge and confidence, arising from their own low qualifications" (p. 133). The issue of low qualifications among college lecturers, however, has been resolved in recent years, as most of the tutors now have relevant undergraduate degrees in their specialist areas.

Again, as noted in Section 5.1.1, one of the main concerns the five Grade 2 teachers expressed when first invited to take part in the study was that they were not sure how the study would affect their pupils' chances of success in public examinations. This related in particular to the Grade 7 examination which determined who entered secondary education. According to Kelly (1991a), the Grade 7 examination emphasised book knowledge at the expense of conceptual knowledge and the ability to solve real life problems, which format and interpretations of how to prepare pupils to take it influenced teaching practice in primary schools. As Kelly (1991a) states:

> [This type of examination] stimulates a type of teaching that emphasizes drilling pupils in factual knowledge and that promotes passive acceptance and reproduction of information as the ideal. It does not encourage teachers to set about developing in their pupils the ability to think independently and flexibly…" (p. 120).

However, the data collected mainly through lesson observations, interviews, and teachers' journal entries suggested that by the end of the study changes had began to appear in the classroom practice of most of the five teachers. For example, in place of passivity on the part of pupils during lessons there was increased interaction both among themselves and between pupils and the teacher. Furthermore, teachers were more likely than previously to encourage children to engage in discussion during lessons; and the pupils began to show not only willingness but also readiness to explain their solution strategies.

This result supports the conclusion reached by Joyce and Showers (1995), regarding how soon during implementation of an innovation at school level, pupils will change their learning habits:

> Many educators believe that school improvement efforts will not have demonstrable effects on students for several years, but the evidence points toward quite a different conclusion. Students respond right away to changes in instruction and begin to accelerate their rates of learning provided that the educational environment is designed to do just that-teach the students to learn more effectively (p. 59).

This study has shown that this was indeed the case.

5.2 Discussion of Findings Concerning Pupil Learning

The study sought to answer one main question relating to Grade 2 pupils. This question, paraphrased, is: Can Grade 2 pupils in Zambia learn to use strategic methods for double-digit addition and subtraction, and would there be a difference in numeracy performance between pupils who learn such methods and their colleagues who do not but continue with the use of procedures as currently taught? What do the results given in Chapter 4 show in this regard?

5.2.1 Pre-treatment Numeracy Achievement of Participating Grade 2 Pupils

5.2.1.1 Determining number of objects in a small set

The data collected has shown that with regard to the ability to determine the numeral representing the number of elements in a small set (Question 1), participating Grade 2 children had attained near masterly levels of achievement, with a success rate of more than 98%. In other words, almost all the Grade 2 pupils who participated in the study could count the number of elements in a set having ten or fewer elements and write down the correct numeral representing that number. However, when the pupils were given a set with concrete objects and asked to draw more objects of the same kind to equal a given numerosity (Question 2), the success rate dropped by about 10 per cent; and dropped even further to about 81% when the requirement was to show objects of a set that equalled a pre-specified number.

Thus, at Grade 2 level pupils can easily handle situations involving identifying concrete objects belonging to a small set (not more than 10), counting them, and writing down the correct numeral representing their number. The high percentage of success recorded here (98 percent) indicates that this aspect of learning about sets is taught well by teachers at this grade level. However, when required to match a set of concrete objects with a pre-determined cardinality, the task becomes much harder. This result seems obvious, in that the cognitive demand of counting what is already there and writing down its number is lower than that of grouping a set of objects to equal a given number.

5.2.1.2 Writing Down a Single-digit Number that is a Specified Number More Than or Less Than Another Single-digit Number

As stated in Chapter 4, Question 3 presented pupils with a single-digit number and asked them to find another single-digit number that is a specified number more than or less than the previous single-digit number. This situation was equivalent to asking one to start with a single digit number and then to add or subtract another single-digit number. The study has shown that Grade 2 pupils could handle this *more than*, *less than* situation better when only smaller numbers such as 2 or 3 were involved than when relatively larger single-digit numbers such as 6, 7, 8 or 9 were involved. Thus for example a success rate of more than 90% was attained in the former case, while in the latter case the success rate was around 81%. Indeed, it is known that in general children, even older ones, can handle better problem situations involving smaller numbers than larger ones (Brown, 1981).

There was also a difference in performance when the *more than* and the *less than* situations were compared. The success rate for the *more than* situation was higher than that for the *less than* situation, which might reflect that it is relatively easier to understand the concept of addition than the concept of subtraction. This is supported by Brown (1981) in a study that investigated secondary school pupils' understanding of the four basic operations-addition, subtraction, multiplication, and division. She found that the pupils grasped better situations involving addition than those relating to subtraction. She also observed that division, because of its use in sharing situations in the home, appeared to be easier for the pupils to understand than multiplication. Nevertheless, in the case of

Question 3 above the success rate seems to have been influenced more by the sizes of the numbers added or subtracted than by whether or not addition or subtraction was involved.

5.2.1.3 Determining Missing Single or Double-digit Numbers on a Number Line

This sub-section refers to two questions, 4 and 10. To appreciate what was involved here it is necessary once again to explain briefly the situation the pupils faced. Two tasks were involved. In the first task (Q. 4) pupils were presented with a number line involving the numbers 0 to 10, well spaced out, and the task was to identify and to write down two missing integers between 0 and 10. The second task (Q. 10) involved a similar number line but in this case the range was 0 to 50, the spaces between consecutive integers were narrower, and the task was the same: to identify and write down two missing integers. Neither of the two required integers was a multiple of 10.

Why were the results higher in the first case than in the second? And why was the rate of success in either case higher when finding the first missing integer than the second? It would seem that the answer does not lie so much in the narrowness or otherwise of the spaces between consecutive integers but in the method pupils used to solve the two problems. In both cases the pupils tried to identify the missing numbers by *counting* individual lines in the number line and checking to see which ones were identified and which ones missing. This was obviously easier to do when only the numbers 0 to 10 were involved than with the range 0 to 50. In the case of the number line which had the numbers 0 to 50, I observed during administration of the test that many pupils counted the single lines in the number line to see which ones were already identified and which ones were not. In this way counting errors could have crept in, which were compounded by the fact that the spaces between consecutive numbers were narrower. In short, the major source of mistakes in identifying the missing numbers on the number line was pupils' use of inefficient counting strategies.

5.2.1.4 Translating Single or Multi-digit Numbers from the Verbal Form to the Symbolic Form; and Writing Down other Numbers One More Than or One Less Than the Given Single or Multi-digit Numbers

Pupils faced these tasks in questions 5, 6, 11 and 12. The tasks were as explained in the sub-heading above. Results obtained show that when the numbers involved were either single digits, multiples of ten or multiples of one hundred, the majority of the pupils (more than 50% in each case) succeeded in translating them correctly from the verbal to the symbolic form, with the highest success rate (86 percent) being *recorded* in the case of single digit numbers. This result was not surprising. In the case of multiples of ten or of one hundred, for example, all the pupils had to do was to write down a single digit and then add the appropriate number of zeros.

An interesting result was obtained in the case of the question (Q. 11), the first part of which required pupils to write down the numeral representing the number *one hundred nine*. Only 35% of the pupils were able to write down the correct numeral. The most popular incorrect answer given by pupils in this regard was 1009, which corresponds to the sound pupils heard when the teacher pronounced the words *one hundred nine*. Before the 1990s the teacher would have said *one hundred* ***and*** *nine*. In the 1990s, the Ministry of Education decided to remove the 'and' so that it became *one hundred nine*. One could argue that this change could have contributed to the low success rate in this case. However, it is not clear to what extent this contributed if at all.

Once pupils had written down the (correct) numeral for the stated single or multi-digit numbers, they were also required to find numbers that were *one more than* or *one less than* the numbers already written down. The study has shown that when single digits were involved this was a relatively easy task, as whether the situation involved *more than* or *less than* about half the total number of pupils who sat for the test succeeded in writing down the correct number. My observation of the way pupils worked out the answers during the test, particularly in the case of questions 5 and 6, suggested that this relatively good level of success was made possible by the fact that many of them used their fingers to count up or count down from the earlier numbers to arrive at the correct result. Other pupils even marked out tallies and counted from one up to the given number, and then proceeded either to add one more or take away one as required, to arrive at the final answer, which they then wrote down in numerical form. The task of writing down numbers that were one more or one less than a given double-digit number (Q. 6) was also done rather well, with about 60% of the pupils succeeding in reaching the correct result.

However, disaster was the result in questions 11 and 12 when, having written down a three-digit number, pupils attempted to find another number, one more or one less than the first number. Only 19 percent could write down the number that is *one more* that one hundred nine; and 7 percent the number that is *one less* than two hundred. Two possibilities explain these poor results. First, I observed that many of the pupils who had succeeded in writing down the numeral for one hundred nine marked out 109 tallies, and then added one more. They then tried to count all the tallies together to get the value of 109 + 1. Similarly, pupils marked out 200 tallies, crossed out one, and then tried to count them all! As one can imagine, marking out that many tallies raises the possibility of ending up with an incorrect number of tallies. There was also the likelihood of miscounting the tallies. In other words, inefficient counting strategies were probably the main cause of pupils' poor performance in this case too. Second, it would seem that in trying to work out the answers many of the Grade 2 pupils considered the relative sizes of 0 and 9 and concluded, rightly, that 9 is greater than 0. Consequently, they could not imagine that a number that is *one more* that one hundred nine could end in a zero. Nor could a number that is *one less* than two hundred end with a nine, since 9 is greater than 0. Regardless of how one looks at it, the pupils' efforts here were doomed to failure.

5.2.1.5 Using Grouping to Perform Multiplication and Division by 5 or 10

As mentioned in Chapter 4, questions 7, 8 and 13 required determining quickly how many apples or cakes were in a given number of boxes or bags, having been told how many each bag or box held. Following this, pupils were also required to tell how many boxes or bags would be needed to pack a certain number of apples or cakes. These questions produced mixed results, depending on the numbers of bags or boxes involved. In Question 7 for example, where pupils were required to work out quickly how many cakes were in one box plus one loose cake the rate of success was 81 percent. But in a similar question [13 (b)], which required determining the total number of apples in four bags plus three loose apples, only 44% were successful in working out the correct answer. These results were thus determined by the sizes of the numbers involved than by whether or not the situation required multiplication or division. We have already noted that, whereas children can cope quite easily with situations involving small numbers, larger numbers tended to confuse them. Indeed as Brown (1981) notes, when children are a little unsure about how to handle operations with small numbers, bigger numbers push them over their threshold.

5.2.1.6 Solving Addition or Subtraction Word Problems Involving Single or Double-digit Numbers

Pupils demonstrated near masterly levels of success (over 90 percent) with regard to solution of word problems involving adding two single-digit numbers to get another single digit number. This was an interesting result because, as the reader will remember, at the beginning of the study (specifically during the Staff Development Workshop) all five teachers in experimental schools were convinced that Grade 2 pupils could not cope with solving word problems. This belief may have been shared and perpetuated by curriculum developers and textbook writers who until June 2008, when new Grade 4 mathematics textbooks were produced, had not included word problems in lower primary mathematics.

This belief, therefore, was in contradiction to research evidence that suggested that even before beginning formal education children can solve simple word problems. Baroody and Standifer (1993) report the following incident involving a new Grade 1 child in a school in a certain country (most probably the United States), an incident which obviously does not represent the intellectual behaviour of young children in that country alone (see Ginsburg and Baron, 1993).

> Alissa, a new first grader, was shown the expression 3 + 2 = ? and asked: "How much is three plus two?" She squirmed in her seat and finally confessed: "I haven't learned that yet". Asked how much three pennies and two more pennies are altogether, Alissa quickly put up three fingers on her left hand and two fingers on her right hand, counted the fingers, and responded cheerfully: "Five" (Baroody and Standifer, 1993, p. 72).

This example shows that what sometimes causes young children to fail to answer teachers' questions in mathematics is not a lack of conceptual understanding but a premature introduction of symbolic language. When teachers do this they fail to connect with children's existing mathematical knowledge. Indeed,

> All children [regardless of where they live in the world] develop in an environment containing a multitude of quantitative phenomena and events. From infancy, children encounter small, discrete objects that can be manipulated, touched, and counted…..The social environments [may] vary considerably from culture to culture. Some cultures offer educational television; others do not. Some cultures are characterized by an everyday written mathematics-as on telephones or houses- and others are not. But almost all known cultures offer a fundamental mathematical system….In brief, throughout development, before and after school, children are normally exposed to physical and social environments rich in mathematical opportunities (Ginsburg and Baron 1993, pp. 4 – 5).

As demonstrated by the high percentage (around 70 percent) of correct answers to the two questions involving word problems, the majority of lower primary children in Zambia *can* solve word problems, provided teachers use smaller numbers such as 2 and 3 and a familiar language in asking the questions.

5.2.1.7 Performing Multiplication by 4

Pupils' performance on numerical problems involving the times table for 4 (Q. 14) showed that multiplication involving small numbers (4 by 2) was more likely to be performed correctly by Grade 2 pupils than multiplication involving slightly larger numbers (e.g. 4 by 5), especially if the final answer is a double-digit number. This again stresses the fact that, regardless of the operation involved young children find situations involving smaller numbers easier to cope with than those involving larger numbers (Brown, 1981).

5.2.1.8 Adding or Subtracting 10 or 100

The results obtained from pupils' performance on questions 15 and 16 show that the ability of participating Grade 2 pupils to add ten or 100 to another number was influenced mainly by two factors: use of an inefficient counting strategy; and not writing down the bigger number first before performing the addition. For example, whereas 72 percent of the pupils succeeded in adding ten to a single digit number to get a result between 10 and 20, only 23 percent were successful in adding ten to 92 to get a three-digit number. These results are explained by the fact that when calculating answers in this regard most of the pupils were seen counting either using their fingers and toes; or first marked out a series of tallies representing the numbers to be added and then counted all the tallies together to obtain the required result. For example, in adding 10 to 92, most of the pupils first marked out 92 tallies on the question paper, added 10 more, and then counted them all to get the final result. In doing this errors were sometimes made in the number of tallies marked out, so that even if adding the tallies was done accurately the final result would still be wrong. Again in adding 100 to 10 most pupils worked as above, that is they marked out 100 tallies and ten more, then attempted to count together all the tallies. The result was that only 14 percent got the right answer. In this case the results would have probably been very different if the pupils had written down the bigger number first and then counted-on from there. As Nunes and Bryant (1996) point out: "The child who sees that she does not laboriously have to re-count the larger set may have realized that this set can be treated as a larger unit which can be combined with a smaller one" (p. 53).

I expected 'taking away' to be more difficult than adding, since other things being equal subtraction is conceptually more difficult to grasp than addition (see Brown, 1981). However, the percentage of success when pupils were asked to 'take away 10 from 50' was high (i.e. 54%), which perhaps reflects the fact that pupils were subtracting 10 from 50, a multiple of 10. A more surprising result was obtained in the question involving taking away one hundred from four hindered, where 45 percent were successful. This result, however, may reflect the fact that in asking the question teachers used a popular Lusaka street language term for *one hundred*, namely one *zali*. This effectively reduced the question to "Take away one *zali* from four *zali*"; which simplified the calculation considerably. But "Take away ten from seven hundred" was beyond the ability of participating Grade 2 pupils in that only 2 of them (representing 1 percent) succeeded in arriving at the correct answer. In general, the performance of participating Grade 2 pupils on questions 15 and 16 show that before the study the pupils were so dependent on counting strategies, mainly the *counting-all,* that their ability to add any two positive numbers was limited to situations where this strategy was effective, namely when dealing with small numbers.

5.2.1.9 Grouping Objects to Show Proportions

As mentioned in the last chapter, Question 17 examined Grade 2 pupils' understanding of the concept of *half* by requiring them physically to encircle half of eight balls shown in a diagram. That only 50 percent of the pupils succeeded in doing this was surprising. To perform the task pupils needed first to verify (by counting) that there were eight balls altogether; and then to encircle half of eight, which is four. Why did 50% of the pupils fail to do this? It is unlikely that the difficulty of the task lay in the process of counting, since in other parts of the test the same pupils demonstrated the ability to count in situations more complex than the one described above. It would seem that the low success rate in this regard indicated the pupils' failure to understand the meaning of the term *half*, which class teachers in administering the test rendered 'half' in Chinyanja. Indeed in a study I conducted with Grade 6 pupils in Zambia (Tabakamulamu, 2001), which investigated difficulties primary school pupils encountered in reading mathematics, I found that most of the mathematical terms (such as *prime number*, *difference*, etc.) which teachers used during mathematics lessons assuming pupils understand them, were in fact words whose meanings the majority of pupils did not know.

Before leaving this section, it is important to summarise the pre-intervention numeracy achievement of participating Grade 2 pupils in experimental and control schools, as this was used as baseline data to measure the effectiveness of the intervention. Table 13 is my attempt to do this. However, to understand the contents of the table requires some background information. The book *Children's Understanding of Mathematics: 11 – 16* edited by Hart (1981), which reports the results of a study in mathematics education conducted by Chelsea College in the UK, uses the term *level of understanding* to describe the hierarchy of understanding in mathematics achieved by participating secondary school pupils in England and Wales. For example, level 0 describes failure to cope with questions focusing on the most basic ideas in a given topic; while level 3 (the highest) relates to a mature performance, that is a performance indicating that pupils had reached a more or less mature understanding of the topic in question. In Table 13, I use *level of understanding* to mean percentages of success as follows: Level 0: 0 to 25 percent; level 1: 26 to 50 percent; level 2: 51 to 75 percent; and Level 3: 76 percent and above. A tick [✔] indicates the level of understanding reached by pupils on a given test item.

Table 13: Grade 2 pupils' pre-treatment levels of understanding in numeracy

Summary of task	Level 0	Level 1	Level 2	Level 3
Q.1 Determining number of items in a small finite set.				✓
Q.2 (a) Adding more items to a small set to equal a given cardinality.				✓
(b) Given the cardinality of a small set, to draw the same number of objects				✓
Q.3 (a) Which number is three more than six?				✓
(b) Which number is seven more than eight?				✓
(c) Which number is three less than eight?				✓
(d) Which number is six less than eight?				✓
Q.4(a_1) Pupils shown a number line with two missing numbers. Range: 0 to 10. Find first (lower) missing number on the number line.				✓
(a_2) Find second (higher) missing number on the number line.				✓
Q.5 (a_1) Write 'nine' in the box next to the tortoise.				✓
(a_2) What number is one more than nine?			✓	
(b_1) Write 'eighty-nine' in the box next to the hut.			✓	
(b_2) What number is one more than eighty-nine?		✓		
Q.6 (a_1) Write 'thirty' in the box next to the boat.				✓
(a_2) What number is one less than thirty?			✓	
(b_1) Write 'seventy-six' in the box next to the car.				✓
(b_2) What number is one less than seventy-six?			✓	
Q.7 Pupils shown boxes containing five cakes each. Task: To find total number of cakes in one box plus one loose cake.				✓
Q.8 Same context as in Q.7 above. Task: To write down the number of boxes needed to pack ten cakes.			✓	
Q.9 Three boys and four girls get on a bus. How many children get on the bus?				✓
Q.10(a_1) Pupils shown a number line with two missing numbers. Range 0 to 50. Find first (lower) missing number on the number line.		✓		
(a_2) Find second (higher) missing number on the number line.	✓			
Q.11(a_1)Write 'one hundred nine' in the box next to the crab.		✓		
(a_2) What number is *one more than* one hundred nine?	✓			
Q.12 (a_1) Write 'two hundred' in the box next to the lorry.			✓	
(a_2) What number is *one less than* two hundred?	✓			
Q.13 (a)Pupils shown bags containing ten apples each. Task: To find total number of apples in three bags.			✓	
(b) Same context as in 13 (a). Task: To find total number of apples in four bags plus three loose apples.		✓		
Q.14 (a) Four times two equals…				✓
(b) Four times five equals…			✓	
Q.15 (a) Add ten to nine			✓	
(b) Add ten to ninety-two	✓			
(c) Take away ten from fifty			✓	
(d) Take away ten from seven hundred	✓			
Q.16 (a) Add one hundred to nine	✓			
(b) Take away one hundred from four hundred		✓		
Q.17 Given eight balls, to encircle half the number of balls		✓		
Q.18 (a) Five people are on a bus. Eight more get on. How many people are on the bus?			✓	
(b) Twelve people are on a bus. Five get off. How many people are on the bus now?				✓

What does Table 13 show about numeracy attainment in participating Grade 2 classes before the study? It shows that participating Grade 2 pupils in both experimental and control schools could deal comfortably with various situations in mathematics involving small numbers between 0 and 10. In this regard they could determine the number of elements in a set; write down correct numerals; identify missing numbers on an incomplete number line; solve word problems; and in general handle situations involving addition, subtraction, multiplication and division. They could also deal relatively easily with situations involving writing down correct numerals representing two-digit numbers and in some cases three-digit numbers, particularly where only multiples of 100 were involved. As already suggested, the main reason why the Grade 2 pupils involved displayed near mastery competency in performing mathematics tasks relating to the situations above is that such situations can be handled using a counting-all strategy, which often included marking out of tallies to represent the numbers involved.

But the table also shows that the pupils experienced particular difficulties when faced with mathematics problems that did not lend themselves easily to the use of a counting-all strategy, such as taking away ten from seven hundred, adding ten to ninety-two, or adding one hundred to nine; and situations that challenged pupils' understanding that 9 is bigger that 0. As an example of this later case, the Grade 2 pupils failed to find the answer to questions such as: "What number is one les than two hundred?", or "What number is one more than one hundred nine?" To get correct answers to such questions the children needed to understand that a number ending in zero can be greater than one ending in a nine, a fact which it seems they had not yet grasped.

5.2.2 Learning to Use Mental Strategies for Double-digit Addition and Subtraction

As mentioned in Chapter 7, to develop proficiency in using strategic methods for two digit addition and subtraction pupils needed to master the underlying pre-requisite skills, namely: counting forward and back in 1s, 2s, 3s, 4s, 5s and 10s; partitioning the single-digit numbers 6, 7, 8, and 9 into 5s and ones, and double-digit numbers such as 79 into a near multiple of 10, 5s, and 1s, and use these in the context of single or multi-digit digit addition and subtraction; and use effectively the empty number line as a model for performing addition and subtraction. To what extent did the pupils in experimental schools develop these skills?

5.2.2.1 Counting Forward and Back in 1s, 2s, 3s, 4s, 5s and 10s

The results of this study have shown that most of the Grade 2 pupils in experimental schools learned to count forward easily in 1s, 2s, 3s, 5s, and 10s, either starting with these numbers or from any other numbers; but had difficulty doing so in 4s. An examination of Grade 1 and Grade 2 mathematics textbooks indicated that, counting forwards in 1s, 2s, 3s, 5s, and 10s was part of the existing mathematics curriculum, whereas counting in 4s was not. This explains why the pupils seemed to have some difficulty mastering counting in fours. An interesting result in this regard was that, whereas pupils could easily guess the next term in a sequence involving the above numbers, for example the sequence 5, 10, 15, 25, and so on, they had difficulty identifying missing numbers when the same sequence was presented as follows: 5, 10, ---, 20, 25, 30, ----, 40. Why was this so? Part of the answer may be that, whereas one can say the numbers in a sequence without understanding their meaning (see Nunes and Bryant, 1996), it is impossible to identify missing numbers in a sequence without understanding how individual numbers are generated.

Compared to counting forward, counting back was not a regular activity in Grade 2 mathematics. Consequently, pupils found counting back more difficult than counting forward. However, after continued encouragement from teachers, pupils eventually developed some facility in counting back in 1s, 2s, 3s, 5s, and 10s; with counting back in 5s and 10s being much easier particularly when multiples of these numbers were involved (for example 90, 80, 70, 60, and so on). This was probably due to the familiarity of multiples of 10 for example, which come up from time to time in the mathematical activities of primary school children in Zambia from Grade 1 upwards. As one would expect, counting back in 4s was again more difficult than counting back in either 1s, or 2s or 3s or 5s or 10s.

An interesting result with regard to counting back in ones was that, pupils simply could not identify the next number when ‘crossing-over’ from a higher decade to the one immediately below it. That is, they failed to see that when counting back in ones from 30 (or 50 or 40, and so on) the next number is 29 (or 49 or 39). Other than that, the pupils were perhaps doing this for the first time. It would seem that the behaviour of the pupils in this regard suggested that they could not imagine a number that is one less than 50, for example, could end in a nine. It is possible that the minds of the pupils were fixed on the fact that 9 is greater than 0, regardless of the size of the numbers involved. Nevertheless, it was clear from observing lessons that in general Grade 2 pupils in experimental schools could learn to count forward and back, which is a key pre-requisite skill required for developing confidence in using strategic methods for double-digit addition and subtraction (Skinner et al., 2001).

5.2.2.2 Partitioning Single and Double-digit Numbers into Multiples of 10, 5s and 1s

Lesson observations, teachers' journal entries and interviews with teachers showed that almost all Grade 2 children in experimental schools could write the single-digit numbers 6, 7, 8 or 9 initially as a sum of any two smaller numbers (e.g. writing 8 as 6 + 2 or 5 + 3, and so on) and later in terms of 5s and 1s. This was the form deemed most efficient for purposes of performing two-digit addition and subtraction, as it enabled pupils to avoid the concepts of 'carrying' and 'borrowing, whose logic many primary school pupils do not understand.

With regard to partitioning of two-digit numbers, the study aimed to help pupils realise that it is more efficient when adding or subtracting two-digit numbers first to partition them into a near multiple of 10, 5 and 1s, for example writing 59 as 50 + 5 + 4. However, the majority of the pupils often did not use the above form and partitioned numbers such as 95 for example as 10 + 50 + 10 + 10 + 10 + 5 or 34 as 10 + 10 + 10 + 4. Thus many pupils failed to identify a near multiple of 10 and merely expressed two-digit numbers merely as a sum of smaller numbers, which did not help much when adding and subtracting two-digit numbers.

5.2.2.3 Double-digit Addition and Subtraction Using the 1010 and N10 Methods

As mentioned in Chapter 7, the three main methods used to add and/or subtract whole numbers were based on one's ability to partition numbers as explained above. The first method, called 1010, involved partitioning *both* numbers, adding or subtracting the separate parts, and finally combining the sums obtained; while the second, called the N10 method, required leaving the first (actually the larger) number intact and only partitioning the second number. The final method, namely using the Empty Number Line (ENL) is similar to the N10 method but in addition requires using a number line to work out the final answer. I discuss results relating to use of the 1010 and N10 methods in this section, and leave results relating to pupils' use of the ENL for the next section.

The results of the study showed that more than half the total number of Grade 2 pupils in experimental schools learned easily how to use the 1010 method to add two-digit whole numbers accurately, although as mentioned earlier their partitioning of the two numbers being added did not always conform to the form *near multiple of 10 + 5 + ones*. Use of the N10 method on the other hand proved rather difficult for the pupils and, it would seem, for the teachers as well, many of whom did not even reach the part of the experimental curriculum where the N10 method was discussed. Indeed, as mentioned in Chapter 4, Dutch educators concluded that although the N10 method was the more efficient of the two methods, children tended to favour the 1010 method as it is conceptually easier to learn.

As mentioned in Chapter 4, subtraction using the 1010 method only works well if the digit representing the ones in the first number is lager than that representing ones in the number being subtracted. Unfortunately, many children did not understand this and tried to use the 1010 method throughout even in situations where the N10 method would be more appropriate. Because of this, many pupils usually failed to reach correct answers when undertaking two-digit subtraction.

5.2.2.4 Double-digit Addition and Subtraction Using an Empty Number Line

The results of this study have demonstrated that the ability of Grade 2 pupils to use the empty number line as a model for performing two-digit whole number addition depended on whether or not the teacher him or herself was clear about how the procedure worked. For example, the pupils taught by Teacher D, who seemed to have understood most completely how to implement the intervention, used the empty number line more or less efficiently. They knew which number to write down first and which one to split before drawing the number line; they knew in which direction to go along the number line; and could tell, and voiced their objections, if colleagues seemed to use the empty number line wrongly or inappropriately.

However, when asked to defend their chosen solution method the pupils could only explain the procedure itself and not *why* they thought it worked.

The other Grade 2 pupils, most of whose teachers either had not reached the topic when the study ended or were not very clear in their own minds how to use the empty number line, did not use the empty number line efficiently. When asked to use the empty number line to perform two-digit addition many of these pupils reverted to inefficient counting methods (using both fingers and toes) and only after doing this would they draw empty number lines to show what they had done. Therefore, the pupils were not using the empty number line as a model for performing addition, but merely to demonstrate diagrammatically an action they had already completed using other methods. There was also a tendency on the part of these pupils to include unnecessary numbers on empty number lines, making them look like ordinary (i.e. full) number lines. The main difficulty here, then, was the teachers' own failure to understand how an empty number line could be used as a model for two-digit addition and, consequently, their inability to explain its use to pupils appropriately.

With regard to the Grade 2 pupils' ability to use the empty number line to perform two-digit whole number subtraction even fewer pupils demonstrated proficiency. Indeed, as mentioned earlier, by the end of the study only Teacher D and Teacher E's classes had moved far enough in the experimental curriculum to discuss meaningfully subtraction using the empty number line. Again Teacher D's pupils exhibited better understanding of what to do than did all the other Grade 2 classes from experimental schools. Although their answers in this regard were rather inaccurate, the pupils nevertheless demonstrated that they understood the procedure involved and could explain it.

5.2.3 Quantitative Post-treatment Differences between Experimental and Control Schools

As shown in Chapter 7, although there was no significant difference in performance on the pre-test between the experimental and control groups, indicating that the two groups were comparable before the experimental group received the treatment, post-test results showed that the mean of the experimental group was significantly greater than that of the control group. This result suggested that teaching mathematics meaningfully to young children, as we tried to do in this study, that is, ensuring that learners did not only absorb mathematical thought (i.e. memorising formulae, facts and figures) but also understand and were personally involved in constructing knowledge, results in better performance in mathematics. The results of the study thus also indicated that strategic methods for double-digit addition and subtraction can be taught.

Related t-tests indicated that both comparison groups had made significant gains before and after the intervention. What does this mean? The answer is suggested by the sizes of the before and after standard deviations for each group. As shown in Chapter 7, the pre-test and post-test standard deviations for the experimental group were respectively 7.69 and 7.35. This suggests that implementation of the experimental mathematics syllabus reduced variation between extreme grades, that is, the lowest grades and the highest grades. In other words, experiencing the experimental syllabus enabled not only high achievers to perform better, but also low achievers. This should be expected since pupils were allowed to use methods that worked for them, something that could have convinced even those who were regarded as low achievers to believe that they had something to contribute during mathematics lessons, thereby improving their interest in learning mathematics. The situation was different with the control group, which had pre-test and post-test standard deviations respectively as 7.61 and 7.91. This suggests that mathematics teaching in control schools, where teachers continued to emphasise the use of vertical (textbook) procedures for double-digit addition and subtraction, increased variation between extreme scores. In other words, low achievers continued to perform poorly while high achievers did even better. There is thus the possibility that mathematics teaching in control schools did not really affect pupil performance, since high achievers could still have excelled despite the teaching.

One could argue that Tables 1 and 2 in Chapter 4 show that teachers in experimental schools were generally older and more experienced than teachers in control schools, which could have affected the post-test performances of the two groups. I would argue that this conclusion might not be true. As I pointed out in Chapter 4, while there were differences in teaching experience in general between teachers in experimental schools and their colleagues in control schools, the actual difference between them with regard to teaching experience at Grade 2 level was not much. In fact, at the time of the study the longest serving teacher in experimental schools (i.e. Teacher E) had taught at this level for less than a year. Furthermore, in Zambia length of teaching experience does not necessarily mean more efficient teaching. It sometimes means the reverse. Because of poor conditions of service (for example poor salaries) older teachers tend to be more frustrated than younger teachers, so that it is the younger teachers who show more enthusiasm for the job, spend more time on lesson preparation and, in general, are more available to help pupils when needed.

5.2.4 Qualitative Post-treatment Differences between Experimental and Control Schools

In terms of qualitative differences between the experimental and control groups, the results of this study have shown that when young children are taught mathematics meaningfully they develop more positive attitudes towards mathematics and rates of absence from mathematics lessons are drastically reduced. Instead of trying to dodge mathematics lessons or learning the subject just because they are required to do so, pupils choose to attend the lessons and begin to show genuine interest in their own progress in the subject.

As shown in Chapter 7, the significance of these gains only become apparent when one looks at how the situation was before the study. Before the study very little learning took place in mathematics; and rates of absenteeism were high (Maimbolwa-Sinyangwe and Chilangwa, 1995). Since the study lasted only one school term, and seemed to reverse the above characteristics of the ‘normal’ primary school mathematics classroom, one wonders what would happen if it had taken longer (perhaps one year), covered a larger number of schools, and all participating teachers were qualified at diploma level. These questions suggest areas for further research.

5.3 Chapter Summary

This Chapter discussed the effect the study had on Grade 2 teachers and their pupils. It showed that asking teachers to implement change at school level initially generates all sorts of fears and concerns in that it often meant overturning settled ways of doing ones' duties gained over a long period of time. Unless such fears and concerns are identified and addressed, not much change would occur. The fears and concerns entertained by teachers in experimental schools at the beginning of the study were influenced by the beliefs they had concerning mathematics teaching and learning, particularly the belief that young children entered school with no mathematical experiences of their own and needed to be taught the finished products of mathematics for them to become knowledgeable in the subject. Such beliefs were reflected in the teachers' classroom practices, which were not helped by the fact that although professionally trained the teachers themselves had an insecure knowledge of mathematics. This made the teachers over reliant on teaching handbooks, whose contents were passed on to pupils usually in the exact form. However, by the end of the study it was clear that most of the teachers in experimental schools had accepted at least mentally that their previous teaching practices inhibited pupils' mathematical learning and needed to be revised in line with the teaching approaches being promoted in the study.

This chapter also showed that, in general the Grade 2 pupils in the sample were over reliant on inefficient counting strategies. This meant that they experienced difficulty solving any mathematics problems that did not readily allow the use of such strategies. Nevertheless, the study indicated that Grade 2 pupils had the ability to learn to use strategic methods for double-digit addition and subtraction, and were only limited in this regard by their teachers' ability to help them develop the necessary pre-requisite skills. The next chapter focuses on the conclusions, recommendations, and further research that seem to arise from this study.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.0 Introduction

This study set out to assess the extent to which teachers in Zambia could learn to foster the use of strategies for mental calculation relating to double-digit addition and subtraction in the early primary grades; and to determine the resulting impact of their teaching on the performance of pupils in mathematics. Data relating to teacher learning in this regard were collected using a variety of qualitative methods, including individual and group interviews, lesson observations, and teachers' journal entries. Corresponding data on pupil learning in the classroom was assessed mainly by means of two numeracy tests: one administered at the beginning of the study, and the other at the end of the study. The aim of this chapter is to summarise the main findings of the study. In view of this, the rest of the chapter is structured as follows. Section 6.1 draws the conclusions based on the findings of the study; Section 6.2 the gives recommendations of the study, mainly for consideration by policy-makers and planners in the Ministry of Education; and Section 6.3 proposes areas of further research arising from the findings of the study.

6.1 Conclusions

6.1.1 The Teachers

This study has confirmed that, asking teachers at whatever level of education to adopt new ways of teaching, particularly if such ways of teaching constituted radical departures from what the teachers are used to doing, generates all sorts of fears and concerns in teachers' minds. While such fears and concerns might look different on the surface, their underlying cause is the importance teachers attach to ensuring that their pupils eventually pass the relevant public examinations, as their colleagues and society in general assessed their effectiveness on this basis. In the present study, the five teachers from experimental schools wanted to ensure that their pupils, although only in Grade 2 at the time of the study, eventually passed the Grade 7 terminal examination that determined who proceeded to enter secondary education. The teachers did not want anything to jeopardise their pupils' chances in this regard.

Although by the end of the study four of the five teachers from experimental schools who implemented the study had overcome their initial concerns in this regard, the fact that they experienced them initially indicated that when introducing change at school level the focus should not only be on how change benefits pupil learning. It is also important to consider how teachers participating in the change process are affected emotionally, and to do something to address those fears and concerns. Change agents could address teacher concerns in this regard through careful questioning, to enable teachers to verbalise these concerns, and then to confront them. Nevertheless, as this study has shown, doing this does not necessarily mean that *all* participating teachers will overcome their initial concerns during the process of change, as some might in fact pick up new ones along the way.

Teachers' concerns when implementing change at school level are influenced and somewhat intertwined with the beliefs they hold about the teaching and learning of particular school subjects. All five participating Grade 2 teachers from experimental schools initially believed that young children entered school with no knowledge of mathematics whatsoever, and relied on their teachers to give them that knowledge. Consequently, they considered it their duty to *transmit* to pupils' minds the finished products of mathematics as given in official textbooks, handbooks, and teacher' guides. In view if this, the teachers expected only correct answers from pupils and saw no need to investigate further pupils' answers, whether right or wrong. Such views of mathematics teaching and learning were influenced largely by the nature of the initial teacher education programmes the teachers underwent. These training programmes tended to focus too much on the need for student teachers to transmit knowledge to pupils and to ensure that pupils passed public examinations. The programmes also filled student teachers' working days with so much to learn from manuals and handbooks that student teachers had no room to exercise personal initiative.

Despite the fact that all participating teachers in experimental schools were professionally qualified to teach in primary school, their knowledge of mathematics at the beginning of the study and throughout the study was rather insecure. This made it difficult for many of the five teachers from experimental schools first to understand the aims and objectives of the study, and to implement the study with fidelity. Only one teacher among the five, who at the time of the study held both the Primary Teachers' Certificate and the Primary Teachers' Diploma and had participated in previous school change programmes as a trainers of trainers, seemed to understand what the study was about and implemented it more or less as it was meant to be implemented. Thus participation in previous school innovations and holding of further teaching qualifications, were more likely to help teachers have the confidence to foster the use of strategies for mental calculation relating to double-digit addition and subtraction.

The beliefs about mathematics teaching and learning held by the five teachers from experimental schools, the lack of initiative created by their initial teacher education programmes, and the fact that these training programmes did not enable them to develop a secure knowledge of mathematics for teaching at Grade 2 level, had a more or less predictable effect on the teachers' classroom practices. They all initially did not use children's mathematical thinking when planning lessons but stuck to contents of textbooks and teachers' handbooks, and entertained only correct answers from pupils. The pupils had nothing to offer in terms of understanding of mathematics subject matter, let alone express their own ideas and views about mathematics.

Nevertheless, this study showed even for a relatively short period of about 10 weeks, it was possible for four of the five teachers from experimental schools to begin moving towards the kinds of teaching practices that supported the use of strategies for mental calculation in early primary mathematics. Indeed, by the end of the study the four teachers' appeared to be making genuine efforts to adopt a social constructivist approach to mathematics teaching and learning, which compares mathematics learning in the classroom to participation in a cultural activity. In such classrooms the views and mathematical ideas of individual pupils are shared and debated by fellow pupils and by the teacher, so that at the end of the day what individual pupils take away from the classroom are negotiated meanings. The results of this study suggest that the four teachers' movement in this direction was only limited by the relatively short duration of the study.

6.1.2 The Pupils

For pupils the focus of the study was to determine the impact of using strategies for mental calculation relating to double-digit addition and subtraction on their classroom performance in mathematics. As mentioned in Section 6.0, this assessment was mainly based on a comparison of the performance of the pupils on the pre-test, which was used as a baseline for measuring the effectiveness of the treatment implemented in experimental schools, and on the post-test. The performance of the pupils on the pre-test indicated no significant differences between control and experimental schools, suggesting that the two groups were initially comparable. However, post-test results showed a significant difference in performance between the two comparison groups, with an effect size of 0.3 in favour of the experimental schools. While such an effect size might appear small, the literature on school innovations indicates that quasi-experimental designs rarely produce higher effects, suggesting that the difference in performance was due to the treatment implemented in experimental schools.

It was not only the performance in experimental schools that was significantly better than that in control schools. The pre-test and post-test standard deviations of the two comparison groups suggested strongly that the experimental numeracy development approach reduced variation among scores in experimental schools. That is, both low and high achievers improved their performance on the post-test, which was another welcome effect of this study. On the other hand, in control schools variation among grades increased, suggesting that the teaching of standard procedures widened the achievement gap between low achievers and high achievers. It is possible, therefore, to argue that the teaching of standard procedures did not really affect pupil learning, since high achievers would always do better regardless of the teaching.

There were also unexpected post-treatment *qualitative* differences in behaviour between the control and experimental groups. The experimental numeracy development approach inculcated more positive attitudes towards learning mathematics among pupils in experimental schools and significantly reduced levels of absenteeism from mathematics lessons. However, one might argue that since the control schools were not assessed in this regard, it is not possible to compare them with experimental schools. I do not think this is a valid argument, for the following reasons. Before the study, national surveys conducted by the Ministry of Education on learning achievement in selected subjects (including mathematics) at primary school level, *always* identified absenteeism from lessons as a serious problem that negatively affected pupil learning at this level of education. This study reversed the trend in experimental schools. Therefore, an important achievement of the study was to show that high rates of absenteeism in primary (mathematics) lessons were linked to the way mathematics was often taught, and to the amount of time teachers took to prepare themselves for lessons.

6.2 Recommendations

The results of this study point to the following recommendations:

- All the five Grade 2 teachers who implemented the experimental curriculum had an inadequate understanding of the mathematics they were supposed to be teaching. It maybe necessary, therefore, to design some way of measuring the mathematics content knowledge of primary school teachers, and supplementing it where necessary, so that all those who teach mathematics at this level of education can do so with confidence.

- Change or reform at school level, whether in mathematics or in any other school subject, should not focus only on benefits to learners but should determine how the teachers involved are affected emotionally. This is especially the case when such change requires teachers to think differently about the way they perform their duties.

- Mathematics educators in primary colleges of education should aim to acquaint teachers with a deeper understanding of basic ideas in mathematics, and also introduce them to theoretical issues influencing thinking in mathematics education at a given time.

- It is important for teachers, when teaching standard (i.e. textbook) solution procedures in lower primary mathematics, to make it clear to pupils that such procedures arose from and were a refinement of pupils' own solution strategies. This would help children realise that their own mathematical views were valued, which would increase their receptivity of standard procedures and ownership of the learning process, and might help pupils to work harder to understand the application of such procedures.

6.3 Further Research

The results of this study suggest that more research is needed in the following areas:

- The extent to which both Grade 2 teachers and pupils would adopt the use of strategies for mental calculation relating to double-digit addition and subtraction in the early primary grades was investigated for about 10 weeks only. How would the results change if a longer period of time was involved, and the study covered a larger part of Zambia? And what would the results be like, if the use of strategies for mental calculation relating to multiplication and subtraction were also involved?

- Grade 2 children were generally so reliant on inefficient counting strategies to work out answers to questions in the numeracy tests that wrong answers were likely whenever they faced questions that did not readily allow the use of this method. Is the use of such counting strategies inevitable at Grade 2 level, or did prevailing teaching practices have something to do with it?

- More than 80% of Grade 2 pupils in experimental schools experienced difficulty counting back from 30 to 29 (or from 40 to 39, 70 to 69, and so on). Why was this so? The pupils also had difficulty translating number names for numbers with more than two digits from the verbal to the symbolic form. Was this a question of conceptual understanding or merely a lack of understanding of number names?

- Throughout the study Grade 2 pupils experienced difficulty explaining *why* their solution methods were valid, despite the fact that they used Zambian languages as media of communication? Since the pupils could communicate easily in these languages, why did they experience this difficulty?

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APPENDIX 1

STAFF DEVELOPMENT WORKSHOP TRAINING MANUAL

Teaching Mental Calculation Strategies for Double-digit Whole Number Addition and Subtraction

Staff Development Training Manual for Grade 2 Teachers in Experimental Schools

16 – 27 August 2004

Prepared by

Muhau Tabakamulamu

School of Education

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TABLE OF CONTENTS

1. WHAT ARE MENTAL CALCULATION STRATEGIES

Many adults in Zambia will be familiar with the term *mental arithmetic*. It referred to mathematics questions which teachers gave orally with the strict requirement that pupils worked out answers 'in the head'. Writing anything on a piece of paper to facilitate calculation was not allowed.

The term *mental calculation* as used here includes both working 'in the head' and making jottings on a piece of paper to find the answer. Because of this difference, some mathematics educators describe mental arithmetic as 'working *in* the head', while mental calculation is viewed as 'working *with* the head'. When mathematics educators use the term *mental calculation strategies* they mean *informal* solution methods usually developed by children which they use to solve a variety of arithmetic problems. These methods are strategic in that they involve an element of choice. That is, children have to *decide* which method or strategy will work best in a given situation.

Why should teachers be interested in children's mental strategies? Research has indicated that children who use mental calculation strategies to perform, say, double-digit addition and subtraction are more likely to produce correct answers than their peers who use standard (vertical) procedures. This is mainly because standard calculation procedures often involve the *column value* aspect of place value (e.g. seeing 37 as 3 tens and 7 ones). On the other hand mental calculation uses the *quantity value* aspect of place value, for example seeing 37 as 30 and 7; it thus involves working with *complete* numbers rather than with single digits as is the case with standard procedures (Thompson, 2001). But how does this affect the likelihood that children will get correct answers? As Nunes and Bryant (1996) have observed,

[The] system of signs that…children use when they calculate…allows them to calculate and to think of the values they are working with at the same time. When tens and hundreds are added, for example, they are spoken of as tens and hundreds. In contrast, in written arithmetic we set the meaning of the numbers aside during calculation. We operate with digits and speak about them as if they were all units, following the same rules as we move from units to tens and hundreds. This approach seems to detach the children from the meaning of what they are trying to calculate and thereby makes it easier for bugs [i.e. errors] to appear in their solutions (p. 107).

This booklet has been prepared to achieve two main objectives:

• To acquaint you with the range of children's mental calculation strategies for double-digit addition and subtraction; and

• To help you develop an understanding of how you might foster the use of these strategies in early primary mathematics.

2. SINGLE-DIGIT ADDITION AND SUBTRACTION

Research indicates that initially young children use two main strategies to perform single-digit addition and subtraction, namely *counting-all* and *counting-on,* which strategies are eventually superseded by the use of familiar *number combinations* (Carpenter et all, 1999). We illustrate these ideas below:

• To add the elements of two sets children will together the items to be counted, and, using their fingers or physical objects, they will *count-all* items starting with those in the first set and ending with elements of the second set.

• As children gain more experience they begin to use a more efficient strategy. They begin with the total of one set (preferably the larger) and *count-on* from there to obtain the sum of the two sets.

• Counting-all and counting-on persist for sometime, but are eventually replaced by the use of number combinations. For example, to find 5 + 7 a child might first find the familiar 5 + 5 = 10 and then add 2 to arrive at the final answer.

To solve subtraction problems young children rely on a number of strategies, mainly *separating from*, *counting-down*, and *counting-up*.

• Separating from involves removing from the larger set the required number of items and them counting the remainder to find the answer.

• Counting-down means starting with the larger number and counting back a number of times equal to the amount being taken away, announcing the last number counted as the amount left.

• To count-up one starts with the smaller number (e.g. 2 in 5 – 2 = ?) and counting forward until the lager number is reached, noting the number of steps in the forward count, which indicates the answer.

• Ultimately children start using number combinations to solve subtraction problems. For example, to find 13 – 7 a child can say: "7 + 6 = 13. Therefore, 13 – 7 = 6".

3. DOUBLE-DIGIT ADDITION AND SUBTRACTION

Children's mental methods for multi-digit addition and subtraction are an extension of the methods they use for single-digit addition and subtraction. Four main methods have been identified (Fuson et al, 1997).

• Beginning with one number and moving up or down by tens and ones. For example, to find 27 + 36 one might begin with 27 and count-up in tens and ones to arrive at 63 (i.e. 27, 37, 47, 57, 58, 59, 60, 61, 62, 63). The subtraction 55 – 33 would be accomplished by counting down in tens and ones: 55, 45, 35, 25, 24, 23, 22.

• Decomposing tens and ones and adding or subtracting the tens and the ones separately: In this case to find 27 + 36 one might proceed as follows: 20 + 30 = 50, 7 + 6 = 13, and finally saying 50 + 13 = 63.

• Changing both numbers. This involves compensating, that is changing the numbers involved to ones that are easier to work with, and then making adjustments to compensate for the changes made. Thus to add 27 + 36 a child might work as follows: 30 + 35 = 65, taking away 3 and adding 1 gives 63.

• Mixing strategies. This simply means combining the above methods.

4. 1010, N10 AND THE EMPTY NUMBER LINE

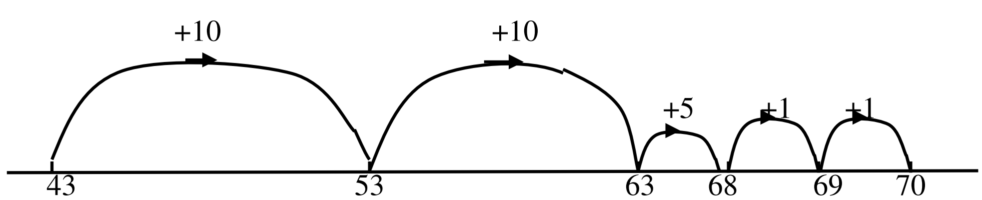
Mathematics educators in the Netherlands have grouped children's mental methods for double-digit addition and subtraction into two main methods, which they have called **1010** and **N10** (Beishuizen, 1993).

• The 1010 method involves partitioning or splitting numbers into multiple of ten and ones, and then adding or subtracting the parts separately. To find 27 + 16 one might work as follows: 20 + 10 = 30; 7 + 6 = 5 + 2 + 5 + 1 = 5 + 5 +2 + 1 = 10 + 3 = 13. The final result is found by adding the two parts together to get 43.

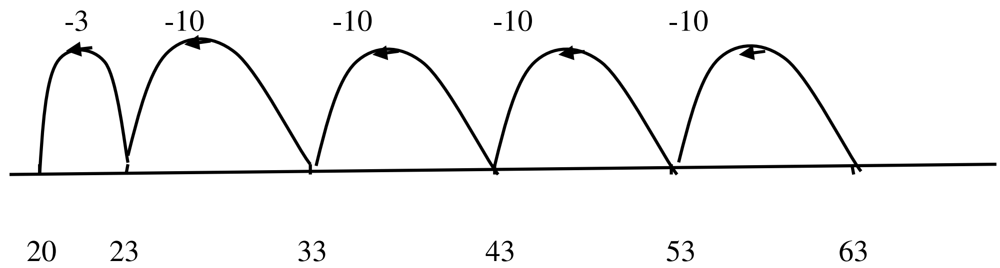
• With the N10 method, one number (usually the larger) is not split and is used as the starting point, as happens when one is counting-up or counting-down. For example, 27 + 16 = 27 + 10 + 5 + 1 = 43.

Dutch mathematics educators recommend that the N10 method should be used in combination with what is known as the Empty Number Line (ENL), a model for performing addition and subtraction. As its name suggests, the ENL is a number line without any markings on it, except the ones children choose to put on it to facilitate calculation. The examples below illustrate how the ENL can be used to perform double-digit addition and subtraction:

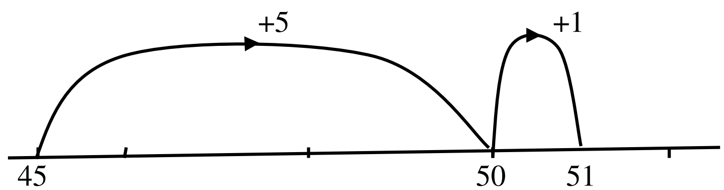
(a) 43 + 27 = 70



(b) 63 – 43 = 20



(c) 51 – 45 = 5 + 1 = 6



In using the ENL as a model for subtraction, whether one counts-on from the smaller number to the larger number or counts-down (or back) from the larger number to the smaller number depends on how close together the two numbers are. Experience has shown that it is more efficient to count-on from the smaller to the larger number if the two numbers are close together in terms of magnitude, as shown in Question (c) above. We count-back from the larger to the smaller number if there is a big difference in the sizes of the two numbers as illustrated in Question (b) above.

5. TEACHING MENTAL CALCULATION

As mentioned earlier, children *can* develop their own mental calculation strategies. However, many educators agree that teaching mental strategies can benefit children in that it makes their methods more efficient; it also allows individual children access to a variety of methods through participation in classroom discussion (Thompson, 2001). Indeed, Robinson (2002) states that "teaching mental calculations supports children's learning in all aspects of the mathematics curriculum, contributing to:

• Deeper understanding of number structure and properties;

• More creative and independent thinking: using their understanding to develop ingenious ways of manipulating numbers;

• Skills and strategies associated with problem solving and computational estimation” (p. 61).

The question, then, is not whether to teach mental calculations but how. It has been suggested that to become efficient at mental calculation children need to master the following pre-requisite skills (Askew, Robinson and Mosley, 2001; Mosley, Ebbutt and Askew, 2001).

• Counting on and back in 1s, 2s, 3s, 4s, and so on up to 10.

• Partitioning the single-digit numbers 6, 7, 8, and 9 into ‘5 and ones’; and double-digit numbers into multiples of 10, 5 and ones.

• Adding and subtracting single or multi-digit numbers using the splitting methods as shown above.

• Using known factors such as 10 + 10 = 20, 7 + 7 = 14 and 7 + 3 = 10 and similar patterns to work out the values of additions like 30 + 70.

• Reorganising the order in which numbers appear to facilitate calculation. For example, instead of 7 + 45 they can write 45 + 7, and then count-on.

• Using relationships such as 3 + 4 = 7 to work out the value of 7 – 4.

• Using an Empty Number line to perform addition and subtraction.

• Having some understanding of place value but also realizing, as Anghileri (2001) points out, that numbers can be partitioned in ways different from the usual ‘tens’ and ‘units’ partition associated with place value.

FINALLY…

> "[Mental calculation requires] teaching that is not just about exposition and practice. [Lessons] should provide opportunities for children to develop, refine and extend their thinking skills through discussion, practical work, problem solving and investigative work if they are to fully realize their potential as mental calculators" (Robinson, 2002, p. 61).

This is what we shall be trying to achieve in the next two weeks.

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APPENDIX 2

STAFF DEVELOPMENT WORKSHOP PROGRAMME (SCHOOL X)

TEACHING MENTAL CALCULATION STARTEGIES

DATES: 30 AUGUST – 3 SEPTEMEBER 2004

DAY/DATE	09 00 – 10 40	10 40 – 11 00	11 – 12 30
MONDAY 30/08/04	• Registration • Aims of the study/Roles of participants • Theoretical Framework	Tea Break	• Children's mathematical thinking/Informal mathematical knowledge • Practical work
TUESDAY 31/08/04	• What Is mental calculation? How does it differ from mental arithmetic? • Practical work	Tea Break	• Children's mental strategies for single and double-digit addition and subtraction • Practical work
WEDNESDAY 01/09/04	• Meaning of 1010 and N10 methods • Addition and subtraction using the 1010 method • Practical work	Tea Break	• Using the N10 method to add and subtract two-digit numbers • Using the Empty Number Line (ENL) to add and subtract two-digit numbers • Practical work
THURSDAY 02/09/04	• Promoting/ managing classroom discussion during in the primary mathematic classroom • Practical work	Tea Break	• Developing a possible curriculum for teaching/learning mental calculation strategies for double-digit addition and subtraction • Practical work
FRIDAY 03/09/04	• Lesson planning: Integrating children's mathematical thinking and constructivist perspectives • Practical work	Tea Break	• Workshop evaluation/Group interview • Closing remarks

STAFF DEVELOPEMNT **WORKSHOP PROGRAMME (SCHOOL Y)**

TEACHING MENTAL CALCULATION STARATEGIES

WEEK 1: DATES: 16 – 20 AUGUST 2004

DAY/DATE	**08 30 – 10 00**	**10 00 - 10 30**	**10 30 – 12 30**
MONDAY **16/08/04**	• Registration • Aims of the study/Roles of participants • Theoretical framework	Tea Break	• Children's mathematical thinking/Informal calculation strategies • Practical work
TUESDAY **17/08/04**	• What is mental calculation? How does it differ from mental arithmetic • Practical work	Tea Break	• Children's mental strategies for single and double-digit addition and subtraction • Practical work
WEDSNESDAY **18/08/04**	• Double-digit addition and subtraction using the 1010 method • Practical work	Tea Break	• Double-digit addition and subtraction using the N10 method • Practical work
THURSDAY **19/08/04**	• Using the Empty number line to add or subtract double-digit numbers • Practical work	Tea Break	• A brief look at mental calculation strategies for multiplication and dividsion • Practical work
FRIDAY **20/08/04**	• Promoting/managing classroom discussion in the primary mathematics classroom: Teachers' role and importance of teacher knowledge	Tea Break	• Review of work covered so far

TEACHING MENTAL CALCULATION STRATEGIES (CONTINUED)

WEEK 2: 23-27 AUGUST 2004

DAY/DATE	**08 30 – 10 00**	**10 00 - 1030**	**10 30 - 12 30**
MONDAY **23/08/04**	• Promoting/managing classroom discussion in the primary mathematics classroom • Practical work	Tea Break	• Developing a possible curriculum for teaching/learning mental calculation strategies for double-digit addition and subtraction • Practical work
TUESDAY **24/08/04**	• Developing a possible curriculum for teaching/learning mental calculation strategies for double-digit addition and subtraction • Practical work	Tea Break	• Lesson planning: Integrating children's mathematical thinking and constructivist perspectives • Practical work
WEDNESDAY **25/08/09**	• Lesson planning: Integrating children's mathematics thinking and constructivist perspectives • Practical work	Tea Break	• Review of work done: Using the 1010 method to add and subtract double-digit numbers ENL) • Practical work
THURSDAY **26/08/04**	• Review of work done: Using the N10 method and the Empty Number Line • Practical work	Tea Break	• Further review of work done: 1010, N10 and Empty number Line • Practical work
FRIDAY **27/08/04**	• Workshop evaluation/Group interview	Tea Break	• Closing remarks

APPENDIX 3

FORM FOR RECORDING FIELD NOTES

Teacher's name:
School name:
Class:
Date:
Time:

APPENDIX 4

GROUP INTERVIEW GUIDE

MENTAL CALCULATION STRATEGIES FOR DOUBLE-DIGIT ADDITION AND SUBTRACTION

INTRODUCTION

Today marks the end of the staff development workshop that has been running for the last two weeks. I am interested to know what we have learned from it and what we feel in general about it about the workshop. The interview will be tape recorded.

1. What skills do you think you have attained as a result of participation in the workshop?

Probe: Things you could not do well but now can do better; how to conduct lessons; etc.

2. How has the training affected what you believed about mathematics teaching and learning?

Probe: For example, what did you think is the most effective way to teach mathematics? The most effective way to learn mathematics? Do you think differently about it now? Why?

3. How will your experience during the workshop affect what you do in future mathematics lessons with your pupils?

Probe: You can mention here how you will plan future mathematics lessons; things you plan to do during lessons; and so on.

4. Is there anything you did not like about the workshop, something you feel should not have been there or should be changed if the workshop is repeated?

Probe: The way the workshop was organised; what was discussed; and so on.

5. Finally, if officials from the Ministry of Education consulted you to find out whether or not you think this training should be extended to all teachers in the country, what would you tell them? Why?

Probe: Are there any benefits in undergoing this training? Would it make primary mathematics teaching more effective or lees effective?

Thank you very much for agreeing to discuss these issues with me. I can assure you that all the information you have provided will be used solely for purposes of this research.

APPENDIX 5

POST-OBSERVATION INTEVIEW GUIDE

MENTAL CALCULATION STRATEGIES FOR DOUBLE-DIGIT ADDITION AND SUBTRACTION

INTRODUCTION

The aim of this interview is to give you the opportunity to talk about the lesson I observed. I am particularly interested to know what you liked about the lessons; anything that surprised you about what pupils' did or said. In short what you think you learned from it and how it will affect the way you do your job as a teacher.

1. What did you like most about you mathematics lesson today?

   Probe: Ask for specific examples of what happened, reasons why, and so on.

2. Tell me about children's use of mental calculation strategies during the lesson. To what extent did children use their own strategies?

   Probe: Ask for specific examples of classroom events and what the teacher felt about them.

3. How will this experience affect the way you approach mathematics teaching in future.

   Probe: For example the way you will relate with pupils. Any changes you will make for example in the way you prepare and teach mathematics at this level of education? Etc.

4. How would you describe your level of success with regard to using pupils' questions to encourage classroom discussion?

   Probe: Encourage extended articulation of the teacher's evaluation of her performance in this regard.

5. It has been said that pupils know more mathematics than many teachers expect. In relation to pupils' use of mental calculation strategies during the lesson, did you see or hear anything from any one of them that you think supports this idea?

   Probe: Did a pupil for example surprise you with the originality of how he/she worked out answers or what he/she said?

6. In the light of your experiences during the lesson, how would you describe your role as a teacher in a classroom focusing on the use mental calculation strategies?
   Probe: Encourage the teacher to talk about what she thinks she should be doing during lessons. This might suggest changing beliefs about mathematics teaching and learning.

7. Is there anything else you would like to tell me about your lesson today that I have not asked about?

Thank you very much for allowing me to learn about your classroom experiences. As I said at the beginning of the study, I will be meeting you from time to time so that we can continue discussing like this how your lessons are progressing. See you next time.

APPENDIX 6

POST-TREATMENT INTERVIEW GUIDE

MENTAL CALCULATION STRATEGIES FOR DOUBLE-DIGIT ADDITION AND SUBTRACTION

INTRODUCTION:
We have now come to the end of the study. My main aim in this interview is to learn from you how you feel your life as a teacher of primary mathematics has been affected by your participation in the study. I am also interested to know how you feel the study has affected your pupils' work in mathematics. The interview is being tape-recorded.

1. To start with, how did you feel when I first invited you to participate in the study?

   Probe: Where you disturbed? Were you happy? How did you feel?

2. Do you still feel the same or do you now feel differently about it?

3. In the light of this, what changes if any if any do you plan to make in your teaching of mathematics?

   Probe: Encourage the teacher to talk about specific examples of what will change in her teaching of mathematics, e.g. in lesson planning, classroom management, etc.

4. How would you describe pupils' feelings about using mental calculation strategies?

   Probe: Did they like it? Did they find it easy or difficult to use the strategies? Etc.

5. What effect would you say learning to use mental calculation strategies has had on the way pupils approach mathematics tasks during lessons?

   Probe: tell me any thing you have observed that is different, for example in the way pupils go about working out answers, the types of questions they ask, their morale to learn, and so on.

6. What about pupils' performance in mathematics? Has anything changed?

7. One of the important things we have been trying to achieve during the study is to encourage pupils to share their mathematical ideas with classmates during lessons. To explain their methods; to defend them; and so on. How would you describe your pupils' performance in this regard?

   Probe: Encourage the teacher to talk about any changes she has observed in this regard.

8. To what extent would you say you have been successful in using pupils' questions to encourage classroom discussion in general?

   Probe: Encourage the teacher to evaluate herself in this regard, citing specific examples if possible.

9. Did you find the ideas on mental calculation that you were teaching difficult or easy to understand?

   Probe: For example, did you find it easy to implement the aims of the study? Was it easy or difficult for you to see what to do in the next lesson? And so on.

10. As I have been asking all along, suppose Ministry of Education officials asked you to tell them whether or not they should extend the use of mental calculation strategies to the rest of the country, how would you respond?

Probe: Would you recommend that other primary school teachers at this level of education should experience what you have experienced? Why?

11. Is there anything else concerning your experiences during the past several weeks that you would like to talk about?

Thank you very much for taking the time to talk with me. You have been a tremendous help during the study and I am very grateful for your participation. I will produce typed copies of our interview so that you can see if I have represented your views fairly.

APPENDIX 7

NUMERACY ASSESSMENT TEST

(PRE-TEST)

GRADE 2

TEACHER'S INSTRUCTIONS

Start by saying:

- Please fill in the form at the front of the booklet
- Write your name in full
- Then write the name of the school
- Class…
- Today's date is…
- Now open your booklet

NOTE:

1. The above instructions should be given orally to pupils in English and/or in a familiar Zambian language.
2. Ensure that all children have followed the instructions.
3. Ask children to look at you after writing the answer to each question, so that you can tell when they are ready to move on to the next question.
4. Read each question in English and then translate as necessary into a familiar Zambian language, using terms most familiar to the children.

SECTION 1

1. How many?

Some bicycles have been drawn on your page. How many are there?

- Write your answer in the box.

2. Flowers

(a) There is a flower pot with five flowers in it.

- Draw some more flowers in the flower pot so that there are eight flowers altogether.

(b) There is another flower pot in your booklet with no flowers in it. The number written on it tells you how many flowers should be in this flower pot.

- Look at the number carefully and draw that number of flowers in the pot.

3. Which number?

I am going to ask you to write down the answers to some sums. Here is a practice question first:

Which number is two more than four?

- Write your answer in the box nest to sun.
- When you have written down the answer look at me.

Collect children's responses to establish that 'more than' requires them to add the two numbers together.

(a) Which number is three more than six?

- Write your answer in the box next to the clouds

(b) Which number is seven more than eight?

- Write your answer in the box next to the cow.

Now the next one. Again I am going to give you a practice question first:

(c) Which number is one less than eight?

- Write your answer in the box nest to the pineapple.
- When you have written down the answer look at me.

Collect children's responses and establish that 'less than' requires them to subtract or take away one number from the other.

(d) Which number is three less than eight?

- Write your answer in the box next to the key

(e) Which number is six less than eight?

- Write your answer in the box next to the shoe.

4. Find the number

There is a number line in your booklet with numbers from zero to ten written on it. Some of the numbers are missing.

- Write the missing numbers in the boxes.

5. One more

(a) Write 'nine' in the box nest to the tortoise.

What number is one more than nine?

- Write your answer on the line next to the tortoise

(b) Write 'eighty-nine' in the box next to the hut.

What number is one more than eighty-nine?

- Write your answer on the line next to the hut.

6. One less

(a) Write 'thirty' in the box next to the boat.

What number is one less than thirty?

- Write your answer on the line next to the boat.

(b) Write 'seventy-six' in the box next to the car.

What number is one less than seventy-six?

- Write your answer on the line next to the car.

7. School Tuck Shop

The school tuck shop sold cakes. The cakes were put in boxes and each box contained five cakes. This is how the cakes in a box looked **(Show picture).** There were some loose cakes and the loose cakes looked like this **(point to picture).**

I am going to show you a picture of the cakes that Natasha bought and I want you to write down how many. I will show the picture for a short time, so look carefully. Remember there are five cakes in each box. **(Display picture for 4 seconds).**

How many cakes did Natasha buy?

- Write your answer in the box drawn in your booklet.

8. At home

Mubita bought some cakes to take home. I am going to tell you how many cakes he bought and I want you to write down how many boxes. Remember, there are five cakes in each box.

Mubita bought ten cakes. How many boxes did he buy?

- Write your answer in the box drawn in your booklet.

9. Number story

I am going to read you a number story and I want you to answer the question at th end of the story.

Three boys and four girls get on a bus. How many children get on the bus?

- Write your answer in the box next to the beetle.

This is the end of Section 1

SECTION 2

10.Find the number

There is a ruler shown in your booklet with numbers from zero to fifty written on it. There are also two boxes and two arrows shown.

- Write down in the box above each arrow the number that the arrow points to.

11. One more

Write 'one hundred nine' in the box next to the crab.
What number is one more than one hundred nine?

- Write your answer on the line next to the crab.

12.One less

Write two hundred in the box next to the truck.
What number is one less than two hundred?

- Write your answer on the line next to the truck.

13. School Tuck Shop

The school tuck shop sold apples. The apples were put into bags of ten. This is how one bag of ten apples looked like **(show picture).**

I am going to show you pictures of apples some children bought and I want you to write down how many. I am going to show you these for a short amount of time so you might not have time to count each apple. Remember there are ten apples in each bag.

(a) How many apples did Mulenga buy? (Display picture for 5 seconds).

- Write your answer in the box.

(b) How many apples did Mutinta buy? (Display picture for 10 seconds).

Write your answer in the box.

14. Times

I am going to ask you to write the answers to these questions in the boxes in your booklet.
(a) Four times two equals…

- Write your answer in the box next to the cat.

(b) Four times five equals…

- Write your answer in the box next to the chick

15. Ten

(a) Add ten to nine.

- Write your answer in the box next to the rat.

(b) Add ten to ninety-two.

- Write your answer in the box next to the fish.

(c) Take away ten from fifty.

- Write your answer in the box next to the clock.

(d) Take away ten from seven hundred.

- Write your answer in the box next to the star.

16. One hundred

(a) Add one hundred to nine.

- Write your answer in the box next to the chair.

(b) Take away one hundred from four hundred.

- Write your answer in the box next to the butterfly.

17.Tennis balls

Kasongo and Mweene play a game involving old tennis balls. Kasongo wins half of these balls.

- Draw a ring around the balls Kasongo has won.

18. Number stories

I am going to read you two stories and I want you to answer the questions at the end of each story.

(a) Five people are on a bus and eight more get on. How many people are on the bus?

- Write your answer in the box next to the tree.

(b) Twelve people are on a bus and five get off. How many people are on the bus now?

- Write your answer in the box next to the aero plane.

APPENDIX 8

NUMERACY ASSESSMENT TEST

(POST-TEST)

GRADE 2

TEACHER'S INSTRUCTIONS

Start by saying:

- Please fill in the form at the front of the booklet
- Write your name in full
- Then write the name of the school
- Class…
- Today's date is…
- Now open your booklet

NOTE:

5. The above instructions should be given orally to pupils in English and/or in a familiar Zambian language.
6. Ensure that all children have followed the instructions.
7. Ask children to look at you after writing the answer to each question, so that you can tell when they are ready to move on to the next question.
8. Read each question in English and then translate as necessary into a familiar Zambian language, using terms most familiar to the children.

SECTION 1

Repeat instructions as necessary for each question

1. How many?

Some bicycles have been drawn on your page. Each bicycle has two wheels. How many wheels are there altogether?

- Write your answer in the box.

2. Flowers

(a) There is a flower pot with five flowers in it.

- Draw some more flowers in the flower pot so that there are eight flowers altogether.

(b) There is another flower pot in your booklet with no flowers in it. The number written on it tells you how many flowers should be in this flower pot.

- Look at the number carefully and draw that number of flowers in the pot.

3. Which number?

I am going to ask you to write down the answers to some sums. Here is a practice question first:

Which number is three more than four?

- Write your answer in the box nest to sun.
- When you have written down the answer look at me.

Collect children's responses to establish that 'more than' requires them to add the two numbers together.

(a) Which number is four more than seven?

- Write your answer in the box nest to the clouds

(b) Which number is eight more than nine?

- Write your answer in the box next to the cow.

Now the next one. Again I am going to give you a practice question first:

(c) Which number is one less than nine?

- Write your answer in the box nest to the pineapple.
- When you have written down the answer look at me.

Collect children's responses and establish that 'less than' requires them to subtract or take away one number from the other.

(d) Which number is four less than nine?

- Write your answer in the box next to the key

(e) Which number is eight less than ten?

- Write your answer in the box next to the shoe.

4. Find the number

There is a number line in your booklet with numbers from zero to ten written on it. Some of the numbers are missing.

- Write the missing numbers in the boxes.

5. One more

(a) Write 'nineteen' in the box nest to the tortoise.

What number is one more than nineteen?

- Write your answer on the line next to the tortoise

(b) Write 'ninety-nine' in the box next to the hut.

What number is one more than ninety-nine?

- Write your answer on the line next to the hut.

6. One less

(a) Write 'fifty' in the box next to the boat.

What number is one less than fifty?

- Write your answer on the line next to the boat.

(b) Write 'eighty-seven' in the box next to the car.

What number is one less than eighty-seven?

- Write your answer on the line next to the car.

7. School Tuck Shop

The school tuck shop sold cakes. The cakes were put in boxes and each box contained five cakes. This is how the cakes in a box looked **(Show picture).** There were some loose cakes and the loose cakes looked like this **(point to picture).**

I am going to show you a picture of the cakes that Natasha bought and I want you to write down how many. I will show the picture for a short time, so look carefully. Remember there are five cakes in each box. **(Display picture for 7 seconds).**

How many cakes did Natasha buy?

- Write your answer in the box drawn in your booklet.

8. At home

Mubita bought some cakes to take home. I am going to tell you how many cakes he bought and I want you to write down how many boxes. Remember, there are five cakes in each box.

Mubita bought fifteen cakes. How many boxes did he buy?

- Write your answer in the box drawn in your booklet.

9. Number story

I am going to read you a number story and I want you to answer the question at the end of the story.

Five boys and six girls get on a bus. How many children get on the bus?

- Write your answer in the box next to the beetle.

This is the end of Section 1

SECTION 2

10. Find the number

There is a ruler shown in your booklet with numbers from zero to fifty written on it. There are also two boxes and two arrows shown.

- Write down in the box above each arrow the number that the arrow points to.

11. One more

Write ‘two hundred nine’ in the box next to the crab.
What number is one more than two hundred nine?

- Write your answer on the line next to the crab.

12. One less

Write three hundred in the box next to the truck.
What number is one less than three hundred?

- Write your answer on the line next to the truck.

13. School Tuck Shop

The school tuck shop sold apples. The apples were put into bags of ten. This is how one bag of ten apples looked like **(show picture).**

I am going to show you pictures of apples some children bought and I want you to write down how many. I am going to show you these for a short amount of time so you might not have time to count each apple. Remember there are ten apples in each bag.

(a) How many apples did Mulenga buy? **(Display picture for 7 seconds).**

- Write your answer in the box.

(b) How many apples did Mutinta buy? **(Display picture for 10 seconds).**

Write your answer in the box.

14. Times

I am going to ask you to write the answers to these questions in the boxes in your booklet.
(a) Four times three equals…

- Write your answer in the box next to the cat.

(b) Four times six equals…

- Write your answer in the box next to the chick

15. Ten

(a) Add ten to eleven.

- Write your answer in the box next to the rat.

(b) Add ten to ninety-six.

- Write your answer in the box next to the fish.

(c) Take away ten from sixty.

- Write your answer in the box next to the clock.

(d) Take away ten from six hundred.

- Write your answer in the box next to the star.

16. One hundred

(a) Add one hundred to seven.

- Write your answer in the box next to the chair.

(b) Take away one hundred from five hundred.

- Write your answer in the box next to the butterfly.

17. Tennis balls

Kasongo and Mweene play a game involving old tennis balls. Kasongo wins half of these balls.

- Draw a ring around the balls Kasongo has won.

18. Number stories

I am going to read you two stories and I want you to answer the questions at the end of each story.

(a) Thirteen people are on a bus and eight more get on. How many people are on the bus?

- Write your answer in the box next to the tree.

(b) Seventeen people are on a bus and nine get off. How many people are on the bus now?

- Write your answer in the box next to the aero plane.

APPENDIX 9

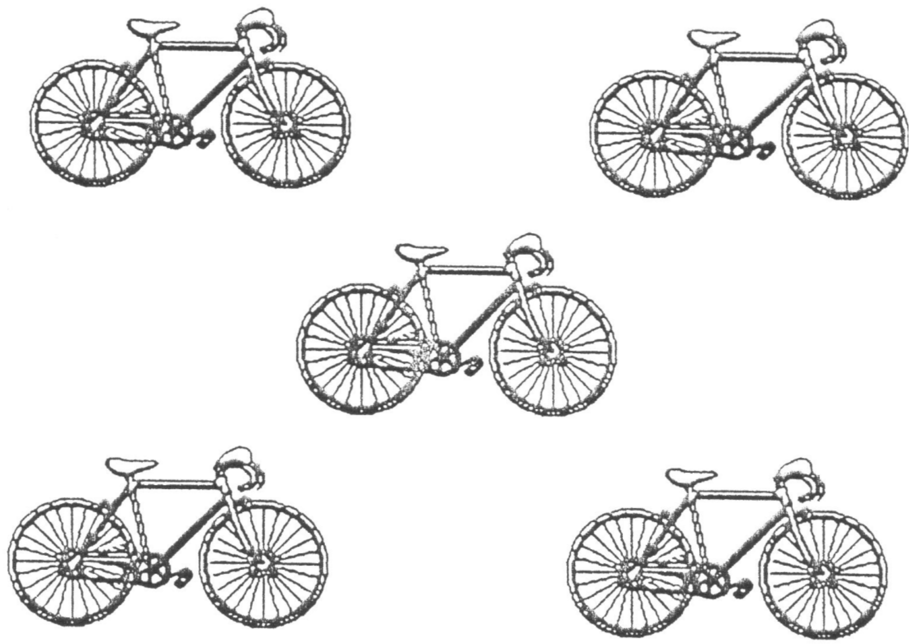
PUPILS' ANSWER BOOKLET

Numeracy assessment test
Pupil's booklet

Grade 2

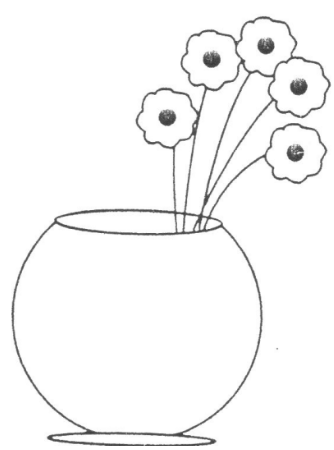
Name
School
Class
Date

1. How many?



2. Flowers

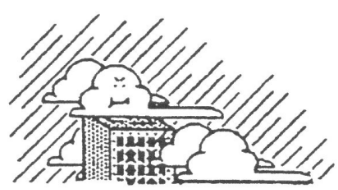
a)



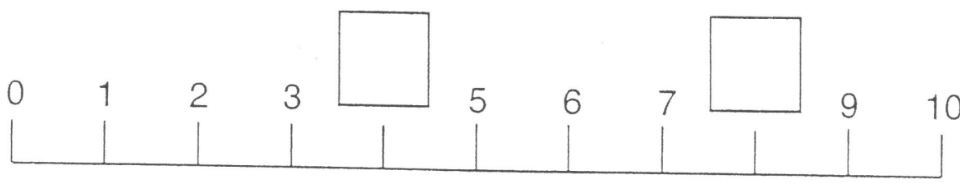
2. Flowers

b)

3. Which number?

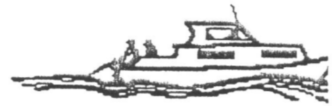


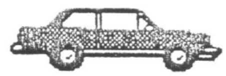
4. Find the number



5. One more

6. One less





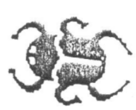
7. School Tuck Shop

Natasha bought ☐ cakes.

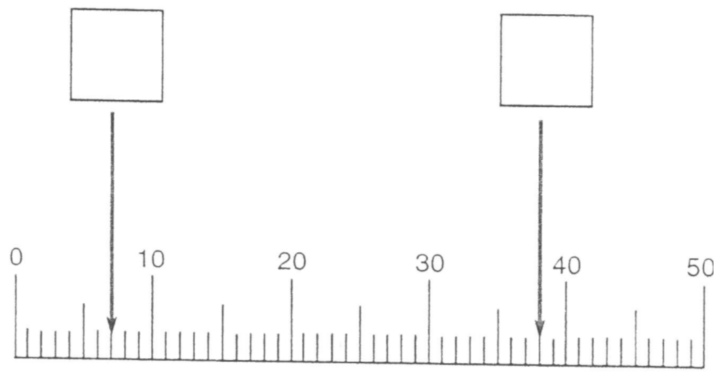
8. At home

Mubita bought ☐ boxes of cakes

9. Number story

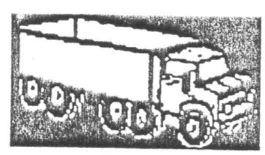


10. Find the number



11. One more

12. One less



13. School Tuck shop

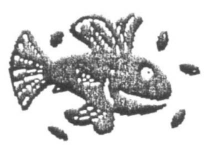
(a) Mulenga bought ☐ apples.

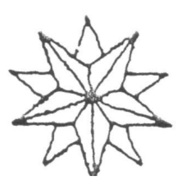
(b) Mutinta bought ☐ apples.

14. Times

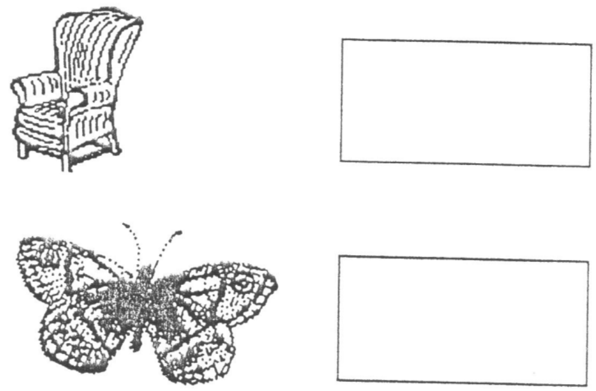
---

15. Ten



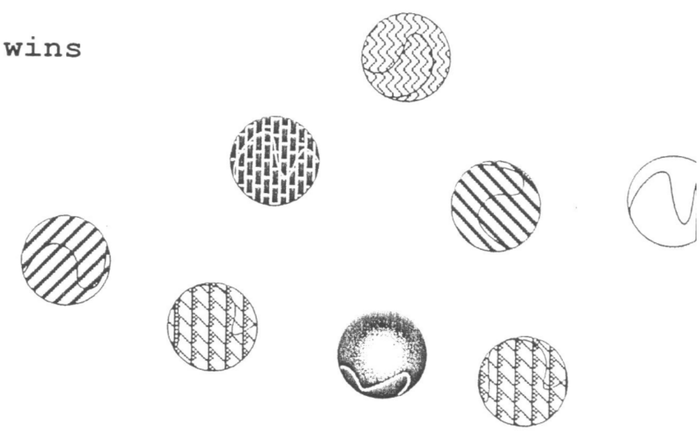


16. One hundred

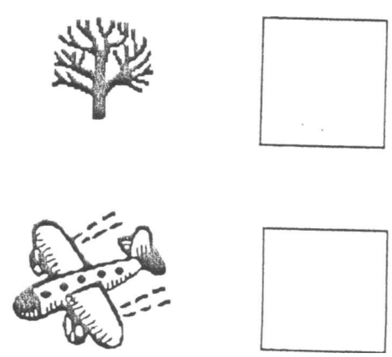


17. Tennis balls

Kasongo wins



18. Number stories



APPENDIX 10

LETTER SUBMITTED TO DISTRICT EDUCATIONAL ADMINISTARTORS FOR PERMISSION TO CONDUCT THE STUDY IN LUSAKA SCHOOLS

THE UNIVERSITY OF ZAMBIA
SCHOOL OF EDUCATION

Telephone: 291381/291777
Telegram: UNZA, LUSAKA
Telex: UNZALU ZA 44370
Fax: + 260-1-253952

P O Box 32379
Lusaka, Zambia

Your Ref.:

Our Ref.:

REPUBLIC OF ZAMBIA
MINISTRY OF EDUCATION
DISTRICT EDUCATION BOARD
-4 AUG 2004
LUSAKA URBAN
BOARD SECRETARY
P.O. BOX 50297, LUSAKA

Permission Granted

4 August 2004

The District Education Board Secretary
Lusaka District

Dear Sir

PERMISSION TO ENTER LUSAKA SCHOOLS TO CONDUCT PhD RESEARCH

I am writing to request permission to carry out a research study in Lusaka schools, involving Grade 2 children and their teachers.

The study is entitled ***Feasibility of Using Mental Calculation Strategies for Addition and Subtraction in Primary Mathematics in Zambia at Grade 2 Level: The Case of Lusaka Schools***. It assesses the appropriateness of developing numeracy in young children in Zambia using mental calculation strategies, that is children's informal solution methods, and whether or not this leads to better learning results in two-digit addition and subtraction than does the use of standard procedures as currently taught.

The study, which is being performed in fulfilment of the requirements of the researcher's PhD in mathematics education at the University of Zambia, has two phases: a staff development workshop for participating teachers; and the main study, when the teachers will implement workshop ideas in the classroom. During the main study I will conduct lesson observations, interviews with both teachers and children, and also administer a test intended to measure children's number sense.

Thank you in anticipation for your assistance.

Yours faithfully

Muhau Tabakamulamu
Lecturer in Mathematics Education and PhD Candidate

APPENDIX 11

INVITATION LETTER TO GARDE 2 TEACHERS IN EXPRIMENTAL SCHOOLS TO PARTICIPATE IN THE STUDY

THE UNIVERSITY OF ZAMBIA
SCHOOL OF EDUCATION

Telephone: 291381/291777
Telegram: UNZA, LUSAKA
Telex: UNZALU ZA 44370
Fax: + 260-1-253952

P O Box 32379
Lusaka, Zambia

Your Ref.:

Our Ref.:

11 August 2004

LUSAKA

Dear

INVITATION TO PARTICIPATE IN A RESEARCH PROJECT

Following the discussion I had with you when I visited your school a few days ago, I would like to invite you formally to participate in the study I am currently involved in, which assesses the appropriateness of developing numeracy in young children in Zambia at Grade 2 level using *mental calculation strategies*, that is children's informal solution methods.

As I explained to you the study, which I am undertaking in fulfilment of the requirements of a PhD in mathematics education at the University of Zambia, has two phases: a staff development workshop; and the main study, when participating teachers will implement workshop ideas in the classroom. The aim of the staff development workshop is twofold: To allow participants to experience the range of mental calculation strategies used by children to solve whole number two-digit addition and subtraction problems; and to have the opportunity to think about and discuss ways to foster the use of mental calculation strategies in the mathematics classroom. During the main study I will conduct lesson observations, interviews with both teachers and children, and also administer a test intended to measure children's number sense.

Your participation in the study will provide useful information that cannot be obtained otherwise. The information you provide during the study will be treated confidentially and will be used for research purposes only. In the final report pseudonyms will be used, so that it will not be possible for people not involved in the study to connect participants' names to the information reported.

Yours sincerely

Muhau Tabakamulamu
Lecturer in Mathematics Education and PhD Candidate
Enc. Provisional Programme of Activities