MISCONCEPTIONS AND ERRORS IN ALGEBRA AT GRADE 11 LEVEL: THE CASE OF TWO SELECTED SECONDARY SCHOOLS IN PETAUKE DISTRICT

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DECLARATION

I, Titus Luka Mbewe, hereby declare that this dissertation represents my own work and that it has not been previously submitted for a degree at this or any other University.

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This dissertation of Titus Luka Mbewe is approved as fulfilling the requirements for the award of the degree of Master of Education in Mathematics Education of the University of Zambia.

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ABSTRACT

This study investigated secondary school pupils' errors and misconceptions in algebra among grade 11 pupils in two selected schools in Petauke district in Eastern Province of Zambia, with a view to exposing the nature and origin of these errors and make suggestions for classroom teaching. The study employed a quantitative and qualitative approach to data collection process, involving the use of pencil-and-paper test and interviews.

In the quantitative data, a test was given to 60 pupils which was pre-tested for its validity and reliability. After two weeks, five pupils were interviewed to identify their misconceptions and reasoning processes.

Data analysis was largely done through descriptive statistics, and incorporated elements of inferential statistics such as Analysis of Variance (ANOVA), Post-hoc tests and the Chi-Square tests for in-depth analysis and deeper interpretation of data. The main reason for misconceptions was lack of understanding of basic concepts of the variable. Under algebraic expressions, the main reason for misconception was that they were abstract in nature in that there was not much context attached to it. As for equations, the cause of misconception was the inadequate understanding and misuse of the equal sign which hindered solving equations correctly.

The main conclusions drawn from this study is that the misconceptions which pupils experience in algebra are attributed to lack of conceptual knowledge and understanding. The study also found that misconceptions were robust, this simply meant that they could not easily be dislodged and occurred frequently. This was evident from the interviews conducted that pupils appeared to overcome a misconception only to have the similar misconceptions appear later.

The study recommended that teachers and pupils should openly talk about misconceptions in the classroom during teaching and learning process. Individual attention to pupils should be given by the teacher in order to understand pupils' mathematical reasoning.

DEDICATION

My help cometh from above

I dedicate this work firstly to my wife Esnala and

my daughters Faith and Talandila. Secondly to my late parents

Luka Phillip Mbewe

and

Agness Banda

May Their Souls Rest in Eternal Peace of Our Lord.

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ABBREVIATIONS AND ACRONYMS

ANOVA : Analysis of Variance

UNZA : University of Zambia

CHAPTER 1

INTRODUCTION

1.0 Background of the study.

During my sixteen-year career as a secondary school teacher of mathematics, I observed that many pupils have serious problems pertaining to algebra. The pupils seem to have mastered concepts in arithmetic such as addition and subtraction and are able to solve lengthy arithmetic problems, but are hesitant when it comes to using algebraic methods.

It is with this in mind that I became interested in mathematics education and particularly the core topic of Algebra. Later, through observing pupils in an informal and unsystematic way I found that they had a number of misconceptions, for example, making the same errors over a period of time which they seem to acquire during the process of learning. Also, through discussions with my fellow teachers of mathematics, it was realised that their explanations for these types of behaviours were consistent with mine. The reasons for this were not clear to me . Whatever the reasons may be, there could be a way to identify and offer remedy to these problems. For pupils, Algebra is somehow very difficult to learn (Hercovics, 1989; Kieran, 1992).

The problem of Algebra in terms of pupil errors and misconceptions seems to be common to pupils at both junior and senior secondary school levels. Pupils' errors on paper are seen when they do classroom work or answer test questions. There were no opportunities to listen to pupils' explanations except for a few informal discussions with them.

To the best of my knowledge, no study has been conducted in Zambia which focused on pupils' errors and misconceptions in algebra. It is because of this background that this study was conducted in Zambia.

1.1 Statement of the problem.

Basic mathematics concepts or operations, like addition and subtraction of whole numbers, may involve complicated cognitive processes. Since teachers are already familiar with these basic concepts, this leads them to ignore or underestimate the complexity by taking a naïve approach to teaching these concepts (Schoenfeld, 1983). Without adequate knowledge about pupils' learning of basic mathematics concepts or operations, teachers could underestimate the complexity of the individual learning process of mathematics.

Although there are many causes of pupils' difficulties in mathematics, the lack of support from research fields for teaching and learning is noticeable. If research could characterise pupils' errors and misconceptions, it would be possible to design effective instructions to avoid those situations.

In mathematics education field, research on pupils' errors and misconceptions is not well documented. In the past error analysis in the mathematics education focused more on procedural analysis and less on misconception analysis. Furthermore, pupils' errors and misconceptions about variables, expressions and equations, which are fundamental in the learning of algebra lack systematic research. Thus the question still remains unanswered: How can errors and misconceptions in algebra be minimized?

1.2 Purpose of the study.

The goal of this study was to investigate secondary school pupils' errors and misconceptions in algebra, with a view to exposing the nature and origin of these errors and misconceptions. In order to reach this goal, the study focused on the nature of pupils' learning basic concepts by analysing their errors in solving well-designed problems used to assess those concepts.

Resnick (1982), attributed pupils' learning difficulties to concept learning. In order to explore such issues, the three basic algebra concepts: variables, expressions and equations were chosen to analyse pupils' errors and misconceptions. Through focusing on these three fundamental algebraic concepts, it was hoped that more general principles of understanding and learning difficulties could be illustrated by these cases.

The study examined three areas of inquiry. Firstly it looked at the errors and misconceptions which pupils made when solving problems related to variables, expressions and equations. These are reported and analysed in detail. Secondly, this study looked at the differences among pupils when solving algebraic expressions, variables and equations. Finally, the study assessed pupils' problem solving processes and reasoning in algebra.

1.3 Research questions.

This study sought answers to and was structured around three main research questions. These:

(a) What errors and misconceptions do secondary school pupils make when solving problems related to variables, algebraic expressions and equations?

(b) Why are there differences among pupils when solving algebraic expressions, variables and equations ?

(c) What can be learned from pupils' problem solving processes and reasoning in algebra?

1.4 Significance of the study.

The study of pupils' errors and misconceptions in algebra was worth carrying out as the results of the study could inform teachers, curriculum planners and developers, textbook writers and other stakeholders, such as parents, to broaden their understanding of how errors and misconceptions in algebra can be noticed and thoughtfully engaged. It will also help teachers and researchers to design effective methods and approaches to improve pupils' understanding in algebra. Such detailed information about pupils misconception in learning variables, algebraic expressions and equations could contribute to teachers' classroom instruction.

Therefore, if research focuses on identifying pupils' misconceptions and errors in algebra, it will be easier to categorise these errors and misconceptions that grade 11 pupils commit when performing algebraic problems so that they can be eliminated through properly organised instructional methods.

1.5 Theoretical framework.

1.5.1 Overview of the constructivism theory.

Since the aim of the study was to determine pupils' errors and misconceptions in algebra, my theoretical framework was based on constructivism, which seems to be consistent and appropriate for studying pupils' thinking. The constructivist perspective, developed as a part of the work of Piaget asserts that conceptual knowledge cannot be transferred from one person to another but is constructed by the individual him or herself (Piaget, 1970).

Piaget was interested in examining children's answers in their tests. The basic assumption of constructivism is that pupils are active learners and must construct knowledge for themselves. For instance in mathematics, for a student to completely understand the concepts, the pupil must rediscover the mathematical principles. Therefore, constructivism emphasises the role of prior knowledge in learning

Although terms such "radical constructivism" and "social constructivism" provide some orientation for the discussion of this study, there is a diversity of epistemological perspectives even within these categories (Steffe and Gale, 1995). Radical constructivists consider knowledge as an individual construction while social constructivists believe that knowledge production is a result of social interaction.

Constructivism places emphasis on learning, it states that knowledge can only happen by relating the unknown to the known. Thus, all learning depends on the prior knowledge of the learner, which serves as a schema, into which the new information is fitted (Smith et al.,1993). When existing schemas are not adequate to absorb new knowledge, new schemas are constructed by the learner during the process of learning (Skemp, 1987). Since knowledge schemata are personal and individual, learners generate unique links between new and old information (Wittrock, 1986). Therefore, it is not surprising to find that different learners construct alternative conceptions of the phenomena.

Therefore in mathematics, persistently fixed nature of incorrect schemas in pupils' minds made them to formulate wrong rules (Demby, 1977). Constructivist theories suggest that in order for pupils to be successful in solving a problem, they should select and apply correct solving schemas. There are situations where pupils apply incorrect schemas while having the correct ones in their heads. One possible explanation is that pupils probably had the correct methods in their long-term memory but they could not recall the information (Matlin, 2005).

1.5.2 Radical constructivism versus social constructivism.

Radical constructivists hold that knowledge is constructed by an individual while social constructivists believe that knowledge is constructed as a result of social interactions. The radical constructivists focus on the individual construction, thus taking a cognitive perspective (Wertsch and Toma, 1995). However, although social interaction is viewed as an important context for learning in this perspective, the focus is still on the resulting reorganization of the individual cognition. On the other hand, social constructivists view higher mental processes as socially mediated. It is with this view that sociocultural processes are given the priority in understanding individual mental functioning (Wertsch and Toma, 1995).

1.5.3 Criticisms of constructivism.

Other schools of thought argue that if everyone had a different experiential world, no one could agree on any knowledge. They argue that constructivism is a stance that denies reality (Kilpatrick, 1987). The constructivists reply to this argument by saying that, agreement on social and scientific issues does not prove that what we experience has objective reality. The models that we construct about something are our own constructs that are accessible to us (von Glaserfeld, 1991).

A similar criticism is that if everyone is able to construct their own knowledge, then everyone's constructs must be equally valid. The constructivists reply to this by saying that the constructive process does not happen in isolation, but is subject to social influences. The constructs of knowledge is both social and individual (Kitchener, 1986).

Finally, both individual as well as social construction of knowledge are important if we are to think of a combined and complete notion of constructivism.

1.6 Operational definition of terms.

- **Misconception:** Incorrect features of pupil knowledge that are repeatable and explicit (Leinhardt, Zaslavsky, and Stein, 1990).
- **Cognition**: In this study cognition shall mean an action of knowing (Matlin, 2005).
- **Strategy**: A strategy is considered as a goal-directed procedure that facilitates both problem solving and acquisition of domain-specific knowledge (English, 1996).

- **Error:** In this study an error shall be regarded as a mistake in the process of solving a mathematical problem procedurally or by any other method (Young and O'Shea, 1981).
- **Schema**: Is a mechanism in human memory that allows for the storage, synthesis, generalization and retrieval of similar experiences (Marshall, 1995).

1.7 Limitations.

Limitations include uncontrollable aspects of the study that weaken its design and thus weaken the generalisability of its findings.

One of the limitations of this study could be that a relatively small sample was used. The study only represents portraits of selected pupils in two schools in Petauke district. Therefore, it cannot claim to have captured the entire performance on algebra. In order to overcome this, a multidimensional approach to data collection was used in an attempt to ensure richness and triangulation.

Another limitation was with the test items. The items given to the pupils must be consistent with the course descriptions and adopted textbooks, the researcher had limited control over the test questions. Some test questions needed a stronger measure of pupils' ability to follow prescribed rules than of a pupil's comprehension of underlying algebraic ideas or pupil's creative problem-solving abilities.

In order to overcome this limitation, four experienced teachers were consulted about the suitability of the test items and also interview sessions with the pupils were used to investigate more aspects of pupils' errors and misconceptions.

CHAPTER 2

REVIEW OF RELATED LITERATURE.

2.0 Introduction.

Although there are many causes of pupils' difficulties in learning mathematics, the lack of enough support from research fields for teaching and learning is an important one. If research could characterise pupils' learning difficulties, it would be possible to design effective instructions to help pupils learning. As Booth (1988, 20) pointed out, "one way of trying to find out what makes algebra difficult is to identify the kind of errors pupils commonly make in algebra and then investigate the reasons for these errors". The research on pupils' errors and misconceptions is a way to provide such support for both teachers and pupils.

In this section the researcher reviews literature from three aspects. Firstly the nature of algebra and secondly pupils' problem solving mental strategies and finally the errors and possible misconceptions under the three main areas understudy.

2.1 Nature of algebra

There are many conceptions about algebra in literature. It is correct to say many historically developed concepts about algebraic expressions are present in the current secondary and high school algebra curricula throughout the world. This means that the inclusion of algebra in secondary and high schools shows how important this branch of mathematics is and how it is related to other branches of mathematics. These conceptions are important when selecting algebraic concepts in a test for any grade level in secondary and high schools.

There are four conceptions in algebra and these are; algebra as generalized arithmetic, algebra as a study of procedures for solving certain kinds of problems, algebra considered to be the study of relationship among quantities and finally algebra as a the study of structures (Usiskin,1988).

The first conception of algebra as generalized arithmetic considers a variable as a pattern generaliser. For example, the arithmetic expressions such as *-2x5= -10*
-3x5= -15 could be generalized to give properties such as *–axb*=-*ab.*

The second conception suggests that algebra is a study of procedures for solving certain kinds of problems and finding a generalization for a question and solving it for the unknown. For example, we consider the problem: "When 3 is added to 5 times a certain number, the sum is 40. Find the answer'' (Usiskin 1988: 12). The first task is to translate the algebraic language into an equation of the form "$5x + 3= 40$". Therefore in this conception, variables are either unknowns or constants. The most important thing here is to "simplify and solve".

The third conception is algebra is considered as the study of relationships among quantities. This implies that variables tend to vary. For example, the formula for the volume of a cuboid is $V= LBH$. In this relationship there are four quantities and hence no unknown, instead all values V, L, B and H can take as many values as possible (Booth, 1988).

The last conception looks at algebra as the study of structures. Under this conception, the variable is little more than an arbitrary symbol. This is the view of variable found in abstract algebra for example in the problem “$3x^2+ 4ax-132a^2$”. Usiskin (1988), points out the relationship that exists between the various conceptions of algebra and the different uses of letters in teaching, represented in Table 1 below:

Table 1: **Uses of letters in algebra.**

Conception of Algebra	Use of Variables
Generalised Arithmetic	Pattern Generaliser
Means to Solve certain problems	(Translate, Generalise)
Study relationships	Unknowns, constants (Solve, Simplify)
Structure	Arguments ,parameters (relate, graph) Arbitrary marks on paper (manipulate, Justify)

Source : Usiskin,(1988).

Further it has been noted from the literature that the nature of algebra has epistemological roots and has been at the centre of discussions of what algebra is. Kieran (1990) discusses a serious debate among British mathematicians in the first of the nineteenth century. One side took the position that algebra is generalised arithmetic in this sense, algebra deals with quantities and operations on them. The other side took the position that algebra is purely a system of arbitrary symbols and is purely ruled by arbitrary principles making it difficult for the pupils to understand it

Algebra is being viewed as "extension or completion" of mathematics. Arithmetic cannot live without the help of algebra because it needs real numbers for functioning Algebra therefore as generalized arithmetic may not be appropriate from a school pedagogical perspective (Wheeler, 1996).

2.2 Pupils' problem solving strategies and mental strategies.

Pupils' construction of knowledge in mathematical problems solving is influenced in their use of strategies as they attempt to master a problem situation. Various stages of the solving process will bring different sets of challenges to them.

According to Polya (1957), a problem solving strategy follows a four-phase heuristic process. The stages under this model include: understanding the problem, devising a plan, carrying out the plan and looking back. According to Polya (1957), a pupil has to strictly follow the outlined stages in order to fully understand the problem and get a correct solution. Polya advocates a linear type of approach to problem solving strategy.

On the other hand Schoenfeld (1983), devised a model for analysing problem solving that was derived from Polya's model. This model describes a mathematical problem in five levels and these include: reading, analysis, exploration, Planning/ implementation and verification. In applying this framework, Schoenfeld discovered that expert mathematicians returned several times to different heuristics episodes. For instance, in one case a problem solver engaged in the following sequence of heuristics: read, analyse, plan/ implement, verify, analyse, explore, plan/implement and verify. Therefore, according to Schoenfeld(1983), the problem solving model is rather cyclic than linear.

Confrey (1991), presented a simple model to describe the construction of cognitive structures in problem solving. This model has three stages and these include: identifying the problem, acting on them and finally reflecting on the results of those actions to create operations. The model is followed by checks to determine whether the problem was resolved satisfactorily by reflecting on the problem again, thereby making the process cyclic. This is represented diagrammatically in the figure below:

Figure 1 : Stages of problem Solving (Confrey, 1991, p.119)

Comparing and contrasting the models devised by Polya (1957), Schoenfeld (1983) and Confrey (1991), it is evident that, although the number of steps in the problem solving process is different for each model, almost all of them contain similar basic aspects.

2.3 Philosophy underpinning algebraic concepts.

As earlier on alluded to, one of the main debates going round today by mathematics educators are whether algebra should be presented as generalized arithmetic governed by the laws or those concerning computations on plain numbers. The other side of the argument is that instead of working with specific numbers, the letters which represent numbers in algebra should be treated as a separate symbolic system based on formal rules (Kilpatrick and Izsak, 2008). There are proposers and opposers of the two views.

There is a direct link between arithmetic and algebraic concepts. A good example to illustrate this is the manipulation of algebraic expressions having integers and overgeneralization of dividing procedures (fraction errors) have their source in arithmetic misconceptions, and the incomplete understandings and the failure to transfer arithmetic understandings to algebraic contexts (Norton and Irvin, 2007). Pupils who are not comfortable in computing numbers will be less disposed to manipulate symbols because computational procedures with fractions provide a natural entry into symbolic use.

Many algebraic problems are difficult for pupils to comprehend, because solving them may require an understanding of the conceptual aspects of fractions, decimals, negatives numbers, percentages or rates. For pupils to have conceptual understanding they need to understand the structure or rules of algebra or rules of arithmetic such as associative, commutative, transitivity and the closure property. For example, students should understand that $\frac{3-1}{8}$ can be separated as $\frac{3}{8}$ - $\frac{1}{8}$ in the same way as they understand the reverse process. There is a connection between arithmetic and algebra as students sometimes assume incorrect rules when solving problems involving algebra. One misconception which students make is on the comparison of equal quantities. For example when students were asked which quantity was larger, smaller, or equal: $\frac{25x}{5}$ or $5x$? They said that $\frac{25x}{5}$ was larger because it had larger quantities (Norton and Irvin, 2007).

On the other hand, algebra and arithmetic are different (Lee and Wheeler, 1989). They suggested that lack of numerical support for algebraic reasoning was the main reason why some students perceived the world of algebra and arithmetic to be disconnected. There are many covert signs in secondary school algebra which has its own rules, not necessarily deducible from the rules of arithmetic. For example two digits in the number 12 have there own place value where as in algebra xz means x times z. The result of this confusion leaves many pupils unsure of the grounds that justify particular algebraic transformations.

Booth (1984) distinguished some properties of arithmetic strategies which hinder the development of algebraic understanding. He noted that arithmetic strategies were intuitive, primitive and context-bound. They usually involve the basic operations: addition, subtraction, multiplication and division. He further noted that arithmetical problems are connected so that the pupil can reason from the known to the unknown directly. However, on the contrary, algebraic problems are labeled as “disconnected” because they require reasoning with unknowns. Hence, arithmetical and algebraic reasoning appear to be essentially different and thus could cause serious obstacles from the passage of arithmetic to algebra.

One area where pupils' ideas on arithmetic can influence their performance in algebra is the use of parentheses. Kieran (1979), records that students believe that the written sequence of operations determines the order in which the computation should be performed, In addition many students think that the value of an expression remains unchanged even if the order of calculation is varied. Booth (1984, p.55) gives an example of Keith, thirteen years old, computing 18x 27 + 19 having just calculated 27+ 19 x18 from left to right.

Keith: Do ….27 +19 x *18. It's the same as the last one…its just the other way around.*

"I": Right, well, suppose I came along and thought it meant multiply 18 x 27, and then add 19. Would I get the same answer?

"K": Yes.

"I": Which way would you do it?

"K": Either! Either way. Depends what comes in to my mind at that time.

"I": But would it matter which way you did it?

"K": No, you would still get the same answer.

The rules used to solve the problems in algebra are closely associated with the procedural and conceptual (structural) aspects of algebra. For example substituting different values for the variable in a simple equation until a correct value is found is a process that is procedural. The student does not need to understand the underlying principles of the structure of algebra to solve such problems. However in terms of algebra, pupils have to understand how and why these rules or properties work in order for them to explain the application of these rules. In most cases, pupils fail to explain the rationale behind applying these rules because they lack conceptual understanding.

2.4 Errors and misconceptions in algebra.

Research has shown that systematic errors have documented that pupils hold mini-theories about scientific and mathematical ideas. Research has also shown that pupils have many naïve theories, preconceptions or misconceptions about mathematics that interfere with their learning processes (Posamentier, 1998). However because pupils have actively constructed their misconceptions from their experiences, they are very much attached to them. Pupils' erroneous thinking is an important part of the learning process (Olivier, 1992). Similarly, pupils' errors and misconceptions contribute to the process of learning. Errors and misconceptions do not originate in a consistent conceptual frame work based on earlier acquired knowledge but rather are usually out grow of an already acquired system of concepts and beliefs wrongly applied to an extended domain (Nesher, 1987).

According to Rodatz (1979), there are four categories of errors namely iconic representation, errors due to deficiencies of mastery pre-requisite skills, facts and concepts, errors due to incorrect associations or rigidity of thinking leading to inadequate flexibility in decoding new information and the inhibition of processing and errors due to the application of irrelevant rules or strategies. Some errors caused by lack of meaning can be differentiated into three stages: algebraic errors originating in arithmetic, use of formulas or procedural rules inadequately, and errors due to the properties themselves of algebraic language (structural errors) (Barrera, Medina, and Robayna, 2004).

It is true that quite often, intuitive background knowledge hinders the formal interpretation or use of algorithmic procedures as seen in pupils misinterpretations of $(a+b)^2$ as $a^2 + b^2$ can be categorized as evolving from the application of the distributive law intuitively. In some instances, solving schema is applied inadequately because of superficial similarities in disregard of formal similarities. Sometimes, solving schema is deeply rooted in the student's mind is mistakenly applied despite correct, intuitive understanding (Fischbein and Barash, 1993).

More often than not, the intuitive interpretation is based on a primitive, limited but strongly rooted in individual experience that annihilates the formal control or the requirements of the algorithmic solution, and thus distorts or hinders a correct mathematical reaction. The solving procedures, acting as generalized models, may sometimes lead to wrong solutions in disregard of the corresponding formal constraints. For instance, students usually write $sin(x+y) = sinx + siny$. Obviously the property of distributive law of multiplication over addition $n(x+y) = nx + ny$ does not apply in the above situations (Fischbein and Barash, 1993).

Matz (1980) extended the research on pupils' errors to behaviour in a rule based problem with a view to building a generative theory that accounts for as many errors as possible that pupils make in problem solving. The theory states two extrapolation mechanisms for generating algebraic errors. They are the use of a known rule in a new situation where it is appropriate, and incorrectly adapting a known rule so that it can be used to solve a new problem. This is true for overgeneralization of the distributive law (Kirshner, 1985).

In some instances, errors are logically consistent and rule based rather than random (Ben-Zeev, 1998). He further discussed the need to have a clearer distinction among various stages of the problem solving process such as execution of errors and encoding them.

2.4.1 Variables in algebra.

Research shows that novice algebra students do not understand the meaning of letters and commonly interpret them as standing for objects or words (Macgregor and Stacey, 1997). Even once pupils are able to accept that letters are standing for numbers, they have a tendency to associate letters with their positions in alphabet (Watson,1990). Some pupils do not understand that multiple occurrences of the same letter represent the same number (Kieran,1988). After these misconceptions are addressed, students still view letters in algebra as representing specific unknown values, as $3+y=9$,rather than for numbers in general as in $a+b=b+a$. When letters are present in algebraic entities, this is seemingly difficult for students. Kieran (1990), explains that in arithmetic 12m can mean 12 meters, that is 12 times 1meter.The same in algebra can mean 12 times some unknown number of meters. Therefore, the letter carries two different meanings depending on the context.

Phillip (1999) used seven categories to group variables with examples to illustrate the uses of them. These were letters as labels as f and y in 3f =1y to denote 3feet in 1yard, as constants π, e and c, as unknowns to denote x in $5x - 9 = 11$, as generalised numbers to denote $x + y=y + x$, as varying quantities to denote a, b in $b = 3a + 5$, parameters to denote m, c in $y= mx + c$ and as abstract symbols e, x in $e*x = x$

A detailed classification about children's interpretation of letters was given by Kuchemann (2001) who administered a 51-item paper and pencil test to 3000 British secondary school students. Using a category originally developed by Callis in 1975 , Kuchemann categorized each item in the test to six levels ; letter evaluated, letter ignored, letter as an object, letter as a specific unknown, letter as a generalized number and letter as a variable. The first category had an example such as "what can you say about x if $x + 3 = 5$?. Then the second and third categories had examples such as "if $a - 230 = 512$, then what is a -230 and simplify $4a + 66 + 3a$. The fourth and fifth categories had examples such as "which is larger $2y$ or $y + 2$?"

The results indicated that pupils' interpretations of letters were partly dependant on the nature and complexity of the question. It was further observed that a small percentage of 13-15 year old pupils interpreted the letter as a generalized number and also as specific unknown. Nevertheless, 73% of 13 year olds , 59% of 14year olds and 15 year olds either treated letters as concrete objects when they were not or they ignored the existence of the letters completely.

Other researches have shown that students of up to the age of fifteen failed to interpret algebraic letters as generalized numbers or specific unknowns. Most of the pupils ignored the letters and replaced then with numerical values. The explanation for these errors in literature is that there is a general link to levels of cognitive development (Macgregor and Stacey, 1997).

However, an alternative explanation for origins of misinterpretations are given and these include: intuitive assumptions and pragmatic reasoning about a new notation, analogies with familiar symbols, interference from new learning in mathematics and the effects of misleading materials. An example of such analogy is the ancient Roman numeral system in which *X1* means one more than ten and *1X* means one less than ten. This analogy causes students to apply their experience in one number system where it is inapplicable (Macgregor and Stacey, 2007).

Further more, conversions stated 3m=300cm are read as “3m are equivalent to 300 centimetres. Pupils use this knowledge to read algebraic equations such as 6P=S as 6 professors are equal to 1 student (Booth, 1988). Intuitively this implies that there are 6 times as many professors as there are students. However, algebraically this equation is representing the exact opposite. This conversion could cause students to incorrectly translate word sentences into algebraic equations. In the reverse of the task above, namely symbolizing that there are six times as many students as professors, the most common error is writing the equation “6S=P known as the reversal,”(Wollman, 2003 p.55). This translation, however, would make sense to the student who reads it as a conversion statement, “six students are equal to one professor.”

2.4.2 Algebraic expressions.

Letters are very important in algebra as they are used to build up an algebraic expression. Either one letter or a combination of letters may be used in an algebraic expression. One of the major misleading factors is where pupils interpret expressions such as 6a as a short form for “6 apples”. This is further transformed into an algebraic expression for $3a + 5a$ as 3 apples added to 5 apples (Agnieska, 2007).

Other problems which pupils face relate to conjoining algebraic expressions. A good example is an expression such as $6x + 5$ which causes students to have a misconception by forcing to simplify the expression to $11x$ when $6x + 5$ is actually a final answer. This is a feeling pupil draw from arithmetic problem where a final answer is always a single digit (Tall and Thomas, 2007).

Due to similar meanings of ‘and’ and ‘plus’ in natural language students may consider xy to mean the same as $x + y$. Pupils may erroneously draw on previous learning from other subjects that do not necessarily differentiate between conjoining and adding. For example, in chemistry, adding oxygen to hydrogen produces H_2O (Tall and Thomas, 2001).

Sometimes pupils’ experience difficulty in accepting the lack of closure property of algebraic letters and perceive open algebraic expressions as incomplete and try to finish them by carrying an oversimplification. An example is when pupils consider an answer such as $x + y$ as incomplete and try to simplify it to xy. A typical explanation for this misconception is that many arithmetic problems tend to have a final solution as a single digit (Booth; 1988, Tall and Thomas, 2001).

Many pupils also make common errors in simplifying algebraic expressions. This is due to the fact that pupils retrieve correct information but use inappropriate rules. For instance, pupils incorrectly misapply $\frac{ax}{bx} = \frac{a}{b}$ in to an expression like $\frac{a+x}{b+x}$ (Matz, 2000). Another example, is given by Schoenfeld (1985) who showed that students write $x(yz) = xy + xz$ by considering the transformation $x(y + z) = xy + xz$. The pupils apply the distributive law which is not correct to this situation.

2.4.3 Equations.

An equation is an algebraic expression which is combined together with an equal sign. In order for a pupil to solve a problem correctly, one must know the application of rules of simplifying algebraic expressions. An equal sign is used to express the equivalence between the two sides. This is what seems to give pupils a big problem.

The mathematical interpretation given to an equal sign by the pupils is sometimes different from the accepted meaning. The two interpretations attributed to the equal sign are symmetric and transitive relation. The symmetric relation indicates that the two quantities on both sides of the equal sign are equal. On the other hand the transitive relation indicates that a quantity on one side can be transformed to the other side using rules. In elementary school, the equal sign is used more to announce a result than to express a symmetric or transitive relation. Some of the sources for errors in equations are caused by the misuse of the equal sign (Kieran et al., 1990). This means that when pupils use the equal sign as a 'step marker' to indicate the next step of the procedure, they do not really consider the equivalence property of if.

In some instances the procedures required to solve some equations involve transformations that are different from normal operations that pupils use. The procedure for equation solving rests on the procedure that adding the same number to or subtracting the same number from both sides of the equation conserves the equality (Filloy and Rojano, 1984). This principle is equally applicable to multiplying or dividing both sides by the same number. Equations that have the variable ‘x’ on one side such as $x + a = b$, $ax + b = c$, can be solved by those methods. However the problems occur an with equation of the form $bx + c = dx + e$. The procedure required here to solve the equation of this type involves transformations that are different from mere subtraction of bx or dx from both sides.

Further, pupils usually have difficulties in solving linear systems of equations with two unknowns such as: $y = 2x + 3$, $y = 4x + 1$ despite the unknown being represented by a letter (the y), it has also been represented by an expression that involves another unknown (the letter x). In this case, the students will have to operate the unknowns with a second level representation of variables to them (Filloy, Rojano and Puig, 2007).

2.5 Summary

In this section I began the discussion with the nature of mathematical understanding in general and algebraic thinking in particular. Further, error categories under the three main areas of the research were explained. The main cognitive obstacles that students encounter in solving algebraic problems according to literature include difficulties in transforming from arithmetic to algebra, difficulties in understanding the procedural and structural aspects of algebra, use of incorrect mini-theories and difficulties in processing iconic representations.

The other non-mathematical factors include over confidence, lack of motivation, carelessness and lack of attention hinder pupils' progress. In the past, research on student errors and misconceptions has been limited to the study of isolated conceptions such as variables, equations, inequalities, or expressions. However, comparatively fewer attempts have been made to understand the combined effects of misconceptions and their interrelatedness pertaining to a number of areas. Algebraic concepts like other areas in mathematics are closely related so that student misconceptions could better be viewed if we could study those concepts together in a study and examine the interrelationship among the error patterns.

It is also evident from the review of literature that research which has been conducted so far in the past has been limited to a single method that is qualitative or quantitative. I feel that if a mixed method approach were used deeper understanding of the problem would be possible (Creswell, 2003). It is from this background that looking at the gaps that have been explained above that I felt that the research involving errors and misconceptions in algebra is worth carrying out.

CHAPTER 3

RESEARCH METHODOLOGY

3.0 Introduction

In this section, the methodology that was employed at various stages of the study is discussed and thereafter offers a summary of the research. This section also highlights and reviews the methods that were used at different stages of the study. The issues of validity, reliability, sampling procedures and data collection instruments are also discussed.

3.1 Mixed Methods Research.

Research involves systematic investigations undertaken to resolve a problem. According to Brew (2001), the general purpose of research is to contribute to the body of knowledge that shapes and guides academic disciplines. There are two main approaches to research namely, scientific and naturalistic. In the scientific approach quantitative research methods are employed in an attempt to establish general laws. This approach assures that social reality is objective and external to individuals. Alternatively the naturalistic approach emphasizes the importance of the subjective experience of individuals with a focus on qualitative analysis (Burns, 2000).

The division between quantitative and qualitative research is still noticed . In order to strike a balance between quantitative and qualitative methods, the mixed methods systematically combines ideas from both quantitative and qualitative methods (Johnson and Christensen, 2008). Mixed methods researchers strongly believe that by mixing quantitative and qualitative methods, the researcher can get richer data and stronger evidence of knowledge than using a single method (Creswell, 2003).

The idea is further reinforced by the belief that social phenomena are complex and one needs to understand them better, there is need to employ mixed methods. Johnson and Christensen (2008), list five purposes of a mixed design approach as shown in Table 2.

Table 2 : Purposes to select a mixed study.

Purpose	Explanation
Triangulation	Seeks convergence, corroboration, correspondence of results from different methods
Complementarity	Seeks elaboration, enhancement, illustration, clarification of the results from one method with the results from the other method.
Development	Seeks to use the results from one method to help develop or inform the other method. Where development is broadly construed to include sampling and implementation, as well as measurement decisions.
Initiation	Seeks the discovery of paradox and contradiction, new perspectives of frame works ,the recasting of questions or results from one method with questions or results from the other method
Expansion	Seeks to extend the breadth and range of inquiry by using different inquiry method

Source : Johnson and Christensen (2008, p. 451).

The researcher employed all the above five methods as these improved the focus of the research. Triangulation is the term used to indicate the use of multiple pieces of evidence to claim results with confidence (Johnson and Christensen, 2008). For instance, the researcher used pupils' written work, interview transcripts and researcher notes to triangulate the data and arrive at valid conclusions about pupils' misconceptions and errors in algebra. The term complementarity is used to discuss in detail and understand the different aspects of phenomena. In order to clarify and further elaborate the results, the interviews were used to enrich the data. In addressing the developmental purpose of the mixed method approach the quantitative phase was used to inform the qualitative phase. For example pupils selected for interviews were based on the test results. The term "expansion" promotes the breadth and range of inquiry by using different methods of inquiry.

3.2 Research design.

The purpose of the study was to identify pupils' errors and misconceptions in algebra pertaining to: variables, algebraic expressions and equations. The sequential explanatory design in which the collection and analysis of quantitative data was followed by the collection and analysis of qualitative data (Creswell, 2003). The main purpose of using the sequential explanatory design was to use qualitative results to assist in explaining and interpreting the findings of primarily the quantitative design. In addressing the developmental purpose of the mixed method inquiry, the pupils were selected using the quantitative phase to inform the qualitative phase.

In the quantitative method, the test instrument was used to identify and classify pupils' errors. Thereafter, interviews were used to identify pupils' reasoning for their misconceptions and errors in the qualitative phase of the study. In this study qualitative study was used to explain the quantitative data. The findings of the quantitative study were used to determine the type of data to be collected in the qualitative phase (Gay, Mills, and Airasian, 2006).

In this study, qualitative data was used to explain quantitative data. Secondly, qualitative data was used to explore quantitative data in depth. Since the overall design of my research was more exploratory than descriptive, the case study method was used extensively. Multiple data sources such as pupils' written work, pupils' interview transcripts and researchers' notes were used.

As a general frame work for interviews, I adopted the interview format developed by Newman (1987). The questions in this format were divided into three main areas thus: input stage, process and output. The input stage had the component of reading the problem, interpreting it and selecting a strategy to solve it .the process stage contained solving the problem using a selected strategy. The output stage contained verification of questions from the interviewer. Interviews were strictly used to explore the students' thinking process.

3.3 Target population

The study targeted grade 11 secondary school pupils in Petauke district of the eastern province of Zambia.

3.4 Study sample and sampling techniques.

According to de Vos (2002, p.145), in a survey research, the researcher must first identify the research population after which data collection methods may be used to gather information. In this study, the population was all grade 11 pupils in Petauke district. As this is definitely a very large population to handle, it was therefore necessary to work with a sample of the population. A sample is a group of subjects or persons selected from the target population. This is a group of individuals with the same characteristics as the target population and is trusted to provide the relevant data as it would be obtained from the whole population (Vos, 2002).

For this study, 60 grade 11 pupils participated from two secondary schools, hereafter designated school A and school B. School A is located in a growing town and is a boarding secondary school where pupils are fed from a dinning hall and have access to compulsory study time in the evenings. It has a total population of 1145 pupils. Whereas school B is located in an agricultural rural area and is a day school. For pupils who come from distant places, the school has provided a simple shelter and pupils do self-catering. It has a total population of 448 pupils.

The two schools were selected using purposive sampling technique which involves selecting units or cases based on a specific purpose rather than randomly. The power of purposive sampling lies in selecting information which can help in the analysis of the results of the study (Kombo and tromp, 2010).

Pupils to sit for the test were selected using stratified random sampling, which ensures that specific groups are represented proportionally. The population was separated into two groups (strata), one for boys and another for girls. There after, independently selected a random sample from each group (stratum). This provided the quantitative sample of the study.

The purposive sampling technique was used to select eight pupils to be interviewed. Four pupils were selected from each of the two schools. There are different types of purposive sampling techniques such as extreme case sampling, critical case sampling and snowball sampling. For the purpose of this study, the extreme case sampling was used because it focuses on information which is unusual or special in some way (Kombo and Tromp, 2010). The results of the test were used to select the qualitative sample of the pupils.

3.5 Pilot study.

A pilot study is a “small study conducted prior to a larger piece of research to determine whether the methodology, sampling, the instruments and analysis are adequate and appropriate” (bless and Higson-smith, 2000, p.155). This mini-research is intended to expose deficiencies of the measuring instruments or the procedure to be followed in the actual project. In this study, the piloting was achieved by consulting experienced teachers of mathematics. The researcher in this particular study, referred to the supervisor for advice particularly on the wording of the statements on the items in the pupils’ tasks. The purpose of this was to get a clearer picture of the demands of the investigation with regards to time, finance and transport costs. The amount of time within which the pupils’ tasks might be completed was observed during piloting.

The pilot stage was used to refine the instruments and identify other possible unforeseen problems that might emerge during the main investigation. Therefore the pilot stage was important as it allowed corrections to be made to the test before administering the main test.

Some of the changes made included questions 4, 5(a), 5(b), 7, 9(a), 9(b) and 9(c) which were deleted because their facility value was greater than 0.8. Question 2 was deleted because similar concept was tested in item 3. For question5(d), the wording was changed from simplify to expand. As for question 6 it was deleted because the facility value was below 0.3.

Features such as overall structure of the test, suitability, item coherence, appropriateness and face validity of the test were discussed with teachers of mathematics. The pilot test contained 13 items under the 3 main areas of algebra: variables, expressions and equations. Each item in the test belonged to one category only. The items were not mutually exclusive as an item could belong to more than one category. However, the major concept that was expected to test using the item was considered as the one that made up for that category.

3.6 The facility value

The pilot study was conducted with a group of 30 pupils in two selected secondary schools in Petauke district. The test items were given and marked and the facility index was calculated using the formula: facility index = $\frac{P}{n}$, where p is the number of pupils who answered a test question correctly and n is the total number of pupils in the sample (McAlpine, 2002). The test items that were easy were to give fewer pupils' errors. The response rate was too low for the items that were difficult to answer. Therefore, a reasonable facility value in between 0.3 and 0.8 was selected for the items that were to be included in the final test.

3.7 Pilot interviews

Pilot interviews were conducted on two pupils chosen randomly. This was important as it enabled the researcher to understand the right kind of questions to be asked and to decide on suitable pace for interviewing pupils. The interviews were tape-recorded. By listening to the interviews the researcher decided to make some adjustments to the questioning technique. Probing was more important for me than asking lengthy questions. In some instances were the researcher felt he was quick, the pace was slowed to give the interviewee time to think and answer.

3.8 Administration of the main test.

After the pilot test, the final version of the test was prepared with a total of 12 items. The test comprised all the three categories already discussed. The pupils were given instructions to use algebraic methods when solving the questions. The main test was administered to 60 pupils from the two schools by the researcher with the help of teachers of mathematics. There were 30 pupils selected from each school. After that, the papers were marked by the researcher and categorised for errors. Pupils' answers from the test were carefully analysed and grouped into various error types. The researcher assembled the same errors that appeared in different questions into one category with their percentages.

For each category, the percentage of occurrence of a particular error in that category was calculated. For example, the number of pupils who made the same error was divided by the total number of pupils who attempted the question. These percentages were used to calculate the mean number of errors for each conceptual area.

3.9 Pupil interviews

In this study, interviews were used to explore pupils' thinking. The interview method was preferred here because according to Cohen and manion (1994), interviews are used, for instance to follow up unexpected results, to validate other methods or to go deeper into motivating the respondents and their reasons for responding as they did. In this case pupils answers from a pencil-and –paper test were used to get a deeper understanding of the pupils' errors and misconceptions in Algebra.

The interviews were clinical in nature. Clinical interviews also known as "flexible interviewing" was available for the purpose of assessing any kind of mathematical problem solving ability (Cohen and Manion, 1994). Being, flexible, responsive and open-ended in nature, the interview questions were structured based on the items in the test. The pupils selected for the interviews were those who had shown some misconceptions, misinterpretations or had shown an peculiar way of answering some questions in the test given.

According to Brink (1996), unstructured interviews produce more in-depth information on subjects' beliefs and attitudes that cannot be obtained through any other data gathering methods. During the interview process, the interviewees were encouraged to explain what they were doing as they attempted to solve the problem. However, short intervening questions were asked during the process in order to probe their thinking thoroughly. Each interview lasted between twenty to thirty minutes. The interviews were tape-recorded and transcribed verbatim. Out of the eight pupils earmarked to be interviewed only five turned up. The reasons for the absenteeism were not clear, however the interviews went ahead with the five pupils.

3.10 Reliability of the test.

As a researcher, one must ensure that test items are reliable. Measurements are said to be reliable if they reflect the true aspects but not the chance aspects of what is going to be measured. This means that a study is said to be reliable if another researcher carrying out the same research is likely to replicate your findings (Gilbert, 1989; Wisker, 2001).

Literature has outlined several forms of reliability. Nunnally (1972) suggested that these include: alternate-form reliability, test-retest reliability and split-half reliability. The alternate-form reliability is one which involves correlating scores of students obtained by administering two alternate forms of the same test to the same group of pupils. The test-retest reliability involves giving the same test on two occasions while the split-half reliability method needs the same test to be divided into two equal parts and their correlation is then found (Gay, Mills, and Airasian, 2006).

For the purpose of this study, the split-half method was used to check the reliability of the test instrument, because it is a "more efficient way of testing reliability" and it is less time consuming (Durrheim, 1999a,p.90). The split-half method requires the construction of a single test consisting of a number of items. These items are then divided (or split) into two parallel halves (usually, making use of the even-odd item criterion). Pupils scores from these halves are then correlated using the spearman-brown formula: r total test = $\frac{2rsplit-half}{1+rsplit-half}$.

The value of the reliability coefficient ranges between -1 and 1. The split-half reliability coefficient for the preliminary trial was 0.64 and the reliability coefficient for the whole test using the above formula was 0.76. Since this shows an adequate level of reliability, the test was considered to be reliable.

3.11 Validity of the test.

A test is said to be valid if it serves its intended function well. According to Remmers (1965) there are four main types of validity namely content, concurrent, predictive and construct. Content validity addresses how well the content of the test samples the subject matter. Concurrent validity measures how well the test scores correspond to already accepted measures of performance. Predictive validity deals with how well predictions made from the test are confirmed by subsequent evidence. Construct validity is about what psychological quantities a test measures (Remmers, 1965).

In order to preserve the issues of content validity, the test was prepared by consulting the Zambian secondary junior mathematics syllabus as a basis. The content of the test was discussed with four selected teachers of mathematics and their suggestions were included prior to the administration of the test.

3.12 Data analysis.

3.12.1 Introduction.

The study followed a sequential mixed design approach in which the collection and analysis of quantitative data was followed by collection and analysis of qualitative data. Analysis is a stage in research in which data collected is carefully scrutinised in order to help the researcher arrive at conclusions, suggestions and recommendations.

3.12.2 Quantitative analysis.

The pencil-and-paper test comprised the quantitative data of the study. Durrheim (1999b, p.96) argues that "statistical procedures are used to analyse quantitative data''. Basically, statistical analysis in educational research is of two types; descriptive and inferential data analyses (Daramora, 1998, Durrheim, 1999b). Descriptive analysis seeks to organise and describe the data by investigating how scores are distributed on each construct, by determining whether the scores on different constructs are related to each other (Durrheim, 1999b). In this study the means, frequency charts and bar charts were used to analyse, describe and compare sets of quantitative data in this study.

However, descriptive analysis of data does not allow the researcher to extend conclusions beyond the sample data. Inferential data analysis, by contrast, allows the researcher to extend the knowledge obtained from the sample data to whole population (Kothari, 2012).

3.12.3 Qualitative analysis.

Secondly, the detailed qualitative analysis was discussed with an emphasis on pupils' interviews. Qualitative data analysis is both an iterative and an on-going process. According to Glesne (1999, p.130), qualitative data analysis is "working with the data, you describe, create explanations, pose hypotheses, develop theories, and link your story to other stories." Although there are many ways of analysing qualitative data, it becomes a common sense in qualitative inquiry that analysis of data is a continuous task from the beginning of research.

The interview was audio-taped and later transcribed verbatim. It might not be easy to conduct such an interview because in most cases the interviewees became cautious. In order to avoid this the purpose of the interview was made known to the interviewee before the interview.

The transcripts were analysed with reference to pupils conceptions, misconceptions and errors. The analysis began by listening to the taped interviews and editing the transcripts. The interview transcripts were analysed by person, and by item. The transcribed interviews were read over a number of time so as to keep as close as possible to the data source.

3.13 Ethical issues.

Informed consent from head teachers and parents were obtained using relevant documentation. The documents included informed invitation letters to the head teachers to conduct research in their schools, informed invitation letters to pupils for their participation and consent forms to parents for their children's participation in the study. Only pupils whose parents granted permission were tested and interviewed. Participation was voluntary and during the reporting and discussion of data, none of the participants, schools and communities were identified.

CHAPTER 4

PRESENTATION OF FINDINGS

4.0 Introduction.

This chapter presents the main findings of the study. In the first phase a detailed analysis of the quantitative data were presented using both descriptive and inferential statistics. In specific terms, various descriptive statistics such as frequency distributions, charts, measures of central tendency and construction of rubrics were used. Further the use of statistical tests such the Analysis of Variance (ANOVA) and the Chi-square were used to analyse quantitative data so as to extend knowledge from a sample to the whole population. Secondly, the detailed qualitative analyses were discussed with an emphasis on pupil interviews. The interviews were audio-taped and later transcribed verbatim and analysed qualitatively.

The researcher adopted the method explained by Johnson and Onwuegbuzie (2004) for mixed method research data analysis, which comprised three stages, discovery of patterns (induction), testing of patterns and hypothesis (deduction), and uncovering the best set of explanations for constructing meaning related to findings (abduction). Since the aim of the research was to identify pupils' misconceptions and errors in Algebra, I justified, whenever necessary, how pupils wrong answers exposed their misconceptions.

4.1 What errors and misconceptions do secondary school pupils make when solving problems related to variables?

There were four questions in the test which asked for pupils' understanding of variables and these were questions 1, 3 8 and 12. The rubric for errors and possible misconceptions for variables was constructed. The type of error or possible misconception identified under variables included: assigning labels, values or verbs for variables, assigning labels for constant, misinterpreting the product of two variables, misjudging the magnitudes of variables, lack of understanding of variables as generalised numbers, lack of understanding of the unitary concept when dealing with the variables and forming wrong equations as answers.

Individual percentages were given against each error or possible misconception. Since the aim of the study was to identify pupils' errors underlying misconceptions, the 'correct answers' and 'no answers' were eliminated. There were different forms of incorrect answers and sometimes, there were no visible reasons for such responses and they were classified into separate group. Finally, the error groups were carefully examined again to combine similar groups together or separate different groups. The percentages for each error type were calculated based on the number of pupils who answered the question.

Errors and misconceptions on question 1.

Question 1 was meant to test if the pupils had a proper understanding of the concept of a variable. Question read:

Titus sells x sugar canes. Luka sells twice as many sugar canes as Titus. A sugar cane costs K1000.

(a) Name a variable in this problem

(b) Name another variable in the problem

(c) Name something which is not a variable

The errors and possible misconceptions are shown in Table 3.

Table 3 : Pupils errors or possible misconception in question 1.

Type of error or possible misconception.	Expected answer(correct response)	Pupils incorrect response(s)	Frequency (incorrect responses) N= 60	Percentage of incorrect responses (%)
Assigning labels for variables.	Variable = x and another variable is $2x$	2 times or 2.	5	8
Assigning values for variables.	Variable = x and another variable is $2x$	Cost k500.	6	10
Assigning verbs for variables.	One variable is x and another one is $2x$.	Sells	3	5
Assigning constants for variables.	One variable is x and another one is $2x$	Titus or Luka.	7	12
Forming wrong equations as answers.	$2x + x = 500$ and $2x \times x = 500$.	$2x + x = 500$ and $2x \times x = 500$.	8	13

From Table 3, it can be seen that pupils misinterpreted a variable as a 'label' or even as a verb such as 'sells'. They did not perceive the correct interpretation of the variable as the 'number of a thing'. It was difficult for pupils to distinguish between a variable and a non-variable. The pupils were confused with and viewed variables as constants or vice-versa. This error was noticed when pupils were asked to name something in the question that was not a variable, the answer such as 'Titus' or 'Luka' were given. In a general sense, these answers may be considered as incorrect in the context of the given problem since there was a variable or a given number attached to it. Therefore, these words had meanings in the given context when they are taken together with those variables.

Errors and misconceptions on question 2

Question 2 was meant to find out if pupils perceived the product of two variables as two separate variables when combined by a sign. Question 2 was:

What does *xy* mean?. Write your answer in words. The error categories on question 2 are shown in Table 4.

Table 4 : Pupils errors or possible misconception in question 2.

Type of error or possible misconception.	**Expected answer(correct response)**	**Pupils' incorrect response(s)**	**Frequency of incorrect response(s)**	**Percentage of incorrect responses (%)**
Misinterpreting the product of two variables	*xy* means *x* multiplied by y	*xy* means a variable	6	10
Misinterpreting the product of two variables.	*xy* means *x* multiplied by *y*	*xy* means the variable represents part of the question	4	7

From Table 4, it can be observed that pupils had difficulties to perceive the product of two variables as two separate variables when combined by a sign. They viewed the product as one variable.

Errors and misconceptions on question 8

Question 8 was meant to find out if pupils had the correct understanding of the magnitude of variables. The question was: Which is larger than the other y or x in y = 2x + 3. Explain ?

Table 5 : Pupils errors or possible misconception in question 8.

Type of error or possible Misconception	**Expected answer(correct response)**	**Pupils' incorrect response(s)**	**Frequency of incorrect responses. N= 60**	**Percentage of incorrect responses (%)**
Misjudging the magnitudes of variables.	You cannot know because they are two different variables either one could be bigger depending on the number.	*x* is larger because it has a bigger value beside it	7	12
Misjudging the magnitudes of the variables		*x* is larger because you can multiply by 2	3	5
		I do not, they are both variable.	6	10

It can be observed from Table 5, that pupils misjudged the magnitude of two variables by examining their coefficients when they were in an equation such as $y = 2x + 3$. Since *x* has a larger value beside it, they thought that *2x* is larger than y in the equation. This comparison is correct when comparing two like terms such as *2x* and *x* but inapplicable when comparing unlike terms. The highest percentage in this category 12% in which pupils gave the solution that *x* was larger than *y* because it had a bigger value beside it. Pupils were able to recongnise that both x and y are variables but did not realise that these variables could take more than one value.

Errors and misconceptions on question 12.

The question was meant to find out if pupils had the correct understanding of the unitary concept of the variables. Question 12 was:

Shirts cost s dollars each and pants cost p dollars a pair. If I buy 4 shirts and 3 pairs of pants, explain what 4s + 3p represents ?. The error categories on question 12 are shown on Table 6.

Table 6: Pupils' errors or possible misconception in question 12.

Type of error possible misconception.	**Expected answer(correct response).**	**Pupils incorrect response.**	**Frequency. N = 60.**	**Percentage of incorrect responses (%)**
Assigning labels for variables.	It represents the total cost of 4 shirts and 3pairs of pants	It represents buying 4 shirts and 3 pants.	3	5
Assigning labels for variables	It represents the total cost of 4 shirts and 3 pairs of pants	4 shirts plus 3 pairs of pants as well as 4 dollars plus 3dollara equals 7 dollars	4	7
Lack of understanding of the unitary concept when dealing with variables	It means the total cost of 4 shirts and 3 pairs of pants	It means you bought 4 shirts for s dollars and 3 pairs of pants for p dollars.	5	8

From Table 6 it can be observed that another possible misconception for some pupils was that they had difficulties in understanding the unitary concept when multiplying a variable with a constant(8%). When the price of a shirt was given as s dollars and when they had to find out the price of 4 shirts , they could have understand that the unit price s had to be multiplied by 4. This is basic arithmetic concept. The only difference was that the price was given as a variable. It was again evident that pupils considered s as the label for ‘shirts’, rather than the unit price of a shirt and at the same time considered s as the unit price (8%).

4.2 What errors and misconceptions do secondary school pupils make when Solving problems related to algebraic expressions?

In this study, algebraic expressions had the highest number of pupil errors. The error were classified into eight major groups. These included : incomplete simplification, incorrect cross multiplication, converting algebraic expressions as answers into equations and over simplification. Others are oversimplification, invalid distribution and incorrect quantitative comparison.

Errors misconceptions on question 3(c)

Question 3 was given as: Simplify $B(\frac{1}{B})$ .The question was given in order to find out if pupils were able to simplify algebraic expression. The error categories on question 3 (c) are shown in Table 7.

Table 7: Pupils errors and possible misconception in 3(c).

Type of error or possible misconception.	**Accepted answer (correct response)**	**Pupils answers (incorrect response)**	**Frequency N =60**	**Percentage of incorrect responses (%)**
Incomplete simplification.	1	$\frac{1B}{B}$	8	13
	1	$\frac{B}{B}$	14	23
	1	$\frac{B1}{1B}$	2	3
Incorrect cross multiplication.	1	$\frac{1B}{B^2}$	22	37
Converting algebraic expressions as answers in to equations	1	$B \times B \times 1$	3	5
	1	B = 0	4	7

From Table 7, it can be observed that pupils had problems in the simplification of algebraic expressions. An answer was categorised as incomplete when some pupils terminated the simplification somewhere in the middle of the process without reaching the final or accepted solution. Some invalid cross multiplication was observed during the categorization of errors for algebraic expressions. When pupils multiplied an algebraic fraction with a letter $[B(\frac{1}{B})]$, they often multiplied both the denominator and the numerator of that fraction by the letter ($\frac{1B}{B^2}$).

Errors and misconceptions on question 3 (c)

Question 3(c) was: Expand $(x + y)^2$. The question was meant to find out if pupils were able to expand the binomial expression. The error categories on question 3 (c) are presented in Table 8.

Table 8 : Pupils errors or possible misconception in question 3(c).

Type of error or possible misconception.	**Expected answer(correct response)**	**Pupils incorrect response(s)**	**Frequency of incorrect responses. N = 60.**	**Percentage of incorrect responses (%)**
Incomplete simplification.	$x^2 + 2xy + y^2$	$(x + y)(x + y)$	7	12
Invalid distribution.	$x^2 + 2xy + y^2$	$x^2 + y^2$	17	28
Invalid distribution	$x^2 + 2xy + y^2$	$x^2 + x^2y^2 + y^2$ $x^2 + y^2 + xy^2$	6	10
Invalid distribution	$x^2 + 2xy + y^2$	x^2y^2, xy^2	9	15

It can be observed from Table 8 that pupils were unable to expand the algebraic expression $(x + y)^2$. About 12% of the pupils gave the response to the question as $(x + y)(x + y)$ which was categorised as incomplete simplification. Another error which pupils committed on this question was invalid distribution. There were many answers under the category of invalid distribution such as: $x^2 + y^2$, $x^2 + x^2y^2 + y^2$ and $x^2 + y^2 + xy^2$.

Errors and misconceptions on question 4 (c)

Question 4(c) was given as: $\frac{ax + bx}{x + cx}$. The question was meant to test pupils' ability to simplify algebraic expression using factorisation method.

Table 9 shows error categories on question 4 (c).

Table 9 : Pupils errors or possible misconception in question 4(c).

Type of error or possible misconception.	**Expected solution(correct answer).**	**Pupils answer(incorrect solution).**	**Frequency**	**Percentage of incorrect responses (%)**
Over simplification	$\frac{a+b}{1+c}$	$\frac{abx^2}{cx^2}$	27	45
		$\frac{a+b}{c}$	6	10
		$\frac{ab}{c}$	7	12
		$\frac{2abx}{2cx}$	5	8
		$\frac{abx}{cx}$	2	3

From Table 9, it can be observed that for question 4(c) pupils experienced a number of difficulties regarding the simplification of the problem. There was a wide range of answers which were oversimplified. The highest error category being 45% who gave the answer $\frac{abx}{cx^2}$.

Errors and misconceptions on question 6.

Question 6 was Multiply a + 2 by 4. This question was given in order to find out if pupils had were able to multiply an algebraic expression by a number. The error categories for question 6 are shown on Table 10.

Table 10 : Pupils' errors or possible misconception in question 6.

Type of error or possible misconception	**Accepted answer(correct solution)**	**Pupils answers(incorrect solution)**	**Frequency**	**Percentage of correct responses (%)**
Over simplification	4a + 8	8a	5	8
		2 ×4(a)	6	10
		4a ×2	2	3
Incomplete simplification	4a + 8	4(a+2)	6	10
Invalid distribution	4a + 8	4a + 2 or a+8	8	13

From, Table 10 above, it can be seen that pupils faced a number of difficulties in solving question 6 and these included: over simplification, incomplete simplification and invalid distribution. Under oversimplification pupils gave answers such as 8a (8%), which was against algebraic rules. Concerning incomplete simplification, only one type of error was identified which was 4(a+2) with a frequency of 10%,

whereas invalid distribution included answers such as 4a+2 or a+8 (13%).

Errors and misconceptions on question 5.

Question 5 was given as: Subtract 3*x* from 7. This question was meant to find out if pupils were able to subtract an algebraic expression from a given number. The error categories on question 5 are presented on Table 11.

Table 11 : Pupils errors or possible misconception in question 5.

Type of error or possible misconception	Accepted answer(correct solution)	Pupils answers(incorrect solution)	Frequenc y	Percentage of incorrect responses (%)
Over simplification	$7 - 3x$	4 or $4x$	9	15
Reversal error	$7 - 3x$	$3x - 7$	7	12

From Table 11, it can be seen that pupils oversimplified the algebraic expression $7 - 3x$ to get answers such as 4 or $4x$ (15%). Another error which was noticed was the reversal error in which pupils came up with the solution $3x - 7$ instead of the expected answer $7 - 3x$. (12%).

Errors and misconceptions on question 7.

Question 7 was given as: The letter 'n' represents a natural number. Which one is greater than the other $\frac{1}{n}$ or $\frac{1}{n+1}$?. How do you know ?. This question was given in order to find out if the pupils had a proper understanding of comparison of algebraic expressions. The error categories for question 7 are presented in Table 12.

Table 12: Pupils errors or possible misconception in question 7.

Type of error or possible misconception	Accepted answer (correct solution)	Pupils' answers (incorrect solution)	Frequency	Percentage of incorrect responses (%)
Incorrect quantitative comparisons	$\frac{1}{n}$ is greater than $\frac{1}{n+1}$ or $\frac{1}{n}$ □ $\frac{1}{n+1}$ This is because the reciprocal of a number is greater than the number.	$\frac{1}{n+1}$ is greater than $\frac{1}{n}$ because if we put n = 10, then $\frac{1}{10+1}$ = $\frac{1}{11}$ and for $\frac{1}{n}$ it will be $\frac{1}{10}$, hence $\frac{1}{11}$ is greater than $\frac{1}{10}$	32	53

From Table 12, it can be observed that more than half of the pupils (53%) had an incorrect quantitative comparison given two algebraic fractions. These pupils substituted numbers to the algebraic expressions in order to compare them. After that, they only compared the magnitudes of the denominators instead of comparing the whole fractions thereby arriving at wrong conclusions. They did not realise that the reciprocal of a number is smaller than the number itself.

4.3 What errors and misconceptions do secondary school pupils make when Solving problems related to equations?

There were three questions in the test involving solving algebraic equations and these were question 9, 10 and 11. It is important to mention that some error types appeared more than once in the same question and in different questions. For example, the error type 'add when the equations have to be subtracted or vice versa' appeared in questions10(a), 10(b) and 10(c). These types of errors or possible misconceptions included misinterpreting numbers as labels, misinterpreting the elimination method in solving simultaneous equations, wrong operations when using substitution method, and over simplification.

Errors and misconceptions on question 10.

Question 10 was given as:

Consider solving the linear system of equations : $x + y = 5, x - y = 7$

To eliminate x from both equations, do you add or subtract the two equations ?

To eliminate y from both equations, do you add or subtract the two equations ?

Will you obtain the same answer if you add or subtract the two equations ?

The question was meant to test pupils' ability in solving simultaneous equations. The error categories on question 10 are shown in Table 13.

Table 13 : Pupils errors or possible misconception in question 10.

Type of error or possible misconception.	**Accepted answer(correct solution)**	**Pupils answers (incorrect solution).**	**Frequency N = 60**	**Percentage of incorrect responses (%)**
Misinterpreting the elimination method when solving simultaneous equations.	10(a) subtract 10(b) add	Subtract when the equation has to be added or vice-versa	31	52
Misinterpreting the elimination method when solving simultaneous equations.	10(c) yes, you get the same answer.	You do not get the same answer.	45	75

From, Table 13, it can be seen that some pupils misjudged the operations to be performed. Some of them chose the reverse operation (52%), for example, subtracted when the equations had to be added or vice-versa. In addition, 75% of the pupils said that they would not get the same answer if either addition or subtraction was applied on the equation.

Errors and misconceptions on question 11

Question 11 was given as: Solve the following system of linear equations :

$2x + y = 2, 3x - 2y = 3$

Question 6 was meant to find out pupils' solving ability of simultaneous linear equations. The error categories on question 6 are presented in Table 14

Table 14 : pupils errors or possible misconceptions in question 11.

Type of error or possible misconception	**Accepted answer (correct solution)**	**Pupils answers (incorrect solution)**	**Frequency**	**Percentage (%)**
Wrong operations when using substitution method.	$X = 1$ and $y = 0$	$2x + y - 2 = 3x$-2y- 3.	15	25
Over simplification	$X = 1$ and $y = 0$	Y = 2 – 2x, $\frac{y}{0} = \frac{0x}{0}$	8	13
Over Simplification	$X = 1$and $y = 0$	3- $2y = 3$, $x = 3$	6	10
Misuse of the " change-side, change-sign" rule	$X = 1$ and $y = 0$	$7x = 0$, $x = -7$	10	17
Interference from previously learned method.	$X = 1$ and $y = 0$	$2x + (\frac{2}{x})^2 - (\frac{2}{2})^2 + y = 2\ (2x + 1) - 1 + y = 2$ $(2x + 1) + y = 2 + 1$ $(2x+1)+ y=3$ $2x+y=1$	9	15

In this study, pupils used two methods to solve the simultaneous equations: the substitution and elimination methods. In the substitution method, pupils had to isolate a variable from one equation and substitute its value in the second equation. In this study, they isolated the same variable from both equations and equalized them. However, 25% of the pupils made the right hand side of both equations as zero and equalized them. This method could not work and pupils ended up getting a single equation with two variables which were unsolvable. This was a wrong application of the substitution method.

It was also observed that 14 pupils oversimplified algebraic terms in an illegal way. They operated directly on numbers separating them from adjacent terms. This separation led to situations where answers were in undefined forms (y = 2x, $\frac{y}{0}$ = $\frac{0x}{x}$ (13%).

The misconception of the misuse of the 'change-side, change-sign' rule was observed in the last steps of the equation solving process. Ten (10) pupils carried over the terms to the other side of the equation without properly changing the sign or without executing proper operations (7x = 0, x = -7).

The other error which the pupils committed was when they mistakenly chose a previously learned method which was not applicable for solving simultaneous linear equations. Interestingly, pupils did not realise that they would end up with the original equation when this method was used to solve simultaneous linear equations as this method would only work for finding roots to a quadratic equations.

Errors and misconceptions on question 9.

Question 9 was given as: Solve for x : 4x + 25= 73. The question was meant to find out pupils' ability of solving linear equations. The error categories on question 9 are shown on Table 15.

Table 15 : Pupils errors or possible misconceptions in question 9.

Type of error or possible misconception.	**Accepted answer (correct response)**	**Pupils incorrect response**	**Frequency**	**Percentage. (%)**
Letter as labels.	X = 12	X = 8, 48 + 25 = 73	5	8

From, Table 15, it can be shown that pupils misinterpreted a letter as a label. In order to solve the equation: $4x + 25 = 73$, 8% of the pupils simply pasted the number 8 as a label for x as the answer to the question by not substituting it.

Mean number of errors committed by pupils in three conceptual areas.

Figure 2 shows the mean number of errors committed by the pupils in the three conceptual areas: variables, expressions and equations

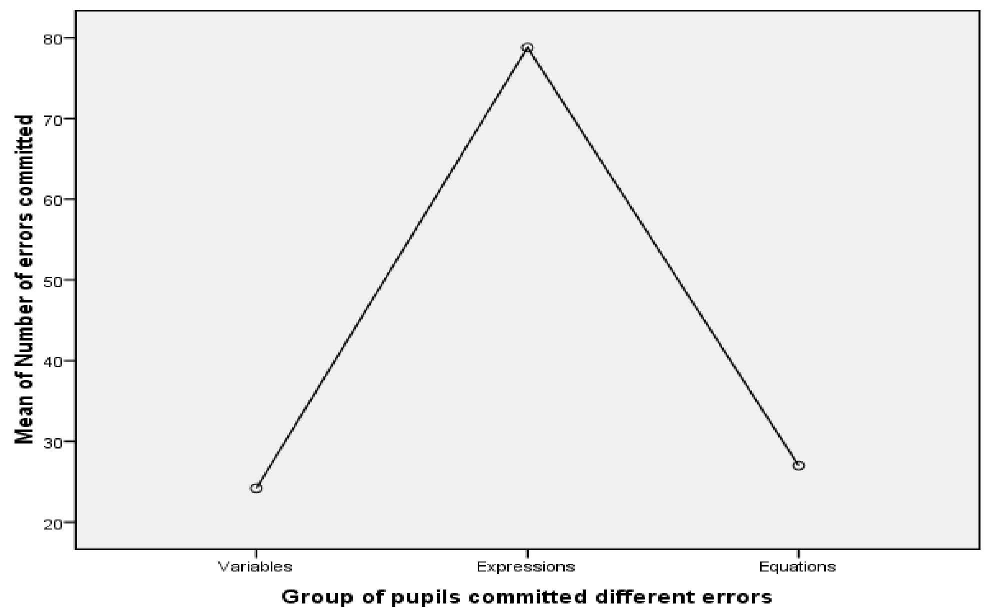

Figure 2: Mean number of errors committed by the pupils.

From Figure 2 above it can be observed that algebraic expressions had the highest percentage of errors at 78%, this was followed by equations at 27% and variables 24.2%. At a glance it can be observed that pupils experienced more problems in solving problems involving algebraic expressions than in equations and variables. Further, it can be observed that the mean percentage difference between variables and equations was 2.8%. The difference in errors committed by pupils between variables and equations was minimal.

4.4 Why are there differences among pupils when they engage in solving problems in variables, algebraic expressions and equations ?

Table 16 shows the mean and standard deviation according to the three conceptual areas : variables, expressions and equations.

Table 16: Descriptive statistics: Mean score in Algebra.

Category	Mean	Std. Deviation
Variables	24.2	11.692
Expressions	78.8	58.264
Equations	27	8.515

From Table 16 it can be seen that pupils committed more errors when solving algebraic expressions questions than when solving questions involving variables and equations. The mean error percentage of algebraic expressions was 78.80% and standard deviation of 58.264%, while variables had a mean error percentage of 24.20% and standard deviation of 11.692% . Equations had a mean error percentage of 27% with a standard deviation of 8.515%.

Performance by pupils on variables, expressions and equations.

A one way Analysis of Variance (ANOVA) was used to test if there was a significance difference in the means involving variables, expressions and equations.

From Table 17, the statistic test of the Anova revealed that there was a significance difference of the means among variables, expressions and equations at the confidence level of p □ 0.05 that is [f(2,12) =3.935, p=0.049)].

Table 17: ANOVA TEST: Comparison of the means among variables, expressions and equations

Group	N	Mean score	Sd	Df	F	Sig
Performance variables Expressions Equations	60 60 60	24.20 78.80 27.00	11.69 2 58.26 4 8.515	2 12	3.935	0.049

significant at p □ 0.05.

The results from the one-way Anova test do not indicate which of the three groups differ from one another, so in many cases it is of interest to follow the analysis with a Post hoc test. Table 17 shows the Scheffe Post-hoc test with all possible pairs of comparisons that can be made.

Table18: Scheffe Post-hoc tests.

Concept	Variables	Expressions	Equations
Variables		*0.028	0.900
Expressions	*0.028		*0.036
Equations	0.900	*0.036	

* The mean difference is significant p □ 0.05.

The Scheffe Post-hoc test show that the means were statistically different between variables and expressions and also between equations and expressions. However, the performance between equations and variables was not statistically different.

Performance in Algebra between boys and girls.

The chi-square test was performed in order to determine if there was a significance difference in performance in algebra between boys and girls. Since our calculated chi-square value has a $p = 0.122$ is greater than 0.05, we can therefore conclude that there was no significant difference in performance between boys and girls.

Table 19:The chi-square test.

	Performance
Chi-square	4.211[a]
Df	2
Calculated p-value	0.122

The results from the chi-square suggest that boys did not perform better than girls or vice-versa.

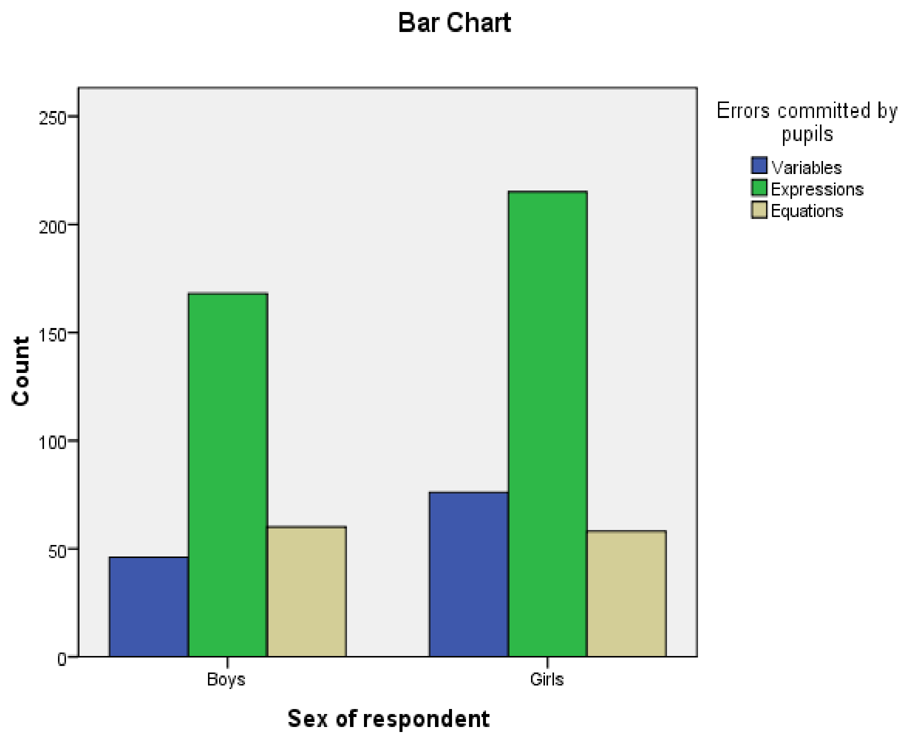

Figure 3: Errors committed by pupils by gender.

Figure 3 shows the performance of boys and girls in the three conceptual areas: variables, expressions and equations. It can be observed that more errors were committed in algebraic expressions than in variables and equations by both boys and girls. This simply implies that the pupils found expressions to be more difficult than variables and equations.

4.5 What can be learned from the pupils' problem solving processes and reasoning in algebra ?

The data for the study was obtained in several ways such as pupils' answers to the test, pupil interviews and notes during the research process. The data obtained from these sources enabled me to discuss pupils errors and misconceptions in algebra. In this section, the interview process with the five pupils is discussed.

4.5.1 The case with pupil 1.

The pupil demonstrated a good understanding of algebra with a few possible misconceptions in expressions. The following excerpt describes pupil 1 interview regarding expressions. R stands for researcher while p stands for the pupil who is the interviewee.

Question 4c : simplify : $\frac{ax+bx}{x+cx}$, the answer in the test was $\frac{a+b}{c}$

R: Would you please read the question ?

P: $ax + bx$ over $x + cx$.

R: Would you please simplify : $\frac{ax+bx}{x+cx}$

P : I cancelled all the x's , so that they are no x's anymore so that I will have $a+b$ over c

R: Can you simplify : $\frac{4\times 3+5\times 3}{3+6\times 3}$

P: I cancel out the 3's as follows: $\frac{4\times \cancel{3}+5\times \cancel{3}}{\cancel{3}+6\times \cancel{3}}$

R: If you use your previous method, I mean crossing out the 3's do you still get the same answer?

P: I do not know…, I am not sure.

However, another way of solving the problem was suggested to see whether she knew the method.

R: Do you know any other method of solving this problem($\frac{ax+bx}{x+cx}$)?

P: No sir.

R: How about by factorizing(taking out common factors out).

P: Yes I can do it like this $x(a + b)$ over $x(1 + c)$ and then cancelling off the x's

R: Do you still get the same answer as your previous one ?.

P: No, its like it is different.

R: What do you think now?

P: This method is right now

4.5.2 The case with pupil 2.

Pupil 2's answer to the question 4(b): Simplify $x(\frac{c}{d})$ was $\frac{cx}{dx}$. The interview to find out the reasoning behind went ahead. The following excerpt describes the interview with pupil 2 .

R: Would you please read the question for me $x(\frac{c}{d})$?

P: x times open brackets c over d.

R: Would you please simplify the expression for me ?

P: We multiply x times c and x multiply by d, then we have $\frac{cx}{dx}$

R: Would you please simplify $2(\frac{3}{5})$.

P: We say $\frac{2}{1}(\frac{3}{5})$ and the answer is $\frac{6}{5}$.

R: You have used two different methods for the two questions. Which one do you think is the correct one ?

P: The one which has over 1. Let me solve first one properly. It will be $\frac{x}{1}(\frac{c}{d})$ which gives us $\frac{cx}{d}$.

The numerical algebraic expression $2(\frac{3}{5})$ helped him to simplify the algebraic expression $(\frac{c}{d})$.

Pupil 2 had a problem with simplifying the expression $\frac{ax + bx}{x + cx}$, his answer to the question was $\frac{x^2 + ab}{x^2 + c}$. The following excerpt describes the interview excerpt:

R: How did you get the x^2 in $(\frac{x^2 + ab}{x^2 + c})$?

P: Add *x* and *x* which gives us x^2.

R: How did you get *ab* ?

P: Again by adding *a* and *b* which gives us ab.

Pupil 2 had a problem with simplification of the algebraic fraction $\frac{A}{B} + \frac{A}{C}$. His answer in the test was $\frac{A}{BC}$. The following was the interview:

R: Would you please simplify the algebraic fraction $\frac{A}{B} + \frac{A}{C}$?

P: Yes it will be *a* over *bc*

R: How did you get bc?

P: Off course by adding *b* and *c*.

R: What would you get if you add A and A ?

P: I get A^2

R: But why do you put *a* only ?

P: Its wrong, its suppose to be a^2.

From the interview, pupil 2 had problems with expressions. He did not know when to add or multiply the letters given in the expression. The rules of Algebra were not important to him.

4.5.3 The case with pupil 3.

Pupil 3 showed that she had problems with simplification of binomial expressions and over simplification of algebraic expressions. The answer to the expansion of $(x + y)^2$ was $(x^2 + y^2)$. The researcher decided to interview her in order to find out more from her. The following excerpt describes the interview with pupil 3:

R : How did you get it ?

P: I…mean I multiplied x by 2 and y by 2.

R: What is x multiplied by 2 ?

P: It is $2x$, sir.

Further, pupil 3 had a misunderstanding of the concept of a variable. This was seen from her answer of question 1(a) name variable in the question, her answer was $x + 2x =500$. This answer was by far contrary to the expected answer. The interview was carried out in order to find out about her understanding about the variable. The following excerpt describes the interview with pupil 3 :

R: What is a variable ?

P: A variable is a letter that represents the unknown in an equation

R: Can a variable take values ?

P: Yes , you can substitute it in an equation.

R: How many values can it take at a time ?

P: One value at a time.

4.5.4 A case with pupil 4.

Pupil 4 had a number of problems. One of his problems was that he had some misconceptions in solving simultaneous linear equations. His answer for question 11 was $2x + y = 2$

$2x + (\frac{2}{2})^2 - (\frac{2}{2})^2 + y = 2$

$(2x + 1) - 1 + y = 2$

$(2x + 1) + y = 2 + 1$

$(2x + 1) + y = 3$

$2x + y = 2$

The interview was carried out in order to find the reasoning over the solving of simultaneous equation. The interview was as follows :

R: Why did you decide to use this method?

P: Because that is the way I know

R: Is there any other method you can use apart from the one you used?

P: No there is isn't

Pupil 4 had a misconception on the concept of variables his answer in the test of question 12 was 4s +3p represents 4 shirts and 3 pairs of pants. The following excerpt describes the interview:

R: Would you please give me an answer to question 12?

P: 4s stands for 4 shirts and 3p stands for 3 pairs of pants

R: Did you say that *s* stands for shirts?

P: Yes, sir.

R: What does *p* stand for ?

P: Off course , for pairs of pants.

R: What then is $4s + 3p$?

P: It gives us the total of 7 items and each item cost one dollar each.

R: How did you come up with this solution?

P: Because 4 shirts represents 4 dollars and 3p represents 3 dollars and in total is 7 dollars.

From, the interview with pupil 4 it can be noticed that naming s for shirts and p for a pair of pants is a clear indication that he was perceiving letters as objects and not as the cost of each item. Pupil 4 also incorrectly assumed that each item costs one dollar and concluded that the number of items in the problem represents the cost of that item.

4.5.5 The case with pupil 5.

Pupil 5, seemed to have problems in understanding the concept of a variable and also he was reluctant to use algebraic methods when solving problems in Algebra. The interview was carried outs on question 8: "which is larger y or x in $y = 2x + 3$?"

His answer in the test was x is larger there are 2 while y is single on the equation provided. The following excerpt describes the interview with pupil 5.

R: Which is larger x or y ?

P: x is larger because there are 2 while y is single on the equation provided.

R: Is it always that $2x$ will always be larger than y ?

P: Yes, sir.

R: What values can x or y take ?

P: Positive numbers

R: How about negative numbers ?

P: Not at all.

From the interview it can be noticed that pupil 5 was comparing two variables y and x hence he was able to conclude that $2x$ was larger than y because variable x had a coefficient 2 while y had a coefficient of 1. He only considered the positive values for x and y.

4.6 Summary.

In this mixed method research design, the findings using quantitative and qualitative analysis were described. The quantitative analysis was done using both descriptive and inferential statistics. Under descriptive statistics tables showing the frequency of errors and misconceptions in variables, expressions and equations were shown. Figure 2 showing the overall mean percentages of errors and misconceptions in variables, expressions and equations was presented. Under inferential statistics the chi-square tests, ANOVA and Scheffe Post-hoc test were used to determine the significance difference in the performance of pupils in Algebra. In the qualitative analysis, the five cases of the interview process were discussed in order to get a full understanding of pupil reasoning when solving algebraic problems.

CHAPTER 5

DISCUSSION OF THE RESEARCH FINDINGS.

5.0 Introduction

The fundamental goal of this research was to explore pupils' errors and misconceptions in the three conceptual areas of algebra and further to expose pupils' reasoning for doing so. In pursuance of this broad goal, this study adopted a multidimensional approach to the data collection process and an organised and systematic process of data analysis was undertaken which was explained in chapter three. This study assumed that a better understanding of pupils' errors and misconceptions in algebra leads to a better understanding of pupils' general understanding of mathematics principles.

In this chapter the researcher discusses the research findings of the errors and possible misconceptions under each conceptual area by carefully relating them to the various existing theories in literature.

5.1. Errors and misconceptions in variables.

The four categories of pupils' misconceptions related to variables are discussed in this section. These include assigning labels for variables, misinterpreting of two variables, lack of understanding of variables as generalised numbers and forming incorrect equations as answers.

5.1.2 Assigning labels and arbitrary values for variables.

Some pupils misinterpreted a variable for a 'label' , or as a verb such as 'buying' rather than as the "number of things". This was common for question 12 for example, when the price of the shirt was s dollars, the students gave a response that

4s stood for "4 shirts" . This was a clear misinterpretation of the algebraic term. This was in consistent with Philip (1999) who explained a similar use of letters as labels as used in $3f = 1y$ to denote 3feet equals 1yard. In this interpretation, f and y stand for 'feet' and 'yard' respectively. The letter in this case was used to denote the name of the unit. Further the use of the letter as a label was found when 3 pupils solved $4x + 25 = 73$, by pasting the number 8 as a label for x but not by substituting it. This is similar to finding the number to satisfy a number using the arithmetic methods.

It was also discovered that, 12% of the pupils found it difficult to differentiate between variables and non-variables. This was evident to question 1, when pupils provided names of persons, things and letters for non-variables. Some of these solutions were correct, but unacceptable under algebraic interpretations. Misinterpretation of letters as labels is a misconception which may lead to many other errors in Algebra. For instance in the famous student-professor problem (clement, 1982), College students made similar interpretations of variables. In the student-professor problem, students used p to represent professors rather than the number of professors and similarly s to represent students rather than the number of students. The result for this was a reversal error, writing the equation as

$6s = p$ instead of $6p = s$.

5.1.2 Misinterpretation of products of two variables.

A number of pupils viewed the product of two variables as one variable. For instance they perceived the product of xy in question 2 as a single variable. In this case the pupils did not take note of the multiplication sign between the letters and simply thought of xy as a number similar to 15. As earlier discussed under literature review, Macgregor and Stacey (1997), attributed this misconception to come from other numerical system such as the roman numeral system in which *iv* is given as five less one. This was consistent with the findings from this study. Pupils who said that there was one variable might have seen *xy* as a conjoined answer.

5.1.3 Lack of understanding of a variable as a generalised number.

This misconception was seen from question 8 during the interview with pupil 5 who did not realise that the two variables x and y can take more than one value in the equation. Further, pupil 5 focused only on the domain of positive numbers to decide which variable was larger than the other and disregarded a zero as a value for substitution. Philipp (1999), categorised seven situations in which a variable can be used and one of these is that a variable is a generalised number. He further explained that it is difficult to understand that a variable can take many values in certain situations. It was also evident from the interviews with pupil 2 and 3, who did not realise the existence of a variable as a generalised number. Macgregor and Stacey (1997) attributed this to what is given in literature that this error has a link with levels of cognitive development. This means that if a pupil has slowed mental development then that pupil might make more errors and misconceptions than a pupil who has normal mental development.

5.1.4 Forming incorrect equations as answers.

This was a very unique misconception as the researcher did not find any supporting evidence from the existing literature. This was evident from one pupil who when asked to name something which was not a variable in question 1 his answer was $x \times 2x = 500$.this was indeed meaningless in the context of the problem. There was a false relationship between variables and constants. however, lack of understanding of basic concepts is a possible reason for this misconception.

5.2 Errors and misconceptions in algebraic expressions.

The category of algebraic expressions had a long list of errors and possible misconceptions. These include the following: incomplete simplification, incorrect cross multiplication, converting algebraic expressions as answers into equations, oversimplification, invalid distribution and incorrect quantitative comparisons. These are presented next.

5.2.1 Incomplete simplification.

An answer was categorised incomplete if a pupil terminated the simplification of an algebraic expression before the accepted answer. In this situation a pupil would start the problem and proceed with one or two steps and abruptly terminates the process before reaching the final answer. Through the written test and the interviews conducted with the pupils it was observed that probably there were two reasons for this. Firstly, it could be that pupils did not know how to proceed with the problem. Secondly, pupils could have thought they had reached the final answer. For the incomplete answers, further simplification was possible to reach the desirable solution. According to Booth (1984) such errors were as a result of students' lack of knowledge or lack of confidence in the problem solving process. The cause of incomplete simplification could have been due to lack of knowledge.

5.2.2 Incorrect cross multiplication.

This misconception was evident when pupils incorrectly used cross multiplication on the simplification of the algebraic expression{question 3(c); B($\frac{1}{B}$). When an algebraic fraction had to be multiplied by a letter, 22 pupils used cross multiplication in carrying out this operation. It was observed this misconception occurred when there was no visible denominator such as 1. It was clearly noticed that pupils disregarded the denominator of 1 or lacked experience of making 1 as a denominator. The pupils did not have problems when solving the numerical problem. This was clear during the interview with pupil 3 who first failed to solve the algebraic expression but later solved it after giving him a numerical problem. This misconception could have been caused by of lack of masterly of pre-requisite facts and concepts (kieran,1992).

5.2.3 Converting algebraic expressions as answers into equations.

This error happened when pupils incorrectly used the 'equal sign' to mean the next step'. This was seen from question 4(b) which read simplify $x(\frac{c}{d})$, 10% of the pupils gave the answer $dx=c$ and also for question (5) which read 'subtract 3x from7, 12% of the pupils gave the answer of $x = 2\frac{1}{3}$. In this situation, the pupils used the equal sign as a step marker. When the pupils put the additional equal sign at the beginning, they tended to follow what until they finish solving the question. This finally leads the pupils to misconstrue the algebraic expression into an equation. This is in consistent with other researchers who claim that most pupils wrongly use the equal sign (Booth,1984; Kieran,1992).

5.2.4 Over simplification.

This is the direct opposite of 'incomplete simplification'. In this category of errors the pupils conjoined, connected or even put together the terms against the accepted algebraic terms. There were many instances of such, but I will cite a few. For instance question 4(d): Simplify $\frac{A}{B}+\frac{A}{C}$, the common answer was $\frac{2A}{BC}$ (19%). Pupils found themselves making this error because of illegal divisions. Multiplication and addition. Pupils often misused the laws of factorisation. This was revealed through the interview with pupil 3. According to Tall and Thomas (1991) the reasons for over simplification were due to similar meanings of 'and' and 'plus' in ordinary language. This later made pupils to have difficulty in interpreting the relations that exists amongst the algebraic expressions, for example '*ab*' is often read as '*a* and *b*' which later gives rise to a misconception, that it is the same as '*a* plus *b*'(Tall & Thomas,1991; Booth,1988).

Pupils perceived an open algebraic expression as 'incomplete' and try to 'finish' them by over simplifying. Pupils, for instance did not consider an algebraic expression such as $(7 + 2x)$ as a final answer, they would rather simplify it to $9x$ or 9 (Booth,1988; Stacey & MacGregor,1994). This misconception is often attributed to arithmetic problems in which a final single-termed digit answer is given (Booth,1988; Tall & Thomas, 1991).

However, from the findings of this study, the oversimplification of Algebra is not consistent with what other researchers have found, in the sense that the reasoning of the pupils interviewed changed due to situations. For example from the interview excerpt, with pupil 2, it was noticed that her reasoning was inconsistent and when she realised that her answer was wrong she quickly changed her reasoning. The other reason was that over simplification was the failure by pupils to differentiate among variables, algebraic expressions and equations. Five percent (5%) pupils perceived an algebraic expression as an equation which led them to over simplify the expression.

5.2.5 Invalid or incomplete distribution.

The misconception of 'invalid distribution' had a variety of forms. One common example of this was the failure of the pupils to retrieve the correct expansion of a binomial such as $(x + y)^2$. The common solution of such a binomial was $x^2 + y^2$ (29%). Others went on further to oversimplify and got answers such as x^2y^2 and $(xy)^2$ (15%). Another category was the incomplete distribution category, where students correctly multiplied on one term leaving the other undone. A good example of this was question 6, which read; multiply $a + 2$ by 4, the answers given were; $4a + 2$ (8%) and others gave the answer of $a + 8$ (4%).

According to literature this error is a result of deficiency in the mastery of prerequisite facts and concepts. Another explanation is that students overgeneralised a correct rule to misapply it in another situation is a result of explicit, declarative knowledge gained from the curriculum(Matz, 1982; Macgregor & Stacey,1991). Finally, the pupils misused the distributive law because of the roots in arithmetic misconceptions. There was misunderstanding of arithmetic concepts or failure to transfer arithmetic understandings to algebraic context (Norton & Irvin, 2007). However from the findings of this study it was found that no single reason is responsible for pupils to have invalid or incomplete distribution but a combination of deficiency in the mastery of prerequisite facts and concepts and overgeneralising a correct rule to misapply it in another situation.

5.2.6 Reversal error.

Under algebraic expressions, a good number of pupils had the reversal error committed in question 5 (11%), where they formed the expression in the reverse order. The question read 'subtract $3x$ from 7'. Eleven percent (11%) of the pupils wrote $3x - 7$ instead of the expected answer of $7 - 3x$. In this question, the pupils had to read the word sentence and translate it into an algebraic form. In reality the reverse order of the answers showed that pupils literally matched the word order given in the problem into algebraic form rather than the actual understanding of the correct relationship among the given variables.

This was in consistent with other previous researchers such as Kieran (1979), who records that students believe that the written sequence of operations determines the correct order in which computation should be performed. In addition, pupils thought that the value of an expression remained unchanged if the order of calculation was varied. The reversal error as recorded in literature over the famous Student-Professor problems, "There are six times as many students as professors". The common error is writing the equation, "6s = p known as the reversal instead of the desired equation of 6p = s.

5.2.7 Incorrect quantitative comparisons.

This error was evident for question 7 in which pupils compared the magnitude of two algebraic fractions, that is $\frac{1}{n}$ and $\frac{1}{n+1}$. Fifty-Three percent (53%) of the pupils compared the two expressions by examining their denominators only, thereby perceiving that $\frac{1}{n+1}$ was larger than $\frac{1}{n}$. In the pupils' view, the larger the denominator , the greater the fraction, which was a misconception. Norton and Ivin (2007), carried out a similar research in which pupils where asked to find out which was larger between $\frac{25x}{5}$ and $5x$. Most of the pupils found that $\frac{25x}{5}$ was larger than $5x$ because it had a larger quantity such as 25.

From the research findings of this study, it was found that pupils had problems with the reciprocal of numbers. They did not understand that the reciprocal of a number is smaller than the number. For instance $\frac{1}{2}$ is smaller than $\frac{2}{1}$ despite the latter having a smaller denominator than the former. The use of numerical examples can help the pupils understand comparisons of algebraic expressions easily.

5.3 Errors and misconceptions in Equations.

There were six categories of misconceptions under the category of equation solving these included numbers as labels, misinterpreting the elimination method, wrong use of the substitution method, misuse of the "change-side, change-sign" rule, interference from previously learned methods and misinterpreting the equal sign.

5.3.1 Numbers as labels.

There were 3 pupils who made this error. In this case a number was used as a label for a letter. The pupils had to solve the question which stated: "Solve for x : $4x + 25 = 73$." The pupils simply pasted the number 8 into the position of x to have a complete equation as “48 + 25 = 73”. This seemed to have made sense as it showed that the pupil understood the equivalence property as he pasted the correct number to make the equivalence work, although he did not follow the normal procedure. This answer was not expected from the grade 11s. The only reason for this error could be that they might have used the previous knowledge of arithmetic of number equations to insert a number to satisfy the numerical equation.

5.3.2 Misinterpreting the elimination method in solving equations.

Pupils misconstrued the elimination method when solving the linear systems of simultaneous equations. Pupils often misjudged the operation to be performed and chose a reverse operation, that is they added when they needed to subtract or vice-versa (52%). Other misunderstandings of simultaneous equations were that pupils concentrated on one equation of the system. Both misconceptions showed that pupils’ had incomplete understanding of the elimination method of solving linear systems of simultaneous equations.

This was further evidenced from the reluctance of pupils to solve the equations using the elimination method as most of them used the substitution method to solve question 11. One main problem faced was lack of understanding when starting to solve the problem. Another observation was that pupils’ had difficulty in arriving at the conclusions intuitively. It was difficult for pupils to deduce whether the solution was the correct one.

5.3.3 Wrong use of the substitution method.

The pupils wrong use of the substitution method was used by pupils when solving question 11. This question did not restrict the pupils of which method to use. The pupils were at liberty to use elimination or substitution method. Most of the pupils preferred to use the substitution method to the elimination method. However for those who attempted to use the substitution method, 26% of them were unable to use the method properly. Apart from misunderstanding the substitution method, there was lack of monitoring of the solution process. If pupils examined the solution carefully, they would have realised that substituting for a zero would not eliminate any variable from the equations. Instead, it gave a single equation with two variables.

5.3.4 Misuse of the "change-side, change-sign" rule.

The misuse of the "change-side, change-sign" rule was a common error in solving equations as evidenced in this study. The common one is that pupils forget to change the sign whenever they carried over the terms to the other side of the equation or sometimes applied wrong operations to the terms. At one stage of the solving process, 17% of the pupils misused the "change-side, change-sign rule" were the pupils showed that $7x= 0, x= -7$. This happened because they attempted to separate the letter and consonants in an algebraic term. The main reason for this is the lack of understanding of the basic features of algebra. It was also observed that even if pupils understood those properties, they committed these errors unknowingly.

5.3.5 Interference from previously learned methods.

The pupils mistakenly modified and applied a previously learned rule to a new problem situation. This was evident to question 11 where pupils in the process of solving simultaneous equations wrongly used the quadratic method. This interference often comes without pupils realising that an error had occurred (case of interview with pupil 4). As recorded in literature previously learned Roman Numeral system interfered with pupils' learning of variables (Macgregor and Stacey, 2007). Based on this reason, it is appropriate to say that pupils misuse the previously learned procedures and rules in situations where they are not appropriate.

5.3.6 Summary of algebraic errors and misconceptions.

A total of 21error types were discussed in this study. They were seven new error types that were discovered in this study and these included misinterpreting the product of two variables, giving answers in the form of equations, incomplete simplification, incorrect cross multiplication, incorrect quantitative comparisons, numbers as labels and misinterpreting the elimination method when solving equations.

The symbols in Algebra have different meanings and interpretations in different situations. The overall picture that emerged from the findings was that the pupils had difficulties in understanding the various uses of letters and signs in different situations. The misunderstanding of the concept of the variable did have a clear bearing on their errors and misconceptions.

With regard to algebraic expressions, pupils' problems increased due to lack of understanding of the basic concept of the variable. The problems of algebraic expressions were the most difficult ones for the grade 11 pupils. What was observed was that there was lack of understanding of the structural features of algebraic expressions which led pupils to use many illegal procedures. To understand algebraic expressions, pupils should have a good understanding of the structure and properties of Algebra. This is because many algebraic expressions are made of letters and signs and most often they do not involve words. When letters, numbers and signs are put together to produce algebraic expressions, these entities should be manipulated according to accepted rules.

The other problem which the pupils encountered in algebraic expressions was that they modified or misapplied rules or procedures which were inappropriate in certain situations. Finally, the other problem which the pupils had was that there was unclear reasoning that was unaccounted for by any accepted rules or procedures.

With regards to equation solving, the pupils' problems were that they misused the equal sign out of its accepted meaning. They mostly used the equal sign as something to do the operation to the left and get the answer on the right or vice-versa. Others misused the equal sign as a step marker which was not right for particular situations. To avoid building up algebraic equations, they used other methods such as arithmetical methods or trial and error. They often misused the elimination and substitution methods.

5.4 Research question 2.

Why are there differences among learners when they engage in solving problems on variables, algebraic expressions and equations ?

In order to answer research question 2, I will refer to the descriptive and inferential statistics presented in the previous chapter.

It was shown in Figure 1 that the mean number of errors in expressions was 78%, variables (24.2%) and for equations it was 27%. This was later tested using the statistical test of Analysis of Variance (ANOVA), which showed that there was a significance difference in performance in at least two conceptual areas under study namely: variables, expressions and equations at the confidence level P □ 0.05 that is [(2,12) = 3.935, P = 0.049]. However, the ANOVA test was not adequate as it did not tell which of the means were different. I further conducted a post-Hoc test to ascertain which of the means were different. The post-hoc test showed that there was a significance difference in the means between variables and expressions and equations and expressions. However there was no significance difference between variables and equations.

The question is why are their variations in performance among the three conceptual areas under study, i.e., variables, expressions and equations?. Nickson(2000), identified three obstacles frequently met by pupils in making sense of algebraic expressions and these include, the pairing obstacle, the expected answer obstacle and the process-product obstacle. To some pupils , the "+" sign signals that they have to do the calculation in order to produce an answer. this is what is referred to as the 'expected answer obstacle'. The way we read from left to right is also noted to influence pupils to interpret for example $5 + 3x$ as saying 'add 5 and 3 and then multiply by x'. This further makes pupils not to accept $5 + 3x$ as a final answer. They fail to appreciate the dual nature of algebraic expressions. Structural operations refer to a set of operations carried out on an algebraic expression. For instance simplifying an algebraic expression such as 4p + q – 3p to obtain an equivalent expression of p + q is a structural operation. This is inconsistent with earlier research which provide evidence that simplification of algebraic expressions create serious difficulties in many pupils (Linchevski and Herscovics, 1996). In this study it was found that there were more errors and misconceptions in expressions than in variables and equations.

It was also noted that many questions under "expressions" were abstract in nature in the sense that there was not much content attached to them. As was realised from the literature study, indeed pupils experience serious problems when they have to deal with letters in expressions. The problems were that algebraic expressions were in symbolic forms and the most challenging part for pupils was to find the correct method of solving the problem. Pupils had to choose the correct method from a wide range of possible strategies which included but not limited to factoring, building expressions, simplifications and comparisons. Many of the incomplete solutions that were observed in pupils responses bear evidence that they could not select the correct strategy or the strategy they selected was inadequate to solve the problem. Some pupils obtained the correct answers, they oversimplified them to reach the final wrong answers. Others lacked confidence in the problem solving process thereby abruptly ending the solving process before reaching the very end. What we can say is that masterly of skills and knowledge is required for pupils to correctly manipulate of algebraic expressions.

In most cases solving of algebraic equations involves structural operations. For instance in an equation $p + q = 15$. In order to obtain 15 we need to substitute, p and q with an integer which is a procedural operation. Simplifying an expression such as $4p + q - 3p$ to yield an equivalent algebraic expression of $p + q$ is a structural operation (Kieran, 1992). This was consistent with the findings from this study in the sense that there were more errors in expressions than in variable and equations, this could be that the simultaneous equations in the test just required pupils to follow certain procedures.

On the other hand questions under 'variables' were easy as most of them did not need a pupil to follow any procedure. Some of the questions did not require deep thinking strategies. The questions were mostly about pupils' knowledge of basic definitions. The low incorrect response rate in this conceptual area showed that pupils found these questions easier than in other conceptual areas.

The Chi-square statistic test was used to test if there was significant difference in performance between boys and girls. The results revealed that there was no significant difference in performance between boys and girls. This could be attributed to the fact that both boys and girls received equal treatment during mathematics instruction. Further, this could mean that the teachers of mathematics are not gender biased when teaching and they look at boys and girls as equal partners in mathematics achievement. This was not consistent with findings from the previous researchers who found out that boys performed better than girls in Algebra (Usiskin, 1982).

5.5 Research Question 3: What can be learned from pupils' problem solving processes and reasoning in algebra ?

To answer this research question, the researcher referred to the interview excerpts with the 5 pupils and also to the lists of errors and possible misconceptions from Tables 3, 4 and 5.

The findings from this study showed that misconceptions are robust, this simply meant that they could not easily be dislodged. It is evident from the interview conducted that in many instances pupils appear to overcome a misconception only to have the same misconception later. According to Piaget (1970) when pupils constructed knowledge, they became attached to what they had constructed (radical constructivism). Therefore one major requirement in trying to eliminate these misconceptions is to make sure that pupils actively participate in the process of overcoming their misconceptions. This is not a process that is entirely dependent on a pupil, but the teacher can also help in facilitating the complete elimination of the misconceptions. The teacher can help do so by providing pupils with an enabling classroom environment that can help them develop both procedural and conceptual knowledge such that they can construct correct conceptions from the start.

It was also noted that, the follow-up question which the researcher gave during the interview process to allow the pupils explain their thinking, helped the teacher to get better insight into the pupils' thinking. When teachers listened to the pupils, they would understand the diversity of pupils' understanding. At the same time pupils would revise and refine their own mathematical thinking. Therefore individual attention to the pupil was necessary as it would reveal a lot of inadequacies on the part of the pupils' understanding of algebraic concepts.

As earlier discussed, that during interviews some pupils were able to change their incorrect responses and came up with the correct response (Interview with pupil3). By looking at this revelation, we can decide as teachers to come up with methods to change the misconception and reorganize the incorrect schema. It is important that the teacher should carefully assess the misconception of the pupil and engage with the pupil in such a way that he or she is enabled to rearrange the problem. One way this could be done is to bring pupils' incorrect answers to class for a discussion. This may help pupils get a better understanding of their own errors and misconceptions.

It was also observed that pupils made similar procedural errors in more than one conceptual area. It could be seen from the findings that both knowledge of procedures and concepts are important for pupils to correctly handle questions in Algebra. This means that it is important for pupils not only to have procedural knowledge (how procedures and algorithms work) but they should also develop conceptual knowledge so that they should be able to explain why certain procedures and algorithms work. It is important to note that the two types of knowledge (procedural and conceptual) would lead to an understanding and interconnectedness of these two types of knowledge. The study revealed that both types of knowledge are important to prevent pupils from making many errors and misconceptions. Further, when teaching a new concept, giving examples as well as non-examples is vital as it will help pupils get a better understanding of concepts, facts and procedures.

One notable feature which came out of the interviews was that pupils achieved new ways of thinking. as sometimes pupils gave up their previous, erroneous thinking (case of interview with pupil 2). Occasionally, pupils reflected on their use of methods and identified that mistakes were made. Sometimes, pupils reflected on their previous mistakes and corrected them during the interview process. To a larger extent, the interview session helped the researcher to direct the pupils to explain more or get alternative explanations for the same phenomenon. The errors and misconceptions, therefore served the purpose of constructive and adaptive tools for promoting algebraic understanding.

By committing errors and looking to understand their origins, pupils may achieve a stronger conceptual basis than if they had never committed the errors in the first place. Matz (1980) reinforced this idea by saying that rational errors should not be a hindrance to the mathematical learning process but should serve as constructive and adaptive tools for promoting mathematical understanding. In the process of correcting or searching for the origins of errors, pupils may reach a better understanding of their own mathematical reasoning.

The researcher found out that there were two categories of errors and misconceptions. The first was pupils' lack of understanding of algebraic concepts The second, was some common deficiencies that could happen in any problem solving situation initiated some errors. For instance, pupils' hurriedness to start solving a problem without properly understanding it, using incorrect short-cut methods, not verifying the answers and not being aware of the validity of the answers were obstacles not only to find the correct answer but they could also initiate errors.
Another conceptual area which needs attention was pupils' lack of arithmetic skills in this study it was discovered that some pupils used some arithmetic methods to solve certain algebraic problems. A good example was Question 9 (Solve for x; $4x + 25 = 73$. The answers to this question showed that pupils used to arithmetic methods when they were not necessary. Sometimes, pupils made errors in the algebraic

answers because of their incorrect arithmetic manipulations. Interestingly, most pupils preferred to use arithmetic methods to algebraic methods when solving equations.

Many researchers have attributed the fact that some error patterns associated with algebraic expressions have roots in arithmetic. For example, manipulating algebraic expressions such as over generalization of division procedures (fraction errors) and failure to transfer arithmetic understandings to algebraic contexts (Norton and Irvin, 2007 ; Stacey and Macgregor, 1999). Considering these facts, it can be said that poor arithmetic skills is a factor that contributes to algebraic errors.

In some cases pupils who provided wrong answers in the test recovered in the interview and gave the correct answers to the same questions. One of the possible explanations for this behaviour is test anxiety which makes pupils solve problems incorrectly in a test situation. Test anxiety takes away their confidence and interferes with their thinking in that particular situation. There are two different components of mathematics anxiety: intellectual or cognitive and emotional or affective which affect pupils' solving of algebraic problems (Posamentier, 1998).

The cognitive component of mathematics anxiety involves worrying about failure and its consequences. On the other hand the emotional component involves fear and feeling nervous. In this study, the pupils could not have worried about the consequences of failure because the test was non-evaluative. It is possible that the emotional component could have played a role. In some cases the emotional component has a stronger and more negative impact on pupils' mathematical performance (Posamentier, 1998).

Another possible explanation over the misconceptions made is that pupils probably had correct methods in their long-term memory but they could not recall the information (Matlin, 2005). Pupils probably had both the correct and wrong information in their long-term memory but could only recall the wrong information first. The correct information may have been inhibited by the wrong information.

Comparison of the test solutions and those from the interviews showed that some pupils did not use uniform mental mechanisms when solving algebraic problems. In particular, lack of uniformity between pupil strategies in the pencil-paper test and those used during the interview points to the instability of their thinking process. Since some pupils did not perform consistently, predicting a model to explain their misconceptions was not possible.

5.6 Summary.

In this chapter, the researcher articulated a number of pupil errors and misconceptions based on the findings in the previous chapter. In order to answer the three research questions, detailed explanations about the origins of these errors and misconceptions were given. Later, these explanations were related to the existing literature on the misconceptions in order to connect them with broader theoretical arguments. Finally, the implications of the findings were discussed with suggestions for classroom teaching.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS.

6.0 Introduction.

The fundamental goal of this research was to explore pupils' errors and misconceptions in the three conceptual areas of Algebra and further expose pupils' reasoning for doing so. This study assumed that a better understanding of pupils' errors and misconceptions in algebra leads to a better understanding of pupils' general understanding of mathematics principles.

6.1 Research Questions.

The following research questions, guided the study:

- What errors and misconceptions do secondary school pupils make when solving problems related to variables, expressions and equations?
- Why are there differences among pupils when they engage in solving algebraic expressions, variables and equations?
- What can be learned from pupils' problem solving processes and reasoning in Algebra?

This chapter revisits the research questions, summarise the findings and offer conclusions based on the findings. Recommendations for future research was included. Additionally, a section reflecting on the research process that had been undertaken is also included.

6.2. Research Questions: Summary of findings and conclusions.

6.2.1 Research Question 1: What errors and misconceptions do secondary school pupils make when solving problems related to variables, expressions and equations?

The results of this study show that Grade 11 pupils had several misconceptions in the three conceptual areas of Algebra, i.e. variables, algebraic expressions and equations. Previous research has shown that these misconceptions occur not only at the college level but also at the secondary level as well (Kuchemann, 1981). This research supports previous findings as it seems pupil understanding of Algebra continues to be an issue to this day.

In this study, 17 error types under the three conceptual areas have been discussed. There were seven new error types that came out from this study and these included: misinterpreting the product of two variables, giving answers in the form of equations, incomplete simplification and incorrect cross multiplication. Others were incorrect quantitative comparisons, numbers as labels and misinterpreting of the elimination method when solving equations.

The symbols in Algebra have different meanings and interpretations in different situations. Pupils have incorrect and incomplete perceptions about the letters, numbers and signs. The overall image that emerged from the findings was that the misunderstanding of the concept of the variable did have a clear bearing on their errors and misconceptions.

With regards to algebraic expressions, it was discovered from the study that the main problem which pupils encountered was the lack of understanding of the structural features in this conceptual area which led pupils to use many illegal procedures. Also pupils modified or misapplied rules which were inappropriate in certain situations.

As regards to equation solving, the misuse of the equal sign out of its accepted meaning was obvious. In most cases, pupils' used the 'equal sign' in a single sense, that is, to do the operation to the left and get the answer to the right or vice versa. Finally the pupils misused the elimination and substitution method.

6.2.2 Research Question 2: Why are there differences among pupils when they engage in solving algebraic expressions, variables and equations?

The study revealed that pupils' errors and misconceptions were more in algebraic expressions than in variables and equations. Previous research has confirmed that some error patterns associated with manipulation of algebraic expressions have roots in arithmetic (Stacey and Macgregor,1991). The other problem which pupils experience in algebraic expressions is that they are abstract in nature, thus they use letters instead of numbers (Usiskin, 1982).

As for variables and equations, the questions did not require pupils to apply higher levels of cognitive skills such as synthesis, analysis and evaluation. Questions involving variables such as: Name a variable in the question were mainly at knowledge level of cognitive level of Blooms taxonomy. As for equations the pupils were just required to apply already set rules to come up with the solution.

The findings from this study revealed that pupils had difficulties with algebraic expressions because there were abstract in nature. The fact that algebraic expressions involve the use of letters as opposed to the numbers made pupils feel uncomfortable with the operations. The other factor was that there is interference from the arithmetic methods which they wrongly apply in algebraic expressions.

6.2.3 Research Question 3: What can be learned from pupils' problem solving processes and reasoning in algebra?

Sometimes, solving schema is deeply rooted in the students mind is mistakenly applied despite correct, intuitive understanding (Fischbein and Barash, 1993). In many instances, pupils appear to overcome a misconception only to have some misconceptions resurface later. This was consistent with the findings from this study. This is probably as a result of the fact that, when pupils construct knowledge, they become attached to the notions they have constructed. Therefore, one important requirement in eliminating misconceptions is that pupils must actively participate in the learning process.

Another notable feature of the interview process was that pupils achieved new ways of thinking, sometimes giving up their erroneous methods. Occasionally, pupils reflected on their previous mistakes and corrected them during the interview process. In particular, the lack of uniformity between their strategies in the written test and during interview, points to the instability of their thinking process.

By committing errors and looking at their origin, pupils may have a stronger basis for reasoning correctly than if they never committed the errors in the first place. Matz

(1980), reinforced this idea by saying that rational errors should not be a hindrance to the mathematical learning process, but they serve as constructive tools for promoting learning. Finally, the pupils' errors and misconceptions are largely attributed to the lack of conceptual knowledge in solving algebraic problems.

6.3 Recommendations.

6.3.1 Teachers and pupils to talk about misconceptions.

The question at the end of the "statement of the problem" was posed, 'How can errors and misconceptions be minimized? The answer is that teachers and pupils should talk about these misconceptions in the classroom and try to address them by allowing learners to investigate the truth of some of these statements or expressions. As it has been shown in the literature, misconceptions are not due to pupils' lack of procedural knowledge, but are due to their lack of conceptual understanding of the concepts. According to the findings of this study, misconceptions in Algebra have been attributed to the incomplete treatment of certain concepts or topics were pupils try to fill in the gaps with false generalization.

It is therefore important that teachers of mathematics come together and draw up programmes of action where particular misconceptions are addressed in workshops during which groups are tasked to come up with ways of addressing these misconceptions. This entails brainstorming teaching strategies that focus on instruction of pupils in algebra. This is important because different levels of experiences come together during workshops to share ideas that makes knowledge acquired richer and informative.

6.3.2 Formation of district and cluster mathematics networks.

The formation of district and cluster mathematics networks will assist teachers of Algebra to come and share the problems and challenges in an effort to address them by way of model demonstration lessons. One possibility is to develop a set of video tapes where exemplary teachers teach demonstration lessons. Also the pupils in these district and cluster mathematics networks to meet and share the successes and challenges in mathematics in general and Algebra in particular. This will help pupils improve the mathematical and algebraic concepts in Algebra.

6.3.3 Individual attention to pupils.

When teachers listen to the pupils, they will be able to understand the diversity of pupils' understanding. At the same time pupils will revise and refine their own mathematical reasoning. This was evident during interview sessions when some pupils changed their reasoning by coming up with a correct answer contrary to the wrong answer given in the test. Therefore individual teacher attention to the pupil is necessary as it will address a lot of inconsistencies in terms of pupil reasoning.

6.4 Future research.

My work as a researcher has been a long one and I feel it should not end here. There are a number of issues which came up during the time of the research. There is need to explore this fascinating area further. It must be mentioned here that it was somehow difficult to find out the real causes of errors and misconceptions without having a thorough and careful examination into each of them. There is need that proper identification and micro analysis of individual errors and misconceptions is done. In order to do this future researchers can plan and administer a set of carefully planned questions to identify specific errors and misconceptions under a given concept.

Identification of errors and misconceptions is meaningless unless suggestions to overcome them are made. For future research, lesson plans will be prepared and tested in real classroom situation. Research suggests that, if pupils can visualize abstract algebraic concepts, it will help them to understand them properly.

6.5 Summary.

In this chapter, I articulated a number of pupils' errors and misconceptions based on the findings in Chaptet4. To answer the three research questions set out in this study, I explained in detail the nature and, wherever possible the origins of these errors and misconceptions. The implications of the findings were discussed with suggestions for classroom teaching. Finally the suggestions for further research were given.

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APPENDICES.

Appendix A

Test instrument-Pilot Study-Stage

Pupil Number

Your performance in this assessment will have no bearing on your grades or evaluation in the subject. The assessment is meant to help you with algebra, by assisting your teachers to understand the mistakes you make, as well as why you make them.

Instructions.

(a)Answer all questions.

(b)Use algebraic methods to solve all the problems.

(c)Time: 90 minutes.

1. Titus sells x sugar canes. Luke sells twice as many sugar canes as Titus. A sugar cane costs k1000.

(a) Name a variable in this problem

(b)Name another variable in the problem.

(c)Name something in the question that is not a variable.

2. What does 3x mean? Write your answer in words.

3. What dose xy mean? Write your answer in words.

4. Write 2 pairs of values for x and y to make x= y+2 a true statement.

5. Simply; (a) $\frac{3y}{6y}$ (b) 2y x $\frac{1}{6y}$ (c) $0 \times y$ (d) $(x + y)^2$ (e) $\frac{x}{4} - \frac{2-y}{3}$

(f) $x(\frac{c}{d})$ (g) $\frac{ax + bx}{x + cx}$

6. Write an equivalent expression for $\frac{x}{2y}$

7. Evaluate ; -(2y-x)

8. Simplify; $\frac{x}{y} + \frac{x}{z}$

9. Simply where possible :

(a) 5a + 6b + 10c (b) 10 + 3y (c) x + x + 3b +6Y

10. The n represents a natural number. Which one is greater than the other in the

other $\frac{1}{n}$ or $\frac{1}{n+1}$

11. Consider the following system of simultaneous linear equations:

x+ y= 5

x – y= 7

(a)In order to eliminate x, do you add or subtract the two equations

(b)In order to eliminate y, do you add or subtract the two equations.

(c)If you add or subtract the two equations would you get the same answer?

.Explain.

12. Solve the following system of simultaneous linear equations

a+ b = 4

b = 2a + 4

13. Solve the following system of simultaneous linear equations.

$\frac{a}{2} - \frac{2b}{3} = \frac{7}{3}$

$\frac{3a}{2} + 2b = 5$

Explain why you chose the method which you used?

Appendix B.

Pupil Interview Format.

Process.	**Interview question**
1. Reading	1.Please read the question
2. Comprehension	1.What does the question mean?
3. Strategy selection.	3. How will you solve the question?
4. Process.	4. Work out the question .Tell me what you are doing as you proceed.
5. Encoding.	5. Write down the answer.
6. Consolidation.	6. What does the answer mean?
7. Verification.	7. Is there something you can do to make sure that your answer is correct?
8. Conflict.	8. Is there any conflict? (Here the interviewer will ask some conflicting questions to prove whether pupils have conflicts in the solving process

Appendix C.

Letter to School Head teachers

Dear, Sir / Madam,

I am a postgraduate student of the University of Zambia in the School of Education, who is pursuing a Master of Education in Mathematics Education. My dissertation supervisor is Dr. M. Tabakamulamu a Lecturer in mathematics education in the School of Education. I am hoping to conduct research which examines grade 11 pupils' misconceptions and errors in algebra. I have selected your school as one of two schools Petauke to collect data for my study.

The purpose of my study is to identify pupil difficulties when performing algebraic problems and to suggest some remedial measures to overcome these difficulties. In order to examine students' errors and misconceptions, I wish to administer a test to 60 pupils in three schools. Later, six students will be selected for interviews based on their answers to the test. The test paper will take approximately 90 minutes and each interview will take 30 minutes or so.

I would like to request participation of your school in this study by allowing me to conduct the tests and the interviews. I will not use teachers' or pupils' names or anything else that might identify them in the written work, oral presentations or publications. The information will remain confidential. They are free to change their minds anytime, and withdraw even after they have consented to participate. If you would like to get more information, please contact me by phone on 0964156897.

Titus Mbewe

Appendix D

Parent / Guardian consent letter.

Dear Parent or Guardian,

I am a postgraduate student of the University of Zambia in the School of Education, who is pursuing a Master of Education in Mathematics Education. My dissertation supervisor is Dr M. Tabakamulamu a lecturer in mathematics education in the School of Education. I am conducting a research study which examines grade 11 pupils' difficulties in algebra. I have selected your child's school as one of two schools in Petauke District to collect data for this study.

The purpose of my study is to identify pupils' difficulties when performing algebraic problems and to suggest some remedial measures to overcome these difficulties. In order to examine students' errors and misconceptions, I wish to administer attest instrument to 60 pupils in three schools in grade 10. Your child will be asked to participate in a written test. The test will approximately take 90 minutes. The test will contain 13 short answer items. Based on the results, your child may be asked to participate in an interview to identify his or her difficulties in algebraic problem solving. The interviews will take 30 minutes or so.

I would like to request participation of your child in this study. Participation in this study is voluntary and will affect your child's attendance in class or his or her evaluation by the school.

All the information collected will be anonymous. In a way, the results of this study may help the school as well to identify pupils' difficulties in algebra and propose remedial work.

Please indicate on the attached form whether you permit your child to take part in this study. Your cooperation will be very much appreciated. If you have any questions or you would like to get more information, please contact me by phone on 0964156897.

Thank you

Titus Luka Mbewe.

Appendix E

Test Instrument-Main study

Student Number :........................   Sex :....

Your performance in this assessment will have no bearing on your grades or evaluations in your course of study. The assessment is designed to help you with algebra, by helping your teacher understand the mistakes you make, as well as why you make them.

Instructions.

(1) Answer all questions.

(2) Use algebraic methods to solve all the problems.

(3) Time : One hour.

1. Titus sells x sugar canes . Luka sells twice as many sugar canes as Titus. A sugar cane costs K500.00.

(a) Name a variable in the problem.

(b) Name another variable in the problem.

(c) Name something in the question that is not a variable.

2. What does xy mean ?. Write your answer in words.

3. (a) Simplify: $B(\frac{!}{B})$ (b) evaluate: 0(y) (c) Expand : $(x + y)^2$

(4) Simplify : (a) $\frac{x}{4} - \frac{2 - y}{3}$ (b) $x(\frac{c}{d})$ (c) $\frac{ax + bx}{x + cx}$ (d) $\frac{A}{B} + \frac{A}{c}$

(5) Subtract 3x from 7

(6) Multiply a +2 by 4

(7) The letter n represents a natural number. Which one is greater than the other $\frac{1}{n}$ or $\frac{1}{n+1}$?. How do you know?.

(8) Which is larger than the other y or x in $y= 2x + 3$. Explain.

(9) Solve for x : $4x + 25 = 73$

(10) Consider solving the linear system of equations

$$x + y = 5$$
$$x - y = 7$$

(a) To eliminate x from both equations, do you add or subtract the two equations?

(b) To eliminate y from both equations, do you add or subtract the two equations?

(c) will you obtain the same answer if you add or subtract the two equations ?

(11) Solve the following system of linear equations

$$2x + y = 2$$

$$3x - 2y= 3$$

(12) Shirts cost s dollars each and pants cost p dollars a pair. If I buy 4 shirts and 3 pairs of pants, explain what $4s + 3p$ represents ?

Appendix F

Error responses according to gender.

Question number.	**Number of incorrect responses (Boys)**	**Number of incorrect responses(girls)**	**Percentage (%) (boys)**	**Percentages (%) (girls)**
1(a)	4	7	13	23
1(b)	7	10	23	33
1(c)	8	12	27	40
2	12	24	40	80
3(a)	10	12	33	40
3(b)	12	13	40	43
3(c)	21	19	70	63
4(a)	12	24	40	80
4(b)	10	20	33	67
4(c)	21	27	70	90
4(d)	12	15	40	50
5	22	27	73	90
6	23	28	77	93
7	24	30	80	100
8	9	13	30	43
9	6	6	50	50
10(a)	12	18	40	60
10(b)	14	17	53	57
11	16	17	53	57
12	6	9	40	30