Article

Secondary Teachers' Mathematics Knowledge for Teaching Quadratic Equations: A Case of Selected Secondary Schools in Katete District

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Abstract

The purpose of the study was to explore teachers' mathematical knowledge for teaching quadratic equations. The study had three specific objectives: (1) to determine teachers' subject matter knowledge for teaching quadratic equations, (2) To assess strategies used by teachers in teaching quadratic equations and (3) To examine how teachers address pupils' errors and misconceptions related to quadratic equations. This was a qualitative study involving three participants and data was collected in form of semi-structured interviews, classroom observations and questionnaires. Data was analysed using analytical coding in which the researcher identified the conceptual categories into which the phenomena observed were grouped. The study revealed that secondary school teachers possess adequate subject matter knowledge for teaching quadratic equations which was limited to procedural understanding. Subject matter was more influential in the delivery of the lesson than didactic knowledge and knowledge of learner conceptions. The teachers mathematical knowledge for teaching ability to generate different solutions, to address pupils' difficulties and misconceptions and to choose appropriate examples to teach quadratic equations were based on the depth of the subject matter which played a critical role in their reasoning and decision making in specific contexts. The participants provided a wide range of instances of pupils' errors in learning how to solve quadratic equations but they were unable to detect the root cause of these errors the study recommended that mathematics teachers need to be encouraged to present lessons in such a way that procedural understanding is supplemented by conceptual understanding of concepts.

Keywords: Subject matter, Mathematical knowledge for teaching, Misconceptions, didactic, conceptions, conceptual, procedural.

1.1 Background of the Study

The Zambian government attaches great importance to the teaching and learning of mathematics as this is envisaged in the MoE (2013, P15) which states that "... mathematics prepares and enhances the learners prospect of employment and further education". In spite of the importance attached to mathematics in the Zambian curriculum, performance in mathematics both grades 9 and 12 has persistently been the source of great concern. On average two thirds of the candidates who sat for either junior secondary or school certificate fail mathematics (MoE, 2013). In trying to address the issue of underperformance in mathematics, Jones, Hopper, and Franz (2008, P.308) records, " No matter the psychological or socio-economical reasons , poor mathematical ability has serious consequences, and as educators we must address the question of why so many students are failing. The major component of the Zambian mathematics syllabus comprise algebra which include topics such as equations, inequations, algebraic expressions and quadratic equations among others. This study looked at Quadratic equations which is one of the core topics under algebra. The understanding of quadratic equations is critical for pupils' understanding of further mathematics such as polynomials and calculus (Kotsopoulos, 2007, and Didis, 2011). In the Zambian situation some candidates at school certificate level examinations failed to realise that $(2x-1)(3x-2)=3$ and solved it as an ordinary linear equation as shown below:

$2x-1=30r3x-2=3$

$2x=3+1or3x=3+2$

$x=\frac{4}{2}orx=\frac{5}{3}$

$x=2orx=1\frac{2}{3}$

(Source: Chief Examiners report, 2011)

Since the candidates have problems in answering quadratic equations, it is important to find out more about the teaching and learning aspect. Kieran (2007) notes we tend to know much about how pupils learn algebra than how the teachers teach it. This paradigm shift of research to focus on teacher knowledge was initiated in the mid 1980's by Shulman in which he called the Pedagogical Content Knowledge (PCK) as a missing paradigm (Shulman, 1986).The education policy document puts much emphasis on the fact that teachers should posses essential competencies and masterly of the material that is to be taught (MoE, 1996). Considering the fact that pupils underperform in mathematics at grades 9 and 12 examinations and have particular difficulties in quadratic equations motivated the researcher to conduct this study.

1.2 Statement of the Problem

In the past three decades, various studies have focused on learners understanding of different aspects of equations (Kieran, 1988, Linchevski, and Herscovics, 996, and Li, 2007). The findings from these studies partially demonstrate the complexity of learning and teaching of equations, and this could be a valuable resource for algebra teachers to enrich their knowledge for teaching. Different scholars such (Ball, Bass, Hill and Schiling, 2005)

developed instruments for assessing knowledge needed for teaching mathematics and they have been validated or tested for trustworthiness by research groups (Ferrini, and Mundy, 2005). However, most of the instruments which have been developed are mostly for primary school level mathematics concepts such as number and numeration, multiplication and division. There is lack of systematic research on teachers' mathematical knowledge at secondary school mathematics including in key topics such as quadratic equations.

1.3 Purpose of Study

The purpose of the study was to explore teachers' mathematical knowledge for teaching quadratic equations.

1.4 Research Questions

The study sought to answer three main questions:

(1) What subject matter knowledge for teaching quadratic equations do teachers possess?

(2) What strategies do teachers use in teaching quadratic equations?

(3) How do teachers identify pupils' errors and misconceptions related to quadratic equations?

1.5 Significance of the Study

This study will try to respond to calls for better understanding of mathematics knowledge for teaching (MKT) in specific topics of mathematics at secondary school level. The study on mathematics teachers' knowledge of quadratic equations is important because it may reveal the necessary condition for teaching methodologies and approaches that could help pupils have conceptual understanding of algebra in general and quadratic equations in particular. The studies could contribute to identifying the most crucial features of subject matter knowledge and pedagogical knowledge and support the effective teaching and facilitate substantial growth in pupils' algebraic proficiencies, including the recent emphasis on developing reasons and sense making skills (Graham, Cuoco, and Zimmerman, 2010).

1.6 Theoretical and Conceptual Framework

The theoretical framework that informs the study is drawn from the works of Scheiner (2015) of which he called on the broad perspective of Mathematical Knowledge of Teaching (MKT) to include an epistemological, cognitive, and didactical dimension. Thus the theoretical framework for this study shall be referred to as (Mathematics) Teachers' knowledge: Epistemological, Didactical and cognitive perspective. The three dimensions in the framework thus: epistemological, cognitive and didactical) were considered as useful to be useful lenses in investigating the mathematical knowledge

1.7 Definitions of the Theoretical Framework Terms

(a) Epistemological dimension: This is a thorough understanding of a particular discipline by the teacher and how knowledge is structured within the subject

(b) Didactical dimension: This has to do with teachers' knowledge of the pedagogical form as the content is represented in the instructional media such as visual aids, textbooks and other teaching strategies (Ball et, al., 2008).
(c) Cognitive dimension: This represents teachers' knowledge of cognitive form of the content in learners mental representations.

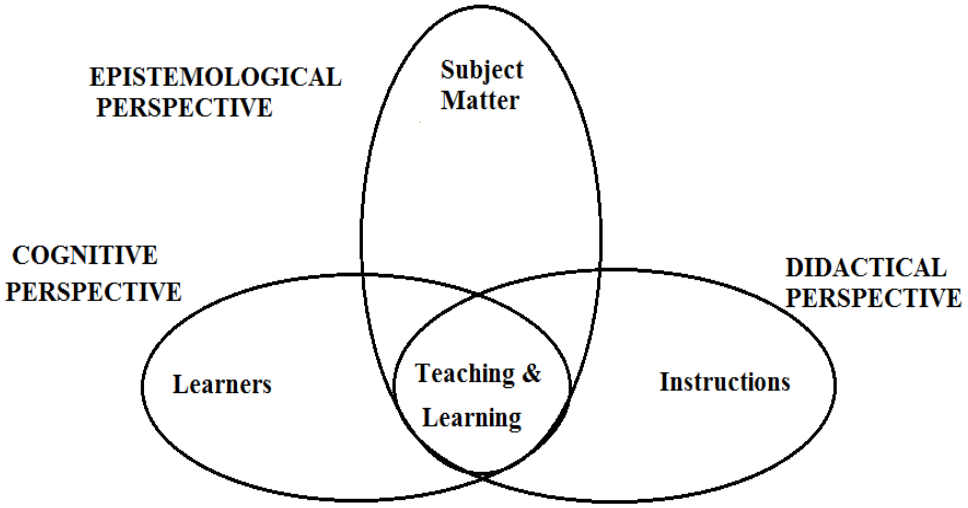


Figure 1 : **(Mathematics) Teachers'knowledge: Epistemological Didaction & Cognitive Perspective. Adapted from Scheiner (2015)**

Table 1: *Concepts drawn from the theoretical framework*

Dimension of the theory	**Concepts**	**How am using the concepts in the study**
Epistemology	Knowledge of Subject Matter	(a) How the teacher presents facts, and the relationships among them.
Didactic Dimension	Knowledge of Instruction	(a) How the teacher plans and organises the work. (b) The type of the teaching strategies the teacher uses.
Cognitive Dimension	Knowledge of leaner conceptions.	(a) How the teacher takes care of pupils' common difficulties and anticipated misconceptions

2. Literature Review

Before the 1980's, education research comprised research studies which were behaviourist in nature. these were referred to as "process-product" oriented studies which described the

relationship between teacher conduct or behaviour and learner achievement (Hill, Rowan, and Ball, 2005). The typical behaviours of teachers which some studies examined included group work of learners. Although there were a lot of studies done by the behaviourist movement, they were not particularly useful in producing data which we would improve pedagogy. Beginning the mid-1980's a major paradigm shift came on the scene with an emphasis on;

2.1 Pedagogical Content Knowledge.

Shulman in1985 made the Presidential address at the American Educational Research Association (AERA) annual meeting. In his address Shulman indicated that there was lack of focus on knowledge of subject matter in the process-product research on teaching and further argued that education research placed much emphasis on issues to do with classroom management and neglecting pedagogical content knowledge (PCK) that teachers needed to have in order to teach effectively. Shulman (1986) identified PCK as a missing paradigm in education research. However, Shulman went further and stated that, "The teacher need not only understand *that* something is so; they must further understand *why* it is so" (Shulman, 1986, p.9). This gives us a type of knowledge unique to the teaching profession. Grossman (1990) investigated subject matter knowledge of teachers of English. It was a case study which involved two teachers of literature in English. The purpose of the study was to find out how subject matter knowledge of teachers of Literature in English affected the interpretation of the texts. The results of the study showed that teachers' knowledge and beliefs about the purpose for teaching a subject had an impact on the instructional beliefs. For instance, one of the participants viewed teaching literature as explaining the given text in depth, where as another participant put emphasis on helping students to relate the text to their own lives. Another important aspect of teacher knowledge include subject matter knowledge which includes teachers' beliefs. (Grossman et al., 1989). Grossman (1989) contended that teachers' mathematical beliefs affected teachers' instructions. For instance some teachers believed that mathematics was about getting the right answer and that pupils should follow an algorithm, thus step-by-step procedures.

2.2 Procedural and Conceptual Knowledge of Teachers

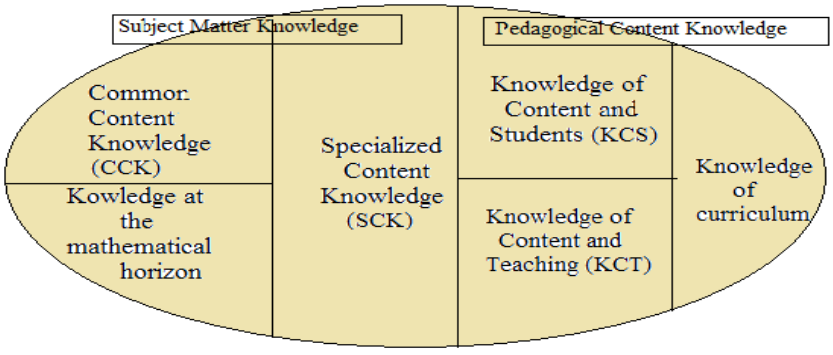
Although, the two orientations of *conceptual* and *procedural* teacher knowledge have been studied for a number of years before the 1980's, the major works in describing these two types of knowledge have been done by Hiebert and Lefevre (1986), who defined *conceptual knowledge* as "a connected web of knowledge" (p.3). *Procedural knowledge* is divided in to two parts thus the recognition of proper "forms" and knowledge of rule, algorithms, and procedures (Hiebert and Lefevre, 1986). In an effort to give a clear understanding of procedural and conceptual knowledge, Hiebert and Leferve (1986), describe the difference between the two is that the former is associated with rote learning and the later is meaningful learning .Star (2007) pointed out that some key notions such as procedural and conceptual understanding, still required more careful characterizations in terms of knowledge and knowledge of quality. Even and Tirosh (1995), examined teachers presentations of certain content in terms of their knowledge of subject matter. The study was based on the idea of generating appropriate representations and explanations for a concept should not be based on

the facts, rules and procedures but also know why they are true. He referred to such knowledge as 'knowing that' and 'knowing why'. In their study, one participant knew that 4 divided by 0 is undefined but did not explain why. Thus Even and Tirosh concluded that teachers' knowledge of the subject matter of students thinking had a strong influence on their pedagogical decisions.

2.3 Mathematical Knowledge for Teaching by Ball and Her Colleagues

One of the scholars who developed keen interest in mathematical knowledge for teaching in the late 1980's was Debora Ball. Her work is believed to have been inspired by Shulman's seminal presentation 1986 and 1987. Ball identified two areas of knowledge of mathematics which she called, knowledge of mathematics and knowledge about mathematics, which she deemed to be important aspects for teachers to adequately know. Ball referred to knowledge of mathematics as *substantive knowledge*. Ball described knowledge of mathematics or substantive knowledge as one which comprised both conceptual and procedural understanding. Ball defined knowledge *about* mathematics as "knowledge of the nature and discourse of mathematics...{which} entails ideas about what is involved in doing mathematics and how truth or validity is established in the domain" (Ball, 1991, p.44). The knowledge of mathematics constitutes procedural understanding. Ball and her colleagues that they were able to develop a model for mathematical knowledge for teaching as shown in Figure 2.

Figure 2. Mathematical Knowledge for Teaching



Mathematical knowledge for teaching (Ball etal. 2008)

The subject matter knowledge has two main categories namely the *Common Content Knowledge(CCK)* and the *Specialised Content Knowledge (SCK)*. CCK is knowledge which any adult person may possess while SCK is mathematical knowledge which unique for teachers (Ball, Thames, Phelps, p.400, 2008). *knowledge of content and students (KCS)* This the knowledge that includes anticipation of learners difficulties and reasoning and knowing common errors and misconceptions that learners may face in a specific topic. The *KCT* systematically combines knowing about teaching with knowing about mathematics.

3. Methodology

3.1. Qualitative

Qualitative researchers hold the view that one needs to get close to the people in order to understand how people think, learn and know the world around them (Bogdan and Biklen,1998). In this study the researcher intends to get closer to the people and collect data through observing people (teachers) in natural setting.

3.2. Research Design: Case Study

Merriam (1998), also explains that qualitative research is an overarching concept that covers several forms of inquiry that help us to understand and explain the meaning of social phenomena with little or no disruption of the natural setting. One form of qualitative inquiry is the case study. Merriam (1998, p.28) asserted the fact that a 'case study is appropriate when the researcher's goal is to is to gain insights, discovery, and interpretations of a particular phenomenon.'

3.3. Participant Selection

Qualitative researchers tend to select each of their cases purposefully that seeks to get information- rich cases that can be studied in depth (Patton, 1990). In this study, the researcher selected the participants purposefully using a combination of maximum variation sampling in which the participants were chosen according to their varying experiences. The study involved three secondary school mathematics teachers. For the purpose of this study, the first criteria for a teacher to participate in the study would be one who has a class in grade eleven because the topic of quadratic equations is taught at this grade level. The other criterion is that the participants was that of varying experiences as follows: One teacher with work experience of less than five years, a second one with work experience of more than five years but less than ten years and the third one with work experience of more than ten years.

3.4. Data Collection Methods

Peoples utterances and actions represent the data of qualitative inquiry and this requires methods that allow the researcher to capture the spoken word and behaviour (Norman and Denzin, 2000). The key ways of capturing data were through interviews, and observations. Data was also supplemented by the questionnaire.

3.5. Data Analysis

The initial step in the analysis of data was data management which involved proper handling and keeping track of data as the study moved along. The researcher transcribed the interviews recorded in audiotapes and transform them into text files that were stored and reviewed. The researcher followed a similar approach with the audio tapes to be recorded during the classroom observations. During the analysis stage, the data was manipulated by creating and applying abstract categories and use the categories to compare, contrast and refine distinguishable thoughts and behaviours. In order to make connections among stories, analysis began with identification of the themes emerging from the raw data, a process sometimes referred to as open coding (Strauss and Corbin, 1990).

4. Presentation of Findings

In this chapter, I present the findings of the study conducted on three mathematics teachers on the teaching and learning of quadratic equations.

Research question 1: What subject matter knowledge do teachers have when teaching quadratic equations?

4.1. Display of Skill for Solving Quadratic Equations

During the lesson observation, it was observed that all the three participants followed the same pattern of lesson presentation and that is the introduction which was a recap of the previous lesson, followed by the presentation the new topic which was usually by way of examples. For Talandila two or three examples were given. Below is an extract one of the lessons which was presented by Talandila on the solving of quadratic equations using factorisation method.

Solve *m(m- 3) = 40*, using the factorisation.

The teacher proceeded as follows:

Teacher: *What is the first step to do?*

Pupils: *In chorus- remove the brackets*

Then the teacher proceeded and expanded the equation by removing the brackets

m^2 *-3m- 40 = 0*

Product=1x -40 = -40

Sum = -3

Teacher: *We look for two numbers if when multiplied we get -40 and when added we get 3.*

The possible factors were listed as follows: (5, -8), (-5, 8), (-2, 20), (-20, 2), (4, -10), (-4, 10). Through inspection with the pupils, the factors which were got were (5, -8) as 5 *x* -8 = -40, and 5 + (-8) = -3.

Teacher: *We rewrite the sum as in this case -3 as indicated below*:

m^2 *- 8m + 5m - 40 = 0*

m (m – 8) +5 (m – 8) = 0

(m + 5) (m - 8) = 0

m + 5 = 0 or m – 8 = 0

m = 0 – 5 or m = 0 + 8

m = -5 or m = 8.

Source :(First lesson observation of Talandila).

The extract above showed that Talandila had the skills to solve quadratic equations and this was observed to the other two participants.

4.1. Prerequisite Knowledge in Relationship to Subject Matter Knowledge

Talandila and Titus responded that they considered integers and linear equations as necessary prerequisite knowledge and skill for teaching quadratic equations. Below is the interview excerpt with Titus :

Researcher*: What prior knowledge or topics do you regard to be important before you teach the topic of quadratic equations ?*

Titus *:The pupils should be conversant with integers (adding and subtraction of integers), solving of linear equations as well as factorisation and be able to locate points on the XOY-* plane(Titus ln # 64-68).

However, Faith considered that coordinate geometry, inequations and inequalities as important prerequisite knowledge for teaching quadratic equations. Below is the interview excerpt with Faith:

Researcher *: What pre-knowledge of the pupils do you regard to be important before teaching quadratic equations ?*

Faith *: They should have the knowledge of coordinate geometry, inequations and inequalities. This will help them plot the coordinates and find the necessary scale.*

Researcher *: Any other pre-knowledge which the learners need to know before they learn quadratic equations ?*

Faith *: They should also have knowledge of ratio and proportion and scale drawing* (Faith ln # 67-75).

Question 8 on the questionnaire asked the participants on the prerequisite knowledge which were necessary before teaching factorisation method of solving quadratic equations shown below:

Table 2: *Prerequisite knowledge for factorisation method*

Name of participant	Combining like terms	The zero-factor theorem	Solving linear equations of the form $ax + b = 0$	The distributive Law	Expressing equation in standard form	Multiplying two binomials
Faith	✓	✓		✓	✓	
Talandila	✓	✓	✓	✓		
Titus	✓	✓	✓		✓	✓

The varying responses on the prerequisite knowledge needed before the teaching of quadratic equations showed the different levels of the subject matter knowledge.

4.2. Research Question 2: What strategies do teachers use when teaching quadratic equation

In the interview session, when the participants where asked if they used real life examples when teaching quadratic equations, they all responded that they included real life examples when teaching quadratic equations. However, when a follow up question was asked to site an example of a real life example they would give when teaching quadratic equations, Talandila and Titus were able to give the examples while Faith did not give any. Below is the interview excerpt with Talandila:

Researcher *: When giving examples, do you include real life examples?.*

Talandila*: Yes, I have.*

Researcher*: Can you give theses examples?*

Talandila: *There are some questions which involve area of a piece of land and because area is squared, it can be used to find side of*

the piece of land (Talandila, ln # 61-65).

Below is an interview excerpt with Faith:

Researcher*: When teaching quadratic equations, do you include real life examples ?*

Faith: *Yes we do. As we conclude the lesson, after teaching all the four methods that is: factorisation, completing the square, formula and graphical. We usually include real life examples.*

Researcher: *Give an example of a real life example which you may give under quadratic equations ?*

Faith*: Mmm....not really* (Faith ln # 62-66).

Teaching strategies and representation of work.

As regards to teaching strategies, the researcher asked strategies the participants used when teaching quadratic equations. Below is the interview excerpt with Faith:

Researcher*: How would you introduce the lesson of solving quadratic equations using factorisation method?*

Faith: *I would first remind the pupils of factorisation of quadratic expressions and linear equations.*

Researcher*: What else would you do ?*

Faith*: Then I will also remind them of the product rule* $a \times b = 0$. *Either a or b =0, implying that a or b represents the factors of the quadratic equations.*

Talandila introduced the lesson by reminding pupils of linear equations and wrote the equation: $x + 2 = 3$. Refer to the following extract below:

Talandila: *Who can come and solve the equation (* $x + 2 = 3$)

Pupil*:* Pupil solved the equation as follows. $x + 2 = 3$, $x = 3 - 2$, $x = 1$

Talandila: *The equation* $x + 2 = 3$ *has the highest power of 1 and today we are going to look at the quadratic equations whose power is 2*. He then wrote the title 'Quadratic Equations' on the chalk board.

Teacher Talandila wrote standard quadratic equation '$ax^2 + bx + c = 0$'. *He told the class that this was a standard quadratic Equation and has the highest power of 2. He further said that there are four methods of solving quadratic equations namely: factorisation , completing the square, formula and graphical method.*

Talandila introduced the product rule and wrote on the chalkboard that if, $A \times B = 0$, then either $A = 0$ or $B = 0$ or both (Talandila lesson observation 1). Titus emphasised that the pupils needed to master the steps leading to the derivation process of the formula. Below is the interview excerpt of Titus to validate the point above:

Researcher*: How do you introduce the topic of solving quadratic equations using the formula ?*

Titus: *The formula is usually taught as the third method. The first one being factorisation, then completing the square and the formula method. We first derive the formula from completing the square method and when that is done it becomes simpler to the learners. This is necessary as pupils will be able to know where the formula is coming from.*

Researcher: *When deriving the formula, is it a must that pupils should master all the steps regarding deriving the formula ?*

Titus: *Yes, they have to know that, because do not know some pupils may ask where the formula is coming from because they may feel that it has been imposed on them* (Titus ln # 38-49).

4.3. Research Question 3: What strategies do teachers use in identifying pupils difficulties and misconceptions.

Identification of possible learner difficulties and misconception. The participants were asked what misconceptions and difficulties they anticipated before teaching the topic of quadratic equations. Talandila and Faith responded that pupils had difficulties in understanding the quadratic formula. This is what Faith had to say:

Researcher: *What possible difficulties do you anticipate that learners may face as you teach quadratic equations?*

Faith: *Where they need to find the roots of* $b^2 - 4ac$ *. If it is not a perfect square root, they will still need a calculator to find the square root.*

Researcher: *Any other difficulties or misconceptions you anticipate your learners to have before you teach quadratic equations?*

Faith: *There is also an issue of understanding the quadratic formula properly. For example in the formula there is negative (b) and if the quadratic equation there is a negative, the pupils tend to miss it* (Faith ln # 76-83).

Below is the interview extract with Titus:

Researcher : *What learner difficulties or misconceptions do you anticipate before you teach the topic of quadratic equations ?*

Titus : *Yes, for example if the solution of the quadratic equation is recurring, pupils like rounding off on the way thereby getting wrong solutions. Failure by the pupils to follow the right step as they solve quadratic equations* (Titus ln # 69-73).

Addresses learners' difficulties and misconceptions

Faith and Titus responded that they would help pupils with difficulties by putting them in groups. Below is the interview extract with Faith which best represents the idea;

Researcher: *How do you assist learners who experience difficulties in quadratic equations?*

Faith: *I divide the pupils in small groups and give them questions so that they can assist each other* (Faith ln # 90-93).

However, on the other hand Talandila responded that he assisted pupils with difficulties and misconceptions in quadratic equations by having a makeup lesson with the pupils. below is the interview extract with Talandila:

Researcher*: How do you assist learners who experience difficulties in quadratic equations?*

Faith*: I divide the pupils in small groups and give them questions so that they can assist each other* (Faith ln # 90-93).

During classroom observation, it was observed that the participants neither divided the learners in small groups nor made make up lessons but instead made the whole class revision. The learners with learning difficulties were not singled out as reflected to during the interview responses. The extract below shows part of the lesson as delivered by Titus:

The teacher wrote the quadratic formula as shown below:

$$y = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Teacher: *Is the formula correct?*

Pupils: *Pupils while chorusing "No....*

Teacher: *Can one person give the reason why the formula is not correct?*

Pupil: *mmm....because the division sign should be extended to where –b is.*

Question 11(b) of the questionnaire asked the participants what strategies they would use in order to overcome certain misconceptions which students faced when solving quadratic equation. Below is the question and the responses given.

(11) A grade 12 pupil in a certain school solved the quadratic *(2x- 1) (3x – 2) = 3* as Illustrated below:

$(2x - 1)\ (3x - 2) = 3$

$(2x - 1)(3x - 2) = 3$

$2x - 1 = 3$ or $3x - 2 = 3$

$2x = 4$ or $3x = 5$

$x = \frac{4}{2}\ Or\ x = \frac{5}{3}$

$x = 2$ Or $1\frac{2}{3}$

Source: Examinations Council of Zambia (Examiners report, 2011).

How do you assist the student to overcome the misconception? Titus explained that he would explain to such a student by reminding him of the zero product property by emphasizing the fact that $ab = 0$ if and only if $a = 0$ and $b = 0$ that the Quadratic Equation has to be of the form $f(x) = 0$ when using the factorisation method of solving quadratic equation. Talandila explained that he would assist such a student by reminding him of the graphical method of solving quadratic equations and that the solution of the quadratic equation are the value of x at the point $y = 0$ or where the graph cuts the $x - axis$. Faith explained that she would remind the student of the standard quadratic formula of the form $ax^2 + bx + c = 0$ and encourage them that the question *(2x- 1) (3x – 2) = 3* was supposed to be by expressing the equation in standard quadratic form.

5. Discussion

The findings of the study revealed that mathematics teachers' knowledge of subject matter consisted of memorised facts and procedures. This was visible when the participants were teaching solving quadratic equations using the formula method. Although all the participants taught the derivation process of the formula using completing the square method, only Titus emphasised the need for the learners to understand the derivation well. Both Faith and Talandila emphasised the desire for the learners to master the formula. The findings of this study are consistent with the previous studies found that teachers lack conceptual understanding of mathematical ideas and procedures and that their mathematical knowledge was primarily limited to mathematical rules (Ball, 1990 and Even and Tirosh, 1995). Ball (1990a) contended that mathematics teachers were unable to make connections between mathematical ideas, this study found out that teachers were able to make sound connections with other mathematical ideas as all the participants were able to show the relationship which existed between the quadratic formula and the completing the square method. The decision of which examples to use in order to teach a particular lesson revealed that the participants did not pay attention to how the examples would facilitate pupils understanding, rather they looked at their surface features such the number of steps involved in solving the quadratic equation. The findings of my study are generally consistent with the findings of other studies which stressed the fact that teachers lack knowledge of instructional strategies and representations of mathematical representations (Ball, 1990a, and Grossman, 1990).

The study also focused on how teachers were able to plan and teach lessons using a variety of teaching strategies and how they engaged learners through anticipation of learner errors. The study found out that mathematics teachers knowledge of teaching strategies were highly dependent on their subject matter knowledge. The teachers in most of the times explained the concepts on how to carry out procedures or apply rules according to the algorithms on order to solve to solve quadratic equations. The findings of the study are in consistent with other scholars Even and Tirosh (1995) who found out that teachers were reluctant to make an attempt to understand the source of learners responses. Through lesson observations, teachers identified a variety of possible learner difficulties and misconceptions in solving quadratic equations which include computational problems, problems with the square root, problems with the negative sign and the problem with the $\pm$ sign. It was observed that such possible misconceptions committed by the learners were related to learners insufficient knowledge and skills in arithmetic.

The teachers teaching strategies revealed although the teachers were able to identify learner possible difficulties and misconceptions, there were unable to recognise the gap in solving of the quadratic equations as they focused on the procedural steps. Teachers attributed learners' misconceptions to their inability to remember and correctly perform procedures. When the participants were asked to explain how they would assist the student who solved the quadratic equation: $(2x- 1)(3x - 2) = 3$ wrongly, both Faith and Talandila explained the fact that the student did not identify that the question was not a quadratic equation and that they would

assist them by revisiting the factorisation method of solving quadratic equations. The responses were not comprehensive and did not address how this would be done. However, it was only Titus who was able to explain how he would be able to handle it by explaining the concept in relation to zero product property and that the quadratic equation was to be in the form $f(x) = o$.The findings of this study are in consistence with what other scholars Ball, Thames, and Phelps (2008) who noted that Specialised Content of Knowledge (SPK) of teachers should also include Knowledge of Content and Students (KCS) which should include the anticipation of learners difficulties common errors and misconceptions that learners may face in a specific topic.

6. Conclusion

The major aim of teaching is to enhance learners' understanding of the subject matter. It is against this background that teachers need to be equipped with various knowledge and skills including mathematical knowledge for teaching in order for them to establish and maintain that goal.

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