THE UNIVERSITY OF ZAMBIA

2019/20 ACADEMIC YEAR FINAL EXAMINATIONS

DPE 2111: FUNDAMENTAL CONCEPTS OF MATHEMATICS

INFORMATION:

- This paper contains **six (6)** questions.
- Each question carries 20 marks.
- Marks for parts of questions are shown in brackets [ ]

INSTRUCTION: Answer any **five (5)** questions.

TIME ALLOWED: **3 hours** **MARKS POSSIBLE:** **100**

---

1. (a) Explain rational numbers. [04]
   (b) Demonstrate how the following numbers satisfy the definition of a rational number.
   i) $-5\frac{1}{6}$ ii) -1 [06]
   (c) Determine if the following are True or False. If True, prove, if False, give a counter example.
   i. The difference of two positive rationals is a positive rational [05]
   ii. The product of two positive rationals is a positive rational [05]

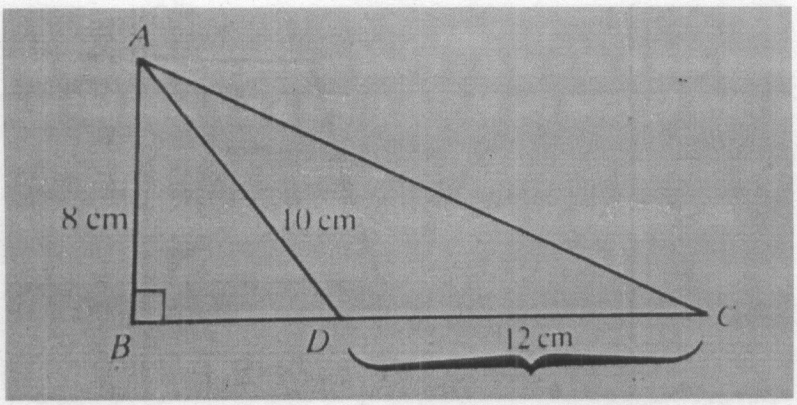
2. (a) Write down **two (2)** real life examples of integers. [04]
   (b) Demonstrate the 'commutative property of addition' over integers. [05]
   (c) Write down a real life situation or story to depict $^{+}8 - {}^{-}3 =$ [05]
   (d) Draw and show on a number line
   (i) $4 + {}^{-}7$ [03]
   (ii) $^{-}3 - {}^{-}6$ [03]

3.(a) (i) A brick is 6.8 cm thick. If 180 bricks were piled on top of each other, how high would the pile be in meters? [03]

(ii) The length of a roll of electrical wire is 2.24 km. The wire is cut into 40 pieces of equal length. What is the length of each piece in metres? [04]

(iii) A rectangular piece of land is 5 m by 4 m. A man intends to keep rabbits on this land and fence it at a cost of K38.50 per metre of fence wire. How much will the fence cost him? [05]

(b) In the figure below, calculate the area of the triangle ABC. [05]



(c) Find the volume of a pyramid with a rectangular base of sides 5cm by 3cm and height 10cm. [03]

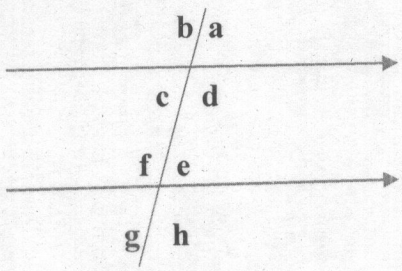
4. (a) Explain briefly what is meant by measures of central tendency. [05]

(b) (i) Prepare a 5 marks question that assesses primary school learners' understanding of the idea of the mean. The question should be based on grouped data. [05]

(ii) Provide a marking key for the question presented in part (b) (i) above. [05]

(c) Describe a strategy which could be used to effectively teach the concept of measures of central tendency to primary school pupils. [05]

5. (a) Calculate the missing angles in the figure below given that angle labelled **e** is equal to 78°, and give a reason for each of your answers. [07]



(b) Explain tessellations in geometry? [04]

(c) The size of an interior angle of a regular polygon is 160°.

(i) What is the size of an exterior angle? [02]

(ii) How many sides does the regular polygon have? [02]

(d) Calculate the number of sides of a regular polygon whose sum of interior angles is 4140°. [05]

6. The grouping of objects according to a common attribute that they have depicts a concept that is fundamental in mathematics.

(a) Identify the mathematics concept. [02]
(b) List **two (2)** groups of five items/objects each and state for each group the common attribute amongst them [08]
(c) Draw a diagram that would clearly show the information in (b) above. [04]
(d) Suppose another group (a third group) is formed which has four members but two of which also appear in each of the existing two groups in (b) above. Explain how this could happen illustrating with examples. [06]

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
2019/2020 ACADEMIC YEAR FINAL EXAMINATIONS

GES 1310: INTRODUCTION TO GEOGRAPHY

TIME: Three Hours

INSTRUCTIONS: Answer Question1 and one question from SECTION A, and one question from SECTION B.

---

1. Write short explanatory notes on ALL of the following:
   a) Hierarchy of settlements (4 Marks)
   b) Settlement patterns (4 Marks)
   c) Usefulness of population pyramids.(4 Marks)
   d) Political Ecology (4 Marks)
   e) Features of igneous intrusions or plutons (4 Marks).

SECTION A

2. Explain the mechanism of spatial diffusion under the following subheadings (20 Marks):
   a) Types of diffusion
   b) Importance of diffusion
   c) Rates of adoption of innovations
3. With the aid of a diagram, describe the various features of the Concentric model of urban growth and explain whether the model is applicable to Lusaka (20 Marks).

SECTION B

4. With the aid of well labelled diagrams, describe the major physiological units of the Earth's surface (20 Marks).
5. Explain what happens during the processes of a river's erosion, transportation and deposition. (20 Marks).

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2019/2020 ACADEMIC YEAR FINAL EXAMINATIONS

GES 2122: ELEMENTS OF CULTURAL GEOGRAPHY

TIME; Three Hours

INSTRUCTIONS: Answer Any Three Questions. All Questions carry equal marks.

Candidates are encouraged to use illustrations.

---

1. Write short explanatory notes on ALL of the following:
   (a) The definition of culture.
   (b) Four functions of culture according to Ali Mazrui (2002).
   (c) The meaning of culture as a Compass according to Ngulube (1997).
   (d) The difference between Invention and Innovation during the Industrial Revolution in England.
   (e) Land tenure and natural resources tenure, according to White (1959) and Birgegard (1993).

2. Using the example given by Wilber Zelinsky with respect to mentifacts and world views within components of culture, construct the following world views and comment on the contemporary direction of these views:
   (a) The American world view.
   (b) The Chinese world view, and
   (c) The African world view.

3. What lessons can Africa learn from the experiences of England and the recently industrializing countries of South East Asia, about the industrialization process?

4. How should the land tenure debate be handled to ensure the empowerment of men, women and the youth, in the sustainable use of land as an investment resource in contemporary Africa?

5. 'Urbanisation refers to a broad-based rural-urban transition involving four main factors'. Discuss.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2019/2020 ACADEMIC YEAR FINAL UNIVERSITY EXAMINATIONS

GES 2130: THE GEOGRAPHY OF AFRICA WITH SPECIAL REFERENCE TO ZAMBIA

TIME: Three Hours

INSTRUCTIONS: Answer question 1, and any other two. All questions carry equal marks. Candidates are advised to make use of illustrations wherever appropriate. Use of the Philips University Atlas is allowed.

---

1. Write short explanatory notes on ALL of the following:
   (a) The concept of Early Man in Africa.

   (b) The impact of rainfall variability on agriculture in Zambia.

   (c) The role of Tourism in economic development in Kenya.

   (d) Any three explanations of Africa's socio-economic crises according to Griffiths and Binns (1988); and their possible solutions.

   (e) Zambia's Vision 2030 and Agenda 2063-The Africa We Want by the African Union Commission.
2. Describe the factors that influence climate on the continent of Africa.

3. With the use of examples explain the benefits of lakes and wetlands to people on the African continent.

4. 'Linguistic diversity in Zambia has been a 'blessing', rather than a 'curse', as the country strives for national unity and socio-economic development in 56 years of political independence'. Discuss.

5. 'Kenya and the Cote D'Ivoire followed similar economic paths in their journeys to achieve socio-economic development after independence'. Explain.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2019/2020 ACADEMIC YEAR FINAL EXAMINATIONS

GES 2210: FUNDAMENTALS OF PHYSICAL GEOGRAPHY

Time: Three hours

Instructions: Answer question one (1) and any other two. Marks for each question are indicated. Candidates are encouraged to make use of illustrations wherever appropriate. The use of an approved calculator is allowed.

---

1. When a parcel of air is rising, it experiences Dry, Wet and Dewpoint Adiabatic Lapse Rates. Use this information to answer the following questions. [40 Marks]
   (a) Show the relationship among these adiabatic rates. Use an appropriate illustration to show the relationship among these adiabatic rates.
   (b) Describe what happens at the lifting condensation level.
   (c) Why is the wet adiabatic lapse rate lower than the dry adiabatic lapse rate?
   (d) Describe the processes that takes place during the formation of a typhoon and a hurricane. What are the similarities and differences?

2. Research has shown that the earth's surface is composed of oceanic and continental plates that float on top of the asthenosphere. [30 Marks]
   (a) Draw a sketch showing the collision between the two types of plates and label the following features that form: mantle, oceanic trench, and rising magma.
   (b) Describe the rock types and physical properties of oceanic plates, continental plates and asthenosphere.

3. Biogeochemical cycles are the natural pathways by which essential elements of living matter are circulated. [30 Marks]
   a) Use nitrogen as an example to illustrate the essential features of a biogeochemical cycle.
   b) What is the role of micro organisms in the nitrogen cycle?
   c) How have human beings modified the nitrogen cycle?

4. Three major types of rocks can be identified as: igneous, sedimentary and metamorphic [30 Marks]
   (a) Sketch the cycle of changes in rock and describe the processes that act within it to form igneous, sedimentary and metamorphic rocks.
   (b) What is the difference between chemical and physical weathering of rocks? Use appropriate examples for each.

5. A student investigating a soil profile identified master soil horizons in the order: O, E, A, B, C and R respectively. Answer the following questions related to this soil profile:

[30 Marks]

(a) Is this order correct or not? Explain the basis of your answer.
(b) Define these master horizons and their properties.
(c) Relate the formation of these master horizons to processes of soil formation.
(d) Draw an ideal soil profile for a Spodosol.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2019/2020 ACADEMIC YEAR FINAL EXAMINATIONS

GES 2322: FUNDAMENTALS OF NATURAL RESOURCES ECONOMICS

TIME: Three (3) Hours

INSTRUCTIONS: Answer Question 1 and any other two questions. All questions carry equal marks. Candidates are encouraged to use illustrations wherever appropriate. Use of an approved calculator is allowed. Graph paper is provided.

---

1. Write short explanatory notes on **all** of the following:
a) Inflation and its effects
b) Types of goods
c) Demand
d) Gross Domestic Product
e) Barriers to entry into monopolistic markets.

2. Study Table 1 and answer the questions that follow:

Table 1. Quantity supplied vs price

Periods	Quantity Supplied	Price (ZMW)
A	200	20
B	150	15
C	100	10
D	70	6
E	40	3
F	30	2
G	10	1

i) On the graph paper provided, plot quantity supplied against price.
ii) Why is the supply curve downward sloping?
iii) On the same graph in (i), show the difference between a *shift in supply* and *a change in quantity supplieddue to price changes*.
iv) Briefly explain how any two factors affect the supply for a normal good.

3. Table 2 shows the quantities of a normal good that are bought at different prices.

Table 2. Quantity demanded vs price

Periods	Quantity demanded	Price (ZMW)
A	500	25
B	250	50
C	125	40
D	100	20

What is the price elasticity of demand when;

i. There is a change from A to B
ii. There is a change from B to C
iii. There is a change from C to D
iv. Describe the elasticity in i, ii and iii.
v. Outline and explain three factors that influence the elasticity of demand of a good.
vi. Give two examples of situations in which information on price elasticity of demand can be utilized by the government.

4. 'A country's natural resources can enhanceitseconomic development'. Discuss

5. Study Table 4 showing the labour demand by a firm that has employed six workers, as well as the marginal productivity for each worker, and then answer the questions that follow.

Table 4. Labour demand by Firm X

Worker	MPP	Price of Product (ZMW)	MRP	MRC
1	200	15		1660
2	167	15		1660
3	143	15		1660
4	111	15		1660
5	82	15		1660
6	61	15		1660

a) Copy Table 4 into your answer booklet and complete it by filling in the Marginal Revenue Product (MRP) column.

b) Does the addition of each extra worker increase or decrease the firm's marginal revenue product? Explain the reasons for the observed trend.

c) The firm decides to hire you to help them estimate the optimum number of workers they need to have in order to remain profitable. How many workers would you recommend the firm have? Explain your answer.

d) What assumptions would you take into consideration when estimating marginal productivity?

e) What factors could determine the demand for the resource produced by firm X?

END OF EXAMINATION

CANDIDATE'S COMPUTER NUMBER: .....

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2019/20 ACADEMIC YEAR FINAL EXAMINATIONS
GES 2411: MAPPING AND FIELD TECHNIQUES IN GEOGRAPHY

PAPER 1I: PRACTICAL

TIME: Three Hours

INSTRUCTIONS: Answer **ALL** questions. The use of an approved calculator is allowed and candidates are encouraged to make use of illustrations wherever appropriate.

MATERIALS PROVIDED: Topographic Map Sheet 1228 C1

.. FOR USE BY EXAMINER

Question	Examiner's Mark	Moderator's Mark
1		
2		
3		
4		
Total		

IMPORTANT

Please read the instructions before attempting any question on this examination paper. Failure to follow instructions will lead to automatic loss of marks.

SECTION A: MAP READING

Answer all questions in this section in the spaces provided on this question paper

1. Use Topographic Map sheet 1228 C1 to answer all parts of this question

(a) Using map evidence only, what non- landscape feature is associated with the Kafue and Musakashi rivers on Map Sheet 1228 C1? [2 marks]

.....

.....

.....

(b) In which district is the settlement of Chambishi located? [2 marks]

.....

.....

.....

(c) If you wanted to have a full view of Chisenga National Forest No. 9, what other map sheets would you require? [2 marks]

.....

.....

.....

(d) Apart from the slimes deposit, what other two pieces of evidence are there on map sheet 1228 C1 to show that there are mining activities in Chambishi? [2 marks]

.....

.....

.....

(e) How far is Chingola by road from the edge of the map in grid square 0805? [2 marks]

.....

.....

.....

(f) Calculate the Grid Bearing of the trigonometrical station in Grid Square 2805 from Grid Reference point 270070. [2 marks]

.....

.....

(g) When was Map Sheet 1228 C1 first published and by whom? [2 marks]

.....

.....

(h) Map Sheet 1228 C1 uses a vertical interval of twenty metres. What does it mean? [2 marks]

.....

.....

.....

.....

(i) In which direction does the Musakashi River generally flow? [2 marks]

.....

.....

(j) What is the latitude and longitude of the north eastern corner of Map Sheet 1228 C1? [2 marks]

.....

.....

.....

(k) Identify and name at least one symbol that is incorrectly used on Map Sheet 1228 C1 and suggest what is it supposed to be? [2 marks]

.....

.....

.....

.....

(l) According to map evidence only, what source of information did the cartographers use in the drawing of the first edition of Map Sheet 1228 C1? [2 marks]

.....

.....

.....

(m) Draw a conventional symbol of a narrow water course with a dam and waterfall as shown on Map Sheet 1228 C1. [2 marks]

(n) If the Kafue River is taken to be a consequent river, what terms would best describe the Itimpe and Chambeshi rivers respectively in the idealised development of a drainage basin model? [2 marks]

.....

.....

.....

.....

(o) According to map evidence only, why is there no cultivation in Grid Square 1294? [2 marks]

.....

.....

.....

.....

.....

(p) Using the squares method, calculate the size of Kansuswa built-up area in square kilometres. [2 marks]

.....

.....

.....

.....

(q) What drainage pattern does the Lukashi River system generally exhibit? [2 marks]

.....

.....

.....

(r) Assuming that a straight line profile is drawn from the road junction in grid square 1600 to the road junction in grid square 2194, would the two road junctions be intervisible? [2 marks]

.....

.....

(s) Determine the Six Figure Grid Reference of the junction of the Kitwe and Mufulira railway lines. [2 marks]

.....

.....

(t) What is the dominant relief feature in Nsato National Forest No. 17? [2 marks]

.....

.....

2. This question does not require the use of topographic Map Sheet1228 C1

(a) Express 1:25,000 as a scale in words. [3 marks]
(Show all the steps of your work)

(b) Express one kilometre to a centimetre as a scale in figures. [4 marks]
(Show all the steps of your work)

(c) Convert the scale 1: 500,000 to a scale in words. [4 marks]
(Show all the steps of your work)

(d) Using a scale of 1:40,000, draw a line scale in metric units given that the maximum space available is 18 centimetres. [5 marks]
(Show all the steps of your work)

(e) Using a scale of 1:10,000 calculate the dimensions of a regular polygon measuring 15kilomtres by 5kilometres (15km x 5km). [4 marks]
(Show all the steps of your work)

3. Assuming that an aerial survey was conducted in the vicinity of Lake Kariba in Siavonga Ditrict, the aeroplane used was flying at a mean height of 7,625 metres above the terrain and the camera used has a focal length of 152.5 millimetres.

(a) What is the mean scale of the air photograph? [4 marks]
(Show all the steps of your work)

(b) What would be the scale at the Zambezi valley bottom located at 152.5 metres below the mean altitude of the terrain? [4 marks]
(Show all the steps of your work)

(c) What would be the scale at the Zambezi escarpment summit located at approximately 152.5 metres above the mean altitude of the terrain? [4 marks]
(Show all the steps of your work)

(d) Assuming that the distance between the two opposite shores of Lake Kariba is 625metres wide and the same camera as in part (a) is used, how long will the distance between the two opposite shores be in the air photograph? [4 marks]
(Show all the steps of your work)

(e) Assume that the fish processing plant building is 0.6 millimetres in the air photograph and the flying height is changed to 2,287.5metres above the ground and the same camera as in part (a) is used, what would be the length of the building on the ground? [5 marks]
(Show all the steps of your work)

4. Answer both questions in the spaces provided.

(a) Using the contour method draw an annotated diagram to show an escarpment with a river flowing on the scarp slope bearing in mind that rivers flow in the direction of the steepest slope and never uphill. [10 marks]

(b) Draw a well labelled diagram to show a theoretical development of a drainage basin with the following: consequent, insequent, subsequent, obsequent and resequent streams. [10 marks]

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2019/2020 ACADEMIC YEAR FINAL YEAR EXAMINATIONS

GES 3151: REGIONAL PLANNING AND DEVELOPMENT

TIME: Three Hours

INSTRUCTIONS: Answer questions 1 and any other TWO. All questions carry equal marks. You are encouraged to use illustrations wherever appropriate.

---

1. Write short explanatory notes on ALL of the following:
   a) Leakage as a transaction in a regional economy
   b) Brandt Map
   c) Occupational structure as a measure of development status of a region
   d) Horizontal decentralisation
   e) Range in Central Place Theory.

2. Explain how factor endowments can be a source of global competitive advantage.

3. Critically examine Neo-liberalism as a development perspective for third world countries.

4. Analyse the assertion that 'the most successful regions have the most endogenous forms of production.'

5. Explain Goldman's (1998) permutations of decentralisation.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCE

2019/2020 ACADEMIC YEAR FINAL EXAMINATIONS

GES 3241: CLIMATOLOGY

TIME: Three Hours

INSTRUCTIONS: Answer Questions **One** and any other **two**. Use of an approved calculator is allowed.

---

1. Write short explanatory notes on ALL of the following:

   a) Occluded front (4 Marks)
   b) Single-cell model of the general circulation (4 Marks)
   c) Frontolysis (4 Marks).
   d) Controls of weather and climate (4 Marks)
   e) Physical laws governing radiation energy transfer. (4 Marks)

2. With the aid of specific examples, explain the positive and negative feedbacks occurring in the earth's climate system. (20 Marks)
3. Study Figure 1 and answer the questions that follow:

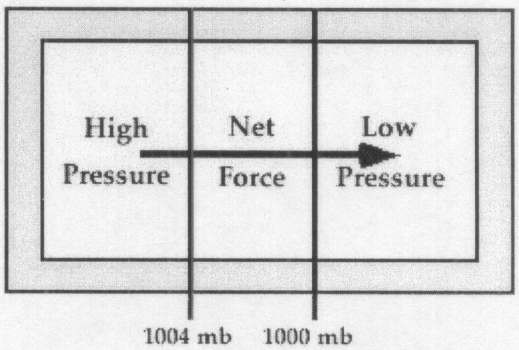


Figure 1. Atmospheric force affecting wind circulation

a) Identify and explain how the atmospheric force presented in Figure 1 affect atmospheric circulation. (4 Marks)

b) Explain how Newton's second law of motion, the law of mass continuity as well as the force in Figure 1 combine to influence local climates in land and sea breezes. (4 Marks)

c) The force in Figure 1 is a real force, however, atmospheric circulation is also influenced by apparent forces. Describe any two of such apparent forces affecting atmospheric circulation. (6 Marks)

d) What is the role of the force in Figure 1 in the Geostrophic and Ekman balances? (6 Marks)

4. Explain the factors affecting radiation transfer from the sun to the earth. (20 Marks).
5. Using specific examples, explain the criteria for naming air masses. (20 Marks)

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2019/2020 ACADEMIC YEAR FINAL EXAMINATIONS

GES 3271: SOILS AND SOIL RESOURCES

TIME: Three hours

INSTRUCTIONS: Answer any three questions. All question carry equal marks. Candidates are encouraged to make use of illustrations wherever appropriate.

---

1. Explain with appropriate examples how temperature affects the physical, biological and chemical processes occurring in a soil.

2. Describe the importance of cohesion, adhesion and capillarity properties of water in soil.

3. Explain the causes for the kind of base saturation you expect to find in the following regions:
   (a) The high rainfall region of Zambia
   (b) The low rainfall region of Zambia.

4. Using appropriate examples, describe the importance of the following living organisms in a soil
   (a) Heterotrophic bacteria
   (b) Fungi.

5. Describe measurable and observable parameters of a soil that are important for it to be classified as an Ustalf.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2019/2020 ACADEMIC YEAR FINAL EXAMINATION

GES 3342: ENVIRONMENTAL MANAGEMENT AND PLANNING

TIME: Three Hours

INSTRUCTIONS: Answer any three questions. All questions carry equal marks. Candidates are encouraged to use illustrations wherever appropriate.

---

1. Explain how humans can cause the total collapse of environmental life support systems through environmental degradation and excessive consumption of resources.

2. The Life Cycle Assessment (LCA) is expressed in terms of potential environmental impacts. Discuss.

3. Write short explanatory notes on ALL of the following:

   (a) The 'commons problem'
   (b) Natural resources
   (c) Collective environmental failures
   (d) Excludability and rivalry in consumption
   (e) Neo-Malthusian views on environmental management.

4. Using Zambia's forestry sector as an example, explain the opportunities and challenges of decentralized natural resources management.

5. Discuss the main issues around common property and open access regimes in the fisheries sector.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SHOOL OF NATURAL SCIENCES

GES 4172: RURAL LAND USE AND LANDSCAPES

2019/2020 ACADEMIC YEAR FINAL EXAMINATIONS

TIME: **THREE HOURS**

INSTRUCTIONS: **ANSWER ANY THREE QUESTIONS.**

---

1. Write short explanatory notes on ALL of the following:
   (a) Rural landscape.
   (b) Sources of energy in rural Zambia.
   (c) Sources of water in rural Africa.
   (d) Types of rural transport in Africa.
   (e) Climate change impacts on food security in Africa.

2. What roles do the following entities play in driving rural development in Africa?
   (a) The government,
   (b) The private sector,
   (c) The Non-governmental organisations; various development agencies and co-operating partners,
   (d) The traditional leaders,
   (e) The individual.

3. Explain the role of infrastructure construction in the integration of space economies as well as in the socio-economic development of Africa.

4. Discuss the role and significance of gender in natural resources management in rural Zambia.

5. 'Water insecurity is an increasing challenge for rural landscapes'. Suggest options for addressing the challenge in rural Zambia.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCE

2019/2020 ACADEMIC YEAR FINAL EXAMINATIONS

GES 4342: ENVIRONMENT AND NATURAL RESOURCES MANAGEMENT

TIME: Three Hours

INSTRUCTIONS: Answer Questions **1** and any other **two**. Candidates are encouraged to use illustrations wherever appropriate.

---

1. Write short explanatory notes on ALL of the following:
   a) Negative and positive externalities (4 Marks)
   b) Benefits and weaknesses between in-situ and ex-situ wildlife conservation strategies (4 Marks)
   c) Wetland (4 Marks)
   d) Principles of Integrated Water Resources Management (IWRM) paradigm (4 Marks)
   e) Integrated Solid Waste Management (4 Marks).
2. Discuss the constraints of managing an effective waste management system in Zambia (20 Marks).
3. Study Figure 1 and answer the questions that follow (20 Marks).

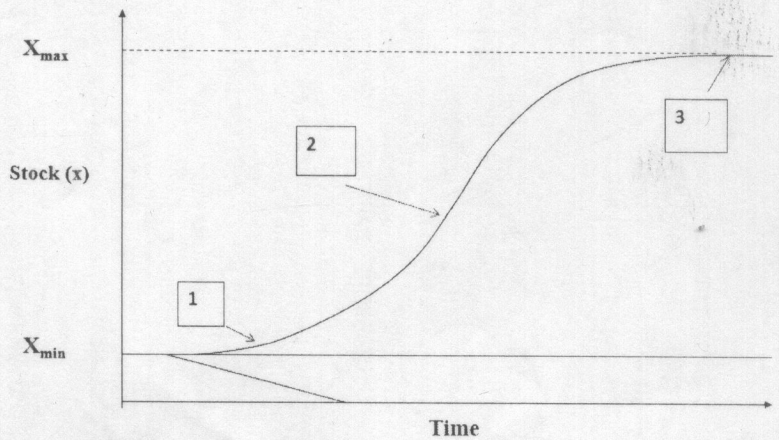


Figure 1.Average relationship between growth of fish population and the size of the fish population

a) Identify and explain the significance of the stage in fisheries growth development marked 2. (3 Marks)

b) What changes are observable at stage 3 of the fisheries development, which are not evident in stages 1 and 2? (4 Marks)

c) Briefly explain how fisheries as a resource can be managed sustainably. (5 Marks)

d) State and outline any four constraints to aquaculture development. (8 Marks)

4. Use Zambia as an example to explain the main physical and hydrological functions and values of wetland ecosystems? (20 Marks)

5. With reference to Zambia's water sector, discuss the major components of water resources management. (20 Marks)

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2019/2020 ACADEMIC YEAR FINAL EXAMINATIONS

GES 4372: TOURISM, ENVIRONMENT AND DEVELOPMENT

TIME: Three Hours

INSTRUCTIONS: Answer Question 1 and any other two questions. All questions carry equal marks. Candidates are encouraged to make use of illustrations wherever appropriate.

---

1. You have just been employed by the Ministry of Tourism and Arts as a planner. Explain the policy adjustments you would recommend in order to improve the tourism sector in Zambia.
2. Write short explanatory notes on **ALL** of the following:
   (a) Myths of tourism
   (b) Cohens typology of tourists
   (c) The importance of accurate statistical measurement of tourism
   (d) Allocentrics and psycho-centrics
   (e) Butler's Model of tourism development.
3. Discuss the contention that the definition of tourism is problematic.
4. Using relevant examples, discuss the impacts of tourism on the environment.
5. Explain the meaning of sustainability and show its relevance to the tourism industry.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2019/2020 ACADEMIC YEAR FINAL EXAMINATIONS

GES 4385: ENVIRONMENTAL HAZARDS AND DISASTERS

TIME: Three Hours

INSTRUCTIONS: Answer Questions **One** and **any other two**. Use of an approved calculator is allowed. Candidates are encouraged to make illustrations wherever possible.

---

1. Explain how each of the following can increase disaster risk:

   a) Urban development (4 Marks)
   b) Poverty and inequality (4 Marks)
   c) Poor governance (4 Marks)
   d) Environmental degradation (4 Marks)
   e) Social norms.(4 Marks)

2. With reference to the Disaster Management Cycle, explain how you would manage flood hazards in informal settlements of Lusaka city. (20 Marks).

3. Discuss any four common types of natural hazards. (20 Marks)

4. The level of economic development influences the vulnerability of a population to environmental hazard risks. Discuss. (20 Marks)

5. Discuss any five factors that could render a community vulnerable to disasters. (20 Marks)

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2019/2020 ACADEMIC YEAR FINAL EXAMINATIONS

GES 4451: CARTOGRAPHY

TIME: **THREE (3) hours**

INSTRUCTIONS: Answer any THREE (3) Questions. All Questions carry equal marks. Candidates are advised to make use of illustrations and examples wherever appropriate.

---

1. Write short explanatory notes on ALL of the following

   a) Figure ground

   b) Letter form

   c) Cartographic conventions

   d) Geospatial data

   e) Map elements

2. Discuss how the level of complexity works or does not work for maps.

3. Discuss the assertion that accuracy is sacrificed for the benefit of generalisation in map making.

4. Outline the general guidelines to consider in map making.

5. 'Maps are neither pictures nor are they diagrams'. Discuss.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2019/2020 ACADEMIC YEAR FINAL EXAMINATIONS

GES 4461: LAND RESOURCES SURVEY

TIME: Three hours

INSTRUCTIONS: Answer any three questions. All questions carry equal marks. Candidates are encouraged to make use of illustrations wherever appropriate.

---

1. Write short explanatory notes on ALL of the following:
   (a) The land use system of Zambia
   (b) Detailed and semi detailed surveys
   (c) Attributes of land utilization types
   (d) Productive Available Moisture of a soil
   (e) Length of the growing period.

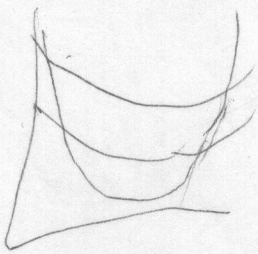
2. Explain the different aspects of precipitation and their importance for agriculture.

3. Outline the procedure for carrying out a Gross Margin Analysis.

4. Using an annotated diagram, describe how you can determine the growing period for a specific region.

5. Outline the principles of land evaluation and explain in detail the principle that aims to optimise the ecological benefits of land uses.

END OF EXAMINATION



The University of Zambia

End of Year Examinations: November/December 2020

ICT 1110: Computer Systems and Architecture

Marks: 100
Time: Three (3) hours
Instructions:

- This examination paper consists of a total of five (5) questions and seven (7) pages.
- Answer all questions from **Section A** and any two (2) questions from **Section B**.
- Show all calculations were applicable.
- You are allowed to use a certified scientific calculator if you wish to do so.
- Page seven (7) has reference tables for MIPS instructions and system calls.

Section A [Questions 1 is compulsory]

Question 1 [50]

Consider a hypothetical scenario in which the MIPS assembler code fragment below is a partial representation of a program—"ICT1110App"—presently loaded into memory

```
addi $10, $0, 2021 # Next elections in the Republic of Zambia
addi $11, $0, 1991 # Multi-party democracy declared in the Republic of Zambia
sub $12, $10, $11
j theJumpAddress
```

a) A typical program execution cycle involves specific program fragments being held in various memory components, with the so-called "Stored Program Concept" being instrumental in determining how contemporary computer systems work
   i) Briefly outline the fundamental idea behind the so-called Stored Program Concept [2]
   ii) Computer memory can be either volatile or non-volatile. Distinguish between volatile and non-volatile memory and provide an example for each type [4]

iii) There are a number of specific types of memory components that are used by computer systems. List six (6) types of memory components typically used in microcomputer and for each, state its role during program execution **[12]**

b) The execution of the program "ICT1110App" is performed in a systematic manner

i) Briefly outline how the program "ICT1110App" is executed, from the point when a user starts the program, by making reference to the high-level functional units of the Von Neumann model and the role of each of the functional units **[5]**

ii) The individual instructions associated with the program "ICT1110App" are executed sequentially, using the so-called instruction execution cycle. Briefly outline how the instruction "addi $10, $0, 2021" is executed, by making reference to the core phases associated with the instruction execution cycle. **[5]**

c) System calls are a crucial part of MIPS assembler programs

i) Briefly outline the role of system calls **[2]**

ii) Provide complete code fragment that will facilitate the graceful exit of the program "ICT1110App" **[2]**

d) The specifics of how program instructions are interpreted is dependent on the Instruction Set Architecture implemented by the computer system. Answer the following questions, assuming the program "ICT1110App" is installed on a computer system based on the MIPS instruction set architecture.

i) Assuming the label "theJumpAddress" is represented by the value 0x3402000A, what is the binary representation of the label "theJumpAddress", represented by the correct binary representation? **[4]**

ii) Assuming the PC value, prior to execution of the "j theJumpAddress" instruction is executed is "0x00400008", what would be the value of the resulting address to jump to once the instruction "j theJumpAddress" is executed? **[4]**

e) Prior to execution of the instructions associated with the program "ICT1110App", the instructions are translated into a form that the computer system is able to understand—binary code

i) Provide four (4) reasons why the program instructions for program "ICT1110App" cannot be written using machine language **[4]**

ii) State the category of software used to convert the "ICT1110App" program instructions into machine language **[1]**

iii) Decode the MIPS instruction "addi $11, $0, 1991" into its equivalent machine language representation **[5]**

Section B [Answer any two questions ONLY]

Question 2 [25]

Abstraction is crucial in understanding how electronic digital computers work. A typical example of how abstraction is used in computing is in the way programming languages are used to represent instructions to be executed by electronic digital computers. Specifically, programming languages are classified as being low-level or high-level.

a) Briefly outline two (2) advantages of abstraction when applied to computing [2]

b) Briefly outline two (2) disadvantages of abstraction when applied to computing [2]

c) Briefly describe the significance of assembly language programs in relation to machine code [1]

d) MIPS assembly language programs are typically composed of core programming elements. State the role of each of the following MIPS assembly language elements and, additionally, provide an example:

   i) Comments [1]

   ii) Directives [1]

   iii) Labels [1]

   iv) Variables [1]

   v) Pseudoinstructions [1]

e) Write a MIPS assembly program that prompts a user to enter an integer value and then prints out 1 if the input value is divisible by 2, otherwise, it prints 0. The program must include all of the following aspects:

   - Use of appropriate comments [1]
   - Prompting for integer input [2]
   - Printing integer output [2]
   - Graceful exiting of program [2]
   - Logical implementation of program [6]
   - Correctness of program, including the use of appropriate directives [2]

Question 3 [25]

Consider the following the MIPS datapath and control representation in Figure 1 and answer the questions below:

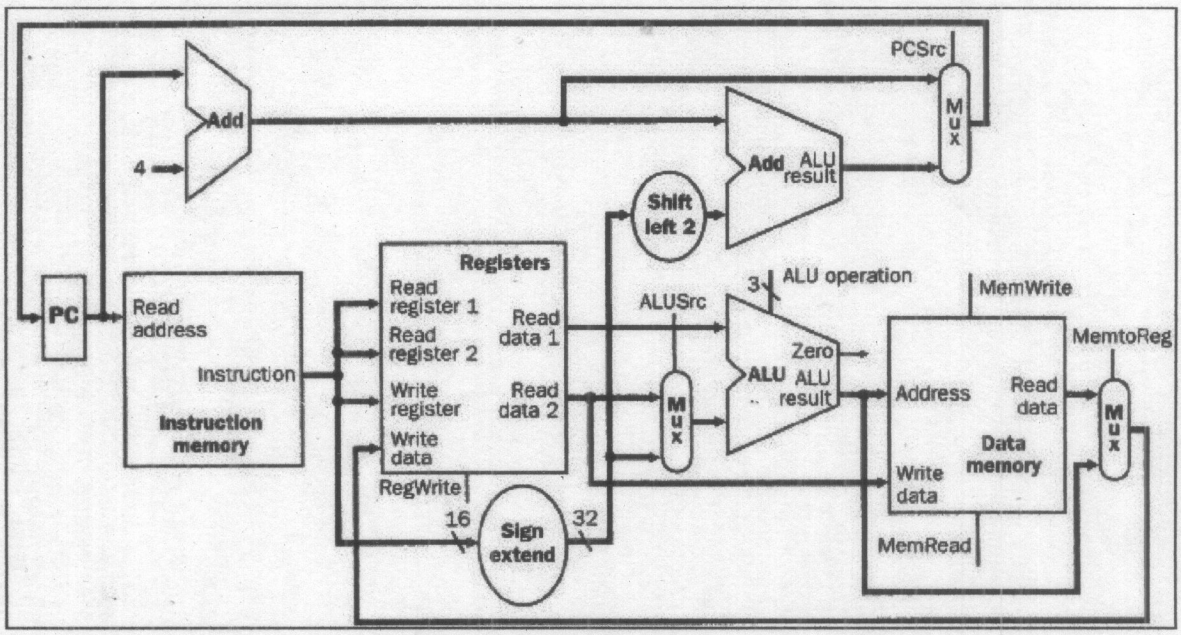


Figure 1: High-Level Representation of the MIPS Datapath and Control

a) The MIPS Instruction Set Architecture (ISA) is composed of three (3) broad types of instructions

   i) List the three (3) types of MIPS instructions and briefly state what they are used for **[3]**

   ii) For each of the three (3) types of MIPS instructions, state the MIPS Datapath and Control elements that are involved during the execution of the instructions **[12]**

b) The multiplexer (MUX) linked to the ALU is in effect a so-called 2-to-1 MUX, with two inputs originating from "Read data 2"—A—and "Sign extend"—B—, a selector input "ALUSrc"—S—and the output "Z".

   i) Derive the algebraic expression for the 2-to-1 MUX **[4]**

   ii) Draw a Truth Table representing the 2-to-1 MUX **[6]**

Question 4 [25]

Data encoding in electronic computer systems is contextual and is typically performed using computer software—program instructions .

a) State two (2) ways that can be used by computer system end users (human beings) to determine the appropriate software to be used to open a given file **[2]**

b) Images are perhaps one of the most widely used types of data on social media platforms. Consider an image file with the following characteristics and answer the questions that follow

- File size: 47306.25 KB
- Colour channel bit depth: 8 bits per channel
- Colour mode: RGB
- Aspect ratio: 16:9

i) Calculate the resolution of the image **[8]**

ii) How long would it take, in minutes, to download the file using a download link speed of 20kbs **[4]**

iii) By default, most modern Smartphones will generate image files that are significantly smaller in size than the theoretical image file size. Explain how this is possible and provide two (2) reasons why this is the default configuration **[4]**

iv) Describe how images are used to derive video footage **[2]**

c) Data compression techniques are classified into so-called lossy and lossless compression techniques

i) Distinguish between lossy and lossless compression **[2]**

ii) Provide a specific scenario when applying lossless compression would be appropriate, in preference to lossy compression **[1]**

iii) State one (1) disadvantage of lossless compression techniques **[1]**

iv) State one (1) disadvantage of lossy compression techniques **[1]**

Question 5 [25]

A group of five third-year students enrolled in the ICT 3020 course are proposing to implement an application called "MAR: Makeup-Assessment Requisition Plugin" as part of the Group Project practical assessment.

MAR will be implemented as a Moodle plugin and is meant to enable students enrolled into the B.ICTs programme to effectively request for make-up assessments, as opposed to the traditional method of sending emails to instructors/lecturers to make such requests. In questions that follow, assume that MAR has been successfully implemented and works as expected.

a) Distinguish between computer software and computer hardware **[1]**

b) Briefly outline the role of computer software **[2]**

c) With the exception of MAR, list at least four (4) software applications that will be needed for the application to be successfully used by students. In addition, state the role of each of the four (4) application software. **[4]**

d) State the names of at least two types of software, including the category of software they fall under, that a typical student would need to have installed on their computer system for them to be able to access the MAR plugin. **[2]**

e) List three (3) advantages of implementing MAR as a plugin. **[3]**

f) List three (3) disadvantage of implementing MAR as a plugin **[3]**

g) There is a broad spectrum of computer systems that would be involved as a typical student interacts with the MAR plugin.

   i) Describe four (4) characteristics that are used to evaluate the relative effectiveness of computer systems **[4]**

   ii) List three (3) types of computer systems involved as a student interacts with the MAR plugin, and for each type, state its role **[6]**

Table 3: MIPS System Calls

Code	Service	Argument	Result
1	Print Integer	$a0=Integer Value	–
4	Print String	$a0=String Address	–
5	Read Integer	–	$v0=Integer Value
8	Read String	$a0=String Address $a1=length	–

Table 4: MIPS Instructions

Instruction	Description	Opcode/FUNCT
add <Rdes>, <Rsrc1>, <Rsrc2>	Addition	$0/20_{HEX}$
addi <Rdes>, <Rsrc1>, <Rsrc2>	Addition	8_{HEX}
sub <Rdes>, <Rsrc1>, <Rsrc2>	Subtraction	$0/22_{HEX}$
j <JumpAddress>	Jump	2_{HEX}
b <label>	Unconditional branching	–
beq <Rsrc1>, <Rsrc2>, <label>	Branch if equal	4_{HEX}
bne <Rsrc1>, <Rsrc2>, <label>	Branch if not equal	5_{HEX}
blt <Rsrc1>, <Rsrc2>, <label>	Branch if less than	–
bgt <Rsrc1>, <Rsrc2>, <label>	Branch if greater than	–

—End of Examination—

The University of Zambia
School of Natural Sciences
Department of Mathematics & Statistics

2019/20 Academic Year Final Examinations

MAT1100 – Foundation Mathematics

17th December, 2020

INSTRUCTIONS: (1) Write down your **Computer number** and the **TG number** on each answer booklet used.

(2) There are seven questions in this examination paper. **Answer any Five (5).**

(3) Show all essential working to score full marks.

(4) Write down the **number** of each question attempted on the cover of the main answer booklet.

(5) Calculators are **not allowed** in this Examination.

TOTAL TIME ALLOWED: Three (3) hours.

Q1. (a) (i) Simplify the set $[(B - A) \cup A']'$. (3)

(ii) Given the subsets $A = [-2,6)$, $B = (-5,3)$, $C = [-1,8]$ of the set of real numbers $\mathbb{R}$, find the set $A \cup (B' \cup C)'$. (4)

(b) (i) Express $(1 + \sqrt{2})^2$ in the form $a + b\sqrt{2}$, where a and b are integers. (3)

(ii) Express $\frac{\sqrt{2}}{(1+\sqrt{2})^2}$ in the form $p + q\sqrt{2}$, where p and q are rational numbers. (4)

(c) (i) Express $2.5\overline{34}$ in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$, in its lowest terms. (3)

(ii) Simplify $5\sqrt{6} - 2\sqrt{24} + \sqrt{294}$. (3)

Q2. (a) Let $*$ be a binary operation on the set of real numbers defined by

$$a * b = ab - (a - b)^2.$$

(i) Determine whether the binary operation $*$ is commutative or not. **(3)**

(ii) Using the definition of $*$ above evaluate $3 * (-1 * 2)$ **(2)**

(b)(i) If $f(x) = 2x - 3$ and $g(x) = \frac{x+1}{2-x}$, find $(fog)(x)$ and state its domain. **(3)**

(ii) Find the turning points of the curve $y = x^3 + x^2 - 4x - 4$, and identify the relative maxima and relative minima **(6)**

(c) Determine whether the function

(i) $f(x) = \frac{x^2-4}{x^3}, x \neq 0$ is even or odd or neither even nor odd. **(3)**

(ii) $f(x) = \frac{3x+2}{x-1}, x \neq 1$ is one – to – one or not. **(3)**

Q3. (a) Given that $x^2 + 4x + r = (x + p)^2 + q$ where p, q and r are constants.

(i) Find the value of p. **(3)**

(ii) Find q in terms of r. **(2)**

(iii) Given that the equation $x^2 + 4x + r = 0$ has distinct real roots, find the range of possible values of r. **(3)**

(b) (i) The polynomial $p(x) = 3x^3 - 2x^2 + px - q$ is divisible by $(x - 1)$ and leaves a remainder of 10 when divided by $(x + 1)$. Find the values of p and q. **(4)**

(ii) Solve the simultaneous equations:

$$y + 2x = 5;$$
$$2x^2 - 3x - y = 16.$$ **(4)**

(c) Solve the equation

$$\sqrt{2x + 1} - \sqrt{x} = 1.$$ **(4)**

Q4. (a) (i) Find the term independent of x in the Binomial expansion of $\left(x^2 - \frac{1}{x^2}\right)^{12}$. **(3)**

(ii) Express $\frac{1}{\sqrt{1-2x}}$ as a series up to and including the term in x^3 and state the range of values of x for which the expansion is valid. **(4)**

(iii) Using part (ii), find the approximate value of $\frac{1}{\sqrt{0.98}}$, giving your answer correct to 7 decimal places. **(3)**

(b) Solve each of the following inequalities:

(i) $\frac{2x-3}{x-2} \geq 2$ **(3)**

(ii) $|k| > |2k - 3|$ **(4)**

(c) Express $z = \frac{1-i}{2i-3}$, in the form $a + ib$, where a and b are real numbers. **(3)**

Q5. (a) (i) Prove the identity:

$$\frac{\cos^2\theta}{\sin\theta + \sin^2\theta} = \frac{1-\sin\theta}{\sin\theta}.$$ **(3)**

(ii) Find the period, amplitude, phase shift and vertical shift for the function

$$f(\theta) = -2 + 3\sin\left(\frac{1}{2}\theta - \pi\right).$$ **(4)**

(b) Solve each of the following equations:

(i) $2\cos^2 x - \sqrt{3}\sin x - 2 = 0$, for $0 \leq x < 360^o$. **(4)**

(ii) $4^x - 6(2^x) - 16 = 0$. **(4)**

(c) (i) Show that $\log_4 x = \frac{1}{2}\log_2 x$. **(1)**

(ii) solve the equation

$$\log_2(2x + 3) - 2\log_4 x = \log_2(2x - 1).$$ **(4)**

Q6. (a) Evaluate each of the following limits:

(i) $\lim_{x\to 2} \frac{x^3-8}{x^2-4}$. (3)

(ii) $\lim_{x\to \infty} \frac{5-x^2-3x^3}{3x^3+5}$. (3)

(b) (i) Given that $y = \frac{(3x^2+2)^3}{e^{2x}}$, find $\frac{dy}{dx}$ at $x = 0$. (3)

(ii) The curve C has equation $y = 6x^3 - 7x^2 + 4$. Find the equation of its normal at the point $(1,3)$. (3)

(c) The curved surface area of a closed cylinder of height $h\ cm$ and radius $r\ cm$ is $2\pi rh\ cm$, and its volume $\pi r^2 h$ is $128\pi\ cm^3$.

(i) Show that the surface area of the cylinder is given by $S = \frac{256\pi}{r} + 2\pi r^2$. (3)

(ii) Find the value of r for which the surface area of the cylinder is minimum. (2)

(iii) For the value of r found in (ii) show that the surface area of the cylinder is minimum. (3)

Q7. (a) Evaluate each of the following integrals:

(i) $\int \left(8x^3 + \frac{3}{\sqrt{x}} + 5\right) dx$ (3)

(ii) $\int \frac{x}{\sqrt{x^2-9}} dx$ (3)

(iii) $\int xe^{2x}\, dx$. (3)

(b) (i) Resolve $\frac{4x+5}{(x+2)(x-1)}$ into partial fractions. (3)

(ii) Find the integral $\int_2^3 \frac{4x+5}{(x+2)(x-1)} dx$. (3)

(c) (i) Sketch the graph of $f(x) = x^2 - 4x - 25$ and the graph of $g(x) = 16x - 4x^2$ on the same coordinate axes. (2)

(ii) Find the area of the region enclosed by the lines $x = 0, x = 4$ and the curves $y = f(x)$ and $y = g(x)$ defined in part (c)(i) above. (3)

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS & STATISTICS

2019/2020 Academic Year
End of Year Final Examinations

30 December 2020 (am)

MAT2001 – Principles of Financial Mathematics

Time allowed : **Three (3) hours** Full marks : **100**

Instructions:
- There are **seven (7)** questions in this paper. Attempt **five (5)** questions.
- **All** questions carry **equal** marks. Mark allocations are shown in brackets.
- **Full credit** will only be given when **necessary work** is shown.
- Indicate your **computer number** on all answer booklets used.

---

1. (a) The rate of interest is 6.2% per annum effective. Calculate

   (i) the annual effective rate of discount, **(1)**

   (ii) the nominal rate of interest per annum convertible quarterly, **(1)**

   (iii) the effective rate of interest over a five year period, **(1)**

   (b) The force of interest, $\delta(t)$, is a function of time and at any time t, measured in years, is given by the formula

$$\delta(t) = \begin{cases} 0.05, & 0 \leq t \leq 10, \\ 0.002t, & t > 10. \end{cases}$$

   (i) Calculate the present value of a unit sum of money due at time $t = 15$. **(4)**

   (ii) Hence determine from $t = 0$ to $t = 15$ the equivalent constant force of interest. **(2)**

(c) Describe the characteristics of the cash flows that are paid/received in respect of an index-linked security. (3)

(d) (i) Calculate the present value of a continuously payable annuity that is initially paid at a rate of K4,800 *pa* but increases linearly to K8,400 *pa* after 12 years, assuming a force of interest of $\delta = 0.032 + 0.002t$, where t is the time from commencement (measured in years). (6)

(ii) Calculate the accumulated value of the annuity after 12 years at the same force of interest. (2)

2. (a) Explain what is meant by the following terms:

(i) equation of value, (1)

(ii) flat rate of interest. (1)

(b) A borrower repays a loan effected on 1 January 2014 by making 54 equal monthly payments starting on 31 July 2014. The flat rate of interest is 13% *pa*.

Explain why the APR for this arrangement would be quite a lot less than 26% (twice the flat rate), which should be the usual first guess. (2)

(c) An investor wishes to find the present value of a stream of property income payments. She proposes to make the following assumptions.

- The level of current payments is K100,000 *pa*, paid half-yearly in advance.
- Payments will remain fixed for 5-year periods. At the end of each 5-year period the payments will rise in line with total inflationary growth over the previous five years.
- Inflation is assumed to be constant at 3% per annum.
- The interest rate for the calculation is 12% per annum effective.

Find the present value of the income stream if the payments continue for 50 years. (6)

(d) A loan is to be repaid by an immediate annuity. The annuity starts at a rate of K1,500 *pa* and increases by K200 per annum. The annuity is paid for 15 years. Repayments are calculated using a rate of interest of 12% *pa* effective.

(i) Calculate the amount of the loan. (3)

(ii) Construct a loan schedule showing the capital and interest elements in and the amount of loan outstanding after the 11th and 12th payments. (5)

(iii) Find the capital and interest element of the last instalment. (2)

3. (a) In respect of an investment project, define

(i) the discounted payback period, **(1)**

(ii) the payback period. **(1)**

(b) State two reasons why both the discounted payback period and the payback period are inferior measures compared with the net present value for determining whether to proceed with an investment project. **(2)**

(c) An insurance company borrows K38 million at an effective interest rate of 13% per annum. The insurance company uses the money to invest in a capital project that pays 6 million per annum payable half-yearly in arrear for 22 years. The income from the project is used to repay the loan. Once the loan has been repaid, the insurance company can earn interest at an effective interest rate of 4% per annum.

(i) Calculate the discounted payback period for this investment. **(4)**

(ii) Calculate the accumulated profit the insurance company will have made at the end of the term of the capital project. **(5)**

(d) A fund had a value of K6.5 million on 1 January 2019. On 1 May 2019, K7.6 million was invested. Immediately before this investment, the value of the fund was K7.2 million. At the close of business on 31 December 2019, the value of the fund was K11 million.

(i) Calculate the annual effective time-weighted rate of return for 2019. **(2)**

(ii) Calculate the annual effective money-weighted rate of return for 2019. **(3)**

(iii) Comment on your answers to parts (i) and (ii). **(2)**

4. (a) Describe what is meant by the term "ex-dividend". **(1)**

(b) A investor, who is taxed at 30% on income, has just purchased 700 shares in a small education company ex-dividend. Dividends are paid annually and the next dividend is due in two month's time. The last dividend was K95 per share and dividends are expected to rise by 2% *pa*. Calculate the price paid by the investor if the expected yield is 10% *pa* effective. **(4)**

(c) A loan of nominal amount K2.21 million is to be issued bearing coupons payable quarterly in arrear at a rate of 6% per annum. Capital is to be redeemed at K102 per K100 nominal on a coupon date between 12 and 18 years inclusive after the date of issue. The date of redemption is at the option of the borrower.

An investor who is liable to income tax at 20% and capital gains tax at 30% wishes to purchase the entire loan at the date of issue.

(i) Carry out the capital gains test to determine if there is a gain on redemption if the investor requires a net effective yield of 3% per annum. **(1)**

(ii) Explain the significance of the redemption date being at the option of the borrower in relation to your calculation in part (i). **(2)**

(iii) Determine the price which the investor should pay to ensure a net effective yield of at least 3% per annum. **(4)**

(d) A fixed interest bond was issued on 1 January 2019 with a term of 20 years and is redeemable at 105%. The security pays a coupon of 4% per annum, payable half-yearly in arrear.

An investor is liable to income tax at the rate of 30% and capital gains tax at the rate of 40%. Income tax is collected on 1 June each year in relation to gross payments made during the previous 12 months, and capital gains tax is collected on 1 June of the final year.

The investor bought K100,000 nominal of the stock at an issue price of K98,000.

(i) Show that the net redemption yield obtained by the investor will be between 3% and 4% per annum effective. **(4)**

(ii) Use linear interpolation to find this net redemption yield. **(1)**

(iii) If the inflation rate over the term of the bond is assumed to be 2% per annum, calculate the real redemption yield that would be obtained by the investor. **(1)**

(iv) Explain, without any further calculations, how your answers to parts (i) and (ii) would alter if the tax were collected on 1 April instead of 1 June each year. **(2)**

5. (a) State what is meant by a "forward contract". Your answer should include reference to the terms "short forward position" and "long forward position". **(3)**

(b) A two-year forward contract on a stock was agreed on 1 February 2019 when the stock price was K92 and the risk-free force of interest was 3% per annum. The stock was expected to pay a dividend of K4 on 1 July 2019 and a dividend of K6 on 1 January 2020. On 1 June 2019, the stock price was K93 and the risk-free force of interest was 3.5% per annum. The dividend expectations were unchanged.

Calculate, showing all working, the value of the contract to the holder of the long forward position on 1 June 2019. **(6)**

(c) Explain the main difference

(i) between options and futures, **(2)**

(ii) between call options and put options. **(2)**

(d) Let f_t denote the one-year effective forward rate of interest over the year from time t to $(t+1)$. Let y_t be the t-year effective spot rate over the period 0 to t.

The annual effective gross redemption yield from an n-year bond which pays coupons of 4% annually in arrears is given by

$$g_n = 0.06 + 0.003n \quad \text{for} \quad n = 1, 2$$

Each bond is redeemed at par and is exactly one year from the next coupon payment.

It is assumed that no arbitrage takes place.

(i) Calculate y_1 and y_2 as percentages to three decimal placese. **(3)**

(ii) Calculate f_0 and f_1 as percentages to three decimal places. **(2)**

(iii) Explain why the one-year forward rates increase more quickly with term than the spot rates. **(2)**

6. (a) State the main differences between a preference share and an ordinary share. **(3)**

(b) (i) Explain what is meant by the term "par yield". **(2)**

(ii) The n-year forward rate for transactions beginning at time t and maturing at time $t+n$ is denoted as $f_{t,n}$. You are given:

$$f_{0,1} = 3.5\% \text{ per annum}$$
$$f_{0,2} = 4.0\% \text{ per annum}$$
$$f_{1,2} = 4.5\% \text{ per annum}$$

Calculate the par yield for a 3-year bond. **(3)**

(c) An insurance company has liabilities of K30 million due in exactly 2 years' time and a further K60 million due in exactly 5 years' time.

The assets held by the insurance company consist of:

- a 1-year zero-coupon bond of nominal amount K33.1742 million; and
- a 7-year zero-coupon bond of nominal amount K62.9568 million

The current rate of interest is 15% per annum effective at all durations.

(i) Show that the first two conditions of Redingtons theory for immunisation against small changes in the rate of interest are satisfied. **(5)**

(ii) Explain, without doing any further calculations, whether the insurance company will be immunised against small changes in the rate of interest. **(2)**

(d) Consider the previous problem in part (c).

(i) Calculate the present value of profit that the insurance company will make if the interest rate decreases immediately to 14.8% per annum effective. **(3)**

(ii) Explain why the insurance company made a profit as a result of the change in the interest rate. **(2)**

7. (a) State the main difference between fixed and varying interest rate models. **(1)**

(b) The annual rates of return on a fund are independent and identically distributed, with mean 9% and standard deviation 15%. Each year the distribution of $1 + i$ is log-normal with parameters μ and σ^2, where i is the annual yield on the fund.

(i) Calculate the values of μ and σ^2 **(4)**

(ii) Calculate the probability that an investment of K300,000 at time 0 will grow to more than K500,000 at time 6. **(4)**

(c) Mr Zimba is aged exactly 58 and he intends to retire in seven years' time and receive an annuity-certain. The annuity will be payable monthly in advance and will cease after 30 years. The annuity will increase at each anniversary of the commencement of payment at the rate of 6% per annum.

Mr Zimba would like the initial level of annuity to be K276,000 per annum. The price of the annuity will be the present value of the payments on the date it commences using an interest rate of 8% per annum effective.

Calculate the price of the annuity. **(3)**

(d) To purchase the annuity described in part (c), Mr Zimba invests K2.9 million on his 58th birthday in a particular fund. The investment return on the fund in any given year is independent of returns in all other years and the annual return is

- 10% with a probability of 90%,
- 21% with a probability of 10%.

Determine, showing all workings, the

(i) expected accumulation of the investment at the time of retirement, **(3)**

(ii) standard deviation of the investment at the time of retirement, **(3)**

(iii) probability that Mr Zimba will have insufficient funds to purchase the annuity **(2)**

END OF PAPER

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

The University of Zambia

School of Natural Sciences

Department of Mathematics & Statistics

2019/2020 Academic Year Final Examinations

MAT 2100 - Analytic Geometry and Calculus

Time allowed : Three **(3)** hrs Full marks : 100

Instructions:

- Indicate your **computer number** on all answer booklets.
- There are seven (7) questions in this examination. Attempt **any five (5)** questions. All questions carry equal marks.
- **Full credit** will only be given when **necessary work** is shown.

This paper consists of 4 pages of questions.

---

1. **(a)** Define the following terms:

   (i) a parabola, [2]

   (ii) an ellipse. [3]

   (b) Find the center, vertices and foci of the ellipse given by the equation

   $$4x^2 + y^2 - 8x + 4y - 56 = 0.$$ [4]

   (c) Convert the equation

   $$r = \frac{8}{2 - 4\cos\theta}$$

   into Cartesian form. [3]

   (d) **(i)** Rotate the coordinate axes to change the equation

   $$x^2 - \sqrt{3}\,xy + 2y^2 = 1$$

   into an equation that has no cross product (xy) term. [6]

   (ii) Identify the graph of the new equation. [2]

Turn Over/...

2. **(a)** The function

$$f(x) = x^{\frac{1}{3}} + x$$

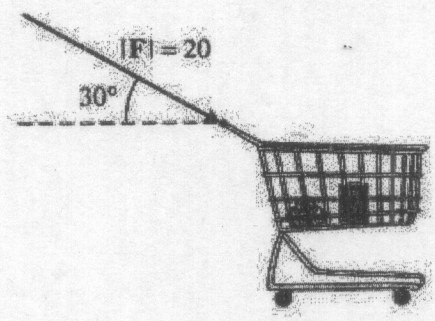
satisfies the hypotheses of the mean value theorem on the interval $[1, 8]$. Find the number $c \in (1, 8)$ that satisfies the conclusion of the theorem. [7]

(b) Evaluate each of the following limits:

(i) $\lim_{x \to 0} \left(\frac{\sin x}{2x^3} - \frac{1}{2x^2} \right)$ [4]

(ii) $\lim_{x \to \infty} [x \ln(5x)]^{\frac{3}{\ln x}}$ [5]

(c) You push a loaded supermarket cart by applying a 20 N force, F, that makes a 30° angle with the horizontal as shown below:



Express F in terms of its horizontal and vertical components. [4]

3. **(a)** Evaluate each of the following indefinite integrals:

(i) $\int \frac{1}{\sqrt{\sqrt{x}+1}}\, dx$ [3]

(ii) $\int \frac{1}{\cos\theta + 1}\, d\theta$ [5]

(b) Find the moment of inertia about the $x-$axis (I_x), of the finite region bounded by the curve $y = \sqrt{x-1}$, the $x-$axis and the line $x = 5$. [4]

(c) Solve each of the following ordinary differential equations:

(i) $\left(y - 2x^{\frac{3}{2}}\right) dx + (2x) dy = 0, \quad y(4) = 8$ [4]

(ii) $\frac{d^2y}{dx^2} + y = \csc x$ [4]

Turn Over/...

4. **(a)** Two lines $\mathcal{L}_1$ and $\mathcal{L}_2$ are contained in a plane $\mathcal{P}$, where

$$\mathcal{L}_1 : x = -2 + 2t,\ y = 3t,\ z = 1 - t$$

and

$$\mathcal{L}_2 : x = -1 - r,\ y = 1 - 2r,\ z = 3 + 3r.$$

(i) Find the point of intersection of $\mathcal{L}_1$ and $\mathcal{L}_2$. [5]

(ii) Hence, or otherwise, find the equation of $\mathcal{P}$. [3]

(b) Given that

$$R(t) = (3\cos t)i + (3\sin t)j + (4t)k,$$

find the principle unit normal vector $\mathcal{N}(t)$. [6]

(c) Determine the nature of the critical points of the function

$$f(x, y) = \tfrac{2}{3}x^3 - x^2 - xy + \tfrac{y^2}{2} + y + 5.$$ [6]

5. **(a)** **(i)** Find the function g, such that

$$f(x, y) = \begin{cases} \frac{x^3+y^3}{x+y}, & x \neq -y \\ g(y), & x = -y \end{cases}$$

is continuous at the point $x = -y$. [3]

(ii) Given that

$$h(x, y) = \frac{1 + xy}{x},$$

where $x = ts^2$ and $y = \ln\left(\frac{t}{s}\right)$, find $\dfrac{\partial h}{\partial s}$. [4]

(b) Given the surface

$$x^2yz + 3y^2 = 2xz^2 - 8z,$$

find the equation of the normal line to the surface at the point $(1, 2, -1)$. Hence, or otherwise, find the coordinates of the point where this normal line meets the xz-plane. [8]

(c) Evaluate the improper integral

$$\int_0^\infty 2xe^{-x^2}\, dx.$$ [5]

Turn Over/...

6. **(a)** Solve each of the following ordinary differential equations:

(i) $\dfrac{dy}{dx} + y = y^3$ [4]

(ii) $y'' - 28y' + 144y = 0$ [3]

(b) Find the power series solution to the ordinary differential equation

$$\frac{dy}{dx} - y = 0.$$ [6]

(c) A curve is given by

$$y = \frac{1}{3}(2x+1)^{\frac{3}{2}}, \quad x \geq 0.$$

Find an intrinsic equation of the curve in the form $s = g(\psi)$, where s is measured from the point $(0, \frac{1}{3})$ and ψ is the angle the tangent to the curve makes with the positive $x-$axis. [7]

7. **(a)** Use the $\varepsilon - \delta$ definition to prove that

$$\lim_{x \to 0} \left(\frac{x^2+x-2}{x+3} \right) = -\frac{2}{3}.$$ [7]

(b) The finite region bounded by the curve $y = (x-1)^3$, the lines $y = 1$ and $x = 1$ is rotated about the $y-$axis. Find the volume of the resulting solid. [7]

(c) The rate at which a bacteria culture is growing is represented by the equation

$$\frac{dN}{dt} - kN = 0,$$

where $N(t)$ is the number of strands of the bacteria in the culture at time t hours, and k is a constant. After 1 hour, 1000 strands of the bacteria are observed in the culture and after 4 hours, 3000 strands of the bacteria are observed. Find the approximate number of strands of the bacteria originally in the culture. [6]

END OF EXAMINATION!

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS & STATISTICS

2019/2020 Academic Year Final Examinations DECEMBER, 2020
MAT 2200 LINEAR ALGEBRA

Total time allowed: Three (3) hours
Instructions:

- This question paper comprises six (6) questions. **Answer any five** (5) **questions**.
- Be as neat and precise as possible to earn full marks.
- The marks to parts of each question are indicated at the end of each question.

---

1. (*a*) Define the following terms:
   (*i*) Characteristic polynomial, $\triangle_A(t)$ of a square matrix A,
   (*ii*) eigenvalue and hence eigenvector of a square matrix A.

   (*b*) (i) State Cayley-Hamilton theorem without proof.
   (ii) Suppose that A and B are two similar matrices, say $B = P^{-1}AP$, where P is invertible, prove that A and B have the same characteristic polynomial.

   (*c*) (i) Apply Cayley-Hamilton theorem to find a polynomial having a root
   $$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}.$$
   (ii) Let $A = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$.
   (*a*) Find eigenvalues and corresponding eigen-vectors,
   (*b*) find an invertible matrix P such that $D = P^{-1}AP$ is diagonal.

[3; 3; 4; 3; 5; 2]

2. (*a*) Define the following terms:

   (*i*) orthogonal complement of a subset S of an inner product space V,

   (*ii*) Fourier coefficient of a vector v with respect to w.

   (*b*) State and prove Cauchy-Schwarz Inequality.

   (*c*) (i) Let V be the vector space of polynomials over $\mathbb{R}$ of degree ≤ 2 with inner product defined by:

$$\langle f, g \rangle = \int_0^1 f(t)g(t)dt.$$

   Find a basis of the subspace W orthogonal to $h(t) = 2t + 1$.

   (ii) Let U be the subspace of $\mathbb{R}^4$ spanned by:

$$u_1 = (1, 1, 1, 1),\ u_2 = (1, -1, 2, 2),\ u_3 = (1, 2, -3, -4).$$

   Apply the Gram-Schmidt algorithm to find an orthogonal and an orthonomal basis for U.

[3; 3; 5; 4; 5]

3. (*a*) Define the following terms:

   (*i*) Linear transformation,

   (*ii*) kernel of a linear transformation.

   (*b*) Let $f : V \to U$ be a linear transformation. Prove that the kernel of f, $\ker(f)$ is a subspace of V.

   (*c*) (i) Show that the following mapping is not linear:

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

   defined by $f(x, y, z) = (x + 2, y + z)$.

   (ii) For the given linear transformation f find a basis and the dimension of the kernel and the image of f:

$$f : \mathbb{R}^3 \to \mathbb{R}^3$$

   defined by $F(x, y, z) = (x + 2y - 3z, 2x + 5y - 4z, x + 4y + z)$.

[3; 3; 5; 3; 6]

4. (*a*) Define the following terms:

   (*i*) minor of an element a_{ij} of a matrix $A = [a_{ij}]$,

   (*ii*) cofactor of an element a_{ij} of a matrix $A = [a_{ij}]$.

   (*b*) State the Laplace expansion theorem without proof.

   (*c*) (i) Find $\det(A)$, $\text{adj}A$ and A^{-1}, where

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}.$$

   (ii) Let $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 1 & 0 & -2 & 3 \\ 3 & -1 & 2 & 5 \\ 4 & -3 & 0 & -1 \end{bmatrix}$. Find the minor and signed minor corresponding to $A(2,3;2,4)$.

[3; 3; 4; 8; 2]

5. (*a*) Define the following terms:

   (*i*) a vector v is a linear combination of vectors $u_1, u_2, \cdots, u_n$,

   (*ii*) basis of a vector space V.

   (*b*) Let S be a subset of a vector space V. Prove that span(S) is a subspace of V that contains S (infact, span(S) is the smallest subspace of V that contains S).

   (*c*) (i) Find a subset of u_1, u_2, u_3, u_4 that gives a basis for $W =$ Span(u_i) of $\mathbb{R}^5$, where $u_1 = (1,1,1,2,3)$, $u_2 = (1,2,-1,-2,1)$, $u_3 = (3,5,-1,-2,5)$, $u_4 = (1,2,1,-1,4)$.

   (ii) Find the rank of the matrix: $A = \begin{bmatrix} 1 & 3 & -2 & 5 & 4 \\ 1 & 4 & 1 & 3 & 5 \\ 1 & 4 & 2 & 4 & 3 \\ 2 & 7 & -3 & 6 & 13 \end{bmatrix}$.

[3; 3; 5; 6; 3]

6. (*a*) Define the following terms:

   (*i*) Augmented matrix of a system of linear equations,

   (*ii*) coefficient matrix of a system of linear equations.

   (*b*) Suppose that A is a matrix with rows $R_1, R_2, \cdots, R_m$. State the elementary row operations that can be applied on matrix A to transform it into another row equivalent matrix.

   (*c*) Solve the following system of linear equations:

$$\begin{aligned} x + 2y - z + 3t &= 3 \\ 2x + 4y + 4z + 3t &= 9 \\ 3x + 6y - z + 8t &= 10. \end{aligned}$$

[3; 3; 4; 10]

END OF EXAM!

THE UNIVERSITY OF ZAMBIA

DEPARTMENT OF MATHEMATICS AND STATISTICS

END OF YEAR EXAMINATIONS FOR THE 2019/2020 ACADEMIC YEAR

MAT 2300 INTRODUCTION TO REAL ANALYSIS

Duration: Three (3) Hours

Instructions:

- There are SIX (6) questions in this paper. Answer any FIVE (5)
- Show all necessary work to earn full marks

---

1. (a) Define the following:
   i. A Dadekind cut.
   ii. A rational cut.

   (b) Prove that the ordered set $\mathbb{R}$ of all Dadekind cuts has the least upper bound property.

   (c) i. Show that the set $\alpha = \{p \in \mathbb{Q} : p < 0\} \cup \{p \in \mathbb{Q} : p \geq 0 \text{ and } p^2 < 3\}$ is a Dedekind cut.
   ii. If α and β Dadekind cuts, prove that the set $\alpha + \beta = \{r + s : r \in \alpha, s \in \beta\}$ is a Dadekind cut.

2. (a) Define the following terms:
   i. A positive class in a field $\mathbb{F}$.
   ii. The absolute value function in an ordered field $\mathbb{F}$.

   (b) Let $\mathbb{F}$ be a field with a positive class P. Prove the following:
   i. $|xy| = |x||y|$ for all $x, y \in \mathbb{F}$
   ii. $|x + y| \leq |x| + |y|$ for all $x, y \in \mathbb{F}$

   (c) i. Show that the set $P = \{\frac{a}{b} : a, b \in \mathbb{Q} : a \geq 0, b > 0\}$ forms a positive class in P.
   ii. Show that the absolute value function in an ordered field may be defined by $|x| = \sqrt{x^2}$.

3. (a) Define the following:
   i. The inverse image of a set under a function.
   ii. An equivalence relation.

(b) Let $f : X \to Y$ be a function and let $A, B \subset X$ and $C, D \subset Y$. Prove the following:

i. $f(A \cup B) = f(A) \cup f(B)$

ii. $f^{-1}(C \cap D) = f^{-1}(C) \cap f^{-1}(D)$

(c) i. Prove that an equivalence relation on a set partitions the set into distinct classes.

ii. Show that the relation $x \equiv y$ if and only if $x - y$ is even defines an equivalence relation on the set $\mathbb{Z}$ of all integers.

4. (a) Define the following terms:

i. A monotone decreasing sequence of real numbers.

ii. A bounded sequence of real numbers.

(b) Give examples to verify the following statements:

i. The sum of two unbounded sequences of real numbers may be bounded or unbounded.

ii. The product of a monotone increasing sequence of real numbers and a monotone decreasing one may be monotone increasing or decreasing.

(c) i. Show that the sequence $x_n = \dfrac{1}{\arctan\left(1 - \frac{1}{n}\right)}$ is monotone decreasing.

ii. Show that the sequence $y_n = \sqrt{n+1} - \sqrt{n}$ is bounded.

5. (a) Define the following

i. A Cauchy sequence of real numbers.

ii. A convergent sequence of real numbers.

(b) Prove that a sequence of real numbers is convergent if and only if it is Cauchy.

(c) i. Show that $x_n = \dfrac{1}{2n-1}$ is a Cauchy sequence.

ii. Show that $\sqrt{n+1} - \sqrt{n}$ is a convergent sequence.

6. (a) Define the following terms:

i. The limit supremum of a sequence of real numbers.

ii. The limit infimum of a sequence of real numbers.

(b) State and prove the Bolzano-Weierstrass Theorem.

(c) Find the limit supremum and infimum for the following sequences:

i. $x_n = (-1)^n \left(2 + \dfrac{1}{n}\right)$

ii. $y_n = (-1)^n \sin\left(\dfrac{\pi n}{2}\right)$

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES

2019 ACADEMIC YEAR
FINAL EXAMINATIONS

MAT2602 INTRODUCTION TO STATISTICS

Time Allowed: Three (3) Hours

Instructions:
1. Answer any **Five (5)** Questions
2. Show All Essential Working
3. Statistical Tables will be provided
4. Calculators are Allowed

---

1. (a) Define the following:
   (i) a parameter
   (ii) descriptive statistics

   (b) The number of hours that 50 employees of a certain organization spend in the office in a week is given in the following grouped cumulative distribution table:

Hours	Cumulative frequency
15 – 17	2
18 – 20	6
21 – 23	13
24 – 26	22
27 – 29	32
30 – 32	44
33 – 35	50

   (i) Construct a grouped frequency distribution table.
   (ii) Construct a histogram on the absolute frequency scale.
   (iii) Construct a frequency polygon on the relative frequency scale.
   (iv) Describe the distribution of the number of hours spent in the office.

   (c) The time taken to run a road race is normally distributed with a mean of 180 minutes and a standard deviation of 14 minutes.
   (i) Find the probability that a randomly selected runner completes the race between 170 and 215 minutes.
   (ii) If 11.9% of the runners complete the race in less than k minutes, find the value of k.

2. (a) Define the following:
   (i) a random sample
   (ii) a stratified sample

   (b) The number of kilometres 30 taxi drivers cover on a given day are:

11	8	26	31	62	19	7	3	14	75
33	30	42	15	18	23	29	13	16	6
50	9	10	17	12	20	22	24	32	25

   (i) Prepare an ordered stem and leaf plot for the data.
   (ii) Construct a boxplot for the data and identify the outliers, if any.

   (c) Delivery times for all food orders at a fast food restaurant during lunch hour are normally distributed with a mean of 8.4 minutes and a standard deviation of 1.8 minutes. Find the probability that the mean delivery time for a random sample of 16 such orders at this restaurant is
   (i) between 8 and 9 minutes.
   (ii) within 1 minute of the population mean.

3. (a) (i) Define response bias
   (ii) State two causes of response bias.
   (iii) Define a probability sample.

   (b) A politician claims that the percentage of female voters exceeds that of male voters by more than 5%. A random sample of 500 female registered voters showed that 300 of them voted in the last presidential election. Another independent random sample of 400 male voters showed that 208 of them voted in the same election.
   (i) Construct a 98% confidence interval for the difference between the true proportions of female and male voters in the last election.
   (ii) Test the politician's claim at the 5% level of significance.

   (c) A medical expert claims that the average age of first-time mothers in Zambia is 25 years or less. The standard deviation of the age of first-time mothers is known to be 4.8 years. Ministry of Health officials take a random sample of 57 first-time mothers and decide the claim should be rejected if the mean age is greater than 27 years.
   (i) Explain how one can commit a type I error in this context.
   (ii) Find the probability of a type I error if the claim is true.
   (iii) Find the probability of a type II error if the true mean age of first-time mothers is 28 years.

4. (a) Define the following:
   (i) a statistical hypothesis
   (ii) a type II error

(b) A consumer agency wanted to find out if the mean time taken by each of three brands of medicine to provide relief from a headache is the same. The following table gives the time (in minutes) taken by each patient to get relief from a headache for patients randomly assigned to the three drugs.

Drug

1	2	3
25	15	44
38	21	39
42	19	54
65	25	58
47		73
52		

   (i) What design was used in the experiment?
   (ii) Write down a model for the above design. **Explain** all the terms in your model. **State** all the assumption(s).
   (iii) Is the mean relief time the same for each of the three drugs? Use $\alpha = 0.05$ level of significance.

(c) A sample of 13 cans of brand A diet soda gave a mean number of 23 calories with a standard deviation of 3 calories. Another independent sample of 11 cans of brand B diet soda had a mean of 25 calories with a standard deviation of 5 calories. Assume the amounts of calories of diet soda are normally distributed.
   (i) Is there a difference in the mean number of calories for the two brands? Carry out a test to answer the question using a 5% level of significance assuming the variances of the two brands are equal.
   (ii) Construct a 90% confidence interval for the ratio of variances of the two brands.

5. (a) (i) State two properties of a normal distribution curve.
(ii) Define a sampling distribution.

(b) The following data give the experience (in years) and annual salaries (in millions of Kwacha) of 9 randomly selected employees of a certain organization.

Experience (x)	14	3	5	6	4	9	18	5	16
Annual salary (y)	62	29	37	43	35	60	67	32	60

(You may use the following summary statistics:

$$\sum_{i=1}^{6} x_i = 80,\ \sum_{i=1}^{6} x_i^2 = 968,\ \sum_{i=1}^{6} y_i = 425,\ \sum_{i=1}^{6} y_i^2 = 21841,\ \sum_{i=1}^{6} x_i y_i = 4404)$$

(i) Draw a scatter diagram. Comment on the pattern.
(ii) Estimate the simple linear regression equation.
(iii) Explain the meaning of the estimated parameters.
(iv) Copy and complete the following ANOVA table.

Source of variation	Sum of squares (SS)	Degrees of freedom (df)	Mean square (MS)	F*
Regression				
Error			35.000	
Total				

(v) Is there a significant linear relationship between experience and annual salary? Use a 5% level of significance.
(vi) Compute a 95% confidence interval for the slope parameter.

6. (a) (i) Define the coefficient of determination.
(ii) State two properties of the correlation coefficient.

(b) Recent recession and bad economic conditions forced many people to hold more than one job to make ends meet. A sample of 500 persons who held more than one job produced the following two way table.

	Marital status		
Gender	Single	Married	Other
Male	65	230	25
Female	40	120	20

Test at the 10% level of significance whether gender and marital status are related for all people who hold more than one job.

(c) A computer company claims that the mean time to learn how to use its new software is not more than 2 hours. A random sample of 12 persons took the following times (in hours) to learn how to use the software:

1.75	2.25	2.40	1.90	1.50	2.75
2.15	2.25	1.80	2.20	3.25	2.60

(i) Test at the 1% level of significance whether the company's claim is true. Assume the times are normally distributed.

(ii) Construct a 98% confidence interval for the population standard deviation of times taken to learn the software. Is it significantly different from 0.5 hour? Explain.

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS
2019/2020 ACADEMIC YEAR FINAL EXAMINATION
MAT3001 – INSURANCE AND ALTERNATIVE RISK TRANSFER

30TH NOVEMBER 2020

Time Allowed: THREE (3) hours

INSTRUCTIONS

1. Answer ALL Questions
2. Show all your workings to earn full credit
3. Indicate your computer number on ALL answer booklets

Question One **[Total 25 Marks]**

(a) List any six of the COSO principles of risk management. [3 marks]

(b) Define insurable interest and outline its four elements. [8 marks]

(c) Define the term indemnity and outline how it is achieved in the following classes of insurance. Your answer should highlight cases of under/over insurance.

(i) Residential Property insurance [4 marks]
(ii) Business Interruption insurance (in light of Covid-19) [3 marks]
(iii) Takaful insurance for motor vehicle [3 marks]

(d) Outline any four benefits of insurance to the society. [4 marks]

Question Two **[Total 25 Marks]**

(a) Outline the roles and processes associated with the following insurance operations.

(i) Underwriting [4 marks]

(ii) Production/marketing [4 marks]

(iii) Claims [4 marks]

(b) Rather surprisingly, actuaries are relatively recent arrivals in the general insurance world. It's clear to us as actuaries that we should have lots to offer this industry, but the industry has, until recently, been a little slow to recognise that.

(i) List the roles that Actuaries perform in a general insurance setting. [3 marks]

(ii) List the steps under methodology of actuarial problem solving based on the Actuarial Control Cycle (ACC). [3 marks]

(iii) Given the case of Travel Insurance under the current Covid-19 pandemic. Present an ACC based solution to a company offering such a product to global travellers. [7 marks]

Question Three **[Total 25 Marks]**

(a) Outline the importance of a policy document in an insurance contract. [2 marks]

(b) Outline the content of the following components of the policy document

(i) Schedule [2 marks]
(ii) Exclusions [3 marks]

(c) Explain how the following strategies of implementing partial insurance leads to lower insurance premiums.

(i) Increase deductible [3 marks]
(ii) Reduce Cap [3 marks]

(d) Outline the rationale of setting up a captive insurance. [5 marks]

(c) Clearly show your workings in the following reinsurance calculations.

(i) A reinsured has a property risk excess of loss treaty with a limit of K2,500,000, excess of K2,500,000 and an event limit of three times its per risk limit. A storm occurs and causes damage to the factories of five different insureds. The amounts paid by the reinsured for each claim are as follows

Claim	Amount
A	5,000,000
B	4,750,000
C	3,000,000
D	4,250,000
E	3,500,000

Show the how much will the reinsurance payment be for all five claims. [2 marks]

(ii) A cedant has a 10-line surplus treaty which is subject to a maximum cession of K1,000,000. Calculate the maximum gross capacity of the cedant, for a single risk, if no other reinsurance coverage is in force. [2 marks]

(iii) An insurance company underwrites a large risk with a sum insured of K50,000,000 It maintains a K500,000 gross retention and fully utilises its five line surplus treaty. The remainder is reinsured under a proportional facultative reinsurance contract. If a loss of K37,500,000 occurs. Calculate how much the facultative reinsurers will pay in settlement? [3 marks]

Question Four **[Total 25 Marks]**

(a) Describe the nature and operation of any two loss sensitive arrangements available to a corporate seeking insurance or reinsurance. [4 marks]

(b) Distinguish between multiple peril and a multiple trigger products [2 marks]

(c) Describe the insurance business transaction process at the Lloyd's of London. [4 marks]

(d) Explain in detail how any three alternative risk transfer solutions have been used to compliment or substitute to conventional reinsurance for the total exposure named catastrophic risks or pandemics. You write up should include actual global examples. [15 marks]

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS
2019/2020 ACADEMIC YEAR EXAMINATION
MAT3011 – PRINCIPLES OF FINANCE AND FINANCIAL REPORTING
25TH NOVEMBER 2020

Time Allowed: THREE (3) hours

INSTRUCTIONS

1. Answer ALL Questions from both Section A and B
2. Show all your workings to earn full credit
3. Indicate your computer number on ALL answer booklets

Question One **[20 Marks] – MULTIPLE CHOICE – [2 marks each]**

1.Which of the following statements best explains the role of the external auditor from an agency perspective?
A The auditor enables the shareholders to rely on the financial statements prepared by the directors.
B The auditor ensures that the company is well managed.
C The auditor provides the directors with independent advice on accounting and financing matters.
D The shareholders can task the external auditor to examine particular areas of concern.

2. Which of the following statements best describes the driver of the market price of a quoted company?
A Expectations of future revenues.
B Expectations of future dividends.
C Historical trends of reported profits.
D Historical trends of dividend payments.

3. Maka has just been admitted to a long established business partnership. He has bought 20% of the partnership equity, although he has not paid for this yet. He will be entitled to 15% of the partnership profit. If the firm incurs any liability, what proportion of that liability will be Maka's legal responsibility?
A 0%
B 15%
C 20%
D 100%

4. When does a bondholder receive the nominal value of a bond?
A At the time of purchase.
B Annually.
C Whenever coupon payments are made.
D At maturity.

5. Which of the following statements describes the agency problem?
A Agents may feel that they cannot trust their principals.

B Agents may have insufficient authority to manage their principals' affairs.
C Principals may feel that they cannot trust their agents.
D Principals may not have the necessary expertise to manage their own businesses.

6. Which of the following statements provides the best explanation for why non-financial factors ought to be considered when evaluating an investment project?
A Non-financial factors can be more easily understood.
B Non-financial factors can give insights into upside risks.
C Non-financial factors are the primary driver of shareholder wealth.
D Only anticipated net cash flows matter.

7. Which of the following is a valid basis for weighting debt and equity in order to calculate the weighted average cost of capital (WACC)?
A Book value of debt and book value of equity.
B Book value of debt and market value of equity.
C Market value of debt and book value of equity.
D Market value of debt and market value of equity.

8. What is the strongest argument in favour of setting a common hurdle rate across a company for all projects?
A All projects in the same industry have the same risk.
B The decision rule is clear and can be applied consistently.
C A common rate will ensure consistency when capital rationing is in effect.
D The hurdle rate is not a material factor in evaluating a project.

9. You have been asked to determine the internal rate of return (IRR) of a project that has an initial cash outflow, followed by seven years of net cash inflows. The project's net present value was + $500,000 when determined at 11% and – $500,000 when determined at 16%.
Which of the following statements concerning the project's IRR is correct?
A The IRR is approximately 13.5%.
B The IRR is exactly 13.5%.
C The IRR is greater than 16%.
D The IRR is less than 11%.

10. Why is it generally expensive to grant a cash discount for prompt settlement of trade receivables?
A Credit customers will be unlikely to take up the cash discount.
B Granting a discount will involve significant and complicated bookkeeping entries.
C It is not possible to incorporate the cost of cash discounts into selling prices.
D The discount must be compounded, generally revealing a high annualised cost.

11. [BONUS QUESTION] A quoted company is planning to make a rights issue on the basis of one new share for every seven shares currently held. The present share price is K5.20. The rights issue will be priced at K4.50. The directors intend to use the funds raised to fund a project that they are confident will increase the company's present market capitalisation by 20%.

Calculate the expected price per share after the rights issue. [2 marks]

Question Two **[Total 20 Marks]**

(a) Describe whether preference shares should be treated as debt rather than equity when evaluating a company's gearing. [4 marks]

(b) Victor is an actuarial consultant who has operated as a sole trader for several years. He has decided to bring Winmore, a colleague, into the business.

Describe the advantages to Victor of a traditional partnership over a limited company for the business arrangement with Winmore. [4 marks]

(c) Explain why an investment opportunity could be rejected when evaluated using the shareholder value approach despite having a positive net present value (NPV). [4 marks]

(d) A listed company is planning a bond issue.

Describe the implications of making the issue convertible or puttable in return for a lower interest rate. [4 marks]

(e) National tax systems often have the objectives that the tax burden is fair and reasonable. Explain how these objectives are achieved. [4 marks]

Question Three **[Total 20 Marks]**

(i) Outline the roles of accounting information of any four users. [4 marks]

(ii) On 1 October 20X9, the owner of the USS Enterprise, Mr Kirk, decided that he will boldly go and keep his records on a double entry system. His assets and liabilities at that date were:

		K
Fixtures and equipment		20,000
Stock including weapons		15,000
Balance at Universe Bank		17,500
Cash		375
Creditors	– Spock	3,175
	– Scott	200
	– McCoy	500

Kirk's transactions during October were as follows:

1. Sold faculty phasers, original cost K500, to Klingon Corp, for cash K5,000
2. Bought Photon Torpedoes (weapons), on credit from Pemba Council K2,500
3. Sold Stocks to Aardvarks, original cost K250, on credit, K1,500
4. Bought Cloaking Device (Fixture and Fittings) from Klingon Corp K3,500
5. Paid the balance owed to Spock at 1 October less a 5% cash discount
6. Paid Pemba Council full amount due by cheque
7. Received full amount due from Aardvarks by cheque
8. Paid Klingon Corp by cheque after deducting 20% trade discount
9. Paid, by bankers order, K10,000 for repairs to Enterprise following disagreement over amount owing to Klingon Corp and faculty phasers.

(i) Open Enterprise's ledger accounts at 1 October, record selected transactions for the month and balance the ledger accounts as at 31 October 20X9 under the following headings.

(a) Bank Account
(b) Cash Account
(c) Stock Account [8 marks]

(iii) Using your own hypothetical figures, present an outline of the structure of the following components of the company's financial statements.

(a) Balance sheet
(b) Income statement [8 marks]

Question Four [Total 20 Marks]

Protégé is an unquoted company which was founded 12 years ago and which has grown steadily since. Its directors are considering an investment opportunity that will increase the productive capacity of the business by 30%.

Protégé was established using the savings of its three founders, who now comprise the company's board of directors. Each of the three founders owns one third of the company's equity. Protégé's growth has been funded using retained earnings and the proposed expansion will be funded in the same way. The company has no debt. None of the founders has any other significant personal assets apart from their shares in Protégé and their family homes.

The founders are considering more sophisticated approaches to the evaluation of capital investment projects. They have asked you to determine Protégé's cost of equity.

They have established the following facts:
- The risk free rate is 4% per annum.
- The equity risk premium is 10% per annum.
- The beta coefficient of a quoted company which is in the same industry as Protégé is 1.4.
- The above quoted company has a debt:equity ratio of 0.6:1.
- The corporation tax rate is 20%.

(i) Calculate Protégé's cost of equity. [6 marks]

(ii) Describe any five steps under the IPO process by which Protégé might issue fresh share capital through the Lusaka securities exchange (LuSE). [5 marks]

(iii) Briefly describe the roles of any three players of an IPO. [3 marks]

(iv) Outline how any three dividend policy theories affect the dividend policy for Protégé's post listing. [6 marks]

Question Five **[Total 20 Marks]**

Ndiyepano is a major quoted company which manufactures highly specialised equipment for use in the mining industry. Ndiyepano has very limited scope for expansion. It dominates the markets for its equipment and there is no real scope to develop sales in alternative markets. Ndiyepano has been highly profitable and it has amassed significant retained earnings and has a large cash balance.

Ndiyepano's board has sought advice about the most efficient use of that cash balance. Three main options are under consideration:

- The first possibility is that the cash might be retained for the foreseeable future, so that Ndiyepano has the necessary funding in place if any new opportunities arise.
- Alternatively, the board could use the funds in order to diversify into a completely different line of business. Ndiyepano's cash reserves are large enough to fund the creation of a significant business in another industry, such as leisure.
- Finally, Ndiyepano could implement a share buyback under which Ndiyepano would buy shares back using the surplus funds.

(i) Discuss the implications of Ndiyepano retaining the surplus cash in order to take advantage of any new opportunity which might arise. [5 marks]

(ii) Discuss the potential advantages and disadvantages of Ndiyepano using the surplus funds in order to diversify. [5 marks]

(iii) Discuss the suitability of a share buyback to Ndiyepano's circumstances. [5 marks]

(iv) Discuss the assertion that a company's optimal gearing ratio could be affected by its line of business. [4 marks]

END OF EXAMINATION

©UNZA2020

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

2019/20 ACADEMIC YEAR

Wednesday, November 18, 2020 (pm)

MAT 3022-INVESTMENT AND PORTFOLIO MANAGEMENT

TIME ALLOWED: Three(3) hours

INSTRUCTIONS TO THE CANDIDATE

1. **Attempt ALL four (4) questions**, beginning your answer to each question on a separate sheet.
2. Candidate should show calculations where this is appropriate
3. Enter all the candidate and examination details as requested on the front of your answer booklet.

Graph paper is NOT required for this paper

AT THE END OF THE EXAMINATION

Hand in your booklet, with any additional sheets firmly attached.

In addition to the paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

This examination paper consist of 5 printed pages, each printed on one side only.

The front cover is page 1.

Question 1 starts on page 2

©2020

Question 1

(a) (i) Define the following measures of investment risk:

(a) variance of return [2]

(b) downside semi-variance of return [2]

(c) shortfall probability [2]

(ii) An investor is contemplating an investment with a return of R, where:

$$R = 250{,}000 - 100{,}000N$$

and N is a Normal [1,1] random variable. Calculate variance of return. [3]

(b) (i) Let $U(w)$ be the utility function of an investor's wealth, w. The first and second derivatives of U with respect to W are positive. Explain what these conditions imply about the investor's economic characteristics. [3]

(ii) You are given the following information on projects A and B:

State of nature	*Probability*	*Return on project A*	*Return on project B*
1	10%	-3%	-2%
2	20%	4%	4.5%
3	50%	6%	7%
4	20%	12%	14%

Explain with reasons which of the two projects the investor in (b) above should choose in order to maximise his expected utility of wealth. [3]

(iii) Describe the forms of the Efficient Markets Hypothesis (EMH). [4]

(c) An investor has the following utility function: $U(w) = 1 - e^{-aw}$

where $w > 0$ is her wealth.

(i) Show what constraints exist on the value of a if she prefers more wealth to less wealth (is non-satiated) and is risk averse. [2]

(ii) Derive her absolute and relative risk aversion functions. [2]

(iii) Explain the implications of the results in (i) and (ii) for the proportion of wealth she will invest in risky assets as her level of investible wealth changes. [2]

[Total:25]

Question 2

(a) (i) Following the financial crisis in 2008 a number of economists were interviewed and claimed that they had been expecting something like the crisis to occur; however, there was no record of them having expressed this opinion before the crisis.

Describe the behaviour these economists were demonstrating. **[3]**

(ii) An investor has been investing their own money for a period of years despite the fact that a better return could have been produced by investing the money in a risk-free investment that produced a modest income.

Outline four behaviours that may have led to the investor continuing to invest as had been done in the past. **[4]**

(b) List any four domestic equities sector categories (giving at least one listed counter under each) that are evident on our local LuSE and briefly describe the fundamental characteristics of each. **[6]**

(c) (i) Distinguish between a passive and an active fund manager and outline one way in which each can be implemented. **[6]**

(ii) Justify the use of technical analysis in markets and outline the tools and techniques to be used. **[6]**

[Total:25]

Question 3

(a) You are given the following information for returns on two stocks I and Z.

	I	Z
α	0.04	0.09
β	1.20	1.50
σ_e	0.25	0.40

$\bar{R}_m = 0.16$ and $\sigma_m = 0.20$

The returns generating process is assumed to be as follows:

$$R = \alpha + \beta R_m + e, \qquad \sigma_m^2 = \beta^2 \sigma_m^2 + \sigma_e^2$$

where R is a random variable representing the return on the stock; R_m is a random variable representing the return on a market index; e is the residual term; σ, σ_m^2 and σ_e^2 are the variances of the stock, index and residual term, respectively. The residual terms of the returns generating processes for stocks I and Z are assumed uncorrelated with each other and have a mean of zero.

Calculate the following:

(i) The mean and variance of the returns of each stock. **[3]**

(ii) The covariance of returns between the stocks. **[2]**

(iii) The beta of an equally-weighted portfolio of the two stocks. **[2]**

(iv) The expected return and variance of an equally-weighted portfolio of the two stocks. **[3]**

(b) Discuss the following statement:

The existence of fund managers who sell their services based on their alleged ability to select over-performing sectors and stocks and so add value to portfolios demonstrates that capital markets are not efficient. **[6]**

(c) (i) Define first and second order stochastic dominance. Illustrate the definitions by sketching cumulative distribution functions of two random variables which represent the returns on two investments, one of which dominates the other. **[5]**

(ii) Explain how an investor's economic characteristics will affect his choice of an investment that:

(a) first order stochastically dominates another **[2]**

(b) second order stochastically dominates another **[2]**

[Total:25]

Question 4

(a) Explain the following terms (stating any assumptions needed) in the context of mean-variance portfolio theory:

(i) opportunity set [2]

(ii) efficient frontier for a portfolio of risky assets [2]

(iii) indifference curves [2]

(iv) optimal portfolio [2]

(b) (i) Describe, using a sketch, the effect on the efficient frontier of introducing a risk-free asset that can be bought or sold in unlimited quantities. [2]

(ii)Explain what the Separation theorem implies about optimal investment strategies. [3]

(iii) Explain the four axioms that are required to derive the expected utility theorem. [6]

(c) (i) Describe systematic and specific risks and returns. [3]

(ii) Explain how the risks of a portfolio can be reduced by diversification, without impacting on expected return. [3]

[Total: 25]

END OF PAPER

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS & STATISTICS
MAT3100: Multivariable Calculus
Final Examination

2019/2020 Academic Year

Total time allowed: **Three (3) hours**

Instructions:

- There are **seven (7)** questions in this paper; **attempt any five (5)**.
- Show detailed working to earn full marks.

1. (a) Given that $f(x,y) = \ln|x - y^2 - 2y + 2|$,
   i. **Find** the domain of $f(x,y)$. [2 marks]
   ii. **Sketch** the domain of $f(x,y)$. [4 marks]

   (b) Find the first four terms in the **Taylor series** expansion of
   $$f(x,y) = \cos(x+y)e^{-x^2}$$ [7 marks]
   about the origin.

   (c) If $z = 2x^2 + 3xy + 4y^2$ with $u = x^2 + y^2$ and $v = x + 2y$,
   i. **Find**
   $$\frac{\partial x}{\partial u}, \frac{\partial x}{\partial v}, \frac{\partial y}{\partial u}, \frac{\partial y}{\partial v}.$$ [8 marks]
   ii. **Find** $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$. [4 marks]

2. (a) Let $f(x,y,z) = x + y + z$, $g(x,y,z) = xy + xz + yz$ and $h(x,y,z) = x^2 + y^2 + z^2$.
   i. Show that the three functions are functionally dependent. [6 marks]
   ii. Find a relation connecting the three functions. [2 marks]

(b) Find the derivative of $z = x \ln y$ at the point $(1, 2)$ in the direction making an of $30°$ with the positive x-axis. [6 marks]

(c) **Find** and **determine the nature** of the stationary point(s) of the function

$$f(x, y) = x^3 - 6xy + y^3$$ [11 marks]

3. (a) i. **Describe** the graph of the function $f(x, y, z) = \sqrt{a^2 - x^2 - y^2}$ where $a > 0$. [2 marks]

ii. **Describe** and **sketch** the level curves of $f(x, y) = y^2 - x^2$.[5 marks]

(b) Let

$$r = 2 \sin \theta,$$

be an equation of a surface in cylindrical coordinate system.

i. Find a rectangular equation for the surface. [2 marks]

ii. Sketch the graph of the equation found in (i) above in $\mathbb{R}^3$. [5 marks]

(c) Given that $y^2 = 3x$ and $y^2 = 4 - x$,

i. **Sketch** the graphs of the two functions on the same axes.[4 marks]

ii. Evaluate

$$\iint_D (x - y)\, dA,$$

where D is the region above the x-axis bounded by $y^2 = 3x$ and $y^2 = 4 - x$. [7 marks]

4. (a) Given that $x^2 + y^2 = 1$ and $z + x = 1$,

i. **Sketch**, in $\mathbb{R}^3$, on the same set of axes the graph of $x^2 + y^2 = 1$ and of $z + x = 1$. [4 marks]

ii. Find the volume, V, of the solid bounded by $x^2 + y^2 = 1$, the xy-plane and the plane $z + x = 1$. [6 marks]

(b) Use polar coordinates to find the area of the region inside the circle $x^2 + y^2 = 9$ and to the right of the line $x = \frac{3}{2}$. [8 marks]

(c) Evaluate

$$\oint_c y\, dx,$$

when c is the circle $x^2 + y^2 = 4$. [7 marks]

5. (a) Given that $f(x) = e^{-x}$:

i. **Find** the sine fourier transform of $f(x)$. [5 marks]

ii. Hence **using** inverse Fourier transform and results from $a)i$, deduce that

$$\int_0^\infty \frac{x \sin mx}{1 + x^2} dx = \frac{\pi}{2} e^{-m}.$$ [5 marks]

(b) Let $f(x)$ be a periodic function of period 2π and be given by $f(x) = x^2$ over the interval $-\pi < x < \pi$.

i. **Sketch** the graph of $f(x)$ in the interval $-3\pi < x < 3\pi$. [2 marks]

ii. **Find** the Fourier coefficients for $f(x)$ namely a_0, a_n and b_n. [6 marks]

iii. **Show** that the Fourier series for $f(x)$ in the interval $-\pi < x < \pi$ is

$$\frac{\pi^2}{6} - 4[\cos x - \frac{1}{2^2}\cos 2x + \frac{1}{3^2}\cos 3x - \cdots] \quad \text{[3 marks]}$$

iv. By using an appropriate value of x, show that

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots . \quad \text{[4 marks]}$$

6. (a) Let $f(x, y, z) = 2x^2 + 8z$ be a function in x, y and z and $\mathbf{c}(t) = (e^t, t^2, t)$, $0 \leq t \leq 1$, a curve in $\mathbb{R}^3$.

i. Calculate $f(\mathbf{c}(t))$ and $ds = ||\mathbf{c}'(t)||dt$. [4 marks]

ii. Evaluate the line integral

$$\int_C f(x, y, z)ds. \quad \text{[5 marks]}$$

(b) i. Find the Laplace transform

$$\mathcal{L}[t\cos\omega t]. \quad \text{[5 marks]}$$

ii. Using your results in $(b), (i)$ above evaluate the integral

$$\int_0^\infty e^{-2t}t\cos t dt. \quad \text{[3 marks]}$$

(c) Using Laplace transforms, find the solution of the differential equation

$$y'' + y = t,$$

satisfying the initial conditions

$$y(0) = y'(0) = 0. \quad \text{[8 marks]}$$

7. (a) i. Find the parametrizations of the portion of the plane S with equation $3x - 4y + z = 1$ where $0 \leq x, y \leq 1$ and calculate the normal vector $\mathbf{n}(x, y) = \mathbf{r}_x \times \mathbf{r}_y$. [4 marks]

ii. Calculate the surface integral

$$\int\int_S \mathbf{F} \cdot dS$$

where S is the portion of the plane $3x - 4y + z = 1$, $0 \leq x, y \leq 1$ and $\mathbf{F} = \langle y, z, x \rangle$. [5 marks]

(b) For the vector field $\mathbf{F} = \langle y^2, 2xy + e^z, ye^z \rangle$

i. Find the $\mathbf{Curl}(\mathbf{F})$. [3 marks]

ii. Determine whether $\mathbf{F}$ is conservative and hence find a potential function of $\mathbf{F}$. [5 marks]

(c) i. State Stoke's theorem. [3 marks]

ii. Use Stoke's theorem to evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle z^2, y^2, x \rangle$ and C is the boundary of the plane $x + y + z = 1$ and $0 \leq x \leq 1, 0 \leq y \leq -x+1$. [5 marks]

END OF EXAMINATION

The University of Zambia
School of Natural Sciences
Department of Mathematics and Statistics

2019/20 Academic Year Examinations

MAT 3110 Engineering Mathematics II

November 30, 2020.

Duration: Three hours

Instructions:

- Do not open this booklet until you are told to do so.
- This paper consists of six questions, each carry 20 marks. Attempt any FIVE questions.
- Show all the essential working to earn full marks.
- Non programmable calculators are allowed.

---

1. (a) Find a power series solution in powers of x of the differential equation

$$(1-x^2)y'' - 2xy' + 2y = 0.$$

(b) Solve the following initial value problem

$$x^2y'' - 5xy' + 8y = 0,\ y(1) = 5,\ y'(1) = 18.$$

(c) Find the general solution to the following system of first order differential value problem.

$$\mathbf{x}' = \begin{pmatrix} -3 & 2 \\ -1 & -5 \end{pmatrix} \mathbf{x}.$$

2. (a) Solve the initial value problem below using Laplace transforms.

$$y'' + 9y = \begin{cases} 8\sin t, & : 0 \le t < \pi \\ 0, & : t \ge \pi. \end{cases} \qquad y(0) = 0,\ y'(0) = 4.$$

(b) Determine the Laplace transform of

$$f(t) = \sinh t \cos t.$$

(c) Determine the inverse Laplace transform of

$$F(s) = \frac{6}{(s+1)^3}.$$

3. (a) Evaluate the following double integrals.

i.

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} \sqrt{x^2+y^2}\, dydx$$

ii.

$$\int_{-3}^{0}\int_{1}^{3} 10 + xy\sin(x^2-y^2)\, dydx$$

(b) Use a double integral to determine the volume of the region bounded by $z = 3 - 2y$, the surface $y = 1 - x^2$ and the planes $y = 0$ and $z = 0$.

(c) Determine the volume that lies under $f(x,y) = 9x^2 + 4xy + 4$ and above the rectangle $R = \{(x,y) : -1 \leq x \leq 1,\ 0 \leq y \leq 2\}$ in the xy-plane.

4. (a) Evaluate the following triple integrals.

i.
$$\int_{-4}^{4}\int_{0}^{\sqrt{16-y^2}}\int_{0}^{6+x} 6yx^2\ dzdxdy$$

ii.
$$\int_{0}^{2}\int_{0}^{4}\int_{z}^{2} yze^{x^3}\ dxdydz$$

(b) Find the volume of the solid that is between $x^2+y^2+z^2 = 4$ and $x^2+y^2+z^2 = 25$, and inside $z = -\sqrt{\frac{1}{3}x^2 + \frac{1}{3}y^2}$.

(c) Convert the triple integral
$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{\sqrt{x^2+y^2}} xyz\ dzdydx$$
into a triple integral in cylindrical coordinates and evaluate.

5. (a) Evaluate
$$\int_C (3x^2 - 2y)\ ds$$
where C is the line segment from $(3,6)$ to $(1,-1)$.

(b) Let $\vec{\mathbf{F}}(x,y) = \left(2xy - 4 - \frac{1}{2}\sin\left(\frac{x}{2}\right)\sin\left(\frac{y}{2}\right)\right)\ \mathbf{i} + \left(x^2 + \frac{1}{2}\cos\left(\frac{x}{2}\right)\cos\left(\frac{y}{2}\right)\right)\ \mathbf{j}$.

i. Prove that $\vec{\mathbf{F}}$ is conservative.

ii. Evaluate
$$\int_C \vec{\mathbf{F}} \cdot d\mathbf{r}$$
where C the portion of the circle centered at the origin, having radius 2 and is in the 1st quadrant and oriented counter clockwise.

(c) Use Green's theorem to evaluate the line integral
$$\int_C \vec{\mathbf{F}} \cdot d\mathbf{r}$$

where $\vec{\mathbf{F}}(x,y) = xy^2\mathbf{i} + (1 - xy^3)\ \mathbf{j}$ and C is the parallelogram with vertices $(-1,2)$,$(-1,-1)$, $(1,1)$ and $(1,4)$. The orientation of C is counterclockwise.

6. (a) Evaluate

$$\int\int_S 6xy\ \mathrm{dS}$$

where S is the portion of the plane $x + y + z = 1$ that lies in the 1st octant and is in front of the yz-plane.

(b) Use Stoke's theorem to evaluate the integral

$$\int\int_S \mathrm{curl}\ \vec{\mathbf{F}} \cdot \mathrm{dS}$$

where $\vec{\mathbf{F}}(x,y,z) = 2y\ \mathbf{i} + 3x\ \mathbf{j} + (z-x)\ \mathbf{k}$ and S is the portion of $y = 11 - 3x^2 - 3z^2$ in front of $y = 5$ with orientation in the positive y-axis direction.

(c) Use the divergence theorem to evaluate

$$\int\int_S \vec{\mathbf{F}} \cdot \mathrm{dS}$$

where $\vec{\mathbf{F}}(x,y,z) = (4x - z^2)\ \mathbf{i} + (x + 3z)\ \mathbf{j} + (6 - z)\mathbf{k}$ and S is the surface of the solid bounded by the cylinder $x^2 + y^2 = 36$ and the plane $z = -2$ and $z = 3$. Note that all the three surfaces of the solid are included in S.

End of Examination

The University of Zambia
School of Natural Sciences
Department of Mathematics & Statistics

2019/2020 ACADEMIC YEAR

FINAL EXAMINATIONS

MAT3601 : Mathematical Statistics

Time Allowed: Three (3) Hours

Instructions:

- There are Five (5) questions in this paper, Attempt any **Four (4)** questions.
- Full credit will only be given where all the necessary work is shown
- Some formulae are given on the last page
- You must write your **Computer Number** on each answer booklet used.

---

1. **a)** Define each of the following:
   (i) Chi-square distribution (no pdf).
   (ii) An F- distribution (no pdf).
   (iii) j^{th}–order statistics

   b) Let X and Y be independent random variables each having a standard normal distribution, i.e means 0 and variances 1.

   Let $Z = X^2 + Y^2$ *and* $U = \frac{X}{Y}$

   (i) Find the joint probability function of Z and U.
   (ii) Find the marginal probability density function of Z and U
   (iii) Are Z and U independent? Justify your answer

   c) Prove each of the following:
   (i) If the random variables $X_1, X_2, X_3, \ldots, X_k$ are independent and $X_j \sim N(\mu_j, \sigma_j^2)$ then the random variable $Y = \sum_{j=1}^{k} \left(\frac{X_j - \mu_j}{\sigma_j}\right)^2$ has a Chi-square distribution with k-degrees of freedom.

   (ii) If X and Y are independent gamma random variables with $X \sim g(\alpha, \lambda)$ and $Y \sim g(\beta, \lambda)$ then the random variable $X + Y \sim g(\alpha + \beta, \lambda)$.

2. **a)** Define the following:
   (i) Mean-squared error of an estimator T of θ.
   (ii) Consistent estimator for θ.
   (iii) Sufficient statistics for θ

   b) Let $X_1, X_2, \ldots X_n$ be a random sample of size n from a uniform distribution on the interval (θ-1, θ+1).

   $f(x, \theta) = \frac{1}{2}, \quad \theta - 1 < x < \theta + 1, \quad \theta > 0$ Find the following:
   (i) Method of moments estimator for θ
   (ii) Maximum likelihood estimator for θ
   (iii) Sufficient statistics for θ

   c) Let X and Y have a joint probability function

   $f(x, y) = 4x(1 - y), \quad x \in (0, 1)$ *and* $y \in (0, 1)$. Find the following:

   (i) $P(2X - Y > 1)$
   (ii) Conditional probability density function of Y given $X = \frac{1}{3}$
   (iii) $E(Y/X = \frac{1}{3})$.

3 **a)** Define the following:
(i) Relative efficient of T_2 with respect to T_1.
(ii) Power function of a test γ.
(iii) Size of a test γ

b) Let X be a random variable with probability density function

$f(x,\theta) = 3\theta x^2 e^{-\theta x^3}$, $x > 0$, $\theta > 0$. Find the following:

(i) Cumulative distribution function (cdf) of X.
(ii) Probability density function of the 2nd –order statistics.
(iii) Probability function of $Y = \theta X^3$
(iv) Cramer-Rao lower bound for θ.

c) Let X_1, X_2 be a random sample of size two from

$$f(x,\theta) = \theta x^{\theta-1}, \quad 0 < x < 1, \qquad \theta > 0$$

Consider the hypotheses H_0 : θ = 1 and H_a θ = 2, find the following
(i) A most powerful test of size α
(ii) Size of the test γ : reject H_0 if $X_1 + X_2 > \frac{3}{2}$
(iii) The power of the test γ in (ii) above

4. a) Define each of the following:
(i) Most powerful test of size α
(ii) Uniformly most powerful test of size α
(iii) Monotone likelihood ratio.

b) (i) State the Neyman - Pearson lemma.
(ii) Prove the Neyman - Pearson lemma.
(iii) Let X_1, X_2, ...X_n be a random sample of size n from

$f(x,\theta) = \frac{\theta}{(x+1)^{\theta+1}}$, $x > 0$, $\theta > 0$

Construct a most powerful test of size α for testing H_0 θ = 2 vs H_a θ = 4.

c) (i) Let X_1, X_2, ... X_n be a random sample of size n from exponential with pdf

$f(x,\theta) = \theta e^{-\theta x}$, $x > 0$, $\theta > 0$. Construct a generalized likelihood ratio test of size α for testing H_0: θ = 2vs H_a: θ≠2.
(ii) Show that the Poisson distribution has the monotone likelihood ratio property
(iii) Let X_1, X_2,.....X_n be a random sample of size n from Poisson with parameter μ. using the result of (ii) above, construct a uniformly most powerful test of size α for testing H_0 : μ = μ_0 vs H_a : μ > μ_0 .

5. a) Define each of the following:
   (i) Simple likelihood ratio test.
   (ii) Generalized likelihood ratio
   (iii) Generalized likelihood ratio test of size α

b) (i) Define a t-distribution without specifying its pdf
   (ii) Derive the probability density function for a t-distribution with m-degrees of freedom.

c) Let $X_1, X_2, \ldots X_n$ be a random sample of size n from

$$f(x,\theta) = \theta(1-\theta)^x, \quad 0 < \theta < 1, \quad x = 0,1,2,3, \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$

   (i) Find the maximum likelihood estimator for θ.
   (ii) Find the Cramer-Rao lower bound for θ

FORMULAE

$$If\ Y \sim \chi^2_V, then \quad f(y) = \frac{y^{\alpha-1}e^{-\frac{y}{2}}}{2^{\alpha}\Gamma(\alpha)}, where\ \alpha = \frac{V}{2}, \quad y > 0, \quad V > 0$$

$$If\ Z \sim N(0,1), then\ f(z) = \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}}, \quad z \in (-\infty, \quad \infty)$$

$$If\ X \sim g(\alpha,\lambda), then \quad f(x) = \frac{x^{\alpha-1}e^{-\frac{x}{\lambda}}}{\lambda^{\alpha}\Gamma(\alpha)}, \quad \lambda > 0, \quad x > 0, \quad \alpha > 0$$

$$M_X(t) = \left(\frac{1}{1-2t}\right)^{\frac{k}{2}} \ mgf\ for\ \chi^2_k, \qquad \int_{-\infty}^{\infty} \frac{1}{x^2+1}dx = 2\pi$$

END OF EXAM

The University of Zambia

School of Natural Sciences

Department of Mathematics and Statistics

2019/20 Academic Year Examinations - November 2020

MAT4001 - Life & Health Insurance Mathematics

Time allowed : 3hrs Total marks : 100

Instructions:

- Attempt **all five (5)** questions.
- Marks available per question are indicated in [ ] at the end of each question. Each questions carries 20 marks.
- Indicate your **Computer Number** on all your booklets.
- **Full credit** will only be given when **necessary work** is shown.

This paper consists of 4 pages.

Some useful formulas

Annuities and assurances

$A^{1}_{x:\overline{n}|} = A_x - v^n \, {}_np_x \, A_{x+n}$

$\ddot{a}_{x:\overline{n}|} = \ddot{a}_x - v^n \, {}_np_x \, \ddot{a}_{x+n}$

$A_x = 1 - d\,\ddot{a}_x$

$A_{x:\overline{n}|} = 1 - d\,\ddot{a}_{x:\overline{n}|}$

1. a) With reference to their defining characteristics, distinguish between the following products:

(i) Whole life assurance

(ii) Term assurance

(iii) Pure endowment

(iv) Endowment assurance

(v) Whole life level annuity

(vi) Temporary level annuity

b) Derive expressions for the expected present value and variance of the payments resulting from the following products for an individual aged x exact. Annotate your derivations.

(i) An n-year temporary life assurance with sum assured S. Assume payments are made on the first birthday after death.

(ii) An n-year temporary immediate unit annuity paid in arrears.

Note: You may assume the premium conversion formula without proof.

[6, 14]

2. a) Explain the terms select and ultimate in the context of mortality tables.

b) Assuming the PMA92Base table applies, calculate ${}_{1.25}p_{50.5}$ using two different methods, clearing stating any assumptions you make.

c) A life office uses a particular mortality table with a select period of two years based on empirical data. The following relationships hold for the mortality rates at ages $x = 20$ and over:

$$q_x = 0.0004x \qquad q_{[x]} = 0.003 \qquad \text{and} \qquad q_{[x]+1} = 0.4q_{x+1} \, .$$

(i) Construct a life table of $l_{[x]+t}$ for ages $x = 30, 31, 32$ and $t = 0, 1, 2$ given that $l_{32} = 100000$.

(ii) What is the probability that a person selected at age 30 exact will be aged 31 last birthday at the date of death?

(iii) What is the probability that a person now aged 32 exact, but was selected one year ago, will survive to age 34?

[4, 4, 12]

3. a) A term assurance contract for a life aged 50 exact for a term of 10 years provides a benefit of K10,000 payable at the end of the year of death. Calculate the expected present value and variance of benefits payable under this contract.

Basis

Mortality: AM92 Select

Interest: 4% per annum

b) A life insurance company issues identical deferred annuities to each of 100 women aged 63 exact. The benefit is K5,000 per annum payable continuously from a woman's 65th birthday, if still alive at that time, and for life thereafter.

(i) Write down an expression for the random variable for the present value of future benefits for one policy at outset.

(ii) Calculate the total expected present value at outset of these annuities.

Basis

Mortality: PFA92C20

Interest: 4% per annum

(iii) Calculate the total variance of the present value at outset of these annuities, using the same basis as in part (ii).

[7, 13]

4. a) A life office issued a 25-year temporary assurance policy to a life aged 30 exact with a sum assured of K75,000 payable at the end of year of death. Premiums are payable annually in advance for 20 years or until earlier death.

(i) Show that the annual net premium for the policy is approximately equal to K104 using the basis given below.

(ii) Calculate the net premium reserve at the start and at the end of the 20th year of the policy.

Basis

Mortality: AM92 Ultimate

Interest: 4% per annum

b) (i) John is aged 40 and would like to retire at age 65 with a whole life pension of K10000 pa, paid on a continuous basis. He asks you to calculate the net annual premium required between now and retirement to fund this pension, assuming he makes annual payments starting immediately.

Basis:

Mortality: AM92 Select

Interest: 4% per annum.

(ii) On reaching age 55, John decides that he would actually like to retire at age 60. Assuming John has paid the net premiums calculated above over the last 15 years, calculate the increase required to his net annual premium going forward to fund the earlier retirement. Using the same basis as above, calculate the existing value of John's benefit using a retrospective policy value approach.

[6, 14]

5. a) For each of the following health and care products listed below, give a definition, an outline of the benefits and the circumstances/nature of claims that arise.

(i) Income protection

(ii) Critical illness

(iii) Long-term care

b) You have been invited to an UNZA Radio programme to discuss the new National Health Insurance Management Authority (NHIMA) Scheme and its co-existence with Private Medical Insurance (PMI). Prepare summary notes for your presentation touching on some of the following points.

A - Why NHIMA?

B - PMI as an optional alternative?

C - PMI an optional complement?

D - PMI as a compulsory alternative?

E - PMI as a compulsory complement?

c) Present comparison of Unit Linked and Non-linked Insurance Plans.

[6, 10, 4]

END!

THE UNIVERSITY OF ZAMBIA
DEPARTMENT OF MATHEMATICS AND STATISTICS
2019/2020 ACADEMIC YEAR EXAMINATION
MAT4011 – OPERATIONAL RISK MANAGEMENT
23RD NOVEMBER 2020

Time Allowed: Three (3) hours

INSTRUCTIONS TO CANDIDATES

1. Attempt ALL FOUR (4) questions
2. Start the answer to each full question on a fresh page of the answer booklet.
3. Write neatly and legibly, showing ALL numbering and workings were necessary

REQUIREMENTS: NONE

Question One **[Total 25 Marks]**

(a) Define four major risks [apart from operational risk] which can be effectively modelled quantitatively. Explain the reasons for each choice. [4 marks]

(b) Define four major risks which cannot be effectively modelled quantitatively. Explain the reasons for each choice. [4 marks]

(c) Explain any four challenges of quantitative risk management [8 marks]

(d) Assuming that a company already has an office for a Chief Risk Officer. Describe the six implications of the definition of enterprise risk management (ERM) and outline the ERM framework practices that a firm could introduce to reduce risk without incurring significant additional direct or indirect cost to the firm as a whole. [9 marks]

Question Two **[Total 25 Marks]**

(a) Describe the nature and elements of the Top down models approach in assessing operational risks. [7 marks]

(b) Outline the key elements of a typical operational risk management policy. [6 marks]

(c) Consider a risk manager addressing the road accident risk for the company truck fleet. S/he wants to represent the three parameters (exposure, occurrence, and impact) of the vulnerability distribution using an Actuarial Approach/Bayesian network.

Given that exposure is measured by; truck driver qualification {Low (10%), Moderate (60%), High (30%)}, road used {Freeway (70%), Urban motorway (10%), Minor Road (20%)} and Speed {Low (70%), Moderate (20%), High (10%)}. Accident impact depends solely on speed with estimated liabilities at {Low (15000), Moderate (55 000), High (150 000)}

Construct a probability tree diagram for this case and compute the expected cost of road accidents. [7 marks

(d) You are given information on a medium-sized general insurance company and a medium-sized bank in Table 1.

Table 1: Gross Income/Premiums for Bank T and Insurer C

Year	Bank T	Insurer C
2017	700000	-
2018	900000	900000
2019	850000	850000

You are given that:
Alpha = 15%
NL = 75000 and
ORCI = 2%*{max(GP_1,NL) + max(0, $|GP_1 - GP_0|$ - 0.2 * GP_0)}

Calculate the operational risk capital charge under the Basel II basic indicator approach and solvency II. Comment on your results. Define all notation used. [5 marks]

Question Three **[Total 25 Marks]**

(a) In the course of creating a single European market for financial services and in the wake of two financial crises, regulatory frameworks in the financial services industry in the European Union have undergone significant change. One of the major reforms has been the transition from static rules-based systems towards principles-based regulation with the intent to better capture the risk situation of an undertaking. For insurance companies, the regulatory framework Solvency II implementation began after 2013. At the same time, the regulatory regime for banking, Basel II, has been revised in response to the financial crisis; the new version is Basel III.

Present a summative structured comparative assessment of similarities and differences under Basel II/III and Solvency II showing at **least three** broad criterions under each of the pillars I, II and III. [15 marks]

(b) Describe any one application of extreme value theory (EVT) in insurance and give reference to key historical events. [2 marks]

(c) Consider the *Gamma*(0.5, 0.005) and *Pa*(4, 300) distributions, both of which have a mean of 100 and a variance of 20,000. Use the 'limiting density ratio' method to compare these two distributions. [3 marks]

NB: Given the PDFs: $f(x) = \frac{\alpha\lambda^{\alpha}}{(\lambda + x)^{\alpha+1}}$ and $f(x) = \frac{\lambda^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}$

(d) In a feasibility of a Wind Energy Farm with a consultant has suggested assessing the risk of damage by using the generalised Pareto distribution (GPD) to model the likelihood of wind speeds of above 150 miles per hour, using the country level wind data.

Describe how this would be done and any challenges. [5 marks]

Question Four **[Total 25 Marks]**

(a) Read the following IFoA response to Basel Committee on Banking Supervision: Consultative Document Standardised Measurement Approach for Operational Risk

1. The Institute and Faculty of Actuaries (IFoA) welcomes the opportunity to respond to the Basel Committee on Banking Supervision's (BCBS) consultative document on Standardised Measurement Approach (SMA) for Operational Risk. The IFoA's Risk Management Board has had responsibility for drafting this response. Under that Board's oversight sits the IFoA's Operational Risk Working Party, which has contributed much of the content of the response. General Comments

2. The IFoA is disappointed the Committee has decided to discard the Advanced Measurement Approach (AMA). Our disappointment is for the following reasons:

- Many banks have invested considerable resources in developing plans to meet the requirements for AMA approval;
- There will be no regulatory capital benefit from this investment;
- The withdrawal of the AMA will probably reduce investment in risk modelling which, in turn, may have an adverse effect on risk management; and
- This decision removes an element of flexibility from the regulatory capital regime, particularly where standardised requirements are not proportionate.

3. One example of where retaining AMA could have provided regulatory benefits would be for banks, with high net interest margins and/or fees, which would have had no need to introduce further adjustments to an already complex calculation.

4. The IFoA agrees with the Committee's view that the variability of modelling approaches adopted for AMA could exacerbate the variability of Risk-Weighted Asset (RWA) figures for operational risk. We believe, due to the idiosyncratic nature of operational risk, there will always be variability in the underlying operational risk exposures. These will vary in accordance with different business models and control frameworks. The operational risk exposure of a global private bank catering to high net worth clients will be different from a retail bank operating within one particular country. Naturally both of these different banking models will differ significantly from an investment bank.

(i) Explain how the implementation of AMA leads to the issues raised above. [6 marks]

(ii) Describe how the adoption a model risk management framework may assist in the reversal of the Basel Committee's stance towards AMA. [5 marks]

(b) Define stress-testing and outline the key inputs for such an exercise for a typical Zambia life insurer in light of the COVID-19 pandemic effects. [7 marks]

(c) Write short notes on the following operational risk process models components.

(i) specific tools [4 marks]

(ii) management actions [3 marks]

END OF EXAMINATION

The University of Zambia

School of Natural Sciences

Department of Mathematics and Statistics

2019-20 Academic Year

End of Year University Examinations

MAT4022: Pension Design and Valuation

Time: Three hours. [Maximum Marks : 100]

1. Attempt All questions. All questions carry equal marks.
2. Write clearly your computer number on each answer booklet.
3. Show all essential working

Some Useful Formulas:

Probabilities

Joint life

$$ {}_tp_{x:y} = {}_tp_x {}_tp_y \quad , \quad {}_tq_{x:y} = {}_tq_x + {}_tq_y - {}_tq_{\overline{x:y}} $$

Last survivor

$$ {}_tq_{\overline{x:y}} = {}_tq_x {}_tq_y \quad , \quad {}_tp_{\overline{x:y}} = {}_tp_x + {}_tp_y - {}_tp_{x:y} $$

Joint Assurances and annuities

$$ A_{\overline{x:y}} = A_x + A_y - A_{x:y} \quad , \quad \ddot{a}_{\overline{x:y}} = \ddot{a}_x + \ddot{a}_y - \ddot{a}_{x:y} $$

Contingent Assurances

$$ A^1_{x:y} + A_{x:y}^{\ 1} = A_{x:y} $$

Reversionary annuities

$$ \ddot{a}_{x|y} = \ddot{a}_y - \ddot{a}_{x:y} \cong a_y - a_{x:y} = a_{x|y} $$

Independent and Dependent joint life functions

$$ \begin{aligned} (aq)^k_x &= \frac{(ad)^k_x}{(al)_x} \\ (al)_{x+1} &= (al)_x - (ad)^k_x - (ad)^{-k}_x \\ (aq)^k_x &= \frac{\mu^k_x}{\sum_j \mu^j_x}\left(1 - e^{\sum_j \mu^j_x}\right) \\ q^k_x &= \frac{(aq)^k_x}{1 - \frac{1}{2}(aq)^{-k}_x} \qquad \text{where } -k \text{ indicates the state not } k \\ (aq)^k_x &= q^k_x \left[1 - \frac{1}{2}\sum_{i\neq k} q^i_x + \frac{1}{3}\sum_{i\neq k}\sum_{i\neq j\neq k} q^i_x q^j_x + \cdots\cdots\right] \\ (aq)^k_x &= \int_0^1 {}_t(ap)_x (a\mu)^k_{x+t} dt \end{aligned} $$

QUESTION ONE

(a) (i) Define the term "sponsor covenant". [2]

(ii) Explain the circumstances under which the sponsor covenant should/should not be considered. [3]

(b) A large company that operates a final salary pension scheme is considering a proposal to provide employees with an income in the event of their being unable to work due to ill health.

(i) Describe how such a benefit might be provided through the scheme. Your answer should include comments on the consistency of the benefit with other scheme benefits. [3]

(ii) Outline the conditions and safeguards which you might expect to see included in the Trust Deed and Rules of the scheme in relation to the payment of an ill-health pension. [2]

(c) The government of a country is reviewing its State provided retirement benefits scheme, which was first set up 20 years ago. The review has been prompted as the cost of the current scheme is proving to be an increasing burden on the States finances.

The Scheme provides a flat rate pension at age 60, which is payable for life and available to all citizens who have paid at least ten years' State retirement contributions (at a rate of 3% of wages).

The benefit was set at a level equivalent to 40% of national average wages when the system was set up; since then the flat rate has been adjusted annually in line with the State measure of price inflation.

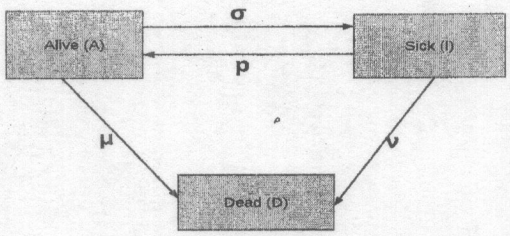
(i) Suggest possible reasons for the increasing cost burden. [4]

(ii) Discuss the measures the State could introduce to alleviate the problem highlighting any difficulties that might be encountered in implementing the proposed new measures. [6]

[Total 20 marks]

QUESTION TWO

(a) A three-state transition model is shown in the following diagram:



Assume that the transition probabilities are constant at all ages with $\rho = 1\%$, $\mu = 2\%$, $\nu = 4\%$, and $\sigma = 5\%$.

Calculate the present value of a sickness benefit of K2,000 p.a. paid continuously to a life now aged 40 exact and sick, during this period of sickness, discounted at 4% p.a. and payable to a maximum age of 60 exact. [7]

(b) A life office issues an assurance to a married couple, husband aged 65 exact and wife aged 60 exact. The sum assured of K100, 000 will be paid at the end of the year of the second death. The couple pay a lump sum for this assurance. Calculate the net premiums payable, assuming: PMA92C20 for the husband and PFA92C20 for the wife, interest rates of 4% per annum. [6]

(c) A reversionary annuity is payable continuously beginning on the death of a life aged x to an annuitant aged y .

(i) Derive an expression for the present value of the reversionary annuity using random variables for the future lifetimes. [4]

(ii) Derive an expression for the expected present value of the reversionary annuity in terms of assurance functions. [3]

[Total 20 marks]

QUESTION THREE

(a) Describe three distinct methods of averaging salary that might be defined in the scheme rules of a pension fund. [3]

(b) A population is subject to two modes of decrement α and β where

$$q_x^{\beta} = \frac{1}{3} + \frac{1}{4} q_x^{\alpha}$$

Using the assumptions

- Equal forces in the multiple and single decrement tables
- Uniform distribution of all decrements across year of age

Derive from first principles $(aq)_x^{\beta}$.

[5]

(c) The table extract below shows the death statistics in a population due to cancer and other causes.

Age (x)	Population $(al)_x$	Cancer $(ad)_x^c$	Others $(ad)_x^o$
61	10,000	55	248
62	9,697	175	533
63	8,989		

Recent changes have resulted in an estimate that the annual independent cancer death rate is reduced by 20% than that previously used.
Calculate a revised table assuming no changes to the other independent death rates.

(i) Calculate dependent rates $\left[(aq)_x^k\right]$ from the information given. [2]

(ii) Calculate approximate independent rates $\left[q_x^k\right]$ [3]

(iii) Make the necessary adjustment given and calculate approximate dependent rates $\left[(aq)_x^k\right]$ from the adjusted indepndent rates. [4]

(iv) Reconstruct the Revised Table. [3]

[Total 20 marks]

QUESTION FOUR

(a) Give definations of a unit fund and a sterling fund. Discuss how the two funds differ. [5]

(b) A life insurance company issues a three-year unit-linked endowment assurance contract to a male life aged 62 exact under which level annual premiums of K10,000 are payable in advance throughout the term of the policy or until earlier death. 85% of each year's premium is invested in units at the offer price.

There is a bid-offer spread in unit values, with the bid price being 95% of the offer price.

There is an annual management charge of 1.25% of the bid value of units. Management charges are deducted at the end of each year, before death or maturity benefits are paid.

On the death of the policyholder during the term of the policy, there is a benefit payable at the end of the year of death of K20,000, or the bid value of the units allocated to the policy, if greater. On maturity, 115% of the full bid value of the units is payable.

The company uses the following assumptions in carrying out profit tests of this contract:

Mortality: AM92 Ultimate

Initial expenses: K600

Renewal expenses: K100 at the start of each of the second and third policy years

Unit fund growth rate: 8% per annum

Non-unit fund interest rate: 4% per annum

Non-unit fund reserve basis: AM92 Ultimate mortality, interest 4% per annum

Risk discount rate : 15% per annum

Calculate the profit margin on the contract. [15]

[Total 20 marks]

QUESTION FIVE

(a) Outline the criteria used to appraise funding methods. [5]

(c) Using the following assumptions and data:

- the discount rate ($i = 6\%$pa);
- the rate at which earnings grow ($e = 4\%$pa);
- the rate at which pensions increase in payment ($p = 3\%$);
- the rate at which leaving service benefits are revalued ($r = 3\%$);
- the age at which members retire from the scheme ($R = 65$);
- the age at which members enter the scheme (entry age $E = 30$); and
- the rate at which new members enter the scheme (new entrant rate) $\rho = 0.275$.
- the accrual rate is 60ths, and
- the value of annuity payable from age R (allowing for any contigent, spouse's pension or pension increases) is $a'_R = 15$
- Assume that salaries are payable continuously throughout the year.

(ignore all pre-retirement decrements, such as mortality, withdrawal, ill-health, etc).

(i) Calculate the Standared contribution rate and the Actuarial Liability for a scheme that contains only these three members under the Attained age method and Entry age method and comment on your results.

* 25 year old, no past service, salary 20,000
* 40 year old, 15 years past service, salary 15,000
* 55 year old, 30 years past service, salary 30,000

[15]

[Total 20 marks]

======END OF MAT4022 EXAMINATION ======

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS & STATISTICS

2019/2020 Academic Year
End of Year Final Examinations

19 November 2020 (am)

MAT4032 – Financial Engineering

Time allowed : **Three (3) hours** Full marks : **100**

Instructions:
- There are **six (6)** questions in this paper. Attempt **any four (4)** questions
- **All** questions carry **equal** marks. Mark allocations are shown in brackets.
- **Full credit** will only be given when **necessary work** is shown.
- Indicate your **computer number** on all answer booklets used.

This paper consists of 5 pages of questions..

---

1. (a) Let B_t be a standard Brownian motion, and let $F_t = \sigma(B_s, 0 \leq s \leq t)$ be its natural filtration.

   (i) Derive the conditional expectations $E\left[B_t^2|F_s\right]$ and $E\left[B_t^4|F_s\right]$ where $s \leq t$.

   You may assume that the third and fourth moments of a random variable with distribution $N\left(0, \sigma^2\right)$ are 0 and $3\sigma^4$, respectively. **(4)**

   (ii) Hence construct a martingale out of B_t^2. Justify your answer using Ito's lemma. **(3)**

   (b) (i) Outline the three types of credit risk models. **(4)**

   (ii) Describe how the Merton model can be used to estimate the risk-neutral probability of default. **(2)**

(c) Let r be the constant continuously compounded risk-free rate and d be the constant recovery rate for a defaultable zero-coupon bond in a two state model for credit rating with a deterministic transition intensity.

(i) State the formula for the bond price. **(1)**

(ii) Determine the risk-neutral default intensity if the zero-coupon bond price is given by:

$$P(t,T) = e^{-(T-t)}\left[1-(1-\delta)\left(1-e^{-\frac{T^3-t^3}{6}}\right)\right].$$ **(2)**

(iii) Calculate the fair price of an insurance contract which pays K1,200,000 after two years if the bond defaults in the first year and the continuously compounded risk-free rate is 2% p.a. **(4)**

(d) (i) State Lévy's Theorem. **(1)**

(ii) By applying Lévy's Theorem, show that the scaled Wiener process is a Wiener process in its own right. **(4)**

2. (a) Define a Weiner process $\{W_t : t \geq 0\}$. **(4)**

(b) (i) State the stochastic differential equation that implies that the price of an asset at time t, $S(t)$, follows a geometric Brownian motion. **(1)**

(ii) Describe and discuss the plausibility of the assumptions behind this equation. **(5)**

(iii) Solve the stochastic differential equation in part (i). **(5)**

(c) The price S_t of a share at time t is modelled as geometric Brownian motion. If $\mu = 25\%$ p.a. and $\sigma = 20\%$ p.a., calculate the probability that the share price will exceed 45 in four months' time given that its current price is 38. **(5)**

(d) Under the real-world probability measure P, the process W_t is a standard Brownian motion and the price of a stock at time t is given by

$$S_t = S_0 e^{\sigma W_t + (\mu - \frac{1}{2}\sigma^2)t}.$$

The continuously compounded risk-free rate of interest is r and a zero coupon bond with maturity T has price $B_t = e^{-r(T-t)}$. Suppose that in the market any contract which pays $f(S_T)$ at time T is valued at

$$p_t = \mathrm{E}_P\left[e^{-r(T-t)} f(S_T)\Lambda_T | F_t\right],$$

where $\Lambda_t = e^{\mu W_t - \frac{1}{2}\mu^2 t}$ for all $t \leq T$ and for some real number μ.

(i) Prove, using Ito's formula, that Λ_t is a martingale. **(3)**

(ii) Show that $\mathrm{E}\left[e^{\mu W_t}\right] = e^{\frac{1}{2}\mu^2 t}$. **(2)**

3. (a) The following unconventional model has been suggested for short-term interest rates, r:

$$\mathrm{d}r = \mu r\,\mathrm{d}t + \sigma r\,\mathrm{d}Z\,,$$

where Z is a Wiener process and μ and σ are fixed parameters. Outline the properties of the model and comment on its appropriateness. **(5)**

(b) Outline the properties of the following two models for interest rates:

(i) the one-factor Vasicek model, **(2)**

(ii) the Cox–Ingersoll–Ross model. **(2)**

(c) Let W_t be a standard Brownian motion, and define $X_t = e^{W_t}$ for all $t \in [0, \infty)$.

(i) Find $\mathrm{P}(W_1 > 2 | W_2 = 1)$. **(3)**

(ii) Find $\mathrm{E}(X_t)$ for all $t \in [0, \infty)$. **(2)**

(iii) Find $\mathrm{var}(X_t)$ for all $t \in [0, \infty)$. **(3)**

(iv) Find $\mathrm{cov}(X_s, X_t)$ where $0 \leq s \leq t$. **(4)**

(d) Explain, with the aid of examples, what is meant by the terms "credit event" and "recovery rate". **(4)**

4. (a) State Ito's lemma as it applies to a stochastic process $\{X_t : t \geq 0\}$ and a function $f(X_t)$ which is not explicitly dependent on t. **(2)**

(b) (i) Apply Ito's lemma with $f(x) = x^4$ to calculate the stochastic differential $\mathrm{d}W_t^4$, where W_t is a Weiner process. **(3)**

(ii) Hence express the Ito integral $\int_0^t W_s^3\,\mathrm{d}W_s$ in terms of W_t and of an ordinary integral involving W_s. **(3)**

(c) State the Cameron–Martin–Girsanov theorem. **(3)**

(d) (i) State the martingale representation theorem, including conditions for its application, defining all terms used. **(3)**

(ii) Let S_t denote the price of an underlying security at time t; r denotes the risk free rate of return expressed in continuously compounded form, B_t represents an accumulated bank account at time t that earns the risk free rate of return.

Let X be any derivative payment contingent on F_T, payable at some fixed future time T, where F_T is the sigma algebra generated by S_u for $0 \leq u \leq T$.

You may assume that, under the equivalent measure Q, the process $D_t = e^{-rT} S_t$ is a martingale and that $\mathrm{d}S_t = B_t(rD_t\,\mathrm{d}t + \mathrm{d}D_t)$.

Show that the value of this derivative at time $t < T$ is $V_t = e^{-r(T-t)}\mathrm{E}_Q[X|F_t]$. **(11)**

5. (a) Consider a trading position which involves

- a short position in a call option with a strike price $K = 60$ and a price $c = 15$,
- a short position in a put option with a strike price $K = 60$ and price $p = 12$.

Both options have the same underlying stock and the same expiration date. Find and draw the payoff diagram for this position as a function of S_T. **(4)**

(b) A share with price S_t at time t is currently worth K140. The continuously-compounded risk-free rate is 4% per annum.

(i) Calculate the fair price of a forward contract written on the share at time $t = 0$ with expiry at time $t = 6$. **(2)**

(ii) An investor takes a long position in the forward contract at time 0. At time 2 the share price has fallen to K120. Calculate the value to the investor of the forward contract at time $t = 2$. **(3)**

(iii) At time $t = 3$ the share unexpectedly pays a one-off dividend. Explain, with reasons, how the forward price might change as a result of the one-off dividend. **(2)**

(c) Let p_t denote the value at time t of a European put option on a non-dividend paying share S_t with maturity T and a strike price K. The risk-free rate of interest is r.

(i) Derive the lower bound for p_t in terms of S_t and K. **(3)**

(ii) Explain how the lower bound would change if p_t were an American put option. **(1)**

(iii) Explain the conditions necessary for p_t to approach its upper bound. **(2)**

(d) Consider a 30-day, at-the-money call option on a non-dividend-paying share currently valued at K100 pa. The volatility of the share is 24% and the continuously compounded risk-free rate of interest is 4% pa.

The outputs of a computer model used to value the option are:

Option value	K7.12	Theta	−K0.0514/day
Delta	0.1202	Vega	K0.182/%
Gamma	K0.001032	Rho	K0.049/%

(i) After one day the share price increases by K9, volatility is reassessed to be 27% pa and the risk-free rate moves to 3.5% pa.

Estimate the new price of the option. **(6)**

(ii) Explain why a low value of gamma is desirable when using options in a delta-hedged portfolio. **(2)**

6. (a) Explain what is meant by a "risk-neutral probability measure" and state mathematically what it implies about the pricing of derivatives relative to the price of the underlying asset. **(2)**

(b) A company share price is to be modelled using a 7-step recombining binomial tree, with each step in the tree representing one day. Each day, it is assumed that the share price rises by 4%, or falls by 6%. You may assume that the force of interest is $\delta = 4\%$ pa and that there are 365 days in a year. No dividends are to be paid over the next seven days.

(i) Calculate the risk-neutral probability of an up-step on any given day. **(2)**

(ii) Calculate the fair price of a seven-day at-the-money call option on K600,000 worth of shares in this company. **(5)**

(iii) A special option is available where the payoff after seven days is given by

$$\max\left\{S_7^* - K, 0\right\},$$

where S_7^* is the arithmetic average share price recorded at the end of each of the 7 days and K is the strike price.

Calculate the fair price of a 7-day special option (strike price $K = 1.08S_0$) on K600,000 worth of shares in this company. **(5)**

(iv) Explain whether the at-the-money option in part (ii) is likely to have a higher value of vega than the special option in part (iii). **(3)**

(c) Consider a call option on a non-dividend paying stock when the stock price is K300, the exercise price is K250, the continuously compounded risk-free interest rate is 4% per annum, the volatility is 20% per annum and the time to maturity is 9 months.

(i) Calculate the price of the option using the Black-Scholes model. **(4)**

(ii) Determine the (risk neutral) probability of the option expiring in the money. **(1)**

(iii) A special option called a "digital cash-or-nothing" option has a payoff in nine months' time of

$$\begin{cases} 1 & \text{if } S_T > K, \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the price of the digital option. **(1)**

(d) Describe two limitations of the Black-Scholes model. **(2)**

End of Paper

The University of Zambia

School of Natural Sciences

Department of Mathematics & Statistics

2019/2020 Academic Year

End of Year Final Examinations

MAT 4119 - Engineering Mathematics III

Time allowed : **Three (3)** hours **Full marks : 100**

Instructions:

- Indicate your **computer number** on all answer booklets.
- There are six (6) questions in this examination. Attempt **any five (5)** questions. All questions carry equal marks.
- **Full credit** will only be given when **all necessary work** is shown.

This question paper consists of 5 pages.

---

1. **(a)** Regard the value of π in your calculator as true value of π.

   (i) Find the absolute error in approximating π by 3.1.

   (ii) Find the relative error in approximating π by 3.1.

   (b) The $(n-1)^{th}$ order Taylor polynomial and the associated error term for the function $f(x) = e^x$ at $x_0 = 0$ is given by

   $$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... + \frac{x^{n-1}}{(n-1)!} + \frac{x^n}{n!}e^{\xi}, \quad 0 < \xi < x.$$

   We wish to estimate value of e. Find the value of n which allows to estimate e from the above Taylor polynomial with relative error less than $\frac{1}{2}10^{-3}$.

   (c) **(i)** Find the 3rd order Taylor polynomial for function $\cos x$ about $x_0 = \frac{\pi}{4}$.

Turn Over/...

(ii) Use the polynomial of part (i) to approximate $\cos 42^o$ to an accuracy of 10^{-4}.

[4, 8, 8]

2. (a) Show that the equation

$$\cos\left(\frac{\pi(x+1)}{8}\right) + 0.148x - 0.9062 = 0$$

has a root in the interval $(-1, 0)$. Perform one iteration of Newton-Raphson method with initial approximation $x_0 = -0.5$ to improve the approximation.

(b) (i) Show on a graph that the equation $e^x = \frac{1}{x}$ has only one real root.

(ii) Given that the root of the equation $xe^x = 1$ lies between 0 and 1, approximate the root by performing four iterations of the Bisection method.

(c) A root of equation $f(x) = 0$ is approximated by the Bisection method. The method is initialized by the interval (a_1, b_1). Given that $\frac{1}{2^n}(b_1 - a_1)$ is an upper bound on the absolute error at the nth iteration, find how many iterations will be needed to approximate the root to an accuracy of 10^{-5} using $a_1 = 1$ and $b_1 = 2$. [8, 8, 4]

3. (a) Consider the following system of linear equations:

$$x_1 + 0.07x_2 + 0.31x_3 = 2$$

$$0.07x_1 + x_2 + 0.55x_3 = 2$$

$$0.31x_1 + 0.55x_2 + x_3 = 2.$$

(i) Using the Jacobi's method, write the system in the form

$$X^{(k)} = TX^{(k-1)} + C$$

for some fixed matrices T and C.

(ii) Using $X^{(0)} = (0, 0, 0)^t$, find the approximate solution $X^{(2)}$ to the system.

(iii) Given that the eigenvalues of T are $\lambda_1 = -0.662952$, $\lambda_2 = 0.603268$ and $\lambda_3 = 0.0596843$, does the sequence $\{X^{(k)}\}_{k=0}^{\infty}$, generated by the Jacobi's method for the system above converge to a unique solution? Give reason(s).

Turn Over/...

(b) **(i)** If the system of non-linear equations

$$3yx^2 - 10x + 7 = 0$$

$$y^2 - 5y + 4 = 0$$

is written in the form

$$x = g_1(x, y) = \frac{1}{10}(3yx^2 + 7)$$

$$y = g_2(x, y) = \frac{1}{5}(y^2 + 4),$$

show that $G(X) = (g_1, g_2)^t$, where $X = G(X)$, $X = (x, y)^t$, has a fixed point on $D = \{X = (x, y)^t : -1 \leq x, y \leq 1\}$.

(ii) Starting with $X^{(0)} = (0.5, 0.5)^t$, approximate the solution $X^{(k)}$ to the system until

$$\frac{||X^{(k)} - X^{(k-1)}||_\infty}{||X^{(k)}||_\infty} < 5 \times 10^{-1}.$$

(c) Consider the following table of data:

x	2.00	2.01	2.02
$f(x)$	0.48045	0.48739	0.49435

(i) Use the three-point formulas

$$f'(x_0) = \frac{1}{2h}[-3f(x_0) + 4f(x_0 + h) - f(x_0 + 2h)] + \frac{h^2}{3}f'''(\xi)$$

$$f'(x_0) = \frac{1}{2h}[f(x_0 + h) - f(x_0 - h)] - \frac{h^2}{6}f'''(\xi)$$

to approximate $f'(x_0)$ at $x_0 = 2$.

(ii) Find the actual error if the data given in part (i) were taken from the function $f(x) = (\ln x)^2$.

[8, 6, 6]

Turn Over/...

4. **(a)** A reservoir discharging through sluices at a depth h below the water surface has a surface area A for various values of h as given below:

$h(\text{in}ft)$	10	11	12	13	14
$A(\text{in}ft^2)$	950	1070	1200	1350	1530

Use the Newton backward-divided difference formula to estimate surface area at depth 13.52 ft.

(b) **(i)** Estimate the value of

$$\int_0^{\sqrt{\pi}} \sin(x^2)\, dx$$

using the composite Simpson's rule with $h = \frac{\sqrt{\pi}}{4}$

(ii) Find an upper bound for the error in part (i).

NOTE: Error function is $E(f) = -\frac{b-a}{180}h^4 f^{(4)}(\mu)$

(c) Given the initial-value problem

$$\frac{dy}{dt} = (1+t)\sqrt{y},\ \ 0 \le t \le 1,\ \ y(0) = 1,$$

use Euler's method with $h = 0.25$ to estimate $y(0.75)$.

[7, 8, 5]

5. **(a)** The table below shows the voltage across the capacitor measured for 5 seconds:

t	1	2	3	4	5
V	9.4	7.31	5.15	3.55	2.81

By fitting an exponential curve $V = V_0\, e^{-(\frac{1}{RC})t}$ to the data, find the capacitance C, when $R = 2000\,\Omega$.

(b) Transform the second-order initial-value problem

$$t^2y'' - 2ty' + 2y = t^3 \ln t,\ \ 1 \le t \le 2,\ \ y(1) = 1,\ \ y'(1) = 0$$

into a system of first order initial-value problems.

Turn Over/...

(c) Use the Runge-Kutta method of order four to solve the initial value problem in part (b) at $y(1.1)$ with $h = 0.1$

NOTE:

$K_{1,i} = hf_i(t_j, W_{1,j}, W_{2,j}, ..., W_{m,j})$

$K_{2,i} = hf_i(t_j + \frac{h}{2}, W_{1,j} + \frac{1}{2}K_{1,1}, W_{2,j} + \frac{1}{2}K_{1,2}, ..., W_{m,j} + \frac{1}{2}K_{1,m})$

$K_{3,i} = hf_i(t_j + \frac{h}{2}, W_{1,j} + \frac{1}{2}K_{2,1}, W_{2,j} + \frac{1}{2}K_{2,2}, ..., W_{m,j} + \frac{1}{2}K_{2,m})$

$K_{4,i} = hf_i(t_j + h, W_{1,j} + K_{3,1}, W_{2,j} + K_{3,2}, ..., W_{m,j} + K_{3,m}),$

for $i = 1, 2, ..., m$

$W_{i,j+1} = W_{i,j} + \frac{1}{6}(K_{1,i} + 2K_{2,i} + 2K_{3,i} + K_{4,i})$

[8, 4, 8]

6. (a) Taylor's theorem can be used to express the three-point midpoint formula as

$$f'(x_0) = \frac{1}{2h}[f(x_0 + h) - f(x_0 - h)] - \frac{h^2}{6}f^{(3)}(x_0) - \frac{h^4}{120}f^{(5)}(x_0) - ....$$

Find an approximation of order $\mathcal{O}(h^6)$ for $f'(1.5)$ when $f(x) = \sqrt{x^{2e^x}}$ and $h = 0.5$.

NOTE:

$$N_j(h) = \frac{4^{j-1}N_{j-1}(\frac{h}{2}) - N_{j-1}(h)}{4^{j-1} - 1}, \quad j = 2, 3, 4...$$

(b) A clamped cubic spline with conditions $S'(1) = 2$ and $S'(3) = 1$ is given by

$$S(x) = \begin{cases} 2 + a(x-1) - \frac{5}{2}(x-1)^2 + b(x-1)^3 & x \in [1, 2] \\ c + \frac{3}{2}(x-2) + d(x-2)^2 - \frac{3}{2}(x-2)^3 & x \in [2, 3] \end{cases}$$

where a, b, c and d are real constants. Find the values of a, b, c and d.

(c) Use Taylor's method of order two with $N = 4$ to solve the initial-value problem

$$y' = y - t^2 + 1, \quad 0 \leq t \leq 2, \quad y(0) = 0.5$$

at $y(1.5)$

[8, 6, 6]

END OF EXAMINATION!

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS & STATISTICS

2019/2020 Academic Year Final Examinations DECEMBER, 2020
MAT 4201 STRUCTURE AND REPRESENTATION OF GROUPS

Total time allowed: Three (3) hours
Instructions:

- This question paper comprises seven (7) questions. **Answer any five (5) questions.**
- Show detailed working to earn full marks.

---

1. (*a*) Define the following terms:
   (*i*) Subnormal series.
   (*ii*) Normal series.
   (*iii*) Composition series.
   (*b*) Construct a composition series for S_4.
   (*c*) (*i*) State the Jordan -Hölder theorem without proof.
   (*ii*) Give any application of this theorem.
   (*iii*) What is meant by the term refinement of a subnormal series? Comment on the Jordan Hölder theorem with regards to refinement.
   (*d*) Prove that a composition series is a subnormal series of maximal length.

2. (*a*) Define the following terms:
   (i) Solvable group.
   (*ii*) Nilpotent group.
   (*b*) Prove that the following conditions are equivalent:
   (i) G is solvable.
   (*ii*) G has a normal series with abelian factors.
   (*iii*) G has a subnormal series with abelian factors.
   (*c*) (i) Prove that a subgroup of a solvable group is solvable..
   (*ii*) Show that if $n \geq 5$, then S_n is not solvable.

3. (a) (i) Show that any finite $p-$ group is nilpotent.
   (ii) Hence or otherwise show that D_8 is nilpotent.
   (b) Show that all abelian groups are nilpotent.
   (c) (i) Give an example of a solvable group that is not nilpotent.
   (ii) Prove that a nilpotent group is solvable.

4. (a) Define the following terms:
   (i) Extension of two groups N and G.
   (ii) Automorphism group of a group G.
   (iii) Semidirect product of N by G.
   (b) Show that $\mathrm{Aut}(\mathbb{Z}) \cong \mathbb{Z}_2$.
   (c) Show that the dihedral group D_{2n} is a semidirect product.
   (d) Prove that if G is an elementary abelian group of order p^n, then $\mathrm{Aut}(G) \cong \mathrm{GL}(n,p)$.

5. (a) Define the following terms:
   (i) Matrix represntation of a finite group G.
   (ii) Sum of two representations.
   (b) State without proof Schur's lemma.
   (c) If ρ is an irreducible representation of degree n of a group G over an algebraically closed field $\mathbb{F}$, then the only matrices that commute with $\rho(g)$, $g \in G$, are the scalar matrices aI_n, $a \in \mathbb{F}$. (Hint apply Schur's lemma).

6. (a) Define the following terms:
   (i) A character afforded by a representation ρ.
   (ii) Class function.
   (iii) A reducible character χ of G.
   (b) If $\chi = \sum_{i=1}^{k} \lambda_i \chi_i$, where χ_i are distinct irreducible characters of G and λ_i are non negative integers, show that $< \chi, \chi >= \sum_{i=1}^{k} {\lambda_i}^2$.
   (c) Construct the character table of S_4.

7. (a) Define the following terms:

   (i) Restriction of a character χ of G to a subgroup H of G.

   (ii) The induced character ϕ^G of the character ϕ of H to G.

   (b) Let $G = S_4$ and $H =< (123) > \cong \mathbb{Z}_3$.

   If the set of irreducible characters of G is denoted and given by $\text{Irr}(G) = \{\chi_1, \chi_2, \chi_3, \chi_4, \chi_5\}$, find the restricted character values on the conjugacy classes of H.

   (c) Construct all the irreducible characters of a cyclic group of order 5.

END OF EXAM!

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS & STATISTICS

2019/2020 Academic Year Final Examinations NOVEMBER, 2020
MAT 4212 MODULE AND FIELD THEORY

Total time allowed: Three (3) hours
Instructions:

- This question paper comprises six (6) questions. **Answer any five** (5) **questions**.
- Show detailed working to earn full marks.

---

1. (a) Define the following terms:
   (i) Cyclic $R-$ module.
   (ii) Annihilator I_0 of the module M.
   (b) Prove the following:
   (i) If x generates a cyclic module M over the commutative ring R; then $I_x = I_0$, so that $M \cong R/I_0$ (I_0 in this case is said be the order ideal of M).
   (ii) Two cyclic $R-$ modules over a commutative ring are isomorphic if and only if they have the same annihilator.
   (c) Define the following terms:
   (i) A free $R-$ module.
   (ii) Give an example of a free $R-$ module. Justify your answer.
   (d) State without proof the fundamental decomposition theorem for modules

2. (*a*) Define the following terms:
   (i) A field extension E of F.
   (*ii*) Algebraic element $\alpha \in E$.
   (*iii*) Minimal polynomial of α over F denoted by $\min(\alpha, F)$.
   (*iv*) Degree of a field extension E over F denoted by $[E{:}F]$.
   (*b*) Find the minimal polynomial of $\sqrt{7}$ over the rationals $\mathbb{Q}$.
   (*c*) Prove that if E is a finite extension of F, then E is an algebraic extension of F.
   (*d*) Find a splitting field K for $f(X) = X^2 - 2X + 4$ over $\mathbb{Q}$ and determine the degree of F over $\mathbb{Q}$.

3. (*a*) Define the following terms:
   (i) Splitting field E of F.
   (ii) Algebraic closure C of F.
   (*b*) Find the splitting field for $f(X) = X^2 - 4X + 4$ over $\mathbb{Q}$.
   (*c*) Prove that if C is a field, the following conditions are equivalent:
   (i) Every non-constant polynomial $f \in C[X]$ has atleast one root in C.
   (ii) Every non-constant polynomial $f \in C[X]$ splits over C.
   (iii) Every irreducible polynomial $f \in C[X]$ is linear.
   (iv) C has no proper algebraic extensions.

4. (*a*) Define the following terms:
   (i) $R-$ Module.
   (ii) $R-$ Submodule.
   (iii) A module homomorphism or $R-$ homomorphism f from M to N.
   (*b*) Give an example of an $R-$ module M with non -zero elements $r \in R$ and $x \in M$ such that $rx = 0$.
   (*c*) Prove that $\ker(f)$ is always a submodule of M.

5. (*a*) Define the following terms:
   (i) Galois group $G = Gal(E/F)$ of the extension E/F.
   (ii) Fixed field of $H \leq G$.
   (iii) Fixing group H of an intermediate field K.
   (*b*) (*i*) When is an extension E/F said to be Galois?

(*ii*) Suppose that E is a splitting field of the separable polynomial f over F, the Galois group of f is the Galois group of the extension E/F. Find the Galois group of $f(X) = X^3 - 2$ over the rational field.

(*c*) (*i*) Let f be a polynomial with distinct roots $x_1, x_2, \cdots, x_n$ in a splitting field E over F. Define the discriminant of f.

(*ii*) Hence or otherwise find the discriminant of

$$f(X) = X^2 + bX + c.$$

(*d*) State without proof the Fundamental Theorem of Galois Theory.

6. (*a*) State and prove the second isomorphism theorem for modules.

(*b*) Given that M is an $R-$module, prove the following properties of M:

(*i*) $r0_M = 0_M$.

(*ii*) $0_R x = 0_M$ for all $x \in M$.

(*c*) (*i*) What is meant by a normal extension of an algebraic extension E of F?

(*ii*) Give an example of a field extension that is normal and one that is not normal. Justify your answer.

(*iii*) Prove that the finite extension E/F is normal iff E is a splitting field for some polynomial $f \in F[X]$.

END OF EXAM!

The University of Zambia

School of Natural Sciences

Department of Mathematics & Statistics

2019/20 Academic Year Final Examinations

MAT 4300 – ELEMENTS OF FUNCTIONAL ANALYSIS

20th November, 2020

INSTRUCTIONS: 1. Answer any **Five (5)** Questions Only out of Six.

2. All questions carry equal marks.

3. Indicate the question number for each question attempted on the cover of the main answer book.

TIME ALLOWED: **Three (3)** hours.

---

1. (a) (i) Define an l^p space, for $1 \leq p < \infty$ and $p = \infty$. [1,1]

(ii) Let $1 \leq p < \infty$. Prove that if $a, b \in l^p$, where $a = \{a_n\}_{n=1}^{\infty}$, $b = \{b_n\}_{n=1}^{\infty}$, then $a + b \in l^p$.

[5]

(b) (i) State without proof, the Minkowski's inequality in l^p spaces, for $1 \leq p < \infty$ and $p = \infty$. [1,1]

(ii) Let $x = \{x_n\}_{n=1}^{\infty}$ and $y = \{y_n\}_{n=1}^{\infty}$ be sequences of all real numbers in l^p. Prove that for $1 \leq p < \infty$ the function

$$d(x, y) = \left(\sum_{n=1}^{\infty} |x_n - y_n|^p \right)^{\frac{1}{p}}$$

is a metric on l^p.

[5]

(c) Let $1 \leq p < q < \infty$. Prove that $l^p \subset l^q$ and $l^p \neq l^q$.

[6]

2. (a) Let (X,d) be a metric space and A a subset of X. Give the definition of a dense set and of a nowhere dense set in X.

[1,1]

(b) Let $d : l^p \times l^p \to \mathbf{R}$ be a metric defined by $d(x,y) = \left(\sum_{i=1}^{\infty} |x_i - y_i|^p \right)^{\frac{1}{p}}$. Let $N \in \mathbf{N}$ and let $E_N = \left\{ \{x_n\}_{n=1}^{\infty} \in l^p : x_n = 0 \ \forall \ n > N \right\}$. Prove that

(i) E_N is nowhere dense in l^p.

[5]

(ii) $E = \bigcup_{N \in \mathbf{N}} E_N$ is a dense set in l^p.

[5]

(c) (i) Construct the Cantor set.

[3]

(ii) Prove that the cantor set is closed.

[5]

3. (a) (i) When is a metric space (X,d) said to be complete?

[1]

(ii) Let (X,d) be a metric space. If $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ are Cauchy sequences in X, prove that $\{d(x_n, y_n)\}_{n=1}^{\infty}$ converge in $\mathbb{R}$ with usual metric, $d(x,y) = |x - y|$.

[5]

(b) State and prove the Baire's Category theorem.

[1,6]

(c) (i) State, without proof, the Bolzano-Weiestrass property.

[1]

(ii) Prove that a compact metric space is sequentially compact.

[6]

4. (a) (i) Let X be a vector space. When are two norms said to be equivalent in X?

[1]

(ii) Show that the norms $\|f\|_1$ and $\|f\|_2$ are not equivalent in $C[0,1]$.

[5]

(b) (i) Let $(X, \|\cdot\|)$ be a normed vector space and let $x, y \in X$. Show that

$$|\|x\| - \|y\|| \leq \|x - y\|.$$

[4]

(ii) Suppose $\{x_n\}_{n=1}^{\infty}$ is a Cauchy sequence in a normed vector space $(X, \|\cdot\|)$. Show that the norms $\|x_n\|$ form a convergent sequence.

[3]

(c) (i) Consider the space $X = C[0,1]$ and the sequence $f_n(x) = \sqrt{n}x^n$. Show that $f_n \to 0$ in $(X, \|\cdot\|_1)$. Is this true in $(X, \|\cdot\|_2)$?

[4]

(ii) Define a closed subset of a normed space $(X, \|\cdot\|)$.

[1]

(iii) Consider the unit sphere $S = \{x \in X : \|x\| = 1\}$. Show that S is closed in every vector space $(X, \|\cdot\|)$.

[2]

5. (a) (i) Let X and Y be normed spaces and $T: D(T) \to Y$ a linear operator, where $D(T) \subset X$. When is T said to be bounded?

[1]

(ii) Consider the linear operator

$$T: (\mathbb{R}^2, \|\cdot\|_1) \to (\mathbb{R}^2, \|\cdot\|_\infty)$$

which is defined by

$$T(x_1, x_2) = (2x_1 + 3x_2, 4x_2).$$

Find its operator norm. [6]

(b) Show that $\|S \circ T\| \leq \|S\| \cdot \|T\|$ whenever $S, T \in L(X, X)$.

[6]

(c) (i) Define a sublinear functional on a vector space X.

[1]

(ii) State, without proof, the Hahn-Banach theorem for real vector spaces.

[1]

(iii) State and prove, the Hahn-Banach theorem for complex vector spaces.

[5]

6. (a) Let X be an inner product space and $x, y \in X$. Show that

(i) $|\langle x, y\rangle| \leq \|x\|\|y\|$

[4]

(ii) $\|x + y\| \leq \|x\| + \|y\|$.

[4]

(b) (i) Define an orthogonal system in an inner product space.

[1]

(ii) Prove that if $x_1, x_2, \cdots, x_n$ are orthogonal vectors in an inner product space, then $\|\sum_{k=1}^{n} x_k\|^2 = \sum_{k=1}^{n} \|x_k\|^2$.

[5]

(c) (i) Let X be an inner product space and S be a nonempty subset of X. Define an orthogonal complement of S. [1]

(ii) Prove that if f is a nontrivial bounded linear functional on a Hilbert space H, then $\dim N(f)^{\perp} = 1$.

[5]

END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS & STATISTICS

2019/20 ACADEMIC YEAR
FINAL EXAMINATIONS

MAT4600: MULTIVARIATE STATISTICAL ANALYSIS

TIME ALLOWED: Three (3) Hours

INSTRUCTIONS: **(i)**All questions are of the same weight. (ii) Answer any Five (5) Questions.
(iii) Statistical tables will be provided.

1. (a) Let X be a p dimensional random vector with $E(X) = \mu$.

(i) Define covariance matrix of X

(ii) Show that $COV(X) = E(XX') - \mu\mu'$.

(b) The joint probability function of random variables X_1 and X_2 is given in the following table:

X_2 \ X_1	– 1	0	2
– 1	·1	·02	·08
0	·2	·2	·4

Let $X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$. Find

(i) E(X).

(ii) COV(X).

(iii) COV(AX) where $A = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$.

(c) Environmental Council of Zambia (ECZ) carried out a study to estimate the gross weight of solid waste generated per year in Lusaka. Five years were randomly selected from 2000 to 2018 period and amounts attributed to X_1 = paper and paperboard waste and X_2 = plastic and glass waste in millions of tons were analyzed. The following matrix represents the sample observations.

$$\begin{bmatrix} 12 & 17 \\ 18 & 20 \\ 14 & 16 \\ 20 & 18 \\ 16 & 19 \end{bmatrix}$$

Define $Y_1 = \frac{X_1 + X_2}{2}$ (mean annual waste)

$Y_2 = X_2 - X_1$ (comparison between two sources of solid waste) and $Y = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$

Find the

(i) Sample mean of Y

(ii) Sample variance covariance of Y

(iii) Sample covariance of Y_1 and Y_2

2. (a) Let $X_1, X_2, \ldots, X_{100}$ be a random sample from a population whose mean vector is $\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix}$ and 4x4 covariance matrix is Σ. Let $\bar{X}$ denote the sample mean. Find the following:

(i) $E(\bar{X})$

(ii) $COV(\bar{X})$

(iii) $E(\sum_{j=1}^{100}(X_j - \bar{X})(X_j - \bar{X})')$

(b) Let $X_1, \ldots, X_{10}$ to be a random sample from $N_4(\mu, \Sigma)$. Specify the distribution of each of the following:

(i) $\bar{X}$

(ii) $\sqrt{10}\,(\bar{X} - \mu)$

(iii) $10\,(\bar{X} - \mu)'\,\Sigma^{-1}(\bar{X} - \mu)$

(c) Let X be a two dimensional random vector with mean μ and covariance matrix Σ. Let Σ be a positive definite matrix with eigen values λ_1 and λ_2 and the corresponding orthonormalized eigen vectors e_1 and e_2.

(i) Show that Σ^{-1} is positive definite.

(ii) Show that $\frac{1}{\lambda_1}$ is an eigen value of Σ^{-1} corresponding to eigen vector e_1.

(iii) Write the spectral decomposition of Σ^{-1}.

3. (a) Let a random vector X be distributed as $N_2(\mu, \Sigma)$ where Σ is a positive definitive matrix. Show that $(X-\mu)' \Sigma^{-1} (X-\mu)$ is a chi – square variate with two degrees of freedom.

(b) The following table gives the age X_1, measured in years and the selling price X_2, measured in thousands of kwacha for 10 used cars.

x_1	3	4	5	6	7	7	8	9	10	11
x_2	23	19	18	17	13	10	10	4·5	7	3

Construct a Q – Q plot for observations on variable X_1. Does the data support normality assumption? Comment.

(c) Let ρ denote the coefficient of correlation of variables defined in part (b). The sample value r of ρ using the data of part (b) is $r = -\cdot 96$. Assume that X_1 and X_2 form a bivariate normal population. Test $H_0 : \rho = -\cdot 9$ against $H_1 : \rho < -\cdot 9$ at 0·05 level of significance.

4. (a) Suppose that the first two components of a three dimensional random vector X are independent random variables X_1 and X_2, each with the same variance σ^2 and that the third component of X is given by $X_3 = X_1 + X_2$. Find the covariance matrix for X.

(b) Let $X = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$ be a normal random vector with mean vector μ *and covariance matrix* Σ. Assume X_1, X_2, X_3 are mutually independent and the moment generating function of X is $M_X(t) = e^{t'\mu + \frac{1}{2} t'\Sigma t}$, $t \in l_3$

If $t'\mu + \frac{1}{2} t'\Sigma t = t_1 - t_2 + 2t_3 + t_1^2 + \frac{1}{2} t_2^2 + 2t_3^2$.

(i) Find μ *and* Σ.
(ii) Find the distribution of $2X_1 - 3X_2 + X_3$
(iii) Find b such that $P[2X_1 - 3X_2 + X_3 > b] = 0 \cdot 95$.

(c) Given independent random vectors X and Y with distributions $N_4(\mu_x, \Sigma)$ and $N_4(\mu_y, \Sigma)$ respectively.

(i) Write the distribution of $Z = \begin{pmatrix} Y \\ X \end{pmatrix}$

(ii) Find the distribution of $W = \begin{pmatrix} W_1 \\ W_2 \end{pmatrix}$ *where* $W_1 = Y - X$ *and* $W_2 = Y + X$.

5. (a) Hearing loss in decibels at 2,000 cps in the left and right ears was measured for 37 normal males and 52 men with hearing disabilities. The correlations between the left and right ears hearing losses were 0·72 and 0·86 for the normal men and those with hearing disabilities respectively. Test at $\alpha = 0{\cdot}05$ level, the hypothesis that the population correlations are equal against the alternative that the group with hearing disabilities has a higher correlation.

You may use the test statistic

$$d = \frac{Z_1 - Z_2}{\sqrt{\frac{1}{N_1 - 3} + \frac{1}{N_2 - 3}}}$$ having $N(0,1)$ where Z represents

Fisher's transformation defined as $Z = \frac{1}{2} \ln \frac{1+r}{1-r}$.

(b) Let X_1, X_2, X_3 be mutually independent random vectors with X_j distributed as $N_2(\mu, \Sigma)$, j = 1, 2, 3. Consider two random vectors Y_1 and Y_2 defined as

$$Y_1 = \frac{1}{2} X_1 + \frac{1}{2} X_3$$

$$Y_2 = X_1 + 2X_2 + X_3$$

Find (i) $E(Y_1)$

(ii) $COV(Y_2)$

(iii) $COV(Y_1, Y_2)$

(c) A wildlife ecologist measured X_1 (tail length in millimeters) and X_2 (wing length in millimeters) on a random sample of 90 female kites. The sample mean vector and covariance matrix were computed as:

$$\bar{x} = \begin{pmatrix} 169{\cdot}5 \\ 281 \end{pmatrix}, \quad s = \begin{pmatrix} 27{\cdot}3 & 18{\cdot}2 \\ 18{\cdot}2 & 21 \end{pmatrix}$$

(i) Write an inequality representing the 95% confidence ellipse for the population means μ_1 and μ_2. (Do not perform matrix multiplication).

(ii) Are $\mu_1 = 190$mm and $\mu_2 = 275$mm plausible values for the mean tail length and mean wing length for the female kites? Explain without calculations.

6. (a) The height, chest circumference and upper arm circumference, all measured in centimeters, of six two years old boys were extracted from UNZA clinic records. The sample statistics was as follows.

$$\bar{x}' = (82{\cdot}0,\ 62{\cdot}2,\ 14{\cdot}5),\quad s = \begin{pmatrix} 31{\cdot}6 & 8{\cdot}04 & {\cdot}5 \\ 8{\cdot}04 & 3{\cdot}1 & 1{\cdot}3 \\ {\cdot}5 & 1{\cdot}3 & 1{\cdot}9 \end{pmatrix}$$

$$s^{-1} = \frac{1}{23{\cdot}1}\begin{pmatrix} 4{\cdot}3 & -14{\cdot}6 & 8{\cdot}9 \\ & 59{\cdot}7 & -37{\cdot}3 \\ & & 36 \end{pmatrix}$$

Assuming the joint distribution of above considered variables normal,

(i) Test the hypothesis

$$H_0:\ \mu = \begin{pmatrix} 90 \\ 58 \\ 16 \end{pmatrix}$$ at 0·01 level of significance.

(ii) State the alternative hypothesis.

(b) Using the data of part (a), without computation, write;

(i) 90% one at a time confidence interval for μ_h and μ_c where μ_h represents mean height and μ_c represents mean chest circumference.

(ii) 90% simultaneous confidence interval for μ_h and μ_c.

(iii) 90% Bonferonni simultaneous confidence intervals for μ_h and μ_c.

(c) Consider three response variables Y_1, Y_2, Y_3 and three predictor variables Z_1, Z_2, Z_3. Ten independent trials were run. Write the multivariate regression model in the matrix form, stating the elements of each of the matrix in detail using general naming. State the assumptions made in the model.

END OF EXAM

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

2019/20 ACADEMIC YEAR FINAL EXAMINATIONS

MAT 4615: NONPARAMETRIC METHODS

INSTRUCTIONS:
1. Answer any FIVE (5) questions.
2. Calculators are allowed.
3. Statistical tables are provided.
4. Show all your work to earn full marks.

TIME: THREE (3) Hours

Question 1

(a) (i) List four advantages nonparametric tests have over parametric ones.

(ii) List four disadvantages nonparametric tests have over parametric ones.

(b) Corrosion of different metals is a problem in many mechanical devices. Two sealers used to help retard the corrosion of metals were tested to see whether there was any difference between them. Samples of fifteen different metal compositions were treated with each of the two sealers and the amount of corrosion was measured after exposure to the some environmental conditions for 1 month, the data are given below.

	Sealer		
Metal	I	II	Difference
1	4.6	4.2	0.4
2	7.2	6.4	0.8
3	3.4	3.5	-0.1
4	6.2	5.3	0.9
5	8.4	6.5	1.9
6	5.6	4.8	0.8
7	3.7	3.8	-0.1
8	6.1	6.2	-0.1
9	4.9	4.1	0.8
10	4.4	4.2	0.2
11	7.2	6.5	0.7
12	3.3	3.2	0.1
13	6.4	5.3	1.1
14	4.5	5.2	-0.7
15	6.2	5.4	0.8

(i) Discuss the reason why this experiment qualifies as a paired study design.

(ii) Using the Wilcoxon signed rank statistic, test whether there is any difference between the two sealants at 5% level of significance.

Question 2

There are N individuals competing in a race and each has been assigned a number between 1 and N, inclusively, for identification purposes. Let R_j be the position (rank) the jth individual will take when the race is completed. The probability that the jth individual takes position k after the race is given by:

$\Pr(R_j = k) = \frac{1}{N}$, where k = 1, 2, . . . ,N.

(a) (i) Find $E(R_j)$, the mean of the rank for the jth individual.

(ii) Find the variance of R_j.

(b) In the race, there are three individuals who belong to your team. Let $S = \sum_{j=1}^{3} R_j$, be the sum of the ranks of their positions when the race is completed.

(i) Find E(S), the mean of the sum of their ranks.

(ii) Find the variance of S given that $\mathrm{Cov}(R_r, R_k) = -\frac{N+1}{12}$ for any given two ranks such that $R_r \neq R_k$.

Question 3

(a) Assume that in a completely randomized design two treatments are available. One treatment is assigned to group 1 and the other to group 2. A test of the difference between the two treatments will involve the sum of the ranks of one group, the smaller of the two. Group 1 has n_1 observations and group 2 has n_2 observations. The two groups are combined and ranking is done to the combined group.

Let:

- $N = n_1 + n_2$
- $w_k = \begin{cases} 1 & \text{if the kth rank is assigned to group 1} \\ 0 & \text{if the kth rank is assigned to group 2} \end{cases}$
- k = 1, 2, . . . , N.
- $\bar{R}_1 = \frac{\sum_{k=1}^{N} k w_k}{n_1}$, be the mean of the ranks for group 1.

(i) Determine the expected value of w_k

(ii) Determine the expected value of $\bar{R}_1$.

(b) A stock market analyst produced at the beginning of the year a list of stocks to buy and another list of stocks to sell. For a random sample of 10 stocks from the "buy list," and another independent random sample of 10 stocks from the "sell list," had the following percentage returns over the year:

Buy list	9.6	5.8	13.8	17.2	11.6	4.2	3.1	11.7	13.9	12.3
Sell list	22.7	6.2	8.9	11.3	2.1	3.9	22.4	1.3	7.9	10.2

Using the Wilcoxon rank sum statistic, test whether the mean returns for the "Buy list" is higher than the mean returns for the "Sell list", at 5% level of significance.

Question 4

The Zambia Demographic and Health Survey Report of 2018 has information on the percent distribution of HIV-positive females aged 15-59 years by self-reported antiretroviral therapy (ART) status and by selected demographic characteristics. The data are shown in the table below.

Province	Unaware of HIV status	Aware of HIV status but not on ART	Aware of HIV status but on ART
Central	31.9	7.9	60.2
Copperbelt	31.2	11.1	57.7
Eastern	24.3	13.4	62.3
Luapula	42.8	6.5	50.7
Lusaka	29.2	11.9	58.9
Muchinga	37.9	5.2	56.9
Northern	40.3	19.5	40.2
North-Western	42.5	8.0	49.4
Southern	23.5	7.1	69.3
Western	40.8	11.2	48.0

(a) Briefly, state what your observations are regarding the three levels of HIV status for the females aged 15 to 59 years of age.

(b) Using the Friedman's statistic, test whether the average percentages are the same for the three levels of awareness of HIV status at 5% level of significance. No pair-wise tests are necessary.

Question 5

(a) An agricultural experiment is being planned in which p treatments (fertilizers) are to be compared using a completely randomized design.

(i) Briefly discuss the role of randomization in the experiment, use bullet format in presenting your response.

(ii) When is it necessary to carry out pair-wise analyses in a such an experiment? Be brief and use the bullet format in your response.

(b) Three suppliers provide parts in shipments of 500 units. Random samples of six shipments from each of the three suppliers were carefully checked, and the numbers of parts not conforming to standards were recorded. These numbers are listed in the following table:

Supplier A	Supplier B	Supplier C
28	22	33
37	27	29
34	29	39
29	20	33
31	18	37
33	30	38

Test the null hypothesis that the means of the number of parts not conforming to standards are the same for the three suppliers, use $\alpha = 0.05$.

Question 6

(a) Suppose that X is a continuous random variable with the exponential distribution EXP(θ), defined by: $f(x) = \theta e^{-x\theta}$ for $0 < x < \infty$.

Evaluate $F(t) = Pr(X \leq t)$.

(b) Gross and Clark (1975, page 105) consider the following relief times in hours of patients receiving analgesic (medication that alleviates pain), 15 of the times are shown below together with some relevant statistics.

Patient	x_i (time)	$\hat{F}_n(x_i) = \frac{i}{15}$	$F_{H_0}(x_i)$	$D = \hat{F}_n(x_i) - F_{H_0}(x_i)$
1	1.1	0.067	0.442	
2	1.2	0.133	0.471	
3	1.3	0.200	0.498	
4	1.4	0.267		
5	1.5	0.333		
6	1.6	0.400		
7	1.7	0.467		
8	1.8	0.533		
9	1.9	0.600		
10	2.0	0.667		
11	2.2	0.733		
12	2.3	0.800		
13	2.7	0.867		
14	3.0	0.933		
15	4.1	1.000		

(i) Complete the last two columns of the table to 3 decimal places where $F_{H_0}(x_i) = \Pr(X \leq x_i)$ and f (x) = Exp(0.53)

(ii) Test the null hypothesis H_0: f = Exp(0.53) versus H_a: f ≠Exp(0.53) at 5% level of significance using the Kolmogorov-Smirnov statistic.

END OF EXAMINATION

©TSA2020

THE UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

2019/20 ACADEMIC YEAR

Monday, November 30, 2020 (14:00-17:00)

MAT 4645-TIME SERIES ANALYSIS

TIME ALLOWED: Three(3) hours

INSTRUCTIONS TO THE CANDIDATE

1. **Attempt any four (4) questions**, beginning your answer to each question on a separate answer sheet.
2. Full credit will only be given when necessary work is shown.
3. Enter all the candidate and examination details as requested on the front of your answer booklet.

Graph paper is NOT required for this paper

AT THE END OF THE EXAMINATION

Hand in your booklet, with any additional sheets firmly attached.

In addition to the paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

This examination paper consist of 6 printed pages, each printed on one side only.

The front cover is page 1.

Question 1 starts on page 2

©TSA2020

Question 1

(a) (i) Calculate the autocovariance function $\{\gamma_k: k \geq 0\}$ and autocorrelation function $\{\rho_k: k \geq 0\}$ of a first-order Moving Average process : $X_t = \mu + e_t + \beta_1 e_{t-1}$,
where $\{e_t: t \geq 0\}$ is a sequence of uncorrelated, zero-mean random variables with common variance σ^2 [4]

(ii) Find the range of values of β_1 such that the process is invertible. [2]

(iii) A sequence of observations $\{x_1, x_2, \ldots, x_n\}$ has sample variance $\hat{\gamma}_0 = 14.5$, sample lag-1 autocovariance $\hat{\gamma}_1 = 5.0$. Show that there is more than one first-order moving average process which can be fitted to these data, but verify that only one of the fitted processes is invertible. [5]

(b) A stationary second-order autoregressive process $\boldsymbol{X}$, which may be assumed to be in equilibrium at time 0, is defined by

$$X_t = \mu + \alpha_1(X_{t-1} - \mu) + \alpha_2(X_{t-2} - \mu) + e_t ,$$

where $\{e_t: t \geq 1\}$ is a sequence of independent, zero-mean Normal random variables, each with variance σ_e^2. Let γ_k be the kth autocorrelation function of the process.

(i) Obtain an equation for γ_1 in terms of γ_0 and γ_2 [2]

(ii) Derive similar equations for γ_2 and γ_0. [2]

(iii) State the autocorrelation function ρ_k of $\boldsymbol{X}$ for $k = 0, 1, 2$. [2]

(iv) Suppose that the equations derived in (i) for ρ_1 and ρ_2 are used as the basis of an estimation procedure: estimates $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are defined to be the solutions of those equations when ρ is replaced by a suitably defined sample autocorrelation function r. Solve these equations. [2]

(c) A univariate $AR(2)$ process has defining equation:

$$X_n = 0.7X_{n-1} + 0.1X_{x-2} + e_n$$

where e_n is a white noise process with mean 0 and variance σ^2.
Explain whether the process is:

(i) Stationary [2]

(ii) Invertible [1]

(iii) purely indeterministic [1]

(iv) Markov. [2]

[Total: 25]

©TSA2020

Question 2

(a) (i) Explain the concept of cointegrated time series. [3]

(ii) Give two examples of circumstances when it is reasonable to expect that two processes may be cointegrated. [2]

(b) (i) Describe how a strictly stationary time series process differs from a weakly stationary time series process. [3]

(ii) Consider the following pair of equations:

$$X_t = 0.5X_{t-1} + \beta Y_t + \varepsilon_t^1$$
$$Y_t = 0.5Y_{t-1} + \beta X_t + \varepsilon_t^2$$

where ε_t^1 and ε_t^2 are independent white noise processes.

(a) Determine the matrices M and N such that these equations can be represented as:

$$M\begin{pmatrix} X_t \\ Y_t \end{pmatrix} = N\begin{pmatrix} X_{t-1} \\ Y_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \end{pmatrix}$$

[5]

(b) Find the values of β for which these equations represent a stationary bivariate time series model. [4]

(c) (i) Show that the following set of equations represents a $VAR(p)$ (vector autoregressive) process, by specifying the order and the relevant parameters:

$$X_t = \alpha X_{t-1} + \alpha Y_{t-1} + \varepsilon_t^1$$
$$Y_t = \beta X_{t-1} - \beta X_{t-2} + \varepsilon_t^2$$

[3]

(ii) Define, giving relevant equations, the three main linear models used for modeling stationary time series. [5]

[Total: 25]

Question 3

(a) (i) Define Partial autocorrelation function (PACF). Also briefly mention how it can be derived by minimizing the expected value of "square of difference between X_t and a linear functions of X_{t-1}, X_{t-2}, ..., X_{t-k}". [2]

(ii) The movements of a consumer price index are to be subjected to time series analysis with the aim of forecasting future behaviour. The index is calculated monthly. Explain whether you would expect to fit a model which included (a) a trend term, (b) a seasonal effect. [4]

(b) The time series X_t is assumed to follow $ARIMA(p,d,q)$ process defined by $X_t = 1 + \frac{11}{6}X_{t-1} - X_{t-2} + \frac{1}{6}X_{t-3} + \epsilon_t$, where ϵ_t are $N(0,1)$ random variables. One of the roots of the characteristic equation is 1.

(i) find the other roots of the characteristic equation. [3]

(ii) state whether the process is stationary. [1]

(iii) determine p, d and q. [1]

(iv) what is the mean of $X_t - X_{t-1}$? [2]

(v) what is the mean of X_t? [1]

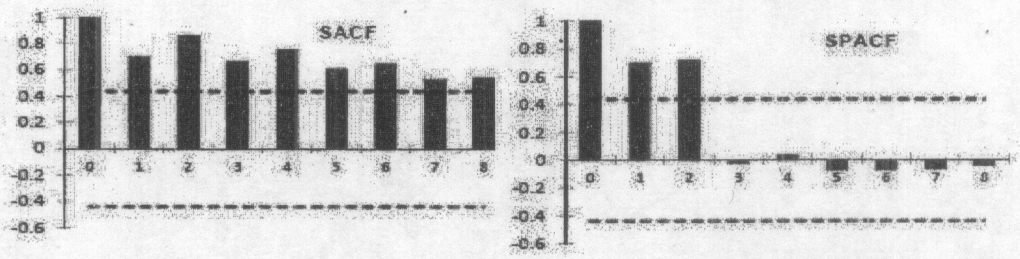
(vi) if $X_{98} = 2.4$, $X_{99} = 1.6$ and $X_{100} = 1.2$, forecast X_{101}. [2]

(c) A company observes its quarterly utility bills over the last 5 years $\{q_t: t = 1, 2, \ldots, 20\}$. The company decomposes the time series as follows: $q_t = \mu_t + \theta_t + y_t$ where y_t is the underlying zero-mean stationary time series.

(i) The time series exhibits quarterly seasonal variation, θ_t. Describe one method of removing this variation, giving any formula or filter used. [3]

(ii) The time series also exhibits a linear trend $\mu_t = a + bt$. Describe one method of removing this trend. [3]

(iii) The seasonal variation and linear trend are removed and the sample ACF, r_k, and sample PACF, $\hat{\phi}_k$, of the resultant zero-mean stationary time series, z_t, are obtained:



State, with reasons, an appropriate time series to model the observations z_t. [3]

[Total: 25]

Question 4

(a) Derive expressions for ρ_1 and ρ_2, the autocorrelation function of X at lags 1 and 2, in the case that X is a stationary process satisfying the recursion:

$$X_t = \alpha X_{t-1} + e_t + \beta e_{t-1},$$

where $\{e_t : t = 1, 2, \ldots\}$ is a sequence of uncorrelated random variables with mean 0, variance σ^2. **[5]**

(b) A company.s monthly sales figures, corrected for trend and seasonal factors, exhibit sample autocorrelation function at lags 1 and 2 of $r_1 = 0.5$, $r_2 = 0.4$. Find method of moments estimators of α and β for the model in (a). **[3]**

(c) (i) List the main steps in the Box-Jenkins approach to fitting an ARIMA time series to observed data. **[3]**

Observations $x_1, x_2, \ldots, x_{200}$ are made from a stationary time series and the following summary statistics are calculated:

$$\sum_{t=1}^{200} x_t = 83.7 \qquad \sum_{t=1}^{200} (x_t - \hat{\mu})^2 = 35.4 \qquad \sum_{t=2}^{200} (x_t - \hat{\mu})(x_{t-1} - \hat{\mu}) = 28.4$$

$$\sum_{t=3}^{200} (x_t - \hat{\mu})(x_{t-2} - \hat{\mu}) = 17.1$$

(ii) Calculate the first two values of the partial correlation function φ_1 and φ_2. **[6]**

The following model is proposed to fit the observed data:

$$X_t - \mu = \alpha_1 (X_{t-1} - \mu) + e_t$$

where e_t is a white noise process with variance σ^2.

(iii) Estimate the parameters μ, α_1 and σ^2 in the proposed model. **[5]**

After fitting the model in part (iii) the 200 observed residual values $e\hat{}t$ were calculated. The number of turning points in the residual series was 110.

(iv) Carry out a statistical test at the 95% significance level to test the hypothesis that $\hat{e}_t$ is generated from a white noise process. **[3]**

[Total: 25]

©TSA2020

Question 5

(a) Consider the autoregressive process given by

$$Y_t = \alpha Y_{t-2} + e_t$$

e_t being white noise with mean zero and variance σ^2.

(i) What is the range of values of the real valued parameter α so that the process is stationary? **[2]**

(ii) Obtain a representation of Y_t as $\sum_{j=0}^{\infty} a_j e_{t-j}$, by specifying $a_0, a_1, \ldots$ explicitly. **[3]**

(iii) Using part (b) or otherwise, find an expression for the variance of Y_t in terms of α and σ^2. **[2]**

(iv) Compare the result of part (iii) with the variance of an AR(1) process and explain any similarity or dissimilarity. **[2]**

(b) The sample ACF and PACF values at lags 1 to 10 of a time series of length 500, are as given below.

lag	1	2	3	4	5	6	7	8	9	10
SACF	−0.7793	0.6180	−0.4824	0.386	−0.341	0.3172	−0.2989	0.2728	−0.2181	0.163
SPACF	−0.7793	0.0275	0.0188	0.0232	−0.084	0.0538	−0.0289	0.0004	0.0616	−0.0301

(i) Determine through a statistical test whether the time series can be regarded as white noise. **[5]**

(ii) Indicate, with reasons, if an $AR(p)$ or an $MA(q)$ model may be appropriate for this time series, and if so, what could be the model order. **[4]**

(c) (i) An $ARCH(1)$ process X is defined by the equation:

$$X_t = 4 + e_t\sqrt{5 + (X_{t-1} - 4)^2}$$

where e is a white noise process with variance σ^2.

Calculate the mean of the process X and describe the behaviour of this process. **[4]**

(ii) State the meaning of a white noise process (in words, not formulae), and why it is an important concept in the context of modelling financial time series. **[3]**

[Total: 25]

END OF EXAMINATION PAPER

THE UNIVERSITY OF ZAMBIA

2019/2020 ACADEMIC YEAR FINAL EXAMINATIONS

MSE 9030: ADVANCED MATHEMATICS EDUCATION

TIME:	**Three (3) hours**	**Marks possible:**	**100**
INFORMATION:	1. There are **five (5)** questions in this paper.		
	2. Each question carries 25 marks.		
	3. Marks for parts of questions are shown in square brackets [ ].		
INSTRUCTION:	Attempt **four (4)** questions of your choice in this paper.		

Question 1

(a) It is argued by some mathematics educators that a possible side effect of mathematics summative examinations is the temptation of *teaching to the test*.

(i) What is meant by *teaching to the test*? [04]

(ii) Give **one (1)** negative result of teaching to the test, and explain a way in which mathematics summative examinations could enhance the attainment of mathematics curricular objectives. [06]

(iii) Describe a strategy which you could use to minimise the occurrence of the negative effect of teaching to the test specified in part (a) (ii) above. [06]

(b) Explain **three (3)** factors which have an impact on the reliability of a mathematics assessment. [09]

Question 2

(a) Give **three (3)** reasons that justify a school mathematics club as a necessary co-curricular engagement in the context of effective teaching and learning of mathematics. [06]

(b) Outline **three (3)** ways in which failure to review a mathematics curriculum could negatively affect the teaching and learning of mathematics. [06]

(c) Discuss **three (3)** factors that are essential to consider when reviewing a mathematics curriculum. [13]

Question 3

(a) One of the responsibilities of a head of a mathematics department at school level is to appraise teachers of mathematics.

(i) Explain briefly what is meant by teacher appraisal. [03]

(ii) Discuss **three (3)** benefits of teacher appraisal as it relates to professional development of mathematics teachers. [09]

(b) Use **two (2)** points to support the necessity of teachers' subject matter knowledge of mathematics for effective motivation of learners of mathematics. [06]

(c) Outline **three (3)** reasons that could justify the inclusion of *Ethnomathematics* in the teaching and learning process of mathematics. [07]

Question 4

(a) Suggest **two (2)** advantages and **two (2)** disadvantages of allowing learners to use scientific calculators when studying mathematics. [08]

(b) Consider a scenario where a teacher of mathematics has requested you to devise a mathematical investigation which can facilitate learners' relational understanding of the idea of the first derivative under Calculus.

(i) Explain briefly what is meant by a mathematical investigation. [03]

(ii) Provide a comprehensive description of a mathematical investigation which you could recommend for the realisation of the aspiration by the mathematics teacher intimated in the hypothesised scenario. [05]

(c) Explain **three (3)** reasons why *mathematics projects* should be part of the process of teaching and learning mathematics. [09]

Question 5

(a) Recommend, with justification, **two (2)** aspects of mathematics teacher training requiring modification at the University of Zambia. [06]

(b) By reflecting on the topics trigonometry and introductory calculus as prescribed in the Zambian senior secondary school mathematics syllabus,

(i) Describe how a specific information communication technology (ICT) could be used to teach the idea of a trigonometric function. [03]

(ii) Formulate a two parts question that assesses leaners' understanding of any one of the following concepts: *a tangent function*, *an equation of a normal to a curve*, or *an equation of a tangent to a curve*. [06]

(iii) Provide a marking key for the question in part (b) (ii) above showing the distribution of marks. [06]

(iv) Explain briefly how you could help a learner who demonstrates a specific misconception in the process of drawing the graph of the tangent function. [04]

----------**END OF THE EXAMINATION**----------

THE UNIVERSITY OF ZAMBIA

2019/2020 ACADEMIC YEAR FINAL EXAMINATIONS

MSE 9060: ADVANCED CHEMISTRY TEACHING METHODS

Duration: Three **(3)** Hours **Marks:** 100

INFORMATION:

1. There are seven **(7)** questions in this paper.
2. Each question has possible maximum marks of **20.**
3. The intended marks for part of the question are shown in brackets [ ]

INSTRUCTION:

Answer any five **(5)** questions

---

1. With reference to Zambia, UNESCO (2016) identifies the following policy issues: Low teacher training capacity and inadequate teacher qualifications; Inadequate continuous professional development (CPD) programmes; Lack of facilities and resources and weak capacity and qualifications of staff at teacher education institutions and; Low status, morale, and professionalism of teachers among other factors. The ministry responsible for education has initiated several programs aimed at mitigating issues of quality, lack of teaching materials and many other issues related to provision of quality education. Several programmes rolled out over the years and with substantial government and donor funding include: ZAMSTEP; SHAPE; ZEPIU; AIEMS; SMASTE; BEDMAS; SPRINT; lesson study and FTTEP.
   a) Identify two factors past and present that have led to the resurgence of the same challenges over a period of 30 years. [04]
   b) How would have the past governments managed these changes or transitions in the education ministry in order to have lasting impact on the quality of chemistry and other science teachers? [08]
   c) Generate an action research that will be informed by the current/past failures and success and how these can inform chemistry teaching in 2021. [08]

---

2. a) Design an experiment on *Chemical kinetics* that has a direct appeal to a real-life situation of a Zambian pupil, using low cost materials. [04]
   b) Explain **four** important points of teaching about the allotropes of carbon. These points should make the teaching about allotropes an unforgettable experience to the learners. [04]
   c) What are the **two** possible reasons why chemistry teachers in general opt not to perform experiments? [02]
   d) As important as chemistry for national building is, most of the products of chemistry are imported from outside the country. Where is the education system getting it wrong in inculcating productive chemistry values, knowledge and skills in the learners? [06]
   e) Which curriculum model can we utilize to redesign the curriculum in order to begin to see some changes in the way chemistry is taught in schools. Justify your answer. [04]

---

3. a) The leachate from Kalingalinga landfills and pit latrines may leak into groundwater aquifers due to rainfalls, spread into the adjacent river system by groundwater flow and pollute the surrounding environment. However, this process does not stop even after the landfill (or pit latrines) activities have stopped receiving solid waste. Hence, it is very essential to keep assessing and monitoring the surroundings of decommissioned landfill sites. A water sample from a UNZA tap deriving its source from a borehole near Kalingalinga is believed to contain the following contaminants: Aluminium, zinc, copper, chromium, and iron III.
Design a scheme that will help you separate and qualitatively analyse the ions involved. [08]

b) The amount of chloride present in water samples can be determined by this reaction below. The formation of crystals which are later analysed pose real practical issues, many of which are not mentioned at high school level. These issues can cause too little or an exaggerated mass of the precipitate. Explore two issues which can affect the quality and quantity of the precipitates. [02]

$Ag^+(aq) + Cl^-(aq) \rightarrow AgCl(s)$ (White)

c) As a result of the mishap, we have 0.7209 g of a mysterious mixture containing $MgCl_2$ and $NaNO_3$. We would like to know the relative amount of each compound in our mixture, which is fully dissolved in water. We add an excess of our precipitating agent silver(I) nitrate, $AgNO_3(aq)$ and observe the formation of a precipitate, $AgCl(s)$. Once the precipitate is filtered and dried, we find that the mass of the solid is 1.032g.

i. Calculate the mass percent of $MgCl_2$ in the original mixture? Use two methods to arrive at the same answer of which one of them should be the gravimetric method. [06]

ii. Which method involves fewer steps? [01]

iii. Which method would you recommend to your pupils and why? [03]

---

4. a) Using concepts from the Johnstone triangle explain how you can simplify the teaching of polymerization with reference to the confusion which is mostly encountered regarding the identifying of fabrics by their chemistry names (e.g nylon, polyester etc). [08]

b) Name the following ions

i. H_2S (aq)

ii. P^{3-} [02]

iii. OF_2 [02]

c) Write the formula of

i. hydroselenic acid [02]

ii. hydrogen oxide [02]

iii. Mercury (II) [02]

d) Why is it important to teach about the language of chemistry? [02]

---

5. In a secondary school, organizing and managing a science department is often closely linked to effective and efficient use of resources in the department.

a) i) Identify the **three** major resources commonly associated with the natural sciences department and the chemistry section in particular. [06]

ii) Suggest a reason why, as Head of Department (HOD), it will be essential to organize and manage the said resources prudently. [02]

iii) Which type of resource would the chemicals and apparatus represent in the department? [02]

iv) Explain how, as the HOD, you would engage the teachers in this process of organizing and managing the department successfully. [04]

b) Briefly explain what is expected of the HOD for the Natural Sciences Department, with regards to:

i) Professional development for science teachers; [02]

ii) leadership [02]

iii) management [02]

---

6. a) Provide specific scenarios when the following reasoning types can be applied in a chemistry class:

i) inductive reasoning; [04]

ii) deductive reasoning; [04]

iii) abductive reasoning. [04]

b) Which type of reasoning is encouraged in a chemistry class? Justify your answer. [08]

---

7. a) The following multiple choice question appeared in one of the examination and it was found that the question has a discrimination value of +0.51.

i) What can you comment about this value with regards the quality of the question? [02]

ii) Of these responses, (a) to (e), which response would have scored the +0.51 discrimination index. [02]

The question

Which name/formula combination is **WRONG**?

(a) phosphorous acid / H_3PO_4
(b) nitrogen oxide / NO
(c) acetate ion / CH_3COO^-
(d) sodium chromate / Na_2CrO_4
(e) calcium hypobromite / $Ca(BrO)_2$

iii) What value, to the teacher, was achieved by doing this type of analysis? [04]

iv) Would you repeat this question in subsequent examination? Justify your answer. [04]

b) It has been argued that destroying any nation does not need the use of atomic bombs or missiles...it only requires a compromised quality of education. In what ways is this statement relevant to the Zambian examination system? [08]

END OF EXAMINATION

The Periodic Table of the Elements

1																	2
H Hydrogen 1.00794																	**He** Helium 4.003
3	4											5	6	7	8	9	10
Li Lithium 6.941	**Be** Beryllium 9.012182											**B** Boron 10.811	**C** Carbon 12.0107	**N** Nitrogen 14.00674	**O** Oxygen 15.9994	**F** Fluorine 18.9984032	**Ne** Neon 20.1797
11	12											13	14	15	16	17	18
Na Sodium 22.989770	**Mg** Magnesium 24.3050											**Al** Aluminum 26.981538	**Si** Silicon 28.0855	**P** Phosphorus 30.973761	**S** Sulfur 32.066	**Cl** Chlorine 35.4527	**Ar** Argon 39.948
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
K Potassium 39.0983	**Ca** Calcium 40.078	**Sc** Scandium 44.955910	**Ti** Titanium 47.867	**V** Vanadium 50.9415	**Cr** Chromium 51.9961	**Mn** Manganese 54.938049	**Fe** Iron 55.845	**Co** Cobalt 58.933200	**Ni** Nickel 58.6934	**Cu** Copper 63.546	**Zn** Zinc 65.39	**Ga** Gallium 69.723	**Ge** Germanium 72.61	**As** Arsenic 74.92160	**Se** Selenium 78.96	**Br** Bromine 79.904	**Kr** Krypton 83.80
37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
Rb Rubidium 85.4678	**Sr** Strontium 87.62	**Y** Yttrium 88.90585	**Zr** Zirconium 91.224	**Nb** Niobium 92.90638	**Mo** Molybdenum 95.94	**Tc** Technetium (98)	**Ru** Ruthenium 101.07	**Rh** Rhodium 102.90550	**Pd** Palladium 106.42	**Ag** Silver 107.8682	**Cd** Cadmium 112.411	**In** Indium 114.818	**Sn** Tin 118.710	**Sb** Antimony 121.760	**Te** Tellurium 127.60	**I** Iodine 126.90447	**Xe** Xenon 131.29
55	56	57	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86
Cs Cesium 132.90545	**Ba** Barium 137.327	**La** Lanthanum 138.9055	**Hf** Hafnium 178.49	**Ta** Tantalum 180.9479	**W** Tungsten 183.84	**Re** Rhenium 186.207	**Os** Osmium 190.23	**Ir** Iridium 192.217	**Pt** Platinum 195.078	**Au** Gold 196.96655	**Hg** Mercury 200.59	**Tl** Thallium 204.3833	**Pb** Lead 207.2	**Bi** Bismuth 208.98038	**Po** Polonium (209)	**At** Astatine (210)	**Rn** Radon (222)
87	88	89	104	105	106	107	108	109	110	111	112	113	114				
Fr Francium (223)	**Ra** Radium (226)	**Ac** Actinium (227)	**Rf** Rutherfordium (261)	**Db** Dubnium (262)	**Sg** Seaborgium (263)	**Bh** Bohrium (262)	**Hs** Hassium (265)	**Mt** Meitnerium (266)	(269)	(272)	(277)						

58	59	60	61	62	63	64	65	66	67	68	69	70	71
Ce Cerium 140.116	**Pr** Praseodymium 140.90765	**Nd** Neodymium 144.24	**Pm** Promethium (145)	**Sm** Samarium 150.36	**Eu** Europium 151.964	**Gd** Gadolinium 157.25	**Tb** Terbium 158.92534	**Dy** Dysprosium 162.50	**Ho** Holmium 164.93032	**Er** Erbium 167.26	**Tm** Thulium 168.93421	**Yb** Ytterbium 173.04	**Lu** Lutetium 174.967
90	91	92	93	94	95	96	97	98	99	100	101	102	103
Th Thorium 232.0381	**Pa** Protactinium 231.03588	**U** Uranium 238.0289	**Np** Neptunium (237)	**Pu** Plutonium (244)	**Am** Americium (243)	**Cm** Curium (247)	**Bk** Berkelium (247)	**Cf** Californium (251)	**Es** Einsteinium (252)	**Fm** Fermium (257)	**Md** Mendelevium (258)	**No** Nobelium (259)	**Lr** Lawrencium (262)

1995 IUPAC masses and Approved Names from http://www.chem.qmw.ac.uk/iupac/AtWt/
masses for 107-111 from C&EN, March 13, 1995, p. 35
112 from http://www.gsi.de/z112e.html

The University of Zambia
Physics Department
University Examinations 2019-20
PHY 1010: Introductory Physics

All questions carry equal marks. The marks are shown in brackets. Question 1 is compulsory. Attempt four more questions.

Time: Three hours. **Maximum marks = 100.**

Do not forget to write your computer number clearly on the answer booklets as well as on the answer sheet for Question 1. Tie all your answer scripts together with the provided string.

==========

Wherever necessary use:

$g = 9.8 m/s^2$ 1 metric ton = 1000 kg

$P_A = 1.01 \times 10^5$ N/m²

1 cal. = 4.18 J

C_{water} = 4184 J/kg.°C

$R = 8314$ J/kmol.K

$k = 1.381 \times 10^{-23}$ J/K (Bolzmann's constant)

$\varepsilon_0 = 8.85 \times 10^{-12}$ C²/Nm²

$c = 3 \times 10^8$ m/s

$h = 6.63 \times 10^{-34}$ J-s

1 pascal =1 N/m²

$1eV = 1.6 \times 10^{-19}$ J

$e = 1.6 \times 10^{-19}$ C

$m_e = 9.11 \times 10^{-31}$ kg

$k = 9 \times 10^9$ Nm²/C²

$\mu_0 = 4\pi \times 10^{-7}$ N/A²

$\rho_{water} = 1000$ kg/m³

$G = 6.67 \times 10^{-11}$ Nm²/kg²

Question 1: For each correct answer, 2 marks will be given. For each wrong answer, 0.67 will be deducted. For no answer, zero mark will be given. The minimum total mark for Question 1 is zero.

(A) The only modulus that applies to liquids is:

a) Young's modulus:
b) bulk modulus
c) shear modulus
d) modulus of rigidity

(B) An ideal gas is heated at constant volume from 300 K to 600 K. If the original pressure was 1.5 atmospheres, the new pressure will be:

a) doubled
b) halved
c) less than doubled
d) the same

(C) At constant temperature the pressure and volume of a gas are:

a) inversely proportional
b) independent of one another
c) directly proportional
d) constant

(D) A modulus of elasticity is a ratio of:

a) stress to deformation
b) stress to strain
c) strain to stress
d) elongation to deformation

(E) The product of force and the perpendicular distance between some point and the line of application of the force is:

a) the moment of inertia acting on the object
b) the angular momentum of the object
c) torque
d) lever arm

(F) In an elastic collision:

a) momentum is conserved and not kinetic energy
b) neither momentum nor kinetic energy is conserved
c) momentum and kinetic energy are both conserved
d) kinetic energy is conserved and not momentum

The University of Zambia
Department of Physics
University Examinations 2019/20
Introductory Physics

Answer sheet for Question 1

Computer Number .....

Q1. Put a cross (×) or tick mark (√) in the appropriate box. If it is on the dividing line, it will not be counted.

	a	b	c	d
A				
B				
C				
D				
E				
F				
G				
H				
I				
J				

☆

Attach this sheet firmly with the main answer book. If this sheet gets lost, you may lose the marks for Question 1

Do NOT write here. For official use only :

	Number of parts N	Factor f	Marks f × N
Correct		2	
Wrong		–(0.67)	
	Net Marks :		

(G) The first law of thermodynamics is concerned with the conservation of:

a) number of moles
b) number of molecules
c) energy
d) temperature

(H) An acceleration which is constant in time results in a velocity wh:

a) depends inversely on time
b) is a quadratic function of time
c) changes linearly with time
d) remains constant

(I) A vector quantity defined as a change in velocity of a moving object during a given time interval divided by the time interval is:

a) average speed
b) average velocity
c) instantaneous acceleration
d) average acceleration

(J) Two boys are standing at the edge of a vertical bridge. One boy fires an air-gun pellet horizontally at the same time the second boy drops a pellet at the edge of the bridge. The shot pellet and the dropped pellet start at the same height from the top of the bridge. The time taken for the pellets to reach the ground:

a) is the same for both
b) depends on the initial velocity of the one shot off the bridge
c) is shorter for the one shot off the bridge horizontally
d) is greater for the one shot off the bridge horizontally

Attempt any four questions from the following:

Q 2 (a) Add vector **A** and vector **B**, given that vector **A** is 70 N in magnitude and directed at angle of 30^0 with respect to the positive *x*-axis and vector **B** is 50 N in magnitude at angle of 110^0 with respect to the positive *x*-axis. Find:

i) the resultant **R**, its magnitude and direction, and [10]
ii) sketch the resultant vector **R** on an *x-y* plane. [2]

(b) A boy standing on a bridge drops a stone from the bridge. After an interval of 2.5 seconds the stone hits the water. How high is the bridge from the water? [5]

(c) A force of 2000 N pulls a boat through a canal. The angle between the force and the displacement is 25^0. Calculate the work done. [3]

Q.3 (a) A 200 g block moving on a frictionless horizontal surface at a speed of 0.8 m/s strikes a 300 g block which is at rest. Clay on the sides of the two blocks causes them to stick together and move as a unit. Calculate the speed of the two blocks after their collision. [9]

(b) A steel wire is 50 cm long and has a diameter of 0.5 mm. Young's modulus of steel is 20×10^{10} N/m^2. What force is required to increase the length of this wire by 1.0 mm? [6]

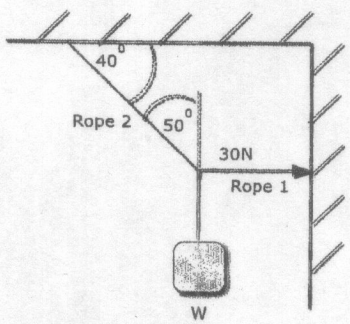
(c) How much heat must be added to 240 g of aluminium to change it from a solid at 27 ^{0}C to a liquid at 660 ^{0}C? Given that the specific heat capacity of aluminium is 880 kJ/kg.^{0}C and the heat of fusion is 397 kJ/kg. [5]

Q.4 (a) A ball is whirled (rotated) at the end of a string in a horizontal circle 50 cm in radius at the rate of 1 revolution (rev) every 2 seconds. Find the ball's centripetal acceleration. [5]

(b) The speed limit along a certain road is 80 km/h. To stop at a junction, the driver applied excessive force on the brakes and skidded for 55 m before stopping. The coefficient of kinetic friction μ_k between the tyres and the road is 0.75. Determine if the driver exceeded the speed limit. [10]

(c) A Carnot engine absorbs 1 million joules (1MJ) of heat from a cold reservoir at 300 ^{0}C and exhausts heat to a reservoir at 150 ^{0}C. Find the work it does. [5]

Q.5 (a) The figure below shows a weight suspended by two ropes, the tension in rope 1 is 30 N. Draw a free body diagram showing the resolution of forces and find the weight *w* of the object. [9]



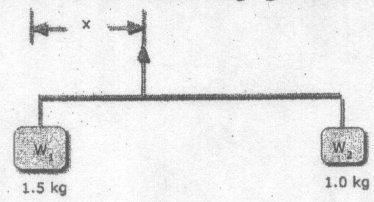
(b) A fish at a depth of 15 m in Lake Kariba exhales an air bubble whose volume is 0.5 cm^3. Find the volume of the bubble just before it reaches the surface. Assume that the temperature remains constant. Given that atmospheric pressure is 1.01×10^5 Pa. [8]

(c) Explain briefly the three modes of heat transfer. [3]

Q.6 (a) A young boy shoots a stone from a catapult at a speed of 15 m/s. The catapult is pointed 40^0 above the horizontal.

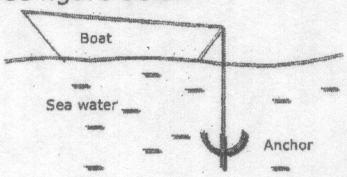
i) How far (horizontally) from the catapult does the stone travel when it falls to the same level as the catapult?
ii) How long does the stone take to reach this level? [9]

(b) A thin rod 1 m long has masses of 1 kg and 1.5 kg hanging at its ends. Find the point at which the rod can be lifted without the rod tending to rotate. Assume the weight of the rod to be negligible. See figure below. [9]



(c) State the condition of translational

Q.7 (a) How much force is needed to support a 100 kg iron anchor when it is immersed in sea water? The density of iron is 7.8×10^3 kg/m^3 and that of sea water is 1.03×10^3 kg/m^3. See figure below. [8]

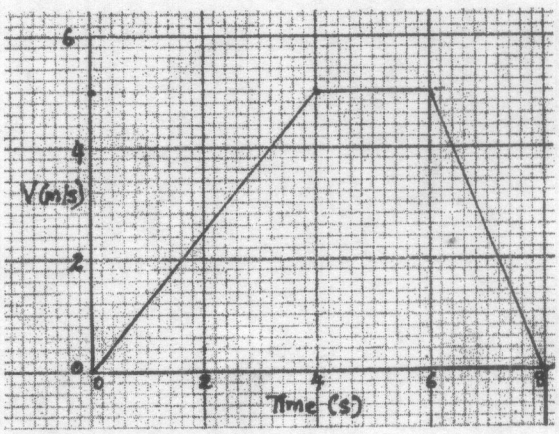


(b) A car travelling at a speed of 54 km/h is brought to rest in 90 s. Find:

i) retardation (deceleration), and
ii) the distance travelled by the car before coming to rest. [8]

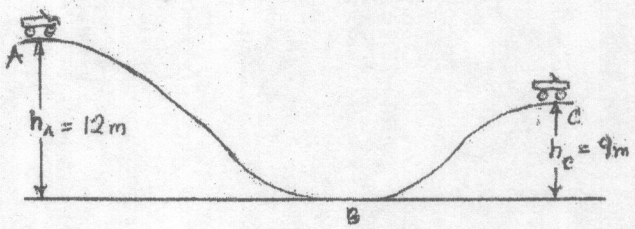
(c) The motion of a toy car is plotted on a velocity-time graph as shown below. The toy car covers a total distance of 13 m. Determine:

i) the distance covered during the first 4 s of the motion.
ii) the distance covered during the last 2 s of the motion. [4]



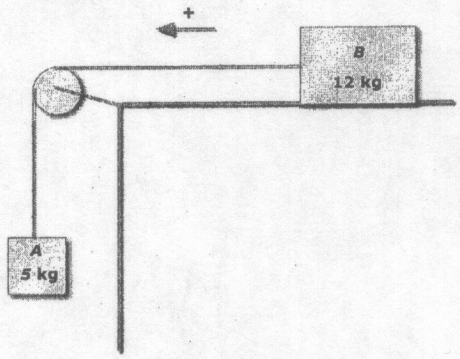
Q.8 (a) A roller coaster car starts from rest at point *A*. It moves to point *C* through point *B*. Assume that friction forces are negligible. See diagram below.

i) What will be its speed at point *B*?
ii) What will be its speed when it reaches point *C*? [10]



(b) The figure below shows a 5 kg block *A* which hangs from a string which passes over a frictionless pulley and is joined to a 12 kg block *B* which lies on frictionless table.

i) Find the acceleration of the two blocks.
ii) Find the tension in the string [10]



END OF EXAMINATION

Some Useful Equations

Uniformly accelerated motion:

$x = \bar{v}t \qquad \bar{v} = \frac{1}{2}(v_f + v_i) \qquad v_f = v_i + at \qquad v_f^2 = v_i^2 + 2ax \qquad x = v_i t + \frac{1}{2}at^2$

Projectile motion:

$v_x = v_i \cos\theta_i = \text{constant} \qquad v_y = v_i \sin\theta_i - gt \qquad y = (v_i \sin\theta_i)t - \frac{1}{2}gt^2$

$y = (\tan\theta_i)x - \left[\frac{g}{2v_i^2(\cos^2\theta_i)}\right]x^2 \qquad R = \frac{v_i^2}{g}\sin 2\theta \qquad t = \frac{2v_i \sin\theta}{g}$

Force and motion:

$F = ma \qquad w = mg \qquad F_{AB} = -F_{BA} \qquad F_f = \mu F_N$

Energy:

$PE = wh = mgh \qquad KE = \frac{1}{2}mv^2 \qquad W = Fx\cos\theta \qquad P = \frac{W}{t} = Fv\cos\theta$

Linear momentum:

$p = mv \qquad F\Delta t = \Delta mv$

Circular motion and gravitation:

$T = \frac{2\pi r}{v} \qquad a_c = \frac{v^2}{r} \qquad F_c = \frac{mv^2}{r} \qquad F_{grav} = G\frac{m_A m_B}{r^2} \qquad 1rev = 360^0 = 2\pi\, rad$

Rotational motion and angular momentum:

$\theta = \frac{s}{r} = \left(\frac{\omega_i + \omega_f}{2}\right)t \qquad \omega = \frac{\theta}{t} \qquad \theta = \omega_i t + \frac{1}{2}\alpha t^2 \qquad \omega_f = \omega_i + \alpha t$

$v = \omega r \qquad \omega_f^2 = \omega_i^2 + 2\alpha\theta \qquad \alpha = \frac{\Delta\omega}{\Delta t} = \frac{a_T}{r} \qquad I = \sum mr^2 \qquad I = mk^2$

$KE_{rot} = \frac{1}{2}I\omega^2 \qquad \tau = FL = I\alpha \qquad W = \tau\theta \qquad P = \tau\omega \qquad L = I\omega$

Properties of matter:

$\rho = \frac{m}{V} \qquad F = -kx \qquad \frac{\Delta L}{L_i} = \frac{1}{Y}\frac{F}{A} \qquad \phi = \frac{s}{d} = \frac{1}{s}\frac{F}{A} \qquad B = -\frac{\Delta P}{\Delta V / V_0}$

$W_{app} = W\left(1 - \frac{\rho_{fluid}}{\rho}\right) \qquad F_B = \rho V g$, submerged object $\qquad F_B = Mg$ floating M

Thermal Properties of matter:

$PV = nRT$: $\Delta Q = mc\Delta T = nC\Delta T$: $\Delta L = \alpha L \Delta T$: $L_t = L_0(1+\alpha t)$: $\Delta V = \gamma V \Delta T$: $\Delta W = P\Delta V$

$(\Delta Q / \Delta t) = (kA\Delta T)/\Delta L$ $\Delta Q = \Delta U + \Delta W$

Thermodynamics:

$\Delta Q = \Delta U + \Delta W$: $P_1V_1^{\gamma} = P_2V_2^{\gamma}$ $PV = nRT$ $W = p.\Delta V$

$COP_{ref} = \frac{Q_C}{W}$ $COP_{heat\ pump} = \frac{Q_H}{W}$

$e = 1 - \frac{T_c}{T_h}$

Waves and Sound:

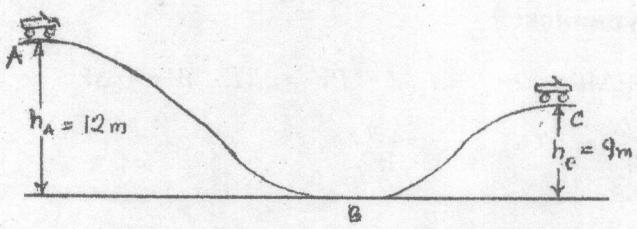
$f = \frac{1}{\tau}$ $v = \pm\sqrt{\frac{k}{m}(x_0^2 - x^2)}$ $v = \sqrt{\frac{T}{m/L}}$ $\tau = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$ $a = -\left(\frac{k}{m}\right)x$ $v = \sqrt{\frac{Y}{\rho}}$

$v = \sqrt{\frac{B}{\rho}}$ $f' = f\,\frac{v \pm v_L}{v \mp v_S}$ (dB) = 10. log $\frac{I}{I_0}$

Volume of a sphere $= (4/3)\pi r^3$; Surface area of a sphere = $4\pi r^2$

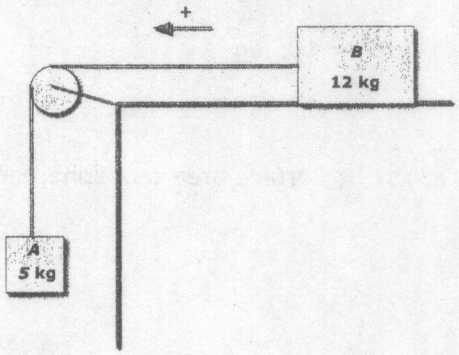
Q.8 (a) A roller coaster car starts from rest at point *A*. It moves to point *C* through point *B*. Assume that friction forces are negligible. See diagram below.

i) What will be its speed at point *B*?
ii) What will be its speed when it reaches point *C*? [10]



(b) The figure below shows a 5 kg block *A* which hangs from a string which passes over a frictionless pulley and is joined to a 12 kg block *B* which lies on frictionless table.

i) Find the acceleration of the two blocks.
ii) Find the tension in the string [10]



END OF EXAMINATION

THE UNIVERSITY OF ZAMBIA
School of Natural Sciences
Department of Physics
2019–2020 Academic Year
Final Examinations

PHY2112 – ATOMIC AND MODERN PHYSICS

Duration: 3 hours Maximum Marks: 100

Instructions

- This examination paper contains 7 questions. Attempt any 5 questions.
- Each question carries 20 marks. Marks allocated for each question are indicated in brackets [ ].
- Show all your working clearly. Omission of essential work will result in loss of marks.
- Follow carefully instructions written on the answer booklets.
- Additional information that you may require is found at the end of this examination paper.

---

1. (a) State Stefan's law and write an equation for it. [4]

(b) Consider a black body of surface area 20.0 cm^2 and temperature 5 000 K. How much power does it radiate? [4]

(c) Find the peak wavelength of the blackbody radiation emitted by the human body when the skin temperature is 35°C. [4]

(d) A 2.00-kg block is attached to a massless spring that has a force constant of $k = 25.0$ N/m. The spring is stretched 0.400 m from its equilibrium position and released from rest.

i. Find the total energy of the system and the frequency of oscillation according to classical calculations. [4]

ii. Assuming the energy of the oscillator is quantized, find the quantum number n for the system oscillating with this amplitude. [4]

---

2. (a) State the photoelectric effect. [2]

(b) A sodium surface is illuminated with light having a wavelength of 300 nm. Given that the work function for sodium metal is 2.46 eV,

i. find the maximum kinetic energy of the ejected photoelectrons, [4]

ii. find the cutoff wavelength λ_c for sodium. [5]

(c) Electrons are ejected from a metallic surface with speeds of up to 4.60×10^5 m/s when light with a wavelength of 625 nm is used.

i. What is the work function of the surface? [4]

ii. What is the cutoff frequency for this surface? [5]

---

3. (a) State the principle of complementarity. [2]

(b) X-rays of wavelength $\lambda_0 = 0.200$ nm are scattered from a block of material. The scattered x-rays are observed at an angle of 45.0° to the incident beam. Calculate their wavelength. [6]

(c) Find the ratio of the de Broglie wavelength for an electron moving at 1.00×10^7 m/s to that of a rock of mass 50 g thrown with a speed of 40 m/s. [8]

(d) Considering the answer to the previous question (2c), state why the wave properties of large-scale objects cannot be observed. [4]

---

4. (a) State the Heisenberg uncertainty principle and provide an equation for this principle. [4]

(b) The speed of an electron is measured to be 5.00×10^3 m/s to an accuracy of 0.003%. Find the minimum uncertainty in determining the position of this electron. [8]

(c) Atoms have quantized energy levels similar to those of Planck's oscillators, although the energy levels of an atom are usually not evenly spaced. When an atom makes a transition between states, energy is emitted in the form of a photon.

Although an excited atom can radiate at any time from $t = 0$ to $t = \infty$, the average time interval after excitation during which an atom radiates is called the lifetime τ. If $\tau = 1.0 \times 10^{-8}$ s, use the uncertainty principle to compute the line width Δf produced by this finite lifetime. [8]

---

5. (a) The electron in a hydrogen atom makes a transition from the $n = 2$ energy level to the ground level ($n = 1$). Find the wavelength and frequency of the emitted photon. [8]

(b) In interstellar space, highly excited hydrogen atoms called Rydberg atoms have been observed. Find the wavelength to which radio astronomers must tune to detect signals from electrons dropping from the $n = 273$ level to the $n = 272$ level. [4]

(c) What is the radius of the electron orbit for a Rydberg atom for which $n = 273$? [4]

(d) How fast is the electron moving in a Rydberg atom for which $n = 273$? [4]

---

6. (a) State the correspondence principle. [2]

(b) For a hydrogen atom, determine the allowed states corresponding to the principal quantum number $n = 2$ and calculate the energies of these states. [6]

(c) Calculate the most probable value of r for an electron in the ground state of the hydrogen atom. [6]

(d) Calculate the probability that the electron in the ground state of hydrogen will be found outside the first Bohr radius. [6]

---

7. (a) Define half-life of a radioactive substance. [2]

(b) A sample of the isotope ^{131}I, which has a half-life of 8.04 days, has an activity of 5.0 mCi at the time of shipment. Upon receipt of the sample at a medical laboratory, the activity is 2.1 mCi. How much time has elapsed between the two measurements? [4]

(c) At time $t = 0$, a radioactive sample contains 3.50 μg of pure $^{11}_{6}$C, which has a half-life of 20.4 min.

i. Determine the number N_0 of nuclei in the sample at $t = 0$. [3]

ii. What is the activity of the sample initially and after 8.00 h? [3]

(d) A piece of charcoal containing 25.0 g of carbon is found in some ruins of an ancient city. The sample shows a ^{14}C activity R of 250 decays/min. How long has the tree from which this charcoal came been dead? [4]

(e) The isotope carbon-14, $^{14}_{6}$C, is radioactive and has a half-life of 5 730 years. If you start with a sample of 1 000 carbon-14 nuclei, how many nuclei will still be undecayed in 25 000 years? [4]

End of Examination

ADDITIONAL INFORMATION

CONSTANTS THAT MAY BE USEFUL

- $e = 1.602 \times 10^{-19}$ C
- $k_e = 8.99 \times 10^{9}$ N.m^2/C^2
- $m_p = 1.67 \times 10^{-27}$ kg
- $m_e = 9.11 \times 10^{-31}$ kg
- $\varepsilon_o = 8.85 \times 10^{-12}$ C^{2}N^{-1}m^{-2}
- $h = 6.63 \times 10^{-34}$ Js
- $N_{\text{Av}} = 6.02 \times 10^{23}$
- $\mu_o = 4\pi \times 10^{-7}$ Ns^{2}C^{-2}
- $R_H = 1.097 \times 10^{7}$/m
- $a_0 = 0.0529$ nm
- $c \cong 3.0 \times 10^{8}$ m/s
- 1Ci $\equiv 3.7 \times 10^{10}$ decays/s $= 3.7 \times 10^{10}$ Bq

FORMULAE THAT MAY BE USEFUL

- $\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)$
- $\lambda = \frac{h}{p}$

- $\lambda_{\max} T = 2.898 \times 10^{-3}$ m.K
- $K_{\max} = hf - \phi$
- $\frac{1}{\lambda} = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$
- $r_n = n^2 a_0$
- $v^2 = \frac{k_e e^2}{m_e r}$
- $E_n = -\frac{13.606}{n^2}$ eV
- $P_{1s}(r) = \left(\frac{4r^2}{a_0^3} \right) e^{-2r/a_0}$
- $E_L = -(Z-1)^2 \left(\frac{13.6}{2^2} \right)$ eV
- $E_K \approx -Z^2(13.6)$ eV
- $R = |dN/dt| = \lambda N = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$
- $T_{1/2} = \frac{\text{In2}}{\lambda} = \frac{0.693}{\lambda}$
- $N = N_0 \left(\frac{1}{2} \right)^n$



THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
Department of Physics
2019/2020 ACADEMIC YEAR
End of Year University Examinations
PHY 2231 - Thermodynamics and Properties of Matter

Duration : 3 Hours
Total marks: 100
INSTRUCTIONS
This question paper contains seven questions.
Answer any five questions. Each question carries 20 total marks.
Write clearly your computer number on the answer booklet provided.
All working should be shown clearly.
Omission of essential work will lead to loss of marks.
Additional information can be found at the end of this question paper.

QUESTION 1

(a) Define the terms viscosity and coefficient of viscosity for a liquid. [2]

(b) Derive Poiseuille's formula. [10]

(c) A cylindrical vessel of diameter 10 cm has at its bottom a horizontal capillary tube of length 20 cm and internal radius 0.5 mm. If the vessel is filled with water, find the time in which the water level becomes half the initial height. Viscosity of water $= 10^{-3}$ Ns/m^{-3}. [8]

QUESTION 2

(a) A copper wire (Y=11.5x10^{10} N/m^2) with a cross section area of 0.005 cm^2 is to be used in lifting a 2 kg box. How fast can the object be accelerated if the wire is to stretch no more than 0.1 %? [6]

(b) If a cantilever is pulled down and released, it will move up and down. Show that such motion is simple harmonic motion. [6]

(c) Two cylindrical shafts have the same length, and are made from the same material. One is solid and the other is hollow with the external radius twice the internal radius. Compare the torsional rigidities produced by equal twisting torques. [8]

QUESTION 3

(a) Describe Jaeger's method for determining the surface tension of a liquid. [7]

(b) Derive an expression for the excess pressure inside a spherical

bubble blown within a liquid. [7]

(c) Calculate the excess pressure inside a bubble of radius 0.01 m if the surface tension of a soap solution is 5.5×10^{-2} N/m. [3]

(d) Estimate the height to which water of surface tension of 7×10^{-2} N/m will rise in a glass capillary tube of radius 1 mm. [3]

QUESTION 4

(a) A copper wire 2 m long is stretched by 1 mm. If the energy stored in the stretched wire is converted into heat, calculate the rise in temperature of the wire. Take $Y_{Cu} = 1.2 \times 10^{11}$ N/m^2, $c_{Cu} = 0.1$ J/kg, and $\rho_{Cu} = 9000$ kg/m^2. [7]

(b) A composite flat sheet is made of 1.2 cm thickness of the material of conductivity (K_1) 3×10^{-3} cal/cm°C and thickness 0.8 cm of another material of conductivity (K_2) 6×10^{-4} cal/cm°C joined together. If the open face of the first material is at a temperature of -30 °C and the open face of the second material is at 40 °C, calculate the temperature of the interface between the two materials. [5]

(c) Along the axis of a long uniform cylindrical shape of inner and outer radii 4 and 12 cm respectively, there is a wire in which 7 watts of energy is developed per cm length. If the conductivity of the material is 0.3 J/cm°Cs and it is at a steady state temperature of 20 °C for outer surface, find the temperature at the inner face of the cylinder and at r = 8 cm. [8]

QUESTION 5

(a) The opposite faces of a rectangular block of iron are kept in contact with a block of ice and steam at 100 °C. The area of cross section of the iron block is 4 cm^2, and the thickness is 1 cm. Find the

mass of ice that melts in 20 minutes. Take H_f to be equal to 333 kJ/kg and K for iron to be equal to 59 W/m.k. [4]

(b) Show that the total depression (y) of cantilever whose beam's weight is negligible, and with an external load of W at the free end is given by

$$y = \frac{WL^3}{3YI}.$$

Where Y is the Young modulus of the material from which the cantilever is made, L is the length of the cantilever and I is the moment of inertia. [8]

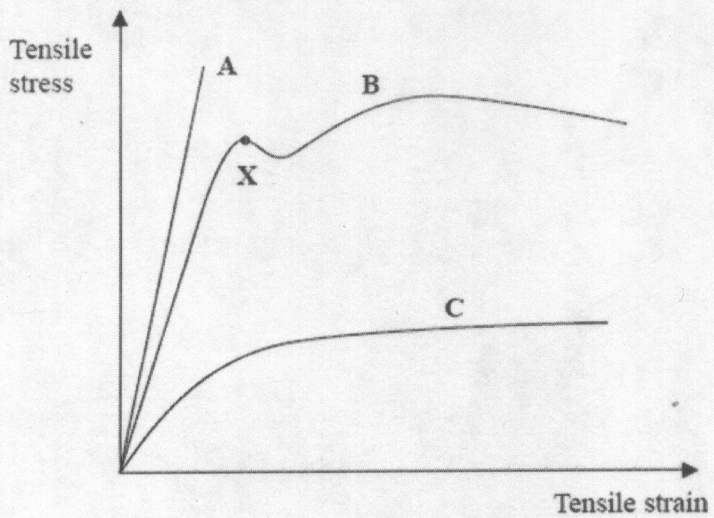
(c) One mole of a gas is isothermally expanded at 27 °C until it's volume is doubled. Then it is adiabatically compressed to its original volume. Find the total work done by the gas. Take $\gamma = 1.4$ and R = 8.314 J/K. [8]

.....

QUESTION 6

.....

(a) The graph shows the stress-strain curves for three materials A, B, and C up to the point of fracture.



(i) Identify which of the materials A, B or C is, a brittle material,

a ductile material, the strongest material, and the least stiff material. [4]

(ii) If the three materials are copper, glass and steel, identify which graph refers to each material. [2]

(iii) State the name for the point marked X on graph B and explain its significance. [2]

(b) Briefly explain what is meant by a variable state and a steady state. [2]

(c) Calculate the increase in entropy when 2 kg of ice at -30 °C is converted into steam at 100 °C. Take specific heat of ice to be 0.5 kcal/kg°C, latent heat of fusion of ice to be 340 kJ/kg and latent heat of vaporisation to be 2268 kJ/kg.. [10]

.....

QUESTION 7

.....

(a) State

(i) three fundamental assumptions of the kinetic theory of a gas. [3]

(ii) the Zeroth, First, and Second Laws of thermodynamics. [3]

(b) If internal energy (U) is a function of temperature (T) and pressure (P) i.e., U = f(P,T), show that

$$dQ = \left[\left(\frac{\delta U}{\delta T}\right)_P + P\left(\frac{\delta V}{\delta T}\right)_P\right] dT + \left[\left(\frac{\delta U}{\delta P}\right)_T + P\left(\frac{\delta V}{\delta P}\right)_T\right] dP.$$

[7]

(c) What considerations led van der Waals to modify the equation of state? Deduce the van der Waals gas equation $(P+\frac{a}{v^2})(V\text{-}b) = RT$. What are the dimensions of a and b? [7]

——— END OF EXAMINATIONS ———

ADITIONAL INFORMATION

Planck's constant h = 6.62x10^{-34} Js,
electron charge e = 1.602x10^{-19} C,
electron mass m_e = 9.11x10^{-31} kg,
proton mass m_p = 1.67x10^{-27} kg,
neutron mass m_n = 1.67x10^{-27} kg,
Bohr magneton μ_B =9.274x10^{-24} Am^2
Avogadro's constant N_A = 6.022x10^{23} mol^{-1},
Boltzmann constant k_B = 1.38x10^{-23} JK^{-1},
Permitivity of free space ϵ_0 = 8.85x10^{-12}s^{2}m^{-2},
permeability of free space $\mu_0 = 4\pi$x10^{-7},
Bohr magneton μ_B =9.274x10^{-24} Am^2

$$dQ = dU + PdV; dQ = TdS; Tds = dU + PdV; W = -\int_{V_1}^{V_f} PdV$$

$$C_V - C_P = R; PV = nRT; \left[P + \frac{a}{v^2}\right][v - b] = RT; PV^\gamma = constant$$

$$\eta = 1 - \frac{T_2}{T_1}; L = L_0[1 + \alpha\Delta T]; A = A_0[1 + \beta\Delta T]; V = V_0[1 + \gamma\Delta T]$$

$$Y = \frac{F/A}{\Delta L/L_0}; \eta = \frac{F/A}{\Theta}; W = \frac{1}{2}kx^2; U_e = \frac{1}{2}XstressXstrain; \sigma = \frac{\beta}{\alpha}$$

$$C = \frac{\pi\eta}{2L}\left[r_0^4 - r_i^4\right]; \rho = \frac{m}{V}; \tau = YI_g\frac{d^2y}{dx^2}; \gamma = \frac{F}{2L}; \Delta P = 2\gamma\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

$$P = \frac{2\gamma}{R}; P = \frac{4\gamma}{R}; \gamma = \frac{gR}{2}[\rho_i h_i - \rho h]; V = \frac{\pi Pr^4}{8\eta l}; \gamma = \frac{r[h + \frac{r}{3}]\rho g}{2cos\Theta}$$

$$K = -\frac{dP}{\Delta V/V}$$

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF PHYSICS
2019/2020 ACADEMIC YEAR
END YEAR UNIVERSITY EXAMINATIONS
PHY-2511: CLASSICAL MECHANICS

Time allowed: **3 Hours**

Instructions

- **This examination paper contains 7 questions. Each question carries 20 marks. Attempt any 5 questions out of the 7 questions given.**
- **This paper has a total of 100 marks**
- **Show all your work clearly. Omission of essential work will result in loss of marks**
- **Write your computer number clearly on the answer sheet**

Where necessary, you may use the following:

In plane polar coordinates

$x = r\cos\theta, \qquad y = r\sin\theta$

$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} \qquad \vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\theta}$

In cylindrical coordinates

$x = \rho\cos\phi \qquad y = \rho\sin\phi \qquad z = z$

$\vec{v} = \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi} + \dot{z}\hat{z} \qquad \vec{a} = \left(\ddot{\rho} - \rho\dot{\phi}^2\right)\hat{\rho} + \left(\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi}\right)\hat{\phi} + \ddot{z}\hat{z}$

In spherical coordinates

$x = \rho\cos\phi \qquad y = \rho\sin\phi \qquad z = z$

$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + \left(r\dot{\phi}\sin\theta\right)\hat{\phi}$

$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2 - r\sin^2\theta\dot{\phi}^2\right)\hat{r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\sin\theta\cos\theta\dot{\phi}^2\right)\hat{\theta} + \left(r\sin\theta\ddot{\phi} - 2\dot{r}\dot{\phi} + 2r\dot{\theta}\dot{\phi}\cos\theta\right)\hat{\phi}$

$\int\cos x dx = \sin x + C \qquad \int(\alpha + bx)^{-\frac{1}{2}}dx = \frac{2}{b}(\alpha + bx)^{\frac{1}{2}}$

$\int\frac{xdx}{\sqrt{\alpha^2 - x^2}} = -\sqrt{\alpha^2 - x^2}$

$\int\sin x dx = -\cos x + C \qquad e^{i\theta} = \cos\theta + i\sin\theta$

$$Curl\ \overline{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

Q1 The velocities of a particle along and perpendicular to the radius (that is, radial and transverse velocities) from a fixed origin are λr^2 and $\mu\theta^2$ respectively, λ and μ are constants. [**Hint**: In Cartesian polar coordinates, $\hat{r} = \dot{\theta}\hat{\theta}$]

(a) Show that the radial component of the acceleration of the particle is

$$\left(2\lambda^2 r^3 - \frac{\mu\theta^4}{r}\right)$$ **[9 marks]**

(b) Show that the angular component of the acceleration of the particle is

$$\left(\frac{2\mu^2\theta^3}{r} - \lambda\mu r\theta^2\right)$$ **[9 marks]**

where $\hat{r}$ and $\hat{\theta}$ are unit vectors along and perpendicular to the radius vector respectively.

(c) Write down the complete vector expression for the acceleration **[2 marks]**

Q2(a) A particle of mass m moves in a straight line under the action of a force whose potential energy is

$$U = ax^2(b - x)$$

where a and b are positive constants

(i) Find an expression for the force. **[4 marks]**

(ii) Give the two conditions under which the force can be zero **[2 marks]**

(b) Find the kinetic energy of a particle of mass 20kg moving with a velocity $\bar{v} = 3\hat{i} - 5\hat{j} + 4\hat{k}$ **[4 marks]**

(c) A particle moves in a plane elliptical orbit described by the position vector

$$\bar{r}(t) = 2b\sin(\omega t)\hat{i} + b\cos(\omega t)\hat{j}$$

(i) Find the expression for the acceleration of the particle at any time t. **[4 marks]**

(ii) Find the acceleration of the particle when $t = \frac{\pi}{2\omega}$. **[6 marks]**

Q3(a) Find the value of a such that the force field given by

$$\bar{F} = xy\hat{i} - ax^2\hat{j} + z^3\hat{k}$$

is conservative. **[8 marks]**

(b) By invoking the principle of energy conservation, show that if the force kx on a particle of mass m causes it to oscillate without loss of energy, then the velocity of the particle at any time can be written as

$$\dot{x} = \pm\left(\frac{2E}{m} - \frac{k}{m}x^2\right)^{1/2}$$ **[6 marks]**

where E is the total energy of the particle.

(c) A bead slides on a wire bent into the form of a helix. The motion of the bead is given in cylindrical coordinates by $\rho = b$, $\phi = \omega t$, $z = ct$, where b, ω, c are constants.

Determine the

(i) velocity and **[3 marks]**

(ii) acceleration of the bead. **[3 marks]**

Q4 A particle of mass m starts from rest and moves under a constant force given by $\bar{F}(x) = k\hat{i}$ resulting in a potential $V(x) = kx$

(i) Show that the time taken to move any distance x in general is given by

$$t = \frac{m}{k}\left[\frac{2}{m}(E + kx)\right]^{1/2} + T$$ **[12 marks]**

where the constant of integration T can be found from the initial conditions ($t = 0$, $v = v_0$) to be

$$T = -\frac{mv_0}{k}$$

(ii) Show that if at $t = 0$, $v = v_0$, the position at any time t may be written as

$$x = v_0 t + \frac{1}{2}at^2$$ **[8 marks]**

where a is the acceleration and is given by $a = k/m$.

Q5(a) The equation of a "free" simple harmonic oscillator is given by

$$m\ddot{x} = -kx .$$

Assuming an exponential solution of the form $x = e^{\lambda t}$

(i) Show that $\lambda = \pm i\omega$ **[6 marks]**

(ii) Write down the expression of the general exponential solution in terms of the two roots of λ and some two constants A and B **[2 marks]**

(iii) By defining $x_0 \sin\delta = A + B$ and $x_0 \cos\delta = i(A + B)$, show that its general solution can be written as $x = x_0 \sin(\omega t + \delta)$ when it starts at the positive extreme of its motion at time $t = 0$ **[6 marks]**

(b) For a simple harmonic oscillator of mass m, the displacement is given by

$$x = A\sin(\omega_0 t) .$$

Show that the maximum kinetic energy of such an oscillator is

$$K = T = \frac{1}{2}kA^2$$

where $\omega_0 = \sqrt{k/m}$. **[6 marks]**

Q6 An object of mass m falls vertically downwards under a constant gravitational acceleration g. The resistive force acting on the object is mkv where $k = 0.10s^{-1}$ and v is the velocity at any time t.

(a) Show that the expression for the velocity at any time t in terms of g and k is given by

$$v = \frac{g}{k}\left(1 - e^{-kt}\right).$$ **[12 marks]**

(b) Find the expression for the distance travelled by the object at any time t. **[4 marks]**

(c) It the object hits the ground after 2 seconds, calculate the velocity of the object as it strikes the ground and the initial height of the object at the start of the motion. (Assume $v = v_0 = 0$ at $t = 0$). **[4 marks]**

Q7 A particle of mass m starts from rest at $x_0 (>0)$ in an attractive inverse-cube force field $F = -\frac{k}{x^3}$ (k is a positive constant).

(i) Show that the equation of motion results in

$$v(x) = \frac{dx}{dt} = -\sqrt{\frac{k}{mx_0^2}} \frac{\sqrt{x_0^2 - x^2}}{x}$$

for particle moving in the negative x-direction, that is $v(x) \leq 0$. **[10 marks]**

(ii) From the velocity expression given in (i) above or otherwise, show that the trajectory of the particle is given by

$$x(t) = x_0 \sqrt{\left(1 - \frac{k}{mx_0^4} t^2\right)}$$ **[7 marks]**

(iii) Deduce the time required for the particle to reach the origin. **[3 marks]**

**********END OF PHY 2511 EXAMINATION**********

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF PHYSICS
2019/2020 ACADEMIC YEAR
END YEAR UNIVERSITY EXAMINATIONS
PHY-2522: ANALYTICAL MECHANICS AND SPECIAL THEORY OF RELATIVITY

Time allowed: **3 Hours**

Instructions

- **This examination paper contains 7 questions. Each question carries 20 marks. Attempt any 5 questions out of the 7 questions given.**
- **This paper has a total of 100 marks.**
- **Show all your working clearly. Omission of essential work will result in loss of marks**
- **Write your computer number clearly on the answer sheets.**

Where necessary, you may use the following:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right)-\frac{\partial L}{\partial q_k}=Q_k, \qquad \dot{q}_k=\frac{\partial H}{\partial p_k}, \qquad \frac{\partial H}{\partial t}=-\frac{\partial L}{\partial t}, \qquad -\dot{p}_k=\frac{\partial H}{\partial q_k}, \qquad x'=\gamma(x-vt),$$

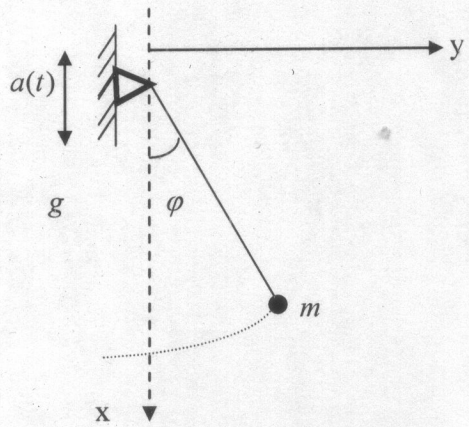
$$t'=\gamma\left(t-\frac{Vx}{c^2}\right), \qquad y'=y, \quad z'=z, \qquad \gamma=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}, \text{ speed of light } c=3\times10^8\,\text{m/s}$$

$$\frac{\partial^2 u(x,t)}{\partial t^2}=\frac{T}{\mu}\frac{\partial^2 u(x,t)}{\partial x^2}, \qquad \gamma L=L_0, \quad T=\gamma T_0$$

$$T=\gamma T_0$$

$$\int\frac{xdx}{\sqrt{\alpha^2-x^2}}=-\sqrt{\alpha^2-x^2}$$

Q1 A pendulum with a periodically moving support is depicted in the figure below. The length of the string connecting the mass m is inextensible and mass less. If the point of support moves according to the equation $x = a(t)$,



(i) write down the x and y coordinates of the mass. **[2 marks]**

(ii) Obtain the expression for the kinetic energy. **[3 marks]**

(iii)Find the potential energy **[3 marks]**

(iv)Use the Lagrangian formalism to obtain the equation of motion **[9 marks]**

(v) Show that your equation in (iv) above reduces to the equation of a simple pendulum oscillating with very small angles when the support is not moving. **[3 marks]**

Q2 (a) Consider a projectile motion under gravity in two dimensions. If the projectile is fired at an angle θ to the horizontal, employ polar coordinates to

(i) Obtain the Lagrangian. **[6 marks]**

(ii) Show that the resulting equations of motion employing are:

$$r\dot{\theta}^2 - g\sin\theta - \ddot{r} = 0$$ **[5 marks]**

and

$$-gr\cos\theta - 2r\dot{r}\dot{\theta} - r^2\ddot{\theta} + = 0$$ **[5 marks]**

(b) Explain and give reasons why it is more convenient to use Cartesian coordinates in the projectile motion problem. **[4 marks]**

Q3(a) The relativistic expression for the kinetic energy of a moving particle is given by

$$T = mc^2 - m_0c^2$$

Show that for velocities v much lower than the speed of light c, this expression reduces to the Newtonian one. **[6 marks]**

(b) One of a pair of identical twins sets off on a space voyage at the age of 30 years and his spaceship travels at 90% the speed of light. He comes back and tells his brother that he is 36 years old. How old will his brother say he is? **[4 marks]**

(c) In a stationary laboratory the length of a meter rule moving longitudinally along its length is found to be 0.64m. How fast in terms of the speed of light ($c = 3.0\times10^8$ m/s) is the meter rule moving with respect to the observer in the laboratory? **[4 marks]**

(d) A wave in a string has its displacement y in meters represented by

$$y(x,t) = 0.1\cos\left[\left(\frac{\pi}{2}\right)x - (100\pi)t\right]$$

(i) Obtain the wavelength λ of the waves on the string and **[3 marks]**

(ii) the frequency f of the waves on the string. **[3 marks]**

Q4(a) Explain the meaning of an ignorable coordinate in the Lagrangian or Hamiltonian of a dynamical problem. **[3 marks]**

(b) A particle of mass m moves in a plane under a central force field which gives rise to a potential energy function $U(r)$ for the particle.

(i) Obtain the expression for the Lagrangian for the particle. **[6 marks]**

(ii) Find the equation of motion of the $r-$component. **[4 marks]**

(iii) Find the equation of motion of the $\theta-$component. **[4 marks]**

(iv) What conclusion can be made about the $\theta-$equation in terms of conservation? **[3 marks]**

Q5(a). A string of length L and linear mass density μ lies horizontally and is fixed at both ends. If the string is slightly pulled upward in the middle and allowed to vibrate, show that the total energy in the string at any time is given by

$$E = \frac{\mu}{2}\left[\int_0^L \left\{\left(\frac{dy}{dt}\right)^2 + v^2\left(\frac{dy}{dx}\right)^2\right\}\right]dx$$ **[10 marks]**

(b)(i) Show that the sum of the waves $u_1(x,t) = f(x-vt) = A\sin(kx-\omega t)$ and $u_2(x,t) = f(x+vt) = A\sin(kx+\omega t)$ travelling in a medium in opposite directions is

$$u(x,t) = 2A\sin(kx)\cos(\omega t)$$ **[4 marks]**

(iii) Show if any of the following is a solution of the one dimensional wave equation

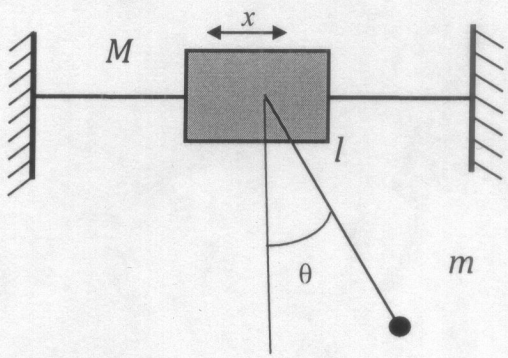
$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

$$y = \sin(2x)\cos(vt)$$ **[3 marks]**

and

$$y = (x-vt)^2$$ **[3 marks]**

Q6 A mass M is free to slide along a frictionless rail. A pendulum of length l and mass m hangs from M as shown in the figure below



(i) Find the Lagrangian of the system. **[15 marks]**

(ii) What is the ignorable coordinate in the system and what does it signify? **[2+3 marks]**

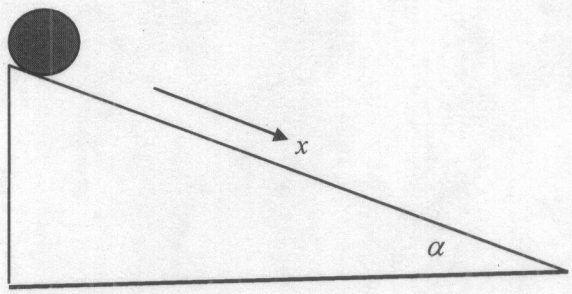
Q7(a) A solid sphere of mass M and radius a is rolling without slipping down the incline as shown in the figure below.

(i) Obtain the Hamiltonian of the system **[8 marks]**

(ii) Use your result in (i) above to show that the sphere has an acceleration of

$$\ddot{x} = \frac{5}{7} g \sin\alpha \qquad \textbf{[5 marks]}$$

down the incline and g is the acceleration due to gravity.



(b) By employing the Lagrangian formalism, obtain the equation of motion for a mass m attached to one end of a spring of spring constant k that is performing simple harmonic motion on a horizontal frictionless table in one dimension. **[7 marks]**

**********END OF PHY 2522 EXAMINATION**********

THE UNIVERSITY OF ZAMBIA

Department of Physics

University Examinations: 2019/2020 Academic Year

PHY 2611: ELECTRICITY AND MAGNETISM

Time duration: **Three (3)** hours.

Instructions: Answer any **five (5)** questions only. All questions carry equal marks. Total marks 100. All working must be shown.

Some formulas you may find useful:

$Z_L = j\omega L$	$Z_C = \frac{1}{j\omega C}$
$\omega = 2\pi f$	$\mathcal{E} = Blv$
$V = IR$	$\mathcal{E} = -\frac{d}{dt}\varphi$
$\varphi = BA$	$qE = qvB$
$q = \sigma A$	$\frac{1}{C_{eq}} = \sum \frac{1}{C_i}$
$\oint_0^R \vec{B}.\vec{dl} = \mu_o \int_0^R \vec{j}.d\vec{dS}$	$C_{eq} = \sum_1^n C_i$
$\int \vec{E}.d\vec{dS} = \varepsilon_o \int \rho dV$	$I_{rms}^2 = \frac{1}{T}\int_0^T i^2 dt$
$dV = 4\pi r^2 dr$	$dA = 2\pi r dr$
$i = qf$	

Q1 (a) A series RLC circuit consists of an 8.00 Ω resistor, a 5.00 μF capacitor, and a 50.00mH inductor. A variable frequency source applies an emf of 400-V (rms) across the combination.

Determine the following:

(i) The resonance frequency ω_o [2]

(ii) The capacitive and inductive reactances at resonance [4]

(iii) The total impedance of the circuit at resonance [2]

(iv) The power delivered to the circuit when the source frequency is half the resonance frequency [7]

(b) Consider the arrangement shown in Figure 1. Assume that $R = 6.00\ \Omega$, $l = 1.20$ m and a uniform 2.5 T magnetic field is directed into the page. At what speed must the bar be moved to produce a current of 0.5 A in the resistor? [3]

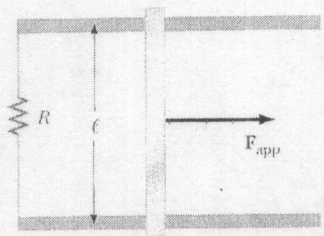


Figure 1

(c) It is found that an electron makes 10^{15} revolutions around the proton in one second. Determine the current generated by this electronic motion. [2]

Q2 (a) Study the following diagram, Figure 2 below, and answer the following questions:

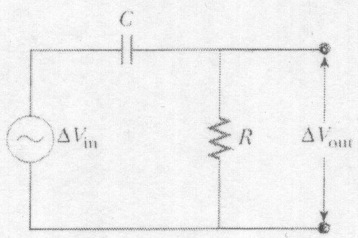


Figure 2

(i) State, giving reasons, what kind of filter this is and [2]

(ii) show that its transmission function, T is given by [6]

$$T = \frac{R}{\sqrt{R^2 + \left(1/j\omega C\right)^2}}.$$

(b) Show that the r.m.s value for the saw-tooth voltage shown below is [12]

$$V_{rms} = \frac{V_{max}}{\sqrt{3}}.$$

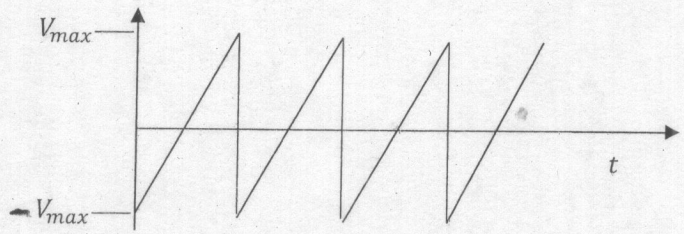


Figure 3

Hint: First derive the mathematical function for the voltage.

Q3 (a) The magnetic field B in a 20 turn rectangular loop of wire measuring 10 cm by 20 cm is varying as

$$B(t) = at^3 + bt \text{ where } a = 3s^{-3} \text{ and } b = 4s^{-1}.$$

Given that the resistance of the coil is 20 Ω, calculate the current in the wire at $t =$ 5.0 sec.

Take the magnetic field to be oriented normal to the plane of the loop.

[7]

(b) A boy wishes to push a magnet towards a coil of wire in a quest to generate current in the coil.

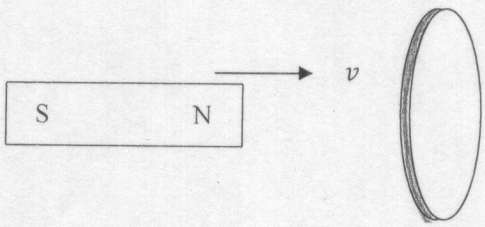


Figure 4

(i) Show that, according to Lenz's law, the right side of the loop must exhibit a south - pole. [6]

(ii) Indicate the direction of the induced current in the coil. [2]

(iii) Draw the sketch of magnetic field lines threading the coil. [2]

(c) Calculate the velocity of ions that pass un-deflected through crossed electric and magnetic fields for which $E = 7.7k$V/m and $B = 0.14$ T. [3]

Q4 (a) A solenoid wound with 2000 turns/m is supplied with current that varies according to the following equation:

$$I = 4\,sin(120\pi t),$$

where I is in amps and t is in seconds.

Now a small coaxial coil of 40 turns and radius $r = 5.00$ cm is located inside the solenoid near its centre. Derive an expression that describes the manner in which the e.m.f. in the small coil varies with time. [10]

(b) A long, cylindrical conductor of radius R carries a current I as shown in figure 5.

However, the current density, J, is not uniform over the cross-section of the conductor but is a function of the radius according to $J = br^2$ where b is a constant.

(i) What are the units of b? [2]

(ii) Find an expression for the magnetic field B at a distance $r < R$ [6]

(c) Use Ampere's law to explain the reason why the magnetic field, B inside a hollow copper tube must be zero. [2]

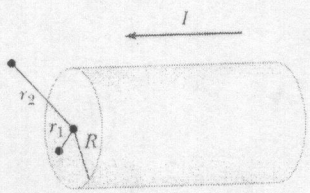


Figure 5

Q5 (a) The charge density λ, on a non-conducting rod varies as

$$\lambda = \lambda_o x + kx^2.$$

(i) State the units of k [2]

(ii) Find an expression for the total charge on the rod. [5]

(b) The area charge density, σ on a non-conducting disk of radius R varies as $\sigma = \sigma_o r^3$.
Find the total charge on the disk. [6]

(c) A 0.1μF capacitor is charged using a battery of 2 V and then isolated. It is then connected across an uncharged capacitor of value 0.3μF. Compute the potential difference across the combination and the charge stored by each of the capacitors. [7]

Q6 (a) A circuit connected to a resistance R is driven by a power source with e.m.f = $\mathscr{E}$ whose internal resistance is r has a current of 0.5 A when R has a value of 10-Ω and current of 0.27 A when R has a value of 20-Ω. Find the values of $\mathscr{E}$ and r. [8]

(b) In the following circuit network, determine the values of I_1, I_2 and I_3 when the switch k is closed. [12]

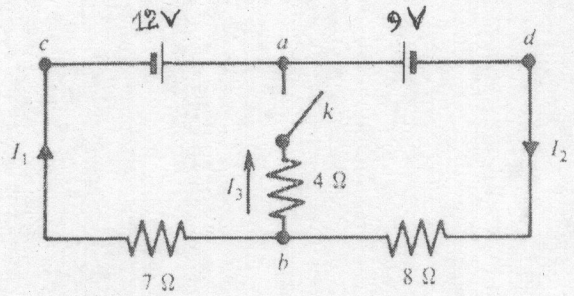


Figure 6

Q7 (a) A spherical region of space of radius a is filled with positive charge of volume density ρ which varies with distance r from the centre of the sphere according to the equation

$$\rho = \rho_o \frac{r}{a}$$

but is constant for given r. Find an expression for the electric field **E** at a distance r from the centre of the sphere. [8]

(b) Three charges, each of magnitude $= +Q$, are situated at the corners of a square of side a. Find an expression for the electric field at the fourth (vacant) corner of the square. [12]

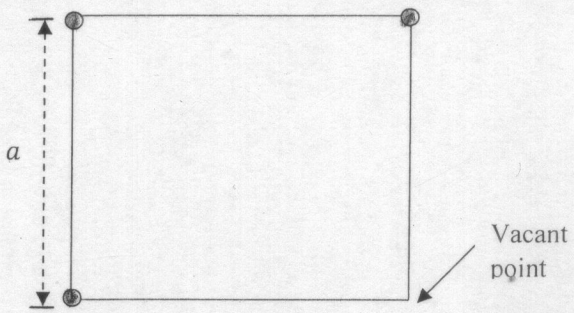


Figure 7

END OF EXAMINATION

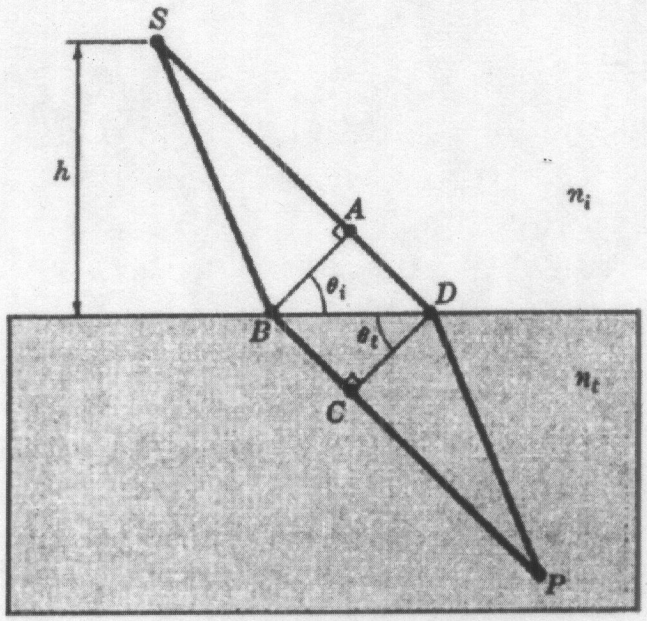
THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
Department of Physics
2019/2020 ACADEMIC YEAR
End of Year University Examinations
PHY 2712 - OPTICS

Duration : 3 Hours
Total marks: 100
INSTRUCTIONS
This question paper contains seven questions.
Answer any five questions. Each question carries 20 total marks.
Write clearly your computer number on the answer booklet provided.
All working should be shown clearly.
Omission of essential work will lead to loss of marks.
Additional information can be found at the end of this question paper.

QUESTION 1

(a) State Fermats principle. [2]

(b) With the aid of the figure below, which shows two rays (optical path lengths) going from S to P, and the alternative of Fermats formulation derive Snells law without Calculus. [8]



(c) Light from a sodium lamp $\lambda_0 = 589nm$ passes through a tank of gycerin (of refractive index 1.47) 20 m long in a time t_1. If it takes the time t_2 to traverse the same tank when filled with carbon disulfide (of refractive index 1.63), determine the difference t_2-t_2. [5]

(d) Why do the sky appear blue at mid-day, while it is red at sun rise and sun set. [5]

QUESTION 2

(a) Define the following terms

(i) optical path. [1]

(ii) optical density. [1]

(b) Imagine an electromagnetic plane wave in a vacuum whose E-Field in SI units is given by

$$E_x = 10^2 sin\pi(3 \times 10^6 z - 9 \times 10^{14} t), E_y = 0, E_z = 0.$$

Determine the

(i) speed, [2]

(ii) wavelength, [2]

(iii) frequency, [2]

(iv) period, [2]

(v) amplitude, and [2]

(vi)polarisation [2]

of this plane wave.

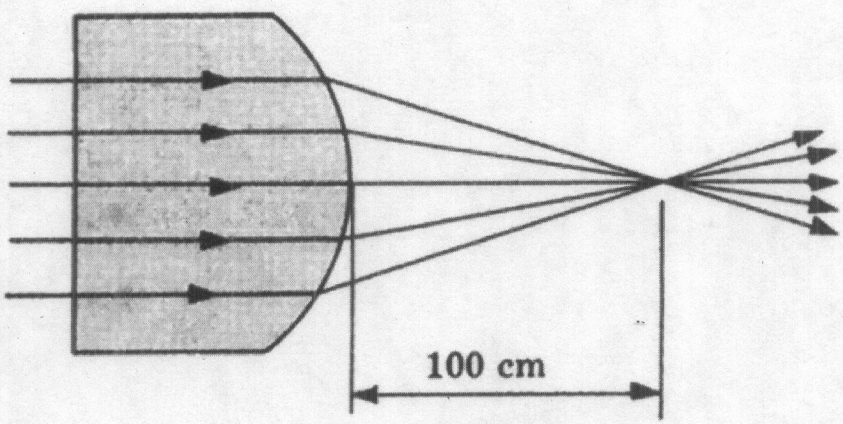
(c) Write an expression for magnetic field associated with the wave of

$$E_x = 10^2 sin\pi(3 \times 10^6 z - 9 \times 10^{14} t), E_y = 0, E_z = 0.$$

[6]

QUESTION 3

(a) What must be the radius of curvature of the rod's right end in the figure below if the parrallel bundle opf rays is to come to focus 100 cm from the vertex? The glass rod (n=1.46) is immersed in ethyl alcohol (n=1.86). [5]



(b) An object 2 cm high is placed 30 cm from a concave mirror with a radius of curvature of 10 cm. Find the position and size of the image. [5]

(c) What is dispersive power? [2]

(d) The table below shows Fraunhofers' designationations, elements source wavelength and refractive index for four optical glasses.

Designation	Chemical element	Wavelength	Spectacle crown	Light flint	Dense flint	Extra dense flint
C	H	6563	1.52042	1.57208	1.6665	1.71301
D	Na	5892	1.52300	1.57600	1.67050	1.7200
F	H	4861	1.52933	1.58606	1.68059	1.73780
G	H	4340	1.53435	1.59441	1.68882	1.75324

Calculate the

(i) dispersive power for the four glasses. [6]

(ii) constringence (ν) for each of the four glasses. [2]

.....

QUESTION 4

.....

(a) Differentiate an isotropic material from an anisotropic material. [2]

(b) In the refraction of rays at the interface between an isotropic medium and anisotropic medium,

$$tan\Theta_r = \frac{n_0 n_1 sin\Theta_i}{n_e\sqrt{n_e^2 - n_1^2 sin^2\Theta_i}},$$

where Θ_r is the angle of refraction and Θ_i is the angle of incidence.

(i) Calculate the angle of refraction for a ray of light travelling from air into calcite at an angle of incidence of 45°, given that for calcite $n_0 = 1.65838$ and $n_e = 1.48641$. [3]

(ii) For $n_0 = n_e = n$, show that

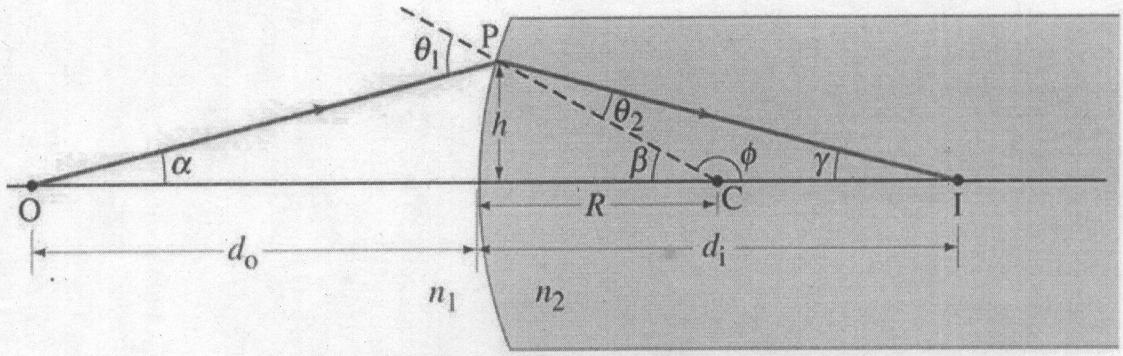
$$tan\Theta_r = \frac{n_0 n_1 sin\Theta_i}{n_e\sqrt{n_e^2 - n_1^2 sin^2\Theta_i}},$$

reduces to Snells law. [5]

(c) For the figure shown below, show that

$$\frac{n_2}{d_o} + \frac{n_1}{d_i} = \frac{n_2 - n_1}{R}$$

[6]



(d) State and briefly describe the two categories of diffraction phenomena. [4]

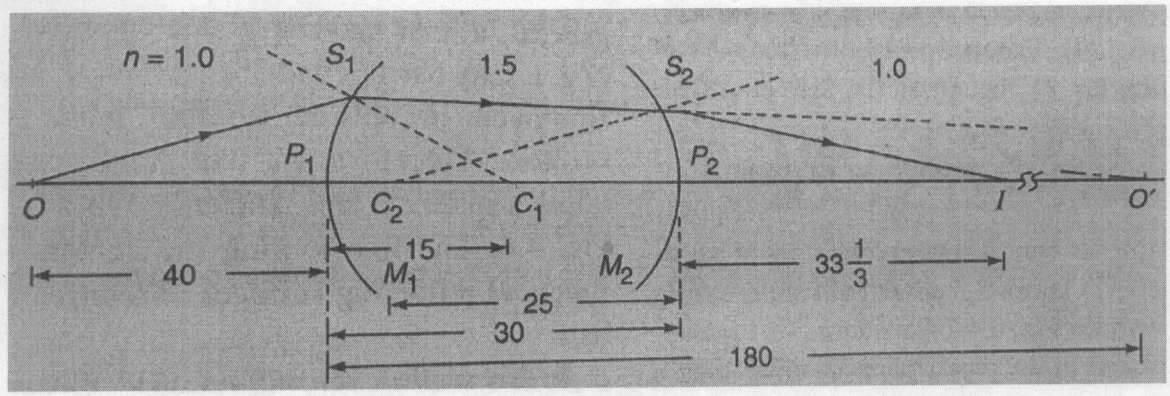
QUESTION 5

(a) Looking into the Michelson interferometer, one sees a dark central disc surrounded by concentric bright and dark rings. One arm of the device is 2 cm longer than the other arm and $\lambda_0 = 500$ nm. Determine

(i) the order of the central disc. [4]

(ii) the order of the sixth dark ring. [2]

(b) Consider a medium of refractive index 1.5 bounded by two curved spherical surfaces $S_1P_1M_1$ and $S_2P_2M_2$ as shown in the figure below. The radii of curvature of the surfaces are 15 cm and 25 cm with their centres being C_1 and C_2. There is an object (O) at a distance 40 cm from P_1 on the line joining C_1 and C_2. Determine the position of the paraxial image. [10]



(c) (i)What is a laser? [1]

(ii) What are the three main components of any laser device? [3]

.....

QUESTION 6

.....

(a) List the five coefficients for specifying primary aberrations. [5]

(b) A chromatic doublet of focal length 20 ccm is to be made by placing a convex lens made of borosilicate crown glass in contact with a diverging lens of dense flint glass. Assuming $n_r = 1.51462$, $n_b = 1.52264$, $n'_r = 1.61216$ and $n'_b = 1.62901$. Calculate the focal length of each lens.[Here, unprimed and primed quantities refer to borosilicate crown glass and dense flint glass respectively] [10]

(c) Define the following terms:

(i) Zeeman effect. [1]

(ii) Inverse Zeeman effect. [1]

(iii) Faraday effect. [1]

(iv) Kerr Magneto-optical effect. [1]

(v) Stark effect. [1]

QUESTION 7

(a) What are photons? [1]

(b) By considering an atom having two states N_1 and N_2, and taking advantage of Einstein coefficients, derieve Plancks radiation law in terms of light matter interaction. [7]

(d) State atleast nine basic principles involved in operation of most lasers. [9]

(c) Who built the first Laser? What type of laser was it? [3]

END OF EXAMINATIONS

ADITIONAL INFORMATION

$e = 1.602\text{x}10^{-19}$ C, $m_e = 9.11\text{x}10^{-31}$ kg, $m_p = 1.67\text{x}10^{-27}$ kg, $m_n = 1.67\text{x}10^{-27}$ kg, $N_A = 6.022\text{x}10^{23}$ mol^{-1}, $k_B = 1.38\text{x}10^{-23}$ JK^{-1}, $\epsilon_0 = 8.85\text{x}10^{-12}s^2m^{-2}$, $\mu_0 = 4\pi\text{x}10^{-7}$, $\mu_B = 9.274\text{x}10^{-24}$ Am^2, $h = 6.62\text{x}10^{-34}$ J/s, $c = 3\times 10^8$m/s, $n_{water} = 1.33$, $n_{glass} = 1.5$, $n_{air} = 1$

$$n_1 sin\Theta_1 = n_2 sin\Theta_2; \frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i}; \frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}; D = P = \frac{1}{f}$$

$$\nu = \frac{1}{v} = \frac{n_D - 1}{n_F - n_c}; c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}; M_T = \frac{-s_i}{s_o}; n = \frac{sin\Theta_i}{sin\Theta_r}; f_0 = \frac{n_1}{n_2 - n_1} R;$$

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right); v = \nu\lambda; \omega = ck; k = \frac{2\pi}{\lambda}; \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$-\frac{2}{R} = \frac{1}{s_o} + \frac{1}{s_i}; n = \frac{c}{v} = \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}}$$

The University of Zambia
School of Natural Sciences
Department of Physics
2019/2020 Academic Year
End of Year Examinations
PHY 3032 - Computational Physics I

Duration: 3 hours **Maximum Marks: 100**

Instructions

1. Write clearly your computer number on the answer booklets.
2. This examination paper contains six (6) questions.
3. Attempt any four (4) questions.
4. This examination paper has a total of 100 marks.
5. All questions carry equal marks.
6. Marks allocated for each question are indicated in brackets.

Question 1

(a) Find the root of the equation

$$f(x) = \cos x + 1 - x = 0$$

with the bisection method in the interval $[a, b] = [0.8, 1.6]$. Show that the computed midpoints are $c_o = 1.2$, $c_1 = 1.4$, $c_2 = 1.3$, $c_3 = 1.25$ and $c_4 = 1.28$. **[10 marks]**

(b) Solve the equation

$$f(x) = x^2 - 2x - 1$$

with the Newton-Raphson method. Starting with $x_o = 3.5$, show that $x_1 = 2.65$, $x_2 = 2.431$ and $x_3 = 2.414$. **[7 marks]**

(c) Find the root of the equation

$$f(x) = e^{-x} - x = 0$$

with the secant method. Starting with $x_o = 3.0$ and $x_1 = 2.6$, show that $x_2 = 0.220$, $x_3 = 0.666$, $x_4 = 0.574$ and $x_5 = 0.567$ **[8 marks]**

TOTAL: 25 MARKS

Question 2

(a) Show using the Gauss elimination method that the following linear system of equations **[8 marks]**

$$\begin{aligned} 2x_1 + 4x_2 - 6x_3 &= -4 \\ x_1 + 5x_2 + 3x_3 &= 10 \\ x_1 + 3x_2 + 2x_3 &= 5 \end{aligned}$$

is equivalent to the upper-triangular linear system

$$\begin{aligned} 2x_1 + 4x_2 - 6x_3 &= -4 \\ 3x_2 + 6x_3 &= 12 \\ 3x_3 &= 3 \end{aligned}$$

(b) Solve the upper triangular linear system shown in (a) **[5 marks]**

(c) The linear system of equations given below can be solved using the direct LU decomposition method.

$$\mathbf{AX} = \mathbf{B}$$

$$\begin{bmatrix} 1 & 1 & 6 \\ -1 & 2 & 9 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \\ 10 \end{bmatrix}$$

(i) Show that square matrix **A** can be factorized as **[6 marks]**

$$\mathbf{A} = \mathbf{LU}$$

where **L** is a lower-triangular linear system and **U** is an upper-triangular linear system

$$\begin{bmatrix} 1 & 1 & 6 \\ -1 & 2 & 9 \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 6 \\ 0 & 3 & 15 \\ 0 & 0 & 12 \end{bmatrix}$$

(ii) Solve the following triangular linear systems **[6 marks]**

$$\mathbf{LY} = \mathbf{B}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \\ 10 \end{bmatrix}$$

$$\mathbf{UX} = \mathbf{Y}$$

$$\begin{bmatrix} 1 & 1 & 6 \\ 0 & 3 & 15 \\ 0 & 0 & 12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

TOTAL: 25 MARKS

Table 1: Relative thermal conductivity k of sodium

T (°C)	79	190	357	524	690
k	1.00	0.932	0.839	0.759	0.693

Table 2: Relative radiation γ from a radiative substance

t (yr)	0	1	2	3	4	5
γ	1.00	0.990	0.979	0.972	0.967	0.956

Question 3

(a) Table 1 shows the variation of relative thermal conductivity k of sodium with temperature in degrees Celsius.

(i) Show that the best fit straight line to the sample data in the least square sense has the form $k = 1.03 - (5 \times 10^{-4})T$. **[8 marks]**

(ii) Determine the relative thermal conductivity k of sodium at $T = 250°C$ from the best fit line. **[2 marks]**

(b) The intensity of radiation from a radioactive substance was measured at one year intervals. The results are given in Table 2 where γ is the radiative intensity of radiation and t is time in years.

(i) Knowing that radiation decays exponentially according to $\gamma = ae^{-bt}$. Show that this exponential $\gamma = ae^{-bt}$ can be linearized as follows

$$y = a_o + a_1 x$$

where $y = \ln \gamma$, $x = t$, $a_1 = -b$ and $a_o = \ln a$ **[4 marks]**

(ii) Determine the constants a and b by fitting a best fit straight line in the least square sense. **[8 marks]**

(iii) Estimate the half-life $t_{0.5}$ of this unknown radioactive substance using the formula **[3 marks]**

$$t_{0.5} = \frac{\ln 0.5}{-b}$$

TOTAL: 25 MARKS

Table 3: Free fall experiment data

t (s)	0.2	0.4	0.8	1.0
h (m)	0.20	0.79	3.14	4.91

Question 4

(a) Find the parabola $y = Ax^2 + Bx + C$ that passes through the points (1,4), (2,7) and (3,14) using Lagrange interpolation. **[13 marks]**

(b) A first year physics student collected experimental data shown in Table 3 for a free fall experiment due to gravity designed using the formula

$$h(t) = \frac{1}{2}gt^2$$

(i) Find the Newton polynomials $P_1(t)$ and $P_2(t)$ that can be used to approximate $h(t)$ based on the nodes that pass through the points in Table 3. **[9 marks]**

(ii) Estimate the value of $P_1(t)$ and $P_2(t)$ at $t = 0.6$ and compare with $h(t = 0.6)$ **[3 marks]**

Question 5

(a) Let $f(x) = \dfrac{\sin x}{x}$, where x is in radians.

(i) Calculate the approximation of $f'(1.57)$ and $f''(1.57)$ for $h = 0.001$ using the first central difference approximations formulas. **[6 marks]**

(ii) Calculate the approximation of $f'(1.57)$ and $f''(1.57)$ for $h = 0.01$ using the second central difference approximations formulas. **[6 marks]**

(b) Write a Python 3 program that performs the following tasks on a user's bank account. **[13 marks]**

(1.) Prompt the user for an opening balance with string "Enter your opening balance: ",

(2.) Store the opening balance entered by the user in a variable **balance**.

(3.) Convert the variable **balance** into a *float* data type.

(4.) Prompt the user for a transaction using the string "Do you want to perform a transaction. Type Y for Yes and N for No: ",

(5.) Store the response to prompt in (4) in variable **transact**.

(6.) Change the case of the user's response stored in variable **transact** into lowercase

(7.) Display the opening balance correct to 2 decimal places,

(8.) While **transact** is equal to "y" or "yes", do the following tasks:

(i) Prompt the user for a transaction type using the string "What kind of transaction do you what to perform. Type D for Debit or C for Credit: "

(ii) Store the response to prompt in (8)(i) in variable **transactionType**

(iii) Change the case of response stored in variable **transactionType** to lowercase.

(iv) Prompt the user for a transaction amount using the string " Enter transaction amount: "

(v) Store the transaction amount in a variable **transactionAmount**

(vi) Convert the **transactionAmount** into a *float* data type.

(vii) If **transactionType** is equal to "d" or "debit", then

(a) Debit the account with transaction amount.

(b) Display the transaction amount and balance prefixed with strings "Debit : " and "Balance : " up to 2 decimal places.

(viii) Else if **transactionType** is equal to "c" or "credit", then

(a) Credit the account with transaction amount.

(b) Display the transaction amount and balance prefixed with strings "Credit : " and "Balance : " up to 2 decimal places.

(ix) Else display the message ""Invalid transaction type entered".

(x) Prompt the user for another transaction using the string "Do you want to perform another transaction.Type Y for Yes and N for No): "

(xi) Store the response to prompt in (8)(x) in variable **transact**.

(xii) Change the case of the users response stored in variable **transact** into lowercase

(9) Print out the closing balance

Question 6

(a) Solve the integral

$$I = \int_0^1 \sin(\pi x) dx$$

using

(i) trapezoidal rule with a step size of $h = \frac{1}{3}$ **[5 marks]**

(ii) Simpson's one-third rule **[5 marks]**

(b) Solve the integral

$$I = \int_0^4 x^2 e^{-x} dx$$

using

(i) Simpson's one-third rule **[5 marks]**

(ii) Simpson's three-eighth rule **[5 marks]**

(iii) Boole's rule **[5 marks]**

TOTAL: 25 MARKS

USEFUL FORMULAS AND ALGORITHMS

Bisection Method

To approximate a root of $f(x) = 0$ which lies in the interval [a,b]

1. Check if $f(a) \cdot f(b) < 0$, then

   (i) Calculate the midpoint of the interval c as

   $$c = \frac{a+b}{2}$$

   (ii) Next, check the sign of the following condition $f(a) \cdot f(c)$

   if $f(a) \cdot f(c) = 0$, then make

   $$a = c \quad , \quad b = c$$

   else if $f(a) \cdot f(c) > 0$, then make

   $$a = c \quad \text{and} \quad f(a) = f(c)$$

   else

   $$b = c \quad \text{and} \quad f(b) = f(c)$$

   (iii) Re-calculate a new c and go back to step II.

   (iv) Once the required number of iterations k is met or the bracket is small enough, calculate a final root as

   $$c = \frac{a+b}{2}$$

Newton-Raphson Iteration Method

To approximate a root of $f(x) = 0$ given one initial approximation x_0 and using the iteration

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} \qquad \text{for} \quad k = 0, 1, 2, ...$$

Secant Method

To approximate a root of $f(x) = 0$ given two initial approximations x_0 and x_1. We use the secant iteration formula

$$x_{k+1} = x_k - \frac{f(x_k)(x_{k-1} - x_k)}{f(x_{k-1}) - f(x_k)} \qquad \text{for} \quad k = 1, 2, 3...$$

Direct LU Decomposition Method

To solve a system of linear equations $\mathbf{AX} = \mathbf{B}$ with the direct LU decomposition method. Let $\mathbf{A}$ be the coefficient matrix, $\mathbf{X}$ the column matrix of unknowns, and $\mathbf{B}$ the column matrix of known values.

(i) LU decomposition

$$\mathbf{AX} = \mathbf{B}$$

$$\mathbf{AX} = \mathbf{LUX} = \mathbf{B}$$

where $\mathbf{A} = \mathbf{LU}$

(ii) Forward substitution

Let $\mathbf{UX} = \mathbf{Y}$

Solve for $\mathbf{Y}$

$$\mathbf{LY} = \mathbf{B}$$

(iii) Back substitution

Solve for $\mathbf{X}$

$$\mathbf{UX} = \mathbf{Y}$$

Least Square Fit Regression

Linear Least Square Fit

To fit a straight line $y = a_0 + a_1 x$ to a sample of N paired data points (x_i, y_i) where $i = 1, 2, 3, ...N$, we calculate the coefficients a_0 and a_1 as follows

$$a_1 = \frac{N\sum x_i y_i - \sum x_i \sum y_i}{N\sum x_i^2 - \left(\sum x_i\right)^2}$$

$$a_0 = \bar{y} - a_1 \bar{x}$$

$$\bar{y} = \frac{\sum y_i}{N} \quad , \quad \bar{x} = \frac{\sum x_i}{N}$$

Least Square Power Fit

To fit a line $y = Ax^M$ where M is known to a sample of N paired data points (x_i, y_i) where $i = 1, 2, 3, ...N$, we calculate the coefficient A as follows

$$A = \frac{\sum x_i^M y_i}{\sum x_i^{2M}}$$

Interpolation and Extrapolation Formulas

Lagrange Polynomials

A Lagrange interpolating polynomial $P_N(x)$ of degree at most N that passes through $N+1$ points (x_i, y_i) for $i = 0, 1, 2, 3..., N$ can be constructed as follows

$$P_N(x) = \sum_{k=0}^{N} y_k \cdot L_{N,k}(x)$$

where $L_{N,k}(x)$ is the Lagrange coefficients polynomial based on nodes x_o, x_1, x_2,x_3,...,x_N and is defined as

$$L_{N,k}(x) = \frac{\prod_{j=0, j\neq k}^{N}(x - x_j)}{\prod_{j=0, j\neq k}^{N}(x_k - x_j)}$$

Newton Polynomials

Suppose that x_i, for $i = 0, 1, 2, 3, ..., N$ are $N + 1$ distinct numbers in interval $[a, b]$. There exists a series of polynomials $P_N(x)$ of degree at most N.

$$P_o = a_o$$

$$P_1(x) = P_o(x) + a_1(x - x_o)$$

$$P_2(x) = P_1(x) + a_2(x - x_o)(x - x_1)$$

where

$$a_o = f[x_o] \quad , \quad a_1 = f[x_o, x_1] \quad , \quad a_2 = f[x_o, x_1, x_2]$$

Table 4: Divided Difference Table

x_k	$f[x_k]$	$f[x_{k-1}, x_k]$	$f[x_{k-2}, x_{k-1}, x_k]$
x_o	$f[x_o]$		
x_1	$f[x_1]$	$f[x_o, x_1]$	
x_2	$f[x_2]$	$f[x_1, x_2]$	$f[x_o, x_1, x_2]$

where

$$f[x_k] = f(x_k)$$

$$f[x_{k-1}, x_k] = \frac{f[x_k] - f[x_{k-1}]}{x_k - x_{k-1}}$$

$$f[x_{k-2}, x_{k-1}, x_k] = \frac{f[x_{k-1}, x_k] - f[x_{k-2}, x_{k-1}]}{x_k - x_{k-2}}$$

Numerical Differentiation Formulas

First Central Difference Approximations

$$f'(x) \simeq \frac{f_1 - f_{-1}}{2h}$$

$$f''(x) \simeq \frac{f_1 - 2f_o + f_{-1}}{h^2}$$

where

$$f_1 = f(x+h)\ ,\ f_o = f(x)\ ,\ f_{-1} = f(x-h)$$

Second Central Difference Approximations

$$f'(x) = \frac{-f_2 + 8f_1 - 8f_{-1} + f_{-2}}{12h}$$

$$f''(x) = \frac{-f_2 + 16f_1 - 30f_o + 16f_{-1} - f_{-2}}{12h^2}$$

where

$$f_2 = f(x+2h)\ ,\ f_1 = f(x+h)\ ,\ f_o = f(x)\ ,\ f_{-1} = f(x-h)\ ,\ f_{-2*} = f(x-2h)$$

Quadrature Formulas for Numerical integration

Trapezoidal rule

The integral

$$\int_a^b f(x)\ dx \simeq \frac{h}{2}\left[f(a) + f(b)\right]$$

where $h = b - a$

Composite Trapezoidal rule

The integral

$$\int_a^b f(x)\ dx \simeq \frac{h}{2}\left[f(x_o) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)\right]$$

where

n is the number of segments in interval $[a, b]$

$$h = \frac{b-a}{n}$$

$$x_0 = a, \quad x_1 = a + h, \quad x_2 = a + 2h, ..., \ x_n = a + nh = b$$

Simpson's one-third rule

The integral

$$\int_a^b f(x)\ dx \simeq \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right]$$

where

$$h = \frac{b-a}{2}, \quad x_0 = a, \quad x_1 = \frac{a+b}{2}, \quad x_2 = b$$

Simpson's three-eighth rule

The integral

$$\int_a^b f(x)\ dx \simeq \frac{3h}{8}\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right]$$

where

$$h = \frac{b-a}{3}, \quad x_0 = a, \quad x_1 = a + h, \quad x_2 = a + 2h, \quad x_3 = a + 3h = b$$

Boole's Rule

The integral

$$\int_a^b f(x)\,dx \simeq \frac{2h}{45}\left[7f(x_0) + 32f(x_1) + 12f(x_2) + 32f(x_3) + 7f(x_4)\right]$$

where

$$h = \frac{b-a}{4}, \quad x_0 = a, \quad x_1 = a+h, \quad x_2 = a+2h, \quad x_3 = a+3h, \quad x_4 = a+4h = b$$

THE UNIVERSITY OF ZAMBIA
School of Natural Sciences
Department of Physics
2019–2020 Academic Year
Final Examinations

PHY3621 – ELECTROMAGNETIC THEORY

Duration: 3 hours Maximum Marks: 100

Instructions

- This examination paper contains 6 questions. Attempt any 4 questions.
- Each question carries 25 marks. Marks allocated for each question are indicated in brackets [ ].
- Show all your working clearly. Omission of essential work will result in loss of marks.
- Follow carefully instructions written on the answer booklets.

CONSTANTS THAT MAY BE USEFUL

$e = 1.602 \times 10^{-19}$ C
$m_p = 1.67 \times 10^{-27}$ kg
$m_e = 9.11 \times 10^{-31}$ kg
$\varepsilon_o = 8.85 \times 10^{-12}$ $C^2N^{-1}m^{-2}$
$h = 6.63 \times 10^{-34}$ Js
$\mu_o = 4\pi \times 10^{-7}$ Ns^2C^{-2}
$c \cong 3.0 \times 10^8$ m/s

1. (a) There are currently just four known basic forces two of which are: strong and weak forces. Name the remaining two basic forces. [2]

(b) By using the definition of ∇, show that the divergence of the gradient of a function f results in the Laplacian of the function f:

$$\nabla \cdot (\nabla f) = \nabla^2 f.$$

[3]

(c) A rod of length L, has a uniform positive charge per unit length λ and a total charge Q. Calculate the electric field at a point P that is located along the long axis of the rod and a distance d from one end. [10]

(d) A rod of length L, located along the x axis (with one end at the origin) has a total charge Q and a uniform linear charge density λ. Find the electric potential at a point P located on the y axis a distance d from the origin. [10]

---

2. (a) By using the equality of cross derivatives:

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right),$$

show that the curl of the gradient vanishes:

$$\nabla \times (\nabla f) = 0.$$

[4]

(b) Find the electric field a distance z above the midpoint of a straight line segment of length $2L$, which carries a uniform line charge λ. [5]

(c) A ring of radius R carries a uniformly distributed positive total charge Q. Calculate the electric field due to the ring

at a point P lying a distance x from its center along the central axis perpendicular to the plane of the ring. [8]

(d) Write down the Maxwell's equations and give the names given to each one of them. [8]

---

3. (a) By using the vector identity

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C},$$

show that the curl of the gradient is always zero:

$$\nabla \cdot (\nabla \times \mathbf{v}) = 0.$$ [3]

(b) A disk of radius R has a uniform surface charge density σ. Calculate the electric field at a point P that lies along the central perpendicular axis of the disk and a distance x from the center of the disk. [8]

(c) Find

i. an expression for the electric potential at a point P located on the perpendicular central axis of a uniformly charged ring of radius R and total charge Q. [4]

ii. an expression for the magnitude of the electric field at point P. [4]

(d) Two infinite parallel planes carry equal but opposite uniform charge densities $\pm\sigma$. Find the field in each of the three regions:

i. to the left of both planes [2]

ii. between the planes [2]

iii. to the right of both planes [2]

---

4. (a) By using the definition of ∇, show that the curl of the curl of a vector function $\mathbf{v}$ is:

$$\nabla \times (\nabla \times \mathbf{v}) = \nabla(\nabla \cdot \mathbf{v}) - \nabla^2 \mathbf{v}.$$ [5]

(b) Find the potential inside and outside of a uniformly charged spherical shell of radius R. [10]

(c) A uniformly charged disk has radius R and surface charge density σ.

i. Find the electric potential at a point P along the perpendicular central axis of the disk. [5]

ii. Find the x component of the electric field at a point P along the perpendicular central axis of the disk. [5]

---

5. (a) Using spherical polar coordinates, find the volume of a sphere of radius R. [5]

(b) An insulating solid sphere of radius R has a uniform volume charge density ρ and carries a total positive charge Q.

i. Calculate the magnitude of the electric field at a point outside the sphere. [5]

ii. Find the magnitude of the electric field at a point inside the sphere. [5]

(c) Two spherical conductors of radii r_1 and r_2 are separated by a distance much greater than the radius of either sphere. The spheres are connected by a conducting wire. The charges on the spheres in equilibrium are q_1 and q_2 , respectively, and they are uniformly charged. Find the ratio of the magnitudes of the electric fields at the surfaces of the spheres. [10]

---

6. (a) Define a conservative force. [1]

(b) State the mathematical form of Gauss's law. [2]

(c) Given that $\mathbf{A} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$ and $\mathbf{B} = B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}} + B_z\hat{\mathbf{z}}$, use properties of unit vectors $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ to show that

i. the dot product can be written as $\mathbf{A} \cdot \mathbf{B} = A_xB_x + A_yB_y + A_zB_z$, [3]

ii. the cross product can be written as

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}. \quad [3]$$

(d) Find the electric field due to an infinite plane of positive charge with uniform surface charge density σ. [8]

(e) Find the electric field a distance d from a line of positive charge of infinite length and constant charge per unit length λ. [8]

End of Examination

The University of Zambia

School of Natural Sciences
Department of Physics
2019/20 Academic Year
End of Year University Examinations
PHY4031: Computational Physics II

Answer <u>any four</u> questions. All questions carry equal marks. The marks are shown in brackets.

Time: Three (3) hours. **Maximum marks = 100.**

Write clearly your computer number on the answer book.
Show your working clearly. Omission of essential work will lead to loss of marks.

A question requiring Fortran (or Python) programming may be answered using Python (or Fortran) programming. However, in such cases, 20% of the total marks will be deducted.

Use of `COMMENTS` **to explain** `CODEBLOCKS` **is mandatory. Omission of comments in computer codes will lead to loss of marks.**

SOME USEFUL PHYSICAL CONSTANTS

$g = 9.81\ \text{ms}^{-2}$	$R = 8.314\ \text{Jmol}^{-1}\text{K}^{-1}$
$\rho_{\text{mercury}} = 13600\ \text{kg/m}^3$	$\rho_{\text{water}} = 1000\ \text{kg/m}^3$
$G = 6.674\times10^{-11}\ \text{m}^3\text{kg}^{-1}\text{s}^{-2}$	$\sigma = 5.672\times10^{-8}\ \text{Wm}^{-2}\text{K}^{-4}$
$h = 6.6261\times10^{-34}\ \text{Js}$	$k_B = 1.38066\times10^{-23}\ \text{J/K}$
$c = 2.9979\times10^{8}\ \text{m/s}$	$c_{water} = 4184\ \text{Jkg}^{-1}\text{K}^{-1}$

Q1. (a) Define the following terms:

(i) an array [2]

(ii) rank/dimension of an array [2]

(iii) shape of an array [2]

(iv) extent of an array [2]

(v) size of an array [2]

(b) A matrix n – rows and m – columns can be expressed as

$$\mathbf{A} = \left[a_{ij} \right] = \begin{bmatrix} a_{11} & a_{12} & . & . & . & a_{1m} \\ a_{21} & a_{22} & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ a_{n1} & a_{n2} & . & . & . & a_{nm} \end{bmatrix},$$

where a_{ij} for $i = 1,2,3,...,n$ are rows while $j = 1,2,3,...,\mathrm{m}$ are columns. Consider a matrix whose elements are given by

$$a_{ij} = \exp\left(-\frac{x_i^2}{10} - \frac{y_j^2}{5} \right),$$

where $x_i = 3i$ for $i = 1,2,3,...,n$ and $y_j = j/4$ for $j = 1,2,3,...,m$.

Write a **Fortran 95** code which can

(i) populate the matrix $\mathbf{A}$ for $n = 10$ and $m = 8$, [4]

(ii) slices the 3rd and 7th column and save the values into *1-dimensional* arrays whose variable names are **col_3** and **col_7**, respectively, [2]

(iii) compute the sum of all elements of $\mathbf{A}$, [2]

(iv) determines the dot product of **col_3** and **col_7**, [2]

(v) determines the transpose of $\mathbf{A}$, [2]

(vi) compute the product $\mathbf{A} \times$ **row_3**. [3]

Q2. (a) Starting from first principles, and with the aid of a diagram, show that the Trapezoidal rule, with has n subintervals is given by

$$\int_{x_{\min}}^{x_{\max}} f(x)\,dx \approx \frac{dx}{2}[V+2W],$$

where $$V = f(x_{\min}) + f(x_{\max}),$$

$$W = \sum_{i=1}^{n-1} f(x_i),$$

$$x_i = x_{\min} + i\,dx \text{ for } i = 0,1,2,3,....,n$$

and $$dx = \frac{x_{\max} - x_{\min}}{n}.$$ **[8]**

(b) The ground-state of eigenfunction for the harmonic oscillator is

$$\psi_o(x) = A\exp\left(-\frac{k_1 x^2}{2}\right)$$

where $k_1 = m^2\omega/\hbar$ and $A = k_1/\pi m$, $\hbar = h/2\pi$, $h = 6.62606891\times10^{-34}$ Js being the Planck's constant, $m = 9.10938211\times10^{-31}$ kg the mass of an electron and $\omega = 10^{14}$ rad/s is the Einstein frequency. The motion of a harmonic oscillator takes place over the interval $a \le x \le b$.

Given that $a = -10$, $b = 10$ and 1000 – evenly spaced numbers between a and b, write a quad precision **Fortran 95** code with a module which

(i) contains a routine or function called

`linspace(x_start,x_end,x_len,x_array)`

where `x_start` is the (floating point) starting value of an interval,
`x_end` is the (floating point) stopping value of an interval
`x_len` is the (integer) number of samples to be generated between `x_start` and `x_end`,
`x_array` is an array containing `x_len` – evenly spaced numbers between `x_start` and `x_end`, **[7]**

Hint: Take `x_start` $= a$, `x_end` $= b$ and `x_len` $= 1000$.

(ii) uses `x_array` generated in (i) to compute values of $\psi_o(x)$ for $a \le x \le b$ and save them in a 1-dimensional array, called **`phi_array`**. **[2]**

(iii) implements the Trapezoidal rule with $10,000,000$ subintervals to estimate, correct to 4 decimal places, the integral

$$\int_a^b |\psi_o(x)|^2\,dx.$$ **[8]**

Q3. The Bessel function of order k, where $k = 0,1,2,3,4,...$, arises frequently in physics, engineering and mathematics and can be expressed as

$$J_k(u) = \int_0^{2\pi} f(x)\,dx,$$

where the generally complex integrand is given by

$$f(x) = \frac{i^{-k}}{2\pi} e^{i(kx + u\cos(x))},$$

such that $i = \sqrt{-1}$ while x and $u \geq 0$ are real numbers, but $k = 0,1,2,3,4,...$ is an integer.

Write a **Fortran 95** code which implements the Simpson's rule to estimate $J_0(u)$ and $J_1(u)$ for $0.0 \leq u \leq 2.9$ in steps of 0.1. In particular, your code should have a

(i) user defined complex function $f(x)$, **[6]**

(ii) routine implementing the Simpson's rule with 20000 subintervals, **[9]**

(iii) loop that uses the routine in (ii) to estimate values of $J_0(u)$ and $J_1(u)$ for $0.0 \leq u \leq 2.9$ in steps of 0.1, **[8]**

(iv) Display results in a file called `Bessel.txt`. **[2]**

Q4. For a given radioactive species,

$$p = \exp(-\lambda t),$$

where $p = N/N_0$ is the ratio of the amount of radioactive nuclei $N = N(t)$ at any time t to the initial amount of radioactive nuclei N_0 while λ, the decay constant, is positive.

Since N_0 is greater than he initial $N_0 \geq N$, it follows that $0 \leq p \leq 1$. Here, p is the probability that the remaining radio nuclei N will decay. This probability can be modelled using a pseudo random number between 0 and 1.

(a) Write an algorithm for estimating the number of remaining radioactive nuclei, using Monte Carlo method, at an arbitrary time t using a step size dt. **[9]**

(b) The decay constant of carbon 14 is $\lambda = 0.00012097$ / year. Given a sample of 10,000 carbon 14 radioactive nuclei, write a **Python 3** code that implements the algorithm in (a) to estimate the number of remaining carbon 14 nuclei in a sample over a period of $100,000$ years using time steps of 1 year. In particular, the code should

(i) generate different pseudo random numbers at every instant, **[5]**

(ii) estimate the value of the remaining number of radioactive nuclei N in the sample at a given time t for $0 \leq t \leq 500$s in steps sizes $dt = 0.01$s, **[5]**

(iii) evaluates the percentage error between the Monte Carlo simulation results in (ii) and the theoretical value, namely

$$N = N_0 \exp(-\lambda t).$$ **[6]**

Q5. (a) Ten pseudo-random numbers rnd in the interval $0 \le \text{rnd} < 1$ were generated by a computer code and shown in Table 1.

Table 1: Ten Pseudo-random numbers rnd between 0 and 1 generated by a computer code.

	1	2	3	4	5	6	7	8	9	10
rnd	0.5675	0.3801	0.7847	0.5648	0.5545	0.9808	0.6999	0.3657	0.2123	0.3922

(i) Using these pseudo-random numbers, generate ten pseudo-random numbers x, correct to 4 decimal places, in the interval $0 \le x < 2\pi$, **[5]**

(ii) Compute values of $f(x)$, correct to 4 decimal places, for each value of x given that

$$f(x) = \exp\left(-x^2/10\right), \qquad \textbf{[4]}$$

(iii) Show via Monte Carlo implementation that, correct to 4 decimal places,

$$\int_0^{2\pi} f(x)dx \approx 2.2954. \qquad \textbf{[3]}$$

(b) Using the Monte Carlo integration, with 1,000,000 pseudo random numbers, to write a **Python 3** code that

(i) generates different values of pseudo-random numbers each time a the program is executed, **[4]**

(ii) stores the pseudo-random numbers generated in (i) in an array u, **[2]**

(iii) uses the array u in (i) to implements the Monte Carlo method and estimate

$$\int_0^{2\pi} u^{3/2} \exp(-u/2)du. \qquad \textbf{[7]}$$

Q6. A cylindrical tank 1.5 meters high stands on its circular base of diameter 1.00 m and is initially filled with water. At the bottom of the tank there is a hole of diameter 1.00cm, which is opened at some instant, so that the water starts draining under the influence of gravity. The rate at which the water level h in the tank decreases can be modelled by the initial value problem

$$\frac{dh}{dt} = -0.002656\sqrt{h} \quad ; \qquad h(t_0) = 1.5000\text{m}.$$

(a) Apply the Euler – Cauchy algorithm to the above initial value problem, choosing a step size of $\Delta t = 1.0$s seconds, and show that the water levels in the tank after 5.0s is $h_5 \approx 1.4805$m.

Note: $t = t_0 + n\Delta t$ for $n = 0,1,2,3,4,5,...$ while $t_0 = 0$. **[10]**

(b) Write a **Python 3** code, that

(i) implements the Runge–Kutta algorithm, via the `numpy` module, to estimate the water levels in the cylindrical tank as a function of time, in time steps of 0.01s, until the tank is empty, **[7]**

(ii) stores the values of h and t is a file called `rk4.txt`, **[4]**

(iii) uses the `matlotlib` module to plot the graph of h against t. **[4]**

FORMULAE THAT MAY BE USEFUL

1. TAYLOR SERIES EXPANSION

$$f(x)=\sum_{n=0}^{\infty}\frac{f^n(x_o)}{n!}[x-x_o]^n.$$

2. LAGRANGE INTERPOLATION

For a set of $n+1$ data points $\{(x_i,y_i)|i=1,2,3,....,n\}$, the Lagrange Interpolating Polynomial of degree n is generally

$$p_n(x)=\sum_{k=0}^{n}L_k(x)y_k\,,\quad \text{where}\quad L_k(x)=\prod_{\substack{i=0\\ i\neq k}}^{n}\frac{x-x_i}{x_k-x_i}\quad \text{for } k=0,1,2,.....,n.$$

3. EULER-CAUCHY ALGORITHM

$$y_{i+i}=y_i+idx.$$

4. RUNGE-KUTA ALGORITHM

$$k_1=hf(x_i,y_i);\qquad k_2=hf\left(x_i+\tfrac{1}{2}h,y_i+\tfrac{1}{2}k_1\right);\qquad k_3=hf\left(x_i+\tfrac{1}{2}h,y_i+\tfrac{1}{2}k_2\right);$$

$$k_4=hf(x_i+h,y_i+k_3);\quad x_{i+1}=x_i+h;\qquad y_{i+1}=y_i+\frac{1}{6}[k_1+2k_2+2k_3+k_4].$$

5. SIMPSONS RULE

$$\int_{x_{\min}}^{x_{\max}}f(x)dx\approx\frac{h}{3}[U+2V+4W];\ h=\frac{x_{\max}-x_{\min}}{N};\qquad U=f(x_{\min})+f(x_{\max});$$

$$V=\sum_{i=1}^{N-1}f(x_i)\ ;\ W=\sum_{i=2}^{N-2}f(x_i)\ \text{ and }\ N=2m,\ \text{where } m \text{ is a positive integer.}$$

6. MONTE CARLO ALGORITHM

For n randomly distributed points $\{x_i|i=1,2,3,...,n\}$ and $a\le x_i<b$, we have

$$\int_a^b f(x)dx\approx[b-a]\hat{f}\,,\ \text{where}\ \ \hat{f}=\frac{1}{n}\sum_{i=1}^{n}f(x_i).$$

UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF PHYSICS

2019/2020 End of Year Final Examination

PHY3411: Analogue Electronics I

==========

Instructions: Answer any **four** questions only. This examination paper has six questions. Each one of the **six questions** carries 25 marks. All essential working must be shown.

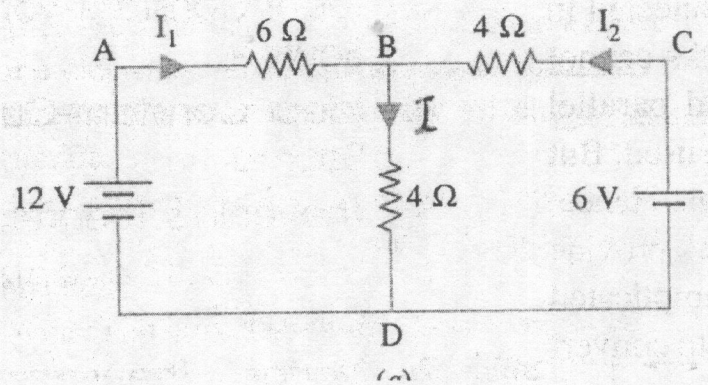
Time duration: three hours.

Some formulae that you might find useful:

$np = n_i^2$	$v = \eta V_T \ln\left(\frac{i}{I_s} + 1\right)$	$i = I_s\left(e^{v/\eta V_T} - 1\right)$	$r = \eta \frac{V_T}{I_Q}$
$V_{eff} = V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\, dt}$	$V_{av} = \frac{1}{T}\int_0^T v(t) dt$	$T = \frac{1}{\sqrt{1 + \left(\frac{1}{\omega RC}\right)^2}}$	$T = \frac{1}{\sqrt{1 + (\omega RC)^2}}$
$I_E = I_C + I_B$	$I_C = \beta I_B$	$R = \rho \frac{L}{A}$	$f(E_C) = \frac{1}{1 + e^{E_g/2K_BT}}$
$\frac{n}{N} = e^{-E_g/k_BT}$	$\sigma \approx n\mu_n q$	$\sigma \approx p\mu_p q$	$n \approx \frac{n_i^2}{N_A}$
$V_T = \frac{T}{11586}$	$\rho = \frac{1}{\sigma}$	$W = \frac{1}{2} LI^2$	$X_C = \frac{1}{\omega C}$
$f(E_C) = \frac{1}{1 + e^{E_g/2K_BT}}$	$p \approx \frac{n_i^2}{N_D}$	$r = \eta \frac{V_T}{I_Q}$	$\sigma = (n\mu_e + p\mu_h)$
$I = \frac{E_g}{R_L + R_i}$	$\sum_{x=1}^{n} I_x = 0$	$\sum_{y=1}^{n} V_y = 0$	$V = IR$, $v = iR$

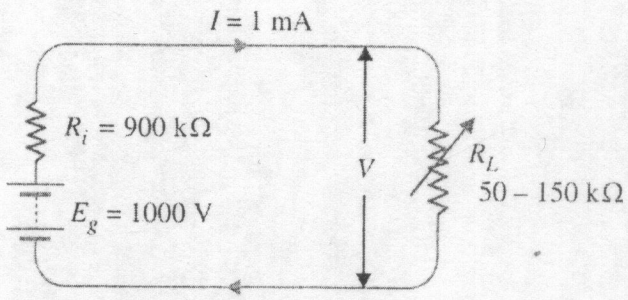
1. a) State Kirchhoff's current law. [1]

b) Using the principle of superposition, calculate the current in each branch of the network shown below. [20]



c) Draw the full-wave bridge rectifier circuit showing a transformer diagram as input and the load resistance as output. [4]

2. a) A *d.c.* source generating 500 V has an internal resistance of 1000 Ω and is connected to a variable resistor in a similar set-up as shown in the circuit below. Find the load current if the load resistance is set to (i) 50 Ω, (ii) 100 Ω and (iii) 150 Ω. [2 +2 + 2]

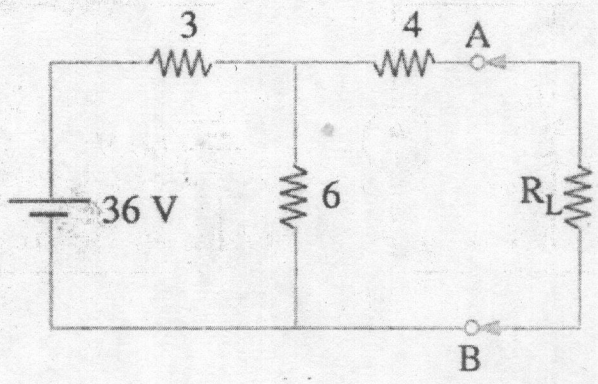


b) For the circuit given below,

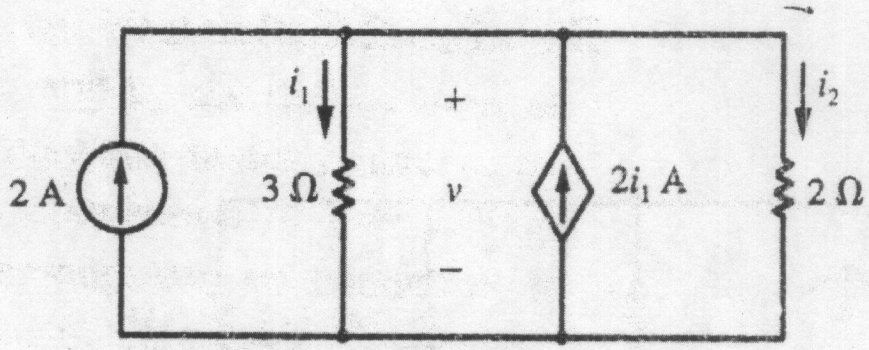
(i) draw a circuit that would enable you to perform the maximum power transfer theorem calculations, [1]

(ii) find the value of the load resistance (R_L) to be connected across terminals A and B which would extract maximum power from the circuit, and also [4]

(iii) evaluate the value of this maximum power transferred to the load resistance. [6]



c) In the following circuit, the value of the dependent current source depends on the current i_1 through the 3-Ω resistor, the value of the dependent source is $2i_1$ with units of amperes. Find the currents i_1 and i_2, and also the value of the dependent current source. [8]

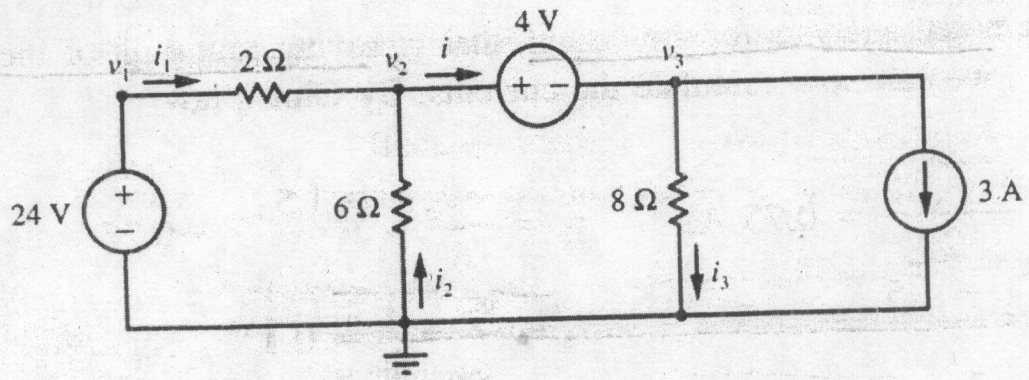


3. a) State Norton's Theorem. [1]

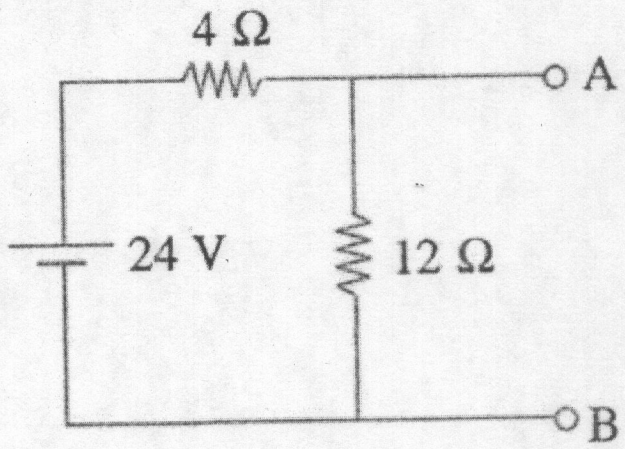
b) For the circuit shown below, the 4 V source is connected between two nonreference nodes. Find the

(i) node voltages v_1, v_2 and v_3, and hence the [15]

(ii) labeled currents i_1, i_2, i_3 and i. [1 + 1 + 1 + 1]



c) For the circuit shown below,



Determine

(i) the current I_N, and also [2]

(ii) the resistance R_N [3]

using the Norton theorem.

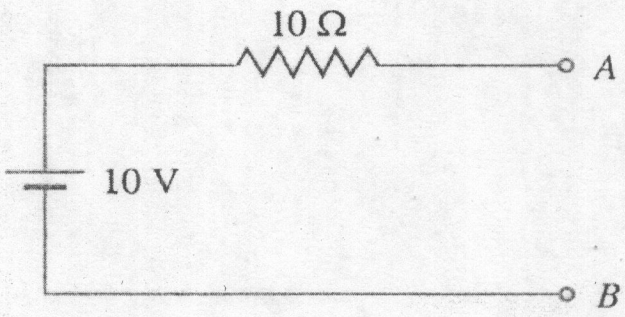
4. a) State Kirchhoff's voltage law. [1]

b) Convert the constant voltage source shown in the figure below into a constant current source by

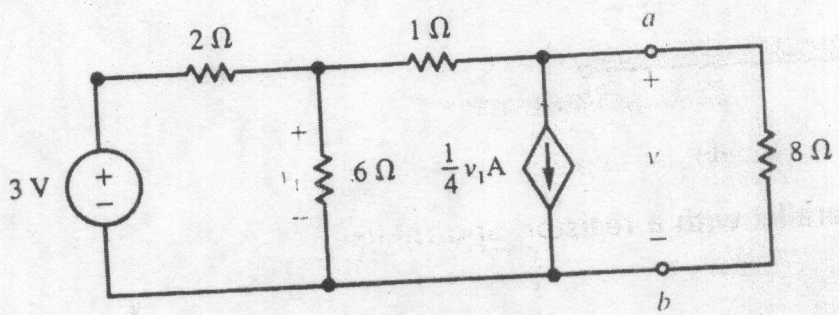
(i) calculating the short-circuit current, [2]

(ii) finding the resistance between terminals A and B, and [1]

(iii) drawing the current source equivalent circuit [1]



c) For the circuit below, use Norton's Theorem to calculate the short-circuit current (i_{sc}) and the output resistance (R_o) by performing following steps:



(i) redraw the given circuit replacing the 8-Ω resistor with a short circuit and also highlighting the label v_1 at the node connecting the three resistances. [1]

(ii) show that $v_1 = 0.9$ V by applying KCL at the node associated. [4]

(iii) apply KCL at an appropriate node to determine i_{sc} as equaling 0.675 A. [3]

(iv) redraw the given circuit into a diagram for the determination of R_o showing the new node v_1 label clearly. [1]

(v) apply KCL at this new node to illustrate that this $v_1 = 0.6v_o$. [4]

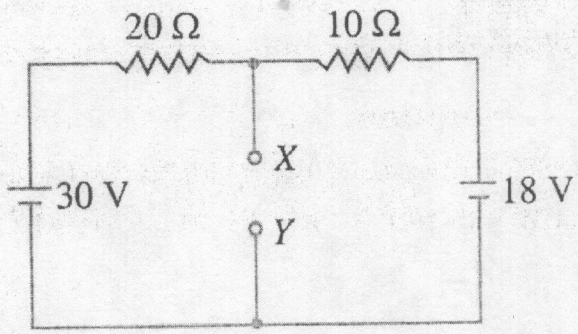
(vi) prove that the output current (i_o) = $0.55v_o$ and then find R_o. [5]

(vii) calculate v_{oc} by effecting Thevenin's Theorem from the appropriate results above. [1]

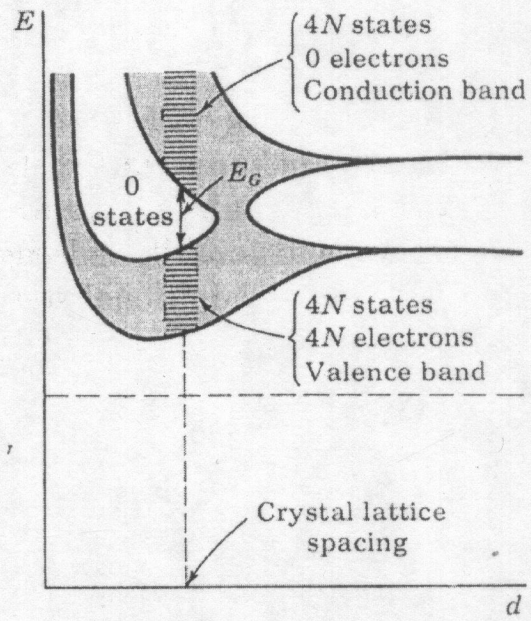
(viii) draw both the Norton equivalent circuit and the Thevenin's equivalent circuit showing the values of each circuit element with the load resistor (R_L). [1]

5. a) State Thevenin's Theorem. [1]

b) Find the Thevinin equivalent circuit at terminals X and Y in the following figure by first finding the Thevenin equivalent circuit parameters and then drawing the resulting circuit. [13]



c) The following diagram represents an electronic energy and separation distance relationship of a solid material of which diodes are a device realisation. Diodes are used in electronic circuits for specific applications like filters.



(i) Draw a line-bar and then label the line-bar on your own as above given figure indicating the part for a conductor, a semiconductor and an insulator. [3]

(ii) Draw the *CLC* or π filter circuit and explain how each of the three circuit elements work together in filtering an input signal from a rectifier. [8]

6. a) Why is a semiconductor referred to as bipolar while a metal is known to be unipolar? [2]

b) To compare the sine-wave current with a direct current, the joule heat produced in a resistor is considered. Given that *ac* current and voltage are

$$i = I_p \sin(wt) \text{ , and}$$

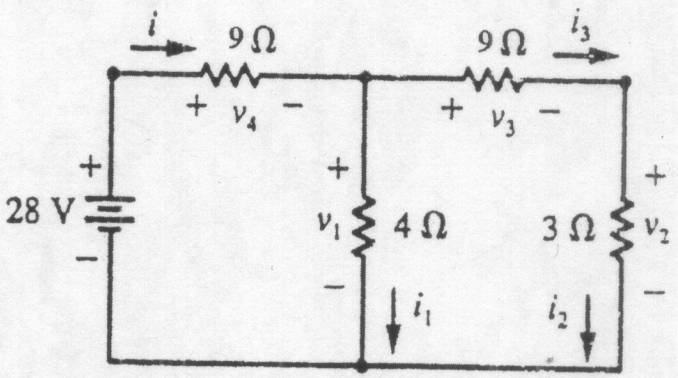
$$v = V_p \sin(wt + \phi)$$

where I_p and V_p are the maximum or peak amplitude values, and ϕ is the phase difference, the average power calculated by averaging the IR losses over a complete cycle is

$$P_{av} = \frac{I_p^2 R}{2}.$$

Find the expression for the effective value, or *rms* value, of an alternating current by comparing the average power above with the joule heating in a resistor by *dc* I^2R. [4]

c) For the circuit shown below,



(i) apply Kirchhoff's current law, Ohm's law, Kirchhoff's voltage law at respective circuit points to write necessary expression of the labelled current and voltages to show that v_2 = 1.75 V, [10]

(ii) use the voltage-divider formula to find v_1. [3]

(iii) calculate i_1, and [3]

(iv) apply the current-divider formula to find i. [3]

END

UNIVERSITY OF ZAMBIA

DEPARTMENT OF PHYSICS

SCHOOL OF NATURAL SCIENCES

2019/2020 End of Academic Year Examinations

PHY3422: Digital Electronics I

Instructions: Answer any **four** questions only. This examination paper has six questions. Each of the six questions carry equal marks. All essential working must be shown. The **positive logic system is considered always** unless it is explicitly mentioned in a particular question that negative logic system must be used.

Time duration: three hours.

Some Boolean algebra, axioms and theorems that you might find useful:

$x \cdot y = y \cdot x$ $x + y = y + x$	*Commutative*	$0 \cdot 0 = 0$	$x \cdot 0 = 0$
$x \cdot (y \cdot z) = (x \cdot y) \cdot z$ $x + (y + z) = (x + y) + z$	*Associative*	$1 + 1 = 1$	$x + 1 = 1$
$x \cdot (y + z) = x \cdot y + x \cdot z$ $x + y \cdot z = (x + y) \cdot (x + z)$	*Distributive*	$1 \cdot 1 = 1$	$x \cdot 1 = x$
$x + x \cdot y = x$ $x \cdot (x + y) = x$	*Absorption Combining*	$0 + 0 = 0$	$x + 0 = x$
$x \cdot y + x \cdot \bar{y} = x$ $(x + y) \cdot (x + \bar{y}) = x$		$x + \bar{x} \cdot y = x + y$ $x \cdot (\bar{x} + y) = x \cdot y$	$x \cdot x = x + x = x$
$\overline{x \cdot y} = \bar{x} + \bar{y}$ $\overline{x + y} = \bar{x} \cdot \bar{y}$	*de Morgan's theorems*	$0 \cdot 1 = 0$	$x + \bar{x} = 1$
$x \cdot y + y \cdot z + \bar{x} \cdot z = x \cdot y + \bar{x}z$ $(x + y) \cdot (y + z) \cdot (\bar{x} + z) =$ $(x + y) \cdot (\bar{x} + z)$	*Consensus*	$0 + 1 = 1$ $x \cdot \bar{x} = 0$	If $x = 0$, Then $\bar{x} = 1$ and $\bar{\bar{x}} = 0$

1. a) Distinguish between the analogue system and the digital system. [1]

b) (i) Convert $(101101.101)_2$ to its decimal equivalent. [4]

(ii) Convert $(235.165)_{10}$ to its octal equivalent up to 3 octal places from the radix point. [4]

c) (i) Convert the octal number $(273.15)_8$ to its binary equivalent. [4]

(ii) Complete the given table for addition and subtraction of octal numbers by filling in the boxes where the sum of two numbers is located at the intersection point of the augend with the addend. Then add $(2427)_8$ and $(3165)_8$. [12]

Table for addition and subtraction of octal numbers

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**
0	1	2	3	4	5	6	7
1	2	3	4	5		7	10
2	3	4		6	7	10	11
3	4	5	6	7	10	11	12
4	5	6	7	10	11		13
5	6		10		12	13	14
6	7	10	11	12	13		15
7	10	11	12	13	14	15	16

2. a) Discuss the (i) positive logic and (ii) negative logic systems that are implemented in specific digital devices. [4]

b) Using only 2-input NAND Gates, in their minimum number, draw a circuit diagram that implements the basic OR Gate (operation). [5]

c) Design the logic circuit using AND, OR or NOT gates, and where necessary a combination of all the types of gates for the following Boolean expressions:

(i) $(A+B)\cdot\overline{C}$ [4]

(ii) $A\overline{B}+C$ [4]

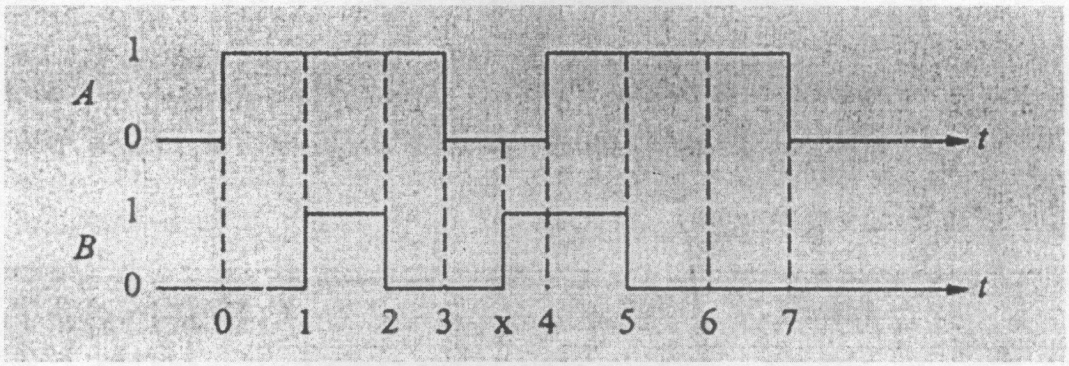
(iii) $Y=A\overline{B}+\overline{A}B$ [4]

(iv) $Y = \overline{P}\overline{Q} + PQ$ [4]

3. a) (i) What is $(A3E4)_{16}$ in binary? [2]

(ii) Add $(110011101)_2$ and $(1011011)_2$. [4]

b) If the following waveforms in the figure, where t is in seconds, are applied at the inputs, A and B, of a 2-input OR gate, determine the would-be Y output waveform from t = 0 by drawing a timing diagram. The default signal at Y is level 0. [7]



c) (i) Implement the following Boolean expression using NAND gates only by drawing the desired logic diagram: [8]

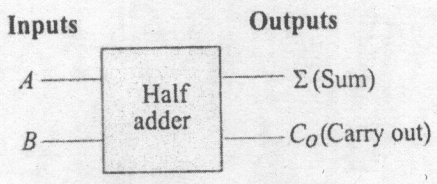
$$Y = A\overline{B}C + ABC + \overline{A}BC$$

(ii) Implement a 2-input (i.e., inputs A and B) AND gate function using only NOR gates by drawing a logic circuit with a minimum number of 2-input NOR gates.

[4]

4. a) What are the two types of complements for the octal number system? [2]

b) The following is a block diagram of (or symbol for) a Half Adder (HA) used to add two binary bits. The circuit would need two inputs and two outputs. Also shown below is the adder's addition table that can be thought of as a truth table.



(*i*)

Truth Table

Inputs		Outputs	
B	A	Σ	C_O
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1
Binary digits to be added		Sum	Carry out
		XOR	AND

(*ii*)

(i) Using the given details, design a circuit diagram of logic gates that produces a HA operation. [5]

(ii) Describe how you would use HAs with a specified basic logic gate to produce a Full Adder that can be used to add two binary digits taking into account also the carry-in digit that may arise. [2]

c) Using Boolean algebra, show that

(i) $ABC + A\overline{B}\overline{C} + AB\overline{C} = A(B + C)$ [6]

(ii) $(A + B)(A + \overline{B})(\overline{A} + C) = AC$ [8]

5. a) Write the 'dual' relationships of the following Boolean expressions:

(i) $A + 1 = 1$ [1]

(ii) $A(B + C) = AB + AC$ [1]

(iii) $A + AB = A$ [1]

(iv) $A + \overline{A}B = A + B$ [1]

(v) $A.1 = A$ [1]

b) (i) Given a 9's-complement number a certain number, how would you compute a 10's-complement number of the certain number? [2]

(ii) What is the 2's-complement of the binary digit 0? [2]

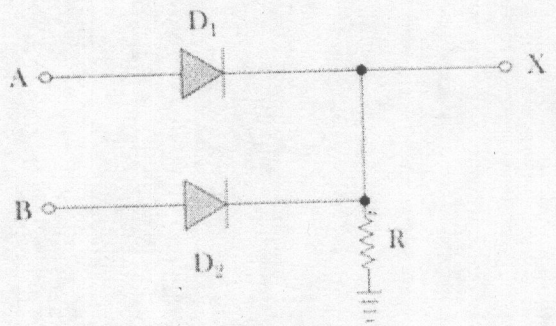
c) Prove the following using Boolean axioms:

(i) $x + x \cdot y = x$ [3]

(ii) $x + \bar{x}y = x + y$ [6]

(iii) $(x + y)(x + z) = x + yz$ [7]

6. a) The figure below shows the diode OR gate circuit with inputs A and B, and output X. Explain how it logically operates when various choice input values are provided. [8]

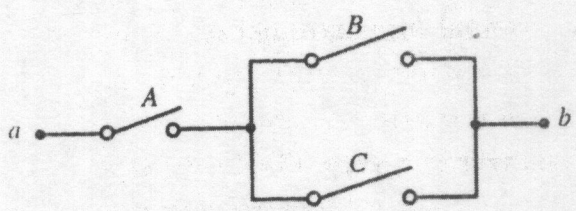


b) (i) Subtract 11011 from 10101 using the 2's complement method. [4]

(ii) Convert the decimal number $(757.546)_{10}$ to its hexadecimal equivalent. [3]

c) (i) What is a flip-flop and what important role is it used for in digital systems? [2]

(ii) Write the truth table for the three-switch connection shown below where A, B and C are inputs and point b is the output. [8]



END

UNIVERSITY OF ZAMBIA
DEPARTMENT OF PHYSICS
UNIVERSITY EXAMINATIONS
2020
PHY3531: INTRODUCTION TO QUANTUM MECHANICS

TIME: THREE HOURS
ANSWER: ANY FOUR QUESTIONS
ALL QUESTIONS CARRY EQUAL MARKS
TOTAL MARKS: 100

Useful information

$$\mathbf{j} = \frac{\hbar}{2mi}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*), \quad \int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

The Hermitian conjugate of A is $A^\dagger$ and is defined through

$$\int \psi^* A\phi d\tau = \int \left(A^\dagger\psi\right)^* A\phi d\tau$$

---

1. (**a**) (**i**) Interpret the wave function $\Psi(x,t)$. [2 marks]
Hence explain why the wave function must have the following properties
(**ii**) $\Psi(x,t)$ is single-valued [2 marks]
(**iii**) $\Psi(x,t)$ is continuous [2 marks]
(**iv**) $\Psi(x,t) \to 0$ as $x \to \pm\infty$. [2 marks]

(**b**) (**i**) Show that if the potential energy of a particle is independent of time, the time-dependent Schroedinger equation

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r})\Psi(\mathbf{r},t) = i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t)$$

separates into two equations. [9 marks]

(**ii**) Show that in such a case the wave function has the form

$$\Psi(\mathbf{r},t) = \psi(\mathbf{r})e^{-i\omega t}$$

[3 marks]

(**iii**) Show that for such a potential, the probability density is time-independent. [3 mark]

(**iv**) Show that the probability current density of the particle is not a function of the time. [2 marks]

2. (**a**) Explain the importance of Hermitian operators in quantum mechanics. [2 marks]

(**b**) Determine which of the operators defined below are Hermitian.
(**i**) $A\ \psi(x) = \sqrt{\psi(x)}$ [2 marks]
(**ii**) $A\ \psi(x) = -\frac{d^2}{dx^2}\psi(x)$ [4 marks]
(**iii**) $A\ \psi(x) = \psi(x)^*$. [2 marks]
Note that the operators act on functions which vanish at infinity.

(**c**) The Hamiltonian of the one-dimensional harmonic oscillator is

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega^2x^2$$

while the ground-state wave function is

$$\psi_0(x,t) = A\exp\left(-bx^2\right)\exp(-i\omega t)$$

(i) Explain why this wave function is acceptable over the range of motion $-\infty < x < \infty$. [2 marks]

(ii) Calculate the normalisation constant A. [5 marks]

(iii) Determine the ground-state energy given that $b = m\omega/2\hbar$ [10 marks]

3. A particle is trapped in a well such that

$$V(x) = \begin{cases} V_0, & x < -a \\ 0, & -a < x < 0 \\ \infty, & x > 0 \end{cases}$$

This means that $E < V_0$.

(i) Sketch the well. [2 marks]

(ii) Show that the wave function is given by

$$\psi(x) = \begin{cases} Ae^{qx}, & < -a \\ B\sin kx, & -a < x < 0 \\ 0, & x > 0 \end{cases}$$

[10 marks]

.(iv) Show that the allowed values of energy are solutions of the equation

$$q \tan ka = -k$$

[9 marks]

(v) Show that when we normalise the the wave function, the normalisation constant B is obtained from

$$|B|^2 \left[\sin^2 kae^{2qa} \int_{-\infty}^{-a} e^{2qx} dx + \int_{-a}^{0} \sin^2 kx dx \right] = 1$$

[4 marks]

4. A particle of mass m is incident from the left on a potential barrier of the form

$$\begin{aligned} V(x) &= 0, \quad x \leq 0 \quad \text{(Region I)} \\ &= V_0, \ 0 < x < a \quad \text{(Region II)} \\ &= 0, \ x \geq a \ , \text{(Region III)} \end{aligned}$$

The energy of the particle is such that $E > V_0$

(i) Sketch the potential [2 marks]

(ii) Show that in the three regions, the eigenfunction takes the following forms:

$$\psi_I(x) = Ae^{ikx} + Be^{-ikx}, \quad k = \sqrt{\frac{2mE}{\hbar^2}} \quad \text{(Region I)}$$

$$\psi_{II}(x) = Ce^{iqx} + De^{-iqx} \quad q = \sqrt{\frac{2m\,(E - V_0)}{\hbar^2}} \quad \text{(Region II)}$$

$$\psi_{III}(x) = Fe^{ikx}, \quad k = \sqrt{\frac{2mE}{\hbar^2}} \quad \text{(Region I)}$$

[8 marks]

(ii) Interpret each of the terms in the expressions above. [3 marks]

(ii) Explain what is meant by the reflection and transmission coefficients R and T and show that they are given by

$$R = \left|\frac{B}{A}\right|^2 \quad \text{and} \quad T = \left|\frac{F}{A}\right|^2$$

[6 marks]

(iii) Show that the equations to be used to eliminate the constants of integration so as to obtain R and T in terms of E and V_0 are

$$A + B = C + D$$

$$k\,(A - B) = q\,(C - D)$$

$$Ae^{ika} + Be^{-ika} = Ce^{iqa} + De^{-iqa}$$

$$k\left(Ae^{ika} - Be^{-ika}\right) = q\left(Ce^{iqa} - De^{-iqa}\right)$$

$$Ce^{iqa} + De^{-iqa} = Fe^{ika}$$

$$q\left(Ce^{iqa} - De^{-iqa}\right) = kFe^{ika}$$

[6 marks]

5. A particle of mass m is trapped in an in infinite square well potential whose walls are at $x = 0$ and $x = L$. Prove that the eigenfunctions of the system are

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L},$$

and the energy eigenvalues are

$$E_n = \frac{\hbar^2 \pi^2 n^2}{2mL^2}, \; n = 1, 2, 3, ...$$

[25 marks]

6. (**a**) The wave function of a certain particle whose motion is confined to the positive x axis (so that $0 \leq x \leq \infty$) is $\Psi(x,t) = Ce^{-\alpha x}e^{-i\omega t}$ where α and C are constants.

(**i**) Determine the constant C. [3 marks]

(**ii**) Obtain the position probability density of the particle. [2 marks]

(**iii**) Obtain the probability that a measurement of the position of the particle finds a value lying between $x = 0$ and $x = 5$. [3 marks]

(**iv**) Obtain the probability that a measurement of the position of the particle finds the particle between $x = 10$ and $x = \left(10 + 10^{-12}\right)$. [2 marks]

(**v**) Obtain the expectation value of the position [4 marks]

(**b**) The time-dependent Schroedinger equation of a free particle is

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t)$$

(**i**) Use separation of variables to show that if the particle is moving in the positive direction it has the wave function

$$\Psi(x,t) = A\exp(i(kx - \omega t)) \text{ with } k = \left(\frac{2mE}{\hbar^2}\right)^{1/2}$$

[9 marks]

(**ii**) Show that the momentum operator of the particle $p = -i\hbar\partial/\partial x$ yields the particle momentum $p = \hbar k$ when it acts on this wave function and hence give an expression for the expectation value of the momentum. [3 marks]

******END OF EXAMINATION*****

UNIVERSITY OF ZAMBIA

SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF PHYSICS
TERM 1 EXAMINATION
2019/2020 ACADEMIC YEAR

PHY4021
MATHEMATICAL METHODS FOR PHYSICS

DURATION:	Three hours.
INSTRUCTIONS:	Answer any **four** questions from the six given. *Each question carries 25 marks, with the marks for parts of questions indicated.*
MAXIMUM MARKS:	100
DATE:	Thursday, 26th November 2020

Formulae that may be needed:

1.
$$u_x = v_y, \qquad u_y = -v_x, \qquad u_r = \frac{v_\theta}{r}, \qquad v_r = -\frac{u_\theta}{r}$$

2.
$$\nabla^2 f(x,y) = 0$$

3.
$$\frac{f(x)}{(ax+b)(cx+d)} = \frac{A}{ax+b} + \frac{B}{cx+d}$$

4.
$$\int_C f(z)\,dz = \int_a^b f[z(t)]\dot{z}(t)\,dt.$$

5.
$$f^{(n)}(z_0) = \frac{n!}{2\pi i}\oint_C \frac{f(z)}{(z-z_0)^{n+1}}\,dz, \qquad (n = 1, 2, \ldots)$$

6.

$$\cosh iz = \cos z, \quad \sinh iz = i \sin z, \quad \cos iz = \cosh z, \quad \sin iz = i \sinh z$$

7.

$$(\cosh z)' = \sinh z, \qquad (\sinh z)' = \cosh z$$

8.

$$\left| \frac{z_{n+1}}{z_n} \right| \leq q < 1, \quad \text{for } n \text{ greater than some } N$$

9.

$$\lim_{n \to \infty} \left| \frac{z_{n+1}}{z_n} \right| = L$$

10.

$$\sqrt[n]{|z_n|} \leq q < 1, \quad \text{for } n \text{ greater than some } N$$

11.

$$\lim_{n \to \infty} \sqrt[n]{|z_n|} = L$$

12.

$$\begin{aligned} R &= \frac{1}{L^*}, \qquad L^* = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| \\ R &= \frac{1}{\tilde{L}}, \qquad \tilde{L} = \lim_{n \to \infty} \sqrt[n]{|a_n|} \\ R &= \frac{1}{\tilde{l}}, \qquad \tilde{l} = \text{largest limit of } \lim_{n \to \infty} \sqrt[n]{|a_n|} \end{aligned}$$

13.

$$\begin{aligned} \frac{1}{1-z} &= \sum_{m=0}^{\infty} z^m, \qquad |q| < 1; \qquad e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!}, \\ \cos z &= \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!}; \qquad \sin z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!} \\ \cosh z &= \sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!}; \qquad \sinh z = \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!} \end{aligned}$$

14.

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n + \sum_{n=1}^{\infty} \frac{b_n}{(z - z_0)^n}$$

15.

$$\operatorname*{Res}_{z=z_0} f(z) = \lim_{z \to z_0} (z - z_0) f(z)$$

16.

$$\operatorname*{Res}_{z=z_0} \frac{p(z)}{q(z)} = \frac{p(z_0)}{q'(z_0)}$$

17.

$$\operatorname*{Res}_{z=z_0} f(z) = \frac{1}{(m-1)!} \lim_{z \to z_0} \left\{ \frac{d^{m-1}}{dz^{m-1}} [(z - z_0)^m f(z)] \right\}$$

18.

$$\oint_C f(z)\,dz = 2\pi i \sum_{j=1}^{k} \operatorname*{Res}_{z=z_j} f(z)$$

19. Integrals of rational trigonometric functions of $\sin\theta$ and $\cos\theta$ (integration taken counterclockwise)

$$\int_C f(z)\,\frac{dz}{iz} = 2\pi i \sum_{j=1}^{k} \operatorname*{Res}_{z=z_j} \left[\frac{f(z)}{iz}\right]_{z=z_0}, \qquad C: \ |z| = 1$$

where $f(z)$ is obtained from $f(\cos\theta, \sin\theta)$ by the substitutions

$$\cos\theta = \frac{1}{2}\left(z + \frac{1}{z}\right), \qquad \sin\theta = \frac{1}{2i}\left(z - \frac{1}{z}\right)$$

20. Improper integrals of rational functions:

$$\int_{-\infty}^{\infty} f(x)\,dx = 2\pi i \sum \mathrm{Res} f(z)$$

21.

$$y(x) = \sum_{m=0}^{\infty} a_m x^m$$

22. Inverse of 2×2 matrix:

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \qquad \mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}, \qquad \det \mathbf{A} = a_{11}a_{22} - a_{21}a_{12}$$

23. For $F(y, \dot{y})$ the Euler-Lagrange equation reduces to

$$F - \dot{y}\frac{\partial F}{\partial \dot{y}} = \text{const}$$

QUESTION 1

(a) Give the definition of complex differentiation. (4 marks)

(b) Test whether or not $f(x) = \bar{z} - z$ is differentiable from first principles, i.e., from the definition of complex differentiation. (8 marks)

(c) Give the definition of an analytic function. (4 marks)

(d) Test whether or not $u(x, y) = y^2 - x^2$ is a harmonic function. If it is, find its conjugate harmonic and hence find the corresponding analytic function $f(z)$. (9 marks)

QUESTION 2

(a) Evaluate the following integral

$$\oint_C (z - z_0)^m \, dz$$

by the method of path, where C is a the circle $|z| = \rho$, m is an integer and z_0 is constant.

(20 marks)

(b) Find the Laurent series of $z^2 e^{1/z}$ with center 0. (5 marks)

QUESTION 3

(a) Is the series

$$\sum_{n=0}^{\infty} \frac{(1+2i)^n}{n!}$$

convergent or divergent. (4 marks)

(b) Find the center and radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2} z^n.$$

(7 marks)

(c) Evaluate the integral

$$\oint_C \frac{z - \frac{1}{4}}{z^2 + 3z + 2} \, dz$$

by the method of residues, where C is $|z| = 1.5$. You must determine the order of the poles before selecting the appropriate residue formula. (14 marks)

QUESTION 4

Diagonalise the matrix

$$A = \begin{bmatrix} 0 & 4 \\ 9 & 0 \end{bmatrix}$$

by first finding its eigenvalues and eigenvectors. (25 marks)

QUESTION 5

Solve the differential equation

$$y'' - 3y' + 2y = 0$$

by the power series method. Express the series solution in terms of two elementary functions of x. Hint: To express your answer in terms of two elementary functions the substitutions $a_0 = A + B$ and $a_1 = A + 2B$ may help. (25 marks)

QUESTION 6

(a) For a function that does not depend on y, but only on $\dot{y}$ and x, i.e., $F = F(\dot{y}, x)$, simplify the Euler-Lagrange equation,

$$\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial \dot{y}}\right) = 0.$$

(5 marks)

(b) Using the calculus of variations, show that the distance between two points in a plane is a straight line. (20 marks)

END

The University of Zambia
Department of Physics
Examination 2019/20
PHY 4121: Nuclear Experimental Techniques

Attempt any four questions. All questions carry equal marks. Attach any graphs to the answer booklet firmly where applicable.

Time: 3 hours. **Maximum marks 100**

Useful Equations:

$$I = I_o e^{-\mu x} \quad : \quad \text{HVL} = \frac{\ln 2}{\mu} \quad : \quad \lambda = \frac{1.24}{h\nu} \quad : \quad \lambda = \frac{\ln 2}{T_{½}}$$

$$\Delta\lambda = 0.00243(1-\cos\phi) \quad : E_{\max} = h\nu_{\max} = \frac{hc}{\lambda_{\min}} \quad : \quad t-\text{value} = \frac{|n_1 - n_2|}{\sqrt{\sigma_1^2 - \sigma_2^2}}$$

$$: A = \phi\sigma_{\text{act}} N\left(1-e^{-\lambda t}\right)e^{-\lambda t_1} : \quad \frac{t_{s+b}}{t_b} = \sqrt{\frac{c_{s+b}}{c_b}} \quad : \quad \sigma = \sqrt{n}$$

$$X^2 = \frac{\sum x^2 - \frac{\left(\sum x\right)^2}{N}}{\bar{x}} \quad : \quad \sigma = \sqrt{\frac{\sum_{x=i}^{N}\left(x_i - \bar{x}\right)^2}{N-1}}$$

Constants:

Avagadro's number = 1.602 x 10^{23}

1R = 2.58 x 10^{-4}C/kg-air

1 Ci = 3.7 x 10^{10} dps

h = 6.62 x 10^{-34}J.s

1 fermi = 10^{-15}m

1u = 1.6605 x 10^{-27}kg

electron mass = 0.00055 u

proton mass = 1.00727 u

neutron mass = 100866 u

1 eV = 1.6022 x 10^{-19} /J

c = 2.998 x 10^{8} m/s

Planck's Constant =6.626 x 10^{-34}J.s

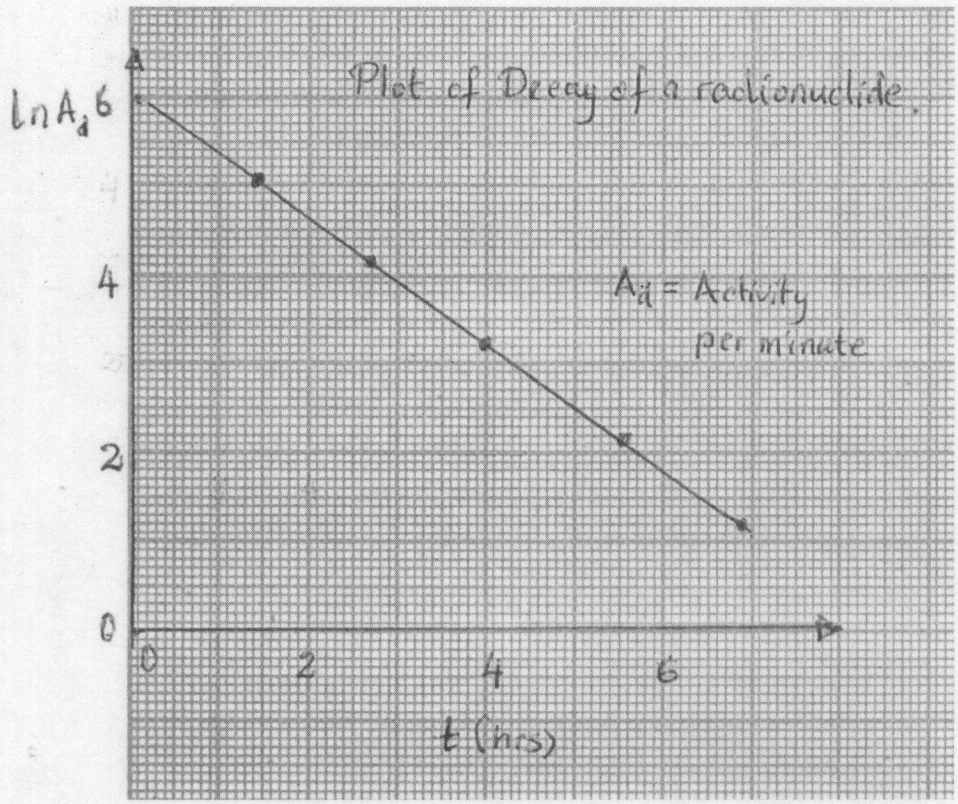
Attachments: Excerpts i) Guide to Chart of Nuclides, ii) Portion of Chart, and Chi Square Table

Q 1.0 a) Explain what is meant by *Half-life* of a radionuclide with aid of a diagram. [4]

b) Write down the relationship between half-life and the decay constant. [2]

c) The graph below shows how ln A_d (where A_d is activity per minute) changes with time, t, during the decay of a radioactive nuclide. Use the graph to find:

i) the initial activity; [3]
ii) the decay constant for the nuclide [3]



d) Derive the decay law from first principles. [10]

e) Complete the following decay equations:

i) ${}^{241}_{95}Am \rightarrow {}^{237}_{?}Np + {}^{?}_{2}\alpha$

ii) ${}^{11}_{6}\mathrm{C} \rightarrow {}^{11}_{?}\mathrm{B} + {}^{?}_{1}\beta$

iii) ${}^{1}_{1}\mathrm{p} + {}^{0}_{?}\mathrm{e} \rightarrow {}^{0}_{1}?+\nu$ [3]

Q 2.0 a) Name the processes by which the following radiations lose energy in matter:

i) Alpha particles [1]
ii) Beta particles [2]
iii) Gamma photons [3]
iv) Neutrons [3]

b) From the excerpt of the Chart of Nuclides provided determine:

i) Whether ^{14}N is stable or unstable
ii) Whether ^{15}N is stable or unstable
iii) The nuclide which decays to ^{14}N by positron emission
iv) The nuclide which decays to ^{15}N by negatron emission
v) The half-life of ^{15}O
vi) The percent abundance of ^{14}N in naturally occurring nitrogen
vii) The isotopic mass of ^{14}N
viii) The energy of beta particles emitted by O^{15} [8]

c) Determine the activity of a sample of ^{128}I ($T_{1/2}$ = 8 d) after four half-lives have elapsed, if the initial activity 1kBq. [4]

d) Find the radioactive constant (λ) when the half-life is 36 minutes. [3]

Q 3.0 a) The set of counts was recorded under identical detector conditions and counting times as shown below:

62, 68, 67, 80, 77, 72, 75, 64, 71, & 64

i) Apply the chi-squared test to determine whether the observed fluctuations are consistent with expectations from Poisson statistics. [8]
ii) Calculate the standard deviation for this data set. [6]
iii) Express the mean of the data set above. [1]

b) Explain the following modes of decay, in addition write the generalized equation for each process.

i) Negatron decay
ii) Electron capture
iii) Positron decay [6]

c) Describe two possible mechanisms by which $^{22}_{11}Na$ could decay to $^{22}_{10}Ne$. [4]

Q 4.0 a) Use the decay law $N = N_o e^{-\lambda t}$ to show that the half-life $T_{1/2} = \frac{0.693}{\lambda}$. [6]

b) At certain instant, a piece of radioactive material contains 1 x 10^{12} atoms. The half-life of the material is 30 days.

i) Calculate the number of disintegrations in first second. [5]
ii) What time will elapse before 1 x 10^4 atoms remain? [3]
iii) What is the count rate at this time? [2]

c) Describe an isotope of a nuclide, give an example and comment on the chemical properties of all isotopes of a nuclide. [5]

d) Write generalized equations for negatron decay, electron capture, and comment about the *N:Z* ratios for these decay processes. [4]

Q 5.0 a) A sample counted in the presence of background gave 2700 counts in 3 minutes. In the absence of the sample the counter gave 300 counts in 3 minutes. Find its:

i) Activity,
ii) Standard deviation, and
iii) Percent deviation.

What portion of time should have been spent on counting the background in order to achieve maximum precision? [10]

b) Describe how you would set an experiment to verify the Inverse Square Law for radiation, list the equipment required, and draw a schematic diagram of the experimental set-up. [9]

c) State the functions of each of the electronic modules used in the set-up described in part *b* of this Question. [6]

Q 6.0 a) A thin gold foil weighing 0.01 g was exposed for 7 days to an average thermal neutron flux of 9×10^6 n/cm^2-sec. What was its activity 3 days after irradiation? Given data for gold: atomic weight of gold is 197, density is 19.3 g/cm^3, activation cross section is 98.8 barns, and half-life is 2.7 days. [6]

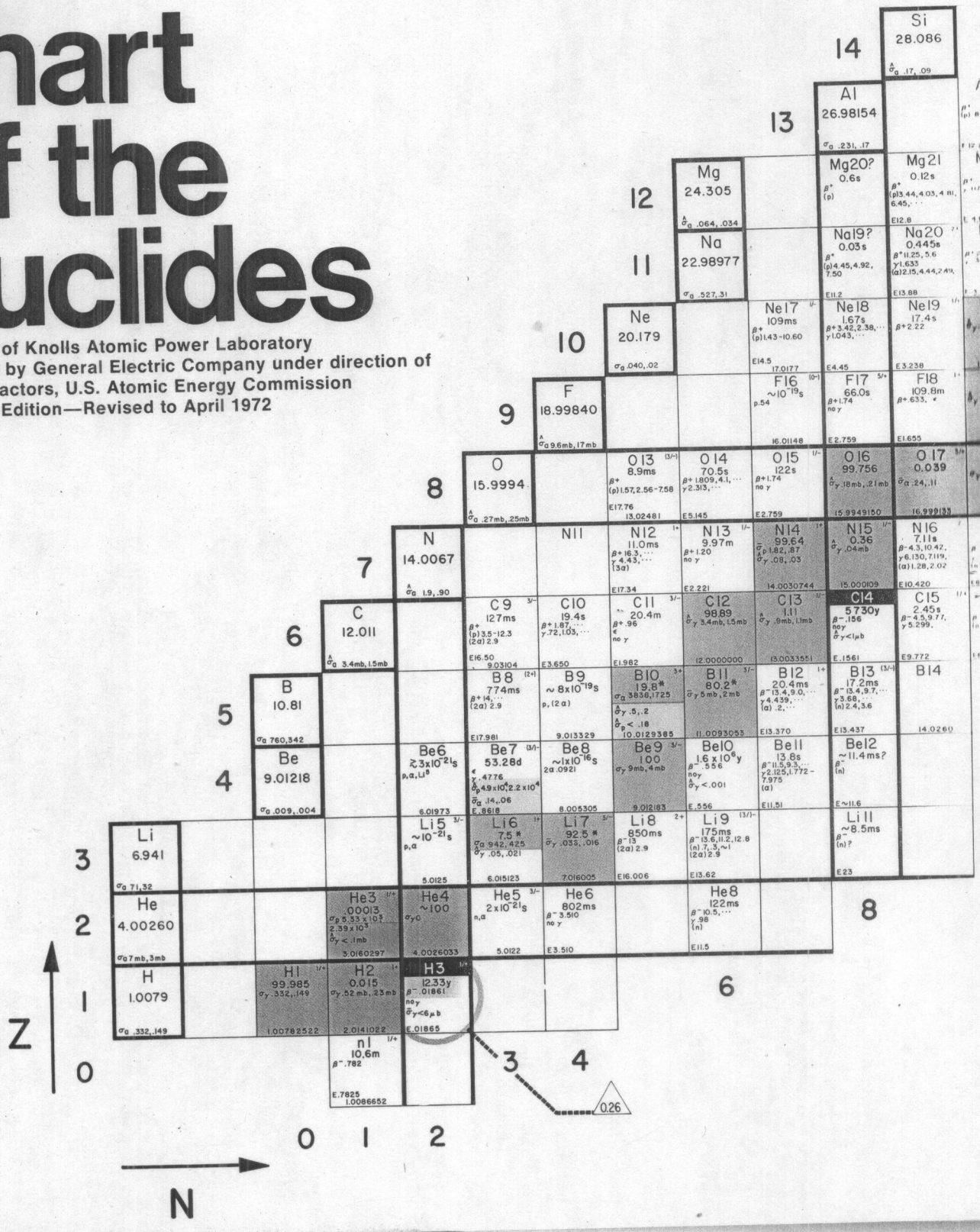
b) Draw a schematic diagram of scintillation counting system and discuss briefly the function of the components. [10]

c) Draw a detailed well labelled diagram of Sodium Iodide Thallium activated detector and discuss its operation. [9]

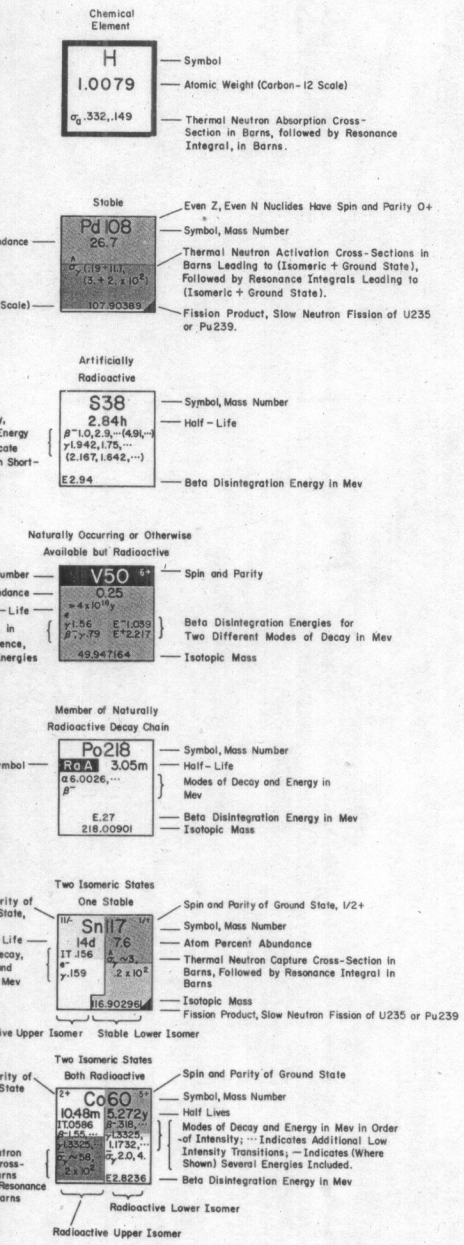
END OF EXAMINATION

chart of the nuclides

Courtesy of Knolls Atomic Power Laboratory
Operated by General Electric Company under direction of
Naval Reactors, U.S. Atomic Energy Commission
Eleventh Edition—Revised to April 1972



uide for using
e chart of the nuclides



Displacements Caused by Nuclear Bombardment Reactions

α, 3n	α, 2n ^{3}He, n	α, n	
p, n	p, γ d, n ^{3}He, np	α, np t, n ^{3}He, p	
p, pn γ, n n, 2n	Original Nucleus n, n	d, p n, γ t, np	t, p
n, t γ, np n, nd	n, d γ, p n, np	n, p t, ^{3}He	
n, α n, n^{3}He	n, ^{3}He n, pd		

Fig. 1

Relative Locations of the Products of Various Nuclear Processes

			^{3}He in	α in
	β$^-$ out	p in	d in	t in
	n out	Original Nucleus	n in	
t out	d out	p out	β$^+$ out ϵ	
α out	^{3}He out			

n = neutron
p = proton
d = deuteron
t = triton (^{3}H)
α = alpha particle
β$^-$ = negative electron
β$^+$ = positron
ϵ = electron capture

Fig. 2

SYMBOLS

RADIATIONS AND DECAY

α	alpha particle
β$^-$	negative electron
β$^+$	positron
γ	gamma ray
n	neutron
p	proton
ϵ	electron capture
IT	isomeric transition
D	radiation delayed
SF	spontaneous fission
E	disintegration energy
e$^-$	conversion electron

TIME

μs	microseconds (10^{-6} s)
ms	milliseconds (10^{-3} s)
s	seconds
m	minutes
h	hours
d	days
y	years

THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
Department of Physics
2019/2020 ACADEMIC YEAR
End of Year University Examinations
PHY 4221 - Solid State Physics I

Duration : 3 Hours
Total marks: 100

INSTRUCTIONS

This question paper contains six questions.
Answer any four questions. Each question has 25 total marks.
Write clearly your computer number on the answer booklet provided.
All working should be shown clearly.
Omission of essential work will lead to loss of marks.
Additional information can be found at the end of this question paper.

QUESTION 1

(a) Consider the model of a one dimensional chain of N atoms, equally spaced with separation a and each with mass m. The force constant coupling each atom to its nearest neighbour is β.

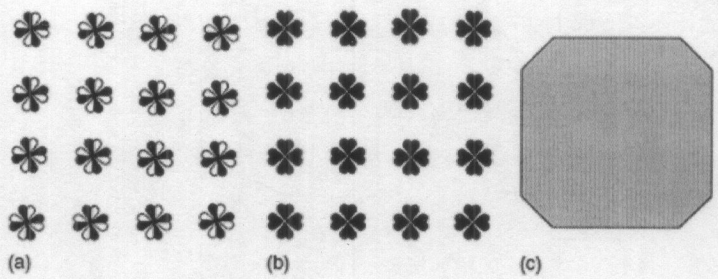
(i) Derive the dispersion relationship for such a system. [7]

(ii) Derive an expression for the group velocity v_g. [2]

(iii) Using the results of part **a(ii)**, evaluate v_g at very small values. of k Briefly discuss the physical significance of this low k velocity. [2]

(iv) Using the results of part **c(ii)**, evaluate v_g for k at the Brillouin zone boundary $[k = \pm\frac{\pi}{a}]$. Briefly discuss the physical significance of this Brillouin zone boubdary group velocity. [2]

(b) In the figure below, two crystals (a and b) and a polygon (c) are



shown. Identify the point group operations of the three objects assuming that the crystals are of infinite size. Show that the point groups of the two crystals are different, and one of them has the equivalent point group as the polygon. [7]

(c) Ultraviolet light of wavelength 4450 Å undergoes scattering from a crystal of refractive index 1.67. Calculate the maximum frequency of the phonon generated and the fractional change in the frequency of the incident radiation given that the velocity of sound in the crystal is 12600 m/s [5]

QUESTION 2

(a) (i)Name the three most important kinds of probes used in diffraction experiments on crystals. [3]

(ii)Discuss the essential condition that must be satisfied if such probes are to be useful in understanding crystal structure. [2]

(b) In a quantitative model of bonding in ionic crystals, the total potential energy is assumed to be of the form (in SI units)

$$U_{tot}(R) = N\frac{A}{R^n} - N\frac{\alpha e^2}{4\pi\epsilon_0 R}$$

where N is the number of positive-negative ion pairs, R is the nearest neighbour distance in the material, α is the Madelung constant for the material, A and n are adjustable empirical parameters to fit the experimental data and ϵ_0 is the permittivity of free space.

(i) The first term in U_{tot} is a repulsive interaction while the second term in U_{tot} is an attractive interaction. Discuss the physical origin of the two terms. [4]

(ii) Sketch U_{tot} potential as a function of R. [3]

(iii) Calculate the equlibrium nearest-neighbour distance R_0 in terms of N, A, ϵ_0 and α. Also find a simple expression of the equilibrium bonding energy per ion pair. [8]

(iv) For an ionic salt NaCl, α=1.75, R_0=5.63 Å, and the measured energy per ion pair is 7.95 eV. Use these numbers to estimate the value of the parameter n for NaCl. [5]

QUESTION 3

(a) State the four basic assumptions of the Drude model. [4]

(b) Deduce a value for the penetration length of the electron wave function outside the metal for electrons of the Fermi energy 5.26 eV, if the work function of the metal is typically 5.26 eV. [5]

(c) The atomic radius of potassium is 2.5 Å. Taking the Bravais lattice of potassium to be bcc, calculate its Fermi energy at 0 K. [5]

(d) Show that the number of energy states available for the electrons in a cubical box of 0.01 m side lying below an energy of 4 eV is 3.62×10^{24}. [6]

(e) The Fermi energy of copper is 7.24 eV at 0 K. Calculate its Fermi energy at 300 K. Comment on the variation of the Fermi energy as a function of temperature. [5]

QUESTION 4

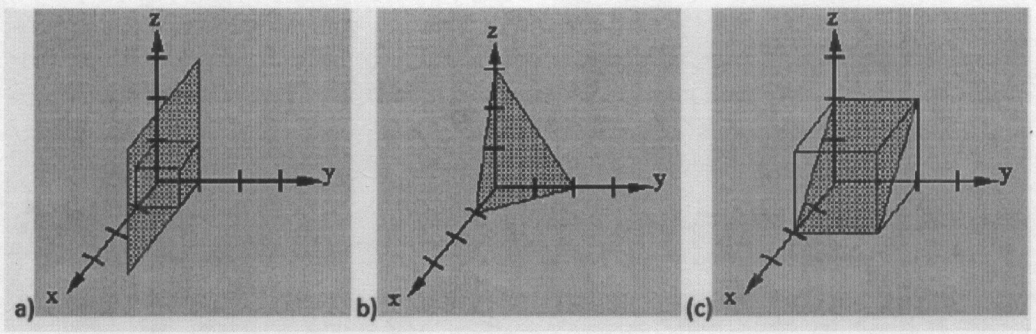
(a) Show that $\mathbf{c/a} = (8/3)^{1/2}$ for hcp close packing of hard spheres. [5]

(b) Powder specimens of three different monoatomic cubic crystals are analyzed with a Debye-Scherrer camera. It is known that one sample is fcc, one is bcc, and one has the diamond structure. The approximate positions of the first four diffraction rings in each case

A	B	C
28.8°	42.2°	42.8°
41.05°	49.2°	73.15°
50.8°	72.2°	89°
59.6°	87.25°	115.2°

are shown in the table above. Identify the crystal structures of a, b and c. [12]

(c) Determine the Miller indices (hkl) of the shaded planes below. [8]



QUESTION 5

(a) Derive the temperature dependence of the lattice specific heat capacity predicted by Einstein model, and deduce its variation at low

and high temperatures. [9]

(b) The Debye temperature and the Fermi energy for copper are 330 K and 8 eV, respectively, while the number of valence electrons per atom is 1. At what temperature are the contributions from the lattice and electronic specific heats equal? [5]

(c) Derive the density of states of a free electron gas in 3D, and sketch it at 0 K and at T > 0 K. [6]

(d) State the two relevant frequencies within the framework of the Drude model. Briefly describe the three spectral regimes that result from such a framework. [5]

QUESTION 6

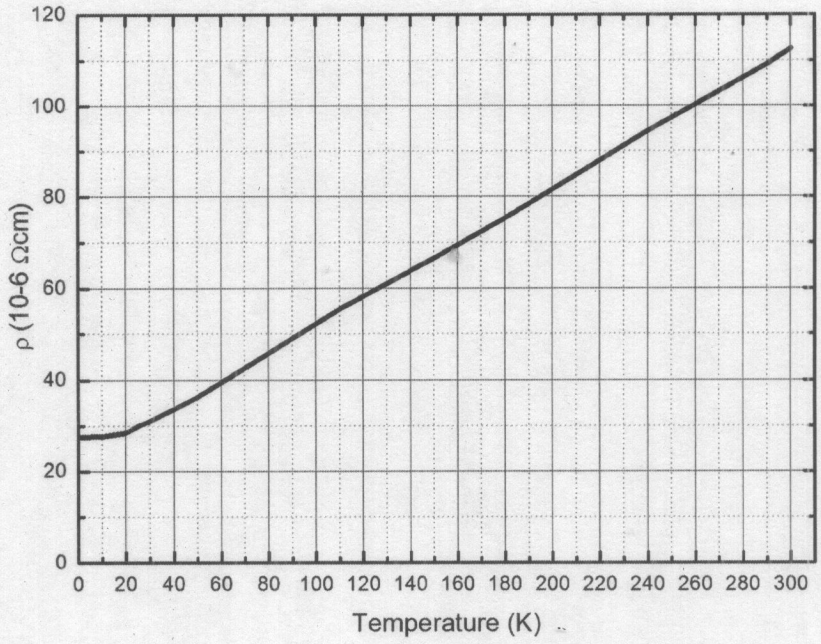
(a) Show that taking the reciprocal of the reciprocal lattice gives the real space lattice again. Also, prove that $|\vec{a}_1.\vec{a}_2 x \vec{a}_3| = V$ (volume of a unit cell). [9]

(b) The figure below shows the electrical resistivity of $ErRhB_4$.

(i) From this figure, is this material ($ErRhB_4$) a metal or an insulator. Give a reason for your answer. [2]

(ii) Describe the physical processes that account for resistivity, and explain the temperature dependence of the resistivity at T very near 0 K, T near 25 K and T near 300 K. [6]

(iii) Estimate the mean free path and the mean free time at T = 0 K



and at T = 300 K [Useful numbers: n = 10^{23}cm^{-3}, e = 5x10^{-10}esu, v_F=10^8cm/s, 1(Ωcm)$^{-1}$=9x10^{11}esu, m=10^{-27}g]. [8]

———END OF EXAMINATIONS———

ADITIONAL INFORMATION

Planck's constant h = 6.62x10^{-34} Js, electron charge e = 1.602x10^{-19} C,

electron mass m_e = 9.11x10^{-31} kg, proton mass m_p = 1.67x10^{-27} kg,

neutron mass m_n = 1.67x10^{-27} kg,

Bohr magneton μ_B =9.274x10^{-24} Am^2

Avogadro's constant N_A = 6.022x10^{23} mol^{-1},

Boltzmann constant k_B = 1.38x10^{-23} JK^{-1},

Permitivity of free space ϵ_0 = 8.85x$10^{-12}s^2m^{-2}$,

permeability of free space $\mu_0 = 4\pi x10^{-7}$,

Bohr magneton μ_B =9.274x10^{-24} Am^2

The first four diffraction peaks correspond to (110), (200), (112), and (220) for bcc; (110), (200), (220), and (311) for fcc; and (111),(220), (311), (400) for diamond structure.

$$\vec{b}_1 = 2\pi\frac{\vec{a}_2 * \vec{a}_3}{\vec{a}_1.\vec{a}_2 * \vec{a}_3}; \vec{b}_2 = 2\pi\frac{\vec{a}_1 * \vec{a}_3}{\vec{a}_1.\vec{a}_2 * \vec{a}_3}; \vec{b}_3 = 2\pi\frac{\vec{a}_1 * \vec{a}_2}{\vec{a}_1.\vec{a}_2 * \vec{a}_3}; < n >= \frac{\sum\limits_s se^{(-s\hbar\omega/k_BT)}}{\sum\limits_s e^{(-s\hbar\omega/k_BT)}};$$

$$\int_0^{+\infty}\frac{x}{e^x-1}dx = 1.51; \int_0^{+\infty}\frac{x^3}{e^x-1}dx = \frac{\pi^4}{15}; \int_{-\infty}^{+\infty}\frac{x^3e^x}{(e^x+1)^2}dx = \frac{\pi^2}{3}$$

Density of states $D(\omega) = \frac{L}{\pi}\frac{dk}{dw}$ in 1 dimension; $D(\omega) = \frac{Vk^2}{2\pi^2}\frac{dk}{dw}$ in 3 dimensions.

The University of Zambia

Department of Physics

Fundamentals of Medical Physics, PHY 4242

Final Examination Academic Year 2019/20

All questions carry equal marks. The marks are shown in brackets.

Time: 3 hours. **Maximum marks = 100**

Instructions:

Attempt any 4 questions, all questions carry equal marks

Useful Equations:

$$\text{SI}=\frac{4500}{\left(v/c\right)^2} \qquad \frac{\text{Radiation energy loss}}{\text{Ionisation energy loss}}=\frac{E_k Z}{820} \qquad I=I_o e^{-\mu x} \qquad \text{HVL}=\frac{\ln 2}{\mu}$$

$$\mu_{\text{en}}=\mu\frac{E_a}{h\nu} \qquad \lambda=\frac{1.24}{h\nu}\ (\text{nm}) \qquad \Delta\lambda=0.00243(1-\cos\phi) \qquad E_{\max}=h\nu_{\max}=\frac{hc}{\lambda_{\min}}$$

$$\Phi=\frac{N}{A} \qquad \phi=\frac{\Phi}{t}=\frac{N}{At} \qquad \Psi=\Phi E=\frac{NE}{A} \qquad I=\psi=\phi E=\frac{NE}{At}$$

$$I=\psi=\sum_{i=1}^{m} f_i\phi E_i \qquad X=\frac{Q}{m}, \qquad \frac{\Phi}{X}=\frac{2.11\times10^{14}}{h\nu\left(\mu_{en}\right)_m} \qquad D(\text{Gy})=\frac{E/m}{1\text{J/kg}}$$

$$\text{DE}(\text{Sv})=D(\text{Gy})\times\text{QF} \qquad \text{ED}(\text{mSv})=D(\text{mGy})\times W_R \qquad E=JW$$

$$\Psi=\frac{JW}{\rho\left(\mu_{en}\right)_m} \qquad \frac{V}{X}=\frac{1.29v}{C+C_e} \qquad \overline{Z}=\left(a_1Z_1^{2.94}+a_2Z_2^{2.94}+.....a_nZ_n^{2.94}\right)^{\frac{1}{2.94}}$$

Constants:

$W=33.85\text{eV}$: Avagadro's number = 6.023 x 10^{23}/mol : 1R = 2.58 x 10^{-4}C/kg-air

1 R = 1electrostatic unit (ESU)/0.001293 g air = 1 ESU/cm^3 air at STP

1 **a)** How many electrons flow from cathode to anode each second in an x-ray tube with a current of 60 mA? If the tube voltage is constant and equals 120 kV, at what rate (J/s) is energy being delivered to the anode?

b) Explain the term half-value layer (HVL) and how it relates to the beam of radiation. [3]

c) A narrow mono-energetic beam containing 12,000 photons is reduced to 6000 photons by a slab of aluminium 2 mm thick. What is the total linear attenuation coefficient of aluminium for the photons? [5]

d) With the aid of a diagram, give a brief description, of how a radiation calorimeter is used to measure dose. [6]

e) A dose of 45 Gy is delivered uniformly to a 270 g mass of water. How much energy in joules is absorbed by each gram of water and by the entire mass? [5]

Q 2 **a)** A focal spot of apparent size 1.2 mm is projected from an X-ray tube. The true focal spot is 6 mm.

i) What is the target angle? [5]

ii) Sketch a pin-hole camera for determining the size of the apparent focal spot of an x-ray tube, label the key features and explain how the determination is made. [10]

b) Show that, regardless of the energy of the incident photon scattered at an angle greater than 60^0 during Compton scattering, the photon cannot undergo pair-production. [10]

Q 3 **a)** An abdominal X-ray beam uses 10^{17} photons to expose film with an area of 30 cm x 40 cm for 0.2 second. Assume all photons have energy of 90 keV. Find:

i) The photon fluence
ii) The photon flux
iii) The energy fluence, and
iv) The intensity I. [8]

b) Describe in detail the operation of a Geiger Muller counter and explain why it is preferred as a routine instrument for detection of radiation. [8]

c) Give a brief explanation of the principles on which the following dosimetry systems are based:

i) Thermolumniscent dosimeter

ii) Fricke dosimeter

iii) Photographic dosimeter [9]

Q 4 **a)** Copper has a density of 8.9 g/cm^3 and a molecular mass of 63.56. The total attenuation of copper is 8.8 x 10^{-24} cm^2/atom for 500 keV photons. What thickness (in cm) of copper is required to attenuate 500 keV photons to a quarter of the original number? [7]

b) A 180 keV photon is scattered at an angle of 75^0 during a Compton interaction. What is the energy of the scattered photon? [6]

c) It is desired to reduce a beam of γ-rays to 1/16 of its intensity. The γ-rays have an energy of 1 MeV and lead will be used as shielding material. How many half-value layers are required? How many centimeters of lead are required given that $(\mu_{en})_{Pb} = 0.431\text{cm}^{-1}$. [7]

d) Find the power dissipated in a diagnostic x-ray tube operating at 80 kV and 400 mA. If the tube is operated for 1.5 seconds at this rating, calculate the number of calories deposited in the anode. (4.18 J = 1 cal) [5]

Q 5 **a)** Draw a well labeled diagram of a free air-ionization chamber and explain its operation. [10]

b) The tenth-value layer is the thickness of a slab of matter necessary to attenuate a beam of x or γ rays to one tenth of the intensity with no attenuator present. Assuming good geometry and mono-energetic photons, show that the tenth-value layer equals 2.30/μ, where μ is the total linear attenuation coefficient. [5]

c) Explain the principles on which solid scintillators, such as thallium activated sodium iodide are based [5]

d) Draw a well labelled diagram of a solid state scintillation detector. [5]

Q 6 **a)** A mono-energetic narrow beam of γ-rays is attenuated by copper in the manner shown below. Plot the data semi-logarithmically and determine the HVL and total linear coefficient of the beam. [6]

cpm	6000	1800	545	166	49
Thickness, cm	0	1	2	3	4

b) Water is 89 % oxygen with gram-atomic mass, 15.999 and 11 % hydrogen with gram-atomic mass, 1.008 by weight. Calculate the effective atomic number of water. [10]

c) Draw a well labelled diagram of a rotating anode x-ray tube showing key components. [9]

End of Exam

THE UNIVERSITY OF ZAMBIA
PHYSICS DEPARTMENT
SECOND TERM EXAMINATIONS 2020
PHY4535 QUANTUM MECHANICS II

TIME: THREE HOURS
ANSWER: ANY FOUR QUESTIONS
MAXIMUM MARKS: 100

$$\int_0^\infty x^n e^{-\mu x} dx = n! \mu^{-n-1}$$

$$\begin{vmatrix} H'_{11} - E^{(1)} & H'_{12} & \cdots & H'_{1\alpha} \\ H'_{21} & H'_{22} - E^{(1)} & \cdots & H'_{2\alpha} \\ \cdots & \cdots & \cdots & \cdots \\ H'_{\alpha 1} & H'_{\alpha 2} & \cdots & H'_{\alpha\alpha} - E^{(1)} \end{vmatrix} = 0$$

---

1. (**a**) (**i**) Show that the expression for the first-order energy correction in non-degenerate time-independent perturbation theory is

$$E_n^{(1)} = \left\langle \psi_n^{(0)} \left| H' \right| \psi_n^{(0)} \right\rangle$$

where H' is the perturbation. [8 marks]

(**ii**) A hydrogen atom in the ground state is perturbed by an electric field in the z direction. Thus, it is acted upon by a potential of the form

$$H' = -e\varepsilon z = -e\varepsilon r \sin\theta \cos\phi$$

Show that the first-order energy change due to this perturbation is zero.[5_marks]

Note that the ground-state hydrogen-atom eigenfunction is

$$\psi_{100}(r,\theta,\phi) = \frac{1}{\sqrt{\pi}} \left(\frac{1}{a_0}\right)^{3/2} e^{-r/a_0}, \quad a_0 = \frac{\hbar^2}{\mu e^2}$$

and that $dV = r^2 \sin\theta dr d\theta d\phi$

(**b**) (**i**) Show that the system with Hamiltonian

$$[H] = \begin{pmatrix} \varepsilon & 0 \\ 0 & \varepsilon \end{pmatrix}$$

has the degenerate eigenvalues $E_1 = E_2 = \varepsilon$ with the eigenfunctions

$$\chi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } \chi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

.. [6 marks]

(**ii**) If this system is perturbed by

$$[H'] = \begin{pmatrix} \mu & -i\mu \\ i\mu & -\mu \end{pmatrix}$$

where $\mu << 1$ is real, obtain the first-order corrections to the energies of the states. [6 marks]

2.(**a**) A certain system is three-dimensional and has the **orthonormal** eigenfunctions $\{\phi_n\}_{n=1}^3$ which satisfy the eigenvalue equation

$$A\phi_n = \alpha n^2 \phi_n$$

where α is a constant.

(i) Show that when the set $\{\phi_n\}$ is used as the basis, the matrix representation of A is

$$[A] = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & 4\alpha & 0 \\ 0 & 0 & 9\alpha \end{pmatrix}$$

(ii) Obtain the eigenvectors $\{[\phi_i]\}$ of $[A]$ and show that they are orthogonal. [10_marks]

(iii) If a certain wave function can be expanded as

$$\psi = C\,[2\phi_1 - \phi_2 + 5\phi_3]$$

where **C** is a normalisation factor, determine the value of C and the matrix representation of the wave function. [6 marks]

(b) Prove that the expression $\langle B\rangle = \langle\psi\,|B|\,\psi\rangle$ for the expectation value of a dynamical variable B can be converted to the matrix-mechanics expression

$$\langle B\rangle = [\psi]^{\dagger}\,[|B|]\,[\psi]$$

[9 marks]

3. (a) The use of the variational method for estimating ground-state energies and eigenfunctions is based on the result $\langle E\rangle \geq E_0$. Prove this result.[6_marks]

(b) A particle of mass m moves in the one-dimensional square-well potential

$$\begin{aligned} V &= 0;\ 0 < x < L \\ V &= \infty;\ x < 0, x > L \end{aligned}$$

(i) Using the variational function $\phi_\alpha(x) = x^\alpha(1-x)^\alpha$, write down an expression for the expectation value $\langle E\rangle_\alpha$ of the energy, but do not attempt to evaluate the integrals. [6 marks]

(ii) The integrals may be evaluated to give

$$\langle E\rangle_\alpha = \frac{\hbar^2}{2mL^2}\,\frac{2\alpha(4\alpha+1)}{2\alpha-1}$$

where only positive values of α are allowed. Use the variational principle to obtain an estimate of the ground-state energy and compare with the exact value.

$$E_1 = \frac{\pi^2\hbar^2}{2mL^2}$$

[13 marks]

4 (**a**) How may we crudely understand intrinsic spin? [2 marks]

(**b**) The components of the operator for spin 1/2 are

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \text{ and } S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

(**i**) Given that the eigenvalues of the z component are $\pm\hbar/2$, explain why the eigenvalues of S_x and S_y should be the same and prove that this is indeed so. [8 marks]

.(ii) Show that the operator for S^2 is

$$S^2 = \frac{3}{4}\hbar^2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

[4 marks]

(**b**) A certain hydrogen atom has the wave function

$$\psi = Bre^{-r/2a_0}\cos\theta$$

where B is a constant and

$$a_0 = \frac{4\pi\varepsilon_0\hbar^2}{\mu e^2}$$

is the Bohr radius.

(**i**) Calculate the normalisation constant B. [6 marks]

(**ii**) Obtain the quantum number m of the z component of the angular momentum. [2 marks]

(**iii**) If ψ is the hydrogen-atom wave function with quantum numbers $(n, l, m) = (2, 1, m)$ enumerate all the other wave functions that have the same energy as ψ. Take spin into account. [3 marks]

Note that

$$L_z = -i\hbar\frac{\partial}{\partial\phi}$$

and

$$dV = r^2\sin\theta dr d\theta d\phi$$

5 The ladder operators for the harmonic oscillator are

$$a_\pm = \frac{1}{\sqrt{2}}[\frac{p}{(m\hbar\omega)^{1/2}} \pm i(\frac{m\omega}{\hbar})^{1/2}x]$$

They satisfy the commutation relations

$$[H, a_+] = \hbar\omega a_+, \text{ and } [H, a_-] = -\hbar\omega a_-$$

(**i**) Prove that the operators are indeed ladder operators. [5 marks]

(**ii**) Explain why the ground-state eigenfunction ψ_0 of the harmonic oscillator satisfies the equation

$$\frac{1}{\sqrt{2}}[-i\left(\frac{\hbar}{m\omega}\right)^{1/2}\frac{d}{dx} - i(\frac{m\omega}{\hbar})^{1/2}x]\psi_0 = 0$$

[3 marks]

(**iii**) Solve the equation and hence show that the normalised ground-state eigenfunction is

$$\psi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\exp(-\frac{m\omega}{2\hbar}x^2)$$

[8 marks]

(**iv**) Explain how to generate all the states of the harmonic oscillator.[4_marks]

(**v**) Given that

$$H = (a_+a_- + \frac{1}{2})\hbar\omega$$

show that the energies of the states are given by

$$E_n = (n + \frac{1}{2})\hbar\omega. \quad n = 0, 1, 2, ...$$

[5 marks]

6. The ladder operators $L_\pm$ for orbital angular momentum are

$$L_+ = L_x + iL_y$$

and

$$L_- = L_x - iL_y$$

and have the explicit forms

$$L_+ = \hbar e^{i\phi}\left(\frac{\partial}{\partial\theta} + i\cot\theta\frac{\partial}{\partial\phi}\right)$$

and

$$L_- = \hbar e^{-i\phi}\left(-\frac{\partial}{\partial\theta} + i\cot\theta\frac{\partial}{\partial\phi}\right)$$

(**i**) Use the commutation relations for angular momentum

$$[L_i, L_j] = i\hbar L_k \quad i, j, k \text{ in cyclic order}$$

to prove that

$$[L_z, L_+] = \hbar L_+ and\, [L_z, L_-] = -\hbar L_-$$

[4 marks]

(**ii**) Derive the action of L_+ on an eigenfunction of L_z and hence justify the fact that $L_+ Y_{ll} = 0$. [8 marks]

Given that for $l = 1$, $Y_{10} = A\cos\theta$.

(**iii**)) Normalise Y_{10}. [4 marks]

(**iv**) Obtain normalised expressions for Y_{11} and $Y_{1,-1}$. [9 marks]

Note that $d\Omega = \sin\theta d\theta d\phi$

******END OF EXAMINATION*****

The University of Zambia

Department of Physics

2019/2020 University Examinations

PHY 4815: Physics of Renewable Energy Resources

Duration: Three (3) Hours **Total Marks: 100**

Instructions

- This paper has **six (6)** questions. Attempt **any four (4)** questions
- Show all your work clearly
- All questions carry equal marks and are indicated in brackets [ ]

Table 1: Some Physical Constants you may find useful

Radius of the Sun, $R_s = 6.96x10^8$ m	Mass of the Sun, $M_s = 1.99x10^{30}$ kg
Radius of the Earth, $R_e = 6.38x10^6$ m	Mass of the Earth, $M_e = 5.97x10^{24}$ kg
1 Astronomical Unit, $1AU = 1.496x10^{11}$ m	Sun's average surface temperature = 5777 K
Solar Constant I_{SC} = 1367 Wm^{-2}	Boltzmann constant, $k = 1.38x10^{-23}$ JK^{-1}
Universal gas constant, $R = 8.314$ J/mol.K	Stefan-Boltzmann constant, $\sigma = 5.67x10^{-8}$ $W.m^{-2}K^{-4}$
Mean molecular mass of air, $\overline{M} = 29.0$ amu	Planck's constant, $h = 6.63x10^{-34}$ J.s
Speed of light (vacuum), $c = 2.9979\times10^8$ $m.s^{-1}$	Electron charge, $e = 1.6x10^{-19}$ C
Specific heat of water, $c_w = 4.186$ kJ/kg.K	Wien's Law: $\lambda_{max}T = 2898$ μ. mK

Figure 1: The Equation of Time in Minutes

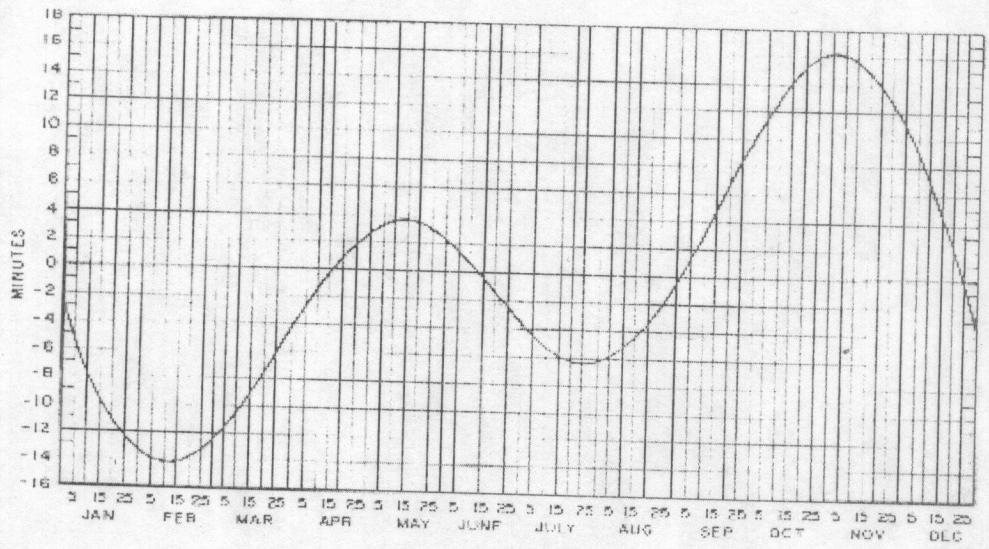


Table 2: The function $f(x)$

$x(\mu m\text{-}K)$	$f(x)$	$x(\mu m\text{-}K)$	$f(x)$	$x(\mu m\text{-}K)$	$f(x)$
1100	0.001	4600	0.580	8100	0.860
1200	0.002	4700	0.594	8200	0.864
1300	0.004	4800	0.608	8300	0.868
1400	0.008	4900	0.621	8400	0.871
1500	0.013	5000	0.634	8500	0.875
1600	0.020	5100	0.646	8600	0.878
1700	0.029	5200	0.658	8700	0.881
1800	0.040	5300	0.669	8800	0.884
1900	0.052	5400	0.680	8900	0.887
2000	0.067	5500	0.691	9000	0.890
2100	0.083	5600	0.701	9100	0.893
2200	0.101	5700	0.711	9200	0.895
2300	0.120	5800	0.720	9300	0.898
2400	0.140	5900	0.730	9400	0.901
2500	0.161	6000	0.738	9500	0.903
2600	0.183	6100	0.746	9600	0.905
2700	0.205	6200	0.754	9700	0.908
2800	0.228	6300	0.762	9800	0.910
2900	0.251	6400	0.770	9900	0.912
3000	0.273	6500	0.776	10000	0.914
3100	0.296	6600	0.783	11000	0.932
3200	0.318	6700	0.790	12000	0.945
3300	0.340	6800	0.796	13000	0.955
3400	0.362	6900	0.802	14000	0.963
3500	0.383	7000	0.808	15000	0.969
3600	0.404	7100	0.814	16000	0.974
3700	0.424	7200	0.819	17000	0.978
3800	0.443	7300	0.824	18000	0.981
3900	0.462	7400	0.830	19000	0.983
4000	0.483	7500	0.834	20000	0.986
4100	0.499	7600	0.840	30000	0.995
4200	0.516	7700	0.844	40000	0.998
4300	0.533	7800	0.848	50000	0.999
4400	0.549	7900	0.852		
4500	0.564	8000	0.856		

Table 3: Some formulae you may find useful

$\delta = 23.45 \sin\left[\frac{360}{365}(n+284)\right]$	$r = \frac{a(1-\varepsilon^2)}{1+\varepsilon\cos\theta}$
$J_o = DT^3 \exp\left(\frac{\varepsilon_g}{kT}\right)$	$\alpha = \exp\left(-\frac{n\kappa s}{\sqrt{n^2-\sin^2\theta_i}}\right)$
$J = \varepsilon\sigma T^4$	$R = Nk$
$Solar\ time = standard\ time + 4(L_{Loc} - L_{St}) + EOT$	$\omega = 15\left[(ST-12) + \frac{MM}{60}\right]$
$F^{dir} = I_{Sc} cos\theta exp\left(-\frac{\tau}{cos\theta_z}\right)$	$cos\gamma_s = \frac{sin\alpha sin\phi - sin\delta}{cos\alpha cos\phi}$
$\omega_s = cos^{-1}(-tan\phi tan\delta)$	$T_{f,e} = T_B - (T_B - T_{f,i}) exp\left[-\frac{U_L L}{\dot{m}\, C_f}\right]$
$d_f = \frac{m\lambda_o}{4n_f}$; m = 1, 3, 5, ...	$N = \frac{2}{15}\omega_s$
$r_p = \left[\frac{n_r^2\cos\theta_i - n_i\sqrt{n_r^2 - n_i^2\sin^2\theta_i}}{n_r^2\cos\theta_i + n_i\sqrt{n_r^2 - n_i^2\sin^2\theta_i}}\right]^2$	$r_s = \left[\frac{n_i^2\cos\theta_i - \sqrt{n_r^2 - n_i^2\sin^2\theta_i}}{n_i^2\cos\theta_i + \sqrt{n_r^2 - n_i^2\sin^2\theta_i}}\right]^2$
$R = r\left[1 + \frac{\alpha^2(1-r)^2}{1-\alpha^2 r^2}\right]$	$T = \frac{\alpha(1-r)^2}{1-\alpha^2 r^2}$

$$\cos\theta_i = \sin\delta\left[\sin\varphi\cos\beta - \cos\varphi\sin\beta\cos\gamma\right] + \cos\delta\cos\omega\left[\cos\varphi\cos\beta + \sin\varphi\sin\beta\cos\gamma\right] + \cos\delta\sin\beta\sin\gamma\sin\omega,$$

Q1. A solar panel with tilt $\beta = 20^{o}$ and facing North is installed in Livingstone (latitude = 17°49'S, longitude = 25°49'E). The standard meridian for Livingstone is 30° East. Find at 11:00 clock time on 5 February

(i) the solar declination [3]

(ii) the solar time [5]

(iii) the hour angle [4]

(iv) the day length [4]

(v) the solar zenith angle [5]

(vi) solar power incident on a horizontal surface of area $2m^2$ (assume there is no atmosphere) [4]

Q2. (a) Define solar constant. [3]

(b) Taking the Sun as a blackbody, show that the solar constant for a planet varies as the inverse of the square of its distance from the Sun. [4]

(c) Using $r_{venus}=1.08x10^{11}$ m, the solar constant for the Earth and the given value of the Earth-Sun distance, find the solar constant for Venus. [4]

(d) An orbiting probe is being launched to orbit Venus. The probe's electronic equipment is to be powered by silicon photovoltaic cells that are 12% efficient. If a total of 20kW of electrical power is required, assuming that photocells always remain normal to the solar beam,

(i) find the area of the photocells necessary (Use $r_{venus}=1.08x10^{11}$ m), [4]

(ii) what will be the area required of the photocells for a similar probe orbiting the earth? [2]

(iii) on which of the two planets is solar power more economical? [2]

(e) A solar panel consists of an absorber plate placed under a glazing. The glazing only transmits wavelength smaller than 1.0 μm. The absorber absorbs all wavelengths. If the collector is $2m^2$ in area and is oriented toward the Sun above the atmosphere, find the heating power produced in the absorber due to the absorption of solar radiation. (Assume the Sun to be a black body at temperature of 6000K). [6]

Q3. (a)

(i) Stating the assumptions made and considering the equilibrium of a thin layer of the atmosphere, obtain an equation for the density profile of the atmosphere. [8]

(ii) assuming an isothermal atmosphere, obtain an equation for the density profile of the atmosphere. [3]

(iii) assuming the atmospheric temperature to be 27°C find the height at which the density of the atmosphere in (ii) above reduces to half its value at the sea level. [4]

(b) On a given day the average optical thickness of the atmosphere is 0.3 for the wave-length range 0.4μm to 1.0μm and 0.2 for the rest of the solar spectrum. Find the direct flux on a horizontal surface when the Sun's zenith angle is 60°. Take the solar constant to be 1367W/m^2 and the Sun's black-body temperature to be 6000K. [10]

Q4. A solar glazing has bulk transmissivity T_g = 0.96 at normal incidence. It has a film deposited on its upper and lower surfaces to reduce surface reflection from 0.04 to 0.0133. Calculate the;

(a) overall transmittances of two such glazings (i) without and (ii) with the coating. [12]

(b) overall reflectances of two such glazings (i) without and (ii) with the coating. Assume normal incidence in both cases. [13]

Q5. (a) (i) What are selective absorber coatings? Explain the use of selective absorber coatings. [2+2]

(ii) Explain the greenhouse effect as it applies to the solar heating panels. [4]

(b) Consider an absorber plate suspended in space above the atmosphere. The plate is blackened and oriented towards the Sun. Assume no convective losses and no back surface losses.

(i) Calculate the steady-state absorber plate temperature. [4]

(ii) If a glazing is placed in front of the absorber in (b), calculate the steady-state absorber plate temperature. (Assume further that $a_\lambda = \epsilon_\lambda = 1$ in both cases) [6]

(c) If the absorber plate in Q5 (b) (i) is coated on the front part with an ideal selective coating with cut-off wavelength $\lambda_c = 4\mu m$, estimate the steady-state temperature of the absorber plate. [Hint: Use $f(x)x^4 = x_o^4$] [7]

Q6. **(a)** A given p-n junction diode is characterized by the constants $D = 0.2A/cm^3K^3$ and $\varepsilon_g = 1.12eV$. Find its reverse saturation current density at a temperature of 300K. Also, find the current density when the voltages of 0.5 and - 0.5V are applied to the diode. [4+4+4]

(b) (i) Applying the Kirchhoff's current rule to the equivalent solar cell diagram, the current density through the load may be expressed as

$$J = \bar{K}F - J_o\left(e^{\frac{eV}{kT}} - 1\right),$$

where the symbols have their usual meaning. Under open circuit condition, show that the open-circuit voltage is given by

$$V_{oc} = \frac{kT}{e}\ln\left(\frac{\bar{K}F}{J_o} + 1\right).$$ [7]

(ii) Hence, evaluate the open-circuit voltage for silicon given that $F = 1Sun$,

$J_o = 5 \times 10^{-10} mA/cm^2$ and $\bar{K} = 25mA/cm^2\ Sun$ at 27°C. [6]

=======End of PHY 4815 Examination =======

THE UNIVERSITY OF ZAMBIA
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION

ACTURIAL ECONOMICS

3 HOURS

Instructions:

SECTION A IS COMPULSORY

ANSWER ANY THREE QUESTIONS FROM SECTION B.

SECTIN A

1. Scarcity exists if:
   A prices are too high. B. economies are not perfectly competitive.
   C human wants cannot be satisfied when price is zero. D there are shortages of some goods.

2. Given the free market price for Good X is at K20. A decrease in the demand for Good X keeping the maximum price fixed at K20 will lead to:

   A. No change in price and a surplus. B. A fall in price and a surplus.
   C. A fall in price and a shortage. D. A fall in price and a fall in the number of units sold.

3. A minimum price is set for Good X at K10 which happens to be below the free market price. A decrease in the supply of Good X keeping the minimum price fixed at K10 will result in:

   A. a rise in price and a surplus of Good X B. a rise in price and a shortage of Good X
   B. a rise in price and a balance between supply and demand for Good
   C. no change in price and a shortage of Good X

4. Which one of the following will shift the supply curve for Good X to the right?

   A. a government subsidy on the production of Good X.
   B. a decrease in labour productivity in industry X.
   C A rise in price of raw materials used to produce Good X.
   D An increase in real wages in industry X.

5. The demand equation for good X is Qd = 15 −0.5P and the supply equation for good X is Qs = 3 + 2P, where P is the price. When the price is K6 there will be:

   A. A surplus of good X and price will rise. B. a shortage of good X and price will fall.
   C. A surplus of good X and price will fall. D a shortage of good X and price will rise.

6. Good X has a cross price elasticity of demand with respect to Good Y that is negative and Good Z has a cross price elasticity of demand with respect to Good Y that is positive. Which of the following is correct?

A. Goods X and Y are substitutes.
B. Goods Y and Z are complements.
C Goods X and Z are substitutes.
D. Goods X and Y are complements.

7. Which of the following best describes the opportunity cost of producing Good X?

A. The cost of producing Good X in money terms
B. The next best alternative use to which the factors of production used to produce Good X could be put
C. The stream of services provided by Good X over its entire lifetime
D. The production of Good X foregone in the previous year to enable Good X to be produced this year.

8. Under what conditions should a profit maximizing firm definitely keep open its production in the short run?

A. When average fixed cost is above average revenue and average fixed costs are less than average variable costs.
B. When average revenue is above average variable cost.
C. When average revenue is above average fixed costs.
D. When total revenue is above total fixed costs and total fixed costs are less than total variable costs.

9. A managing director of a monopoly firm is given the following data:

Marginal Revenue = K9
Marginal Cost = K10
Average Cost = K11
Average Revenue = K15

In order to maximize profits the firm should:

A. reduce price and increase output.
B. reduce price and reduce output.
C. increase price and increase output.
D. increase price and reduce output.

10. Which one of the following statements reflects decreasing returns to scale?

A. If more labor is added to a given amount of capital, the marginal and average product of labor falls.
B. If the ratio of labor to capital doubles, the output of the firm less than doubles.
C. If the input of capital and labor doubles, the output of the firm less than doubles.
D. If the input of capital and labor doubles, the ratio of inputs to output falls.

11. Consider the following table:

Units of Capital	Units of labor	Output
10	1	100
10	2	190
10	3	270
10	4	340
10	5	400

Which of the following is depicted by the above table:

A. increasing returns to scale.
B. constant returns to scale.
C. decreasing returns to scale.
D. diminishing marginal productivity.

12. Which of the following is NOT a feature of monopolistic competition?

A. Equilibrium price will be equal to marginal cost
B. There are no barriers to entry
C. Each firm produces a differentiated product
D. Only normal profits are made in the long run.

13. Following an increase in the price of fuel, there is a 20% increase in the price of air travel tickets that results in a 20% decrease in total revenue. Which of the following statements is TRUE about the demand for air travel?

A. It is price inelastic
B. It is price elastic
C. It is infinitely price elastic
D. It has zero price elasticity.

14. Good Y has a cross elasticity of demand with respect to Good X calculated using the average method of –1. Initially 100 units of Good Y are demanded when Good X costs 20 pence. A rise in the price of Good X to 25 pence will result in a new level of demand for Good Y of:

A. 125 units
B. 95 units.
C. 80 units.
D 70 units.

15. Good X has a cross price elasticity of demand with respect to Good Y that is negative and Good Z has a cross price elasticity of demand with respect to Good Y is positive. Which of the following is correct?

A Goods X and Y are substitutes.
B Goods Y and Z are complements.
C Goods X and Z are substitutes.
D Goods X and Y are complements.

16. The short run supply curve for a firm in a perfectly competitive industry is its:

A. average cost curve.
B. average variable cost curve.
C marginal cost curve above the lowest point of the average total cost curve.
D marginal cost curve above the lowest point of the average variable cost curve.

17. Which of the following would constitute a supply side economic policy for reducing unemployment?

A. Increasing social security benefits
B. Increasing the money supply
C. Reducing corporate and personal taxation
D. Increasing government expenditure aimed at exploiting the multiplier effect.

18. Which one of the following is NOT a feature of perfect competition?
A. Equilibrium price will be equal to marginal cost.
B. There are no barriers to entry.
C. The average revenue curve is greater than the marginal revenue curve.
D. Only normal profits are made in the long run.

19. If the price of Good X has risen and the quantity sold has increased, then there must have been a:
A. rightward shift of the supply curve for Good X.
B. leftward shift of the supply curve for Good X.
C. rightward shift of the demand curve for Good X.

D. leftward shift of the demand curve for Good X.

20. Which one of the following economic statements correctly defines moral hazard in relation to insurance?

A. Moral hazard describes the fact that people who know that they are a particularly bad risk are more inclined to take out insurance.
B. Moral hazard describes the fact that a policyholder may act in a way which makes the insured event more likely.
C. Moral hazard describes the fact that people who know that they are a particularly bad risk are less inclined to take out insurance.
D. Moral hazard describes the fact that a policyholder may act in a way which makes the insured event less likely.

Indicate whether the following statements are true or false.
21. If exports are greater than imports, then the trade balance is not in deficit.
22. If aggregate demand is too low relative to aggregate supply, then inflation and a balance of trade deficit may result.
23. As a form of demand-side policy, Government can boost spending by increasing taxes and interest rates.
24. Inflation refers to the percentage increase in the level of prices over a 12 months period.
25. A change in demand refers to the movement along a demand curve, while change in quantity demanded refers to a shift in the demand curve.

SECTION B. ANSWER ANY THREE QUESTIONS FROM THIS SECTION. ALL QUESTIONS CARRY EQUAL MARKS.

QUESTION ONE

a) State the law of demand. **(2)**
b) What is meant by "change in demand"? How does it differ from change in quantity demanded? **(4)**
c) The labor market for Actuarial Science young professionals is given by the following market Supply and Demand functions:
Qs = 150-3W,
Qd =2W-30, where *W* is the wage rate in thousands (000). Using this information, calculate:

(i) The equilibrium market wage and number of graduates (Quantity) required to clear the labor market in the Actuarial Science market **(4)**

(ii) Draw a graph depicting the equilibrium wage and labor quantity for the situation in b(i) **(3)**

(iii) Owing to the Covid situation in the year 2020, the University only managed to supply 36 Actuarial Science graduates on the market, while the level of demand for these graduates did not change? Calculate the new equilibrium wage for the graduates in this industry in the year 2020, and illustrate any changes in the demand and supply curves, if any, as a result of this change in the number of graduates in the year 2020. **(6)**

(iv) Using the original market equilibrium level in b(i) above, clearly illustrate and explain what would happen if industries in the Actuarial Science decided to impose a wage that is below the market equilibrium wage for labor. **(6)**

QUESTION TWO

a) Outline and explain three determinants of supply **(6)**
Define cross price elasticity of demand. (2)

b) Below is a table depicting changes in prices and quantities of a given commodity. Fill in the missing price elasticity values by indicating whether they elastic, unity or inelastic. **(10)**

Price	Quantity	Price Elasticity of demand	Classification of elasticity
K12.5	0	-∞	Perfectly elastic
K10	20	a	f
K7.5	40	b	g
K5	60	c	h
K2.5	80	d	i
K0	100	e	j

c) Define the following: (i) adverse selection. ii) Information Asymmetry **(4)**

d) Discuss briefly *three (3)* factors that would explain why economies of scale might occur in large firms. **[3]**

QUESTION THREE

a) Consider the following Companies A to E where each is an individual firm operating under a specific market structure.

Company	*Marginal cost*	*Average Cost*	*Average Revenue*	*Marginal Revenue*
A	*10*	*10*	*10*	*10*
B	*12*	*9*	*12*	*12*
C	*15*	*18*	*20*	*15*
D	*9*	*14*	*20*	*14*
E	*25*	*20*	*20*	*10*

(i) State which company has a short run equilibrium output that may be that of a profit maximizing monopolist? [2]
(ii) (ii) Give ALL the companies that are making excess profits. [2]
(iii) Give ALL the companies that could expand their output and increase their profits. [2]
(iv) (iv) Give ALL the companies that could be operating in a perfectly competitive environment. [2]

d) Modernity has increased demand for ripped jeans among female students at UNZA. Assuming that the market for ripped jeans is perfectly competitive, explain, using clearly labelled diagrammes what is likely to happen to market for ripped jeans in the long run. In your explanation, illustrate what would happen to the demand, prices, profit and supply for the commodity. **(10)**
(NB: Explanation and diagram earns you five marks each)

e) Draw a diagram showing a monopolist making a loss in the short run that is not sufficient to make it close down. Include the following curves; marginal revenue (MR), average revenue (AR), average total cost (AC) and average variable cost (AVC). Indicate the loss minimizing price (P1), output (Q1), average variable cost (AVC1) and average total cost (C1). **(7).**

QUESTION FOUR

a) Draw and label a basic Circular flow of income, illustrating the three ways through which income can be determined. **(6)**

b) Mention *three (3)* ways in which Government might boost spending and achieve the demand-side policy, as well as three (3) ways in which Government can influence the level of production and achieve supply side policies. **(6)**

c) Below is scenario presented for two sisters, Wangu and Watipaishe playing a strategic game, with a given payoffs:

		WATIPAISHE		
	STRATEGY	LEFT	MIDDLE	RIGHT
WANGU	UP	4,0	2,3	1,3
	DOWN	0,1	0,2	4,1

(i) What is meant by "Nash Equilibrium" in Game theory? **(2)**
(ii) Indicate the payoffs for all the strategies available for Wangu. **(3)**
(iii) Determine Wangu's best strategy in an event that Watipaishe decides to go right. **(2)**
(iv) Determine Watipaishe's best strategy regardless of whatever strategy Wangu adopts **(3)**
(v) Present the above table information into an extension or tree presentation form. **(3)**

End of exam!
Good luck!